A ONE-DIMENSIONAL MATHEMATICAL MODEL OF LONG-TERM SHORELINE EVOLUTION WITH GROIN SYSTEM USING AN UNCONDITIONALLY STABLE EXPLICIT FINITE DIFFERENCE METHOD

PIDOK UNYAPOTI

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR
THE DEGREE OF MASTER IN SCIENCE (APPLIED MATHEMATICS)
DEPARTMENT OF MATHEMATICS FACULTY OF SCIENCE
KING MONGKUT'S INSTITUTE OF TECHNOLOGY LADKRABANG
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Abstract

Shoreline evolution prediction is used to investigate the beach topography in the future. There are three phenomena give a large effect to the coastal structure such as the erosion, the accretion and the water level changes. In this research, we introduce a governing equation of a one-dimensional shoreline evolution model. The introduced model is a transient one-line model. The manipulation of physical parameters for the model is proposed. The setting method of the initial condition and the boundary conditions techniques are also proposed. The traditional forward time centered space method and the unconditionally stable Saulyev finite difference methods are employed to approximate the shoreline evolution in each year. The proposed numerical models give practically simulation for long-term shoreline evolution investigation.

Keywords : groin system,one-dimension, mathematical model, finite difference method.

Acknowledgements

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Pidok Unyapoti

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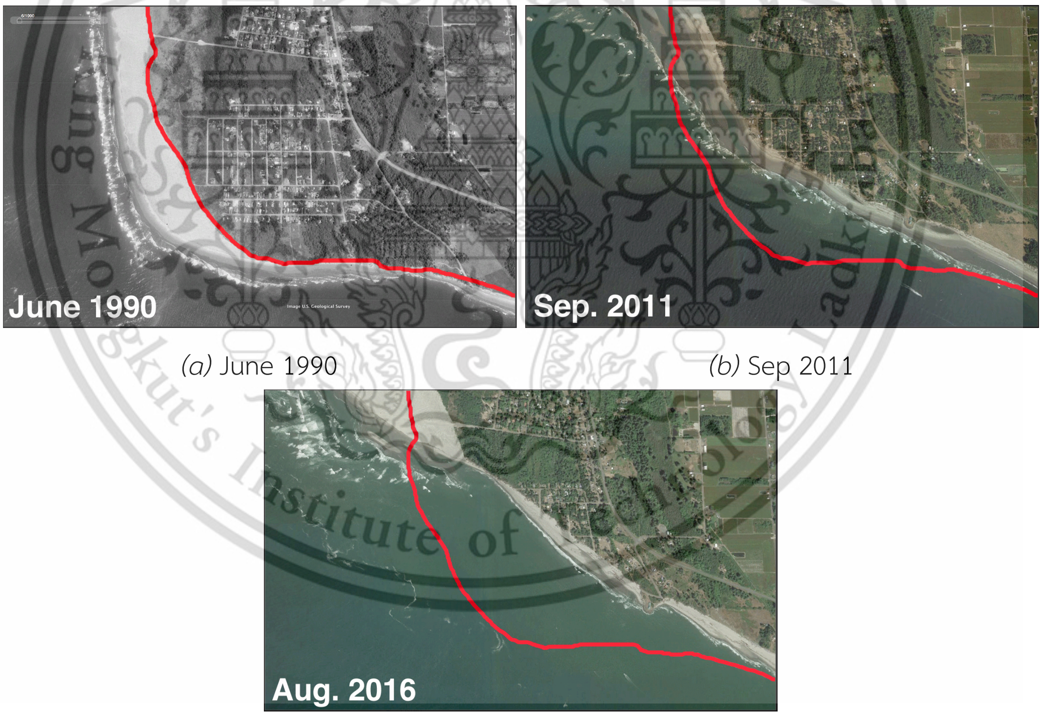
Chapter 1

Introduction

1.1 Research Motivation

Thailand has distance of shoreline is 3,148.23 kilometers. Which has been used in the area of activities such as tourism, fishery, aquaculture and community source. Beach area is important to people's daily life and tourism business. If beach areas are changed, it will affect the people live in the area and tourism business. In 30 years ago the entire coastal area of Thailand has been eroded in a total of 180.8672 square kilometer. The rate of coastal erosion on the Gulf of Thailand and the Andaman average of more than 5.0 meters per year. Beach erosion is important problem. Beach erosion is a natural process which occurs whenever sea waves or wind moves the material off the shoreline. The natural processes affecting coastal erosion is rising sea levels. The erosion of the cliffs has decreased has reduced the amount of material into shoreline, severe wind wave abnormal, water current has changed naturally and the direction of the wave changes.



(a) June 1990 *(b)* Sep 2011

(c) Aug 2016

Figure 1.1*:* Washaway Beach shoreline [1]

In order to prevent beach erosion and beach deposition so it has devised a break-water, seawall and groin.

Figure 1.2: Breakwater [2]

A breakwater is structure built out into the sea with the purpose of creating a safe harbor, marina or anchorage for fishing vessels, and protecting the coast from powerful swells and waves. Breakwater are often constructed near the coast in parallel or perpendicularly.

Figure 1.3*:* seawall [2]

A seawall is a large barrier built along the shoreline to protect coastal communities against flooding and mitigate the effects of erosion.

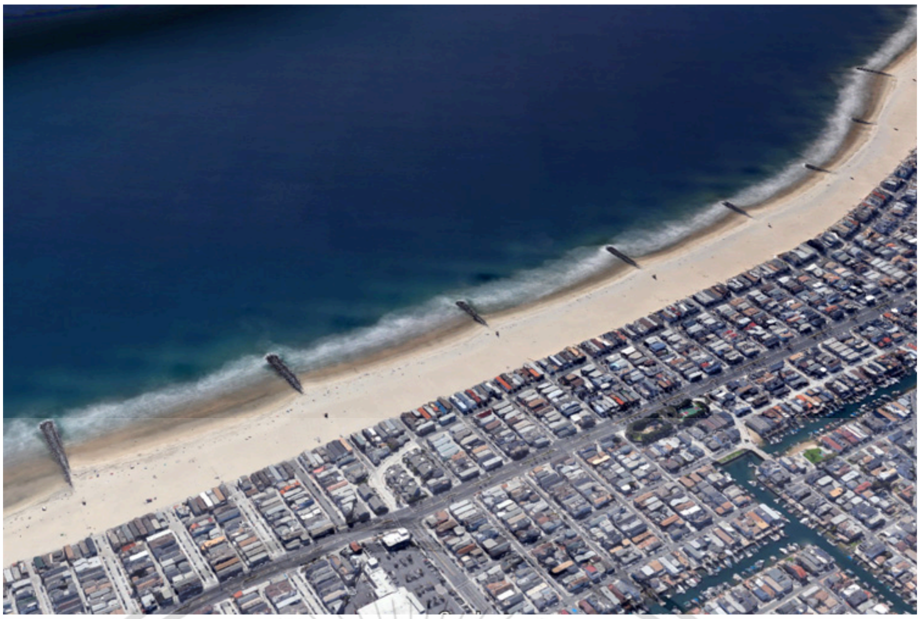


Figure 1.4: Groin [2]

A groin is a medium-sized artificial structure built perpendicular to the shoreline. It is built in series that work together to catch sediments in the surf zone brought by longshore drift.

1.2 Literature Review

In [3], they propose modern approach to functional groin design is demonstrated by using the GENESIS shoreline response model to simulate the action of single and multiple groins. The report predictions are tested in reproducing shoreline change observed at the 15 groins at Westhampton, Long Island, New York. In [4] reported on changes in beach profile due to the construction of single zigzag type of porous groins named GROPOZAG. In [5], they proposed the ONELINE modeling system and show its capabilities through model testing and case studies. This outlines two case studies in which complex configurations of the beach system are simulated. The first one features a groin field at Sea Isle City, New Jersey along the East Coast of the United States. The second is along the Nile Delta Coast in Egypt. In [6], they proposed one-line modeling concept has been extended to achieve long-term coastline evolution predictions, as well as to support and deliver better coastal engineering solutions to control erosion. The model has been applied to two Portuguese northwest coastal stretches: Aveiro and Figueira da Foz. The results allow the key potential consequences of continuing erosion to be evaluated qualitatively.

In order to investigate of beach erosion and beach deposition is needed qualitative understanding of idealized shoreline response to the governing process. Analytical solution originating from a mathematical model which describes the basic physics is the one tool to understanding it. Many authors obtained an analytical solution to shoreline evolution by using a simple mathematical formula. The one-line theory was introduced by many authors, several contributors in the analytical solution of shoreline evolution include Grijm [7], [8], Bakker and Edelman [9], Bakker [10], Le Mahute and Soldate [11], Walton and Chiu [12], and Larson et al. [13]. The analytical solution cannot be expected to provide quantitatively accurate solutions to problems involving complex boundary conditions and wave inputs. In the real situation, a numerical model of shoreline evolution would be more appropriate.

A general expression for the long-shore sand transport rate was developed by the US Army Corp [14]. The empirical predictive formula for the amplitude of the long-shore sand transport rate presented by L. X. Hoan [15]. In [16], they have examined and presented two numerical schemes of shoreline evolution for simplified configuration beach. In [17], [18], they have used the conditionally stable explicit finite difference methods to approximate their model solutions.

In this research, the governing equation of a one-dimension shoreline evolution model is introduced. We will also propose techniques of physical parameters, the initial condition and boundary conditions setting. Finite difference techniques will be used to approximate the model solution.

1.3 Objectives of the study

1) We will introduce a shoreline evolution model.
2) We will introduce a shoreline evolution model in long period of time.
3) We will introduce parameters setting.
4) We will propose numerical methods that is used to approximate solution of an introduced shoreline evolution in long period of time.
5) Several shoreline evolution scenarios will be simulated.

1.4 Scopes of the study

1) One-dimensional in space model of shoreline evolution is focused.
2) The average berm height of the considered shoreline are given.
3) The average closure depth of the considered shoreline are given.
4) The amplitude of the long-shore sand transport rate of the considered shoreline are given.
5) Sand transport rate function of the considered shoreline are given.
6) The measured initial shoreline function are given.
7) The measured groin system dimension are given.

1.5 Methodology

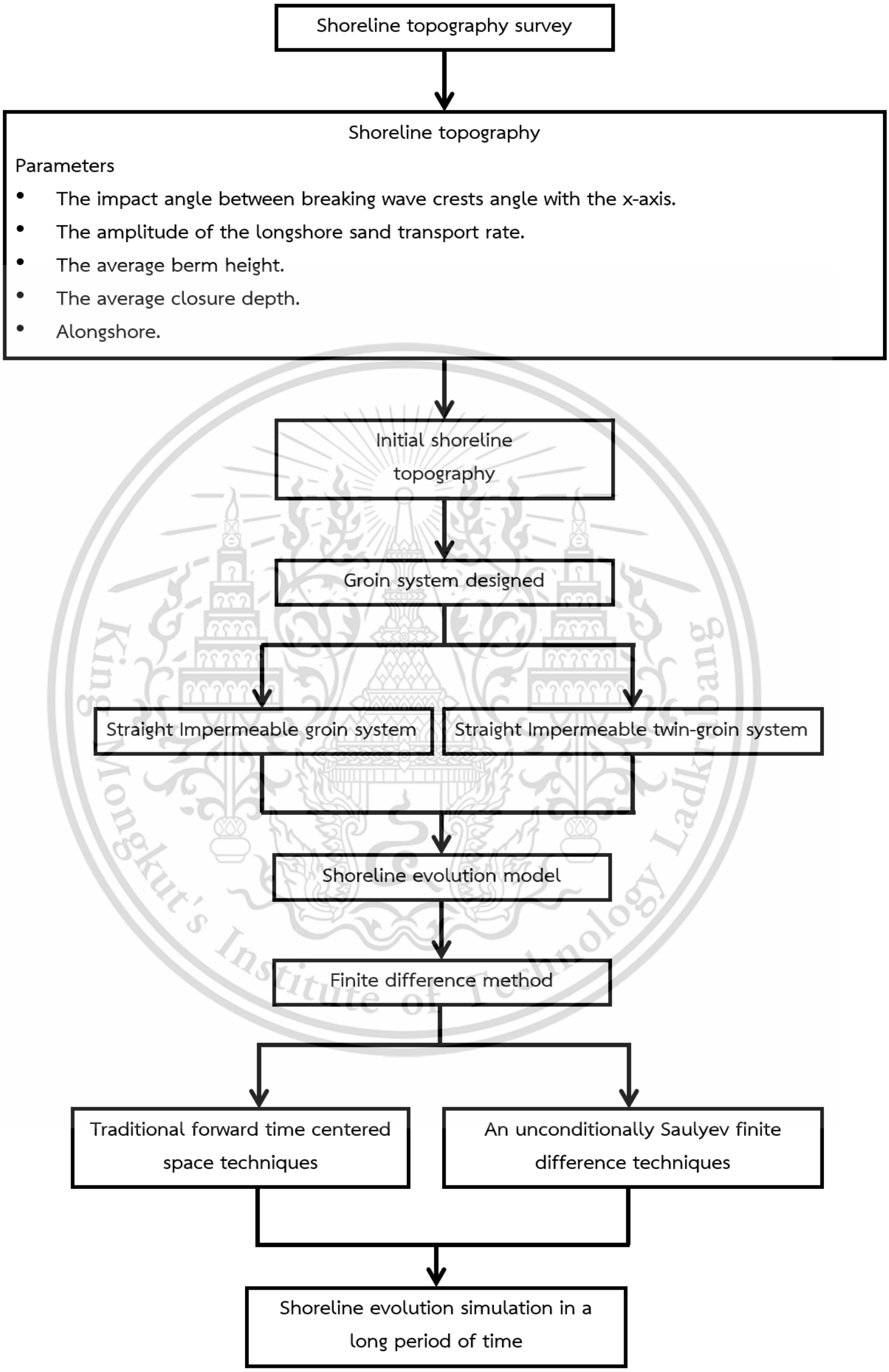
1) To review related research articles.
2) To focus the meaning of related parameters.
3) To employ numerical method such as finite difference method.
4) MATLAB programing will be construct.
5) The simulation of shoreline evolution in different scenarios are tested.
6) Numerical solution visualization will be illustrated.

1.6 Benefits of the study

1) We can measure shoreline evolution in long run.
2) We can forecast value of groin system.

1.7 Plan of the thesis

Chapter 1 : Research motivation and literature review.
Chapter 2 : Shoreline evolution model.
Chapter 3 : Numerical methods.
Chapter 4 : Shoreline evolution simulation.
Chapter 5 : Discussion and conclusion.


Shoreline topography survey
Shoreline topography
Parameters
The impact angle between breaking wave crests angle with the x-axis.
The amplitude of the longshore sand transport rate.
The average berm height.
The average closure depth.
Alongshore.
Initial shoreline topography
Groin system designed
Straight Impermeable groin system
Straight Impermeable twin-groin system
Shoreline evolution model
Finite difference method
Traditional forward time centered space techniques
An unconditionally Saulyev finite difference techniques
Shoreline evolution simulation in a long period of time

Chapter 2

Shoreline evolution model

2.1 Governing equation

In the one-line model, the beach profile is assumed to move landward and seaward while retaining the same shape, implying that all bottom contours are parallel. Consequently, under this assumption it is sufficient to specify the horizontal location of the profile with respect to baseline, and one contour line can be used to describe changes in the beach plan shape and volume as the beach erodes and accretes. The major assumption of the model is the sand is transported alongshore between two well-defined limiting elevations on the profile. One contribution to the volume change results if there is a difference in the alongshore sand transport rate at the lateral sides of the section and the associated the sand continuity. The principles of mass conservation must apply to the system at all times. By considering above definitions, the following differential equation for shoreline evolution is obtained,

$$\frac{\partial y}{\partial t} = \frac{1}{D_B + D_C}\left(-\frac{\partial Q}{\partial x}\right), \tag{2.1}$$

where x is the alongshore coordinate (m); y is the shoreline positions (m) and perpendicular to x-axis; t is time (day); Q is the long-shore sand transport rate (m^3/day); D_B is the average berm height (m) and D_C is the closure depth(m) as show in Fig 2.2.

In order to solve the Eq (2.1), necessary to specify an expression for the longshore sand transport rate (Q). This quantity is considered to be generated by the wave obliquely incident to the shoreline. A general expression for the long-shore sand transport rate was developed by the US Army Corp [14] :

$$Q = Q_0 \sin(2\alpha_b), \tag{2.2}$$

where Q_0 is the amplitude of the long-shore sand transport rate. α_b is the angle between breaking wave crest impact angle and local shoreline. The empirical predictive formula for the amplitude of the long-shore sand transport rate is [15] :

$$Q_0 = \frac{\rho}{16}(H_b^2 c_{gb})\frac{K}{(\rho_s - \rho)(1-n)}, \tag{2.3}$$

where the subscript b denotes value at the point of breaking, $c_g$is the wave group velocity, H is the wave height, ρ_s is the density of the sediment (kg/m^3), ρ is the density of the sea water, n is the porosity and K is the dimensionless coefficient which is a function of particle size. The quantity α_b is the angle between breaking wave crest and local shoreline, and may be written as :

$$\alpha_b = \alpha_0 - \tan^{-1}\left(\frac{\partial y}{\partial x}\right), \tag{2.4}$$

where α_0 is the angle between breaking wave crests and the x-axis. For beaches with mild slope, it can be assumed that breaking wave angle to the shoreline is small.
In this case,

$$\sin(2\alpha_b) \approx 2\alpha_b, \tag{2.5}$$

and

$$\tan^{-1}\left(\frac{\partial y}{\partial x}\right) \approx \frac{\partial y}{\partial x}. \tag{2.6}$$

Substituting Eq (2.4) into the Eq (2.2), and assuming the beach with mild slope yields :

$$Q = Q_0\left(2\alpha_b - 2\frac{\partial y}{\partial x}\right), \tag{2.7}$$

Substituting Eq (2.7) into the Eq (2.1) and neglecting the sources or sinks along the coast gives :

$$\frac{\partial y}{\partial x} = D\frac{\partial^2 y}{\partial x^2}, \tag{2.8}$$

for all$(x, t) \in (L, T)$, where

$$D = \frac{2Q_0}{D_B + D_C}. \tag{2.9}$$

Eq (2.8) is analogous to the one-dimensional heat diffusion equation, it can be solved analytically for various initial and boundary conditions.

2.2 Physical parameters

Physical parameter of the model can be illustrated as show in Figures 2.1-2.2 that are listed below.
α_0 is the impact angle between breaking wave crests angle with the x-axis.
Q_0 is the amplitude of the long-shore sand transport rate.
D_B is the average berm height.
D_C is the average closure depth.
L is alongshore.
T is time of simulation.

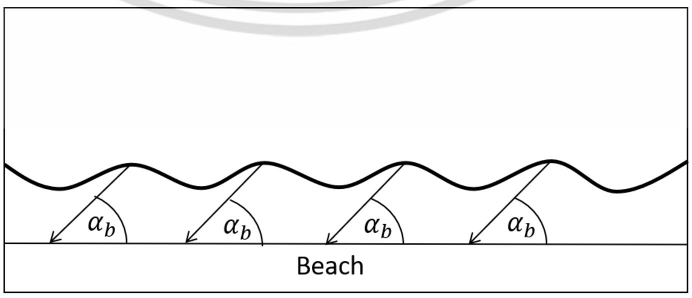


Figure 2.1*:* Breaking wave crests impact angle

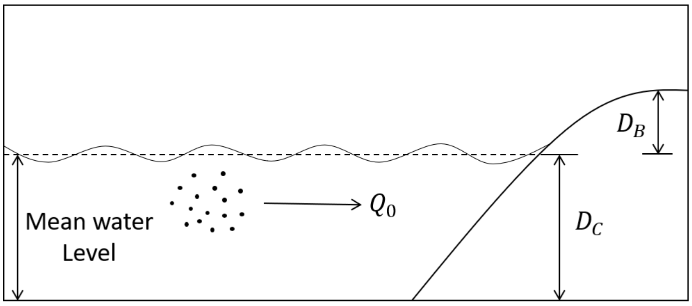


Figure 2.2: Shoreline physical parameters

2.3 Physical parameters setting techniques

Assuming that the density of the sediment (ρ_s) [19], the density of sea water (ρ) [20], the porosity (n) [21], the dimensionless coefficient which is a function of particle size (K) [22], the averaged berm height (D_B) and the closure depth (D_C) are listed belows.

Table 2.1: Parameters of sand transport rate [19], [20], [21], [22].

The density of the sediment (ρ_s (kg/m^3))	1700
The density of sea water (ρ (kg/m^3))	1020
The porosity (n)	0.406
The dimensionless coefficient which is a function of particle size (K)	0.375
The averaged berm height (D_B (m))	2
The closure depth (D_C (m))	28

The wave group velocity (c_g) and the wave height (H) in each month along a year is measured by field data at gulf of Thailand such that data are collected by Geo-Informatics and Space Technology Development Agency (Public Organization) (GISTDA) [23] as listed belows.

Table 2.2*:* The wave group velocity and the wave height [23].

Month	c_g (m/day)	H (m)
Jan 2019	8951.04	1.5
Feb 2019	6998.4	1.5
Mar 2019	5866.56	0.5
Apr 2019	6920.64	1.5
May 2019	5719.68	0.5
Jun 2019	5546.88	0.5
Jul 2018	8225.28	1.5
Aug 2018	9357.12	1.5
Sep 2018	13711.68	1.5
Oct 2018	15085.44	2.5
Nov 2018	10877.76	1.5
Dec 2018	11396.16	1.5

The amplitude of the long-shore transport rates (Q_0) are obtained by Eq (2.3) and the long-shore transport rates (D) are obtained by Eq (2.9) as listed belows.

Table 2.3*:* The amplitude of the long-shore transport rates and the long-shore transport rates.

Month	Q_0 (m/day)	D (m/day)
Jan 2019	1191.99	79.4659
Feb 2019	931.96	62.1307
Mar 2019	86.80	5.7869
Apr 2019	921.61	61.4403
May 2019	84.63	5.6420
Jun 2019	82.07	5.4716
Jul 2018	1095.34	73.0227
Aug 2018	1246.07	83.071
Sep 2018	1825.95	121.7301
Oct 2018	5580.26	372.017
Nov 2018	1448.57	96.5710
Dec 2018	1517.60	101.1233

We setting parameters for use in the Gulf of Thailand.

2.4 Initial and boundary conditions setting

The initials shoreline is assumed to be parallel to the x-axis. The boundary conditions can be classified into two cases.

2.4.1 Straight Impermeable groin system

Straight Impermeable groin system as show in Figure 2.3. Assuming that the braking wave angle to the shoreline is as show in Figure 2.4. It follows that the sand transport rate along the shoreline is uniform. The groin is instantaneously added at as show in Figure 2.4. These means that the initial condition becomes

$$y(x,0) = h_1(x), \tag{2.10}$$

boundary conditions are also assumed by

$$y(0,t) = f_1(t), \tag{2.11}$$

and

$$y(L,t) = g_1(t), \tag{2.12}$$

when $h(x)$, $f_1(t)$ and $g_1(t)$ are given functions.

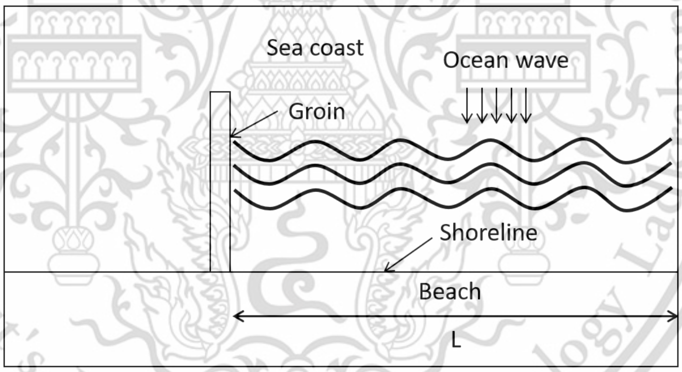


Figure 2.3*:* Initial shoreline with configuration straight impermeable groin

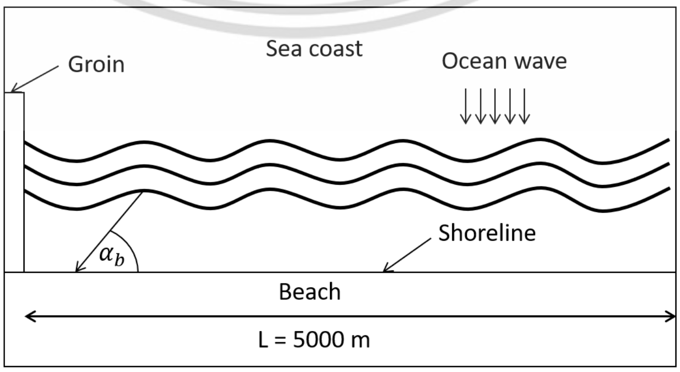


Figure 2.4*:* Initial shoreline

2.4.2 Straight Impermeable twin-groin system

Straight Impermeable twin-groin system as show in Figure 2.5. Assuming that the braking wave angle to the shoreline is as show in Figure 2.6. It follows that the sand transport rate along the shoreline is uniform. The groin is instantaneously added at as show in Figure 2.6. These means that the initial condition becomes

$$y(x,0) = h_2(x), \tag{2.13}$$

boundary conditions are also assumed by

$$\frac{\partial y(0,t)}{\partial x} = -\tan(\alpha_0), \tag{2.14}$$

and

$$\frac{\partial y(L,t)}{\partial x} = -\tan(-\alpha_0), \tag{2.15}$$

when $h_2(x)$ are given functions.

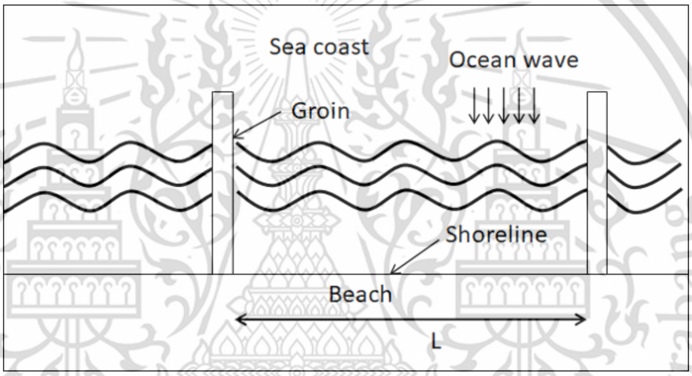


Figure 2.5*:* Initial shoreline with configuration straight impermeable twin-groin

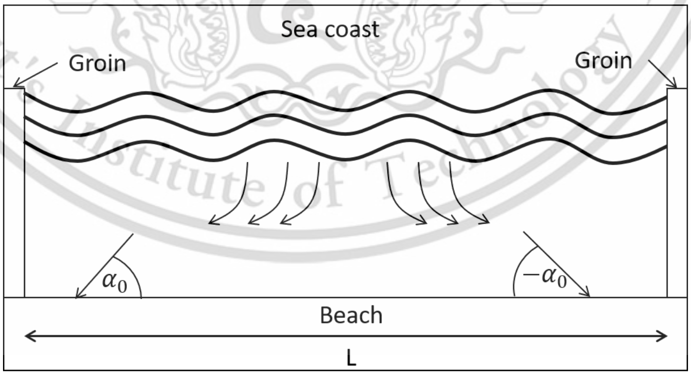


Figure 2.6*:* Initial shoreline

2.5 Numerical techniques

As in other mathematical problems, PDEs can be solved either analytically or numerically. Due to the complex nature of PDEs, various analytical techniques such as the method of separation of variables and the integral transforms can only be applied to some simple cases. Table 2.4 shows the nature of the analytical and numerical solutions.

Table 2.4*:* A comparison between analytical and numerical solution [25].

Analytical solution	Numerical solution
Solution is continuous, i.e. solution at any values of the independent variables can be found.	Solution can only be obtained at discrete grid points. Interpolations are necessary to obtain the solution between the grid points.
Exact or very accurate.	Approximate numerical errors have to be controlled properly for accuracy.
Provide physical insights into the problems. For instance, frequencies and mode shapes of vibrations can be obtained easily.	Physical information is more difficult to be obtained.
Do not exist for most of today's practical problems due to their complexities.	Can be obtained for complicated problems due to the advance in computer technology and the availability of more sophisticated numerical algorithms

The last difference, together with today's more economical computer technology, has made numerical methods important and indispensable. There are two main groups of numerical methods for solving PDEs - namely the finite-difference method and the finite-element method. The former is for general purpose while the latter is primarily for structural analysis problems although it was recently extended to other non-structural applications such as fluid flow problems and electromagnetism problems.

In our context, the finite-difference methods are mainly considered with the inclusion of the classical method of characteristics for hyperbolic equations which is neither a finite-difference method nor a finite-element method.

2.5.1 Even Grid System

Consider an even grid-spacing system in which Δx =constant= h and Δy =constant= k as shown in Figure 2.7

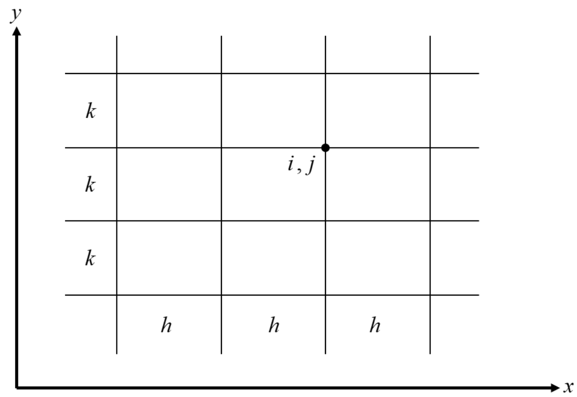


Figure 2.7: An even grid-spacing system.

The Taylor series for a function $u(x, y)$ expanded about x_i at $(x_i + h)$ and $(x_i - h)$ are respectively,

$$u(x+h,y) = u(x,y) + h\frac{\partial u(x,y)}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{h^3}{3!}\frac{\partial^3 u(x,y)}{\partial x^3} + ... \quad (2.16)$$

$$u(x-h,y) = u(x,y) - h\frac{\partial u(x,y)}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u(x,y)}{\partial x^2} - \frac{h^3}{3!}\frac{\partial^3 u(x,y)}{\partial x^3} + ... \quad (2.17)$$

where h, called the grid size, grid spacing or step size, is sufficiently small for the series to be convergent.

Introducing the double-subscript notation in which the first subscript denotes the x-position and the second subscript denotes the y-position, the above expressions can be written as

$$u_{i+1,j} = u_{i,j} + h\frac{\partial u_{i,j}}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u_{i,j}}{\partial x^2} + \frac{h^3}{3!}\frac{\partial^3 u_{i,j}}{\partial x^3} + ... \quad (2.18)$$

$$u_{i-1,j} = u_{i,j} - h\frac{\partial u_{i,j}}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u_{i,j}}{\partial x^2} - \frac{h^3}{3!}\frac{\partial^3 u_{i,j}}{\partial x^3} + ... \quad (2.19)$$

From Eq 2.18,

$$\frac{\partial u_{i,j}}{\partial x} = \frac{u_{i+1,j} - u_{i,j}}{h} - \frac{h}{2!}\frac{\partial^2 u_{i,j}}{\partial x^2} - \frac{h^2}{3!}\frac{\partial^3 u_{i,j}}{\partial x^3} + ... \quad (2.20)$$

$$\frac{\partial u_{i,j}}{\partial x} = \frac{u_{i+1,j} - u_{i,j}}{h} + O(h) \quad (2.21)$$

Likewise, from Eq 2.19,

$$\frac{\partial u_{i,j}}{\partial x} = \frac{u_{i,j} - u_{i-1,j}}{h} - \frac{h}{2!}\frac{\partial^2 u_{i,j}}{\partial x^2} - \frac{h^2}{3!}\frac{\partial^3 u_{i,j}}{\partial x^3} + ... \quad (2.22)$$

$$\frac{\partial u_{i,j}}{\partial x} = \frac{u_{i,j} - u_{i-1,j}}{h} + O(h) \quad (2.23)$$

Hence, if the terms containing the second-order and higher-order derivatives are truncated in these expressions, we get the forward difference and backward difference approximations respectively for the first-order derivative. As h must be sufficiently small so that the series converge, the second and other truncated terms are much smaller than the first truncated term and all truncated terms are thus written in terms of the order of magnitude of the first truncated term. Therefore the approximation errors, hence known as the truncation errors, in Eq 2.21 and Eq 2.23 are of the order of h and are written as

$O(h)$. This implies that the truncation errors are approximately proportional to h because all derivatives are fixed for a given problem. The truncation errors are thus approximately halved if h is halved. These finite-difference expressions are said to be first-order accurate. Physically, the truncation error represents the difference between the exact value of the derivative and its finite-difference value.

If we take Eq 2.18 - Eq 2.19 and rearrange, we get the central difference

$$\frac{\partial u_{i,j}}{\partial x} = \frac{u_{i+1,j} - u_{i-1,j}}{2h} - \frac{h^2}{3!}\frac{\partial^3 u_{i,j}}{\partial x^3} + ... \tag{2.24}$$

$$\frac{\partial u_{i,j}}{\partial x} = \frac{u_{i+1,j} - u_{i-1,j}}{2h} + O(h^2) \tag{2.25}$$

and the truncation error is $O(h^2)$ and is approximately proportional to h^2. The central difference is second-order accurate. In this case, halving the grid size h would approximately reduce the truncation error to a quarter of the previous error. The central difference is thus more accurate than the forward or backward differences which may be seen geometrically in Figure 2.8 in which the central difference is closest to the actual tangent representing the first-order derivative.

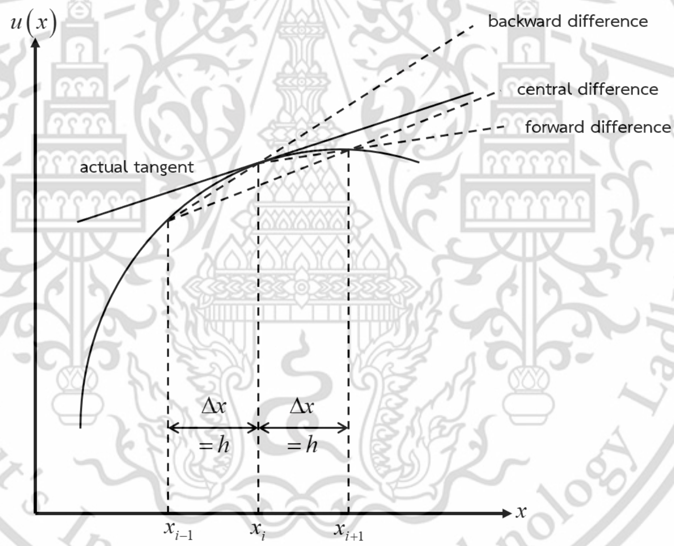


Figure 2.8: Direct approximations of derivatives.

If we take Eq 2.18 + Eq 2.19 and rearrange, we get the central difference for the second-order derivative

$$\frac{\partial^2 u_{i,j}}{\partial x^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{2h^2}{4!}\frac{\partial^4 u_{i,j}}{\partial x^4} - ... \tag{2.26}$$

$$\frac{\partial^2 u_{i,j}}{\partial x^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + O(h^2) \tag{2.27}$$

and the truncation error is $O(h^2)$.

Similarly,for the y-derivatives, we have

$$\frac{\partial u_{i,j}}{\partial y} = \frac{u_{i,j+1} - u_{i,j}}{k} + O(k) \tag{2.28}$$

$$\frac{\partial u_{i,j}}{\partial y} = \frac{u_{i,j} - u_{i,j-1}}{k} + O(k) \tag{2.29}$$

$$\frac{\partial u_{i,j}}{\partial y} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} + O(k^2) \tag{2.30}$$

$$\frac{\partial^2 u_{i,j}}{\partial y^2} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} + O(k^2) \tag{2.31}$$

2.5.2 Traditional forward time centered space techniques

The forward time centered space schemes is employed. Consequently, the finite difference approximation becomes [24]

$$y \cong y_i^n, \tag{2.32}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{2.33}$$

$$\frac{\partial y}{\partial x} \cong \frac{y_{i+1}^n - y_{i-1}^n}{2\Delta x}, \tag{2.34}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}, \tag{2.35}$$

2.5.3 An unconditionally Saulyev finite difference techniques

The Saulyev scheme is employed. Consequently, the finite difference approximation becomes [17]

$$y \cong y_i^n, \tag{2.36}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{2.37}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - y_i^n - y_i^{n+1} + y_{i-1}^{n+1}}{(\Delta x)^2}, \tag{2.38}$$

Chapter 3

A long term shoreline evolution with a single groin structure and their numerical precision testing

This chapter proposes shoreline evolution model. We take two numerical methods, the traditional forward time centered space and the Saulyev methods, to compare with the analytical solution that forms the governing equation by boundary condition straight impermeable groin system.

3.1 Shoreline evolution model

The one-dimensional shoreline evolution model :

$$\frac{\partial y}{\partial x} = D\frac{\partial^2 y}{\partial x^2}, \tag{3.1}$$

for all$(x,t) \in (L,T)$ .where $D = \frac{2Q_0}{D_B+D_C}$.

3.2 Numerical techniques for shoreline evolution model

3.2.1 Traditional forward time centered space techniques applied to shoreline evolution model

The forward time centered space schemes is employed. Consequently, the finite difference approximation becomes

$$y \cong y_i^n, \tag{3.2}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{3.3}$$

$$\frac{\partial y}{\partial x} \cong \frac{y_{i+1}^n - y_{i-1}^n}{2\Delta x}, \tag{3.4}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}, \tag{3.5}$$

where $A = \frac{D\Delta t}{(\Delta x)^2}$.
Substituting Eqs (3.2)-(3.5) into Eq (3.1), we obtain,

$$\frac{y_i^{n+1} - y_i^n}{\Delta t} = D\left(\frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}\right), \tag{3.6}$$

for $1 \le i \le M-1$ and $0 \le n \le N-1$. Eq (3.6) can be written in an explicit form of finite difference as follows,

$$y_i^{n+1} = Ay_{i+1}^n + (1-2A)y_i^n + Ay_{i-1}^n, \tag{3.7}$$

for $1 \le i \le M-1$ and $0 \le n \le N-1$.

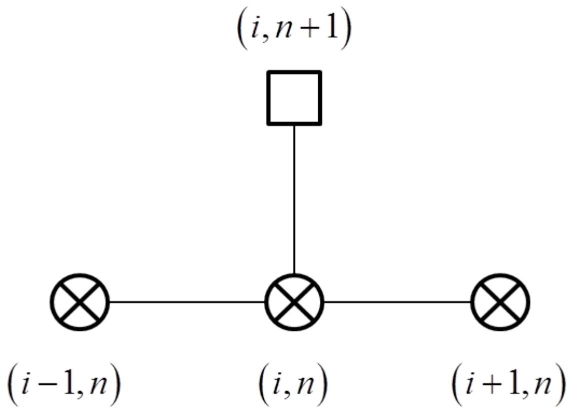


Figure 3.1*:* Stencil diagram of forward central space finite difference technique

3.2.2 An unconditionally Saulyev finite difference techniques applied to shoreline evolution model

The Saulyev method is employed. Consequently, the finite difference approximation becomes

$$y \cong y_i^n, \tag{3.8}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{3.9}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - y_i^n - y_i^{n+1} + y_{i-1}^{n+1}}{(\Delta x)^2}, \tag{3.10}$$

where $A = \frac{D\Delta t}{(\Delta x)^2}$. Substituting Eqs (3.8)-(3.10) into Eq (3.1), we obtain

$$\frac{y_i^{n+1} - y_i^n}{\Delta t} = D\left(\frac{y_{i+1}^n - y_i^n - y_i^{n+1} + y_{i-1}^{n+1}}{(\Delta x)^2}\right), \tag{3.11}$$

for $1 \leq i \leq M-1$ and $0 \leq n \leq N-1$. Eq (3.11) can be written in an explicit form of finite difference as follows,

$$y_i^{n+1} = \frac{1}{1+A}(Ay_{i+1}^n + (1-A)y_i^n + Ay_{i-1}^{n+1}), \tag{3.12}$$

for $1 \leq i \leq M-1$ and $0 \leq n \leq N-1$.

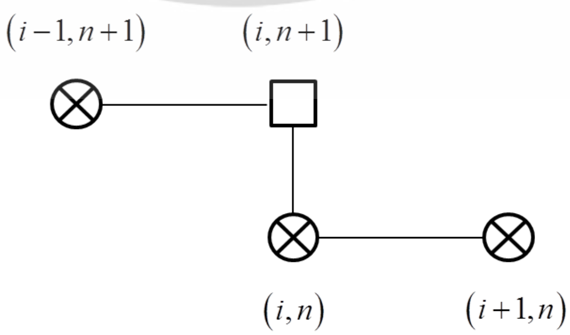


Figure 3.2*:* Stencil diagram of Saulyev finite difference technique

3.3 Initial and boundary conditions setting

The initials shoreline is assumed to be parallel to the x-axis.

3.3.1 Straight Impermeable groin system

Straight Impermeable groin system as show in Figure 3.3.

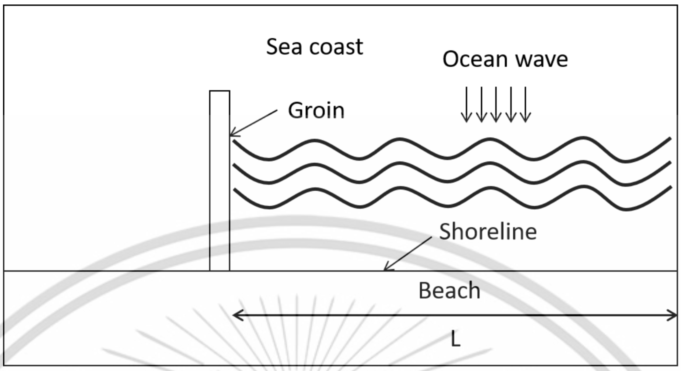


Figure 3.3: Initial shoreline with configuration straight impermeable groin

Assuming that the braking wave angle to the shoreline is as show in Figure 3.4. It follows that the sand transport rate along the shoreline is uniform. The groin is instantaneously added at as show in Figure 3.4.

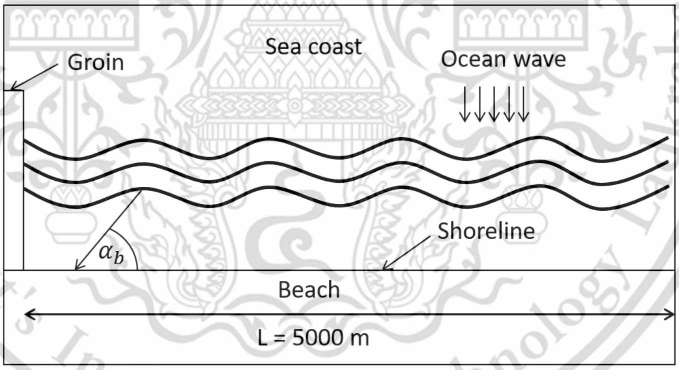


Figure 3.4: Initial shoreline

These means that the initial condition becomes

$$y(x,0) = h_1(x), \tag{3.13}$$

boundary conditions are also assumed by

$$y(0,t) = f(t), \tag{3.14}$$

and

$$y(L,t) = g(t), \tag{3.15}$$

when $h_1(x)$, $f(t)$ and $g(t)$ are given functions as show in Figure 3.5

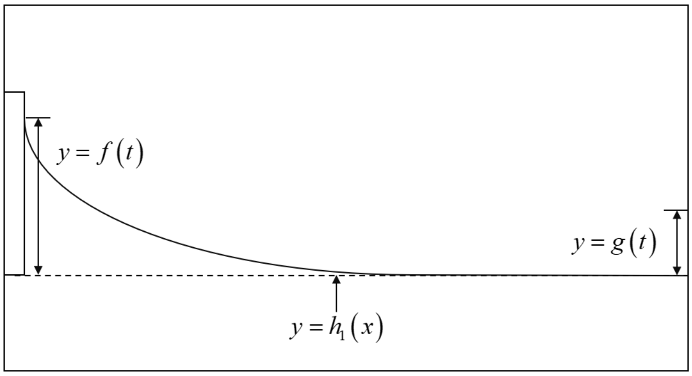


Figure 3.5*:* Initial shoreline

Their given functions in Eqs (3.13) - (3.15) are archived by a numerical interpolation method such as the lagrange interpolation techniques, etc as show in the Figure 3.6.

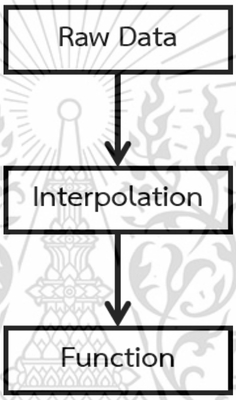


Figure 3.6*:* Function

3.4 Numerical experiments

In order to investigate the shoreline evolution in the long-term scale. The numerical results of the different beach situation are considered and the solution the idealized problem is presented. During the simulations, assuming that the length of considered shoreline is $L = 5000\ m.$ The amplitude of the long-shore transport rate is $Q_0 = 7500\ m^2/day.$ the averaged berm height $D_B = 2\ m.$ the closure depth $D_C = 28\ m.$ the breaking wave impact angle $\alpha_0 = 0.02$ The simulation setting is illustrated in Figure 3.4.

We will employ the traditional forward time centered space techniques Eq (3.7) and the Saulyev finite difference techniques Eq (3.12) to approximate the model solution. The calculated results are shown in Figure 3.7-3.8 and Table 3.2-3.3. The exact solutions of the simulation is [16]

$$y(x,t) = \tan\alpha_0\sqrt{\frac{4Dt}{\pi}}\left(e^{\frac{x^2}{4Dt}} - \frac{x\sqrt{\pi}}{2\sqrt{Dt}}erfc\left(\frac{x}{2\sqrt{Dt}}\right)\right), \qquad (3.16)$$

where the values of them are show in Table 3.1. The approximated solutions of the traditional forward time centered space techniques gives approximated solutions in Table 3.2. Saulyev techniques gives approximated solutions in Table 3.3.

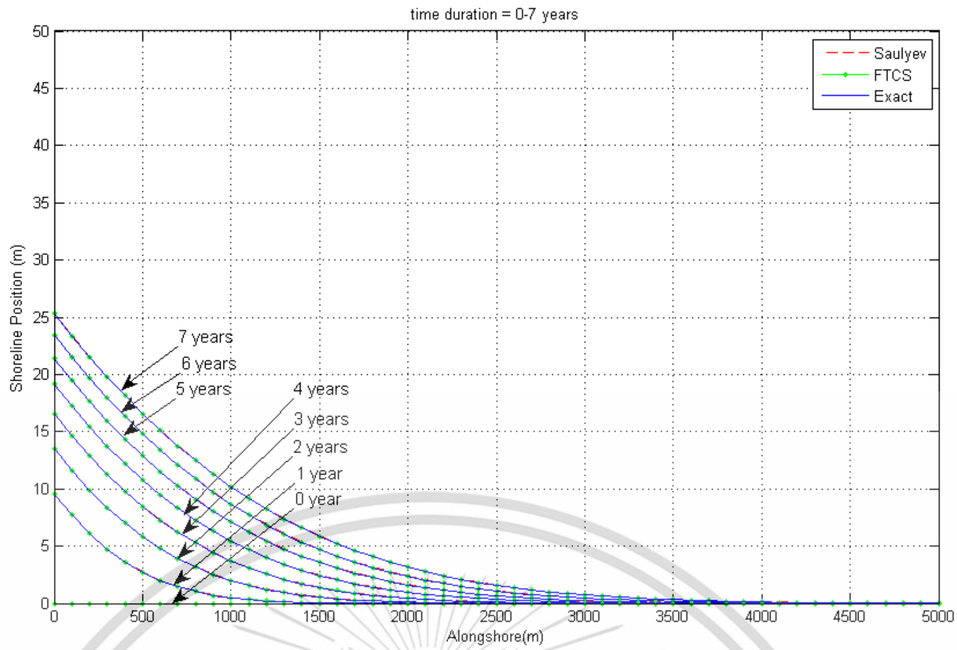


Figure 3.7*:* Shoreline evolution 0-7 years

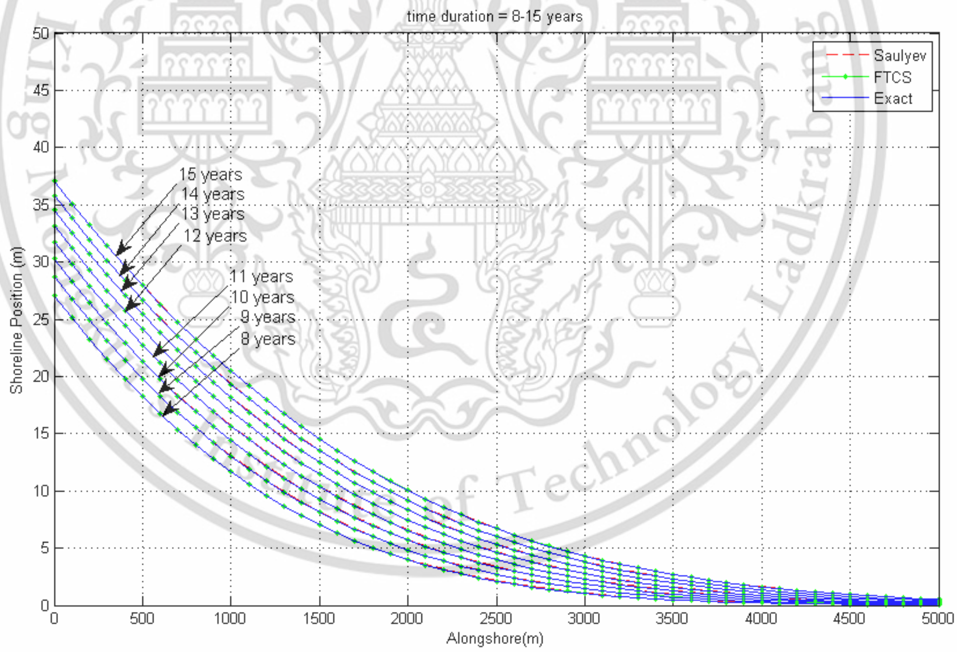


Figure 3.8*:* Shoreline evolution 8-15 years

Table 3.1: Theoretical solution of shoreline evolution.

Time (years)	Distance(m)					
	0	1000	2000	3000	4000	5000
0	0.00	0.00	0.00	0.00	0.00	0.00
1	9.56	0.47	0.00	0.00	0.00	0.00
2	13.53	1.99	0.10	0.00	0.00	0.00
3	16.58	3.72	0.43	0.02	0.00	0.00
4	19.15	5.43	0.95	0.10	0.01	0.00
5	21.41	7.09	1.60	0.24	0.02	0.00
6	23.45	8.68	2.35	0.45	0.06	0.01
7	25.33	10.20	3.14	0.72	0.12	0.01
8	27.08	11.65	3.98	1.05	0.21	0.03
9	28.72	13.04	4.83	1.43	0.33	0.06
10	30.28	14.39	5.70	1.84	0.48	0.10
11	31.76	15.68	6.57	2.29	0.66	0.15
12	33.17	16.93	7.44	2.77	0.86	0.22
13	34.52	18.14	8.31	3.27	1.09	0.31
14	35.83	19.32	9.17	3.78	1.34	0.41
15	37.08	20.46	10.03	4.31	1.61	0.52

Table 3.2: Approximated shoreline evolution along 15 years using the traditional forward time centered space techniques ($\Delta x = 100\ (m)$, $\Delta t = 1\,(day)$).

Time (years)	Distance(m)					
	0	1000	2000	3000	4000	5000
0	0.00	0.00	0.00	0.00	0.00	0.00
1	9.56	0.48	0.00	0.00	0.00	0.00
2	13.53	1.99	0.11	0.00	0.00	0.00
3	16.58	3.72	0.43	0.02	0.00	0.00
4	19.15	5.44	0.95	0.10	0.01	0.00
5	21.41	7.09	1.61	0.24	0.02	0.00
6	23.45	8.68	2.35	0.45	0.06	0.01
7	25.33	10.20	3.15	0.72	0.12	0.01
8	27.08	11.65	3.98	1.05	0.21	0.03
9	28.72	13.04	4.83	1.43	0.33	0.06
10	30.28	14.39	5.70	1.84	0.48	0.10
11	31.76	15.68	6.57	2.29	0.66	0.15
12	33.17	16.93	7.44	2.77	0.86	0.22
13	34.52	18.15	8.31	3.27	1.09	0.31
14	35.83	19.32	9.17	3.78	1.34	0.41
15	37.08	20.46	10.03	4.31	1.61	0.52

Table 3.3*:* Approximated shoreline evolution along 15 years using the Saulyev techniques ($\Delta x = 100\ (m)$, $\Delta t = 1\,(day)$).

Time (years)	Distance(m)					
	0	1000	2000	3000	4000	5000
0	0.00	0.00	0.00	0.00	0.00	0.00
1	9.56	0.50	0.00	0.00	0.00	0.00
2	13.53	2.01	0.11	0.00	0.00	0.00
3	16.58	3.74	0.45	0.03	0.00	0.00
4	19.15	5.46	0.97	0.10	0.01	0.00
5	21.41	7.12	1.63	0.25	0.02	0.00
6	23.45	8.70	2.37	0.46	0.06	0.01
7	25.33	10.22	3.17	0.74	0.12	0.01
8	27.08	11.67	4.00	1.07	0.22	0.03
9	28.72	13.06	4.86	1.45	0.34	0.06
10	30.28	14.41	5.72	1.86	0.49	0.10
11	31.76	15.70	6.59	2.31	0.67	0.15
12	33.17	16.95	7.46	2.79	0.87	0.22
13	34.52	18.16	8.33	3.29	1.10	0.31
14	35.83	19.34	9.19	3.81	1.36	0.41
15	37.08	20.48	10.05	4.34	1.63	0.52

Table 3.4: Absolute error of shoreline evolution when the traditional forward time centered space techniques and Sualyev techniques are used 1-9 years.

Time (years)	Distance (m)	Absolute error	
		FTCS	Saulyev
1	1000	0.5300×10^{-2}	0.2420×10^{-1}
	2000	0.3277×10^{-3}	0.1100×10^{-2}
	3000	0.7868×10^{-6}	0.2748×10^{-5}
	4000	0.1224×10^{-9}	0.7012×10^{-9}
2	1000	0.3782×10^{-2}	0.2883×10^{-1}
	2000	0.1876×10^{-2}	0.8632×10^{-2}
	3000	0.1314×10^{-3}	0.4934×10^{-3}
	4000	0.1872×10^{-5}	0.6734×10^{-5}
3	1000	0.2594×10^{-2}	0.2783×10^{-1}
	2000	0.2587×10^{-2}	0.1580×10^{-1}
	3000	0.5741×10^{-3}	0.2671×10^{-2}
	4000	0.4017×10^{-4}	0.1625×10^{-3}
4	1000	0.1891×10^{-2}	0.2617×10^{-1}
	2000	0.2674×10^{-2}	0.2041×10^{-1}
	3000	0.1059×10^{-2}	0.5963×10^{-2}
	4000	0.1654×10^{-3}	0.7785×10^{-3}
5	1000	0.1450×10^{-2}	0.2458×10^{-1}
	2000	0.2527×10^{-2}	0.2319×10^{-1}
	3000	0.1417×10^{-2}	0.9403×10^{-2}
	4000	0.3576×10^{-3}	0.1942×10^{-2}
6	1000	0.1155×10^{-2}	0.2317×10^{-1}
	2000	0.2314×10^{-2}	0.2480×10^{-1}
	3000	0.1636×10^{-2}	0.1252×10^{-1}
	4000	0.5646×10^{-3}	0.3494×10^{-2}
7	1000	0.9473×10^{-3}	0.2195×10^{-1}
	2000	0.2097×10^{-2}	0.2571×10^{-1}
	3000	0.1747×10^{-2}	0.1516×10^{-1}
	4000	0.7477×10^{-3}	0.5216×10^{-2}
8	1000	0.7946×10^{-3}	0.2089×10^{-1}
	2000	0.1895×10^{-2}	0.2617×10^{-1}
	3000	0.1784×10^{-2}	0.1733×10^{-1}
	4000	0.8890×10^{-3}	0.6929×10^{-2}
9	1000	0.6787×10^{-3}	0.1996×10^{-1}
	2000	0.1714×10^{-2}	0.2634×10^{-1}
	3000	0.1771×10^{-2}	0.1906×10^{-1}
	4000	0.9852×10^{-3}	0.8517×10^{-2}

Table 3.5: Absolute error of shoreline evolution when the traditional forward time centered space techniques and Sualyev techniques are used 10-15 years.

Time (years)	Distance (m)	Absolute error	
		FTCS	Saulyev
10	1000	0.5881×10^{-3}	0.1913×10^{-1}
	2000	0.1554×10^{-2}	0.2632×10^{-1}
	3000	0.1726×10^{-2}	0.2043×10^{-1}
	4000	0.1041×10^{-2}	0.9919×10^{-2}
11	1000	0.5156×10^{-3}	0.1840×10^{-1}
	2000	0.1413×10^{-2}	0.2617×10^{-1}
	3000	0.1659×10^{-2}	0.2147×10^{-1}
	4000	0.1062×10^{-2}	0.1111×10^{-1}
12	1000	0.4562×10^{-3}	0.1774×10^{-1}
	2000	0.1413×10^{-2}	0.2593×10^{-1}
	3000	0.1659×10^{-2}	0.2224×10^{-1}
	4000	0.1062×10^{-2}	0.1209×10^{-1}
13	1000	0.4063×10^{-3}	0.1713×10^{-1}
	2000	0.1175×10^{-2}	0.2562×10^{-1}
	3000	0.1492×10^{-2}	0.2279×10^{-1}
	4000	0.1034×10^{-2}	0.1287×10^{-1}
14	1000	0.3637×10^{-3}	0.1658×10^{-1}
	2000	0.1074×10^{-2}	0.2527×10^{-1}
	3000	0.1401×10^{-2}	0.2314×10^{-1}
	4000	0.9966×10^{-3}	0.1347×10^{-1}
15	1000	0.3265×10^{-3}	0.1610×10^{-1}
	2000	0.9811×10^{-3}	0.2490×10^{-1}
	3000	0.1300×10^{-2}	0.2330×10^{-1}
	4000	0.9499×10^{-3}	0.1390×10^{-1}

Table 3.6: The comparison of the stability in each grid spacing sizes.

Δx	Δt	Stability	
		FTCS	Saulyev
0.50	25	stable	stable
	50	stable	stable
	100	stable	stable
1.00	25	unstable	stable
	50	stable	stable
	100	stable	stable
2.00	25	unstable	stable
	50	stable	stable
	100	stable	stable
4.00	25	unstable	stable
	50	unstable	stable
	100	stable	stable

Chapter 4

A long term shoreline evolution with a twin groins structure and their parameter setting techniques

This chapter proposes shoreline evolution model. We take field data at gulf of Thailand to compare the long-shore transport rate (D) in each month along a year. We take two numerical methods, the traditional forward time centered space and the Saulyev methods by boundary condition straight impermeable twin-groin system.

4.1 Shoreline evolution model

In the one-line model is obtained,

$$\frac{\partial y}{\partial t} = \frac{1}{D_B + D_C}\left(-\frac{\partial Q}{\partial x}\right), \tag{4.1}$$

where x is the alongshore coordinate (m); y is the shoreline positions (m) and perpendicular to x-axis; t is time (day); Q is the long-shore sand transport rate (m^3/day); D_B is the average berm height (m) and D_C is the closure depth(m) as show in Fig 2.2.

In order to solve the Eq (4.1), necessary to specify an expression for the longshore sand transport rate, Q . This quantity is considered to be generated by the wave obliquely incident to the shoreline. A general expression for the long-shore sand transport rate was developed by the US Army Corp [14] :

$$Q = Q_0 \sin(2\alpha_b), \tag{4.2}$$

where Q_0 is the amplitude of the long-shore sand transport rate. α_b is the angle between breaking wave crest impact angle and local shoreline. The empirical predictive formula for the amplitude of the long-shore sand transport rate is [15] :

$$Q_0 = \frac{\rho}{16}(H_b^2 c_{gb})\frac{K}{(\rho_s - \rho)(1-n)}, \tag{4.3}$$

where the subscript b denotes value at the point of breaking, $c_g$is the wave group velocity, H is the wave height, ρ_s is the density of the sediment (kg/m^3), ρ is the density of the sea water, n is the porosity and K is the dimensionless coefficient which is a function of particle size. The quantity α_b is the angle between breaking wave crest and local shoreline, and may be written as :

$$\alpha_b = \alpha_0 - \tan^{-1}\left(\frac{\partial y}{\partial x}\right), \tag{4.4}$$

where α_0 is the angle between breaking wave crests and the x-axis. For beaches with mild slope, it can be assumed that breaking wave angle to the shoreline is small.
In this case,

$$\sin(2\alpha_b) \approx 2\alpha_b, \tag{4.5}$$

and

$$\tan^{-1}\left(\frac{\partial y}{\partial x}\right) \approx \frac{\partial y}{\partial x}. \tag{4.6}$$

Substituting Eq (4.4) into the Eq (4.2), and assuming the beach with mild slope yields :

$$Q = Q_0\left(2\alpha_b - 2\frac{\partial y}{\partial x}\right), \tag{4.7}$$

Substituting Eq (4.7) into the Eq (4.1) and neglecting the sources or sinks along the coast gives :

$$\frac{\partial y}{\partial x} = D\frac{\partial^2 y}{\partial x^2}, \tag{4.8}$$

for all$(x,t) \in (L,T)$, where

$$D = \frac{2Q_0}{D_B + D_C}. \tag{4.9}$$

4.2 Numerical techniques for shoreline evolution model

4.2.1 Traditional forward time centered space techniques applied to shoreline evolution model

The forward time centered space schemes is employed. Consequently, the finite difference approximation becomes

$$y \cong y_i^n, \tag{4.10}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{4.11}$$

$$\frac{\partial y}{\partial x} \cong \frac{y_{i+1}^n - y_{i-1}^n}{2\Delta x}, \tag{4.12}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}, \tag{4.13}$$

where $A = \frac{D\Delta t}{(\Delta x)^2}$.

Substituting Eqs (4.10)-(4.13) into Eq (4.8), we obtain,

$$\frac{y_i^{n+1} - y_i^n}{\Delta t} = D\left(\frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}\right), \tag{4.14}$$

for $1 \le i \le M-1$ and $0 \le n \le N-1$. Eq (4.14) can be written in an explicit form of finite difference as follows,

$$y_i^{n+1} = Ay_{i+1}^n + (1-2A)y_i^n + Ay_{i-1}^n, \tag{4.15}$$

for $1 \le i \le M-1$ and $0 \le n \le N-1$.

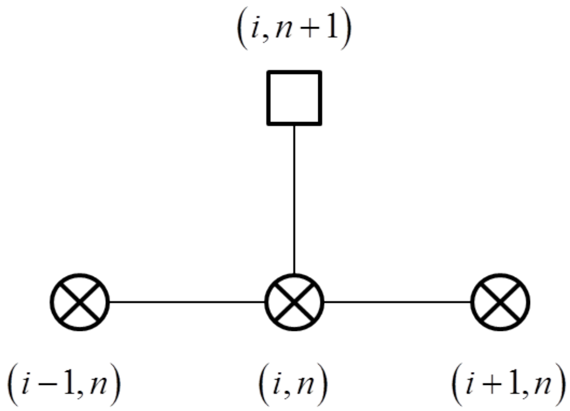


Figure 4.1*:* Stencil diagram of forward central space finite difference technique

4.2.2 An unconditionally Saulyev finite difference techniques applied to shoreline evolution model

The Saulyev method is employed. Consequently, the finite difference approximation becomes

$$y \cong y_i^n, \tag{4.16}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{4.17}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - y_i^n - y_i^{n+1} + y_{i-1}^{n+1}}{(\Delta x)^2}, \tag{4.18}$$

where $A = \frac{D\Delta t}{(\Delta x)^2}$. Substituting Eqs (4.16)-(4.18) into Eq (4.8), we obtain

$$\frac{y_i^{n+1} - y_i^n}{\Delta t} = D\left(\frac{y_{i+1}^n - y_i^n - y_i^{n+1} + y_{i-1}^{n+1}}{(\Delta x)^2}\right), \tag{4.19}$$

for $1 \leq i \leq M-1$ and $0 \leq n \leq N-1$. Eq (4.19) can be written in an explicit form of finite difference as follows,

$$y_i^{n+1} = \frac{1}{1+A}(Ay_{i+1}^n + (1-A)y_i^n + Ay_{i-1}^{n+1}), \tag{4.20}$$

for $1 \leq i \leq M-1$ and $0 \leq n \leq N-1$.

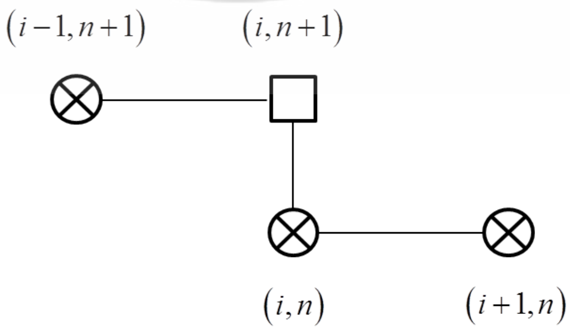


Figure 4.2*:* Stencil diagram of Saulyev finite difference technique

4.2.3 The employment of traditional forward time centered space techniques to the left and the right boundary conditions

The forward time centered space method is employed. Consequently, the finite difference approximation becomes

$$y \cong y_i^n, \tag{4.21}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{4.22}$$

$$\frac{\partial y}{\partial x} \cong \frac{y_{i+1}^n - y_{i-1}^n}{2\Delta x}, \tag{4.23}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}, \tag{4.24}$$

where $A = \frac{D\Delta t}{(\Delta x)^2}$.

Substituting Eqs (4.21)-(4.24) into Eq (4.8), we obtain,

$$\frac{y_i^{n+1} - y_i^n}{\Delta t} = D\left(\frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}\right), \tag{4.25}$$

For $i = 0$, substitution of the approximate unknown value of the left boundary by a traditional central difference approximation with the known derivative the left boundary condition gives

$$y_{-1}^n = y_1^n - 2(\Delta x)(-\tan(\alpha_0)), \tag{4.26}$$

Substituting Eq (4.26) into Eq (4.25), we obtain

$$y_i^{n+1} = (1-2A)y_i^n + 2Ay_{i+1}^n - 2A(\Delta x)(-\tan(\alpha_0)), \tag{4.27}$$

For $i = M$, substitution of the approximate unknown value of the right boundary by a traditional central difference approximation with the known derivative the right boundary condition gives

$$y_{M+1}^n = y_{M-1}^n + 2(\Delta x)(-\tan(-\alpha_0)), \tag{4.28}$$

Substituting Eq (4.28) into Eq (4.25), we obtain

$$y_i^{n+1} = 2Ay_{i-1}^n + (1-2A)y_i^n + 2A(\Delta x)(-\tan(-\alpha_0)), \tag{4.29}$$

The Eq (4.27) and Eq (4.29) can be used to calculate the values y_i^n on grid points of the solution domain.

4.3 Initial and boundary conditions setting

The initials shoreline is assumed to be parallel to the x-axis.

4.3.1 Straight Impermeable twin-groin system

Straight Impermeable twin-groin system as show in Figure 4.3.

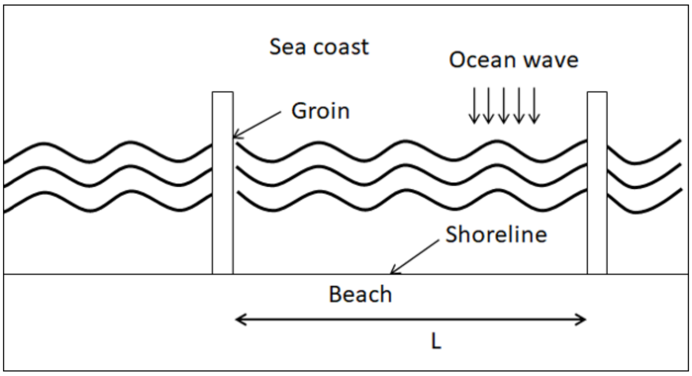


Figure 4.3*:* Initial shoreline with configuration straight impermeable twin-groin

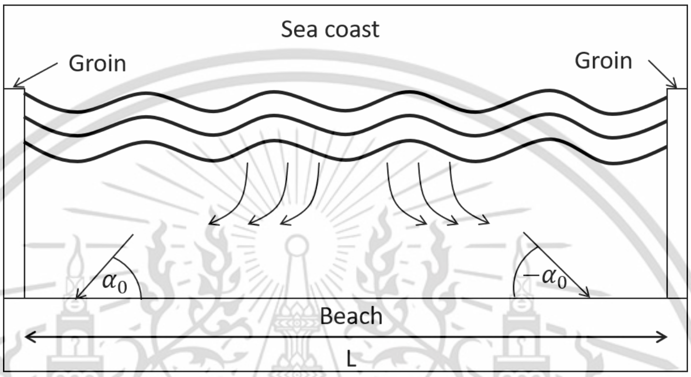


Figure 4.4*:* Initial shoreline

Assuming that the braking wave angle to the shoreline is as show in Figure 4.5. It follows that the sand transport rate along the shoreline is uniform. The groin is instantaneously added at as show in Figure 4.5.

These means that the initial condition becomes

$$y(x,0) = h_2(x), \tag{4.30}$$

boundary conditions are also assumed by

$$\frac{\partial y(0,t)}{\partial x} = -\tan(\alpha_0), \tag{4.31}$$

and

$$\frac{\partial y(L,t)}{\partial x} = -\tan(-\alpha_0), \tag{4.32}$$

when $h_2(x)$ are given functions as show in Figure 4.5

4.4 Physical parameters setting techniques

Assuming that the density of the sediment (ρ_s) [19], the density of sea water (ρ) [20], the porosity (n) [21], the dimensionless coefficient which is a function of particle size (K) [22], the averaged berm height (D_B) and the closure depth (D_C) are listed belows.

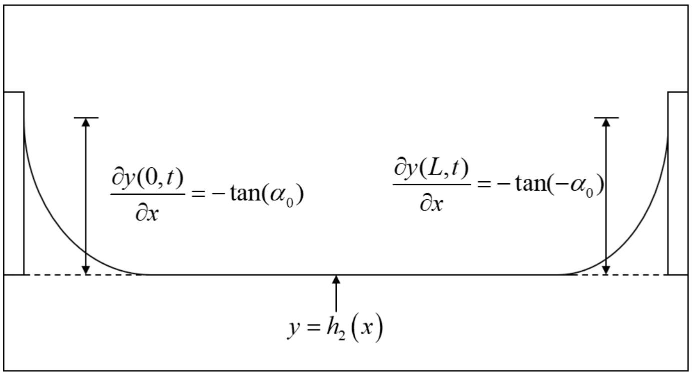


Figure 4.5*:* Initial shoreline

Table 4.1*:* Parameters of sand transport rate [19], [20], [21], [22].

The density of the sediment (ρ_s (kg/m^3))	1700
The density of sea water (ρ (kg/m^3))	1020
The porosity (n)	0.406
The dimensionless coefficient which is a function of particle size (K)	0.375
The averaged berm height (D_B (m))	2
The closure depth (D_C (m))	28

The wave group velocity (c_g) and the wave height (H) in each month along a year is measured by field data at gulf of Thailand such that data are collected by Geo-Informatics and Space Technology Development Agency (Public Organization) (GISTDA) [23] as listed belows.

Table 4.2*:* The wave group velocity and the wave height [23].

Month	c_g (m/day)	H (m)
Jan 2019	8951.04	1.5
Feb 2019	6998.4	1.5
Mar 2019	5866.56	0.5
Apr 2019	6920.64	1.5
May 2019	5719.68	0.5
Jun 2019	5546.88	0.5
Jul 2018	8225.28	1.5
Aug 2018	9357.12	1.5
Sep 2018	13711.68	1.5
Oct 2018	15085.44	2.5
Nov 2018	10877.76	1.5
Dec 2018	11396.16	1.5

The amplitude of the long-shore transport rates (Q_0) are obtained by Eq (4.3) and the long-shore transport rates (D) are obtained by Eq (4.9) as listed belows.

Table 4.3: The amplitude of the long-shore transport rates and the long-shore transport rates.

Month	$Q_0\ (m/day)$	$D\ (m/day)$
Jan 2019	1191.99	79.4659
Feb 2019	931.96	62.1307
Mar 2019	86.80	5.7869
Apr 2019	921.61	61.4403
May 2019	84.63	5.6420
Jun 2019	82.07	5.4716
Jul 2018	1095.34	73.0227
Aug 2018	1246.07	83.071
Sep 2018	1825.95	121.7301
Oct 2018	5580.26	372.017
Nov 2018	1448.57	96.5710
Dec 2018	1517.60	101.1233

4.5 Numerical experiments

In order to investigate the shoreline evolution in the long-term scale. The numerical results of the different beach situation are considered and the solution the idealized problem is presented. During the simulations, assuming that the length of considered shoreline is $L = 100,\ 200,\ 300$ and $400\ m.$ the breaking wave impact angle $\alpha_0 = 0.02$ The simulation setting is illustrated in Figure 4.5.

We will employ the traditional forward time centered space techniques Eq (4.15) and the Saulyev finite difference techniques Eq (4.20) to approximate the model solution. The calculated results $L = 100,\ 200,\ 300$ and $400\ m$ are shown in Figure 4.6-4.13

The approximated solutions of the traditional forward time centered space techniques gives approximated solutions in Table 4.4-4.7. Saulyev techniques gives approximated solutions in Table 4.8-4.11.

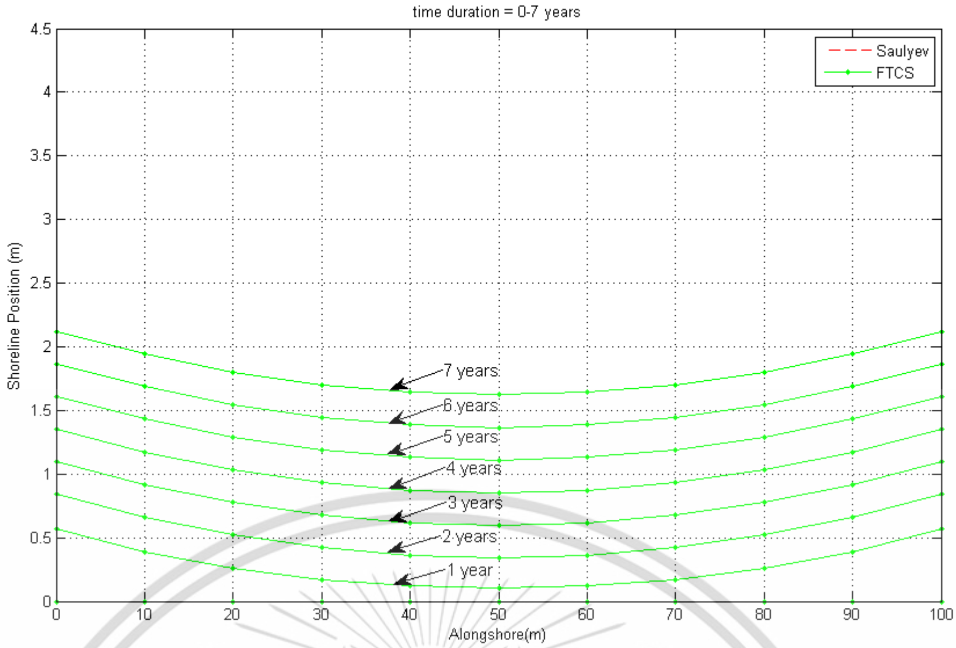


Figure 4.6: Shoreline evolution with distance between groin $L = 100\ m$ 0-7 years

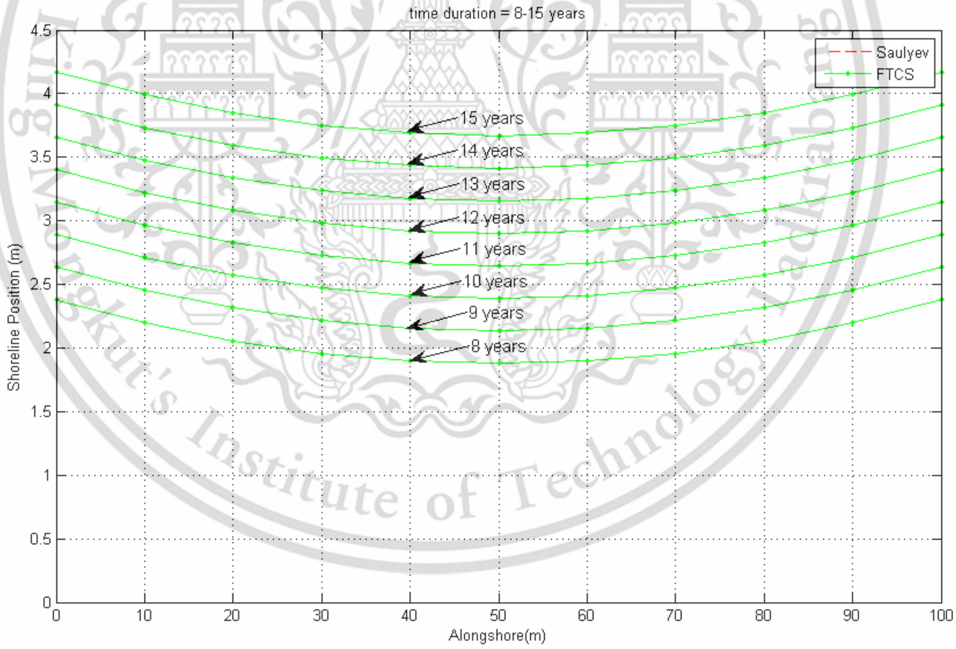


Figure 4.7: Shoreline evolution with distance between groin $L = 100\ m$ 8-15 years

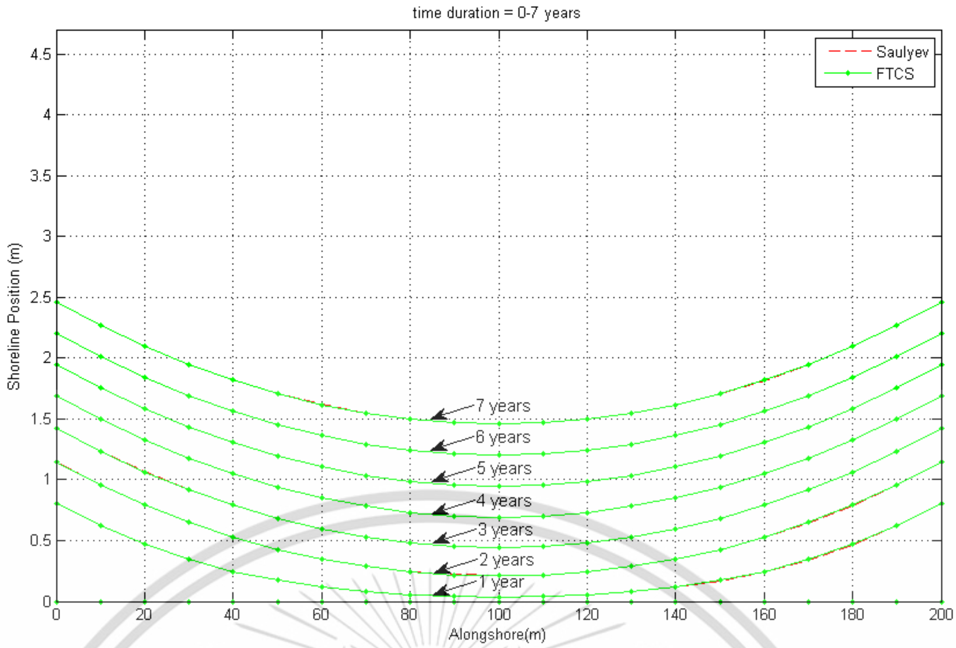


Figure 4.8*:* Shoreline evolution with distance between groin $L = 200\ m$ 0-7 years

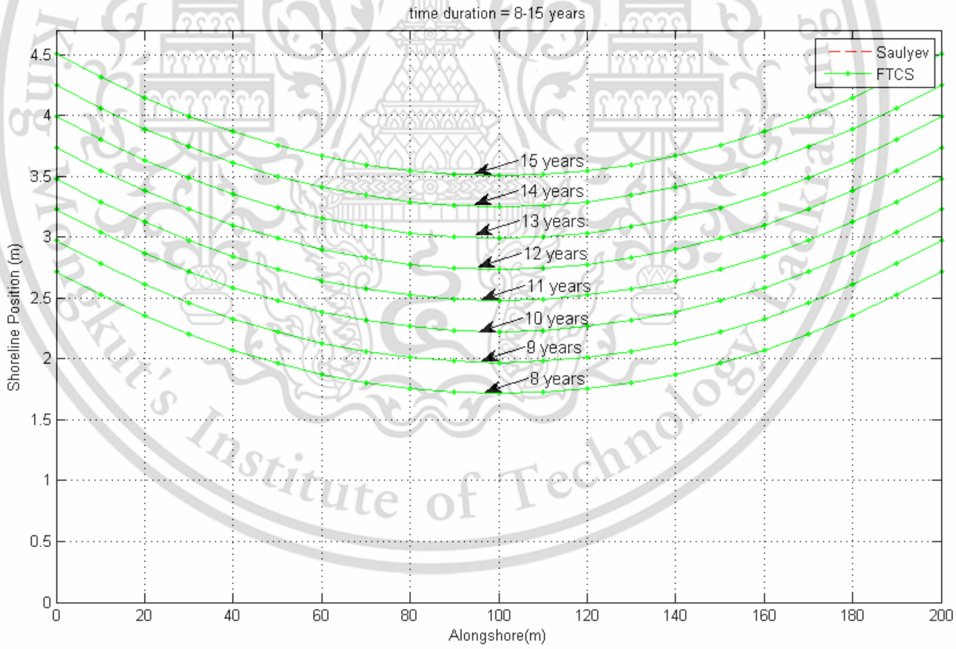


Figure 4.9*:* Shoreline evolution with distance between groin $L = 200\ m$ 8-15 years

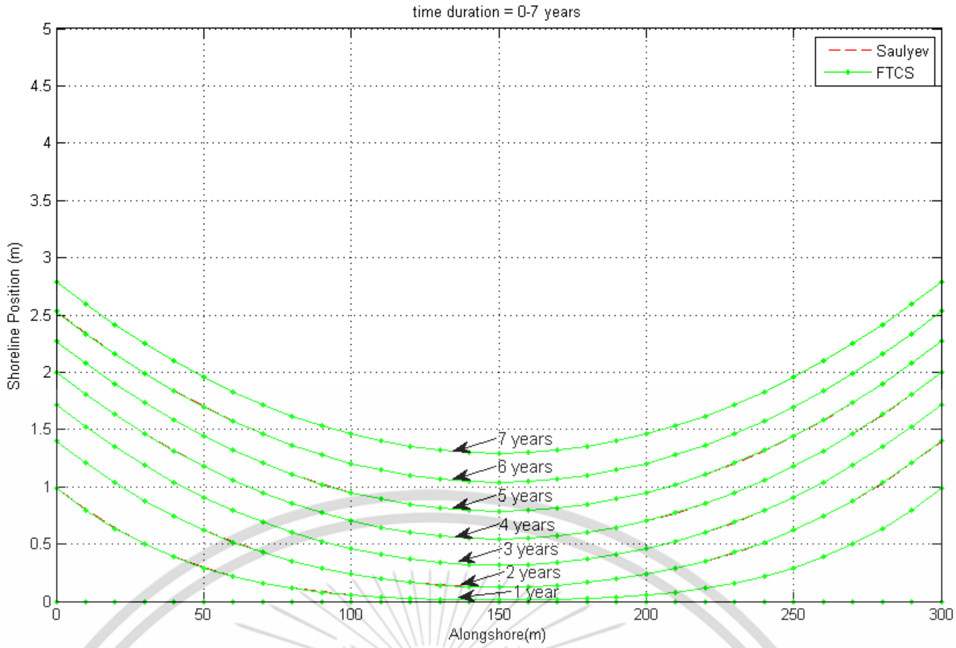


Figure 4.10*:* Shoreline evolution with distance between groin $L = 300\ m$ 0-7 years

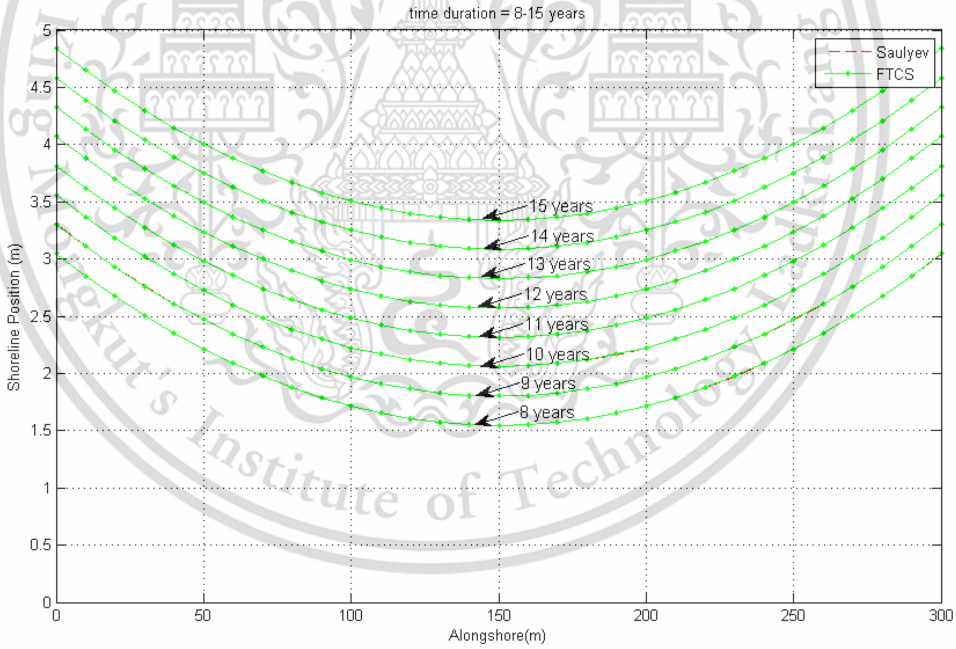


Figure 4.11*:* Shoreline evolution with distance between groin $L = 300\ m$ 8-15 years

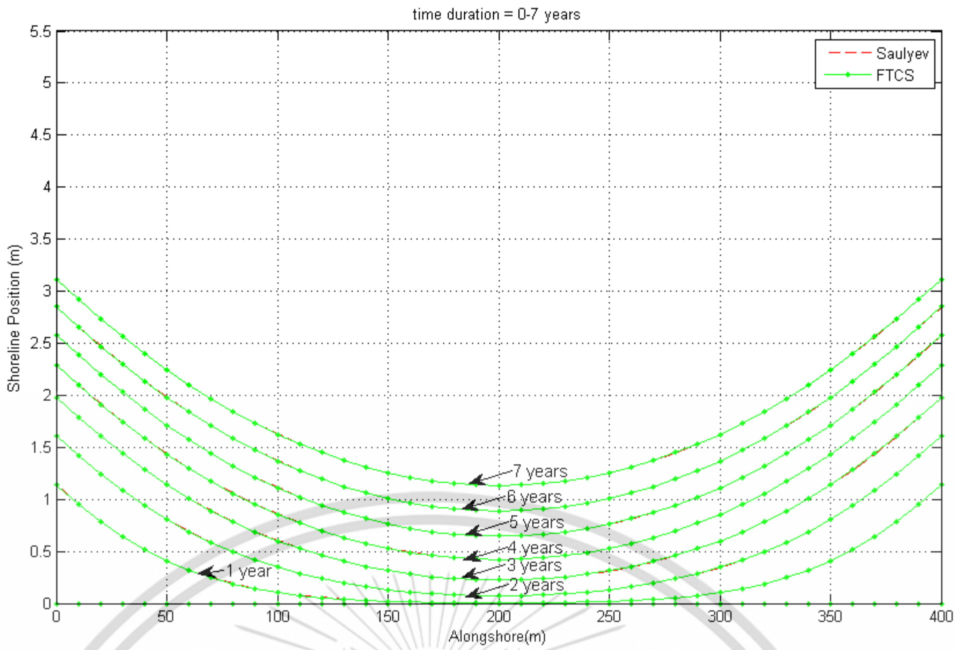


Figure 4.12*:* Shoreline evolution with distance between groin $L = 400\ m$ 0-7 years

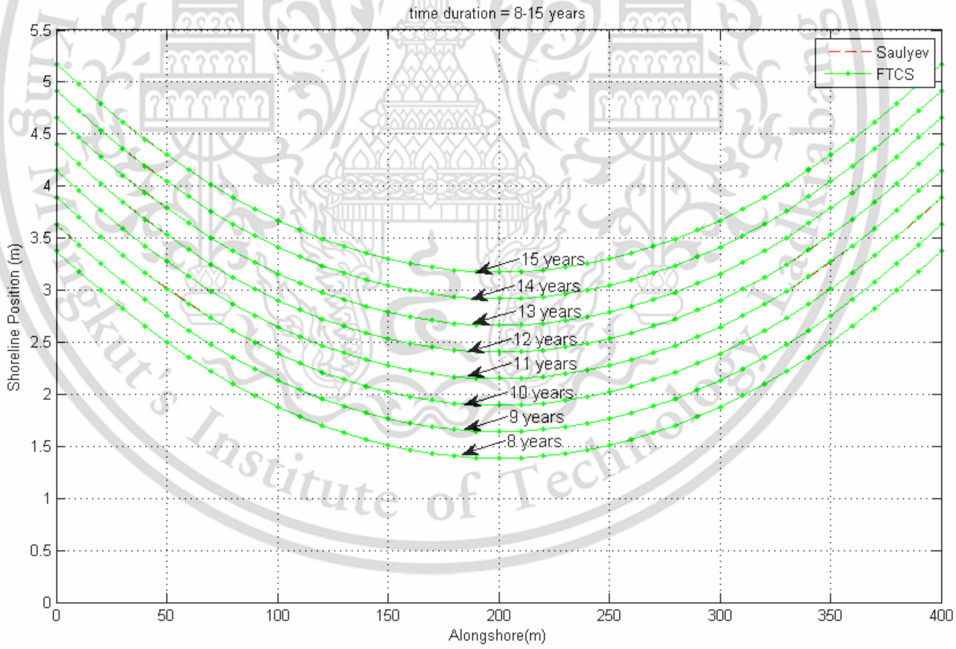


Figure 4.13*:* Shoreline evolution with distance between groin $L = 400\ m$ 8-15 years

Table 4.4: Approximated shoreline evolution along 15 years using the traditional forward time centered space techniques L is 100 m.

Time	Distance(m)					
(years)	0	20	40	60	80	100
1	0.5679	0.2604	0.1206	0.1206	0.2604	0.5679
5	1.6102	1.2902	1.1302	1.1302	1.2902	1.6102
10	2.8905	2.5705	2.4104	2.4104	2.5705	2.8905
15	4.1708	3.8507	3.6907	3.6907	3.8507	4.1708

Table 4.5: Approximated shoreline evolution along 15 years using the Saulyev finite difference techniques L is 100 m.

Time	Distance(m)					
(years)	0	20	40	60	80	100
1	0.5682	0.2607	0.1208	0.1206	0.2602	0.5676
5	1.6103	1.2903	1.1303	1.1302	1.2902	1.6103
10	2.8906	2.5705	2.4105	2.4105	2.5705	2.8906
15	4.1708	3.8508	3.6908	3.6908	3.8508	4.1708

Table 4.6: Approximated shoreline evolution along 15 years using the traditional forward time centered space techniques L is 200 m.

Time	Distance(m)					
(years)	0	20	40	60	80	100
1	0.8036	0.4664	0.2455	0.1176	0.0551	0.0369
5	1.9445	1.5846	1.3050	1.1054	0.9858	0.9459
10	3.2256	2.8655	2.5855	2.3854	2.2654	2.2254
15	4.5058	4.1458	3.8657	3.6657	3.5457	3.5057
Time	Distance(m)					
(years)	120	140	160	180	200	
1	0.0551	0.1176	0.2455	0.4664	0.8036	
5	0.9858	1.1054	1.3050	1.5846	1.9445	
10	2.2654	2.3854	2.5855	2.8655	3.2256	
15	3.5457	3.6657	3.8657	4.1458	4.5058	

Table 4.7: Approximated shoreline evolution along 15 years using the Saulyev finite difference techniques L is 200 m.

Time (years)	Distance(m)					
	0	20	40	60	80	100
1	0.8043	0.4671	0.2463	0.1182	0.0555	0.0369
5	1.9449	1.5850	1.3053	1.1057	0.9859	0.9460
10	3.2257	2.8656	2.5856	2.3855	2.2655	2.2255
15	4.5059	4.1459	3.8658	3.6658	3.5458	3.5058
Time (years)	Distance(m)					
	120	140	160	180	200	
1	0.0549	0.1172	0.2449	0.4656	0.8029	
5	0.9857	1.1053	1.3047	1.5843	1.9442	
10	2.2655	2.3855	2.5855	2.8655	3.2255	
15	3.5458	3.6658	3.8658	4.1458	4.5059	

Table 4.8: Approximated shoreline evolution along 15 years using the traditional forward time centered space techniques L is 300 m.

Time (years)	Distance(m)							
	0	20	40	60	80	100	120	140
1	0.9857	0.6370	0.3858	0.2179	0.1145	0.0560	0.0143	0.0143
5	2.2700	1.8974	1.5796	1.3162	1.1067	0.9503	0.7947	0.7947
10	3.5593	3.1860	2.8660	2.5993	2.3860	2.2260	2.0661	2.0661
15	4.8398	4.4664	4.1463	3.8796	3.6663	3.5062	3.3462	3.3462
Time (years)	Distance(m)							
	160	180	200	220	240	260	280	300
1	0.0264	0.0560	0.1145	0.0143	0.2179	0.3858	0.6370	0.9857
5	0.8464	0.9503	1.1067	0.7947	1.3162	1.5796	1.8974	2.2700
10	2.1194	2.2260	2.3860	2.0661	2.5993	2.8660	3.1860	3.5593
15	3.3996	3.5062	3.6663	3.3462	3.8796	4.1463	4.4664	4.8398

Table 4.9: Approximated shoreline evolution along 15 years using the Saulyev finite difference techniques L is 300 m.

Time (years)	Distance(m)							
	0	20	40	60	80	100	120	140
1	0.9869	0.6382	0.3869	0.2190	0.1153	0.0567	0.0268	0.0145
5	2.2709	1.8983	1.5805	1.3170	1.1073	0.9508	0.8468	0.7948
10	3.5597	3.1863	2.8663	2.5996	2.3863	2.2263	2.1196	2.0662
15	4.8399	4.4665	4.1465	3.8798	3.6664	3.5064	3.3997	3.3463
Time (years)	Distance(m)							
	160	180	200	220	240	260	280	300
1	0.0143	0.0261	0.0556	0.1138	0.2170	0.3847	0.6359	0.9846
5	0.7946	0.8462	0.9499	1.1062	1.3156	1.5789	1.8967	2.2692
10	2.0661	2.1194	2.2259	2.3859	2.5991	2.8657	3.1857	3.5591
15	3.3463	3.3996	3.5062	3.6663	3.8796	4.1463	4.4663	4.8397

Table 4.10: Approximated shoreline evolution along 15 years using the traditional forward time centered space techniques L is 400 m.

Time (years)	Distance(m)						
	0	20	40	60	80	100	120
1	1.1392	0.7836	0.5136	0.3198	0.1886	0.1051	0.0553
5	2.5781	2.1998	1.8647	1.5723	1.3220	1.1127	0.9434
10	3.8917	3.5117	3.1718	2.8721	2.6125	2.3929	2.2134
15	5.1733	4.7933	4.4533	4.1532	3.8932	3.6732	3.4932
Time (years)	Distance(m)						
	140	160	180	200	220	240	260
1	0.0274	0.0130	0.0064	0.0046	0.0064	0.0130	0.0274
5	0.8130	0.7207	0.6656	0.6473	0.6656	0.7207	0.8130
10	2.0738	1.9741	1.9143	1.8944	1.9143	1.9741	2.0738
15	3.3532	3.2532	3.1932	3.1732	3.1932	3.2532	3.3532
Time (years)	Distance(m)						
	280	300	320	340	360	380	400
1	0.0553	0.1051	0.1886	0.3198	0.5136	0.7836	1.1392
5	0.9434	1.1127	1.3220	1.5723	1.8647	2.1998	2.5781
10	2.2134	2.3929	2.6125	2.8721	3.1718	3.5117	3.8917
15	3.4932	3.6732	3.8932	4.1532	4.4533	4.7933	5.1733

Table 4.11: Approximated shoreline evolution along 15 years using the Saulyev finite difference techniques L is 400 m

Time (years)	Distance(m)						
	0	20	40	60	80	100	120
1	1.1407	0.7852	0.5152	0.3212	0.1899	0.1061	0.0560
5	2.5796	2.2012	1.8661	1.5737	1.3232	1.1138	0.9443
10	3.8924	3.5124	3.1726	2.8728	2.6131	2.3935	2.2138
15	5.1737	4.7937	4.4536	4.1536	3.8935	3.6735	3.4935
Time (years)	Distance(m)						
	140	160	180	200	220	240	260
1	0.0280	0.0134	0.0066	0.0046	0.0064	0.0128	0.0270
5	0.8138	0.7213	0.6659	0.6474	0.6655	0.7203	0.8124
10	2.0741	1.9744	1.9145	1.8944	1.9142	1.9739	2.0735
15	3.3534	3.2534	3.1933	3.1733	3.1932	3.2532	3.3531
Time (years)	Distance(m)						
	280	300	320	340	360	380	400
1	0.0546	0.1042	0.1874	0.3184	0.5121	0.7821	1.1376
5	0.9425	1.1117	1.3208	1.5711	1.8634	2.1984	2.5768
10	2.2130	2.3925	2.6120	2.8716	3.1712	3.5110	3.8910
15	3.4931	3.6730	3.8930	4.1530	4.4530	4.7930	5.1731

Chapter 5

Discussion and Conclusion

5.1 Discussion

In chapter 3, the shoreline evolution in each year can be obtained by the traditional forward time centered space method and the Saulyev method. The approximated shoreline evolutions of both numerical methods are closed. However, the precision of the traditional forward time centered space method is quite better than the Saulyev method as shown by the absolute error in Table 3.4, 3.5. Although, the Saulyev method gives better the stability condition than the traditional forward time centered space method.

These mean that the Saulyev method gives a practical numerical simulation due to the precision requirement of the displacement when the time increment becomes large.

In chapter 4, we measure the long-shore sand transport rate (D) in each month along a year by field data. We obtain the long-shore sand transport rate(D) from Equation (4.9). The averaged berm height (D_B), the closure depth (D_C), the density of the sediment(ρ_s), the density of sea water (ρ), the porosity (n) and the dimensionless coefficient which is a function of particle size (K) as show in Table 4.1. The wave group velocity (c_g) and the wave height (H) in each month along a year is measured by field data as show in Table 4.2. The amplitude of the long-shore transport rates (Q_0) from Equation (4.3) and the long-shore sand transport rate (D) as show in Table 4.3.

The shoreline evolution in each year can be obtained by the traditional forward time centered space techniques and the Saulyev finite difference techniques in the length of considered shoreline is 100 m as show in Tables 4.4, 4.5 and Figures 4.6, 4.7. The distance from the farthest shoreline evolution is 4.1708 m. The shortest distance from the shoreline evolution is 3.6707 m.

The length of considered shoreline is 200 m as show in Tables 4.6, 4.7 and Figures 4.8, 4.9. The distance from the farthest shoreline evolution is 4.5059 m. The shortest distance from the shoreline evolution is 3.5707 m.

The length of considered shoreline is 300 m as show in Tables 4.8, 4.9 and Figures 4.10, 4.11. The distance from the farthest shoreline evolution is 4.8398 m. The shortest distance from the shoreline evolution is 3.3396 m.

The length of considered shoreline is 400 m as show in Tables 4.10, 4.11 and Figures 4.12, 4.13. The distance from the farthest shoreline evolution is 5.1737 m. The shortest distance from the shoreline evolution is 3.1732 m.

The approximated shoreline evolutions of both numerical methods in 4 length of considered shoreline are closed.

5.2 Conclusion

In chapter 3, a governing equation of a one-dimensional shoreline evolution model is introduced the manipulation of physical parameters for shoreline evolution in practical simulations are proposed such as the impact angle between breaking wave crests with x-axis, the amplitude of the long-shore sand transport rate, the averaged berm height and the closure depth. The setting of the initial condition and the boundary conditions techniques are also introduced the traditional forward time centered space and the unconditionally Saulyev finite difference methods are used to approximate the model solutions. The approximated solutions of both numerical techniques are compared with the ideal exact solution. the traditional forward time centered space method gives more accurate solutions than the Saulyev approximated solutions. When the time increment increased for the traditional forward time centered space method cannot handle solution for many cases. However, the Saulyev method still can handle numerical solution for any cases due to there are no limitation of the stability condition. These mean that the proposed Saulyev method is a practical numerical method for the shoreline evolution model.

In chapter 4, In this study, when a couple of groins are installed we introduce a governing equation of a one-dimensional model of shoreline evolution. The implemented model is a one-line model, which is transient. The modification is implemented of physical parameters for the model. The initial condition setting approach and the boundary conditions techniques while also proposing some groin structure effect. The classical forward-time centered-space method and the unconditionally stable Saulyev finite differential methods are used to measure the incremental model in each year. The proposed numerical models give practically simulation for long-term shoreline evolution investigation. The simulation proposed can be used to predict the efficiency of constructing a groin network at a local beach. The classical forward-time centered-space method provides more precise solutions than the solutions approximated by the Saulyev. For many cases, when the time increment for the classical forward-time centered-space method has increased, solution can not be handled. However, the Saulyev method can still handle numerical solution for any case because the stability condition is not limited. This mean the Saulyev method proposed is a functional computational approach for the concept of shoreline evolution.

5.3 Further study

1) We will change initial shoreline by interpolation.
2) We will change groin design.
3) We will simulate 2D, 3D shoreline evolution model.

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Appendix

Appendix A

A One-Dimensional Mathematical Model of Long-Term Shoreline Evolution with Groin System using an Unconditionally Stable Explicit Finite Difference Method

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***Abstract* - Beach erosion is a natural process that occurs when movement of sediment away from the shoreline is not balanced by depositing new material on the shoreline. This is a problem that is causing beach areas to decline. To avoid beach erosion and flooding, a sea wall and groin are built. Shoreline evolution prediction is used to investigate the beach topography in the future. Three phenomena affect the coastal structure: erosion, accretion and water level changes. In order to investigate beach erosion and beach deposition we need qualitative understanding of idealized shoreline response to the governing forces. In this research, we introduce a governing equation of a one-dimensional shoreline evolution model when a groin is added. The introduced model is a transient one-line model. The manipulation of physical parameters for the model is introduced. The setting method of the initial condition and the boundary conditions techniques are also proposed. The traditional Forward Time Centered Space, FTCS, method and the unconditionally stable Saulyev finite difference methods are employed to approximate the shoreline evolution in each year. The proposed numerical models give practical simulation for long-term shoreline evolution investigation. The proposed simulation can be used to predict the efficiency of a groin system construction in a local beach.**

Keywords - shoreline evolution, groin system, one-dimension, mathematical model, finite difference method

I. INTRODUCTION

Beach erosion is a natural process which occurs whenever the movement of material away from the shoreline is not balanced by new material being deposited onto the shoreline. This is a problem that causes a decrease in beach areas. In order to prevent beach erosion and beach deposition a structure is devised consisting of a sea wall and groin. In [9], authors propose a modern approach to functional groin design which is demonstrated by using the GENESIS shoreline response model to simulate the action of single and multiple groins. The report predictions are tested in reproducing shoreline change observed at the 15 groins at Westhampton, Long Island, New York. In [7] reported on changes in beach profile due to the construction of single zigzag type of porous groins named GROPOZAG.

In order to investigate of beach erosion and beach deposition is needed qualitative understanding of idealized shoreline response to the governing process. Analytical solution originating from a mathematical model which describes the basic physics is the one tool to understanding it. Many authors obtained an analytical solution to shoreline evolution by using a simple mathematical formula. The one-line theory was introduced by many authors, several contributors in the analytical solution of shoreline evolution include [3, 4], [2], [1], [6], [12], and [8]. The analytical solution cannot be expected to provide quantitatively accurate solutions to problems involving complex boundary conditions and wave inputs. In the real situation, a numerical model of shoreline evolution would be more appropriate.

A general expression for the long-shore sand transport rate was developed by [10]. The empirical predictive formula for the amplitude of the long-shore sand transport rate presented by [5]. In [11], they have examined and presented two numerical schemes of shoreline evolution for simplified configuration beach. In [13], [14], they have used the conditionally stable explicit finite difference methods to approximate their model solutions.

In [15] this paper an empirical correlation is obtained from established invariant beach physical characteristics according to a theoretical bi-parabolic equilibrium beach profile (EBP) between the wave energy and the equilibrium shoreline location ("Equilibrium energy function", EEF). The tests of the proposed model at Nova Icaria show the same abilities with only one calibration parameter as state-of - the-art models with more than 4 free-parameters. Nonetheless, observations are required at different sites with diverse beach characteristics and wave conditions to validate widely the EEF analytical method and the shoreline evolution model proposed in this paper. In [16], They proposed one-line modeling concept has been extended to achieve long-term coastline evolution predictions, as well as to support and deliver better coastal engineering solutions to control erosion. The model has been applied to two Portuguese northwest coastal stretches: Aveiro and Figueira da Foz. The results allow the key potential consequences of continuing erosion to be evaluated qualitatively. In [17], they proposed a general and replicable chain approach, evaluated at a vulnerable coastal site in southern Italy and based on a joint study of field data, statistical instruments and numerical modeling. This could help to understand the dynamics of the coastal environment fully, identify typical and recurring erosion-accretion processes, and predict possible future patterns that are useful for coastal activity planning. In [18], they proposed probabilistic shoreline changes are computed by using the GenCade and Monte Carlo simulations. GenCade simulation is used to simulate longshore sediment transport and the long-term shoreline changes caused by random offshore waves. Monte Carlo simulation is used to model the shoreline evolution in response to changes in sea level, land subsidence and onshore sand transport. In [19], they proposed a simple behavioral template model for beach profile evolution which is calibrated and tested against a 6-year shoreline position time series derived from a coastal imaging system at the Gold Coast, Australia. Testing the calibrated model on unseen data shows that it is capable of reproducing the dominant seasonal shoreline succession observed at this site and up to 77% of the shoreline variability. In [20], they proposed the ONELINE modeling system and show its capabilities through model testing and case studies. This outlines two case studies in which complex configurations of the beach system are simulated. The first one features a groin field at Sea Isle City, New Jersey along the East Coast of the United States. The second is along the Nile Delta Coast in Egypt. In [21], they proposed the comparison between the analytic and numerical solutions in the idealized wave condition for four different shoreline configurations.

In this research, the governing equation of a one-dimensional shoreline evolution model is introduced. We will also propose techniques of physical parameters, the initial condition and boundary conditions setting. Finite difference techniques will be used to approximate the model solution.

II. GOVERNING EQUATIONS

A. Shoreline Evolution Model

In the one-line model, the beach profile is assumed to move landward and seaward while retaining the same shape, implying that all bottom contours are parallel.

Consequently, under this assumption it is sufficient to specify the horizontal location of the profile with respect to baseline, and one contour line can be used to describe changes in the beach plan shape and volume as the beach erodes and accretes. The major assumption of the model is the sand is transported alongshore between two well-defined limiting elevations on the profile. One contribution to the volume change results if there is a difference in the alongshore sand transport rate at the lateral sides of the section and the associated the sand continuity. The principles of mass conservation must apply to the system at all times. By considering above definitions, the following differential equation for shoreline evolution is obtained:

$$\frac{\partial y}{\partial t} = \frac{1}{D_B + D_C}\left(-\frac{\partial Q}{\partial x}\right), \tag{1}$$

where x is the alongshore coordinate (m), y is the shoreline positions (m) and perpendicular to x-axis, t is time (day), Q is the long-shore sand transport rate (m^3/day), D_B is the average berm height (m) and D_C is the closure depth (m).

In order to solve the Eq (1), necessary to specify an expression for the longshore sand transport rate, Q. This quantity is considered to be generated by the wave obliquely incident to the shoreline. A general expression for the long-shore sand transport rate was developed by [11]:

$$Q = Q_0 \sin\left(2\alpha_b\right), \tag{2}$$

where Q_0 is the amplitude of the long-shore sand transport rate. The empirical predictive formula for the amplitude of the long-shore sand transport rate is [6]:

$$Q_0 = \frac{\rho}{16}\left(H_b^2 c_{gb}\right)\frac{K}{(\rho_s - \rho)(1-n)}, \tag{3}$$

where the subscript b denotes value at the point of breaking, c_g is the wave group velocity, H is the wave height, ρ_s is the density of the sediment (kg/m^3), ρ is the density of the sea water, n is the porosity and K is the dimensionless coefficient which is a function of particle size. The quantity α_b the impact angle between breaking wave crests angle with local shoreline, and may be written as:

$$\alpha_b = \alpha_0 - \tan^{-1}\left(\frac{\partial y}{\partial x}\right), \tag{4}$$

where α_0 is the angle between breaking wave crests and the $x-axis$. For beaches with mild slope, it can be assumed that breaking wave angle to the shoreline is small.

Assuming that, $\sin\left(2\alpha_b\right) \approx 2\alpha_b$, and $\tan^{-1}\left(\frac{\partial y}{\partial x}\right) \approx \left(\frac{\partial y}{\partial x}\right)$. Substituting Eq. (4) into the Eq. (2), and assuming the beach with mild slope yields:

$$Q = Q_0\left(2\alpha_b - 2\frac{\partial y}{\partial x}\right), \tag{5}$$

Substituting Eq. (5) into the Eq. (1) and neglecting the sources or sinks along the coast gives:

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$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2}, \tag{6}$$

for all $(x,t)\in(L,T)$, where $D = \frac{2Q_0}{D_B + D_C}$.

Eq (6) is analogous to the one-dimensional heat diffusion equation, it can be solved analytically for various initial and boundary conditions.

B. Physical Parameters

Physical parameter of the model can be illustrated as show in Figures 1-2 that are listed below.

α_0 is the impact angle between breaking wave crests angle with the x-axis.

Q_0 is the amplitude of the long-shore sand transport rate.

D_B is the average berm height.

D_C is the closure depth.

L is Alongshore.

T is Time of simulation.

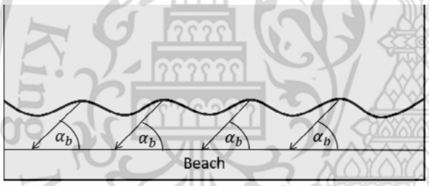

Figure 1. Breaking wave crests impact angle.

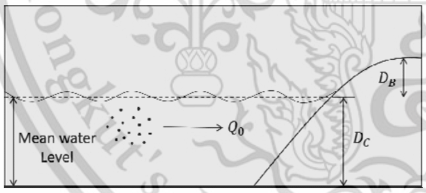

Figure 2. Shoreline physical parameters.

C. The Initial and Boundary Conditions

Straight Impermeable groin system: The initials shoreline is assumed to be parallel to the x-axis. Assuming that, the braking wave angle to the shoreline is α_0 as show in Figure 3. It follows that the sand transport rate along the shoreline is uniform. The groin is instantaneously added at $x=0$ as show in Figure 3. These means that the initial condition becomes

$$y(x,0) = 0, \tag{7}$$

Boundary conditions are also assumed by:

$y(0,t) = f(t)$,

and

$y(L,t) = g(t)$,

where $f(t)$ and $g(t)$ are given functions.

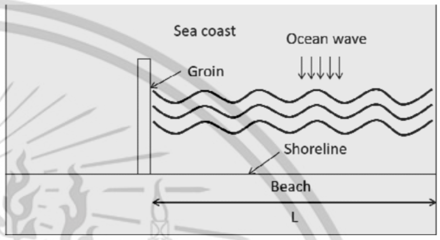

Figure 3. Initial shoreline with configuration straight impermeable groin.

III. NUMERICAL TECHNIQUES

A. Grid Spacing

We now discretize Eq (6) by dividing the interval $[0,L]$ into M subintervals such that $M\Delta x = L$ and the interval $[0,T]$ into M subintervals such that $N\Delta t = T$. We then approximate $y(x_i,t_n)$ by y_i^n , at the point $x_i = i\Delta x$ and $t_n = n\Delta t$, where $0 \le i \le M$ and $0 \le n \le N$ in which M and N are positive integers.

B. Traditional Forward Time Centered Space Techniques

The forward time centered space method is employed. Consequently, the finite difference approximation becomes [22]:

$$y \cong y_i^n, \tag{8}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{9}$$

$$\frac{\partial y}{\partial x} \cong \frac{y_{i+1}^n - y_{i-1}^n}{2\Delta x}, \tag{10}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}, \tag{11}$$

where $A = \frac{D\Delta t}{(\Delta x)^2}$ Substituting Eqs (8)-(11) into Eq (6), we obtain,

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$$\frac{y_i^{n+1}-y_i^n}{\Delta t} \cong D\left(\frac{y_{i+1}^n-2y_i^n+y_{i-1}^n}{(\Delta x)^2}\right), \quad (12)$$

for $1 \le i \le M-1$ and $0 \le n \le N-1$. Eq (12) can be written in an explicit form of finite difference as follows,

$$y_i^{n+1} \cong Ay_{i+1}^n + (1-2A)y_i^n + Ay_{i-1}^n, \quad (13)$$

for $1 \le i \le M-1$ and $0 \le n \le N-1$.

C. An Unconditionally Saulyev Finite Difference Techniques

The Saulyev finite diffence method will be also employed. We can obtain that the finite difference approximatetion is:

$$y \cong y_i^n, \quad (14)$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1}-y_i^n}{\Delta t}, \quad (15)$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n-y_i^n-y_i^{n+1}+y_{i-1}^{n+1}}{(\Delta x)^2}, \quad (16)$$

where $A=\frac{D\Delta t}{(\Delta x)^2}$. Substituting Eqs (14)-(16) into Eq (6), we obtain

$$\frac{y_i^{n+1}-y_i^n}{\Delta t} \cong D\left(\frac{y_{i+1}^n-y_i^n-y_i^{n+1}+y_{i-1}^{n+1}}{(\Delta x)^2}\right), \quad (17)$$

for $1 \le i \le M-1$ and $0 \le n \le N-1$. Eq (17) can be written in an explicit form of finite difference as follows,

$$y_i^{n+1} \cong \frac{1}{(1+A)}\left(Ay_{i+1}^n + (1-A)y_i^n + Ay_{i-1}^{n+1}\right), \quad (18)$$

for $1 \le i \le M-1$ and $0 \le n \le N-1$.

IV. NUMERICAL EXPERIMENTS

In order to investigate the shoreline evolution in the long-term scale. The numerical results of the different beach situation are considered and the solution the idealized problem is presented. During the simulations, assuming that the length of considered shoreline is L=5000 m. The amplitude of the long-shore transport rate is $Q_0 = 7500$ m^3/day. the averaged berm height $D_B = 2$ m. the closure depth $D_C = 28$ m. the breaking wave impact angle $\alpha_0 = 0.02$. The simulation results are illustrated in Figure 4.

We then employed the FTCS Eq (13) and the Saulyev finite difference techniques Eq (18) to approximate the model solution.

The calculated results are shown in Fig 5-7 and Table I-III. The exact solutions of the simulation is:

$$y(x,t) \cong \tan\alpha_0 \sqrt{\frac{4Dt}{\pi}}\left(e^{\frac{x^2}{4Dt}} - \frac{x\sqrt{\pi}}{2\sqrt{Dt}}\,\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)\right), \quad (19)$$

where their values are shown in Table I. The approximated solutions of FTCS method are given in Table II, while Saulyev method approximated solutions are given in Table III.

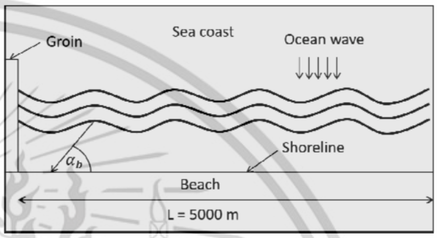

Figure 4. Initial shoreline

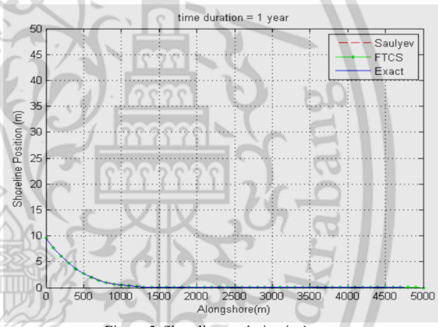

Figure 5. Shoreline evolution in 1 year

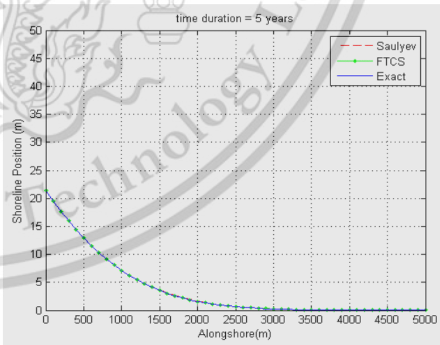

Figure 6. Shoreline evolution in 5 years

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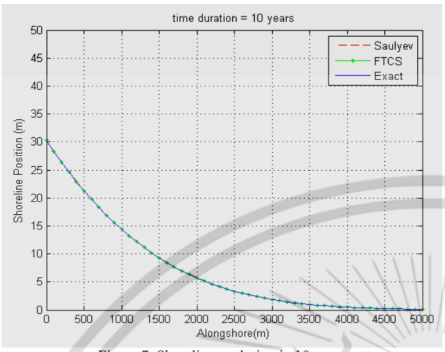

Figure 7. Shoreline evolution in 10 years

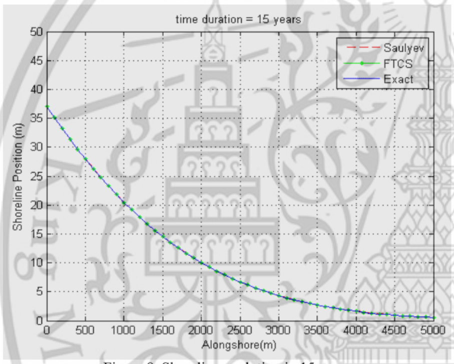

Figure 8. Shoreline evolution in 15 years

TABLE I. THEORCTICAL SOLUTION OF SHORELINE EVOLUTION.

Time (years)	*Distance (m)*					
	0	*1000*	*2000*	*3000*	*4000*	*5000*
0	0.00	0.00	0.00	0.00	0.00	0.00
1	9.56	0.47	0.00	0.00	0.00	0.00
5	21.41	7.09	1.60	0.24	0.02	0.00
10	30.28	14.39	5.70	1.84	0.48	0.10
15	37.08	20.46	10.03	4.31	1.61	0.52

TABLE II. APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING FTCS

Time (years)	**Distance (m)**					
	0	***1000***	***2000***	***3000***	***4000***	***5000***
0	0.00	0.00	0.00	0.00	0.00	0.00
1	9.56	0.48	0.00	0.00	0.00	0.00
5	21.41	7.09	1.61	0.24	0.02	0.00
10	30.28	14.39	5.70	1.84	0.48	0.10
15	37.08	20.46	10.03	4.31	1.61	0.52

TABLE III. APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING SUALYEV

Time (years)	**Distance (m)**					
	0	***1000***	***2000***	***3000***	***4000***	***5000***
0	0.00	0.00	0.00	0.00	0.00	0.00
1	9.56	0.48	0.00	0.00	0.00	0.00
5	21.41	7.12	1.63	0.25	0.02	0.00
10	30.28	14.41	5.72	1.86	0.49	0.10
15	37.08	20.48	10.05	4.34	1.63	0.52

TABLE IV. ABSOLUTE ERROR OF SHORELINE EVOLUTION WHEN THE FTCS AND SUALYEV METHOD ARE USED.

Time (years)	**Distance (m)**	**Absolute error**	
		FTCS	***Saulyev***
1	1000	0.5300×10^{-2}	0.2420×10^{-1}
	2000	0.3277×10^{-3}	0.1100×10^{-2}
	3000	0.7868×10^{-6}	0.2748×10^{-5}
	4000	0.1224×10^{-9}	0.7012×10^{-9}
5	1000	0.1500×10^{-2}	0.2460×10^{-1}
	2000	0.2500×10^{-2}	0.2320×10^{-1}
	3000	0.1400×10^{-2}	0.9400×10^{-2}
	4000	0.3577×10^{-3}	0.1900×10^{-2}
10	1000	0.1500×10^{-2}	0.1910×10^{-1}
	2000	0.1600×10^{-2}	0.2630×10^{-1}
	3000	0.1700×10^{-2}	0.2040×10^{-1}
	4000	0.1000×10^{-2}	0.9900×10^{-2}
15	1000	0.3265×10^{-3}	0.1610×10^{-1}
	2000	0.9811×10^{-3}	0.2490×10^{-1}
	3000	0.1300×10^{-2}	0.2330×10^{-1}
	4000	0.9499×10^{-3}	0.1390×10^{-1}

TABLE V. THE COMPARISON OF THE STABILITY IN EACH GRID SPACING SIZES.

Δt	Δx	**Stability**	
		FTCS	***Saulyev***
0.50	25	stable	stable
	50	stable	stable
	100	stable	stable
1.00	25	unstable	stable
	50	stable	stable
	100	stable	stable
2.00	25	unstable	stable
	50	stable	stable
	100	stable	stable
4	25	unstable	stable
	50	unstable	stable
	100	stable	stable

V. DISCUSSION

The shoreline evolution in each year can be obtained by the FTCS method and the Saulyev method. The approximated shoreline evolutions of both numerical

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methods are close. However, the precision of the FTCS method is quite better than the Saulyev method as shown by the absolute error in Table IV. Although, the Saulyev method gives a better stability condition than the FTCS method.

These mean that the Saulyev method gives a practical numerical simulation due to the precision requirement of the displacement when the time increment becomes large.

VI. CONCLUSION

The governing equation of a one-dimensional shoreline evolution model was introduced for the manipulation of physical parameters for shoreline evolution and practical simulations were proposed to include: i) the impact angle between breaking wave crests with the x-axis, ii) the amplitude of the long shore sand transport rate, iii) the averaged berm height and (iv) the closure depth. The setting of the initial conditions and the boundary conditions techniques were also introduced for the traditional FTCS and the unconditionally Saulyev finite difference methods were used to approximate the model solutions.

The approximated solutions of both numerical techniques were compared with the ideal exact solution. The FTCS method gave more accurate solutions than the Saulyev approximated solutions. When the time increment increased we found the FTCS method could not handle the solution for many cases.

However, the Saulyev method still could handle numerical solution for any case due to no limitation of the stability condition. These mean that the proposed Saulyev method is a practical numerical method for the shoreline evolution model.

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Academic Publication(s)

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