MATHEMATICAL MODELS OF WATER QUAILITY MEASUREMENT AFTER DAM-BREAK PROBLEMS

KANAWOOT SUBKLAY

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF DOCTOR OF PHILOSOPHY (APPLIED MATHEMATICS)

DEPARTMENT OF APPLIED MATHEMATICS, FACULTY OF SCIENCE
KING MONGKUT'S INSTITUTE OF TECHNOLOGY LADKRABANG

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หัวข้อวิทยานิพนธ์	ตัวแบบเชิงคณิตศาสตร์ของการวัดคุณภาพน้ำหลังเกิดปัญหาพนังกั้นน้ำพังทลาย
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คำสำคัญ: พนังกั้นน้ำพังทลาย แบบจำลองการแพร่กระจาย คุณภาพของน้ำ ระเบียบวิธีผลต่างจำกัด

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ABSTRACT

In this research we used the dam-break (the breaking of sandbag dike) simulation to estimate the concentration of the water pollutant in a long term flooding area. The couple of the dispersion equations and the dam-break shallow water are involved and formulated. As there are no exact solution method in solving this source of problem. We then search for many finite difference method with the explicit and implicit formula. The results showed the group of explicit formula give more reasonable accepted solution to the problem and it is reasonable agree with the former work in the references.

Keywords: Dam-break Dispersion Model water-quality finite difference methods

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Kanawoot Subklay

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Chapter 1
Introduction

1.1 Water pollution after dike failure

In the present, world environment has increasingly changed, for instance, dry and hot water weather is getting longer than usual. On the other hand, if there is too much rain and water, affected until flooding.

In 2012, Thailand has been confronted a largest flooding. The mass of water has been drenched from many main and branch rivers to cover wide areas. The residents who lived in the flooding area have to build a manmade sandbag dike to protect their village. The flooding has been taken for a long time meanwhile the flooding water becomes contaminated. There are some residents in their flooding area want to drain their contaminated water to a nearest area. They have been destroyed their sandbag dike. Consequently, the dispute among residents is occurred. The simulation of water-quality is required to compromise the problem. The simulation process of water-quality model is require the input as the water flow velocities after the villagers dike has been destroyed but there is lack of field data on the flooding period.

Figure 1.1 The residents must build a manmade sandbag dike to protect their village.

Figure 1.2 The residents of another village want to drain the polluted water to another village. The debate among villager is occurred.

1.2 Literature review

In the recent years, there has been many research magnitude on the evolution of numerical models to simulate dam-break flows. For example, Garcia et al. [1] used Lax-Friendrichs, MacCormack and MacCormack TVD to compare exact solution with a one dimension (1D) dam-break flow simulation.

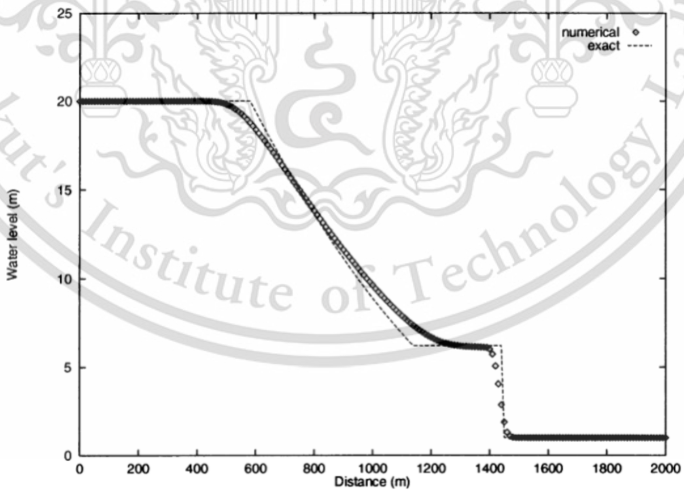


Figure 1.3 Dam break problem with Lax-Friendrichs versus exact solution [1].

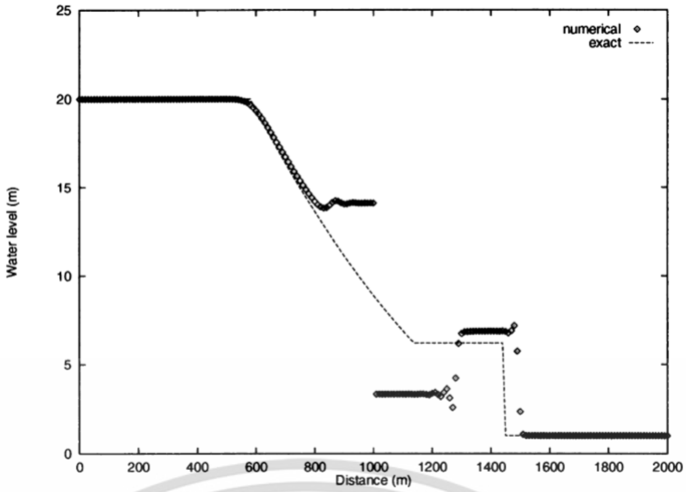


Figure 1.4 Dam break problem with MacCormack versus exact solution [1].

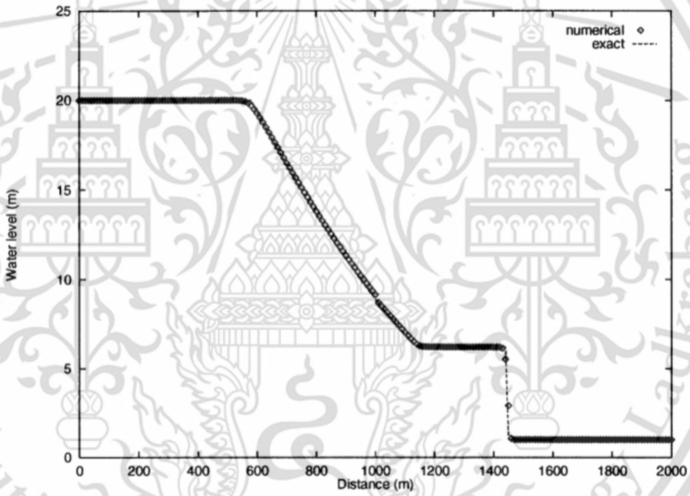


Figure 1.5 Dam break problem with TVD MacCormack versus exact solution [1].

Gottardi et al. [2] used central scheme for 1D and two dimension (2D) dam-break simulation.

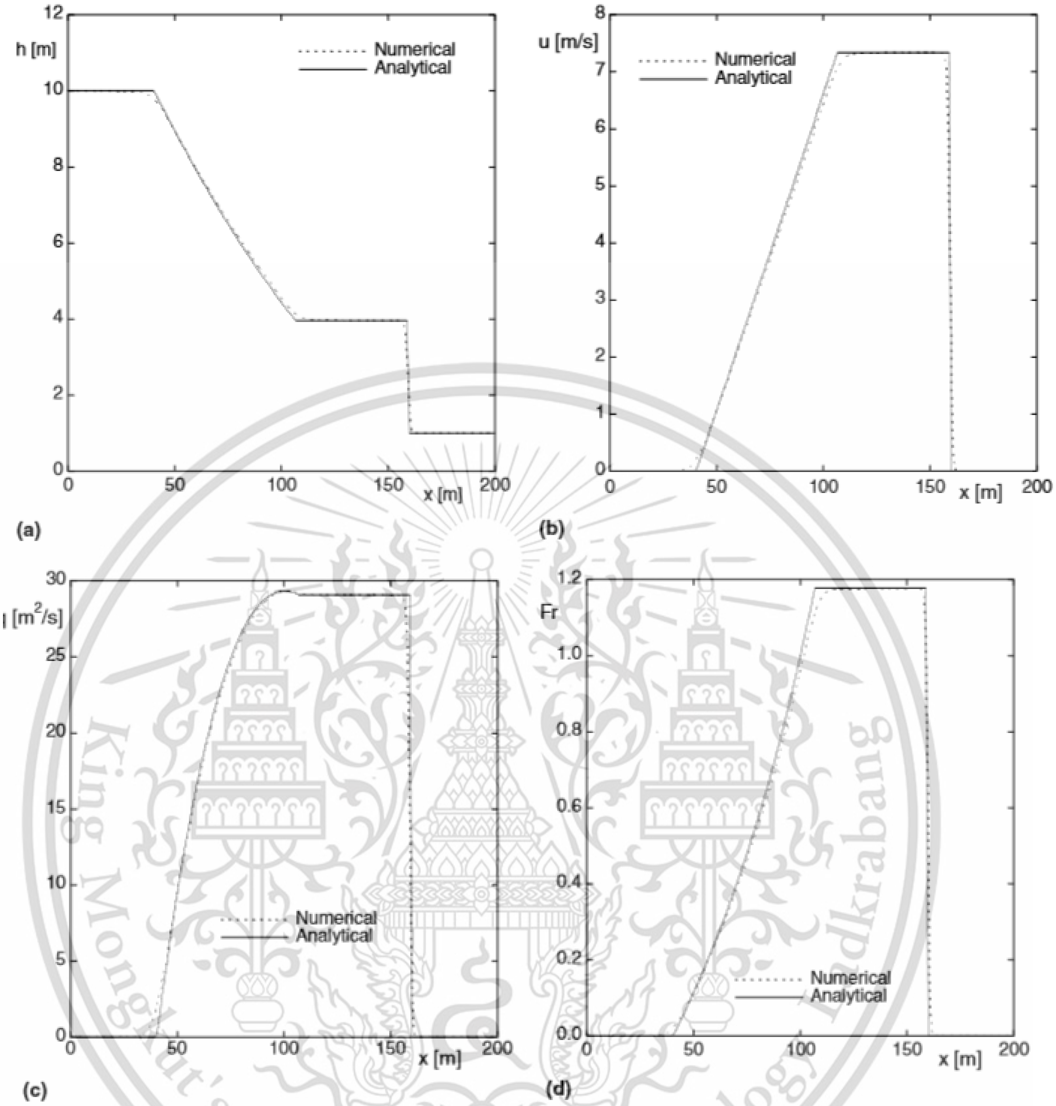


Figure 1.6 Comparison of the numerical and analytical solutions at t=6 s :(a) depth h; (b) velocity u; (c) unit discharge q and, (d) Froude number Fr [2].

Fayssal et al. [3] used finite volume method for numerical solution of shallow water equations in dam-break with flat topography.

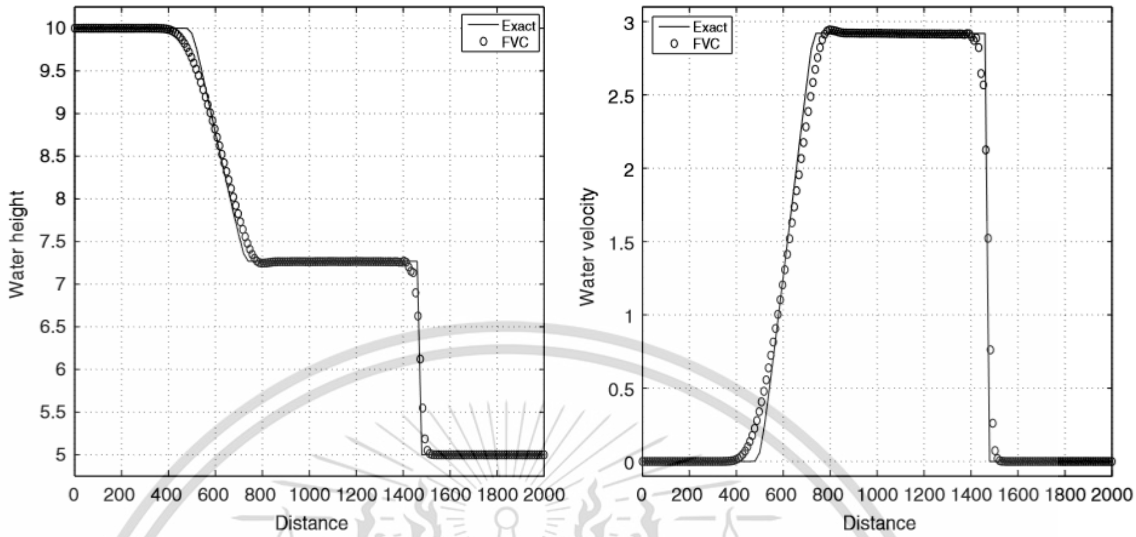


Figure 1.7 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at $T = 50s$ using $\Delta x = 10m$, $h_l/h_r = 0.5$ [3].

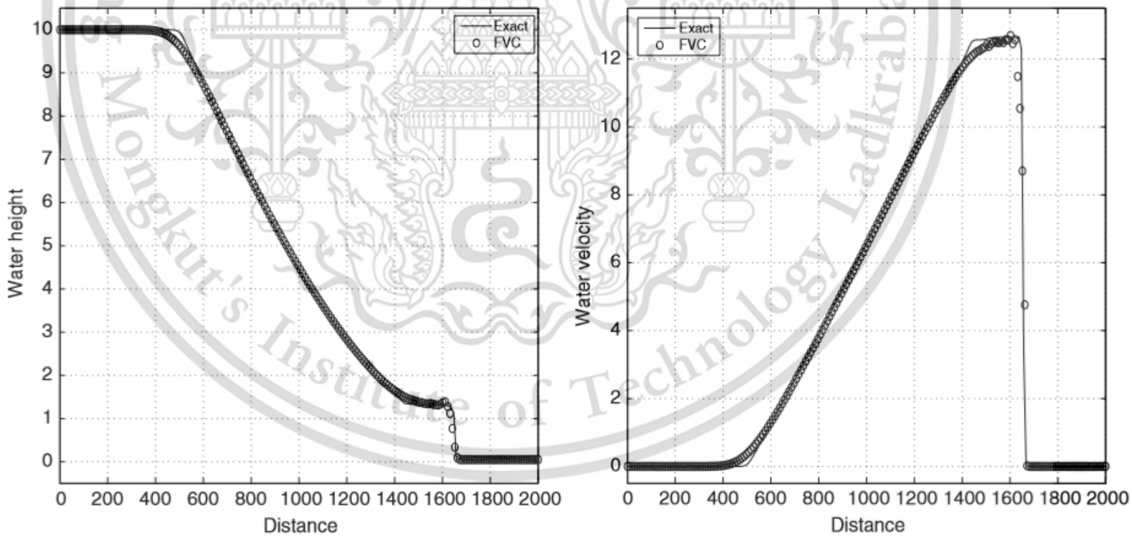


Figure 1.8 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at $t = 50s$ using $\Delta x = 10m$, $h_l/h_r = 0.005$ [3].

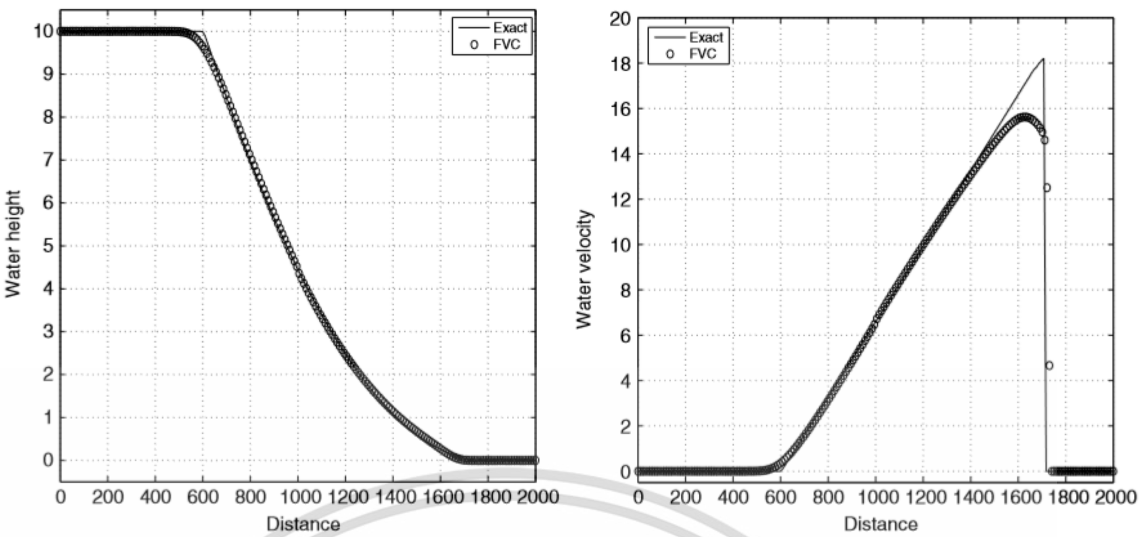


Figure 1.9 Water height (left plot) and water velocity (right plot) for dam-break on dry bed at $t = 40s$ using $\Delta x = 10m$, $h_l / h_r = \infty$ [3].

Pirotton et al. [4] used 2D finite volume multiblock flow solver. The model is based on Flux Vector Splitting method.

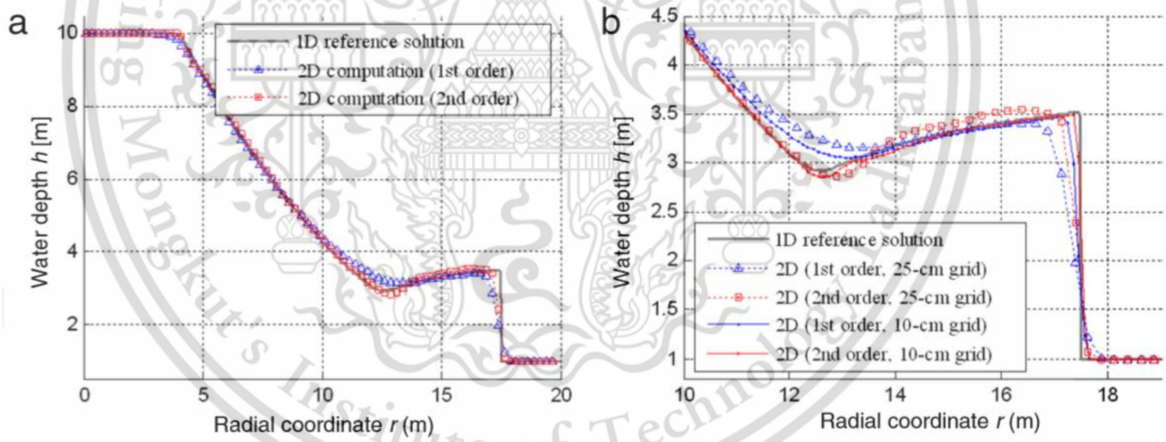


Figure 1.10 Computed water depths after 0.69s for the flow induced by the collapse of a circular dam. 1D reference solution as well as 2D solutions obtained with schemes first and second order accurate in space: (a) 0.25m cells; (b) details of 0.25m and 0.10m cells [4].

Baghlani [5] used a robust and effective flux-vector splitting method to simulate dam-break problem base on finite volume method on a cartesian grid.

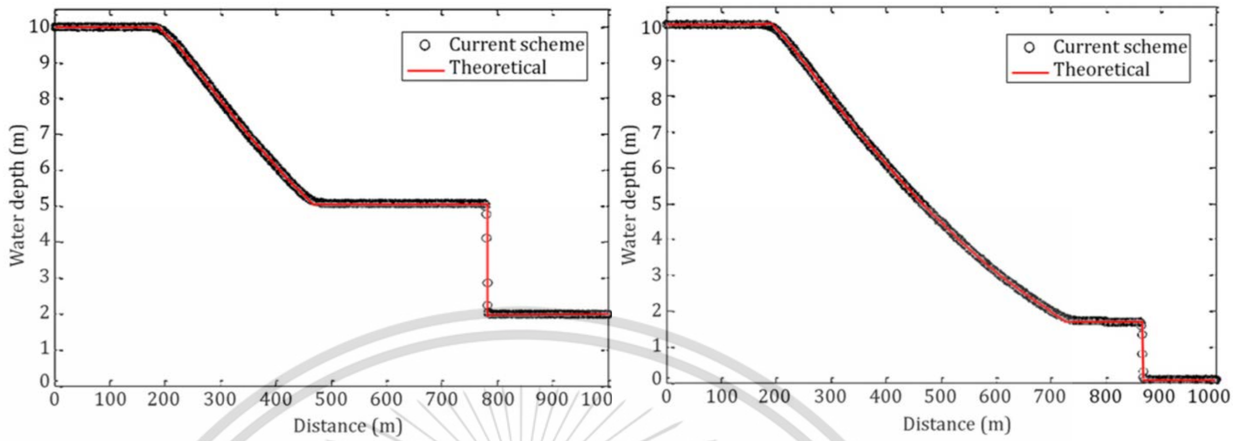


Figure 1.11 Comparison between theoretical and numerical of 1D dam-break problem for depth ratio equa to 0.2 (left) and 0.01 (right) [5].

Hong-Ming Kao et al. [6] used smoothed particle hydrodynamics (SPH) to solve shallow-water dam break flow in open channels.

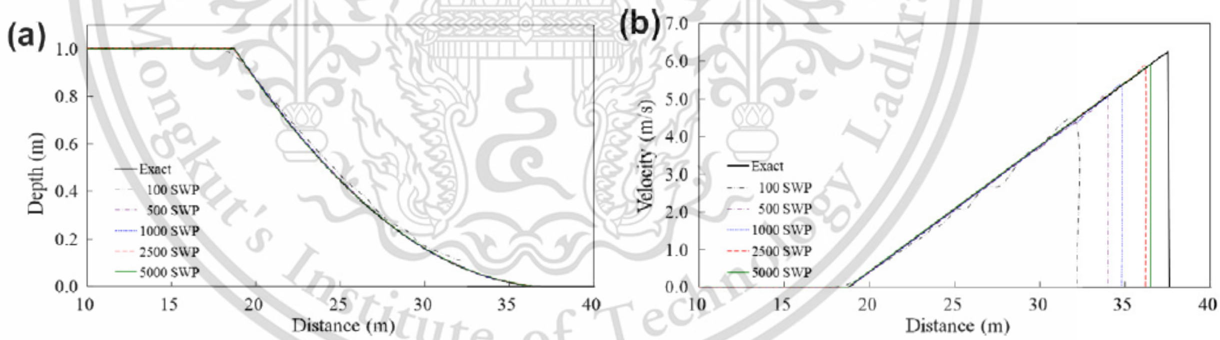


Figure 1.12 Comparison of the exact solutions with the simulated (a) water depths and (b) velocities for various numbers of slice water particles under water depths ratio of zero [6].

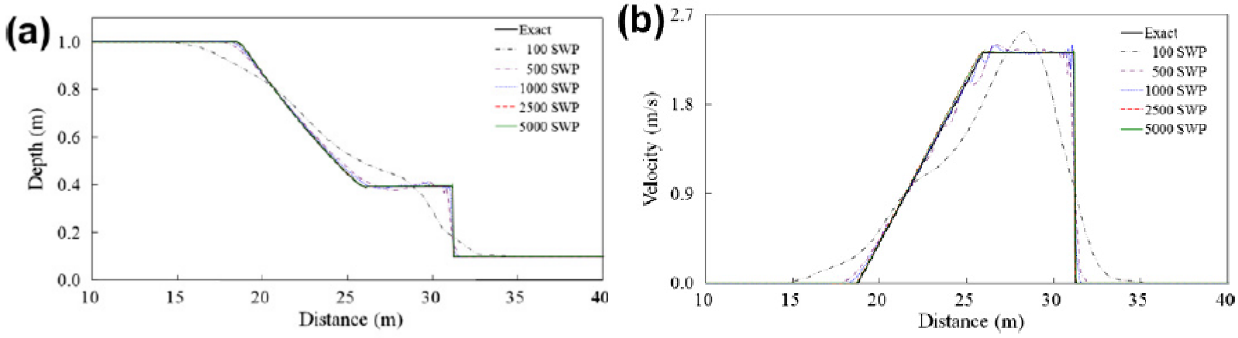


Figure 1.13 Comparison of the exact solutions with the simulated (a) water depths and (b) velocities for various numbers of slice water particles under water depths ratio of 0.1 [6].

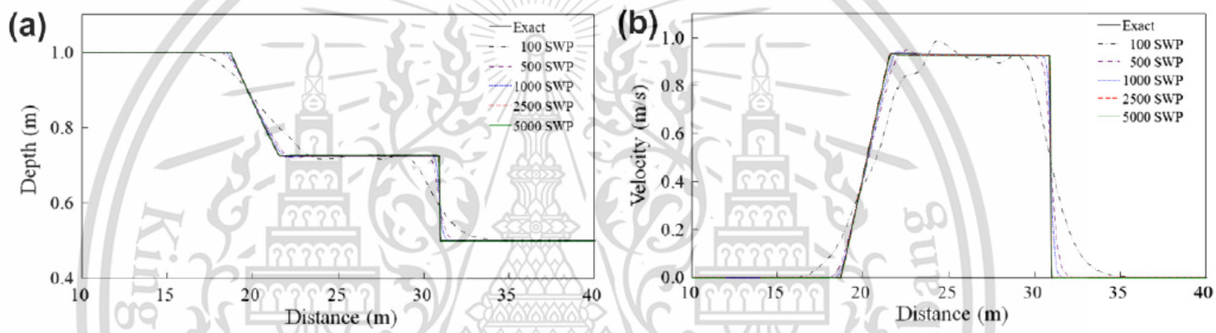


Figure 1.14 Comparison of the exact solutions with the simulated (a) water depths and (b) velocities for various numbers of slice water particles under water depths ratio of 0.5 [6].

Touma et al. [7] used a new well-balanced unstraggered central finite volume scheme for 1D and 2D dam break over a rectangular bump.

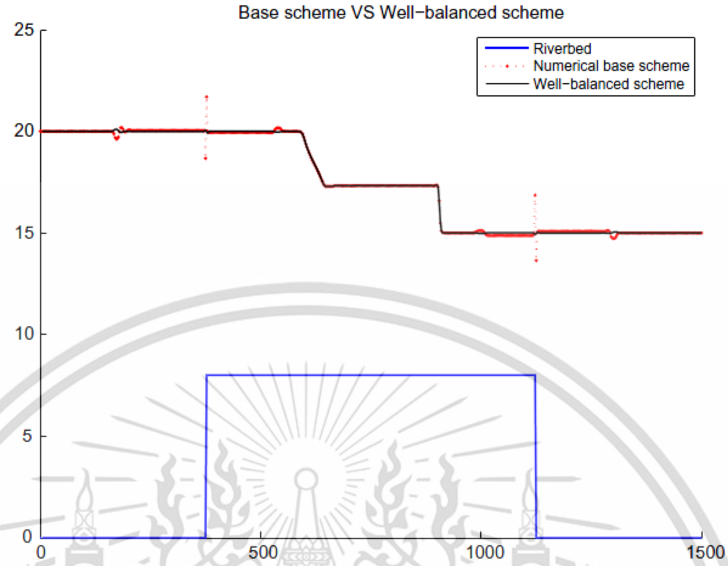


Figure 1.15 Dam break problem: water height obtained at time t = 15 using the numerical base scheme (dotted line) and the well-balanced scheme (solid line) [7].

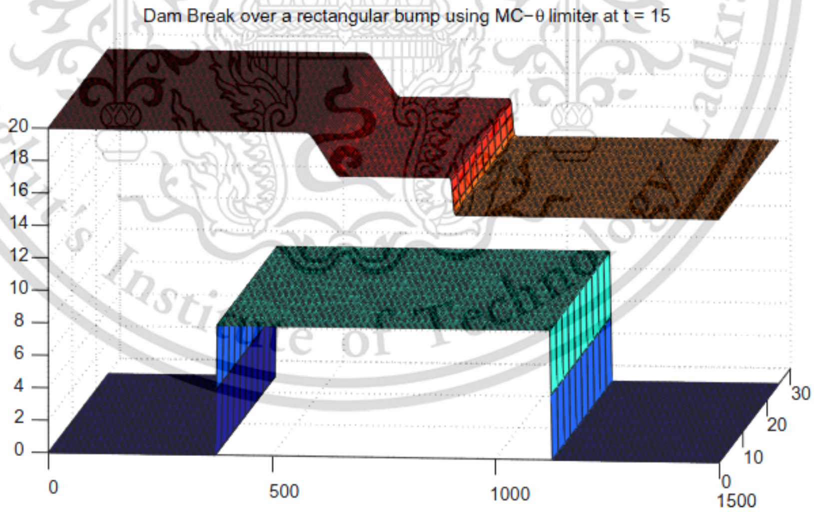


Figure 1.16 Two-dimensional dam break over a rectangular bump: profile of the water height obtained at the final time t = 15 [7].

Berthon et al. [8] used well-balanced hydrostatic upwind schemes for dam-break approximations. The dam-break model is used to Explain unsteady dike failure flow.

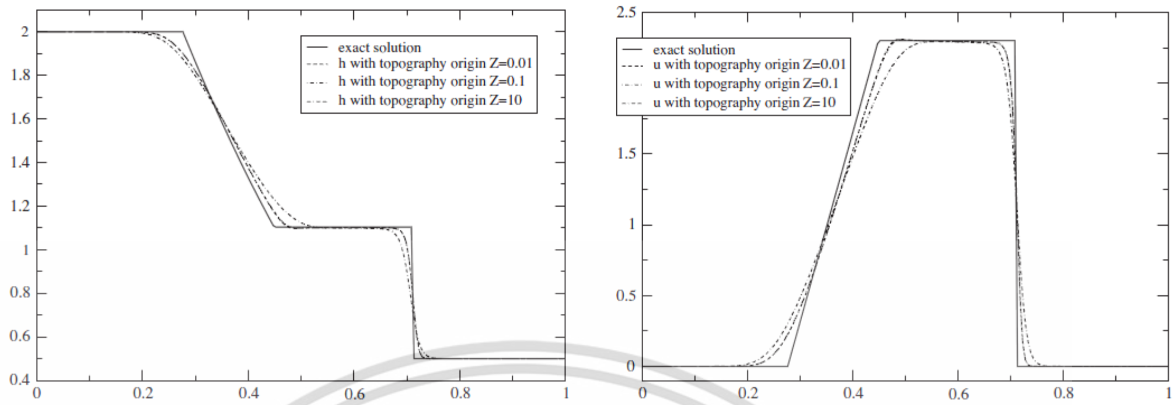


Figure 1.17 First-order wet dam-break [8].

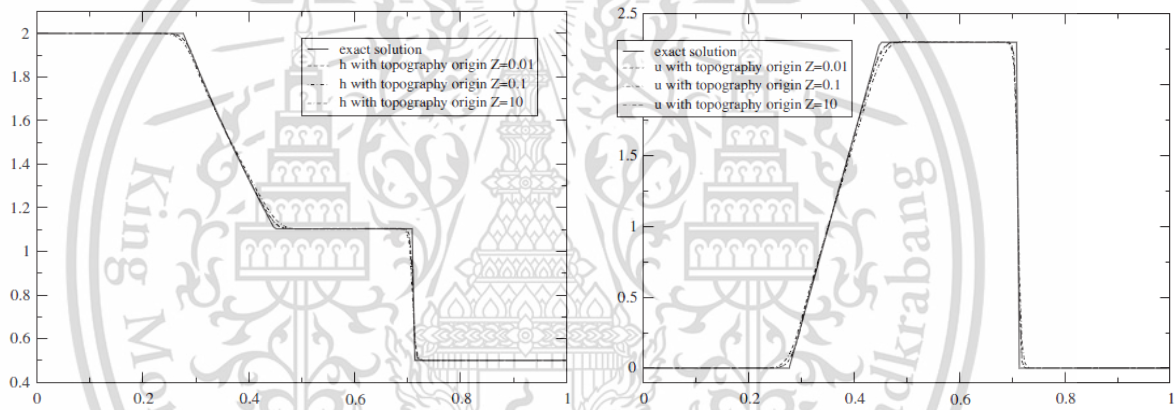


Figure 1.18 Second-order wet dam-break [8].

Akbari and Firoozi [9] used Implicit (PriessMan) and Explicit (Lax Diffusive) methods for Saint-Venent Equations to Simulate Flood wave in Natural Rivers. In this work, the Lax-diffusive technique is used to solve dike failure problem.

The water becomes more dirty due to the fact that all residents must stop flooding for long time period. The one dynamic advection-dispersion-reaction (ADREs) model and the one dynamic advection-dispersion (ADEs) model used to explain pollutant. For example, Guoyuan Li and C. Rhett Jackson [10] compared concentration with different methods in the one dynamic advection-dispersion-reaction equations

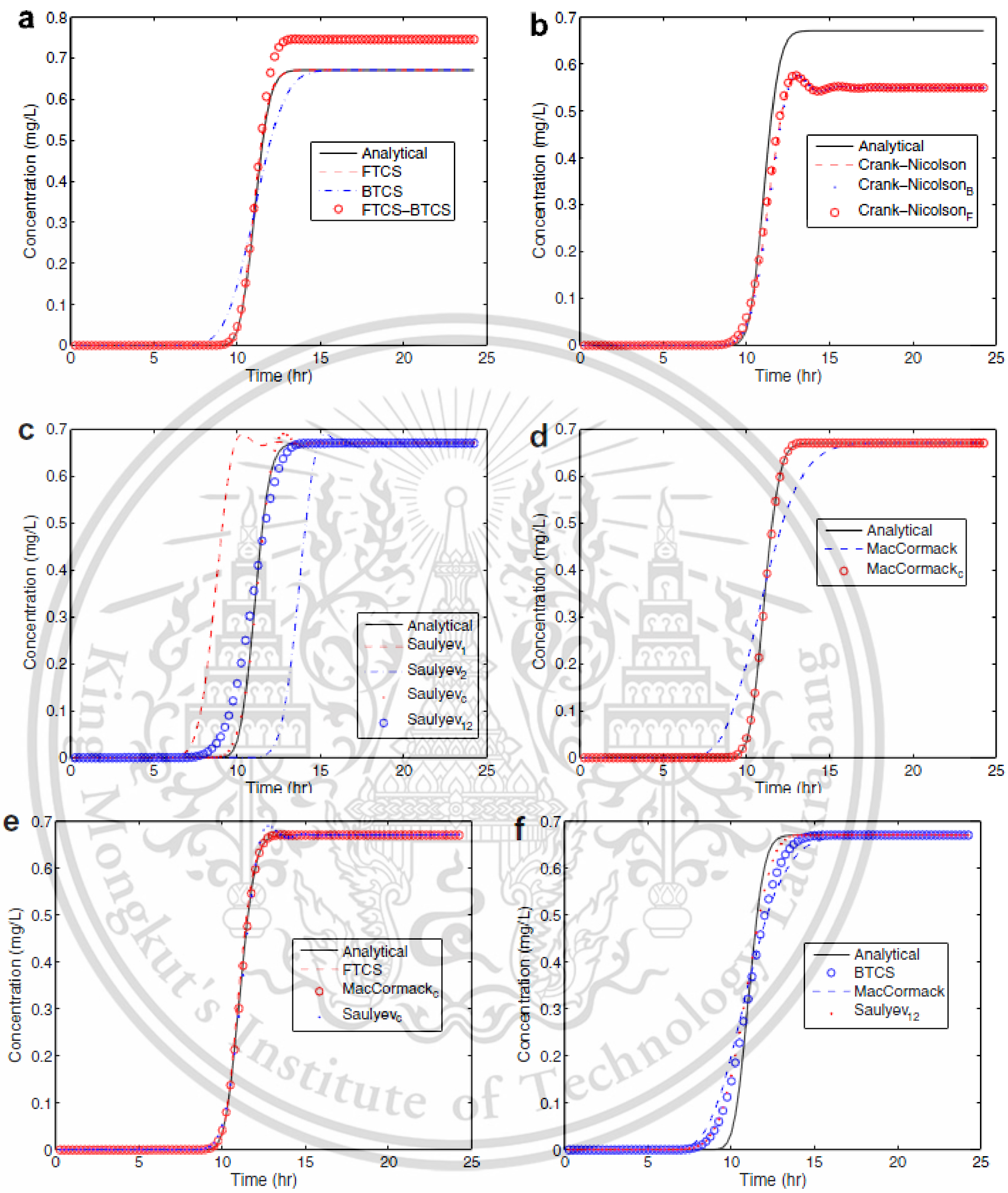


Figure 1.19 Performances of finite difference schemes under step input scenario: (a) FTCS, BTCS schemes, (b) Crank–Nicolson schemes, (c) Saulyev schemes, (d) MacCormack schemes, (e) more accurate schemes, (f) more stable schemes [10].

Nopparat Pochai [11] compared concentration at two different time instants of the FTCS and Sauyev methods in the one dynamic advection-dispersion-reaction equations.

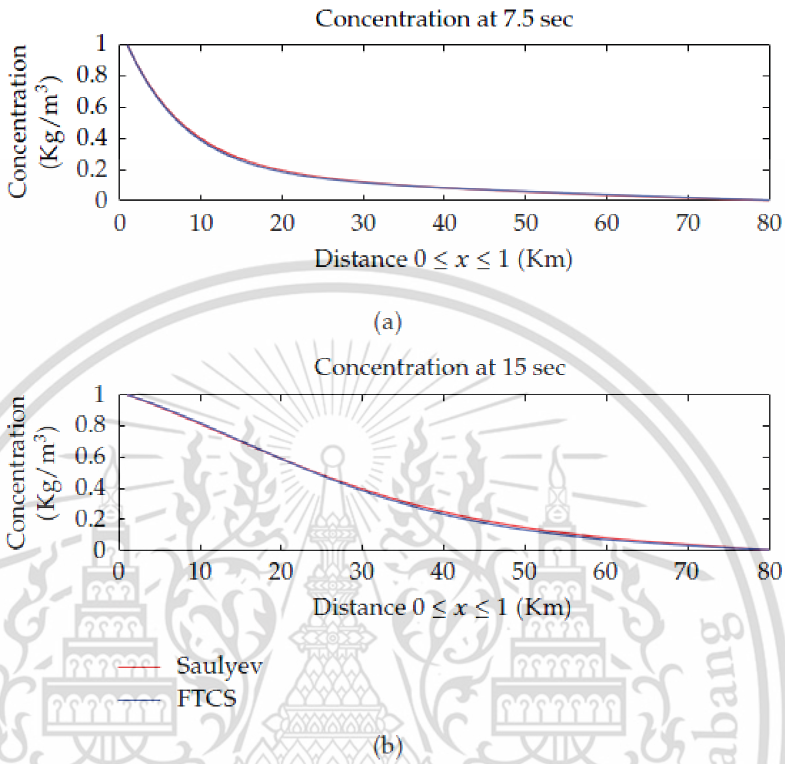


Figure 1.20 The comparison of concentration at two different time instants of the FTCS and Saulyev methods [11].

Mehdi Dehgan [12] compared concentration by weighted finite difference techniques for one-dimensional advection-diffusion equations.

The main idea of thesis, to find pollutant of water after dike failure by various finite difference methods from input variable velocity field data not constant.

1.3 Objectives

1.3.1) To proposed the numerical simulations to one-dimensional dike failure model.

1.3.2) To simulate water elevation and water velocity when the dike is broken.

1.3.3) To proposed the numerical simulations to the water-quality measurement when the dike is broken.

1.3.4) To compare the simulation results of the water-quality measurement after dike-break.

1.4 Scope of the thesis

1.4.1) To find numerical solution of one-dimensional dike failure problem by using modified Lax-Diffusive method.

1.4.2) Simulating water elevation and water velocity under dry bed and wet bed conditions.

1.4.3) To find numerical solution of one-dimensional-advection-dispersion-reaction equations and one-dimensional-advection-diffusion equations by using finite different methods.

1.4.4) To compare the calculated solutions of water quality model by implicit and explicit methods.

1.5 Plan of the thesis

There are two mathematical models used to simulate water quality in a non-uniform water flow systems. The first is the hydrodynamic model that provides the velocity field and the elevation of water. The second is the dispersion model that gives the pollutant concentration field. A couple of models are formulated in one-dimensional equations. At each step, the calculated flow velocity fields of the first model are input into the second model as the field data.

The first step, to study one-dimensional shallow water equations, one-dimensional advection-dispersion-reaction equations and one-dimensional advection-dispersion equations.

The second step, to study finite difference methods for solving one-dimensional hydrodynamic model and one-dimensional dispersion model.

The first part will study the basic knowledge about the shallow water and mathematical model for water quality, measurement and control by defining the domain of problem in thesis and domain of study case.

The third step, to compute the unsteady water pollutant after the dike failure. Two mathematical models are used to simulate pollutant concentration due to sewage from the water barrier. The first model is the one-dimensional shallow water equations that provides the water velocity and water height. The second model is the one-dimensional advection-dispersion-reaction equations that gives the pollutant concentration. In the process of computation, modify the Lax–diffusive method for solving the first model and the second model are used various finite different methods.

The forth step, to compute unsteady polluted water after the dike failure with the advection-diffusion equations. The second model need to input the velocity fields from the first model. We use the traditional methods and combine methods for the second model.

Finally, to compare numerical solutions from many finite difference methods in one-dimensional advection-dispersion-reaction equations or one-dimensional advection-diffusion equations.

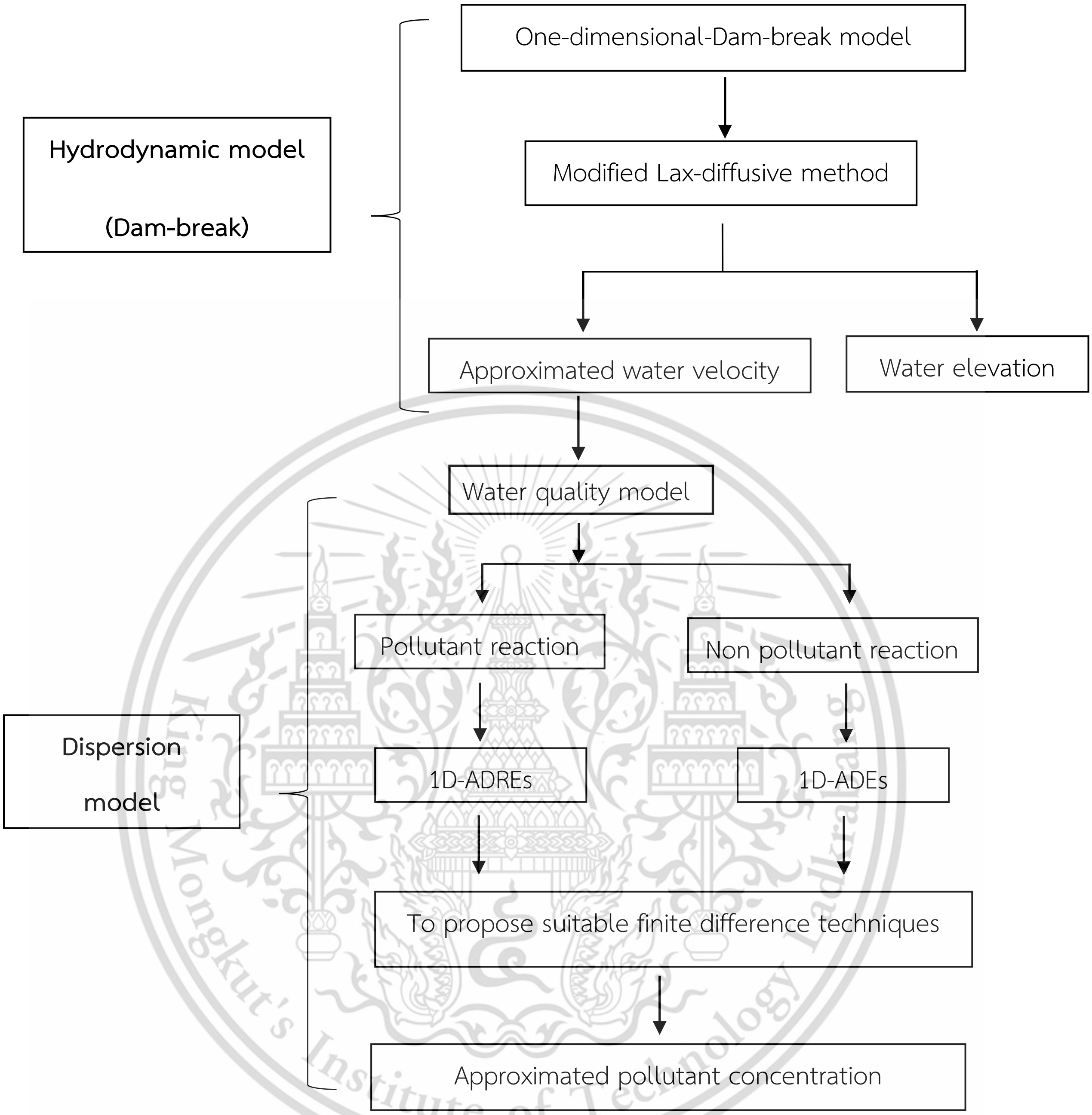


Figure 1.13 Flowchart of dam-break model and water quality model.

1.6 Expected results

The expected results of thesis are the applied couple model of water pollutant concentration and the dam-break model to explain the phenomenon of real world problems.

Chapter 2

Basic Concepts and Preliminaries

2.1 The one-dimensional hydrodynamic model

The one-dimensional hydrodynamic equations are obtained by integrating the Navier-Stokes equations over the flow depth under the assumptions as hydrostatic pressure distribution and small bottom slope. The dam-break flows are high velocity and can be considered as advection-dominated shallow water flows. Therefore, the eddy viscosity terms can be neglected. The governing equations on conversation and vector form can be written in the system of partial differential equations as [3]

$$\partial_x \begin{pmatrix} h \\ hu \end{pmatrix} + \partial_t \begin{pmatrix} hu \\ hu^2 + \frac{1}{2} gh^2 \end{pmatrix} = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix}, \qquad (2.1)$$

where x is the longitudinal distance along a stream (m), t is time (s), h(x,t) is the elevation of the water above the bottom (m/s), z(x) is the function characterizing the bottom topography (m), and u(x,t) is the velocity components (m/s), for all $x \in [0, L]$.

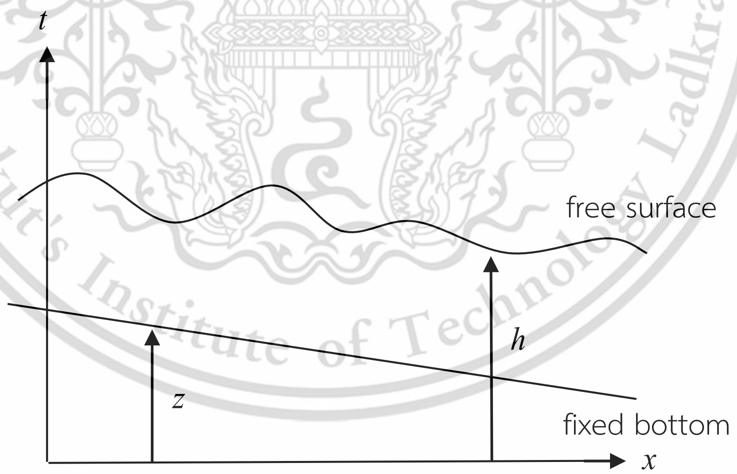


Figure 2.1 The shallow water system.

2.2 The Riemann problem of a dam-break

In this section, we will explain the characteristic structure of a dam-break. A dam will divide two sections of different water elevation at time $t = 0$. The initial condition is given by

$$h(x,0) = \begin{cases} h_l & if\ x < 0, \\ h_r & if\ x > 0, \end{cases} \tag{2.2}$$

$$u(x,0) = 0. \tag{2.3}$$

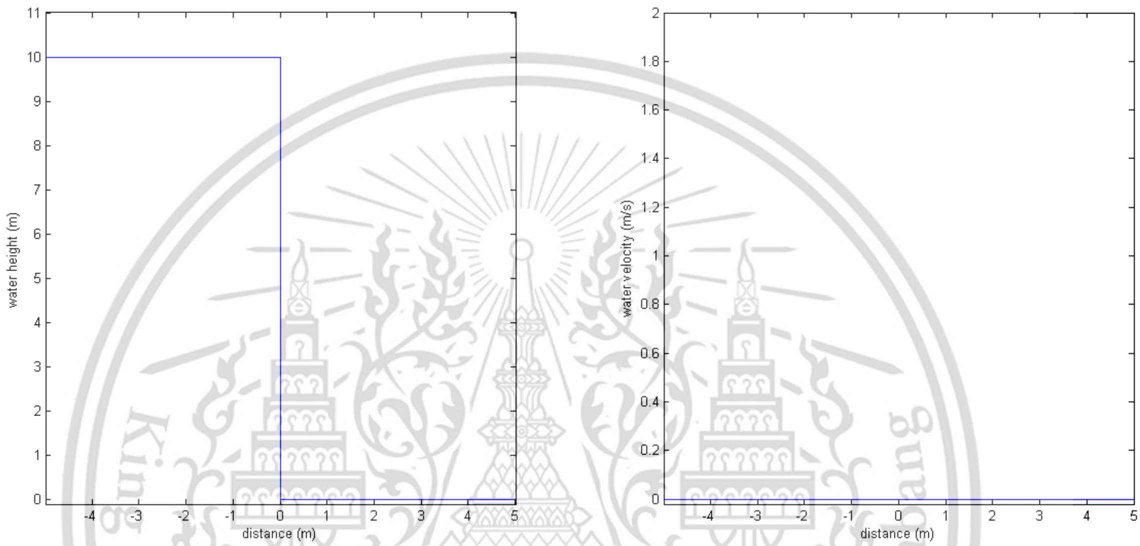


Figure 2.2 The water elevation (left plot) and water velocity (right plot) of initial condition in the dam-break problem.

2.3 The dam-break on wet bed and dry bed

The dam-break on wet bed is mean, the water elevation of flooding area more than zero $(h_r > 0)$. Otherwise, the dam-break on dry bed is non-flooding area $(h_r = 0)$.

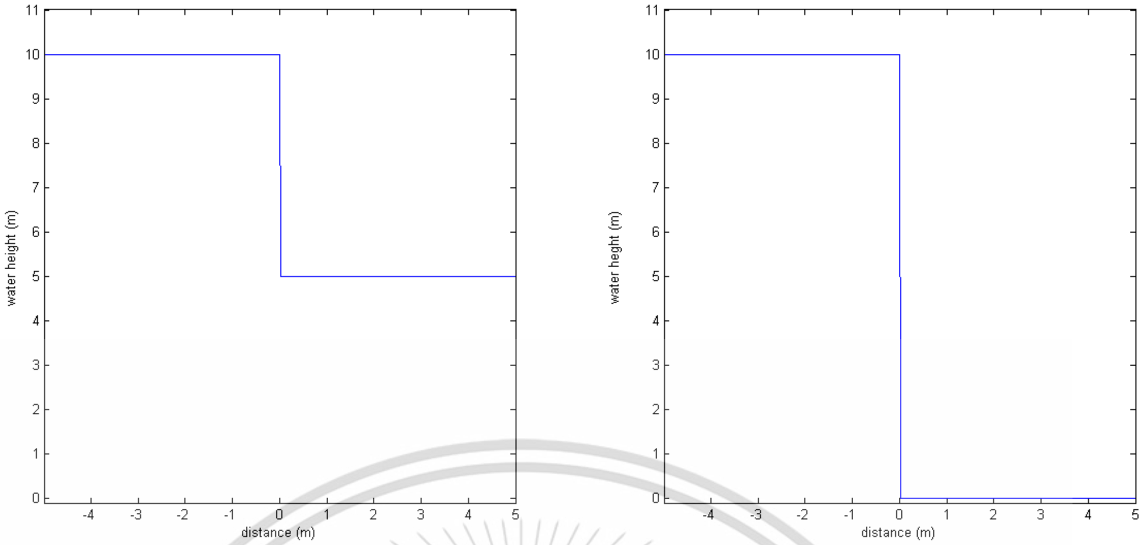


Figure 2.3 The dam-break on wet bed (left plot) and the dam-break on dry bed (right plot).

2.4 The fluid flow

Reynold number

The Reynold number is defined as

$$\mathrm{Re} = \frac{\rho UL}{\mu} = \frac{UL}{\nu}, \qquad (2.4)$$

where U is velocity (m/s),

L is characteristic linear dimension (m),

μ is dynamic viscocity of fluid (kg/ms),

ν is kinematic viscocity (m^2/s),

ρ is density of the fluid (kg/m^3).

The Reynold number can classify the fluid flow to 3 types.

Laminar flow

In fluid dynamics, laminar flow (or streamline flow) occurs when a fluid flows in parallel layers, with no disruption between the layers. At low velocities, the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards. There are no cross-currents perpendicular to the direction of flow. Laminar flow is a flow before regime characterized by high momentum diffusion and low momentum convection. Laminar flow is required less than 2000 Reynold number.

Turbulent flow

Turbulence or turbulent flow is a flow regime characterized by chaotic property changes. This includes low momentum diffusion, high momentum convection, and rapid variation of pressure and flow velocity in space and time. Turbulence is required more than 4000 Reynold number.

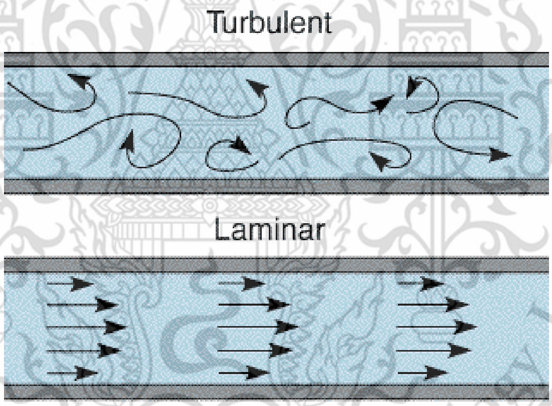


Figure 2.4 The turbulent flow and the laminar flow of the fluid flow.

Transition flow

The transition flow is occurs between laminar flow and turbulent flow, meanwhile between 2000 to 4000 Reynold number.

2.5 Governing equations of water quality measurement

Numerous types of water motion transport matter within natural waters. Wind energy and gravity import motion to the water that leads to mass transport. In the present context within system motion can be divided into two general categories: advection and diffusion.

Advection results from flow that is unidirectional and does not change the identity of the substance being transported. Advection moves matter from one position in space to another.

Diffusion refers to the movement of mass due to random water motion or mixing. Such transport causes the dye patch depicted to spread out and dilute over time with negligible not movement of its center of mass. Diffusion is the movement molecules from an area of higher concentration to one lower concentration.

Reactions refers to the mass is gained or lost by transformations of the substances within the volume. Reactions either add mass by changing another constituent into the substance being modeled or remove mass by transforming the substance into another constituent.

Dispersion is a related process that also causes pollution to spread. Dispersion is the result of velocity differences in space.

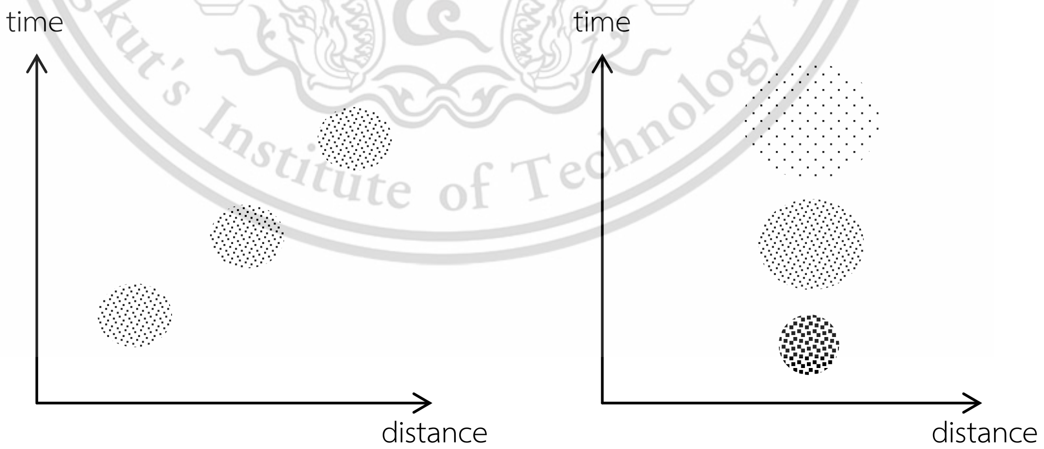


Figure 2.5 The transport of a dye splotch in space and time via (left plot) Advection and (right plot) Diffusion.

Dispersion model

Advection-Diffusion equations

In a stream water quality model, the governing equations are the dynamic one-dimensional advection-diffusion equations (ADEs) and the dynamic one-dimensional advection-dispersion-reaction equations (ADREs). A simplified representation by averaging the equation over the depth as

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D\frac{\partial^2 C}{\partial x^2}. \tag{2.5}$$

Advection-Dispersion-reaction equations

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D\frac{\partial^2 C}{\partial x^2}-KC, \tag{2.6}$$

where $C(x,t)$ is the concentration averaged in depth,
D is the diffusion coefficient,
K is the mass decay rate,
$u(x,t)$ is the velocity component.

CHAPTER 3

NUMERICAL SIMULATIONS OF WATER QUALITY MODEL IN A FLOODING STREAM DUE TO DAM-BREAK PROBLEM USING IMPLICIT AND EXPLICIT METHODS

In this chapter, we propose a modified Lax-diffusive for solving a dam-break model. The first model give water height and water velocity. The second model is dispersion model, need to input water velocity from the first model. The traditional finite difference methods are used to find and compare the pollutant concentration.

3.1 A modified Lax-diffusive method for the dam-break model

In this section the Modified Lax-diffusive is proposed. The domain of the dam-break problem is covered by a mesh of grid-lines $M\Delta x = L$ by dividing the interval $[0, L]$ into M subintervals and $N\Delta t = T$ by dividing the interval $[0, T]$ into N subintervals. We define $h(x_m, t_n)$ by h_m^n for solution of $h(x,t)$ at point $x = m\Delta x$ and $t = n\Delta t$, where $0 \le m \le M$ and $0 \le n \le N$, and same defined for u_m^n. The grid point (x_m, t_n) is defined by $x_m = m\Delta x$ for all $m = 0,1,2,\ldots,M$ and $t_n = n\Delta t$ for all $n = 0,1,2,\ldots,N$ in which M and N are positive integers.

The traditional Lax-diffusive method is modified by changing f^*, from two points average to three points average as

$$f_x = \frac{f_{m+1}^n - f_{m-1}^n}{2\Delta x}, \tag{3.1}$$

$$f_t = \frac{f_m^{n+1} - f_m^*}{\Delta t}, \tag{3.2}$$

where
$$f^* = \frac{f_{m+1}^n + f_m^n + f_{m-1}^n}{3}. \tag{3.3}$$

We transform Eq. (2.1) to matrix form as

$$A_t + B_x + C = 0, \tag{3.4}$$

where
$$A = \begin{pmatrix} h \\ hu \end{pmatrix}, B = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix}. \tag{3.5}$$

Followed in Eq. (3.5) can be written by the uniform spatial grids as

$$A_m^n = \begin{pmatrix} h_m^n \\ h_m^n u_m^n \end{pmatrix},\ B_m^n = \begin{pmatrix} h_m^n u_m^n \\ h_m^n \left(u_m^n\right)^2 + \frac{1}{2} g \left(h_m^n\right)^2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ -g h_m^n \partial_x \left(z_m^n\right) \end{pmatrix}. \tag{3.6}$$

Substituting the finite difference approximations of Eq. (3.1-3.2) and Eq. (3.3) into Eq.(3.4), we get

$$A_m^{n+1} = \frac{\Delta t}{2\Delta x}\left(B_{m-1}^n - B_{m+1}^n\right) + A^*. \tag{3.7}$$

where $A^* = \begin{pmatrix} h^* \\ (hu)^* \end{pmatrix}$ Substituting Eq. (3.6) into Eq. (3.7), we get

$$\begin{pmatrix} h_m^{n+1} \\ h_m^{n+1} u_m^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x} \begin{pmatrix} h_{m-1}^n u_{m-1}^n - h_{m+1}^n u_{m+1}^n \\ h_{m-1}^n \left(u_{m-1}^n\right)^2 - h_{m+1}^n \left(u_{m+1}^n\right)^2 + \frac{1}{2} g \left(\left(h_{m-1}^n\right)^2 - \left(h_{m+1}^n\right)^2\right) \end{pmatrix} + \frac{1}{3} \begin{pmatrix} h_{m-1}^n + h_m^n + h_{m+1}^n \\ h_{m-1}^n u_{m-1}^n + h_m^n u_m^n + h_{m+1}^n u_{m+1}^n \end{pmatrix}. \tag{3.8}$$

For all $1 \le m < M$ and $0 \le n \le N-1$. For upper boundary, where m = 0, fix the known value of the left boundary by $u_{-1}^n = u_0^n$ and $h_{-1}^n = h_0^n$ into Eq.(3.8) in the right-hand side, we obtain

$$\begin{pmatrix} h_1^{n+1} \\ h_1^{n+1} u_1^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x} \begin{pmatrix} h_0^n u_0^n - h_1^n u_1^n \\ h_0^n \left(u_0^n\right)^2 - h_1^n \left(u_1^n\right)^2 + \frac{1}{2} g \left(\left(h_0^n\right)^2 - \left(h_1^n\right)^2\right) \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 2h_0^n + h_1^n \\ 2h_0^n u_0^n + h_1^n u_1^n \end{pmatrix}. \tag{3.9}$$

For lower boundary, where m = M, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $u_{M+1}^n = u_M^n$ and $h_{M+1}^n = h_M^n$ by rearranging, we obtain

$$\begin{pmatrix} h_M^{n+1} \\ h_M^{n+1} u_M^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x} \begin{pmatrix} h_{M-1}^n u_{M-1}^n - h_M^n u_M^n \\ h_{M-1}^n \left(u_{M-1}^n\right)^2 - h_M^n \left(u_M^n\right)^2 + \frac{1}{2} g \left(\left(h_{M-1}^n\right)^2 - \left(h_M^n\right)^2\right) \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 2h_{M-1}^n + h_M^n \\ 2h_{M-1}^n u_{M-1}^n + h_M^n u_M^n \end{pmatrix}. \tag{3.10}$$

The stability condition of the scheme needed CFL number as,

$$C_n = u_{max}\left(\frac{\Delta t}{\Delta x}\right) \le 1. \tag{3.11}$$

3.2 Numerical solution of dam-break problem

First, we show some example by using initial and boundary conditions from Hong-Ming Kao et al. [6]. We consider the dam-break problem with flat topography, $z(x)=0.$ The length of channel is 40 meter, the space meshing $\Delta x = 0.4$ meter and the time meshing $\Delta t = 0.04$. We use Neumann boundary for boundary conditions.

Simulation 1: High wet bed scenario

$$h(x,0)=\begin{cases} 1, & if \quad x \le 20 \\ 0.5, & if \quad 20 < x \le 40 \end{cases} \tag{3.12}$$

$$u(x,0)=0,\ 0 \le t \le 2. \tag{3.13}$$

$$\begin{aligned} h_x(0,t) &= h_x(L,t)=0. \\ u_x(0,t) &= u_x(L,t)=0. \end{aligned} \tag{3.14}$$

Table 3.1 The velocity of water flow $u(x,t)\ m/s$, depth ratio 0.5.

t\x	0	4	8	12	16	20	24	28	32	36	40
0.4	0.0000	0.0000	0.0000	0.0000	0.0000	0.6027	0.0000	0.0000	0.0000	0.0000	0.0000
0.8	0.0000	0.0000	0.0000	0.0000	0.0488	0.7513	0.0929	0.0000	0.0000	0.0000	0.0000
1.2	0.0000	0.0000	0.0000	0.0008	0.2135	0.8173	0.4030	0.0008	0.0000	0.0000	0.0000
1.6	0.0000	0.0000	0.0000	0.0205	0.3816	0.8530	0.6824	0.0207	0.0000	0.0000	0.0000
2.0	0.0000	0.0000	0.0008	0.0864	0.5150	0.8747	0.8274	0.1188	0.0003	0.0000	0.0000

Table 3.2 The elevation of water flow $h(x,t)\ m/s$, depth ratio 0.5.

t\x	0	4	8	12	16	20	24	28	32	36	40
0.4	1.0000	1.0000	1.0000	1.0000	1.0000	0.7770	0.5000	0.5000	0.5000	0.5000	0.5000
0.8	1.0000	1.0000	1.0000	1.0000	0.9844	0.7587	0.5207	0.5000	0.5000	0.5000	0.5000
1.2	1.0000	1.0000	1.0000	0.9997	0.9323	0.7480	0.5918	0.5002	0.5000	0.5000	0.5000
1.6	1.0000	1.0000	1.0000	0.9934	0.8804	0.7409	0.6606	0.5047	0.5000	0.5000	0.5000
2.0	1.0000	1.0000	0.9998	0.9725	0.8405	0.7361	0.6985	0.5268	0.5001	0.5000	0.5000

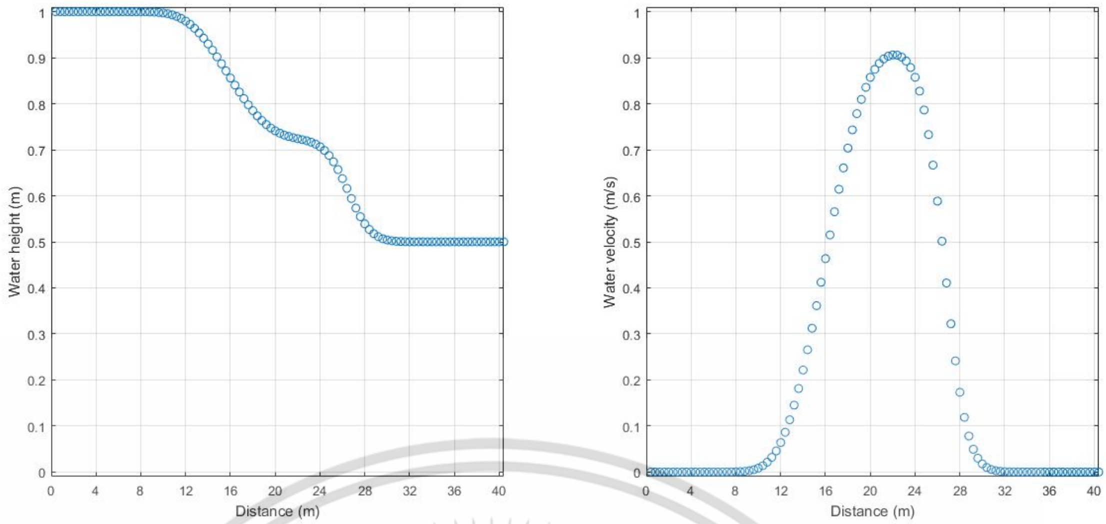


Fig. 3.1 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at t=2s. $\left(h_r/h_l = 0.5\right)$

Simulation 2: low wet bed scenaio

$$h(x,0)=\begin{cases}1, & if \quad x\le 20\\ 0.1, & if \quad 20<x\le 40\end{cases} \tag{3.12}$$

$$u(x,0)=0,\ 0\le t\le 2. \tag{3.13}$$

$$\begin{aligned}h_x(0,t)=h_x(L,t)=0.\\ u_x(0,t)=u_x(L,t)=0.\end{aligned} \tag{3.14}$$

Table 3.3 The velocity of water flow $u(x,t)\ m/s,$ depth ratio 0.1.

t\x	0	4	8	12	16	20	24	28	32	36	40
0.4	0.0000	0.0000	0.0000	0.0000	0.0000	1.0474	0.0000	0.0000	0.0000	0.0000	0.0000
0.8	0.0000	0.0000	0.0000	0.0000	0.0736	1.3199	0.2434	0.0000	0.0000	0.0000	0.0000
1.2	0.0000	0.0000	0.0000	0.0013	0.3056	1.4486	1.2214	0.0004	0.0000	0.0000	0.0000
1.6	0.0000	0.0000	0.0000	0.0298	0.5373	1.5267	1.8836	0.0194	0.0000	0.0000	0.0000
2.0	0.0000	0.0000	0.0012	0.1193	0.7253	1.5815	2.0991	0.2370	0.0000	0.0000	0.0000

Table 3.4 The elevation of water flow $h(x,t)\ m/s,$ depth ratio 0.1.

t\x	0	4	8	12	16	20	24	28	32	36	40
0.4	1.0000	1.0000	1.0000	1.0000	1.0000	0.6145	0.1000	0.1000	0.1000	0.1000	0.1000
0.8	1.0000	1.0000	1.0000	1.0000	0.9764	0.5870	0.1200	0.1000	0.1000	0.1000	0.1000
1.2	1.0000	1.0000	1.0000	0.9996	0.9033	0.5687	0.2208	0.1000	0.1000	0.1000	0.1000
1.6	1.0000	1.0000	1.0000	0.9905	0.8328	0.5552	0.3297	0.1018	0.1000	0.1000	0.1000
2.0	1.0000	1.0000	0.9996	0.9621	0.7778	0.5448	0.3826	0.1202	0.1000	0.1000	0.1000

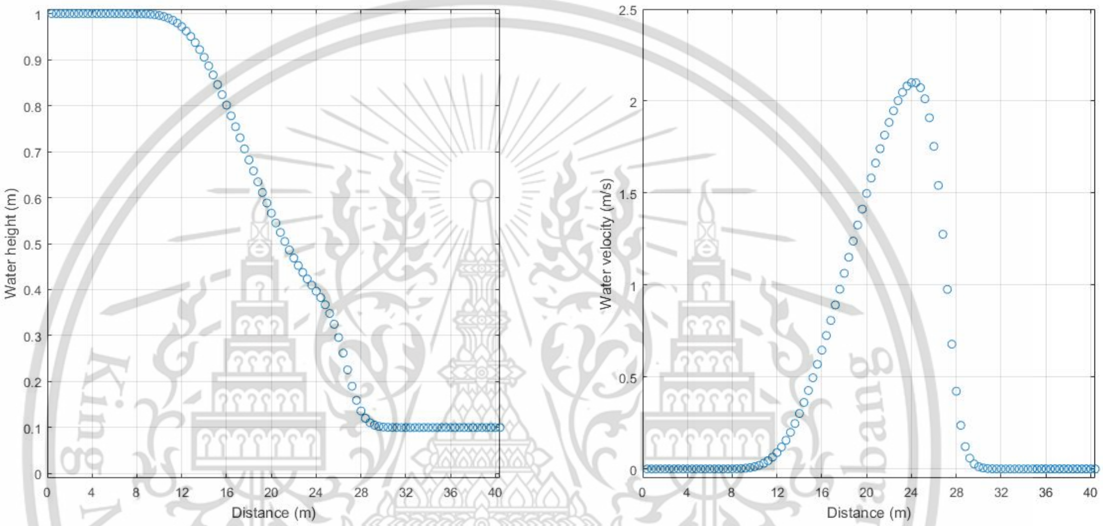


Fig. 3.2 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at t=2s. $\left(h_r/h_l = 0.1\right)$

Simulation 3: dry bed scenario

$$h(x,0) = \begin{cases} 1, & if \quad x \le 20 \\ 0, & if \quad 20 < x \le 40 \end{cases} \tag{3.12}$$

$$u(x,0) = 0,\ 0 \le t \le 2. \tag{3.13}$$

$$\begin{aligned} h_x(0,t) &= h_x(L,t) = 0. \\ u_x(0,t) &= u_x(L,t) = 0. \end{aligned} \tag{3.14}$$

Table 3.5 The velocity of water flow $u(x,t)\ m/s$, depth ratio ∞.

t\x	0	4	8	12	16	20	24	28	32	36	40
0.4	0.0000	0.0000	0.0000	0.0000	0.0000	1.9948	0.0000	0.0000	0.0000	0.0000	0.0000
0.8	0.0000	0.0000	0.0000	0.0000	0.0000	2.0335	4.8676	0.0000	0.0000	0.0000	0.0000
1.2	0.0000	0.0000	0.0000	0.0000	0.0382	2.0485	4.2398	0.0000	0.0000	0.0000	0.0000
1.6	0.0000	0.0000	0.0000	0.0000	0.4344	2.0566	3.7029	5.3853	0.0000	0.0000	0.0000
2.0	0.0000	0.0000	0.0000	0.0000	0.7508	2.0618	3.3812	4.7120	0.0000	0.0000	0.0000

Table 3.6 The elevation of water flow $h(x,t)\ m/s$, depth ratio ∞.

t\x	0	4	8	12	16	20	24	28	32	36	40
0.4	1.0000	1.0000	1.0000	1.0000	1.0000	0.4613	0.0000	0.0000	0.0000	0.0000	0.0000
0.8	1.0000	1.0000	1.0000	1.0000	1.0000	0.4543	0.0009	0.0000	0.0000	0.0000	0.0000
1.2	1.0000	1.0000	1.0000	1.0000	0.9878	0.4516	0.1002	0.0000	0.0000	0.0000	0.0000
1.6	1.0000	1.0000	1.0000	1.0000	0.8659	0.4501	0.1647	0.0116	0.0000	0.0000	0.0000
2.0	1.0000	1.0000	1.0000	1.0000	0.7744	0.4492	0.2101	0.0580	0.0000	0.0000	0.0000

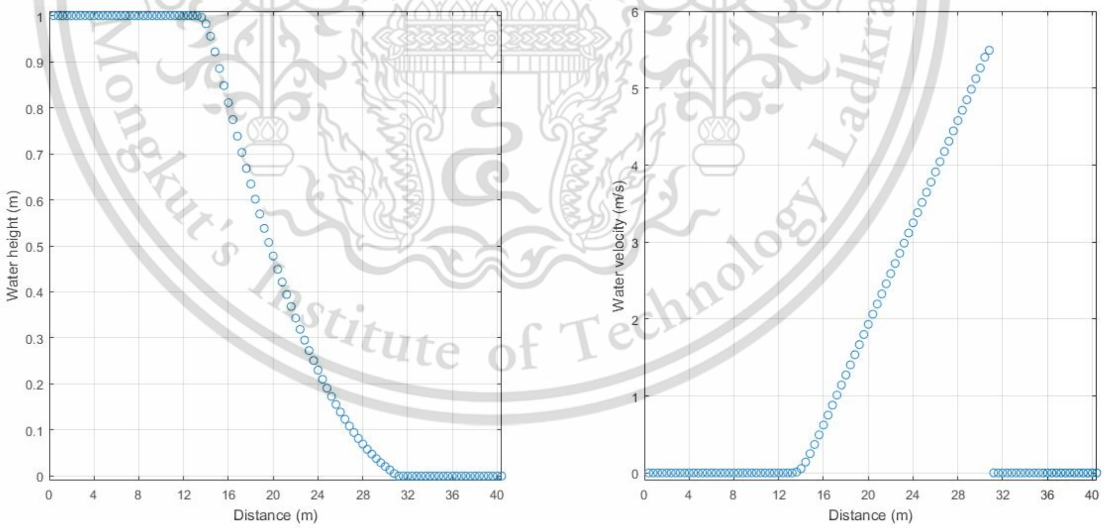


Fig. 3.3 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at t=2s. $\left(h_r/h_l=\infty\right)$

Next, we consider the dam-break problem with flat topography, $z(x)=0$. The length of channel is 2000 meter, the space meshing $\Delta x = 10$ meter. We use Neumann boundary for boundary conditions and initial conditions [3] are given in 3 cases.

Case A: high wet bed scenario

given $$h(x,0)=\begin{cases}10, & if \quad x\le 1000\\ 5, & if \quad 1000 < x \le 2000\end{cases} \tag{3.12}$$

$$u(x,0)=0,\ 0\le t\le 50. \tag{3.13}$$

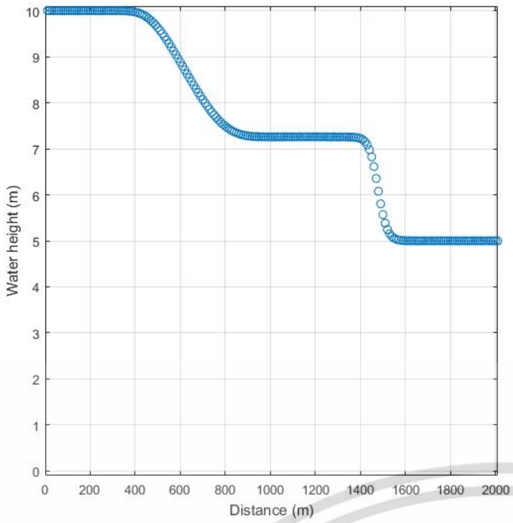
$$\begin{aligned} &h_x(0,t)=h_x(L,t)=0.\\ &u_x(0,t)=u_x(L,t)=0.\end{aligned} \tag{3.14}$$

Table 3.7 The velocity of water flow $u(x,t)\ m/s$, depth ratio 0.5.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.7676	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.5432	2.8906	1.0240	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0134	1.722	2.9128	2.8879	0.0015	0.0000	0.0000
40	0.0000	0.0000	0.0000	0.0000	0.4606	2.3804	2.9171	2.9101	0.6703	0.0000	0.0000
50	0.0000	0.0000	0.0421	1.2290	2.6967	2.9177	2.9122	2.8402	0.0023	0.0000	0.0000

Table 3.8 The elevation of water flow $h(x,t)\ m/s$, depth ratio 0.5.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	10.0000	10.0000	10.0000	10.0000	10.0000	7.3648	5.0000	5.0000	5.0000	5.0000	5.0000
20	10.0000	10.0000	10.0000	10.0000	9.4569	7.2716	5.7426	5.0000	5.0000	5.0000	5.0000
30	10.0000	10.0000	10.0000	9.9864	8.3284	7.2576	7.2357	5.0011	5.0000	5.0000	5.0000
40	10.0000	10.0000	10.0000	9.5393	7.7305	7.2569	7.2561	5.4835	5.0000	5.0000	5.0000
50	10.0000	10.0000	9.9575	8.7937	7.4523	7.2583	7.2589	7.1994	5.0017	5.0000	5.0000



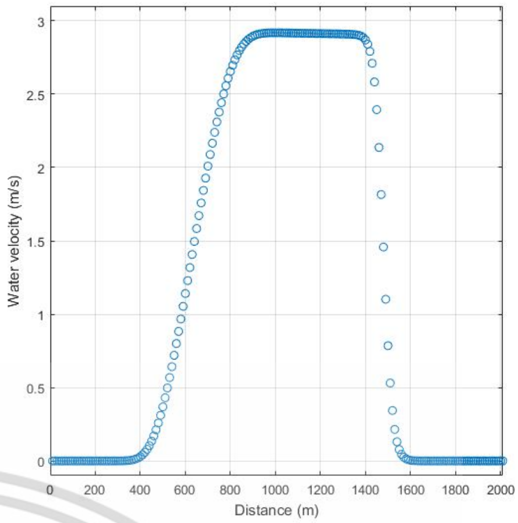


Fig. 3.4 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at t=50s. $\left(h_r/h_l = 0.5\right)$

Case B: low wet bed scenario

given
$$h(x,0) = \begin{cases} 10, & if \quad x \le 1000 \\ 0.05, & if \quad 1000 < x \le 2000 \end{cases} \quad (3.15)$$
$$u(x,0) = 0,\ 0 \le t \le 50.$$

$$h_x(0,t) = h_x(L,t) = 0. \quad (3.16)$$
$$u_x(0,t) = u_x(L,t) = 0.$$

Table 3.9 The velocity of water flow $u(x,t)\ m/s$, depth ratio 0.005.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	5.3233	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.7082	5.7794	10.8515	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0187	2.1748	5.9784	9.8770	0.0056	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0001	0.5590	3.1218	6.0936	9.1105	11.7065	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0542	1.4388	3.7391	6.1699	8.6256	10.9812	9.3860	0.0000	0.0000

Table 3.10 The elevation of water flow $h(x,t)\ m/s$, depth ratio 0.005.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	10.0000	10.0000	10.0000	10.0000	10.0000	5.1902	0.0500	0.0500	0.0500	0.0500	0.0500
20	10.0000	10.0000	10.0000	10.0000	9.2941	4.9232	1.4900	0.0500	0.0500	0.0500	0.0500
30	10.0000	10.0000	10.0000	9.9811	7.9105	4.8073	2.3477	0.0502	0.0500	0.0500	0.0500
40	10.0000	10.0000	9.9999	9.4421	7.0772	4.7404	2.8107	1.4811	0.0500	0.0500	0.0500
50	10.0000	10.0000	9.9453	8.5949	6.5603	4.6961	3.1096	1.8630	0.5364	0.0500	0.0500

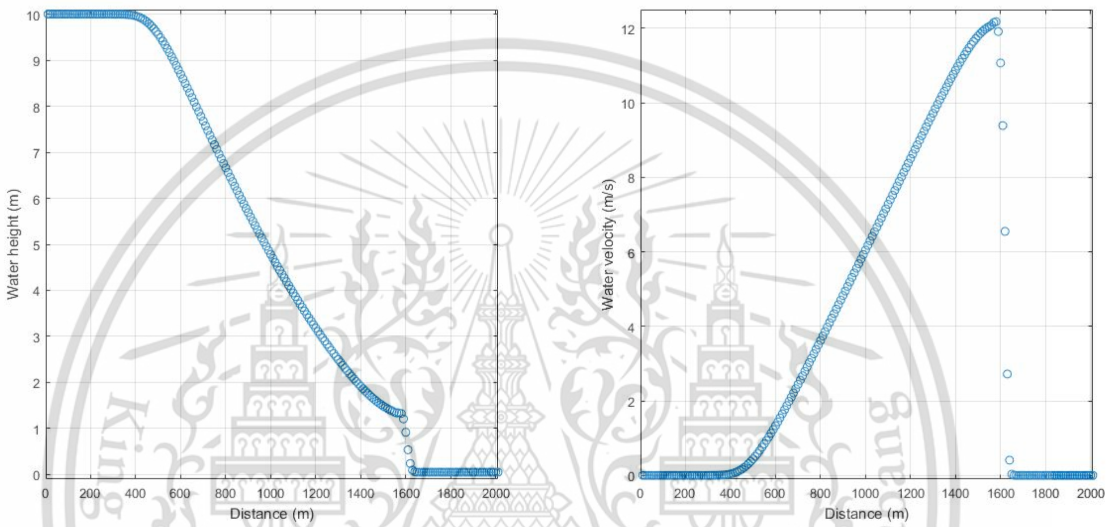


Fig. 3.5 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at t=50s. $\left(h_r/h_l = 0.005\right)$

Case C: dry bed scenario

given $$h(x,0)=\begin{cases}10, & if \quad x \le 1000 \\ 0, & if \quad 1000 < x \le 2000\end{cases} \tag{3.17}$$
$$u(x,0)=0,\ 0\le t \le 40.$$

$$h_x(0,t)=h_x(L,t)=0.$$
$$u_x(0,t)=u_x(L,t)=0. \tag{3.18}$$

Table 3.11 The velocity of water flow $u(x,t)\ m/s$, depth ratio ∞.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
8	0.0000	0.0000	0.0000	0.0000	0.0000	5.3387	0.0000	0.0000	0.0000	0.0000	0.0000
16	0.0000	0.0000	0.0000	0.0000	0.7091	5.7917	11.4513	0.0000	0.0000	0.0000	0.0000
24	0.0000	0.0000	0.0000	0.0188	2.1767	5.9884	9.9854	11.0343	0.0000	0.0000	0.0000
32	0.0000	0.0000	0.0001	0.5595	3.124	6.1021	9.1596	12.3166	8.6766	0.0000	0.0000
40	0.0000	0.0000	0.0543	1.4396	3.7415	6.1772	8.6552	11.1796	13.8577	0.0000	0.0000

Table 3.12 The elevation of water flow $h(x,t)\ m/s$, depth ratio ∞.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
8	10.0000	10.0000	10.0000	10.0000	10.0000	5.1775	0.0000	0.0000	0.0000	0.0000	0.0000
16	10.0000	10.0000	10.0000	10.0000	9.2932	4.9139	1.3036	0.0000	0.0000	0.0000	0.0000
24	10.0000	10.0000	10.0000	9.9811	7.9088	4.7999	2.2892	0.0393	0.0000	0.0000	0.0000
32	10.0000	10.0000	9.9999	9.4416	7.0753	4.7343	2.7827	1.2304	0.0000	0.0000	0.0000
40	10.0000	10.0000	9.9452	8.5941	6.5583	4.6909	3.0922	1.7712	0.6834	0.0000	0.0000

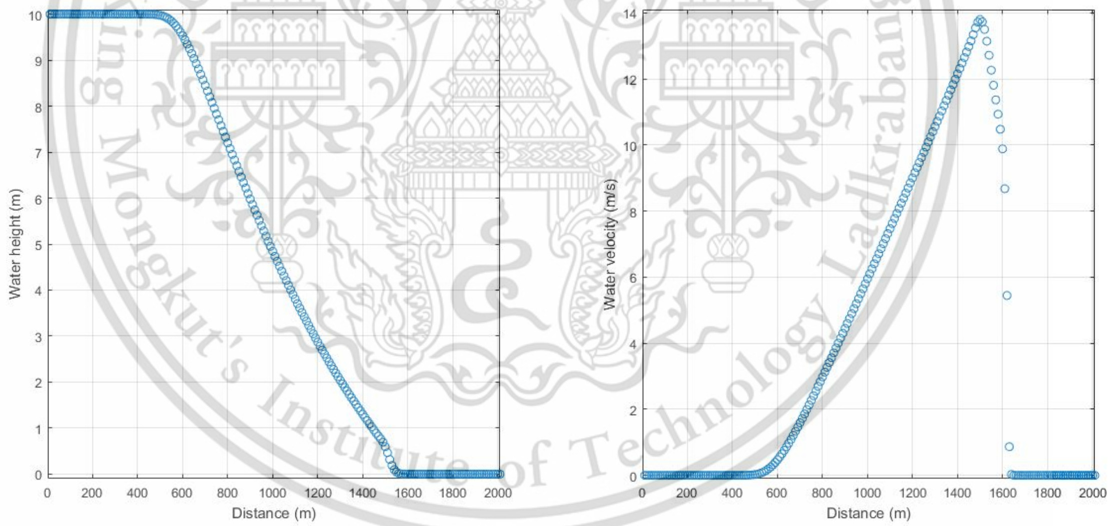


Fig. 3.6 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at t=40s. $\left(h_r/h_l=\infty\right)$

3.3 The finite difference methods for the dispersion model

3.3.1 *The implicit methods*

First of all, we consider the implicit schemes for the advection-dispersion-reaction equation. The backward time central space scheme approximates the temporal and spacial derivatives and the decay in Eq.(2.6).

3.3.1.1 Backward time central space method

We can then approximate $C(x_m,t_n)$ by C_m^n, the value of the difference approximation of $C(x,t)$ at point $x=m\Delta x$ and $t=n\Delta t$, where $0\le m\le M$ and $0\le n\le N$. The grid point (x_m,t_n) is defined by $x_m=m\Delta x$ for all $m=0,1,2,...,M$ and $t_n=n\Delta t$ for all $n=0,1,2,...,N$ in which *M* and *N* are positive integers.

Taking the backward time forward space technique [10] into Eq.(2.6), we get the following discretization:

$$C\cong C_m^{n+1}, \tag{3.19}$$

$$\frac{\partial C}{\partial t}\cong\frac{\left(C_m^{n+1}-C_m^n\right)}{\Delta t}, \tag{3.20}$$

$$\frac{\partial C}{\partial x}\cong\frac{\left(C_{m+1}^{n+1}-C_{m-1}^{n+1}\right)}{2\Delta x}, \tag{3.21}$$

$$\frac{\partial^2 C}{\partial x^2}\cong\frac{\left(C_{m+1}^{n+1}-2C_m^{n+1}+C_{m-1}^{n+1}\right)}{(\Delta x)^2}, \tag{3.22}$$

$$u\cong u_m^n. \tag{3.23}$$

Note that u_m^n are obtained by the modified Lax-diffusive method with the dam-break model of Eqs.(3.8-3.10). Substituting Eqs.(3.19-3.23) into Eq.(2.6), we get

$$\frac{C_m^{n+1}-C_m^n}{\Delta t}+u_m^n\left(\frac{C_{m+1}^{n+1}-C_{m-1}^{n+1}}{2\Delta x}\right)=D\left(\frac{C_{m+1}^{n+1}-2C_m^{n+1}+C_{m-1}^{n+1}}{\left(\Delta x\right)^2}\right)-KC_m^{n+1}. \tag{3.24}$$

For all $1<m<M$ and $0\le n\le N$. If we let $\alpha_m^n=u_m^n\left(\frac{\Delta t}{\Delta x}\right)$ and $\beta=D\frac{\Delta t}{\left(\Delta x\right)^2}$, then, Eq.(3.24) becomes,

$$-(\beta+\frac{\alpha}{2})C_{m-1}^{n+1}+(1+2\beta+K\Delta t)C_m^{n+1}-(\beta-\frac{\alpha}{2})C_{m+1}^{n+1}=C_m^n. \tag{3.25}$$

For upper boundary, where $m=1$, plug the known value of the left boundary by $C_0^{n+1}=C_0^n$ into Eq.(3.25) in the right-hand side, we obtain

$$(1+2\beta+K\Delta t)C_1^{n+1}-(\beta-\frac{\alpha}{2})C_2^{n+1}=C_1^n+(\beta+\frac{\alpha}{2})C_0^{n+1}. \tag{3.26}$$

For lower boundary, where $m=M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^{n+1}=2C_M^{n+1}-C_{M-1}^{n+1}$ by rearranging, we can obtain

$$-\alpha C_{M-1}^{n+1}+(1+\alpha+K\Delta t)C_M^{n+1}=C_M^n. \tag{3.27}$$

The backward time central space scheme is an unconditionally stable scheme [10].

3.3.1.2 Crank-Nicolson method

Consequently, we consider the Crank-Nicolson scheme for the advection-dispersion-reaction equation. It is also used to approximates the temporal and spacial derivatives and the decay in Eq.(2.6). We will use the Crank-Nicolson technique [10] to Eq.(2.6), it can be obtained the following discretization:

$$C \cong \frac{C_m^{n+1}+C_m^n}{2}, \tag{3.28}$$

$$\frac{\partial C}{\partial t} \cong \frac{\left(C_m^{n+1}-C_m^n\right)}{\Delta t}, \tag{3.29}$$

$$\frac{\partial C}{\partial x} \cong \frac{C_{m+1}^{n+1}-C_{m-1}^{n+1}+C_{m+1}^n-C_{m-1}^n}{4\Delta x}, \tag{3.30}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{C_{m+1}^{n+1}-2C_m^{n+1}+C_{m-1}^{n+1}+C_{m+1}^n-2C_m^n+C_{m-1}^n}{2(\Delta x)^2}, \tag{3.31}$$

$$u \cong u_m^n. \tag{3.32}$$

Substituting Eqs.(3.28-3.32) into Eq.(2.6), we obtain

$$\begin{aligned}\frac{C_m^{n+1}-C_m^n}{\Delta t}+u_m^n\left(\frac{C_{m+1}^{n+1}-C_{m-1}^{n+1}+C_{m+1}^n-C_{m-1}^n}{4\Delta x}\right)\\ =D\left(\frac{C_{m+1}^{n+1}-2C_m^{n+1}+C_{m-1}^{n+1}+C_{m+1}^n-2C_m^n+C_{m-1}^n}{2(\Delta x)^2}\right)-K\left(\frac{C_m^{n+1}+C_m^n}{2}\right).\end{aligned} \tag{3.33}$$

For all $1<m<M$ and $0\le n\le N$. Let $\alpha_m^n=u_m^n\left(\frac{\Delta t}{\Delta x}\right)$ and $\beta=D\frac{\Delta t}{(\Delta x)^2}$, Eq.(3.33) becomes,

$$\begin{aligned}-(\beta+\frac{\alpha}{2})C_{m-1}^{n+1}+2(1+\beta+K\Delta t)C_m^{n+1}-(\beta-\frac{\alpha}{2})C_{m+1}^{n+1}\\ =(\beta+\frac{\alpha}{2})C_{m-1}^n+2(1-\beta-K\Delta t)C_m^n+(\beta-\frac{\alpha}{2})C_{m+1}^n.\end{aligned} \tag{3.34}$$

For upper boundary, where $m=1$, plug the known value of the left boundary by arranging C_0^{n+1} to the right-hand side, we obtain

$$\begin{aligned}2(1+\beta+K\Delta t)C_1^{n+1}-(\beta-\frac{\alpha}{2})C_2^{n+1}\\ =(\beta+\frac{\alpha}{2})\left(C_0^n+C_0^n\right)+2(1-\beta-K\Delta t)C_1^n+(\beta-\frac{\alpha}{2})C_2^n.\end{aligned} \tag{3.35}$$

For lower boundary, where $m=M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^{n+1}=2C_M^{n+1}-C_{M-1}^{n+1}$ and $C_{M+1}^n=2C_M^n-C_{M-1}^n$ into Eq.(3.35) by rearranging, we can obtain

$$-\alpha C_{M-1}^{n+1} + 2(1+\frac{\alpha}{2}+K\Delta t)C_M^{n+1} = \alpha C_{M-1}^{n} + 2(1-\frac{\alpha}{2}-K\Delta t)C_M^{n}. \tag{3.36}$$

The Crank-Nicolson is unconditionally stable [10]. Both implicit methods can be obtained that the technique must be generate many large systems of linear equations. It follows that the technique is not economical computer implementation.

3.3.2 Explicit finite difference methods

3.3.2.1 Forward Time Central Space method

The explicit methods can be obtained that the technique not require to generate any systems of linear equations. We can see that the application of the technique is economical computer implementation. First of all, the forward time central space scheme for the advection-dispersion-reaction equation is introduced. It is also used to approximates the temporal and spacial derivatives and the decay in Eq.(2.6). We will use the forward time central space method technique [10] to Eq.(2.6), it can be obtained the following discretization:

$$C \cong C_m^n, \tag{3.37}$$

$$\frac{\partial C}{\partial t} \cong \frac{C_m^{n+1} - C_m^n}{\Delta t}, \tag{3.38}$$

$$\frac{\partial C}{\partial x} \cong \frac{C_{m+1}^n - C_{m-1}^n}{2\Delta x}, \tag{3.39}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{C_{m+1}^n - 2C_m^n + C_{m-1}^n}{(\Delta x)^2}, \tag{3.40}$$

$$u \cong u_m^n. \tag{3.41}$$

Note that u_m^n are obtained by the modified Lax-diffusive method with the dam-break model of Eqs.(3.8-3.10). Substituting Eqs.(3.37-3.41) into Eq.(2.6), we get

$$\frac{C_m^{n+1} - C_m^n}{\Delta t} + u_m^n\left(\frac{C_{m+1}^n - C_{m-1}^n}{2\Delta x}\right) = D\left(\frac{C_{m+1}^n - 2C_m^n + C_{m-1}^n}{(\Delta x)^2}\right) - KC_m^n. \tag{3.42}$$

For all $1 \le m < M$ and $0 \le n \le N$. If we let $\alpha_m^n = u_m^n\left(\frac{\Delta t}{\Delta x}\right)$ and $\beta = D\frac{\Delta t}{(\Delta x)^2}$, so Eq.(3.42) becomes,

$$C_m^{n+1} = (\beta+\frac{\alpha}{2})C_{m-1}^n + (1-2\beta-K\Delta t)C_m^n + (\beta-\frac{\alpha}{2})C_{m+1}^n. \tag{3.43}$$

For upper boundary, where $m=0$, plug the known value of the left boundary by arranging $C_{-1}^{n}=C_{0}^{n}$ into Eq.(3.43) in the right-hand side, we obtain

$$C_{0}^{n+1}=(\beta+\frac{\alpha}{2})C_{0}^{n}+(1-2\beta-K\Delta t)C_{0}^{n}+(\beta-\frac{\alpha}{2})C_{1}^{n}. \tag{3.44}$$

For lower boundary, where $m=M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^{n}=2C_{M}^{n}-C_{M-1}^{n}$ and by rearranging, we can obtain

$$C_{M}^{n+1}=\alpha C_{M-1}^{n}+(1-\alpha-K\Delta t)C_{M}^{n}. \tag{3.45}$$

This scheme is stable in the following region

$$\beta=D\frac{\Delta t}{(\Delta x)^{2}}<\frac{1}{2}, \tag{3.46}$$

$$\alpha_{m}^{n}=u_{m}^{n}\left(\frac{\Delta t}{\Delta x}\right)<1, \tag{3.47}$$

where β is the diffusion number and α_{m}^{n} is the advection number or Courant number. It can be obtained that the strictly stability requirements are the main disadvantage of this scheme. The truncation error for this method is $O\left\{(\Delta x)^{2},\Delta t\right\}$ [10].

3.3.2.2 MacCormack method

The MacCormack scheme is an explicit scheme [10] with two-step predictor-corrector evaluations. The first step is a modification of forward time central space (FTCS) by changing the central space evaluation at time *n* to a forward space evaluation. This step is a forward time forward space (FTFS) scheme. The FTFS scheme approximates the temporal and special derivatives and the decay in Eq.(2.6) with the following discretization:

$$C\cong C_{m}^{n}, \tag{3.48}$$

$$\frac{\partial C}{\partial t}\cong\frac{C_{m}^{n+1}-C_{m}^{n}}{\Delta t}, \tag{3.49}$$

$$\frac{\partial C}{\partial x}\cong\frac{C_{m+1}^{n}-C_{m}^{n}}{\Delta x}, \tag{3.50}$$

$$\frac{\partial^{2}C}{\partial x^{2}}\cong\frac{C_{m+1}^{n}-2C_{m}^{n}+C_{m-1}^{n}}{(\Delta x)^{2}}, \tag{3.51}$$

$$u \cong u_m^n. \tag{3.52}$$

Substituting Eqs.(3.48-3.52) into Eq.(2.6), and we define slope s_{m_1} by

$$s_{m_1} = -u\left(\frac{C_{m+1}^n - C_m^n}{\Delta x}\right) + D\left(\frac{C_{m+1}^n - 2C_m^n + C_{m-1}^n}{(\Delta x)^2}\right) - KC_m^n. \tag{3.53}$$

$$= \left(\frac{D}{(\Delta x)^2}\right)C_{m-1}^{n+1} + \left(\frac{u_m^n}{\Delta x} - \frac{2D}{(\Delta x)^2} - K\right)C_m^n + \left(-\frac{u_m^n}{\Delta x} + \frac{2D}{(\Delta x)^2}\right)C_{m+1}^n. \tag{3.54}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^n = 2C_M^n - C_{M-1}^n$ into Eq.(3.54) and by rearranging, we can obtain

$$s_{M_1} = -\left(\frac{u_M^n}{\Delta x} + K\right)C_M^n + \left(\frac{u_M^n}{\Delta x}\right)C_{M-1}^n. \tag{3.55}$$

We can obtain McCormack predictor step formulation by Euler's formula,

$$C_m^{n+1} = C_m^n + s_{m_1}\Delta t. \tag{3.56}$$

The second step is a modified BTCS by the following discretization

$$\frac{\partial C}{\partial x} \cong \frac{C_m^{n+1} - C_{m-1}^{n+1}}{\Delta x}. \tag{3.57}$$

Substituting Eq.(3.57) into Eq.(2.6), and we also define slope s_{m_2} by

$$s_{m_2} = \left(\frac{D}{(\Delta x)^2} + \frac{u_m^n}{\Delta x}\right)C_{m-1}^{n+1} - \left(\frac{u_m^n}{\Delta x} + \frac{2D}{(\Delta x)^2} + K\right)C_m^{n+1} + \left(\frac{D}{(\Delta x)^2}\right)C_{m+1}^{n+1}. \tag{3.58}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^{n+1} = 2C_M^{n+1} - C_{M-1}^{n+1}$ into Eq.(3.58) and by rearranging, we can obtain

$$s_{M_2} = -\left(\frac{u_M^n}{\Delta x} + K\right)C_M^{n+1} + \left(\frac{u_M^n}{\Delta x}\right)C_{M-1}^{n+1}. \tag{3.59}$$

From the both steps, the MacCormack scheme takes the following form,

$$C_m^{n+1} = C_m^n + \frac{\left(s_{m_1} + s_{m_2}\right)}{2}\Delta t. \tag{3.60}$$

The McCormack scheme is conditionally stable that the stability requirements for the scheme are [10]

$$\beta = D\frac{\Delta t}{\left(\Delta x\right)^2} < \frac{1}{2}, \tag{3.61}$$

$$\alpha_m^n = u_m^n\left(\frac{\Delta t}{\Delta x}\right) < 0.9, \tag{3.62}$$

where β is the diffusion number (dimensionless) and α_m^n is the advection number or courant number (dimensionless).

3.3.2.3 Saulyev Explicit Finite Difference Scheme

Since the Saulyev scheme is an unconditionally stable scheme [10], we can see that the non-strictly stability requirement of Saulyev scheme is the main of advantage and economical to use. Taking Saulyev technique [10] into Eq.(2.6), it can be obtained the following discretization:

$$C \cong C_m^n, \tag{3.63}$$

$$\frac{\partial C}{\partial t} \cong \frac{C_m^{n+1} - C_m^n}{\Delta t}, \tag{3.64}$$

$$\frac{\partial C}{\partial x} \cong \frac{C_{m+1}^n - C_{m-1}^{n+1}}{2\Delta x}, \tag{3.65}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{C_{m+1}^n - C_m^n - C_m^{n+1} + C_{m-1}^{n+1}}{(\Delta x)^2}, \tag{3.66}$$

$$u \cong u_m^n. \tag{3.67}$$

Substituting Eqs.(3.63-3.67) into Eq.(2.6), we get

$$\frac{C_m^{n+1} - C_m^n}{\Delta t} + u_m^n \left(\frac{C_{m+1}^n - C_{m-1}^{n+1}}{2\Delta x} \right) = D \left(\frac{C_{m+1}^n - C_m^n - C_m^{n+1} + C_{m-1}^{n+1}}{(\Delta x)^2} \right) - KC_m^n. \tag{3.68}$$

For all $1 \le m < M$ and $0 \le n \le N$. If we let $\alpha_m^n = u_m^n \left(\frac{\Delta t}{\Delta x} \right)$ and $\beta = D\frac{\Delta t}{(\Delta x)^2}$, so Eq.(3.68) becomes,

$$C_m^{n+1} = \left(\frac{1}{1+\beta} \right) \left(\left(\beta + \frac{\alpha_m^n}{2} \right) C_{m-1}^{n+1} + (1 - \beta - K\Delta t) C_m^n + \left(\beta - \frac{\alpha_m^n}{2} \right) C_{m+1}^n \right). \tag{3.69}$$

For upper boundary, where $m = 0$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{-1}^{n+1} = C_0^{n+1}$, and substituting them into Eq.(3.69) in the right-hand side, we obtain

$$C_0^{n+1} = \left(\frac{1}{1+\beta} \right) \left(\left(\beta + \frac{\alpha_m^n}{2} \right) C_0^{n+1} + (1 - \beta - K\Delta t) C_0^n + \left(\beta - \frac{\alpha_m^n}{2} \right) C_1^n \right). \tag{3.70}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^n = C_M^n + C_M^{n+1} - C_{M-1}^{n+1}$, we obtain

$$C_M^{n+1} = \left(\frac{1}{1 + \frac{\alpha_M^n}{2}} \right) \left(\alpha_M^n C_{M-1}^{n+1} + \left(1 - \frac{\alpha_M^n}{2} - K\Delta t \right) C_M^n \right). \tag{3.71}$$

where β is the diffusion number (dimensionless) and α_m^n is the advection number or courant number. Using Taylor series expansions on the approximation, [10] has shown The truncation error is $O\left\{(\Delta x)^2+(\Delta t)^2+\left(\frac{\Delta t}{\Delta x}\right)^2\right\}$ or $O\left\{2,2,(1/1)^2\right\}$.

3.4 Numerical solution of dispersion model

Suppose that the measurement of pollutant concentration C in a dam-break flow stream is considered. A stream is aligned with longitudinal distance, 2000 (m) total length. There is a dam-break which discharges waste water into the flooding area at middle point of the domain and the pollutant concentrations at the discharge point are assumed into 3 cases:

Simulation 1: High pollutant concentration at middle point

$$C(x,0)=\begin{cases}1 & ,if \quad x=1000\\ 0.1, & if \;\; 1000<x\le 2000\end{cases} \qquad (3.72)$$

$$C(1000,t)=1 \text{ for all } 0<t\le T \text{ and } C_x(2000,t)=0. \qquad (3.73)$$

Simulation 2: Decease pollutant concentration by linear term

$$C(x,0)=\begin{cases}1 & ,if \quad x=1000\\ 0.1, & if \;\; 1000<x\le 2000\end{cases} \qquad (3.74)$$

$$C(1000,t)=1-t/720 \text{ for all } 0<t\le T \text{ and } C_x(2000,t)=0. \qquad (3.75)$$

Simulation 3: Decease pollutant concentration by quadratic term

$$C(x,0)=\begin{cases}1 & ,if \quad x=1000\\ 0.1, & if \;\; 1000<x\le 2000\end{cases} \qquad (3.76)$$

$$C(1000,t)=1-3.8580\cdot 10^{-5}t^2 \text{ for all } 0<t\le T \text{ and } C_x(2000,t)=0. \qquad (3.77)$$

The elevation and velocity of water are obtained by the dam-break (pollutant discharging) model that we assume the initial and boundary conditions in 2 types: the dam-break with wet bed (flooding area) and the dam-break with dry bed (non-flooding area). We will consider the dam-break in 3 cases as below,

Case A: Dam-break on wet bed (pollutant going to the high level flooding area)

$$h(x,0)=\begin{cases}10, & if \quad x\le 1000\\ 5, & if \quad 1000<x\le 2000\end{cases} \tag{3.78}$$

$$u(x,0)=0, 0\le x\le 2000, \tag{3.79}$$

$$\begin{aligned} h_x(0,t)&=h_x(L,t)=0,\ 0<t\le T,\\ u_x(0,t)&=u_x(L,t)=0,\ 0<t\le T.\end{aligned} \tag{3.80}$$

Case B: Dam-break on wet bed (pollutant going to the low level flooding area)

$$h(x,0)=\begin{cases}10, & if \quad x\le 1000\\ 0.05, & if \quad 1000<x\le 2000\end{cases} \tag{3.81}$$

$$u(x,0)=0, 0\le x\le 2000,$$

$$\begin{aligned} h_x(0,t)&=h_x(L,t)=0,\ 0<t\le T,\\ u_x(0,t)&=u_x(L,t)=0,\ 0<t\le T.\end{aligned} \tag{3.82}$$

Case C: Dam-break on dry bed (pollutant going to the non-flooding area)

$$h(x,0)=\begin{cases}10, & if \quad x\le 1000\\ 0, & if \quad 1000<x\le 2000\end{cases} \tag{3.83}$$

$$u(x,0)=0,\ 0\le x\le 2000,$$

$$\begin{aligned} h_x(0,t)&=h_x(L,t)=0,\ 0<t\le T,\\ u_x(0,t)&=u_x(L,t)=0,\ 0<t\le T.\end{aligned} \tag{3.84}$$

The physical parameters of the polluted system are diffusion coefficient $D=1\left(m^2/s\right)$, and a first-order reaction rate $10^{-2}s^{-1}$. In the analysis conducted in this study, meshes the into 1000 elements with $\Delta x=2$, and time increment is $0.1(s)$ with $\Delta t=0.1$.

Table 3.13 The pollutant concentration $C(x,t)(kg/m^3)$ by BTCS scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.0000	0.7100	0.5041	0.0455	0.0407	0.0407	0.0407	0.0407	0.0407	0.0407	0.0407
180	1.0000	0.7101	0.5042	0.3580	0.2542	0.1590	0.0167	0.0166	0.0166	0.0166	0.0166
270	1.0000	0.7101	0.5043	0.3581	0.2542	0.1805	0.1282	0.0910	0.0261	0.0067	0.0067
360	1.0000	0.7100	0.5041	0.3580	0.2542	0.1805	0.1282	0.0910	0.0646	0.0459	0.0311

Table 3.14 The pollutant concentration $C(x,t)(kg/m^3)$ by Crank-Nicolson scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.0000	0.5115	0.2545	0.0174	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166
180	1.0000	0.5116	0.2547	0.1285	0.0648	0.0304	0.0027	0.0027	0.0027	0.0027	0.0027
270	1.0000	0.5116	0.2547	0.1285	0.0648	0.0327	0.0165	0.0083	0.0017	0.0005	0.0005
360	1.0000	0.5115	0.2546	0.1285	0.0648	0.0327	0.0165	0.0083	0.0042	0.0021	0.0011

Table 3.15 The pollutant concentration $C(x,t)(kg/m^3)$ by FTCS scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.0000	0.7100	0.5041	0.0412	0.0407	0.0407	0.0407	0.0407	0.0407	0.0407	0.0407
180	1.0000	0.7101	0.5042	0.3580	0.2542	0.1771	0.0165	0.0165	0.0165	0.0165	0.0165
270	1.0000	0.7101	0.5043	0.3581	0.2542	0.1805	0.1282	0.0910	0.0216	0.0067	0.0067
360	1.0000	0.7100	0.5041	0.3580	0.2542	0.1805	0.1282	0.0910	0.0646	0.0459	0.0326

Table 3.16 The pollutant concentration $C(x,t)(kg/m^3)$ by MacCormark scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.0000	0.7100	0.5041	0.0428	0.0407	0.0407	0.0407	0.0407	0.0407	0.0407	0.0407
180	1.0000	0.7101	0.5042	0.3580	0.2542	0.1663	0.0166	0.0165	0.0165	0.0165	0.0165
270	1.0000	0.7101	0.5043	0.3581	0.2542	0.1805	0.1282	0.0910	0.0245	0.0067	0.0067
360	1.0000	0.7100	0.5042	0.3580	0.2542	0.1805	0.1282	0.0910	0.0646	0.0459	0.0319

Table 3.17 The pollutant concentration $C(x,t)(kg/m^3)$ by Saulyev scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.0000	0.7100	0.5041	0.0819	0.0385	0.0390	0.0395	0.0400	0.0405	0.0407	0.0407
180	1.0000	0.7101	0.5042	0.3580	0.2542	0.1805	0.0225	0.0146	0.0148	0.0150	0.0152
270	1.0000	0.7101	0.5042	0.3581	0.2542	0.1805	0.1282	0.0910	0.0635	0.0070	0.0055
360	1.0000	0.7100	0.5041	0.3580	0.2542	0.1805	0.1282	0.0910	0.0646	0.0459	0.0326

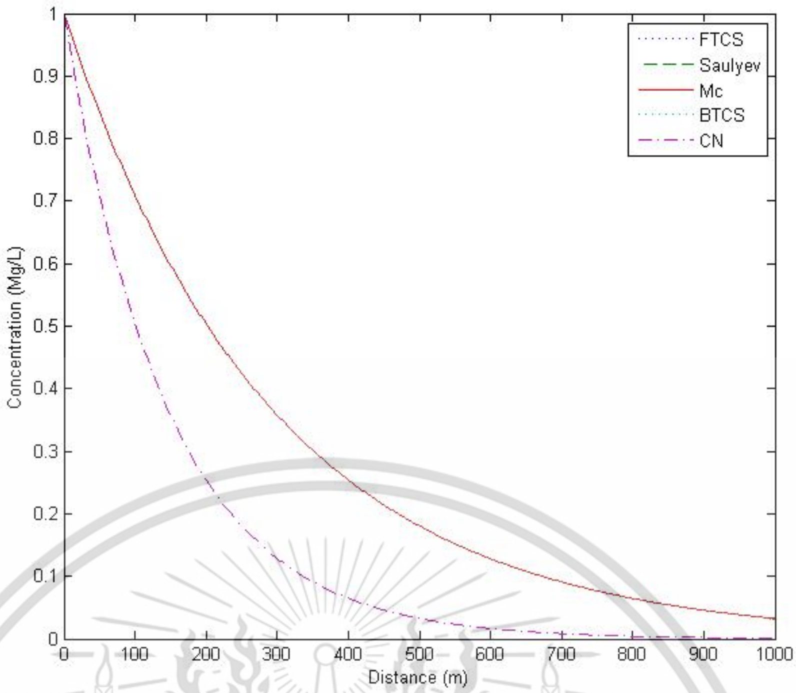


Fig. 3.7 Comparison of finite difference solutions in wet bed (depth ratio 0.5) where $\Delta x = 2,\ \Delta t = 0.1$ and $T = 360s.$

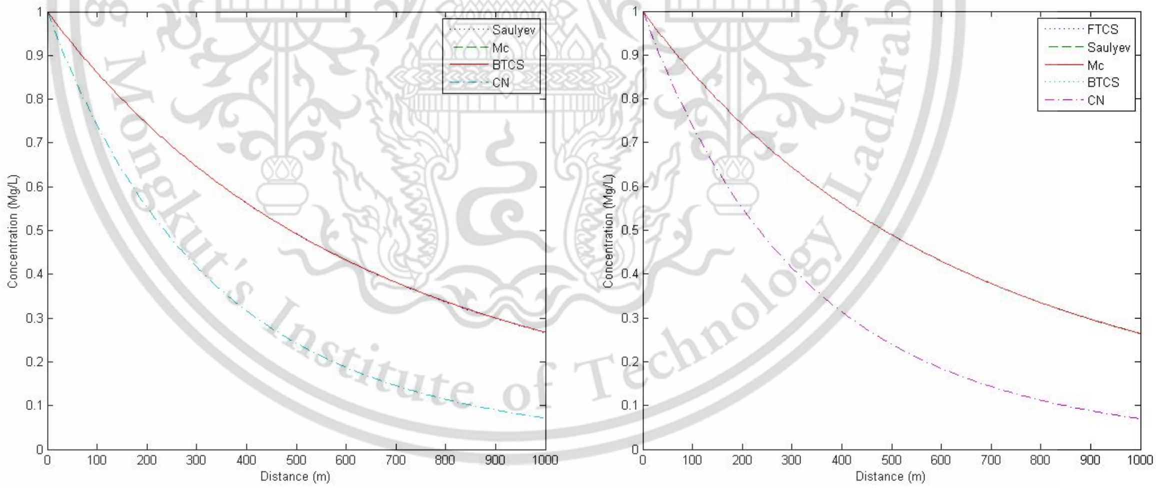


Fig. 3.8 Comparison of finite difference solutions in wet bed (depth ratio 0.005) where $\Delta x = 2,\ \Delta t = 0.1,\ T = 360s$ (left plot) and $\Delta x = 2,\ \Delta t = 0.025,\ T = 360s$ (right plot).

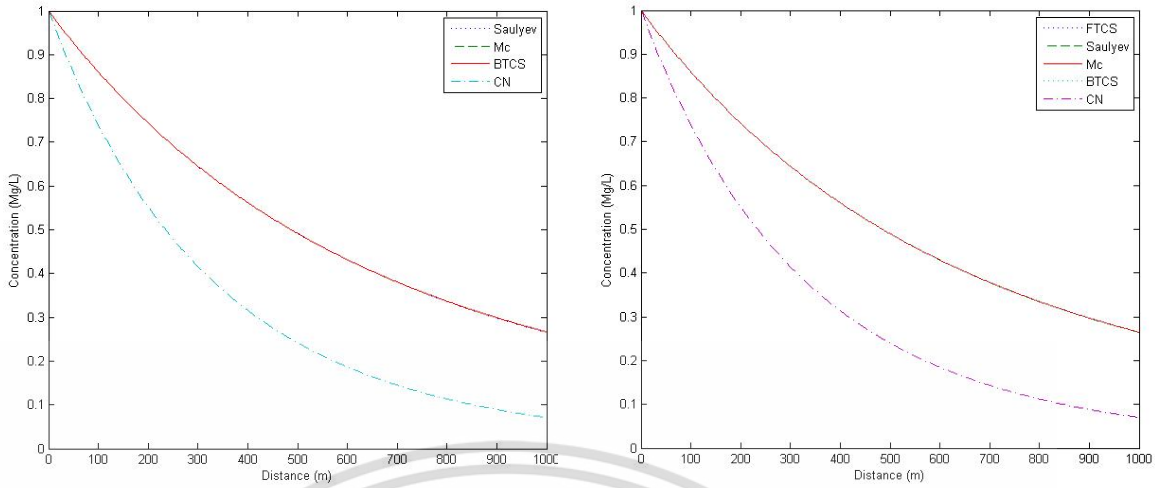


Fig. 3.9 Comparison of finite difference solutions in wet bed (depth ratio ∞) where $\Delta x = 2,\ \Delta t = 0.1,\ T = 360s$ (left plot) and $\Delta x = 2,\ \Delta t = 0.025,\ T = 360s$ (right plot).

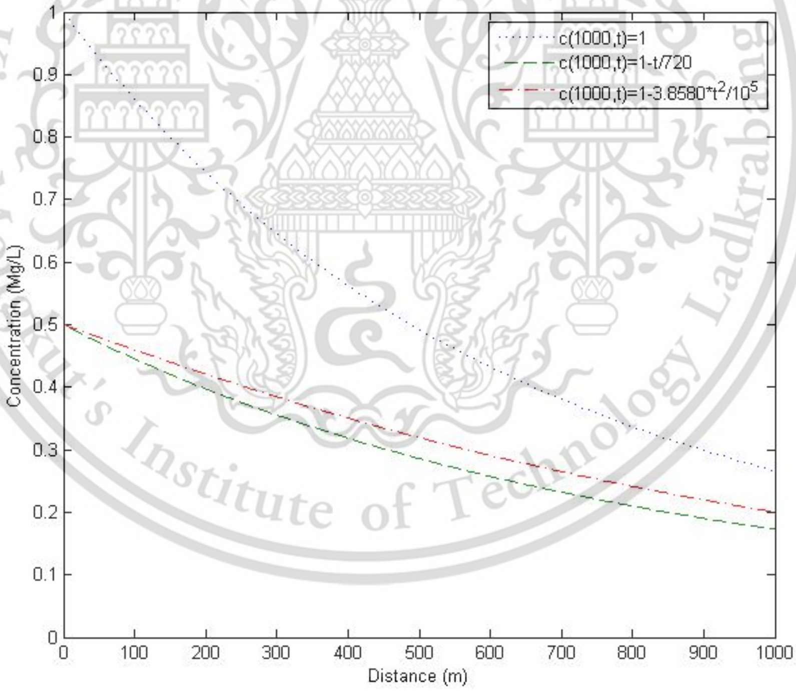


Fig. 3.10 The comparison among difference discharge pollutant levels at the dam-break point on wet bed (depth ratio 0.05) $\Delta x = 2,\ \Delta t = 0.1,\ T = 360s$ by a Saulyev scheme.

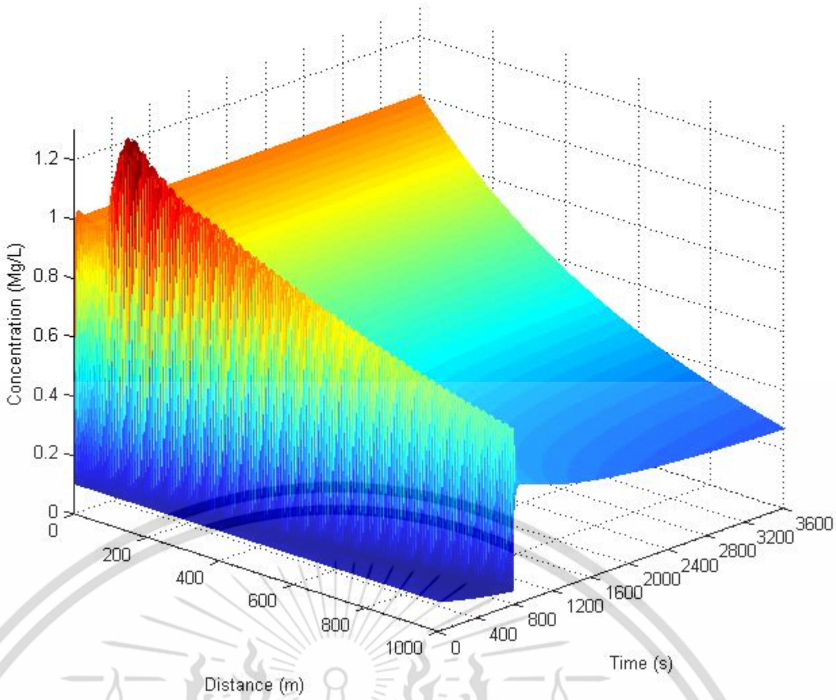


Fig. 3.11 The pollutant concentration on wet bed (depth ratio 0.5) by Saulyev scheme (case 1) where $\Delta x = 2, \Delta t = 0.1$ and $T = 360s$.

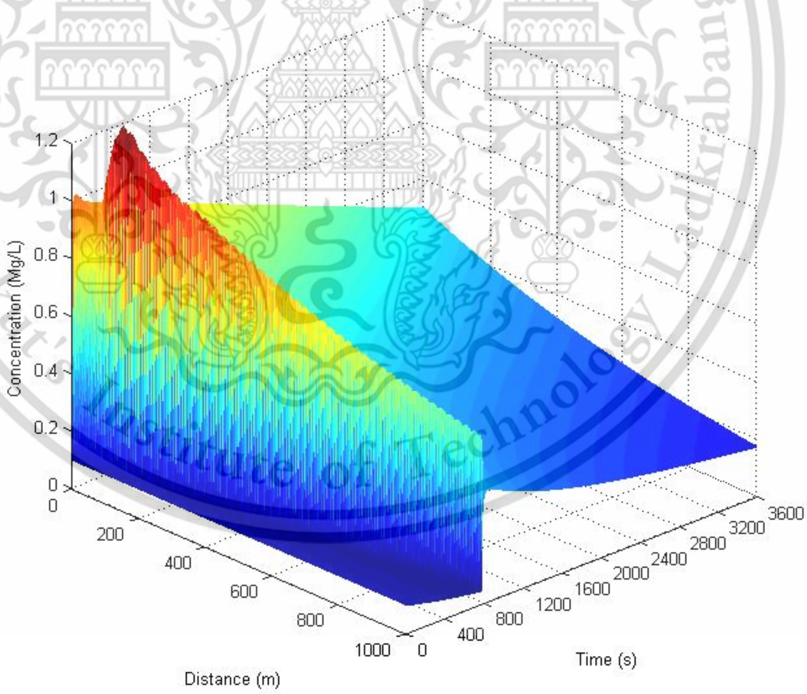


Fig. 3.12 The pollutant concentration on wet bed (depth ratio 0.005) by Saulyev scheme (case 2) where $\Delta x = 2, \Delta t = 0.1$ and $T = 360s$.

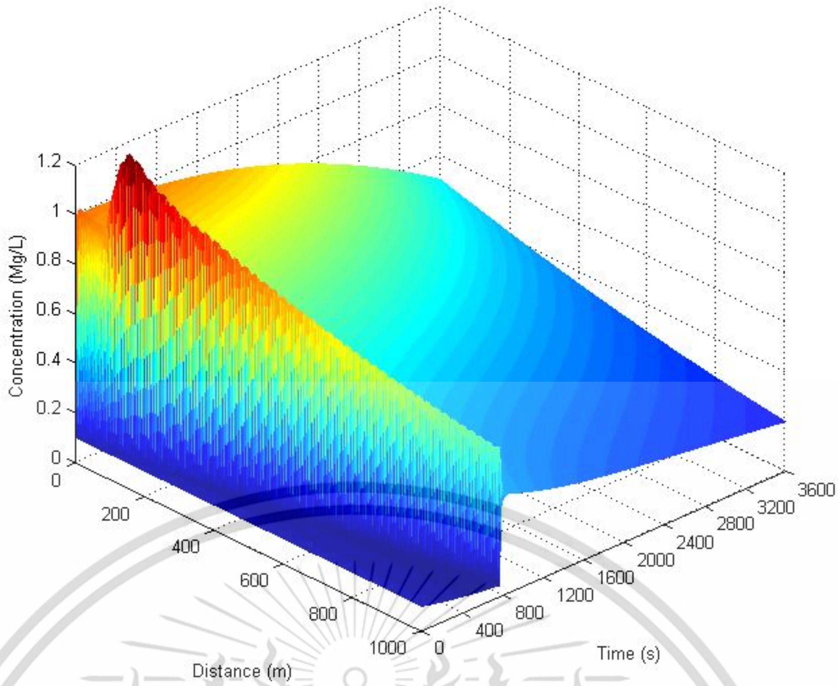


Fig. 3.13 The pollutant concentration on wet bed (depth ratio∞) by Saulyev scheme (case 3) where $\Delta x = 2, \Delta t = 0.1$ and $T = 360s$.

3.5 Discussion

The velocity and elevation of water current are obtained by a modified Lax-diffusive method. The case C of dam failure on dry bed area gives the highest flow velocity. The cases A-B of dam failure on wet bed area give low velocity level. If the residents have encountered flood for longtime, the water pollutant must be increase. The villagers want to drain the water to the other areas by destroying the dike. The other villages that never encounter flood going to get a strong water current while they also receiving the drained-polluted water. The approximation of the pollutant concentrations of the implicit and explicit methods are shown in Tables 3.13-3.17. We can see that the pollutant concentration level on the flooding area is not too high. The real-world problems require a small amount of time interval in obtaining accurate solutions. Unfortunately, the analytical solutions of the dam-break model could not be found over the entire domain. This also implies that the analytical solutions of dispersion model could not work out at any point on the entire domain as well.

We propose a modified Lax-diffusive scheme by editing a simple revision to the traditional Lax-diffusive scheme. The FTCS method is limited by restriction of the stability condition. Then FTCS is not flexible in the real-world situation. The Crank-Nicolson scheme shows excessive dispersion effects for large time and space step lengths, significantly decreasing the efficiency of the Crank-Nicolson scheme. The BTCS still generate a lot of large systems of linear equations. The MacCormack and Saulyev schemes are economical to use. The proposed method show a good agreement in accuracy, the explicit schemes becomes less efficient than the implicit schemes.

Table 3.18 The stability of implicit and explicit scheme. (S=Stable, U=Unstable)

Depth ratio	$(\Delta x, \Delta t)$	CFL no.	Lax Diff.	α	FTCS	Saulyev	MacCormack	BTCS
0.5	(2, 0.1)	0.145	S	0.145	S	S	S	S
	(2, 0.05)	0.0725	S	0.0725	S	S	S	S
	(2, 0.025)	0.0363	S	0.0363	S	S	S	S
0.005	(2, 0.1)	0.6285	S	0.6285	U	S	S	S
	(2, 0.05)	0.3143	S	0.3143	U	S	S	S
	(2, 0.025)	0.1571	S	0.1571	S	S	S	S
Inf.	(2, 0.1)	0.85	U	0.85	-	-	-	-
	(2, 0.05)	0.425	S	0.425	U	S	S	S
	(2, 0.025)	0.2125	S	0.2125	S	S	S	S

Remark: The modified Lax-diffusive method will be stable under empirical condition, $\alpha < 8.5$

Chapter 4

Numerical Simulation of Water-Quality Model on Flooding using Revised Lax-Diffusive and Modified Siemieniuch-Gladwell Methods

In this chapter, we propose the combine models. Although the first model is still shallow water equation, the second model change to advection-diffusion equation. The modified Lax-diffusive are used to calculate the shallow water equation again. The traditional methods and modified Siemieniuch-Gladwell method are used to describe and compare solutions of advection-diffusion equation.

4.1 Revised Lax-diffusive scheme for a hydrodynamic model

From chapter 3, we have a modified Lax-diffusive methods in matrix form for dam-break problem as

$$A_m^{n+1} = \frac{\Delta t}{2\Delta x}\left(B_{m-1}^n - B_{m+1}^n\right) + A^*. \tag{4.1}$$

For all $1 \le m < M$ and $0 \le n \le N-1$ we have a formula in form as

$$\begin{pmatrix} h_m^{n+1} \\ h_m^{n+1} u_m^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{m-1}^n u_{m-1}^n - h_{m+1}^n u_{m+1}^n \\ h_{m-1}^n\left(u_{m-1}^n\right)^2 - h_{m+1}^n\left(u_{m+1}^n\right)^2 + \frac{1}{2}g\left(\left(h_{m-1}^n\right)^2 - \left(h_{m+1}^n\right)^2\right) \end{pmatrix} + \frac{1}{3}\begin{pmatrix} h_{m-1}^n + h_m^n + h_{m+1}^n \\ h_{m-1}^n u_{m-1}^n + h_m^n u_m^n + h_{m+1}^n u_{m+1}^n \end{pmatrix}. \tag{4.2}$$

For upper boundary, where $m = 0$, fix the known value of the left boundary by $u_{-1}^n = u_0^n$ and $h_{-1}^n = h_0^n$ into Eq.(3.8) in the right-hand side, we obtain

$$\begin{pmatrix} h_1^{n+1} \\ h_1^{n+1} u_1^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_0^n u_0^n - h_1^n u_1^n \\ h_0^n\left(u_0^n\right)^2 - h_1^n\left(u_1^n\right)^2 + \frac{1}{2}g\left(\left(h_0^n\right)^2 - \left(h_1^n\right)^2\right) \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 2h_0^n + h_1^n \\ 2h_0^n u_0^n + h_1^n u_1^n \end{pmatrix}. \tag{4.3}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $u_{M+1}^n = u_M^n$ and $h_{M+1}^n = h_M^n$ by rearranging, we obtain

$$\begin{pmatrix} h_M^{n+1} \\ h_M^{n+1}u_M^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{M-1}^n u_{M-1}^n - h_M^n u_M^n \\ h_{M-1}^n\left(u_{M-1}^n\right)^2 - h_M^n\left(u_M^n\right)^2 + \frac{1}{2}g\left(\left(h_{M-1}^n\right)^2 - \left(h_M^n\right)^2\right) \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 2h_{M-1}^n + h_M^n \\ 2h_{M-1}^n u_{M-1}^n + h_M^n u_M^n \end{pmatrix}. \tag{4.4}$$

The stability condition of the scheme needed CFL number as,

$$C_n = u_{max}\left(\frac{\Delta t}{\Delta x}\right) \le 1. \tag{4.5}$$

4.2 Numerical simulation of hydrodynamic model

We consider the dam-break problem with flat topography, $z(x)=0$. The length of channel is 2000 meter, the space meshing $\Delta x = 2$ meters and the time meshing $\Delta t = 0.1$. We separate dam-break problem in 3 cases. The initial conditions and boundary is defined by

case A: Dam-break on wet bed (pollutant going to the high level flooding area)

given $$h(x,0)=\begin{cases} 1.00, & if \quad x \le 1000 \\ 0.75, & if \quad 1000 < x \le 2000 \end{cases} \tag{4.6}$$

$$u(x,0)=0,\ 0 \le x \le 2000, \tag{4.7}$$

$$\begin{aligned} h_x(0,t) &= h_x(L,t) = 0,\ 0 < t \le T, \\ u_x(0,t) &= u_x(L,t) = 0,\ 0 < t \le T. \end{aligned} \tag{4.8}$$

Table 4.1 The velocity of water flow $u(x,t)$ m/s, depth ratio 0.75.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.3856	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.4134	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.4185	0.0000	0.0000	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0000	0.0000	0.0026	0.4196	0.0013	0.0000	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0000	0.0000	0.0301	0.4198	0.0257	0.0000	0.0000	0.0000	0.0000

Table 4.2 The elevation of water flow $h(x,t)$ m/s, depth ratio 0.75.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.8746	0.7500	0.7500	0.7500	0.7500	0.7500
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.8710	0.7500	0.7500	0.7500	0.7500	0.7500
30	1.0000	1.0000	1.0000	1.0000	1.0000	0.8700	0.7500	0.7500	0.7500	0.7500	0.7500
40	1.0000	1.0000	1.0000	1.0000	0.9992	0.8698	0.7504	0.7500	0.7500	0.7500	0.7500
50	1.0000	1.0000	1.0000	1.0000	0.9904	0.8698	0.7571	0.7500	0.7500	0.7500	0.7500

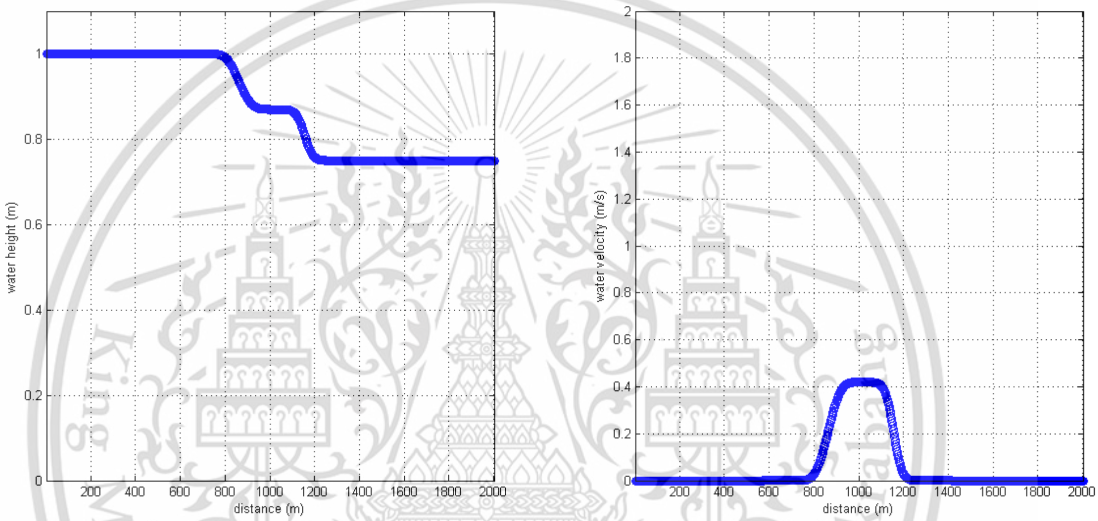


Fig. 4.1 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at t=50s. $(h_r/h_l = 0.75)$

case B: Dam-break on wet bed (pollutant going to the medium level flooding area)

given
$$h(x,0)=\begin{cases}1.00, & if \quad x \le 1000\\ 0.50, & if \quad 1000 < x \le 2000\end{cases} \tag{4.9}$$

$$u(x,0)=0,\ 0 \le x \le 2000, \tag{4.10}$$

$$\begin{aligned}h_x(0,t)=h_x(L,t)=0,\ 0<t\le T,\\ u_x(0,t)=u_x(L,t)=0,\ 0<t\le T.\end{aligned} \tag{4.11}$$

Table 4.3 The velocity of water flow $u(x,t)\ m/s$, depth ratio 0.5.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.7945	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.8762	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.9027	0.0000	0.0000	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0000	0.0000	0.0044	0.9138	0.0006	0.0000	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0000	0.0000	0.0465	0.9189	0.0207	0.0000	0.0000	0.0000	0.0000

Table 4.4 The elevation of water flow $h(x,t)\ m/s$, depth ratio 0.5.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.7491	0.5000	0.5000	0.5000	0.5000	0.5000
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.7353	0.5000	0.5000	0.5000	0.5000	0.5000
30	1.0000	1.0000	1.0000	1.0000	0.9999	0.7294	0.5000	0.5000	0.5000	0.5000	0.5000
40	1.0000	1.0000	1.0000	1.0000	0.9986	0.7269	0.5001	0.5000	0.5000	0.5000	0.5000
50	1.0000	1.0000	1.0000	1.0000	0.9852	0.7259	0.5047	0.5000	0.5000	0.5000	0.5000

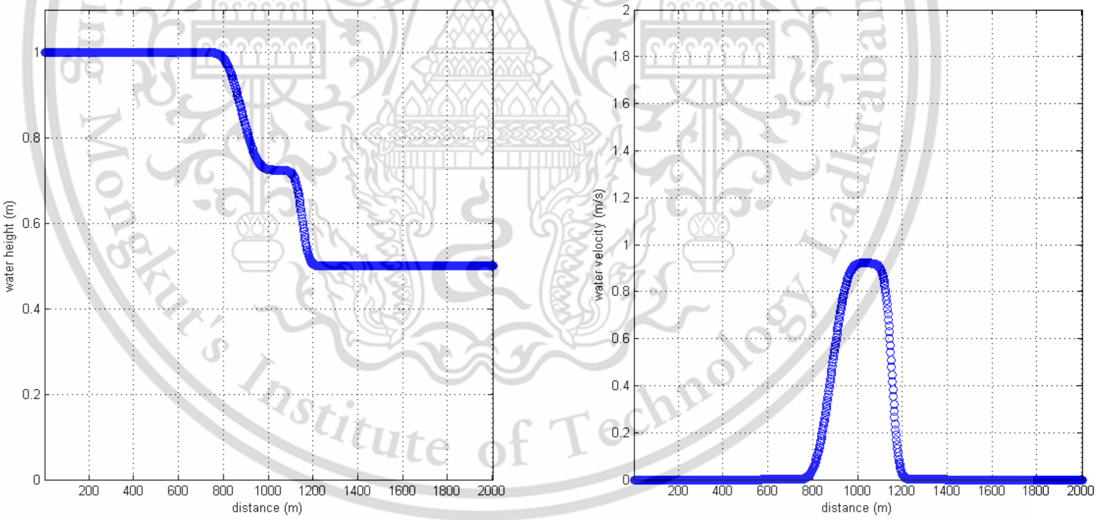


Fig. 4.2 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at t=50s. $\left(h_r/h_l = 0.5\right)$

case C: Dam-break on dry bed (pollutant going to the low level flooding area)

given
$$h(x,0)=\begin{cases}1.00, & if \quad x\le 1000\\ 0.25, & if \quad 1000<x\le 2000\end{cases} \tag{4.12}$$

$$u(x,0)=0,\ \ 0\le x\le 2000, \tag{4.13}$$

$$\begin{aligned}&h_x(0,t)=h_x(L,t)=0,\ 0<t\le T,\\ &u_x(0,t)=u_x(L,t)=0,\ 0<t\le T.\end{aligned} \tag{4.14}$$

Table 4.5 The velocity of water flow $u(x,t)\ m/s$, depth ratio 0.25.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	1.1995	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	1.3517	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0000	1.4222	0.0000	0.0000	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0000	0.0000	0.0055	1.4655	0.0001	0.0000	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0000	0.0000	0.0561	1.4952	0.0076	0.0000	0.0000	0.0000	0.0000

Table 4.6 The elevation of water flow $h(x,t)\ m/s$, depth ratio 0.25.

t\x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.6307	0.2500	0.2500	0.2500	0.2500	0.2500
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.6049	0.2500	0.2500	0.2500	0.2500	0.2500
30	1.0000	1.0000	1.0000	1.0000	1.0000	0.5901	0.2500	0.2500	0.2500	0.2500	0.2500
40	1.0000	1.0000	1.0000	1.0000	0.9982	0.5808	0.2500	0.2500	0.2500	0.2500	0.2500
50	1.0000	1.0000	1.0000	1.0000	0.9821	0.5744	0.2512	0.2500	0.2500	0.2500	0.2500

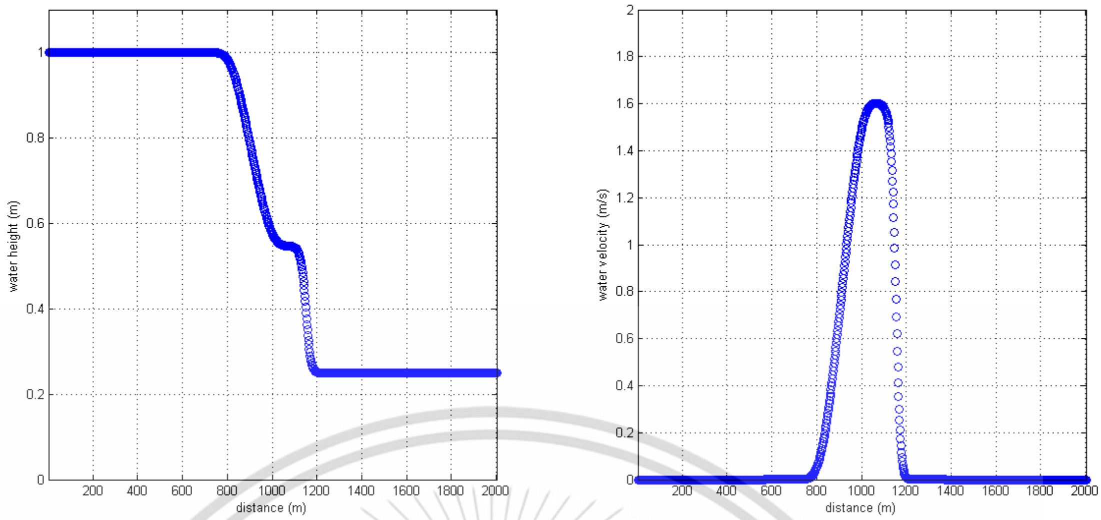


Fig. 4.3 Water height (left plot) and water velocity (right plot) for dam-break on wet bed at t=50s. $\left(h_r/h_l = 0.25\right)$

4.3 Numerical methods for a water quality model in flooding stream

We consider both implicit and explicit methods for solving advection-diffusion equations. The well-known traditional methods are also introduced [12].

4.3.1 The modified Siemieniuch-Gladwell procedure

We can then approximate $C\left(x_m,t_n\right)$ by C_m^n, the value of the difference approximation of $C\left(x,t\right)$ at point $x = m\Delta x$ and $t = n\Delta t$, where $0 \le m < M$ and $0 \le n < N$. The grid point $\left(x_m,t_n\right)$ is defined by $x_m = m\Delta x$ for all $m = 0,1,2,\ldots,M$ and $t_n = n\Delta t$ for all $n = 0,1,2,\ldots,N$ in which M and N are positive integers. Taking the modified Siemieniuch-Gladwell technique [12] into Eq.(2.5), by the following discretization:

$$\frac{\partial C}{\partial t} \cong \frac{\left(2\beta - \alpha_m^n\right)}{4}\frac{\left(C_{m-1}^{n+1} - C_{m-1}^n\right)}{\Delta t} + \frac{\left(2 + \alpha_m^n - 2\beta\right)}{2}\frac{\left(C_m^{n+1} - C_m^n\right)}{\Delta t} + \frac{\left(2\beta - \alpha_m^n\right)}{4}\frac{\left(C_{m+1}^{n+1} - C_{m+1}^n\right)}{\Delta t}, \qquad (4.15)$$

$$\frac{\partial C}{\partial x} \cong \frac{\left(C_{m+1}^n - C_{m-1}^n\right)}{4\Delta x} + \frac{\left(C_{m+1}^{n+1} - C_{m-1}^{n+1}\right)}{4\Delta x}, \qquad (4.16)$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{1}{2}\frac{\left(C_{m+1}^{n+1} - 2C_m^{n+1} + C_{m-1}^{n+1}\right)}{\left(\Delta x\right)^2} + \frac{1}{2}\frac{\left(C_{m+1}^{n} - 2C_m^{n} + C_{m-1}^{n}\right)}{\left(\Delta x\right)^2}, \quad (4.17)$$

$$u \cong u_m^n. \quad (4.18)$$

Remember that u_m^n are obtained by the revised Lax-diffusive method with the first model of Eq.(4.2-4.4). Substituting Eqs.(4.15-4.18) into Eq.(2.5), we have

$$-\alpha_m^n C_{m-1}^{n+1} + \left(2 + \alpha_m^n\right) C_m^{n+1} = 2\beta C_{m-1}^n + \left(2 - 4\beta + \alpha_m^n\right) C_m^n + (2\beta - \alpha_m^n) C_{m+1}^n, \quad (4.19)$$

where $\alpha_m^n = u_m^n \left(\frac{\Delta t}{\Delta x}\right)$ and $\beta = D\frac{\Delta t}{(\Delta x)^2}$ for all $1 \le m < M$ and $0 \le n \le N$. For left boundary, where $m = 0$, the known value on the left boundary are approximated by $C_{-1}^n = C_0^n$ and $C_{-1}^{n+1} = C_0^{n+1}$ into Eq.(4.19) in the right-hand side, we can see that

$$2C_0^{n+1} = \left(2 - 2\beta + \alpha_0^n\right) C_0^n + (2\beta - \alpha_0^n) C_1^n. \quad (4.20)$$

Similarly, the right boundary, where $m = M$, the known value on the left boundary are approximated by $C_{M+1}^n = C_M^n$ and $C_{M+1}^{n+1} = C_M^{n+1}$ into Eq.(4.19) in the right-hand side, we have

$$-\alpha_M^n C_{M-1}^{n+1} + \left(2 + \alpha_M^n\right) C_M^{n+1} = 2\beta C_{M-1}^n + \left(2 - 2\beta\right) C_M^n. \quad (4.21)$$

The stability of modified Siemieniuch-Gladwell procedure is

$$0 < \beta \le \frac{(1 + \alpha_m^n)}{2}. \quad (4.22)$$

4.3.2 The BTCS-type implicit method

Consider the Backward in time center in space(BTCS) scheme for the advection-diffusion equation by the following discretization:

$$\frac{\partial C}{\partial t} \cong \frac{\left(C_m^{n+1} - C_m^n\right)}{\Delta t}, \quad (4.23)$$

$$\frac{\partial C}{\partial x} \cong \frac{\left(C_{m+1}^{n+1} - C_{m-1}^{n+1}\right)}{2\Delta x}, \quad (4.24)$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{\left(C_{m+1}^{n+1} - 2C_m^{n+1} + C_{m-1}^{n+1}\right)}{(\Delta x)^2}, \quad (4.25)$$

$$u \cong u_m^n. \tag{4.26}$$

Substituting Eqs.(4.23-4.27) into Eq.(2.5), we have

$$-(\alpha_m^n + 2\beta)C_{m-1}^{n+1} + 2\left(1+2\beta\right)C_m^{n+1} + (\alpha_m^n - 2\beta)C_{m+1}^{n+1} = 2C_m^n, \tag{4.27}$$

for all $1 \le m < M$ and $0 \le n \le N$. For left boundary, where $m = 0$, the known value on the left boundary are approximated by $C_{-1}^{n+1} = C_0^{n+1}$ into Eq.(4.27) in the right-hand side, we can see that

$$\left(2-\alpha_0^n\right)C_0^{n+1} + (\alpha_0^n - 2\beta)C_1^{n+1} = 2C_0^n. \tag{4.28}$$

Similarly, the right boundary, where $m = M$, the known value on the left boundary are approximated by $C_{M+1}^{n+1} = C_M^{n+1}$ into Eq.(4.27) in the right-hand side, we have

$$-(\alpha_M^n + 2\beta)C_{M-1}^{n+1} + \left(2+\alpha_M^n + 2\beta\right)C_M^{n+1} = 2C_M^n. \tag{4.29}$$

The stability of BTCS scheme is unconditionally stable.

4.3.3 The upwind implicit formula

Consider the upwind implicit scheme for the advection-diffusion equation. We get the following discretization:

$$\frac{\partial C}{\partial t} \cong \frac{\left(C_m^{n+1} - C_m^n\right)}{\Delta t}, \tag{4.30}$$

$$\frac{\partial C}{\partial x} \cong \frac{\left(C_m^{n+1} - C_{m-1}^{n+1}\right)}{\Delta x}, \tag{4.31}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{\left(C_{m+1}^{n+1} - 2C_m^{n+1} + C_{m-1}^{n+1}\right)}{(\Delta x)^2}, \tag{4.32}$$

$$u \cong u_m^n. \tag{4.33}$$

Substituting Eqs.(4.30-4.33) into Eq.(2.5), we have

$$-(\alpha_m^n + \beta)C_{m-1}^{n+1} + \left(1+\alpha_m^n + 2\beta\right)C_m^{n+1} - \beta C_{m+1}^{n+1} = C_m^n, \tag{4.34}$$

for all $1 \le m < M$ and $0 \le n \le N$. For left boundary, where $m = 0$, the known value on the left boundary are approximated by $C_{-1}^{n+1} = C_0^{n+1}$ into Eq.(4.34) in the right-hand side, we can see that

$$\left(1+\beta\right)C_0^{n+1} - \beta C_1^{n+1} = C_0^n , \tag{4.35}$$

For right boundary, where $m = M$, the known value on the left boundary are approximated by $C_{M+1}^{n+1} = C_M^{n+1}$ into Eq.(4.34) in the right-hand side, we have

$$-(\alpha_M^n + \beta)C_{M-1}^{n+1} + \left(1+\alpha_M^n + \beta\right)C_M^{n+1} = C_M^n . \tag{4.36}$$

The stability of upwind implicit scheme is unconditionally stable.

4.3.4 The Crank-Nicolson type technique

Consider the Crank-Nicolson scheme for the advection-diffusion equation. We get the following discretization:

$$\frac{\partial C}{\partial t} \cong \frac{\left(C_m^{n+1} - C_m^n\right)}{\Delta t}, \tag{4.37}$$

$$\frac{\partial C}{\partial x} \cong \frac{\left(C_{m+1}^n - C_{m-1}^n\right)}{4\Delta x} + \frac{\left(C_{m+1}^{n+1} - C_{m-1}^{n+1}\right)}{4\Delta x}, \tag{4.38}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{1}{2}\left(\frac{\left(C_{m+1}^n - 2C_m^n + C_{m-1}^n\right)}{(\Delta x)^2} + \frac{\left(C_{m+1}^{n+1} - 2C_m^{n+1} + C_{m-1}^{n+1}\right)}{(\Delta x)^2}\right), \tag{4.39}$$

$$u \cong u_m^n. \tag{4.40}$$

Substituting Eqs.(4.37-4.40) into Eq.(2.5), we have

$$\begin{aligned} -(\alpha_m^n + 2\beta)C_{m-1}^{n+1} + 4\left(1+\beta\right)C_m^{n+1} + (\alpha_m^n - 2\beta)C_{m+1}^{n+1} \\ = (\alpha_m^n + 2\beta)C_{m-1}^n + 4\left(1-\beta\right)C_m^n + (2\beta - \alpha_m^n)C_{m+1}^n \end{aligned}, \tag{4.41}$$

for all $1 \le m < M$ and $0 \le n \le N$. For left boundary, where $m = 0$, the known value on the left boundary are approximated by $C_{-1}^n = C_0^n$ and $C_{-1}^{n+1} = C_0^{n+1}$ into Eq.(4.41) in the right-hand side, we can see that

$$\left(4-\alpha_0^n + 2\beta\right)C_0^{n+1} + (\alpha_0^n - 2\beta)C_1^{n+1} = \left(4+\alpha_0^n - 2\beta\right)C_0^n + (2\beta - \alpha_0^n)C_1^n , \tag{4.42}$$

Similarly the right boundary, where $m=M$, the known value on the left boundary are approximated by $C_{M+1}^{n}=C_{M}^{n}$ and $C_{M+1}^{n+1}=C_{M}^{n+1}$ into Eq.(4.41) in the right-hand side, we have

$$-(\alpha_m^n+2\beta)C_{m-1}^{n+1}+\left(4+\alpha_m^n+2\beta\right)C_m^{n+1}=(\alpha_m^n+2\beta)C_{m-1}^{n}+\left(4-\alpha_m^n-2\beta\right)C_m^{n}. \quad (4.43)$$

The stability of Crank-Nicolson scheme is unconditionally stable.

4.3.5 The FTCS-type scheme

The traditional forward time central space scheme is considered following discretization:

$$\frac{\partial C}{\partial t}\cong\frac{\left(C_m^{n+1}-C_m^n\right)}{\Delta t}, \quad (4.44)$$

$$\frac{\partial C}{\partial x}\cong\frac{\left(C_{m+1}^{n}-C_{m-1}^n\right)}{\Delta x}, \quad (4.45)$$

$$\frac{\partial^2 C}{\partial x^2}\cong\frac{\left(C_{m+1}^{n}-2C_m^n+C_{m-1}^n\right)}{(\Delta x)^2}, \quad (4.46)$$

$$u\cong u_m^n. \quad (4.47)$$

Substituting Eqs.(4.44-4.47) into Eq.(2.5), we have

$$C_m^{n+1}=\frac{1}{2}(2\beta+\alpha_m^n)C_{m-1}^n+(1-2\beta)C_m^n+\frac{1}{2}\left(2\beta-\alpha_m^n\right)C_{m+1}^n, \quad (4.48)$$

for all $1\le m<M$ and $0\le n\le N$. For left boundary, where $m=0$, the known value on the left boundary are approximated by $C_{-1}^{n}=C_{0}^{n}$ into Eq.(4.48) in the right-hand side, we can see that

$$C_0^{n+1}=(1+\frac{1}{2}\alpha_0^n-\beta)C_0^n+\frac{1}{2}\left(2\beta-\alpha_0^n\right)C_1^n. \quad (4.49)$$

Similarly the right boundary, where $m=M$, the known value on the left boundary are approximated by $C_{M+1}^{n}=C_{M}^{n}$ into Eq.(4.48) in the right-hand side, we have

$$C_m^{n+1}=\frac{1}{2}(2\beta+\alpha_m^n)C_{m-1}^n+(1-\frac{1}{2}\alpha_m^n-\beta)C_m^n. \quad (4.50)$$

The stability of forward time central space scheme is

$$\frac{\alpha^2}{2} \le \beta \le \frac{1}{2}. \tag{4.51}$$

4.3.6 The upwind explicit formula

The upwind explicit scheme is considered by following discretization:

$$\frac{\partial C}{\partial t} \cong \frac{\left(C_m^{n+1} - C_m^n\right)}{\Delta t}, \tag{4.52}$$

$$\frac{\partial C}{\partial x} \cong \frac{\left(C_m^n - C_{m-1}^n\right)}{\Delta x}, \tag{4.53}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{\left(C_{m+1}^n - 2C_m^n + C_{m-1}^n\right)}{(\Delta x)^2}, \tag{4.54}$$

$$u \cong u_m^n. \tag{4.55}$$

Substituting Eqs.(4.52-4.55) into Eq.(2.5), we have

$$C_m^{n+1} = (\beta + \alpha_m^n)C_{m-1}^n + (1 - 2\beta - \alpha_m^n)C_m^n + \beta C_{m+1}^n, \tag{4.56}$$

for all $1 \le m < M$ and $0 \le n \le N$. For left boundary, where $m = 0$, the known value on the left boundary are approximated by $C_{-1}^n = C_0^n$ into Eq.(4.56) in the right-hand side, we can see that

$$C_0^{n+1} = (1-\beta)C_0^n + \beta C_1^n. \tag{4.57}$$

Similarly the right boundary, where $m = M$, the known value on the left boundary are approximated by $C_{M+1}^n = C_M^n$ into Eq.(4.56) in the right-hand side, we have

$$C_M^{n+1} = (\beta + \alpha_M^n)C_{M-1}^n + (1 - \beta - \alpha_M^n)C_M^n. \tag{4.58}$$

The stability of upwind explicit scheme is

$$\frac{\alpha^2 - \alpha}{2} \le \beta \le \frac{1-\alpha}{2}. \tag{4.59}$$

4.3.7 The Lax-Wendroff technique

The Lax-wendroff scheme is considered by the following discretization:

$$\frac{\partial C}{\partial t} \cong \frac{\left(C_m^{n+1} - C_m^n\right)}{\Delta t}, \tag{4.60}$$

$$\frac{\partial C}{\partial x} \cong \alpha_m^n \frac{\left(C_m^n - C_{m-1}^n\right)}{\Delta x} + (1-\alpha_m^n)\frac{\left(C_{m+1}^n - C_{m-1}^n\right)}{2\Delta x}, \tag{4.61}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{\left(C_{m+1}^n - 2C_m^n + C_{m-1}^n\right)}{(\Delta x)^2}, \tag{4.62}$$

$$u \cong u_m^n. \tag{4.63}$$

Substituting Eqs.(4.60-4.63) into Eq.(2.5), we have

$$C_m^{n+1} = \frac{1}{2}\left(2\beta + \alpha_m^n + \left(\alpha_m^n\right)^2\right)C_{m-1}^n + \left(1 - 2\beta - \left(\alpha_m^n\right)^2\right)C_m^n + \frac{1}{2}\left(2\beta - \alpha_m^n + (\alpha_m^n)^2\right)C_{m+1}^n, \tag{4.64}$$

for all $1 \le m < M$ and $0 \le n \le N$. For left boundary, where $m = 0$, the known value on the left boundary are approximated by $C_{-1}^n = C_0^n$ into Eq.(4.64) in the right-hand side, we can see that

$$C_0^{n+1} = \left(1 - \beta + \frac{1}{2}\alpha_0^n - \frac{1}{2}\left(\alpha_0^n\right)^2\right)C_0^n + \frac{1}{2}\left(2\beta - \alpha_0^n + (\alpha_0^n)^2\right)C_1^n. \tag{4.65}$$

Similarly the right boundary, where $m = M$, the known value on the left boundary are approximated by $C_{M+1}^n = C_M^n$ into Eq.(4.64) in the right-hand side, we have

$$C_M^{n+1} = \frac{1}{2}\left(2\beta + \alpha_M^n + \left(\alpha_M^n\right)^2\right)C_{M-1}^n + \left(1 - \beta - \frac{1}{2}\alpha_M^n - \frac{1}{2}\left(\alpha_M^n\right)^2\right)C_M^n. \tag{4.66}$$

The stability of Lax-wendroff scheme is

$$0 < \beta \le \frac{1-\alpha^2}{2}. \tag{4.67}$$

4.4 Numerical simulations of water quality model for flooding stream

Suppose that the measurement of pollutant concentration C in a dam-break flow stream is considered. A stream is aligned with longitudinal distance, 2000 (m) total length. There is a dam-break which discharges waste water into the flooding area at middle point of the domain and the pollutant concentrations at the discharge point are assumed:

Case 1:

$$C(x,0)=\begin{cases}1 & ,if \quad x=1000\\ 0.1, & if \ 1000<x\le 2000\end{cases} \tag{4.68}$$

$$C(1000,t)=\left(1-|\sin t|\right)\left(\frac{T-t}{T}\right) \text{ for all } 0<t\le T, \tag{4.69}$$

$$C_x(2000,t)=0. \tag{4.70}$$

The elevation and velocity of water are obtained by the dam-break model that we assume the initial and boundary conditions by several cases as below,

Case A: Dam-break on wet bed (depth ratio is 0.75)

Table 4.7 The pollutant concentration $C(x,t)(kg/m^3)$ by Modified Siemieniuch-Gladwell scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6411	0.6838	0.1012	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3663	0.4924	0.6715	0.1630	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1741	0.2734	0.4426	0.6334	0.3129	0.1017	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0565	0.1236	0.2382	0.3958	0.5863	0.4518	0.1188	0.1000	0.1000	0.1000	0.1000

Table 4.8 The pollutant concentration $C(x,t)(kg/m^3)$ by BTCS scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6411	0.3926	0.1006	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3663	0.2795	0.3832	0.1320	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1741	0.1552	0.2513	0.3606	0.2069	0.1009	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0565	0.0702	0.1352	0.2247	0.3334	0.2755	0.1096	0.1000	0.1000	0.1000	0.1000

Table 4.9 The pollutant concentration $C(x,t)(kg/m^3)$ by Upwind implicit scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6411	0.7330	0.1056	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3663	0.5540	0.7232	0.2052	0.1002	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1741	0.3079	0.4984	0.6885	0.3632	0.1072	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0565	0.1394	0.2688	0.4463	0.6430	0.4899	0.1422	0.1004	0.1000	0.1000	0.1000

Table 4.10 The pollutant concentration $C(x,t)(kg/m^3)$ by Crank-Nicolson scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6411	0.5044	0.1008	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3663	0.3613	0.4940	0.1435	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1741	0.2007	0.3248	0.4655	0.2472	0.1011	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0565	0.0907	0.1748	0.2904	0.4306	0.3430	0.1130	0.1000	0.1000	0.1000	0.1000

Table 4.11 The pollutant concentration $C(x,t)(kg/m^3)$ by FTCS scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6461	0.7907	0.1010	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3701	0.5836	0.7877	0.1672	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1766	0.3249	0.5248	0.7469	0.3418	0.1016	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0579	0.1475	0.2832	0.4695	0.6930	0.5115	0.1200	0.1000	0.1000	0.1000	0.1000

Table 4.12 The pollutant concentration $C(x,t)(kg/m^3)$ by Upwind explicit scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6461	0.7571	0.1046	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3701	0.5841	0.7553	0.2018	0.1001	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1766	0.3254	0.5257	0.7220	0.3665	0.1065	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0579	0.1480	0.2842	0.4709	0.6757	0.5027	0.1407	0.1003	0.1000	0.1000	0.1000

Table 4.13 The pollutant concentration $C(x,t)(kg/m^3)$ by Lax-wendroff scheme (case A1).

t\x	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6461	0.7898	0.1011	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3701	0.5836	0.7870	0.1680	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1766	0.3249	0.5248	0.7463	0.3424	0.1017	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0579	0.1475	0.2832	0.4695	0.6926	0.5113	0.1204	0.1000	0.1000	0.1000	0.1000

Case A: Dam-break on wet bed (depth ratio is 0.75)

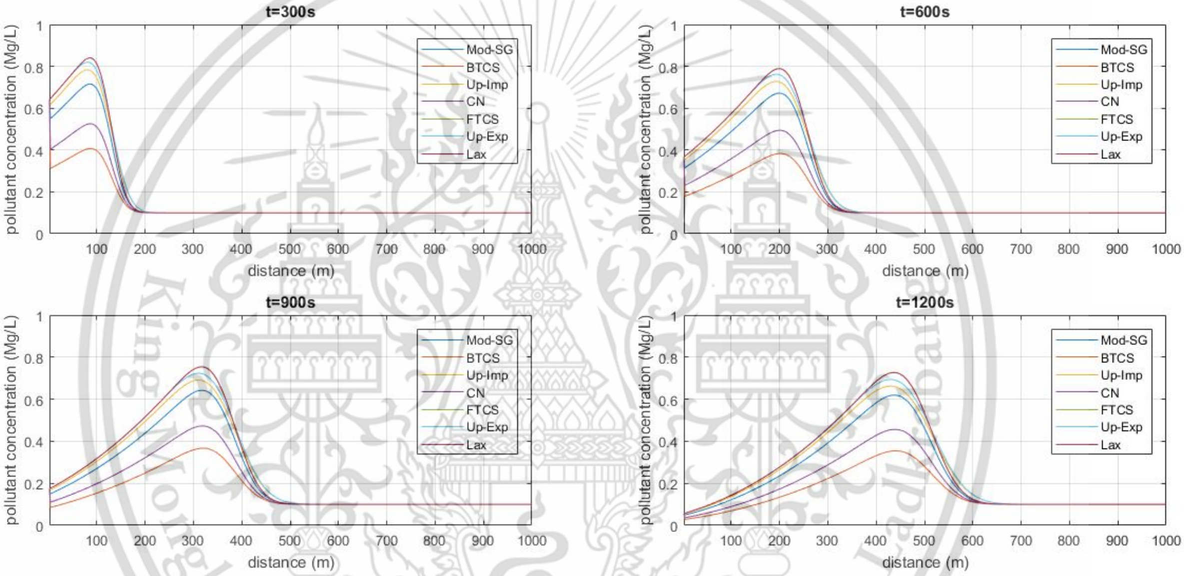


Fig. 4.4 The comparison of pollutant concentration case A (depth ratio 0.75), $\Delta x = 2$, $\Delta t = 0.1$ at difference time.

Case B: Dam-break on wet bed (depth ratio is 0.5)

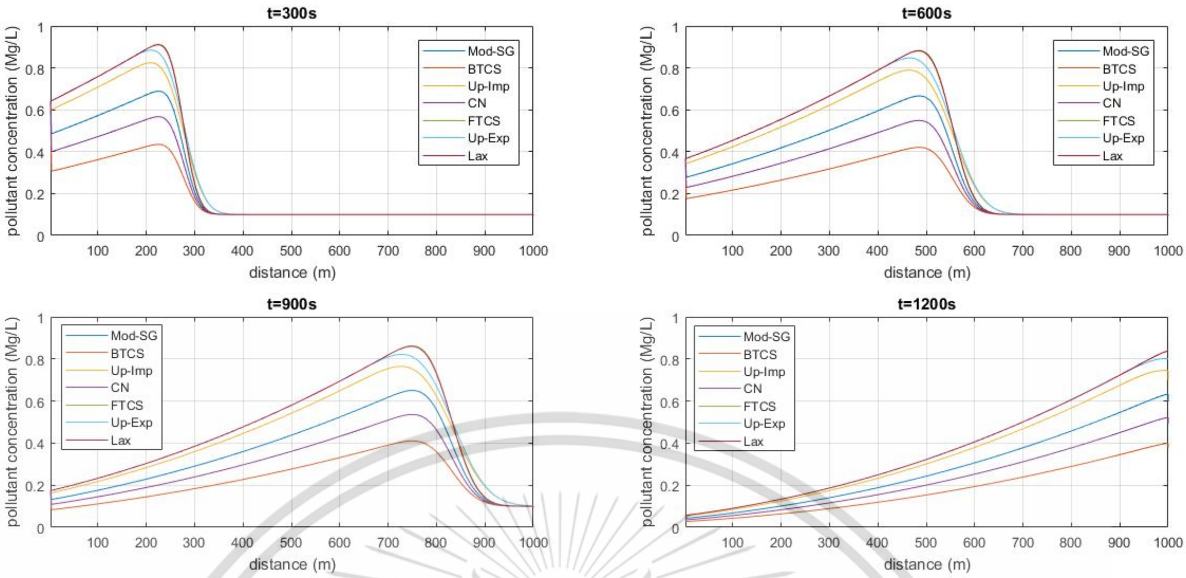


Fig. 4.5 The comparison of pollutant concentration case A (depth ratio 0.5), $\Delta x = 2$, $\Delta t = 0.1$ at difference time.

Case C: Dam-break on dry bed (depth ratio is 0.25)

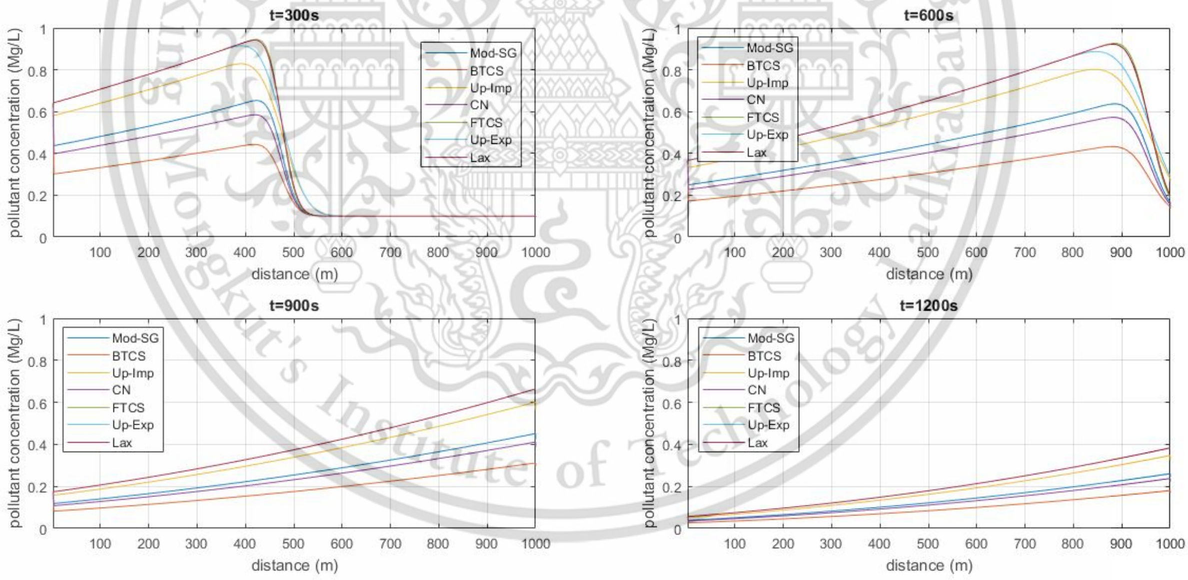


Fig. 4.6 The comparison of pollutant concentration case A (depth ratio 0.25), $\Delta x = 2$, $\Delta t = 0.1$ at difference time.

The physical parameters of the polluted system are diffusion coefficient $D = 1\left(m^2 / s\right)$. In the analysis conducted in this study, meshes the length into 1000 elements with $\Delta x = 2$, and time increment is $0.1(s)$ with $\Delta t = 0.1$.

4.5 Discussion

The elevation and velocity of water current are obtained by a modified Lax-diffusive method. The case C of sandbag-dike failure gives the highest flow velocity. The cases A-B of sandbag-dike failure gives low velocity. If the villagers have received flood for longtime, the water pollutant have to be increase. The residents want to drain the water to the other areas by destroying the sandbag-dike. The other villages that never encounter flood going to receive polluted water. The approximation of the pollutant concentrations of the implicit and explicit methods are shown in Tables 4.7-4.13. We can see that the pollutant concentration level on the flooding area is not too high with the passing of time. The real-world problems require a small amount of time interval in obtaining accurate solutions. Unfortunately, the analytical solutions of the dam-break model could not be found over the entire domain. This also implies that the analytical solutions of dispersion model could not work out at any point on the entire domain as well. We propose a modified Lax-diffusive scheme by editing a simple modification to the traditional Lax-diffusive scheme. The FTCS method is limited by restriction of the stability condition. Then FTCS is not flexible in the real-world situation. The implicit schemes shows excessive dispersion effects for large time and space step lengths, significantly decreasing the efficiency of the implicit schemes. In addition implicit methods still generate a lot of large systems of linear equations. The Upwind explicit and Lax-wendroff schemes are economical to use. The proposed method show a good agreement in accuracy, the implicit schemes becomes less efficient than the explicit schemes.

Chapter 5
Conclusion

We propose a modified Lax-diffusive scheme by editing a simple revision to the traditional Lax-diffusive scheme. For chapter 3, the FTCS method is limited by restriction of the stability condition. Then FTCS is not flexible in the real-world situation. The Crank-Nicolson scheme shows excessive dispersion effects for large time and space step lengths, significantly decreasing the efficiency of the Crank-Nicolson scheme. The BTCS still generate a lot of large systems of linear equations. The MacCormack and Saulyev schemes are economical to use. The proposed method show a good agreement in accuracy, the explicit schemes becomes less efficient than the implicit schemes.

For chapter 4, FTCS is not flexible in the real-world situation. The implicit schemes shows excessive dispersion effects for large time and space step lengths, significantly decreasing the efficiency of the implicit schemes. In addition implicit methods still generate a lot of large systems of linear equations. The Upwind explicit and Lax-wendroff schemes are economical to use. The proposed method show a good agreement in accuracy, the implicit schemes becomes less efficient than the explicit schemes.

In this thesis, the dam-break model and the dispersion model can be combined to approximate the pollutant concentration in a stream when the current reflecting water in the stream is not uniform since the dam becomes failure. The technique constructed in this study is to respond the aspects of the stream in two varied external inputs, which are the level of water and the pollutant concentration at the discharge point. The both of the implicit methods and the explicit methods are utilized by means of the dispersion model because the scheme seems not to be too complicated to implement. In terms of the explicit finite difference formulations, it is believed that the implemented technique is practical and applicable. Inaddition, it seems economical to be employed and used in the real world tasks and problems. It can also be easily used owning to the program simplicity, the straight forward implementation and less time consumption.

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Appendices

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Numerical simulations of a water quality model in a flooding stream due to dam-break problem using implicit and explicit methods

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Numerical simulations of a water quality model in a flooding stream due to dam-break problem using implicit and explicit methods

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Abstract

The heavy monsoon rains have been drenched flooding along the rivers and reservoirs. The residents try to construct the sandbag dike to protect their village. If the residents have encountered flood for long period, the water pollutant must be increase. The villagers want to drain the water to another areas. The residents of another village are not agree to receive the drained-polluted water from them. The simulation of water-quality is required to compromise these problem. The simulation process of water-quality model is require the input as the water flow velocities after the villagers dike has been destroyed but there is lack of field data on the flooding period. In this research, the dam-break model is used to

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describe unsteady dike failure flow. A couple of mathematical models is used to simulate water-quality in the problem. The first model is the dam-break model that provides the velocity fields and elevation of water. The second model is the dispersion model that provides the pollutant concentration fields. A modified Lax-diffusive method for solving a dam-break model is proposed. At each step, the flow velocity fields calculated from the first model are the input into the second model as the field data. The explicit and implicit methods are subsequently employed in the dispersion model. This paper proposes a simply remarkable alteration to the Lax-diffusive method, the implicit methods and the explicit methods so as to make it more accurate without any significant loss of computational efficiency. The numerical techniques indicate that this model can be applied for simulation of water-quality in long-term flooding cases are proposed.

Keywords: Finite differences, Lax-diffusive scheme, Modified Lax-diffusive scheme, One-dimensional, Dam-break model, Shallow water equations, Dispersion model, Advection-dispersion-reaction equations

Mathematics Subject Classification: 65M06, 62P12

1. Introduction

The amount of pollution levels in the a stream can be measured via data selection from a real of field data area. It is somehow rather difficult and complex, and the results obtained tentatively deviate in the assessment from one point in each time/place to another when the water flow in the stream is not uniform. The heavy monsoon rains have been drenched flooding along the rivers and reservoirs. The residents try to construct the sandbag dike to protect their village. If the residents have to encounter flooding in longtime, the water pollutant going to increase. The villagers want to drain the water to the other areas. Although, the residents of another village are not agree to receive the drained-polluted water. The simulation of water-quality is required to compromise the problem. The simulation process of water-quality model is require the input as the water flow velocities after the villagers dike has been destroyed but there is lack of field data on the flooding period.

Numerous numerical techniques for solving such models are available. In [25], the finite element method for solving a steady water pollution control to achieve a minimum cost is presented. The numerical techniques for solving the uniform flow of stream water quality model, especially the one-dimensional advection-dispersion-reaction equation are presented in [14], [21], [24], [15] and [33].

The non-uniform flow model requires data concerned with the velocity of the current at any point and any time in the domain. The hydrodynamics model provides the velocity field and tidal elevation of the water. In [34, 28], [26] and [27], they used the hydrodynamics model and the advection-dispersion equation to approximate the velocity of the water current in bay, uniform reservoir and stream, respectively. Among these numerical techniques, the finite difference methods, including both explicit and implicit schemes, are mostly used for one-dimensional domain such as in longitudinal stream systems [4], [13].

There are two mathematical models used to simulate water quality in a non-uniform water flow systems. The first is the hydrodynamic model that provides the velocity field and the elevation of water. The second is the dispersion model that gives the pollutant concentration field. A couple of models are formulated in one-dimensional equations. The traditional Crank-Nicolson method is used in the hydrodynamic model. At each step, the calculated flow velocity fields of the first model are input into the second model as the field data [26], [27], [29], [30].

Their research on finite difference techniques for the dispersion model have concentrated on computation accuracy and numerical stability. Many complicate numerical techniques, such as the QUICK scheme, the Lax-Wendroff scheme, the Crandall scheme, have been studied to increase performances. These techniques have focused on advantages in terms of stability and higher order accuracy [21].

The simple finite difference schemes become more attractive for model use. The simple explicit methods include the Forward Time-Central Space (FTCS) scheme, the MacCormack scheme, and the Saulyev scheme, and the implicit schemes include the Backward Time-Central Space (BTCS) scheme and the Crank-Nicolson scheme [13]. These scheme are either first-order or second order accurate and have the advantages in programming and computing without losing much accuracy and thus are used for many model applications [21].

A third-order upwind scheme for the advection-diffusion equation using a simple spreadsheets simulation is proposed in [19]. In [22], a new flux splitting scheme is proposed. The scheme is robust and converges as fast as the Roe Splitting. The Godunov Mixed Methods for Advection–Dispersion Equations is introduced in [23]. A time-splitting approach for advection–dispersion equations is also considered. In addition, [16] proposes the time-split methods for multidimensional advection-diffusion equations that advection is approximated by a Godunov-type procedure, and diffusion is approximated by a low-order mixed finite element method. In [2], the flux-limiting solution techniques for simulation of reaction–diffusion–convection system is introduced. A composite scheme to solve the scalar transport equation in a two-dimensional space that accurately resolve sharp profiles in the flow is introduced. The total variation diminishing implicit Runge–Kutta methods for dissipative advection–diffusion problems in astrophysics is introduced by [18]. They derive dissipative space discretizations and demonstrate that together with specially adapted total-variation-diminishing (TVD) or strongly stable Runge-Kutta time discretizations with adaptive step-size control this yields reliable and efficient integrators for the underlying high-dimensional nonlinear evolution equations.

The results from the hydrodynamic model are the data of the water flow velocity for the advection-dispersion-reaction equation which provides the pollutant concentration field. The term of friction forces, due to the drag of sides of the stream, is considered. The theoretical solution of the model at the end point of the domain that guarantees the accuracy of the approximate solution is presented in [26], [27] and [29].

In recent years, there has been a large research important on the development of numerical models to simulate dam-break flows. In [3], they developed a two-dimensional model coupled with a one-dimensional model for simulation of floods due to overtopping and breaching of levees. In [11], they also used a finite volume method for modelling the extreme flood events in natural channels. In [36], they also modeled the Malpasset dam break event using 2D finite volume method.

The dam-break flows can be modeled as unsteady free surface flows over a complex topography using depth-averaged, non-linear shallow water equations (SWEs), also known as Saint-Venant equations. In [37], they develop numerical models for flows generated by a dam failure or levee breaching process using a conservative form of shallow water equations. There are many interest simulations in solving shallow water equations, such as treatment of wet and dry interface, treatment of bed elevation terms, mixed flow regimes. [9] developed a numerical model for dam-break flows and resolved the wetting and drying of irregular terrain to a good extent. In [10], they also develope a two-dimensional numerical model for dam-break flows and achieved a zero mass error by modifying the wetting–drying condition which included the normal velocity to the cell edges. Most of these numerical models are based on Godunov-type schemes or lower order up-winding schemes, which produce significant numerical diffusion and do not necessarily admit steady state solutions in case of natural terrains, where the source terms due to bottom elevation gradient may become very important. In [32], they proposed a two-dimensional dam-break model that numerically solved by using an explicit central upwind, well-balanced and flow depth positivity-preserving, second-order accurate scheme. In [17], they used central scheme for 1D and two dimension (2D) dam-break simulation. In [6], they used finite volume method for numerical solution of shallow water equations in dam-break with flat topography. [31] used 2D finite volume multiblock flow solver. The model is based on Flux Vector Splitting method. [5] used a robust and effective flux-vector splitting method to simulate dam-break problem base on finite volume method on a cartesian grid. [12] used smoothed particle hydrodynamics (SPH) to solve shallow-water dam break flow in open channels. A new well-balanced unstraggered central finite volume scheme for 1D and 2D dam break over a rectangular bump is proposed in [35]. In [7], they used well-balanced hydrostatic upwind schemes for dam-break.

In this paper, the dam-break model and dispersion model are used to describe unsteady-dike-failure water flow and water pollutant concentration. A couple of mathematical models is used to simulate water-quality in the problem. The stream has a simple one space dimension as shown in Fig. 1. The first model is the dam-break model that provides the velocity fields and elevation of water. The second model is the dispersion model that provides the pollutant concentration fields. A modified Lax-diffusive method is used in the dam-break model. At each step, the flow velocity fields calculated from the first model are the input into the second model as the field data. The explicit and implicit methods are subsequently employed in the dispersion model. This paper proposes a simply remarkable alteration to the Lax-diffusive method, the implicit methods and the explicit methods so as to make it more accurate without any significant loss of computational efficiency. The numerical techniques indicate that the model can be applied for simulation of water-quality in long-term flooding cases are proposed.

2. Model Formulation

2.1 *The Dam-break Model*

The one-dimensional shallow water equations are obtained by integrating the Navier-Stokes equations over the flow depth under the assumptions as hydrostatic pressure distribution and small bottom slope. The dam-break flows are high velocity and can be considered as advection-dominated shallow water flows. Therefore, the eddy viscosity terms can

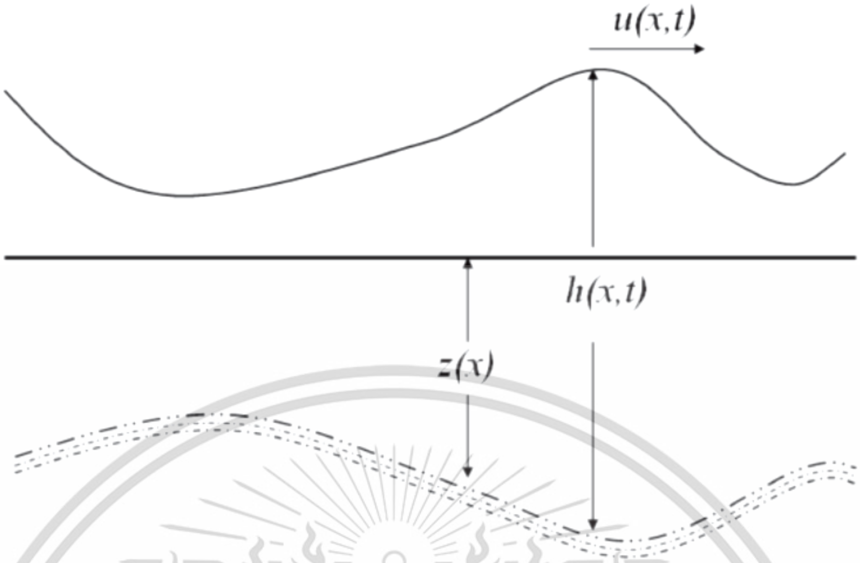


Figure 1

The shallow water system.

be neglected. The governing equations on conversation and vector form can be written in the system of partial differential equations [1], [6] as

$$\partial_x \begin{pmatrix} h \\ hu \end{pmatrix} + \partial_t \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix} = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix}, \tag{1}$$

where x is the longitudinal distance along a stream (m), t is time (s), $h(x, t)$ is the elevation of the water above the bottom (m/s), $z(x)$ is the function characterizing the bottom topography (m), and $u(x, t)$ is the velocity components (m/s), for all $x \in [0, L]$. The initial conditions are given by

$$u(x, 0) = 0 \text{ for all } 0 \le x \le L, \tag{2}$$

and

$$h(x, 0) = \begin{cases} h_l & \text{if } 0 \le x \le \frac{L}{2}, \\ h_r & \text{if } \frac{L}{2} < x \le L. \end{cases} \tag{3}$$

The boundary conditions are also given by

$$u_x(0, t) = u_x(L, t) = 0, \tag{4}$$

$$h_x(0, t) = h_x(L, t) = 0, \tag{5}$$

where $h_l > h_r$ in order to be consistent with the physical phenomenon of a dam-break from the left to the right. At $t = 0$, the dam collapses and the flow problem consists of a shock wave travelling downstream and a rarefaction wave travelling upstream.

2.2 *Dispersion Model*

In a stream water quality model, the governing equations are the dynamic one-dimensional advection-dispersion-reaction equations (ADRE). A simplified representation by averaging the equation over the depth is shown in [14], [21], [24], [27] and [33] as

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2}-KC, \tag{6}$$

where $C(x, t)$ is the concentration averaged in depth at the point x and at time t, D is the diffusion coefficient, K is the mass decay rate, and $u(x, t)$ is the velocity component, for all $x \in [0, L]$. We will consider the model with the following conditions. The initial condition $C(x, 0) = 0$ at $t = 0$ for all $x > 0$. The boundary conditions $C(0, t) = C_0$ at $x = 0$ and $\frac{\partial C}{\partial x}=0$ at $x = 1$ where C_0 is a positive constant.

3. A modified Lax-diffusive method for the dam-break model

The dam-break model provides the velocity field and elevation of the water. Then the calculated results of the model will be the input into the dispersion model which provides the pollutant concentration field. In this section, the modify method of a traditional Lax-diffusive method for the dam-break model of [1] is proposed. We now discretize Eq.(1) by dividing the interval $[0, L]$ into M subintervals such that $M\,\Delta x = L$ and the interval $[0, T]$ into N subintervals such that $N\,\Delta t = T$. We can then approximate $h(x_m, t_n)$ by h^n_m, value of the difference approximation of $h(x, t)$ at point $x = m\Delta x$ and $t = n\Delta t$, where $0 \le m \le M$ and $0 \le n \le N$, and similarly defined for u^n_m. The grid point (x_m, t_n) is defined by $x_m = m\Delta x$ for all $m = 0, 1, 2, ..., M$ and $t_n = n\Delta t$ for all $n = 0, 1, 2, ..., N$ in which M and N are positive integers.

We will modify f^* from the traditional method of [1] to be the three points average. The discretization of Eq.(1) is base on a Lax-diffusive scheme. The semi-discrete scheme is applied to Eq.(1) and using a uniform spatial grid $(x_m, t_n) = (m\Delta x, n\Delta t)$, we can define

$$f_x = \frac{f^n_{m+1}-f^n_{m-1}}{2\Delta x}, \tag{7}$$

$$f_t = \frac{f^{n+1}_m-f^*_m}{\Delta t}, \tag{8}$$

where

$$f^* = \frac{f^n_{m+1}+f^n_m+f^n_{m-1}}{3}. \tag{9}$$

The partial derivative of h and u with respect to x and t are approximated by using Eqs.(7-9), respectively. We can see that Eq.(1) is written in a matrix form as

$$A_t + B_x + C = 0, \tag{10}$$

where

$$A = \begin{pmatrix} h \\ hu \end{pmatrix}, B = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix}. \tag{11}$$

It follows that Eq.(11) can be written by the uniform spatial grids as

$$A_m^n = \begin{pmatrix} h_m^n \\ h_m^n u_m^n \end{pmatrix}, B_m^n = \begin{pmatrix} h_m^n u_m^n \\ h_m^n (u_m^n)^2 + \frac{1}{2}g(h_m^n)^2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ -gh_m^n \partial_x z \end{pmatrix}. \tag{12}$$

Substituting the finite difference approximations of Eqs.(7-8) and Eq.(9) into Eq.(10), we obtain that

$$A_m^{n+1} = \frac{\Delta t}{2\Delta x}(B_{m-1}^n - B_{m+1}^n) + A^* \tag{13}$$

where $A^* = \begin{pmatrix} h^* \\ (hu)^* \end{pmatrix}$. Substituting Eq.(12) into Eq.(13), we can see that

$$\begin{pmatrix} h_m^{n+1} \\ h_m^{n+1} u_m^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{m-1}^n u_{m-1}^n - h_{m+1}^n u_{m+1}^n \\ h_{m-1}^n (u_{m-1}^n)^2 - h_{m+1}^n (u_{m+1}^n)^2 + \frac{1}{2}g((h_{m-1}^n)^2 - (h_{m+1}^n)^2) \end{pmatrix} + \frac{1}{3}\begin{pmatrix} h_{m-1}^n + h_m^n + h_{m+1}^n \\ h_{m-1}^n u_{m-1}^n + h_m^n u_m^n + h_{m+1}^n u_{m+1}^n \end{pmatrix}. \tag{14}$$

for all $1 \le m < M$ and $0 \le n \le N-1$. For upper boundary, where $m = 0$, plug the known value of the left boundary by $u_{-1}^n = u_0^n$ and $h_{-1}^n = h_0^n$ into Eq.(14) in the right-hand side, we obtain

$$\begin{pmatrix} h_1^{n+1} \\ h_1^{n+1} u_1^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_0^n u_0^n - h_1^n u_1^n \\ h_0^n (u_0^n)^2 - h_1^n (u_1^n)^2 + \frac{1}{2}g((h_0^n)^2 - (h_1^n)^2) \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 2h_0^n + h_1^n \\ 2h_0^n u_0^n + h_1^n u_1^n \end{pmatrix}. \tag{15}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $u_{M+1}^n = u_M^n$ and $h_{M+1}^n = h_M^n$ by rearranging, we obtain

$$\begin{pmatrix} h_M^{n+1} \\ h_M^{n+1}u_M^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{M-1}^n u_{M-1}^n - h_M^n u_M^n \\ h_{M-1}^n (u_{M-1}^n)^2 - h_M^n (u_M^n)^2 + \frac{1}{2}g((h_{M-1}^n)^2 - (h_M^n)^2) \end{pmatrix} + \frac{1}{3}\begin{pmatrix} h_{M-1}^n + 2h_M^n \\ h_{M-1}^n u_{M-1}^n + 2h_M^n u_M^n \end{pmatrix}. \tag{16}$$

The stability condition of the scheme needed CFL number as [1],

$$C_n = u_{max}\left(\frac{\Delta t}{\Delta x}\right) \leq 1. \tag{17}$$

4. The finite difference methods for the dispersion model

4.1 *The implicit methods*

First of all, we consider the implicit schemes for the advection-dispersion-reaction equation. The backward time central space scheme approximates the temporal and spacial derivatives and the decay in Eq.(6).

4.1.1 Backward time central space method

We can then approximate $C(x_i, t_n)$ by C_i^n, the value of the difference approximation of $C(x, t)$ at point $x = i\Delta x$ and $t = n\Delta t$, where $0 \leq i \leq M$ and $0 \leq n \leq N$. The grid point (x_i, t_n) is defined by $x_i = i\Delta x$ for all $i = 0, 1, 2, \ldots, M$ and $t_n = n\Delta t$ for all $n = 0, 1, 2, \ldots, N$ in which M and N are positive integers. Taking the backward time forward space technique [13] into Eq.(6), we get the following discretization:

$$C \cong C_m^{n+1}, \tag{18}$$

$$\frac{\partial C}{\partial t} \cong \frac{C_m^{n+1} - C_m^n}{\Delta t}, \tag{19}$$

$$\frac{\partial C}{\partial x} \cong \frac{C_{m+1}^{n+1} - C_{m-1}^{n+1}}{2\Delta x}, \tag{20}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{C_{m+1}^{n+1} - 2C_m^{n+1} + C_{m-1}^{n+1}}{(\Delta x)^2}, \tag{21}$$

$$u \cong \tilde{u}_i^n. \tag{22}$$

Note that $\tilde{u}_i^n$ are obtained by the modified Lax-diffusive method with the dam-break model of Eq.(14). Substituting Eqs.(18-22) into Eq.(6), we get

$$\frac{C_m^{n+1}-C_m^n}{\Delta t}+\tilde{u}_m^n\left(\frac{C_{m+1}^{n+1}-C_{m-1}^{n+1}}{2\Delta x}\right) = D\left(\frac{C_{m+1}^{n+1}-2C_m^{n+1}+C_{m-1}^{n+1}}{(\Delta x)^2}\right)-KC_m^{n+1}, \quad (23)$$

for all $1<m<M$ and $0\le n\le N$. If we let $\alpha_m^n=\tilde{u}_m^n\left(\frac{\Delta t}{\Delta x}\right)$ and $\beta=D\frac{\Delta t}{(\Delta x)^2}$, then, Eq.(23) becomes,

$$-\left(\beta+\frac{\alpha}{2}\right)C_{m-1}^{n+1}+(1+2\beta+K\Delta t)C_m^{n+1}-\left(\beta-\frac{\alpha}{2}\right)C_{m+1}^{n+1} = C_m^n. \quad (24)$$

For upper boundary, where $m=1$, plug the known value of the left boundary by $C_0^{n+1}=C_0^n$ into Eq.(24) in the right-hand side, we obtain

$$(1+2\beta+K\Delta t)C_1^{n+1}-\left(\beta-\frac{\alpha}{2}\right)C_2^{n+1} = C_1^n+\left(\beta+\frac{\alpha}{2}\right)C_0^{n+1}. \quad (25)$$

For lower boundary, where $m=M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^{n+1}=2C_M^{n+1}-C_{M-1}^{n+1}$ by rearranging, we can obtain

$$-\alpha C_{M-1}^{n+1}+(1+\alpha+K\Delta t)C_M^{n+1}=C_m^n. \quad (26)$$

The backward time central space schems is an unconditionally stable scheme [13].

4.1.2 Crank-Nicolson method

Consequently, we consider the Crank-Nicolson scheme for the advection-dispersion-reaction equation. It is also used to approximates the temporal and spacial derivatives and the decay in Eq.(6). We will use the Crank-Nicolson technique [21] to Eq.(6), it can be obtained the following discretization:

$$C\cong\frac{C_m^{n+1}+C_m^n}{2}, \quad (27)$$

$$\frac{\partial C}{\partial t}\cong\frac{C_m^{n+1}-C_m^n}{\Delta t}, \quad (28)$$

$$\frac{\partial C}{\partial x}\cong\frac{C_{m+1}^{n+1}-C_{m-1}^{n+1}+C_{m+1}^n-C_{m-1}^n}{4\Delta x}, \quad (29)$$

$$\frac{\partial^2 C}{\partial x^2}\cong\frac{C_{m+1}^n-2C_m^n+C_{m-1}^n+C_{m+1}^{n+1}-2C_m^{n+1}+C_{m-1}^{n+1}}{2(\Delta x)^2}, \quad (30)$$

$$u\cong\tilde{u}_i^n. \quad (31)$$

Substituting Eqs.(27-31) into Eq.(6), we obtain

$$\frac{C_m^{n+1}-C_m^n}{\Delta t}+\tilde{u}_m^n\left(\frac{C_{m+1}^{n+1}-C_{m-1}^{n+1}+C_{m+1}^n-C_{m-1}^n}{4\Delta x}\right)$$
$$=D\left(\frac{C_{m+1}^n-2C_m^n+C_{m-1}^n+C_{m+1}^{n+1}-2C_m^{n+1}+C_{m-1}^{n+1}}{2(\Delta x)^2}\right)-K\left(\frac{C_m^{n+1}+C_m^n}{2}\right) \quad (32)$$

for all $1<m<M$ and $0\le n\le N$. Let $\alpha_m^n=\tilde{u}_m^n\left(\frac{\Delta t}{\Delta x}\right)$ and $\beta=D\frac{\Delta t}{(\Delta x)^2}$, Eq.(32) becomes,

$$-\left(\beta+\frac{\alpha}{2}\right)C_{m-1}^{n+1}+2(1+\beta+K\Delta t)C_m^{n+1}-\left(\beta-\frac{\alpha}{2}\right)C_{m+1}^{n+1}=$$
$$\left(\beta+\frac{\alpha}{2}\right)C_{m-1}^n+2(1-\beta-K\Delta t)C_m^n+\left(\beta-\frac{\alpha}{2}\right)C_{m+1}^n. \quad (33)$$

For upper boundary, where $m=1$, plug the known value of the left boundary by arranging C_0^{n+1} to the right-hand side, we obtain

$$2(1+\beta+K\Delta t)C_1^{n+1}-\left(\beta-\frac{\alpha}{2}\right)C_2^{n+1}=\left(\beta+\frac{\alpha}{2}\right)(C_0^{n+1}+C_0^{n+1})$$
$$+2(1-\beta-K\Delta t)C_1^{n+1}+\left(\beta-\frac{\alpha}{2}\right)C_2^{n+1}. \quad (34)$$

For lower boundary, where $m=M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^{n+1}=2C_M^{n+1}-C_{M-1}^{n+1}$ and $C_{M+1}^n=2C_M^n-C_{M-1}^n$ into Eq.(34) by rearranging, we obtain

$$-\alpha C_{M-1}^{n+1}+2\left(1+\frac{\alpha}{2}+K\Delta t\right)C_{M-1}^{n+1}=\alpha C_{M-1}^n+2\left(1-\frac{\alpha}{2}-K\Delta t\right)C_{M-1}^n. \quad (35)$$

The Crank-Nicolson is unconditionally stable [13]. Both implicit methods can be obtained that the technique must be generate many large systems of linear equations. It follows that the technique is not economical computer implementation.

4.2 *The explicit methods*

4.2.1 Forward time central space method

The explicit methods can be obtained that the technique not require to generate any systems of linear equations. We can see that the application of the technique is economical computer implementation. First of all, the forward time central space scheme for the advection-dispersion-reaction equation is introduced. It is also used to approximates the temporal and spacial derivatives and the decay in Eq.(6). We will use the forward time central space method technique [21] and [29] to Eq.(6), it can be obtained the following discretization:

$$C \cong C_m^n, \tag{36}$$

$$\frac{\partial C}{\partial t} \cong \frac{C_m^{n+1} - C_m^n}{\Delta t}, \tag{37}$$

$$\frac{\partial C}{\partial x} \cong \frac{C_{m+1}^n - C_{m-1}^n}{2\Delta x}, \tag{38}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{C_{m+1}^n - 2C_m^n + C_{m-1}^n}{(\Delta x)^2}, \tag{39}$$

$$u \cong \tilde{u}_m^n, \tag{40}$$

where $\tilde{u}_i^n$ are obtained by the modified Lax-diffusive method with the dam-break model of Eq.(14). Substituting Eqs.(36-40) into Eq.(6), we obtain

$$\frac{C_m^{n+1} - C_m^n}{\Delta t} + u_m^n\left(\frac{C_{m+1}^n - C_{m-1}^n}{2\Delta x}\right) = D\left(\frac{C_{m+1}^n - 2C_m^n + C_{m-1}^n}{(\Delta x)^2}\right) - KC_m^n, \tag{41}$$

for all $1 \le m < M$ and $0 \le n \le N$. Let $\alpha_m^n = u_m^n\left(\frac{\Delta t}{\Delta x}\right)$ and $\beta = D\frac{\Delta t}{(\Delta x)^2}$, so Eq.(41) becomes,

$$C_m^{n+1} = \left(\beta + \frac{\alpha_m^n}{2}\right)C_{m-1}^n + (1 - 2\beta - K\Delta t)C_m^n + \left(\beta - \frac{\alpha_m^n}{2}\right)C_{m+1}^n. \tag{42}$$

For upper boundary, where $m = 0$, plug the known value of the left boundary by arranging $C_0^n = C_0$ into Eq.(42) on the right-hand side, we obtain

$$C_0^{n+1} = \left(\beta + \frac{\alpha_0^n}{2}\right)C_0^n + (1 - 2\beta - K\Delta t)C_1^n + \left(\beta - \frac{\alpha_0^n}{2}\right)C_2^n. \tag{43}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^n = C_M^n$ and by rearranging, we obtain

$$C_M^{n+1} = \alpha_M^n C_{M-1}^n + (1 - \alpha_M^n - K\Delta t)C_M^n. \tag{44}$$

This scheme is stable [21] in the following region,

$$\begin{aligned} \alpha_m^n &= u_m^n\left(\frac{\Delta t}{\Delta x}\right) < \frac{1}{2}, \\ \beta &= D\frac{\Delta t}{(\Delta x)^2} < 1, \end{aligned} \tag{45}$$

where β is the diffusion number and α_m^n is the advection number or Courant number. It can be obtained that the strictly stability requirements are the main disadvantage of this scheme. The truncation error for this method is $O\{(\Delta x)^2, \Delta t\}$ [30].

4.2.2 MacCormack method

The MacCormack scheme is an explicit scheme [21] with two-step predictor-corrector evaluations. The first step is a modification of forward time central space (FTCS) by changing the central space evaluation at time n to a forward space evaluation. This step is a forward time forward space (FTFS) scheme. The FTFS scheme approximates the temporal and spacial derivatives and the decay in Eq.(6) with the following discretization.

$$C \cong C_m^n, \tag{46}$$

$$\frac{\partial C}{\partial t} \cong \frac{C_m^{n+1} - C_m^n}{\Delta t}, \tag{47}$$

$$\frac{\partial C}{\partial x} \cong \frac{C_{m+1}^n - C_m^n}{\Delta x}, \tag{48}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{C_{m+1}^n - 2C_m^n + C_{m-1}^n}{(\Delta x)^2}, \tag{49}$$

$$u \cong \tilde{u}_m^n. \tag{50}$$

Substituting Eqs.(46-50) into Eq.(6), and we define slope s_{m_1} by

$$s_{m_1} = -U\left(\frac{C_{m+1}^n - C_m^n}{\Delta x}\right) + D\left(\frac{C_{m+1}^n - 2C_m^n + C_{m-1}^n}{(\Delta x)^2}\right) - KC_m^n, \tag{51}$$

$$= \frac{D}{(\Delta x)^2}C_{m-1}^n + \left(\frac{u_m^n}{\Delta x} - \frac{2D}{(\Delta x)^2} - K\right)C_m^n + \left(-\frac{u_m^n}{\Delta x} + \frac{D}{(\Delta x)^2}\right)C_{m+1}^n. \tag{52}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^n = 2C_M^n - C_{M-1}^n$ into Eq.(52) and by rearranging, we obtain

$$s_{M_1} = -\left(\frac{u_m^n}{\Delta x} + K\right)C_M^n + \left(\frac{u_m^n}{\Delta x}\right)C_{M-1}^n. \tag{53}$$

We can obtain MacCormack predictor step formulation by Euler's formula,

$$C_m^{n+1} = C_m^n + s_{m_1} \Delta t. \tag{54}$$

The second step is a modified BTCS by the following discretization

$$\frac{\partial C}{\partial x} \cong \frac{C_m^{n+1} - C_{m-1}^{n+1}}{\Delta x}. \tag{55}$$

Substituting Eq.(55) into Eq.(6), and we also define slope s_{m_2} by

$$s_{m_2} = \left(\frac{D}{(\Delta x)^2} + \frac{u_m^n}{\Delta x} \right) C_{m-1}^{n+1} - \left(\frac{u_m^n}{\Delta x} + \frac{2D}{(\Delta x)^2} + K \right) C_m^{n+1} + \left(\frac{D}{(\Delta x)^2} \right) C_{m+1}^{n+1}. \tag{56}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^{n+1} = 2C_M^{n+1} - C_{M-1}^{n+1}$, and substituting them into Eq.(56), we obtain

$$s_{M_2} = -\left(\frac{u_m^n}{\Delta x} + K \right) C_M^{n+1} + \left(\frac{u_m^n}{\Delta x} \right) C_{M-1}^{n+1}. \tag{57}$$

From the both steps, the MacCormack scheme takes the following form.

$$C_m^{n+1} = C_m^n + \frac{s_{m_1} + s_{m_2}}{2} \Delta t. \tag{58}$$

The MacCormack scheme is conditionally stable that the stability requirements for the scheme are [8]

$$\lambda = \frac{D \Delta t}{(\Delta x)^2} < \frac{1}{2}, \tag{59}$$

$$\gamma_m^n = \frac{\tilde{u}_m^n \Delta t}{\Delta x} < 0.9, \tag{60}$$

where λ is the diffusion number (dimensionless) and γ_m^n is the advection number or Courant number (dimensionless).

4.2.3 Saulyev method

Since the Saulyev scheme is an unconditionally stable scheme [15], we can see that the non-strictly stability requirement of Saulyev scheme is the main of advantage and economical to use. Taking Saulyev technique [15] into Eq.(6), it can be obtained the following discretization,

$$C \cong C_m^n, \tag{61}$$

$$\frac{\partial C}{\partial t} \cong \frac{C_m^{n+1} - C_m^n}{\Delta t}, \tag{62}$$

$$\frac{\partial C}{\partial x} \cong \frac{C_{m+1}^{n} - C_{m-1}^{n+1}}{2\Delta x}, \tag{63}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{C_{m+1}^{n} - C_{m}^{n} - C_{m}^{n+1} + C_{m-1}^{n+1}}{(\Delta x)^2}, \tag{64}$$

$$u \cong u_m^n. \tag{65}$$

Substituting Eqs.(61-65) into Eq.(6), we get

$$\frac{C_m^{n+1} - C_m^n}{\Delta t} + u_m^n\left(\frac{C_{m+1}^{n} - C_{m-1}^{n+1}}{2\Delta x}\right) = D\left(\frac{C_{m+1}^{n} - C_{m}^{n} - C_{m}^{n+1} + C_{m-1}^{n+1}}{(\Delta x)^2}\right) - KC_m^n, \tag{66}$$

for all $1 \le m < M$ and $0 \le n \le N$. Let $\alpha_m^n = u_m^n\left(\frac{\Delta t}{\Delta x}\right)$ and $\beta = D\frac{\Delta t}{(\Delta x)^2}$, Eq.(66) becomes,

$$C_m^{n+1} = \left(\frac{1}{1+\beta}\right)\left(\left(\beta + \frac{\alpha_m^n}{2}\right)C_{m-1}^n + (1-\beta-K\Delta t)C_m^n + \left(\beta - \frac{\alpha_m^n}{2}\right)C_{m+1}^n\right). \tag{67}$$

For upper boundary, where $m = 0$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{-1}^n = C_0$, and substituting them into the right-hand side of Eq.(67), we obtain

$$C_0^{n+1} = \left(\frac{1}{1+\beta}\right)\left(\left(\frac{\alpha_0^n}{2} + 1 - K\Delta t\right)C_0^n + \left(\beta - \frac{\alpha_0^n}{2}\right)C_1^n\right). \tag{68}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $C_{M+1}^n = C_M^n + C_M^{n+1} - C_{M-1}^{n+1}$, we obtain

$$C_M^{n+1} = \left(\frac{1}{1+\frac{\alpha_M^n}{2}}\right)\left(\alpha_M^n C_{M-1}^{n+1} + \left(1 - \frac{\alpha_M^n}{2} - K\Delta t\right)C_M^n\right). \tag{69}$$

where β is the diffusion number and α_m^n is the advection number or Courant number. Using Taylor series expansions on the approximation, [20] has shown the truncation error is $O\left\{(\Delta x)^2 + (\Delta t)^2 + \left(\frac{\Delta t}{\Delta x}\right)^2\right\}$ or $O\{2, 2, (1/1)^2\}$

5. Application to the stream water quality assessment after dike failure over flooding area

Suppose that the measurement of pollutant concentration C in a dam-break flow stream is considered. A stream is aligned with longitudinal distance, 2000 (m) total length. There is a dam-break which discharges waste water into the flooding area at middle point of the domain and the pollutant concentrations at the discharge point are assumed into 3 cases:

Case 1: $C(1000, t) = 1$ kg/m^3 for all $0 \le t \le T$, $c_x(2000, t) = 0$ and

$$C(x, 0) = \begin{cases} 1 & \text{kg/m}^3 \text{ if } \quad x = 1000, \\ 0.1 & \text{kg/m}^3 \text{ if } \quad 1000 < x \le 2000. \end{cases} \tag{70}$$

Case 2: $C(1000, t) = 1 - t/720$ kg/m^3 for all $0 \le t \le T$, $C_x(2000, t) = 0$ and

$$C(x, 0) = \begin{cases} 1 & \text{kg/m}^3 \text{ if } \quad x = 1000, \\ 0.1 & \text{kg/m}^3 \text{ if } \quad 1000 < x \le 2000. \end{cases} \tag{71}$$

Case 3: $C(1000, t) = 1 - 3.8580 \times 10^{-5}\, t^2$ kg/m^3 for all $0 \le t \le T$, $C_x(2000, t) = 0$ and

$$C(x, 0) = \begin{cases} 1 & \text{kg/m}^3 \text{ if } \quad x = 1000, \\ 0.1 & \text{kg/m}^3 \text{ if } \quad 1000 < x \le 2000. \end{cases} \tag{72}$$

The elevation and velocity of water are obtained by the dam-break (pollutant discharging) model that we assume the initial and boundary conditions in 2 types: the dam-break with wet bed (flooding area) and the dam-break with dry bed (non-flooding area). We will consider the dam-break in 3 cases as below,

Case A: Dam-break on wet bed (pollutant going to the high level flooding area)

$$h(x, 0) = \begin{cases} 10.00 & \text{m if } \quad x \le 1000, \\ 5.00 & \text{m if } \quad 1000 < x \le 2000. \end{cases} \tag{73}$$

where $u(x, 0) = 0$ for all $0 \le x \le 2000$ and $u_x(0, t) = u_x(2000, t) = 0$.

Case B: Dam-break on wet bed (pollutant going to the low level flooding area)

$$h(x, 0) = \begin{cases} 10.00 & \text{m if } \quad x \le 1000, \\ 0.05 & \text{m if } \quad 1000 < x \le 2000. \end{cases} \tag{74}$$

where $u(x, 0) = 0$ for all $0 \le x \le 2000$ and $u_x(0, t) = u_x(2000, t) = 0$.

Case C: Dam-break on dry bed (pollutant going to the non-flooding area)

$$h(x,0) = \begin{cases} 10.00 & \text{m if} \quad x \le 1000 \\ 0 & \text{m if} \quad 1000 < x \le 2000. \end{cases} \quad (75)$$

where $u(x, 0) = 0$ for all $0 \le x \le 2000$ and $u_x(0,t) = u_x(2000,t) = 0$. The physical parameters of the polluted system are diffusion coefficient D = 1.00 (m^2/s), and a first-order reaction rate $10^{-2}s^{-1}$. In the analysis conducted in this study, meshes the stream into 1000 elements with $\Delta x = 2$, and time increment is 0.1 (s) with $\Delta t = 0.1$, characterizing a one-dimensional flow. Using the modified Lax-diffusive method Eq.(13) to obtain the velocity and elevation of water when dam is collapsed. We can get the water velocity $u(x, t)$ on Tables 1-3 in 3 cases of the high level flooding area, the low level flooding area and the non-flooding area, respectively. We also get the water elevation $h(x, t)$ on Tables 4-6 and Figs 2-4 of above 3 cases, respectively. Next, it can be plug the approximate water velocity into their implicit and explicit methods: BTCS method, Eq.24, Crank-Nicolson method, Eq.(34), FTCS method, Eq.(41). The approximation of pollutant concentrations C of all schemes are shown in Tables 7, 9, 10 and 11. The comparison of approximated pollutant concentrations of cases 1-3 with dam-break cases A-C are shown in Fig. 8, 9, 10 and 11.

Table 1

The velocity of water flow $u(x, t)$ m/s where $h_l = 10$ m and $h_r = 5$ m (Case A)

t (sec.), x(m)	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	2.7676	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.5432	2.8906	1.0240	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0134	1.7220	2.9128	2.8879	0.0015	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0000	0.4606	2.3804	2.9171	2.9101	0.6703	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0421	1.2290	2.6967	2.9177	2.9122	2.8402	0.0023	0.0000	0.0000

Table 2

The velocity of water flow $u(x, t)$ m/s where $h_l = 10$ m and $h_r = 0.05$ m (Case B)

t (sec.), x(m)	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	5.3233	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.7082	5.7794	10.8515	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0187	2.1748	5.9784	9.8770	0.0056	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0001	0.5590	3.1218	6.0936	9.1105	11.7065	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0542	1.4388	3.7391	6.1699	8.6256	10.9812	9.3860	0.0000	0.0000

Table 3

The velocity of water flow $u(x, t)$ m/s where h_l = 10 m and h_r = 0 m (Case C)

t (sec.), x(m)	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	5.3387	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.7091	5.7917	11.4513	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0188	2.1767	5.9884	9.9854	11.0343	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0001	0.5595	3.1240	6.1021	9.1596	12.3166	8.6766	0.0000	0.0000
50	0.0000	0.0000	0.0543	1.4396	3.7415	6.1772	8.6552	11.1796	13.8577	0.0000	0.0000

Table 4

The elevation of water flow $h(x, t)$ m/s where h_l = 10 m and h_r = 5.00 m (Case A)

t (sec.), x(m)	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	10.0000	10.0000	10.0000	10.0000	10.0000	7.3648	5.0000	5.0000	5.0000	5.0000	5.0000
20	10.0000	10.0000	10.0000	10.0000	9.4569	7.2716	5.7426	5.0000	5.0000	5.0000	5.0000
30	10.0000	10.0000	10.0000	9.9864	8.3284	7.2576	7.2357	5.0011	5.0000	5.0000	5.0000
40	10.0000	10.0000	10.0000	9.5393	7.7305	7.2569	7.2561	5.4835	5.0000	5.0000	5.0000
50	10.0000	10.0000	9.9575	8.7937	7.4523	7.2583	7.2589	7.1994	5.0017	5.0000	5.0000

Table 5

The elevation of water flow $h(x, t)$ m/s where h_l = 10 m and h_r = 0.05 m (Case B)

t(sec.), x(m)	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	10.0000	10.0000	10.0000	10.0000	10.0000	5.1902	0.0500	0.0500	0.0500	0.0500	0.0500
20	10.0000	10.0000	10.0000	10.0000	9.2941	4.9232	1.4900	0.0500	0.0500	0.0500	0.0500
30	10.0000	10.0000	10.0000	9.9811	7.9105	4.8073	2.3477	0.0502	0.0500	0.0500	0.0500
40	10.0000	10.0000	9.9999	9.4421	7.0772	4.7404	2.8107	1.4811	0.0500	0.0500	0.0500
50	10.0000	10.0000	9.9453	8.5949	6.5603	4.6961	3.1096	1.8630	0.5364	0.0500	0.0500

Table 6

The elevation of water flow $h(x, t)$ m/s where h_l = 10 m and h_r = 0 m (Case C)

t (sec.), x(m)	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	10.0000	10.0000	10.0000	10.0000	10.0000	5.1775	0.0000	0.0000	0.0000	0.0000	0.0000
20	10.0000	10.0000	10.0000	10.0000	9.2932	4.9139	1.3036	0.0000	0.0000	0.0000	0.0000
30	10.0000	10.0000	10.0000	9.9811	7.9088	4.7999	2.2892	0.0393	0.0000	0.0000	0.0000
40	10.0000	10.0000	9.9999	9.4416	7.0753	4.7343	2.7827	1.2304	0.0000	0.0000	0.0000
50	10.0000	10.0000	9.9452	8.5941	6.5583	4.6909	3.0922	1.7712	0.6834	0.0000	0.0000

Table 7

The pollutant concentration $C(x, t)$ (Kg/m^3) by BTCS scheme (Case A1)

t (sec.), x(m)	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.00000	0.71004	0.50409	0.04545	0.04072	0.04072	0.04072	0.04072	0.04072	0.04072	0.04072
180	1.00000	0.71010	0.50423	0.35803	0.25420	0.15896	0.01671	0.01656	0.01656	0.01656	0.01656
270	1.00000	0.71012	0.50425	0.35805	0.25424	0.18052	0.12817	0.09097	0.02611	0.00674	0.00674
360	1.00000	0.71003	0.50415	0.35799	0.25420	0.18050	0.12817	0.09101	0.06462	0.04588	0.03108

Table 8

The pollutant concentration $C(x, t)$ (Kg/m^3) by Crank-Nicolson scheme (Case A1)

t (sec.), x(m)	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.00000	0.51152	0.25453	0.01743	0.01656	0.01656	0.01656	0.01656	0.01656	0.01656	0.01656
180	1.00000	0.51161	0.25467	0.12850	0.06483	0.03039	0.00274	0.00274	0.00274	0.00274	0.00274
270	1.00000	0.51163	0.25468	0.12852	0.06485	0.03272	0.01651	0.00833	0.00172	0.00045	0.00045
360	1.00000	0.51151	0.25458	0.12847	0.06483	0.03272	0.01651	0.00833	0.00420	0.00212	0.00105

Table 9

The pollutant concentration $C(x, t)$ (Kg/m^3) by FTCS scheme (Case A1)

t (sec.), x(m)	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.00000	0.71004	0.50406	0.04125	0.04068	0.04068	0.04068	0.04068	0.04068	0.04068	0.04068
180	1.00000	0.71010	0.50423	0.35803	0.25420	0.17707	0.01653	0.01653	0.01653	0.01653	0.01653
270	1.00000	0.71012	0.50425	0.35805	0.25424	0.18052	0.12817	0.09099	0.02156	0.00672	0.00672
360	1.00000	0.71003	0.50415	0.35799	0.25420	0.18050	0.12817	0.09101	0.06462	0.04588	0.03262

Table 10

The pollutant concentration $C(x, t)$ (Kg/m^3) by MaCormack scheme (Case A1)

t(sec.), x(m)	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.00000	0.71004	0.50410	0.04278	0.04070	0.04070	0.04070	0.04070	0.04070	0.04070	0.04070
180	1.00000	0.71011	0.50424	0.35803	0.25420	0.16630	0.01658	0.01655	0.01655	0.01655	0.01655
270	1.00000	0.71012	0.50425	0.35806	0.25424	0.18052	0.12817	0.09099	0.02452	0.00673	0.00673
360	1.00000	0.71003	0.50415	0.35799	0.25420	0.18051	0.12817	0.09101	0.06462	0.04588	0.03192

Table 11

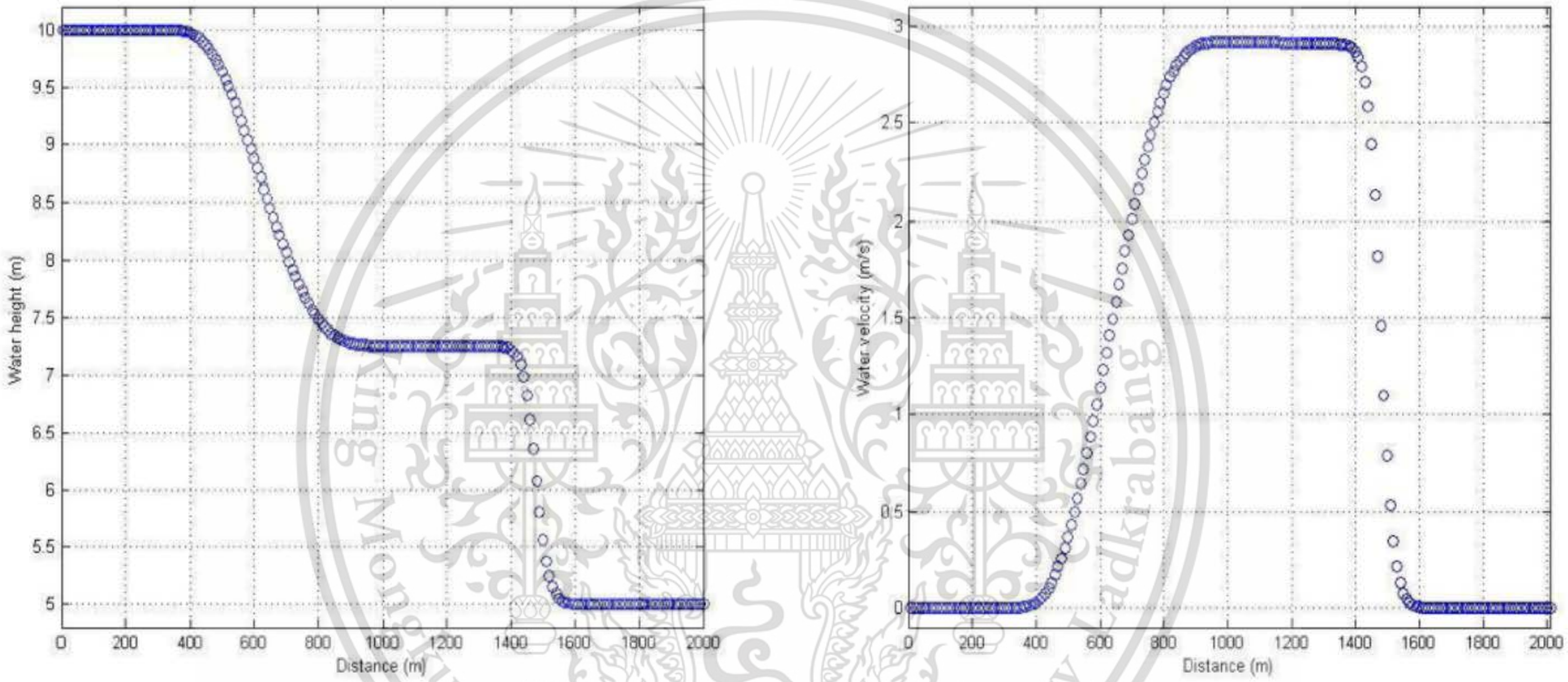
The pollutant concentration $C(x, t)$ (Kg/m^3) by Saulyev scheme (Case A1)

t (sec.), x(m)	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
90	1.00000	0.71004	0.50410	0.08191	0.03847	0.03896	0.03946	0.03996	0.04047	0.04068	0.04068
180	1.00000	0.71010	0.50424	0.35804	0.25421	0.18049	0.02249	0.01459	0.01478	0.01497	0.01516
270	1.00000	0.71012	0.50425	0.35805	0.25424	0.18052	0.12817	0.09100	0.06354	0.00702	0.00554
360	1.00000	0.71003	0.50415	0.35798	0.25420	0.18050	0.12817	0.09101	0.06462	0.04588	0.03257

Table 12

The stability of implicit and explicit schemes (S=Stable, U=Unstable)

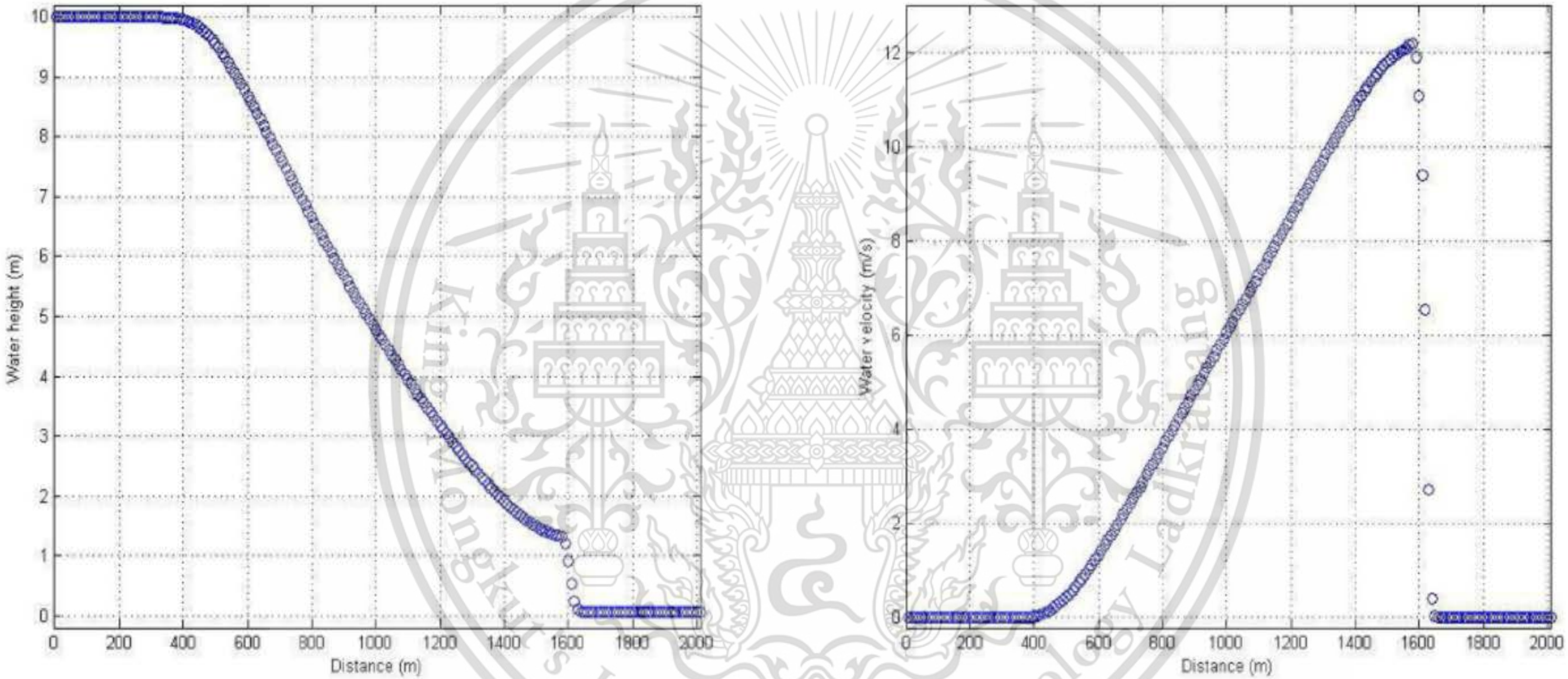
Depth ratio	(Δx, Δt)	CFL no.	Lax Diff	α	BTCS	Crank-Nicolson	FTCS	MacCormack	Sauyev
	(2, 0.1000)	0.14500	S	0.14500	S	S	S	S	S
0.500	(2, 0.0500)	0.07250	S	0.07250	S	S	S	S	S
	(2, 0.0250)	0.03625	S	0.03625	S	S	S	S	S
	(2, 0.1000)	0.62850	S	0.62850	S	S	U	S	S
0.005	(2, 0.0500)	0.31425	S	0.31425	S	S	U	S	S
	(2, 0.0250)	0.15713	S	0.15713	S	S	S	S	S
	(2, 0.1000)	0.85000	U	0.85000	-	-	-	-	-
∞	(2, 0.0500)	0.42500	S	0.42500	S	S	U	S	S
	(2, 0.0250)	0.21250	S	0.21250	S	S	S	S	S



(a) (b)

Figure 2

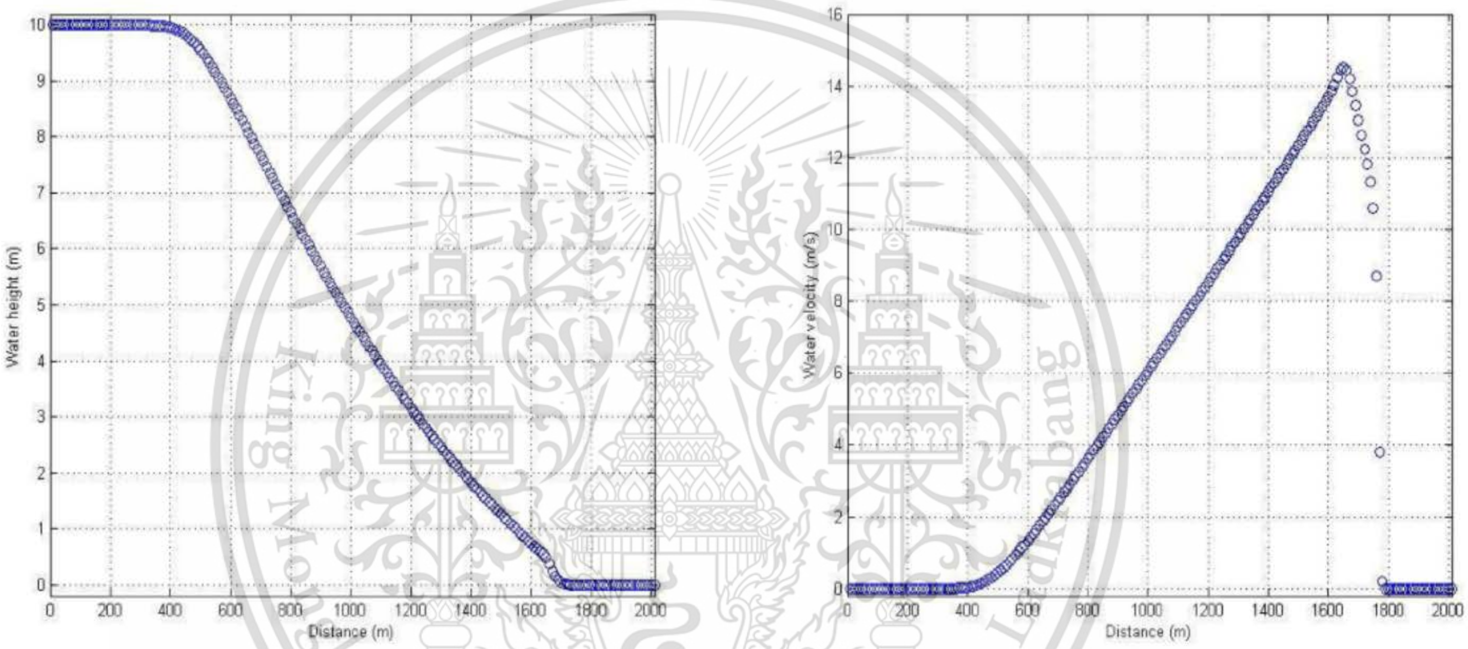
(a) The water elevation *h*(*x*, *t*) (m) and (b) the water velocity *u*(*x*, *t*) (m/sec) of case 1 (wet bed with depth ratio 0.5) at *t* = 50 sec



(a) (b)

Figure 3

(a) The water elevation $h(x, t)$ (m) and (b) the water velocity $u(x, t)$ (m/sec) of case 2 (wet bed with depth ratio 0.005) at t = 50 sec



(a) (b)

Figure 4

(a) The water elevation $h(x, t)$ (m) and (b) the water velocity $u(x, t)$ (m/sec) of case 3 (dry bed with depth ratio ∞) at t = 50 sec

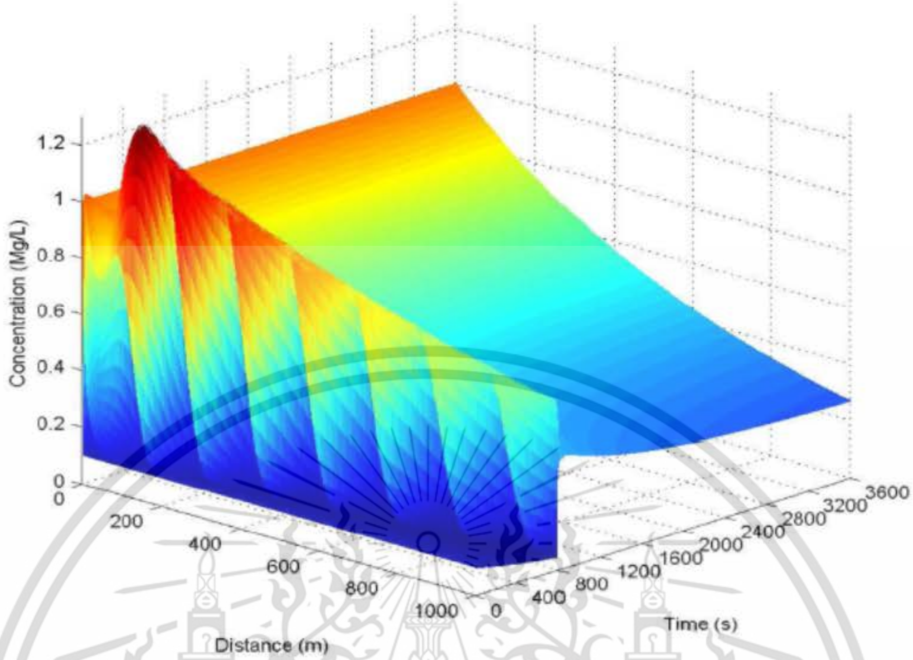


Figure 5

The pollutant concentration on wet bed (depth ratio 0.5), $\Delta x = 2$, $\Delta t = 0.1$ at $T = 360$ sec by Saulyev schemes of Case 1

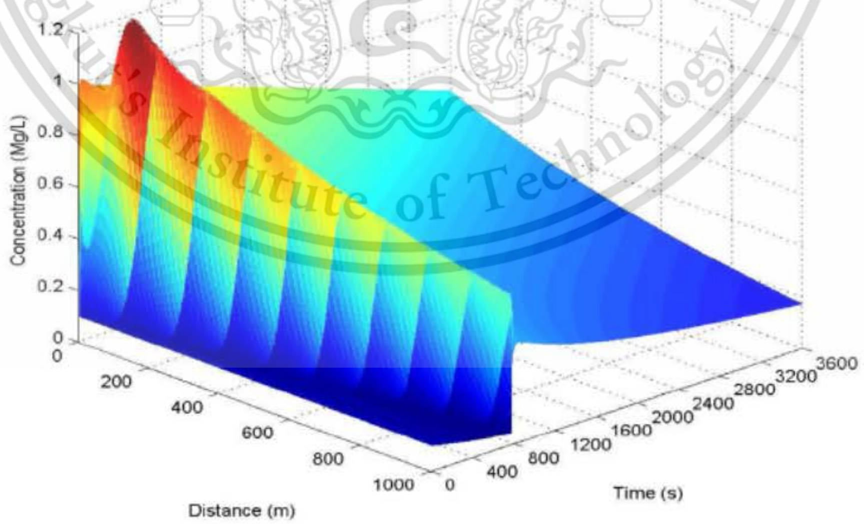


Figure 6

The pollutant concentration on wet bed (depth ratio 0.005), $\Delta x = 2$, $\Delta t = 0.1$ at $T = 360$ sec by Saulyev schemes of Case 2

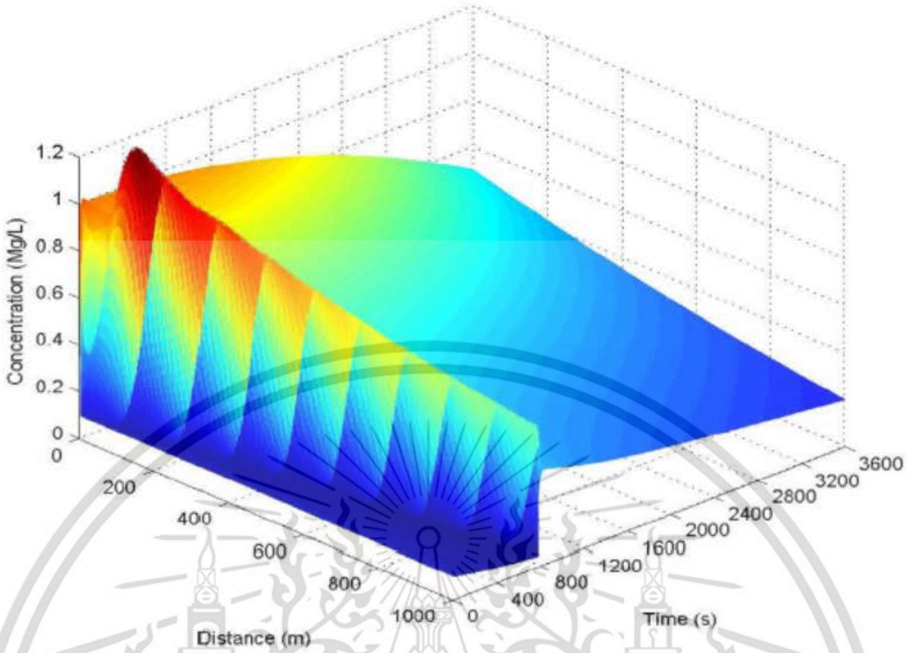


Figure 7

The pollutant concentration on dry bed (depth ratio ∞), $\Delta x = 2$, $\Delta t = 0.1$ at $T = 360$ sec by Saulyev schemes of Case 3

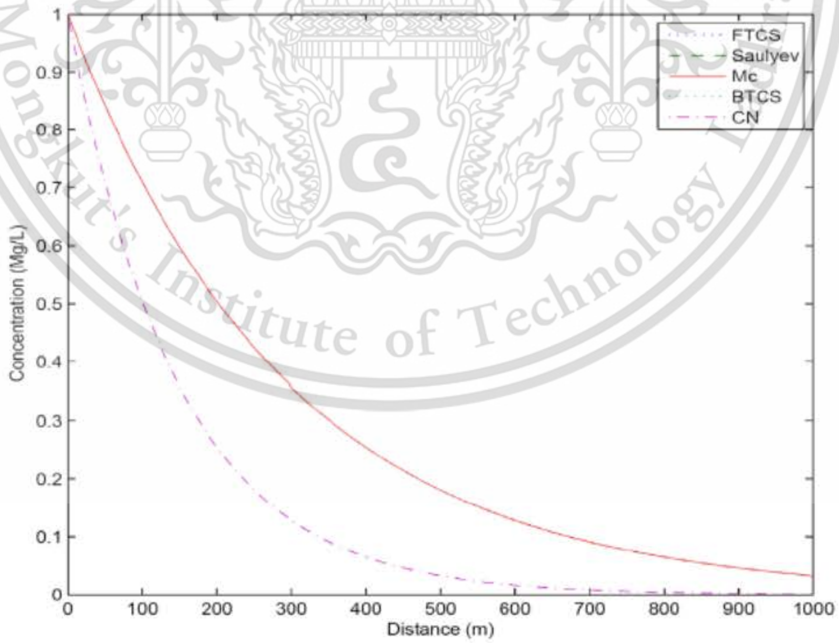


Figure 8

The comparison of pollutant concentration on wet bed (depth ratio 0.5), $\Delta x = 2$, $\Delta t = 0.1$ at $T = 360$ sec by implicit and explicit schemes

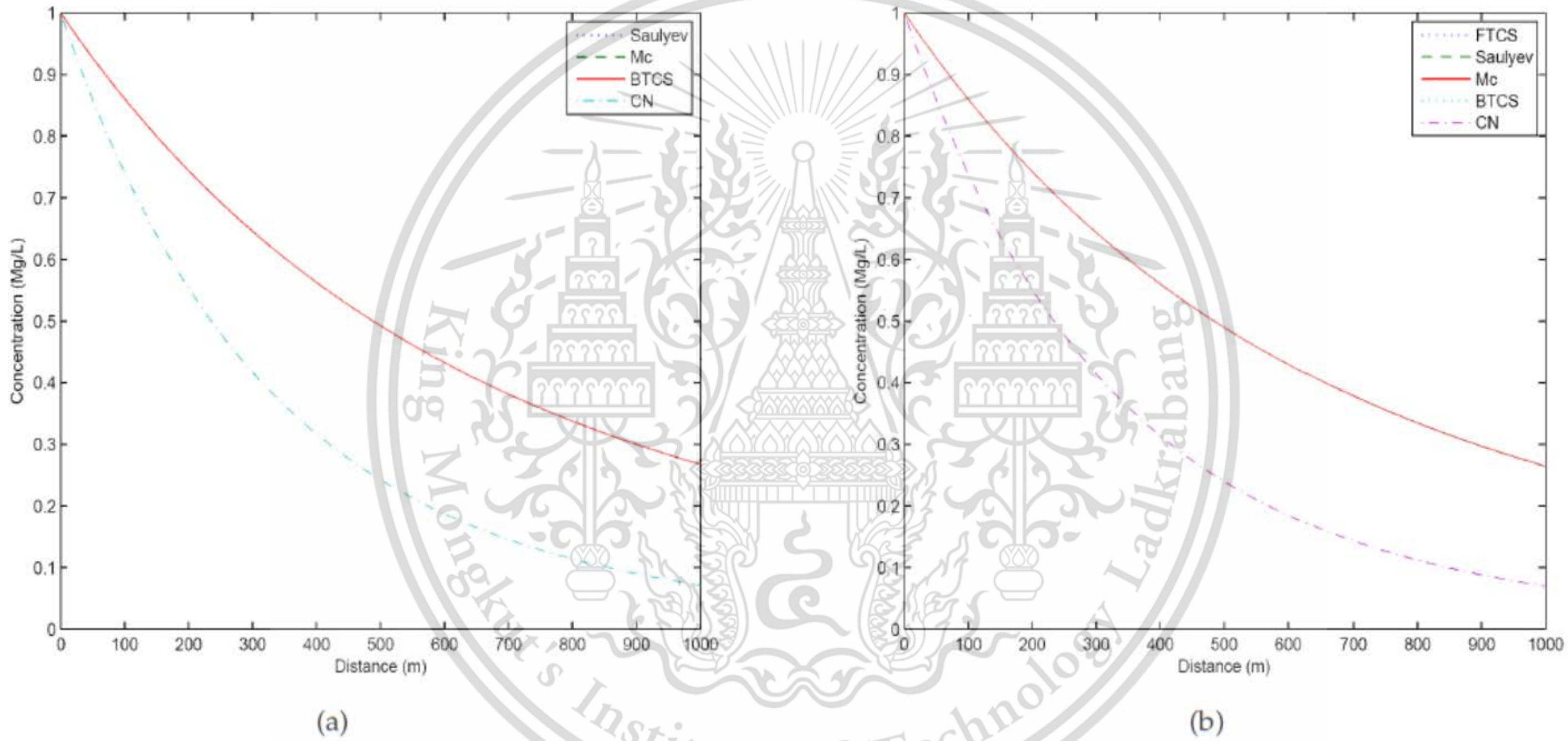
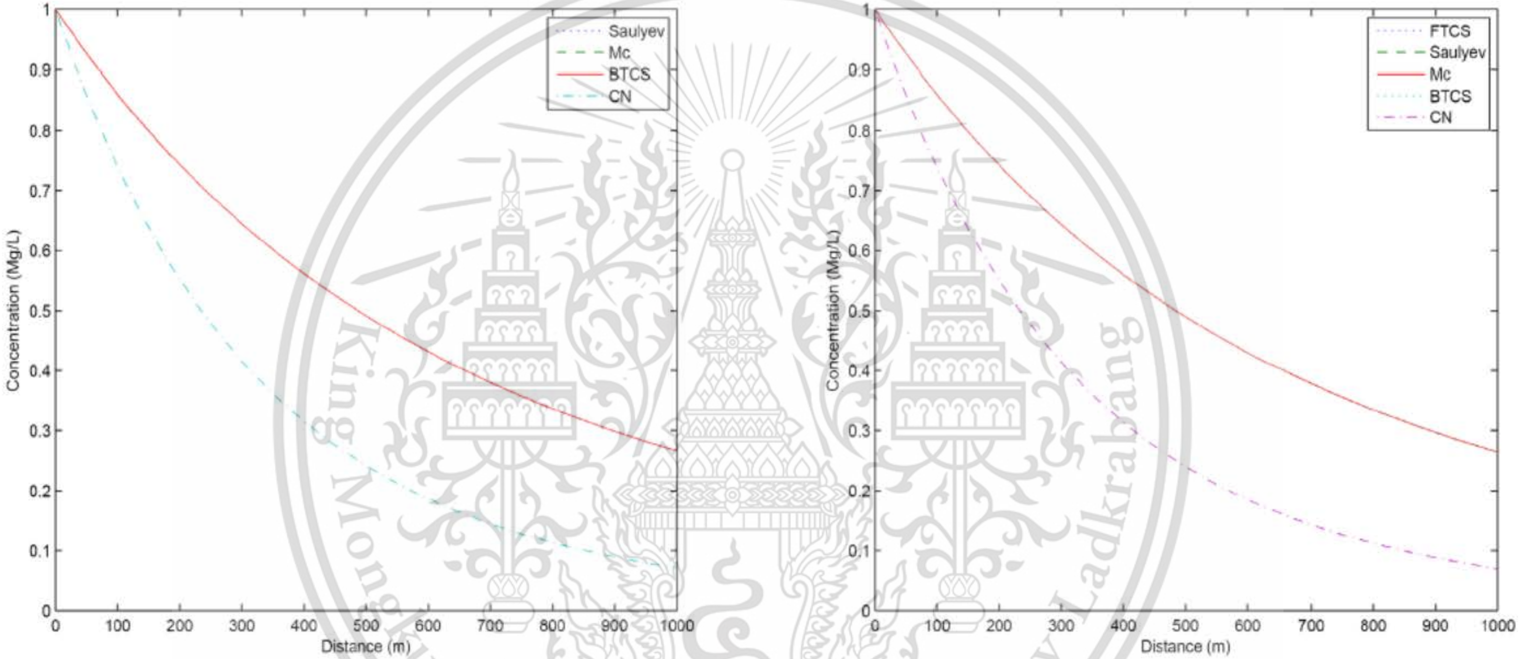


Figure 9

The comparison of pollutant concentration on wet bed (depth ratio 0.005): (a) Δx = 2, Δt = 0.1 (FTCS unstable) (b) Δx = 2, Δt = 0.025 at T = 360 sec



(a) (b)

Figure 10

The comparison of pollutant concentration on dry bed (depth ratio ∞): (a) $\Delta x = 2$, $\Delta t = 0.1$ (FTCS unstable) (b) $\Delta x = 2$, $\Delta t = 0.025$ at $T = 360$ sec

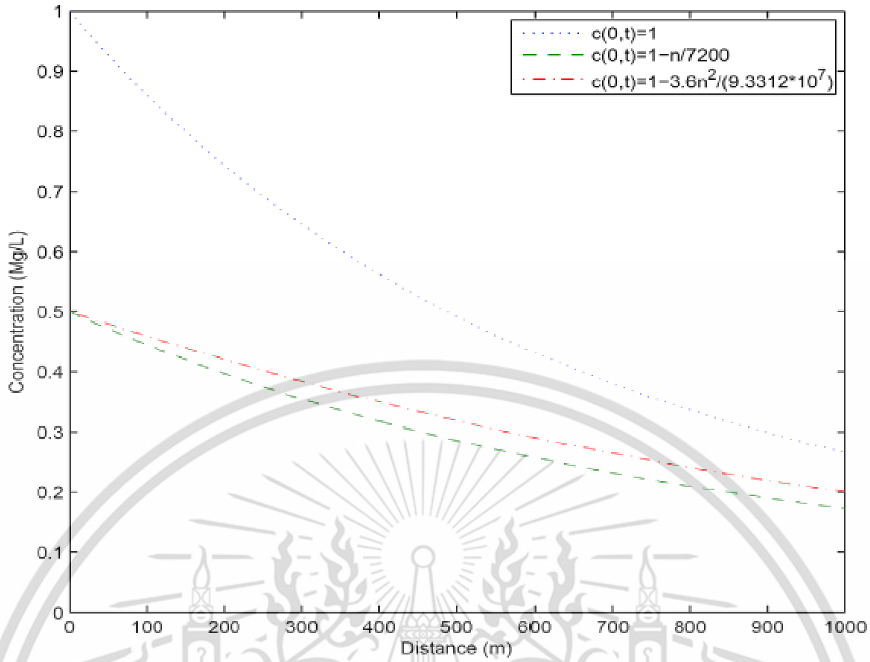


Figure 11

The comparison among difference discharge pollutant levels at the dam-break point on wet bed (depth ratio 0.05), $\Delta x = 2$, $\Delta t = 0.1$ at $T = 360$ sec by a Saulyev scheme

6. Discussion and Conclusions

The velocity and elevation of water current are obtained by a modified Lax-diffusive method. The case C of dam failure on dry bed area gives the highest flow velocity. The cases A-B of dam failure on wet bed area give low velocity level. If the residents have encountered flood for longtime, the water pollutant must be increase. The villagers want to drain the water to the other areas by destroying the dike. The other villages that never encounter flood going to get a strong water current while they also receiving the drained-polluted water. The approximation of the pollutant concentrations of the implicit and explicit methods are shown in Tables 7, 8, 9, 10 and 11. We can see that the pollutant concentration level on the flooding area is not to high. The real-world problems require a small amount of time interval in obtaining accurate solutions. Unfortunately, the analytical solutions of the dam-break model could not be found over the entire domain. This also implies that the analytical solutions of dispersion model could not work out at any point on the entire domain as well.

We propose a modified Lax-diffusive scheme by editing a simple revision to the traditional Lax-diffusive scheme. The FTCS method is limited by restriction of the stability condition. Then FTCS is not flexible in the real-world situation. The Crank-Nicolson scheme shows excessive dispersion effects for large time and space step lengths, significantly decreasing the efficiency of the Crank-Nicolson scheme. The BTCS still generate a lot of large systems of linear equations. The MacCormack and Saulyev schemes are economical to use. The proposed method show a good agreement in accuracy, the explicit schemes becomes less efficient than the implicit schemes.

In this paper, the dam-break model and the dispersion model can be combined to approximate the pollutant concentration in a stream when the current reflecting water in the stream is not uniform since the dam becomes failure. The technique constructed in this study is to respond the aspects of the stream in two varied external inputs, which are the level of water and the pollutant concentration at the discharge point. The both of the implicit methods and the explicit methods are utilized by means of the dispersion model because the scheme seems not to be too complicated to implement. In terms of the explicit finite difference formulations, it is believed that the implemented technique is practical and applicable. What's more, it seems economical to be employed and used in the real-world tasks and problems. It can also be easily used owning to the program simplicity, the straight forward implementation and less time consumption.

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Numerical Simulation of Water-Quality Model on Flooding using Revised Lax-Diffusive and Modified Siemieniuch-Gladwell Methods

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Abstract : In 2011, Thailand has been confronted a largest flooding. The mass of water has been drenched from many main and branch rivers to cover wide areas. The residents who lived in the flooding area have to build a manmade sandbag dike to protect their village. The flooding has been taken for a long time meanwhile the flooding water becomes contaminated. There are some residents in their flooding area want to drain their contaminated water to a nearest area. They have been destroyed their sandbag dike. Consequently, the dispute among residents is occurred. In this research, a mathematical simulation of a water-quality on a long period flooding using a couple of two models is proposed. The first model is the one-dimensional shallow water equations that provide the water elevation and velocity. The second model is a one-dimensional advection-dispersion equation that provides the water pollutant concentrations after the sandbag dike has been destroyed. A revised Lax-diffusive is used to approximate the solution of the first model. Consequently, the numerical solutions of the second model are obtained by using the traditional and modified Siemieniuch-Gladwell schemes.

Keywords : Finite differences; Lax-diffusive scheme; Revised Lax-diffusive scheme; One-dimensional; Dam-break model; Shallow water equations; Dispersion model; Advection-dispersion equation.

2010 Mathematics Subject Classification : 76R50; 39A14; 35Q30; 35L40.

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1 Introduction

Two mathematical models are used to explain the situation. The first model is one-dimensional hydrodynamic model that gives the velocity and elevation of water. The second model is dispersion model that gives the pollutant concentration of water after the sandbag-dike has been destroyed. In the recent years, there has been many research magnitude on the evolution of numerical models to simulate dam-break flows. For example, In [3] used Lax-Friendrichs to compare MacCormack and MacCormack TVD with a one dimension (1D) dam-break flow simulation. In [4] used central scheme for 1D and two dimension (2D) dam-break simulation. In [1] used finite volume method for numerical solution of shallow water equations in dam-break with flat topography. In [5] used 2D finite volume multiblock flow solver. The model is based on Flux Vector Splitting method. In [6] used a robust and effective flux-vector splitting method to simulate dam-break problem base on finite volume method on a cartesian grid. In [7] used smoothed particle hydrodynamics (SPH) to solve shallow-water dam break flow in open channels. In [8] used a new well-balanced unstraggered central finite volume scheme for 1D and 2D dam break over a rectangular bump. In [9] used well-balanced hydrostatic upwind schemes for dam-break approximations. The dam-break model is used to Explain unsteady dike failure flow. In [10] used Implicit (PriessMan) and Explicit (Lax Diffusive) methods for Saint-Venent Equations to Simulate Flood wave in Natural Rivers. In this work, the revised Lax diffusive technique is used to solve dike failure problem.

2 Model Formulation

In this section, the couple of mathematical models are used to describe. The first model is one-dimensional hydrodynamic model that provide water elevation and water velocity. The second model is advection-diffusion-equation that give water pollutant concentration. The second model need to input water velocity from the first model.

2.1 A dam-break Model

The Navier-Stokes equations over the flow depth with two assumptions, the hydrostatic pressure distributions and a small bottom slope, are govern the one-dimensional hydrodynamic equations. Since the dam-break flow the model has the high level velocity can be considered as the advection-dominated shallow water flows. These are the eddy viscosity terms can be omitted. The model equations can be described in the system of partial differential equations:

$$\partial_x \begin{pmatrix} h \\ hu \end{pmatrix} + \partial_t \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix} = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix} \tag{2.1}$$

where x is the longitudinal length along a stream (m), t is time (s) and $h(x,t)$ is the depth of water (m). The velocity of water is defined by $u(x,t)$ (m/s) and $z(x)$

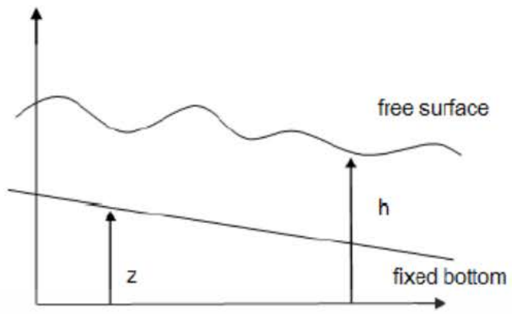


Figure 1: The shallow water system.

is the function of bottom topography (m). g is gravitational constant $(9.8\ m/s^2)$ for all $x \in [0, L]$. The initial conditions are given by

$$u(x,0) = 0, \text{ for all } 0 \le x \le L, \tag{2.2}$$

and

$$h(x,0) = \begin{cases} h_l & \text{if } 0 \le x \le \frac{L}{2}, \\ h_r & \text{if } \frac{L}{2} < x \le L, \end{cases} \tag{2.3}$$

where $h_l > h_r$, the water flow from the upstream to downstream at $t = 0$. The Neumann boundary conditions are also given by

$$u_x(0,t) = u_x(L,t) = 0, \tag{2.4}$$

$$h_x(0,t) = h_x(L,t) = 0. \tag{2.5}$$

2.2 Dispersion model

The governing equations are the one-dimensional advection-diffusion equations (ADE). This equations give a water pollutant concentration and a simplified form is shown in [2] as,

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2}, \tag{2.6}$$

where $C(x,t)$ is the pollutant concentration of water at the point $x(m)$ and time $t(s)$. D is the diffusion coefficient and $u(x,t)$ is the velocity component (m/s) for all $x \in [0, L]$. The initial conditions are given by

$$C(x,0) = C_0 \text{ for all } 0 \le x \le L. \tag{2.7}$$

The boundary conditions are provided by

$$C(0,t) = C_t \text{ for all } t > 0, \tag{2.8}$$

$$C_x(L,t) = C_R \text{ for all } t > 0, \tag{2.9}$$

where $C(t)$ is the function depends on t. C_0 and C_R are constants.

3 Numerical Techniques

The water height and water velocity are obtained by the dam-break model. The dispersion model have to input velocity field from the first model. The second model which provides the pollutant concentration field.

3.1 A revised Lax-diffusive method for a dam-break model

In this section, the revised method of a traditional Lax-diffusive method for the dam-break model of [10] is proposed. We now discretize Eq.(2.1) by dividing the interval $[0, L]$ into M subintervals such that $M\Delta x = L$ and the interval $[0, T]$ into N subintervals such that $N\Delta t = T$. We can then approximate $h(x_m, t_n)$ by h_m^n, value of the difference approximation of $h(x, t)$ at point $x = m\Delta x$ and $t = n\Delta t$, where $0 \leq m \leq M$ and $0 \leq n \leq N$, and similarly defined for u_m^n. The grid point (x_m, t_n) is defined by $x_m = m\Delta x$ for all $m = 0, 1, 2, \ldots, M$ and $t_n = n\Delta t$ for all $n = 0, 1, 2, \ldots, N$ in which M and N are positive integers.

We will modify f^* from two points averaged [10] to be the three points averaged. The discretization of Eq.(2.1) is base on a Lax-diffusive scheme. The semi-discrete scheme is applied to Eq.(2.1) and using a uniform spatial grid $(x_m, t_n) = (m\Delta x, n\Delta t)$, we can define

$$f_x \approx \frac{f_{m+1}^n - f_{m-1}^n}{2\Delta x}, \tag{3.1}$$

$$f_t \approx \frac{f_m^{n+1} - f_m^*}{\Delta t}, \tag{3.2}$$

where

$$f^* = \frac{f_{m+1}^n + f_m^n + f_{m-1}^n}{3}. \tag{3.3}$$

The partial derivative of h and u with respect to x and t are approximated by using Eqs.(3.1-3.3), respectively. We can see that Eq.(2.1) is written in a matrix form as

$$A_t + B_x + C = 0, \tag{3.4}$$

where

$$A = \begin{pmatrix} h \\ hu \end{pmatrix}, B = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix}. \tag{3.5}$$

It follows that Eq.(3.5) can be written by the uniform spatial grids as

$$A_m^n = \begin{pmatrix} h_m^n \\ h_m^n u_m^n \end{pmatrix}, B_m^n = \begin{pmatrix} h_m^n u_m^n \\ h_m^n (u_m^n)^2 + \frac{1}{2} g (h_m^n)^2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ -gh_m^n \partial_x z \end{pmatrix}. \tag{3.6}$$

Substituting the finite difference approximations of Eqs.(3.1-3.2) and Eq.(3.3) into Eq.(3.4), we obtain that

$$A_m^{n+1} = \frac{\Delta t}{2\Delta x}(B_{m-1}^n - B_{m+1}^n) + A^* \tag{3.7}$$

where $A^* = \begin{pmatrix} h^* \\ (hu)^* \end{pmatrix}$. Substituting Eq.(3.6) into Eq.(3.7), we can see that

$$\begin{pmatrix} h_m^{n+1} \\ h_m^{n+1}u_m^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{m-1}^n u_{m-1}^n - h_{m+1}^n u_{m+1}^n \\ h_{m-1}^n (u_{m-1}^n)^2 - h_{m+1}^n (u_{m+1}^n)^2 + \frac{1}{2}g((h_{m-1}^n)^2 - (h_{m+1}^n)^2) \end{pmatrix} + \frac{1}{3}\begin{pmatrix} h_{m-1}^n + h_m^n + h_{m+1}^n \\ h_{m-1}^n u_{m-1}^n + h_m^n u_m^n + h_{m+1}^n u_{m+1}^n \end{pmatrix}. \quad (3.8)$$

for all $1 \le m < M$ and $0 \le n \le N-1$. For upper boundary, where $m = 0$, plug the known value of the left boundary by $u_{-1}^n = u_0^n$ and $h_{-1}^n = h_0^n$ into Eq.(3.8) in the right-hand side, we obtain

$$\begin{pmatrix} h_1^{n+1} \\ h_1^{n+1}u_1^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_0^n u_0^n - h_1^n u_1^n \\ h_0^n (u_0^n)^2 - h_1^n (u_1^n)^2 + \frac{1}{2}g((h_0^n)^2 - (h_1^n)^2) \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 2h_0^n + h_1^n \\ 2h_0^n u_0^n + h_1^n u_1^n \end{pmatrix}. \quad (3.9)$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $u_{M+1}^n = u_M^n$ and $h_{M+1}^n = h_M^n$ by rearranging, we obtain

$$\begin{pmatrix} h_M^{n+1} \\ h_M^{n+1}u_M^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{M-1}^n u_{M-1}^n - h_M^n u_M^n \\ h_{M-1}^n (u_{M-1}^n)^2 - h_M^n (u_M^n)^2 + \frac{1}{2}g((h_{M-1}^n)^2 - (h_M^n)^2) \end{pmatrix} + \frac{1}{3}\begin{pmatrix} h_{M-1}^n + 2h_M^n \\ h_{M-1}^n u_{M-1}^n + 2h_M^n u_M^n \end{pmatrix}. \quad (3.10)$$

The stability condition of the scheme needed CFL number as [10],

$$C_n = u_{max}(\frac{\Delta t}{\Delta x}) \le 1. \quad (3.11)$$

3.2 Numerical method for a dispersion model

We consider both implicit and explicit methods for solving advection-diffusion equations. The well-known traditional methods are also introduced [2].

3.2.1 The modified Siemieniuch-Gladwell implicit procedure

We can then approximate $C(x_i, t_n)$ by C_i^n, the value of the difference approximation of $C(x, t)$ at point $x = i\Delta x$ and $t = n\Delta t$, where $0 \le i < M$ and $0 \le n < N$. The grid point (x_n, t_n) is defined by $x_i = i\Delta x$ for all $i = 0, 1, 2, ..., M$ and $t_n = n\Delta t$ for all $n = 0, 1, 2, ..., N$ in which M and N are positive integers. Taking the modified Siemieniuch-Gladwell technique [2] into Eq.(2.6), by the following discretizations:

$$\frac{\partial C}{\partial t} \approx \frac{(2\beta - \alpha_m^n)}{4}\frac{(C_{m-1}^{n+1} - C_{m-1}^n)}{\Delta t} + \frac{(2 - 2\beta + \alpha_m^n)}{2}\frac{(C_m^{n+1} - C_m^n)}{\Delta t} + \frac{(2\beta - \alpha_m^n)}{4}\frac{(C_{m+1}^{n+1} - C_{m+1}^n)}{\Delta t}, \quad (3.12)$$

$$\frac{\partial C}{\partial x} \approx \frac{(C^n_{m+1} - C^n_{m-1})}{4\Delta x} + \frac{(C^{n+1}_{m+1} - C^{n+1}_{m-1})}{4\Delta x}, \tag{3.13}$$

$$\frac{\partial^2 C}{\partial x^2} \approx \frac{1}{2}\frac{(C^{n+1}_{m+1} - 2C^{n+1}_{m} + C^{n+1}_{m-1})}{(\Delta x)^2} + \frac{1}{2}\frac{(C^{n}_{m+1} - 2C^{n}_{m} + C^{n}_{m-1})}{(\Delta x)^2}, \tag{3.14}$$

$$u \approx \widetilde{u}^n_m, \tag{3.15}$$

where $u = \widetilde{u}^n_m$ are obtained by the revised Lax-diffusive method Eq.(3.8). Substituting Eqs.(3.12-3.15) into Eq.(2.6), we have

$$-\alpha^n_m C^{n+1}_{m-1} + (2+\alpha^n_m)C^{n+1}_m = 2\beta C^n_{m-1} + (2-4\beta+\alpha^n_m)C^n_m + (2\beta-\alpha^n_m)C^n_{m+1}, \tag{3.16}$$

where $\alpha^n_m = u = \widetilde{u}^n_m(\frac{\Delta t}{\Delta x})$ and $\beta = D\frac{\Delta t}{(\Delta x)^2}$ for all $1 \le m < M$ and $0 \le n < N$. For the left boundary condition, $m = 0$, the known value on the left boundary are approximated by $C^n_{-1} = C^n_0$ and $C^{n+1}_{-1} = C^{n+1}_0$, we can see that

$$2C^{n+1}_0 = (2 - 2\beta + \alpha^n_0)C^n_0 + (2\beta - \alpha^n_0)C^n_1. \tag{3.17}$$

Similarly, the right boundary condition, $m = M$, the known value on the left boundary are approximated by $C^n_{M+1} = C^n_M$ and $C^{n+1}_{M+1} = C^{n+1}_M$ into Eq.(3.16) in the right-hand side, we have

$$-\alpha^n_M C^{n+1}_{M-1} + (2 + \alpha^n_M)C^{n+1}_M = 2\beta C^n_{M-1} + (2 - 2\beta)C^n_M. \tag{3.18}$$

The stability of modified Siemieniuch-Gladwell procedure is [2],

$$0 < \beta \le \frac{1 + \alpha^n_m}{2}. \tag{3.19}$$

3.2.2 The BTCS-type implicit method

Consider the Backward in time cecter in space(BTCS) scheme for the advection-diffusion equation by the following discretizations:

$$\frac{\partial C}{\partial t} \approx \frac{(C^{n+1}_m - C^n_m)}{\Delta t}, \tag{3.20}$$

$$\frac{\partial C}{\partial x} \approx \frac{(C^{n+1}_{m+1} - C^{n+1}_{m-1})}{2\Delta x}, \tag{3.21}$$

$$\frac{\partial^2 C}{\partial x^2} \approx \frac{(C^{n+1}_{m+1} - 2C^{n+1}_m + C^{n+1}_{m-1})}{(\Delta x)^2}, \tag{3.22}$$

$$u \approx \widetilde{u}^n_m. \tag{3.23}$$

Substituting Eqs.(3.20-3.23) into Eq.(2.6), we have

$$-(\alpha^n_m + 2\beta)C^{n+1}_{m-1} + 2(1 + 2\beta)C^{n+1}_m + (\alpha^n_m - 2\beta)C^{n+1}_{m+1} = 2C^n_m, \tag{3.24}$$

for all $1 \le m < M$ and $0 \le n < N$. For the left boundary condition, $m = 0$, the known value on the left boundary are approximated $C_{-1}^{n+1} = C_0^{n+1}$, we can see that

$$(2 - \alpha_0^n)C_0^{n+1} + (\alpha_0^n - 2\beta)C_1^{n+1} = 2C_0^n. \tag{3.25}$$

Similarly, the right boundary condition, $m = M$, the known value on the left boundary are approximated $C_{M+1}^{n+1} = C_M^{n+1}$, we have

$$-(\alpha_M^n + 2\beta)C_{M-1}^{n+1} + (2 + 2\beta + \alpha_M^n)C_M^{n+1} = 2C_M^n. \tag{3.26}$$

The stability of BTCS scheme is unconditionally stable[2].

3.2.3 The upwind implicit formula

Consider the upwind implicit scheme for the advection-diffusion equation by the following discretizations:

$$\frac{\partial C}{\partial t} \approx \frac{(C_m^{n+1} - C_m^n)}{\Delta t}, \tag{3.27}$$

$$\frac{\partial C}{\partial x} \approx \frac{(C_m^{n+1} - C_{m-1}^{n+1})}{\Delta x}, \tag{3.28}$$

$$\frac{\partial^2 C}{\partial x^2} \approx \frac{(C_{m+1}^{n+1} - 2C_m^{n+1} + C_{m-1}^{n+1})}{(\Delta x)^2}, \tag{3.29}$$

$$u \approx \tilde{u}_m^n. \tag{3.30}$$

Substituting Eqs.(3.27-3.30) into Eq.(2.6), we have

$$-(\alpha_m^n + \beta)C_{m-1}^{n+1} + (1 + 2\beta + \alpha_m^n)C_m^{n+1} - \beta C_{m+1}^{n+1} = C_m^n, \tag{3.31}$$

for all $1 \le m < M$ and $0 \le n < N$. For the left boundary condition, $m = 0$, the known value on the left boundary are approximated by $C_{-1}^{n+1} = C_0^{n+1}$, we can see that

$$(1 + \beta)C_0^{n+1} - \beta C_1^{n+1} = C_0^n. \tag{3.32}$$

Similarly, the right boundary condition, $m = M$, the known value on the left boundary are approximated $C_{M+1}^{n+1} = C_M^{n+1}$, we have

$$-(\alpha_M^n + \beta)C_{M-1}^{n+1} + (1 + \beta + \alpha_M^n)C_M^{n+1} = C_M^n. \tag{3.33}$$

The stability of upwind implicit scheme is unconditionally stable [2].

3.2.4 The Crank-Nicolson type technique

Consider the Crank-Nicolson scheme for the advection-diffusion equation by the following discretizations:

$$\frac{\partial C}{\partial t} \approx \frac{(C_m^{n+1} - C_m^n)}{\Delta t}, \tag{3.34}$$

$$\frac{\partial C}{\partial x} \approx \frac{(C_m^{n+1} - C_{m-1}^{n+1})}{\Delta x}, \tag{3.35}$$

$$\frac{\partial^2 C}{\partial x^2} \approx \frac{(C_{m+1}^{n+1} - 2C_m^{n+1} + C_{m-1}^{n+1})}{(\Delta x)^2}, \tag{3.36}$$

$$u \approx \tilde{u}_m^n. \tag{3.37}$$

Substituting Eqs.(3.34-3.37) into Eq.(2.6), we have

$$\begin{aligned} &-(\alpha_m^n + 2\beta)C_{m-1}^{n+1} + 4(1+\beta)C_m^{n+1} + (\alpha_m^n - 2\beta)C_{m+1}^{n+1} \\ &= (\alpha_m^n + 2\beta)C_{m-1}^n + 4(1-\beta)C_m^n + (2\beta - \alpha_m^n)C_{m+1}^n, \end{aligned} \tag{3.38}$$

for all $1 \leq m < M$ and $0 \leq n < N$. For the left boundary condition, $m = 0$, the known value on the left boundary are approximated $C_{-1}^{n+1} = C_0^{n+1}$ and $C_{-1}^n = C_0^n$, we can see that

$$(4 + 2\beta - \alpha_0^n)C_0^{n+1} + (\alpha_0^n - 2\beta)C_1^{n+1} = (4 - 2\beta + \alpha_0^n)C_0^n + (2\beta - \alpha_0^n)C_1^n. \tag{3.39}$$

Similarly, the right boundary condition, $m = M$, the known value on the left boundary are approximated $C_{M+1}^{n+1} = C_M^{n+1}$ and $C_{M+1}^n = C_M^n$, we have

$$-(\alpha_M^n + 2\beta)C_{M-1}^{n+1} + (4 + 2\beta + \alpha_M^n)C_M^{n+1} = (\alpha_M^n + 2\beta)C_{M-1}^n + (4 - 2\beta - \alpha_M^n)C_M^n. \tag{3.40}$$

The stability of Crank-Nicolson scheme is unconditionally stable[2].

3.2.5 The FTCS-type scheme

The traditional forward time central space scheme is considered the following discretizations:

$$\frac{\partial C}{\partial t} \approx \frac{(C_m^{n+1} - C_m^n)}{\Delta t}, \tag{3.41}$$

$$\frac{\partial C}{\partial x} \approx \frac{(C_{m+1}^n - C_{m-1}^n)}{2\Delta x}, \tag{3.42}$$

$$\frac{\partial^2 C}{\partial x^2} \approx \frac{(C_{m+1}^n - 2C_m^n + C_{m-1}^n)}{(\Delta x)^2}, \tag{3.43}$$

$$u \approx \tilde{u}_m^n. \tag{3.44}$$

Substituting Eqs.(3.41-3.44) into Eq.(2.6), we have

$$C_m^{n+1} = \frac{1}{2}(2\beta + \alpha_m^n)C_{m-1}^n + (1 - 2\beta)C_m^n + \frac{1}{2}(2\beta - \alpha_m^n)C_{m+1}^n, \tag{3.45}$$

for all $1 \leq m < M$ and $0 \leq n < N$. For the left boundary condition, $m = 0$, the known value on the left boundary are approximated $C_{-1}^n = C_0^n$, we can see that

$$C_0^{n+1} = (1 + \frac{1}{2}\alpha_0^n - \beta)C_0^n + \frac{1}{2}(2\beta - \alpha_0^n)C_1^n. \tag{3.46}$$

Similarly, the right boundary condition, $m = M$, the known value on the left boundary are approximated $C^n_{M+1} = C^n_M$, we have

$$C^{n+1}_M = \frac{1}{2}(2\beta + \alpha^n_M)C^n_{M-1} + (1 - \frac{1}{2}\alpha^n_M - \beta)C^n_M. \tag{3.47}$$

The stability of FTCS scheme is [2]

$$\frac{(\alpha)^2}{2} \leq \beta \leq \frac{1}{2}. \tag{3.48}$$

3.2.6 The upwind explicit formula

The upwind explicit scheme is considered by the following discretizations:

$$\frac{\partial C}{\partial t} \approx \frac{(C^{n+1}_m - C^n_m)}{\Delta t}, \tag{3.49}$$

$$\frac{\partial C}{\partial x} \approx \frac{(C^n_m - C^n_{m-1})}{\Delta x}, \tag{3.50}$$

$$\frac{\partial^2 C}{\partial x^2} \approx \frac{(C^n_{m+1} - 2C^n_m + C^n_{m-1})}{(\Delta x)^2}, \tag{3.51}$$

$$u \approx \tilde{u}^n_m. \tag{3.52}$$

Substituting Eqs.(3.49-3.52) into Eq.(2.6), we have

$$C^{n+1}_m = (\beta + \alpha^n_m)C^n_{m-1} + (1 - 2\beta - \alpha^n_m)C^n_m + \beta C^n_{m+1}, \tag{3.53}$$

for all $1 \leq m < M$ and $0 \leq n < N$. For the left boundary condition, $m = 0$, the known value on the left boundary are approximated $C^n_{-1} = C^n_0$, we can see that

$$C^{n+1}_0 = (1 - \beta)C^n_0 + \beta C^n_1. \tag{3.54}$$

Similarly, the right boundary condition, $m = M$, the known value on the left boundary condition are approximated $C^n_{M+1} = C^n_M$, we have

$$C^{n+1}_M = (\beta + \alpha^n_M)C^n_{M-1} + (1 - \beta - \alpha^n_M)C^n_M. \tag{3.55}$$

The stability of upwind explicit scheme is [2]

$$\frac{\alpha^2 - \alpha}{2} \leq \beta \leq \frac{1 - \alpha}{2}. \tag{3.56}$$

3.2.7 The Lax-Wendroff technique

The Lax-wendroff scheme is considered by the following discretizations:

$$\frac{\partial C}{\partial t} \approx \frac{(C_m^{n+1} - C_m^n)}{\Delta t}, \tag{3.57}$$

$$\frac{\partial C}{\partial x} \approx \alpha_m^n \frac{(C_m^n - C_{m-1}^n)}{\Delta x} + (1-\alpha_m^n)\frac{(C_{m+1}^n - C_{m-1}^n)}{2\Delta x}, \tag{3.58}$$

$$\frac{\partial^2 C}{\partial x^2} \approx \frac{(C_{m+1}^n - 2C_m^n + C_{m-1}^n)}{(\Delta x)^2}, \tag{3.59}$$

$$u \approx \tilde{u}_m^n. \tag{3.60}$$

Substituting Eqs.(3.57-3.60) into Eq.(2.6), we have

$$C_m^{n+1} = \frac{1}{2}(2\beta+\alpha_m^n+(\alpha_m^n)^2)C_{m-1}^n+(1-2\beta-(\alpha_m^n)^2)C_m^n+\frac{1}{2}(2\beta-\alpha_m^n+(\alpha_m^n)^2)C_{m+1}^n, \tag{3.61}$$

for all $1 \leq m < M$ and $0 \leq n < N$. For the left boundary condition, $m = 0$, the known value on the left boundary are approximated $C_{-1}^n = C_0^n$, we can see that

$$C_0^{n+1} = (1 - \beta + \frac{1}{2}\alpha_0^n - \frac{1}{2}(\alpha_0^n)^2)C_0^n + \frac{1}{2}(2\beta - \alpha_0^n + (\alpha_0^n)^2)C_1^n. \tag{3.62}$$

Similarly, the right boundary condition, $m = M$, the known value on the left boundary are approximated $C_{M+1}^n = C_M^n$, we have

$$C_M^{n+1} = \frac{1}{2}(2\beta - \alpha_M^n + (\alpha_M^n)^2)C_{M-1}^n + (1 - \beta - \frac{1}{2}\alpha_M^n - \frac{1}{2}(\alpha_M^n)^2)C_M^n. \tag{3.63}$$

The stability of upwind explicit scheme is [2]

$$0 < \beta \leq \frac{1-\alpha^2}{2}. \tag{3.64}$$

4 Numerical Experiments

Suppose that the measurement of pollutant concentration C in a dam-break flow stream is considered. A stream is aligned with longitudinal distance, 2000 (m) total length. There is a dam-break which discharges waste water into the flooding area at middle point of the domain and the pollutant concentrations at the discharge point are assumed:

$$C(1000, t) = (1 - |\sin t|)\frac{T-t}{T} \text{ for all } 0 \leq t < T, \tag{4.1}$$

$$C_x(2000, t) = 0, \tag{4.2}$$

$$C(x,0) = \begin{cases} 1\ kg/m^3 & \text{if } x = 1000, \\ 0.1\ kg/m^3 & \text{if } 1000 < x \leq 2000. \end{cases} \quad (4.3)$$

The elevation and velocity of water are obtained by the dam-break model that we assume the initial and boundary conditions by several cases as below,
Case A: Dam-break on wet bed (pollutant discharging into the high flooding area)

$$h(x,0) = \begin{cases} 1\ m & \text{if } x = 1000, \\ 0.75\ m & \text{if } 1000 < x \leq 2000, \end{cases} \quad (4.4)$$

where $u(x,0) = 0$ for all $1000 \leq x \leq 2000$ and $u_x(0,t) = u_x(2000,t) = 0$.
Case B: Dam-break on wet bed (pollutant discharging into the medium flooding area)

$$h(x,0) = \begin{cases} 1\ m & \text{if } x = 1000, \\ 0.50\ m & \text{if } 1000 < x \leq 2000, \end{cases} \quad (4.5)$$

where $u(x,0) = 0$ for all $1000 \leq x \leq 2000$ and $u_x(0,t) = u_x(2000,t) = 0$.
Case C: Dam-break on wet bed (pollutant discharging into the low flooding area)

$$h(x,0) = \begin{cases} 1\ m & \text{if } x = 1000, \\ 0.25\ m & \text{if } 1000 < x \leq 2000, \end{cases} \quad (4.6)$$

where $u(x,0) = 0$ for all $1000 \leq x \leq 2000$ and $u_x(0,t) = u_x(2000,t) = 0$. The physical parameters of the polluted system are diffusion coefficient $D = 1.00(m^2/s)$. In the analysis conducted in this study, meshes the stream into 1000 elements with $\Delta x = 2$, and time increment is $0.1(s)$ with $\Delta t = 0.1$, characterizing a one-dimensional flow. Using the modified Lax-diffusive method Eq.(3.7) to obtain the velocity and elevation of water when sandbag-dike is destroyed. We can get the water velocity $u(x,t)$ on Tables 2, 4 and 6 in 3 cases of the high level flooding area, the medium level flooding area and the low level flooding area, respectively. We also get the water elevation $h(x,t)$ on Tables 1, 3 and 5, respectively. Next, it can be plug the approximate water velocity into their implicit and explicit methods such as Modified Siemieniuch-Gladwell method, BTCS method, Upwind implicit method, Crank-Nicolson method, FTCS method, Upwind explicit method, Lax-wendroff method. The approximation of pollutant concentrations C of several schemes are shown in Tables 7-13. The comparison of approximated pollutant concentrations of FTCS, Upwind explicit and Lax-wendroff with several dam-break cases A, B and C are shown in Fig. 5, 6 and 7.

5 Discussion

The elevation and velocity of water current are obtained by a revised Lax-diffusive method. The case C of sandbag-dike failure gives the highest flow velocity. The

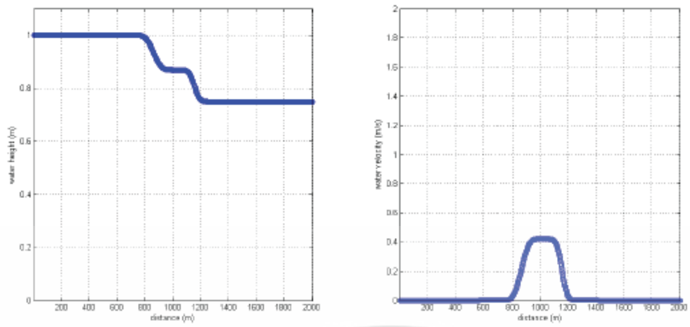


Figure 2: (a) The water elevation $h(x,t)(m)$ and (b) The water velocity $u(x,t)(m/sec)$ of case A (wet bed with depth ratio 0.75) at $t = 50$ sec

Table 1: The water elevation of water flow $h(x,t)m$ where $h_l = 1m$ and $h_l = 0.75m$ (Case A)

$t(sec), x(m)$	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.8746	0.7500	0.7500	0.7500	0.7500	0.7500
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.8710	0.7500	0.7500	0.7500	0.7500	0.7500
30	1.0000	1.0000	1.0000	1.0000	1.0000	0.8700	0.7500	0.7500	0.7500	0.7500	0.7500
40	1.0000	1.0000	1.0000	1.0000	0.9992	0.8698	0.7504	0.7500	0.7500	0.7500	0.7500
50	1.0000	1.0000	1.0000	1.0000	0.9904	0.8698	0.7571	0.7500	0.7500	0.7500	0.7500

Table 2: The water velocity of water flow $u(x,t)m/s$ where $h_l = 1m$ and $h_l = 0.75m$ (Case A)

$t(sec), x(m)$	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.3856	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.4134	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.4185	0.0000	0.0000	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0000	0.0000	0.0026	0.4196	0.0013	0.0000	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0000	0.0000	0.0301	0.4198	0.0257	0.0000	0.0000	0.0000	0.0000

Table 3: The water elevation of water flow $h(x,t)m$ where $h_l = 1m$ and $h_r = 0.5m$ (Case B)

$t(sec), x(m)$	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.7491	0.5000	0.5000	0.5000	0.5000	0.5000
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.7353	0.5000	0.5000	0.5000	0.5000	0.5000
30	1.0000	1.0000	1.0000	1.0000	0.9999	0.7294	0.5000	0.5000	0.5000	0.5000	0.5000
40	1.0000	1.0000	1.0000	1.0000	0.9986	0.7269	0.5001	0.5000	0.5000	0.5000	0.5000
50	1.0000	1.0000	1.0000	1.0000	0.9852	0.7259	0.5047	0.5000	0.5000	0.5000	0.5000

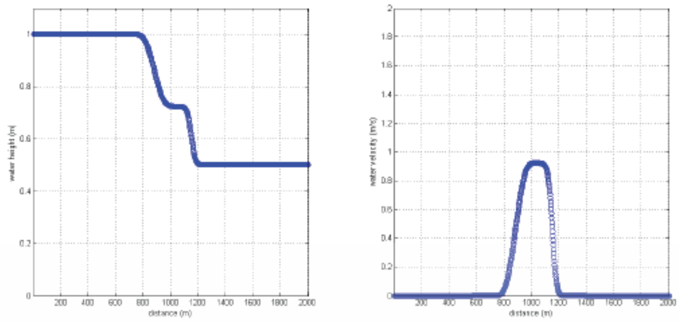


Figure 3: (a) The water elevation $h(x,t)(m)$ and (b) The water velocity $u(x,t)(m/sec)$ of case B (wet bed with depth ratio 0.5) at $t = 50$ sec

Table 4: The water velocity of water flow $u(x,t)m/s$ where $h_l = 1m$ and $h_r = 0.5m$ (Case B)

$t(sec), x(m)$	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.7945	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.8762	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.9027	0.0000	0.0000	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0000	0.0000	0.0044	0.9138	0.0006	0.0000	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0000	0.0000	0.0465	0.9189	0.0207	0.0000	0.0000	0.0000	0.0000

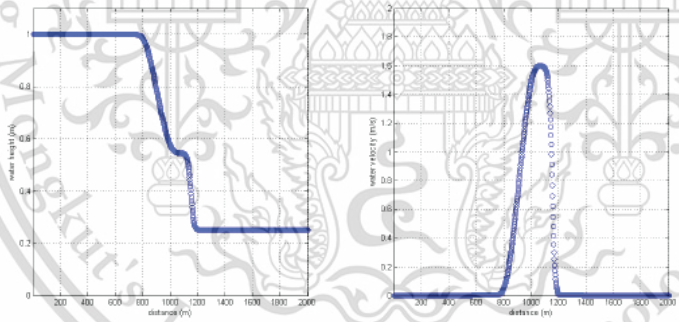


Figure 4: (a) The water elevation $h(x,t)(m)$ and (b) The water velocity $u(x,t)(m/sec)$ of case C (wet bed with depth ratio 0.25) at $t = 50$ sec

Table 5: The water elevation of water flow $h(x,t)m$ where $h_l = 1m$ and $h_r = 0.25m$ (Case C)

$t(sec), x(m)$	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.6307	0.2500	0.2500	0.2500	0.2500	0.2500
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.6049	0.2500	0.2500	0.2500	0.2500	0.2500
30	1.0000	1.0000	1.0000	1.0000	1.0000	0.5901	0.2500	0.2500	0.2500	0.2500	0.2500
40	1.0000	1.0000	1.0000	1.0000	0.9982	0.5808	0.2500	0.2500	0.2500	0.2500	0.2500
50	1.0000	1.0000	1.0000	1.0000	0.9821	0.5744	0.2512	0.2500	0.2500	0.2500	0.2500

Table 6: The water velocity of water flow $u(x,t)m/s$ where $h_l = 1m$ and $h_r = 0.25m$ (Case C)

$t(sec), x(m)$	0	200	400	600	800	1000	1200	1400	1600	1800	2000
10	0.0000	0.0000	0.0000	0.0000	0.0000	1.1995	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	1.3517	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0000	1.4222	0.0000	0.0000	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0000	0.0000	0.0055	1.4655	0.0001	0.0000	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0000	0.0000	0.0561	1.4952	0.0076	0.0000	0.0000	0.0000	0.0000

Table 7: The pollutant concentration $C(x,t)(Kg/m^3)$ by Modified Siemieniuch-Gladwell schemes (Case A)

$t(sec), x(m)$	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6411	0.6838	0.1012	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3663	0.4924	0.6715	0.1630	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1741	0.2734	0.4426	0.6334	0.3129	0.1017	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0565	0.1236	0.2382	0.3958	0.5863	0.4518	0.1188	0.1000	0.1000	0.1000	0.1000

Table 8: The pollutant concentration $C(x,t)(Kg/m^3)$ by BTCS schemes (Case A)

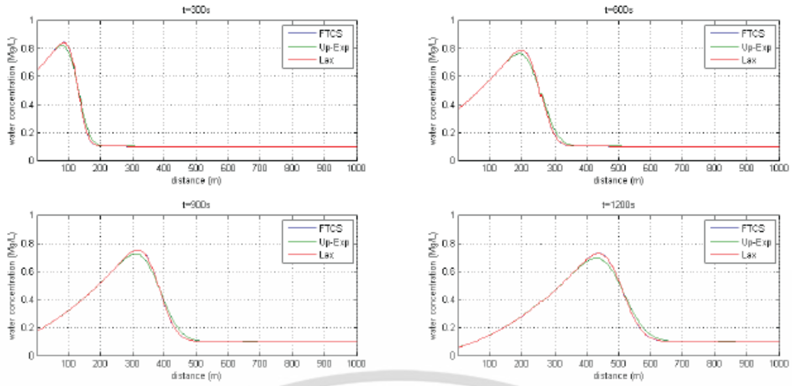
$t(sec), x(m)$	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6411	0.3926	0.1006	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3663	0.2795	0.3832	0.1320	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1741	0.1552	0.2513	0.3606	0.2069	0.1009	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0565	0.0702	0.1352	0.2247	0.3334	0.2755	0.1096	0.1000	0.1000	0.1000	0.1000

Table 9: The pollutant concentration $C(x,t)(Kg/m^3)$ by Upwind implicit schemes (Case A)

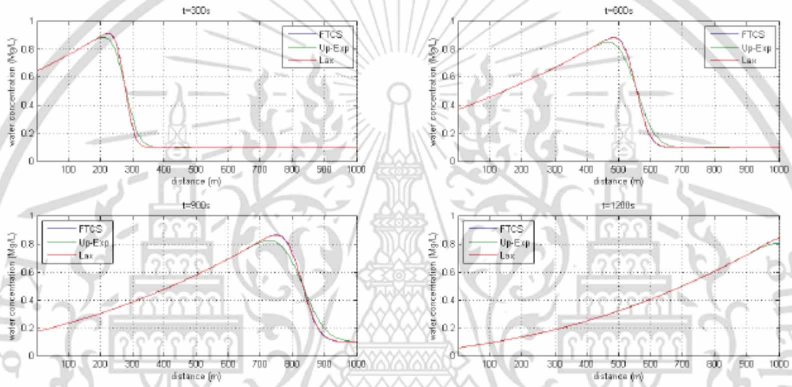
$t(sec), x(m)$	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6411	0.7330	0.1056	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3663	0.5540	0.7232	0.2052	0.1002	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1741	0.3079	0.4984	0.6885	0.3632	0.1072	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0565	0.1394	0.2688	0.4463	0.6430	0.4899	0.1422	0.1004	0.1000	0.1000	0.1000

Table 10: The pollutant concentration $C(x,t)(Kg/m^3)$ by Crank-Nicolson schemes (Case A)

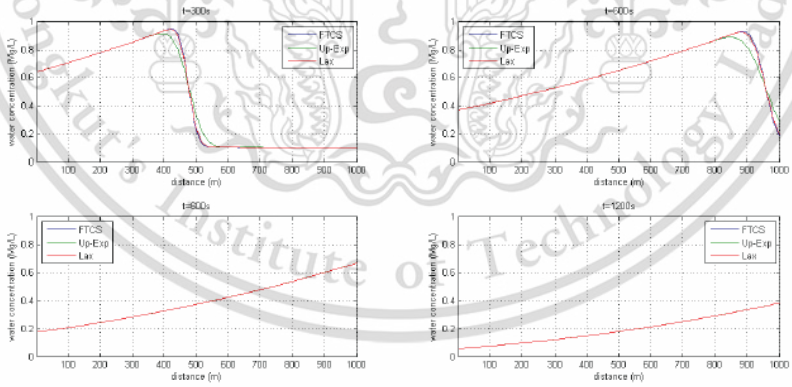
$t(sec), x(m)$	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6411	0.5044	0.1008	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3663	0.3613	0.4940	0.1435	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1741	0.2007	0.3248	0.4655	0.2472	0.1011	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0565	0.0907	0.1748	0.2904	0.4306	0.3430	0.1130	0.1000	0.1000	0.1000	0.1000



(a) case A (depth ratio 0.75)



(b) case B (depth ratio 0.5)



(c) case C (depth ratio 0.25)

Figure 5: The comparison of pollutant concentration for all cases with $\Delta x = 2, \Delta t = 0.1$ at difference time by explicit methods.

Table 11: The pollutant concentration $C(x,t)(Kg/m^3)$ by FTCS schemes (Case A)

$t(sec), x(m)$	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6461	0.7907	0.1010	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3701	0.5836	0.7877	0.1672	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1766	0.3249	0.5248	0.7469	0.3418	0.1016	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0579	0.1475	0.2832	0.4695	0.6930	0.5115	0.1200	0.1000	0.1000	0.1000	0.1000

Table 12: The pollutant concentration $C(x,t)(Kg/m^3)$ by Upwind explicit schemes (Case A)

$t(sec), x(m)$	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6461	0.7571	0.1046	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3701	0.5841	0.7553	0.2018	0.1001	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1766	0.3254	0.5257	0.7220	0.3665	0.1065	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0579	0.1480	0.2842	0.4709	0.6757	0.5027	0.1407	0.1003	0.1000	0.1000	0.1000

Table 13: The pollutant concentration $C(x,t)(Kg/m^3)$ by Lax-wendroff schemes (Case A)

$t(sec), x(m)$	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
300	0.6461	0.7898	0.1011	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
600	0.3701	0.5836	0.7870	0.1680	0.1001	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
900	0.1766	0.3249	0.5248	0.7463	0.3424	0.1017	0.1000	0.1000	0.1000	0.1000	0.1000
1200	0.0579	0.1475	0.2832	0.4695	0.6926	0.5113	0.1204	0.1000	0.1000	0.1000	0.1000

cases A-B of sandbag-dike failure give low velocity. The approximation of the pollutant concentrations of the implicit and explicit methods are shown in Tables 7-13.

6 Conclusion

If the villagers have received flood for longtime, the water pollutant have to be increase. The residents want to drain the water to the other areas by destroying the sandbag-dike. The other villages that never encounter flood going to receive a polluted water. We can see that the pollutant concentration level on the flooding area is not to high with the passing of time. The real-world problems require a small amount of time interval in obtaining accurate solutions. Unfortunately, the analytical solutions of the dam-break model could not be found over the entire domain. This also implies that the analytical solutions of dispersion model could not work out at any point on the entire domain as well. We propose a revised Lax-diffusive scheme by editing a simple modification to the traditional Lax-diffusive scheme. The FTCS method is limited by restriction of the stability condition. Then FTCS is not flexible in the real-world situation. The implicit schemes shows excessive dispersion effects for large time and space step lengths, significantly decreasing the efficiency of the implicit schemes. In addition implicit methods still generate a lot of large systems of linear equations. The Upwind explicit and Lax-wendroff schemes are economical to use. The proposed method show a good agreement in accuracy, the implicit schemes becomes less efficient than the explicit schemes. In this paper, the dispersion model and the dam-break model can be compounded to approximate the pollutant concentration in a stream when the current reflecting water in the stream is not uniform since the sandbag-dike becomes failure. In this paper, the technique developed, the response of the stream to the two different external inputs: the elevation of water and the pollutant concentration at the discharge point, can be obtained. The both of the implicit methods and the explicit methods can be used in the dispersion model since the scheme is very simple to implement. By the explicit finite difference formulations, we obtain that the proposed technique is applicable and economical to be used in the real-world problem due to its simplicity to program and the straight forwardness of the implementation. It is also possible to find tentative better locations and better periods of time of the different discharge points to a stream.

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Academic Publication(s)

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