# NUMERICAL SIMULATIONS OF PRIMARY AND SECONDARY AIR POLLUTANT DISPERSION MODELS FROM POINT SOURCES IN INDUSTRIAL AREAS

PRAVITRA OYJINDA

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN APPLIED MATHEMATICS

DEPARTMENT OF MATHEMATICS

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| **Thesis Title** | Numerical Simulations of Primary and Secondary Air Pollutant Dispersion Models from Point Sources in Industrial Areas |
| **Student Name** | Miss Pravitra Oyjinda |
| **Student ID** | 57605023 |
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| **Thesis Advisor** | Asst. Prof. Dr. Nopparat Pochai |

## Abstract

Air pollution problems occur from rapid industrial growth in big cities. Industrial factories are a main source of toxic fumes released into the atmosphere. These problems affect human health, animal life, and the environment greatly. In this research, a two-dimensional advection-diffusion equation with finite difference techniques is proposed in order to simulate the dispersion of primary pollutant emitted from a chimney. Additionally, the dispersion of secondary pollutants which occur in the atmosphere by chemical reaction is simulated. The numerical solution is the concentration of air pollution, which is used to analyze the air quality regarding the independent air pollution and controlled air pollution emission rates. The model is also used to find the most suitable place for a monitoring point for the affected residential area. From this research, we can apply the concentration of air pollution as an auxiliary instrument to estimate and control the air quality in industrial and residential areas. The expected effect of this model can be used for further analysis, especially before sources of emissions are constructed.

**Keywords** : air pollution emission control, atmospheric diffusion equation, finite difference techniques, primary air pollutant, secondary air pollutant, two-dimensional advection-diffusion equation

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Pravitra Oyjinda

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# Chapter 1
# Introduction

## 1.1 The air pollution problem

Air pollution is a major problem around the world because the various pollutants disperse from sources into the atmosphere. Normally, air pollution is an atmospheric condition which contaminates in a sufficient quantity and a long enough duration that is harmful to the health of human, animals, and plants, or has an effect on buildings. The increase in factories and motor vehicles has stemmed from technological development and rapid industrial growth, which makes everyday life more comfortable. However, the toxic smoke emissions from smokestacks and exhaust pipes have brought about air pollution problems. These pollutants are also hazardous to humans and other living things, including agricultural products. The main cause of air pollution is humanity; in fact, human activities are the source of pollutants, such as industrial factories, traffic, buildings, and farming.

In Thailand, air pollution is a serious problem. The rapid expansion of the industrial sectors is the main reason for these problems, which particularly happen in industrial zones and big cities. The contaminated pollutants are considered using the examination of air pollution values in the atmosphere. The objectives of air pollution measurement are to check and compare the concentration levels and the air quality standards. Then, these measurements are used to study the effect of air pollution from discharging pollutants into the ambient atmosphere. One suitable management method of air pollution problems, which can help reduce these problems, is the air pollution emission control of plants. As a result, there will be less danger to human health, animals, and the environment.

In this research, the study of air pollution emission into the atmosphere is considered. The dispersion of pollutants is simulated by using a mathematical model to evaluate the air quality from the source and around industrial areas, in order to control the emission problem. It is also possible to estimate the effect of air pollution before the source has been constructed.

## 1.2 Mathematical model for air pollution

The Gaussian plume idea, which is a material balance model, is used to simulate the advection-diffusion model of air pollution. This model is created to estimate air pollutant concentrations in the atmosphere. Figure 1.1 shows the element of the Gaussian plume model, where the wind current flows along the $x$-axis. The polluted gas stream, which is usually called a plume, appears to rise from the smokestack, then level off to travel in the $x$-direction and spread in the $y$-direction and $z$-direction as it travels. The plume is assumed to be emitted from a point above the smokestack, where the effective stack height ($H$) is the sum of the physical stack height ($H_s$) and the vertical plume rise ($\Delta h$), owing to the plume's buoyancy.

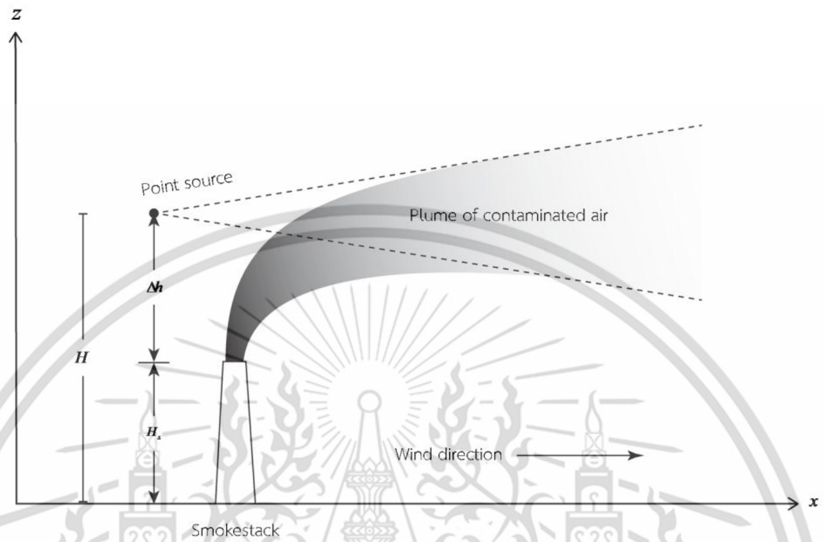



**Figure 1.1** The structure for the Gaussian Plume idea

The mathematical simulation that governs the dispersion of each air pollutants is considered by using the atmospheric diffusion equation (ADE). That is

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}+w\frac{\partial c}{\partial z}=k_x\frac{\partial^2 c}{\partial x^2}+k_y\frac{\partial^2 c}{\partial y^2}+k_z\frac{\partial^2 c}{\partial z^2}+s+r, \tag{1.1}$$

where $c=c(x,y,z,t)$ is the air pollutant concentration at $(x,y,z)$ and time $t$,
$u,v,$ and $w$ are the wind velocity components in $x$-, $y$-, and $z$-directions,
$k_x,k_y,$ and $k_z$ are the diffusivities in $x$-, $y$-, and $z$-directions,
$s$ is the growth of pollutant rate due to sources,
and $r$ is the decaying of pollutant rate due to sinks.

We suppose that equation (1.1) is a laterally averaged atmospheric diffusion equation so that the advection and diffusion terms in the $y$-direction can be eliminated. So, we get

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+w\frac{\partial c}{\partial z}=k_x\frac{\partial^2 c}{\partial x^2}+k_z\frac{\partial^2 c}{\partial z^2}+s+r. \tag{1.2}$$

In the present study, ambient air pollution in industrial areas and residential areas is proposed.

## 1.3 Literature reviews

The study of air pollution problems with a mathematical model is used to describe the dispersion of air pollution. The mathematical model is able to predict the air pollution concentration and the expected effect. Numerical methods are also applied to air pollution problems. The research has developed more and more understanding of air pollution dispersion.

In [1], the researchers studied a mathematical model to consider a delay removal process with variable wind velocity and diffusion coefficient. The air pollutants were emitted from a line source, and the boundary on the ground was dry deposition. The fractional step method was applied by estimating the air pollutant concentration. In [2], a three-dimensional atmospheric diffusion equation, with height-dependent wind speed and eddy diffusivities, was proposed. The analytical solutions for various boundary condition types were derived by applying Green's function concept for multiple source dispersion, where the sources can be set everywhere in the interested area. In [3], a time-dependent two-dimensional advection-diffusion equation was studied to find the approximate solution of air pollutant which was released into the atmosphere. In [4], the researchers studied primary and secondary pollutants by using a two-dimensional advection-diffusion model. The different wind velocities and eddy diffusion coefficients were considered to be realistic values. The concentration of air pollutant due to area source was approximated by using the Crank-Nicolson implicit finite difference technique and upwind difference scheme, which were applied to the advection term. In [5], a three-dimensional mathematical model for studying the dispersion of sulfur dioxide pollutant was presented to analyze the case of a domain without obstacles. In [6], the study of air pollution through simulating a convection-diffusion-reaction equation, with dry deposition on the boundary and wet deposition in the source term, was proposed. The concentration of sulfur and nitrogen oxides was solved by a high-order accurate time-stepping discretization scheme, which was applied by using the Lax and Wendroff technique.

In [7], the dispersion of air pollutant was simulated by a steady-state two-dimensional mathematical model with mesoscale wind under the urban heat island effect. The area source was presented by emitting pollutants from the ground in the urban area, and the removal processes were considered by wet and dry depositions. The approximate solution was solved by using the Crank-Nicolson implicit method. In [8], the variation of sulfur dioxide pollutant emission in China since the year 2000 was presented. The result was estimated by using a technology-based methodology specifically for China; then, the researchers compared the sulfur dioxide emission with a variety of official environment statistics, ground-based measures, satellite observations, and model results of sulfur-related quantities over East Asia. In [9], a two-dimensional diffusion equation with nonlocal boundary conditions was presented. The numerical solution of the Padé schemes demonstrated that these schemes were efficient and gave accurate results. In [10], a mass transport model consisting of stream function, vorticity, and convection-diffusion equation was made. It was considered to be able to predict smoke dispersion from one- and two-point sources with obstacle domain. The finite element and finite difference methods were used as the numerical technique for air pollution solution in two-dimensional space and one-dimensional time, respectively. In [11], the researchers studied a two-dimensional atmospheric diffusion equation to compare the solution of air pollution between two- and three-point sources with obstacle domain. In [12], a land-use regressive (LUR) model was developed to assess air pollution concentration, while mobile monitoring was made to measure the sulfur dioxide concentration between the years 2005 and 2010 in Hamilton, Ontario, Canada. In [13], a two-dimensional air pollution model with mesoscale wind velocities and eddy diffusivity profiles was presented. The researchers studied the removal mechanism of dry deposition, gravitational settling, and chemical reaction, and the primary pollutant which was emitted from an area source was applied. In [14], two-dimensional air pollution models of primary and secondary pollutants were proposed. The researchers studied the effect of dry deposition velocity from an area source with a point source on the boundary. The Crank-Nicolson implicit method was used to calculate the solution of [13] and [14].

In [15], the wind current and air pollutant dispersion problem were simulated to predict the behavior of air pollution within the urban street canyon. Computational Fluid Dynamics (CFD) techniques, which were Reynolds-Averaged Navier-Stokes (RANS), Unsteady RANS (URANS), and Large Eddy Simulation (LES), were presented in order to compare the efficiency of numerical techniques. The results of the air pollution problem explained that LES was observed to produce more accuracy than RANS and URANS. In [16], the modeling and application of the Atmospheric Evaluation and Research Integrated model for Spain (AERIS) were proposed. Presently, AERIS can demonstrate the concentration of nitrogen dioxide, ozone, sulfur dioxide, ammonia ($NH_3$), and particulate matter as a reaction to emission changes in relevant sectors of Spain. The Air Quality Modelling System (AQMS) consisted of the Weather Research and Forecast (WRF), Sparse Matrix Operator Kernel Emissions (SMOKE), and Community Multiscale Air Quality (CMAQ) models, which were used to solve the result by transfer matrices. In [17], the Campania region, Southern Italy was used in order to consider environmental problems. An assessment of a wide and critical area under observation for different air pollution sources was proposed. In [18], air flow and air pollutant dispersion in urban street canyons were simulated by Computation Fluid Dynamic (CFD) using the Large Eddy Simulation (LES) model. The research intended to address the limitation of previous studies by developing Fluctuating Wind Boundary Condition (FWBC). The condition comparison between FWBC and Steady Wind Boundary Condition (SWBC) was investigated. In [19], the researchers studied a three-dimensional advection-diffusion equation with the alteration of atmospheric stability classes and wind velocities. The multiple-point sources were examined by solving the numerical approximation in factory and residential zones which were the considered the domain. The concentration of pollutant in [5], [11], and [19] was approximated by using the fractional step method. In [20], the researchers analyzed the dispersion behavior of $O_z$, $NO_x$, and VOCs in Riyadh, Saudi Arabia, in the summer. The concentrations of $O_z$ and its substrates were estimated by spatial interpolation at 16 different cells for all urban environments of the city. In [21], an air quality monitoring (AQM) station was installed to measure the carbon dioxide in Yongsan, Seoul, South Korea, between the years 1987 and 2013. The concentration of carbon dioxide was investigated by using long-term trend analysis. For this reason, the carbon dioxide concentration decreased from the years 1987 to 2013. In [22], a numerical simulation of a three-dimensional air quality model in an area under the sky train run by Bangkok Transit Systems (BTS) was studied. This consideration of an air pollution problem was presented in different cases regarding the wind inflow with obstacles. In [23], a three-dimensional advection-diffusion equation was considered to approximate the concentration of air pollutant in a heavy traffic area under a Bangkok sky train station. The numerical simulations were studied in three cases, which were the average of source or sink emissions, the moving of source or sink emissions, and the mix of source and sink emissions. The air polluant concentrations of [22] and [23] in the tunnel were solved by using the explicit finite difference scheme.

In this research, a mathematical model is studied to simulate the air pollution dispersion in an industrial area. A two-dimensional advection-diffusion equation, which is transformed into a dimensionless form, is proposed. The finite difference methods (FDM) are used to calculate the approximate solution. The numerical solutions are the air pollution concentrations. These can be efficiently used to analyze the actual problems in the considered area.

## 1.4 Objectives of the thesis

1) To apply a mathematical model with numerical techniques to the air pollution problems for controlling air pollution levels in industrial areas.
2) To estimate air pollutant concentrations which are emitted from one- and two-point sources by using the forward time central space (FTCS) and backward time central space (BTCS) methods.
3) To simulate the actions of primary pollutants to analyze approximate solutions in the cases of controlled and uncontrolled air pollution emissions.
4) To simulate the dispersion of primary and secondary pollutants at each monitoring point to find the proper positions for emission control.

## 1.5 Scopes of the thesis

1) The advection and diffusion in the $y$-direction are laterally averaged.
2) Air flow velocities in the $x$-direction and $z$-direction are horizontally averaged velocity and vertically averaged velocity, respectively.
3) Air pollutant emissions can be shut down over several periods as designed.
4) Primary pollutants can be converted to secondary pollutants by chemical reaction.
5) Secondary pollutants are not converted to other pollutants.
6) Air quality monitoring equipment can check primary and secondary pollutants at the same monitoring point.

## 1.6 Benefits of the thesis

1) To predict the occurrence of air pollution problems before building industrial factories in the target areas.
2) To provide one of the auxiliary instruments for air quality assessment in industrial areas.
3) To control air pollution emissions in order to not affect residential districts near industrial areas.

Chapter 2

# Primary and Secondary Pollutant Concentration Models

## 2.1 Air pollution

The system of air pollution problems can be separated into 3 basic parts (see Figure 2.1), as follows:

(1) The emission source is the genesis of air pollution which is discharged into the atmosphere. The type and quantity of air pollutants depend on the source type and the air pollutant control method.

(2) The atmosphere is a medium of the system, so that air pollutants are discharged into the climate. In addition, atmosphere is also a factor that refers to the dispersion characteristics of air pollutants from the source to the receptor.

(3) The receptor makes contact with air pollutants which cause damage or become hazardous. The effects depend on the type and quantity of the air pollutants. The main receptors are humans, plants, water sources, and so on.

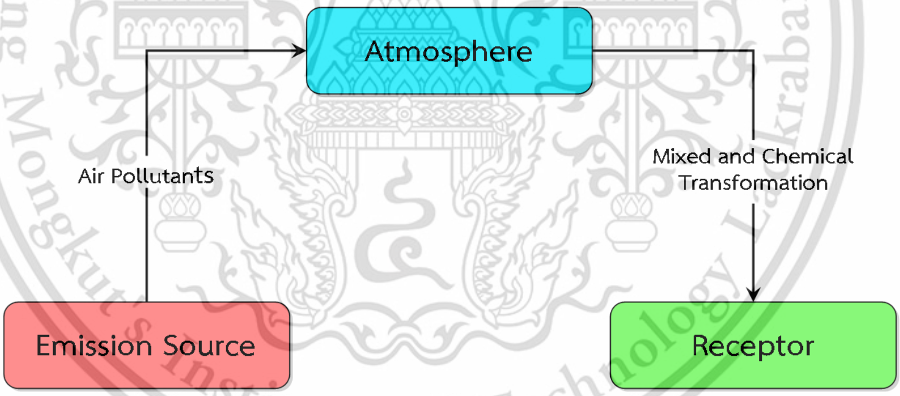



**Figure 2.1** Air pollution system diagram

The six important pollutants, which are known as criteria air pollutants, are particulate matter (PM), carbon dioxide ($CO_2$), sulfur dioxide ($SO_2$), nitrogen dioxide ($NO_2$), ozone ($O_3$), and lead (Pb). Air pollutants can be classified in order of the occurrence into 2 groups:

(1) Primary pollutants mean that air pollutants are directly released from the source into the atmosphere.

(2) Secondary pollutants mean that air pollutants occur by chemical reaction in the atmosphere between the primary pollutants and normal atmospheric substances.

The genesis of air pollution can be currently categorized into 3 types, as follows:

(1) Point source

e.g., industrial processing and power plants.

**Figure 2.2** Air pollution from industrial smokestacks (http://detroit.cbslocal.com/2012/08/10/michigan-ranks-7th-worst-state-for-toxic-air-pollution-from-power-plants/)

(2) Area source

e.g., forest fires.

**Figure 2.3** Air pollution from forest fires (https://blogs.agu.org/geospace/2012/04/30/air-pollutants-hot-spots/)

(3) Mobile source or line source
e.g., highway vehicles and railroad locomotives.

**Figure 2.4** Air pollution from car exhaust pipes
(http://www.hotleasing.com/content_detail.php?content_id=227
http://www.thairath.co.th/content/355050)

Therefore, the emission source classification is suitable for using research in mathematics. It is easier to assess the environmental effects.

## 2.2 Air quality standards

Air quality standard (AQS) refers to the average of air pollutant concentration levels over varied durations in a general atmosphere. Short-term average in 1, 8, and 24 hours is defined to provide protection from acute health effects. Long-term average in 1 month and 1 year is defined to control chronic health effects.

Table 2.1 shows the air quality standard (the value of criteria air pollutants) in Thailand, following the announcement of the promotion and conservation of the National Environmental Quality Act, B.E. 2535. The examination approaches of air pollution often hold to international standards, such as the US-Environment Protection Agency (USEPA), the World Health Organization (WHO), the National Institute of Occupational Safety and Health (NIOSH), and the Occupational Safety and Health Association (OSHA), taking the most proper approach in practice.

Table 2.1 Air quality standard in general areas [24]-[25]

| Pollutant | Duration | Standard value |
|---|---|---|
| Carbon monoxide (CO) | 1 hour<br>8 hours | 30 ppm. (34.2 $mg/m^3$)<br>9 ppm. (10.26 $mg/m^3$) |
| Nitrogen dioxide ($NO_2$) | 1 hour<br>1 year | 0.17 ppm. (0.32 $mg/m^3$)<br>0.03 ppm. (0.057 $mg/m^3$) |
| Ozone ($O_3$) | 1 hour<br>8 hours | 0.10 ppm. (0.20 $mg/m^3$)<br>0.07 ppm. (0.14 $mg/m^3$) |
| Sulfur dioxide ($SO_2$) | 1 hour<br>24 hours<br>1 year | 0.04 ppm. (0.10 $mg/m^3$)<br>0.12 ppm. (0.30 $mg/m^3$)<br>0.3 ppm. (780 $\mu g/m^3$) |
| Lead (Pb) | 1 month | 1.5 $\mu g/m^3$ |
| Particulate matter ($PM_{100}$) | 24 hours<br>1 year | 0.33 $mg/m^3$<br>0.10 $mg/m^3$ |
| Particulate matter ($PM_{10}$) | 24 hours<br>1 year | 0.12 $mg/m^3$<br>0.05 $mg/m^3$ |
| Particulate matter ($PM_{2.5}$) | 24 hours | 0.05 $mg/m^3$ |

### 2.2.1 Air quality index

Air quality index (AQI) is a representative of air quality data that is easy to understand for the general public, in order to be informed of the air pollution situation in each area and whether it will have an impact on health and sanitation or not. AQI in Thailand is calculated to compare the air quality standard in the general atmosphere of air pollutants of 5 types: 1-hour average ozone, 1-hour average nitrogen dioxide, 8-hour average carbon monoxide, 24-hour average sulfur dioxide, and 24-hour average particulate matter ($PM_{10}$).

Table 2.2 Air quality index for Thailand [26]

| AQI | Meaning | Color | Impact Prevention |
|---|---|---|---|
| 0-50 | Good | Blue | Have no health effect |
| 51-100 | Moderate | Green | Have no health effect |
| 101-200 | Unhealthful | Yellow | Everyone should avoid outdoor activities. |
| 201-300 | Very Unhealthful | Orange | Everyone should limit outdoor activities. |
| >300 | Hazardous | Red | Everyone should stay indoors. |

## 2.3 Atmospheric diffusion equation

In this research, we studied the behavior of air pollutants, such as primary and secondary pollutants, in an industrial area and a residential area.

### 2.3.1 Primary pollutant concentration model

Air pollutants are emitted from a smokestack, so the industrial factory is an important factor in the discharge of air pollutants into the atmosphere. The discharge of hazardous air pollutants from the industrial chimney is considered. The primary pollutant concentration is represented by $c_p$. Thus, the air pollutants are considered by the following equation

$$\frac{\partial c_p}{\partial t} + u\frac{\partial c_p}{\partial x} + w\frac{\partial c_p}{\partial z} = k_x\frac{\partial^2 c_p}{\partial x^2} + k_z\frac{\partial^2 c_p}{\partial z^2} + s_p + r_p \tag{2.1}$$

or it can be written as

$$\frac{\partial c_p}{\partial t} + u\frac{\partial c_p}{\partial x} + w\frac{\partial c_p}{\partial z} = k_x\frac{\partial^2 c_p}{\partial x^2} + k_z\frac{\partial^2 c_p}{\partial z^2} + q\delta(x-x_r)\delta(z-z_r) - k_p c_p, \tag{2.2}$$

where $s_p = q\delta(x-x_r)\delta(z-z_r)$ [5] when $q$ is the emission rate of sources and $\delta(\ )$ represents the Dirac's delta function,

and $r_p = -k_p c_p$ [14] when $k_p$ is the first-order chemical interaction rate of primary pollutant $c_p$.

### 2.3.2 Secondary pollutant concentration model

Air pollutants occur in the atmosphere by chemical interaction. The concentration of secondary pollutants is represented by $c_s$. We suppose that secondary pollutants do not convert to other pollutants. Thus, the secondary pollutant equation is

$$\frac{\partial c_s}{\partial t} + u\frac{\partial c_s}{\partial x} + w\frac{\partial c_s}{\partial z} = k_x\frac{\partial^2 c_s}{\partial x^2} + k_z\frac{\partial^2 c_s}{\partial z^2} + s_s \tag{2.3}$$

or we can write as

$$\frac{\partial c_s}{\partial t} + u\frac{\partial c_s}{\partial x} + w\frac{\partial c_s}{\partial z} = k_x\frac{\partial^2 c_s}{\partial x^2} + k_z\frac{\partial^2 c_s}{\partial z^2} + V_g k_s c_p, \tag{2.4}$$

where $s_s = V_g k_s c_p$ [14] when $V_g$ is the mass ratio of secondary pollutants per the primary pollutant and $k_s$ is the first-order chemical interaction rate of secondary pollutants.

## 2.4 Finite difference methods

We introduce a finite-difference approximation for the even grid system. The air pollution concentrations of $x, z,$ and $t$ are presented by $c(x,z,t)$. The approximate solution at each time is denoted by $t = n\Delta t,\ n = 0,1,2,\ldots,P$ when $\Delta t$ is the time increment. Then, the space in the $x$-direction and $z$-direction is meshed by $x = i\Delta x,$ $i = 0,1,2,\ldots,N$ when $\Delta x$ is the step size of the $x$-axis and $z = j\Delta z,\ j = 0,1,2,\ldots,M$ when $\Delta z$ is the step size of the $z$-axis respectively. So, the concentration of air pollution can be written by $c(x,z,t) = c(i\Delta x, j\Delta z, n\Delta t) = c_{i,j}^n$.

The forward difference for the first-order derivative

$$\frac{\partial c_{i,j}^n}{\partial t} = \frac{c_{i,j}^{n+1} - c_{i,j}^n}{\Delta t} + O(h). \tag{2.5}$$

The backward difference for the first-order derivative

$$\frac{\partial c_{i,j}^n}{\partial t} = \frac{c_{i,j}^n - c_{i,j}^{n-1}}{\Delta t} + O(h). \tag{2.6}$$

The central difference for the first-order derivative

$$\frac{\partial c_{i,j}^n}{\partial x} = \frac{c_{i+1,j}^n - c_{i-1,j}^n}{2\Delta x} + O(h^2) \tag{2.7}$$

and

$$\frac{\partial c_{i,j}^n}{\partial z} = \frac{c_{i,j+1}^n - c_{i,j-1}^n}{2\Delta z} + O(h^2). \tag{2.8}$$

The central difference for the secondary-order derivative

$$\frac{\partial^2 c_{i,j}^n}{\partial x^2} = \frac{c_{i+1,j}^n - 2c_{i,j}^n + c_{i-1,j}^n}{(\Delta x)^2} + O(h^2) \tag{2.9}$$

and

$$\frac{\partial^2 c_{i,j}^n}{\partial z^2} = \frac{c_{i,j+1}^n - 2c_{i,j}^n + c_{i,j-1}^n}{(\Delta z)^2} + O(h^2). \tag{2.10}$$

The central difference for the first-order derivative

$$\frac{\partial c_{i,j}^{n}}{\partial x} = \frac{-c_{i+2,j}^{n} + 8c_{i+1,j}^{n} - 8c_{i-1,j}^{n} + c_{i-2,j}^{n}}{12\Delta x} + O(h^{4}) \tag{2.11}$$

and
$$\frac{\partial c_{i,j}^{n}}{\partial z} = \frac{-c_{i,j+2}^{n} + 8c_{i,j+1}^{n} - 8c_{i,j-1}^{n} + c_{i,j-2}^{n}}{12\Delta z} + O(h^{4}). \tag{2.12}$$

The central difference for the secondary-order derivative

$$\frac{\partial^{2} c_{i,j}^{n}}{\partial x^{2}} = \frac{-c_{i+2,j}^{n} + 16c_{i+1,j}^{n} - 30c_{i,j}^{n} + 16c_{i-1,j}^{n} - c_{i-2,j}^{n}}{12(\Delta x)^{2}} + O(h^{4}) \tag{2.13}$$

and
$$\frac{\partial^{2} c_{i,j}^{n}}{\partial z^{2}} = \frac{-c_{i,j+2}^{n} + 16c_{i,j+1}^{n} - 30c_{i,j}^{n} + 16c_{i,j-1}^{n} - c_{i,j-2}^{n}}{12(\Delta z)^{2}} + O(h^{4}). \tag{2.14}$$

# Chapter 3

# Two-dimensional Mathematical Models of Primary Air Pollutant Concentration Measurement and Control

## 3.1 Non-dimensional form of two-dimensional horizontal averaged atmospheric diffusion equation for primary air pollutant

The transformation technique of a variable is applied by converting the dimensional equation to a non-dimensional equation.

$$\frac{\partial c_p}{\partial t}+u\frac{\partial c_p}{\partial x}+w\frac{\partial c_p}{\partial z}=k_x\frac{\partial^2 c_p}{\partial x^2}+k_z\frac{\partial^2 c_p}{\partial z^2}+q\delta(x-x_r)\delta(z-z_r)-k_pc_p\,.$$

The dimensionless variables of this above equation (2.2) are defined by

$$X=\frac{x}{l_x},\ Z=\frac{z}{l_z},\ T=\frac{t}{t_{\max}},\delta(x-x_r)=\frac{\delta(X-X_r)}{l},\ \text{and}\ \delta(z-z_r)=\frac{\delta(Z-Z_r)}{l}\,.$$

We let $c_{\max}=\max\{c_p(x,z,t):0\le x\le l_x,0\le z\le l_z,0\le t\le t_{\max}\}$,

$u_{\max}=\max\{u(x,z,t):0\le x\le l_x,0\le z\le l_z,0\le t\le t_{\max}\}$,

$w_{\max}=\max\{w(x,z,t):0\le x\le l_x,\ 0\le z\le l_z,0\le t\le t_{\max}\}$,

$l=\max\{l_x,l_z\}$ when $l_x=l_z$, and $t_{\max}$ is a stationary time.

A dimensionless variable of primary pollutant concentration is defined by $C_p=c_p/c_{\max}$. We substitute the dimensionless variables into equation (2.2). So, we get

$$\frac{\partial\left(C_pc_{\max}\right)}{\partial\left(Tt_{\max}\right)}+u\frac{\partial\left(C_pc_{\max}\right)}{\partial(Xl)}+w\frac{\partial\left(C_pc_{\max}\right)}{\partial(Zl)}$$
$$=k_x\frac{\partial^2\left(C_pc_{\max}\right)}{\partial(Xl)^2}+k_z\frac{\partial^2\left(C_pc_{\max}\right)}{\partial(Zl)^2}+q\delta(x-x_r)\delta(z-z_r)-k_p\left(C_pc_{\max}\right),$$

$$\left(\frac{c_{\max}}{t_{\max}}\right)\frac{\partial C_p}{\partial T}+\left(\frac{c_{\max}u}{l}\right)\frac{\partial C_p}{\partial X}+\left(\frac{c_{\max}w}{l}\right)\frac{\partial C_p}{\partial Z}$$
$$=\left(\frac{c_{\max}k_x}{l^2}\right)\frac{\partial^2 C_p}{\partial X^2}+\left(\frac{c_{\max}k_z}{l^2}\right)\frac{\partial^2 C_p}{\partial Z^2}+q\delta(x-x_r)\delta(z-z_r)-k_pc_{\max}C_p\,. \qquad (3.1)$$

Equation (3.1) is multiplied by $\dfrac{l}{u_{\max}c_{\max}}$ , then we get

$$\left(\frac{l}{u_{\max}t_{\max}}\right)\frac{\partial C_p}{\partial T}+\left(\frac{u}{u_{\max}}\right)\frac{\partial C_p}{\partial X}+\left(\frac{w}{u_{\max}}\right)\frac{\partial C_p}{\partial Z}$$
$$=\left(\frac{k_x}{u_{\max}l}\right)\frac{\partial^2 C_p}{\partial X^2}+\left(\frac{k_z}{u_{\max}l}\right)\frac{\partial^2 C_p}{\partial Z^2}+\left(\frac{ql}{c_{\max}u_{\max}}\right)\delta(x-x_r)\delta(z-z_r)-\left(\frac{k_p l}{u_{\max}}\right)C_p,$$

$$\frac{1}{ST}\frac{\partial C_p}{\partial T}+U\frac{\partial C_p}{\partial X}+W\frac{\partial C_p}{\partial Z}$$
$$=D_x\frac{\partial^2 C_p}{\partial X^2}+D_z\frac{\partial^2 C_p}{\partial Z^2}+\left(\frac{ql}{c_{\max}u_{\max}}\right)\frac{\delta(X-X_r)\delta(Z-Z_r)}{l^2}-K_pC_p,$$

$$\frac{1}{ST}\frac{\partial C_p}{\partial T}+U\frac{\partial C_p}{\partial X}+W\frac{\partial C_p}{\partial Z}$$
$$=D_x\frac{\partial^2 C_p}{\partial X^2}+D_z\frac{\partial^2 C_p}{\partial Z^2}+\left(\frac{q}{c_{\max}u_{\max}l}\right)\delta(X-X_r)\delta(Z-Z_r)-K_pC_p,$$

$$\frac{1}{ST}\frac{\partial C_p}{\partial T}+U\frac{\partial C_p}{\partial X}+W\frac{\partial C_p}{\partial Z}=D_x\frac{\partial^2 C_p}{\partial X^2}+D_z\frac{\partial^2 C_p}{\partial Z^2}+Q\delta(X-X_r)\delta(Z-Z_r)-K_pC_p. \quad (3.2)$$

Therefore, the dimensionless equation of primary pollutant can be written as

$$\frac{1}{ST}\frac{\partial C_p}{\partial T}+U\frac{\partial C_p}{\partial X}+W\frac{\partial C_p}{\partial Z}=D_x\frac{\partial^2 C_p}{\partial X^2}+D_z\frac{\partial^2 C_p}{\partial Z^2}+S_p+R_p, \quad (3.3)$$

where $ST=\dfrac{u_{\max}t_{\max}}{l}$, $U=\dfrac{u}{u_{\max}}$, $W=\dfrac{w}{u_{\max}}$, $D_x=\dfrac{k_x}{u_{\max}l}$, $D_z=\dfrac{k_z}{u_{\max}l}$,

$S_p=Q\delta(X-X_r)\delta(Z-Z_r)$ when $Q=\dfrac{q}{c_{\max}u_{\max}l}$,

and $R_p=-K_pC_p$ when $K_p=\dfrac{k_p l}{u_{\max}}$.

The non-dimensional form for the initial condition of primary pollutant assumed that

$$C_p(X,Z,0)=0, \quad (3.4)$$

for all $0<X<1$ and $0<Z<1$. For the non-dimensional of boundary conditions, it is supposed that

$$C_p(0,Z,T)=0 \text{ or } \frac{\partial C_p}{\partial X}(0,Z,T)=0, \tag{3.5}$$

$$\frac{\partial C_p}{\partial X}(1,Z,T)=0, \tag{3.6}$$

$$\frac{\partial C_p}{\partial Z}(X,1,T)=0, \tag{3.7}$$

and $$\frac{\partial C_p}{\partial Z}(X,0,T)=f(X,T), \tag{3.8}$$

for all $T>0$ where $f(X,T)$ is a constant function.

## 3.2 Numerical simulation of an air pollution model on industrial areas by considering the influence of multiple point sources

In Figure 3.1, a model of a primary air pollutant dispersion problem is shown. The physical problem is composed of two zones, an industrial zone and an urban zone, with a stable wind along the $x$-axis and $z$-axis. The point sources lie along the $x$-axis. We assume that the primary air pollutant is released from a factory smokestack by a single point source and coupled point sources in the industrial zone. The emissions of air pollution come into contact with the urban zone at the rate of air pollutant absorption. In the numerical experiment, the considered domain of the solutions is shown in Figure 3.2.

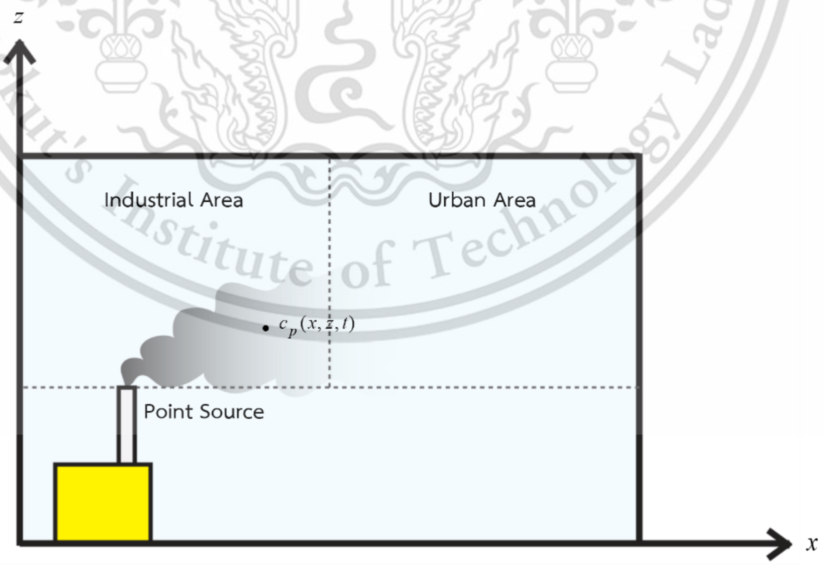



**Figure 3.1** Model of primary air pollutant dispersion problem

**3.2.1 Initial and boundary conditions setting techniques for primary air pollutant dispersion**

The initial condition is assumed under the cold start assumption. That is

$$C_p(X,Z,0)=0, \tag{3.9}$$

for all $0<X<1$ and $0<Z<1$. The boundary conditions are assumed that

$$C_p(0,Z,T)=0, \tag{3.10}$$

$$\frac{\partial C_p}{\partial X}(1,Z,T)=0, \tag{3.11}$$

$$\frac{\partial C_p}{\partial Z}(X,1,T)=0, \tag{3.12}$$

and

$$\frac{\partial C_p}{\partial Z}(X,0,T)=0, \tag{3.13}$$

for all $T>0$. The concentration at the point sources is assumed to be the constant variables as

$$C_p(X_{ps_{no}},0,T)=C_{ps_{no}}, \tag{3.14}$$

for $no=1,2$, where $X_{ps}$ is the position of the point source $no$ in the $x$-direction and $C_{ps}$ is the concentration value at the point source of $no$.

Therefore, we define the air pollutant concentration at a point source that

$$C_p(X_{ps_1},0,T)=C_{ps_1}. \tag{3.15}$$

Then, air pollutant concentration at a two-point source is defined that

$$C_p(X_{ps_1},0,T)=C_{ps_1} \tag{3.16}$$

and

$$C_p(X_{ps_2},0,T)=C_{ps_2}. \tag{3.17}$$

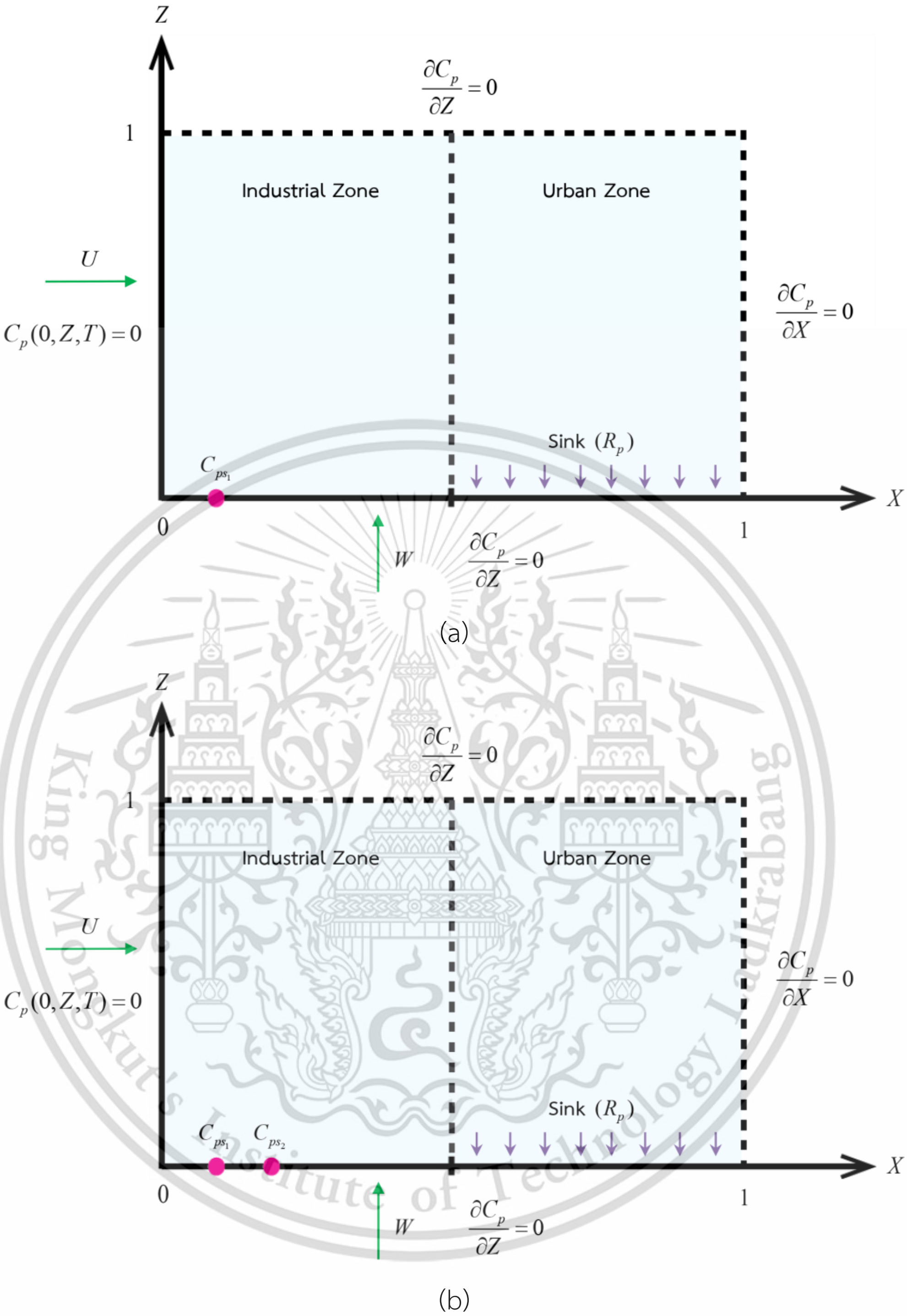



**Figure 3.2** Domain of air pollution solutions (a) one-point source and (b) two-point sources

### 3.2.2 Explicit and implicit finite difference techniques

#### 3.2.2.1 Forward time central space scheme

The first method, we use the forward difference in transient term that is

$$\frac{\partial C_p}{\partial T} = \frac{C_{pi,j}^{n+1} - C_{pi,j}^{n}}{\Delta T}. \tag{3.18}$$

Then, the centered difference for the advection and diffusion in the $x$ - direction and $z$ - direction is applied as follows

$$\frac{\partial C_p}{\partial X} = \frac{C_{pi+1,j}^{n} - C_{pi-1,j}^{n}}{2\Delta X}, \tag{3.19}$$

$$\frac{\partial C_p}{\partial Z} = \frac{C_{pi,j+1}^{n} - C_{pi,j-1}^{n}}{2\Delta Z}, \tag{3.20}$$

$$\frac{\partial^2 C_p}{\partial X^2} = \frac{C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}}{(\Delta X)^2}, \tag{3.21}$$

and

$$\frac{\partial^2 C_p}{\partial Z^2} = \frac{C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}}{(\Delta Z)^2}, \tag{3.22}$$

respectively. We substitute the equations (3.18)-(3.22) into equation (3.3) and then we define that $S_p = 0$. It will be

$$\frac{1}{ST}\left(\frac{C_{pi,j}^{n+1} - C_{pi,j}^{n}}{\Delta T}\right) + U\left(\frac{C_{pi+1,j}^{n} - C_{pi-1,j}^{n}}{2\Delta X}\right) + W\left(\frac{C_{pi,j+1}^{n} - C_{pi,j-1}^{n}}{2\Delta Z}\right)$$
$$= D_x\left(\frac{C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}}{(\Delta X)^2}\right) + D_z\left(\frac{C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}}{(\Delta Z)^2}\right) + R_p,$$

$$\frac{1}{ST}\left(\frac{C_{pi,j}^{n+1} - C_{pi,j}^{n}}{\Delta T}\right) = D_x\left(\frac{C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}}{(\Delta X)^2}\right) + D_z\left(\frac{C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}}{(\Delta Z)^2}\right)$$
$$-U\left(\frac{C_{pi+1,j}^{n} - C_{pi-1,j}^{n}}{2\Delta X}\right) - W\left(\frac{C_{pi,j+1}^{n} - C_{pi,j-1}^{n}}{2\Delta Z}\right) + R_p,$$

$$C_{pi,j}^{n+1} = \frac{ST(\Delta T)D_x}{(\Delta X)^2}\left(C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}\right)$$
$$+\frac{ST(\Delta T)D_z}{(\Delta Z)^2}\left(C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}\right) - \frac{ST(\Delta T)U}{2\Delta X}\left(C_{pi+1,j}^{n} - C_{pi-1,j}^{n}\right)$$
$$-\frac{ST(\Delta T)W}{2\Delta Z}\left(C_{pi,j+1}^{n} - C_{pi,j-1}^{n}\right) + C_{pi,j}^{n} + ST(\Delta T)R_p, \tag{3.23}$$

From equation (3.23), we obtain that

$$
\begin{aligned}
C_{pi,j}^{n+1} =& \frac{ST(\Delta T)D_x}{(\Delta X)^2}C_{pi+1,j}^{n} - \frac{2ST(\Delta T)D_x}{(\Delta X)^2}C_{pi,j}^{n} + \frac{ST(\Delta T)D_x}{(\Delta X)^2}C_{pi-1,j}^{n} \\
&+ \frac{ST(\Delta T)D_z}{(\Delta Z)^2}C_{pi,j+1}^{n} - \frac{2ST(\Delta T)D_z}{(\Delta Z)^2}C_{pi,j}^{n} + \frac{ST(\Delta T)D_z}{(\Delta Z)^2}C_{pi,j-1}^{n} \\
&- \frac{ST(\Delta T)U}{2\Delta X}C_{pi+1,j}^{n} + \frac{ST(\Delta T)U}{2\Delta X}C_{pi-1,j}^{n} - \frac{ST(\Delta T)W}{2\Delta Z}C_{pi,j+1}^{n} \\
&+ \frac{ST(\Delta T)W}{2\Delta Z}C_{pi,j-1}^{n} + C_{pi,j}^{n} + ST(\Delta T)R_p,
\end{aligned}
$$

$$
\begin{aligned}
C_{pi,j}^{n+1} =& \left(\frac{ST(\Delta T)D_x}{(\Delta X)^2} - \frac{ST(\Delta T)U}{2\Delta X}\right)C_{pi+1,j}^{n} + \left(\frac{ST(\Delta T)D_x}{(\Delta X)^2} + \frac{ST(\Delta T)U}{2\Delta X}\right)C_{pi-1,j}^{n} \\
&+ \left(1 - \frac{2ST(\Delta T)D_z}{(\Delta Z)^2} - \frac{2ST(\Delta T)D_x}{(\Delta X)^2}\right)C_{pi,j}^{n} \\
&+ \left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2} + \frac{ST(\Delta T)W}{2\Delta Z}\right)C_{pi,j-1}^{n} + \left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2} - \frac{ST(\Delta T)W}{2\Delta Z}\right)C_{pi,j+1}^{n} \\
&+ ST(\Delta T)R_p. \qquad (3.24)
\end{aligned}
$$

Thus, the forward time central space (FTCS) scheme of the non-dimensional mathematical model is

$$
\begin{aligned}
C_{pi,j}^{n+1} =& (d_x - A_x)C_{pi+1,j}^{n} + (d_x + A_x)C_{pi-1,j}^{n} + (1 - 2d_x - 2d_z)C_{pi,j}^{n} \\
&+ (d_z + A_z)C_{pi,j-1}^{n} + (d_z - A_z)C_{pi,j+1}^{n} + ST(\Delta T)R_p, \qquad (3.25)
\end{aligned}
$$

where $A_x = \frac{ST(\Delta T)U}{2\Delta X}, A_z = \frac{ST(\Delta T)W}{2\Delta Z}, d_x = \frac{ST(\Delta T)D_x}{(\Delta X)^2}$, and $d_z = \frac{ST(\Delta T)D_z}{(\Delta Z)^2}$.

The stability of the forward time central space scheme can be investigated by using von Neumann stability analysis. We can determine that the stability condition is $0 \leqslant 2d_x + A_x + A_z \leqslant 1$.

#### 3.2.2.2 Backward time central space scheme

The second method, we use the backward difference in transient term that is

$$
\frac{\partial C_p}{\partial T} = \frac{C_{pi,j}^{n} - C_{pi,j}^{n-1}}{\Delta T}. \qquad (3.26)
$$

Then, the centered difference for the advection and diffusion in the $x$-direction and $z$-direction is utilized as follows

$$\frac{\partial C_p}{\partial X}=\frac{C^n_{pi+1,j}-C^n_{pi-1,j}}{2\Delta X}, \tag{3.27}$$

$$\frac{\partial C_p}{\partial Z}=\frac{C^n_{pi,j+1}-C^n_{pi,j-1}}{2\Delta Z}, \tag{3.28}$$

$$\frac{\partial^2 C_p}{\partial X^2}=\frac{C^n_{pi+1,j}-2C^n_{pi,j}+C^n_{pi-1,j}}{(\Delta X)^2}, \tag{3.29}$$

and
$$\frac{\partial^2 C_p}{\partial Z^2}=\frac{C^n_{pi,j+1}-2C^n_{pi,j}+C^n_{pi,j-1}}{(\Delta Z)^2}, \tag{3.30}$$

respectively. We substitute the equations (3.26)-(3.30) into equation (3.3) when we define that $S_p=0$. It obtains that

$$\frac{1}{ST}\left(\frac{C^n_{pi,j}-C^{n-1}_{pi,j}}{\Delta T}\right)+U\left(\frac{C^n_{pi+1,j}-C^n_{pi-1,j}}{2\Delta X}\right)+W\left(\frac{C^n_{pi,j+1}-C^n_{pi,j-1}}{2\Delta Z}\right)$$
$$=D_x\left(\frac{C^n_{pi+1,j}-2C^n_{pi,j}+C^n_{pi-1,j}}{(\Delta X)^2}\right)+D_z\left(\frac{C^n_{pi,j+1}-2C^n_{pi,j}+C^n_{pi,j-1}}{(\Delta Z)^2}\right)+R_p,$$

$$\frac{1}{ST}\left(\frac{C^n_{pi,j}-C^{n-1}_{pi,j}}{\Delta T}\right)=D_x\left(\frac{C^n_{pi+1,j}-2C^n_{pi,j}+C^n_{pi-1,j}}{(\Delta X)^2}\right)+D_z\left(\frac{C^n_{pi,j+1}-2C^n_{pi,j}+C^n_{pi,j-1}}{(\Delta Z)^2}\right)$$
$$-U\left(\frac{C^n_{pi+1,j}-C^n_{pi-1,j}}{2\Delta X}\right)-W\left(\frac{C^n_{pi,j+1}-C^n_{pi,j-1}}{2\Delta Z}\right)+R_p,$$

$$C^n_{pi,j}-C^{n-1}_{pi,j}=-\frac{ST(\Delta T)U}{2\Delta X}\left(C^n_{pi+1,j}-C^n_{pi-1,j}\right)-\frac{ST(\Delta T)W}{2\Delta Z}\left(C^n_{pi,j+1}-C^n_{pi,j-1}\right)$$
$$+\frac{ST(\Delta T)D_x}{(\Delta X)^2}\left(C^n_{pi+1,j}-2C^n_{pi,j}+C^n_{pi-1,j}\right)$$
$$+\frac{ST(\Delta T)D_x}{(\Delta Z)^2}\left(C^n_{pi,j+1}-2C^n_{pi,j}+C^n_{pi,j-1}\right)+ST(\Delta T)R_p,$$

$$C^n_{pi,j}+\frac{ST(\Delta T)U}{2\Delta X}C^n_{pi+1,j}-\frac{ST(\Delta T)U}{2\Delta X}C^n_{pi-1,j}+\frac{ST(\Delta T)W}{2\Delta Z}C^n_{pi,j+1}-\frac{ST(\Delta T)W}{2\Delta Z}C^n_{pi,j-1}$$
$$-\frac{ST(\Delta T)D_x}{(\Delta X)^2}C^n_{pi+1,j}+\frac{2ST(\Delta T)D_x}{(\Delta X)^2}C^n_{pi,j}-\frac{ST(\Delta T)D_x}{(\Delta X)^2}C^n_{pi-1,j}$$
$$-\frac{ST(\Delta T)D_z}{(\Delta Z)^2}C^n_{pi,j+1}+\frac{2ST(\Delta T)D_z}{(\Delta Z)^2}C^n_{pi,j}-\frac{ST(\Delta T)D_z}{(\Delta Z)^2}C^n_{pi,j-1}$$
$$=C^{n-1}_{pi,j}+ST(\Delta T)R_p, \tag{3.31}$$

From equation (3.31), we get

$$\left(\frac{ST(\Delta T)U}{2\Delta X}-\frac{ST(\Delta T)D_x}{(\Delta X)^2}\right)C^n_{pi+1,j}-\left(\frac{ST(\Delta T)U}{2\Delta X}+\frac{ST(\Delta T)D_x}{(\Delta X)^2}\right)C^n_{pi-1,j}$$
$$+\left(1+\frac{2ST(\Delta T)D_x}{(\Delta X)^2}+\frac{2ST(\Delta T)D_z}{(\Delta Z)^2}\right)C^n_{pi,j}$$
$$-\left(\frac{ST(\Delta T)W}{2\Delta Z}+\frac{ST(\Delta T)D_z}{(\Delta Z)^2}\right)C^n_{pi,j-1}+\left(\frac{ST(\Delta T)W}{2\Delta Z}-\frac{ST(\Delta T)D_z}{(\Delta Z)^2}\right)C^n_{pi,j+1}$$
$$=C^{n-1}_{pi,j}+ST(\Delta T)R_p. \tag{3.32}$$

Therefore, the backward time central space (BTCS) scheme of primary air pollutant concentration becomes

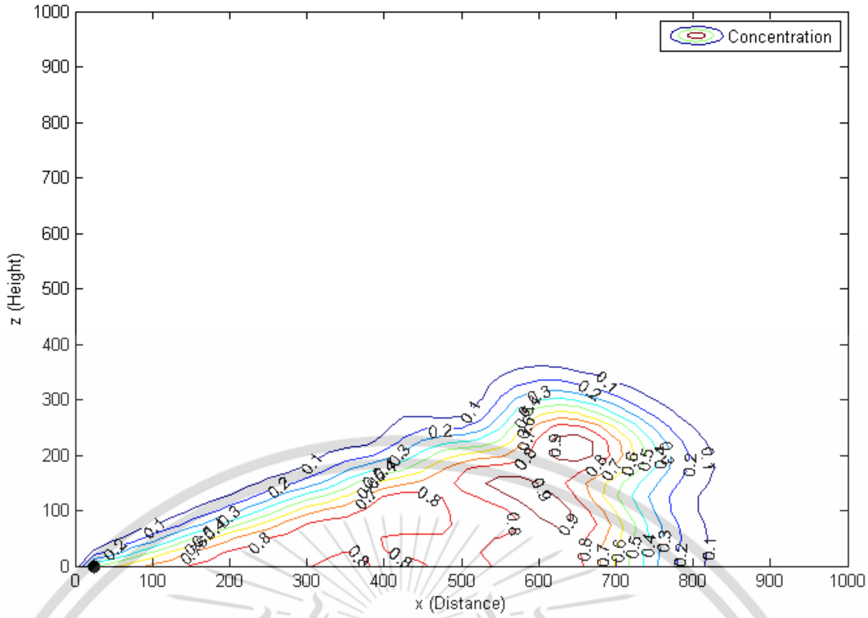
$$(A_x-d_x)C^{n+1}_{pi+1,j}+(A_x+d_x)C^{n+1}_{pi-1,j}+(1+2d_x+2d_z)C^{n+1}_{pi,j}$$
$$-(A_z+d_z)C^{n+1}_{pi,j-1}+(A_z-d_z)C^{n+1}_{pi,j+1}=C^n_{pi,j}+ST(\Delta T)R_p, \tag{3.33}$$

where $A_x=\frac{ST(\Delta T)U}{2\Delta X}, A_z=\frac{ST(\Delta T)W}{2\Delta Z}, d_x=\frac{ST(\Delta T)D_x}{(\Delta X)^2}$, and $d_z=\frac{ST(\Delta T)D_z}{(\Delta Z)^2}$.

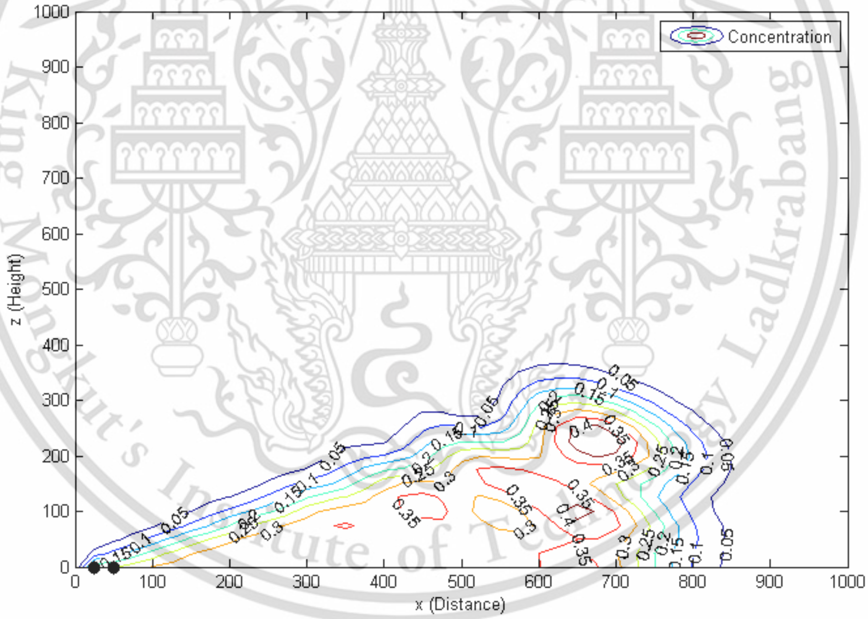
The stability of the implicit backward time central space scheme can be investigated by using von Neumann stability analysis. We can determine that the method is an unconditionally stable method.

### 3.2.3 Numerical experiments and results for primary air pollutant dispersion

A non-dimensional two-dimensional atmospheric diffusion equation (3.3) will be considered. Uniform wind velocities and constant diffusion coefficients are introduced. We choose that the wind velocities in the $x$-direction and $z$-direction are $0.1$ and $0.05$ $m/s$, respectively. The diffusion coefficients in the $x$-direction and $z$-direction are $4.5\times10^{-1}$ and $4.5\times10^{-5}$ $m^2/s$, respectively. The grid spacing is $\Delta x=\Delta z=25$ $m.$ and the time interval is $20$ $s$. In this research, we present two cases. The first case considers a point source when the concentration is $0.5$ $kg/m^3$. The second case considers two-point source when the concentrations are $0.25$ and $0.25$ $kg/m^3$. The air pollutants in equation (3.14) are released into our system. These examples are solved by using the forward time central space and the backward time central space schemes in equations (3.25) and (3.33), respectively, with the initial and boundary conditions (3.9)-(3.13).

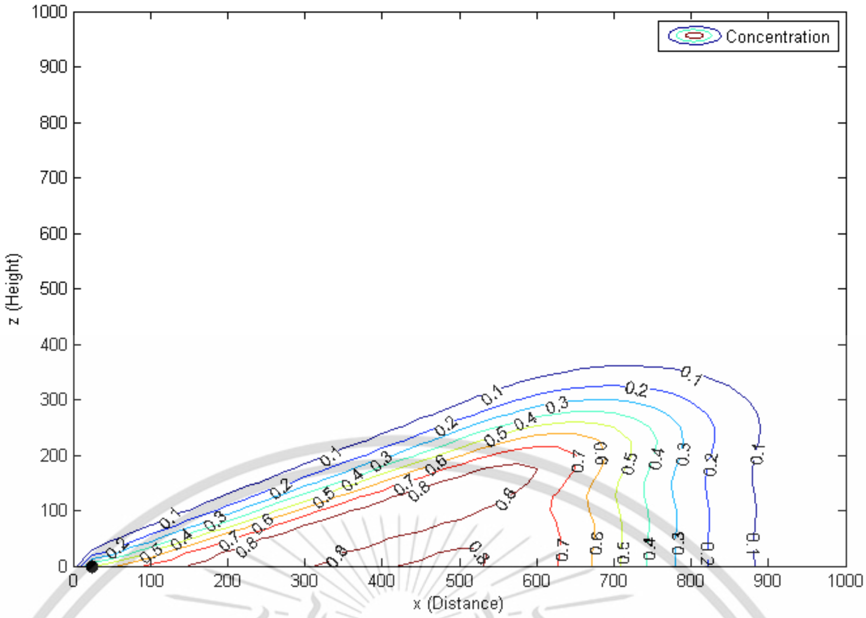



(a)

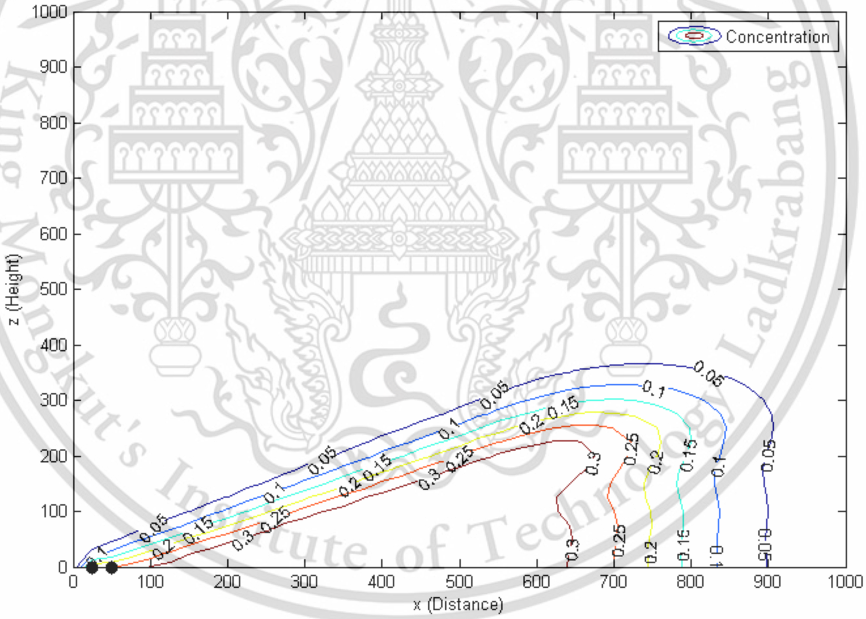



(b)

**Figure 3.3** The air pollutant concentration levels after 2 hours passed which are computed by the forward time central space scheme ($R_p = 0$) (a) one-point source and (b) two-point source

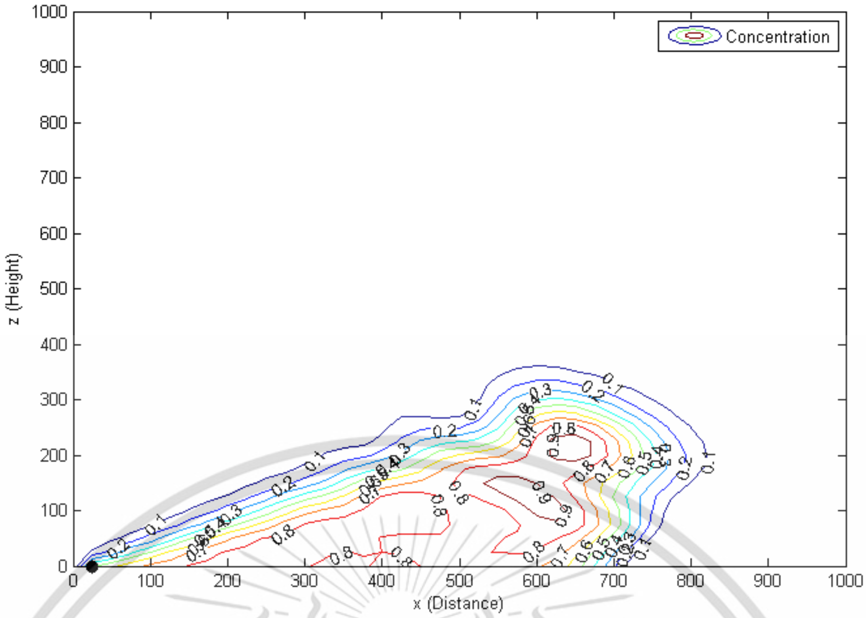



(a)

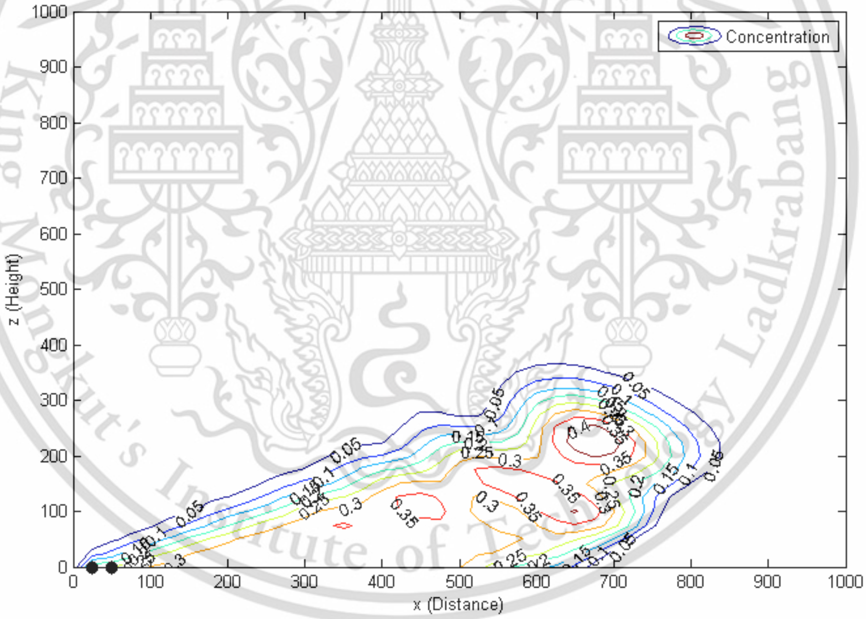



(b)

**Figure 3.4** The air pollutant concentration levels after 2 hours passed which are computed by the backward time central space scheme ($R_p = 0$) (a) one-point source and (b) two-point source

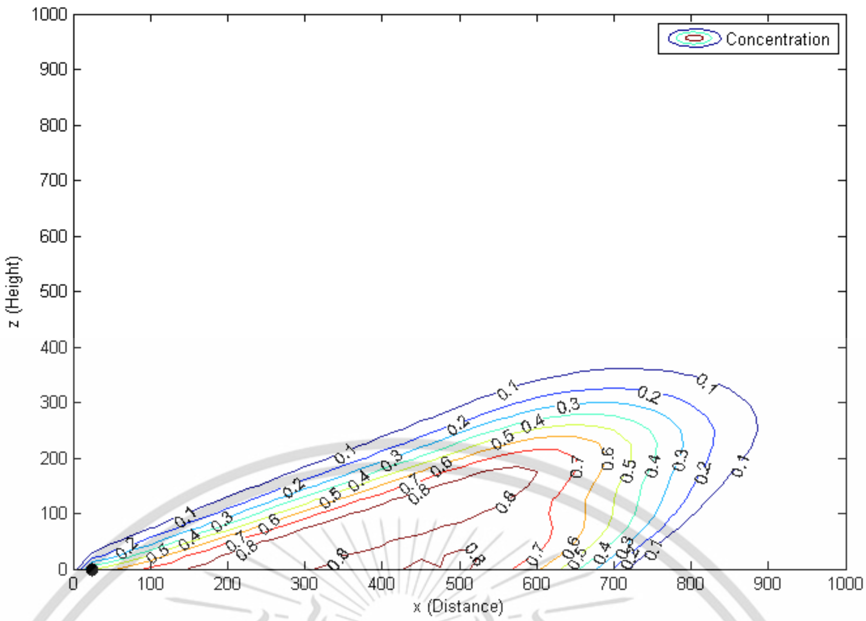



(a)

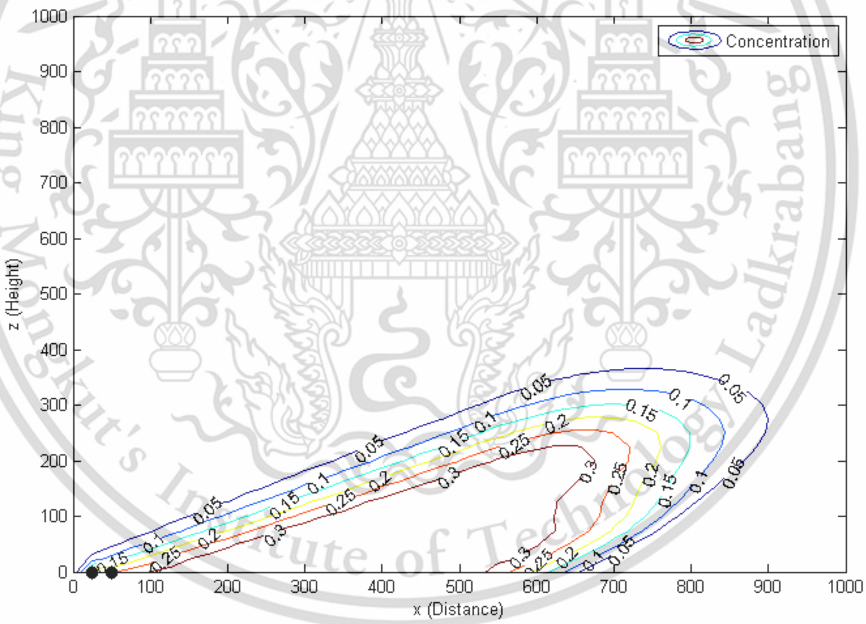



(b)

**Figure 3.5** The air pollutant concentration levels after 2 hours passed which are computed by the forward time central space scheme ($R_p = -10^{-4}$) (a) one-point source and (b) two-point source




(a)




(b)

**Figure 3.6** The air pollutant concentration levels after 2 hours passed which are computed by the backward time central space scheme ($R_p = -10^{-4}$) (a) one-point source and (b) two-point source

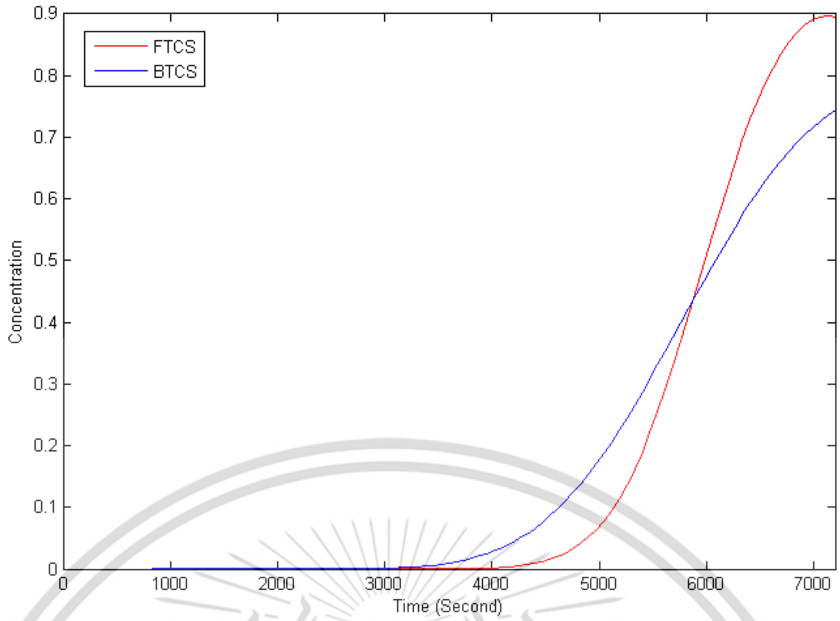



**Figure 3.7** The air pollutant concentration between the forward time central space and the backward time central space schemes ($R_p = 0$) at $z = 0$ m. and $x = 600$ m.

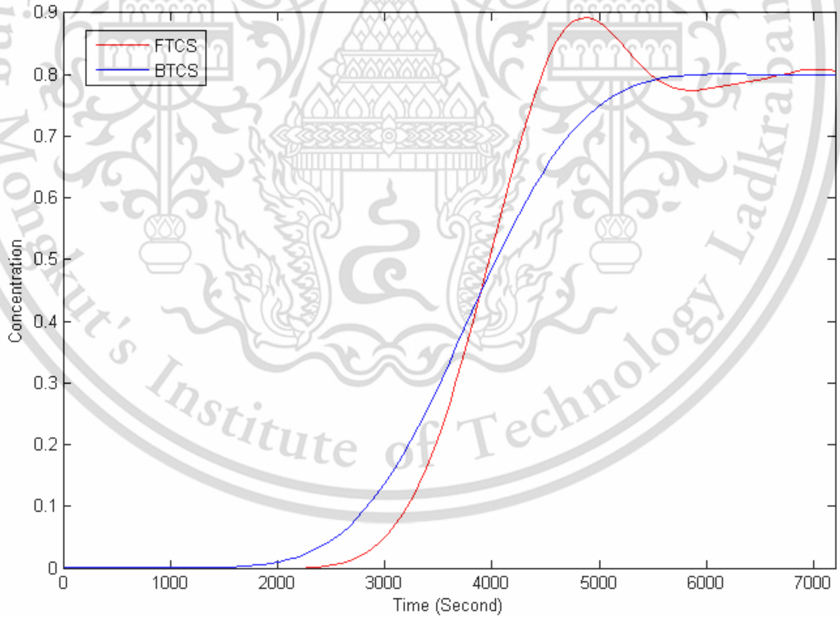



**Figure 3.8** The air pollutant concentration between the forward time central space and the backward time central space schemes ($R_p = -10^{-4}$) at $z = 0$ m. and $x = 400$ m.

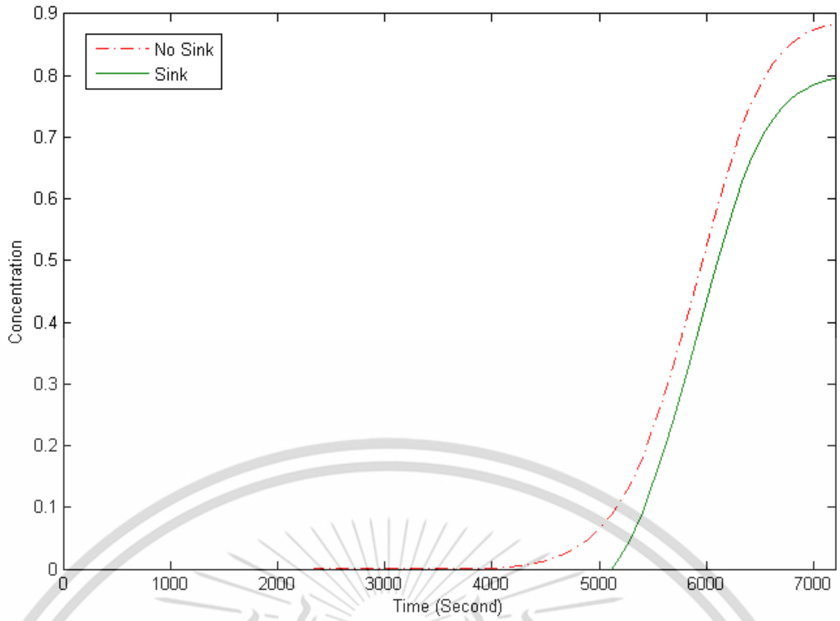



**Figure 3.9** The air pollutant concentration between 2 cases: added sink and without sink (computed by the forward time central space scheme) at $z=25$ m. and $x=600$ m.

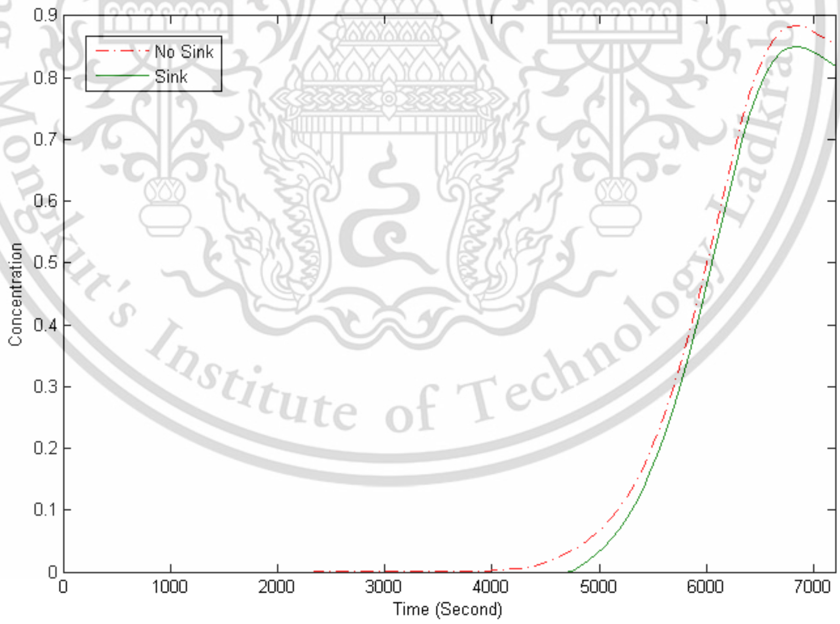



**Figure 3.10** The air pollutant concentration between 2 cases: added sink and without sink (computed by the forward time central space scheme) at $z=50$ m. and $x=600$ m.

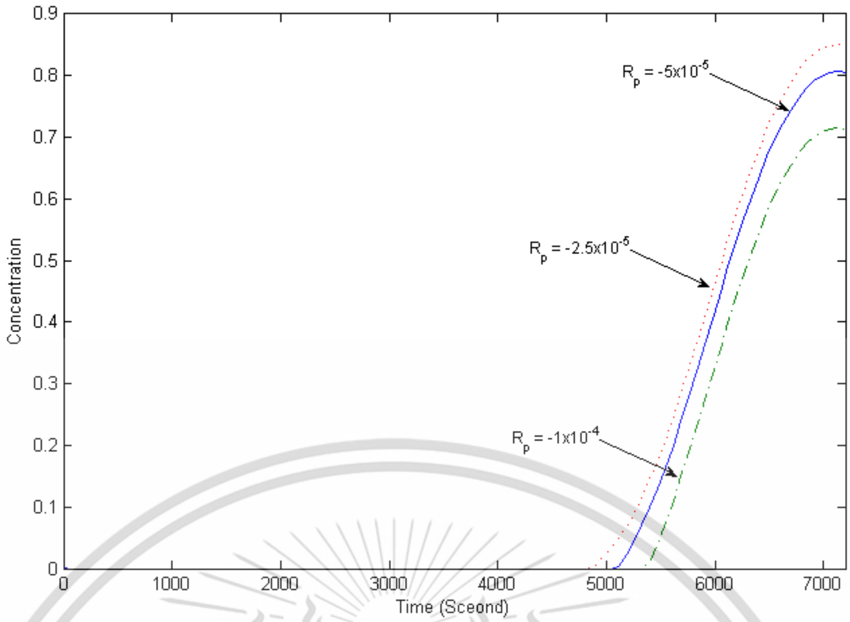



**Figure 3.11** The air pollutant concentration with the variant values of sink rate (computed by the forward time central space scheme) at $z = 0$ m. and $x = 600$ m.

Figure 3.3 and Figure 3.4 compare the air pollutant concentrations between two cases: a single point source and coupled point sources respectively. From the both figures, it is apparent that the results of the forward time central space scheme is close to the results of the backward time central space scheme, when there is no sink of pollutant absorption $(R_p = 0)$. Figure 3.5 and Figure 3.6 illustrate that the sink of pollutant absorption $(R_p = -10^{-4})$ is added into the base of the urban zone. The air pollutant concentration near human habitats goes down, and the two methods also give the close result.

In Figure 3.7 and Figure 3.8, the computed approximate solutions, which are calculated by using the forward time central space and the backward time central space schemes, are compared. We can see that the results of the added sink case and the without sink case are quite similar. These graphs also indicate that the forward time central space scheme gives computed solutions close to the backward time central space scheme. Figure 3.9 and Figure 3.10 demonstrate that the air pollutant concentration at the heights $z = 25$ $m.$ and $z = 50$ $m.$ are solved by using the forward time central space scheme. The added sink case has less concentration than the without sink case.

Figure 3.11 establishes the various concentrations when we take an increased sink rate into our system. Therefore, the sink can lower the overall pollutant levels. The comparison of computing time shows that the forward time central space scheme is faster than the backward time central space scheme, as shown in Table 3.1.

**Table 3.1** Computing time comparison of forward time central space and backward time central space schemes

| Simulation Time | FTCS (sec.) | BTCS (sec.) |
|---|---|---|
| 30 minutes | 1.49 | 22.48 |
| 1 hour | 1.68 | 42.66 |
| 2 hours | 2.05 | 84.18 |

## 3.3 Numerical simulation to air pollution emission control near an industrial zone

A numerical model for an air pollution emission control problem with uniform wind velocities and constant diffusion coefficients is proposed. In this research, an atmospheric diffusion equation is solved by using the finite difference method. This study analyzed the ambient air quality standard of sulfur dioxide that refers to the quantity of sulfur dioxide concentrations in clean air. In Figure 3.12, a model of air pollution emission control problem is presented. This research was designed to study the behavior of the dispersion and the effect of dispersion concentration near the industrial zone. The four monitoring points are set far away from the source. The monitoring points are called M1, M2, M3, and M4, respectively.

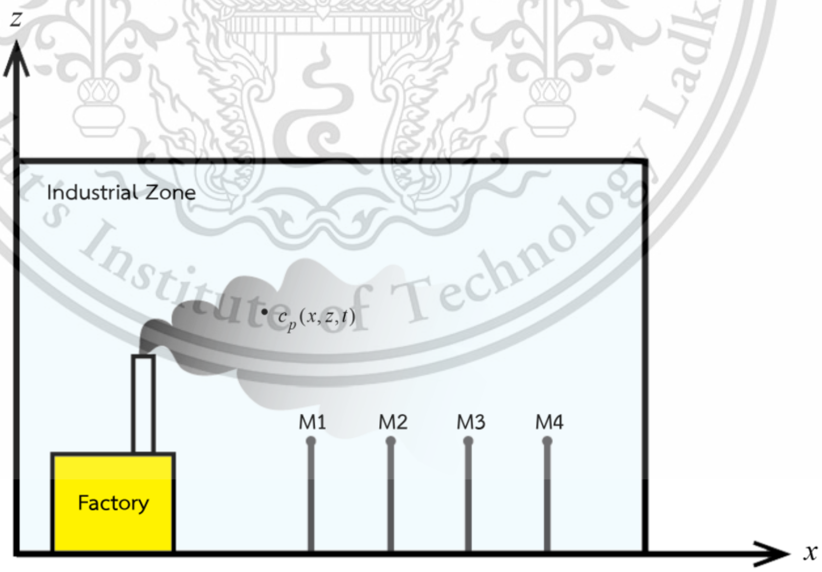



**Figure 3.12** Model of primary air pollutant emission control problem

### 3.3.1 Initial and boundary conditions setting techniques for primary air pollutant control

The non-dimensional of initial condition assumed that

$$C_p(X,Z,0)=0, \tag{3.34}$$

for all $0<X<1$ and $0<Z<1$. For the non-dimensional of boundary, it is assumed that

$$\frac{\partial C_p}{\partial X}(0,Z,T)=0, \tag{3.35}$$

$$\frac{\partial C_p}{\partial X}(1,Z,T)=0, \tag{3.36}$$

$$\frac{\partial C_p}{\partial Z}(X,1,T)=0, \tag{3.37}$$

and

$$\frac{\partial C_p}{\partial Z}(X,0,T)=V_{dp}C_p, \tag{3.38}$$

for all $T>0$. We define $f(X,T)=V_{dp}C_p$ when $V_{dp}$ is the dry deposition velocity of the primary pollutant.

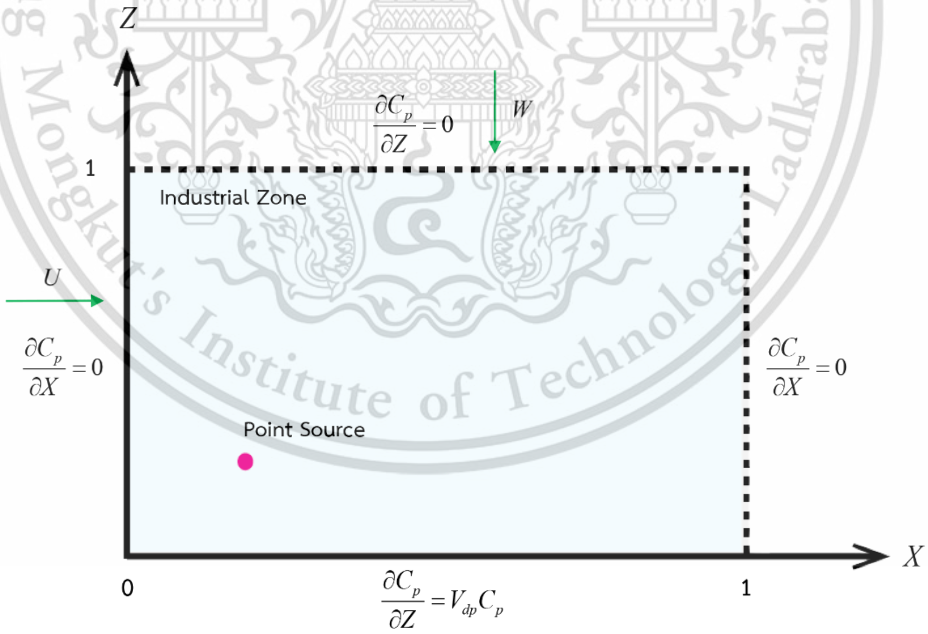



**Figure 3.13** Domain of primary air pollutant solutions

In Figure 3.13, the considered domain for the numerical experiment is shown. Let the height of the point source be $h_{ps}$ *m*. The wind is stable in the $x$-axis and $z$-axis. The concentrations of air pollutants are emitted directly from a continued point source (the chimney) from the industrial factory. The air pollutants are absorbed in a chemical reaction on the ground.

### 3.3.2 Explicit difference technique for primary air pollutant control

This method refers to the non-dimensional model, for which we use the forward time central space (FTCS) scheme. In the transient term, we used the forward difference for

$$\frac{\partial C_p}{\partial T} = \frac{C_{pi,j}^{n+1} - C_{pi,j}^{n}}{\Delta T}. \tag{3.39}$$

The advection and diffusion terms are substituted by using the centered difference in space by

$$\frac{\partial C_p}{\partial X} = \frac{C_{pi+1,j}^{n} - C_{pi-1,j}^{n}}{2\Delta X}, \tag{3.40}$$

$$\frac{\partial C_p}{\partial Z} = \frac{C_{pi,j+1}^{n} - C_{pi,j-1}^{n}}{2\Delta Z}, \tag{3.41}$$

$$\frac{\partial^2 C_p}{\partial X^2} = \frac{C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}}{(\Delta X)^2}, \tag{3.42}$$

and

$$\frac{\partial^2 C_p}{\partial Z^2} = \frac{C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}}{(\Delta Z)^2}, \tag{3.43}$$

respectively. We substitute the equations (3.39)-(3.43) into equation (3.3). So, we get

$$\frac{1}{ST}\left(\frac{C_{pi,j}^{n+1} - C_{pi,j}^{n}}{\Delta T}\right) + U\left(\frac{C_{pi+1,j}^{n} - C_{pi-1,j}^{n}}{2\Delta X}\right) + W\left(\frac{C_{pi,j+1}^{n} - C_{pi,j-1}^{n}}{2\Delta Z}\right)$$
$$= D_x\left(\frac{C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}}{(\Delta X)^2}\right) + D_z\left(\frac{C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}}{(\Delta Z)^2}\right)$$
$$+Q\delta(X - X_r)\delta(Z - Z_r) - K_p C_{pi,j}^{n},$$

where $S_p = Q\delta(X - X_r)\delta(Z - Z_r)$ and $R_p = -K_p C_{pi,j}^{n}$.

$$\frac{1}{ST}\left(\frac{C_{pi,j}^{n+1} - C_{pi,j}^{n}}{\Delta T}\right) = D_x\left(\frac{C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}}{(\Delta X)^2}\right) + D_z\left(\frac{C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}}{(\Delta Z)^2}\right)$$
$$-U\left(\frac{C_{pi+1,j}^{n} - C_{pi-1,j}^{n}}{2\Delta X}\right) - W\left(\frac{C_{pi,j+1}^{n} - C_{pi,j-1}^{n}}{2\Delta Z}\right)$$
$$-K_p C_{pi,j}^{n} + Q\delta(X - X_r)\delta(Z - Z_r), \tag{3.44}$$

Rearrangement of equation (3.44), then we have

$$
\begin{aligned}
C_{pi,j}^{n+1} = {} & \frac{ST(\Delta T)D_x}{(\Delta X)^2}\left(C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}\right) \\
& + \frac{ST(\Delta T)D_z}{(\Delta Z)^2}\left(C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}\right) - \frac{ST(\Delta T)U}{2\Delta X}\left(C_{pi+1,j}^{n} - C_{pi-1,j}^{n}\right) \\
& - \frac{ST(\Delta T)W}{2\Delta Z}\left(C_{pi,j+1}^{n} - C_{pi,j-1}^{n}\right) - ST(\Delta T)K_p C_{pi,j}^{n} + C_{pi,j}^{n} \\
& + ST(\Delta T)Q\delta(X - X_r)\delta(Z - Z_r),
\end{aligned}
$$

$$
\begin{aligned}
C_{pi,j}^{n+1} = {} & \frac{ST(\Delta T)D_x}{(\Delta X)^2}C_{pi+1,j}^{n} - \frac{2ST(\Delta T)D_x}{(\Delta X)^2}C_{pi,j}^{n} + \frac{ST(\Delta T)D_x}{(\Delta X)^2}C_{pi-1,j}^{n} \\
& + \frac{ST(\Delta T)D_z}{(\Delta Z)^2}C_{pi,j+1}^{n} - \frac{2ST(\Delta T)D_z}{(\Delta Z)^2}C_{pi,j}^{n} + \frac{ST(\Delta T)D_z}{(\Delta Z)^2}C_{pi,j-1}^{n} \\
& - \frac{ST(\Delta T)U}{2\Delta X}C_{pi+1,j}^{n} + \frac{ST(\Delta T)U}{2\Delta X}C_{pi-1,j}^{n} - \frac{ST(\Delta T)W}{2\Delta Z}C_{pi,j+1}^{n} \\
& + \frac{ST(\Delta T)W}{2\Delta Z}C_{pi,j-1}^{n} - ST(\Delta T)K_p C_{pi,j}^{n} + C_{pi,j}^{n} \\
& + ST(\Delta T)Q\delta(X - X_r)\delta(Z - Z_r),
\end{aligned}
$$

$$
\begin{aligned}
C_{pi,j}^{n+1} = {} & \left(\frac{ST(\Delta T)D_x}{(\Delta X)^2} - \frac{ST(\Delta T)U}{2\Delta X}\right)C_{pi+1,j}^{n} + \left(\frac{ST(\Delta T)D_x}{(\Delta X)^2} + \frac{ST(\Delta T)U}{2\Delta X}\right)C_{pi-1,j}^{n} \\
& + \left(1 - \frac{2ST(\Delta T)D_z}{(\Delta Z)^2} - \frac{2ST(\Delta T)D_x}{(\Delta X)^2} - ST(\Delta T)K_p\right)C_{pi,j}^{n} \\
& + \left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2} + \frac{ST(\Delta T)W}{2\Delta Z}\right)C_{pi,j-1}^{n} + \left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2} - \frac{ST(\Delta T)W}{2\Delta Z}\right)C_{pi,j+1}^{n} \\
& + ST(\Delta T)Q\delta(X - X_r)\delta(Z - Z_r).
\end{aligned} \qquad (3.45)
$$

Thus, the forward time central space method of primary air pollutant concentration becomes

$$
\begin{aligned}
C_{pi,j}^{n+1} = {} & (d_x - A_x)C_{pi+1,j}^{n} + (d_x + A_x)C_{pi-1,j}^{n} + \left(1 - 2d_x - 2d_z - ST(\Delta T)K_p\right)C_{pi,j}^{n} \\
& + (d_z + A_z)C_{i,j-1}^{n} + (d_z - A_z)C_{i,j+1}^{n} + ST(\Delta T)Q\delta(X - X_r)\delta(Z - Z_r),
\end{aligned} \qquad (3.46)
$$

where $A_x = \frac{ST(\Delta T)U}{2\Delta X}, A_z = \frac{ST(\Delta T)W}{2\Delta Z}, d_x = \frac{ST(\Delta T)D_x}{(\Delta X)^2}$, and $d_z = \frac{ST(\Delta T)D_z}{(\Delta Z)^2}$.

### 3.3.3 Numerical experiments and results for primary air pollutant control

The experiment analyzed the actions of air pollution with the volume of sulfur dioxide emission around the industrial zone. We simulate the air pollution control situation in three case. For the first simulation, the industrial factory released continued air pollutants from the chimney without an emission control system. For the second and the third simulations, the factory discharged sulfur dioxide concentration, which is controlled by the National Air Quality Index.

**Simulation 1 :** Air pollution emission without controlled system

In the first simulation, a two-dimensional advection-diffusion equation (2.2) with an interested domain $1000 \times 400\ m^2$ is considered. The wind velocities in the $x$-direction and $z$-direction are assumed to be $0.11$ and $6.9 \times 10^{-3}\ m/s$, respectively. Sulfur dioxide is released at the chimney height $h_{ps} = 75\ m.$ at coordinates $(100, 75)$ $(m, m)$. The released pollutant concentration is $0.75\ s^{-1}$. The diffusion coefficients in the $x$-direction and $z$-direction are $2$ and $0.45\ m^2/s$, respectively. The grid spacing is $\Delta x = \Delta z = 25\ m.$ and the time interval is $72\ s.$ This simulation is solved by using forward time central space (FTCS) in the equation (3.46) with the initial and boundary conditions (3.34)-(3.38). The numerical solution of air pollutant concentration when $58$ minutes and $1$ hour $36$ minutes had passed are shown in Figure 3.14 and Figure 3.15, respectively. The monitoring points are aligned along: $200$, $300$, $400$, and $500\ m.$ at the same height, $50\ m.$ A comparison of concentrations of different distances is presented in Figure 3.16.

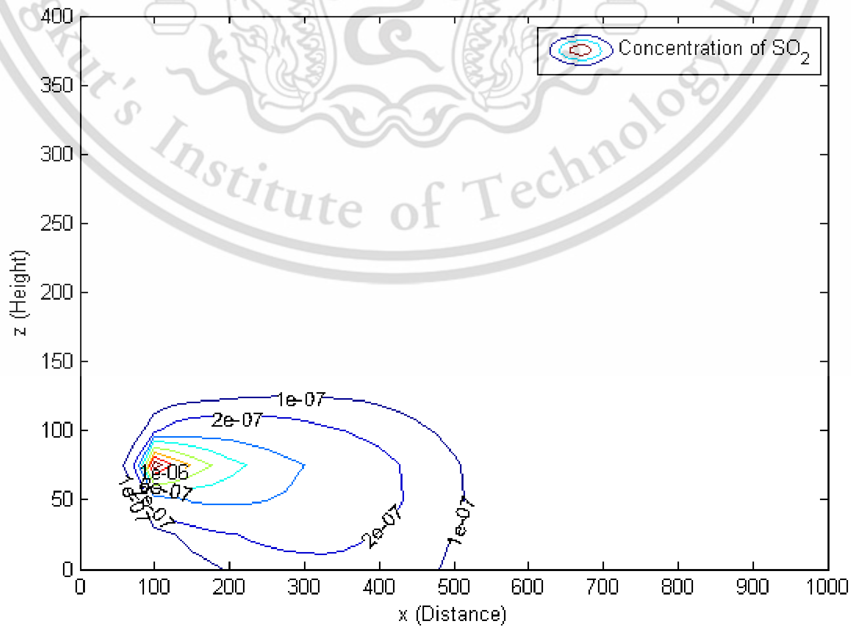



**Figure 3.14** The concentration levels of air pollution after 58 minutes without an emission control system

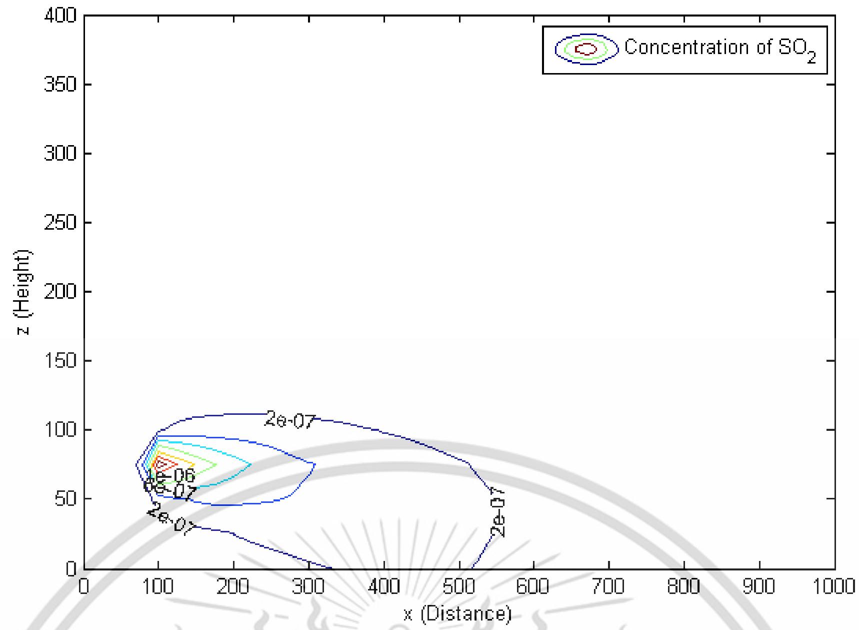



**Figure 3.15** The concentration levels of air pollution after 1 hour 36 minutes without an emission control system

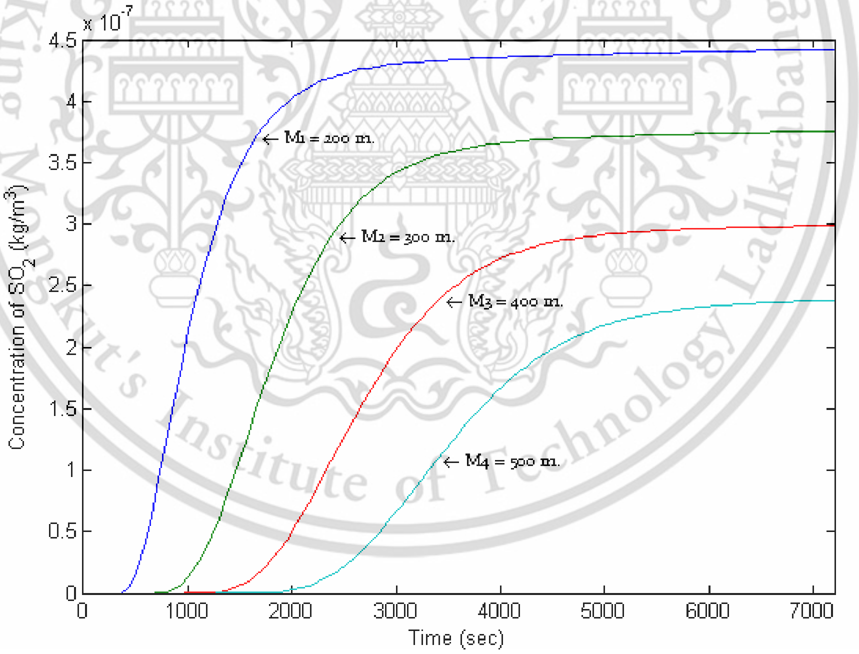



**Figure 3.16** The concentration of air pollution with a different monitoring point at $z = 50$ m. without an emission control system

**Simulation 2 :** Air pollution emission controlled by following the National Air Quality Standard ($3\times10^{-7}\ kg/m^3$)

In the second simulation, a two-dimensional advection-diffusion equation (2.2) with an interested domain $1000\times400\ m^2$ is considered. The wind velocities in the $x$- direction and $z$- direction are assumed to be $0.11$ and $6.9\times10^{-3}\ m/s$, respectively. Sulfur dioxide is released at the chimney height $h_{ps}=75\ m.$ at coordinates $(100,75)\ (m,m)$. The released pollutant concentration is $0.75\ s^{-1}$. The diffusion coefficients in the $x$-direction and $z$-direction are $2$ and $0.45\ m^2/s$, respectively. The grid spacing is $\Delta x=\Delta z=25\ m.$ and the time interval is $72\ s.$ In this simulation, the sulfur dioxide is released by following the United States Environmental Protection Agency (USEPA) air quality standard [27], $3\times10^{-7}\ kg/m^3$. If the approximated pollutant concentration at a monitoring point exceeds the air quality standard, then the chimney is shut down and waits until the concentration goes below $1.5\times10^{-7}\ kg/m^3$. If the pollutant concentration at all monitoring points is below half of the air quality standard, the chimney is opened again. The air pollution emission follows these processes. This example is solved by using forward time central space (FTCS) in equation (3.46) with the initial and boundary conditions (3.34)-(3.38). The results of air pollution emission control are demonstrated as the contour lines of sulfur dioxide concentration in Figure 3.17 and Figure 3.18. The concentration of air pollution at a different distance is shown in Figure 3.19.

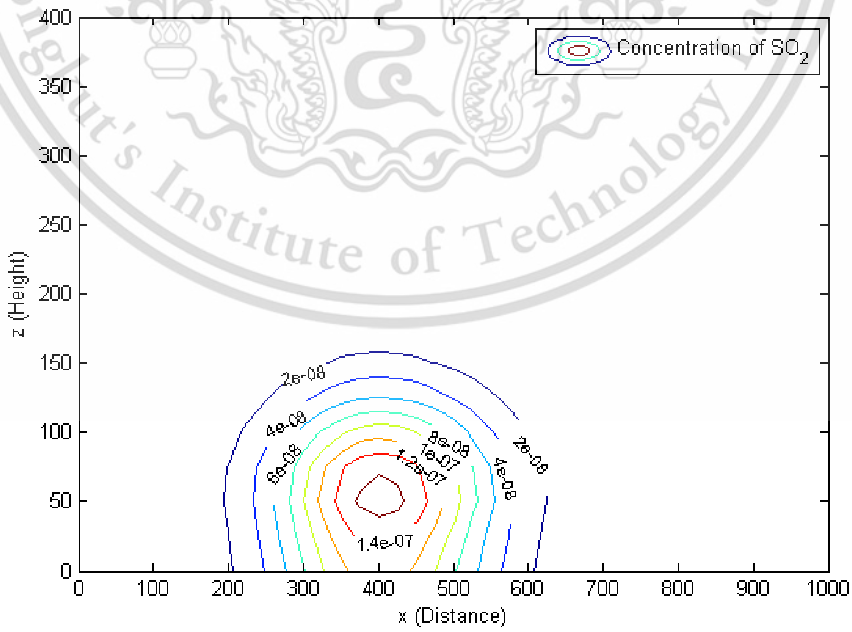



**Figure 3.17** The concentration levels of air pollution after 58 minutes which are controlled by air quality standard ($3\times10^{-7}$ kg/m$^3$)

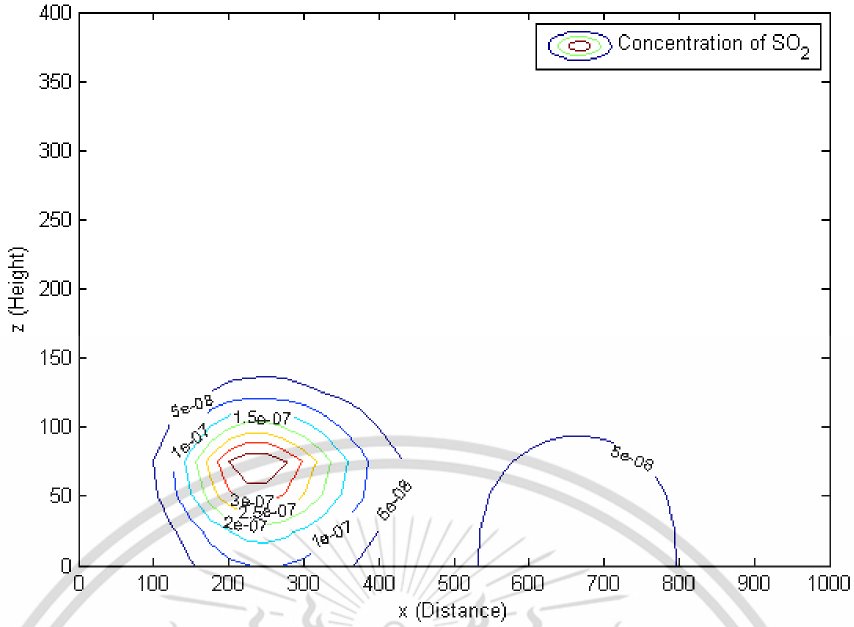



**Figure 3.18** The concentration levels of air pollution after 1 hour 36 minutes which are controlled by air quality standard ($3\times10^{-7}$ kg/m$^3$)

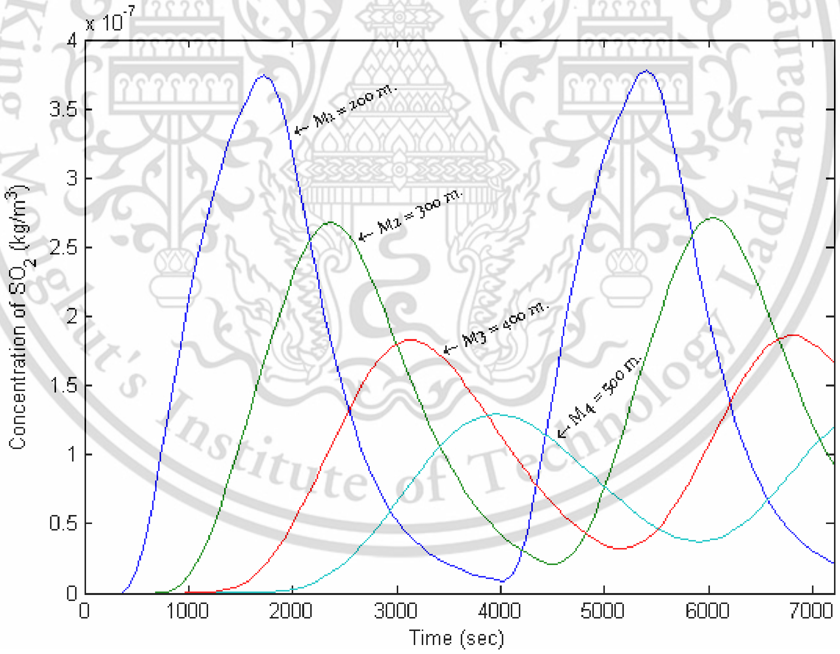



**Figure 3.19** The concentration of air pollution with a different monitoring point at $z = 50$ m. by air quality standard ($3\times10^{-7}$ kg/m$^3$)

**Simulation 3 :** Air pollution emission controlled by following the National Air Quality Standard ($1.5\times10^{-7}\ kg/m^3$)

In the third simulation, a two-dimensional advection-diffusion equation (2.2) with an interested domain $1000\times400\ m^2$ is considered. The wind velocities in the $x$-direction and $z$-direction are assumed to be $0.11$ and $6.9\times10^{-3}\ m/s$, respectively. Sulfur dioxide is released at the chimney height $h_{ps}=75\ m.$ at coordinates $(100,75)\ (m,m)$. The released pollutant concentration is $0.75\ s^{-1}$. The diffusion coefficients in the $x$-direction and $z$-direction are $2$ and $0.45\ m^2/s$, respectively. The grid spacing is $\Delta x=\Delta z=25\ m.$ and the time interval is $72\ s.$ In this simulation, the sulfur dioxide is released by following the USEPA air quality standard, $1.5\times10^{-7}\ kg/m^3$. If the approximated pollutant concentration at a monitoring point exceeds the air quality standard, then the chimney is shut down and waits until the concentration goes below $1.0\times10^{-7}\ kg/m^3$. If the pollutant concentration at all monitoring points is below a third of the air quality standard, the chimney is opened again. The air pollution emission follows these processes. This simulation is solved by using the forward time central space (FTCS) scheme in equation (3.46) with the initial and boundary conditions (3.34)-(3.38). In this emission control case, the concentration of air pollution when 58 minute and 1 hour 36 minutes had passed are shown in Figure 3.20 and Figure 3.21, respectively. The concentration of sulfur dioxide when 2 hours had passed at a different distance is shown in Figure 3.22.

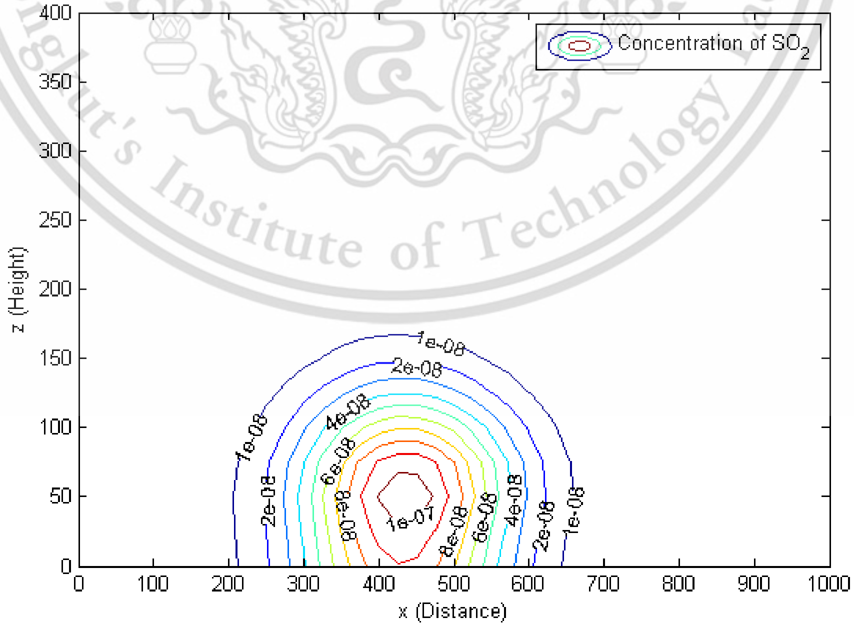



**Figure 3.20** The air pollution concentration levels after 58 minutes which are controlled by air quality standard ($1.5\times10^{-7}$ kg/m$^3$)

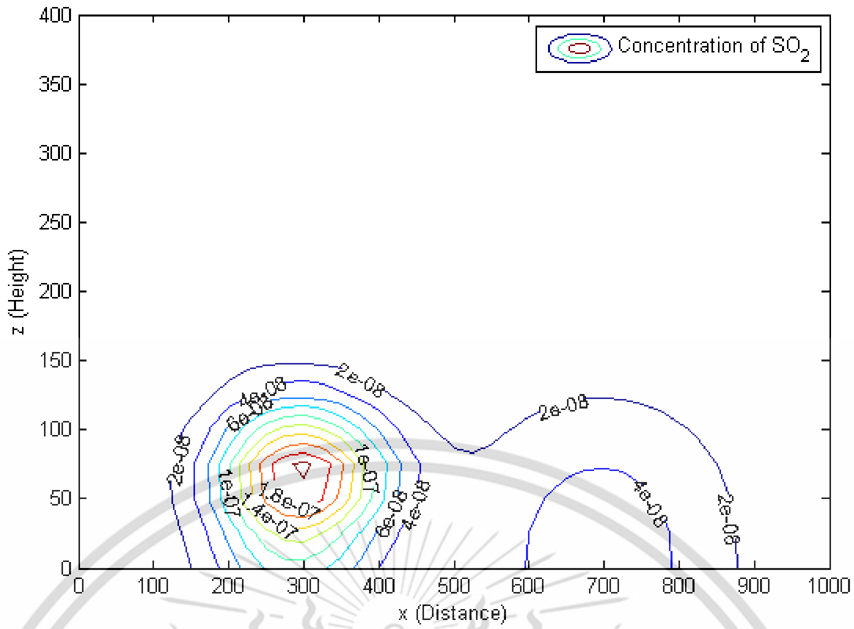



**Figure 3.21** The air pollution concentration levels after 1 hour 36 minutes which are controlled by air quality standard ($1.5\times10^{-7}$ kg/m$^3$)

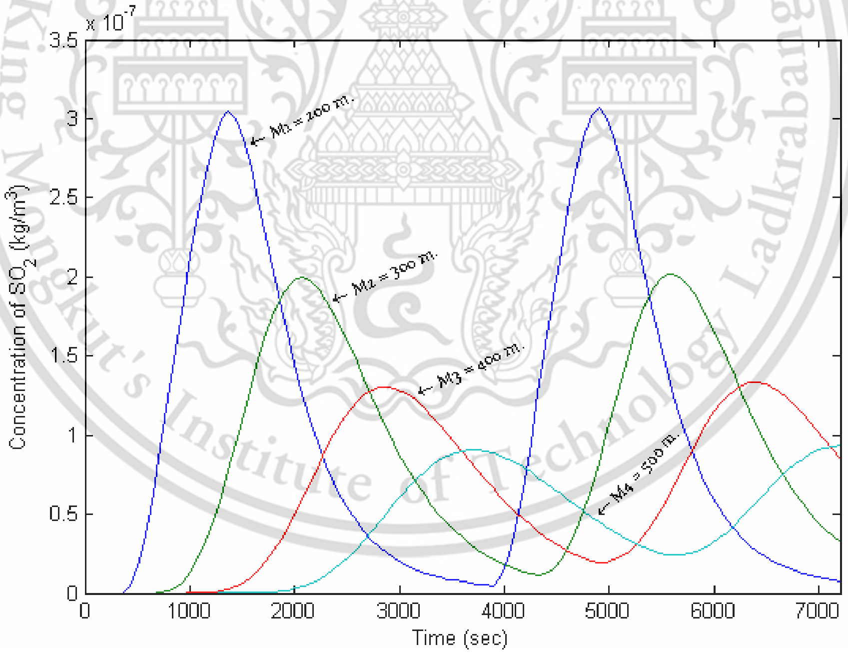



**Figure 3.22** The concentration of air pollution with a different monitoring point at $z=50$ m. by air quality standard ($1.5\times10^{-7}$ kg/m$^3$)

From the above example, the concentration of sulfur dioxide at the height $z = 50\ m.$ and the distance $x = 300\ m.$ (M2) are compared in Figure 3.23.

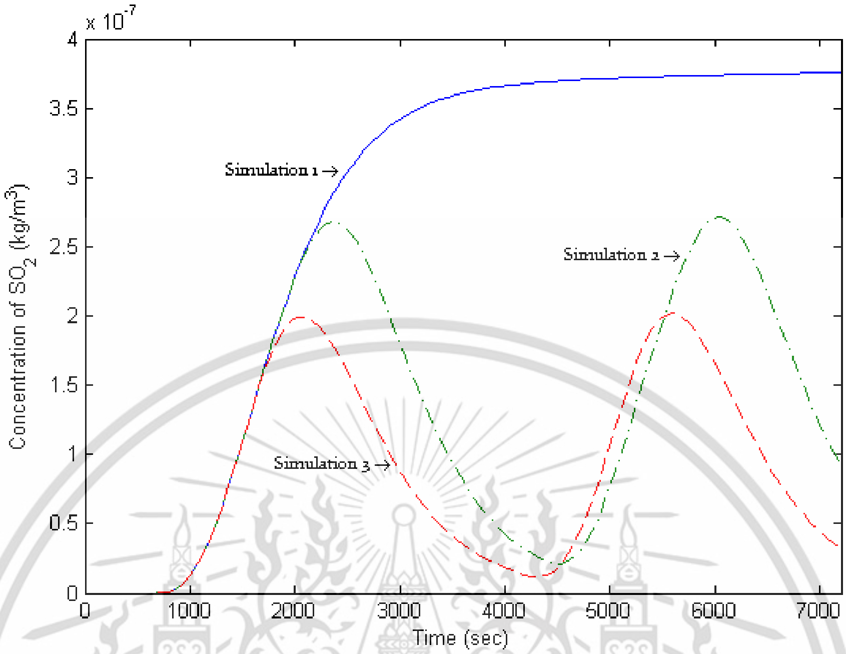



**Figure 3.23** The air pollution concentration at $z = 50$ m. and $x = 300$ m. (M2) in three cases

Chapter 4

# Numerical Simulation to Toxic Smoke Emission Control by Considering Primary and Secondary Pollutant Concentration near an Industrial Zone Using Multiple Air Quality Standards

## 4.1 Non-dimensional form of two-dimensional horizontal averaged atmospheric diffusion equation for secondary air pollutant

For secondary pollutants, we employ the same technique as with primary pollutants. Furthermore, the non-dimensional form of secondary pollutants is presented. From the equation (2.4)

$$\frac{\partial c_s}{\partial t}+u\frac{\partial c_s}{\partial x}+w\frac{\partial c_s}{\partial z}=k_x\frac{\partial^2 c_s}{\partial x^2}+k_z\frac{\partial^2 c_s}{\partial z^2}+V_g k_s c_p.$$

The dimensionless variables are defined by $X=\frac{x}{l_x},\ Z=\frac{z}{l_z},\ T=\frac{t}{t_{\max}}$.

We let $c_{\max}=\max\{c_s(x,z,t):0\le x\le l_x, 0\le z\le l_z, 0\le t\le t_{\max}\}$,

$u_{\max}=\max\{u(x,z,t):0\le x\le l_x, 0\le z\le l_z, 0\le t\le t_{\max}\}$,

$w_{\max}=\max\{w(x,z,t):0\le x\le l_x,\ 0\le z\le l_z, 0\le t\le t_{\max}\}$,

$l=\max\{l_x,l_z\}$ when $l_x=l_z$, and $t_{\max}$ is a stationary time.

We consider the dimensionless variable of secondary pollutant concentration, which is $C_s=c_s/c_{\max}$, and then we substitute the dimensionless variables into the above equation. So, we get

$$\frac{\partial(C_s c_{\max})}{\partial(T t_{\max})}+u\frac{\partial(C_s c_{\max})}{\partial(Xl)}+w\frac{\partial(C_s c_{\max})}{\partial(Zl)}$$
$$=k_x\frac{\partial^2(C_s c_{\max})}{\partial(Xl)^2}+k_z\frac{\partial^2(C_s c_{\max})}{\partial(Zl)^2}+V_g k_s\left(C_p c_{\max}\right),$$

$$\left(\frac{c_{\max}}{t_{\max}}\right)\frac{\partial C_s}{\partial T}+\left(\frac{c_{\max}u}{l}\right)\frac{\partial C_s}{\partial X}+\left(\frac{c_{\max}w}{l}\right)\frac{\partial C_s}{\partial Z}$$
$$=\left(\frac{c_{\max}k_x}{l^2}\right)\frac{\partial^2 C_s}{\partial X^2}+\left(\frac{c_{\max}k_z}{l^2}\right)\frac{\partial^2 C_s}{\partial Z^2}+V_g k_s c_{\max} C_p. \tag{4.1}$$

The equation (4.1) is multiplied by $\dfrac{l}{u_{\max}c_{\max}}$, then we have

$$\left(\frac{l}{u_{\max}t_{\max}}\right)\frac{\partial C_s}{\partial T}+\left(\frac{u}{u_{\max}}\right)\frac{\partial C_s}{\partial X}+\left(\frac{w}{u_{\max}}\right)\frac{\partial C_s}{\partial Z}=\left(\frac{k_x}{u_{\max}l}\right)\frac{\partial^2 C_s}{\partial X^2}+\left(\frac{k_z}{u_{\max}l}\right)\frac{\partial^2 C_s}{\partial Z^2}+V_g\left(\frac{k_s l}{u_{\max}}\right)C_p,$$

$$\frac{1}{ST}\frac{\partial C_s}{\partial T}+U\frac{\partial C_s}{\partial X}+W\frac{\partial C_s}{\partial Z}=D_x\frac{\partial^2 C_s}{\partial X^2}+D_z\frac{\partial^2 C_s}{\partial Z^2}+V_g K_s C_p.$$

Therefore, the non-dimensional of secondary pollutant equation can be rearranged to give

$$\frac{1}{ST}\frac{\partial C_s}{\partial T}+U\frac{\partial C_s}{\partial X}+W\frac{\partial C_s}{\partial Z}=D_x\frac{\partial^2 C_s}{\partial X^2}+D_z\frac{\partial^2 C_s}{\partial Z^2}+S_s, \tag{4.2}$$

where $ST=\dfrac{t_{\max}u_{\max}}{l}$, $U=\dfrac{u}{u_{\max}}$, $W=\dfrac{w}{u_{\max}}$,

$D_x=\dfrac{k_x}{u_{\max}l}$, $D_z=\dfrac{k_z}{u_{\max}l}$, and $K_s=\dfrac{k_s l}{u_{\max}}$.

The non-dimensional of secondary pollutant initial condition assumed that

$$C_s(X,Z,0)=0, \tag{4.3}$$

for all $0<X<1$ and $0<Z<1$. The dimensionless form of boundary conditions assumed that

$$\frac{\partial C_s}{\partial X}(0,Z,T)=0, \tag{4.4}$$

$$\frac{\partial C_s}{\partial X}(1,Z,T)=0, \tag{4.5}$$

$$\frac{\partial C_s}{\partial Z}(X,1,T)=0, \tag{4.6}$$

and $$\frac{\partial C_s}{\partial Z}(X,0,T)=g(X,T), \tag{4.7}$$

for all $T>0$ where $g(X,T)$ is a constant function.

The conduct of air pollution dispersion in the atmosphere is studied by considering the atmospheric diffusion equation. The primary pollutant as sulfur dioxide ($SO_2$), and the secondary pollutants as sulfur trioxide ($SO_3$) and sulfuric acid ($H_2SO_4$), are presented. In this research, constant wind velocities and diffusion coefficients are proposed, and approximate solutions, that are the concentrations of pollutants, are solved by using the finite difference method. The purpose of this study is to examine the concentrations of sulfur dioxide, sulfur trioxide, and sulfuric acid in a multiple air pollution emission control problem. The air quality standard is also used to compare the results of the experiments. Figure 4.1 shows a model of an air pollution emission control problem when the pollutants are emitted from a chimney of a factory. This research was designed to analyze the behavior and effect of primary and secondary pollutant dispersion near an industrial zone. Air quality monitoring equipment can check three pollutants at one monitoring point. The four monitoring points are set far from the source. The monitoring points are called monitoring point no. 1 (M1), monitoring point no. 2 (M2), monitoring point no. 3 (M3), and monitoring point no. 4 (M4), at distances of 200, 300, 400, and 500 $m$, respectively.

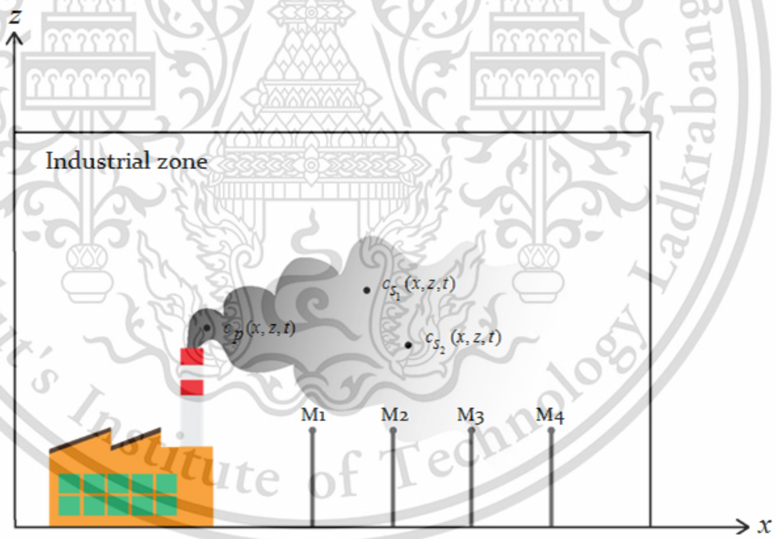



**Figure 4.1** Model of primary and secondary air pollutants emission control problem

## 4.2 Initial and boundary conditions setting techniques for primary and secondary pollutant control

### 4.2.1 Primary pollutant concentration measurement model

The primary pollutant means that the air pollutants are emitted directly from the source. The numerical approximate solution considers the concentration of pollutant that is sulfur dioxide. The chemical formula is $SO_2$.

#### 4.2.1.1 Sulfur dioxide concentration measurement model

Sulfur dioxide is a major gas that is released from the chimney of the factory into the atmosphere. This occurs in the combustion of fuel. In this model, $C_p$ represents the concentration of sulfur dioxide. Therefore, the primary pollutant equation is

$$\frac{1}{ST}\frac{\partial C_p}{\partial T}+U\frac{\partial C_p}{\partial X}+W\frac{\partial C_p}{\partial Z}=D_x\frac{\partial^2 C_p}{\partial X^2}+D_z\frac{\partial^2 C_p}{\partial Z^2}+S_p+R_p. \tag{4.8}$$

The cold start assumption is used for the initial condition. It follows

$$C_p(X,Z,0)=0, \tag{4.9}$$

for all $0<X<1$ and $0<Z<1$. The boundary conditions of sulfur dioxide assumed that

$$\frac{\partial C_p}{\partial X}(0,Z,T)=0, \tag{4.10}$$

$$\frac{\partial C_p}{\partial X}(1,Z,T)=0, \tag{4.11}$$

$$\frac{\partial C_p}{\partial Z}(X,1,T)=0, \tag{4.12}$$

and

$$\frac{\partial C_p}{\partial Z}(X,0,T)=V_{dp}C_p, \tag{4.13}$$

for all $T>0$ where $V_{dp}$ is the dry deposition velocity of the primary pollutant. Sulfur dioxide deposition velocity can be referred to a diffusivity in $z$-direction which is assumed to be an irreversible process.

### 4.2.2 Secondary pollutant concentration measurement models

The secondary pollutant means that the air pollutants occur in the atmosphere by a chemical interaction. There are two considered pollutants in this research. The sulfur trioxide ($SO_3$) is formed from reaction between sulfur dioxide and oxygen, and the sulfuric acid ($H_2SO_4$) is converted from sulfur dioxide, water, and oxygen. Thus, the following chemical reaction equations are

$$\mathrm{SO_2}+\frac{1}{2}\mathrm{O_2}\longrightarrow \mathrm{SO_3}, \tag{4.14}$$

$$\mathrm{SO_2}+\mathrm{H_2O}+\frac{1}{2}\mathrm{O_2}\longrightarrow \mathrm{H_2SO_4}. \tag{4.15}$$

#### 4.2.2.1 Sulfur trioxide concentration measurement model

Sulfur trioxide is a major pollutant, which can be also an agent in other pollutants. The concentration of sulfur trioxide is represented by $C_{s_1}$. Thus, the dispersion of sulfur trioxide is considered by the following equation

$$\frac{1}{ST}\frac{\partial C_{s_1}}{\partial T}+U\frac{\partial C_{s_1}}{\partial X}+W\frac{\partial C_{s_1}}{\partial Z}=D_x\frac{\partial^2 C_{s_1}}{\partial X^2}+D_z\frac{\partial^2 C_{s_1}}{\partial Z^2}+S_{s_1}, \tag{4.16}$$

where $S_{s_1}=V_g K_{s_1} C_p$. when $K_{s_1}$ is the first order chemical interaction rate of sulfur trioxide pollutant. The initial condition is assumed under the cold start assumption. That is

$$C_{s_1}(X,Z,0)=0, \tag{4.17}$$

for all $0<X<1$ and $0<Z<1$. Boundary conditions of sulfur trioxide supposed that

$$\frac{\partial C_{s_1}}{\partial X}(0,Z,T)=0, \tag{4.18}$$

$$\frac{\partial C_{s_1}}{\partial X}(1,Z,T)=0, \tag{4.19}$$

$$\frac{\partial C_{s_1}}{\partial Z}(X,1,T)=0, \tag{4.20}$$

and
$$\frac{\partial C_{s_1}}{\partial Z}(X,0,T)=V_{ds_1}C_{s_1}, \tag{4.21}$$

for all $T>0$ where $V_{ds_1}$ is the dry deposition velocity of the sulfur trioxide air pollutant.

#### 4.2.2.2 Sulfuric acid concentration measurement model

In this research, sulfuric acid, which is one of the most significant chemical compounds, is the product of sulfur dioxide, water, and oxygen. We consider that $C_{s_2}$ is the concentration of sulfuric acid. So, the secondary pollutant of sulfuric acid equation is

$$\frac{1}{ST}\frac{\partial C_{s_2}}{\partial T}+U\frac{\partial C_{s_2}}{\partial X}+W\frac{\partial C_{s_2}}{\partial Z}=D_x\frac{\partial^2 C_{s_2}}{\partial X^2}+D_z\frac{\partial^2 C_{s_2}}{\partial Z^2}+S_{s_2}, \tag{4.22}$$

where $S_{s_2}=V_g K_{s_2} C_p$ when $K_{s_2}$ is the first order chemical interaction rate of sulfuric acid pollutant.

The initial condition is similar to the condition of sulfur trioxide. That is

$$C_{s_2}(X,Z,0)=0, \tag{4.23}$$

for all $0<X<1$ and $0<Z<1$. Then, boundary conditions of sulfuric acid assumed that

$$\frac{\partial C_{s_2}}{\partial X}(0,Z,T)=0, \tag{4.24}$$

$$\frac{\partial C_{s_2}}{\partial X}(1,Z,T)=0, \tag{4.25}$$

$$\frac{\partial C_{s_2}}{\partial Z}(X,1,T)=0, \tag{4.26}$$

and $$\frac{\partial C_{s_2}}{\partial Z}(X,0,T)=V_{ds_2}C_{s_2}, \tag{4.27}$$

for all $T>0$ where $V_{ds_2}$ is the dry deposition velocity of sulfuric acid pollutant.

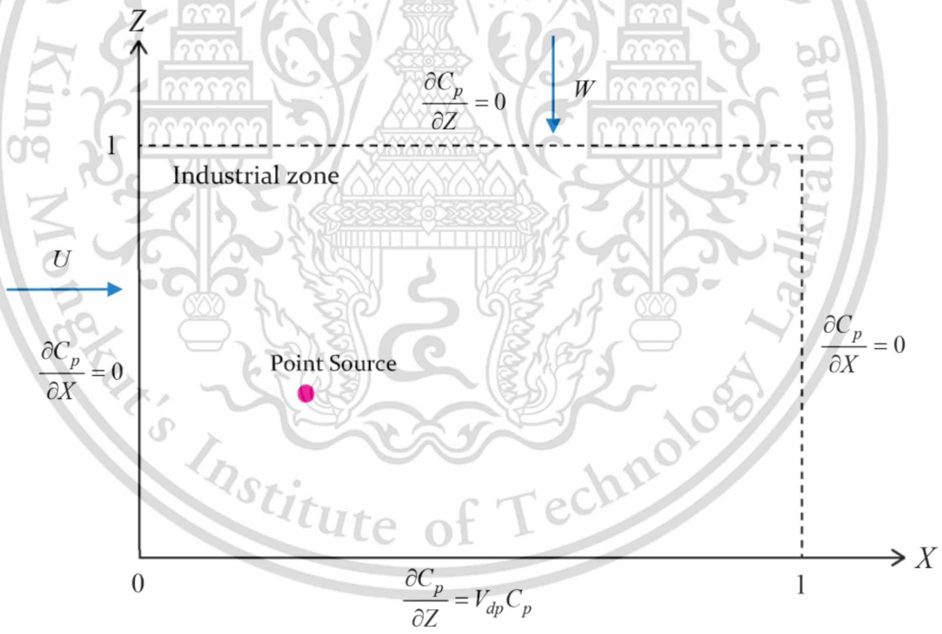



**Figure 4.2** Domain of sulfur dioxide approximate solutions

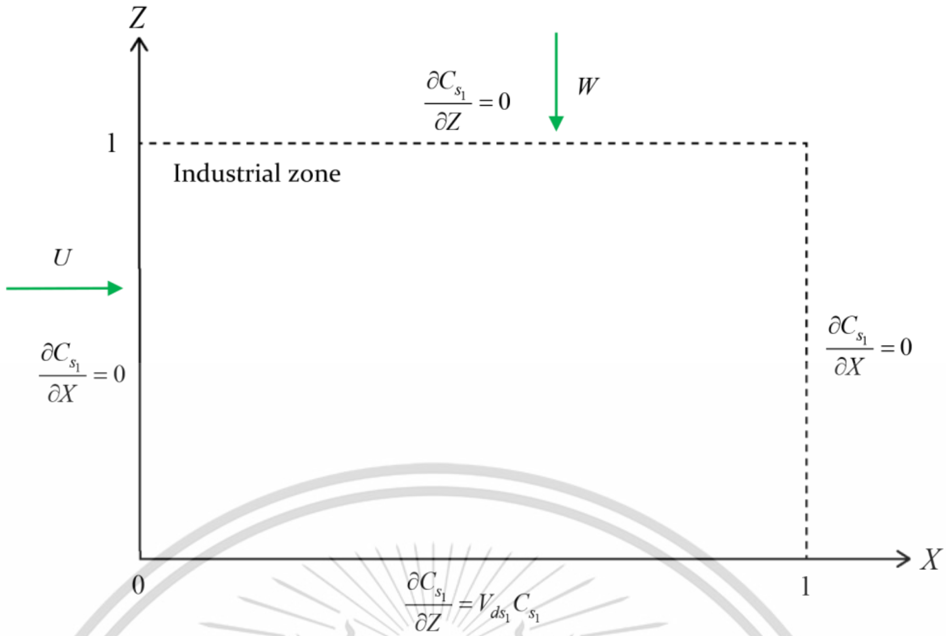



**Figure 4.3** Domain of sulfur trioxide approximate solutions

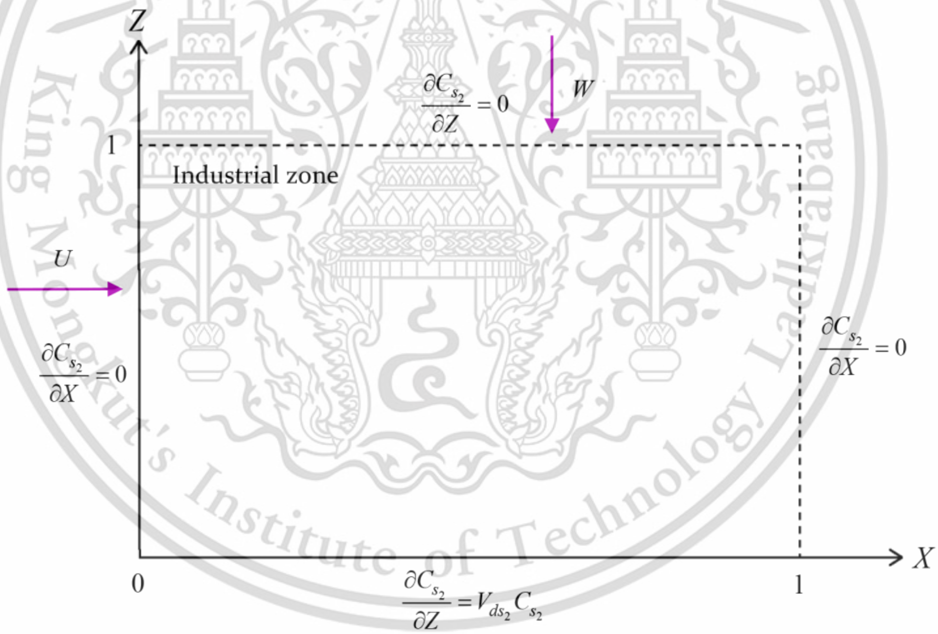



**Figure 4.4** Domain of sulfuric acid approximate solutions

In Figure 4.2, the domain of sulfur dioxide approximate solutions for the numerical experiment is shown. We suppose that the height of the point source is $h_{ps}$ ($m$). The concentration of sulfur dioxide is discharged continuously from the chimney. Figure 4.3 and Figure 4.4 show the domain of sulfur trioxide and sulfuric acid approximate solutions for the numerical experiment. The concentrations of sulfur trioxide and sulfuric acid are converted from the sulfur dioxide pollutant in the system. We assume that the wind and the diffusion coefficient are stable in the $x$-direction and $z$-direction. The sulfur dioxide, sulfur trioxide, and sulfuric acid are absorbed by the removal mechanism from the atmosphere at ground level.

## 4.3 Numerical techniques for primary and secondary pollutant control

The equipment that is utilized to estimate all the solutions is the finite difference method.

### 4.3.1 Explicit difference techniques for primary pollutant model

#### 4.3.1.1 Forward time central space method for sulfur dioxide measurement

We use the forward time central space (FTCS) finite difference scheme for the sulfur dioxide pollutant in a non-dimensional equation (4.8). In the transient term, the method is derived by using the forward difference,

$$\frac{\partial C_p}{\partial T} = \frac{C_{pi,j}^{n+1} - C_{pi,j}^{n}}{\Delta T}. \tag{4.28}$$

The advection and diffusion terms are considered by using the centered difference approximation. We have

$$\frac{\partial C_p}{\partial X} = \frac{C_{pi+1,j}^{n} - C_{pi-1,j}^{n}}{2\Delta X}, \tag{4.29}$$

$$\frac{\partial C_p}{\partial Z} = \frac{C_{pi,j+1}^{n} - C_{pi,j-1}^{n}}{2\Delta Z}, \tag{4.30}$$

$$\frac{\partial^2 C_p}{\partial X^2} = \frac{C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}}{(\Delta X)^2}, \tag{4.31}$$

and

$$\frac{\partial^2 C_p}{\partial Z^2} = \frac{C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}}{(\Delta Z)^2}. \tag{4.32}$$

We substitute the equations (4.28)-(4.32) into equation (4.8). Then, it becomes

$$\begin{aligned}\frac{1}{ST}\left(\frac{C_{pi,j}^{n+1} - C_{pi,j}^{n}}{\Delta T}\right) + U\left(\frac{C_{pi+1,j}^{n} - C_{pi-1,j}^{n}}{2\Delta X}\right) + W\left(\frac{C_{pi,j+1}^{n} - C_{pi,j-1}^{n}}{2\Delta Z}\right) \\ = D_x\left(\frac{C_{pi+1,j}^{n} - 2C_{pi,j}^{n} + C_{pi-1,j}^{n}}{(\Delta X)^2}\right) + D_z\left(\frac{C_{pi,j+1}^{n} - 2C_{pi,j}^{n} + C_{pi,j-1}^{n}}{(\Delta Z)^2}\right) \\ + Q\delta(X - X_r)\delta(Z - Z_r) - K_p C_{pi,j}^{n},\end{aligned}$$

where $S_p = Q\delta(X - X_r)\delta(Z - Z_r)$ and $R_p = -K_p C_{pi,j}^{n}$.

From the above equation, we have

$$\frac{1}{ST}\left(\frac{C_{pi,j}^{n+1}-C_{pi,j}^{n}}{\Delta T}\right)=D_x\left(\frac{C_{pi+1,j}^{n}-2C_{pi,j}^{n}+C_{pi-1,j}^{n}}{(\Delta X)^2}\right)+D_z\left(\frac{C_{pi,j+1}^{n}-2C_{pi,j}^{n}+C_{pi,j-1}^{n}}{(\Delta Z)^2}\right)$$
$$-U\left(\frac{C_{pi+1,j}^{n}-C_{pi-1,j}^{n}}{2\Delta X}\right)-W\left(\frac{C_{pi,j+1}^{n}-C_{pi,j-1}^{n}}{2\Delta Z}\right)$$
$$-K_pC_{pi,j}^{n}+Q\delta(X-X_r)\delta(Z-Z_r),$$

$$C_{pi,j}^{n+1}=\frac{ST(\Delta T)D_x}{(\Delta X)^2}\left(C_{pi+1,j}^{n}-2C_{pi,j}^{n}+C_{pi-1,j}^{n}\right)$$
$$+\frac{ST(\Delta T)D_z}{(\Delta Z)^2}\left(C_{pi,j+1}^{n}-2C_{pi,j}^{n}+C_{pi,j-1}^{n}\right)-\frac{ST(\Delta T)U}{2\Delta X}\left(C_{pi+1,j}^{n}-C_{pi-1,j}^{n}\right)$$
$$-\frac{ST(\Delta T)W}{2\Delta Z}\left(C_{pi,j+1}^{n}-C_{pi,j-1}^{n}\right)-ST(\Delta T)K_pC_{pi,j}^{n}+C_{pi,j}^{n}$$
$$+ST(\Delta T)Q\delta(X-X_r)\delta(Z-Z_r),$$

$$C_{pi,j}^{n+1}=\frac{ST(\Delta T)D_x}{(\Delta X)^2}C_{pi+1,j}^{n}-\frac{2ST(\Delta T)D_x}{(\Delta X)^2}C_{pi,j}^{n}+\frac{ST(\Delta T)D_x}{(\Delta X)^2}C_{pi-1,j}^{n}$$
$$+\frac{ST(\Delta T)D_z}{(\Delta Z)^2}C_{pi,j+1}^{n}-\frac{2ST(\Delta T)D_z}{(\Delta Z)^2}C_{pi,j}^{n}+\frac{ST(\Delta T)D_z}{(\Delta Z)^2}C_{pi,j-1}^{n}$$
$$-\frac{ST(\Delta T)U}{2\Delta X}C_{pi+1,j}^{n}+\frac{ST(\Delta T)U}{2\Delta X}C_{pi-1,j}^{n}-\frac{ST(\Delta T)W}{2\Delta Z}C_{pi,j+1}^{n}$$
$$+\frac{ST(\Delta T)W}{2\Delta Z}C_{pi,j-1}^{n}-ST(\Delta T)K_pC_{pi,j}^{n}+C_{pi,j}^{n}$$
$$+ST(\Delta T)Q\delta(X-X_r)\delta(Z-Z_r),$$

$$C_{pi,j}^{n+1}=\left(\frac{ST(\Delta T)D_x}{(\Delta X)^2}-\frac{ST(\Delta T)U}{2\Delta X}\right)C_{pi+1,j}^{n}+\left(\frac{ST(\Delta T)D_x}{(\Delta X)^2}+\frac{ST(\Delta T)U}{2\Delta X}\right)C_{pi-1,j}^{n}$$
$$+\left(1-\frac{2ST(\Delta T)D_z}{(\Delta Z)^2}-\frac{2ST(\Delta T)D_x}{(\Delta X)^2}-ST(\Delta T)K_p\right)C_{pi,j}^{n}$$
$$+\left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2}+\frac{ST(\Delta T)W}{2\Delta Z}\right)C_{pi,j-1}^{n}+\left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2}-\frac{ST(\Delta T)W}{2\Delta Z}\right)C_{pi,j+1}^{n}$$
$$+ST(\Delta T)Q\delta(X-X_r)\delta(Z-Z_r). \quad (4.33)$$

Thus, the forward time central space method of sulfur dioxide concentration is

$$C_{pi,j}^{n+1}=(d_x-A_x)C_{pi+1,j}^{n}+(d_x+A_x)C_{pi-1,j}^{n}+\left(1-2d_x-2d_z-ST(\Delta T)K_p\right)C_{pi,j}^{n}$$
$$+(d_z+A_z)C_{pi,j-1}^{n}+(d_z-A_z)C_{pi,j+1}^{n}+ST(\Delta T)Q\delta(X-X_r)\delta(Z-Z_r), \quad (4.34)$$

where $A_x = \dfrac{St(\Delta T)U}{2\Delta X}$, $A_z = \dfrac{St(\Delta T)W}{2\Delta Z}$, $d_x = \dfrac{St(\Delta T)D_x}{(\Delta X)^2}$, and $d_z = \dfrac{St(\Delta T)D_z}{(\Delta Z)^2}$.

### 4.3.2 Explicit difference techniques for secondary pollutant model

For the secondary pollutants, we use the finite difference method. In addition, the forward time central space (FTCS) schemes of sulfur trioxide and sulfuric acid are similar to the method of the previous pollutant. The finite difference expressions are proposed.

#### 4.3.2.1 Forward time central space method for sulfur trioxide measurement

The transient term of sulfur trioxide is substituted by using the forward difference

$$\frac{\partial C_{s_1}}{\partial T} = \frac{C^{n+1}_{s_1 i,j} - C^{n}_{s_1 i,j}}{\Delta T}. \tag{4.35}$$

Then, we use the centered difference for the advection and diffusion terms in the $x$-direction and $z$-direction

$$\frac{\partial C_{s_1}}{\partial X} = \frac{C^{n}_{s_1 i+1,j} - C^{n}_{s_1 i-1,j}}{2\Delta X}, \tag{4.36}$$

$$\frac{\partial C_{s_1}}{\partial Z} = \frac{C^{n}_{s_1 i,j+1} - C^{n}_{s_1 i,j-1}}{2\Delta Z}, \tag{4.37}$$

$$\frac{\partial^2 C_{s_1}}{\partial X^2} = \frac{C^{n}_{s_1 i+1,j} - 2C^{n}_{s_1 i,j} + C^{n}_{s_1 i-1,j}}{(\Delta X)^2}, \tag{4.38}$$

and

$$\frac{\partial^2 C_{s_1}}{\partial Z^2} = \frac{C^{n}_{s_1 i,j+1} - 2C^{n}_{s_1 i,j} + C^{n}_{s_1 i,j-1}}{(\Delta Z)^2}. \tag{4.39}$$

Equation (4.16) is substituted by the equations (4.35)-(4.39). We obtain

$$\begin{aligned}\frac{1}{ST}\left(\frac{C^{n+1}_{s_1 i,j} - C^{n}_{s_1 i,j}}{\Delta T}\right) + U\left(\frac{C^{n}_{s_1 i+1,j} - C^{n}_{s_1 i-1,j}}{2\Delta X}\right) + W\left(\frac{C^{n}_{s_1 i,j+1} - C^{n}_{s_1 i,j-1}}{2\Delta Z}\right) \\ = D_x\left(\frac{C^{n}_{s_1 i+1,j} - 2C^{n}_{s_1 i,j} + C^{n}_{s_1 i-1,j}}{(\Delta X)^2}\right) + D_z\left(\frac{C^{n}_{s_1 i,j+1} - 2C^{n}_{s_1 i,j} + C^{n}_{s_1 i,j-1}}{(\Delta Z)^2}\right) \\ + V_g K_{s_1} C^{n}_{p i,j},\end{aligned}$$

From the above equation, we get

$$\frac{1}{ST}\left(\frac{C^{n+1}_{s_1 i,j}-C^{n}_{s_1 i,j}}{\Delta T}\right)=D_x\left(\frac{C^{n}_{s_1 i+1,j}-2C^{n}_{s_1 i,j}+C^{n}_{s_1 i-1,j}}{(\Delta X)^2}\right)+D_z\left(\frac{C^{n}_{s_1 i,j+1}-2C^{n}_{s_1 i,j}+C^{n}_{s_1 i,j-1}}{(\Delta Z)^2}\right)$$
$$-U\left(\frac{C^{n}_{s_1 i+1,j}-C^{n}_{s_1 i-1,j}}{2\Delta X}\right)-W\left(\frac{C^{n}_{s_1 i,j+1}-C^{n}_{s_1 i,j-1}}{2\Delta Z}\right)+V_g K_{s_1} C^{n}_{pi,j},$$

$$C^{n+1}_{s_1 i,j}=\frac{ST(\Delta T)D_x}{(\Delta X)^2}\left(C^{n}_{s_1 i+1,j}-2C^{n}_{s_1 i,j}+C^{n}_{s_1 i-1,j}\right)+\frac{ST(\Delta T)D_z}{(\Delta Z)^2}\left(C^{n}_{s_1 i,j+1}-2C^{n}_{s_1 i,j}+C^{n}_{s_1 i,j-1}\right)$$
$$-\frac{ST(\Delta T)U}{2\Delta X}\left(C^{n}_{s_1 i+1,j}-C^{n}_{s_1 i-1,j}\right)-\frac{ST(\Delta T)W}{2\Delta Z}\left(C^{n}_{s_1 i,j+1}-C^{n}_{s_1 i,j-1}\right)$$
$$+C^{n}_{s_1 i,j}+ST(\Delta T)V_g K_{s_1} C^{n}_{pi,j},$$

$$C^{n+1}_{s_1 i,j}=\frac{ST(\Delta T)D_x}{(\Delta X)^2}C^{n}_{s_1 i+1,j}-\frac{2ST(\Delta T)D_x}{(\Delta X)^2}C^{n}_{s_1 i,j}+\frac{ST(\Delta T)D_x}{(\Delta X)^2}C^{n}_{s_1 i-1,j}$$
$$+\frac{ST(\Delta T)D_z}{(\Delta Z)^2}C^{n}_{s_1 i,j+1}-\frac{2ST(\Delta T)D_z}{(\Delta Z)^2}C^{n}_{s_1 i,j}+\frac{ST(\Delta T)D_z}{(\Delta Z)^2}C^{n}_{s_1 i,j-1}$$
$$-\frac{ST(\Delta T)U}{2\Delta X}C^{n}_{s_1 i+1,j}+\frac{ST(\Delta T)U}{2\Delta X}C^{n}_{s_1 i-1,j}-\frac{ST(\Delta T)W}{2\Delta Z}C^{n}_{s_1 i,j+1}$$
$$+\frac{ST(\Delta T)W}{2\Delta Z}C^{n}_{s_1 i,j-1}+C^{n}_{s_1 i,j}+ST(\Delta T)V_g K_{s_1} C^{n}_{pi,j},$$

$$C^{n+1}_{s_1 i,j}=\left(\frac{ST(\Delta T)D_x}{(\Delta X)^2}-\frac{ST(\Delta T)U}{2\Delta X}\right)C^{n}_{s_1 i+1,j}+\left(\frac{ST(\Delta T)D_x}{(\Delta X)^2}+\frac{ST(\Delta T)U}{2\Delta X}\right)C^{n}_{s_1 i-1,j}$$
$$+\left(1-\frac{2ST(\Delta T)D_z}{(\Delta Z)^2}-\frac{2ST(\Delta T)D_x}{(\Delta X)^2}\right)C^{n}_{s_1 i,j}$$
$$+\left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2}+\frac{St(\Delta T)W}{2\Delta Z}\right)C^{n}_{s_1 i,j-1}+\left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2}-\frac{ST(\Delta T)W}{2\Delta Z}\right)C^{n}_{s_1 i,j+1}$$
$$+ST(\Delta T)V_g K_{s_1} C^{n}_{pi,j}. \tag{4.40}$$

Thus, the forward time central space scheme of the sulfur trioxide concentration becomes

$$C^{n+1}_{s_1 i,j}=\left(d_x-A_x\right)C^{n}_{s_1 i+1,j}+\left(d_x+A_x\right)C^{n}_{s_1 i-1,j}+\left(1-2d_x-2d_z\right)C^{n}_{s_1 i,j}$$
$$+\left(d_z+A_z\right)C^{n}_{s_1 i,j-1}+\left(d_z-A_z\right)C^{n}_{s_1 i,j+1}+ST(\Delta T)V_g K_{s_1} C^{n}_{pi,j}. \tag{4.41}$$

**4.3.2.2 Forward time central space method for sulfuric acid measurement**

Similarly, the transient term of sulfuric acid is represented as

$$\frac{\partial C_{s_2}}{\partial T} = \frac{C^{n+1}_{s_2 i,j} - C^{n}_{s_2 i,j}}{\Delta T}. \tag{4.42}$$

The advection and diffusion terms in the $x$-direction and $z$-direction are as follows

$$\frac{\partial C_{s_2}}{\partial X} = \frac{C^{n}_{s_2 i+1,j} - C^{n}_{s_2 i-1,j}}{2\Delta X}, \tag{4.43}$$

$$\frac{\partial C_{s_2}}{\partial Z} = \frac{C^{n}_{s_2 i,j+1} - C^{n}_{s_2 i,j-1}}{2\Delta Z}, \tag{4.44}$$

$$\frac{\partial^2 C_{s_2}}{\partial X^2} = \frac{C^{n}_{s_2 i+1,j} - 2C^{n}_{s_2 i,j} + C^{n}_{s_2 i-1,j}}{(\Delta X)^2}, \tag{4.45}$$

and

$$\frac{\partial^2 C_{s_2}}{\partial Z^2} = \frac{C^{n}_{s_2 i,j+1} - 2C^{n}_{s_2 i,j} + C^{n}_{s_2 i,j-1}}{(\Delta Z)^2}, \tag{4.46}$$

respectively. Equation (4.22) Becomes

$$\frac{1}{ST}\left(\frac{C^{n+1}_{s_2 i,j} - C^{n}_{s_2 i,j}}{\Delta T}\right) + U\left(\frac{C^{n}_{s_2 i+1,j} - C^{n}_{s_2 i-1,j}}{2\Delta X}\right) + W\left(\frac{C^{n}_{s_2 i,j+1} - C^{n}_{s_2 i,j-1}}{2\Delta Z}\right)$$
$$= D_x\left(\frac{C^{n}_{s_2 i+1,j} - 2C^{n}_{s_2 i,j} + C^{n}_{s_2 i-1,j}}{(\Delta X)^2}\right) + D_z\left(\frac{C^{n}_{s_2 i,j+1} - 2C^{n}_{s_2 i,j} + C^{n}_{s_2 i,j-1}}{(\Delta Z)^2}\right)$$
$$+ V_g K_{s_2} C^{n}_{pi,j},$$

$$\frac{1}{ST}\left(\frac{C^{n+1}_{s_2 i,j} - C^{n}_{s_2 i,j}}{\Delta T}\right) = D_x\left(\frac{C^{n}_{s_2 i+1,j} - 2C^{n}_{s_2 i,j} + C^{n}_{s_2 i-1,j}}{(\Delta X)^2}\right) + D_z\left(\frac{C^{n}_{s_2 i,j+1} - 2C^{n}_{s_2 i,j} + C^{n}_{s_2 i,j-1}}{(\Delta Z)^2}\right)$$
$$- U\left(\frac{C^{n}_{s_2 i+1,j} - C^{n}_{s_2 i-1,j}}{2\Delta X}\right) - W\left(\frac{C^{n}_{s_2 i,j+1} - C^{n}_{s_2 i,j-1}}{2\Delta Z}\right) + V_g K_{s_2} C^{n}_{pi,j},$$

$$C^{n+1}_{s_2 i,j} = \frac{ST(\Delta T)D_x}{(\Delta X)^2}\left(C^{n}_{s_2 i+1,j} - 2C^{n}_{s_2 i,j} + C^{n}_{s_2 i-1,j}\right) + \frac{ST(\Delta T)D_z}{(\Delta Z)^2}\left(C^{n}_{s_2 i,j+1} - 2C^{n}_{s_2 i,j} + C^{n}_{s_2 i,j-1}\right)$$
$$- \frac{ST(\Delta T)U}{2\Delta X}\left(C^{n}_{s_2 i+1,j} - C^{n}_{s_2 i-1,j}\right) - \frac{ST(\Delta T)W}{2\Delta Z}\left(C^{n}_{s_2 i,j+1} - C^{n}_{s_2 i,j-1}\right)$$
$$+ C^{n}_{s_2 i,j} + ST(\Delta T)V_g K_{s_2} C^{n}_{pi,j},$$

From the above equation, we obtain that

$$\begin{aligned}C_{s_2 i,j}^{n+1} &= \frac{ST(\Delta T)D_x}{(\Delta X)^2}C_{s_2 i+1,j}^{n} - \frac{2ST(\Delta T)D_x}{(\Delta X)^2}C_{s_2 i,j}^{n} + \frac{ST(\Delta T)D_x}{(\Delta X)^2}C_{s_2 i-1,j}^{n} \\ &+ \frac{ST(\Delta T)D_z}{(\Delta Z)^2}C_{s_2 i,j+1}^{n} - \frac{2ST(\Delta T)D_z}{(\Delta Z)^2}C_{s_2 i,j}^{n} + \frac{ST(\Delta T)D_z}{(\Delta Z)^2}C_{s_2 i,j-1}^{n} \\ &- \frac{ST(\Delta T)U}{2\Delta X}C_{s_2 i+1,j}^{n} + \frac{ST(\Delta T)U}{2\Delta X}C_{s_2 i-1,j}^{n} - \frac{ST(\Delta T)W}{2\Delta Z}C_{s_2 i,j+1}^{n} \\ &+ \frac{ST(\Delta T)W}{2\Delta Z}C_{s_2 i,j-1}^{n} + C_{s_2 i,j}^{n} + ST(\Delta T)V_g K_{s_2} C_{pi,j}^{n},\end{aligned}$$

$$\begin{aligned}C_{s_2 i,j}^{n+1} &= \left(\frac{ST(\Delta T)D_x}{(\Delta X)^2} - \frac{ST(\Delta T)U}{2\Delta X}\right)C_{s_2 i+1,j}^{n} + \left(\frac{ST(\Delta T)D_x}{(\Delta X)^2} + \frac{ST(\Delta T)U}{2\Delta X}\right)C_{s_2 i-1,j}^{n} \\ &+ \left(1 - \frac{2ST(\Delta T)D_z}{(\Delta Z)^2} - \frac{2ST(\Delta T)D_x}{(\Delta X)^2}\right)C_{s_2 i,j}^{n} \\ &+ \left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2} + \frac{ST(\Delta T)W}{2\Delta Z}\right)C_{s_2 i,j-1}^{n} + \left(\frac{ST(\Delta T)D_z}{(\Delta Z)^2} - \frac{ST(\Delta T)W}{2\Delta Z}\right)C_{s_2 i,j+1}^{n} \\ &+ ST(\Delta T)V_g K_{s_2} C_{pi,j}^{n}.\end{aligned} \tag{4.47}$$

Therefore, the forward time central space scheme of the sulfuric acid concentration can be written as

$$\begin{aligned}C_{s_2 i,j}^{n+1} &= (d_x - A_x)C_{s_2 i+1,j}^{n} + (d_x + A_x)C_{s_2 i-1,j}^{n} + (1 - 2d_x - 2d_z)C_{s_2 i,j}^{n} \\ &+ (d_z + A_z)C_{s_2 i,j-1}^{n} + (d_z - A_z)C_{s_2 i,j+1}^{n} + ST(\Delta T)V_g K_{s_2} C_{pi,j}^{n}.\end{aligned} \tag{4.48}$$

## 4.4 Numerical experiments and results for primary and secondary pollutant control

These experiments analyze the dispersion conduct of air pollution for primary and secondary pollutants, and multiple air pollution emission control from the industrial factory are considered. The National Ambient Air Quality Standards (NAAQS) in [27] are used to manage air quality control in the industrial area and the nearby residential area. Then, air quality criteria in general can be separated standard into two levels: 1) the Primary ambient air quality standard, the standard level to protect the human health, and 2) the Secondary ambient air quality standard, the standard level for human well-being protection or the hazard protection of animals, crops, vegetation, and buildings. In this research, we simulate two situations of decision air pollution emission control in different air quality standards. The factory discharges sulfur dioxide ($SO_2$), while the sulfur trioxide ($SO_3$) and sulfuric acid ($H_2SO_4$) form from sulfur dioxide in the climate. Then, all air pollutants are considered at each monitoring point using the National Air Quality Index. Air quality standards of sulfur dioxide, sulfur trioxide, and sulfuric acid, which appropriate for the three air pollutants, are presented in Table 4.1.

**Table 4.1** Air quality standards of sulfur dioxide, sulfur trioxide, and sulfuric acid concentration measurement

| $SO_2$ [27] ($\times 10^{-8}$) | $SO_3$ ($\times 10^{-8}$) | $H_2SO_4$ ($\times 10^{-8}$) |
|---|---|---|
| 6.5 | 5.45 | 4.25 |

**Simulation 1**: Relaxed Air Quality Standard.

Primary pollution emission is controlled by following the United States Environmental Protection Agency (USEPA) air quality standards, and secondary pollutants controlled by determining the air quality standard. The two-dimensional advection-diffusion equation, (4.8), (4.16), and (4.22), with the interested domain $1000 \times 400\ m^2$, are considered. The proper variables for this simulation suppose that the wind velocities in the $x$-direction and $z$-direction are $0.11$ and $6.9 \times 10^{-3}\ m/s$, respectively. The diffusion coefficients in the $x$-direction and $z$-direction are $2$ and $0.4\ m^2/s$, respectively, and the rate of released pollutant concentration $q$ is $2.1875 \times 10^{-3}\ kg/s$. The height of chimney $h_{ps}$ is $50\ m$. This means a point source emits the sulfur dioxide at the coordinates $(100, 50)\ (m, m)$. The grid spacing is $\Delta x = 25$ and $\Delta z = 25\ m$, and the time interval is $\Delta t = 72\ s$. In this simulation, the sulfur dioxide is released by following the empirical USEPA air quality standard, $(2/3)(6.5 \times 10^{-8}) = 4.33 \times 10^{-8}\ kg/m^3$. The sulfur trioxide and sulfuric acid are accrued by the chemical interaction into the atmosphere, and the empirical air quality standards of sulfur trioxide and sulfuric acid are $(2/3)(5.45 \times 10^{-8}) = 3.63 \times 10^{-8}$ and $(2/3)(4.25 \times 10^{-8}) = 2.83 \times 10^{-8}\ kg/m^3$, respectively. The air pollution emission operates to follow these processes. In the considered pollutants investigated, if the approximate pollutant concentration at a monitoring point is higher than the air quality standards, the chimney is shut down and waits until the concentration of sulfur dioxide, sulfur trioxide, and sulfuric acid is less than $(2/5)(6.5 \times 10^{-8}) = 2.6 \times 10^{-8}$, $(2/5)(5.45 \times 10^{-8}) = 2.18 \times 10^{-8}$, and $(2/5)(4.25 \times 10^{-8}) = 1.7 \times 10^{-8}\ kg/m^3$, respectively. If the concentration of all pollutants at the monitoring point is below two fifths of the air quality standard, then the chimney is opened again to discharge the air pollution. Simulation 1 is solved by using the forward time central space scheme of sulfur dioxide (4.34), sulfur trioxide (4.41), and sulfuric acid (4.48) equations, with the initial and the boundary conditions

(4.9)-(4.13), (4.17)-(4.21) and (4.22)-(4.27), respectively. The numerical solutions of sulfur dioxide at all monitoring points for the decision emission control points at M1, M2, M3, and M4 are shown in Figure 4.5–Figure 4.8. Similarly, the concentrations of sulfur trioxide and sulfuric acid which are controlled by the decision emission control points at M1, M2, M3, and M4 are shown in Figure 4.9–Figure 4.12 and Figure 4.13–Figure 4.16, respectively. The maximums of air pollution concentration at each monitoring point at M1, M2, M3, and M4 are presented in Table 4.2–Table 4.5, respectively.

The air pollution emission control is considered at the monitoring points at M1, M2, M3, and M4. The air quality standards of sulfur dioxide, sulfur trioxide, and sulfuric acid are used to compare the concentrations between the numerical solutions and the standards. (see in Table 4.1).

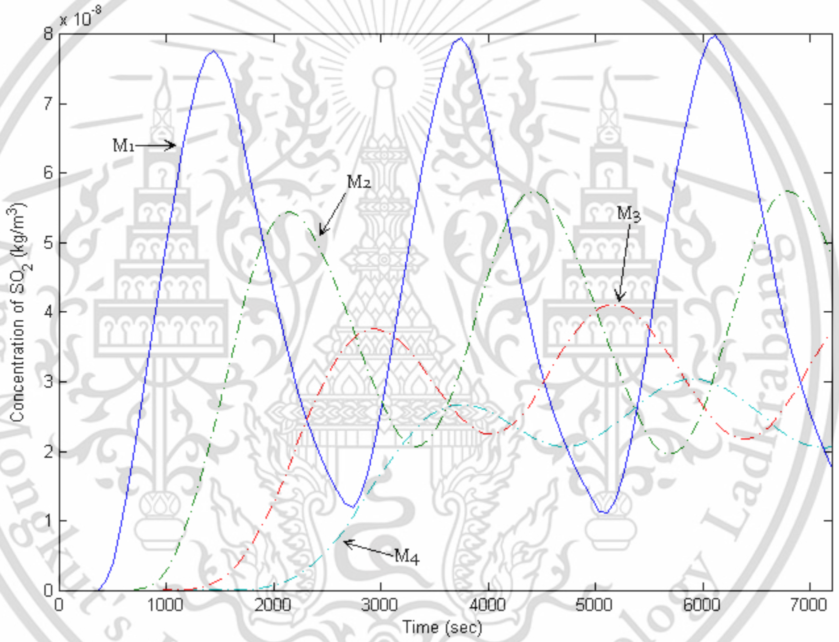



**Figure 4.5** The primary concentration of $SO_2$ at monitoring points by considering the decision emission control point No.1 (M1) with $SO_2$ standard on Simulation 1

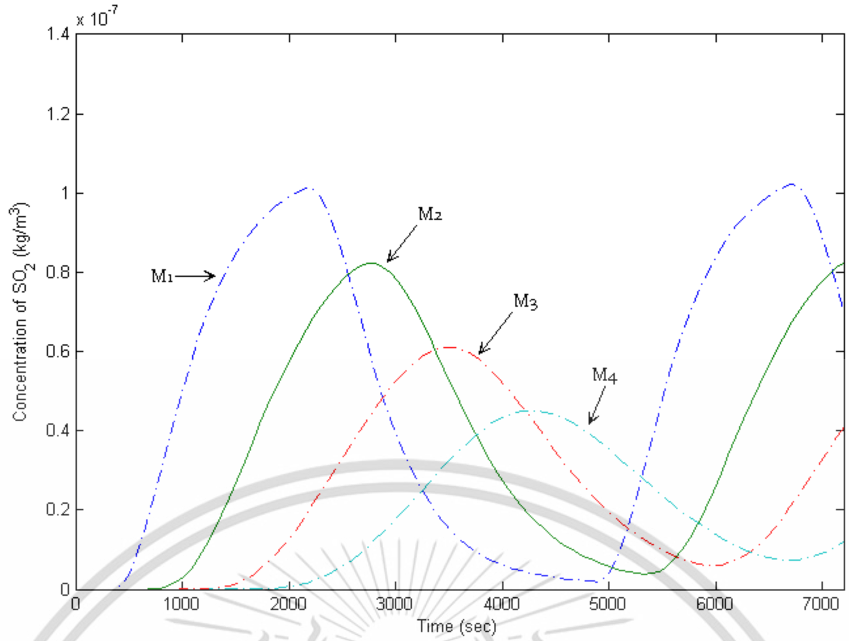



**Figure 4.6** The primary concentration of $SO_2$ at monitoring points by considering the decision emission control point No.2 (M2) with $SO_2$ standard on Simulation 1

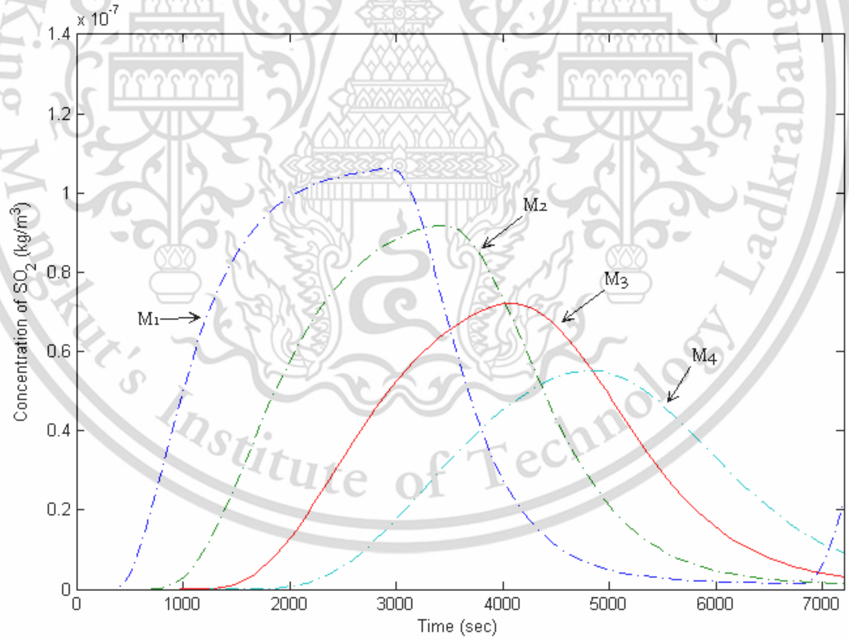



**Figure 4.7** The primary concentration of $SO_2$ at monitoring points by considering the decision emission control point No.3 (M3) with $SO_2$ standard on Simulation 1

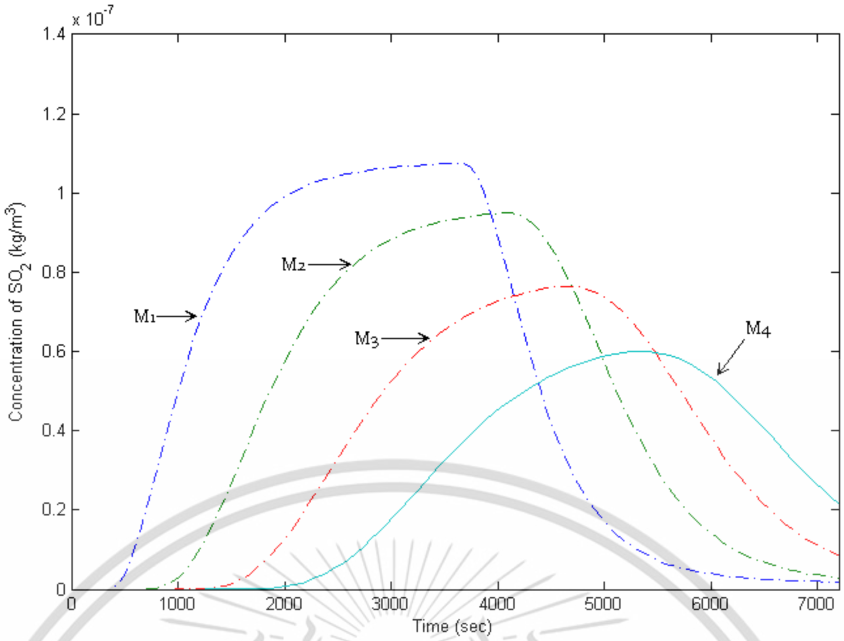



**Figure 4.8** The primary concentration of $SO_2$ at monitoring points by considering the decision emission control point No.4 (M4) with $SO_2$ standard on Simulation 1

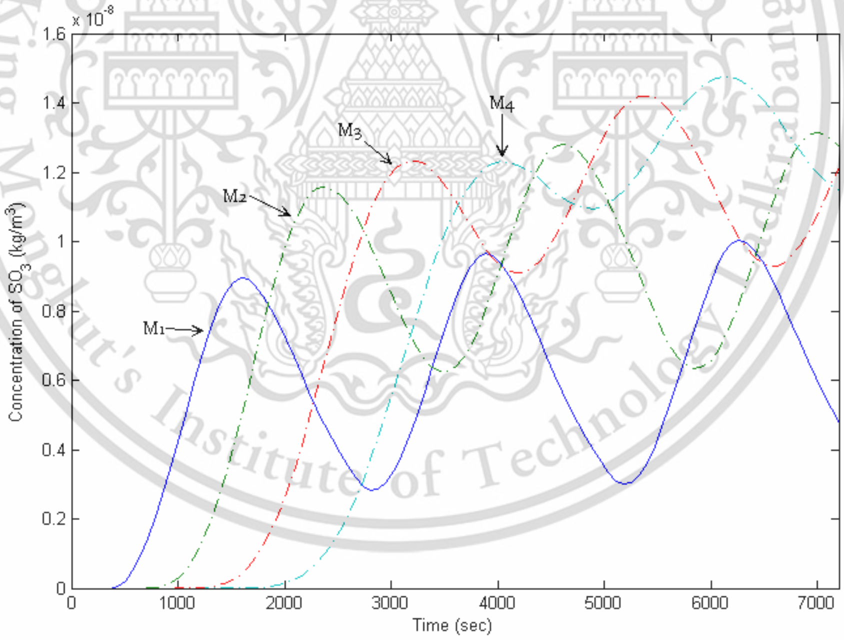



**Figure 4.9** The secondary concentration of $SO_3$ at monitoring points by considering the decision emission control point No.1 (M1) with $SO_3$ standard on Simulation 1

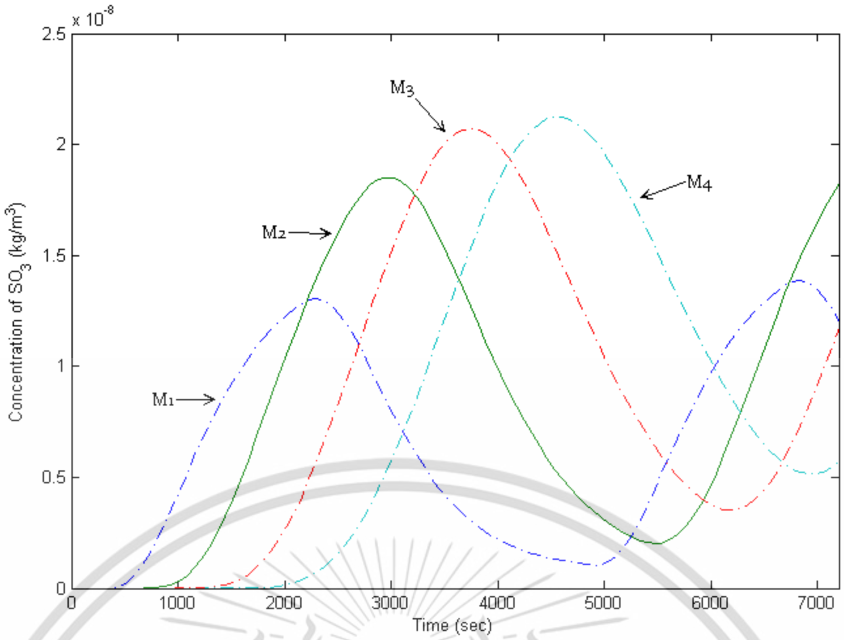



**Figure 4.10** The secondary concentration of $SO_3$ at monitoring points by considering the decision emission control point No.2 (M2) with $SO_3$ standard on Simulation 1

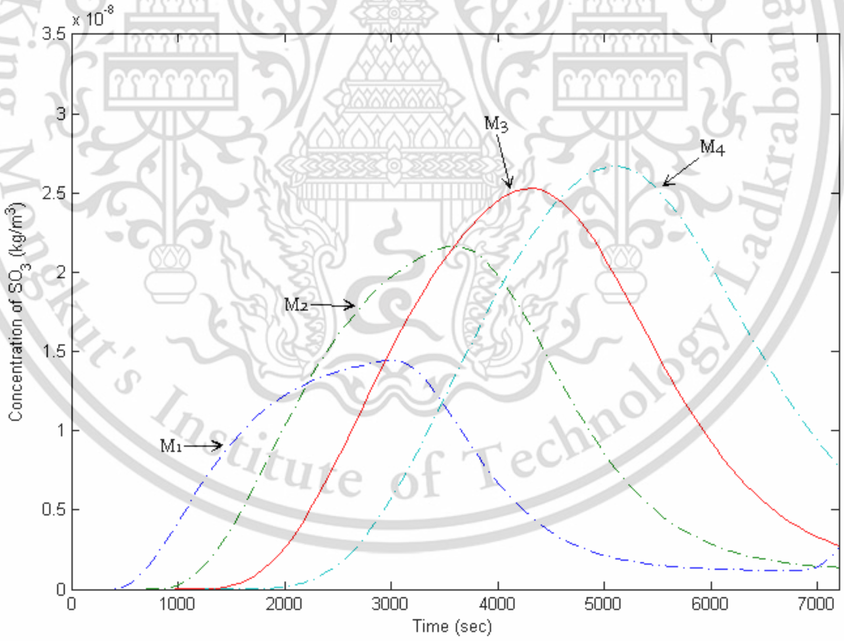



**Figure 4.11** The secondary concentration of $SO_3$ at monitoring points by considering the decision emission control point No.3 (M3) with $SO_3$ standard on Simulation 1

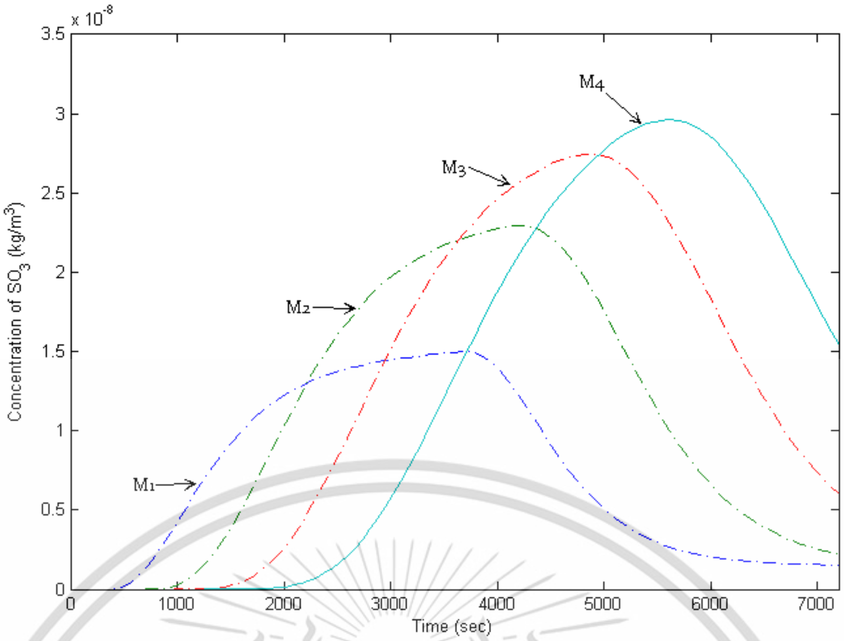



**Figure 4.12** The secondary concentration of $SO_3$ at monitoring points by considering the decision emission control point No.4 (M4) with $SO_3$ standard on Simulation 1

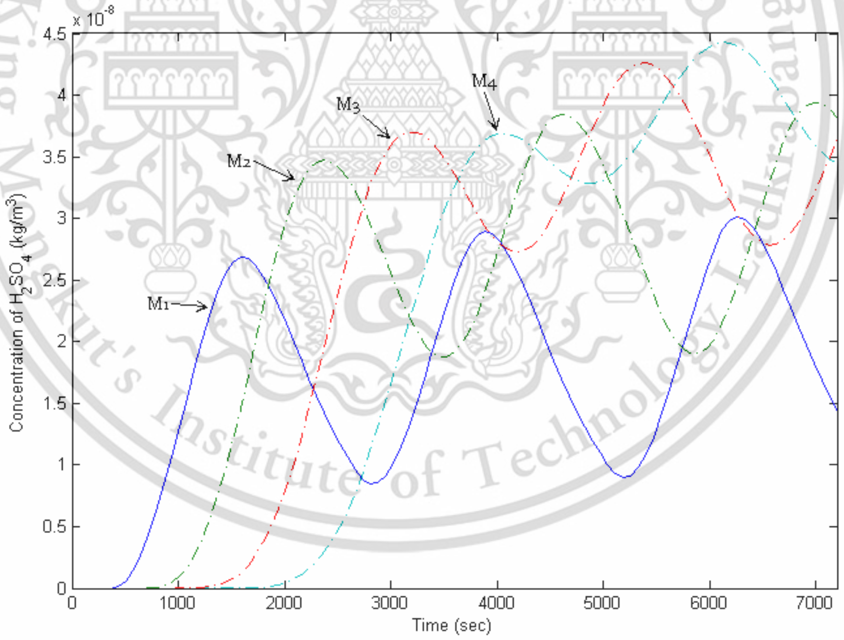



**Figure 4.13** The secondary concentration of $H_2SO_4$ at monitoring points by considering the decision emission control point No.1 (M1) with $H_2SO_4$ standard on Simulation 1

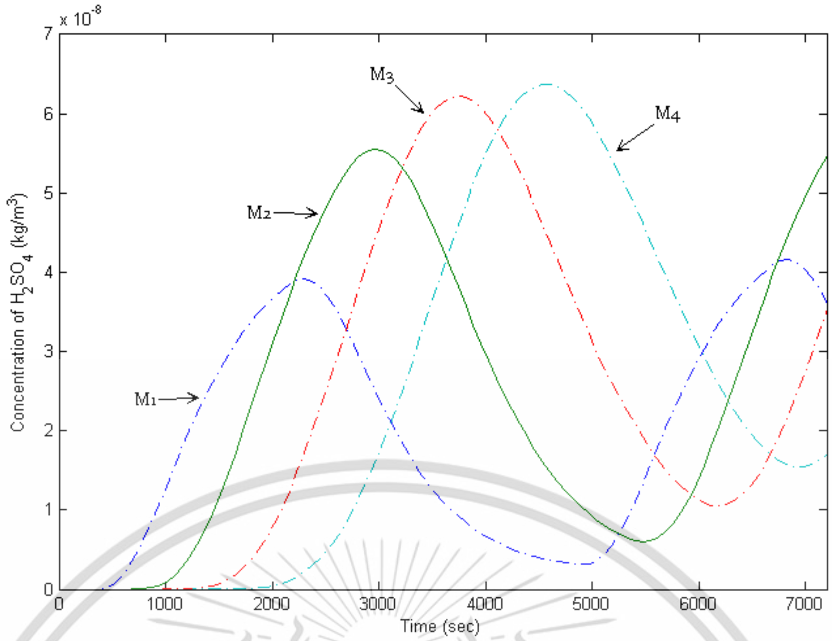



**Figure 4.14** The secondary concentration of $H_2SO_4$ at monitoring points by considering the decision emission control point No.2 (M2) with $H_2SO_4$ standard on Simulation 1

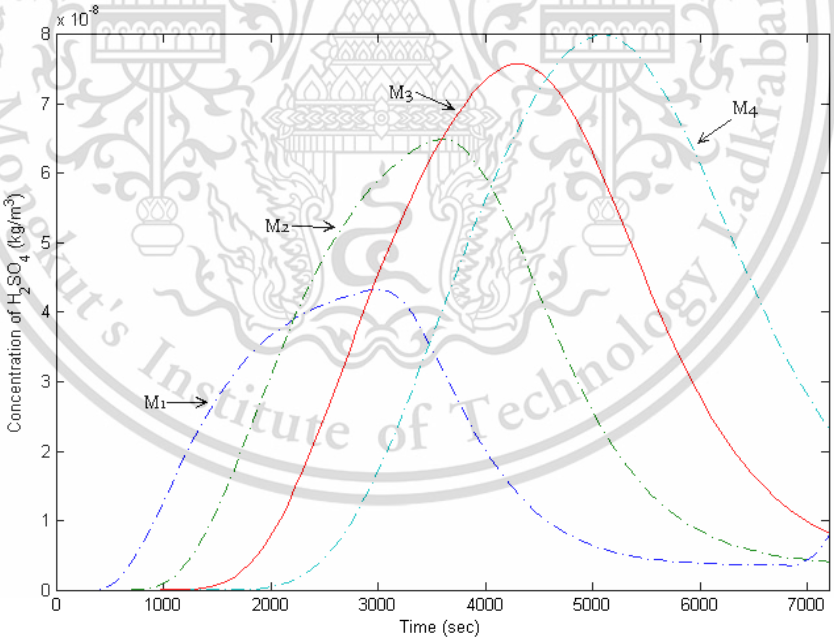



**Figure 4.15** The secondary concentration of $H_2SO_4$ at monitoring points by considering the decision emission control point No.3 (M3) with $H_2SO_4$ standard on Simulation 1

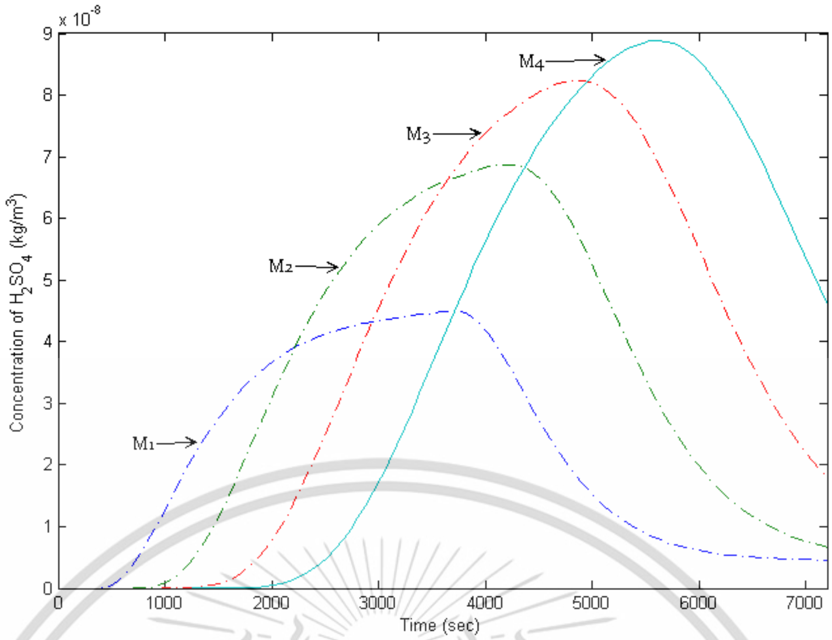



**Figure 4.16** The secondary concentration of $H_2SO_4$ at monitoring points by considering the decision emission control point No.4 (M4) with $H_2SO_4$ standard on Simulation 1

The approximate solutions of Simulation 1 are considered. For the decision emission control point at M1, Figure 4.5 shows that the sulfur dioxide concentration levels at monitoring points M2-M4 are under the air quality standard levels, but the concentration levels at the decision control point exceed the standard levels. Figure 4.6 indicates that the sulfur dioxide concentration levels at M3 and M4 are under the standard levels when the decision emission control point at M2 is considered. For the decision emission control points at M3 and M4, Figure 4.7 and Figure 4.8 show that the concentration levels at M4 only are under the standard levels. The sulfur trioxide concentration levels at all the emission control points are considered. We can see that the calculated concentration levels at all monitoring points are under the air quality standard levels of sulfur trioxide in Figure 4.9– Figure 4.12. For the decision emission control point at M1, the sulfuric acid concentration levels at M1 and M2 are under the sulfuric acid standard levels, but the concentration levels at M3 and M4 exceed the standard levels in Figure 4.13. In addition, Figure 4.14 shows that the concentration levels at M1 are under the standard levels for the decision emission control point at M2, whereas Figure 4.15 and Figure 4.16 indicate that the sulfuric acid concentration levels at all monitoring points exceeds the standard levels by considering the decision emission control points M3 and M4.

Table 4.2 The maximums of concentration (kg/m$^3$) at each monitoring point when the decision emission control monitor is at M1 on Simulation 1

| Monitoring Point | $SO_2$ ($\times 10^{-8}$) | $SO_3$ ($\times 10^{-8}$) | $H_2SO_4$ ($\times 10^{-8}$) |
|---|---|---|---|
| **M1** | 7.9842 | 1.0033 | 3.0096 |
| M2 | 5.7459 | 1.3139 | 3.9412 |
| M3 | 4.1085 | 1.4200 | 4.2590 |
| M4 | 3.0394 | 1.4761 | 4.4267 |

Table 4.3 The maximums of concentration (kg/m$^3$) at each monitoring point when the decision emission control monitor is at M2 on Simulation 1

| Monitoring Point | $SO_2$ ($\times 10^{-8}$) | $SO_3$ ($\times 10^{-8}$) | $H_2SO_4$ ($\times 10^{-8}$) |
|---|---|---|---|
| M1 | 10.2178 | 1.3872 | 4.1612 |
| **M2** | 8.2383 | 1.8511 | 5.5527 |
| M3 | 6.1027 | 2.0727 | 6.2167 |
| M4 | 4.5063 | 2.1230 | 6.3668 |

Table 4.4 The maximums of concentration (kg/m$^3$) at each monitoring point when the decision emission control monitor is at M3 on Simulation 1

| Monitoring Point | $SO_2$ ($\times 10^{-8}$) | $SO_3$ ($\times 10^{-8}$) | $H_2SO_4$ ($\times 10^{-8}$) |
|---|---|---|---|
| M1 | 10.6290 | 1.4443 | 4.3326 |
| M2 | 9.1903 | 2.1639 | 6.4909 |
| **M3** | 7.2011 | 2.5272 | 7.5797 |
| M4 | 5.5217 | 2.6665 | 7.9966 |

Table 4.5 The maximums of concentration (kg/m$^3$) at each monitoring point when the decision emission control monitor is at M4 on Simulation 1

| Monitoring Point | $SO_2$ ($\times 10^{-8}$) | $SO_3$ ($\times 10^{-8}$) | $H_2SO_4$ ($\times 10^{-8}$) |
|---|---|---|---|
| M1 | 10.7587 | 1.4993 | 4.4975 |
| M2 | 9.4933 | 2.2923 | 6.8760 |
| M3 | 7.6424 | 2.7455 | 8.2344 |
| **M4** | 6.0075 | 2.9620 | 8.8828 |

**Simulation 2:** Strictly Air Quality Standard.

Primary pollution emission is controlled by following the USEPA air quality standard, and secondary pollutants controlled by determining the air quality standard. The two-dimensional advection-diffusion equations (4.8), (4.16), and (4.22), with the interested domain $1000 \times 400\ m^2$, are considered. The proper variables for this simulation suppose that the wind velocities in the $x$-direction and $z$-direction are $0.11$ and $6.9 \times 10^{-3}\ m/s$, respectively. The diffusion coefficients in the $x$-direction and $z$-direction are $2$ and $0.4\ m^2/s$, respectively, and the rate of released pollutant concentration $q$ is $2.1875 \times 10^{-3}\ kg/s$. The height of chimney $h_{ps}$ is $50\ m$. This means a point source emits the sulfur dioxide at the coordinates $(100,50)\ (m,m)$. The grid spacing is $\Delta x = 25$ and $\Delta z = 25\ m$, and the time interval is $\Delta t = 72\ s$. In this simulation, sulfur dioxide is released by following the empirical USEPA air quality standard, $(1/2)(6.5 \times 10^{-8}) = 3.25 \times 10^{-8}\ kg/m^3$. Sulfur trioxide and sulfuric acid are accrued by the chemical interaction into the atmosphere, with the empirical air quality standard of sulfur trioxide being $(1/2)(5.45 \times 10^{-8}) = 2.725 \times 10^{-8}\ kg/m^3$ and sulfuric acid being $(1/2)(4.25 \times 10^{-8}) = 2.125 \times 10^{-8}\ kg/m^3$. The air pollution emission operates to follow these processes. If one of the considered pollutants, investigated for the approximate pollutant concentration, at a monitoring point, is higher than the air quality standards, the chimney is shut down and waits until the concentration of sulfur dioxide, sulfur trioxide, and sulfuric acid is less than $2.6 \times 10^{-8}$, $2.18 \times 10^{-8}$, and $1.7 \times 10^{-8}\ kg/m^3$, respectively. If the concentration of all pollutants at the monitoring point is below two fifths of air quality standard, then the chimney is opened again. The numerical solutions are solved by using the forward time central space scheme in (4.34), (4.41), and (4.48), with the initial and the boundary conditions (4.9)-(4.13), (4.17)-(4.21) and (4.22)-(4.27), respectively. The approximate solutions of sulfur dioxide at all monitoring points, which are considered by the decision emission control points at M1, M2, M3, and M4, are shown in Figure 4.17–Figure 4.20. Similarly, the sulfur trioxide and sulfuric acid concentrations for the decision emission control points at M1, M2, M3, and M4 are shown in Figure 4.21–Figure 4.24 and Figure 4.25–Figure 4.28, respectively. The maximums of air pollution concentration at each monitoring point that is controlled by the monitoring points at M1, M2, M3, and M4 are presented in Table 4.6–Table 4.9, respectively.

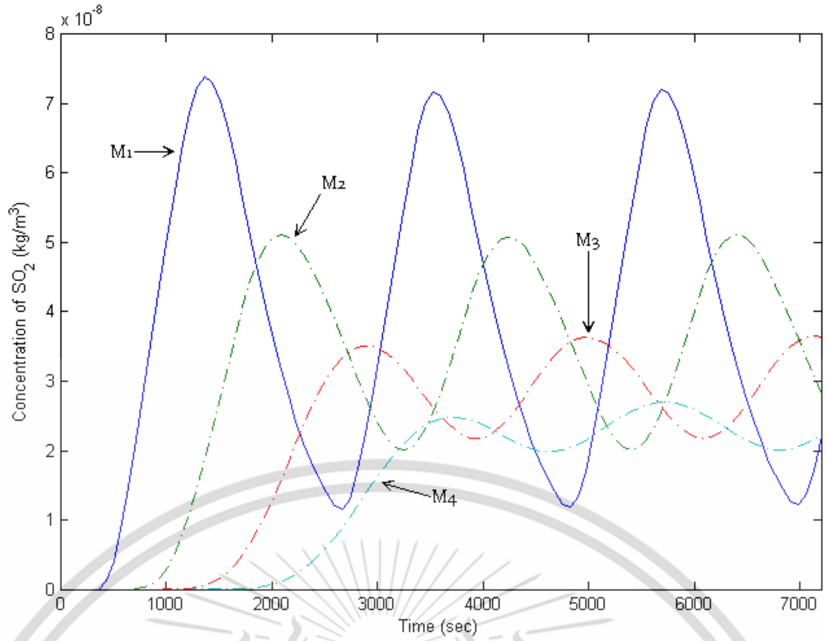



**Figure 4.17** The primary concentration of $SO_2$ at monitoring points by considering the decision emission control point No.1 (M1) with $SO_2$ standard on Simulation 2

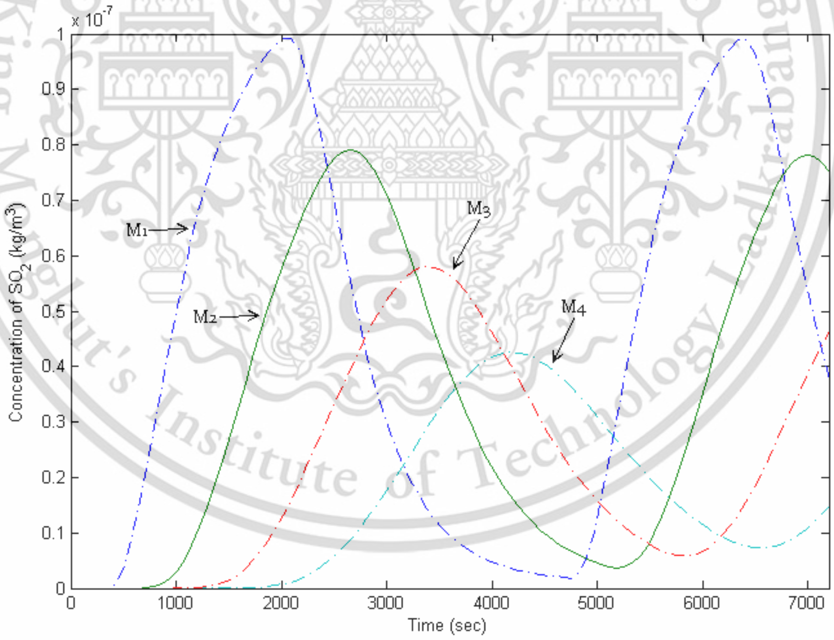



**Figure 4.18** The primary concentration of $SO_2$ at monitoring points by considering the decision emission control point No.2 (M2) with $SO_2$ standard on Simulation 2

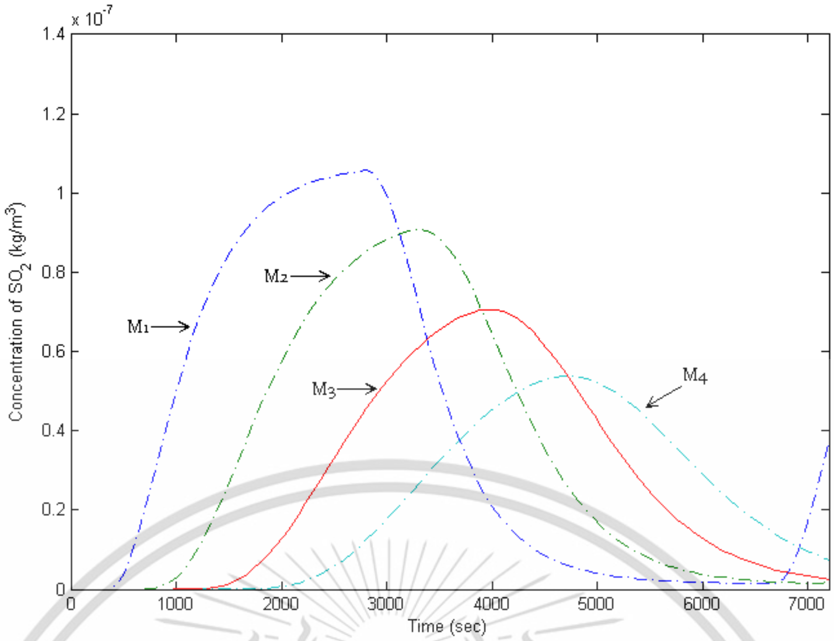



**Figure 4.19** The primary concentration of $SO_2$ at monitoring points by considering the decision emission control point No.3 (M3) with $SO_2$ standard on Simulation 2

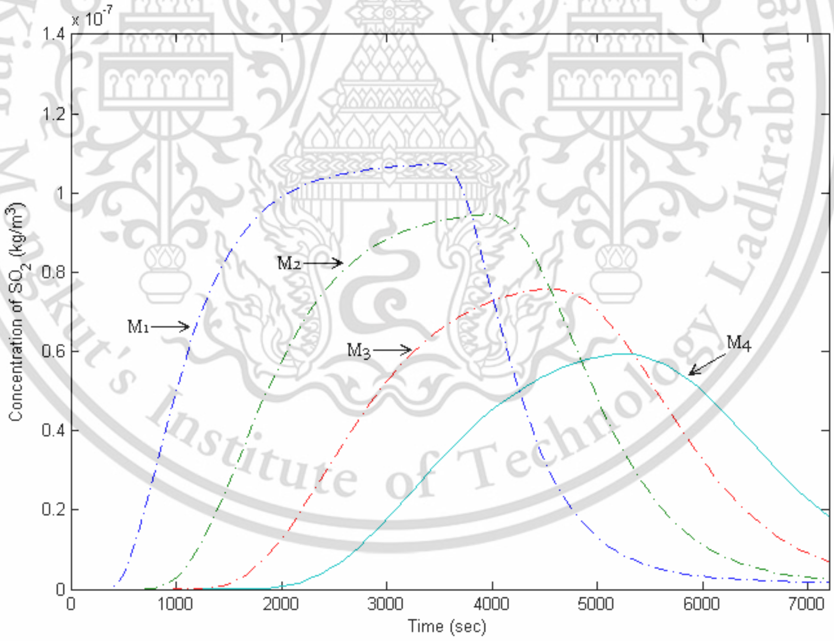



**Figure 4.20** The primary concentration of $SO_2$ at monitoring points by considering the decision emission control point No.4 (M4) with $SO_2$ standard on Simulation 2

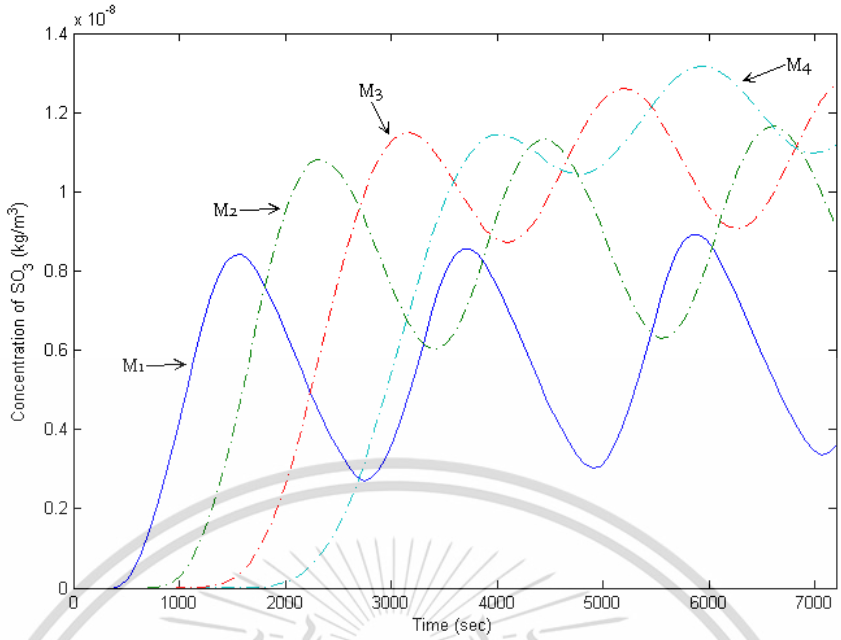



**Figure 4.21** The secondary concentration of $SO_3$ at monitoring points by considering the decision emission control point No.1 (M1) with $SO_3$ standard on Simulation 2

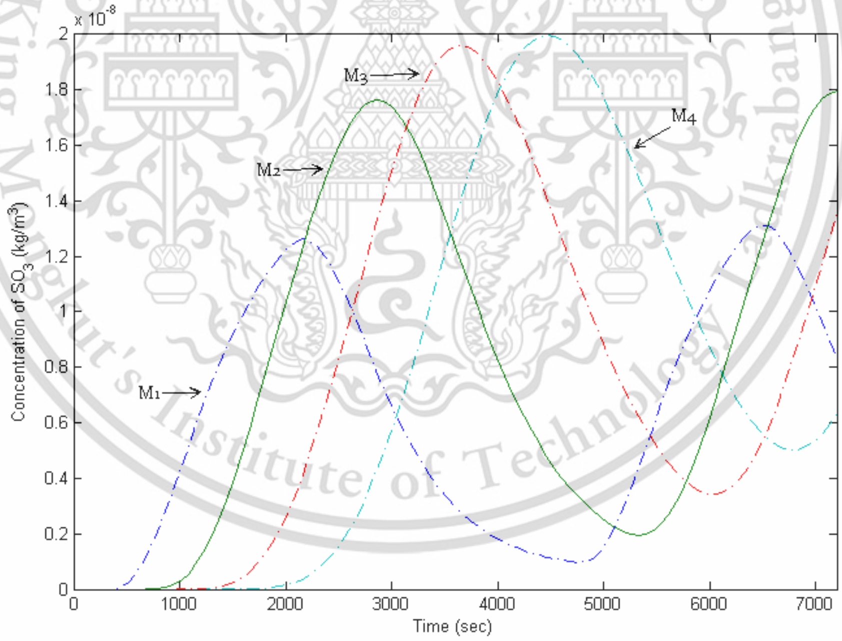



**Figure 4.22** The secondary concentration of $SO_3$ at monitoring points by considering the decision emission control point No.2 (M2) with $SO_3$ standard on Simulation 2

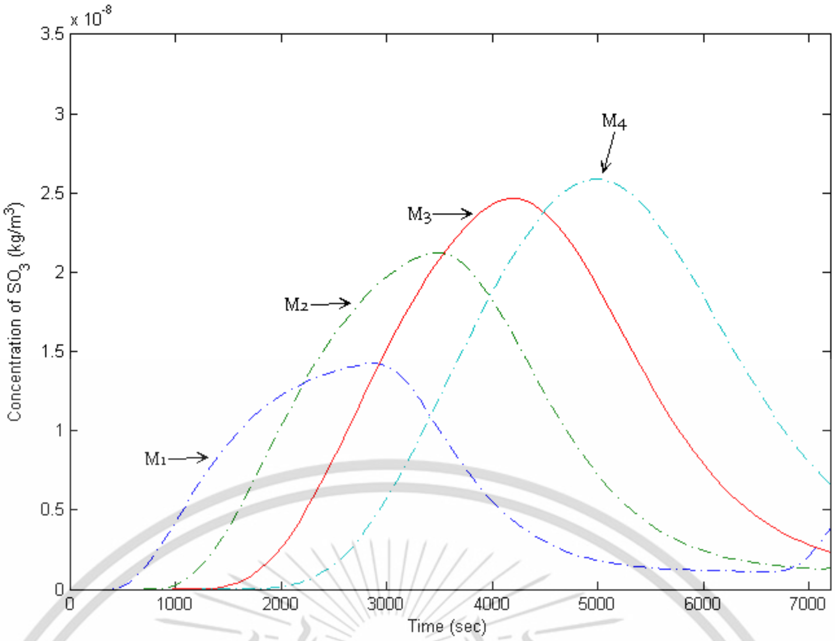



**Figure 4.23** The secondary concentration of $SO_3$ at monitoring points by considering the decision emission control point No.3 (M3) with $SO_3$ standard on Simulation 2

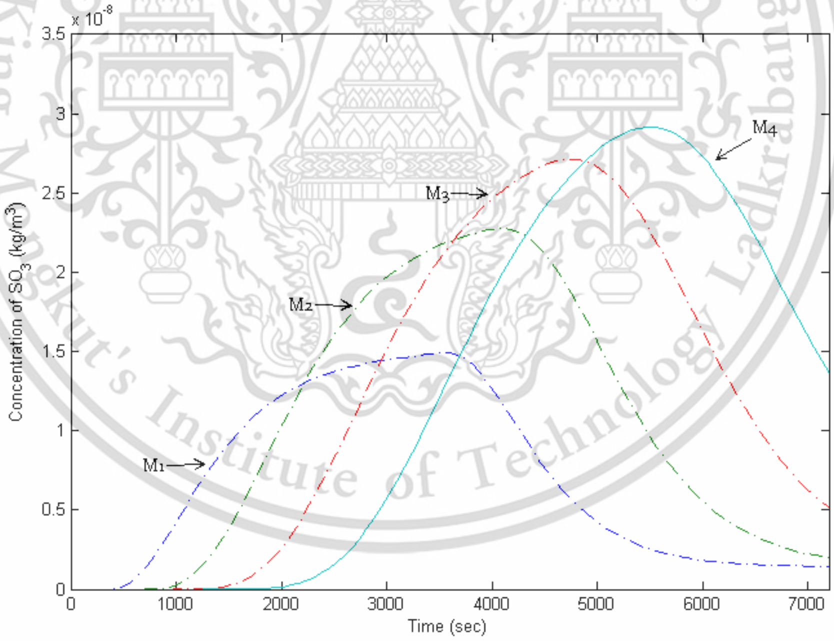



**Figure 4.24** The secondary concentration of $SO_3$ at monitoring points by considering the decision emission control point No.4 (M4) with $SO_3$ standard on Simulation 2

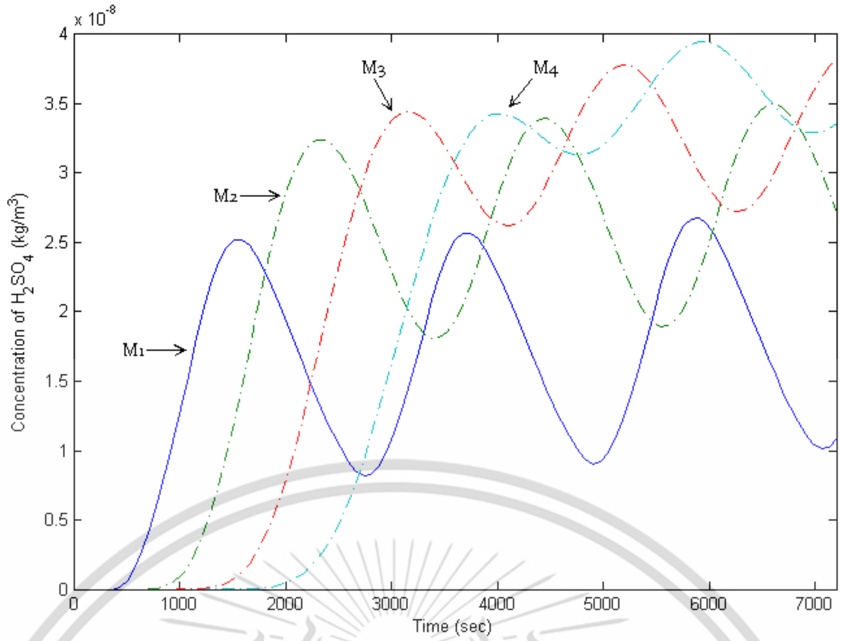



**Figure 4.25** The secondary concentration of $H_2SO_4$ at monitoring points by considering the decision emission control point No.1 (M1) with $H_2SO_4$ standard on Simulation 2

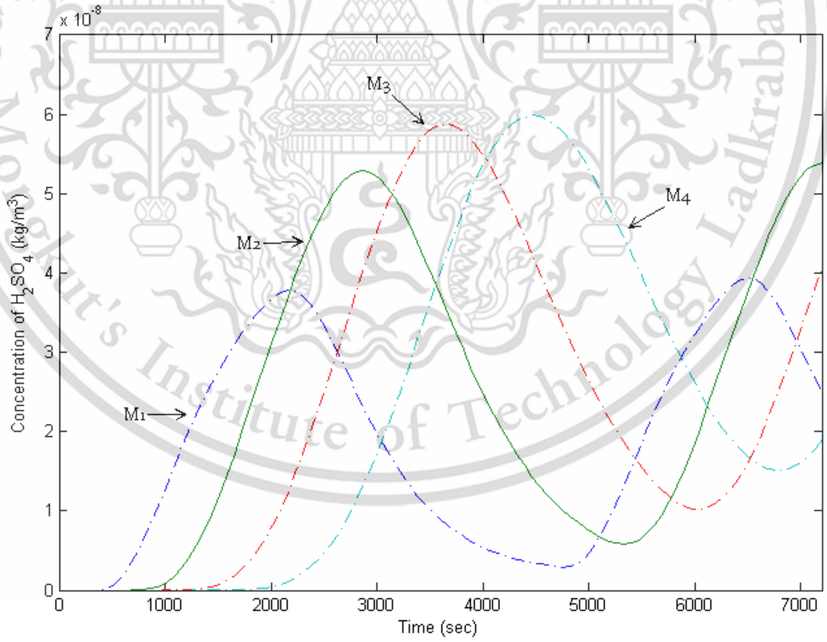



**Figure 4.26** The secondary concentration of $H_2SO_4$ at monitoring points by considering the decision emission control point No.2 (M2) with $H_2SO_4$ standard on Simulation 2

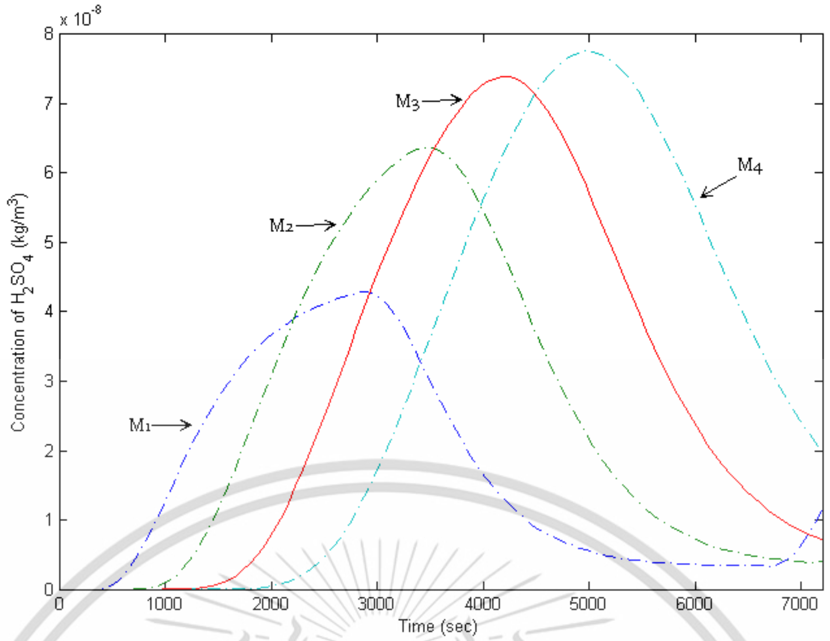



**Figure 4.27** The secondary concentration of $H_2SO_4$ at monitoring points by considering the decision emission control point No.3 (M3) with $H_2SO_4$ standard on Simulation 2

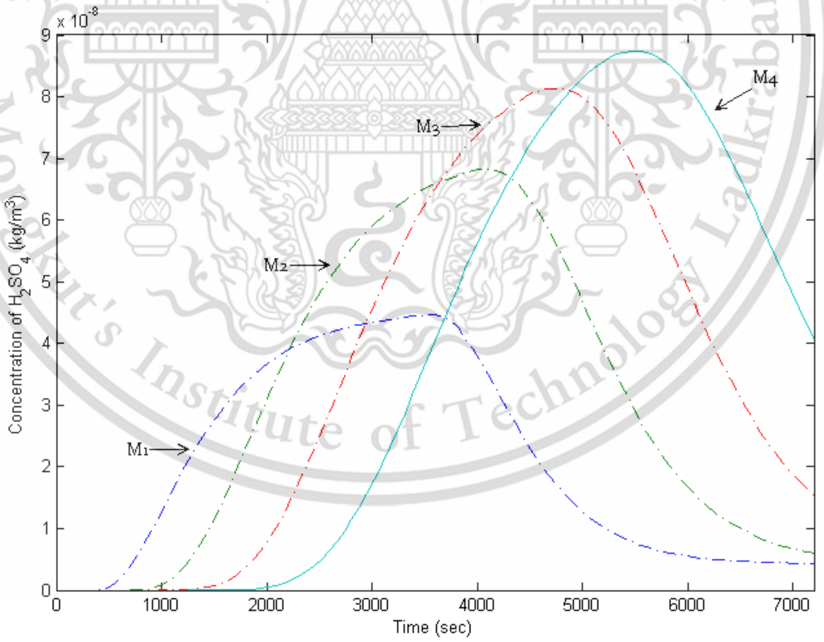



**Figure 4.28** The secondary concentration of $H_2SO_4$ at monitoring points by considering the decision emission control point No.4 (M4) with $H_2SO_4$ standard on Simulation 2

The approximate solutions of Simulation 2 are considered. For the decision emission control points at M1, Figure 4.17 shows that the sulfur dioxide concentration levels at monitoring points M2-M4 are under the air quality standard levels, but the concentration levels at the decision control point exceed the standard levels. Figure 4.18 indicates that the sulfur dioxide concentration levels at M3 and M4 are under the standard levels for the decision emission control point M2. Figure 4.19 and Figure 4.20 show that the concentration levels at M4 only are under the standard levels when the decision emission control points M3 and M4 are considered. For all the decision emission control points in Figure 4.21–Figure 4.24, the sulfur trioxide concentration levels at all monitoring points are under the air quality standard levels. The sulfur acid concentration levels at all monitoring points are under the sulfuric acid standard levels by using the decision emission control point M1, as shown in Figure 4.25. Conversely, Figure 4.26 shows that the concentration levels at M1 only are under the standard levels for the decision emission control point M2. Moreover, Figure 4.27 and Figure 4.28 indicate that the sulfuric acid concentration levels at all monitoring points exceed the standard levels when the decision emission control points are M3 and M4.

**Table 4.6** The maximums of concentration (kg/m$^3$) at each monitoring point when the decision emission control monitor is at M1 on Simulation 2

| Monitoring Point | $SO_2$ ($\times 10^{-8}$) | $SO_3$ ($\times 10^{-8}$) | $H_2SO_4$ ($\times 10^{-8}$) |
|---|---|---|---|
| **M1** | 7.3877 | 0.8911 | 2.6730 |
| M2 | 5.1127 | 1.1659 | 3.4973 |
| M3 | 3.6525 | 1.2751 | 3.8244 |
| M4 | 2.6952 | 1.3153 | 3.9444 |

**Table 4.7** The maximums of concentration (kg/m$^3$) at each monitoring point when the decision emission control monitor is at M2 on Simulation 2

| Monitoring Point | $SO_2$ ($\times 10^{-8}$) | $SO_3$ ($\times 10^{-8}$) | $H_2SO_4$ ($\times 10^{-8}$) |
|---|---|---|---|
| M1 | 9.9247 | 1.3119 | 3.9355 |
| **M2** | 7.9190 | 1.7947 | 5.3835 |
| M3 | 5.8067 | 1.9582 | 5.8734 |
| M4 | 4.2564 | 1.9960 | 5.9861 |

**Table 4.8** The maximums of concentration (kg/m$^3$) at each monitoring point when the decision emission control monitor is at M3 on Simulation 2

| Monitoring Point | $SO_2$ ($\times 10^{-8}$) | $SO_3$ ($\times 10^{-8}$) | $H_2SO_4$ ($\times 10^{-8}$) |
|---|---|---|---|
| M1 | 10.5828 | 1.4281 | 4.2841 |
| M2 | 9.0864 | 2.1241 | 6.3714 |
| **M3** | 7.0630 | 2.4637 | 7.3893 |
| M4 | 5.3781 | 2.5867 | 7.7576 |

**Table 4.9** The maximums of concentration (kg/m$^3$) at each monitoring point when the decision emission control monitor is at M4 on Simulation 2

| Monitoring Point | $SO_2$ ($\times 10^{-8}$) | $SO_3$ ($\times 10^{-8}$) | $H_2SO_4$ ($\times 10^{-8}$) |
|---|---|---|---|
| M1 | 10.7407 | 1.4903 | 4.4705 |
| M2 | 9.4542 | 2.2737 | 6.8201 |
| M3 | 7.5832 | 2.7132 | 8.1376 |
| **M4** | 5.9349 | 2.9149 | 8.7417 |

From both simulations, we consider the maximums of air pollution concentration at each monitoring point. Then, the total monitoring points under air quality standard, which is operated by the monitoring points at M1, M2, M3, and M4, are presented in Table 4.10. The sulfur dioxide, sulfur trioxide, and sulfuric acid concentration levels of suitable cases by the control of monitoring point No.1 (M1) on Simulation 2 are shown in Figure 4.29-Figure 4.34.

**Table 4.10** Number of overall monitoring points which are under USEPA air quality standard when the different decision monitorial node is specified by M1, M2, M3, and M4

| Decision Monitorial Node | Number of Standardized Nodes | |
|---|---|---|
| | Simulation 1 | Simulation 2 |
| M1 | 9 | 11 |
| M2 | 7 | 7 |
| M3 | 5 | 5 |
| M4 | 5 | 5 |

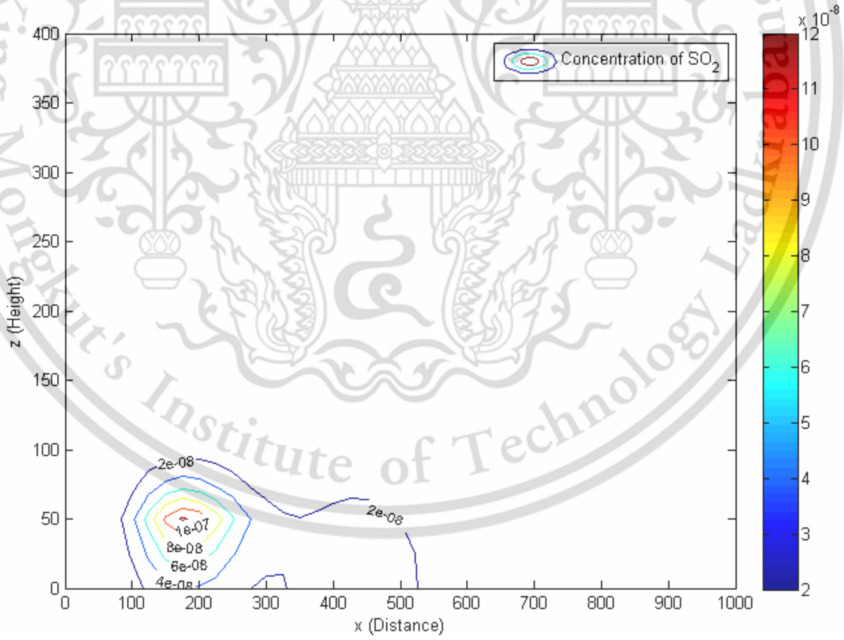



**Figure 4.29** The concentration levels of $SO_2$ pollutant after 58 minutes by the decision emission control point No.1 (M1) on Simulation 2

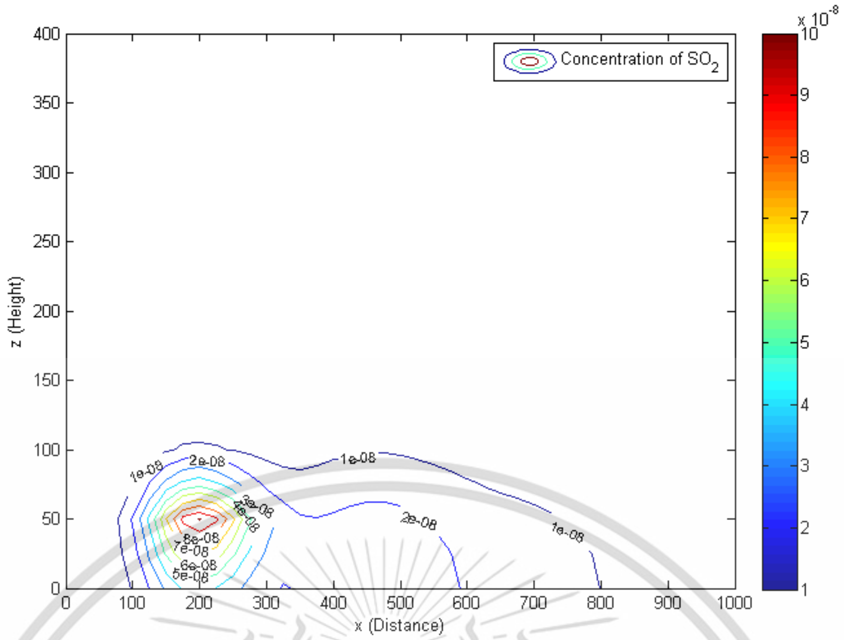



**Figure 4.30** The concentration levels of $SO_2$ pollutant after an hour and 36 minutes by the decision emission control point No.1 (M1) on Simulation 2

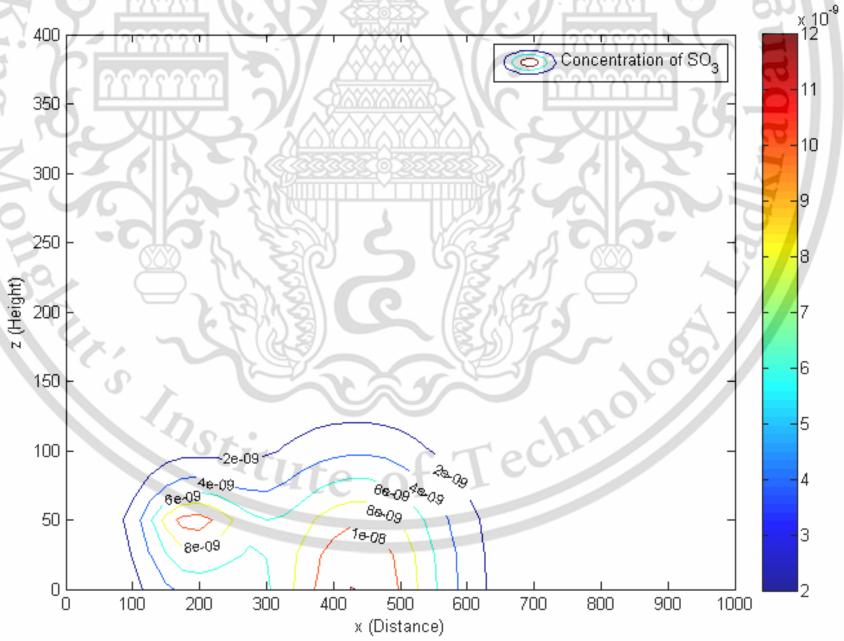



**Figure 4.31** The concentration levels of $SO_3$ pollutant after 58 minutes by the decision emission control point No.1 (M1) on Simulation 2

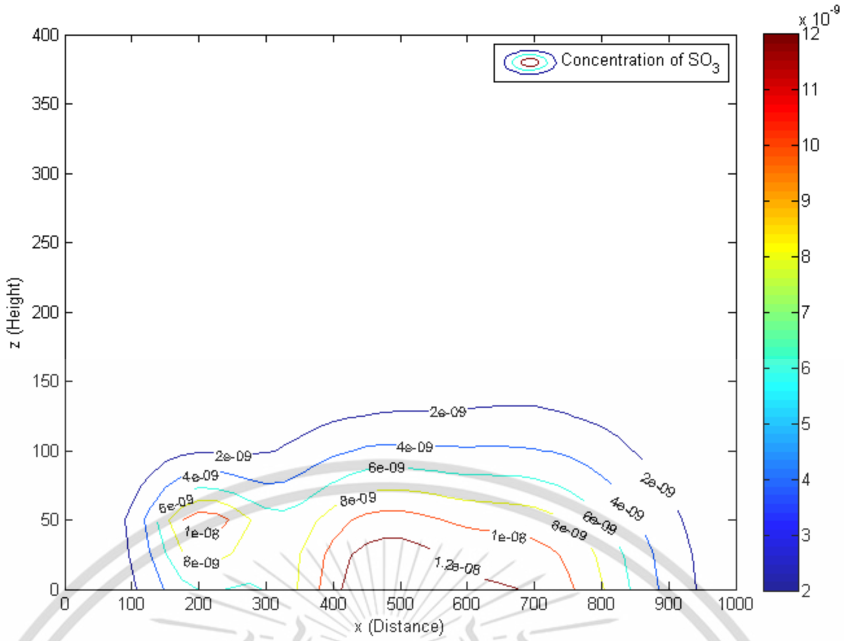



**Figure 4.32** The concentration levels of $SO_3$ pollutant after an hour and 36 minutes by the decision emission control point No.1 (M1) on Simulation 2

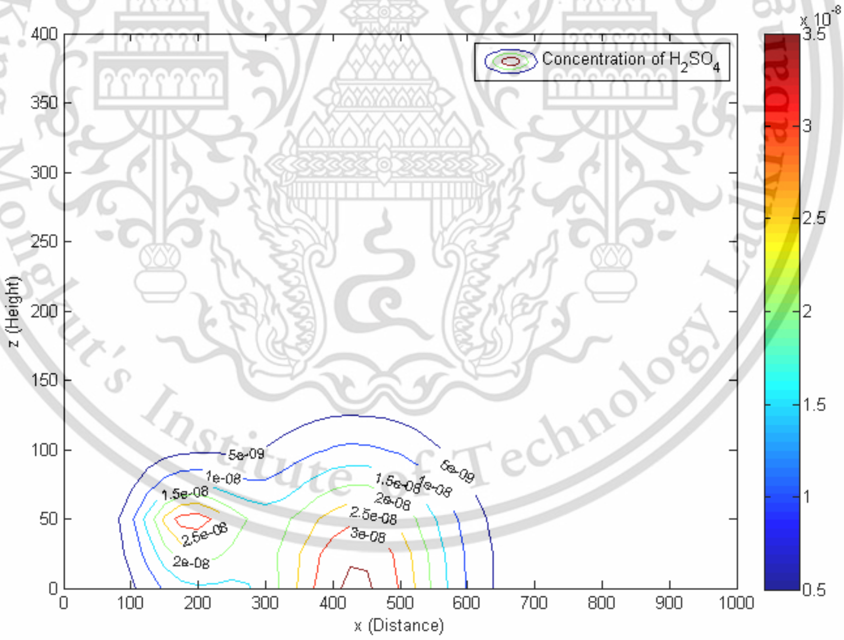



**Figure 4.33** The concentration levels of $H_2SO_4$ pollutant after 58 minutes by the decision emission control point No.1 (M1) on Simulation 2

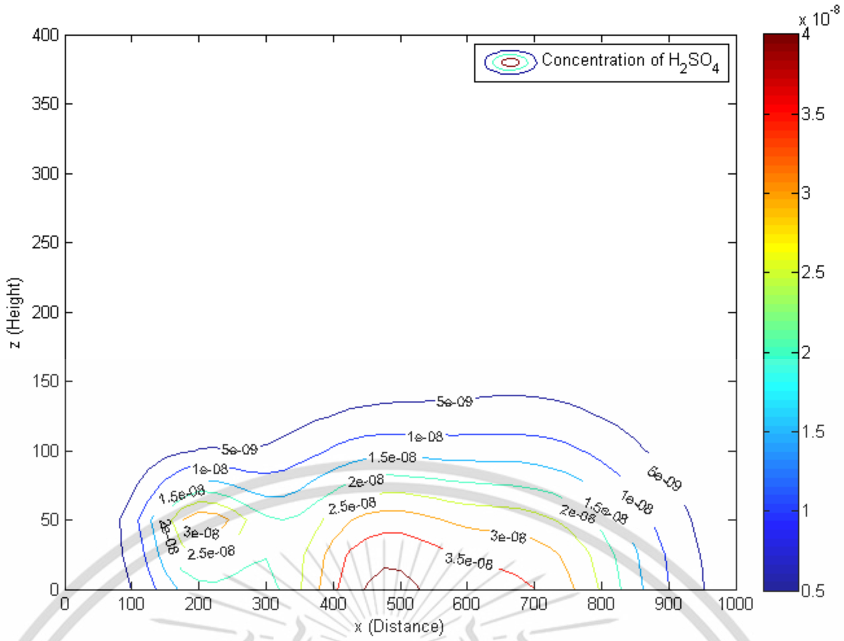



**Figure 4.34** The concentration levels of $H_2SO_4$ pollutant after an hour and 36 minutes by the decision emission control point No.1 (M1) on Simulation 2

Table 4.10 shows the number of standardized nodes which are controlled by M1, M2, M3, and M4. In this research, the maximum number of standardized nodes is 11 points. As a result, the sulfur dioxide, sulfur trioxide, and sulfuric acid air pollution concentrations in Simulation 2 by controlling at M1 give the best air quality standards. The concentration contour of the proper emission control at M1 in Simulation 2 is presented. Figure 4.29 and Figure 4.30 show the actions of sulfur dioxide concentration after 58 minutes and an hour and 36 minutes have passed. Figure 4.31 and Figure 4.32 show the approximate solutions of sulfur trioxide concentrations after 58 minutes and an hour and 36 minutes; similarly, Figure 4.33 and Figure 4.34 show the approximate solutions of sulfuric acid concentrations after 58 minutes and an hour and 36 minutes.

# Chapter 5
# Discussion and Conclusion

## 5.1 Discussion

In the Numerical Simulation of an Air Pollution Model in Industrial Areas by Considering the Influence of Multiple Point Sources, the air pollutant emission from multiple point sources above an industrial zone, to the urban area, is presented. In this research, the finite difference techniques introduced two methods, the forward time central space (FTCS) and backward time central space (BTCS) methods, for calculating air pollutant concentrations. The considered domain is separated into two zones, an industrial and an urban area. We note that air pollutant concentration levels of a two-point source is less than a one-point source. Moreover, the addition of sink rate has an effect on the reduced air pollutant concentrations. From the results, the forward time central space and backward time central space schemes can solve this problem, and that both methods give similar solutions. Therefore, the air pollutant emission in the case of a two-point source with air pollutant absorption is the most efficient in this research.

In the Numerical Simulation to Air Pollution Emission Control near an Industrial Zone, a numerical model for an air pollution emission control problem with uniform wind velocities and constant diffusion coefficients is proposed. In this research, the atmospheric diffusion equation is solved by using the finite difference method. This study analyzed the ambient air quality standard of sulfur dioxide that refers to the quantity of sulfur dioxide concentration in clean air. In Simulation 1, we analyze air pollutant without an emission control system, which is a continuous emission. On the other hand, Simulations 2 and 3 analyze emission control systems under the National Air Quality Standard. In the process of Simulation 2, if the approximated pollutant concentration at a monitoring point exceeds the air quality standards, then the chimney is shut down and waits until the concentration goes below $1.5\times10^{-7}\ kg/m^3$. If the pollutant concentration at all monitoring points are below a half of the air quality standards, the chimney is opened again. Similarly, in the process of Simulation 3, if the approximated pollutant concentration at a monitoring point exceeds the air quality standards, then the chimney is shut down and waits until the concentration goes below $1.0\times10^{-7}\ kg/m^3$. If the pollutant concentration at all monitoring points are below a third of the air quality standards, the chimney is opened again. We can see that the air quality in Simulations 2 and 3 is better than in Simulation 1; especially, Simulation 3 has the best air quality in the considered area.

In the Numerical Simulation to Toxic Smoke Emission Control by Considering Primary and Secondary Pollutant Concentration near an Industrial Zone Using Multiple Air Quality Standards, the primary and secondary pollutant concentrations at the monitoring points that are analyzed by controlling each decision monitoring point are presented. In this research, the forward time central space scheme is used to solve the sulfur dioxide ($SO_2$), sulfur trioxide ($SO_3$), and sulfuric acid ($H_2SO_4$) pollutant concentrations near the industrial zone. We note that the resulting trend of sulfur dioxide concentration in Simulation 2 is similar to that Simulation 1. Then, the results of sulfur trioxide concentrations in both simulations are under the standard levels. As [28] reflects, the quantities of sulfur dioxide and sulfuric acid are more probably to be found in the climate than in the quantity of sulfur trioxide. The results of sulfuric acid concentration in Simulation 2 is similar to those in Simulation 1, except that the concentration using the decision emission control point is at M1 in Simulation 2, and that the quality is better than the air quality in Simulation 1. The decision air pollution emission control at M1 shows that the quantity of sulfur dioxide and sulfuric acid at almost all monitoring points is under the standard levels. On the other hand, all decision air pollution emission controls of sulfur trioxide concentration give a good quality. Therefore, the decision air pollution emission control at M1 in Simulation 2 is a suitable monitorial point for this research. The concentrations of the overall points are under the air quality standards. They do not have an effect on human health and the environment when the distance and the time are increased.

## 5.2 Conclusion

In the Numerical Simulation of an Air Pollution Model on Industrial Areas by Considering the Influence of Multiple Point Sources, simple air pollution measurement models which release air pollutants by a single point source and coupled point sources are proposed. The traditional finite difference methods, such as forward time central space and backward time central space schemes, can be used to approximate the air pollutant levels for each point and time. The results of this study show that the air pollutant concentrations of the forward time central space scheme are close to the air pollutant concentrations of the backward time central space scheme. In the case of a coupled point sources problem, the overall concentration levels of air pollution are less than in a single point source problem. Therefore, the influence of multiple point sources, and the variable rates of sink, are also considered. It is found that a higher sink rate decreases pollutant levels around human habitats. Both finite difference methods are used to compute the numerical solutions of air pollution by MATLAB.

In the Numerical Simulation to Air Pollution Emission Control near an Industrial Zone, the atmospheric diffusion model to describe the released air pollutant concentration by an industrial plant is proposed. The concentration of the sulfur dioxide is approximated by an explicit forward time centered space finite difference technique. The method gives good agreement with approximated solutions. The air quality standards near the industrial zone are controlled by considering the approximated pollutant concentration levels at all monitoring points.

In the Numerical Simulation to Toxic Smoke Emission Control by Considering Primary and Secondary Pollutant Concentration near an Industrial Zone Using Multiple Air Quality Standards, the multiple atmospheric diffusion equations are studied by considering the air pollutant concentration near the industrial zone. The primary pollutant, sulfur dioxide ($SO_2$), is emitted from a chimney into the atmosphere. Then, the secondary pollutants, sulfur trioxide ($SO_3$) and sulfuric acid ($H_2SO_4$), are converted from primary pollutants by chemical reaction. The approximated sulfur dioxide, sulfur trioxide, and sulfuric acid concentrations are calculated by using the forward time central space (FTCS) scheme. Techniques of air pollution emission controls of sulfur dioxide, sulfur trioxide, and sulfuric acid concentration are proposed. The concentrations at considered monitoring points are compared with the air quality standards to find the appropriate emission control point. The research proposes that an effective monitoring point position is close to an industrial area. The chosen monitoring point gives the best overall air quality for emission control methods around industrial factories and residential areas.

We can summarize that an air pollution concentration is calculated by using the finite difference methods. The forward time central space scheme has advantages, in that the method requires less computing time than the backward time central space scheme. On the other hand, the forward time central space scheme also has disadvantages, in that there is a limitation of the grid spacing due to the stability condition. Additionally, the air pollution control simulations demonstrate that industrial plants need to shut down their chimneys for periods. Then, the position of collection for each monitoring point affects the air quality in the air pollution control technique. The air pollution management can also control the quantity of air pollution near industrial areas.

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# Appendices

# Appendix A
# Research Papers

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# Numerical Simulation of an Air Pollution Model on Industrial Areas by Considering the Influence of Multiple Point Sources

**Pravitra Oyjinda**[†,1] **and Nopparat Pochai**[†,‡]

[†]Department of Mathematics, Faculty of Science,
King Mongkut's Institute of Technology Ladkrabang,
Bangkok 10520, Thailand
e-mail : pravitra.o@gmail.com
[‡]Centre of Excellence in Mathematics
CHE, Si Ayutthaya Rd., Bangkok 10400, Thailand
e-mail : nop_math@yahoo.com

**Abstract :** A numerical simulation on a two-dimensional atmospheric diffusion equation of an air pollution measurement model is proposed. The considered area is separated into two parts such as an industrial zone and an urban zone. In this research, the air pollution measurement by releasing the pollutant from multiple point sources above an industrial zone to the other area is simulated. The governing partial differential equation of air pollutant concentration is approximated by using a finite difference technique. The approximate solutions of the air pollutant concentration on both areas are compared. The air pollutant concentration levels that influenced by multiple point sources are also analyzed.

**Keywords :** multiple point sources; finite difference technique; air pollutant concentration; industrial zone; urban zone

**2000 Mathematics Subject Classification :** 35K15; 65M06; 76R50

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[1]Corresponding author email: pravitra.o@gmail.com

# 1 Introduction

The rapid industrial growth can explain the air pollution affects the health of human being who lives around industrial areas. The air pollution has become a major problem for human life and environment. The purpose of this research is to study the air pollution assessment problem in two adjacent zones: industrial and urban zones by using the atmospheric diffusion model. In [1], the mathematical model is presented and used to study the dispersion of sulfur dioxide with constant and reference atmospheric stability. In [2], the smoke dispersion model in two-dimensional space with two and three point sources is proposed. The fractional step method, Carlson's method, and Crank-Nicolson method are used to solve the approximate solutions in [1] and [2]. They study a two-dimensional mathematical model of primary and secondary pollutants of an area source with chemical reaction and dry deposition by considering point source on the boundary in [4]. The finite difference method is used as Crank-Nicolson Implicit method. In [3], the air-quality model in three-dimensional with variations of the atmospheric stability classes and wind velocities from multiple sources is analyzed.

The source that is smokestack of industrial factory or power plant emitted the air pollution into the system. The genesis of air pollution is the cause of problems. In this research, the simple finite difference methods are used for solving the atmospheric diffusion equation.

# 2 Governing Equation

## 2.1 The Atmospheric Diffusion Equation

The diffusion model generally use Gaussian plume idea, which is the well-known atmospheric diffusion equation. It is to represent the behavior of air pollution in industrial areas. The dispersion of pollutant concentration from multiple point sources is described by a three-dimensional advection-diffusion equation [1],[2],[3], [4] following

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} = k_x\frac{\partial^2 c}{\partial x^2} + k_y\frac{\partial^2 c}{\partial y^2} + k_z\frac{\partial^2 c}{\partial z^2} + s, \tag{2.1}$$

where $c = c(x, y, z, t)$ is the concentration of air pollutant at $(x, y, z)$ and time $t$ $(kg/m^3)$, $u, v$, and $w$ are the wind velocity component $(m/sec)$ in $x, y, z$-direction respectively, $k_x, k_y$, and $k_z$ are the diffusion coefficient $(m^2/sec)$ in $x, y, z$-direction respectively, and $s$ is the sink rate of air pollutants $(sec^{-1})$.

The assumptions of equation (2.1) are defined that the concentrations of air pollutant are emitted from continued point sources. The advection and diffusion in $y$-direction are laterally averaged. By the assumption, we can also eliminate the term in $y$-direction. Therefore, the governing equation can be written as

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + w\frac{\partial c}{\partial z} = k_x\frac{\partial^2 c}{\partial x^2} + k_z\frac{\partial^2 c}{\partial z^2} + s. \tag{2.2}$$

The initial condition is assumed under the cold start assumption. That is

$$c(x, z, 0) = 0, \tag{2.3}$$

for all $x > 0$ and $z > 0$. The boundary conditions are assumed that

$$c(0, z, t) = 0, \tag{2.4}$$

$$\frac{\partial c}{\partial x}(L, z, t) = \frac{\partial c}{\partial z}(x, 0, t) = \frac{\partial c}{\partial z}(x, H, t) = 0, \tag{2.5}$$

for all $t > 0$ where $L$ is the length of the domain in $x$-direction and $H$ is the height of the inversion layer. The concentration at the point sources is assumed to be the constant variables as

$$c(x_p, 0, t) = c_{s_p}, \tag{2.6}$$

for $p = 1, 2$ where $x_p$ is the position of the point source $p$ in the $x$-direction and $c_{s_p}$ is the concentration value at the point source of $p$.

## 2.2 The Non-dimensional Form Equation

Now, we introduce the dimensionless form of equation (2.2). The non-dimensional variables are denoted by letting $C = c/c_{\max}$, $X = x/l_x$, $Z = z/l_z$, $T = t/t_{\max}$, $D_x = k_x/l_x u_{\max}$, $D_z = k_z/l_z u_{\max}$, $U = u/u_{\max}$, and $W = \beta w_{\max}/u_{\max}$ when $\beta = w/w_{\max}$. We define $c_{\max} = \max\{c(x, z, t) \colon 0 \leqslant x \leqslant L, 0 \leqslant z \leqslant H, 0 \leqslant t \leqslant t_{\max}\}$, $u_{\max} = \max\{u(x, z, t) \colon 0 \leqslant x \leqslant L, 0 \leqslant z \leqslant H, 0 \leqslant t \leqslant t_{\max}\}$, $w_{\max} = \max\{w(x, z, t) \colon 0 \leqslant x \leqslant L, 0 \leqslant z \leqslant H, 0 \leqslant t \leqslant t_{\max}\}$, and $t_{\max}$ is a stationary time. Thus the non-dimensional equation of air pollution as follows:

$$\frac{1}{St}\frac{\partial C}{\partial T} + U\frac{\partial C}{\partial X} + W\frac{\partial C}{\partial Z} = D_x\frac{\partial^2 C}{\partial X^2} + D_z\frac{\partial^2 C}{\partial Z^2} + S, \tag{2.7}$$

where $l = \max\{l_x, l_z\}$ and $St = \dfrac{t_{\max} u_{\max}}{l}$.

# 3 Numerical Methods

We use the finite difference methods for calculating the non-dimensional form of the atmospheric diffusion equation. In equation (2.7), we get the concentration of $C$ at each time $T_{n+1}$ from $T_n$ when $\Delta T$ is a time increment. The solution of concentration at $(X, Z, T)$ is denoted by $C(X_i, Z_j, T_n) = C^n_{i,j}$. The domain is divided by the grid spacing in $X$ and $Z$-direction, $\Delta X$ and $\Delta Z$ respectively where $X_i = i\Delta X$ and $Z_j = j\Delta Z$ . The approximate solutions are obtained by using the following methods:

### 3.1 Forward Time Central Space Scheme

The first method refer to the non-dimensional of the model, which we use the forward time central space (FTCS) scheme. Thus, the formula of equation (2.7) is

$$\begin{aligned} C_{i,j}^{n+1} =& (d_x - A_x)C_{i+1,j}^{n} + (d_x + A_x)C_{i-1,j}^{n} + (1 - 2d_x - 2d_z)C_{i,j}^{n} \\ &+ (d_z + A_z)C_{i,j-1}^{n} + (d_z - A_z)C_{i,j+1}^{n} + St(\Delta T)S, \end{aligned} \quad (3.1)$$

where $A_x = \dfrac{St(\Delta T)U}{2\Delta X}, A_z = \dfrac{St(\Delta T)W}{2\Delta Z}, d_x = \dfrac{St(\Delta T)D_x}{(\Delta X)^2}, d_z = \dfrac{St(\Delta T)D_z}{(\Delta Z)^2}$.

### 3.2 Backward Time Central Space Scheme

The second method, we calculated by using the backward time central space (BTCS) scheme. It is used to discretize the governing equation. The finite difference equation can also be obtained

$$\begin{aligned} (A_x - d_x)C_{i+1,j}^{n+1} + (A_x + d_x)C_{i-1,j}^{n+1} + (1 + 2d_x + 2d_z)C_{i,j}^{n+1} \\ -(A_z + d_z)C_{i,j-1}^{n+1} + (A_z - d_z)C_{i,j+1}^{n+1} = C_{i,j}^{n} + St(\Delta T)S. \end{aligned} \quad (3.2)$$

## 4 Numerical Experiment

The two-dimensional atmospheric diffusion equation (2.7) with a dimension $1 \times 1\ km^2$ will be considered. The uniform wind velocities and constant diffusion coefficients are introduced. We choose the wind velocities in $x$ and $z$-direction are 0.1 and 0.05 $m/sec$ respectively. The diffusion coefficients in $x$ and $z$-direction are $4.5\times10^{-1}$ and $4.5\times10^{-5}$ $m^2/sec$ respectively. The grid spacing: $\Delta x = \Delta z = 25$ m. and the time interval is 20 *sec*. In this research, we consider two cases. The first case, there is a point source, which the concentration is 0.5 $kg/m^3$. The second case, there are two point sources, which the concentration are 0.25 and 0.25 $kg/m^3$. The air pollutants in equation (2.6) are released into our system. This example is solved by using FTCS and BTCS in equation (3.1) and (3.2) respectively with the initial and boundary conditions (2.3) to (2.5).

In Fig.1, model of the problem is shown. The physical problem composed of two zones: an industrial zone and an urban zone with the stable wind along the $x$ and $z$-axis. The point sources are lied along the $x$-axis. We assume that the primary air pollutants are released by a factory smokestack as a single point source and a couple point sources from industrial zone. The emissions of air pollution are influenced on the urban zone by the rate of air pollutant absorption. In the numerical experiment, the considered domain of solutions is shown in Fig.2.

## 5 Discussion

The air pollutant emission from multiple point sources above an industrial zone to the urban area is presented. The finite difference technique introduced

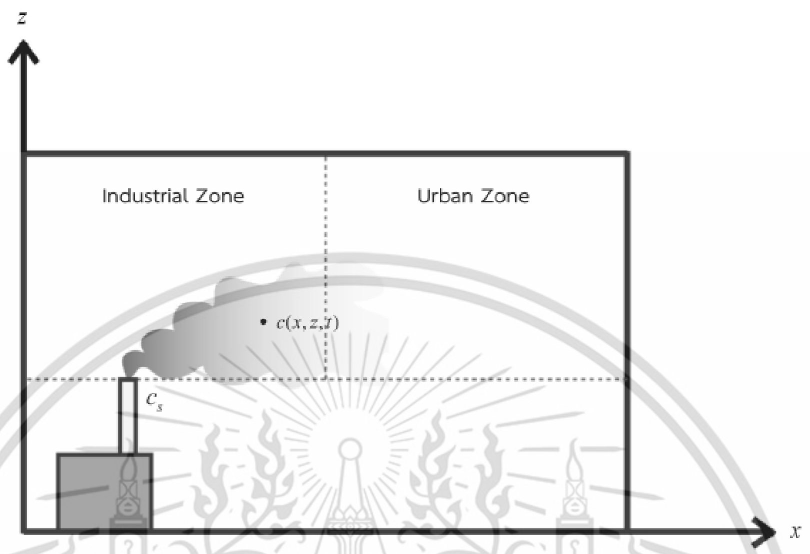



Figure 1: Model of the problem

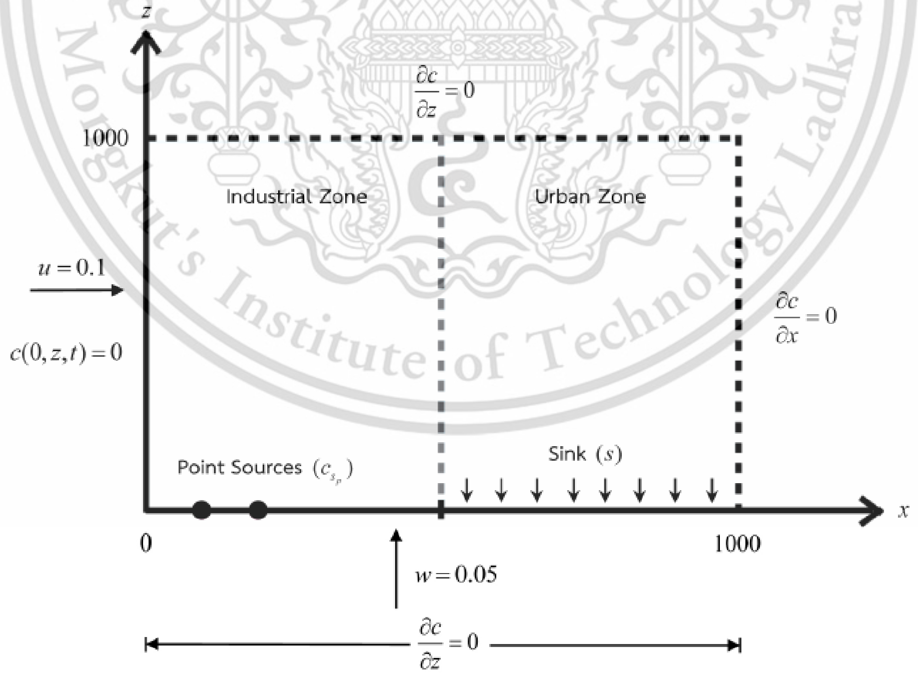



Figure 2: Domain of solutions

two methods for calculating the air pollutant concentrations. Figs.3 and 4 are to compare the air pollutant concentrations between two cases: a single point source and a couple point sources, respectively. From the both figures, it was apparent that the results of FTCS is close to the results of BTCS, when there is no sink of pollutant absorption ($s = 0$). Figs.5 and 6 are to illustrate that the sink of pollutant absorption ($s = -10^{-4}$) are added in the base of urban zone. The air pollutant concentration near human living goes down and the two methods also give the close result. In Figs.7 and 8, the computed approximate solution that obtained by using FTCS and BTCS are compared. We can see that the results of added sink case and without sink case are quite similar. These graphs also indicate that the FTCS gives the computed solutions close to the BTCS.

From Figs.9 and 10 demonstrate that the air pollutant concentration at the height $z = 25$ m. and $z = 50$ m. are solved by using FTCS. The case of added sink is less concentration than the case of without sink. Therefore, the sink can lower the overall pollutant levels. Fig.11 establishes the variant concentration when we take more sink rate into our system. The comparison of computing time shows that FTCS is faster than BTCS in Table 1.

Table 1: Computing time comparison of FTCS and BTCS

| Simulation Time | FTCS | BTCS |
|---|---|---|
| 30 min. | 1.49 sec. | 22.48 sec. |
| 1 hr. | 1.68 sec. | 42.66 sec. |
| 2 hrs. | 2.05 sec. | 84.18 sec. |

## 6 Conclusion

The simple air pollution measurement models, which air pollutants are released by a single point source and couple point sources are proposed. The traditional finite difference methods such as FTCS and BTCS can be used to approximate the air pollutant levels for each points and times. The results of this study show that the air pollutant concentrations of FTCS are close to the air pollutant concentrations of BTCS. In the case of a couple point sources problem, the overall concentration levels of air pollution are less than a single point source problem. Therefore, the influence of multiple point sources and the variable rate of sink are also considered. It obtains that the higher sink rate does decrease pollutant levels around human living. The both finite difference methods are used to compute the numerical solutions of air pollution by MATLAB. In this experiment, FTCS gives less computing time than BTCS.

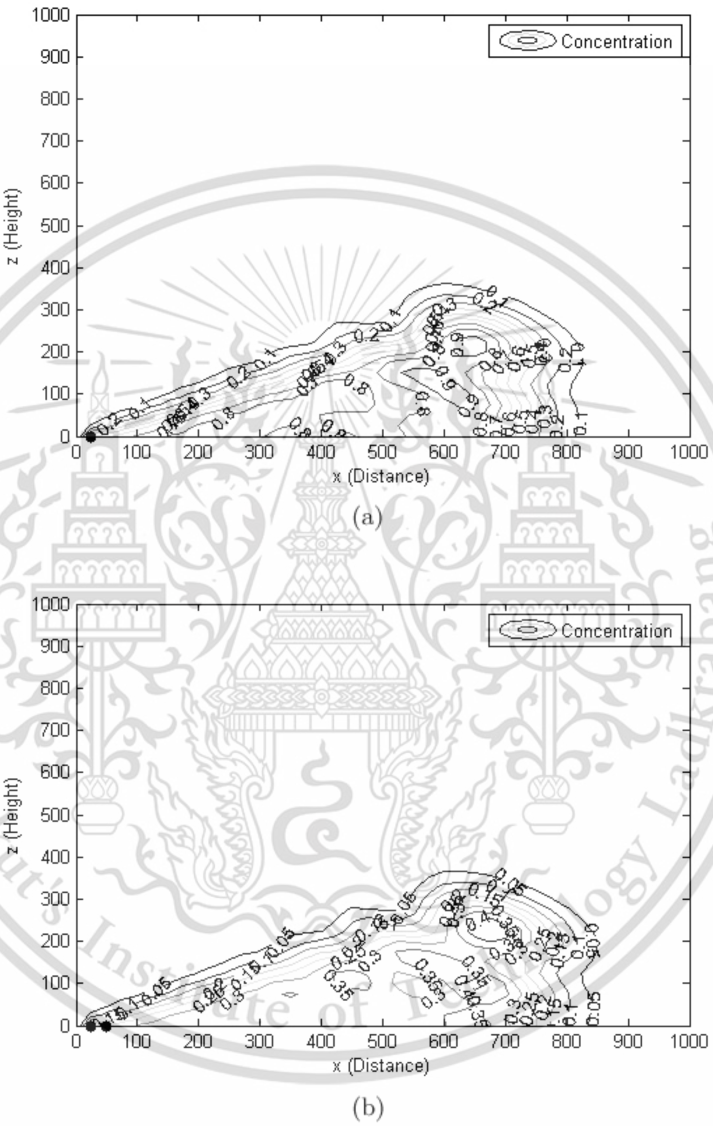
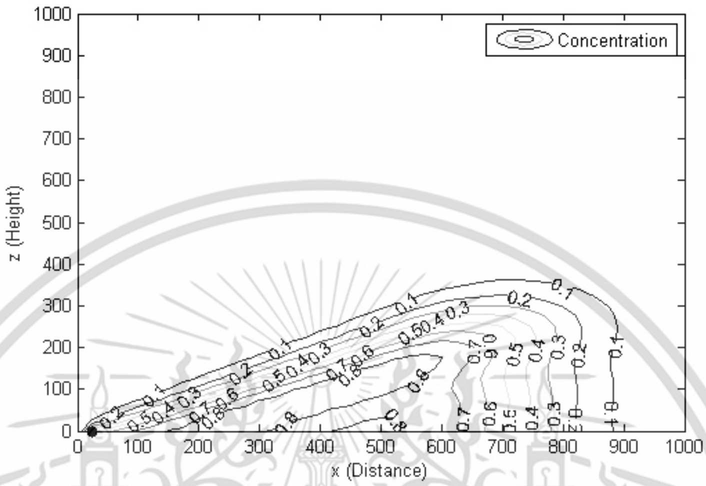
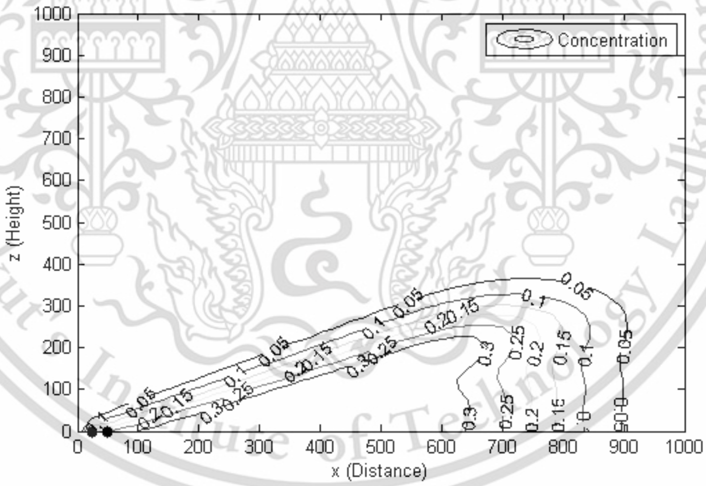



Figure 3: The air pollutant concentration levels after 2 hours passed which are computed by FTCS (There is no sink of pollutant absorption $s = 0$) (a) one point source and (b) two point sources




(a)




(b)

Figure 4: The air pollutant concentration levels after 2 hours passed which are computed by BTCS (There is no sink of pollutant absorption $s = 0$) (a) one point source and (b) two point sources

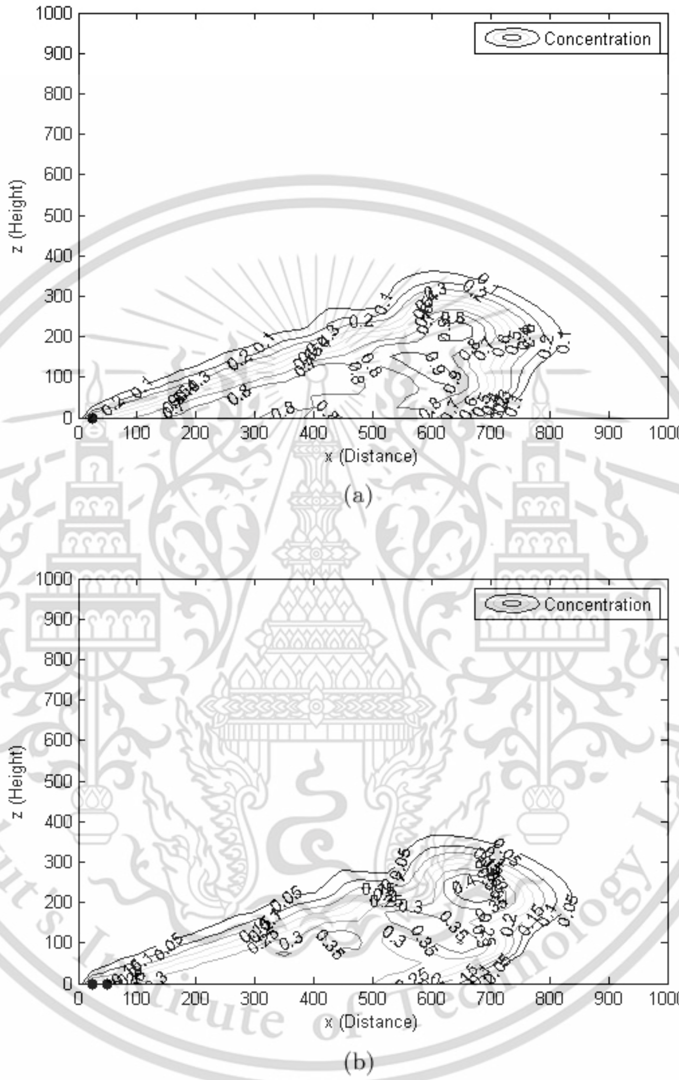



Figure 5: The air pollutant concentration levels after 2 hours passed which are computed by FTCS (There are sink of pollutant absorption $s = -10^{-4}$) (a) one point source and (b) two point sources

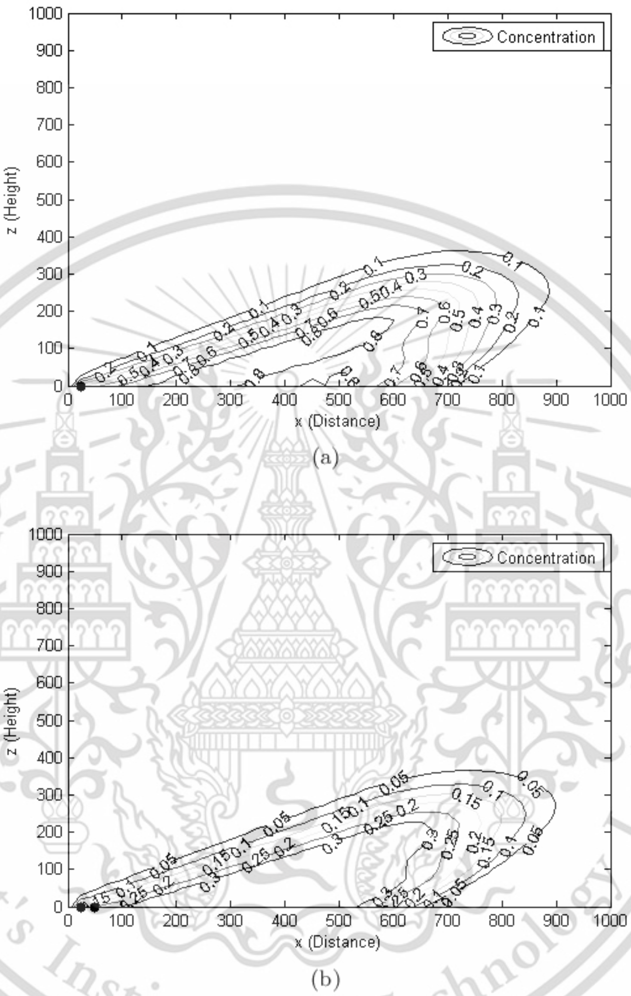



Figure 6: The air pollutant concentration levels after 2 hours passed which are computed by BTCS (There are sink of pollutant absorption $s = -10^{-4}$) (a) one point source and (b) two point sources

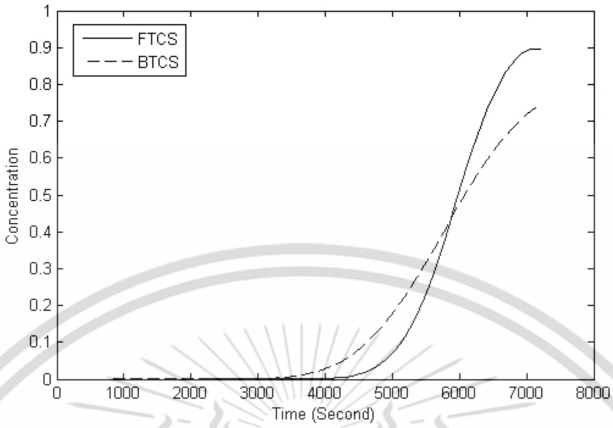



Figure 7: The air pollutant concentration between FTCS and BTCS (There is no sink of pollutant absorption $s = 0$) at $z = 0$ m. and $x = 600$ m.

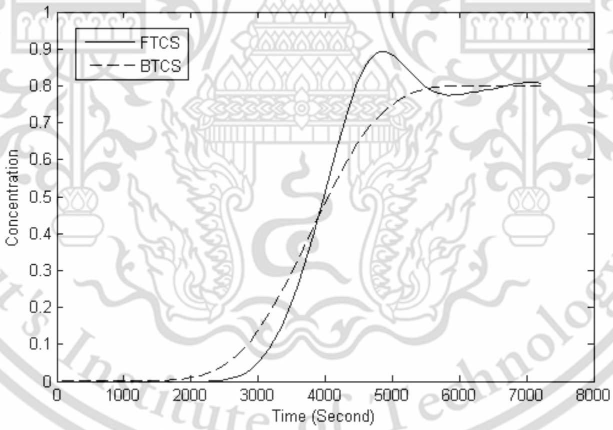



Figure 8: The air pollutant concentration between FTCS and BTCS (There are sink of pollutant absorption $s = -10^{-4}$) at $z = 0$ m. and $x = 400$ m.

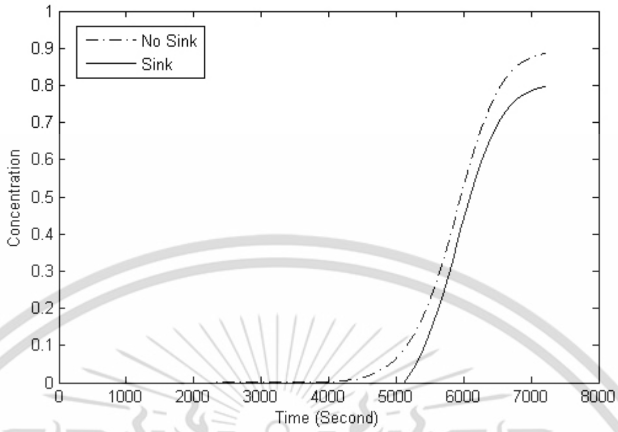



Figure 9: The air pollutant concentration between 2 case: added sink and without sink (computed by FTCS) at $z = 25$ m. and $x = 600$ m.

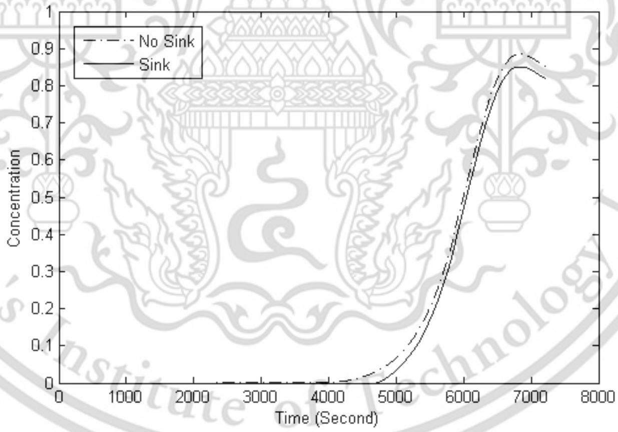



Figure 10: The air pollutant concentration between 2 case: added sink and without sink (computed by FTCS) at $z = 50$ m. and $x = 600$ m.

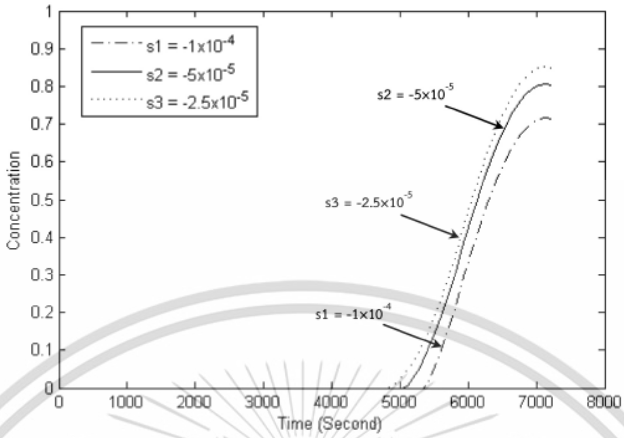



Figure 11: The air pollutant concentration with the variant values of sink rate (computed by FTCS) at $z = 0$ m. and $x = 600$ m.

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*Research Article*

# Numerical Simulation to Air Pollution Emission Control near an Industrial Zone

**Pravitra Oyjinda[1,2] and Nopparat Pochai[1,2]**

[1]*Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand*
[2]*Centre of Excellence in Mathematics, CHE, Si Ayutthaya Rd., Bangkok 10400, Thailand*

Correspondence should be addressed to Nopparat Pochai; nop_math@yahoo.com

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A rapid industrial development causes several environment pollution problems. One of the main problems is air pollution, which affects human health and the environment. The consideration of an air pollutant has to focus on a polluted source. An industrial factory is an important reason that releases the air pollutant into the atmosphere. Thus a mathematical model, an atmospheric diffusion model, is used to estimate air quality that can be used to describe the sulfur dioxide dispersion. In this research, numerical simulations to air pollution measurement near industrial zone are proposed. The air pollution control strategies are simulated to achieve desired pollutant concentration levels. The monitoring points are installed to detect the air pollution concentration data. The numerical experiment of air pollution consisted of different situations such as normal and controlled emissions. The air pollutant concentration is approximated by using an explicit finite difference technique. The solutions of calculated air pollutant concentration in each controlled and uncontrolled point source at the monitoring points are compared. The air pollutant concentration levels for each monitoring point are controlled to be at or below the national air quality standard near industrial zone index.

## 1. Introduction

Nowadays, the air pollution is a major problem in the world because industrial areas grew rapidly. The pollution emission of factories into the atmosphere will have an effect on human health and the environment. The purpose of this research is to study the problem of air pollution emission control. The approximate solution is considered by using the atmospheric diffusion model.

In [1], an atmospheric transport diffusion model with wind velocity profile and diffusion coefficient was considered to study the system of delayed removal. The air pollutant was emitted from a line source with the dry deposition on the ground. The fractional step method was used for computing the air pollutant concentration. In [2], the atmospheric diffusion equation with multiple sources and wind speed and eddy diffusivities was studied to derive the analytical solutions for many boundary condition types. The Green's function concept was used to solve the three-dimensional analytical solutions everywhere in the region of interest. In [3], the finite difference method was used for solving the two-dimensional advection-diffusion equation with a point source. In [4], a time dependent mathematical model of primary and secondary pollutants was studied for approximating the concentration from area source. The wind velocities and eddy diffusion coefficients are considered to be the realistic value. The researchers solved the problem by using Crank-Nicolson implicit finite difference technique and upwind difference scheme which is applied to the diffusion term. In [5], the researchers studied the three-dimensional mathematical model for the sulfur dioxide concentration without obstacles domain.

In [6], the researchers studied a three-dimensional convection-diffusion-reaction equation for sulfur and nitrogen oxides. The model was solved by using a high order accurate time-stepping discretization scheme as Lax and Wendroff technique. A steady state two-dimensional mathematical model of urban heat island was used to describe the dispersion of air pollution with mesoscale wind velocity and meteorological parameters in [7]. The genesis of air pollution was area source emitted from the ground. The removal mechanism was considered by wet and dry depositions. The concentration of air pollutant was approximated by using Crank-Nicolson implicit method. In [8], the mass transport model was considered to simulate the smoke dispersion from one and two point sources with obstacle domain. The model consisted of three equations: a stream function, vorticity, and convection-diffusion equation. The results of air pollution in two-dimensional space and one-dimensional time were calculated by using the finite element method and finite difference method, respectively. In [9], the two-dimensional smoke dispersion model was studied in the cases of two and three point sources with obstacles domain. In [10], the researchers studied a spatial autoregressive model for sulfur dioxide concentration. The evaluation of sulfur dioxide was assessed by the land use regression (LUR) model. The mobile monitoring was used for collecting concentration data in Hamilton, Ontario, Canada.

In [11], the dispersion of primary pollutant was studied in a two-dimensional air pollution model with mesoscale wind. The primary air pollutant was emitted from an area source and the researchers considered removal mechanisms such as dry deposition, gravitational settling, and chemical reaction. The two-dimensional advection-diffusion models of the primary and secondary pollutants are presented in [12]. The researchers studied the air pollutant emitted from area source with removal mechanisms by considering point source on the boundary. The Crank-Nicolson implicit method is used as the finite difference technique in [11, 12]. The design and application of Atmospheric Evaluation and Research Integrated model for Spain (AERIS) are proposed in [13]. The air pollutant concentrations of $NO_2$, $O_3$, $SO_2$, $NH_3$, and PM as a reaction to emission variations of significant sectors in Spain are obtained by AERIS. The results of the model are estimated by using transfer matrices based on an air quality modelling system (AQMS). The system consists of the Weather Research and Forecast (WRF), Sparse Matrix Operator Kernel Emissions (SMOKE), and Community Multiscale Air Quality (CMAQ) models. In [14], the researchers studied air flow and dispersion of pollutant in urban street canyons. The Computational Fluid Dynamics (CFD) were simulated by using Large Eddy Simulation (LES). A velocity comparison between Fluctuating Wind Boundary Conditions (FWBC) and Steady Wind Boundary Conditions (SWBC) was investigated. In [15], the researchers used the three-dimensional air quality model. The considered domain contained three buildings (obstacles) divided into two zones: a factory zone and a residential zone. The modifications of atmospheric stability classes and wind velocities from multiple point sources were also analyzed. The approximate solutions in [5, 9, 15] were solved by using the fractional step method.

A numerical model for air pollution emission control problem with the uniform wind velocities and constant diffusion coefficients is proposed. In this research, the atmospheric diffusion equation is solved by using the finite difference method. This study analyzed the ambient air quality standard of sulfur dioxide that refers to the quantity of sulfur dioxide concentration in clean air.

## 2. Governing Equation

*2.1. The Atmospheric Diffusion Equation.* The diffusion model is used to represent the behavior of air pollutant concentration in industrial areas. The Gaussian plume idea is used as the governing equation. It is the well-known atmospheric diffusion equation. We introduced the three-dimensional advection-diffusion equation as follows:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} = k_x\frac{\partial^2 c}{\partial x^2} + k_y\frac{\partial^2 c}{\partial y^2} + k_z\frac{\partial^2 c}{\partial z^2} + S + R, \quad (1)$$

where $c = c(x, y, z, t)$ is the air pollutant concentration at $(x, y, z)$ and time $t$ (kg/m$^3$), $u$, $v$, and $w$ are the wind velocity components (m/s) in $x$-, $y$-, and $z$-direction, respectively (m/s), $k_x$, $k_y$, and $k_z$ are the diffusion coefficients in $x$-, $y$-, and $z$-direction, respectively (m$^2$/s), $S$ is the growth of pollutant rate due to sources (sec$^{-1}$), and $R$ is the decaying of pollutant rate due to sinks (sec$^{-1}$).

In this research, we considered only the primary pollutant concentration as sulfur dioxide. The chemical formula is $SO_2$. The assumption of (1) defined that the advection and diffusion in $y$-direction are laterally averaged. By the assumption, we can also eliminate the term in $y$-direction. Therefore, the primary pollutant equation can be written as

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + w\frac{\partial c}{\partial z} = k_x\frac{\partial^2 c}{\partial x^2} + k_z\frac{\partial^2 c}{\partial z^2} + S + R. \quad (2)$$

The initial condition is assumed under the cold start assumption. That is,

$$c(x, z, 0) = 0, \quad (3)$$

for all $x > 0$ and $z > 0$. The boundary conditions assumed that

$$\frac{\partial c}{\partial x}(0, z, t) = \frac{\partial c}{\partial x}(L, z, t) = 0,$$
$$\frac{\partial c}{\partial z}(x, H, t) = 0, \quad (4)$$
$$\frac{\partial c}{\partial z}(x, 0, t) = v_d c,$$

for all $t > 0$, where $L$ is the length of the domain in $x$-direction, $H$ is the height of the inversion layer, and $v_d$ is the dry deposition velocity of the primary pollutant (m/s). Sulfur dioxide deposition velocity can be related to a diffusion coefficient $k_z$ which is assumed to be an irreversible process.

In Figure 1, model of air pollution emission control problem is presented. This research was designed to study the behavior of dispersion and effect of dispersion concentration near the industrial zone. The four monitoring points are set far away from the source. Each monitoring point is called $M1$, $M2$, $M3$, and $M4$ respectively. In Figure 2, the considered domain for the numerical experiment is shown. Let the height

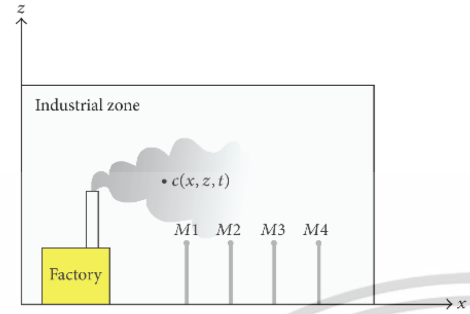


FIGURE 1: Model of air pollution emission control problem.

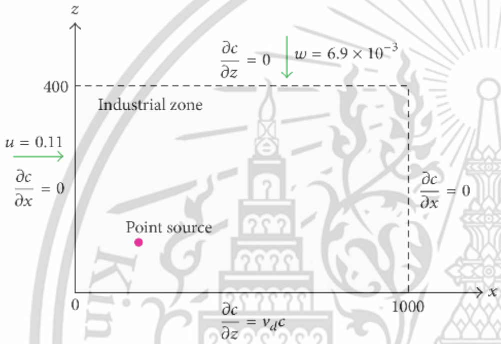


FIGURE 2: Domain of approximate solutions.

of point source be $z = h_s$ m. The wind is stable in $x$- and $z$-axis. The concentrations of air pollutant are emitted directly from a continued point source (chimney) from industrial factory. The air pollutants are absorbed from the chemical reaction on the ground.

*2.2. The Nondimensional Form Equation.* From (2), we present the nondimensional form of air pollution. The following dimensionless variables are defined by $C = c/c_{\max}$, $X = x/l_x$, $Z = z/l_z$, $T = t/t_{\max}$, $D_x = k_x/l_x u_{\max}$, $D_z = k_z/l_z u_{\max}$, $U = u/u_{\max}$, and $W = \beta w_{\max}/u_{\max}$ when $\beta = w/w_{\max}$. We let $c_{\max} = \max\{c(x,z,t) : 0 \le x \le L,\ 0 \le z \le H,\ 0 \le t \le t_{\max}\}$, $u_{\max} = \max\{u(x,z,t) : 0 \le x \le L,\ 0 \le z \le H,\ 0 \le t \le t_{\max}\}$, and $w_{\max} = \max\{w(x,z,t) : 0 \le x \le L,\ 0 \le z \le H,\ 0 \le t \le t_{\max}\}$, and $t_{\max}$ is a stationary time. Therefore, the nondimensional atmospheric diffusion equation can be rearranged to give

$$\frac{1}{St}\frac{\partial C}{\partial T} + U\frac{\partial C}{\partial X} + W\frac{\partial C}{\partial Z} = D_x\frac{\partial^2 C}{\partial X^2} + D_z\frac{\partial^2 C}{\partial Z^2} + S - k_p C, \quad (5)$$

where $l = \max\{l_x, l_z\}$, $St = t_{\max}u_{\max}/l$, and $k_p$ is the chemical interaction rate of primary pollutant equation. For the nondimensional form of initial condition, it is assumed that

$$C(X, Z, 0) = 0, \quad (6)$$

for all $X > 0$ and $0 \le Z \le H$. For the nondimensional form of boundary, it is assumed that

$$\frac{\partial C}{\partial X}(0, Z, T) = \frac{\partial C}{\partial X}(L, Z, T) = 0, \quad (7)$$

$$\frac{\partial C}{\partial Z}(X, H, T) = 0, \quad (8)$$

$$\frac{\partial C}{\partial Z}(X, 0, T) = v_d C, \quad (9)$$

for all $T > 0$.

## 3. Numerical Method

The prediction of primary pollutant from a stationary source can be calculated to solve the air pollution problem in the industrial areas. In (5), we get the concentration of $C$ at each time $T_{n+1}$ from $T_n = n\Delta T$, $n = 0, 1, 2, \ldots, P$, when $\Delta T$ is a time increment. The solution of sulfur dioxide concentration at $(X, Z, T)$ is denoted by $C(X_i, Z_j, T_n) = C^n_{i,j}$. The considered domain is meshed by the grid spacing $\Delta X$ and $\Delta Z$ where $X_i = i\Delta X$, $i = 0, 1, 2, \ldots, N$, and $Z_j = j\Delta Z$, $j = 0, 1, 2, \ldots, M$. The finite difference method is chosen as proper equipment for estimating solutions. The method refers to the nondimensional model, for which we use the forward time central space (FTCS) scheme. In the transient term, we used the forward difference for

$$\frac{\partial C}{\partial T} = \frac{C^{n+1}_{i,j} - C^n_{i,j}}{\Delta T}. \quad (10)$$

The advection and diffusion terms are substituted by using the centered difference in space by

$$\frac{\partial C}{\partial X} = \frac{C^n_{i+1,j} - C^n_{i-1,j}}{2\Delta X}, \quad (11)$$

$$\frac{\partial C}{\partial Z} = \frac{C^n_{i,j+1} - C^n_{i,j-1}}{2\Delta Z}, \quad (12)$$

$$\frac{\partial^2 C}{\partial X^2} = \frac{C^n_{i+1,j} - 2C^n_{i,j} + C^n_{i-1,j}}{(\Delta X)^2}, \quad (13)$$

$$\frac{\partial^2 C}{\partial Z^2} = \frac{C^n_{i,j+1} - 2C^n_{i,j} + C^n_{i,j-1}}{(\Delta Z)^2}, \quad (14)$$

respectively. The formula of (5) becomes

$$\begin{aligned}\frac{1}{St}\left(\frac{C^{n+1}_{i,j} - C^n_{i,j}}{\Delta T}\right) + U\left(\frac{C^n_{i+1,j} - C^n_{i-1,j}}{2\Delta X}\right) \\ + W\left(\frac{C^n_{i,j+1} - C^n_{i,j-1}}{2\Delta Z}\right) \\ = D_x\left(\frac{C^n_{i+1,j} - 2C^n_{i,j} + C^n_{i-1,j}}{(\Delta X)^2}\right) \\ + D_z\left(\frac{C^n_{i,j+1} - 2C^n_{i,j} + C^n_{i,j-1}}{(\Delta Z)^2}\right) + S - k_p C^n_{i,j}.\end{aligned} \quad (15)$$

Thus, the finite difference form of the advection-diffusion equation becomes

$$\begin{aligned} C_{i,j}^{n+1} &= (d_x - A_x) C_{i+1,j}^{n} + (d_x + A_x) C_{i-1,j}^{n} \\ &\quad + \left(1 - 2d_x - 2d_z - k_p \Delta T (St)\right) C_{i,j}^{n} \\ &\quad + (d_z + A_z) C_{i,j-1}^{n} + (d_z - A_z) C_{i,j+1}^{n} \\ &\quad + St(\Delta T) S, \end{aligned} \quad (16)$$

where $A_x = St(\Delta T)U/2\Delta X, A_z = St(\Delta T)W/2\Delta Z, d_x = St(\Delta T)D_x/(\Delta X)^2$, and $d_z = St(\Delta T)D_z/(\Delta Z)^2$.

## 4. Air Pollution Controlled Simulations

The experiment analyzed the action of air pollution with the volume of sulfur dioxide emission around an industrial zone. We will simulate the air pollution control situation in three cases. For the first simulation, an industrial factory released continued air pollutant from a chimney without emission control system. For the second and the third simulations, the factory will discharge the sulfur dioxide, which is controlled by the national air quality index.

*4.1. Simulation 1: Air Pollution Emission without Controlled System.* In the first simulation, the two-dimensional advection-diffusion equation (5) with a domain of interest of $1000 \times 400\,m^2$ is considered. The wind velocities in $x$- and $z$-direction are assumed to be 0.11 and $6.9 \times 10^{-3}$ m/s, respectively. The sulfur dioxide is released at the chimney height $h_s = 75$ m at coordinate (100, 75) (m, m). The released pollutant concentration is $0.75\,sec^{-1}$. The diffusion coefficients in $x$- and $z$-direction are 2 and $0.45\,m^2/s$, respectively. The grid spacing is $\Delta x = \Delta z = 25$ m and time interval is 72 sec. This simulation is solved by using FTCS in (16) with the initial and boundary conditions (6)–(9). The numerical solutions of air pollutant concentration when 58 minutes and 1 hour and 36 minutes have passed are shown in Figures 3 and 4, respectively. The monitoring points are aligned along 200, 300, 400, and 500 m in the same height, 50 m. The comparison of concentrations of different distances is presented in Figure 5.

*4.2. Simulation 2: Air Pollution Emission Controlled by following the National Air Quality Standard ($3\times10^{-7}$ kg/m$^3$).* In the second simulation, the two-dimensional advection-diffusion equation (5) with a domain of interest of $1000 \times 400\,m^2$ is considered. The wind velocities in $x$- and $z$-direction are assumed to be 0.11 and $6.9\times10^{-3}$ m/s, respectively. The sulfur dioxide is released at the chimney height $h_s = 75$ m at the coordinate (100, 75) (m, m). The released pollutant concentration is $0.75\,sec^{-1}$. The diffusion coefficients in $x$- and $z$-direction are 2 and $0.45\,m^2/s$, respectively. The grid spacing is $\Delta x = \Delta z = 25$ m and time interval is 72 sec. In this simulation, the sulfur dioxide is released by following the United States Environmental Protection Agency (USEPA) air quality standard [16], $3 \times 10^{-7}\,kg/m^3$. If the approximated pollutant concentration at a monitoring point becomes higher than the

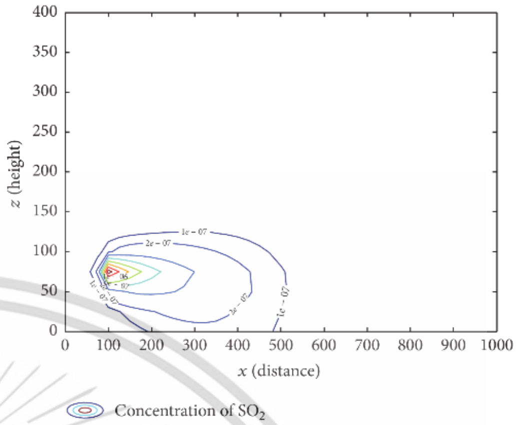


FIGURE 3: The concentration levels of air pollution after 58 minutes have passed without control system.

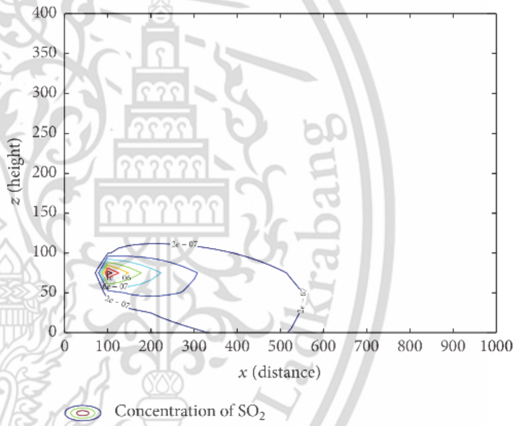


FIGURE 4: The concentration levels of air pollution after 1 hour and 36 minutes have passed without control system.

air quality standard, then the chimney will be shut down and wait until the concentration goes below $1.5 \times 10^{-7}\,kg/m^3$. If the pollutant concentration at all monitoring points is below a half of the air quality standard, the chimney will be opened again. The air pollution emission will be following these processes. This example is solved by using FTCS in (16) with the initial and boundary conditions (6)–(9). The results of air pollution emission control are demonstrated as the contour lines of sulfur dioxide concentration in Figures 6 and 7. The concentration of air pollution in the different distance is shown in Figure 8.

*4.3. Simulation 3: Air Pollution Emission Controlled by following the National Air Quality Standard ($1.5 \times 10^{-7}$ kg/m$^3$).* In the third simulation, the two-dimensional advection-diffusion equation (5) with a domain of interest of $1000\times 400\,m^2$ is considered. The wind velocities in $x$- and $z$-direction

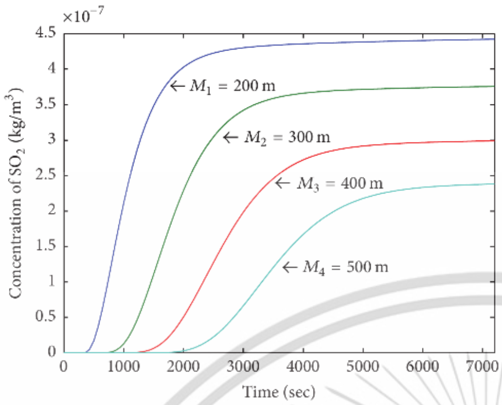


FIGURE 5: The concentration of air pollution with the different monitoring point at $z = 50$ m without control system.

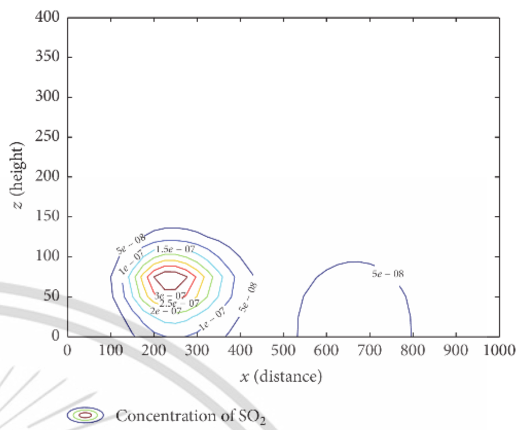


FIGURE 7: The concentration levels of air pollution after 1 hour and 36 minutes have passed which are controlled by air quality standard ($3 \times 10^{-7}$ kg/m$^3$).

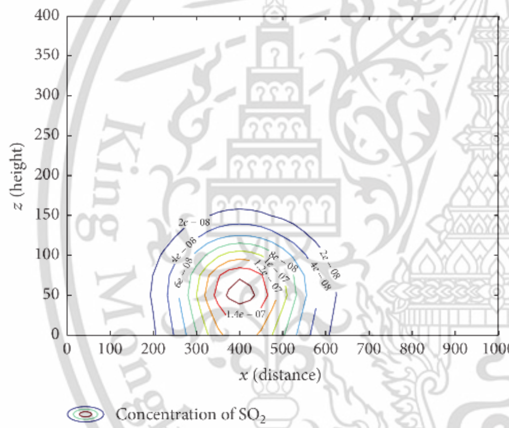


FIGURE 6: The concentration levels of air pollution after 58 minutes have passed which are controlled by air quality standard ($3 \times 10^{-7}$ kg/m$^3$).

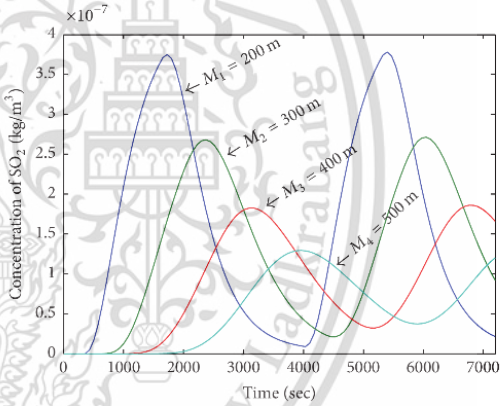


FIGURE 8: The concentration of air pollution with the different monitoring point at $z = 50$ m by air quality standard ($3 \times 10^{-7}$ kg/m$^3$).

are assumed to be 0.11 and $6.9 \times 10^{-3}$ m/s, respectively. The sulfur dioxide is released at the chimney height $h_s = 75$ m at the coordinate (100, 75) (m, m). The released pollutant concentration is 0.75 sec$^{-1}$. The diffusion coefficients in $x$- and $z$-direction are 2 and 0.45 m$^2$/s, respectively. The grid spacing is $\Delta x = \Delta z = 25$ m and time interval is 72 sec. In this simulation, the sulfur dioxide is released by following the USEPA air quality standard, $1.5 \times 10^{-7}$ kg/m$^3$. If the approximated pollutant concentration at a monitoring point becomes higher than the air quality standard, then the chimney will be shut down and wait until the concentration goes below $1.0 \times 10^{-7}$ kg/m$^3$. If the pollutant concentration at all monitoring points is below a third of the air quality standard, the chimney will be opened again. The air pollution emission will be following these processes. This simulation is solved by using FTCS in (16) with the initial and boundary conditions (6)–(9). In this emission control case, the concentrations of air pollution when 58 minutes and 1 hour and 36 minutes have passed are shown in Figures 9 and 10, respectively. The concentration of $SO_2$ when 2 hours have passed with the different distance is shown in Figure 11.

From Simulations 1, 2, and 3, the concentrations of $SO_2$ at the height $z = 50$ m and the distance $x = 300$ m ($M2$) are compared in Figure 12.

## 5. Conclusion

The atmospheric diffusion model to describe the released air pollutant concentration by an industrial plant is proposed.

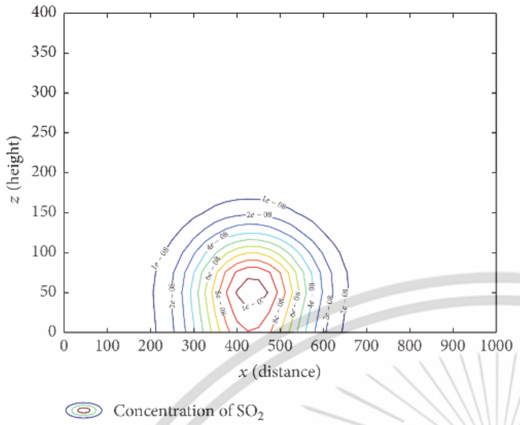


FIGURE 9: The air pollution concentration levels after 58 minutes have passed which are controlled by air quality standard ($1.5 \times 10^{-7}$ kg/m$^3$).

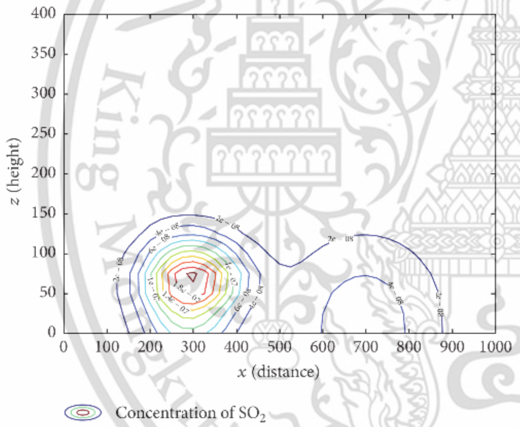


FIGURE 10: The air pollution concentration levels after 1 hour and 36 minutes have passed which are controlled by air quality standard ($1.5 \times 10^{-7}$ kg/m$^3$).

The concentration of the sulfur dioxide is approximated by an explicit forward time centered space finite difference technique. The method gives good agreement of approximated solutions. The air quality standard near industrial zone is controlled by considering the approximated pollutant concentration levels at all monitoring points. The proposed air pollution controlled simulations demonstrated that the industrial plants need to shut down their chimneys for a while.

## Conflicts of Interest

The authors declare no conflicts of interest.

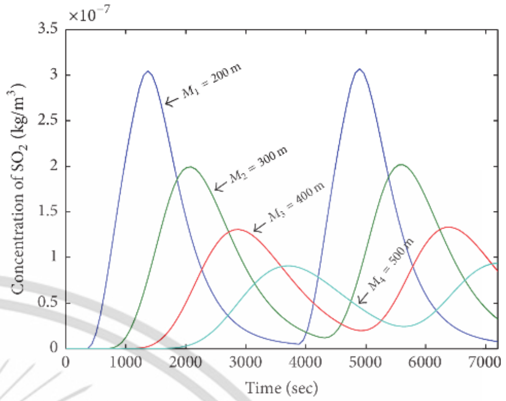


FIGURE 11: The concentration of air pollution with the different monitoring point at $z = 50$ m by air quality standard ($1.5 \times 10^{-7}$ kg/m$^3$).

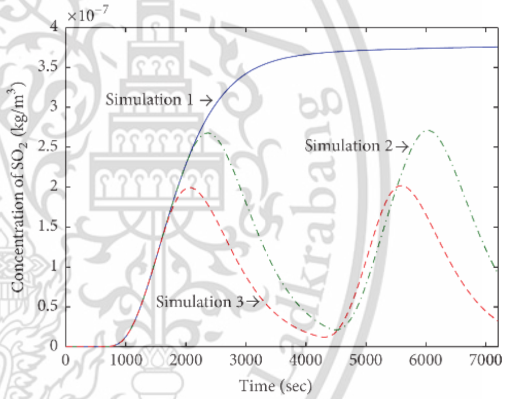


FIGURE 12: The air pollution concentration at $z = 50$ m and $x = 300$ m ($M2$) in three cases.


## Acknowledgments

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Gmail **Pravitra Oyjinda <pravitra.o@gmail.com>**

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# [Thai J Math] Editor Decision: Accepted paper

1 message

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Cc: Nop Pochai <konoppar@kmitl.ac.th>, Nop Pochai <nop_math@yahoo.com>

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*Research Article*

# Numerical Simulation to Air Pollution Emission Control near an Industrial Zone

**Pravitra Oyjinda[1,2] and Nopparat Pochai[1,2]**

[1] *Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand*
[2] *Centre of Excellence in Mathematics, CHE, Si Ayutthaya Rd., Bangkok 10400, Thailand*

Correspondence should be addressed to Nopparat Pochai; nop_math@yahoo.com

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A rapid industrial development causes several environment pollution problems. One of the main problems is air pollution, which affects human health and the environment. The consideration of an air pollutant has to focus on a polluted source. An industrial factory is an important reason that releases the air pollutant into the atmosphere. Thus a mathematical model, an atmospheric diffusion model, is used to estimate air quality that can be used to describe the sulfur dioxide dispersion. In this research, numerical simulations to air pollution measurement near industrial zone are proposed. The air pollution control strategies are simulated to achieve desired pollutant concentration levels. The monitoring points are installed to detect the air pollution concentration data. The numerical experiment of air pollution consisted of different situations such as normal and controlled emissions. The air pollutant concentration is approximated by using an explicit finite difference technique. The solutions of calculated air pollutant concentration in each controlled and uncontrolled point source at the monitoring points are compared. The air pollutant concentration levels for each monitoring point are controlled to be at or below the national air quality standard near industrial zone index.

## 1. Introduction

Nowadays, the air pollution is a major problem in the world because industrial areas grew rapidly. The pollution emission of factories into the atmosphere will have an effect on human health and the environment. The purpose of this research is to study the problem of air pollution emission control. The approximate solution is considered by using the atmospheric diffusion model.

In [1], an atmospheric transport diffusion model with wind velocity profile and diffusion coefficient was considered to study the system of delayed removal. The air pollutant was emitted from a line source with the dry deposition on the ground. The fractional step method was used for computing the air pollutant concentration. In [2], the atmospheric diffusion equation with multiple sources and wind speed and eddy diffusivities was studied to derive the analytical solutions for many boundary condition types. The Green's function concept was used to solve the three-dimensional analytical solutions everywhere in the region of interest. In [3], the finite difference method was used for solving the two-dimensional advection-diffusion equation with a point source. In [4], a time dependent mathematical model of primary and secondary pollutants was studied for approximating the concentration from area source. The wind velocities and eddy diffusion coefficients are considered to be the realistic value. The researchers solved the problem by using Crank-Nicolson implicit finite difference technique and upwind difference scheme which is applied to the diffusion term. In [5], the researchers studied the three-dimensional mathematical model for the sulfur dioxide concentration without obstacles domain.

In [6], the researchers studied a three-dimensional convection-diffusion-reaction equation for sulfur and nitrogen oxides. The model was solved by using a high order accurate time-stepping discretization scheme as Lax and Wendroff technique. A steady state two-dimensional mathematical model of urban heat island was used to describe the dispersion of air pollution with mesoscale wind velocity and meteorological parameters in [7]. The genesis of air pollution was area source emitted from the ground. The removal mechanism was considered by wet and dry depositions. The concentration of air pollutant was approximated by using Crank-Nicolson implicit method. In [8], the mass transport model was considered to simulate the smoke dispersion from one and two point sources with obstacle domain. The model consisted of three equations: a stream function, vorticity, and convection-diffusion equation. The results of air pollution in two-dimensional space and one-dimensional time were calculated by using the finite element method and finite difference method, respectively. In [9], the two-dimensional smoke dispersion model was studied in the cases of two and three point sources with obstacles domain. In [10], the researchers studied a spatial autoregressive model for sulfur dioxide concentration. The evaluation of sulfur dioxide was assessed by the land use regression (LUR) model. The mobile monitoring was used for collecting concentration data in Hamilton, Ontario, Canada.

In [11], the dispersion of primary pollutant was studied in a two-dimensional air pollution model with mesoscale wind. The primary air pollutant was emitted from an area source and the researchers considered removal mechanisms such as dry deposition, gravitational settling, and chemical reaction. The two-dimensional advection-diffusion models of the primary and secondary pollutants are presented in [12]. The researchers studied the air pollutant emitted from area source with removal mechanisms by considering point source on the boundary. The Crank-Nicolson implicit method is used as the finite difference technique in [11, 12]. The design and application of Atmospheric Evaluation and Research Integrated model for Spain (AERIS) are proposed in [13]. The air pollutant concentrations of $NO_2$, $O_3$, $SO_2$, $NH_3$, and PM as a reaction to emission variations of significant sectors in Spain are obtained by AERIS. The results of the model are estimated by using transfer matrices based on an air quality modelling system (AQMS). The system consists of the Weather Research and Forecast (WRF), Sparse Matrix Operator Kernel Emissions (SMOKE), and Community Multiscale Air Quality (CMAQ) models. In [14], the researchers studied air flow and dispersion of pollutant in urban street canyons. The Computational Fluid Dynamics (CFD) were simulated by using Large Eddy Simulation (LES). A velocity comparison between Fluctuating Wind Boundary Conditions (FWBC) and Steady Wind Boundary Conditions (SWBC) was investigated. In [15], the researchers used the three-dimensional air quality model. The considered domain contained three buildings (obstacles) divided into two zones: a factory zone and a residential zone. The modifications of atmospheric stability classes and wind velocities from multiple point sources were also analyzed. The approximate solutions in [5, 9, 15] were solved by using the fractional step method.

A numerical model for air pollution emission control problem with the uniform wind velocities and constant diffusion coefficients is proposed. In this research, the atmospheric diffusion equation is solved by using the finite difference method. This study analyzed the ambient air quality standard of sulfur dioxide that refers to the quantity of sulfur dioxide concentration in clean air.

## 2. Governing Equation

*2.1. The Atmospheric Diffusion Equation.* The diffusion model is used to represent the behavior of air pollutant concentration in industrial areas. The Gaussian plume idea is used as the governing equation. It is the well-known atmospheric diffusion equation. We introduced the three-dimensional advection-diffusion equation as follows:

$$\begin{aligned}\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} \\ = k_x\frac{\partial^2 c}{\partial x^2} + k_y\frac{\partial^2 c}{\partial y^2} + k_z\frac{\partial^2 c}{\partial z^2} + S + R,\end{aligned} \tag{1}$$

where $c = c(x, y, z, t)$ is the air pollutant concentration at $(x, y, z)$ and time $t$ (kg/m$^3$), $u$, $v$, and $w$ are the wind velocity components (m/s) in $x$-, $y$-, and $z$-direction, respectively (m/s), $k_x$, $k_y$, and $k_z$ are the diffusion coefficients in $x$-, $y$-, and $z$-direction, respectively (m$^2$/s), $S$ is the growth of pollutant rate due to sources (sec$^{-1}$), and $R$ is the decaying of pollutant rate due to sinks (sec$^{-1}$).

In this research, we considered only the primary pollutant concentration as sulfur dioxide. The chemical formula is $SO_2$. The assumption of (1) defined that the advection and diffusion in $y$-direction are laterally averaged. By the assumption, we can also eliminate the term in $y$-direction. Therefore, the primary pollutant equation can be written as

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + w\frac{\partial c}{\partial z} = k_x\frac{\partial^2 c}{\partial x^2} + k_z\frac{\partial^2 c}{\partial z^2} + S + R. \tag{2}$$

The initial condition is assumed under the cold start assumption. That is,

$$c(x, z, 0) = 0, \tag{3}$$

for all $x > 0$ and $z > 0$. The boundary conditions assumed that

$$\begin{aligned}&\frac{\partial c}{\partial x}(0, z, t) = \frac{\partial c}{\partial x}(L, z, t) = 0,\\ &\frac{\partial c}{\partial z}(x, H, t) = 0,\\ &\frac{\partial c}{\partial z}(x, 0, t) = v_d c,\end{aligned} \tag{4}$$

for all $t > 0$, where $L$ is the length of the domain in $x$-direction, $H$ is the height of the inversion layer, and $v_d$ is the dry deposition velocity of the primary pollutant (m/s). Sulfur dioxide deposition velocity can be related to a diffusion coefficient $k_z$ which is assumed to be an irreversible process.

In Figure 1, model of air pollution emission control problem is presented. This research was designed to study the behavior of dispersion and effect of dispersion concentration near the industrial zone. The four monitoring points are set far away from the source. Each monitoring point is called $M1$, $M2$, $M3$, and $M4$ respectively. In Figure 2, the considered domain for the numerical experiment is shown. Let the height

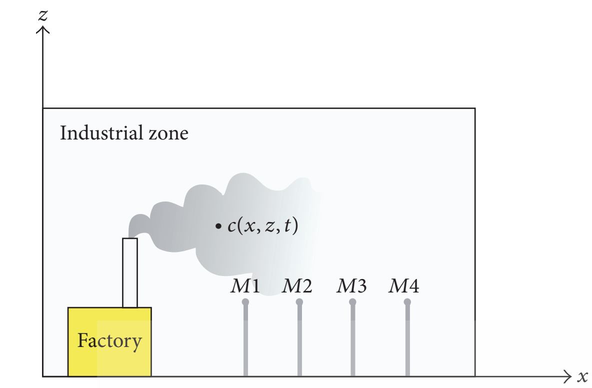


FIGURE 1: Model of air pollution emission control problem.

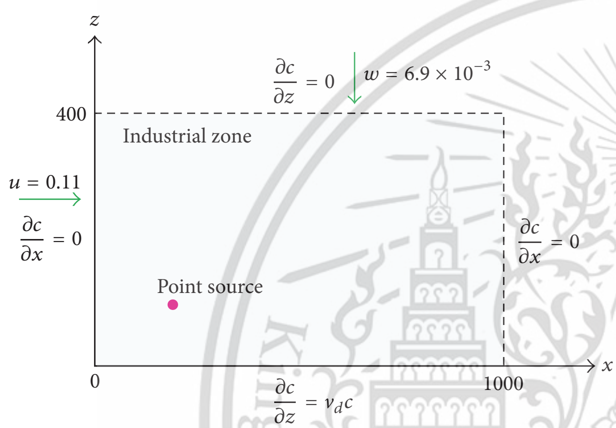


FIGURE 2: Domain of approximate solutions.

of point source be $z = h_s$ m. The wind is stable in $x$- and $z$-axis. The concentrations of air pollutant are emitted directly from a continued point source (chimney) from industrial factory. The air pollutants are absorbed from the chemical reaction on the ground.

*2.2. The Nondimensional Form Equation.* From (2), we present the nondimensional form of air pollution. The following dimensionless variables are defined by $C = c/c_{\max}$, $X = x/l_x$, $Z = z/l_z$, $T = t/t_{\max}$, $D_x = k_x/l_x u_{\max}$, $D_z = k_z/l_z u_{\max}$, $U = u/u_{\max}$, and $W = \beta w_{\max}/u_{\max}$ when $\beta = w/w_{\max}$. We let $c_{\max} = \max\{c(x,z,t) : 0 \le x \le L,\ 0 \le z \le H,\ 0 \le t \le t_{\max}\}$, $u_{\max} = \max\{u(x,z,t) : 0 \le x \le L,\ 0 \le z \le H,\ 0 \le t \le t_{\max}\}$, and $w_{\max} = \max\{w(x,z,t) : 0 \le x \le L,\ 0 \le z \le H,\ 0 \le t \le t_{\max}\}$, and $t_{\max}$ is a stationary time. Therefore, the nondimensional atmospheric diffusion equation can be rearranged to give

$$\frac{1}{St}\frac{\partial C}{\partial T} + U\frac{\partial C}{\partial X} + W\frac{\partial C}{\partial Z} = D_x\frac{\partial^2 C}{\partial X^2} + D_z\frac{\partial^2 C}{\partial Z^2} + S - k_p C, \tag{5}$$

where $l = \max\{l_x, l_z\}$, $St = t_{\max}u_{\max}/l$, and $k_p$ is the chemical interaction rate of primary pollutant equation. **For** the nondimensional form of initial condition, it is assumed that

$$C(X, Z, 0) = 0, \tag{6}$$

for all $X > 0$ and $0 \le Z \le H$. For the nondimensional form of boundary, it is assumed that

$$\frac{\partial C}{\partial X}(0, Z, T) = \frac{\partial C}{\partial X}(L, Z, T) = 0, \tag{7}$$

$$\frac{\partial C}{\partial Z}(X, H, T) = 0, \tag{8}$$

$$\frac{\partial C}{\partial Z}(X, 0, T) = v_d C, \tag{9}$$

for all $T > 0$.

## 3. Numerical Method

The prediction of primary pollutant from a stationary source can be calculated to solve the air pollution problem in the industrial areas. In (5), we get the concentration of $C$ at each time $T_{n+1}$ from $T_n = n\Delta T$, $n = 0, 1, 2, \ldots, P$, when $\Delta T$ is a time increment. The solution of sulfur dioxide concentration at $(X, Z, T)$ is denoted by $C(X_i, Z_j, T_n) = C^n_{i,j}$. The considered domain is meshed by the grid spacing $\Delta X$ and $\Delta Z$ where $X_i = i\Delta X$, $i = 0, 1, 2, \ldots, N$, and $Z_j = j\Delta Z$, $j = 0, 1, 2, \ldots, M$. The finite difference method is chosen as proper equipment for estimating solutions. The method refers to the nondimensional model, for which we use the forward time central space (FTCS) scheme. In the transient term, we used the forward difference for

$$\frac{\partial C}{\partial T} = \frac{C^{n+1}_{i,j} - C^n_{i,j}}{\Delta T}. \tag{10}$$

The advection and diffusion terms are substituted by using the centered difference in space by

$$\frac{\partial C}{\partial X} = \frac{C^n_{i+1,j} - C^n_{i-1,j}}{2\Delta X}, \tag{11}$$

$$\frac{\partial C}{\partial Z} = \frac{C^n_{i,j+1} - C^n_{i,j-1}}{2\Delta Z}, \tag{12}$$

$$\frac{\partial^2 C}{\partial X^2} = \frac{C^n_{i+1,j} - 2C^n_{i,j} + C^n_{i-1,j}}{(\Delta X)^2}, \tag{13}$$

$$\frac{\partial^2 C}{\partial Z^2} = \frac{C^n_{i,j+1} - 2C^n_{i,j} + C^n_{i,j-1}}{(\Delta Z)^2}, \tag{14}$$

respectively. The formula of (5) becomes

$$\frac{1}{St}\left(\frac{C^{n+1}_{i,j} - C^n_{i,j}}{\Delta T}\right) + U\left(\frac{C^n_{i+1,j} - C^n_{i-1,j}}{2\Delta X}\right) + W\left(\frac{C^n_{i,j+1} - C^n_{i,j-1}}{2\Delta Z}\right) = D_x\left(\frac{C^n_{i+1,j} - 2C^n_{i,j} + C^n_{i-1,j}}{(\Delta X)^2}\right) + D_z\left(\frac{C^n_{i,j+1} - 2C^n_{i,j} + C^n_{i,j-1}}{(\Delta Z)^2}\right) + S - k_p C^n_{i,j}. \tag{15}$$

Thus, the finite difference form of the advection-diffusion equation becomes

$$\begin{aligned} C_{i,j}^{n+1} &= \left(d_x - A_x\right) C_{i+1,j}^{n} + \left(d_x + A_x\right) C_{i-1,j}^{n} \\ &\quad + \left(1 - 2d_x - 2d_z - k_p \Delta T \left(St\right)\right) C_{i,j}^{n} \\ &\quad + \left(d_z + A_z\right) C_{i,j-1}^{n} + \left(d_z - A_z\right) C_{i,j+1}^{n} \\ &\quad + St\left(\Delta T\right) S, \end{aligned} \tag{16}$$

where $A_x = St(\Delta T)U/2\Delta X$, $A_z = St(\Delta T)W/2\Delta Z$, $d_x = St(\Delta T)D_x/(\Delta X)^2$, and $d_z = St(\Delta T)D_z/(\Delta Z)^2$.

## 4. Air Pollution Controlled Simulations

The experiment analyzed the action of air pollution with the volume of sulfur dioxide emission around an industrial zone. We will simulate the air pollution control situation in three cases. For the first simulation, an industrial factory released continued air pollutant from a chimney without emission control system. For the second and the third simulations, the factory will discharge the sulfur dioxide, which is controlled by the national air quality index.

*4.1. Simulation 1: Air Pollution Emission without Controlled System.* In the first simulation, the two-dimensional advection-diffusion equation (5) with a domain of interest of $1000 \times 400\,\text{m}^2$ is considered. The wind velocities in $x$- and $z$-direction are assumed to be 0.11 and $6.9 \times 10^{-3}$ m/s, respectively. The sulfur dioxide is released at the chimney height $h_s = 75$ m at coordinate (100, 75) (m, m). The released pollutant concentration is $0.75\,\text{sec}^{-1}$. The diffusion coefficients in $x$- and $z$-direction are 2 and $0.45\,\text{m}^2/\text{s}$, respectively. The grid spacing is $\Delta x = \Delta z = 25$ m and time interval is 72 sec. This simulation is solved by using FTCS in (16) with the initial and boundary conditions (6)–(9). The numerical solutions of air pollutant concentration when 58 minutes and 1 hour and 36 minutes have passed are shown in Figures 3 and 4, respectively. The monitoring points are aligned along 200, 300, 400, and 500 m in the same height, 50 m. The comparison of concentrations of different distances is presented in Figure 5.

*4.2. Simulation 2: Air Pollution Emission Controlled by following the National Air Quality Standard ($3 \times 10^{-7}$ kg/m$^3$).* In the second simulation, the two-dimensional advection-diffusion equation (5) with a domain of interest of $1000 \times 400\,\text{m}^2$ is considered. The wind velocities in $x$- and $z$-direction are assumed to be 0.11 and $6.9 \times 10^{-3}$ m/s, respectively. The sulfur dioxide is released at the chimney height $h_s = 75$ m at the coordinate (100, 75) (m, m). The released pollutant concentration is $0.75\,\text{sec}^{-1}$. The diffusion coefficients in $x$- and $z$-direction are 2 and $0.45\,\text{m}^2/\text{s}$, respectively. The grid spacing is $\Delta x = \Delta z = 25$ m and time interval is 72 sec. In this simulation, the sulfur dioxide is released by following the United States Environmental Protection Agency (USEPA) air quality standard [16], $3 \times 10^{-7}\,\text{kg/m}^3$. If the approximated pollutant concentration at a monitoring point becomes higher than the air quality standard, then the chimney will be shut down and wait until the concentration goes below $1.5 \times 10^{-7}\,\text{kg/m}^3$. If the pollutant concentration at all monitoring points is below a half of the air quality standard, the chimney will be opened again. The air pollution emission will be following these processes. This example is solved by using FTCS in (16) with the initial and boundary conditions (6)–(9). The results of air pollution emission control are demonstrated as the contour lines of sulfur dioxide concentration in Figures 6 and 7. The concentration of air pollution in the different distance is shown in Figure 8.

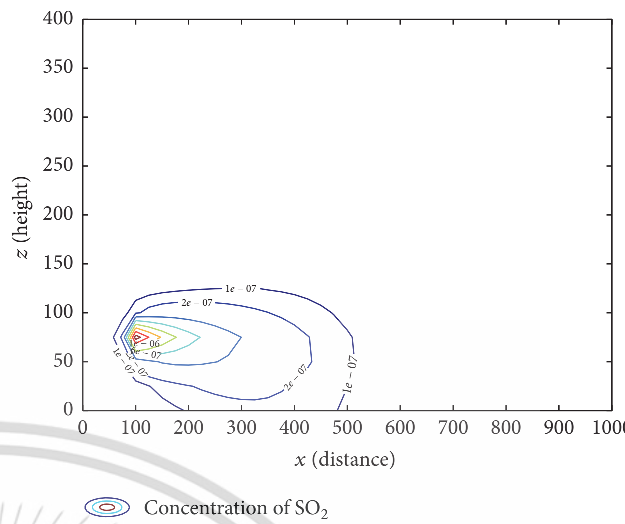


Figure 3: The concentration levels of air pollution after 58 minutes have passed without control system.

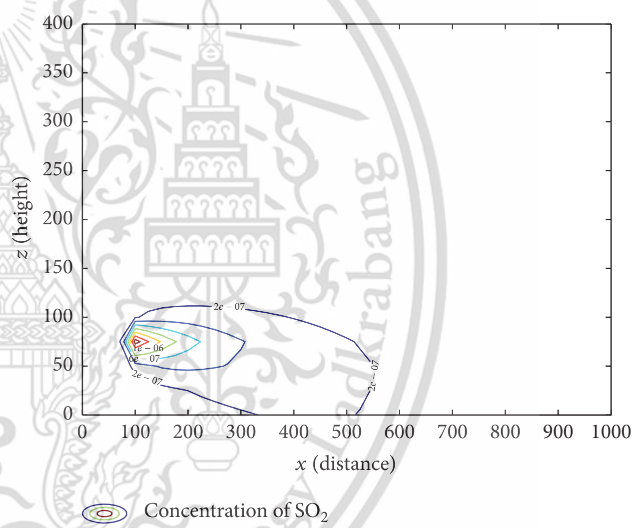


Figure 4: The concentration levels of air pollution after 1 hour and 36 minutes have passed without control system.

*4.3. Simulation 3: Air Pollution Emission Controlled by following the National Air Quality Standard ($1.5 \times 10^{-7}$ kg/m$^3$).* In the third simulation, the two-dimensional advection-diffusion equation (5) with a domain of interest of $1000 \times 400\,\text{m}^2$ is considered. The wind velocities in $x$- and $z$-direction

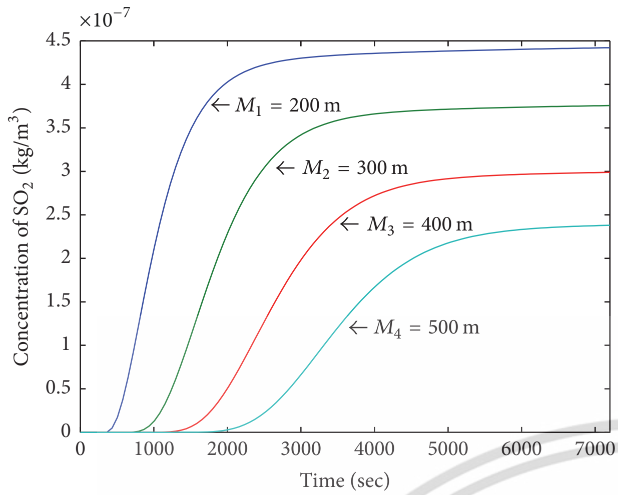


FIGURE 5: The concentration of air pollution with the different monitoring point at $z = 50$ m without control system.

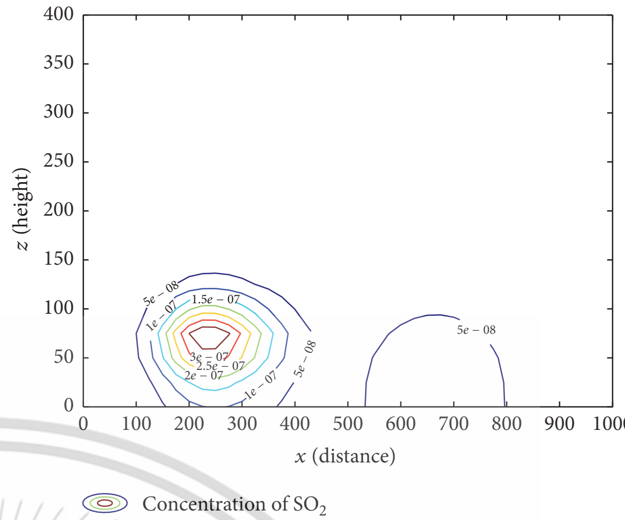


FIGURE 7: The concentration levels of air pollution after 1 hour and 36 minutes have passed which are controlled by air quality standard ($3 \times 10^{-7}$ kg/m$^3$).

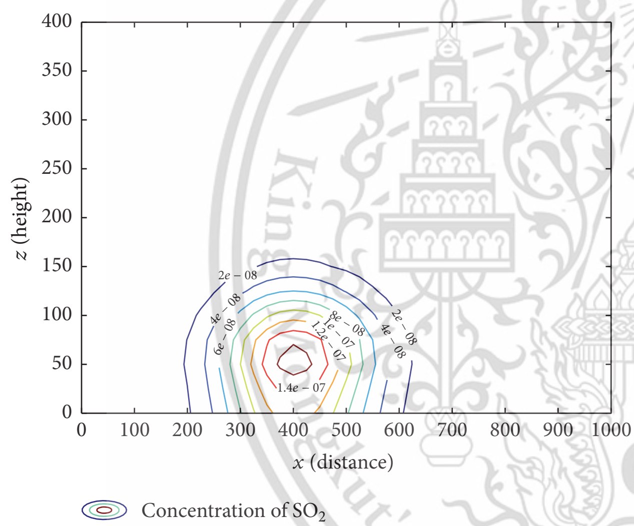


FIGURE 6: The concentration levels of air pollution after 58 minutes have passed which are controlled by air quality standard ($3 \times 10^{-7}$ kg/m$^3$).

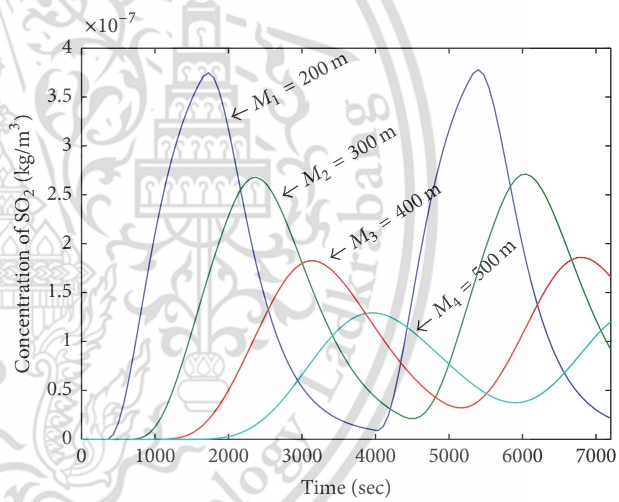


FIGURE 8: The concentration of air pollution with the different monitoring point at $z = 50$ m by air quality standard ($3 \times 10^{-7}$ kg/m$^3$).

are assumed to be 0.11 and $6.9 \times 10^{-3}$ m/s, respectively. The sulfur dioxide is released at the chimney height $h_s = 75$ m at the coordinate $(100, 75)$ (m, m). The released pollutant concentration is 0.75 sec$^{-1}$. The diffusion coefficients in $x$- and $z$-direction are 2 and 0.45 m$^2$/s, respectively. The grid spacing is $\Delta x = \Delta z = 25$ m and time interval is 72 sec. In this simulation, the sulfur dioxide is released by following the USEPA air quality standard, $1.5 \times 10^{-7}$ kg/m$^3$. If the approximated pollutant concentration at a monitoring point becomes higher than the air quality standard, then the chimney will be shut down and wait until the concentration goes below $1.0 \times 10^{-7}$ kg/m$^3$. If the pollutant concentration at all monitoring points is below a third of the air quality standard, the chimney will be opened again. The air pollution emission will be following these processes. This simulation is solved by using FTCS in (16) with the initial and boundary conditions (6)–(9). In this emission control case, the concentrations of air pollution when 58 minutes and 1 hour and 36 minutes have passed are shown in Figures 9 and 10, respectively. The concentration of $SO_2$ when 2 hours have passed with the different distance is shown in Figure 11.

From Simulations 1, 2, and 3, the concentrations of $SO_2$ at the height $z = 50$ m and the distance $x = 300$ m ($M2$) are compared in Figure 12.

## 5. Conclusion

The atmospheric diffusion model to describe the released air pollutant concentration by an industrial plant is proposed.

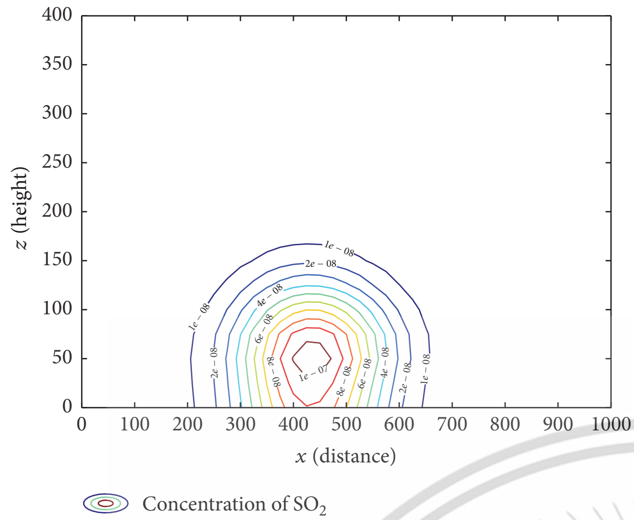


FIGURE 9: The air pollution concentration levels after 58 minutes have passed which are controlled by air quality standard ($1.5 \times 10^{-7}$ kg/m$^3$).

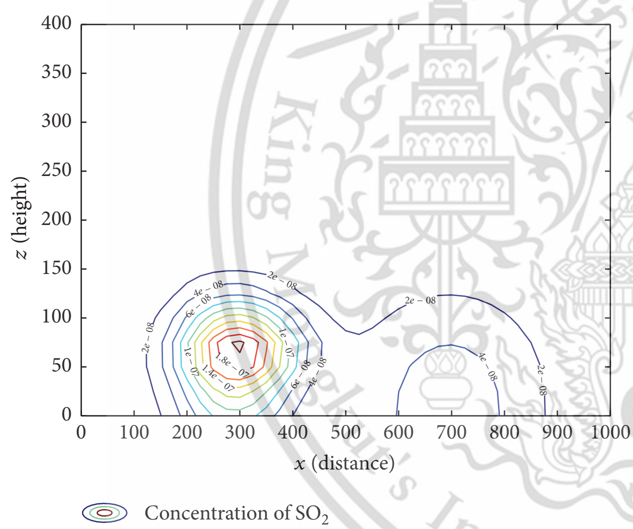


FIGURE 10: The air pollution concentration levels after 1 hour and 36 minutes have passed which are controlled by air quality standard ($1.5 \times 10^{-7}$ kg/m$^3$).

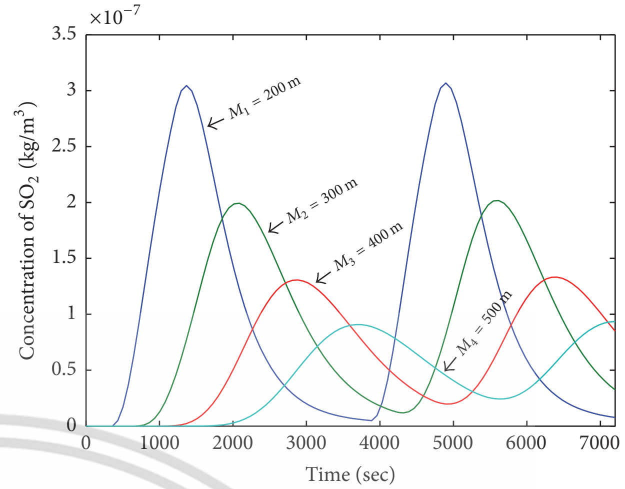


FIGURE 11: The concentration of air pollution with the different monitoring point at $z = 50$ m by air quality standard ($1.5 \times 10^{-7}$ kg/m$^3$).

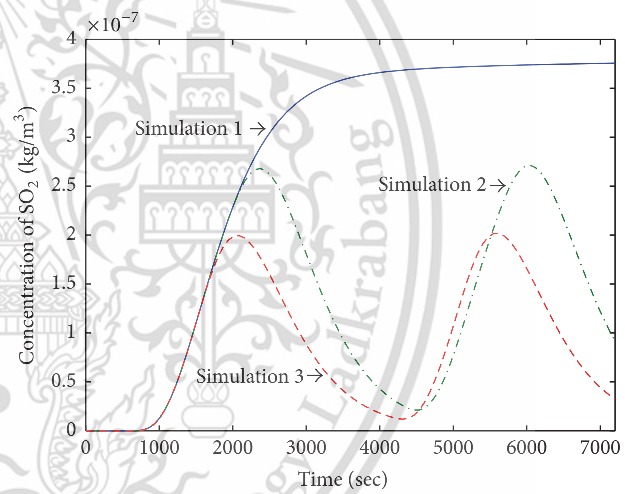


FIGURE 12: The air pollution concentration at $z = 50$ m and $x = 300$ m ($M2$) in three cases.

The concentration of the sulfur dioxide is approximated by an explicit forward time centered space finite difference technique. The method gives good agreement of approximated solutions. The air quality standard near industrial zone is controlled by considering the approximated pollutant concentration levels at all monitoring points. The proposed air pollution controlled simulations demonstrated that the industrial plants need to shut down their chimneys for a while.

## Conflicts of Interest

The authors declare no conflicts of interest.

## Acknowledgments

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# Numerical Simulation of an Air Pollution Model on Industrial Areas by Considering the Influence of Multiple Point Sources

**Pravitra Oyjinda**[†,1] **and Nopparat Pochai**[†,‡]

[†]Department of Mathematics, Faculty of Science,
King Mongkut's Institute of Technology Ladkrabang,
Bangkok 10520, Thailand
e-mail : pravitra.o@gmail.com
[‡]Centre of Excellence in Mathematics
CHE, Si Ayutthaya Rd., Bangkok 10400, Thailand
e-mail : nop_math@yahoo.com

**Abstract :** A numerical simulation on a two-dimensional atmospheric diffusion equation of an air pollution measurement model is proposed. The considered area is separated into two parts such as an industrial zone and an urban zone. In this research, the air pollution measurement by releasing the pollutant from multiple point sources above an industrial zone to the other area is simulated. The governing partial differential equation of air pollutant concentration is approximated by using a finite difference technique. The approximate solutions of the air pollutant concentration on both areas are compared. The air pollutant concentration levels that influenced by multiple point sources are also analyzed.

**Keywords :** multiple point sources; finite difference technique; air pollutant concentration; industrial zone; urban zone

**2000 Mathematics Subject Classification :** 35K15; 65M06; 76R50

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[1]Corresponding author email: pravitra.o@gmail.com

# 1 Introduction

The rapid industrial growth can explain the air pollution affects the health of human being who lives around industrial areas. The air pollution has become a major problem for human life and environment. The purpose of this research is to study the air pollution assessment problem in two adjacent zones: industrial and urban zones by using the atmospheric diffusion model. In [1], the mathematical model is presented and used to study the dispersion of sulfur dioxide with constant and reference atmospheric stability. In [2], the smoke dispersion model in two-dimensional space with two and three point sources is proposed. The fractional step method, Carlson's method, and Crank-Nicolson method are used to solve the approximate solutions in [1] and [2]. They study a two-dimensional mathematical model of primary and secondary pollutants of an area source with chemical reaction and dry deposition by considering point source on the boundary in [4]. The finite difference method is used as Crank-Nicolson Implicit method. In [3], the air-quality model in three-dimensional with variations of the atmospheric stability classes and wind velocities from multiple sources is analyzed.

The source that is smokestack of industrial factory or power plant emitted the air pollution into the system. The genesis of air pollution is the cause of problems. In this research, the simple finite difference methods are used for solving the atmospheric diffusion equation.

# 2 Governing Equation

## 2.1 The Atmospheric Diffusion Equation

The diffusion model generally use Gaussian plume idea, which is the well-known atmospheric diffusion equation. It is to represent the behavior of air pollution in industrial areas. The dispersion of pollutant concentration from multiple point sources is described by a three-dimensional advection-diffusion equation [1],[2],[3], [4] following

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} = k_x\frac{\partial^2 c}{\partial x^2} + k_y\frac{\partial^2 c}{\partial y^2} + k_z\frac{\partial^2 c}{\partial z^2} + s, \tag{2.1}$$

where $c = c(x, y, z, t)$ is the concentration of air pollutant at $(x, y, z)$ and time $t$ $(kg/m^3)$, $u, v$, and $w$ are the wind velocity component $(m/sec)$ in $x, y, z$-direction respectively, $k_x, k_y$, and $k_z$ are the diffusion coefficient $(m^2/sec)$ in $x, y, z$-direction respectively, and $s$ is the sink rate of air pollutants $(sec^{-1})$.

The assumptions of equation (2.1) are defined that the concentrations of air pollutant are emitted from continued point sources. The advection and diffusion in $y$-direction are laterally averaged. By the assumption, we can also eliminate the term in $y$-direction. Therefore, the governing equation can be written as

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + w\frac{\partial c}{\partial z} = k_x\frac{\partial^2 c}{\partial x^2} + k_z\frac{\partial^2 c}{\partial z^2} + s. \tag{2.2}$$

The initial condition is assumed under the cold start assumption. That is

$$c(x,z,0)=0, \tag{2.3}$$

for all $x>0$ and $z>0$. The boundary conditions are assumed that

$$c(0,z,t)=0, \tag{2.4}$$

$$\frac{\partial c}{\partial x}(L,z,t)=\frac{\partial c}{\partial z}(x,0,t)=\frac{\partial c}{\partial z}(x,H,t)=0, \tag{2.5}$$

for all $t>0$ where $L$ is the length of the domain in $x$-direction and $H$ is the height of the inversion layer. The concentration at the point sources is assumed to be the constant variables as

$$c(x_p,0,t)=c_{s_p}, \tag{2.6}$$

for $p=1,2$ where $x_p$ is the position of the point source $p$ in the $x$-direction and $c_{s_p}$ is the concentration value at the point source of $p$.

### 2.2 The Non-dimensional Form Equation

Now, we introduce the dimensionless form of equation (2.2). The non-dimensional variables are denoted by letting $C=c/c_{\max}, X=x/l_x, Z=z/l_z, T=t/t_{\max}, D_x=k_x/l_xu_{\max}, D_z=k_z/l_zu_{\max}, U=u/u_{\max}$, and $W=\beta w_{\max}/u_{\max}$ when $\beta=w/w_{\max}$. We define $c_{\max}=\max\{c(x,z,t)\colon 0\leqslant x\leqslant L, 0\leqslant z\leqslant H, 0\leqslant t\leqslant t_{\max}\}$, $u_{\max}=\max\{u(x,z,t)\colon 0\leqslant x\leqslant L, 0\leqslant z\leqslant H, 0\leqslant t\leqslant t_{\max}\}$, $w_{\max}=\max\{w(x,z,t)\colon 0\leqslant x\leqslant L, 0\leqslant z\leqslant H, 0\leqslant t\leqslant t_{\max}\}$, and $t_{\max}$ is a stationary time. Thus the non-dimensional equation of air pollution as follows:

$$\frac{1}{St}\frac{\partial C}{\partial T}+U\frac{\partial C}{\partial X}+W\frac{\partial C}{\partial Z}=D_x\frac{\partial^2 C}{\partial X^2}+D_z\frac{\partial^2 C}{\partial Z^2}+S, \tag{2.7}$$

where $l=\max\{l_x,l_z\}$ and $St=\dfrac{t_{\max}u_{\max}}{l}$.

## 3 Numerical Methods

We use the finite difference methods for calculating the non-dimensional form of the atmospheric diffusion equation. In equation (2.7), we get the concentration of $C$ at each time $T_{n+1}$ from $T_n$ when $\Delta T$ is a time increment. The solution of concentration at $(X,Z,T)$ is denoted by $C(X_i,Z_j,T_n)=C^n_{i,j}$. The domain is divided by the grid spacing in $X$ and $Z$-direction, $\Delta X$ and $\Delta Z$ respectively where $X_i=i\Delta X$ and $Z_j=j\Delta Z$ . The approximate solutions are obtained by using the following methods:

### 3.1 Forward Time Central Space Scheme

The first method refer to the non-dimensional of the model, which we use the forward time central space (FTCS) scheme. Thus, the formula of equation (2.7) is

$$C_{i,j}^{n+1} = (d_x - A_x)C_{i+1,j}^{n} + (d_x + A_x)C_{i-1,j}^{n} + (1 - 2d_x - 2d_z)C_{i,j}^{n} + (d_z + A_z)C_{i,j-1}^{n} + (d_z - A_z)C_{i,j+1}^{n} + St(\Delta T)S, \quad (3.1)$$

where $A_x = \dfrac{St(\Delta T)U}{2\Delta X}, A_z = \dfrac{St(\Delta T)W}{2\Delta Z}, d_x = \dfrac{St(\Delta T)D_x}{(\Delta X)^2}, d_z = \dfrac{St(\Delta T)D_z}{(\Delta Z)^2}$.

### 3.2 Backward Time Central Space Scheme

The second method, we calculated by using the backward time central space (BTCS) scheme. It is used to discretize the governing equation. The finite difference equation can also be obtained

$$(A_x - d_x)C_{i+1,j}^{n+1} + (A_x + d_x)C_{i-1,j}^{n+1} + (1 + 2d_x + 2d_z)C_{i,j}^{n+1} - (A_z + d_z)C_{i,j-1}^{n+1} + (A_z - d_z)C_{i,j+1}^{n+1} = C_{i,j}^{n} + St(\Delta T)S. \quad (3.2)$$

## 4 Numerical Experiment

The two-dimensional atmospheric diffusion equation (2.7) with a dimension $1 \times 1\ km^2$ will be considered. The uniform wind velocities and constant diffusion coefficients are introduced. We choose the wind velocities in $x$ and $z$-direction are 0.1 and 0.05 $m/sec$ respectively. The diffusion coefficients in $x$ and $z$-direction are $4.5\times10^{-1}$ and $4.5\times10^{-5}$ $m^2/sec$ respectively. The grid spacing: $\Delta x = \Delta z = 25$ m. and the time interval is 20 $sec$. In this research, we consider two cases. The first case, there is a point source, which the concentration is 0.5 $kg/m^3$. The second case, there are two point sources, which the concentration are 0.25 and 0.25 $kg/m^3$. The air pollutants in equation (2.6) are released into our system. This example is solved by using FTCS and BTCS in equation (3.1) and (3.2) respectively with the initial and boundary conditions (2.3) to (2.5).

In Fig.1, model of the problem is shown. The physical problem composed of two zones: an industrial zone and an urban zone with the stable wind along the $x$ and $z$-axis. The point sources are lied along the $x$-axis. We assume that the primary air pollutants are released by a factory smokestack as a single point source and a couple point sources from industrial zone. The emissions of air pollution are influenced on the urban zone by the rate of air pollutant absorption. In the numerical experiment, the considered domain of solutions is shown in Fig.2.

## 5 Discussion

The air pollutant emission from multiple point sources above an industrial zone to the urban area is presented. The finite difference technique introduced

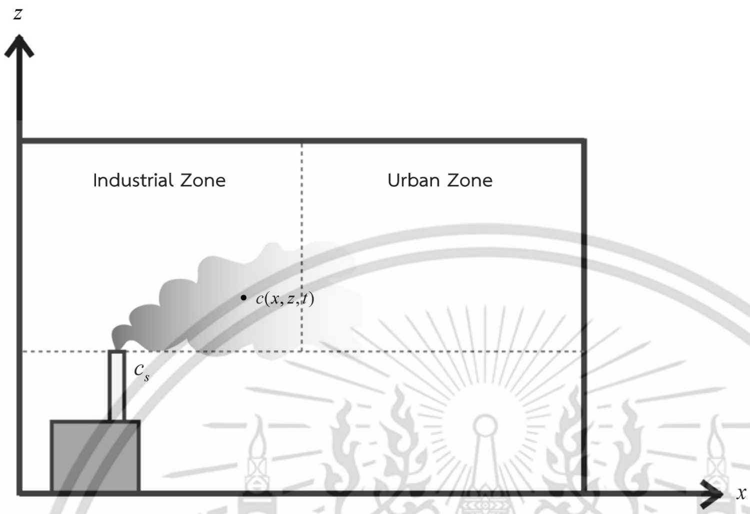



Figure 1: Model of the problem

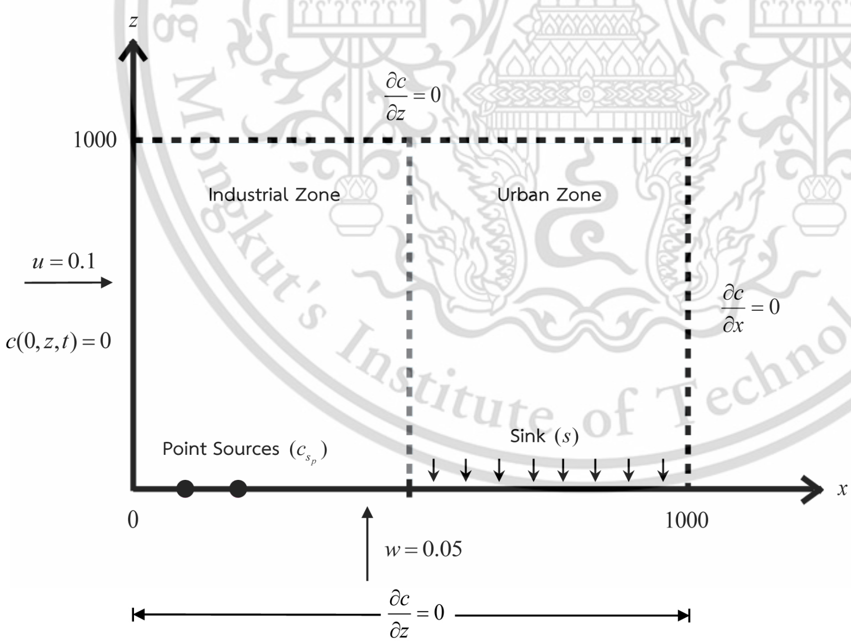



Figure 2: Domain of solutions

two methods for calculating the air pollutant concentrations. Figs.3 and 4 are to compare the air pollutant concentrations between two cases: a single point source and a couple point sources, respectively. From the both figures, it was apparent that the results of FTCS is close to the results of BTCS, when there is no sink of pollutant absorption ($s = 0$). Figs.5 and 6 are to illustrate that the sink of pollutant absorption ($s = -10^{-4}$) are added in the base of urban zone. The air pollutant concentration near human living goes down and the two methods also give the close result. In Figs.7 and 8, the computed approximate solution that obtained by using FTCS and BTCS are compared. We can see that the results of added sink case and without sink case are quite similar. These graphs also indicate that the FTCS gives the computed solutions close to the BTCS.

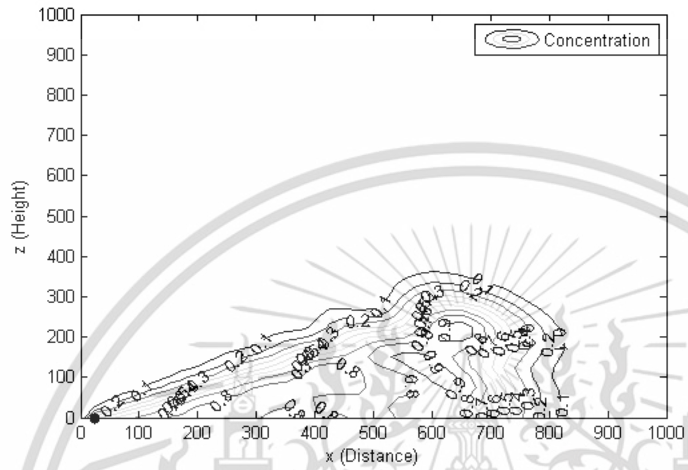
From Figs.9 and 10 demonstrate that the air pollutant concentration at the height $z = 25$ m. and $z = 50$ m. are solved by using FTCS. The case of added sink is less concentration than the case of without sink. Therefore, the sink can lower the overall pollutant levels. Fig.11 establishes the variant concentration when we take more sink rate into our system. The comparison of computing time shows that FTCS is faster than BTCS in Table 1.

Table 1: Computing time comparison of FTCS and BTCS

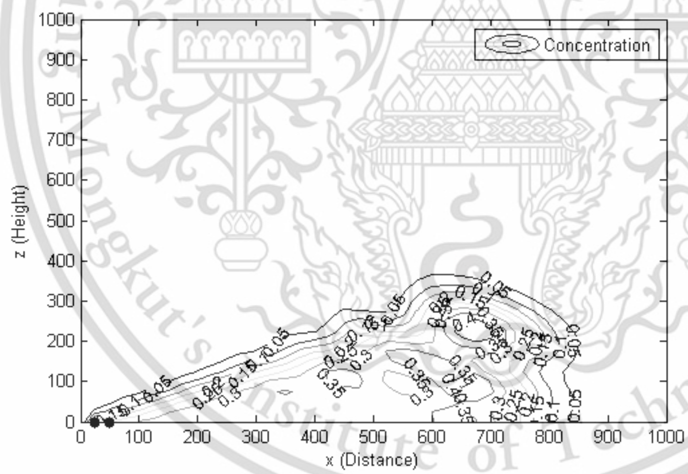
| **Simulation Time** | **FTCS** | **BTCS** |
|---|---|---|
| 30 min. | 1.49 sec. | 22.48 sec. |
| 1 hr. | 1.68 sec. | 42.66 sec. |
| 2 hrs. | 2.05 sec. | 84.18 sec. |

## 6 Conclusion

The simple air pollution measurement models, which air pollutants are released by a single point source and couple point sources are proposed. The traditional finite difference methods such as FTCS and BTCS can be used to approximate the air pollutant levels for each points and times. The results of this study show that the air pollutant concentrations of FTCS are close to the air pollutant concentrations of BTCS. In the case of a couple point sources problem, the overall concentration levels of air pollution are less than a single point source problem. Therefore, the influence of multiple point sources and the variable rate of sink are also considered. It obtains that the higher sink rate does decrease pollutant levels around human living. The both finite difference methods are used to compute the numerical solutions of air pollution by MATLAB. In this experiment, FTCS gives less computing time than BTCS.

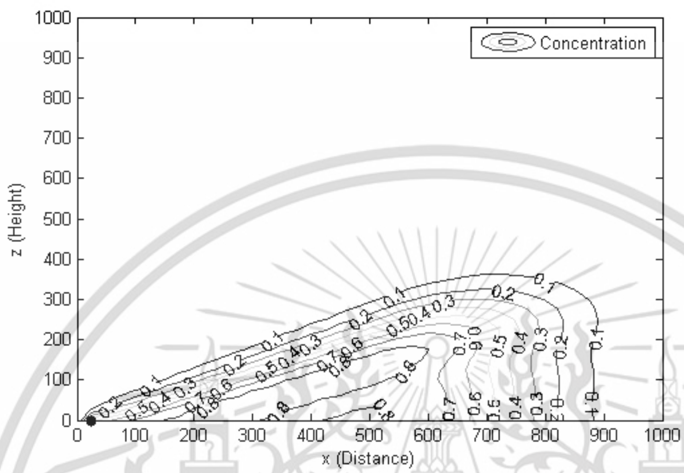



(a)

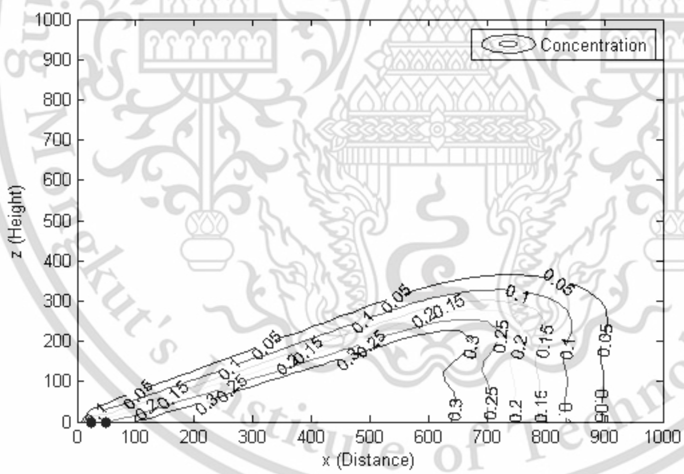



(b)

Figure 3: The air pollutant concentration levels after 2 hours passed which are computed by FTCS (There is no sink of pollutant absorption $s = 0$) (a) one point source and (b) two point sources

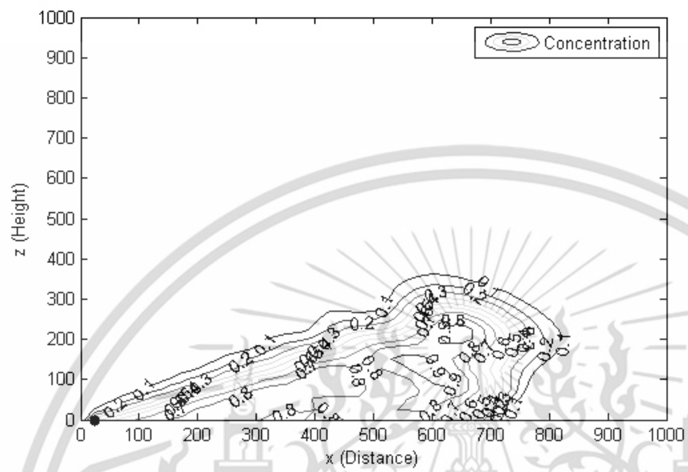



(a)

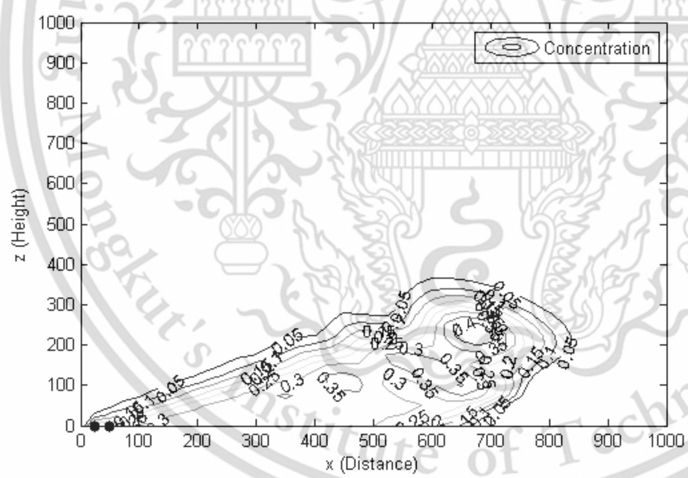



(b)

Figure 4: The air pollutant concentration levels after 2 hours passed which are computed by BTCS (There is no sink of pollutant absorption $s = 0$) (a) one point source and (b) two point sources

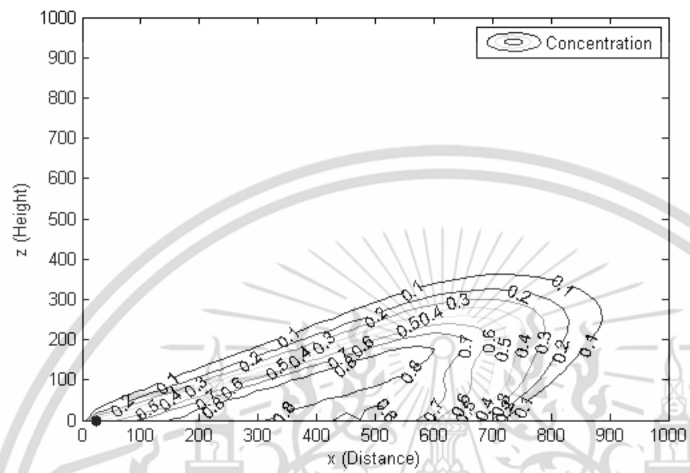



(a)

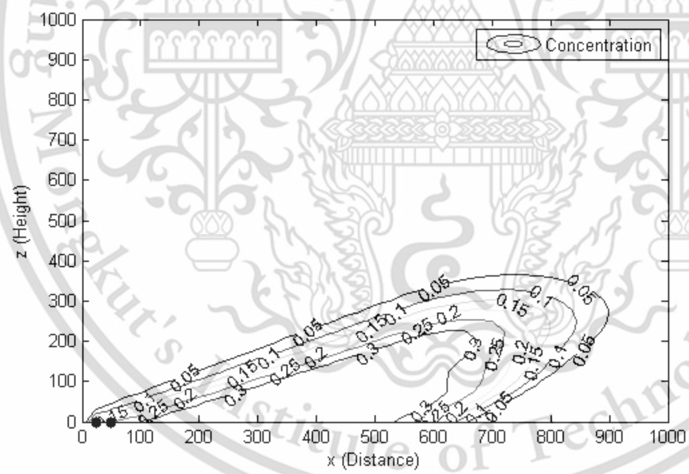



(b)

Figure 5: The air pollutant concentration levels after 2 hours passed which are computed by FTCS (There are sink of pollutant absorption $s = -10^{-4}$) (a) one point source and (b) two point sources




(a)




(b)

Figure 6: The air pollutant concentration levels after 2 hours passed which are computed by BTCS (There are sink of pollutant absorption $s = -10^{-4}$) (a) one point source and (b) two point sources

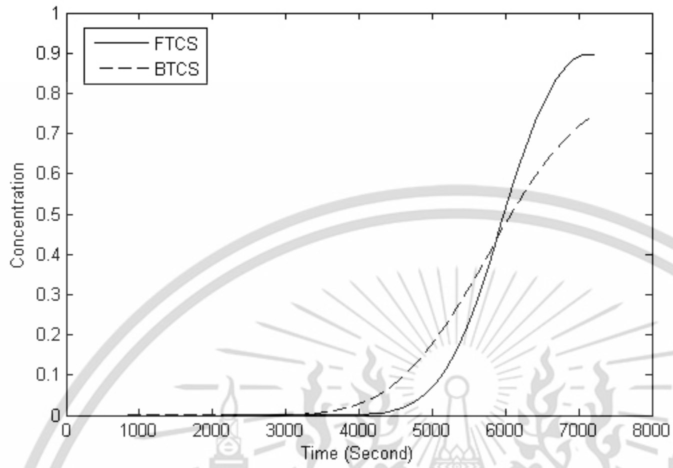



Figure 7: The air pollutant concentration between FTCS and BTCS (There is no sink of pollutant absorption $s = 0$) at $z = 0$ m. and $x = 600$ m.

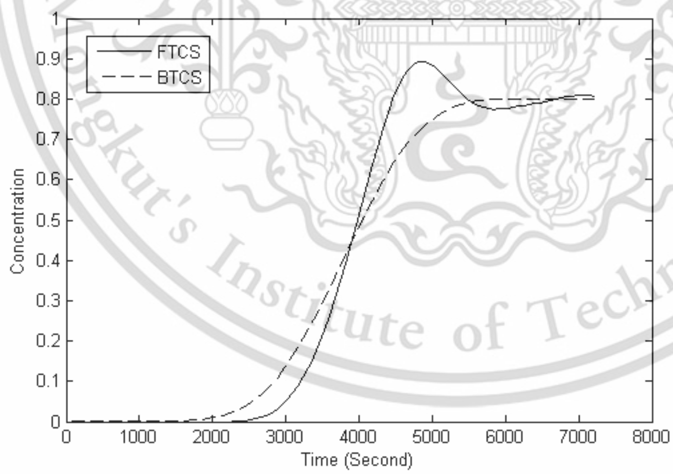



Figure 8: The air pollutant concentration between FTCS and BTCS (There are sink of pollutant absorption $s = -10^{-4}$) at $z = 0$ m. and $x = 400$ m.

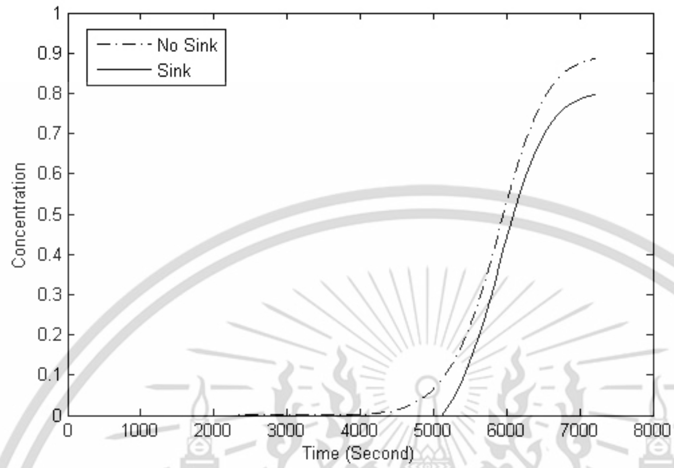



Figure 9: The air pollutant concentration between 2 case: added sink and without sink (computed by FTCS) at $z = 25$ m. and $x = 600$ m.

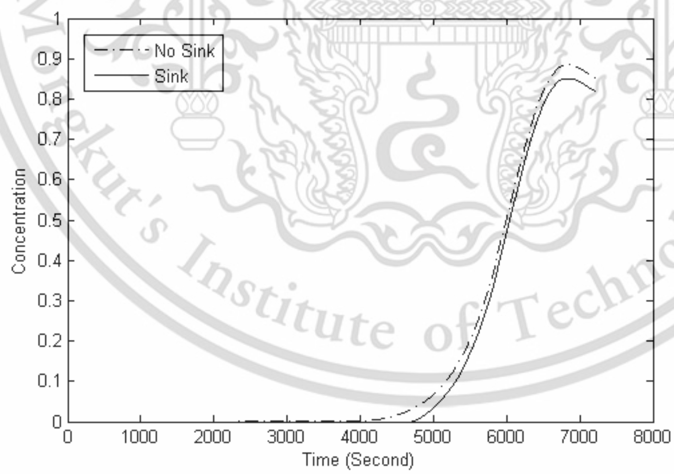



Figure 10: The air pollutant concentration between 2 case: added sink and without sink (computed by FTCS) at $z = 50$ m. and $x = 600$ m.

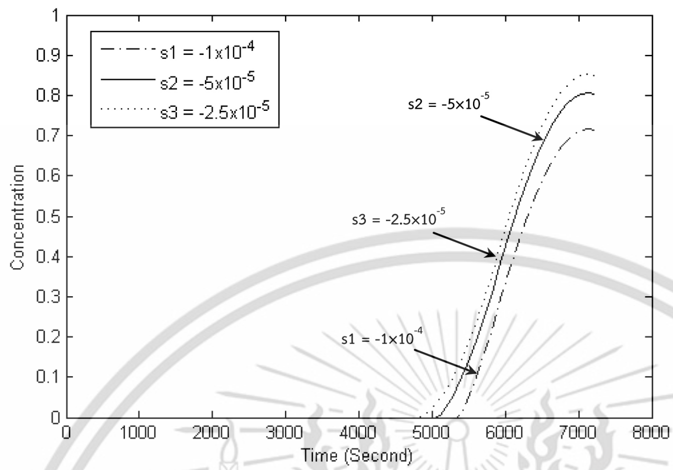



Figure 11: The air pollutant concentration with the variant values of sink rate (computed by FTCS) at $z = 0$ m. and $x = 600$ m.

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Thai J. Math. Online @ http://thaijmath.in.cmu.ac.th

# Author Biography

**Name** Miss Pravitra Oyjinda

**Date of Birth** $4^{th}$ October 1988

**Address** 53/62 Moo 9, Muang Thong Thani, Chaengwattana Road, Bangpood Sub-district, Pakkret District, Nonthaburi Province, 11120, Thailand

**Education**

2010. Bachelor of Science in Applied Mathematics. GPA 3.40
King Mongkut's Institute of Technology Ladkrabang

2013. Master of Science in Applied Mathematics. GPA 3.75
King Mongkut's Institute of Technology Ladkrabang

2018. Doctor of Philosophy in Applied Mathematics. GPA 4.00
King Mongkut's Institute of Technology Ladkrabang

**Scholarship**

2014-2016. Ph.D. science student scholarship, Faculty of science
King Mongkut's Institute of Technology Ladkrabang

2017. Research assistance scholarship, Centre of Excellence in Mathematics (CEM), Commission on Higher Education (CHE), Thailand

**Academic Publications**

1. Pravitra Oyjinda and Nopparat Pochai, " Numerical Simulation to Air Pollution Emission Control near an Industrial Zone," Advances in Mathematical Physics, vol. 2017, Article ID 5287132, 7 pages, 2017. https://doi.org/10.1155/2017/5287132.