MATHEMATICAL MODELS FOR THE EPIDEMIC OF DENGUE VIRUS BY *AEDES AEGYPTI* AND *AEDES ALBOPICTUS* MOSQUITOES

RATTIYA SUNGCHASIT

A THESIS SUBMITTED IN FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY IN APPLIED MATHEMATICS

DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
KING MONGKUT'S INSTITUTE OF TECHNOLOGY LADKRABANG

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หัวข้อวิทยานิพนธ์	แบบจำลองทางคณิตศาสตร์สำหรับการแพร่ระบาดของเชื้อไวรัสเดงกีด้วยยุงลายบ้านและยุงลายสวน
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บทคัดย่อ

โรคไข้เลือดออกเป็นโรคติดต่อชนิดหนึ่งมีสาเหตุมาจากไวรัสเดงกี่ (Dengue Virus) ซึ่งเป็นเชื้อไวรัสที่มียุงเป็นพาหะนำโรคที่พบการแพร่ระบาดได้มากที่สุด ในงานวิจัยนี้ได้สร้างแบบจำลองทางคณิตศาสตร์เพื่อศึกษาการแพร่ระบาดของโรคไข้เลือดออกโดยมียุงที่เป็นพาหะในการแพร่ระบาดของโรคนี้อยู่ 2 สปีชีส์ (ยุงลายบ้านและยุงลายสวน) ซึ่งข้อมูลดังกล่าวได้มาจากรายงานการเฝ้าระวังโรคกรมควบคุมโรค กระทรวงสาธารณสุข ในปี พ.ศ. 2545 – 2554 โดยแบบจำลองของโรคไข้เลือดออกนี้ได้พัฒนามาจากแบบจำลองทางคณิตศาสตร์ของ Esteva และ Vargas (1998) ซึ่งได้แบ่งประชากรออกเป็นประชากรคนและประชากรยุง ในกลุ่มประชากรคนจะมียุงทั้งสองสปีชีส์เป็นพาหะ ส่วนในกลุ่มประชากรยุงจะแบ่งยุงออกเป็นสองกลุ่ม คือ ยุงลายบ้าน (*Aedes aegypti*) และยุงลายสวน (*Aedes albopictus*) แบบจำลองที่ได้เป็นสมการเชิงอนุพันธ์ไม่เชิงเส้น และใช้แบบจำลองเชิงพลวัตมาตรฐาน(Standard Dynamical Modeling) จากนั้นหาเงื่อนไขสำหรับค่าพารามิเตอร์ที่ทำให้เกิดเสถียรภาพของจุดสมดุลภายใต้สภาวะไร้โรค (Disease free state) และสภาวะการระบาดเรื้อรัง (Endemic state) หลังจากนั้นได้นำผลเฉลยเชิงตัวเลขมาแสดงเพื่อใช้ในการสนับสนุนผลลัพธ์เชิงทฤษฎีและนำมาวิเคราะห์การควบคุมโรคไข้เลือดออก และยังได้แบ่งกลุ่มประชากรคนออกเป็นสองกลุ่มคือกลุ่มเด็กและกลุ่มผู้ใหญ่ ซึ่งผลจากการวิจัยครั้งนี้จะเป็นแนวทางในการลดประชากรคนที่ติดเชื้อจากโรคนี้ได้

คำสำคัญ : ไข้เลือดออก ความเสถียรภาพ จุดสมดุลภายใต้สภาวะไร้โรค แบบจำลอง SIR ยุงลายบ้าน ยุงลายสวน สภาวะการระบาดเรื้อรัง

Thesis Title	Mathematical Models for the Spread of Dengue Virus by *Aedes aegypti* and *Aedes albopictus* Mosquito
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ABSTRACT

Dengue disease is a virus infection caused by dengue virus. The transmission vector for this disease is *Aedes* mosquitoes. In this research, we use mathematical models to study the transmission of dengue disease of human and two species of vectors (*Aedes aegypti* and *Aedes albopictus*). We obtain data of a number of dengue cases reported to the Bureau of Epidemiology from the Ministry of Public Health, Thailand, during 2002 – 2011. We developed the mathematical model proposed by Esteva and vargas (1998). The human population was separated into two groups: the human and two species of vector mosquitoes. Each group is subdivided into three subclasses: susceptible, infectious and recovered subclasses. The vector population was separated into two groups: susceptible and infectious in vectors. The mathematical model is explained by the system of nonlinear differential equations. The standard dynamical modeling method is used for determining the behavior of solutions for each model. Then we find the conditions of parameters for the disease - free equilibrium state and disease endemic - equilibrium states. Numerical solutions are shown to support the theoretical predictions. Furthermore, the human population was divided into two groups, children and adults. The results of our models introduce the ways for controlling the transmission of dengue disease.

Keywords: Dengue disease, stability, disease free steady state, SIR model, *Aedes aegypti, Aedes albopictus*, endemic steady state.

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Chapter I
Introduction

1.1 Statement and Significance of the Problems

Dengue virus is the most important arboviral disease. It belongs to the genus *Flavivirus*, family Flavivirdae. There are 4 serotypes of dengue virus namely DEN – 1, DEN – 2, DEN – 3 and DEN – 4 [1,2,3]. Humans are the primary vertebrate hosts of all four serotypes. Infection range from simple fever to much more severe and sometimes fatal. Dengue Haemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS) are the severe forms of this disease. This disease has emerged as an international public health problem, now endemic in more than 100 countries and affecting about 40 % of the world population (2.5 billion people) living in tropical and subtropical regions and in urban /urban areas from Asia to the Pacific [4]. Therefore, their infections may also be due to chikungunya, a coinfection of both, or even other similar viruses. The outbreaks are often described as explosive. Dengue infection is common in the rainy season (approximately May to September in Thailand) when *Aedes* mosquito is abundant [5,6,7].

The transmission of dengue disease is caused by two females *Aedes* mosquito; *Aedes aegypti* (*Linnaeus*) and *Aedes albopictus* (*Skuse*) in urban areas. We observed that the spread of *Aedes albopictus* in the southern coastal states of the United States indicates that the expansion appeared to be occurring at the expense of *Aedes aegypti*. In Southeast Asia, the apparent spread of *Aedes aegypti* has been caused by increased urbanization which favours breeding of this species, which is also prevalent in indoor larval habitats, whereas Aedes albopictus breeds more successfully in suburban and rural areas and tends not to colonize in indoor water collections [8,9]

Thailand has experienced a three-fold increase of dengue infections. The data of 2013, found that there were 154,773 cases and 136 deaths. India also reported more infections with 3,952 cases during the first quarter of 2013 compared to 1,579 cases during the same period in 2012. However, the number of dengue-related death rates decreased to 7 from 12 in 2012. Most of those affected were between 14-24 years old, and advised people to destroy mosquito larvae and monitor their surroundings for conditions favorable to mosquito breeding. In 2013, they expect 150,000 to 200,000 cases. Up to 200 people could die in a worse - case scenario.

Even if dengue cases surpassed the 1987 record, the death toll of that year more than 1,000 was unlikely to be exceeded owing to improved health care and preventive measures. There was still no vaccine for dengue, a mosquito-borne disease. Health officials claimed that unseasonably wet and warm weather had made the situation worse, allowing mosquitoes to reproduce at a rapid rate. In May, the country reported 16,500 dengue cases, almost three times as many as in the previous month. In June, there could be up to 30,000 cases, for the local communities to take more responsibility in combating the disease, especially during the annual monsoon season from May to October when mosquito activity flourishes [6,7,10,11].

As mentioned above, we are interested in the transmission of dengue virus by two *Aedes* mosquitoes. We analyze the reported data from the Bureau of Epidemiology, Ministry of Public Health, Thailand, during 2002 – 2011[12]. The objectives of this study are to modify and to analyze the mathematical model for dengue virus by two *Aedes* mosquitoes.

1.2 Objectives of the study

The objectives of this thesis are to study and develop a mathematical model of dengue virus by *Aedes aegypti* and *Aedes albopictus*. More specifically, for that reason, the following objectives for this study are:

1) To study the dengue disease transmission and develop the mathematical model from SIR model of Esteva and Vargas (1998)[13].
2) To study the differences between the transmission of dengue virus by *Aedes aegypti* and *Aedes albopictus*.
3) To analyze the local stability and global stability and find conditions of parameters for stability.
4) To investigate the relationships between susceptible, exposed, infectious, recovered classed in human and susceptible, infective of *Aedes aegypti* and *Aedes albopictus* during outbreaks of dengue virus.
5) To use the results of the mathematical model the analyzing, and predicting the spread of the disease, and preventing the spread and reducing the outbreak of dengue virus.

1.3 Scope of the study

The scopes of the study are as follows:

1) study and Analyze the data from the reported of the Bureau of Epidemiology, Ministry, of Public Health, Thailand during 2002 – 2011.
2) Analyze the mathematical model for transmission of dengue virus by *Aedes aegypti* and *Aedes albopictus.* We suppose that the number of population is constant. The transmission rates of the disease are seasonal and the transmission rate of the disease is in the children and adult. The transmission rates of disease that depend on *Aedes* mosquitoes is different.
3) Analyze the results from the model by the conditions of parameter in the mathematical model.

1.4 Process of the study

The process of the study is follows:

1) Study the data from the report of the Bureau of Epidemiology, Ministry, of Public Health, Thailand during 2002 – 2011.
2) Study the definition and theoretical background and review the descried literatures.
3) Formulate the mathematical models.
4) Analyze the mathematical models.
5) Develop the mathematical models.
6) Compare the results and discuss them in the research.

1.5 Benefits of the study

The analysis of this research is a guideline to reduce the spread of dengue virus.

We are research in these models. We consider the factors causing the outbreak, by following this model.

Chapter II
Literature Reviews

In this chapter, to be discussed dengue disease by the two types of female mosquitoes (*Aedes aegypti* and *Aedes albopictus*).

2.1 Dengue virus and Mosquitoes

2.1.1 Dengue Virus

Dengue virus is a member of the virus family *Flaviviridae* ahnnd the genus Flavivirus – a family which includes other medically important vector – borne viruses. The Flavivirus genus derives its name from the Latin is "yellow" or "flavivirus,"and includes the dengue, West Nile, Tick – borne Encephalitis, and Yellow Fever viruses, the latter of which are febrile and jaundice [14,15,16]. Dengue viruses are arboviruses that are transmitted to people through the biting of the infected female *Aedes* mosquitos. Dengue virus is now believed to be the most common arthropod-borne disease in the world. Dengue is mainly found in the tropical and subtropical worldwide that originated in Africa because the mosquitoes require a warm climate. A major fear of epidemiologists is that the mosquitoes will develop resistance to cooler climates and then be able to infect people in the United States and other temperate climates [17,18,19].

The four dengue viruses serotypes (DEN 1 to 4), are members of the flaviviridae family, genus flavivirus. All four serotypes have emerged from sylvatic strains in the forests of South-East Asia. The four serotypes can cause a wide range of diseases in humans. Dengue Infection may also be asymptomatic. The diseases range in severity from undifferentiated acute febrile illness, classical dengue fever (Dengue Fever - DF), to the life-threatening conditions DHF/DSS [6,10,11,20,21]. Dengue illness was previously categorized on a I–IV grade scale, but a simplified categorization for dengue case classification has been proposed by WHO's Special Program for Research and Training in Tropical Diseases (TDR) in 2009 where DHF and DSS cases are grouped together as 'severe dengue to avoid false-negative DHF(Dengue Hemorrhagic Fever) / DSS (Dengue Shock Syndrome) diagnosis. DHF was first documented only in the 1950s during epidemics in the Philippines and Thailand. This did not happen until 1981 when large numbers of DHF cases began to appear in the Carribean and Latin America, where highly effective *Aedes* control programs had been in place until the early 1970s[20,21,22,23,24,25].

2.1.2 The Virus

The virus is transmitted when *Aedes* mosquitoes bite an individual infected with dengue virus. The virus in the blood of the infected individual then infects the mosquito and travels from the mosquitoes stomach to its salivary glands were the virus multiplies[22,23]. The virus is then injected into another person when the mosquito vaccinates anticoagulants that prevent blood clotting when the mosquito is feeding. The mosquito remains able to transmit dengue for its entire life. Each year, 100 million people become infected with dengue virus. The first recognized Dengue epidemics occurred almost simultaneously in Asia, Africa, and North America in the 1780s, shortly after the identification and naming of the disease in 1779, but it was most likely present long before it first appeared in literature. However, the majority of deaths that result from dengue infection result from Dengue Hemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS). People who develop DHF have a 5% chance of death but if they go on to develop DSS then the mortality rate can rise as high as 40% of the world's population living in areas where there is a risk of dengue transmission, Dengue spread to more than 100 countries in Asia, the Pacific, the Americas, Africa, and the Caribbean[13,17,19,22,24].

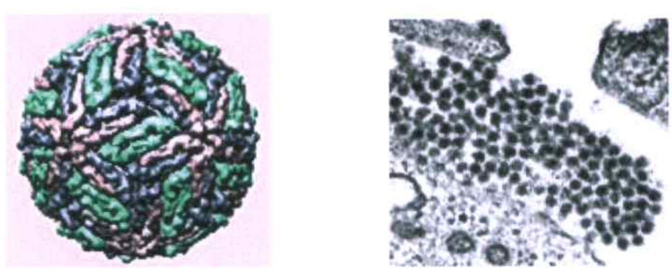

Figure 2.1 Dengue virus particle and microscopic picture of dengue viruses[25].

2.1.3 Transmission of the Dengue Virus

Dengue virus is transmitted between the humans through biting infective females by the mosquitoes *Aedes aegypti* and *Aedes albopictus*, which are found throughout the world. The mosquitoes generally acquire the virus while feeding on the blood of an infected person. Symptoms of infection usually begin 4 - 13 days after the mosquito bite and typically last 3 - 10 days. Infected mosquito is capable of transmitting the virus for the rest of its life. There is no way to tell if a mosquito is carrying the dengue virus. Infected female mosquitoes may also transmit the virus to their offspring by transovarial (via the eggs) transmission, but the role of this in sustaining transmission of the virus to humans has not yet been defined. Infected humans are the main carriers and multipliers of the virus, and serving as a source of the virus for uninfected mosquitoes. This period usually begins a little time before the person become symptomatic. Some people never have significant symptoms but he /she can still infect this disease. After the mosquito enter in the blood meal, the virus will require an additional 8-12 days incubation before it can then be transmitted to another human[22,23,24,25]. The mosquito remains infected for the remainder of its life. In rare cases, dengue virus can be transmitted in organ transplants or blood transfusions from infected donors, and there is evidence of transmission from an infected pregnant mother to her fetus. But in the vast majority of infections. In many parts of the tropics and subtropics, dengue is endemic, that is, it occurs every year, usually during a season when *Aedes* mosquito populations are high, often when rainfall is optimal for breeding. These areas are, however, additionally at periodic risk for dengue epidemics, when large numbers of people become infected during a short period. Dengue epidemics require a coincidence of large numbers of vector mosquitoes, large numbers of people with no immunity to one of the four virus types (DEN 1, DEN 2, DEN 3, DEN 4), and the opportunity for contact between the two species of *Aedes* mosquito. *Aedes* mosquitoes may acquire the virus when they feed on an individual during this period. In parts of South East Asia and Africa, the transmission cycle may also involve jungle primates that act as a reservoir for the virus [10,11,12,13,23,24].

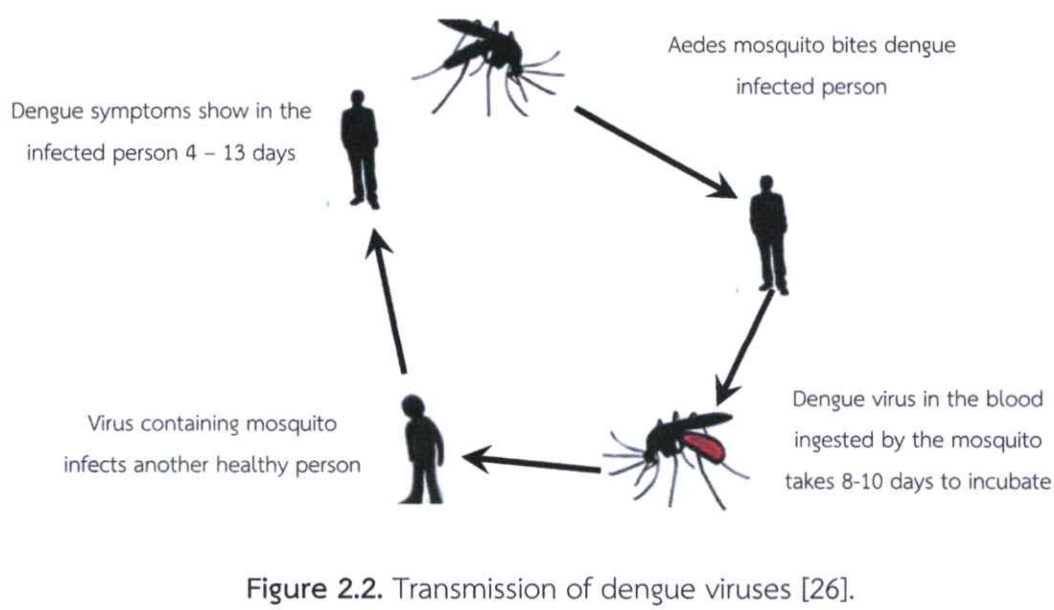

Figure 2.2. Transmission of dengue viruses [26].

Dengue virus is transmitted by *Aedes aegypti* mosquito. *Aedes albopictus* mosquito and other *Aedes species* also transmit disease in specific areas. *Aedes polynesiensis*, *Aedes scutellaris* and *Aedes pseudoscutallaris* in the Pacific Islands and New Guinea. *Aedes polynesiensis* in the Society Islands and *Aedes niveus* in the Philippines. The *Aedes* mosquito prefers to breed in water-filled receptacles, usually close to human habitation. They often rest in dark rooms and breed in small pools that collect in discarded human waste. Although they are most active during daylight hours, biting from dawn to dusk, mosquitoes will feed throughout the day indoors and during overcast weather.

2.1.4 The Vector

The *Aedes* mosquito is the main vector that transmits the virus that causes dengue fever. The virus is passed onto humans through biting of infective female *Aedes* mosquito, which mainly acquires the viruses while feeding on the blood of the infected person. Two mosquitoes species have recently been found in several areas and there is potential patient for them to spread the virus into other areas of the world. They are named *Aedes aegypti* (the yellow fever mosquito) and *Aedes albopictus* (the Asia tiger mosquito). Unlike the other mosquito species. *Aedes aegypti* and *Aedes albopictus* bite during the day. Both species are small black mosquitoes with white stripes on their back and on their legs. They can lay eggs in any small artificial or natural container that holds water. Two mosquitoes species have the potential to transmit several viruses, including dengue, chikungunya, and yellow fever. None of these viruses are currently known to be transmitted within California, but thousands of people are infected with these viruses in other parts of the world, including Mexico, Central and South America, the Caribbean, and Asia.

2.1.5 The species of mosquitoes.

At present, it is found that in this world, there are about 3,450 mosquito species. In Thailand, there are at least 412 species. A name based on a simple language by following in table 2.1.

Table 2.1. Mosquito that caused the disease

Vectors	Disease
Anopheles	Malaria
Culex	Encephalitis
Aedes - *Aedes aegypti* - *Aedes albopictus*	Dengue fever, Dengue hemorrhagic fever, Dengue shock syndrome, chikungunya fever, yellow fever
Mansonia	Filarisis, Scrub Typhus

2.1.6 Aedes aegypti

Aedes aegypti mosquito is the primary vector of dengue disease. The virus is transmitted to human through biting of infected female mosquitoes. *Aedes aegypti* is usually found between latitudes 35°N and 35°S [5,6,7,11] during the warm season. It corresponds to a winter isotherm of 10°C. The biting behavior of *Aedes aegypti* occurs during the day. This species is most active for approximately two hours after sunrise and several hours before sunset, but it can bite at night in some areas. After 4–10 days of virus incubation, an infected mosquito is capable of transmitting the virus for the rest of its life. Infected humans are the main carriers and multipliers of the virus, serving as a source of the virus for uninfected mosquitoes. Patients who are already infected with dengue virus can transmit the virus (for 4–5 days; maximum 12) via *Aedes* mosquitoes after their first symptoms appear. The biting behavior of *Aedes aegypti* occurs during the day. This species is most active for approximately two hours after sunrise and several hours before sunset, but it can bite at night in some areas. Mosquitoes can bite people without being noticed because it approaches from behind and bites on the ankles and elbows [22,23,24,25]. *Aedes aegypti* prefers to bite people but it also bites dogs and other domestic animals, mostly mammals. Only female mosquitoes bite humans to obtain blood in order to lay eggs. This medical importance is found in tropical and subtropical areas of the world, *Aedes aegypti* historically is considered to be a primary vector of viral diseases such as the dengue fever, chikungunya and yellow fever. The seasonal abundance of adults peaked in February, May – June and again in September. Increasing numbers of eggs were found in February and again in July with larval densities peaking one month later. [25,27,28,29,30,33]

Figure 2.3. *Aedes aegypti* [28].

2.1.7 *Aedes albopictus*

Aedes albopictus received the nick-name "Tiger mosquito" a secondary dengue vector in south east Asia, has spread to North America and Europe largely due to the international trade in used types of mosquito (a breeding habitat) and other goods. *Aedes albopictus* will be habitat in temperate and northern latitudes, and they also survive. The biting behavior of *Aedes albopictus* occurs during daytime. Its peak feeding times are during the early morning and late afternoon. They are strongly attracted to biting humans, but feed on cats, dogs, squirrels, deer and other mammals, as well as birds. The bites occur on any exposed skin surface. These mosquitoes can use natural locations, habitats and artificial containers with water to lay their eggs. About four or five days after feeding on blood, the female mosquito lays her eggs just above the surface of the water. The entire immature or aquatic cycle can occur about 7-9 days. The life span for adult mosquitoes is around 3 weeks. *Aedes albopictus* mosquitoes remain alive through the winter in the egg stage in temperate climates (areas with four seasons) but are active throughout the year in tropical and subtropical locations. The medical importance of *Aedes albopictus* is most well known for transmitting dengue and chikungunya viruses. The Asian tiger mosquito is similar, in terms of their close socialization with humans, to the common house mosquito (*Culex pipiens*). Among other differences in their biology, *Culex pipiens* prefers larger breeding waters and is more tolerant to cold. In this respect, there is not any significant competition or suppression between the two species. The seasonal abundance is depend on temperature and the availability of food and water in a particular geographical area. Higher temperatures speed up larval development, increasing the number of adult populations, the autumnal development of immatures and consequently the rates of egg overwintering. Oviposition takes place from mid-April to December, with the numbers of eggs peak from mid-July to the end of the autumn, and significantly increased during mild and rainy weather[27,28,29,30,31,33].

Chikungunya fever is a viral disease transmitted to humans by the biting of infected mosquitoes. Chikungunya virus was first isolated from the blood of a febrile patient in Tanzania in 1953, and has since been cited as the cause of numerous human epidemics in many areas of Africa and Asia, and most recently in a limited area of Europe. Humans become infected with chikungunya virus by the bite of an infected mosquito. *Aedes aegypti,* a household container breeder and aggressive daytime biter which is attracted to humans, is the primary vector of chikungunya virus to humans. *Aedes albopictus* has also played a role in human transmission [52,53]

Figure 2.4. *Aedes albopictus*[32].

2.1.8 The comparison between dengue disease and chikungunya

Table2.2. Disease caused by mosquito two species [26,27,28,29,30,31,32,33,34,35, 52,53].

Disease caused	Dengue disease	chikungunya
Transmission	Dengue virus are transmitted to humans though the biting of infected *Aedes* mosquito.	Chikungunya is a viral disease (genus *Alphavirus*) which is transmitted to humans by infected mosquitoes.
Mosquitoes vectors	*Aedes aegypti* and *Aedes albopictus*,	*Aedes albopictus* (the Asian tiger mosquito)
Disease outbreaks	Dengue has emerged as a worldwide problem only since the 1950s. Although dengue rarely occurs in the continental United States, it is endemic in Puerto Rico and in	Chikungunya occurs in Africa, Asia and the Indian subcontinent. Human infections in Africa have been at relatively low levels for a number of years, but in 1999–2000

	many popular tourist destinations in Latin America, Southeast Asia and the Pacific islands	there was a large outbreak in the Democratic Republic of the Congo, and in 2007 there was an outbreak in Gabon.
Symptoms	Symptoms, which usually begin four to six days after infection and last for up to 10 days, may include - Sudden, high fever - Severe headaches - Pain behind the eyes - Severe joint and muscle pain - Nausea - Vomiting - Skin rash, which appears three to four days after the onset of fever - Mild bleeding (such a nose bleed, bleeding gums, or easy bruising)	Symptoms appear between 4 and 7 days after the patient has been bitten by the infected mosquito and these include: High fever (40°C/ 104°F) - Joint pain (lower back, ankle, knees, wrists or phalanges) - Joint swelling - Rash - Headache - Muscle pain - Nausea - Fatigue

2.1.9 *Aedes* Mosquito Life Cycle

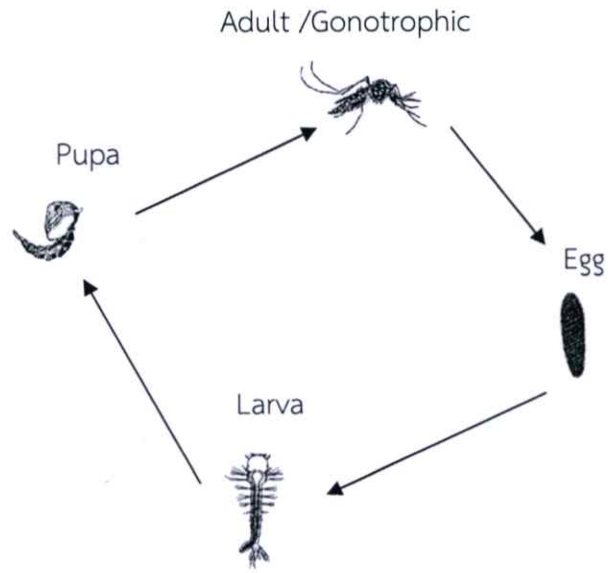

Figure 2.5. life cycle of mosquitoes[34].

The mosquito has a complex life cycle with dramatic changes in shape and habitat. Female *Aedes mosquitoes* commonly lay their eggs on the inner walls of artificial containers. When the containers filled with water, mosquito larvae hatch from the eggs. After developing through four larval stages, the larvae metamorphose into pupas. Like the larval stage, the pupal stage is also aquatic. After two days, a fully developed adult mosquito forms and breaks through the skin of the pupa. The adult mosquito can fly and has a terrestrial habitat. The entire life cycle lasts 8-10 days at room temperature, depending on the level of feeding. Thus, there is an aquatic phase (larvae, pupae) and a terrestrial phase (eggs, adults) in the life-cycle. Mosquitoes have a complicated life cycle, as they change their shapes and habitats. Female mosquitoes generally lay their eggs above the water line inside containers that hold water [26,35]. These containers include tires, buckets, birdbaths, water storage jars, and flower pots. Mosquito larvae hatch the eggs when the containers fill with water, in many cases after a rainfall. The larvae are aquatic, meaning that they live in the water and feed on microorganisms found in the water. Larvae go through developmental stages in which they molt, or shed their skin, three times. These larval stages are called the first to fourth instars. This stage of the mosquito's life is also aquatic. After two days, the fully developed adult mosquito forms and breaks through the skin of the pupa. The adult mosquito is able to fly and there is no longer aquatic. It has a terrestrial habitat. This adaptation has made it very difficult to eliminate mosquito populations completely. In many areas of the world, dengue outbreaks occur every year during the rainy season, when conditions are perfect for mosquito breeding. Dengue can pose a particular threat in highly populated regions because epidemics are more likely where there are large numbers of people in contact with large numbers of mosquito vectors than in more isolated areas, and in countries in the equatorial area that experience tropical monsoon seasons, such as Indonesia, India, Brazil, Thailand, Sri Lanka, and Myanmar[34,35].

2.1.10 Dengue infected patients

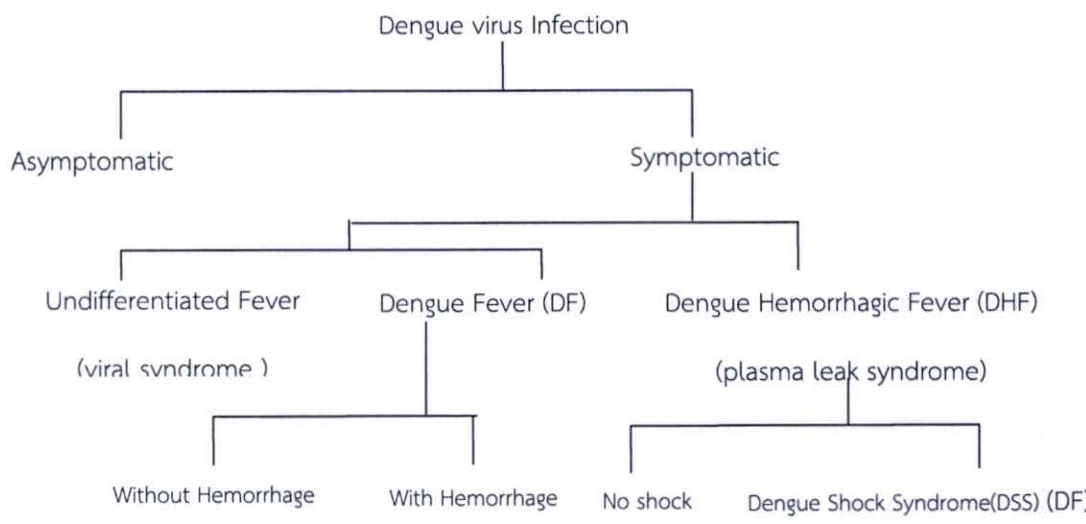

Figure 2.6. Dengue virus infection [26].

The dengue infected individuals are asymptomatic; they have no clinical signs or symptoms of disease. The first clinical course is a relatively benign scenario where the patient experiences fever with mild non-specific symptoms that can mimic any number of other acute febrile illnesses. The criteria for DF of the non-specific presentation of symptoms make positive diagnosis difficult based on physical exam and routine tests alone. For the majority of these patients, unless dengue diagnostic serological or molecular testing is performed, the diagnosis will remain unknown. These patients are typically young children or people who experience the first infection. They recover fully without need for hospital care. The second clinical presentation occurs when a patient develops DF with or without hemorrhage. These patients are typically older children or adults and they present with two to seven days of high fever (occasionally biphasic) and two or more of the following symptoms: severe headache, retro-orbital eye pain, myalgias, arthralgias, a diffuse erythematous maculo-papular rash, and mild hemorrhagic manifestation. Subtle, minor epithelial hemorrhage, in the form of petechiae, are often found on the lower extremities (but may occur on buccal mucosa, hard and soft palates and or subconjunctivae as well), easy bruising on the skin, or the patient may have a positive tourniquet test. Children may also present with nausea and vomiting. Patients with DF do not develop substantial plasma leak (hallmark of DHF and DSS, see below) or extensive clinical hemorrhage. Serological testing for anti-dengue IgM antibodies or molecular testing for dengue viral RNA or viral isolation can confirm the diagnosis, but these tests often provide only retrospective confirmation, as results are typically not available until well after the patient has recovered. Dengue Hemorrhagic Fever (DHF) or Dengue Shock Syndrome (DSS): The third clinical presentation results in the development of DHF, which in some patients progresses to DSS. Vigilant is critical for identifying warning signs of progressing illness and early symptoms of DHF which are very similar to those of DF.

2.1.11 Grading severity of Dengue hemorrhagic fever

Dengue hemorrhagic fever is classified into four grades of severity, where grades III and IV are considered to be DSS [54].

Grade I: Fever accompanied by non – specific constitutional symptoms; the only hemorrhagic manifestation is a positive toumiquet test and/or easy bruising.

Grade II: Spontaneous bleeding in addition to the manifestations of Grade I patients, usually in the forms of skin or other haemorrhages.

Grade III: Circulatory failure manifested by a rapid, wesk pulse and narrowing of pulse pressure or hypotension, with presence of cold, clammy skin and restlessness.

Grade IV: Profound shock with undetectable blood pressure or pulse.

In 2012 and 2013, The Rare and Imported Pathogens Laboratory (RIPL) is now incorporated into the functions of Public Health England (PHE) reported an increase in imported cases of dengue fever in England, Wales and Northern Ireland. RIPL provides a clinical diagnostic service for rare and/or imported pathogens such as

pathogenic arboviruses, haemorrhagic fever viruses and a number of Hazard Group 3 bacterial pathogens including rickettsiae, *Coxiella burnetii* and *Bacillus anthracis*.
RIPL is the frontline laboratory providing diagnostics for the Imported Fever Service following its inception in June 2012. The increasing of cases from Thailand and Barbados was noted. There was also a cluster of cases from Sri Lanka, Brazil and Jamaica. Cases were also reported in residents of the Autonomous Region of Madeira, Portugal.

2.2 Prior Mathematical Modeling Studies

In 1998, Esteva and Vargas [13] proposed a model for the transmission of dengue fever in a constant human population and variable vector population. The SIR model applied to the human; the susceptible, the infective and recovered; SI model applied to vector population; the susceptible and the infective, since the vector is infected for life. This model is divided into two classes; N_h is the human population and N_v is the vector population. The human population has constant size, if birth rate and death rate are constants and equivalent to μ_h . For the mosquito population, recruitment rate μ_h is death rate. μ_v is the mortality rate of mosquito. The dynamic transmission of human and vector are shown in figure2.7.

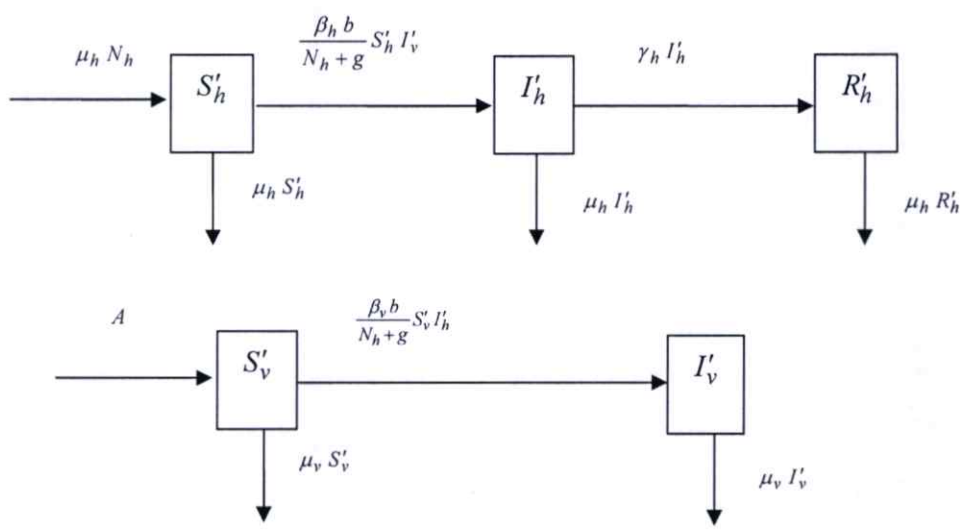

Figure 2.7 The dynamic transmission of population and vector [13].

In above diagram, S'_h , I'_h , R'_h are the number of susceptibles, infective and recovered in the human population; and S'_v, I'_v are the number of susceptible and infectives in the vector population, Because mosquito never recovers from infection. The dynamic from susceptibles to infective for each species depend on the biting rate (b) of vector; the average number of bites per vector per day, the transmission is the probability of an infectious bite produces, where β_h is the transmission probability of dengue virus from vector to human and β_v is the transmission probability from human to vector, g is the number of alternative hosts available as blood sources and the probability a human individual as a host by $\frac{\beta_h}{N_h+g}$. Then, the number of bites that a human receives per unit of time by $b(\frac{N_v}{N_h})(\frac{N_h}{N_h+g})$, a vector takes $\frac{bN_h}{N_h+g}$ human blood meals per unit of time. Then, the infection rates per susceptible human by $\beta_h b(\frac{N_v}{N_h})(\frac{N_h}{N_h+g})(\frac{I'_v}{N_v}) = (\frac{\beta_h b}{N_h+g})I'_v$ and susceptible mosquito by $\beta_v b(\frac{N_v}{N_h})(\frac{N_h}{N_h+g})(\frac{I'_h}{N_h}) = (\frac{\beta_h b}{N_h+g})I'_h$. The infected humans recover at a constant rate γ_h. The model is described by the following system of differential equations:

$$\frac{dS'_h}{dt} = \mu_h N_h - \frac{\beta_h b}{N_h+g} S'_h I'_v - \mu_h S'_h \tag{2.1}$$

$$\frac{dI'_h}{dt} = \frac{\beta_h b}{N_h+g} S'_h I'_v - \mu_h I'_h - \gamma_h I'_h \tag{2.2}$$

$$\frac{dR'_h}{dt} = -\mu_h R'_h + \gamma_h I'_h \tag{2.3}$$

$$\frac{dS'_v}{dt} = A - \frac{\beta_v b}{N_h+g} S'_v I'_h - \mu_v S'_v \tag{2.4}$$

$$\frac{dI'_v}{dt} = \frac{\beta_v b}{N_h+g} S'_v I'_h - \mu_v I'_v \tag{2.5}$$

with two conditions: $N_h = S'_h + I'_h + R'_h$ and $N_v = S'_v + I'_v$.

They used standard dynamic analysis to determine the conditions on the value of parameter to establish the global stability of the endemic equilibrium (global asymptotically stable), then the Routh – Hurwitz conditions for the polynomial is locally asymptotically stable. The basic reproductive number $R_0 = \sqrt{R_0}$ of the disease, since it represents the average number of secondary cases that one case can produce if introduced into a susceptible population. They used the results for $R_0 \le 1$, the disease free equilibrium is globally asymptotically stable and $R_0 > 1$, oscillate to the endemic state. Using competitive system and stability of period orbits asymptotically stable. The control for vector population can be explained in terms of the basic reproductive number, when the outbreak of dengue starts in an endemic region, and decreasing the carrying of the environment for mosquito by frequent reduction of the vector breeding sites, to be a more effective way to control the disease.

In 2003, Pongsumpun and Tang [36]. formulated the transmission of dengue hemorrhagic fever (DHF) described SIR model in the human population. They separated human into an adults classes and juvenile classes with the transmission of the disease being different in the two classes. Two equilibrium states are found and condition for stability of one of these states, the disease free state and endemic state of this model is discussed. The period of fluctuations in the number of individuals in each class is much shorter in absence of age structure. The SIR model with age structure, the dynamic of each component of the human is given by

$$\frac{dS'_J}{dt} = \lambda' N_T - \frac{b\beta_J}{N_T + m} S'_J I'_v S'_J - (\mu_h + \delta) S'_J \tag{2.6}$$

$$\frac{dR'_J}{dt} = r I'_J - (\mu_h + \delta) R'_J \; \frac{d I'_J}{dt} = \frac{b \beta_J}{N_T + m} S'_J I'_v - (\mu_h + \delta + r) I'_J \tag{2.7}$$

$$\frac{dS'_A}{dt} = \delta(S'_J + I'_J + R'_J) - \varepsilon\beta_J \frac{b}{N_T + m} S'_A I'_v - \mu_h S'_A \tag{2.8}$$

$$\frac{dR'_A}{dt} = r I'_A - \mu_h R'_A \tag{2.9}$$

and

$$\frac{dI'_A}{dt} = \varepsilon\beta_J \frac{b}{N_T + m} S'_A I'_v - (\mu_h + r) I'_A \tag{2.10}$$

where $S'_{J(A)}$, $I'_{J(A)}$ and $R'_{J(A)}$ are the numbers of susceptible juveniles (adults), infected juveniles (adults), and recovered juveniles (adults), respectively; N_T the total population (taken to be constant); m, the number of the other animals the mosquitoes can bites; b, the average number of bites a mosquito takes per day; λ', the birth rate; μ_h, the death rate; β_J, the probability of the virus surviving in the juvenile after being bitten by an infected mosquito.

The dynamics of the mosquitoes is described by

$$\frac{dS'_v}{dt} = A - \frac{b\beta_v}{N_T + m} S'_v (I'_J + I'_A) - \mu_v S'_v \tag{2.11}$$

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$$\frac{dI'_v}{dt} = \frac{b\beta_v}{N_T + m} S'_v (I'_J + I'_A) - \mu_v I'_v \tag{2.12}$$

Where S'_v and I'_v the number of susceptible and infected mosquitoes, respectively; μ_v is the death rate of the mosquitoes; A is the carrying capacity of the environment (for the mosquitoes) and β_v is the probability that a dengue virus transmitted to the mosquito from an infected human. The conditions for the stability, show that age structure in the simplified model reduces the periods of oscillations in the SIR in human population and SI in mosquito population for dengue virus.

In 2007, Pongsumpun and Kongnuy [37], formulated the transmission model of dengue disease with the effect of extrinsic incubation period of dengue virus in mosquito caused the seasonality. They compared the seasonality transmission model of dengue disease (without symptomatic and asymptomatic classes). They found that the symptomatic and asymptomatic classes reduced the period of oscillation in the human population.

In 2008, Supriatna, Soewono and Gils [38] formulated A two - age - classes dengue transmission model with vaccination. They divide the human population into two age classes by a vaccination is usually concentrated in one class and a constant rate of individuals in the child - class is vaccinated. They analyze a threshold number which is equivalent to the reproduction number. If there is an undeliberate vaccination to infectious children, which worsens their condition as the time span of being infectious increases, then paradoxically, vaccination can be counter productive. The paradox, stating that vaccination makes the basic reproduction number even bigger, can occur if the worsening effect is greater than a certain threshold, a function of the human demographic and epidemiological parameters, which is independent of the level of vaccination. If the increasing virulence so that one will develop symptoms, then the vaccination is always productive. Use these assumptions and notions, the dynamics of the human population is given by

$$\frac{dS_1(t)}{dt} = B - \alpha S_1(t) - \mu S_1(t) - b_1 p_1 I_v(t-\tau_i) \frac{S_1(t-\tau_i)}{N(t-\tau_i)} - sqS_1(t) \tag{2.13}$$

$$\frac{dS_2(t)}{dt} = \alpha S_1(t) - \mu S_2(t) - b_2 \rho_2 I_v(t-\tau_i) \frac{S_2(t-\tau_i)}{N(t-\tau_i)} \tag{2.14}$$

$$\frac{dI_1(t)}{dt} = \sigma b_1 \rho_1 I_v(t-\tau_i) \frac{S_1(t-\tau_i)}{N(t-\tau_i)} - \mu I_1(t) - (\gamma - q\omega) I_1(t) - \alpha I_1(t) \tag{2.15}$$

$$\frac{d I_2(t)}{dt} = \sigma b_2 \rho_2 I_v(t-\tau_i)\frac{S_2(t-\tau_i)}{N(t-\tau_i)} - \mu I_2(t) - \gamma I_2(t) + \alpha I_1(t) \tag{2.16}$$

$$\frac{d D_1(t)}{dt} = [1-\sigma]\, b_1 \rho_1 I_v(t-\tau_i)\frac{S_1(t-\tau_i)}{N(t-\tau_i)} - \mu D_1(t) - \gamma_d D_1(t) - \alpha D_1(t) \tag{2.17}$$

$$\frac{d D_2(t)}{dt} = [1-\sigma]\, b_2 \rho_2 I_v(t-\tau_i)\frac{S_2(t-\tau_i)}{N(t-\tau_i)} - \mu D_2(t) - \gamma_d D_2(t) + \alpha D_1(t) \tag{2.18}$$

$$\frac{d R(t)}{dt} = sqS_1(t) + (\gamma - q\omega) I_1(t) + \gamma I_2(t) + \gamma_d D_1(t) + \gamma_d D_2(t) - \mu R(t) \tag{2.19}$$

The mosquito population dynamic by

$$\frac{d S_v(t)}{dt} = B_v - [b_1 p_v I_1(t) + b_2 p_v I_2(t)\ (t-\tau_e)]\ \frac{S_v(t-\tau_e)}{N(t-\tau_e)} e^{-\mu\tau_e} - \mu_v S_v(t) \tag{2.20}$$

$$\frac{d I_v(t)}{dt} = [b_1 p_v I_1(t) + b_2 p_v I_2(t)\ (t-\tau_e)]\ \frac{S_v(t-\tau_e)}{N(t-\tau_e)} e^{-\mu\tau_e} - \mu_v I_v(t) \tag{2.21}$$

The number of individuals in the child-class and the adults-class is denoted by N_1 and N_2, respectively. Let us also assume that the total number of individual in age class j at time t, i.e. $N_j(t), j=1,2$ is composed of the number of susceptible individuals, $S_j(t)$ the number of asymptomatic infective individuals, $I_j(t)$ the number of symptomatic or severely infective individuals, $D_j(t)$ and the number of recovered or immune individual, $R_j(t)$ The constant recruitment rates for humans and mosquitoes are B and B_v, respectively. The total number of mosquitoes, $N_v(t)$ consists of the susceptible, $S_v(t)$,and the infectious, $I_v(t)$, individuals. Furthermore, there is an extrinsic incubation period τ_e and intrinsic incubation period τ_i experienced by infected mosquitoes and in infected humans, respectively, before they become infectious. If there is an undeliberate vaccination of asymptomatic infectious children that effectively enlarges the infectious period, then a paradox of vaccination might occur. The threshold is a function of the human demographic and epidemiological parameters, which might be independent of the level of vaccination. Although the region of the realistic parameters in which the vaccination might happen is regarded as a small region, still this paradox must be avoided.

In 2010, Erickson, Presley et.al.[39]. A dengue model with a dynamic of *Aedes albopictus* vector population is considered. It depends on the factors such as socioeconomic, the local environment and biology of mosquito. They created a Susceptible, Exposed, Infectious, Recovered human and Susceptible, Exposed, Infectious mosquito. They consider the effect of temperature. When, they examined the role of temperature as a vector population driver. They found that the maximum dengue outbreaks occurred when the average daily temperature allowed for larger mosquito population.

In 2012, Tasman, Supriatna, et.al. [40] constructed a dengue vaccination model for immigrants in a two – age class population. They formulated the transmission of dengue model, by classifying the human population into child and adult classes. They considered the transmission of disease, according to age structure and each population into separated susceptible, infected and recovered groups. They include immigration and emigration factors into the model, the vector population is divided into susceptible, exposed, and infectious classes. They derived a threshold parameter of existence and stability of disease – free and endemic equilibrium states depending on the reproductive ratio.

In 2012, Chen and Hsieh[41] constructed the transmission dynamics of dengue fever; implications of temperature effects are depend on entomological parameters of *Aedes aegypti*. The biting rate and the initial mosquito population, were adapted to observe features of the epidemic. They showed that the climate factor was indeed important and influenced the system modeling of human and vector interactions.

In 2013, Oki and Yamamoto [42] analyze the probable vector density that caused the largest dengue outbreak in Nagasaki in 1942, using SEIR model in various assumptions. The parameters were applied, such as proportion of symptomatic cases, vector mortality, and human biting rate and longest vector survival of Aedes albopictus. Then, the high vector density due to wartime practices, and the tradition lifestyle were responsible for the earlier dengue outbreak, the outbreak occurs on the environmental and human behavior have change. Equations for the changes in each class of human and vector populations are follows:

$$\frac{dS_h}{dt}=N_h d_h - S_h(a_{vh} q r_{fm}\frac{I_v}{N_h}+d_h) \quad (2.22)$$

$$\frac{dE_h}{dt}= S_h a_{vh} q r_{fm}\frac{I_v}{N_h} - E_h(r_{iip}+d_h) \quad (2.23)$$

$$\frac{dI_h}{dt}=E_h r_{iip} - I_h(r_{recovery}+d_h) \quad (2.24)$$

$$\frac{dR_h}{dt}=I_h(r_{recovery}-R_h d_h) \quad (2.25)$$

$$\frac{dS_v}{dt} = MPPN_h \sin[0.016882(t+240)]d_v - S_v(a_{hv}qr_{fm}\frac{I_{h\_index}+I_h}{N_h} + d_v) \quad (2.26)$$

$$\frac{dE_v}{dt} = S_v(a_{hv}qr_{fm}\frac{I_{h\_index}+I_h}{N_h} - E_v(r_{eip} + d_v) \quad (2.27)$$

$$\frac{dI_v}{dt} = E_v r_{eip} - I_v d_v \quad (2.28)$$

The parameters in their model, d_h , Host death rate ; r_{iip} , Viral development rate in humans; r_{eip} ,Viral development rate in the vector bodies; $r_{recovery}$, Recovery rate of humans; r_{fm} ,development rate of adult female mosquito ; MPP , Number of female mosquitoes per person; I_{h_index} ,Virus introduction; a_{hv} ,Transmission probability (host to vector); a_{vh} ,Transmission probability (vector to host), respectively. we could not directly compare the vector densities between the earlier outbreak and the present, since the actual density of A. albopictus in the natural environment still remains unknown. Although conducting entomological field surveillance is beyond the scope of this study, it is one of the study's limitations; it is important to investigate if the vector density in present-day Japan is definitively lower than the threshold for the prevention of the re-emergence of dengue.

In 2013, Chong, Tchuenche and Smith. [43]. formulated a mathematical model of avian influenza with half – saturated incidence. The virus spread from birds to humans and from human to human. The mathematical model of avian influence for both bird and human populations on the transmission dynamic of disease is investigated, by as essential role in determining the basic reproduction number of this model. By increasing the half – saturation constant for the mutant strain, in addition to other protection measures such as vaccination and personal protection, we can make the disease – free equilibrium globally stable and hence theoretically eradicate the disease.

In this research, we formulate the transmission models for dengue disease considered the influence of two species mosquitoes i.e. *Aedes aegypti* and *Aedes albopictus.* First model, we construct the SIR model, and season for the outbreak of this disease. Second model, we formulate the SIR model with the non construct rate of biting. The third model, we include the incubation to our first model by age group from the second model. The fourth model, we separated the human group. Last model, we study the global behavior of two equilibrium states from the second model by using Lyapunov functions.

In chapter III, we described a set of differential equations for explaining the spread of dengue disease. The different contact rates of three seasons are considered by SIR model (S = Susceptible, I = Infective, R = Recovered) used in this study. We analyze the dengue model of seasonality compartment (rainy season, winter season, summer season). We determine the stability conditions of equilibrium points from Jacobian matrix.

In chapter IV, we study transmission of dengue disease in a population containing two mosquitoes species (*Aedes aegypti* and *Aedes albopictus*) into SIR model of the mathematical model. We determine the conditions on the values of the parameters for the equilibrium states to be local asymptotically stable. The results of the model are obtained by numerical simulations.

In chapter V, we add the incubation rates for each season into SEIR model of the mathematical model in the model III, The results are obtained by numerical simulations.

In chapter VI, we modify the model in chapter IV by separating the human population into two classes, a child class and an adult class, each class being described by SIR model. We determine the conditions for the local asymptotically stability. We then present the results obtained by numerical simulations.

In chapter VII, we study the global behavior of two equilibrium states from model IV by using Lyapunov functions.

In chapter VIII, we conclude and suggest the results of our mathematical models and propose the method to reduce the outbreak of dengue virus considering two species vector of (*Aedes aegypti* and *Aedes albopictus*).

Chapter III

Transmission Model of Dengue Disease with the Different Contact rates of three Seasons in Thailand

3.1 Mathematical model

Dengue disease can not be directly transmitted between the people. The infected female *Aedes* mosquito is the primary vector for this disease. The development of the virus and the mosquito are affected by the climatic factors. The effect of extrinsic incubation period of dengue virus caused the seasonality transmission of this disease[2]. When a vector bites someone who be infected with dengue virus, the virus is transferred to that mosquito and it become infected mosquito. After the infected vector bites the susceptible human then the virus move into the human bloodstream, and it spread throughout the body. Symptoms appeared about eight to ten days after the bitting from an infected mosquito. Symptoms are flu-like illness and can include high fever, nausea, vomiting, body aches, and headache. The moisture content, temperature, season and rainfall are influence to the mosquito development. Dengue infection is endemic in Thailand. From the data of Dengue cases in 2013, we can see that most dengue patients are occurred in rainy season as shown in figure 3.1.

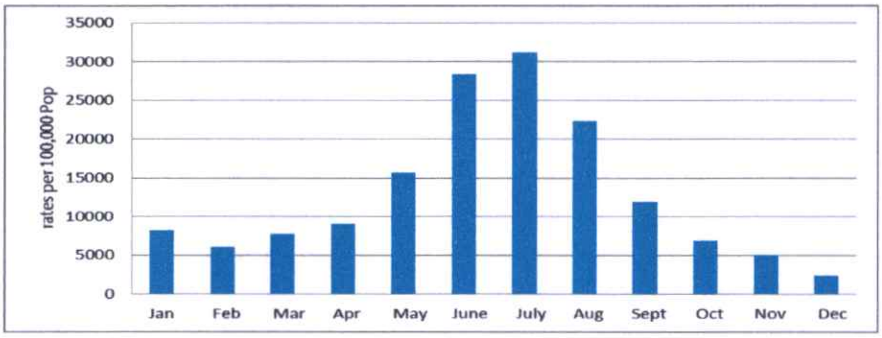


Figure 3.1 Reported cases of Dengue disease per 100,000 population in Thailand in 2013 [12].

The transmission rates are depend on the climatic factors, temperature, environment and the behaviors of human.

Transmission model of dengue disease with the different contact rates of three seasons in Thailand. In this model, summer season start from January to April. From May to September was rainy season and October to December was winter season.

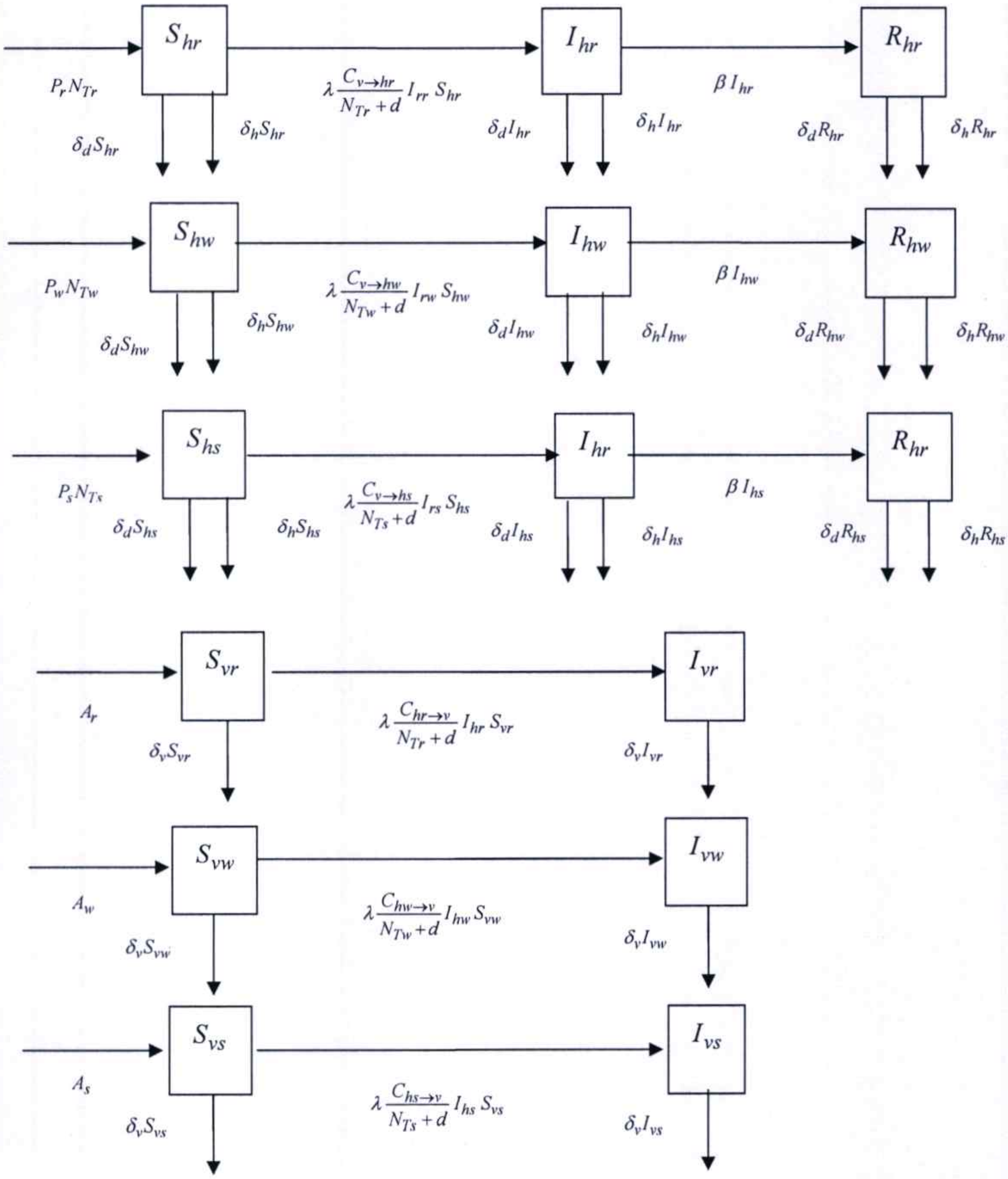


Figure 3.2. Transmission model of dengue disease with the different contact rates of three seasons in Thailand.

The dynamics of human population are given by

For rainy season,

$$\frac{d}{dt}S_{hr} = P_r N_{Tr} - \delta_d S_{hr} - \delta_h S_{hr} - \lambda \frac{C_{v\to hr}}{N_{Tr}+d} I_{vr} S_{hr} \tag{3.1}$$

$$\frac{d}{dt}I_{hr} = \lambda \frac{C_{v\to hr}}{N_{Tr}+d} I_{vr} S_{hr} - \delta_d I_{hr} - \delta_h I_{hr} - \beta I_{hr} \tag{3.2}$$

$$\frac{d}{dt}R_{hr} = \beta R_{hr} - \delta_d R_{hr} - \delta_h R_{hr} \tag{3.3}$$

For winter season,

$$\frac{d}{dt}S_{hw} = P_w N_{Tw} - \delta_d S_{hw} - \delta_h S_{hw} - \lambda \frac{C_{v\to hw}}{N_{Tw}+d} I_{vw} S_{hw} \tag{3.4}$$

$$\frac{d}{dt}I_{hw} = \lambda \frac{C_{v\to hw}}{N_{Tw}+d} I_{vw} S_{hw} - \delta_d I_{hw} - \delta_h I_{hw} - \beta I_{hw} \tag{3.5}$$

$$\frac{d}{dt}R_{hw} = \beta R_{hw} - \delta_d R_{hw} - \delta_h R_{hw} \tag{3.6}$$

For summer season.

$$\frac{d}{dt}S_{hs} = P_s N_{Ts} - \delta_d S_{hs} - \delta_h S_{hs} - \lambda \frac{C_{v\to hs}}{N_{Ts}+d} I_{vs} S_{hs} \tag{3.7}$$

$$\frac{d}{dt}I_{hs} = \lambda \frac{C_{v\to hs}}{N_{Ts}+d} I_{vs} S_{hs} - \delta_d I_{hs} - \delta_h I_{hs} - \beta I_{hs} \tag{3.8}$$

$$\frac{d}{dt}R_{hs} = \beta R_{hs} - \delta_d R_{hs} - \delta_h R_{hs} \tag{3.9}$$

We define the variables and parameter for above model in table 3.1.

Table 3.1 Definitions of variables and parameters for equation (3.1) – (3.9).

variable/ parameter	definition
S_{hr}	the number of susceptible human population in rainy season
I_{hr}	the number of infectious human population in rainy season
R_{hr}	the number of recovered human population in rainy season
S_{hw}	the number of susceptible human population in winter season
I_{hw}	the number of infectious human population in winter season
R_{hw}	the number of recovered human population in winter season
S_{hs}	the number of susceptible human population in summer season
I_{hs}	the number of infectious human population in summer season
R_{hs}	the number of recovered human population in summer season

The dynamics of the mosquito population can be described as the following equations:

$$\frac{d}{dt}S_{vr} = A_r - \lambda\frac{C_{hr\to v}}{N_{Tr}+d}I_{hr}S_{vr} - \delta_v S_{vr} \tag{3.10}$$

$$\frac{d}{dt}I_{hr} = \lambda\frac{C_{hr\to v}}{N_{Tr}+d}I_{hr}S_{vr} - \delta_v I_{vr} \tag{3.11}$$

For rainy season,

$$\frac{d}{dt}S_{vw} = A_w - \lambda\frac{C_{hw\to v}}{N_{Tw}+d}I_{hw}S_{vw} - \delta_v S_{vw} \tag{3.12}$$

$$\frac{d}{dt}I_{hw} = \lambda\frac{C_{hw\to v}}{N_{Tw}+d}I_{hw}S_{vw} - \delta_v I_{vw} \tag{3.13}$$

For winter season,

$$\frac{d}{dt}S_{vs} = A_s - \lambda\frac{C_{hs\to v}}{N_{Ts}+d}I_{hs}S_{vs} - \delta_v S_{vs} \tag{3.14}$$

$$\frac{d}{dt}I_{hs} = \lambda\frac{C_{hs\to v}}{N_{Ts}+d}I_{hs}S_{vs} - \delta_v I_{vs} \tag{3.15}$$

For summer season.

We define the variables and parameter for above model in table 3.2.

Table 3.2 Definitions of variables and parameters for equation (3.10) – (3.15).

variable/ parameter	definition
S_{vr}	the number of susceptible mosquito population in rainy season
I_{vr}	the number of infectious mosquito population in rainy season
S_{vw}	the number of susceptible mosquito population in winter season
I_{vw}	the number of infectious mosquito population in winter season
S_{vs}	the number of susceptible mosquito population in summer season
I_{vs}	the number of infectious mosquito population in summer season

Where the parameters are defined as follows:

Table 3.3 Definitions of variables and parameters for model (3.1) – (3.15).

variable/ parameter	**definition**
N_{Tr}	the total human population in rainy season
N_{Tw}	the total human population in winter season
N_{Ts}	the total human population in summer season
N_{Vr}	the total mosquito population in rainy season
N_{Vw}	the total mosquito population in winter season
N_{Vs}	the total mosquito population in summer season
δ_h	the natural death rate of human population
δ_d	the death rate of human population due to the disease
δ_v	the death rate of mosquito population
P	the birth rate of human population
$C_{v\to hr}$	the transmission probability of dengue disease from mosquito to human in rainy season
$C_{v\to hw}$	the transmission probability of dengue disease from mosquito to human in winter season
$C_{v\to hs}$	the transmission probability of dengue disease from mosquito to human in summer season
$C_{hr\to v}$	the transmission probability of dengue disease from human to mosquito in rainy season
$C_{hw\to v}$	the transmission probability of dengue disease from human to mosquito in winter season
$C_{hs\to v}$	the transmission probability of dengue disease from human to mosquito in summer season
β	the recovery rate of human population
λ	the biting rate of mosquito population
d	the number of other animals available as blood sources

We suppose that

$$N_{Hr} = S_{hr} + I_{hr} + R_{hr}\ ,\ N_{Hw} = S_{hw} + I_{hw} + R_{hw}\ ,\ N_{Hs} = S_{hs} + I_{hs} + R_{hs}\ ,$$

$$N_{Vr} = S_{vr} + I_{vr}\ ,\ N_{Vw} = S_{vw} + I_{vw} \text{ and } N_{Sv} = S_{vs} + I_{vs}\ .$$

We normalize equations (3.1) – (3.15) by defining new variables.

$$\overline{S}_{hr} = \frac{S_{hr}}{N_{Tr}},\ \overline{I}_{hr} = \frac{I_{hr}}{N_{Tr}},\ \overline{R}_{hr} = \frac{R_{hr}}{N_{Tr}},$$

$$\overline{S}_{hw} = \frac{S_{hw}}{N_{Tw}},\ \overline{I}_{hw} = \frac{I_{hw}}{N_{Tw}},\ \overline{R}_{hw} = \frac{R_{hw}}{N_{Tw}},$$

$$\overline{S}_{hs} = \frac{S_{hs}}{N_{Ts}},\ \overline{I}_{hs} = \frac{I_{hs}}{N_{Ts}},\ \overline{R}_{hs} = \frac{R_{hs}}{N_{Ts}},$$

$$\overline{S}_{vr} = \frac{S_{vr}}{N_{Vr}},\ \overline{S}_{vw} = \frac{S_{vw}}{N_{Vw}},\ \overline{S}_{vs} = \frac{S_{vs}}{N_{Vs}},$$

$$\overline{I}_{vr} = \frac{I_{vr}}{N_{Vr}},\ \overline{I}_{vw} = \frac{I_{vw}}{N_{Vw}},\ \overline{I}_{vs} = \frac{I_{vs}}{N_{Vs}}$$

The total human and mosquito populations have constant sizes, thus rates of change for total human and mosquito populations equal to zero. Thus, the birth rate of human population and death rates are equivalent for human population, the total mosquito population equals to $\frac{A_r}{\delta_v}$ in rainy season, $\frac{A_w}{\delta_v}$ in winter season, $\frac{A_s}{\delta_v}$ in summer season.

The set of dynamic equations (3.1) – (3.9) and (3.10) – (3.15) is reduced from fifteen equations to nine equations.

$$\frac{d}{dt}\overline{S}_{hr} = (\delta_d + \delta_h) - (\delta_d + \delta_h + \lambda \frac{C_{v\to hr}}{N_{Tr} + d}\overline{I}_{vr}N_{Vr})\overline{S}_{hr} \tag{3.16}$$

$$\frac{d}{dt}\overline{I}_{hr} = \lambda \frac{C_{v\to hr}}{N_{Tr} + d} I_{vr}N_{Vr}\overline{S}_{hr} - (\delta_d + \delta_h + \beta)\overline{I}_{hr} \tag{3.17}$$

$$\frac{d}{dt}\overline{S}_{hw} = (\delta_d + \delta_h) - (\delta_d + \delta_h + \lambda \frac{C_{v\to hw}}{N_{Tw} + d}\overline{I}_{vw}N_{Vw})\overline{S}_{hw} \tag{3.18}$$

$$\frac{d}{dt}\overline{I}_{hw} = \lambda \frac{C_{v\to hw}}{N_{Tw} + d} I_{vw}N_{Vw}\overline{S}_{hw} - (\delta_d + \delta_h + \beta)\overline{I}_{hw} \tag{3.19}$$

$$\frac{d}{dt}\overline{S}_{hs} = (\delta_d + \delta_h) - (\delta_d + \delta_h + \lambda \frac{C_{v\to hs}}{N_{Ts} + d}\overline{I}_{vs}N_{Vs})\overline{S}_{hs} \tag{3.20}$$

$$\frac{d}{dt}\overline{I}_{hs} = \lambda \frac{C_{v\to hs}}{N_{Ts} + d} I_{vs}N_{Vs}\overline{S}_{hs} - (\delta_d + \delta_h + \beta)\overline{I}_{hs} \tag{3.21}$$

$$\frac{d}{dt}\bar{I}_{vr} = \lambda \frac{C_{hr\to v}}{N_{Tr}+d}\bar{I}_{hr}N_{Tr}(1-\bar{I}_{vr}) - \delta_v I_{vr} \tag{3.22}$$

$$\frac{d}{dt}\bar{I}_{vw} = \lambda \frac{C_{hw\to v}}{N_{Tw}+d}\bar{I}_{hw}N_{Tw}(1-\bar{I}_{vw}) - \delta_v I_{vw} \tag{3.23}$$

$$\frac{d}{dt}\bar{I}_{vs} = \lambda \frac{C_{hs\to v}}{N_{Ts}+d}\bar{I}_{hs}N_{Ts}(1-\bar{I}_{vs}) - \delta_v I_{vs} \tag{3.24}$$

R_{hr}, R_{hw}, R_{hs} and S_{hr}, S_{hw} S_{hs} can be obtained from conditions

$S_{hr}+I_{hr}+R_{hr} = 1$, $S_{hw}+I_{hw}+R_{hw} = 1$, $S_{hs}+I_{hs}+R_{hs} = 1$ and

$S_{vr}+I_{vr} = 1$, $S_{vw}+I_{vw} = 1$ and $S_{vs}+I_{vs} = 1$.

3.2 Analysis of The mathematical model

3.2.1 The Steady State Solutions

The equilibrium points are found by setting the right hand side of (3.16) – (3.24) equal to zero. Let $E=(S_{hr}^*, I_{hr}^*, I_{vr}^*, S_{hw}^*, I_{hw}^*, I_{vw}^*, S_{hs}^*, I_{hs}^*, I_{vs}^*)$

$$(\delta_d+\delta_h)-(\delta_d+\delta_h+\lambda\frac{C_{v\to hr}}{N_{Tr}+d}\bar{I}_{vr}N_{Vr})\bar{S}_{hr} = 0 \tag{3.25}$$

$$\lambda\frac{C_{v\to hr}}{N_{Tr}+d}I_{vr}N_{Vr}\bar{S}_{hr} - (\delta_d+\delta_h+\beta)\bar{I}_{hr} = 0 \tag{3.26}$$

$$(\delta_d+\delta_h)-(\delta_d+\delta_h+\lambda\frac{C_{v\to hw}}{N_{Tw}+d}\bar{I}_{vw}N_{Vw})\bar{S}_{hw} = 0 \tag{3.27}$$

$$\lambda\frac{C_{v\to hw}}{N_{Tw}+d}I_{vw}N_{Vw}\bar{S}_{hw} - (\delta_d+\delta_h+\beta)\bar{I}_{hw} = 0 \tag{3.28}$$

$$(\delta_d+\delta_h)-(\delta_d+\delta_h+\lambda\frac{C_{v\to hs}}{N_{Ts}+d}\bar{I}_{vs}N_{Vs})\bar{S}_{hs} = 0 \tag{3.29}$$

$$\lambda\frac{C_{v\to hs}}{N_{Ts}+d}I_{vs}N_{Vs}\bar{S}_{hs} - (\delta_d+\delta_h+\beta)\bar{I}_{hs} = 0 \tag{3.30}$$

$$\lambda\frac{C_{hr\to v}}{N_{Tr}+d}\bar{I}_{hr}N_{Tr}(1-\bar{I}_{vr}) - \delta_v I_{vr} = 0 \tag{3.31}$$

$$\lambda\frac{C_{hw\to v}}{N_{Tw}+d}\bar{I}_{hw}N_{Tw}(1-\bar{I}_{vw}) - \delta_v I_{vw} = 0 \tag{3.32}$$

and $\lambda \frac{C_{hs\to v}}{N_{Ts}+d}\bar{I}_{hs}N_{Ts}(1-\bar{I}_{vs})-\delta_v I_{vs} = 0$ (3.33)

we get for S^*_{hr}

$$S^*_{hr}=\frac{(\delta_d+\delta_h)}{(\delta_d+\delta_h)+\cfrac{\lambda C_{v\to hr}N_{Vr}}{(N_{Tr}+d)+\cfrac{(N_{Tr}+d)^2\delta_v}{\lambda C_{hr\to v}I^*_{hr}N_{Tr}}}} \tag{3.34}$$

For I^*_{vr}, we have

$$I^*_{vr}=\frac{1}{1+\cfrac{(N_{Tr}+d)\delta_v}{\lambda C_{hr\to v}I^*_{hr}N_{Tr}}} \tag{3.35}$$

For S^*_{hw} ,we have

$$S^*_{hw}=\frac{(\delta_d+\delta_h)}{(\delta_d+\delta_h)+\cfrac{\lambda C_{v\to hw}N_{Vw}}{(N_{Tw}+d)+\cfrac{(N_{Tw}+d)^2\delta_v}{\lambda C_{hw\to v}I^*_{hw}N_{Tw}}}} \tag{3.36}$$

For I^*_{vw} ,we have

$$I^*_{vw}=\frac{1}{1+\cfrac{(N_{Tw}+d)\delta_v}{\lambda C_{hw\to v}I^*_{hw}N_{Tw}}} \tag{3.37}$$

For S^*_{hs} ,we have

$$S^*_{hs}=\frac{(\delta_d+\delta_h)}{(\delta_d+\delta_h)+\cfrac{\lambda C_{v\to hs}N_{Vs}}{(N_{Ts}+d)+\cfrac{(N_{Ts}+d)^2\delta_v}{\lambda C_{hs\to v}I^*_{hs}N_{Ts}}}} \tag{3.38}$$

For I^*_{vs} ,we have

$$I^*_{vs}=\frac{1}{1+\cfrac{(N_{Ts}+d)\delta_v}{\lambda C_{hs\to v}I^*_{hs}N_{Ts}}} \tag{3.39}$$

Substituting equations (3.34) and (3.35) into equation (3.26),

$$I^*_{hr}=\frac{(\delta_d+\delta_h)(-(N_{Tr}+d)^2(\beta+\delta_d+\delta_h)\delta_v+N_{Tr}N_{Vr}\lambda^2C_{v\to hr}C_{hr\to v})}{N_{Tr}(\beta+\delta_d+\delta_h)\lambda C_{hr\to v}((N_{Tr}+d)(\delta_d+\delta_h)+N_{Vr}\lambda C_{v\to hr})} \tag{3.40}$$

Substituting equations (3.36) and (3.37) in equation (3.28),

$$I^{*}_{hw}=\frac{(\delta_d+\delta_h)(-(N_{Tw}+d)^2(\beta+\delta_d+\delta_h)\delta_v+N_{Tw}N_{Vw}\lambda^2C_{v\to hw}C_{hw\to v})}{N_{Tw}(\beta+\delta_d+\delta_h)\lambda C_{hw\to v}((N_{Tw}+d)(\delta_d+\delta_h)+N_{Vw}\lambda C_{v\to hw})} \tag{3.41}$$

Substituting equations (3.34) and (3.35) in equation (3.26)

$$I^{*}_{hs}=\frac{(\delta_d+\delta_h)(-(N_{Ts}+d)^2(\beta+\delta_d+\delta_h)\delta_v+N_{Ts}N_{Vs}\lambda^2C_{v\to hs}C_{hs\to v})}{N_{Ts}(\beta+\delta_d+\delta_h)\lambda C_{hs\to v}((N_{Ts}+d)(\delta_d+\delta_h)+N_{Vs}\lambda C_{v\to hs})} \tag{3.42}$$

One of the solutions to equations (3.41) to (3.42) are $I^{*}_{hr}=0$, $I^{*}_{hw}=0$, $I^{*}_{hs}=0$. This solution yields the disease free equilibrium point $(1,0,0,1,0,0,1,0,0)$.

3.2.2 Stability Analysis

3.2.2.1 Local Stability

The local stability of each equilibrium point is determined from linearizing equation (3.16) – (3.24) about equilibrium point examining the eigenvalues of the resulting Jacobian matrix. We now consider the eigenvalues of the Jacobian matrix at each equilibrium point. From equation (3.16)-(3.24), we can write in the matrix form as follows:

$$J_{E_1}=\begin{pmatrix} -(\frac{\lambda C_{v\to hr}}{(N_{Tr}+d)}I^{*}_{vr}N_{Vr}+\delta_d+\delta_h) & 0 & -(\frac{\lambda C_{v\to hr}}{(N_{Tr}+d)}N_{Vr}S^{*}_{hr}) \\ \frac{\lambda C_{v\to hr}}{(N_{Tr}+d)}I^{*}_{vr}N_{Vr} & -(\beta+\delta_d+\delta_h) & (\frac{\lambda C_{v\to hr}}{(N_{Tr}+d)}N_{Tr}S^{*}_{hr}) \\ 0 & (\frac{\lambda C_{hr\to v}}{(N_{Tr}+d)}N_{Tr}S^{*}_{vr}) & -\delta_v \end{pmatrix} \tag{3.43}$$

in rain season

$$J_{E_2}=\begin{pmatrix} -(\frac{\lambda C_{v\to hw}}{(N_{Tw}+d)}I^{*}_{vw}N_{Vw}+\delta_d+\delta_h) & 0 & -(\frac{\lambda C_{v\to hw}}{(N_{Tw}+d)}N_{Vw}S^{*}_{hw}) \\ \frac{\lambda C_{v\to hw}}{(N_{Tw}+d)}I^{*}_{vw}N_{Vw} & -(\beta+\delta_d+\delta_h) & (\frac{\lambda C_{v\to hw}}{(N_{Tw}+d)}N_{Tw}S^{*}_{hw}) \\ 0 & (\frac{\lambda C_{hw\to v}}{(N_{Tw}+d)}N_{Tw}S^{*}_{vw}) & -\delta_v \end{pmatrix} \tag{3.44}$$

in winter season

$$J_{E_1} = \begin{pmatrix} -(\frac{\lambda C_{v\to hs}}{(N_{Ts}+d)} I^*_{vs} N_{Vs} + \delta_d + \delta_h) & 0 & -(\frac{\lambda C_{v\to hs}}{(N_{Ts}+d)} N_{Vs} S^*_{hs}) \\ \frac{\lambda C_{v\to hs}}{(N_{Ts}+d)} I^*_{vs} N_{Vs} & -(\beta+\delta_d+\delta_h) & (\frac{\lambda C_{v\to hs}}{(N_{Ts}+d)} N_{Ts} S^*_{hs}) \\ 0 & (\frac{\lambda C_{hs\to v}}{(N_{Ts}+d)} N_{Ts} S^*_{vs}) & -\delta_v \end{pmatrix} \quad (3.45)$$

in summer season

The eigenvalues (A) are the solution of the Characteristic equation [13,36]

$$\det(J - AI_3) = 0$$

where J is the Jacobian matrix evaluated at the equilibrium point . I_3 is the identity matrix.

3.2.2.2 Disease free state

Equilibrium point of disease free state $E_0 = (1,0,0,1,0,0,1,0,0)$ has eigenvalues as follows:

$$A_1 = -\delta_d - \delta_h,$$

$$A_2 = \frac{1}{2}\left(-\beta - \delta_d - \delta_h - \delta_v + \sqrt{\frac{(N_{Tr}+d)^2(\beta+\delta_d+\delta_h)^2 + 4N^2_{Tr} S_{vr} \lambda^2 C_{v\to hr} C_{hr\to v}}{(N_{Tr}+d)}} \right)$$

$$A_3 = \frac{1}{2}\left(-\beta - \delta_d - \delta_h - \delta_v - \sqrt{\frac{(N_{Tr}+d)^2(\beta+\delta_d+\delta_h)^2 + 4N^2_{Tr} S_{vr} \lambda^2 C_{v\to hr} C_{hr\to v}}{(N_{Tr}+d)}} \right)$$

$$A_4 = -\delta_d - \delta_h,$$

$$A_5 = \frac{1}{2}\left(-\beta - \delta_d - \delta_h - \delta_v + \sqrt{\frac{(N_{Tw}+d)^2(\beta+\delta_d+\delta_h)^2 + 4N^2_{Tw} S_{vw} \lambda^2 C_{v\to hw} C_{hw\to v}}{(N_{Tw}+d)}} \right)$$

$$A_6 = \frac{1}{2}\left(-\beta - \delta_d - \delta_h - \delta_v + \sqrt{\frac{(N_{Tw}+d)^2(\beta+\delta_d+\delta_h)^2 + 4N^2_{Tw} S_{vw} \lambda^2 C_{v\to hw} C_{hw\to v}}{(N_{Tw}+d)}} \right)$$

$$A_7 = -\delta_d - \delta_h,$$

$$A_8 = \frac{1}{2}\left(-\beta - \delta_d - \delta_h - \delta_v + \sqrt{\frac{(N_{Ts}+d)^2(\beta+\delta_d+\delta_h)^2 + 4N^2_{Ts} S_{vs} \lambda^2 C_{v\to hs} C_{hs\to v}}{(N_{Ts}+d)}} \right)$$

$$A_9 = \frac{1}{2}\left(-\beta - \delta_d - \delta_h - \delta_v - \sqrt{\frac{(N_{Ts}+d)^2(\beta+\delta_d+\delta_h)^2 + 4N^2{}_{Ts}S_{vs}\lambda^2 C_{v\to hs}C_{hs\to v}}{(N_{Ts}+d)}} \right)$$

From our evaluations, all eigenvalues have negative real parts when $R_0 < 1$. So this disease free equilibrium point is local stability when $R_0 < 1$. We define

$$R_0 = \frac{\dfrac{\dfrac{\lambda^2 C_{hs\to v}C_{v\to hs}N_{Vs}N_{Ts}}{(N_{Ts}+d)^2} + \dfrac{\lambda^2 C_{hw\to v}C_{v\to hw}N_{Vw}N_{Tw}}{(N_{Tw}+d)^2}}{(\beta+\delta d+\delta h)} + \dfrac{\lambda^2 C_{hr\to v}C_{v\to hr}N_{Vr}N_{Tr}}{(N_{Tr}+d)^2(\beta+\delta d+\delta h)}}{\delta v}$$

3.2.2.3 Endemic disease state

The endemic disease equilibrium point $E_1 = (S^*_{hr}, I^*_{hr}, I^*_{vr}, S^*_{hw}, I^*_{hw}, I^*_{vw}, S^*_{hs}, I^*_{hs}, I^*_{vs})$ has eigenvalues as follows:

$$A_1 = -\beta - \delta_d - \delta_h,$$

$$A_2 = -\delta_v$$

$$A_3 = \left[\delta_h + \frac{I^*{}_{hr}b_1}{b_7 b_2} + \frac{b_7 b_2^3 \delta_d b_3 + b_2(I^*_{hr} b_2 N_{Vr}\delta_d - b_7 N_{Tr}^2 S_{vr} b_3^2 b_4)b_6 + I^*_{hr} N_{Vr} N_{Tr}(N_{vr} - N_{Tr})S_{vr} b_3 b_4 b_5}{b_2^2\left(b_7 b_2 b_3 + I^*_{hr} b_1\right)}\right] / \left(-1 + \frac{b_7 b_4 b_3 b_6 N_{Tr}^2 S_{vr}}{b_2(b_7 b_3 b_2 + I^*_{hr} b_1)}\right)$$

$$A_4 = -\beta - \delta_d - \delta_h,$$

$$A_5 = -\delta_v$$

$$A_6 = \left[\delta_h + \frac{I^*{}_{hw}f_1}{f_7 f_2} + \frac{f_7 f_2^3 \delta_d f_3 + f_2(I^*_{hw} f_2 N_{Vw}\delta_d - b_7 N_{Tw}^2 S_{vw} f_3^2 f_4)f_6 + I^*_{hw} N_{Vw} N_{Tw}(N_{vw} - N_{Tw})S_{vw} f_3 f_4 f_5}{f_2^2\left(f_7 f_2 f_3 + I^*_{hw} f_1\right)}\right] / \left(-1 + \frac{f_7 f_4 f_3 f_6 N_{Tw}^2 S_{vw}}{f_2(f_7 f_3 f_2 + I^*_{hw} f_1)}\right)$$

$$A_7 = -\beta - \delta_d - \delta_h,$$

$$A_8 = -\delta_v$$

$$A_9 = \left[\delta_h + \frac{I^*{}_{hs}g_1}{g_7 g_2} + \frac{g_7 g_2^3 \delta_d g_3 + g_2(I^*_{hs} g_2 N_{Vs}\delta_d - g_7 N_{Ts}^2 S_{vs} g_3^2 g_4)g_6 + I^*_{hs} N_{Vs} N_{Ts}(N_{vs} - N_{Ts})S_{vs} g_3 g_4 g_5}{g_2^2\left(g_7 g_2 g_3 + I^*_{hs} g_1\right)}\right] / \left(-1 + \frac{g_7 g_4 g_3 g_6 N_{Ts}^2 S_{vs}}{g_2(g_7 g_3 g_2 + I^*_{hs} g_1)}\right)$$

From this model, we known the points local stability about equilibrium point examining the eigenvalue of the results Jacobian matrix. Which we have two equilibrium points (equilibrium point of disease free state and the endemic disease equilibrium point).

3.3 Numerical results

We consider the numerical solutions for dengue virus transmission. The parameters in this study are determined by the real life observations. The values of the parameters are as follows: $\delta_h = 1/(365*70)$ day^{-1}, corresponding to life expectancy of 70 years for human. $\delta_d = 1/7.5$ corresponding to death rate due to the disease of human. The other parameters are arbitrary chosen. Numerical solutions of (3.25) – (3.33) are shown in the following figures.

Cases1. $R_0 < 1$;

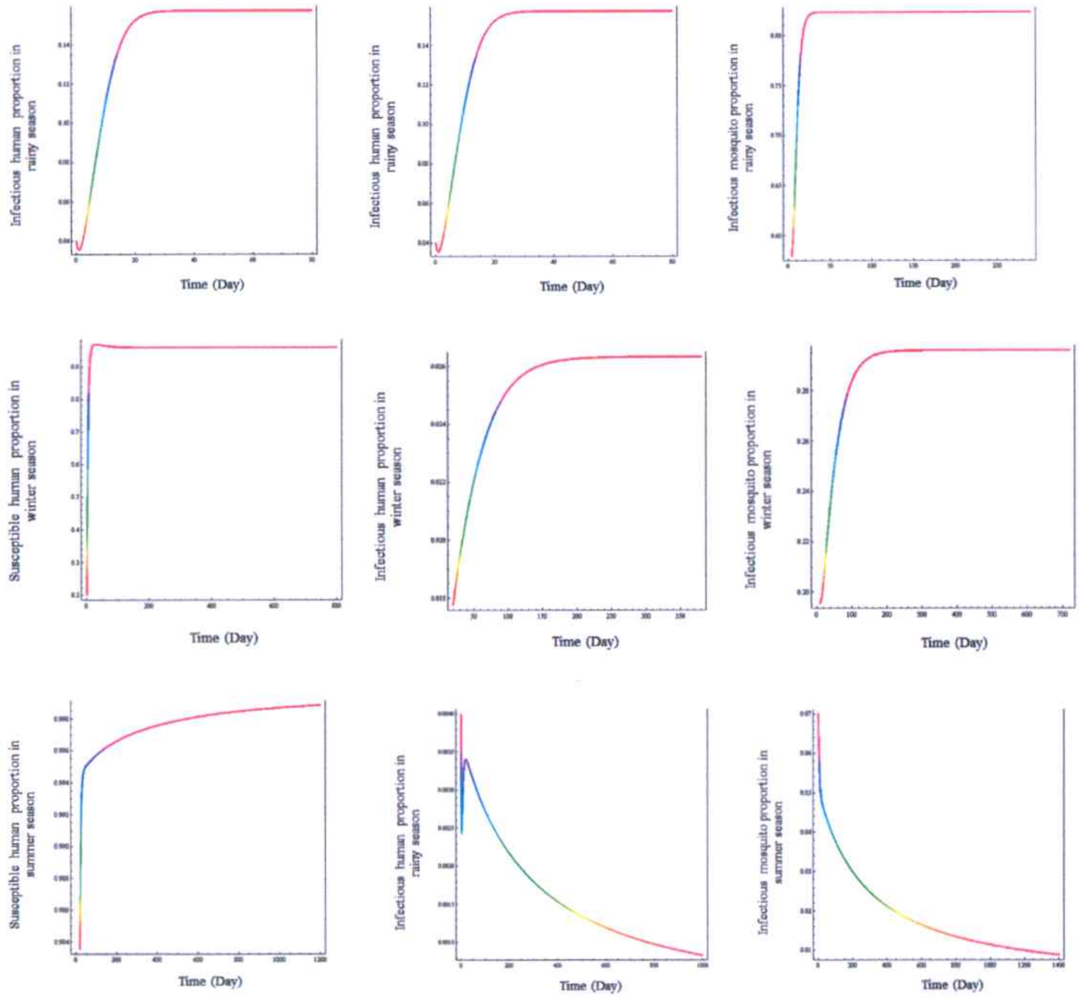


Figure 3.3. Times series solutions of susceptible, infectious human and infectious mosquito population in rainy season, susceptible, infectious human and infectious mosquito population in winter season, and susceptible, infectious human infectious mosquito population in summer season respectively. The parameters are $\delta_h = 1/(365*70)^{-1}$ day, $\delta_d = 1/7.5$, $\lambda = 1/17$, $c_{v \to hr} = 0.076$, $c_{v \to hw} = 0.065$, $c_{v \to hs} = 0.04$, $c_{hr \to v} = 0.142$, $c_{hw \to v} = 0.076$, $c_{hs \to v} = 0.067$, $N_{tr} = 70{,}000$, $N_{tw} = 51{,}000$, $N_{ts} = 13{,}000$, $N_{vr} = 5{,}000$, $N_{vw} = 2{,}000$, $N_{vs} = 1{,}300$, $\beta = 0.143$, $\delta_v = 0.076$ The solutions converge to the disease free states (1,0, 0, 1, 0, 0, 1, 0, 0)

Cases2. $R_0 > 1$;

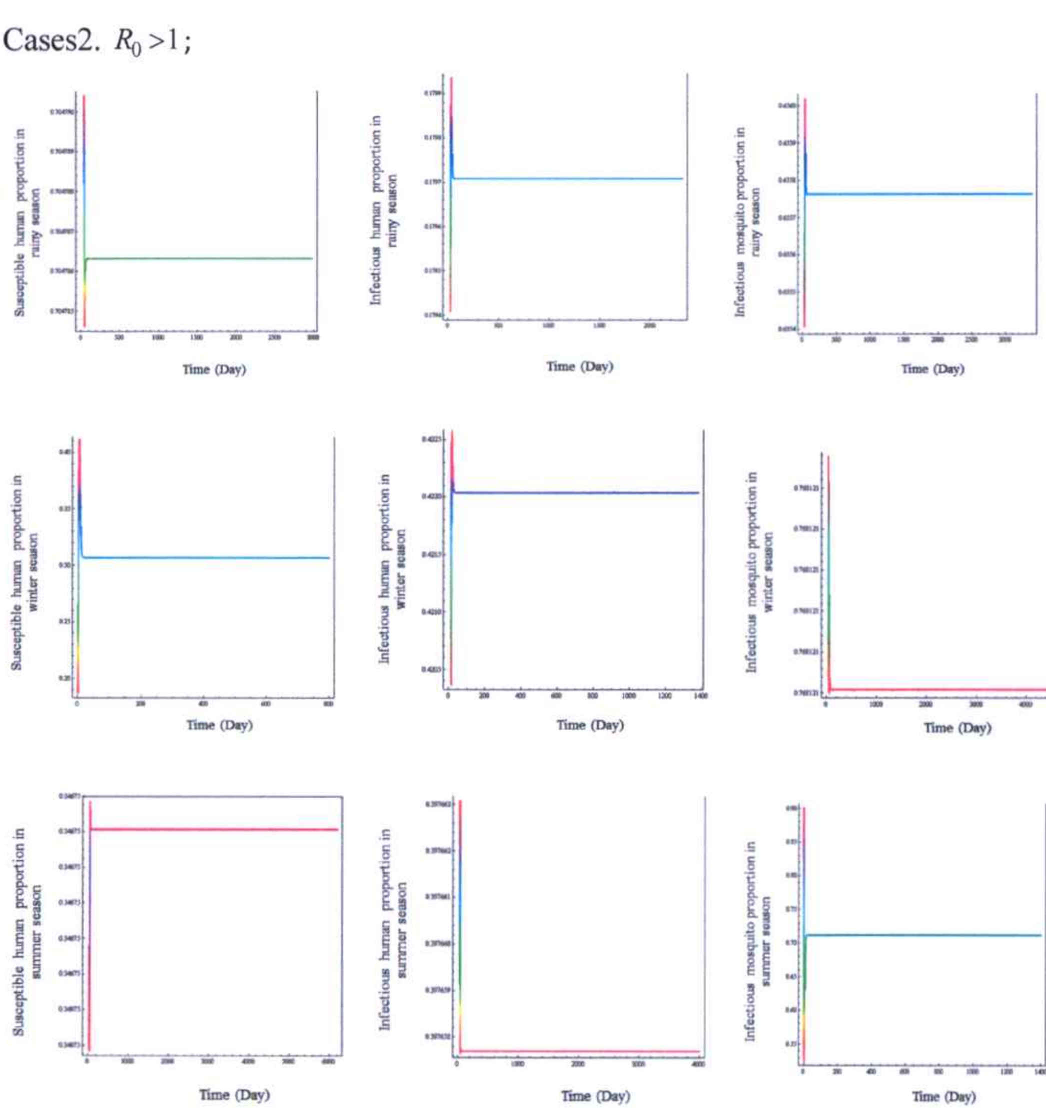


Figure 3.4. Times series solutions of susceptible, infectious human and infectious mosquito population in rainy season, susceptible, infectious human and infectious mosquito population in winter season, and susceptible, infectious human infectious mosquito population in summer season respectively. The parameters are $\delta_h = 1/(365*70)^{-1}$ day, $\delta_d = 1/7.5$, $\lambda = 1/8$, $c_{v\to hr} = 0.143$, $c_{v\to hw} = 0.33$, $c_{v\to hs} = 0.25$, $c_{hr\to v} = 0.33$, $c_{hw\to v} = 0.25$, $c_{hs\to v} = 0.2$, $N_{tr} = 160{,}000$, $N_{tw} = 62{,}000$, $N_{ts} = 33{,}000$, $N_{vr} = 20{,}000$, $N_{vw} = 13{,}000$, $N_{vs} = 10{,}000$, $\beta = 0.143$, $\delta_v = 0.25$ The solutions converge to the disease free states (0.704, 0.179, 0.655, 0.094, 0.551, 0.802, 0.643,0.217,0.578)

After our calculations, we found that all eigenvalues have negative real parts when $R_0 < 1$. This means that the endemic disease state is local stability [13] for $R_0 > 1$.

$$R_0 = \frac{\dfrac{\dfrac{\lambda^2 C_{hs\to v} C_{v\to hs} N_{Vs} N_{Ts}}{(N_{Ts}+d)^2} + \dfrac{\lambda^2 C_{hw\to v} C_{v\to hw} N_{Vw} N_{Tw}}{(N_{Tw}+d)^2}}{(\beta+\delta d+\delta h)} + \dfrac{\lambda^2 C_{hr\to v} C_{v\to hr} N_{Vr} N_{Tr}}{(N_{Tr}+d)^2(\beta+\delta d+\delta h)}}{\delta v}.$$

When we change the value of $c_{v\to hr}$, $c_{v\to hw}$, $c_{v\to hs}$, $c_{hr\to v}$, $c_{hw\to v}$ and $c_{hs\to v}$, the basic reproduction number will be changed. The transmission propability, of dengue disease in each season is influence to the basic reproduction number.

Chapter IV
Transmission Model of Dengue virus by *Aedes aegypti* and *Aedes albopictus*

4.1. Analysis of Data

In this chapter, we analyze the mathematical modeling by formulating the differential equations for describing the transmission of dengue virus among human and mosquitoes. An outbreak of this disease started in the Philippines in 1953, subsequently in Thailand with 150,000 to 200,000 cases in 1978, dengue outbreak in Thailand have periodically. Dengue infection is endemic in Thailand and many other tropical and subtropical countries. The incidence rate of dengue disease reported to the Division of Epidemiology, Ministry of Public Health classified by month, Thailand between the period 2002 -2011 are studied. In 2010, DHF was first recognized as the epidemic disease, there were 60,770 cases with 35 deaths reported. DF was the epidemic disease, there were 53,149 cases with 1 deaths. DSS was the epidemic disease, there were 3,028 cases with 103 deaths. Which that, the incidences are increased during May 2010 – October 2010. There were 7,616 cases with 7 deaths in May 2010, The incidences are increased to 23,462 cases with 27 deaths generally peak in August 2010. Figure 4.1 shows the reported cases of Dengue disease per 100,000 population in Thailand during 2003 and 2011 (month - by - month)[12].

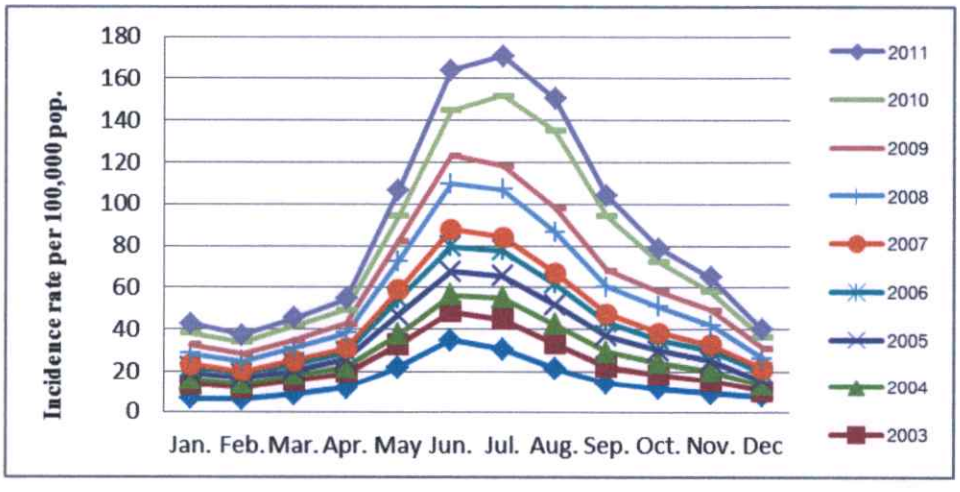


Figure 4.1. Reported cases of Dengue disease per 100,000 population in Thailand during 2003 and 2011 (month - by - month)[12].

The considerations of a dengue epidemic from several observations. Firstly, in certain tropical and subtropical areas, environments, climatic variables such as temperature, humidity, epidemics occurs on a yearly basis. Secondly, the disease and rain cycles are correlated; the epidemic starts at the beginning of rainy, spring - summer season, peaks 3 - 4 months afterward and ends in the beginning of the dry autumn - winter season (F.A.B.). The epidemic exhibited bi - yearly over - winter, appearing every other year, peaking in the summer and disappearing in the winter. During period, epidemic were contained. It is transmission to the human by biting of the infected female *Aedes* mosquitoes as the primary mosquitoes vector. These changes can have a great impact on the densities of *Aedes* mosquitoes, by creating more larval breeding habitats for dengue mosquitoes. Two species of *Aedes* mosquitoes, *Aedes aegypti* (Linnaeus) and *Aedes albopictus* (Skuse) are known to be important dengue virus vectors in Thailand. *Aedes aegypti* is a principal vector in urban areas, *Aedes albopictus* serves as an important vector in the rural and undeveloped areas (Pant, Halstead,) Although different, the preferred breeding habitats of theses 2 species that could facilitate the Aedes control of disease in Thailand. Figure 4.2 shows monthly distribution of *Aedes aegypti* and *Aedes albopictus* from January to December, 2010.

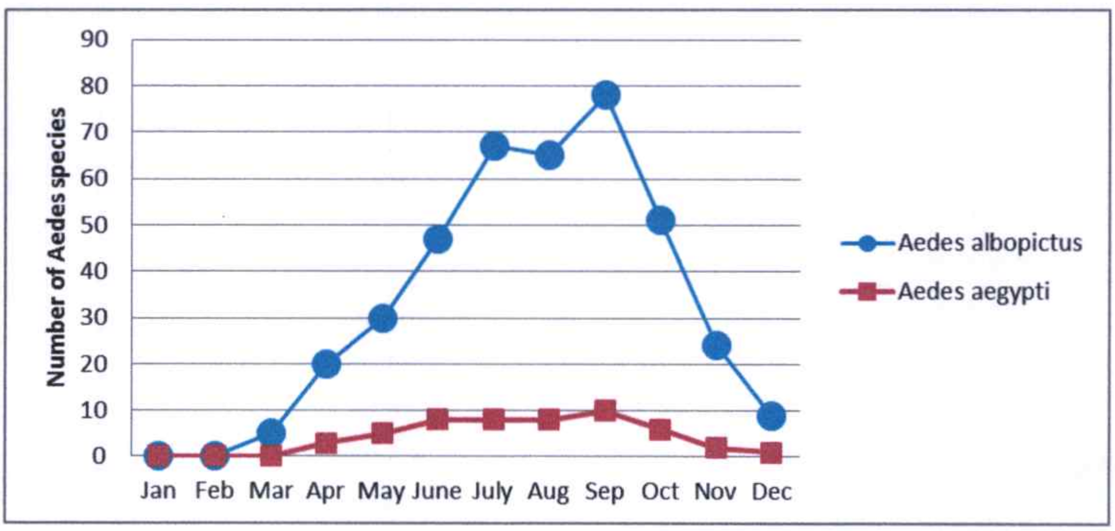


Figure 4.2. Monthly distribution of *Aedes aegypti* and *Aedes albopictus* from January to December, 2010 [15].

4.2. Formulating the Mathematical Model

We propose a model to study the transmission of dengue virus infection by incorporating the different behaviors of *Aedes aegypti* and *Aedes albopictus* into the SIR model. We categorize the human population and mosquito (*Aedes aegypti* and *Aedes albopictus*), we assume that human and mosquito population have constant sizes. The human population is divided into three classes: susceptible, infected and recovered human populations. The vector population is divided into two groups: susceptible and infected classes because the mosquito never recover from infection. The mathematical model for this transmission is based on the transmission diagram as shown in figure 4.3.

This model, we develop from the first model by Esteva and Vargas [13]. The first model described the transmission of dengue disease by SIR model. We separate the transmission of disease through the season (rainy, winter and summer) in Thailand. We propose a mathematical model to study the transmission dengue infection by introducing mosquito with a variable human population size into the SIR model. We classify the human population. Each group has constant in size and it is divided into three classes, susceptible, infectious and recovered human population. The mosquito population is divided into two groups, susceptible and infectious classes. The mosquito do not recover. The transmission diagram of dengue disease between human population and two species of vector population (Aedes aegypti and Aedes albopictus) is show in figure 4.3. We see specific characteristic of figure 4.1 and figure 4.2, the outbreak of the disease and the distribution of two species of vector according to the sine function.

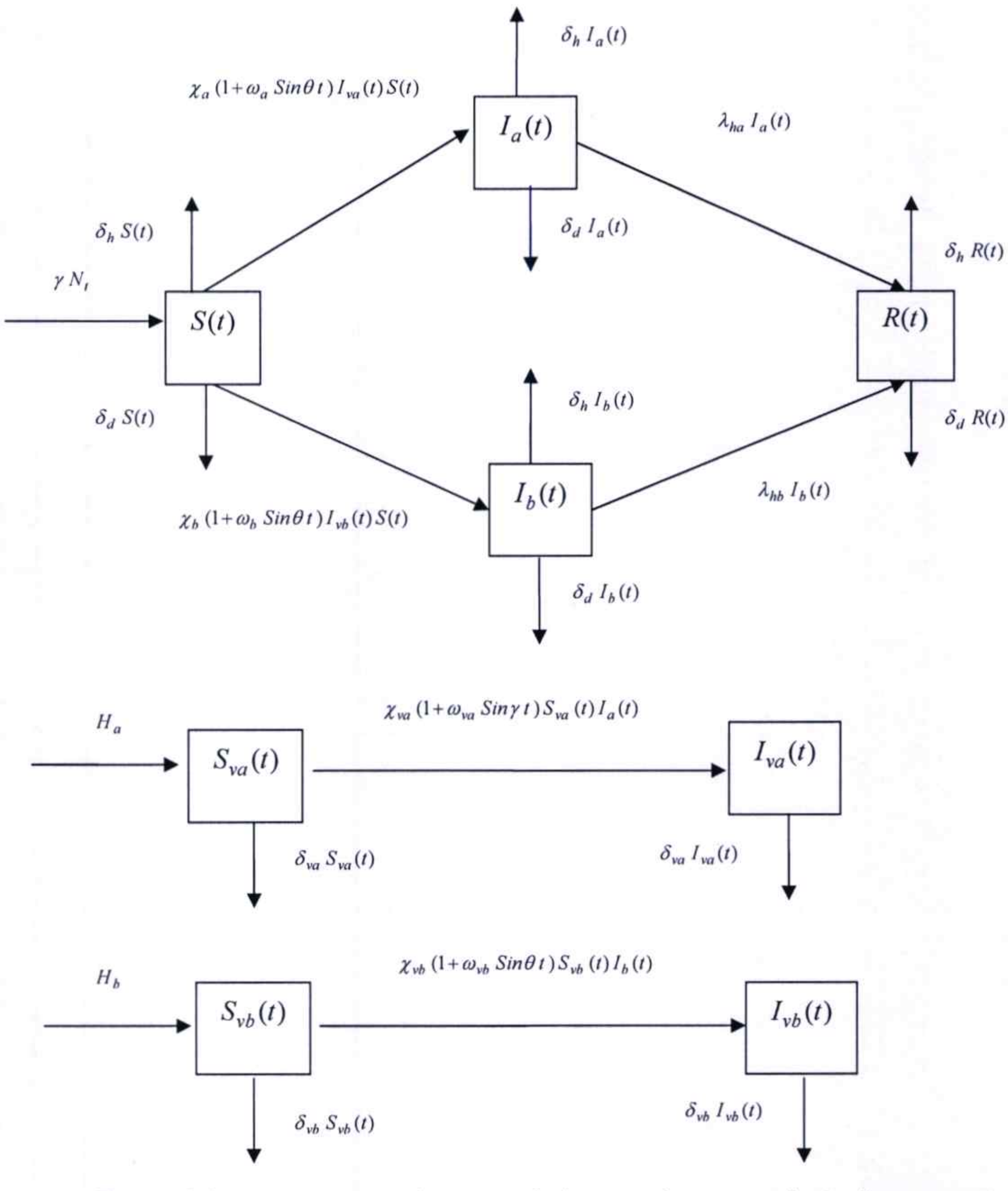


Figure 4.3. Transmission diagram of dengue disease with *Aedes aegypti* and *Aedes albopictus.*

The block diagram of the model is given in figure 4.3. Using diagram in figure 4.3, the dynamical system for human population and vector population are described by the following three system of ordinary differential equations:

$$\frac{d}{dt}S(t) = \gamma N_t - \chi_a(1+\omega_a Sin\theta t)I_{va}(t)S(t) - \delta_d S(t) - \delta_h S(t) - \chi_b(1+\omega_b Sin\theta t)I_{vb}(t)S(t) \quad (4.1)$$

$$\frac{d}{dt}I_a(t) = \chi_a(1+\omega_a Sin\theta t)I_{va}(t)S(t) - \delta_d I_a(t) - \delta_h I_a(t) - \lambda_{ha}I_a(t) \quad (4.2)$$

$$\frac{d}{dt}I_b(t) = \chi_b(1+\omega_b Sin\gamma t)I_{vb}(t)S(t) - \delta_d I_b(t) - \delta_h I_b(t) - \lambda_{hb}I_b(t) \quad (4.3)$$

$$\frac{d}{dt}R(t) = -\delta_d R(t) - \delta_h R(t) - \lambda_{ha}I_a(t) - \lambda_{hb}I_b(t) \quad (4.4)$$

Equations (4.1) – (4.4) describe the transmission of dengue virus with a variable human population size with the condition

$N_t = S(t) + I_a(t) + I_b(t) + R(t)$.

Let

$S(t)$ be the number of susceptible human population,

$I_a(t)$ be the number of infected human population who be infected with *Aedes aegypti*,

$I_b(t)$ be the number of infected human population who be infected with *Aedes albopictus* ,

$R(t)$ be the number of recovered human population,

N_t be the total human population.

The parameter defined as follows:

Table 4.1. Definitions of variables and parameters for model (4.1) – (4.4).

variable/ parameter	definition
γ	birth rate of human population
N_t	the total human population
χ_a	the biting rate of *Aedes aegypti* population
χ_b	the biting rate of *Aedes albopictus* population
ω_a	a measure of influence on the transmission process from human population to *Aedes aegypti*
ω_b	a measure of influence on the transmission process from human population to *Aedes albopictus*
λ_{ha}	the recovery rate of human population who be infected with of *Aedes aegypti*
λ_{hb}	the recovery rate of human population who be infected with of *Aedes albopictus*
δ_h	the natural death rate of human population
δ_d	the death rate of human population due to the disease

We assume that the total population remains constant. Note that $\frac{d}{dt}N_t = \frac{d}{dt}S(t) + \frac{d}{dt}I_a(t) + \frac{d}{dt}I_b(t) + \frac{d}{dt}R(t)$. If we add the RHS (right – hand side) of equation (4.1) – (4.4), then we obtain the following equations:

$$\frac{d}{dt}N_t = \frac{d}{dt}S(t) + \frac{d}{dt}I_a(t) + \frac{d}{dt}I_b(t) + \frac{d}{dt}R(t) \tag{4.5}$$

$$\begin{aligned} \frac{d}{dt}N_t = {} & (\gamma N_t - \chi_a(1+\omega_a Sin\theta t)I_{va}(t)S(t) - \delta_d S(t) - \delta_h S(t) - \chi_b(1+\omega_b Sin\theta t)I_{vb}(t)S(t)) \\ & +(\chi_a(1+\omega_a Sin\theta t)I_{va}(t)S(t) - \delta_d I_a(t) - \delta_h I_a(t) - \lambda_{ha}I_a(t)) \\ & +(\chi_b(1+\omega_b Sin\gamma t)I_{vb}(t)S(t) - \delta_d I_b(t) - \delta_h I_b(t) - \lambda_{hb}I_b(t)) \\ & +(-\delta_d R(t) - \delta_h R(t) - \lambda_{ha}I_a(t) - \lambda_{hb}I_b(t)) \end{aligned} \tag{4.6}$$

$$\frac{d}{dt}N_t = \gamma N_t - (\delta_d + \delta_h)(S(t) + I_a(t) + I_b(t) + R(t)) \tag{4.7}$$

Since the total human population is constant, i.e. $\frac{d}{dt}N_t = 0$, Thus the birth rate would have to be equal to death rate, $\gamma = \delta_d + \delta_h$.

The dynamical change of mosquito population are described by:

$$\frac{d}{dt}S_{va}(t) = H_a - \chi_{va}(1+\omega_{va}Sin\,\theta t)S_{va}(t)I_a(t) - \delta_{va}S_{va}(t) \tag{4.8}$$

$$\frac{d}{dt}I_{va}(t) = \chi_{va}(1+\omega_{va}Sin\,\theta t)S_{va}(t)I_a(t) - \delta_{va}I_{va}(t) \tag{4.9}$$

$$\frac{d}{dt}S_{vb}(t) = H_b - \chi_{vb}(1+\omega_{vb}Sin\,\theta t)S_{vb}(t)I_b(t) - \delta_{vb}S_{vb}(t) \tag{4.10}$$

$$\frac{d}{dt}I_{vb}(t) = \chi_{vb}(1+\omega_{vb}Sin\,\theta t)S_{vb}(t)I_{vb}(t) - \delta_{vb}I_{vb}(t) \tag{4.11}$$

Equations (4.8) – (4.11) describe the transmission of the dengue virus with a variable vector population size with the conditions

$N_{va} = S(t) + I_{va}(t)$

and $N_{vb} = S(t) + I_{vb}(t)$.

Let

$S_{va}(t)$, $I_{va}(t)$ be the number of susceptible and infected *Aedes aegypti* mosquitoes,

$S_{vb}(t)$, $I_{vb}(t)$ be the number of susceptible and infected *Aedes albopictus* mosquitoes.

Table 4.2. Definitions of variables and parameters for equations (4.8) – (4.11).

variable/ parameter	definition
δ_{va}	the death rate of *Aedes aegypti* population
δ_{vb}	the death rate of *Aedes albopictus* population
χ_{va}	the transmission probability of dengue disease from vector (*Aedes aegypti*) to human population
χ_{vb}	the transmission probability of dengue disease from vector (*Aedes albopictus*) to human population
H_a	the constant recruitment rate of vector population(*Aedes aegypti*)
H_b	the constant recruitment rate of vector population (*Aedes albopictus*)

The number of mosquitoes population is assumed to be constant.

Note that

$$\frac{d}{dt}N_{va}=\frac{d}{dt}S_{va}(t)+\frac{d}{dt}I_{va}(t)$$ for *Aedes aegypti*

and

$$\frac{d}{dt}N_{vb}=\frac{d}{dt}S_{vb}(t)+\frac{d}{dt}I_{vb}(t)$$ for *Aedes albopictus* .

It we add the RHS (right – hand side) of equation (4.8) – (4.11), then we obtain the following equations:

$$\frac{d}{dt}N_{va}=\frac{d}{dt}S_{va}(t)+\frac{d}{dt}I_{va}(t) \tag{4.12}$$

$$\begin{aligned}\frac{d}{dt}N_{va}= &\ (H_a-\chi_{va}(1+\omega_{va}Sin\theta t)S_{va}(t)I_a(t)-\delta_{va}S_{va}(t)\ )\\ &+(\chi_{va}(1+\omega_{va}Sin\theta t)S_{va}(t)I_a(t)-\delta_{va}I_{va}(t))\end{aligned} \tag{4.13}$$

$$\frac{d}{dt}N_{va}=H_a-\ \delta_{va}(S_{va}(t)+I_{va}(t))\cdot \tag{4.14}$$

Since the total mosquitoes population is constant, i.e. $\frac{d}{dt}N_{va}=0$ for *Aedes aegypti.*

$$\frac{d}{dt}N_{vb}=\frac{d}{dt}S_{vb}(t)+\frac{d}{dt}I_{vb}(t) \tag{4.15}$$

$$\begin{aligned}\frac{d}{dt}N_{vb}= &\ (H_b-\chi_{vb}(1+\omega_{vb}Sin\theta t)S_{vb}(t)I_b(t)-\delta_{vb}S_{vb}(t))\\ &+(\chi_{vb}(1+\omega_{vb}Sin\theta t)S_{vb}(t)I_{vb}(t)-\delta_{vb}I_{vb}(t))\end{aligned} \tag{4.16}$$

$$\frac{d}{dt}N_{vb}=H_b-\ \delta_{vb}(S_{vb}(t)+I_{vb}(t))\cdot \tag{4.17}$$

Since the total mosquitoes population is constant, i.e. $\frac{d}{dt}N_{vb}=0$ for *Aedes albopictus.*

Thus we have $N_{va}=\dfrac{H_a}{\delta_{va}}$ and $N_{vb}=\dfrac{H_b}{\delta_{vb}}$.

The rate of human contribute to the population of the pathogen, it is set to the γ - periodic seasonality function. That periodic functions of time with a common period, $\gamma=365$ days, or 1 year.

We follow the equations outlined by Esteva and Vargas [13] and introduce the new variables as follows:

$$S'(t)=\frac{S(t)}{N_t} \qquad I'_a(t)=\frac{I_a(t)}{N_t} \qquad I'_b(t)=\frac{I_b(t)}{N_t} \qquad R'(t)=\frac{R(t)}{N_t}$$

$$S'_{va}(t)=\frac{S_{va}(t)}{N_{va}} \qquad I'_{va}(t)=\frac{I_{va}(t)}{N_{va}} \qquad S'_{vb}(t)=\frac{S_{vb}(t)}{N_{vb}} \qquad I'_{vb}(t)=\frac{I_{vb}(t)}{N_{vb}},$$

where N_{va} is the total vector population (*Aedes aegypti*)

N_{vb} is the total vector population (*Aedes albopictus*).

with conditions

$S'(t)+I'_a(t)+I'_b(t)+R'(t)=1$, $S'_{va}(t)+I'_{va}(t)=1$ and $S'_{vb}(t)+I'_{vb}(t)=1$.

We assume $N_t>0$, $N_{va}>0$ and $N_{vb}>0$, for all $t>0$.

Then system (4.1) – (4.11) can be written as:

$$\frac{d}{dt}S'(t)=(\delta_d+\delta_h)-[\delta_d+\delta_h+\chi_a(1+\omega_a Sin\theta t)I'_{va}(t)(N_{va})+\chi_b(1+\omega_b Sin\theta t)I'_{vb}(t)(N_{vb})]S'(t) \tag{4.18}$$

$$\frac{d}{dt}I'_a(t)=\chi_a(1+\omega_a Sin\theta t)I'_{va}(t)(N_{va})S'(t)-(\delta_d+\delta_h+\lambda_{ha})I'_a(t) \tag{4.19}$$

$$\frac{d}{dt}I'_b(t)=\chi_b(1+\omega_b Sin\theta t)I'_{vb}(t)(N_{vb})S'(t)-(\delta_d+\delta_h+\lambda_{hb})I'_b(t) \tag{4.20}$$

$$\frac{d}{dt}I'_{va}(t)=\chi_{va}(1+\omega_{va}Sin\,\theta t)(1-I'_{va}(t))I'_a(t)N_t-\delta_{va}I'_{va}(t) \tag{4.21}$$

$$\frac{d}{dt}I'_{vb}(t)=\chi_{vb}(1+\omega_{vb}Sin\,\theta t)(1-I'_{vb}(t))I'_b(t)N_t-\delta_{vb}I'_{vb}(t) \tag{4.22}$$

with

$$0\le S'(0)\le 1,\ 0\le I'_a(0)\le 1, 0\le I'_b(0)\le 1, 0\le I'_{va}(0)\le 1 \text{ and } 0\le I'_{vb}(0)\le 1. \tag{4.23}$$

Let

$$\Omega''=\{(S',I'_a,I'_b,I'_{va},I'_{vb}):0\le S'(0)\le 1, 0\le I'_a(0)\le 1, 0\le I'_b(0)\le 1, 0\le I'_{va}(0)\le 1, 0\le I'_{vb}(0)\le 1\}, \tag{4.24}$$

then Ω'' is positively invariant for system (4.18) – (4.22).

Since $R'(t)=1-S'(t)-I'_a(t)-I'_b(t)$ and we have $\frac{d}{dt}R'(t) = -(\frac{d}{dt}S'(t)+\frac{d}{dt}I'_a(t)+\frac{d}{dt}I'_b(t))$ and $S'_{va}(t)=1-I'_{va}(t)$, and $\frac{d}{dt}S'_{va}(t) = -\frac{d}{dt}I'_{va}(t)$. We further have $S'_{vb}(t)=1-I'_{vb}(t)$, we have $\frac{d}{dt}S'_{vb}(t)=-\frac{d}{dt}I'_{vb}(t)$, repectively.

4.3. Analysis of the Mathematical Model

4.3.1. The Steady State Solutions

The equilibrium points $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ are obtained by setting the right hand side of equation (4.18) – (4.22) equal to zero.

Let $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ be an equilibrium point of the following equations.

$$(\delta_d+\delta_h)-[\delta_d+\delta_h+\chi_a(1+\omega_a Sin\theta t)I'_{va}(t)(N_{va})+\chi_b(1+\omega_b Sin\theta t)I'_{vb}(t)(N_{vb})]S'(t) = 0 \quad (4.25)$$

$$\chi_a(1+\omega_a Sin\theta t)I'_{va}(t)(N_{va})S'(t)-(\delta_d+\delta_h+\lambda_{ha})I'_a(t) = 0 \quad (4.26)$$

$$\chi_b(1+\omega_b Sin\theta t)I'_{vb}(t)(N_{vb})S'(t)-(\delta_d+\delta_h+\lambda_{hb})I'_b(t) = 0 \quad (4.27)$$

$$\chi_{va}(1+\omega_{va} Sin\theta t)(1-I'_{va}(t))I'_a(t)N_t-\delta_{va}I'_{va}(t) = 0 \quad (4.28)$$

$$\chi_{vb}(1+\omega_{vb} Sin\theta t)(1-I'_{vb}(t))I'_b(t)N_t-\delta_{vb}I'_{vb}(t) = 0 \quad (4.29)$$

We get for S^*,

$$S^* = \frac{\delta_d+\delta_h}{\delta_d+\delta_h+\chi_a(1+\omega_a Sin\theta t)I^*{}_{va}(t)(N_{va})+\chi_b(1+\omega_b Sin\theta t)I^*{}_{vb}(t)(N_{vb})} \quad (4.30)$$

Substituting equation (4.30) in equation (4.27), for I_b^*

$$I_b^* = \frac{I_{vb}^* N_{vb}\chi_b(1+\omega_a\, Sin\,\theta t)(\rho_1)}{(\lambda_{hb}+\delta_d+\delta_h)(\rho_2)(\rho_3)} \quad (4.31)$$

Substituting equation (4.31) in equation (4.29), for I_{vb}^*

$$I_{vb}^*((\frac{-(\rho_2)(\rho_3)(\lambda_{hb}+\delta_d+\delta_h)\delta_{vb}+\rho_1 N_t N_{vb}\lambda_{vb}\chi_b+\rho_1 N_t N_{vb}\lambda_{vb}\chi_b Sin(\theta t)(\omega_b+\chi_{vb}+\omega_b\chi_{vb}Sin(\theta t))}{\rho_1 N_t N_{vb}\lambda_{vb}\chi_b(1+\omega_a\, Sin\theta t)(1+\omega_{va}\, Sin\theta t)})*I_{vb}^*) = 0 \quad (4.32)$$

The solutions I_{vb}^{*} of equation (4.32) are $I_{vb}^{*}=0$ and

$$I_{vb}^{*}=\frac{(-(\rho_5)(\delta_d+\delta_h+I_{va}^{*}N_{va}\chi_a)+N_tN_{vb}\chi_b\rho_6+Sin(\theta t)(-I_{va}^{*}N_{va}\chi_a\omega_a\rho_5+N_tN_{vb}(\omega_b+\omega_{vb})\chi_b\rho_6+N_tN_{vb}\omega_b\,\omega_{vb}\chi_b\rho_6Sin(\theta t)))}{(N_{vb}\chi_b(1+\omega_b\,Sin(\theta t))(N_t\rho_6+\rho_5+N_t\omega_{vb}\rho_6\,Sin(\theta t)))} \quad (4.33)$$

Substituting equation (4.33) in equation (4.30), for S^{*}

$$S^{*}=\frac{\rho_1}{\rho_2}\ . \quad (4.34)$$

Substituting equation (4.34) in equation (4.26), for I_a^{*}

$$I_a^{*}=\frac{I_{va}^{*}N_{va}\,\chi_a(1+\omega_a Sin\theta t)(\rho_1)}{(\lambda_{ha}+\delta_h+\delta_d)(\rho_2)(\rho_3)}\ . \quad (4.35)$$

Substituting equation (4.35) into equation (4.28), we obtain that I_{va}^{*} , a solution of the following equation:

$$\begin{aligned}
&I_{va}^{*}(([2(N_{va}\chi_{va}(2+\omega_a\omega_{va})\rho_5\chi_a+\rho_6(-2\,\rho_4+N_t\,N_{va}\omega_{va}(2+\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})\chi_a)-N_{vb}\chi_{vb}(2+\omega_b\,\omega_{vb})\chi_b\,\rho_4)\\
&\quad -2\,(N_{va}\chi_{va}(N_t\chi_{vb}(\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\delta_h+\delta_d)+\omega_a\omega_{va}\delta_5)\chi_a-N_{vb}\chi_{vb}\omega_b\,\omega_{vb}\chi_b\,\rho_4)Cos(2\theta t)\\
&\quad +(4N_{va}\chi_{va}(\omega_a+\omega_{va})\rho_5\chi_a+\rho_6(-4\,\omega_{vb}\rho_4+N_t\,N_{va}\chi_{va}(4(\omega_a+\omega_{va})+(4+3\,\omega_a\omega_{va})\omega_{vb})\chi_a)\\
&\quad -4\,N_{vb}\chi_{vb}(\omega_b+\omega_{vb})\rho_4\rho_6)Sin(\theta t)-N_t\,N_{va}\chi_{va}\omega_a\omega_{va}\omega_{vb}\rho_6\chi_a\,Sin\,(3\theta t)]/[2\,N_{va}\chi_a(1+\omega_a\,Sin(\theta t))(N_t\,\chi_{va}\chi_{vb}\\
&\quad (2+\omega_{va}\omega_{vb})(\delta_h+\delta_d)+2\,\chi_{vb}\,\rho_4+2\,\chi_{va}\,\rho_5-N_t\,\chi_{va}\omega_{va}\omega_{vb}\rho_6\,Cos(\theta t)+(\,N_t\chi_{va}(\omega_{va}+\omega_{vb})\rho_6\\
&\quad +\chi_{vb}\,\omega_{vb}\rho_4+\chi_{va}\,\omega_{va}\,\rho_5)\,Sin(\theta t))])*I_{va}^{*})\ =\ 0
\end{aligned} \quad (4.36)$$

The solution of equation (4.31) are $I_{va}^{*}=0$ and

$$\begin{aligned}
&I_{va}^{*}=([2(N_{va}\chi_{va}(2+\omega_a\omega_{va})\rho_5\chi_a+\rho_6(-2\,\rho_4+N_t\,N_{va}\omega_{va}(2+\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})\chi_a)-N_{vb}\chi_{vb}(2+\omega_b\,\omega_{vb})\chi_b\,\rho_4)\\
&\quad -2\,(N_{va}\chi_{va}(N_t\chi_{vb}(\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\delta_h+\delta_d)+\omega_a\omega_{va}\delta_5)\chi_a-N_{vb}\chi_{vb}\omega_b\,\omega_{vb}\chi_b\,\rho_4)Cos(2\theta t)\\
&\quad +(4N_{va}\chi_{va}(\omega_a+\omega_{va})\rho_5\chi_a+\rho_6(-4\,\omega_{vb}\rho_4+N_t\,N_{va}\chi_{va}(4(\omega_a+\omega_{va})+(4+3\,\omega_a\omega_{va})\omega_{vb})\chi_a)\\
&\quad -4\,N_{vb}\chi_{vb}(\omega_b+\omega_{vb})\rho_4\rho_6)Sin(\theta t)-N_t\,N_{va}\chi_{va}\omega_a\omega_{va}\omega_{vb}\rho_6\chi_a\,Sin\,(3\theta t)]/[2\,N_{va}\chi_a(1+\omega_a\,Sin(\theta t))(N_t\,\chi_{va}\chi_{vb}\\
&\quad (2+\omega_{va}\omega_{vb})(\delta_h+\delta_d)+2\,\chi_{vb}\,\rho_4+2\,\chi_{va}\,\rho_5-N_t\,\chi_{va}\omega_{va}\omega_{vb}\rho_6\,Cos(\theta t)+(\,N_t\chi_{va}(\omega_{va}+\omega_{vb})\rho_6\\
&\quad +\chi_{vb}\,\omega_{vb}\rho_4+\chi_{va}\,\omega_{va}\,\rho_5)\,Sin(\theta t))])
\end{aligned} \ . \quad (4.37)$$

where

$$\rho_1=N_t\chi_{vb}(\delta_h+\delta_d)+(\lambda_{hb}+\delta_h+\delta_d)\delta_{vb}+N_t\chi_{vb}\omega_{vb}(\delta_h+\delta_d)Sin(\theta t)\,,$$

$$\rho_2=(\delta_h+\delta_d+N_{va}\,\chi_a+N_{vb}\chi_b+(N_{va}\,\chi_a\omega_a\,N_{va}\,\chi_a+N_{vb}\omega_b\chi_b)\,Sin(\theta t))\,,$$

$$\rho_3 = N_t \chi_{vb} (1+\omega_{vb} Sin(\theta t)),$$

$$\rho_4 = (\lambda_{ha} + \delta_h + \delta_d)\delta_{va},$$

$$\rho_5 = (\lambda_{hb} + \delta_h + \delta_d)\delta_{vb},$$

and

$$\rho_6 = \chi_{vb}(\delta_h + \delta_d).$$

If we substitute, $I_{vb}^* = 0$, $I_{va}^* = 0$, into equations (4.25) - (4.29) , we obtain equilibrium points. This solution yields the disease free state $P_0 = (1,0,0,0,0)$ and the endemic disease equilibrium point $P_1 = (S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$

where

$$S^* = \frac{\rho_1}{\rho_2} \tag{4.38}$$

$$I_a^* = \frac{I_{va}^* N_{va}\, \chi_a (1+\omega_a Sin\theta t)(\rho_1)}{(\lambda_{ha} + \delta_h + \delta_d)(\rho_2)(\rho_3)} \tag{4.39}$$

$$I_b^* = \frac{I_{vb}^* N_{vb}\, \chi_b (1+\omega_a\, Sin\, \theta\, t)(\rho_1)}{(\lambda_{hb} + \delta_d + \delta_h)(\rho_2)(\rho_3)} \tag{4.40}$$

$$\begin{aligned} I_{va}^* = {} & ([2(N_{va}\chi_{va}(2+\omega_a\omega_{va})\rho_5\chi_a + \rho_6(-2\,\rho_4 + N_t\, N_{va}\omega_{va}(2+\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})\chi_a) - N_{vb}\chi_{vb}(2+\omega_b\,\omega_{vb})\chi_b\,\rho_4) \\ & -2\,(N_{va}\chi_{va}(N_t\chi_{vb}(\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\delta_h+\delta_d) + \omega_a\omega_{va}\delta_5)\chi_a - N_{vb}\chi_{vb}\omega_b\,\omega_{vb}\chi_b\,\rho_4)Cos(2\theta t) \\ & +(4N_{va}\chi_{va}(\omega_a+\omega_{va})\rho_5\chi_a + \rho_6(-4\,\omega_{vb}\rho_4 + N_t\, N_{va}\chi_{va}(4(\omega_a+\omega_{va})+(4+3\,\omega_a\omega_{va})\omega_{vb})\,\chi_a) \\ & -4\,N_{vb}\chi_{vb}(\omega_b+\omega_{vb})\rho_4\rho_6)Sin(\theta t) - N_t\, N_{va}\chi_{va}\omega_a\omega_{va}\omega_{vb}\rho_6\chi_a\, Sin\,(3\theta t)]/[2\,N_{va}\chi_a(1+\omega_a\, Sin(\theta t))(N_t\,\chi_{va}\chi_{vb} \\ & (2+\omega_{va}\omega_{vb})(\delta_h+\delta_d) + 2\,\chi_{vb}\,\rho_4 + 2\,\chi_{va}\,\rho_5 - N_t\,\chi_{va}\omega_{va}\omega_{vb}\rho_6\, Cos(\theta t) + (\,N_t\chi_{va}(\omega_{va}+\omega_{vb})\,\rho_6 \\ & + \chi_{vb}\,\omega_{vb}\rho_4 + \chi_{va}\,\omega_{va}\,\rho_5)\,Sin(\theta t))]) \end{aligned} \tag{4.41}$$

$$I_{vb}^* = \frac{(-(\rho_5)(\delta_d+\delta_h+I_{va}^* N_{va}\chi_a) + N_t N_{vb}\chi_b\rho_6 + Sin(\theta t)(-I_{va}^* N_{va}\chi_a\omega_a\rho_5 + N_t N_{vb}(\omega_b+\omega_{vb})\,\chi_b\rho_6 + N_t N_{vb}\omega_b\,\omega_{vb}\chi_b\rho_6 Sin(\theta t)))}{(N_{vb}\chi_b(1+\omega_b\, Sin(\theta t))(N_t\rho_6+\rho_5+N_t\omega_{vb}\rho_6\, Sin(\theta t)))} \tag{4.42}$$

We define

$$R_0 = \frac{2(N_{va}\chi_{va}(N_t\,\chi_{vb}\,(\omega_a\omega_{va}+(\omega_a+\omega_{va})\,\omega_{vb})(\eta_h+\eta_d) + \omega_a\omega_{va}\rho_5)\chi_a + 2\chi_{vb}\,\rho_4\,(2(\eta_h+\eta_d) + N_{vb}(2+\omega_b\,\omega_{vb})\chi_b)}{N_{va}\chi_{va}(2N_t\,\chi_{va}(2+\omega_a\,\omega_{va}+(\omega_a+\omega_{va})\,\omega_{vb})(\eta_h+\eta_d) + 2(2+\omega_a\,\omega_{va})\rho_5\,\chi_a + 2N_{vb}\,\chi_{vb}\omega_b\,\omega_{vb}\rho_4\chi_b} \tag{4.43}$$

The square root of this number resents the average number of secondary cases that one cases can produce if introduction into the susceptible population. This model, we interesting in the transmission of dengue disease between human population and two species mosquitoes (*Aedes aegypti* and *Aedes albopictus*). The infective human introduction into the susceptible human is bitten by χ_a of *Aedes aegypti* and χ_b of *Aedes albopictus*. Multiplying this number ω_a a measure of influence on the transmission process from human population to *Aedes aegypti* and ω_b a measure of influence on the transmission process from human population to *Aedes albopictus*. The multiplication between the number of infected human population and the number of infected two species mosquitoes (*Aedes aegypti* and *Aedes albopictus*) during the life time of the infectious. The simulation model shows that a reproductive ratio R_0 that conceptualizes the rate of spread of a dengue disease and determines a threshold: when $R_0 < 1$, a typical infective give rise the average number of the value to less than one secondary infection, and the disease will die out. Otherwise when $R_0 <1$, the disease will persist in the population.

4.3.2. Stability Analysis

From Equations (4.25) – (4.29), we obtain the following Jacobian matrix at each equilibrium point.

$$J(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*) =$$

$$\begin{bmatrix} -[\delta_d+\delta_h+\chi_a(1+\omega_a Sin\theta t)I^*_{va}(t)(N_{va})+\chi_b(1+\omega_b Sin\theta t)I^*_{vb}(t)(N_{vb})] & 0 & 0 & (-\chi_a(1+\omega_a Sin(\theta))N_{va})^*S^*(t) & (-\chi_b(1+\omega_b Sin(\theta))N_{vb})^*S^*(t) \\ \chi_a(1+\omega_a Sin\theta t)I^*_{va}(t)(N_{va}) & -(\delta_d+\delta_h+\lambda_{ha}) & 0 & (\chi_a(1+\omega_a Sin(\theta))N_{va})^*S^*(t) & 0 \\ \chi_b(1+\omega_b Sin\theta t)I^*_{vb}(t)(N_{vb}) & 0 & -(\delta_d+\delta_h+\lambda_{hb}) & 0 & (\chi_b(1+\omega_b Sin(\theta))N_{vb})^*S^*(t) \\ 0 & \chi_{va}(1+\omega_{va} Sin\theta t)(1-I^*_{va}(t))N_t & 0 & -\delta_{va} & 0 \\ 0 & 0 & \chi_{vb}(1+\omega_{vb} Sin\theta t)(1-I^*_{vb}(t))N_t & 0 & -\delta_{vb} \end{bmatrix} \quad (4.44)$$

4.3.2.1. For the disease free equilibrium point $P_0 = (1,0,0,0,0)$

$$J_{P_0} = \begin{bmatrix} -(\delta_d+\delta_h) & 0 & 0 & (-\chi_a(1+\omega_a Sin(\theta t))N_{va}) & (-\chi_b(1+\omega_b Sin(\theta t))N_{vb}) \\ 0 & -(\delta_d+\delta_h+\lambda_{ha}) & 0 & (\chi_a(1+\omega_a Sin(\theta t))N_{va}) & 0 \\ 0 & 0 & -(\delta_d+\delta_h+\lambda_{hb}) & 0 & (\chi_b(1+\omega_b Sin(\theta t))N_{vb}) \\ 0 & \chi_{va}(1+\omega_{va} Sin\theta t)N_t & 0 & -\delta_{va} & 0 \\ 0 & 0 & \chi_{vb}(1+\omega_{vb} Sin\theta t)N_t & 0 & -\delta_{vb} \end{bmatrix} \quad (4.45)$$

The eigenvalues are solutions of the characteristic equation, $\det(J_{P_0} - \beta I_5) = 0$. Where I_5 is the identity matrix dimension 5x5. If all eigenvalues for the equilibrium point have negative real parts then that equilibrium point is locally stable.

For $\theta = 0$, we obtain the following characteristic equation

$$(\beta+\delta_h+\delta_d)(\beta^4+a_1\beta^3+a_2\beta^2+a_3\beta+a_4) = 0 \quad (4.46)$$

where

$$a_1 = \chi_{ha} + \chi_{hb} + 2\delta_h + 2\delta_d + \delta_{va} + \delta_{vb} , \quad (4.47)$$

$$a_2 = (\delta_h + \delta_d)(\delta_h + \delta_d + 2\delta_{va}) + (2(\delta_h + \delta_d) + \delta_{va})\delta_{vb} + \chi_{hb}(\delta_h + \delta_d + \delta_{va} + \delta_{vb}) + \chi_{ha}(\chi_{hb} + \delta_h + \delta_d + \delta_{va} + \delta_{vb}) - N_t(N_{va}\chi_{va}\chi_a + N_{vb}\chi_{vb}\chi_b) , \quad (4.48)$$

$$a_3 = (\chi_{ha} + \delta_h + \delta_d)(\chi_{hb} + \delta_h + \delta_d)\delta_{va} + ((\chi_{ha} + \delta_h + \delta_d)((\chi_{hb} + \delta_h + \delta_d) + (\chi_{ha} + \chi_{hb} + 2(\delta_h + \delta_d))\delta_{va})\delta_{vb} - N_t(N_{va}\chi_{va}(\chi_{hb} + \delta_h + \delta_d + \delta_{vb})\chi_a + N_{vb}\chi_{vb}(\chi_{ha} + \delta_h + \delta_d + \delta_{va})\varphi_b) \quad (4.49)$$

$$a_4 = ((\chi_{ha} + \delta_h + \delta_d)\delta_{va} - N_t N_{va}\chi_{va}\chi_a)((\chi_{hb} + \delta_h + \delta_d)\delta_{vb} - N_t N_{vb}\chi_{vb}\chi_b) \quad (4.50)$$

There are five eigenvalues corresponding to (4.46). we can see that one eigenvalue is $\beta = -\delta_h - \delta_d$.

we check the sign of other four eigenvalues by solving the equation

$$(\beta^4 + a_1\beta^3 + a_2\beta^2 + a_3\beta + a_4) = 0 . \quad (4.51)$$

The remaining four eigenvalues have negative real parts if they satisfy Routh-Hurwitz criteria (4.47) - (4.50), each equilibrium point is locally asymptotically stable if the following conditions are satisfied,

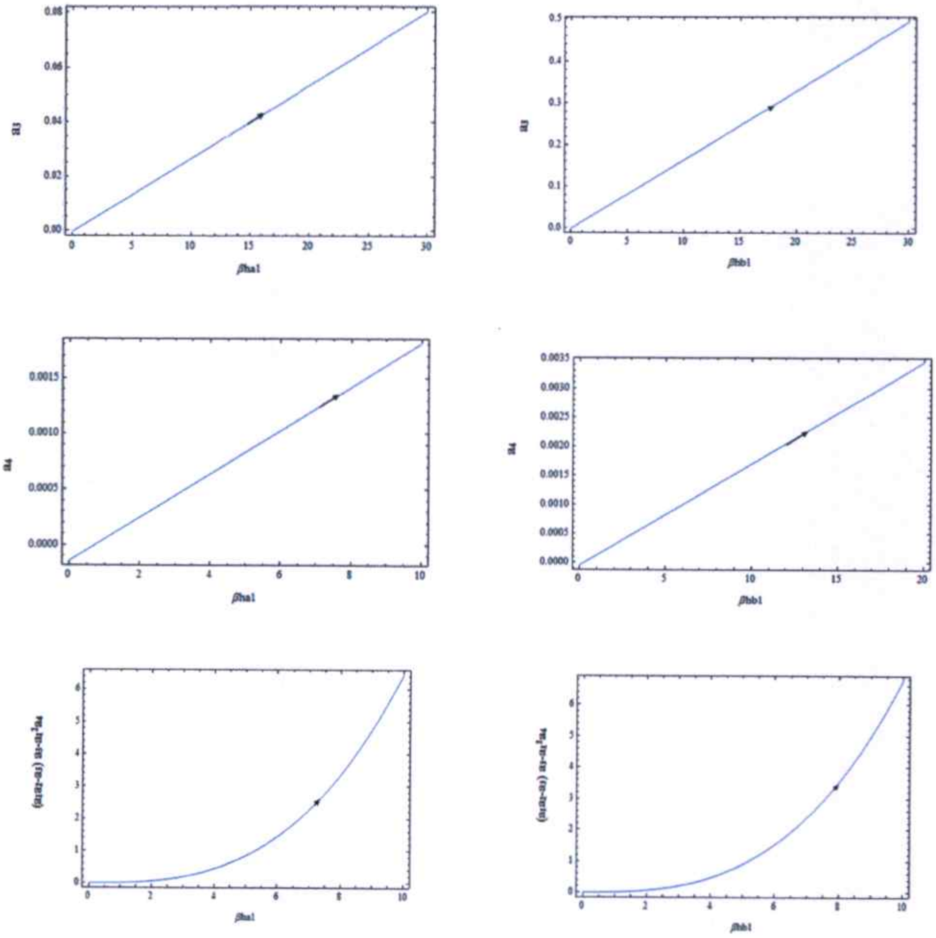
i) $a_1 > 0$ (4.56)

ii) $a_3 > 0$ (4.57)

iii) $a_4 > 0$ (4.58)

iv) $a_1 a_2 a_3 > a_3^2 + a_1^2 a_4$. (4.59)

Consider the above conditions, we can see that a_1 is always positive. For the remaining conditions given by (4.57) – (4.59). They are shown in figure 4.4:



4a) 4b)

Figure 4.4. The parameter spaces for the disease free equilibrium point which satisfies the Routh-Hurwitz criteria. The parameter are given as follows:

4.4a) $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.0000000179$, $\chi_{vb} = 0.000000000346$, $\delta_{va} = 1/40$, $\delta_{vb} = 1/35$, $\chi_a = 1/15$, $\chi_b = 1/12$, $\omega_a = 0.089$, $\omega_b = 0.065$, $\omega_{va} = 0.87$, $\omega_{vb} = 0.57$, $N_{va} = 1000$, $N_{vb} = 2700$, $N_t = 20{,}000$ and $\chi_{hb} = 1/(19/2)$.

4.4b) $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.0000000179$, $\chi_{vb} = 0.000000000346$, $\delta_{va} = 1/40$, $\delta_{vb} = 1/35$, $\chi_a = 1/15$, $\chi_b = 1/12$, $\omega_a = 0.089$, $\omega_b = 0.065$, $\omega_{va} = 0.87$, $\omega_{vb} = 0.57$, $N_{va} = 1000$, $N_{vb} = 2700$, $N_t = 20000$ and $\chi_{ha} = 1/(17/2)$.

From the above figures, the Routh-Hurwitz conditions are satisfies for $R_0 < 1$. Thus, the disease free equilibrium point is locally stable when $R_0 < 1$. In this model, we are interested in the transmission of dengue virus of two species (*Aedes aegypti* and *Aedes albopictus*).

4.3.2.2. For disease endemic equilibrium point $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$

From equations (4.25) - (4.29), The Jacobian matrix evaluated at $P_1 = (S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ given by

$$J_{P_1} =$$

$$\begin{bmatrix} -[\delta_d+\delta_h+\chi_a(1+a_{tr}Sin\theta t)I^*_{va}(t)(N_{va})+\chi_b(1+a_{tr}Sin\theta t)I^*_{vb}(t)(N_{vb})] & 0 & 0 & (-\chi_a(1+a_{tr}Sin(\theta))N_{va})*S^*(t) & (-\chi_b(1+a_{tr}Sin(\theta))N_{vb})*S^*(t) \\ \chi_a(1+a_{tr}Sin\theta t)I^*_{va}(t)(N_{va}) & -(\delta_d+\delta_h+\lambda_{ha}) & 0 & (\chi_a(1+a_{tr}Sin(\theta))N_{va})*S^*(t) & 0 \\ \chi_b(1+a_{tr}Sin\theta t)I^*_{vb}(t)(N_{vb}) & 0 & -(\delta_d+\delta_h+\lambda_{hb}) & 0 & (\chi_b(1+a_{tr}Sin(\theta))N_{vb})*S^*(t) \\ 0 & \chi_{va}(1+a_{t_{va}}Sin\theta t)(1-I^*_{va}(t))N_t & 0 & -\delta_{va} & 0 \\ 0 & 0 & \chi_{vb}(1+a_{t_{vb}}Sin\theta t)(1-I^*_{vb}(t))N_t & 0 & -\delta_{vb} \end{bmatrix} \tag{4.60}$$

Substituting the endemic disease equilibrium point (4.38) – (4.42) in (4.60).

The eigenvalues are solutions of the characteristic equation, $\det(J_{P_1}-\beta I_5)=0$. We consider two cases.

For $\theta = 0$, the eigenvalues are solutions of the characteristic equation. The behaviours of the solutions are same as the disease free equilibrium point.

For $\theta \neq 0$, we obtain the following characteristic equation:

$$(\beta^5 + a_1\beta^4 + a_2\beta^3 + a_3\beta^2 + a_4\beta + a_5) = 0 \tag{4.61}$$

where $S^*, I_a^*, I_b^*, I_{va}^*$ and I_{vb}^* are given by equation (4.38) – (4.42). The characteristic equation for the Jacobian matrix evaluated at the equilibrium point, given by (4.25) – (4.29). It can be seen that eigenvalues, obtained by solving $(\beta^5 + a_1\beta^4 + a_2\beta^3 + a_3\beta^2 + a_4\beta + a_5) = 0$,

where

$$a_1 = \frac{\gamma_{11}}{\gamma_5} + \frac{1}{\gamma_9}((\chi_{ha} + \chi_{hb} + 3\delta_h + 3\delta_d + \delta_{va} + \delta_{vb})\,\gamma_9 + N_{vb}\chi_b(-\gamma_{10}\rho_5 + N_t N_{vb}\chi_b\rho_6)), \tag{4.62}$$

$$\begin{aligned} a_2 = {} & \frac{\gamma_{11}(\chi_{ha} + \chi_{hb} + 2\delta_h + 2\delta_d + \delta_{va} + \delta_{vb} + N_t\chi_{va}\gamma_{12})}{\gamma_5} + \frac{1}{\gamma_9}(\gamma_9(3(\delta_h + \delta_d)(\delta_h + \delta_d + \delta_{va}) \\ & + (3(\delta_h + \delta_d) + \delta_{va})\delta_{vb} + \chi_{hb}(2\delta_h + 2\delta_d + \delta_{va} + \delta_{vb}) + \\ & \chi_{ha}(\chi_{hb} + 2\delta_h + 2\delta_d + \delta_{va} + \delta_{vb}) - N_t(N_{va}\chi_{va}\chi_a + N_{vb}\chi_{vb}\chi_b)\gamma_{12}) \\ & + N_{vb}\chi_b(\chi_{ha} + \chi_{hb} + 2\delta_h + 2\delta_d + \delta_{va} + \delta_{vb} + N_t\chi_{va}\gamma_{12})(-\gamma_{10}\rho_5 + N_t N_{vb}\chi_b\rho_6)) \end{aligned} \tag{4.63}$$

$$
\begin{aligned}
a_3 = \frac{1}{\gamma_5\gamma_9}&(\gamma_{11}\gamma_9((\delta_h+\delta_d)(\delta_h+\delta_d+2\delta_{va})+(2\,(\delta_h+\delta_d)+\delta_{va})\delta_{vb}+\ \chi_{hb}\,(\delta_h+\delta_d+\delta_{va}+\delta_{vb})\\
&+\chi_{ha}(\chi_{hb}+\delta_h+\delta_d+\delta_{va}+\delta_{vb})+N_t(\chi_{va}(\chi_{hb}+2\,(\delta_h+\delta_d)+\delta_{vb})-\\
&N_{vb}\chi_{vb}\chi_b)\gamma_{12})+N_tN_{vb}(\chi_{va}+\chi_{vb})\chi_b\gamma_{12}(-\gamma_{10}\rho_5+N_tN_{vb}\chi_b\rho_6))+\gamma_5(\gamma_9(\chi_{hb}(\delta_h+\delta_d)(\delta_h+\delta_d+2\delta_{va})\\
&+\ \lambda_{hb}\,(2(\delta_h+\delta_d)+\delta_{va})\delta_{vb})+(\delta_h+\delta_d)((\delta_h+\delta_d)(\delta_h+\delta_d+3\delta_{va})+3\rho_5)+\chi_{ha}((\delta_h+\delta_d)(\delta_h+\delta_d+2\delta_{va})\\
&+(2(\delta_h+\delta_d)+\delta_{va})\delta_{vb}+\chi_{hb}(\delta_h+\delta_d+\delta_{va}+\delta_{vb}))-N_t(N_{va}\chi_{va}(\chi_{hb}+2(\delta_h+\delta_d)+\delta_{vb})\chi_a+\\
&+N_{vb}\chi_{vb}(\chi_{ha}+2(\delta_h+\delta_d)+\delta_{va})\chi_b)\gamma_{12})+N_{vb}\chi_b((\delta_h+\delta_d)(\delta_h+\delta_d+2\delta_{va})+2(\delta_h+\delta_d)+\delta_{va})\delta_{vb}\\
&+\chi_{hb}(\delta_h+\delta_d+\delta_{va}+\delta_{vb})+\chi_{ha}(\chi_{hb}+\delta_h+\delta_d+\delta_{va}+\delta_{vb})\\
&+N_t(\chi_{vb}(\chi_{ha}2(\delta_h+\delta_d)+\delta_{va})-N_{va}\alpha_{va}\chi_a)\gamma_{12})(-\gamma_{10}\rho_5+N_tN_{vb}\chi_b\rho_6)))
\end{aligned}
\tag{4.64}
$$

$$
\begin{aligned}
a_4 = \frac{1}{\gamma_5\gamma_9}&(\gamma_{11}\gamma_9((\chi_{hb}+\delta_h+\delta_d)\rho_4+(\chi_{ha}+\delta_h+\delta_d)(\chi_{hb}+\delta_h+\delta_d)+(\chi_{ha}+\chi_{hb}+2\,(\delta_h+\delta_d))\delta_{va})\\
&+\ \delta_{vb}+N_t\,\gamma_{12}(\chi_{va}\,(\delta_h+\delta_d))\delta_{vb}-\ N_{vb}\chi_{vb}(\chi_{ha}+\delta_h+\delta_d+\delta_{va})\chi_b\ -N_t\,N_{vb}\,\chi_{va}\chi_{vb}\chi_b\,\gamma_{12}))\\
&+N_t\,N_{vb}\chi_b\,\gamma_{12}(\chi_{vb}(\chi_{ha}+\delta_h+\delta_d+\delta_{va})+\chi_{va}(\chi_{hb}+\delta_h+\delta_d+\delta_{vb})+\\
&N_t\chi_{va}\chi_{vb}\gamma_{12})(-\gamma_{10}\rho_5+N_tN_{vb}\chi_b\rho_6))+\gamma_5(\gamma_9((\delta_h+\delta_d)(\chi_{hb}+\delta_h+\delta_d)\rho_4+((\delta_h+\delta_d)\\
&(\chi_{ha}+\delta_h+\delta_d)(\chi_{hb}+\delta_h+\delta_d)+(\chi_{ha}(\chi_{hb}+\ 2(\delta_h+\delta_d))+(\delta_h+\delta_d)(2\chi_{hb}+3(\delta_h+\delta_d)))\delta_{va})\delta_{vb}\\
&+N_t\,\gamma_{12}(-N_{va}\chi_{va}(\chi_{hb}\,(\delta_h+\delta_d+\delta_{vb})+(\delta_h+\delta_d)(\delta_h+\delta_d+2\delta_{vb}))\chi_a-\\
&N_{vb}\chi_{vb}(\chi_{ha}(\delta_h+\delta_d+\delta_{va})+(\delta_h+\delta_d)(\delta_h+\delta_d+2\delta_{va})\chi_b+N_tN_{va}N_{vb}\chi_{va}\chi_{vb}\chi_a\chi_b\gamma_{12}))\\
&+N_{vb}\chi_b((\chi_{hb}+\delta_h+\delta_d)\,\rho_4+((\chi_{ha}+\delta_h+\delta_d)(\chi_{hb}+\delta_h+\delta_d)+\\
&+\ (\chi_{ha}+\chi_{hb}\ +2\,(\delta_h+\delta_d))\delta_{va})\delta_{vb}+\ N_t\gamma_{12}(\rho_6(\chi_{ha}+\delta_h+\delta_d)+\chi_{vb}(\chi_{ha}+2(\delta_h+\delta_d))\delta_{va}\\
&-N_{va}\chi_{va}(\chi_{hb}\ +\chi_h+\chi_d+\chi_{vb})\chi_a\ -N_tN_{va}\chi_{va}\ \ \chi_{vb}\chi_a\gamma_{12}))(-\gamma_{10}\rho_5+N_tN_{vb}\chi_b\rho_6)))
\end{aligned}
\tag{4.65}
$$

$$
\begin{aligned}
a_5 = \frac{1}{\gamma_5\gamma_9}&(\gamma_5(\rho_4-N_tN_{va}\chi_{va}\chi_a\gamma_{12})((\delta_h+\delta_d)\gamma_9(\rho_5-N_tN_{vb}\chi_{vb}\chi_b\gamma_{12})+N_{vb}\chi_b(\rho_5+N_t\chi_{vb}(\delta_h+\delta_d)\gamma_{12}\\
&(-\gamma_{10}\rho_5+N_tN_{vb}\chi_b\rho_6))+\gamma_{11}(\gamma_9(\rho_4+N_t\chi_{va}(\delta_h+\delta_d)\gamma_{12}(\rho_5-N_tN_{vb}\chi_{vb}\chi_b\gamma_{12})+N_tN_{vb}\chi_b\gamma_{12}(\chi_{vb}\rho_4+\chi_{va}\rho_5\\
&+N_t\chi_{va}\chi_{vb}(\delta_h+\delta_d)\gamma_{12}(-\gamma_{10}\rho_5+N_tN_{vb}\chi_b\rho_6)))
\end{aligned}
\tag{4.66}
$$

with

$$\gamma_1 = N_t\chi_{vb}(\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\delta_h+\delta_d)+\omega_a\omega_{va}\,\rho_5$$

$$\gamma_2 = N_{vb}\chi_{vb}\,\omega_b\omega_{vb}\rho_4\,\chi_b$$

$$\gamma_3 = N_t\chi_{va}\chi_{vb}\omega_{va}\omega_{vb}(\delta_h+\delta_d)$$

$$\gamma_4 = N_tN_{va}\chi_{va}(2+\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})\chi_a$$

$$\gamma_5 = -\gamma_3+N_t\chi_{va}\,\chi_{vb}(2+\omega_{va}\omega_{vb})(\delta_h+\delta_d)+2\chi_{vb}\rho_4+2\chi_{va}\,\rho_5$$

$$\gamma_6 = N_{va}\chi_{va}(2+\omega_a\omega_{va})\rho_5\chi_a$$

$$\gamma_7 = N_{vb}\chi_{vb}(2+\omega_b\omega_{vb})+\rho_4\chi_b$$

$$\gamma_8 = N_{va}\chi_{va}(\gamma_1+\omega_a\omega_{va}\,\rho_5)\chi_a$$

$$\gamma_9 = N_{vb}(N_t\rho_6+\rho_5)\chi_b$$

$$\gamma_{10} = (\delta_h+\delta_d+\frac{-2(\gamma_2+\gamma_8)+2(\gamma_6-\gamma_7+(\gamma_4-\rho_4)\rho_6}{\gamma_5})$$

$$\gamma_{11} = -2(N_{vb}\chi_{vb}\chi_a\gamma_1 - \gamma_2) + 2(\gamma_6 - \gamma_7 + (\gamma_4 - 2\rho_4)\rho_6)$$

$$\gamma_{12} = \frac{\gamma_5(\rho_5 + N_t\rho_6)}{N_t\chi_{vb}((\delta_d + \delta_h + N_{vb}\chi_b)\gamma_5 + \gamma_{11}}$$

From the characteristic equation (4.61). The eigenvalues are found by solving (4.61).These five eigenvalues have negative real parts if they satisfy Routh – Hurwitz criteria which are:

i) $a_1 > 0$ (4.67)

ii) $a_2 > 0$ (4.68)

iii) $a_3 > 0$ (4.69)

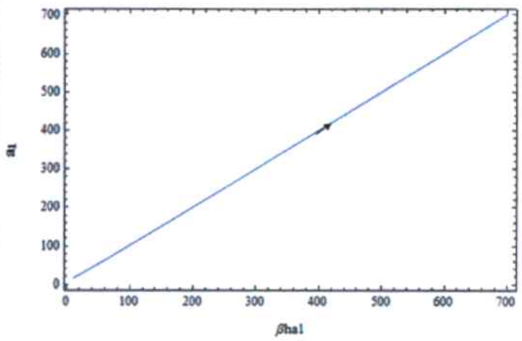
iv) $a_4 > 0$ (4.70)

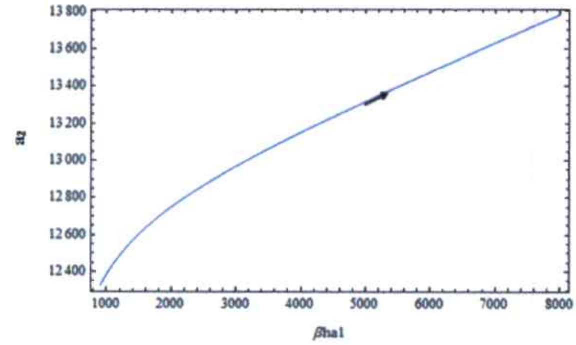
v) $a_5 > 0$ (4.71)

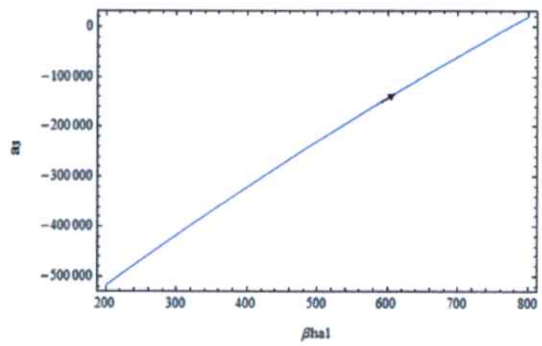
vi) $a_1a_2a_3 > a_3^2 + a_1^2a_4$ (4.72)

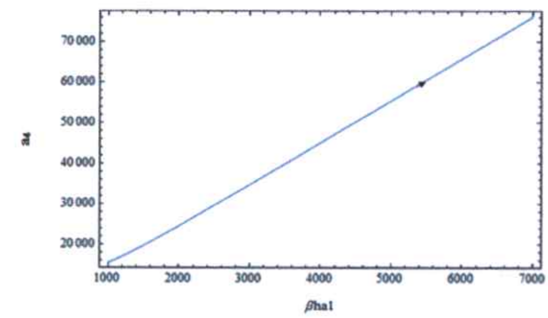
vii) $(a_1a_4 - a_5)(a_1a_2a_3 - a_3^2 - a_1^2a_4) - a_5(a_1a_2 - a_3)^2 - a_1a_5^2 > 0$. (4.73)

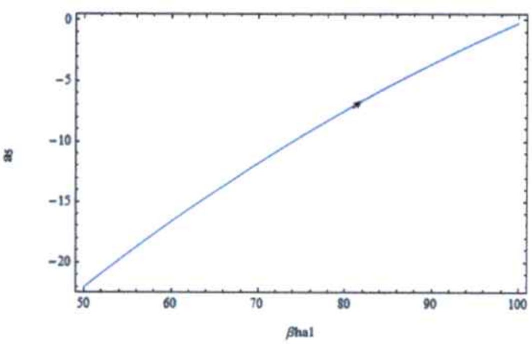
We check the stability of endemic equilibrium point by using conditions (4.67) – (4.73). We show the above conditions are met for $R_0 > 1$. They are shown in figure 4.5:

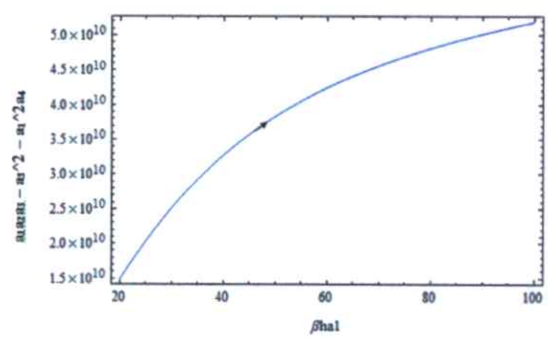

a1
βhal


a2
βhal


a3
βhal


a4
βhal


a5
βhal


a1a2a3 − a3^2 − a1^2a4
βhal

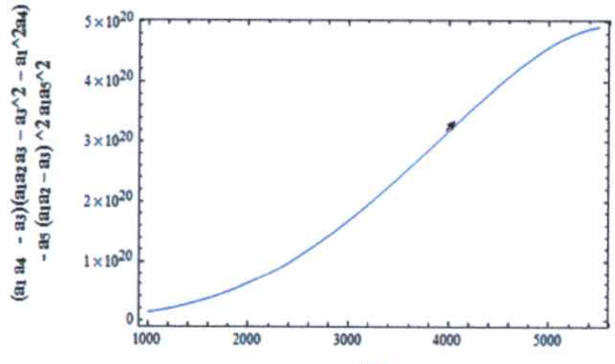

(a1 a4 − a3)(a1a2 a3 − a3^2 − a1^2a4)
− a5 (a1a2 − a3) ^2 a1a5^2
βhal

Figure 4.5a) The parameter spaces for endemic disease equilibrium point which satisfies the Routh-Hurwitz criteria with the value of parameters: respectively, for with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000079$, $\chi_{vb} = 0.0000000046$, $\delta_{va} = 1/12$, $\delta_{vb} = 1/18$, $\chi_a = 1/5$, $\chi_b = 1/8$, $\omega_a = 0.07$, $\omega_b = 0.04$, $\omega_{va} = 0.6$, $\omega_{vb} = 0.03$, $N_{va} = 8000$, $N_{vb} = 3000$, $N_t = 20000$ and $\chi_{hb} = 1/(19/2)$.

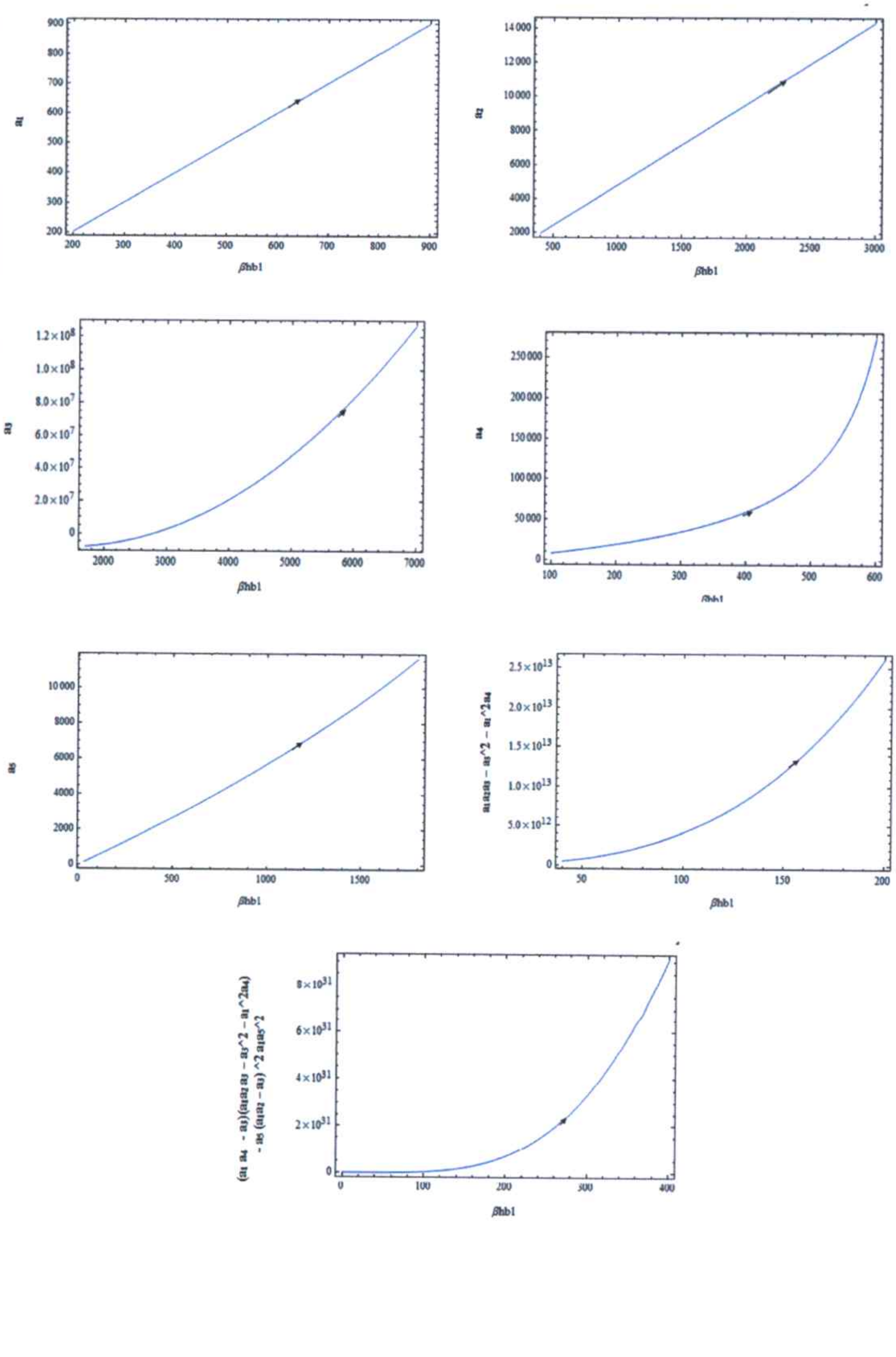


Figure 4.5b) The parameter spaces for endemic disease equilibrium point which satisfies the Routh-Hurwitz criteria with the value of parameters: respectively, for with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000079$, $\chi_{vb} = 0.0000000046$, $\delta_{va} = 1/12$, $\delta_{vb} = 1/18$, $\chi_a = 1/5$, $\chi_b = 1/8$, $\omega_a = 0.07$, $\omega_b = 0.04$, $\omega_{va} = 0.6$, $\omega_{vb} = 0.03$, $N_{va} = 8000$, $N_{vb} = 3000$, $N_t = 20000$ and $\chi_{ha} = 1/(17/2)$.

From the above figures, the Routh-Hurwitz conditions are satisfies for $R_0 > 1$. The endemic equilibrium point is locally stable when $R_0 > 1$. In this model, we are interested in the transmission of dengue virus of two species (*Aedes aegypti* and *Aedes albopictus*).

4.4. Numerical results

The numerical results are used for analyzing behaviors of above seasonality transmission models. θ is the seasonality influence on the transmission process. We use θ as an index parameter. The values of the parameters used in this study are $\delta_h = 1/(365*70)$ per day corresponding to life expectancy of 70 years in human. $\delta_d = 1/4.5$ corresponding to death rate due to the disease of human. $\chi_{ha} = 1/(8.5)$ and $\chi_{hb} = 1/(9.5)$ corresponding to the recovery rate of human population due to biting of *Aedes aegypti* and *Aedes albopictus*, respectively. The transmission probability of *Aedes aegypti* (χ_{va}) and *Aedes albopictus* (χ_{vb}) are arbitrary chosen. We assume that no alternative host. The other parameters are arbitrarily chosen. We present numerical solutions of (4.25) – (4.29) as follows:

First case, $\theta = 0$:

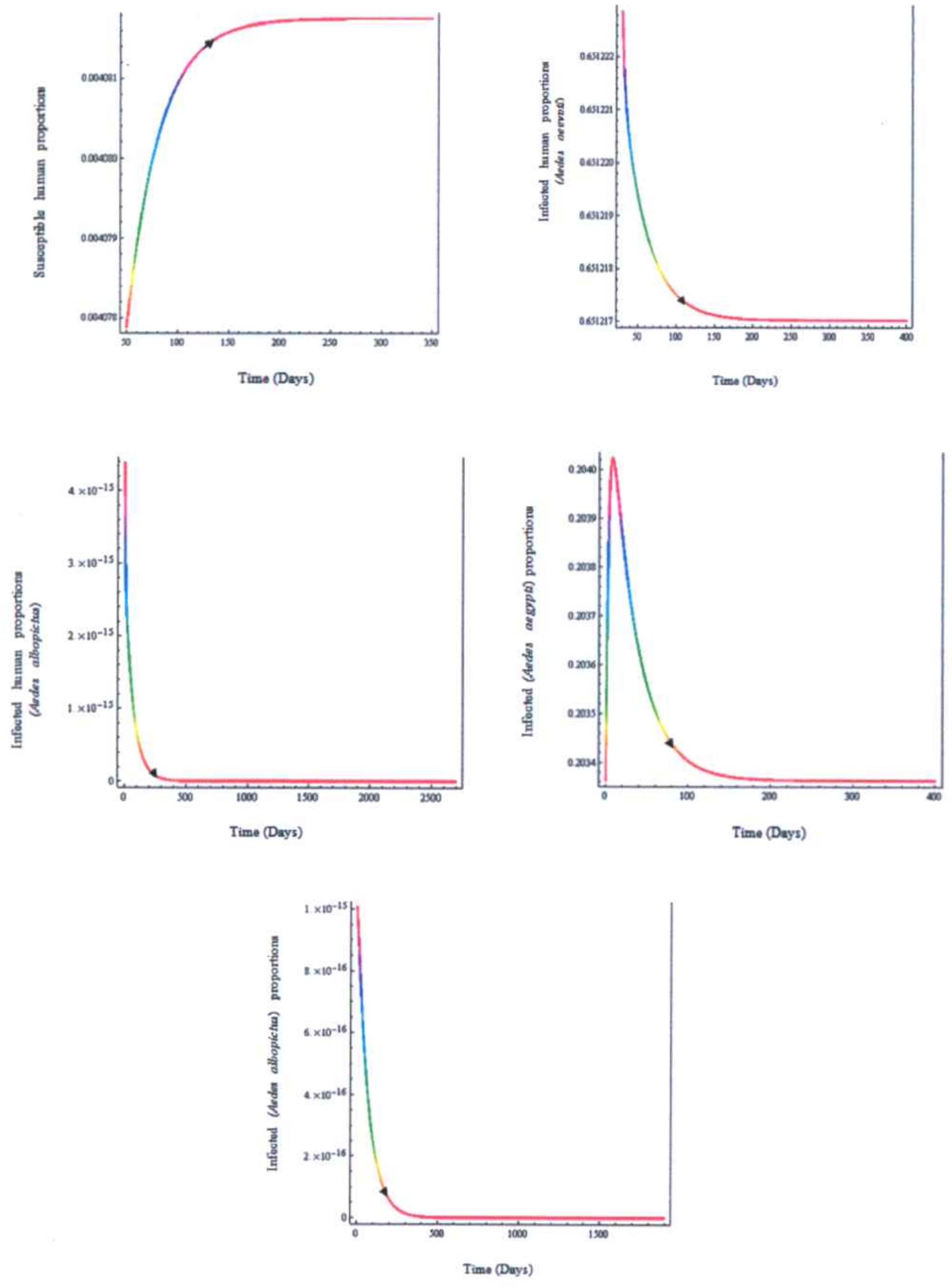


Figure 4.6. Numerical solutions of (4.25) – (4.29), demonstrate the solution trajectories, projected of S' , I'_a , I'_b , I'_{va} and I'_{vb} respectively . For $R_0 < 1$, when R_0 = 0.33878 with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000049$, $\chi_{vb} = 0.00000026$, $\delta_{va} = 1/40$, $\delta_{vb} = 1/35$, $\chi_a = 1/15$, $\chi_b = 1/11$, $\omega_a = 0.07$, $\omega_b = 0.04$, $\omega_{va} = 0.08$, $\omega_{vb} = 0.03$, $N_{va} = 4000$, $N_{vb} = 2300$, $N_t = 20000$, $\chi_{ha} = 1/(17/2)$ and $\chi_{hb} = 1/(19/2)$ .The fractions of populations (S' , I'_a , I'_b , I'_{va} and I'_{vb}) approach to the disease free equilibrium point (1,0,0,0,0).

Second case, $\theta \neq 0$:

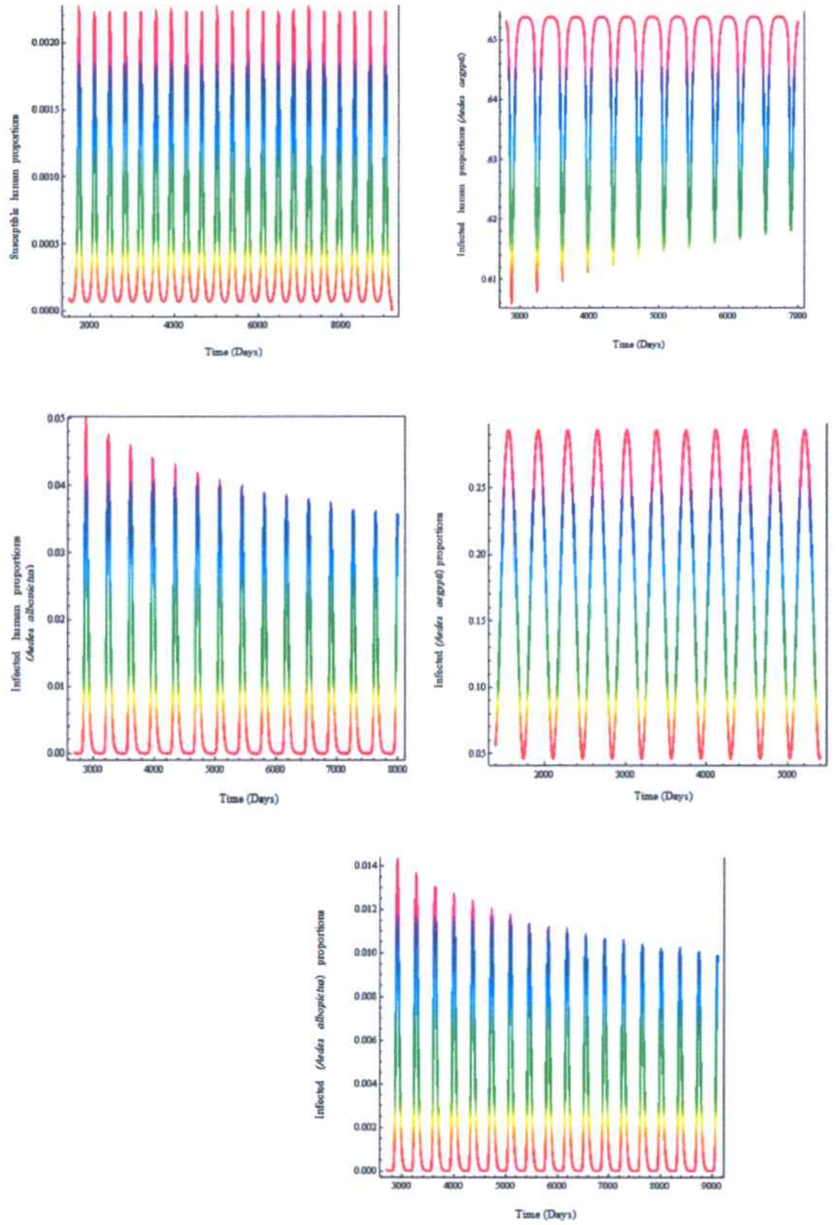


Figure 4.7. Numerical solutions the times series of $S^*, I_a^*, I_b^*, I_{va}^*$ and I_{vb}^* for $R_0 > 1$,respectively. For $R_0 > 1$, when $\theta = \frac{2\pi}{365}$ with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000059$, $\chi_{vb} = 0.00000036$, $\delta_{va} = 1/10$, $\delta_{vb} = 1/16$, $\chi_a = 1/4$, $\chi_b = 1/6$, $\omega_a = 0.7$, $\omega_b = 0.4$, $\omega_{va} = 0.8$, $\omega_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\chi_{ha} = 1/(17/2)$ and $\chi_{hb} = 1/(19/2)$. The behaviors of ($S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*$) are limit cycles.

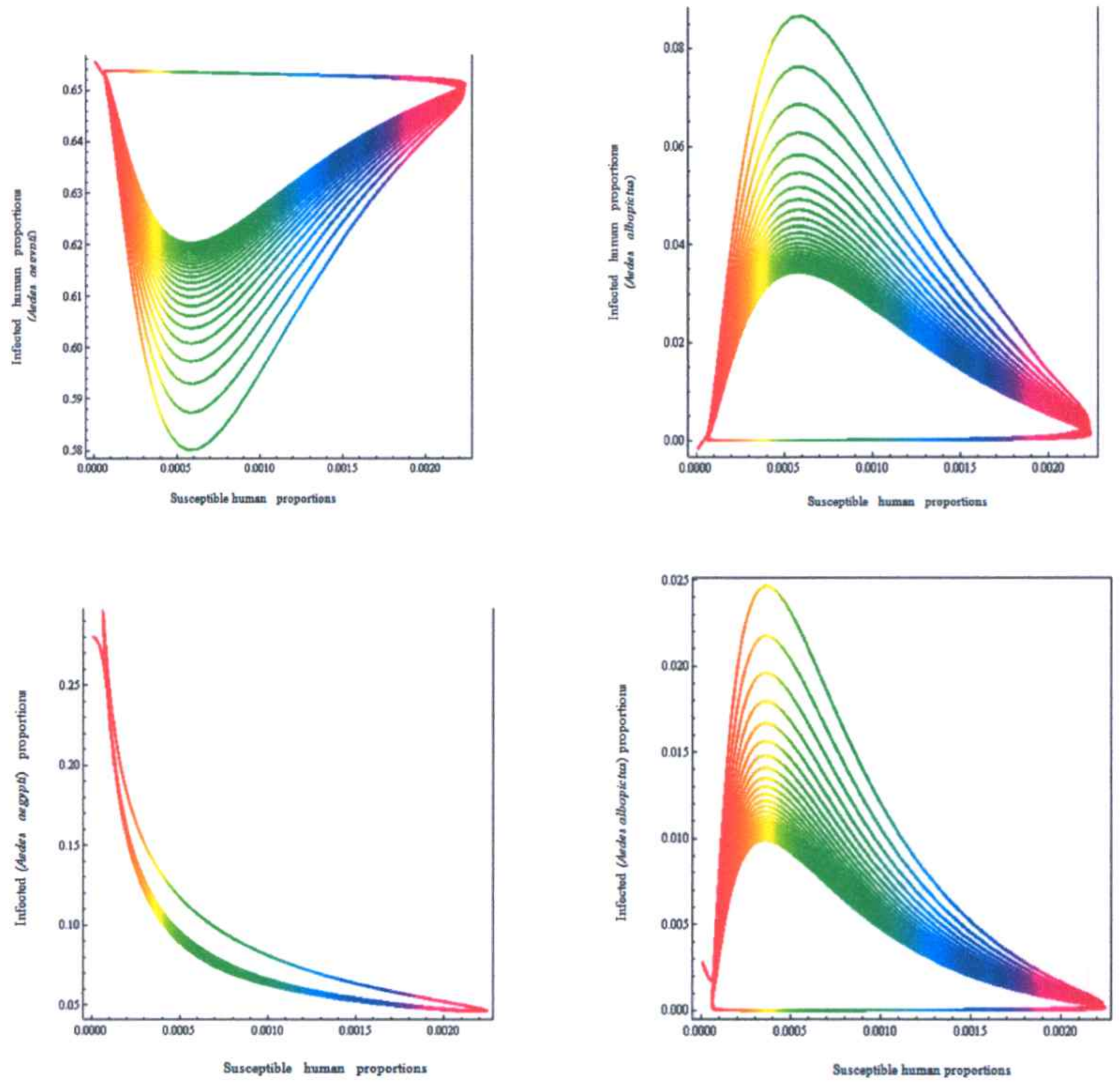


Figure 4.8. Numerical solutions of (4.25) – (4.29), demonstrate the solution trajectories, projected onto (S^*, I_a^*), (S^*, I_b^*), (S^*, I_{va}^*) and (S^*, I_{vb}^*), respectively. For $R_0 > 1$, when $\theta = \frac{2\pi}{365}$ with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000059$, $\chi_{vb} = 0.00000036$, $\delta_{va} = 1/10$, $\delta_{vb} = 1/16$, $\chi_a = 1/4$, $\chi_b = 1/6$, $\omega_a = 0.7$, $\omega_b = 0.4$, $\omega_{va} = 0.8$, $\omega_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\chi_{ha} = 1/(17/2)$ and $\chi_{hb} = 1/(19/2)$. The behaviors of $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ are limit cycles.

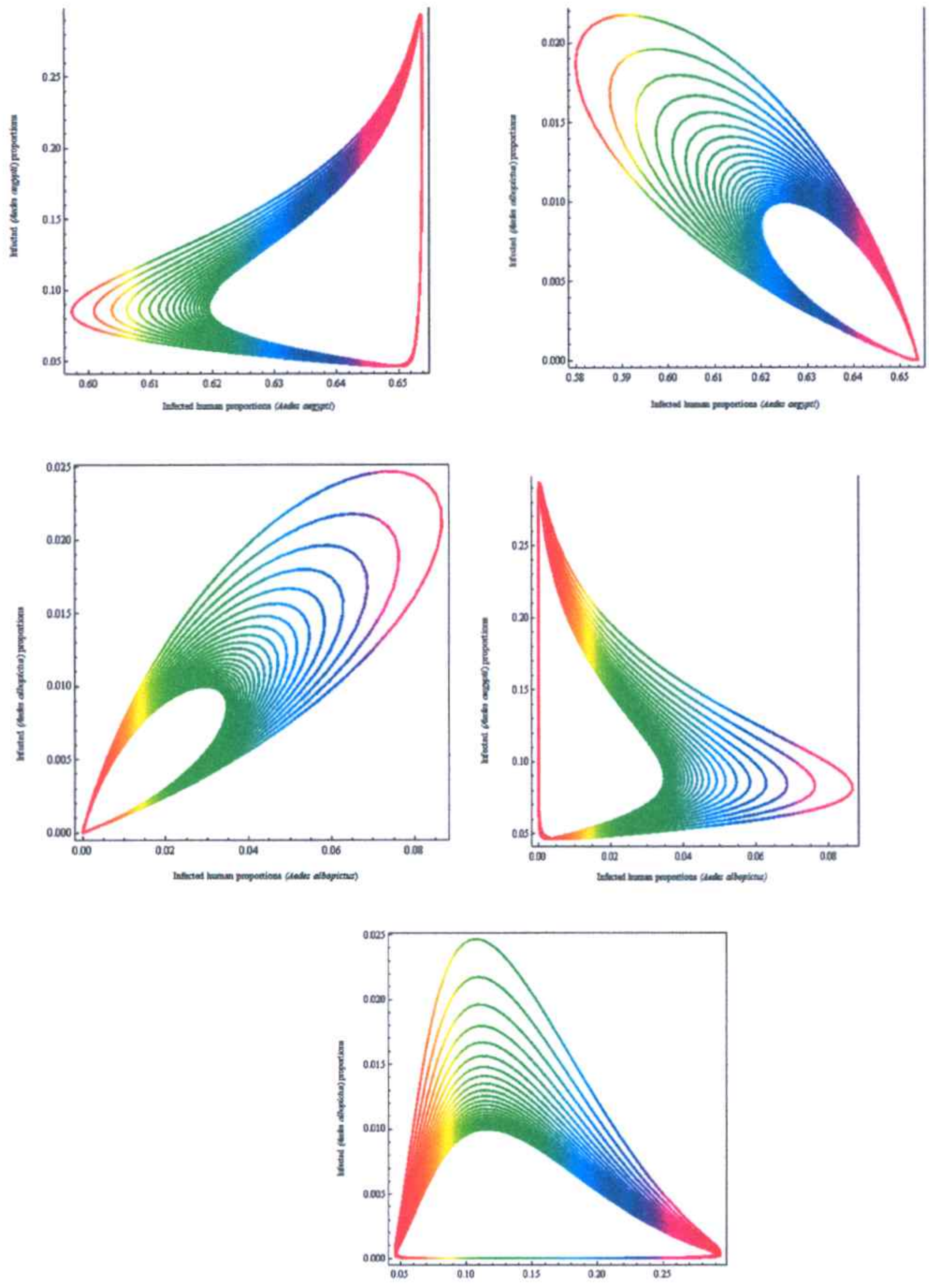


Figure 4.9. Numerical solutions of (4.25) – (4.29), demonstrate the solution trajectories, projected onto (I_a^*, I_b^*), (I_a^*, I_{va}^*), (I_a^*, I_{vb}^*), (I_b^*, I_{vb}^*), (I_b^*, I_a^*), (I_b^*, I_{va}^*) and (I_{va}^*, I_{vb}^*), respectively. For $R_0 > 1$, when $\theta = \frac{2\pi}{365}$ with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000059$, $\chi_{vb} = 0.00000036$, $\delta_{va} = 1/10$, $\delta_{vb} = 1/16$, $\chi_a = 1/4$, $\chi_b = 1/6$, $\omega_a = 0.7$, $\omega_b = 0.4$, $\omega_{va} = 0.8$, $\omega_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\chi_{ha} = 1/(17/2)$ and $\chi_{hb} = 1/(19/2)$.

The behaviors of $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ are limit cycles.

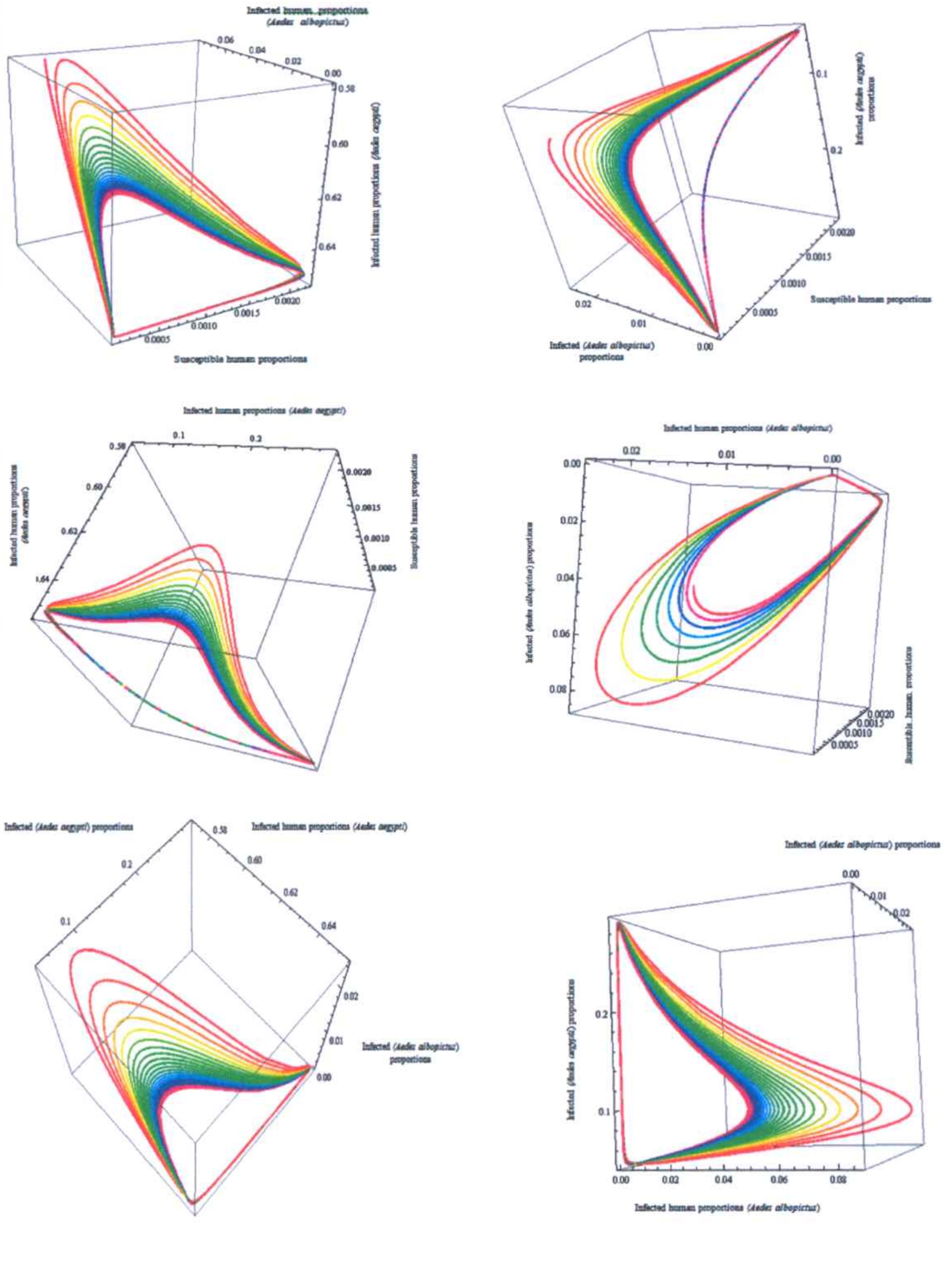

Infected human proportions (Aedes albopictus)
Infected human proportions (Aedes aegypti)
Susceptible human proportions
Infected (Aedes albopictus) proportions
Infected (Aedes aegypti) proportions
Infected human proportions (Aedes aegypti)
Susceptible human proportions
Infected human proportions (Aedes albopictus)
Infected (Aedes albopictus) proportions
Susceptible human proportions
Infected (Aedes aegypti) proportions
Infected human proportions (Aedes aegypti)
Infected (Aedes albopictus) proportions
Infected (Aedes albopictus) proportions
Infected (Aedes aegypti) proportions
Infected human proportions (Aedes albopictus)

Figure 4.10. Numerical solutions of (4.25) – (4.29), demonstrate the solution trajectories, projected onto $(S^*, I_a^*, I_b^*), (S^*, I_{va}^*, I_{vb}^*), (S^*, I_a^*, I_{va}^*), (S^*, I_b^*, I_{vb}^*), (I_a^*, I_{va}^*, I_{vb}^*)$ and $(I_b^*, I_{vb}^*, I_{va}^*)$,respectively. For $R_0 > 1$, when $\theta = \frac{2\pi}{365}$ with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000059$, $\chi_{vb} = 0.00000036$, $\delta_{va} = 1/10$, $\delta_{vb} = 1/16$, $\chi_a = 1/4$, $\chi_b = 1/6$, $\omega_a = 0.7$, $\omega_b = 0.4$, $\omega_{va} = 0.8$, $\omega_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\chi_{ha} = 1/(17/2)$ and $\chi_{hb} = 1/(19/2)$. The behaviors of $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ are limit cycles.

We present the numerical results from solving equation (4.25) – (4.29). Here , we will discuss the all of figures.

In figure 4.6. numerical solutions of (4.25) – (4.29), demonstrate the solution trajectories, projected of S' , I_a' , I_b' , I_{va}' and I_{vb}' respectively . For $R_0 < 1$, when $R_0 = 0.33878$ with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000049$, $\chi_{vb} = 0.00000026$, $\delta_{va} = 1/40$, $\delta_{vb} = 1/35$, $\chi_a = 1/15$, $\chi_b = 1/11$, $\omega_a = 0.07$, $\omega_b = 0.04$, $\omega_{va} = 0.08$, $\omega_{vb} = 0.03$, $N_{va} = 4000$, $N_{vb} = 2300$, $N_t = 20000$, $\chi_{ha} = 1/(17/2)$ and $\chi_{hb} = 1/(19/2)$.
The fractions of populations (S' , I_a' , I_b' , I_{va}' and I_{vb}') approach to the disease free equilibrium point (1,0,0,0,0).

In figure 4.7. numerical solutions the times series of $S^*, I_a^*, I_b^*, I_{va}^*$ and I_{vb}^* for $R_0 > 1$,respectively. For $R_0 > 1$, when $\theta = \frac{2\pi}{365}$ with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000059$, $\chi_{vb} = 0.00000036$, $\delta_{va} = 1/10$, $\delta_{vb} = 1/16$, $\chi_a = 1/4$, $\chi_b = 1/6$, $\omega_a = 0.7$, $\omega_b = 0.4$, $\omega_{va} = 0.8$, $\omega_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\chi_{ha} = 1/(17/2)$ and $\chi_{hb} = 1/(19/2)$. The behaviors of $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ are limit cycles.

In figure 4.8. numerical solutions of (4.25) – (4.29), demonstrate the solution trajectories, projected onto $(S^*, I_a^*), (S^*, I_b^*), (S^*, I_{va}^*)$ and (S^*, I_{vb}^*), respectively. For $R_0 > 1$, when $\theta = \frac{2\pi}{365}$ with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000059$, $\chi_{vb} = 0.00000036$, $\delta_{va} = 1/10$, $\delta_{vb} = 1/16$, $\chi_a = 1/4$, $\chi_b = 1/6$, $\omega_a = 0.7$, $\omega_b = 0.4$, $\omega_{va} = 0.8$, $\omega_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\chi_{ha} = 1/(17/2)$ and $\chi_{hb} = 1/(19/2)$.
The behaviors of $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$are limit cycles.

In figure 4.9. numerical solutions of (4.25) – (4.29), demonstrate the solution trajectories, projected onto $(I_a^*, I_b^*), (I_a^*, I_{va}^*), (I_a^*, I_{vb}^*), (I_b^*, I_{vb}^*), (I_b^*, I_a^*), (I_b^*, I_{va}^*)$ and (I_{va}^*, I_{vb}^*), respectively. For $R_0 > 1$, when $\theta = \frac{2\pi}{365}$ with $\delta_h = 1/(365*70)$ day^{-1}, $\delta_d = 1/4.5$ day^{-1}, $\chi_{va} = 0.00000059$, $\chi_{vb} = 0.00000036$, $\delta_{va} = 1/10$, $\delta_{vb} = 1/16$, $\chi_a = 1/4$, $\chi_b = 1/6$,

ω_a =0.7 , ω_b =0.4 , ω_{va} =0.8 , ω_{vb} =0.03 , N_{va} = 28000 , N_{vb} =18000 , N_t = 60000 , χ_{ha} =1/(17/2) and χ_{hb} =1/(19/2) . The behaviors of ($S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*$) are limit cycles.

In figure 4.10. numerical solutions of (4.25) – (4.29), demonstrate the solution trajectories, projected onto (S^*, I_a^*, I_b^*), $(S^*, I_{va}^*, I_{vb}^*)$, (S^*, I_a^*, I_{va}^*), (S^*, I_b^*, I_{vb}^*), $(I_a^*, I_{va}^*, I_{vb}^*)$ and $(I_b^*, I_{vb}^*, I_{va}^*)$,respectively. For $R_0 > 1$, when $\theta = \frac{2\pi}{365}$ with δ_h =1/(365*70) day^{-1}, δ_d =1/4.5 day^{-1}, χ_{va} = 0.00000059 , χ_{vb} = 0.00000036 , δ_{va} =1/10 , δ_{vb} =1/16 , χ_a =1/4 , χ_b =1/6 , ω_a =0.7 , ω_b =0.4 , ω_{va} =0.8 , ω_{vb} =0.03 , N_{va} = 28000 , N_{vb} =18000 , N_t = 60000 , χ_{ha} =1/(17/2) and χ_{hb} =1/(19/2) . The behaviors of ($S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*$) are limit cycles.

When $\theta \neq 0$, the endemic equilibrium point $P_1 = (S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ is locally asymptotically stable for $R_0 > 1$. These behaviors correspond to figure 4.7 to figure 4.10. We can see that limit cycle occurs in this case. The fraction of *Aedes aegypti* population oscillates between 0.2961 and 0.04587. The fraction of *Aedes albopictus* population oscillates between 0.01026 and 0.00002. This is an example of numerical solutions for limit cycles.

Chapter V

Dengue transmission model with the different incubation rate for each season

In this section, we modified from model 1 by adding the incubation rate for each season.

5.1 Mathematical Model

In Thailand, the annual estimations of dengue fever are depend on the season. The *Aedes aegypti* is the principal transmitter of Dengue fever in Thailand but it also transmits Chikungunya fever, yellow fever and Filariasis among other diseases. The Aedes aegypti prefers feed during daylight hours. Thailand's rainy season, starting from May through September, is also the high risk period for dengue fever, a potentially serious condition is the most prevalent in tropical countries[8]. The moisture content, temperature, season and rainfall are influence to the mosquito development. Dengue infection is endemic in Thailand. From the data of Dengue cases in 2005 – 2010, we can see that most dengue patients are occurred in rainy season. We can see as shown in figure 5.1.

This model is to incorporate this feature into the SEIR model. Models keep track of an individual's infection-age for particular diseases.

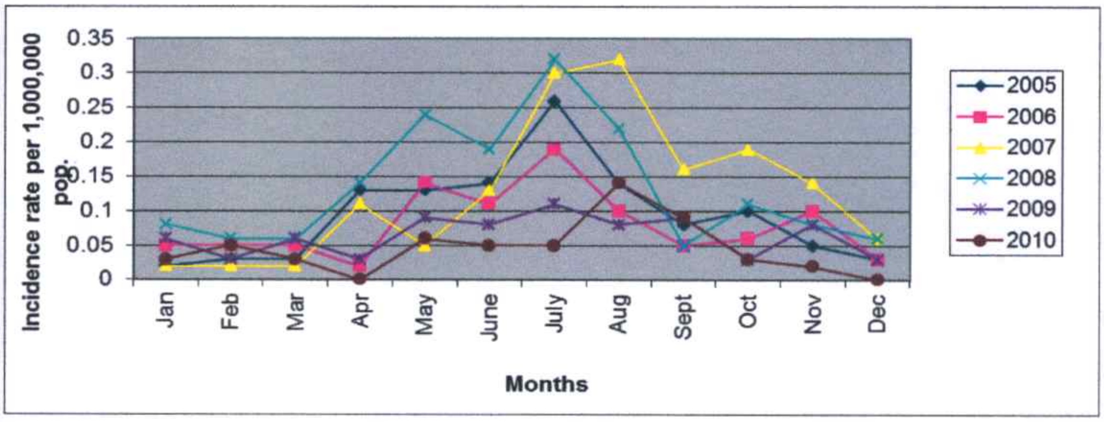


Figure 5.1. Reported cases of Dengue disease per 100,000 population in Thailand during year 2005 and 2010 [12].

We constructed the mathematical model of dengue disease by considering as shown in figure 5.1.

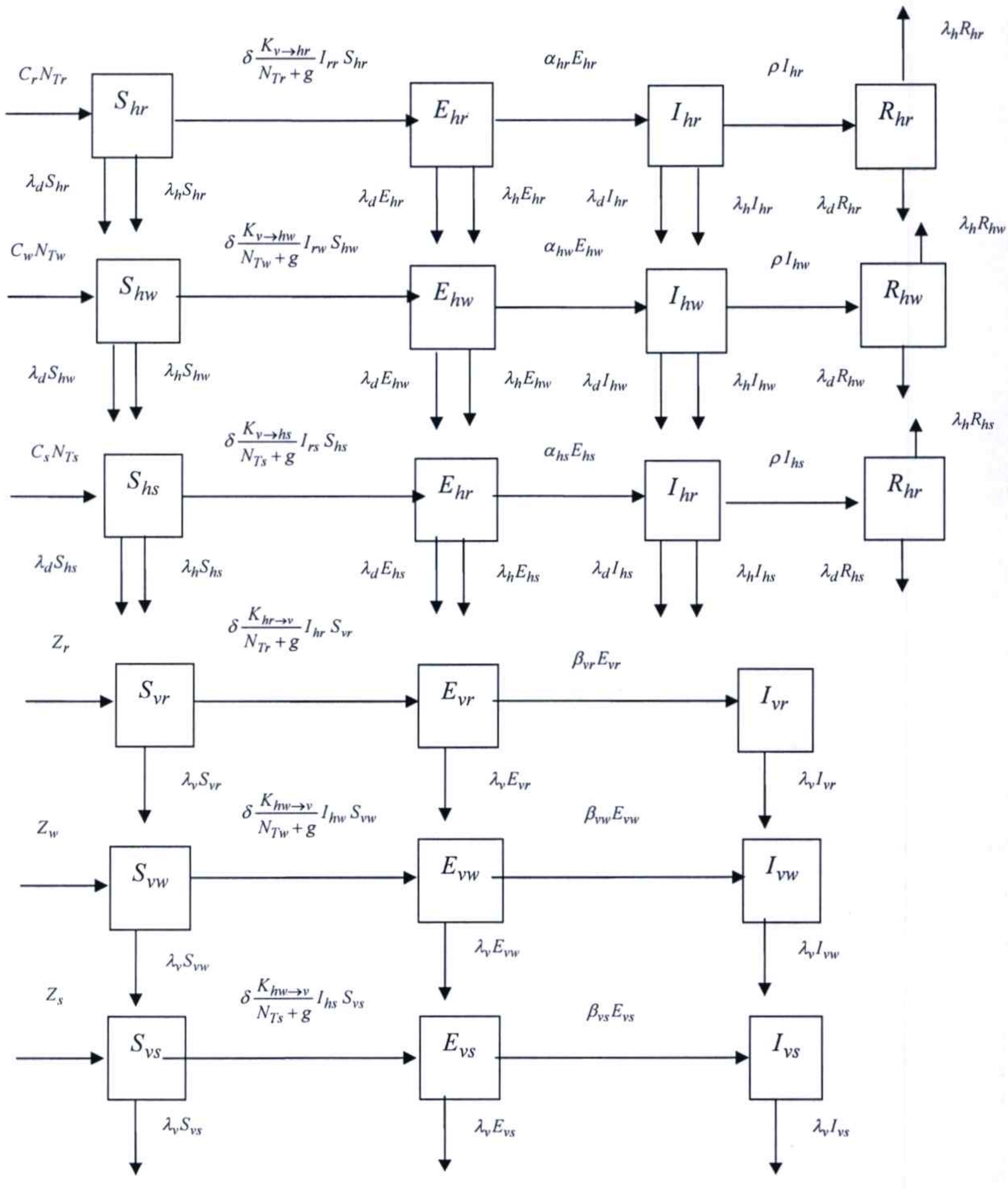


Figure 5.2. The incubation rate for each season is considered.

The dynamics of human population are given by

$$\frac{d}{dt}S_{hr} = C_r N_{Tr} - \lambda_d S_{hr} - \lambda_h S_{hr} - \delta \frac{K_{v \to hr}}{N_{Tr} + g} I_{vr} S_{hr} \quad (5.1)$$

$$\frac{d}{dt}E_{hr} = \delta \frac{K_{v \to hr}}{N_{Tr} + g} I_{vr} S_{hr} - \lambda_d E_{hr} - \lambda_h E_{hr} - \alpha_{hr} E_{hr} \quad (5.2)$$

$$\frac{d}{dt}I_{hr} = \alpha_{hr} E_{hr} - \lambda_d I_{hr} - \lambda_h I_{hr} - \rho I_{hr} \quad (5.3)$$

$$\frac{d}{dt}R_{hr} = \rho I_{hr} - \lambda_d R_{hr} - \lambda_h R_{hr} \quad (5.4)$$

$$\frac{d}{dt}S_{hw} = C_w N_{Tw} - \lambda_d S_{hw} - \lambda_h S_{hw} - \delta \frac{K_{v \to hw}}{N_{Tw} + g} I_{vw} S_{hw} \quad (5.5)$$

$$\frac{d}{dt}E_{hw} = \delta \frac{K_{v \to hw}}{N_{Tw} + g} I_{vw} S_{hw} - \lambda_d E_{hw} - \lambda_h E_{hw} - \alpha_{hw} E_{hw} \quad (5.6)$$

$$\frac{d}{dt}I_{hw} = \alpha_{hw} E_{hw} - \lambda_d I_{hw} - \lambda_h I_{hw} - \rho I_{hw} \quad (5.7)$$

$$\frac{d}{dt}R_{hw} = \rho I_{hw} - \lambda_d R_{hw} - \lambda_h R_{hw} \quad (5.8)$$

$$\frac{d}{dt}S_{hs} = C_s N_{Ts} - \lambda_d S_{hs} - \lambda_h S_{hs} - \delta \frac{K_{v \to hs}}{N_{Ts} + g} I_{vs} S_{hs} \quad (5.9)$$

$$\frac{d}{dt}E_{hs} = \delta \frac{K_{v \to hs}}{N_{Ts} + g} I_{vs} S_{hs} - \lambda_d E_{hs} - \lambda_h E_{hs} - \alpha_{hs} E_{hs} \quad (5.10)$$

$$\frac{d}{dt}I_{hs} = \alpha_{hs} E_{hs} - \lambda_d I_{hs} - \lambda_h I_{hs} - \rho I_{hs} \quad (5.11)$$

$$\frac{d}{dt}R_{hs} = \rho I_{hs} - \lambda_d R_{hs} - \lambda_h R_{hs} \quad . \quad (5.12)$$

We define the variables and parameter for above model in table 5.1

Table 5.1. Definitions of variables and parameters for equations (5.1) – (5.12).

variable/ parameter	definition
S_{hr}	the number of susceptible human population in rainy season
E_{hr}	the number of exposed human population in rainy season
I_{hr}	the number of infectious human population in rainy season
R_{hr}	the number of recovered human population in rainy season
S_{hw}	the number of susceptible human population in winter season
E_{hw}	the number of exposed human population in winter season
I_{hw}	the number of infectious human population in winter season
R_{hw}	the number of recovered human population in winter season
S_{hs}	the number of susceptible human population in summer season
E_{hs}	the number of exposed human population in summer season
I_{hs}	the number of infectious human population in summer season
R_{hs}	the number of recovered human population in summer season

The dynamics of the mosquito population are given by:

$$\frac{d}{dt}S_{vr} = Z_r - \delta\frac{K_{hr\to v}}{N_{Tr}+g}I_{hr}S_{vr} - \lambda_v S_{vr} \tag{5.13}$$

$$\frac{d}{dt}E_{vr} = \delta\frac{K_{hr\to v}}{N_{Tr}+g}I_{hr}S_{vr} - \lambda_v E_{vr} - \beta_{vr}E_{vr} \tag{5.14}$$

$$\frac{d}{dt}I_{vr} = \beta_{vr}E_{vr} - \lambda_v I_{vr} \tag{5.15}$$

$$\frac{d}{dt}S_{vw} = Z_w - \delta\frac{K_{hw\to v}}{N_{Tw}+g}I_{hw}S_{vw} - \lambda_v S_{vw} \tag{5.16}$$

$$\frac{d}{dt}E_{vw} = \delta\frac{K_{hw\to v}}{N_{Tw}+g}I_{hw}S_{vw} - \lambda_v E_{vw} - \beta_{vw}E_{vw} \tag{5.17}$$

$$\frac{d}{dt}I_{vw} = \beta_{vw}E_{vw} - \lambda_v I_{vw} \tag{5.18}$$

$$\frac{d}{dt}S_{vs} = Z_s - \delta\frac{K_{hs\to v}}{N_{Ts}+g}I_{hs}S_{vs} - \lambda_v S_{vs} \tag{5.19}$$

$$\frac{d}{dt}E_{vs} = \delta\frac{K_{hs\to v}}{N_{Ts}+g}I_{hs}S_{vs} - \lambda_v E_{vs} - \beta_{vs}E_{vs} \tag{5.20}$$

$$\frac{d}{dt}I_{vs} = \beta_{vs}E_{vs} - \lambda_v I_{vs}\,. \tag{5.21}$$

We define the variables and parameter for above model in table 5.2.

Table 5.2. Definitions of variables and parameters for equations (5.13) – (5.21).

variable/ parameter	definition
S_{vr}	the number of susceptible mosquito population in rainy season
E_{vr}	the number of exposed mosquito population in rainy season
I_{vr}	the number of infectious mosquito population in rainy season
S_{vw}	the number of susceptible mosquito population in winter season
E_{vw}	the number of exposed mosquito population in winter season
I_{vw}	the number of infectious mosquito population in winter season
S_{vs}	the number of susceptible mosquito population in summer season
E_{vs}	the number of exposed mosquito population in summer season
I_{vs}	the number of infectious mosquito population in summer season

Where the parameters are defined as follows:

Table 5.3 Definitions of variables and parameters for model (5.1) – (5.21).

variable/ parameter	definition
N_{Tr}	the total human population in rainy season
N_{Tw}	the total human population in winter season
N_{Ts}	the total human population in summer season
N_{Vr}	the total mosquito population in rainy season
N_{Vw}	the total mosquito population in winter season
N_{Vs}	the total mosquito population in summer season
λ_h	the natural death rate of human population
λ_d	the death rate of human population due to the disease
λ_v	the death rate of mosquito population
K	the birth rate of human population
$K_{v \to hr}$	the transmission probability of dengue disease from mosquito to human in rainy season
$K_{v \to hw}$	the transmission probability of dengue disease from mosquito to human in winter season
$K_{v \to hs}$	the transmission probability of dengue disease from mosquito to human in summer season
$K_{hr \to v}$	the transmission probability of dengue disease from human to mosquito in rainy season
$K_{hw \to v}$	the transmission probability of dengue disease from human to mosquito in winter season
$K_{hs \to v}$	the transmission probability of dengue disease from human to mosquito in summer season
α_{hr}	the incubation rate of human population in rainy season
α_{hw}	the incubation rate of human population in winter season
α_{hs}	the incubation rate of human population in summer season
α_{vr}	the incubation rate of mosquito population in rainy season
α_{vw}	the incubation rate of mosquito population in winter season
α_{vs}	the incubation rate of mosquito population in summer season
ρ	the recovery rate of human population
δ	the biting rate of mosquito population
g	the number of other animals available as blood sources

We suppose that $N_{Hr} = S_{hr} + E_{hr} + I_{hr} + R_{hr}$, $N_{Hw} = S_{hw} + E_{hw} + I_{hw} + R_{hw}$,

$N_{Hs} = S_{hs} + E_{hs} + I_{hs} + R_{hs}$, $N_{Vr} = S_{vr} + E_{vr} + I_{vr}$, $N_{Vw} = S_{vw} + E_{vw} + I_{vw}$ and $N_{Vs} = S_{vs} + E_{vs} + I_{vs}$.

We assume that total population remains constant and each group of population (human and mosquito) also remains constant, we have

$\frac{dN_{Hr}}{dt} = 0$, $\frac{dN_{Vr}}{dt} = 0$ in rainy season,

$\frac{dN_{Hw}}{dt} = 0$, $\frac{dN_{Vw}}{dt} = 0$ in winter season,

and $\frac{dN_{Hs}}{dt} = 0$, $\frac{dN_{Vs}}{dt} = 0$ in summer season.

i) $$\frac{dN_{Hr}}{dt} = \frac{d}{dt}(S_{hr} + E_{hr} + I_{hr} + R_{hr})$$

$$= C_r N_{Tr} - (\lambda_d + \lambda_h) N_{Hr}$$

So that $C_r N_{Tr} = (\lambda_d + \lambda_h) N_{Hr} \Rightarrow N_{Hr} = \frac{C_r N_{Tr}}{(\lambda_d + \lambda_h)}$.

The results of above equation indicates that the total of human population equals to the ratio between the summation of the constant recruitment rate of human and their constant death rate in rainy season.

ii) $$\frac{dN_{Hw}}{dt} = \frac{d}{dt}(S_{hw} + E_{hw} + I_{hw} + R_{hw})$$

$$= C_w N_{Tw} - (\lambda_d + \lambda_h) N_{Hw}$$

So that $C_w N_{Tw} = (\lambda_d + \lambda_h) N_{Hw} \Rightarrow N_{Hw} = \frac{C_w N_{Tw}}{(\lambda_d + \lambda_h)}$.

The results of above equation indicates that the total human population equals to the ratio between the summation of the constant recruitment rate of human and their constant death rate in winter season.

iii) $$\frac{dN_{Hs}}{dt} = \frac{d}{dt}(S_{hs} + E_{hs} + I_{hs} + R_{hs})$$

$$= C_s N_{Ts} - (\lambda_d + \lambda_h) N_{Hs}$$

So that $C_s N_{Ts} = (\lambda_d + \lambda_h) N_{Hs} \Rightarrow N_{Hs} = \frac{C_s N_{Ts}}{(\lambda_d + \lambda_h)}$.

The results of above equation indicates that the total human population equals to the ratio between the summation of the constant recruitment rate of human and their constant death rate in summer season.

IV) $\frac{dN_{Vr}}{dt} = \frac{d}{dt}(S_{vr}+E_{vr}+I_{vr})$

$= Z_r - \lambda_v N_{Vr}$

This shows that $Z_r - \lambda_v N_{Vr} \Rightarrow N_{Vr} = \frac{Z_r}{\lambda_v}$.

The results of above equation indicates that the number of mosquitoes equals to the ratio between the constant recruitment rate of the mosquitoes and their death rate in rainy season.

V) $\frac{dN_{Vw}}{dt} = \frac{d}{dt}(S_{vw}+E_{vw}+I_{vw})$

$= Z_w - \lambda_v N_{Vw}$

This shows that $Z_w - \lambda_v N_{Vw} \Rightarrow N_{Vw} = \frac{Z_w}{\lambda_v}$.

The results of above equation indicates that the number of mosquitoes to the ratio between the constant recruitment rate of the mosquitoes and their death rate in winter season.

VI) $\frac{dN_{Vs}}{dt} = \frac{d}{dt}(S_{vs}+E_{vs}+I_{vs})$

$= Z_s - \lambda_v N_{Vs}$

This shows that $Z_s - \lambda_v N_{Vs} \Rightarrow N_{Vs} = \frac{Z_s}{\lambda_v}$.

The results of above equation indicates that the number of mosquitoes equals to the ratio between the constant recruitment rate of the mosquitoes and their death rate in summer season.

Introducing the normalized parameter

$$\overline{S_{hr}} = \frac{S_{hr}}{N_{Tr}},\ \overline{E_{hr}} = \frac{E_{hr}}{N_{Tr}},\ \overline{I_{hr}} = \frac{I_{hr}}{N_{Tr}},\ \overline{R_{hr}} = \frac{R_{hr}}{N_{Tr}},$$

$$\overline{S_{hw}} = \frac{S_{hw}}{N_{Tw}},\ \overline{E_{hw}} = \frac{E_{hw}}{N_{Tw}},\ \overline{I_{hw}} = \frac{I_{hw}}{N_{Tw}},\ \overline{R_{hw}} = \frac{R_{hw}}{N_{Tw}},$$

$$\overline{S_{hs}} = \frac{S_{hs}}{N_{Ts}}, \overline{E_{hs}} = \frac{E_{hs}}{N_{Ts}}, \overline{I_{hs}} = \frac{I_{hs}}{N_{Ts}}, \overline{R_{hs}} = \frac{R_{hs}}{N_{Ts}},$$

$$\overline{S_{vr}} = \frac{S_{vr}}{N_{Vr}}, \overline{S_{vw}} = \frac{S_{vw}}{N_{Vw}}, \overline{S_{vs}} = \frac{S_{vs}}{N_{Vs}},$$

$$\overline{E_{vr}} = \frac{E_{vr}}{N_{Vr}}, \overline{E_{vw}} = \frac{E_{vw}}{N_{Vw}}, \overline{E_{vs}} = \frac{E_{vs}}{N_{Vs}},$$

$$\overline{I_{vr}} = \frac{I_{vr}}{N_{Vr}}, \overline{I_{vw}} = \frac{I_{vw}}{N_{Vw}}, \overline{I_{vs}} = \frac{I_{vs}}{N_{Vs}}.$$

Then we obtain normalizing equations (5.13) – (5.21);

$$\frac{d}{dt}\bar{S}_{hr} = (\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta \frac{K_{v \to hr}}{N_{Tr} + g} \bar{I}_{vr} N_{Vr}) \bar{S}_{hr} \quad (5.22)$$

$$\frac{d}{dt}\bar{E}_{hr} = \delta \frac{K_{v \to hr}}{N_{Tr} + g} \bar{I}_{vr} N_{Vr} \bar{S}_{hr} - (\lambda_d + \lambda_h + \alpha_{hr}) \bar{E}_{hr} \quad (5.23)$$

$$\frac{d}{dt}\bar{I}_{hr} = \alpha_{hr} \bar{E}_{hr} - (\lambda_d + \lambda_h + \rho) \bar{I}_{hr} \quad (5.24)$$

$$\frac{d}{dt}\bar{S}_{hw} = (\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta \frac{K_{v \to hw}}{N_{Tw} + g} \bar{I}_{vw} N_{Vw}) \bar{S}_{hw} \quad (5.25)$$

$$\frac{d}{dt}\bar{E}_{hw} = \delta \frac{K_{v \to hw}}{N_{Tw} + g} \bar{I}_{vw} N_{Vw} \bar{S}_{hw} - (\lambda_d + \lambda_h + \alpha_{hw}) \bar{E}_{hw} \quad (5.26)$$

$$\frac{d}{dt}\bar{I}_{hw} = \alpha_{hw} \bar{E}_{hw} - (\lambda_d + \lambda_h + \rho) \bar{I}_{hw} \quad (5.27)$$

$$\frac{d}{dt}\bar{S}_{hs} = (\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta \frac{K_{v \to hs}}{N_{Ts} + g} \bar{I}_{vs} N_{Vs}) \bar{S}_{hs} \quad (5.28)$$

$$\frac{d}{dt}\bar{E}_{hs} = \delta \frac{K_{v \to hs}}{N_{Ts} + g} \bar{I}_{vs} N_{Vs} \bar{S}_{hs} - (\lambda_d + \lambda_h + \alpha_{hs}) \bar{E}_{hs} \quad (5.29)$$

$$\frac{d}{dt}\bar{I}_{hs} = \alpha_{hs} \bar{E}_{hs} - (\lambda_d + \lambda_h + \rho) \bar{I}_{hs} \quad (5.30)$$

$$\frac{d}{dt}\bar{E}_{vr} = \delta \frac{K_{hr \to v}}{N_{Tr} + g} \bar{I}_{hr} N_{Tr} (1 - \bar{E}_{vr} - \bar{I}_{vr}) - (\lambda_v + \beta_{vr}) \bar{E}_{vr} \quad (5.31)$$

$$\frac{d}{dt}\bar{I}_{vr} = \beta_{vr} \bar{E}_{vr} - \lambda_v \bar{I}_{vr} \quad (5.32)$$

$$\frac{d}{dt}\overline{E}_{vw} = \delta\frac{K_{hw\to v}}{N_{Tw}+g}\overline{I}_{hw}\,N_{Tw}(1-\overline{E}_{vw}-\overline{I}_{vw})-(\lambda_v+\beta_{vw})\overline{E}_{vw} \tag{5.33}$$

$$\frac{d}{dt}\overline{I}_{vw} = \beta_{vw}\overline{E}_{vw}-\lambda_v\overline{I}_{vw} \tag{5.34}$$

$$\frac{d}{dt}\overline{E}_{vs} = \delta\frac{K_{hs\to v}}{N_{Ts}+g}\overline{I}_{hs}\,N_{Ts}(1-\overline{E}_{vs}-\overline{I}_{vs})-(\lambda_v+\beta_{vs})\overline{E}_{vs} \tag{5.35}$$

$$\frac{d}{dt}\overline{I}_{vs} = \beta_{vs}\overline{E}_{vs}-\lambda_v\overline{I}_{vs}\,. \tag{5.36}$$

R_{hr}, R_{hw}, R_{hs} and S_{hr}, S_{hw}, S_{hs} can be obtained from conditions

$S_{hr}+E_{hr}+I_{hr}+R_{hr}=1$, $S_{hw}+E_{hw}+I_{hw}+R_{hw}=1$, $S_{hs}+E_{hs}+I_{hs}+R_{hs}=1$,

$S_{vr}+E_{vr}+I_{vr}=1$, $S_{vw}+E_{vw}+I_{vw}=1$ and $S_{vs}+E_{vs}+I_{vs}=1$.

5.2. Analysis of The Mathematical Model

5.2.1. The steady state solution

The equilibrium points

$(S^*_{hr}, E^*_{hr}, I^*_{hr}, E^*_{vr}, I^*_{vr}, S^*_{hw}, E^*_{hw}, I^*_{hw}, E^*_{vw}, I^*_{vw}, S^*_{hs}, E^*_{hs}, I^*_{hs}, E^*_{vs}, I^*_{vs})$
are found by setting the right hand side of (5.22) – (5.36) equal to zero. Then we have

$$(\lambda_d+\lambda_h)-(\lambda_d+\lambda_h+\delta\frac{K_{v\to hr}}{N_{Tr}+g}\overline{I}_{vr}\,N_{Vr})\overline{S}_{hr} = 0 \tag{5.37}$$

$$\delta\frac{K_{v\to hr}}{N_{Tr}+g}\overline{I}_{vr}\,N_{Vr}\,\overline{S}_{hr}-(\lambda_d+\lambda_h+\alpha_{hr})\overline{E}_{hr} = 0\,, \tag{5.38}$$

$$\alpha_{hr}\overline{E}_{hr}-(\lambda_d+\lambda_h+\rho)\overline{I}_{hr} = 0\,, \tag{5.39}$$

$$(\lambda_d+\lambda_h)-(\lambda_d+\lambda_h+\delta\frac{K_{v\to hw}}{N_{Tw}+g}\overline{I}_{vw}\,N_{Vw})\overline{S}_{hw} = 0\,, \tag{5.40}$$

$$\delta\frac{K_{v\to hw}}{N_{Tw}+g}\overline{I}_{vw}\,N_{Vw}\,\overline{S}_{hw}-(\lambda_d+\lambda_h+\alpha_{hw})\overline{E}_{hw} = 0\,, \tag{5.41}$$

$$\alpha_{hw}\overline{E}_{hw}-(\lambda_d+\lambda_h+\rho)\overline{I}_{hw} = 0\,, \tag{5.42}$$

$$(\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta \frac{K_{v \to hs}}{N_{Ts} + g} \bar{I}_{vs} N_{Vs}) \bar{S}_{hs} = 0 , \tag{5.43}$$

$$\delta \frac{K_{v \to hs}}{N_{Ts} + g} \bar{I}_{vs} N_{Vs} \bar{S}_{hs} - (\lambda_d + \lambda_h + \alpha_{hs}) \bar{E}_{hs} = 0 , \tag{5.44}$$

$$\alpha_{hs} \bar{E}_{hs} - (\lambda_d + \lambda_h + \rho) \bar{I}_{hs} = 0 , \tag{5.45}$$

$$\delta \frac{K_{hr \to v}}{N_{Tr} + g} \bar{I}_{hr} N_{Tr} (1 - \bar{E}_{vr} - \bar{I}_{vr}) - (\lambda_v + \beta_{vr}) \bar{E}_{vr} = 0 , \tag{5.46}$$

$$\beta_{vr} \bar{E}_{vr} - \lambda_v \bar{I}_{vr} = 0 , \tag{5.47}$$

$$\delta \frac{K_{hw \to v}}{N_{Tw} + g} \bar{I}_{hw} N_{Tw} (1 - \bar{E}_{vw} - \bar{I}_{vw}) - (\lambda_v + \beta_{vw}) \bar{E}_{vw} = 0 , \tag{5.48}$$

$$\beta_{vw} \bar{E}_{vw} - \lambda_v \bar{I}_{vw} = 0 , \tag{5.49}$$

$$\delta \frac{K_{hs \to v}}{N_{Ts} + g} \bar{I}_{hs} N_{Ts} (1 - \bar{E}_{vs} - \bar{I}_{vs}) - (\lambda_v + \beta_{vs}) \bar{E}_{vs} = 0 , \tag{5.50}$$

$$\beta_{vs} \bar{E}_{vs} - \lambda_v \bar{I}_{vs} = 0 \tag{5.51}$$

Doing this, we get two equilibrium points

i) The disease free equilibrium point

$(1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0)$.

ii)The endemic disease equilibrium point

$(S^*_{hr}, E^*_{hr}, I^*_{hr}, E^*_{vr}, I^*_{vr}, S^*_{hw}, E^*_{hw}, I^*_{hw}, E^*_{vw}, I^*_{vw}, S^*_{hs}, E^*_{hs}, I^*_{hs}, E^*_{vs}, I^*_{vs})$

where

$$S^*_{hr} = \frac{(\lambda_d + \lambda_h)}{(\lambda_d + \lambda_h) + \frac{I^*_{hr} N_{Tr} N_{Vr} \beta_{vr} \delta K_{hr \to v} \delta K_{v \to hr}}{(N_{Tr} + g)(\beta_{vr} + \lambda_v)(I^*_{hr} N_{Tr} \delta K_{hr \to v} + (N_{Tr} + g)\lambda_v)}} , \tag{5.52}$$

$$E^*_{hr} = \frac{I^*_{hr} N_{Vr} \beta_{vr} N_{Tr} \delta K_{hr \to v} \delta K_{v \to hr} (\lambda_d + \lambda_h)}{(N_{Tr} + g)(\alpha_{hr} + \lambda_d + \lambda_h)(\beta_{vr} + \lambda_v)(I^*_{hr} N_{Tr} \delta K_{hr \to v} + (N_{Tr} + g)\lambda_v)(\lambda_d + \lambda_h + \frac{I^*_{hr} N_{Vr} \beta_{vr} N_{Tr} \delta K_{hr \to v} \delta K_{v \to hr}}{(N_{Tr} + g)(\beta_{vr} + \lambda_v)(I^*_{hr} N_{Tr} \delta K_{hr \to v} + (N_{Tr} + g)\lambda_v)}))} , \tag{5.53}$$

$$I^*_{hr} = \frac{(\lambda_d + \lambda_h)(-(N_{Tr} N_{Vr} \alpha_{hr} \beta_{vr} \delta K_{hr \to v} \delta K_{v \to hr} + (N_{Tr} + g)^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h)\lambda_v (\beta_{vr} + \lambda_v)))}{(N_{Tr} \delta K_{hr \to v} (\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h)(N_{Vr} \beta_{vr} \delta K_{v \to hr} + (N_{Tr} + g)(\lambda_d + \lambda_h)(\beta_{vr} + \lambda_v)))} , \tag{5.54}$$

$$E^*_{vr} = \frac{I^*_{hr} N_{Tr} \delta K_{hr\to v} \lambda_v}{(\beta_{vr}+\lambda_v)(I^*_{hr} N_{Tr} \delta K_{hr\to v} + (N_{Tr}+g)\lambda_v)}, \tag{5.55}$$

$$I^*_{vr} = \frac{I^*_{hr} N_{Tr} \delta K_{hr\to v} \beta_{vr}}{(\beta_{vr}+\lambda_v)(I^*_{hr} N_{Tr} \delta K_{hr\to v} + (N_{Tr}+g)\lambda_v)}, \tag{5.56}$$

$$S^*_{hw} = \frac{(\lambda_d+\lambda_h)}{(\lambda_d+\lambda_h) + \dfrac{I^*_{hw} N_{Tw} N_{Vw} \beta_{vw} \delta K_{hw\to v} \delta K_{v\to hw}}{(N_{Tw}+g)(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw} \delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)}}, \tag{5.57}$$

$$E^*_{hw} = \frac{I^*_{hw} N_{Vw} \beta_{vw} N_{Tw} \delta K_{hw\to v} \delta K_{v\to hw} (\lambda_d+\lambda_h)}{(N_{Tw}+g)(\alpha_{hw}+\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw} \delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)(\lambda_d+\lambda_h + \dfrac{I^*_{hw} N_{Vw} \beta_{vw} N_{Tw} \delta K_{hw\to v} \delta K_{v\to hw}}{(N_{Tw}+g)(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw} \delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)}))}, \tag{5.58}$$

$$I^*_{hw} = \frac{(\lambda_d+\lambda_h)(-(N_{Tw} N_{Vw} \alpha_{hw} \beta_{vw} \delta K_{hw\to v} \delta K_{v\to hw} + (N_{Tw}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)))}{(N_{Tw} \delta K_{hw\to v}(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)(N_{Vw} \beta_{vw} \delta K_{v\to hw} + (N_{Tw}+g)(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v)))}, \tag{5.59}$$

$$E^*_{vw} = \frac{I^*_{hw} N_{Tw} \delta K_{hw\to v} \lambda_v}{(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw} \delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)}, \tag{5.60}$$

$$I^*_{vw} = \frac{I^*_{hw} N_{Tw} \delta K_{hw\to v} \beta_{vw}}{(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw} \delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)}, \tag{5.61}$$

$$S^*_{hs} = \frac{(\lambda_d+\lambda_h)}{(\lambda_d+\lambda_h) + \dfrac{I^*_{hs} N_{Ts} N_{Vs} \beta_{vs} \delta K_{hs\to v} \delta K_{v\to hs}}{(N_{Ts}+g)(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts} \delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)}}, \tag{5.62}$$

$$E^*_{hs} = \frac{I^*_{hs} N_{Vs} \beta_{vs} N_{Ts} \delta K_{hs\to v} \delta K_{v\to hs} (\lambda_d+\lambda_h)}{(N_{Ts}+g)(\alpha_{hs}+\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts} \delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)(\lambda_d+\lambda_h + \dfrac{I^*_{hs} N_{Vs} \beta_{vs} N_{Ts} \delta K_{hs\to v} \delta K_{v\to hs}}{(N_{Ts}+g)(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts} \delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)}))}, \tag{5.63}$$

$$I^*_{hs} = \frac{(\lambda_d+\lambda_h)(-(N_{Ts} N_{Vs} \alpha_{hs} \beta_{vs} \delta K_{hs\to v} \delta K_{v\to hs} + (N_{Ts}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)))}{(N_{Ts} \delta K_{hs\to v}(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)(N_{Vs} \beta_{vs} \delta K_{v\to hs} + (N_{Ts}+g)(\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v)))}, \tag{5.64}$$

$$E^*_{vs} = \frac{I^*_{hs} N_{Ts} \delta K_{hs\to v} \lambda_v}{(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts} \delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)}, \tag{5.65}$$

$$I^*_{vs} = \frac{I^*_{hs} N_{Ts} \delta K_{hs\to v} \beta_{vs}}{(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts} \delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)}. \tag{5.66}$$

5.2.2. Stability analysis

The stability of each equilibrium point is determined from linearizing equations (5.22) – (5.36) about equilibrium point examining the eigenvalues of the resulting Jacobian matrix. We now consider the eigenvalues of the Jacobian matrix at each equilibrium point. From equations (5.22) – (5.36), we can write in the matrix form as follows:

$$J_{E_1}=\begin{pmatrix} -(\frac{\delta K_{v\to hr}}{(N_{Tr}+g)}I^*_{vr}N_{Vr}+\lambda_d+\lambda_h) & 0 & 0 & 0 & -(\frac{\delta K_{v\to hr}}{(N_{Tr}+g)}N_{Vr}S^*_{hr}) \\ \frac{\delta K_{v\to hr}}{(N_{Tr}+g)}I^*_{vr}N_{Vr} & -(\lambda_d+\lambda_h+\alpha_{hr}) & 0 & 0 & (\frac{\delta K_{v\to hr}}{(N_{Tr}+g)}N_{Vr}S^*_{hr}) \\ 0 & \alpha_{hr} & -(\lambda_d+\lambda_h+\rho) & 0 & 0 \\ 0 & 0 & (\frac{\delta K_{hr\to v}}{(N_{Tr}+g)}N_{Tr}S^*_{Vr}) & -(\lambda_v+\beta_{Vr}) & 0 \\ 0 & 0 & 0 & \beta_{Vr} & -\lambda_v \end{pmatrix} \quad (5.67)$$

$$J_{E_2}=\begin{pmatrix} -(\frac{\delta K_{v\to hw}}{(N_{Tw}+g)}I^*_{vw}N_{Vw}+\lambda_d+\lambda_h) & 0 & 0 & 0 & -(\frac{\delta K_{v\to hw}}{(N_{Tw}+g)}N_{Vw}S^*_{hw}) \\ \frac{\delta K_{v\to hw}}{(N_{Tw}+g)}I^*_{vw}N_{Vw} & -(\lambda_d+\lambda_h+\alpha_{hw}) & 0 & 0 & (\frac{\delta K_{v\to hw}}{(N_{Tw}+g)}N_{Vw}S^*_{hw}) \\ 0 & \alpha_{hw} & -(\lambda_d+\lambda_h+\rho) & 0 & 0 \\ 0 & 0 & (\frac{\delta K_{hw\to v}}{(N_{Tw}+g)}N_{Tw}S^*_{Vw}) & -(\lambda_v+\beta_{Vw}) & 0 \\ 0 & 0 & 0 & \beta_{Vw} & -\lambda_v \end{pmatrix} \quad (5.68)$$

$$J_{E_3}=\begin{pmatrix} -(\frac{\delta K_{v\to hs}}{(N_{Ts}+g)}I^*_{vs}N_{Vs}+\lambda_d+\lambda_h) & 0 & 0 & 0 & -(\frac{\delta K_{v\to hs}}{(N_{Ts}+g)}N_{Vs}S^*_{hs}) \\ \frac{\delta K_{v\to hs}}{(N_{Ts}+g)}I^*_{vs}N_{Vs} & -(\lambda_d+\lambda_h+\alpha_{hs}) & 0 & 0 & (\frac{\delta K_{v\to hs}}{(N_{Ts}+g)}N_{Vs}S^*_{hs}) \\ 0 & \alpha_{hs} & -(\lambda_d+\lambda_h+\rho) & 0 & 0 \\ 0 & 0 & (\frac{\delta K_{hs\to v}}{(N_{Ts}+g)}N_{Ts}S^*_{Vs}) & -(\lambda_v+\beta_{Vs}) & 0 \\ 0 & 0 & 0 & \beta_{Vs} & -\lambda_v \end{pmatrix} \quad (5.69)$$

The eigenvalues (p) are the solution of the characteristic equation

$$\det(J-PI_5)=0$$

where J is the Jacobian matrix evaluated at the equilibrium point . I_5 is the identity matrix.

i) At the disease free equilibrium state

$E_{1,2,3}$ = (1, 0, 0, 0,0, 1, 0, 0,0, 0,1, 0, 0,0,0) has eigenvalues as follows:

$$(-p-\lambda_d-\lambda_h)(-\frac{\delta^2 K_{hr\to v}K_{v\to hr}N_{Tr}S_{Vr}\alpha_{hr}\beta_{vr}}{(N_{Tr}+g)^2}+(-p-\rho-\lambda_d-\lambda_h)(-p-\alpha_{hr}-\lambda_d-\lambda_h)(-p-\beta_{vr}-\lambda_v)).$$

The eigenvalues are

$p=-\lambda_d-\lambda_h$ and the remaining 4 eigenvalues are the solutions of

$$(-\frac{\delta^2 K_{hr\to v}K_{v\to hr}N_{Tr}S_{Vr}\alpha_{hr}\beta_{vr}}{(N_{Tr}+g)^2}+(-p-\rho-\lambda_d-\lambda_h)(-p-\alpha_{hr}-\lambda_d-\lambda_h)(-p-\beta_{vr}-\lambda_v))=0$$

$$(-p-\lambda_d-\lambda_h)(-\frac{\delta^2 K_{hr\to v}K_{v\to hr}N_{Tr}S_{Vr}\alpha_{hr}\beta_{vr}}{(N_{Tr}+g)^2}+(-p-\rho-\lambda_d-\lambda_h)(-p-\alpha_{hr}-\lambda_d-\lambda_h)(-p-\beta_{vr}-\lambda_v))\ .$$

The eigenvalues are

$p = -\lambda_d - \lambda_h$ and the remaining 4 eigenvalues are the solutions of

$$(-\frac{\delta^2 K_{hw\to v}K_{v\to hw}N_{Tw}S_{Vw}\alpha_{hw}\beta_{vw}}{(N_{Tw}+g)^2}+(-p-\rho-\lambda_d-\lambda_h)(-p-\alpha_{hw}-\lambda_d-\lambda_h)(-p-\beta_{vw}-\lambda_v))=0$$

$$(-p-\lambda_d-\lambda_h)(-\frac{\delta^2 K_{hw\to v}K_{v\to hw}N_{Tw}S_{Vw}\alpha_{hw}\beta_{vw}}{(N_{Tw}+g)^2}+(-p-\rho-\lambda_d-\lambda_h)(-p-\alpha_{hw}-\lambda_d-\lambda_h)(-p-\beta_{vw}-\lambda_v))\ .$$

The eigenvalues are

$p = -\lambda_d - \lambda_h$ and the remaining 4 eigenvalues are the solutions of

$$(-\frac{\delta^2 K_{hs\to v}K_{v\to hs}N_{Ts}S_{Vs}\alpha_{hs}\beta_{vs}}{(N_{Ts}+g)^2}+(-p-\rho-\lambda_d-\lambda_h)(-p-\alpha_{hs}-\lambda_d-\lambda_h)(-p-\beta_{vs}-\lambda_v))=0$$

$$(-p-\lambda_d-\lambda_h)(-\frac{\delta^2 K_{hs\to v}K_{v\to hs}N_{Ts}S_{Vs}\alpha_{hs}\beta_{vs}}{(N_{Ts}+g)^2}+(-p-\rho-\lambda_d-\lambda_h)(-p-\alpha_{hs}-\lambda_d-\lambda_h)(-p-\beta_{vs}-\lambda_v))$$

or $\lambda^4 + A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A = 0$

where

$$A_3=\rho+\alpha_{hr}+\beta_{vr}+2\lambda_d+2+2\lambda_v$$

$$A_2=(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v+\lambda_v^2+\beta_{vr}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hr}(\rho+\beta_{vr}+\lambda_d+\lambda_h+2\lambda_v)$$

$$A_1=\beta_{vr}(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)+(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vr}(\rho+2\lambda_d+2\lambda_h)+\alpha_{hr}(\beta_{vr}+2(\rho+\lambda_d+\lambda_h)))\lambda_v+(\rho+\alpha_{hr}+2\lambda_d+2\lambda_h)\lambda_v^2$$

$$A=\frac{1}{(N_{Tr}+g)^2}\left(-\delta^2K_{hr\to v}K_{v\to hr}N_{Tr}N_{Vr}S_{Vr}\alpha_{hr}\beta_{vr}+(N_{Tr}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)\right)$$

$$A_3=\rho+\alpha_{hw}+\beta_{vw}+2\lambda_d+2+2\lambda_v$$

$$A_2=(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v+\lambda_v^2+\beta_{vw}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hw}(\rho+\beta_{vw}+\lambda_d+\lambda_h+2\lambda_v)$$

$$A_1=\beta_{vw}(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)+(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vw}(\rho+2\lambda_d+2\lambda_h)+\alpha_{hw}(\beta_{vw}+2(\rho+\lambda_d+\lambda_h)))\lambda_v+(\rho+\alpha_{hw}+2\lambda_d+2\lambda_h)\lambda_v^2$$

$$A=\frac{1}{(N_{Tw}+g)^2}\left(-\delta^2K_{hw\to v}K_{v\to hw}N_{Tr}N_{Vw}S_{Vw}\alpha_{hw}\beta_{vw}+(N_{Tw}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)\right)$$

$$A_3=\rho+\alpha_{hs}+\beta_{vs}+2\lambda_d+2+2\lambda_v$$

$$A_2=(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v+\lambda_v^2+\beta_{vs}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hs}(\rho+\beta_{vs}+\lambda_d+\lambda_h+2\lambda_v)$$

$$A_1=\beta_{vs}(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)+(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vs}(\rho+2\lambda_d+2\lambda_h)+\alpha_{hs}(\beta_{vs}+2(\rho+\lambda_d+\lambda_h)))\lambda_v+(\rho+\alpha_{hs}+2\lambda_d+2\lambda_h)\lambda_v^2$$

$$A=\frac{1}{(N_{Ts}+g)^2}\left(-\delta^2K_{hs\to v}K_{v\to hs}N_{Ts}N_{Vs}S_{Vs}\alpha_{hs}\beta_{vs}+(N_{Ts}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)\right)$$

Equilibrium point is local stability its eigenvalues have negative real parts when they are according to the Routh – Hurwitz criteria:

i) $A_3 > 0$ (5.70)

ii) $A_1 > 0$ (5.71)

iii) $A > 0$ (5.72)

iv) $A_3A_2A_1 - A_1^2 - A_3^2A > 0$ (5.73)

After, we consider condition of Routh – Hurwitz criteria as shown with parameters above. For the conditions given by (5.70) – (5.73) are always positive. So this disease free state is local stability for $R_0 < 1$ when

$$R_0 = [(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{Tr} N_{Vr} \alpha_{hr} \beta_{vr}}{(N_{Tr}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)})$$

$$+ (\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{Tw} N_{Vw} \alpha_{hw} \beta_{vw}}{(N_{Tw}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)})$$

$$+ (\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{Ts} N_{Vs} \alpha_{hs} \beta_{vs}}{(N_{Ts}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)})]$$

ii) At the endemic state

$$E_e = (S^*_{hr}, E^*_{hr}, I^*_{hr}, E^*_{vr}, I^*_{vr}, S^*_{hw}, E^*_{hw}, I^*_{hw}, E^*_{vw}, I^*_{vw}, S^*_{hs}, E^*_{hs}, I^*_{hs}, E^*_{vs}, I^*_{vs}).$$

The characteristic equation is:

$$\lambda^5 + A_4\lambda^4 + A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A = 0$$

where

For rainy season,

$$A_4 = \rho + \alpha_{hr} + 3\lambda_d + 3\lambda_h + 2\lambda_v + \beta_{vr} + (1 + \frac{\delta^2 I^*_{hr} K_{hr\to v} K_{v\to hr} N_{Tr} N_{Vr}}{(N_{Tr}+g)(\beta_{vr}+\lambda_v)(\delta I^*_{hr} K_{hr\to v} N_{Tr} + (N_{Tr}+g)\lambda_v)}$$

$$\begin{aligned}
A_3 = &-\frac{1}{(N_{Tr}+g)(\beta_{vr}+\lambda_v)(\delta I^*_{hr} K_{hr\to v} N_{Tr} + (N_{Tr}+g)\lambda_v)} \\
&(-(N_{Tr}+g)^2\lambda_v(\beta_{vr}+\lambda_v)((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)+2(\rho+3\lambda_d+3\lambda_h)\lambda_v \\
&+\beta_{vr}(\rho+3\lambda_d+3\lambda_h+\lambda_v)+\alpha_{hr}(\rho+\beta_{vr}+2\lambda_d+2\lambda_h+2\lambda_v))-\delta I^*_{hr} K_{hr\to v} N_{Tr} \\
&(\delta K_{v\to hr} N_{Vr}\beta_{vr}(\rho+\alpha_{hr}+\beta_{vr}+2\lambda_d+2\lambda_h+2\lambda_v)+(N_{Tr}+g)(\beta_{vr}+\lambda_v)((\lambda_d+\lambda_h) \\
&(2\rho+3\lambda_d+3\lambda_h)+2(\rho+3\lambda_d+3\lambda_h)\lambda_v+\lambda_v^2+\beta_{vr}(\rho+3\lambda_d+3\lambda_h+\lambda_v)+ \\
&\alpha_{hr}(\rho+\beta_{vr}+2\lambda_d+2\lambda_h+2\lambda_v))))
\end{aligned}$$

$$\begin{aligned}
A_2 = &-\frac{1}{(N_{Tr}+g)(\beta_{vr}+\lambda_v)(\delta I^*_{hr} K_{hr\to v} N_{Tr} + (N_{Tr}+g)\lambda_v)} \\
&(-(N_{Tr}+g)^2\lambda_v(\beta_{vr}+\lambda_v)((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h)+2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v+ \\
&(\rho+3\lambda_d+3\lambda_h)\lambda_v^2+\beta_{vr}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)+(\rho+3\lambda_d+3\lambda_h)\lambda_v)+ \\
&\alpha_{hr}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v)+\lambda_v^2+\beta_{vr}(\rho+2\lambda_d+2\lambda_h+\lambda_v)))- \\
&\delta I^*_{hr} K_{hr\to v} N_{Tr}(\delta K_{v\to hr} N_{Vr}\beta_{vr}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v)+\lambda_v^2+ \\
&\beta_{vr}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hr}(\rho+\beta_{vr}+\lambda_d+\lambda_h+2\lambda_v))+(N_{Tr}+g)(\beta_{vr}+\lambda_v) \\
&((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h)+2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v+(\rho+3\lambda_d+3\lambda_h)\lambda_v^2+ \\
&\beta_{vr}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)+(\rho+3\lambda_d+3\lambda_h)\lambda_v)+\alpha_{hr}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+ \\
&2(\rho+2\lambda_d+2\lambda_h)\lambda_v)+\lambda_v^2+\beta_{vr}(\rho+2\lambda_d+2\lambda_h+\lambda_v)))))
\end{aligned}$$

$$
\begin{aligned}
A_1 = {} & -\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{Tr} N_{Vr} \alpha_{hr} \beta_{vr}}{(N_{Tr}+g)^2} + \rho\alpha_{hr}\beta_{vr}\lambda_d + \alpha_{hr}\beta_{vr}\lambda_d^2 + \rho\alpha_{hr}\beta_{vr}\lambda_h + 2\alpha_{hr}\beta_{vr}\lambda_d\lambda_h \\
& + \alpha_{hr}\beta_{vr}\lambda_h^2 - \frac{1}{(N_{Tr}+g)^2}\beta_{vr}(-(N_{Tr}+g)^2(\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h) - \delta^2 I_{hr}^* K_{hr\to v} K_{v\to hr} N_{Tr} N_{Vr} \\
& (\rho+\alpha_{hr}+2\lambda_d+2\lambda_h)) + (\alpha_{hr}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vr}(\rho+2\lambda_d+2\lambda_h)) + (\lambda_d+\lambda_h) \\
& (2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vr}(2\rho+3\lambda_d+3\lambda_h)))\lambda_v + (\alpha_{hr}(\rho+2\lambda_d+2\lambda_h) + (\lambda_d+\lambda_h) \\
& (2\rho+3\lambda_d+3\lambda_h))\lambda_v^2 + \frac{\delta I_{hr}^* K_{hr\to v} N_{Tr} K_{v\to hr} N_{Vr} \beta_{vr}^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)}{(N_{Tr}+g)(-\delta I_{hr}^* K_{hr\to v} N_{Tr} + (N_{Tr}+g)\beta_{vr}(\beta_{vr}+\lambda_v)} + \\
& (\delta^2 I_{hr}^* K_{hr\to v} N_{Tr} K_{v\to hr} N_{Vr} \beta_{vr}((N_{Tr}+g)^2 \beta_{vr}(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h) + \delta^2 {I_{hr}^*}^2 K^2{}_{hr\to v} N_{Tr}^2 \\
& (\rho+\alpha_{hr}+2\lambda_d+2\lambda_h) - \delta I_{hr}^* K_{hr\to v}(N_{Tr}+g) N_{Tr}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vr}(\rho+2\lambda_d+2\lambda_h) \\
& \alpha_{hr}(\beta_{vr}+2(\rho+\lambda_d+\lambda_h))))))/((N_{Tr}+g)^2(-\delta I_{hr}^* K_{hr\to v} N_{Tr} + (N_{Tr}+g)\beta_{vr}) \\
& (\delta I_{hr}^* K_{hr\to v} N_{Tr} + (N_{Tr}+g)\lambda_v)) + (\delta^3 K^2{}_{hr\to v} N_{Tr} K_{v\to hr} N_{Vr} \beta_{vr} \alpha_{hr}(\delta K_{v\to hr} N_{Vr} \beta_{vr} + \\
& (N_{Tr}+g)(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v)))/((N_{Tr}+g)^2(N_{Tr}+g)^2(\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v) + \\
& \delta I_{hr}^* K_{hr\to v} N_{Tr}(\delta K_{v\to hr} N_{Vr} \beta_{vr} + (N_{Tr}+g)(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v))))
\end{aligned}
$$

$$
\begin{aligned}
A = {} & (\lambda_v(\delta^2 {I_{hr}^*}^2 K^2{}_{v\to hr} N_{Tr}^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)(\delta K_{v\to hr} N_{Vr} \beta_{vr} + (N_{Tr}+g)(\lambda_d+\lambda_h) \\
& (\beta_{vr}+\lambda_v))^2 + (N_{Tr}+g)^2(\lambda_d+\lambda_h)^2 \lambda_v(\beta_{vr}+\lambda_v)(-\delta K_{hr\to v} K_{v\to hr} N_{Tr} N_{Vr} \alpha_{hr} \beta_{vr} + (N_{Tr}+g)^2 \\
& (\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)) + \delta I_{hr}^* K_{hr\to v}(N_{Tr}+g) N_{Tr}(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v) \\
& (-\delta^2 K_{hr\to v} K_{v\to hr} N_{Tr} N_{Vr} \alpha_{hr} \beta_{vr}(\lambda_d+\lambda_h) + 2(N_{Tr}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v \\
& (\delta K_{v\to hr} N_{Vr} \beta_{vr} + (N_{Tr}+g)(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v))))))/((N_{Tr}+g)(\delta I_{hr}^* K_{hr\to v} N_{Tr} + (N_{Tr}+g)\lambda_v) \\
& (N_{Tr}+g)^2(\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v) + \delta I_{hr}^* K_{hr\to v} N_{Tr}(\delta K_{v\to hr} N_{Vr} \beta_{vr} + (N_{Tr}+g)(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v))))
\end{aligned}
$$

For winter season,

$$A_4 = \rho + \alpha_{hw} + 3\lambda_d + 3\lambda_h + 2\lambda_v + \beta_{vw} + (1 + \frac{\delta^2 I^*_{hw} K_{hw\to w} K_{v\to vw} N_{Tw} N_{Vw}}{(N_{Tw}+g)(\beta_{vw}+\lambda_v)(\delta I^*_{hw} K_{hw\to v} N_{Tw} + (N_{Tw}+g)\lambda_v)}$$

$$\begin{aligned}
A_3 = & -\frac{1}{(N_{Tw}+g)(\beta_{vw}+\lambda_v)(\delta I^*_{hw} K_{hw\to v} N_{Tw} + (N_{Tw}+g)\lambda_v)} \\
& (-(N_{Tw}+g)^2 \lambda_v (\beta_{vw}+\lambda_v)((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)+2(\rho+3\lambda_d+3\lambda_h)\lambda_v \\
& +\beta_{vw}(\rho+3\lambda_d+3\lambda_h+\lambda_v)+\alpha_{hw}(\rho+\beta_{vw}+2\lambda_d+2\lambda_h+2\lambda_v))-\delta I^*_{hw} K_{hw\to v} N_{Tw} \\
& (\delta K_{v\to hw} N_{Vw} \beta_{vw}(\rho+\alpha_{hw}+\beta_{vw}+2\lambda_d+2\lambda_h+2\lambda_v)+(N_{Tw}+g)(\beta_{vw}+\lambda_v)((\lambda_d+\lambda_h) \\
& (2\rho+3\lambda_d+3\lambda_h)+2(\rho+3\lambda_d+3\lambda_h)\lambda_v+\lambda_v^2+\beta_{vw}(\rho+3\lambda_d+3\lambda_h+\lambda_v)+ \\
& \alpha_{hw}(\rho+\beta_{vw}+2\lambda_d+2\lambda_h+2\lambda_v))))
\end{aligned}$$

$$\begin{aligned}
A_2 = & -\frac{1}{(N_{Tw}+g)(\beta_{vw}+\lambda_v)(\delta I^*_{hw} K_{hw\to v} N_{Tw} + (N_{Tw}+g)\lambda_v)} \\
& (-(N_{Tw}+g)^2 \lambda_v (\beta_{vw}+\lambda_v)((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h)+2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v+ \\
& (\rho+3\lambda_d+3\lambda_h)\lambda_v^2+\beta_{vr}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)+(\rho+3\lambda_d+3\lambda_h)\lambda_v)+ \\
& \alpha_{hw}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v)+\lambda_v^2+\beta_{vw}(\rho+2\lambda_d+2\lambda_h+\lambda_v)))- \\
& \delta I^*_{hw} K_{hw\to v} N_{Tr}(\delta K_{v\to hw} N_{Vw} \beta_{vw}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v)+\lambda_v^2+ \\
& \beta_{vw}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hw}(\rho+\beta_{vw}+\lambda_d+\lambda_h+2\lambda_v))+(N_{Tw}+g)(\beta_{vw}+\lambda_v) \\
& ((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h)+2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v+(\rho+3\lambda_d+3\lambda_h)\lambda_v^2+ \\
& \beta_{vw}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)+(\rho+3\lambda_d+3\lambda_h)\lambda_v)+\alpha_{hw}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+ \\
& 2(\rho+2\lambda_d+2\lambda_h)\lambda_v)+\lambda_v^2+\beta_{vw}(\rho+2\lambda_d+2\lambda_h+\lambda_v)))))
\end{aligned}$$

$$\begin{aligned}
A_1 = & -\frac{\delta^2 K_{hw\to w} K_{v\to hw} N_{Tw} N_{Vw} \alpha_{hw} \beta_{vw}}{(N_{Tw}+g)^2}+\rho\alpha_{hw}\beta_{vw}\lambda_d+\alpha_{hw}\beta_{vw}\lambda_d^2+\rho\alpha_{hw}\beta_{vw}\lambda_h+2\alpha_{hw}\beta_{vw}\lambda_d\lambda_h \\
& +\alpha_{hw}\beta_{vw}\lambda_h^2 - \frac{1}{(N_{Tw}+g)^2}\beta_{vw}(-(N_{Tw}+g)^2(\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h)-\delta^2 I^*_{hw} K_{hw\to w} K_{v\to hw} N_{Tw} N_{Vw} \\
& (\rho+\alpha_{hw}+2\lambda_d+2\lambda_h))+(\alpha_{hw}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vw}(\rho+2\lambda_d+2\lambda_h))+(\lambda_d+\lambda_h) \\
& (2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vw}(2\rho+3\lambda_d+3\lambda_h)))\lambda_v+(\alpha_{hw}(\rho+2\lambda_d+2\lambda_h)+(\lambda_d+\lambda_h) \\
& (2\rho+3\lambda_d+3\lambda_h))\lambda_v^2+\frac{\delta I^*_{hw} K_{hw\to w} N_{Tw} K_{v\to hw} N_{Vw} \beta_{vw}^2(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)}{(N_{Tw}+g)(-\delta I^*_{hw} K_{hw\to w} N_{Tw}+(N_{Tw}+g)\beta_{vw}(\beta_{vw}+\lambda_v)}+ \\
& (\delta^2 I^*_{hw} K_{hw\to w} N_{Tw} K_{v\to hw} N_{Vw} \beta_{vw}((N_{Tw}+g)^2\beta_{vw}(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)+\delta^2 I^{*2}_{hw} K^2{}_{hw\to w} N_{Tw}^2 \\
& (\rho+\alpha_{hw}+2\lambda_d+2\lambda_h)-\delta I^*_{hw} K_{hw\to w}(N_{Tw}+g)N_{Tw}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vw}(\rho+2\lambda_d+2\lambda_h) \\
& \alpha_{hw}(\beta_{vw}+2(\rho+\lambda_d+\lambda_h))))))/((N_{Tw}+g)^2(-\delta I^*_{hw} K_{hw\to w} N_{Tw}+(N_{Tw}+g)\beta_{vw}) \\
& (\delta I^*_{hw} K_{hw\to w} N_{Tw}+(N_{Tw}+g)\lambda_v))+(\delta^3 K^2{}_{hw\to w} N_{Tw} K_{v\to hw} N_{Vw} \beta_{vw} \alpha_{hw}(\delta K_{v\to hw} N_{Vw} \beta_{vw}+ \\
& (N_{Tw}+g)(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v)))/((N_{Tw}+g)^2(N_{Tw}+g)^2(\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)+ \\
& \delta I^*_{hw} K_{hw\to w} N_{Tw}(\delta K_{v\to hw} N_{Vw} \beta_{vw}+(N_{Tw}+g)(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v))))
\end{aligned}$$

$$
\begin{aligned}
A = {} & (\lambda_v(\delta^2 I_{hw}^{*\,2} K^2_{v\to hw} N_{Tw}^2(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)(\delta K_{v\to hw} N_{Vw}\beta_{vw}+(N_{Tw}+g)(\lambda_d+\lambda_h) \\
& (\beta_{vw}+\lambda_v))^2+(N_{Tw}+g)^2(\lambda_d+\lambda_h)^2\lambda_v(\beta_{vw}+\lambda_v)(-\delta K_{hw\to v}K_{v\to hw}N_{Tw}N_{Vw}\alpha_{hw}\beta_{vw}+(N_{Tw}+g)^2 \\
& (\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v))+\delta I_{hw}^* K_{hw\to v}(N_{Tw}+g)N_{Tw}(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v) \\
& (-\delta^2 K_{hw\to v}K_{v\to hw}N_{Tw}N_{Vw}\alpha_{hw}\beta_{vw}(\lambda_d+\lambda_h)+2(N_{Tw}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v \\
& (\delta K_{v\to hw}N_{Vw}\beta_{vw}+(N_{Tw}+g)(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v))))) / ((N_{Tw}+g)(\delta I_{hw}^* K_{hw\to v}N_{Tw}+(N_{Tw}+g)\lambda_v) \\
& (N_{Tw}+g)^2(\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)+\delta I_{hw}^* K_{hw\to v}N_{Tw}(\delta K_{v\to hw}N_{Vw}\beta_{vw}+(N_{Tw}+g)(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v)))
\end{aligned}
$$

For summer season.

$$
A_4 = \rho+\alpha_{hs}+3\lambda_d+3\lambda_h+2\lambda_v+\beta_{vs}+(1+\frac{\delta^2 I_{hs}^* K_{hs\to v}K_{v\to hs}N_{Ts}N_{Vs}}{(N_{Ts}+g)(\beta_{vs}+\lambda_v)(\delta I_{hs}^* K_{hs\to v}N_{Ts}+(N_{Ts}+g)\lambda_v)}
$$

$$
\begin{aligned}
A_3 = {} & -\frac{1}{(N_{Ts}+g)(\beta_{vs}+\lambda_v)(\delta I_{hs}^* K_{hs\to v}N_{Ts}+(N_{Ts}+g)\lambda_v)} \\
& (-(N_{Ts}+g)^2\lambda_v(\beta_{vs}+\lambda_v)((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)+2(\rho+3\lambda_d+3\lambda_h)\lambda_v \\
& +\beta_{vs}(\rho+3\lambda_d+3\lambda_h+\lambda_v)+\alpha_{hs}(\rho+\beta_{vs}+2\lambda_d+2\lambda_h+2\lambda_v))-\delta I_{hs}^* K_{hs\to v}N_{Ts} \\
& (\delta K_{v\to hs}N_{Vs}\beta_{vs}(\rho+\alpha_{hs}+\beta_{vs}+2\lambda_d+2\lambda_h+2\lambda_v)+(N_{Ts}+g)(\beta_{vs}+\lambda_v)((\lambda_d+\lambda_h) \\
& (2\rho+3\lambda_d+3\lambda_h)+2(\rho+3\lambda_d+3\lambda_h)\lambda_v+\lambda_v^2+\beta_{vs}(\rho+3\lambda_d+3\lambda_h+\lambda_v)+ \\
& \alpha_{hs}(\rho+\beta_{vr}+2\lambda_d+2\lambda_h+2\lambda_v))))
\end{aligned}
$$

$$
\begin{aligned}
A_2 = {} & -\frac{1}{(N_{Ts}+g)(\beta_{vs}+\lambda_v)(\delta I_{hs}^* K_{hs\to v}N_{Ts}+(N_{Ts}+g)\lambda_v)} \\
& (-(N_{Ts}+g)^2\lambda_v(\beta_{vs}+\lambda_v)((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h)+2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v+ \\
& (\rho+3\lambda_d+3\lambda_h)\lambda_v^2+\beta_{vs}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)+(\rho+3\lambda_d+3\lambda_h)\lambda_v)+ \\
& \alpha_{hs}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v)+\lambda_v^2+\beta_{vs}(\rho+2\lambda_d+2\lambda_h+\lambda_v)))- \\
& \delta I_{hs}^* K_{hs\to v}N_{Ts}(\delta K_{v\to hs}N_{Vs}\beta_{vs}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v)+\lambda_v^2+ \\
& \beta_{vs}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hs}(\rho+\beta_{vs}+\lambda_d+\lambda_h+2\lambda_v))+(N_{Ts}+g)(\beta_{vs}+\lambda_v) \\
& ((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h)+2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v+(\rho+3\lambda_d+3\lambda_h)\lambda_v^2+ \\
& \beta_{vs}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)+(\rho+3\lambda_d+3\lambda_h)\lambda_v)+\alpha_{hs}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+ \\
& 2(\rho+2\lambda_d+2\lambda_h)\lambda_v)+\lambda_v^2+\beta_{vs}(\rho+2\lambda_d+2\lambda_h+\lambda_v)))))
\end{aligned}
$$

$$
\begin{aligned}
A_1 = {} & \frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{Ts} N_{Vs} \alpha_{hs} \beta_{vs}}{(N_{Ts}+g)^2} + \rho\alpha_{hs}\beta_{vs}\lambda_d + \alpha_{hs}\beta_{vr}\lambda_d^2 + \rho\alpha_{hs}\beta_{vs}\lambda_h + 2\alpha_{hs}\beta_{vs}\lambda_d\lambda_h \\
& + \alpha_{hs}\beta_{vs}\lambda_h^2 \frac{1}{(N_{Ts}+g)^2}\beta_{vs}(-(N_{Ts}+g)^2(\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h) - \delta^2 I_{hs}^{*} K_{hs\to v} K_{v\to hs} N_{Ts} N_{Vs} \\
& (\rho+\alpha_{hs}+2\lambda_d+2\lambda_h)) + (\alpha_{hs}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vs}(\rho+2\lambda_d+2\lambda_h)) + (\lambda_d+\lambda_h) \\
& (2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vs}(2\rho+3\lambda_d+3\lambda_h)))\lambda_v + (\alpha_{hs}(\rho+2\lambda_d+2\lambda_h) + (\lambda_d+\lambda_h) \\
& (2\rho+3\lambda_d+3\lambda_h))\lambda_v^2 + \frac{\delta I_{hs}^{*} K_{hs\to v} N_{Ts} K_{v\to hs} N_{Vs} \beta_{vs}^2(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)}{(N_{Ts}+g)(-\delta I_{hs}^{*} K_{hs\to v} N_{Ts} + (N_{Ts}+g)\beta_{vs}(\beta_{vs}+\lambda_v)} + \\
& (\delta^2 I_{hs}^{*} K_{hs\to v} N_{Ts} K_{v\to hs} N_{Vs} \beta_{vs}((N_{Ts}+g)^2 \beta_{vs}(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h) + \delta^2 I_{hs}^{*\,2} K^2{}_{hs\to v} N_{Ts}^2 \\
& (\rho+\alpha_{hs}+2\lambda_d+2\lambda_h) - \delta I_{hs}^{*} K_{hs\to v}(N_{Ts}+g) N_{Ts}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vs}(\rho+2\lambda_d+2\lambda_h) \\
& \alpha_{hs}(\beta_{vs}+2(\rho+\lambda_d+\lambda_h)))))) / ((N_{Ts}+g)^2(-\delta I_{hs}^{*} K_{hs\to v} N_{Ts} + (N_{Ts}+g)\beta_{vs}) \\
& (\delta I_{hs}^{*} K_{hs\to v} N_{Ts} + (N_{Ts}+g)\lambda_v)) + (\delta^3 K^2{}_{hs\to v} N_{Ts} K_{v\to hs} N_{Vs} \beta_{vs} \alpha_{hs}(\delta K_{v\to hs} N_{Vs} \beta_{vs} + \\
& (N_{Ts}+g)(\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v))) / ((N_{Ts}+g)^2(N_{Ts}+g)^2(\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v) + \\
& \delta I_{hs}^{*} K_{hs\to v} N_{Ts}(\delta K_{v\to hs} N_{Vs} \beta_{vs} + (N_{Ts}+g)(\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v))))
\end{aligned}
$$

$$
\begin{aligned}
A = {} & (\lambda_v(\delta^2 I_{hs}^{*\,2} K^2{}_{v\to hs} N_{Ts}^2(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)(\delta K_{v\to hs} N_{Vs}\beta_{vs} + (N_{Ts}+g)(\lambda_d+\lambda_h) \\
& (\beta_{vs}+\lambda_v))^2 + (N_{Ts}+g)^2(\lambda_d+\lambda_h)^2\lambda_v(\beta_{vs}+\lambda_v)(-\delta K_{hs\to v} K_{v\to hs} N_{Ts} N_{Vs}\alpha_{hs}\beta_{vs} + (N_{Ts}+g)^2 \\
& (\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)) + \delta I_{hs}^{*} K_{hs\to v}(N_{Ts}+g) N_{Ts}(\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v) \\
& (-\delta^2 K_{hs\to v} K_{v\to hs} N_{Ts} N_{Vs}\alpha_{hs}\beta_{vs}(\lambda_d+\lambda_h) + 2(N_{Ts}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v \\
& (\delta K_{v\to hs} N_{Vs}\beta_{vs} + (N_{Ts}+g)(\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v)))))) / ((N_{Ts}+g)(\delta I_{hs}^{*} K_{hs\to v} N_{Ts} + (N_{Ts}+g)\lambda_v) \\
& (N_{Ts}+g)^2(\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v) + \delta I_{hs}^{*} K_{hs\to v} N_{Ts}(\delta K_{v\to hs} N_{Vs}\beta_{vs} + (N_{Ts}+g)(\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v)))) .
\end{aligned}
$$

The eigenvalues of endemic disease state have negative real parts, when they are according to the Routh – Hurwitz criteria:

i) $A_i > 0 \quad (i = 1,2,3,4)$ (5.74)

ii) $A > 0$ (5.75)

iii) $A_4 A_3 A_2 - A_2^2 - A_4^2 A_1 > 0$ (5.76)

iv) $(A_4 A_1 - A)(A_4 A_3 A_2 - A_2^2 - A_4^2 A_1) - A(A_4 A_3 - A_2)^2 - A_4^2 > 0$ (5.77)

We check the conditions of Routh – Hurwitz criteria as shown with parameters above. For the conditions given by (5.74) – (5.77). We show the above conditions are met for $R_0 > 1$. They are shown in figure 5.3.

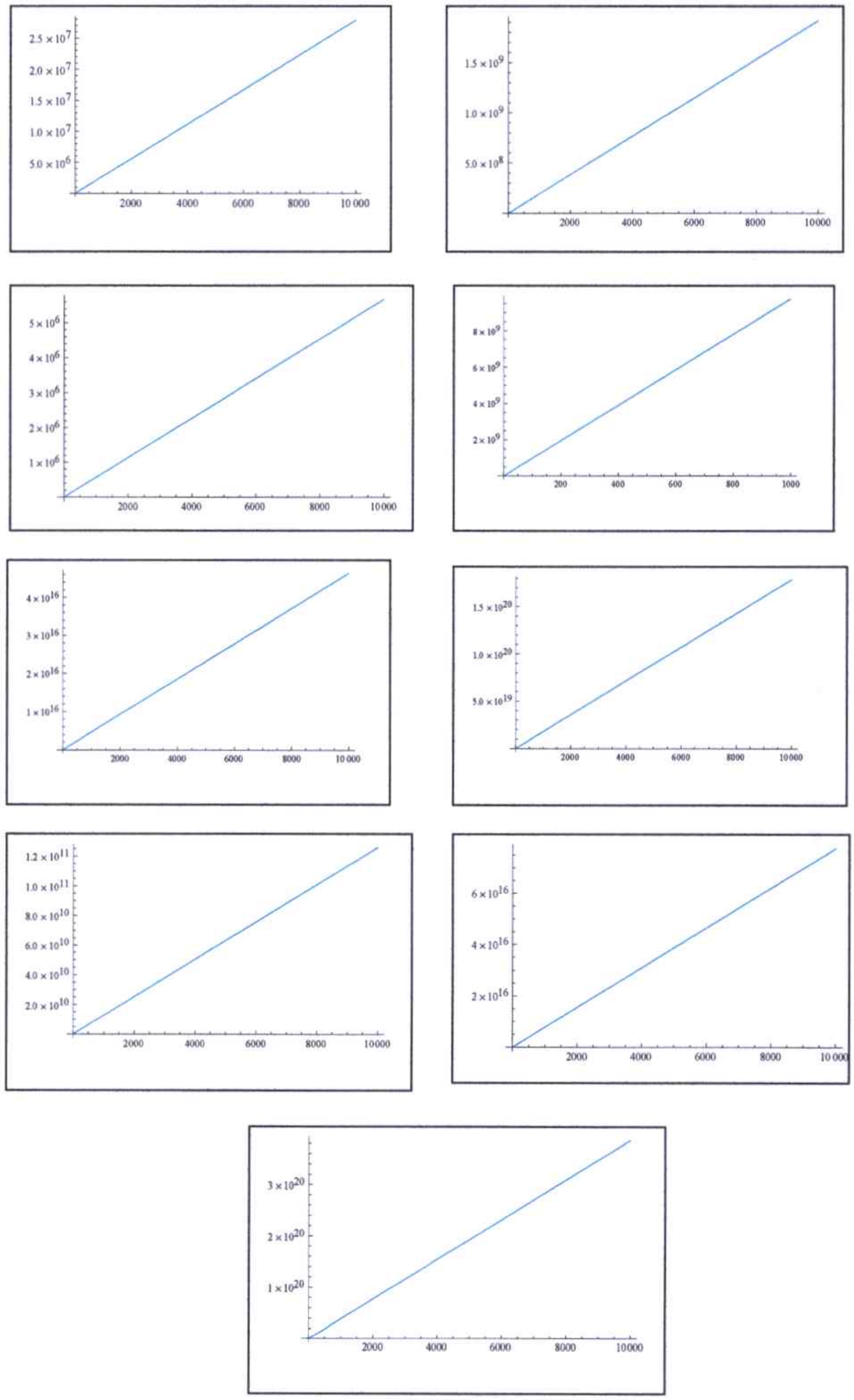


Figure 5.3. Routh – Hurwitz criteria (5.74) – (5.77) for endemic disease state for rainy season, winter season and summer season, respectively.

We found that all eigenvalues have negative real parts when $R_0 > 1$. This means that the endemic disease state is local stability [13] for $R_0 > 1$.

when we adding the incubation rates for each season, R_0 is defined:

$$R_0 = [(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{Tr} N_{Vr} \alpha_{hr} \beta_{vr}}{(N_{Tr}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)})$$

$$+(\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{Tw} N_{Vw} \alpha_{hw} \beta_{vw}}{(N_{Tw}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)})$$

$$+(\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{Ts} N_{Vs} \alpha_{hs} \beta_{vs}}{(N_{Ts}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)})].$$

We can see that the incubation rates for each season influence to the basic reproduction number.

Chapter VI

Transmission model of Dengue virus by age group of human and two species of vectors

6.1. Mathematical model of dengue virus

Dengue is a vector-borne disease. It is transmitted to humans by the biting of the Aedes aegypti and Aedes albopictus mosquitoes. The human population is separated into two classes, a child class and an adult class, each class being described by a SIR model. The transmission rates of the two mosquito species are different and depend on what class the humans belong to. We develop a single model taking into account the presence of two type of mosquitoes and two age classes and apply it to dengue fever. The model shows how it is possible for the maximum level of infected human to be reached in a short time.

We analyze the SIR (Susceptible – Infected – Recovered) equations for human and SI (Susceptible – Infected) equations for mosquitoes. The model will apply empirically on data of dengue patients reports by Ministry of Public Health ,Thailand (2002 – 2012) as shown in figure 6.1[12]. This model, we study of the age structural model of dengue disease incorporated the influence of *Aedes aegypti* and *Aedes albopictus.*

6.1.1. Mathematical Model

In Thailand, the data of dengue virus cases during 2002 – 2012. Which have the number of dengue patients in shown figure 6.1. The highest proportion of cases by age group were 0 – 4 years old, 5 – 9 years old, 10 – 14 years old, 15 – 24 years old, 25 – 34 years old, 35 – 44 years old, 45 – 54 years old, 55 – 64 years old, more than 65 years old, respectively. There is the different distribution of this model about disease in each age group of two species mosquitoes (*Aedes aegypti* and *Aedes albopictus*). We propose a mathematical model to study the transmission of dengue virus by introducing age structure into the SIR model.

The SIR and SI simulates the spread of dengue virus between host and vector populations. The model is based on the Susceptible, Infected, and Recovered (SIR) model of infected disease epidemiology. The age structure is introduced into a model, ie. children and adults, then we modify it by incorporating the different behaviors of *Aedes aegypti* and *Aedes albopictus*. In figure 6.1, we show the age distribution of the incidence rates in one province in thailand during 2002 – 2012 epidemic. As we see in figure 6.1, most cases occur in children under the age of 15. However, a small number of cases do occur in older people. Similar distributions are seen in the other provinces in the country.

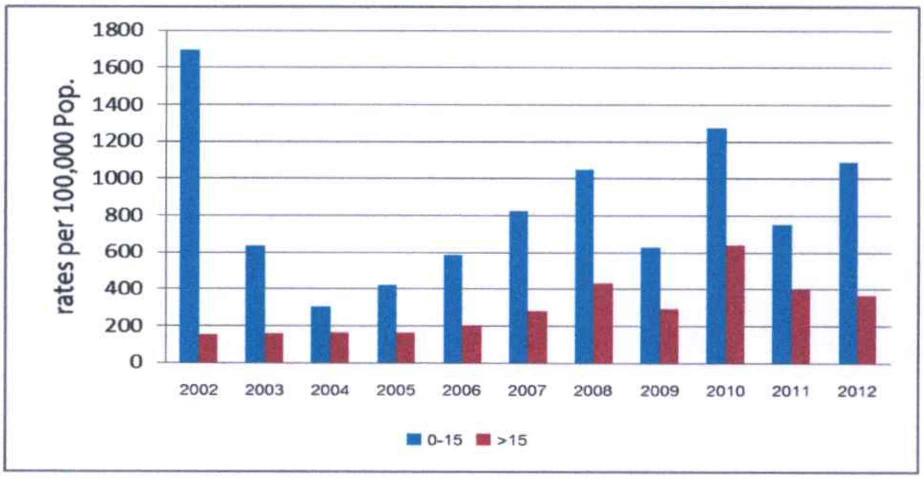


Figure 6.1. Age distribution of the 2002 – 2012 dengue fever incidence rates in thailand [12].

This model with age structure, the dynamics of each component of the human is given by

$$\frac{dS_c}{dt} = D_c N_{tc} - \alpha_{ac}(1+\gamma_a Sin\,\beta t) I_{va1} S_c - \alpha_{bc}(1+\gamma_b Sin\,\beta t) I_{vb1} S_c - \eta_d S_c \tag{6.1}$$

$$\frac{dI_{c1}}{dt} = \alpha_{ac}(1+\gamma_a Sin\beta t)\, I_{va1} S_c - \omega_{c1} I_{c1} - \eta_d I_{c1} \tag{6.2}$$

$$\frac{dI_{c2}}{dt} = \alpha_{bc}(1+\gamma_b Sin\,\beta t) I_{vb1} S_c - \omega_{c2} I_{c2} - \eta_d I_{c2} \tag{6.3}$$

$$\frac{dR_c}{dt} = \omega_{c1} I_{c1} + \omega_{c2} I_{c2} - \eta_d R_c \tag{6.4}$$

$$\frac{dS_a}{dt} = D_a N_{ta} - \alpha_{aa}(1+\gamma_a Sin\beta t) I_{va2} S_a - \alpha_{ba}(1+\gamma_b Sin\,\beta t) I_{vb2} S_a - \eta_d S_a \tag{6.5}$$

$$\frac{dI_{a1}}{dt} = \alpha_{aa}(1+\gamma_a Sin\beta t)\, I_{va2} S_a - \omega_{a1} I_{a1} - \omega_d I_{a1} \tag{6.6}$$

$$\frac{dI_{a2}}{dt} = \alpha_{ba}(1+\gamma_b\, Sin\,\beta t) I_{vb2} S_a - \omega_{a2} I_{a2} - \eta_d I_{a2} \tag{6.7}$$

$$\frac{dR_a}{dt} = \omega_{a1} I_{a1} + \omega_{a2} I_{a2} - \eta_d R_a \,. \tag{6.8}$$

Where the variables and parameters are defined in table 6.1.

Table 6.1. Parameters for equations (6.1) – (6.8) and their definitions.

variable/parameter	definition
S_c, I_{c1}, I_{c2}, R_c	the number of susceptible children, infected from *Aedes aegypti* and *Aedes albopictus* in children and recovered children,
S_a, I_{a1}, I_{a2}, R_a	the number of susceptible adult, infected from *Aedes aegypti* and *Aedes albopictus* in adult, and recovered adult,
N_t	the total population ,
N_{tc} and N_{ta}	the total population in children and the total population in adult , they are constant variables ,
D_c and D_a	the birth rate of children and adult human,
α_{ac}	the transmission probability of dengue virus from *Aedes aegypti* to children,
α_{bc}	the transmission probability of dengue virus from *Aedes albopictus* to children,
α_{aa}	the transmission probability of dengue virus from *Aedes aegypti* to adult ,
α_{ba}	the transmission probability of dengue virus from *Aedes albopictus* to adult,
ω_{c1}	the rate at which the infected children from *Aedes aegypti* can recover,
ω_{c2}	the rate at which the infected children from *Aedes albopictus* can recover,
ω_{a1}	the rate at which the infected adult from *Aedes aegypti* can recover,
ω_{a2}	the rate at which the infected adult from *Aedes albopictus* can recover,
η_d	the natural death rate of human,
γ_a	the measure of influence on the transmission process from *Aedes aegypti* mosquito to human,
γ_b	the measure of influence on the transmission process from

	Aedes albopictus to human,
ρ_{va}	the measure of influence on the transmission process from *Aedes aegypti* to human,
ρ_{vb}	the measure of influence on the transmission process from *Aedes albopictus* to human.

It we add equations (6.1) – (6.8) together, we get

$$\frac{dN_t}{dt} = \frac{dN_{tc}}{dt} + \frac{dN_{ta}}{dt}$$

$$= (S_c + I_{c1} + I_{c2} + R_c) + (S_a + I_{a1} + I_{a2} + R_a).$$

The total children and adult populations are supposed to have constant sizes , i.e., $\frac{dN_{tc}}{dt}=0$ and $\frac{dN_{ta}}{dt}=0$, the birth rate would have to be equivalent to the death rate, $P_c = P_a = \mu_d$ in children and adult ,respectively.

Where N_{tc} is the total number of children and is equivalent to $S_c + I_{c1} + I_{c2} + R_c$,

N_{ta} is the total population in adult and is equal to $S_a + I_{a1} + I_{a2} + R_a$.

The dynamics of mosquitoes is described by

$$\frac{dS_{va1}}{dt} = A_{v_a} - \alpha_{va1}(1 + \gamma_{va} Sin\ \beta t) I_{c1} S_{va1} - \eta_{v_a} S_{va1} \tag{6.9}$$

$$\frac{dI_{va1}}{dt} = \alpha_{va1}(1 + \gamma_{va} Sin\ \beta t) I_{c1} S_{va1} - \eta_{v_a} I_{va1} \tag{6.10}$$

$$\frac{dS_{va2}}{dt} = A_{v_a} - \alpha_{va2}(1 + \gamma_{va} Sin\ \beta t) I_{a1} S_{va2} - \eta_{v_a} S_{va2} \tag{6.11}$$

$$\frac{dI_{va2}}{dt} = \alpha_{va2}(1 + \gamma_{va} Sin\ \beta t) I_{a1} S_{va2} - \eta_{v_a} I_{va2} \tag{6.12}$$

$$\frac{dS_{vb1}}{dt} = A_{v_b} - \alpha_{vb1}(1 + \gamma_{vb} Sin\ \beta t) I_{c2} S_{vb1} - \eta_{v_b} S_{vb1} \tag{6.13}$$

$$\frac{dI_{vb1}}{dt} = \alpha_{vb1}(1 + \gamma_{vb} Sin\ \beta t) I_{c2} S_{vb1} - \eta_{v_b} I_{vb1} \tag{6.14}$$

$$\frac{dS_{vb2}}{dt} = A_{v_b} - \alpha_{vb2}(1 + \gamma_{vb} Sin\ \beta t) I_{a2} S_{vb2} - \eta_{v_b} S_{vb2} \tag{6.15}$$

$$\frac{dI_{vb2}}{dt} = \alpha_{vb2}(1 + \gamma_{vb} Sin\ \beta t) I_{a2} S_{vb2} - \eta_{v_b} I_{vb2}. \tag{6.16}$$

We define children as people who age 0 – 15 years old. Adult is the people who age greater than 15 years old.

The variable and parameters are given in table 6.2.

Table 6.2. Parameters for equations (6.9) – (6.16) and their definitions.

variable/ parameter	definition
S_{va1} and I_{va1}	the number of susceptible and infected *Aedes aegypti* mosquitoes who be infected from children,
η_{va}	the death rate of *Aedes aegypti* mosquito,
A_{v_a}	the carrying capacity of the environment for *Aedes aegypti*,
α_{va1}	the probability that a dengue virus transmitted to the *Aedes aegypti* from an infected children,
S_{va2} and I_{va2}	the number of susceptible and infected *Aedes aegypti* mosquitoes who be infected from adult,
η_{va}	the death rate of *Aedes aegypti* mosquito,
A_{v_a}	the carrying capacity of the environment for *Aedes aegypti* mosquito,
α_{va2}	the probability that a dengue virus transmitted to the *Aedes aegypti* from an infected adult human,
S_{vb1} and I_{vb1}	the number of susceptible and infected *Aedes albopictus* mosquitoes who be infected from children,
η_{vb}	the death rate of *Aedes albopictus* mosquito,
A_{v_b}	the carrying capacity of the environment for *Aedes albopictus* mosquito,
α_{vb1}	the probability that a dengue virus transmitted to the *Aedes albopictus* from an infected human in children,
S_{vb2} and I_{vb2}	the number of susceptible and infected *Aedes albopictus* mosquitoes who be infected from adult human,
η_{vb}	the death rate of *Aedes albopictus* mosquito,
A_{v_b}	the carrying capacity of the environment for *Aedes albopictus* mosquito,
α_{vb2}	the probability that a dengue virus transmitted to the *Aedes albopictus* from an infected adult.

If we add equations (6.9) – (6.16) together, we get

$$\frac{d}{dt}(S_{va1}+I_{va1}) = A_{v_a}-\eta_{v_a}N_{va1} \quad (6.17)$$

$$\frac{d}{dt}(S_{va2}+I_{va2}) = A_{v_a}-\eta_{v_a}N_{va2} \quad (6.18)$$

$$\frac{d}{dt}(S_{vb1}+I_{vb1}) = A_{v_b}-\eta_{v_b}N_{vb1} \quad (6.19)$$

$$\frac{d}{dt}(S_{vb2}+I_{vb2}) = A_{v_b}-\eta_{v_b}N_{vb2}. \quad (6.20)$$

Where N_{va1} and N_{va2} are the numbers of *Aedes aegypti* in children and adult respectively, which is equal to $S_{va1}+I_{va1}$ and $S_{va2}+I_{va2}$. N_{vb1} and N_{vb2} are the numbers of *Aedes albopictus* in children and adult respectively, which is equal to $S_{vb1}+I_{vb1}$ and $S_{vb2}+I_{vb2}$. If the numbers of mosquitoes are also constant each other (6.17) – (6.20) gives

$N_{va1}=A_{v_a}/\eta_{v_a}$, $N_{va2}=A_{v_a}/\eta_{v_a}$, $N_{vb1}=A_{v_b}/\eta_{v_b}$ and $N_{vb2}=A_{v_b}/\eta_{v_b}$.

We normalize parameter (6.1) – (6.8) and (6.9) – (6.16) by writing

$S'_c=\frac{S_c}{N_{tc}}$, $I'_{c1}=\frac{I_{c1}}{N_{tc}}$, $I'_{c2}=\frac{I_{c2}}{N_{tc}}$, $R'_c=\frac{R_c}{N_{tc}}$ in children and

$S'_a=\frac{S_a}{N_{ta}}$, $I'_{a1}=\frac{I_{a1}}{N_{ta}}$, $I'_{a2}=\frac{I_{a2}}{N_{ta}}$, $R'_a=\frac{R_a}{N_{ta}}$ in adult.

$S'_{va1}=\frac{S_{va1}}{N_{va1}}$, $I'_{va1}=\frac{I_{va1}}{N_{va1}}$, $S'_{va2}=\frac{S_{va2}}{N_{va2}}$, $I'_{va2}=\frac{I_{va2}}{N_{va2}}$,

$S'_{vb1}=\frac{S_{vb1}}{N_{vb1}}$, $I'_{vb1}=\frac{I_{vb1}}{N_{vb1}}$, $S'_{vb2}=\frac{S_{vb2}}{N_{vb2}}$ and $I'_{vb2}=\frac{I_{vb2}}{N_{vb2}}$, then the reduced equations become

$$\frac{d}{dt}S'_c=\eta_d-\alpha_{ac}(1+\gamma_a Sin\beta t)I'_{va1}N_{va1}S'_c-\alpha_{bc}(1+\gamma_b Sin\beta t)I'_{vb1}N_{vb1}S'_c-\eta_d S'_c \quad (6.21)$$

$$\frac{d}{dt}I'_{c1}=\alpha_{ac}(1+\gamma_a Sin\beta t)I'_{va1}N_{va1}S'_c-\omega_{c1}I'_{c1}-\eta_d I'_{c1} \quad (6.22)$$

$$\frac{d}{dt}I'_{c2}=\alpha_{bc}(1+\gamma_b Sin\beta t)I'_{vb1}N_{vb1}S'_c-\omega_{c2}I'_{c2}-\eta_d I'_{c2} \quad (6.23)$$

$$\frac{d}{dt}S'_a=\eta_d-\alpha_{aa}(1+\gamma_a Sin\beta t)I'_{va2}N_{va2}S'_a-\alpha_{ba}(1+\gamma_b Sin\beta t)I'_{vb2}N_{vb2}S'_a-\eta_d S'_a \quad (6.24)$$

$$\frac{d}{dt}I'_{a1} = \alpha_{aa}(1+\gamma_a Sin\beta t)\, I'_{va2}\, N_{va2}S'_a - \omega_{a1}I'_{a1} - \eta_d I'_{a1} \quad (6.25)$$

$$\frac{d}{dt}I'_{a2} = \alpha_{ba}(1+\gamma_b Sin\beta t) I'_{vb2}N_{vb2}\, S'_a - \omega_{a2}I'_{a2} - \eta_d I'_{a2} \quad (6.26)$$

$$\frac{d}{dt}I'_{va1} = \alpha_{va1}(1+\gamma_{va}\, Sin\,\beta t) I'_{c1}\, N_{tc}S'_{va1} - \eta_{v_a} I'_{va1} \quad (6.27)$$

$$\frac{d}{dt}I'_{va2} = \alpha_{va2}(1+\gamma_{va}\, Sin\,\beta t) I'_{a1}\, N_{ta}S'_{va2} - \eta_{v_a} I'_{va2} \quad (6.28)$$

$$\frac{d}{dt}I'_{vb1} = \alpha_{vb1}(1+\gamma_{vb}\, Sin\,\beta t) I'_{c2}\, N_{tc}S'_{vb1} - \eta_{v_b} I'_{vb1} \quad (6.29)$$

$$\frac{d}{dt}I'_{vb2} = \alpha_{vb2}(1+\gamma_{vb}\, Sin\,\beta t) I'_{a2}\, N_{ta}S'_{vb2} - \eta_{v_b} I'_{vb2} \quad . \quad (6.30)$$

Where

$S'_c + I'_{c1} + I'_{c2} + R'_c = 1$,

$S'_a + I'_{a1} + I'_{a2} + R'_a = 1$,

$S_{va1} + I_{va1} = 1$,

$S'_{va2} + I'_{va2} = 1$,

$S'_{vb1} + I'_{vb1} = 1$ and

$S'_{vb2} + I'_{vb2} = 1$.

6.2 Analysis of Model

6.2.1. Equilibrium states

The equilibrium points $(S^*_c, I^*_{c1}, I^*_{c2}, S^*_a, I^*_{a1}, I^*_{a2}, I^*_{va1}, I^*_{vb1}, I^*_{va2}, I^*_{vb2})$ are obtained by setting the right hand side of (6.21) – (6.30) to zero. Doing this, we get four equilibrium points,

1) The two group disease free equilibrium point

$$S_0 = (1,0,0,1,0,0,0,0,0,0) \ .$$

i) The disease free equilibrium point

$S_{oc} = (S_c^*, I_{c1}^*, I_{c2}^*, I_{va1}^*, I_{vb1}^*) = (1,0,0,0,0)$ in children.

ii) The disease free equilibrium point

$S_{oa} = (S_a^*, I_{a1}^*, I_{a2}^*, I_{va2}^*, I_{vb2}^*) = (1,0,0,0,0)$ in adult.

2) The two group endemic equilibrium point

$\hat{S} = (S_c^*, I_{c1}^*, I_{c2}^*, S_a^*, I_{a1}^*, I_{a2}^*, I_{va1}^*, I_{vb1}^*, I_{va2}^*, I_{vb2}^*)$.

i) The endemic point

$S_{1c} = (S_c^*, I_{c1}^*, I_{c2}^*, I_{va1}^*, I_{vb1}^*)$ in children,

where

$$S_c^* = \frac{(N_{tc}\eta_d\alpha_{vb1} + (\omega_{c2}+\eta_d)\eta_{vb1} + N_{tc}\eta_d\alpha_{vb1}\gamma_{vb} Sin\beta t)}{(N_{tc}\alpha_{vb1}(I_{va1}^* N_{va1}\alpha_{ac} + N_{vb1}\alpha_{bc} + \mu_d + (I_{va1}^* N_{va1}\alpha_a\beta_{ac} + N_{vb1}\alpha_b\beta_{bc}) Sin\beta t)(1+\gamma_{vb} Sin\beta t)))} \tag{6.31}$$

$$I_{c1}^* = \frac{I_{va1}^* N_{va1}\alpha_{ac}(1+\gamma_a Sin\,\beta\,t)(N_{tc}\eta_d\alpha_{vb1} + (\omega_{c2}+\eta_d)\eta_{vb} + N_{tc}\eta_d\alpha_{vb1}\gamma_{vb} Sin\,\beta\,t))}{(N_{tc}\alpha_{vb1}(\omega_{c1}+\eta_d)(I_{va1}^* N_{va1}\alpha_{ac} + N_{vb1}\alpha_{bc} + \eta_d + (I_{va1}^* N_{va1}\gamma_a\alpha_{ac} + N_{vb1}\gamma_b\alpha_{bc}) Sin\,\beta\,t)(1+\gamma_{vb} Sin\,\beta\,t))} \tag{6.32}$$

$$I_{c2}^* = \frac{(I_{vb1}^* N_{vb1}\alpha_{bc}(1+\gamma_b Sin\beta\,t)(N_{tc}\alpha_{vb1}\eta_d + (\omega_{c2}+\eta_d)\eta_{vb} + N_{tc}\alpha_{vb1}\eta_d\gamma_{vb} Sin\,\beta t))}{(N_{tc}\alpha_{vb1}(\omega_{c2}+\eta_d)(I_{va1}^* N_{va1}\alpha_{ac} + I_{vb1}^* N_{vb1}\alpha_{bc} + \eta_d + (I_{va1}^* N_{va1}\gamma_a\alpha_{ac} + N_{vb1}\gamma_b\gamma_{bc}) Sin\beta\,t)(1+\gamma_{vb} Sin\beta\,t))} \tag{6.33}$$

$$\begin{aligned}
I_{va1}^* = {} & [(2(-2\alpha_{vb1}\eta_d(\omega_{c1}+\eta_d)\eta_{va} + 2N_{va1}\alpha_{ac}\alpha_{va1}(\omega_{c2}+\eta_d)\eta_{vb} \\
& + N_{va1}\gamma_a\alpha_{ac}\alpha_{va1}(\omega_{c2}+\eta_d)\eta_{vb}\gamma_{va} - N_{vb1}\alpha_{bc}\lambda_{vb1}(\omega_{c1}+\eta_d)\eta_{va}(2+\gamma_b\gamma_{vb}) \\
& + N_{tc}N_{va1}\alpha_{ac}\alpha_{va1}\alpha_{vb1}\eta_d(2+\gamma_{va}\gamma_{vb}+\gamma_a(\gamma_{va}+\gamma_{vb}))) \\
& - 2(N_{va1}\gamma_a\alpha_{ac}\alpha_{va1}(\omega_{c2}+\eta_d)\eta_{vb}\gamma_{va} \\
& - N_{vb1}\alpha_b\alpha_{bc}\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{va}\gamma_{vb} + N_{tc}N_{va1}\alpha_{ac}\alpha_{va1}\alpha_{vb1}\eta_d \\
& (\gamma_{va}\gamma_{vb} + \alpha_a(\gamma_{va}+\gamma_{vb}))) Cos\,2\beta\,t + (4(N_{va1}\alpha_{ac}\alpha_{va1}(\omega_{c2}+\eta_d)\eta_{vb}(\gamma_a+\gamma_{va}) \\
& - \alpha_{vb1}\eta_d(\omega_{c1}+\eta_d)\eta_{va} - N_{vb1}\alpha_{bc}\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{va}(\gamma_b+\gamma_{vb})) \\
& + N_{tc}N_{va1}\alpha_{ac}\alpha_{va1}\alpha_{vb1}\eta_d(4(\gamma_{va}+\gamma_{vb}) + \gamma_a(4+3\gamma_{va}\gamma_{vb}))) Sin\,\beta\,t \\
& - N_{tc}N_{va1}\gamma_a\alpha_{ac}\alpha_{va1}\alpha_{vb1}\eta_d\gamma_{va}\gamma_{vb} Sin\,\beta\,t)] \\
& / [(2N_{va1}\alpha_{ac}(1+\gamma_a Sin\,\beta\,t)(2\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{va} + 2\alpha_{va1}(\omega_{c2}+\eta_d)\eta_{vb}(\omega_{c2}+\eta_d)\eta_{vb} \\
& + N_{tc}\eta_d\alpha_{va1}\alpha_{vb1}(2+\gamma_{va}\gamma_{vb}) - N_{tc}\eta_d\alpha_{va1}\alpha_{vb1}\gamma_{va}\gamma_{vb} Cos\;2\;\beta\,t \\
& + 2(\alpha_{va1}(\omega_{c2}+\eta_d)\eta_{vb}\gamma_{va} + \alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{vb}\gamma_{vb} \\
& + N_{tc}\eta_d\alpha_{va1}\alpha_{vb1}(\gamma_{va}+\gamma_{vb})) Sin\;\beta\;t))]
\end{aligned} \tag{6.34}$$

$$I^{*}_{vb1} = [(N_{tc} N_{vb1} \alpha_{ac} \alpha_{vb1} \eta_d - (I^{*}_{va1} N_{va1} \alpha_{ac} + \eta_d)(\omega_{c2} + \eta_d) \eta_{vb}$$

$$+ Sin\beta t (-I^{*}_{va1} N_{va1} \gamma_a \alpha_{ac} (\omega_{c2} + \eta_d) \eta_{vb} + \tag{6.35}$$

$$N_{tc} N_{vb1} \alpha_{bc} \alpha_{vb1} \eta_d (\gamma_b + \gamma_{vb}) + N_{tc} N_{vb1} \gamma_b \alpha_{bc} \alpha_{vb1} \eta_d \gamma_{vb} \; Sin\beta t))]$$

$$/[N_{vb1} \alpha_{bc} (1 + \gamma_b \; Sin\,\beta t)(N_{tc} \alpha_{vb1} \eta_d + (\omega_{c2} + \eta_d) \eta_{vb} + N_{tc} \alpha_{vb1} \eta_d \gamma_{vb} \; Sin\beta t))]$$

ii) the endemic point

$$S_{1a} = (S^{*}_{a}, I^{*}_{a1}, I^{*}_{a2}, I^{*}_{va2}, I^{*}_{vb2}) \quad \text{in adult, where}$$

$$S^{*}_{a} = \frac{(N_{ta} \eta_d \alpha_{vb2} + (\omega_{a2} + \eta_d) \eta_{vb} + N_{ta} \eta_d \alpha_{vb2} \gamma_{vb} \; Sin\,\beta t)}{(N_{ta} \alpha_{vb2} (I^{*}_{va2} N_{va2} \alpha_{aa} + N_{vb2} \alpha_{ba} + \eta_d + (I^{*}_{va2} N_{va2} \gamma_a \alpha_{aa} + N_{vb2} \gamma_b \alpha_{ba}) \; Sin\,\beta t)(1 + \gamma_{vb} \; Sin\,\beta t)))} \tag{6.36}$$

$$I^{*}_{a1} = \frac{I^{*}_{va2} N_{va2} \alpha_{aa} (1 + \gamma_a \; Sin\beta t)(N_{ta} \eta_d \alpha_{vb2} + (\omega_{a2} + \eta_d) \eta_{vb} + N_{ta} \eta_d \alpha_{vb2} \gamma_{vb} \; Sin\,\beta t))}{(N_{ta} \alpha_{vb2} (\omega_{a1} + \eta_d)(I^{*}_{va2} N_{va2} \alpha_{aa} + N_{vb2} \alpha_{ba} + \eta_d + (I^{*}_{va2} N_{va2} \gamma_a \alpha_{aa} + N_{vb2} \gamma_b \alpha_{ba}) \; Sin\beta t)(1 + \gamma_{vb} \; Sin\beta t))} \tag{6.37}$$

$$I^{*}_{a2} = [(I^{*}_{vb2} N_{vb2} \alpha_{ba} (1 + \gamma_b \; Sin\beta t)(N_{ta} \alpha_{vb2} \eta_d + (\omega_{a2} + \eta_d) \eta_{vb} + N_{ta} \alpha_{vb2} \eta_d \gamma_{vb} \; Sin\,\beta t))/$$

$$N_{ta} \alpha_{vb2} (\omega_{a2} + \eta_d)(I^{*}_{va2} N_{va2} \alpha_{aa} + I^{*}_{vb2} N_{vb2} \alpha_{ba} + \eta_d + (I^{*}_{va2} N_{va2} \gamma_a \alpha_{aa} \tag{6.38}$$

$$+ N_{vb2} \gamma_b \alpha_{ba}) \; Sin\,\beta t)(1 + \gamma_{vb} \; Sin\,\beta t))]$$

$$
\begin{aligned}
I^{*}_{va2} = & [(2(-2\alpha_{vb2}\eta_d(\omega_{a1}+\eta_d)\eta_{va} + 2N_{va2}\alpha_{aa}\alpha_{va2}(\omega_{a2}+\eta_d)\eta_{vb} \\
& + N_{va2}\alpha_a\alpha_{aa}\alpha_{va2}(\omega_{a2}+\eta_d)\eta_{vb}\gamma_{va} - N_{vb2}\alpha_{ba}\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{va}(2+\alpha_b\gamma_{vb}) \\
& + N_{ta}N_{va2}\alpha_{aa}\alpha_{va2}\alpha_{vb2}\eta_d(2+\gamma_{va}\gamma_{vb}+\alpha_a(\gamma_{va}+\gamma_{vb}))) \\
& -2(N_{va2}\gamma_a\alpha_{aa}\alpha_{va2}(\omega_{a2}+\eta_d)\eta_{vb}\gamma_{va} - N_{vb2}\gamma_b\alpha_{ba}\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{va}\gamma_{vb} \\
& + N_{ta}N_{va2}\alpha_{aa}\alpha_{va2}\alpha_{vb2}\eta_d(\gamma_{va}\gamma_{vb}+\gamma_a(\gamma_{va}+\gamma_{vb})))\, Cos 2\beta t + \\
& (4(N_{va2}\alpha_{aa}\alpha_{va2}(\omega_{a2}+\eta_d)\eta_{vb}(\gamma_a+\gamma_{va}) - \alpha_{vb2}\eta_d(\omega_{a1}+\eta_d)\eta_{va} \\
& - N_{vb2}\alpha_{ba}\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{va}(\gamma_b+\gamma_{vb})) + \\
& N_{ta}N_{va2}\alpha_{aa}\alpha_{va2}\alpha_{vb2}\eta_d(4(\gamma_{va}+\gamma_{vb})+\gamma_a(4+3\gamma_{va}\gamma_{vb})))\, Sin\beta t \\
& - N_{ta}N_{va2}\gamma_a\alpha_{aa}\alpha_{va2}\alpha_{vb2}\eta_d\gamma_{va}\gamma_{vb}\, Sin\beta t)] / \\
& [(2N_{va2}\alpha_{aa}(1+\gamma_a Sin\beta t)(2\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{va} \\
& + 2\alpha_{va2}(\omega_{a2}+\eta_d)\eta_{vb}(\omega_{a2}+\eta_d)\eta_{vb} + N_{ta}\eta_d\alpha_{va2}\alpha_{vb2}(2+\gamma_{va}\gamma_{vb}) \\
& - N_{ta}\eta_d\alpha_{va2}\alpha_{vb2}\gamma_{va}\gamma_{vb}\, Cos 2\beta t + 2(\alpha_{va2}(\omega_{a2}+\eta_d)\eta_{vb}\gamma_{va} \\
& + \alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{va}\gamma_{vb} + N_{ta}\eta_d\alpha_{va2}\alpha_{vb2}(\gamma_{va}+\gamma_{vb}))\, Sin\beta t))]
\end{aligned}
\tag{6.39}
$$

$$
\begin{aligned}
I^{*}_{vb2} = & [(N_{ta}N_{vb2}\alpha_{aa}\alpha_{vb2}\eta_d - (I^{*}_{va2}N_{va2}\alpha_{aa}+\eta_d)(\omega_{a2}+\eta_d)\eta_{vb} \\
& + Sin\beta t(-I^{*}_{va2}N_{va2}\gamma_a\alpha_{ad}(\omega_{a2}+\eta_d)\eta_{vb} + \\
& N_{ta}N_{vb2}\alpha_{ba}\alpha_{vb2}\eta_d(\alpha_b+\rho_{vb}) + N_{ta}N_{vb2}\gamma_b\alpha_{ba}\alpha_{vb2}\eta_d\rho_{vb}\, Sin\beta t))] / \\
& [N_{vb2}\alpha_{bd}(1+\gamma_b Sin\beta t)(N_{ta}\alpha_{vb2}\eta_d + (\omega_{a2}+\eta_d)\mu_{vb} \\
& + N_{ta}\alpha_{vb2}\eta_d\gamma_{vb}\, Sin\beta t))]
\end{aligned}
\tag{6.40}
$$

6.3. Local Asymptotically Stability

The local stability of each equilibrium point is determined from Jacobian matrix of right hand side of the above set of differential equations evaluated at the equilibrium point.

Proposition 6.3.1 If $E_0 < 1$, $E_{0c} < 1$ and $E_{0a} < 1$ when $\beta = 0$, then the disease free equilibrium point E_{0c} in children and E_{0a} in adult are locally asymptotically stable, where

$$E_0 = \max \left\{ \frac{2N_{vb1}\gamma_b\alpha_{bc}\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{va}\gamma_{vb}+N_{va1}\alpha_{ac}\alpha_{va1}(2(N_{tc}\alpha_{vb1}\eta_d+(\omega_{c2}+\eta_d)\eta_{vb})(2+\gamma_a\gamma_{va})+2N_{tc}\alpha_{vb1}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{va}(2\eta_d+N_{vb1}\alpha_{bc}(2+\gamma_b\gamma_{vb}))+2N_{va1}\alpha_{ac}\alpha_{va1}(\alpha_a(\omega_{c2}+\eta_d)\eta_{vb}\gamma_{va}+N_{tc}\alpha_{vb1}\eta_d(\gamma_a\gamma_{va}+(\gamma_a+\gamma_{va})\gamma_{vb})))}, \right.$$

$$\left. \frac{2N_{vb2}\gamma_b\alpha_{ba}\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{va}\gamma_{vb}+N_{va2}\alpha_{aa}\alpha_{va2}(2(N_{ta}\alpha_{vb2}\eta_d+(\omega_{a2}+\eta_d)\eta_{vb})(2+\gamma_a\gamma_{va})+2N_{ta}\alpha_{vb2}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{va}(2\eta_d+N_{vb2}\alpha_{ba}(2+\gamma_b\gamma_{vb}))+2N_{va2}\alpha_{aa}\alpha_{va2}(\gamma_a(\omega_{a2}+\eta_d)\eta_{vb}\gamma_{va}+N_{ta}\alpha_{vb2}\eta_d(\gamma_a\gamma_{va}+(\gamma_a+\gamma_{va})\gamma_{vb})))} \right\}$$

$$E_{0c} = \frac{2N_{vb1}\gamma_b\alpha_{bc}\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{va}\gamma_{vb}+N_{va1}\alpha_{ac}\alpha_{va1}(2(N_{tc}\alpha_{vb1}\eta_d+(\omega_{c2}+\eta_d)\eta_{vb})(2+\gamma_a\gamma_{va})+2N_{tc}\alpha_{vb1}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{va}(2\eta_d+N_{vb1}\alpha_{bc}(2+\gamma_b\gamma_{vb}))+2N_{va1}\alpha_{ac}\alpha_{va1}(\gamma_a(\omega_{c2}+v_d)\eta_{vb}\gamma_{va}+N_{tc}\alpha_{vb1}\eta_d(\gamma_a\gamma_{va}+(\gamma_a+\gamma_{va})\gamma_{vb})))}$$

in children.

$$E_{0a} = \frac{2N_{vb2}\gamma_b\alpha_{ba}\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{va}\gamma_{vb}+N_{va2}\alpha_{aa}\alpha_{va2}(2(N_{ta}\alpha_{vb2}\eta_d+(\omega_{a2}+\eta_d)\eta_{vb})(2+\gamma_a\gamma_{va})+2N_{ta}\alpha_{vb2}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{va}(2\eta_d+N_{vb2}\alpha_{ba}(2+\gamma_b\gamma_{vb}))+2N_{va2}\alpha_{aa}\alpha_{va2}(\gamma_a(\omega_{a2}+\eta_d)\eta_{vb}\gamma_{va}+N_{ta}\alpha_{vb2}\eta_d(\gamma_a\gamma_{va}+(\gamma_a+\gamma_{va})\gamma_{vb})))}$$

in adult.

Proof.

For the disease free equilibrium point in children $E_{oc} = (1,0,0,0,0)$ and in adult $E_{oc} = (1,0,0,0,0)$.

The system defined by equations (6.21) – (6.30), the Jacobian matrix evaluated at E_{0c} and E_{0a}, respectively given by

$$J_c = \begin{bmatrix} -(\eta_d) & 0 & 0 & -\alpha_{ac}(1+\gamma_a Sin(\beta t))N_{va1} & -\alpha_{bc}(1+\gamma_b Sin(\beta t))N_{vb1} \\ 0 & -(\omega_{c1}+\eta_d) & 0 & \alpha_{ac}(1+\gamma_a Sin(\beta t))N_{va1} & 0 \\ 0 & 0 & -(\omega_{c2}+\eta_d) & 0 & \alpha_{bc}(1+\gamma_b Sin(\beta t))N_{vb1} \\ 0 & \alpha_{va1}(1+\gamma_{va} Sin(\beta t)) & 0 & -\eta_{va1} & 0 \\ 0 & 0 & \alpha_{vb1}(1+\gamma_{vb} Sin(\beta t)) & 0 & -\eta_{vb1} \end{bmatrix} \quad (6.41)$$

$$J_a = \begin{bmatrix} -(\eta_d) & 0 & 0 & -\alpha_{aa}(1+\gamma_a Sin(\beta t))N_{va2} & -\alpha_{ba}(1+\gamma_b Sin(\beta t))N_{vb2} \\ 0 & -(\omega_{a1}+\eta_d) & 0 & \alpha_{aa}(1+\gamma_a Sin(\beta t))N_{va2} & 0 \\ 0 & 0 & -(\omega_{a2}+\eta_d) & 0 & \alpha_{ba}(1+\gamma_b Sin(\beta t))N_{vb2} \\ 0 & \alpha_{va2}(1+\gamma_{va} Sin(\beta t)) & 0 & -\eta_{va2} & 0 \\ 0 & 0 & \alpha_{vb2}(1+\gamma_{vb} Sin(\beta t)) & 0 & -\eta_{vb2} \end{bmatrix} \quad (6.42)$$

The eigenvalues are obtained by solving the characteristic equations, $\det|\eta I_5 - J_i| = 0$. Where I_5 is the 5x5 identity matrix and $J_i (J_i; i = c, a)$ is the Jacobian matrix for (6.41) and (6.42), respectively. To evaluate the determinant, we get the following characteristic equations:

$$(\lambda + \eta_d)(\lambda^4 + W_1\lambda^3 + W_2\lambda^2 + W_3\lambda + W_4) = 0 \quad (6.43)$$

$$(\lambda + \eta_d)(\lambda^4 + F_1\lambda^3 + F_2\lambda^2 + F_3\lambda + F_4) = 0 \quad (6.44)$$

where

$$W_1 = \omega_{c1} + \omega_{c2} + 2\eta_d + \eta_{v_a} + \eta_{v_b} \quad , \tag{6.45}$$

$$\begin{aligned} W_2 = &-N_{va1}\,\alpha_{ac}\,\alpha_{va1} - N_{vb1}\,\alpha_{bc}\,\alpha_{vb1}) + \eta_d^2 + 2\eta_d\,\eta_{v_a} + 2\eta_d\,\eta_{v_b} + \eta_{v_a}\,\eta_{v_b} + \\ &\omega_{c2}(\eta_d + \eta_{v_a} + \eta_{v_b}) + \omega_{c1}(\omega_{c2} + \eta_d + \eta_{v_a} + \eta_{v_b}) \end{aligned} \quad , \tag{6.46}$$

$$\begin{aligned} W_3 = &(\omega_{c1}+\eta_d)(\omega_{c2}+\eta_d)\eta_{v_a} - N_{vb1}\alpha_{bc}\alpha_{vb1}(\omega_{c1}+\eta_d+\eta_{v_a}) + ((\omega_{c1}+\eta_d)(\omega_{c2}+\eta_d) \\ &+(\omega_{c1}+\omega_{c2}+2\eta_d)\eta_{v_a})\eta_{v_b} - N_{va1}\alpha_{ac}\alpha_{va1}(\omega_{c2}+\eta_d+\eta_{v_b}) + (\alpha_{ac}\alpha_{va1}(\omega_{c2}+\eta_d+\eta_{v_b})) \end{aligned} \quad , \tag{6.47}$$

$$W_4 = (N_{va1}\alpha_{ac}\alpha_{va1} - (\omega_{c1}+\eta_d)\eta_{v_a})(N_{vb1}\alpha_{bc}\alpha_{vb1} + (\omega_{c2}+\eta_d)\eta_{v_b}) \quad , \tag{6.48}$$

$$F_1 = \omega_{a1} + \omega_{a2} + 2\eta_d + \eta_{v_a} + \eta_{v_b} \quad , \tag{6.49}$$

$$\begin{aligned} F_2 = &-N_{va2}\alpha_{aa}\alpha_{va2} - N_{vb2}\alpha_{ba}\alpha_{vb2}) + \eta_d^2 + 2\eta_d\eta_{v_a} + 2\eta_d\eta_{v_b} + \eta_{v_a}\eta_{v_b} \\ &+\omega_{a2}(\eta_d + \eta_{v_a} + \eta_{v_b}) + \omega_{a1}(\omega_{a2} + \eta_d + \eta_{v_a} + \eta_{v_b}) \end{aligned} \quad , \tag{6.50}$$

$$\begin{aligned} F_3 = &(\omega_{a1}+\eta_d)(\omega_{a2}+\eta_d)\eta_{v_a} - N_{vb2}\alpha_{ba}\alpha_{vb2}(\omega_{a1}+\eta_d+\eta_{v_a}) + ((\omega_{a1}+\eta_d)(\omega_{a2}+\eta_d) \\ &+(\omega_{a1}+\omega_{a2}+2\eta_d)\eta_{v_a})\eta_{v_b} - N_{va2}\alpha_{aa}\alpha_{va2}(\omega_{a2}+\eta_d+\eta_{v_b}) \\ &+(\alpha_{aa}\alpha_{va2}(\omega_{a2}+\eta_d+\eta_{v_b})) \end{aligned} \quad , \tag{6.51}$$

$$F_4 = (N_{va2}\alpha_{aa}\alpha_{va2} - (\omega_{a1}+\eta_d)\eta_{v_a})(N_{vb2}\alpha_{ba}\alpha_{vb2} + (\omega_{a2}+\eta_d)\eta_{v_b}) \tag{6.52}$$

From the characteristic equations (6.43) – (6.44), we see that one eigenvalue of (6.43) and (6.44) is $\lambda = -\eta_d$. The sign of other four eigenvalues can be ascertained by solving equation $(\lambda^4 + W_1\lambda^3 + W_2\lambda^2 + W_3\lambda + W_4) = 0$ and $(\lambda^4 + F_1\lambda^3 + F_2\lambda^2 + F_3\lambda + F_4) = 0$. The remaining four eigenvalues have negative real parts if they satisfy Routh-Hurwitz criteria (6.53) - (6.56)[13], each equilibrium point is locally asymptotically stable if the following conditions are satisfied,

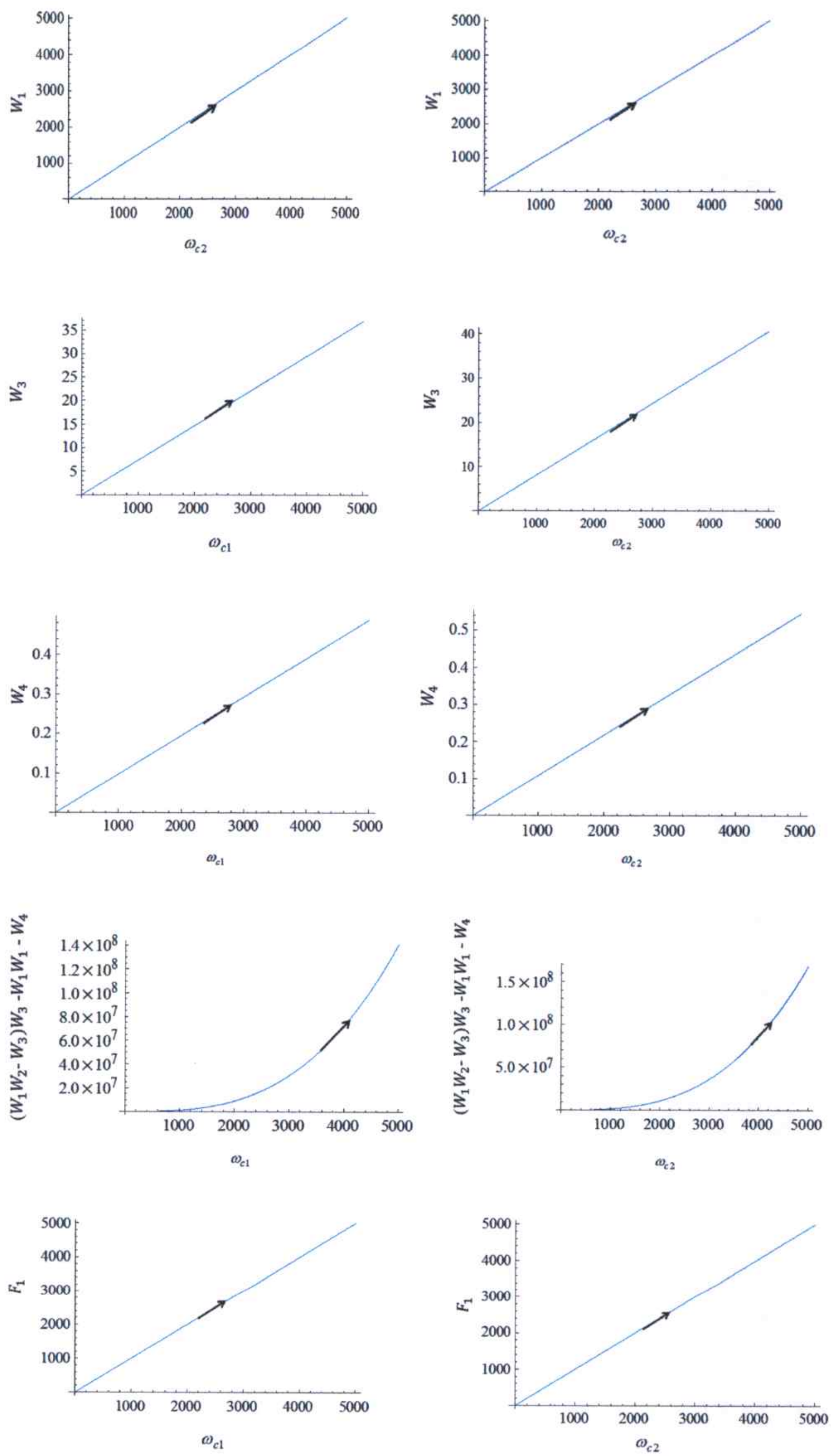
i) $W_1\ and\ F_1 > 0$ (6.53)

ii) $W_3\ and\ F_3 > 0$ (6.54)

iii) $W_4\ and\ F_4 > 0$ (6.55)

iv) $(W_1W_2 - W_3)W_3 + W_1^2W_4 > 0\ and\ (F_1F_2 - F_3)F_3 + F_1^2F_4 > 0$ (6.56)

After we use *Mathematica* to show the conditions of locally asymptotically stable, we can see that W_1 and F_1 are always positive. For the equations given by (6.54) – (6.56), we show these conditions by using the following figures,



W_1
ω_{c2}
W_1
ω_{c2}
W_3
ω_{c1}
W_3
ω_{c2}
W_4
ω_{c1}
W_4
ω_{c2}
$(W_1W_2 - W_3)W_3 - W_1W_1 - W_4$
ω_{c1}
$(W_1W_2 - W_3)W_3 - W_1W_1 - W_4$
ω_{c2}
F_1
ω_{c1}
F_1
ω_{c2}

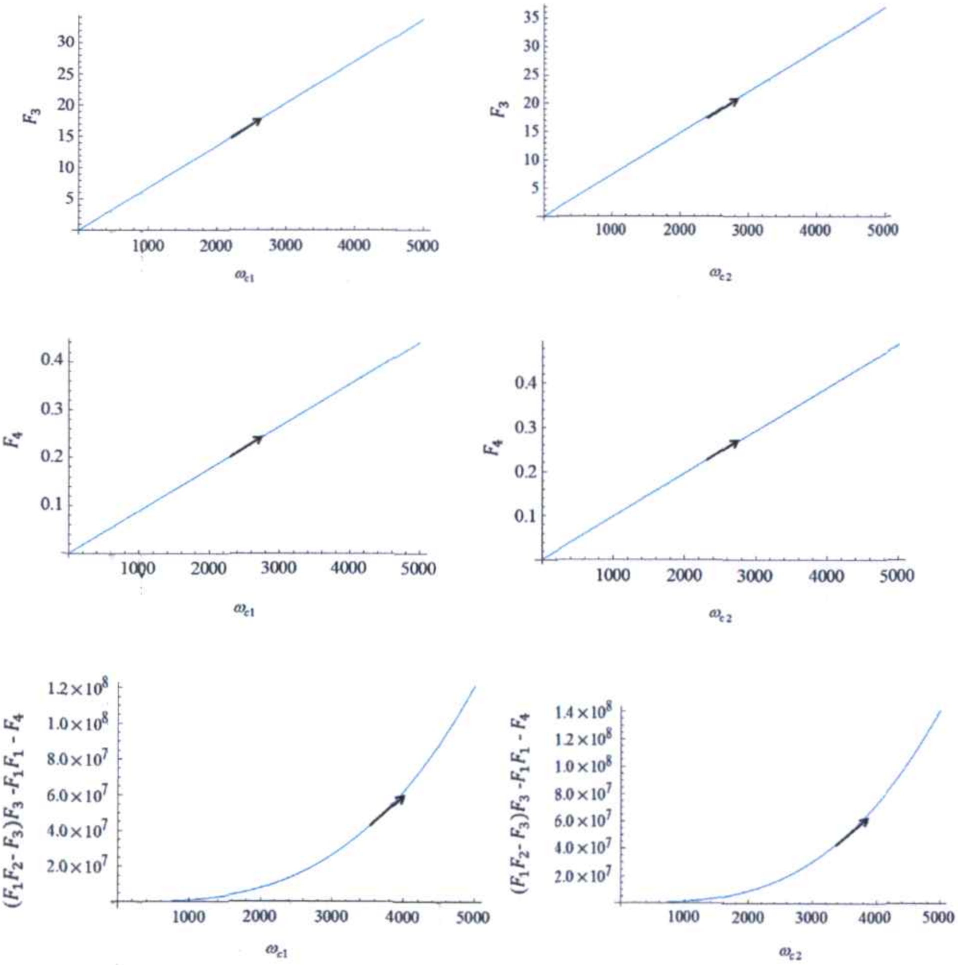


Figure 6.2. The parameter spaces for the disease free equilibrium point, which satisfies the Routh-Hurwitz conditions, show onto

(ω_{c1}, W_3), (ω_{c2}, W_3), (ω_{a1}, F_3), (ω_{a2}, F_3), (ω_{c1}, W_4), (ω_{c2}, W_4), (ω_{a1}, F_4), (ω_{a2}, F_4), $(\omega_{c1}, ((W_1W_2-W_3)W_3-W_1^2W_4))$, $(\omega_{c1}, ((W_1W_2-W_3)W_3-W_1^2W_4))$, $(\omega_{a1}, ((F_1F_2-F_3)F_3-F_1^2F_4))$, $(\omega_{a2}, ((F_1F_2-F_3)F_3-F_1^2F_4))$, respectively. The values of parameter are follows:

$\omega_{c1} = 1/(17/2)$, $\omega_{c2} = 1/(19/2)$, $\eta_d = 1/(365*74.6)$ day^{-1}, $N_{tc} = 9000$, $N_{va1} = 4000$, $N_{vb1} = 5500$, $\alpha_{ac} = 0.00769$, $\alpha_{bc} = 0.000246$, $\alpha_{va1} = 0.00000576$, $\alpha_{vb1} = 0.00000335$, $\gamma_a = 0.07$, $\gamma_b = 0.067$, and $N_t = 100000$, $\omega_{a1} = 1/(19/2)$, $\omega_{a2} = 1/(21/2)$, $\eta_d = 1/(365*74.6)$ day^{-1}, $N_{ta} = 6000$, $N_{va2} = 3000$, $N_{vb2} = 4100$, $\alpha_{aa} = 0.000045$, $\alpha_{ba} = 0.000067$, $\alpha_{va2} = 0.0066$, $\alpha_{vb2} = 0.00235$, $\gamma_a = 0.07$, $\gamma_b = 0.067$ and $N_t = 100000$.

From the above figures 6.3, the Routh-Hurwitz conditions are satisfied for $E_0 > 1$.

6.4. Endemic Disease State

Proposition 6.4.1 If $E_0 > 1$, when $\beta = 0$,then the equilibrium state $\hat{E} = (S_c^*, I_{c1}^*, I_{c2}^*, I_{va1}^*, I_{vb1}^*, S_a^*, I_{a1}^*, I_{a2}^*, I_{va2}^*, I_{vb2}^*)$ is locally asymptotically stable.

Proof. For the endemic disease equilibrium point $S_{1c} = (S_c^*, I_{c1}^*, I_{c2}^*, I_{va1}^*, I_{vb1}^*)$ in children and $S_{1a} = (S_a^*, I_{a1}^*, I_{a2}^*, I_{va2}^*, I_{vb2}^*)$ in adult, we obtain the characteristic equation:

$$(\lambda^5 + D_1\lambda^4 + D_2\lambda^3 + D_3\lambda^2 + D_4\lambda + D_5) = 0 \qquad \text{for children} \tag{6.57}$$

$$(\lambda^5 + G_1\lambda^4 + G_2\lambda^3 + G_3\lambda^2 + G_4\lambda + G_5) = 0 \qquad \text{for adult} \tag{6.58}$$

where

$$D_1 = N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \omega_{c1} + \omega_{c2} + 3\eta_d + \eta_{va} + \eta_{vb} , \tag{6.59}$$

$$\begin{aligned} D_2 =\ & \omega_{c1}\omega_{c2} + 2\omega_{c1}\eta_d + 2\omega_{c2}\eta_d + 3\mu_d^2 + \omega_{c1}\eta_{va} + \omega_{c2}\eta_{va1} + 3\eta_d\eta_{va} + \\ & (\omega_{c1} + \omega_{c2} + 3\eta_d + \eta_{va1})\eta_{vb} + N_{va1}\alpha_{ac}(-(\alpha_{va1}\eta_d / N_{va1}\alpha_{ac}\theta_1 \\ & + N_{vb1}\alpha_{bc}\theta_2\eta_d) + \theta_1(\omega_{c1} + \omega_{c2} + 2\eta_d + (\alpha_{va1}\eta_d / N_{va1}\alpha_{ac}\theta_1 \\ & + N_{vb1}\alpha_{bc}\theta_2\eta_d) + \eta_{va} + \eta_{vb})) + N_{vb1}\alpha_{bc}(-(\alpha_{vb1}\eta_d / N_{va1}\alpha_{ac}\theta_1 \\ & + N_{vb1}\alpha_{bc}\theta_2\eta_d) + \theta_2(\omega_{c1} + \omega_{c2} + 2\eta_d + (\alpha_{vb1}\eta_d / N_{va1}\alpha_{ac}\theta_1 \\ & + N_{vb1}\alpha_{bc}\theta_2\eta_d)\eta_{va} + \eta_{vb})) \end{aligned} , \tag{6.60}$$

$$\begin{aligned} D_3 =\ & \frac{1}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d}(N_{va1}^2\alpha_{ac}^2\theta_1^2(\mu_d^2 + 2\eta_d\eta_{va1} + 2\eta_d\eta_{vb} + \eta_{va}\eta_{vb} \\ & + \omega_{c2}(\eta_d + \eta_{va} + \eta_{vb}) + \omega_{c1}(\omega_{c2} + \eta_d + \eta_{va} + \eta_{vb})) + N_{vb1}^2\alpha_{bc}^2\theta_2^2(\eta_d^2 \\ & + 2\eta_d\eta_{va} + 2\eta_d\eta_{vb} + \eta_{va}\eta_{vb} + \omega_{c2}(\eta_d + \eta_{va} + \eta_{vb}) \\ & + \omega_{c1}(\omega_{c2} + \eta_d + \eta_{va} + \eta_{vb})) + \eta_d(\omega_{c2}(\eta_d^2 + \eta_{va}\eta_{vb} + 2\eta_d(\eta_{va} \\ & + \eta_{vb})) + \eta_d(\eta_d^2 + 3\eta_{va}\eta_{vb} + 3\eta_d(\eta_{va} + \eta_{vb})) + \\ & \omega_{c1}(\eta_d^2 + \eta_{va}\eta_{vb} + 2\eta_d(\eta_{va} + \eta_{vb}) + \omega_{c2}(\eta_d + \eta_{va} + \eta_{vb}))) \\ & + N_{vb1}\alpha_{bc}(-\alpha_{vb1}\eta_d(\omega_{c1} + 2\eta_d + \eta_{va}) + \theta_2(\eta_d(2\eta_d \\ & (\omega_{c2} + \alpha_{vb1} + \eta_d) + (3\omega_{c2} + \alpha_{vb1} + 5\eta_d)\eta_{va}) + (\omega_{c2}(3\eta_d + \eta_{va}) \\ & + \eta_d(5\eta_d + 4\eta_{va}))\eta_{vb} + \omega_{c2}(2\eta_d + \eta_{va} + \eta_{vb})))) + \\ & N_{va1}\alpha_{ac}(-\alpha_{va1}\eta_d(\omega_{c2} + 2\eta_d + \eta_{vb}) + \theta_1(\omega_{c2}\eta_d(\alpha_{va1} + 2\eta_d + 3\eta_{vb}) \\ & + \omega_{c2}(3\eta_d + \eta_{va})\eta_{vb} + \eta_d(2\eta_d(\alpha_{va1} + \eta_d) + 5\eta_d\eta_{va} + (\alpha_{va1} + 5\eta_d + 4\eta_{va})\eta_{vb}) \\ & + \omega_{c1}(2\eta_d^2 + \eta_{va}\eta_{vb} + 3\eta_d(\eta_{va} + \eta_{vb}) + \omega_{c2}(2\eta_d + \eta_{va} + \eta_{vb}))) + \\ & N_{vb1}\alpha_{bc}(-\theta_2\alpha_{va1}\eta_d + \theta_1(-\alpha_{vb1}\eta_d + \theta_2(\eta_d(\alpha_{va1} + \alpha_{vb1} \\ & + 2\eta_d + 4\eta_{va}) + 2(2\eta_d + \eta_{vb})\eta_{vb} + 2\omega_{c2}(\eta_d + \eta_{va} + \eta_{vb}) \\ & + 2\omega_{c1}(\omega_{c2} + \eta_d + \eta_{va} + \eta_{vb})))))) \end{aligned} , \tag{6.61}$$

$$
\begin{aligned}
D_4 =& \omega_{c1}\omega_{c2}\eta_d\eta_{v_a} + \omega_{c1}\eta_d^2\eta_{v_a} + \omega_{c2}\eta_d^2\eta_{v_a} + \eta_d^3\eta_{v_a} + \omega_{c1}\omega_{c2}\eta_d\eta_{v_b} \\
& + \omega_{c1}\eta_d^2\eta_{v_b} + \omega_{c2}\eta_d^2\eta_{v_b} + \eta_d^3\eta_{v_b} + \omega_{c1}\omega_{c2}\eta_{v_a}\eta_{v_b} + 2\omega_{c1}\eta_d\eta_{v_a}\eta_{v_b} \\
& + 2\omega_{c2}\eta_d\eta_{v_a}\eta_{v_b} + 3\eta_d^3\eta_{v_a}\eta_{v_b} + \frac{1}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d} \\
& N_{vb1}\alpha_{bc}((-1+\theta_2)\alpha_{vb1}\eta_d^2(\eta_d + 2\eta_{v_a}) + \theta_2(N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d) \\
& (\eta_d(\omega_{c2}+\eta_d)\eta_{v_a} + (\eta_d(\omega_{c2}+\eta_d) + (\omega_{c2}+2\eta_d)\eta_{v_a})\eta_{v_b}) \\
& + \omega_{c1}((-1+\theta_2)\alpha_{vb1}\eta_d(\eta_d+2\eta_{v_a}) + \theta_2(N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d)((\omega_{c2}+\eta_d)\eta_{v_a} + \\
& (\omega_{c2}+\eta_d+\eta_{v_a})\eta_{v_b}))) + N_{va1}\alpha_{ac}(\frac{\omega_{c2}\alpha_{va1}\eta_d^2}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d} + \frac{\theta_1\omega_{c2}\alpha_{va1}\eta_d^2}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d} - \\
& \frac{\lambda_{va1}\mu_d^3}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d} + \frac{\theta_1\lambda_{va1}\mu_d^3}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d} + \theta_1\omega_{c1}\omega_{c2}\eta_{v_a} + \theta_1\omega_{c1}\eta_d\eta_{v_a} \\
& + \theta_1\omega_{c2}\eta_d\eta_{v_a} + \theta_1\eta_d^2\eta_{v_a} + \theta_1\omega_{c1}\omega_{c2}\eta_{v_b} + \theta_1\omega_{c1}\eta_d\eta_{v_b} + \theta_1\omega_{c2}\eta_d\eta_{v_b} + \theta_1\eta_d^2\eta_{v_b} \\
& \frac{\omega_{c2}\alpha_{va1}\eta_d\eta_{v_b}}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d} + \frac{\theta_1\omega_{c2}\alpha_{va1}\eta_d\eta_{v_b}}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d} - \frac{2\alpha_{va1}\eta_d^2\eta_{v_b}}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d} + \frac{2\theta_1\alpha_{va1}\eta_d^2\eta_{v_b}}{N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d} \\
& + \theta_1\omega_{c1}\eta_{v_a}\eta_{v_b} + \theta_1\omega_{c2}\eta_{v_a}\eta_{v_b} + 2\theta_1\eta_d\eta_{v_a}\eta_{v_b} + \frac{1}{(N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d)^2}N_{vb1}\alpha_{bc}\eta_d(N_{va1}\alpha_{ac}\theta_1 \\
& (-\theta_1\alpha_{vb1}(\omega_{c1}+\eta_d+\eta_{v_a}) + \theta_2((-1+\theta_1)\omega_{c2}\alpha_{va1} - \alpha_{va1}(\eta_d+\eta_{v_b}) + \theta_1(\alpha_{vb1}(\omega_{c1}+\eta_d+\eta_{v_a}) + \alpha_{va1}(\eta_d+\eta_{v_b})))) \\
& + N_{vb1}\alpha_{bc}\theta_2(-\theta_1\alpha_{vb1}(\omega_{c1}+\eta_d+\eta_{v_a}) + \theta_2((-1+\theta_1)\omega_{c2}\alpha_{va1} - \alpha_{va1}(\eta_d+\eta_{vb1}) + \theta_1(\alpha_{vb1}(\omega_{c1}+\eta_d+\eta_{v_a}) + \alpha_{va1}(\eta_d+\eta_{v_b})))) \\
& + \eta_d(-\alpha_{vb1}(-\alpha_{va1} + \theta_1(\omega_{c1} + \alpha_{va1} + \eta_d + \eta_{v_a})) + \theta_2((-1+\theta_1)\omega_{c2}\alpha_{va1} - \alpha_{va1}(\alpha_{vb1}+\eta_d+\eta_{v_b}) + \\
& \theta_1(\omega_{c1}\alpha_{vb1} + \alpha_{vb1}(\eta_d+\eta_{v_a}) + \alpha_{va1}(\alpha_{vb1}+\eta_d+\eta_{v_b})))))
\end{aligned}
\quad , (6.62)
$$

$$
\begin{aligned}
D_5 =& \frac{1}{(N_{va1}\alpha_{ac}\theta_1 + N_{vb1}\alpha_{bc}\theta_2 + \eta_d)^2}(N_{va1}^3\alpha_{ac}^3\theta_1^3(\omega_{c1}+\eta_d)(\omega_{c2}+\mu_d)\eta_{va1}\eta_{v_b} \\
& + (N_{vb1}\alpha_{bc}\theta_2 + \eta_d)(\omega_{c1}+\eta_d)\eta_{v_a}(N_{vb1}\alpha_{bc}(-1+\theta_2)\alpha_{vb1}\eta_d^2 + (N_{vb1}\alpha_{bc}\theta_2 + \eta_d)^2(\omega_{c2}+\eta_d)\eta_{v_b}) \\
& + N_{va1}^2\alpha_{ac}^2\theta_1(N_{vb1}\alpha_{bc}\theta_1(-1+\theta_2)\alpha_{vb1}\eta_d(\omega_{c1}+\eta_d)\eta_{v_a} + (N_{vb1}\alpha_{bc}\theta_2 + \eta_d)(\omega_{c2}+\eta_d) \\
& ((-1+\theta_1)\alpha_{va1}\eta_d + 3\theta_1(\omega_{c1}+\eta_d)\eta_{v_a})\eta_{v_b}) + N_{va1}\alpha_{ac}(\eta_d^2(\omega_{c2}+\eta_d) \\
& ((-1+\theta_1)\alpha_{va1}\eta_d + 3\theta_1(\omega_{c1}+\eta_d)\eta_{v_a})\eta_{v_b}) + N_{vb1}^2\alpha_{bc}^2\theta_2(\theta_1(-1+\theta_2)\alpha_{vb1}\eta_d(\omega_{c1}+\eta_d)\eta_{v_a} \\
& + \theta_2(\omega_{c2}+\eta_d)((-1+\theta_1)\alpha_{va1}\eta_d + 3\theta_1(\omega_{c1}+\eta_d)\eta_{v_a})\eta_{v_b}) + N_{vb1}\alpha_{bc}\eta_d((-1+\theta_2)\alpha_{vb1}\eta_d + 2\theta_1(\omega_{c1}+\eta_d)\eta_{v_a}) \\
& + 2\theta_2(\omega_{c2}+\eta_d)((-1+\theta_1)\alpha_{va1}\eta_d + 3\theta_1(\omega_{c1}+\eta_d)\eta_{v_a})\eta_{v_b})))
\end{aligned}
\quad , (6.63)
$$

$$
G_1 = N_{va2}\alpha_{aa}\theta_3 + N_{vb2}\alpha_{ba}\theta_4 + \omega_{a1} + \omega_{a2} + 3\eta_d + \eta_{v_a} + \eta_{v_b} \quad , \quad (6.64)
$$

$$
\begin{aligned}
G_2 =& \omega_{a1}\omega_{a2} + 2\omega_{a1}\eta_d + 2\omega_{a2}\eta_d + 3\eta_d^2 + \omega_{a1}\eta_{v_a} + \omega_{a2}\eta_{v_a} + 3\eta_d\eta_{v_a} + \\
& (\omega_{a1} + \omega_{a2} + 3\eta_d + \eta_{v_a})\eta_{v_b} + N_{va2}\alpha_{aa}(-(\alpha_{va2}\eta_d / N_{va2}\alpha_{aa}\theta_3 + N_{vb2}\alpha_{ba}\theta_4\eta_d) \\
& + \theta_3(\omega_{a1} + \omega_{a2} + 2\eta_d + (\alpha_{va2}\eta_d / N_{va2}\alpha_{aa}\theta_3 + N_{vb2}\alpha_{ba}\theta_4\eta_d) + \eta_{v_a} + \eta_{v_b})) \\
& + N_{vb2}\alpha_{ba}(-(\alpha_{vb2}\eta_d / N_{va2}\alpha_{aa}\theta_3 + N_{vb2}\alpha_{ba}\theta_4\eta_d) + \theta_4(\omega_{a1} + \omega_{a2} + 2\eta_d + (\alpha_{vb2}\eta_d \\
& / N_{va2}\alpha_{aa}\theta_3 + N_{vb2}\alpha_{ba}\theta_4\eta_d)\eta_{v_a} + \eta_{v_b}))
\end{aligned}
\quad , \quad (6.65)
$$

$$G_3=\frac{1}{N_{va2}\,\alpha_{aa}\,\theta_3+N_{vb2}\,\alpha_{ba}\,\theta_4+\eta_d}(N_{va2}^2\,\alpha_{aa}^2\,\theta_3^2(\eta_d^2+2\eta_d\,\eta_{v_a}+2\eta_d\,\eta_{v_b}$$
$$+\eta_{va2}\,\eta_{v_b}+\omega_{a2}(\eta_d+\eta_{v_a}+\eta_{v_b})+\omega_{a1}(\omega_{a2}+\eta_d+\eta_{v_a}+\eta_{v_b}))+N_{vb2}^2\alpha_{ba}^2\theta_4^2(\eta_d^2+2\eta_d\,\eta_{v_a}$$
$$+2\eta_d\,\eta_{v_b}+\eta_{v_a}\,\eta_{v_b}+\omega_{a2}(\eta_d+\eta_{v_a}+\eta_{v_b})+\omega_{a1}(\omega_{a2}+\eta_d+\eta_{v_a}+\eta_{v_b}))+\eta_d(\omega_{a2}(\eta_d^2+\eta_{v_a}\,\eta_{v_b}$$
$$+2\eta_d(\eta_{v_a}+\eta_{v_b}))+\eta_d(\eta_d^2+3\eta_{v_a}\eta_{v_b}+3\eta_d(\eta_{v_a}+\eta_{v_b}))+\omega_{a1}(\eta_d^2+\eta_{v_a}\,\eta_{v_b}$$
$$+2\eta_d(\eta_{v_a}+\eta_{v_b})+\omega_{a2}(\eta_d+\eta_{v_a}+\eta_{v_b})))+N_{vb2}\,\alpha_{ba}(-\alpha_{vb2}\,\eta_d(\omega_{a1}+2\eta_d+\eta_{v_a})$$
$$+\theta_4(\eta_d(2\eta_d(\omega_{a2}+\alpha_{vb2}+\eta_d)+(3\omega_{a2}+\alpha_{vb2}+5\eta_d)\eta_{v_a})+(\omega_{a2}(3\eta_d+\eta_{v_a})+\eta_d(5\eta_d+4\eta_{v_a}))\eta_{v_b} \quad , (6.66)$$
$$+\omega_{a2}(2\eta_d+\eta_{v_a}+\eta_{v_b})))+N_{va2}\,\alpha_{aa}(-\alpha_{va2}\,\eta_d(\omega_{a2}+2\eta_d+\eta_{v_b})+\theta_3(\omega_{a2}\eta_d(\alpha_{va2}$$
$$+2\eta_d+3\eta_{v_a})+\omega_{a2}(3\eta_d+\eta_{v_a})\eta_{vb2}+\eta_d(2\eta_d(\alpha_{va2}+\eta_d)+5\eta_d\,\eta_{v_a}+(\alpha_{va2}+5\eta_d+4\eta_{v_a})\,\eta_{v_b})$$
$$+\omega_{a1}(2\eta_d^2+\eta_{v_a}\eta_{v_b}+3\eta_d(\eta_{v_a}+\eta_{v_b})+\omega_{a2}(2\eta_d+\eta_{v_a}+\eta_{v_b})))+N_{vb2}\,\alpha_{ba}(-\theta_4\,\alpha_{va2}\,\eta_d$$
$$+\theta_3(-\alpha_{vb2}\,\eta_d+\theta_4(\eta_d(\alpha_{va2}+\alpha_{vb2}+2\eta_d+4\eta_{v_a})+2(2\eta_d+\eta_{v_b})\eta_{v_b}+2\omega_{a2}(\eta_d+\eta_{v_a}+\eta_{v_b})$$
$$+2\omega_{a1}(\omega_{a2}+\eta_d+\eta_{va2}+\eta_{v_b}))))))$$

$$G_4=\omega_{a1}\omega_{a2}\,\eta_d\eta_{v_a}+\omega_{a1}\eta_d^2\,\eta_{v_a}+\omega_{a2}\eta_d^2\,\eta_{v_a}+\eta_d^3\,\eta_{v_a}+\omega_{a1}\omega_{a2}\eta_d\eta_{v_b}$$
$$+\omega_{a1}\eta_d^2\eta_{v_b}+\omega_{a2}\eta_d^2\eta_{v_b}+\eta_d^3\eta_{v_b}+\omega_{a1}\omega_{a2}\eta_{v_a}\eta_{v_b}+2\omega_{a1}\eta_d\,\eta_{v_a}\eta_{v_b}$$
$$+2\omega_{a2}\,\eta_d\eta_{v_a}\eta_{v_b}+3\eta_d^2\,\eta_{v_a}\eta_{v_b}+\frac{1}{N_{va2}\alpha_{aa}\theta_3+N_{vb2}\alpha_{ba}\theta_4+\eta_d}N_{vb2}\alpha_{ba}((-1+\theta_4)$$
$$\alpha_{vb2}\eta_d^2(\eta_d+2\eta_{v_a})+\theta_4(N_{va2}\alpha_{aa}\theta_3+N_{vb2}\alpha_{ba}\theta_4+\eta_d)(\eta_d(\omega_{a2}+\eta_d)\eta_{v_a}+(\eta_d(\omega_{a2}+\eta_d)$$
$$+(\omega_{a2}+2\eta_d)\eta_{v_a})\eta_{v_b})+\omega_{a1}((-1+\theta_4)\alpha_{vb2}\eta_d(\eta_d+2\eta_{v_a})+\theta_4(N_{va2}\alpha_{aa}\theta_3$$
$$+N_{vb2}\alpha_{ba}\theta_4+\eta_d)((\omega_{a2}+\eta_d)\eta_{v_a}+(\omega_{a2}+\eta_d+\eta_{v_a})\eta_{v_b})))+N_{va2}\alpha_{aa}$$
$$(\frac{\omega_{a2}\,\alpha_{va2}\eta_d^2}{N_{va2}\alpha_{aa}\theta_3+N_{vb2}\alpha_{ba}\theta_4+\eta_d}+\frac{\theta_3\omega_{a2}\,\alpha_{va2}\,\eta_d^2}{N_{va2}\,\alpha_{aa}\,\theta_3+N_{vb2}\alpha_{ba}\theta_4+\eta_d}\frac{\alpha_{va2}\,\eta_d^3}{N_{va2}\alpha_{aa}\,\theta_3+N_{vb2}\alpha_{ba}\,\theta_4+\eta_d}$$
$$+\frac{\theta_3\alpha_{va2}\eta_d^3}{N_{va2}\alpha_{aa}\theta_3+N_{vb2}\,\alpha_{ba}\,\theta_4+\eta_d}+\theta_3\,\omega_{a1}\,\omega_{a2}\,\eta_{v_a}+\theta_3\,\omega_{a1}\eta_d\eta_{v_a}+\theta_3\,\omega_{a2}\eta_d\eta_{v_a}$$
$$+\theta_3\,\eta_d^2\eta_{v_a}+\theta_3\,\omega_{a1}\omega_{a2}\eta_{v_b}+\theta_2\,\omega_{a1}\,\eta_d\eta_{v_b}+\theta_3\omega_{a2}\,\eta_d\eta_{v_b}+\theta_3\,\eta_d^2v_{v_b}\,\frac{\omega_{a2}\,\alpha_{va2}\eta_d\eta_{v_b}}{N_{va2}\alpha_{aa}\theta_3+N_{vb2}\alpha_{ba}\theta_4+\eta_d}+$$
$$\frac{\theta_3\,\omega_{a2}\alpha_{va2}\eta_d\,\eta_{v_b}}{N_{va2}\,\alpha_{aa}\,\theta_3+N_{vb2}\,\alpha_{ba}\theta_4+\eta_d}\frac{2\alpha_{va2}\eta_d^2\eta_{v_b}}{N_{va2}\alpha_{aa}\theta_3+N_{vb2}\alpha_{ba}\theta_4+\eta_d}+\frac{2\theta_3\,\alpha_{va2}\eta_d^2\eta_{v_b}}{N_{va2}\,\alpha_{aa}\theta_3+N_{vb2}\,\alpha_{ba}\theta_4+\eta_d}$$
$$+\theta_3\omega_{a1}\eta_{v_a}\,\eta_{v_b}+\theta_3\,\omega_{a2}\eta_{v_a}\eta_{v_b}+2\theta_3\eta_d\,\eta_{v_a}\eta_{v_b}+\frac{1}{(N_{va2}\,\alpha_{aa}\,\theta_3+N_{vb2}\alpha_{ba}\theta_4+\eta_d)^2}$$
$$N_{vb2}\alpha_{ba}\eta_d(N_{va2}\alpha_{aa}\theta_3(-\theta_3\,\alpha_{vb2}(\omega_{a1}+\eta_d+\eta_{v_a})+\theta_4((-1+\theta_3)\omega_{a2}\alpha_{va2}-\alpha_{va2}(\eta_d+\eta_{v_b}) \quad , (6.67)$$
$$+\theta_3(\alpha_{vb2}(\omega_{a1}+\eta_d+\eta_{v_a})+\alpha_{va2}(\eta_d+\eta_{v_b}))))+N_{vb2}\,\alpha_{ba}\theta_4(-\theta_3\,\alpha_{vb2}(\omega_{a1}+\eta_d+\eta_{v_a})+\theta_4((-1+\theta_3)\omega_{a2}\alpha_{va2}$$
$$-\alpha_{va2}(\eta_d+\eta_{v_b})+\theta_3(\alpha_{vb2}(\omega_{a1}+\eta_d+\eta_{v_a})+\alpha_{va2}(\eta_d+\eta_{v_b}))))+\eta_d(-\alpha_{vb2}(-\alpha_{va2}+\theta_3(\omega_{a1}+\alpha_{va2}$$
$$+\eta_d+\eta_{v_a}))+\theta_4((-1+\theta_3)\omega_{a2}\alpha_{va2}-\alpha_{va2}(\alpha_{vb2}+\eta_d+\eta_{v_b})+\theta_3(\omega_{a1}\,\alpha_{vb2}+\alpha_{vb2}(\eta_d+\eta_{v_a})+\alpha_{va2}(\alpha_{vb2}+\eta_d+\eta_{v_b})))))$$

$$G_5=\frac{1}{(N_{va2}\,\alpha_{aa}\theta_3+N_{vb2}\,\alpha_{ba}\,\theta_4+\eta_d)^2}(N_{va2}^3\,\alpha_{aa}^3\theta_3^3(\omega_{a1}+\eta_d)(\omega_{a2}+\eta_d)\,\eta_{v_a}\eta_{v_b}+(N_{vb2}\,\alpha_{ba}\,\theta_4$$
$$+\eta_d)(\omega_{a1}+\eta_d)\eta_{v_a}(N_{vb2}\,\alpha_{ba}(-1+\theta_4)\,\alpha_{vb2}\eta_d^2+(N_{vb2}\,\alpha_{ba}\,\theta_4+\eta_d)^2(\omega_{a2}+\eta_d)\,\eta_{v_b})$$
$$+N_{va2}^2\,\alpha_{aa}^2\,\theta_3(N_{vb2}\,\alpha_{ba}\,\theta_3(-1+\theta_4)\,\alpha_{vb2}\eta_d(\omega_{a1}+\eta_d)\,\eta_{v_a}+(N_{vb2}\,\alpha_{ba}\,\theta_4+\eta_d)$$
$$(\omega_{a2}+\eta_d)((-1+\theta_3)\,\alpha_{va2}\,\eta_d+3\theta_3(\omega_{a1}+\eta_d)\eta_{v_a})\eta_{v_b})+N_{va2}\,\alpha_{aa}(\eta_d^2(\omega_{a2}+\eta_d) \quad , (6.68)$$
$$((-1+\theta_3)\,\alpha_{va2}\,\eta_d+3\theta_3(\omega_{a1}+\eta_d)\eta_{v_a})\,\eta_{v_b})+N_{vb2}^2\alpha_{ba}^2\,\theta_4\,(\theta_3(-1+\theta_4)\,\alpha_{vb2}\,\eta_d\,(\omega_{a1}+\eta_d)\,\eta_{v_a}$$
$$+\theta_4(\omega_{a2}+\eta_d)((-1+\theta_3)\,\alpha_{va2}\,\eta_d+3\theta_3(\omega_{a1}+\eta_d)\,\eta_{v_a})\eta_{v_b})+N_{vb2}\,\alpha_{ba}\eta_d((-1+\theta_4)\,\alpha_{vb2}\,\eta_d$$
$$+2\theta_3(\omega_{a1}+\eta_d)\eta_{v_a})+2\theta_4(\omega_{a2}+\eta_d)((-1+\theta_3)\,\alpha_{va2}\eta_d+3\theta_3\,(\omega_{a1}+\eta_d)\,\eta_{v_a})\eta_{v_b})))$$

where

$$\theta_1=\frac{(N_{tc}N_{va1}\alpha_{ac}\alpha_{va1}\alpha_{vb1}\eta_d-\alpha_{vb1}(N_{vb1}\alpha_{bc}+\eta_d)(\omega_{c1}+\eta_d)\eta_{v_a}+N_{va1}\alpha_{ac}\alpha_{va1}(\omega_{c2}+\eta_d)\eta_{v_b})}{(N_{va1}\alpha_{ac}(N_{tc}\alpha_{va1}\alpha_{vb1}\eta_d+\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{v_a}+\alpha_{va1}(\omega_{c2}+\eta_d)\eta_{v_b}))} \quad , (6.69)$$

$$\theta_2=\frac{N_{tc}N_{vb1}\alpha_{bc}\alpha_{vb1}\eta_d-(N_{va1}\alpha_{ac}\theta_1+\mu_d)(\omega_{c2}+\eta_d)\eta_{v_b}}{N_{vb1}\alpha_{bc}(N_{tc}\alpha_{vb1}\eta_d+(\omega_{c2}+\eta_d)\eta_{v_b})} \quad , (6.70)$$

$$\theta_3=\frac{(N_{ta}N_{va2}\alpha_{aa}\alpha_{va2}\alpha_{vb2}\eta_d-\alpha_{vb2}(N_{vb2}\alpha_{ba}+\eta_d)(\omega_{a1}+\eta_d)\eta_{v_a}+N_{va2}\alpha_{aa}\alpha_{va2}(\omega_{a2}+\eta_d)\eta_{v_b})}{(N_{va2}\alpha_{aa}(N_{ta}\alpha_{va2}\alpha_{vb2}\eta_d+\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{v_a}+\alpha_{va2}(\omega_{a2}+\eta_d)\eta_{v_b}))} \quad ,(6.71)$$

$$\theta_4=\frac{N_{ta}N_{vb2}\alpha_{ba}\alpha_{vb2}\eta_d-(N_{va2}\alpha_{aa}\theta_3+\mu_d)(\omega_{a2}+\eta_d)\eta_{v_b}}{N_{vb2}\alpha_{ba}(N_{ta}\alpha_{vb2}\eta_d+(\omega_{a2}+\eta_d)\eta_{v_b})} \quad . (6.72)$$

From the characteristic equation (6.57) – (6.58), the eigenvalues are found by solving $(\lambda^5+D_1\lambda^4+D_2\lambda^3+D_3\lambda^2+D_4\lambda+D_5)=0$ for children and for adult $(\lambda^5+G_1\lambda^4+G_2\lambda^3+G_3\lambda^2+G_4\lambda+G_5)=0$,when $T_1=D_1$ and G_1 for children and adult, $T_2=D_2$ and G_2 , for children and adult, $T_3=D_3$ and G_3 for children and adult $T_4=D_4$ and G_4 for children and adult, $T_5=D_5$ and G_5 for children and adult. The five eigenvalues have negative real parts if they satisfy Routh-Hurwitz criteria (6.73) - (6.77)[49], each equilibrium state is locally asymptotically stable, when it satisfies the following conditions.

i) $\det H_1 = T_1 > 0$ (6.73)

ii) $\det H_2 = T_1T_2 - T_3 > 0$ (6.74)

iii) $\det H_3 = T_1T_2T_3 - T_3^2 - T_1^2T_4 > 0$ (6.75)

iv) $\det H_4 = T_1T_2T_3T_4 - T_3^2T_4 - T_1^2T_4^2 > 0$ (6.76)

v) $\det H_5 = T_1T_2T_3T_4T_5 - T_3^2T_4T_5 - T_1^2T_4^2T_5 - T_1T_2^2T_5^2 + T_2T_3T_5^2 + 2T_1T_4T_5^2 - T_5^3 > 0$ (6.77)

We check the stability condition of endemic equilibrium state by using the Routh-Hurwitz conditions (6.73) – (6.74), the results are given in figure 6.3 and figure 6.4.

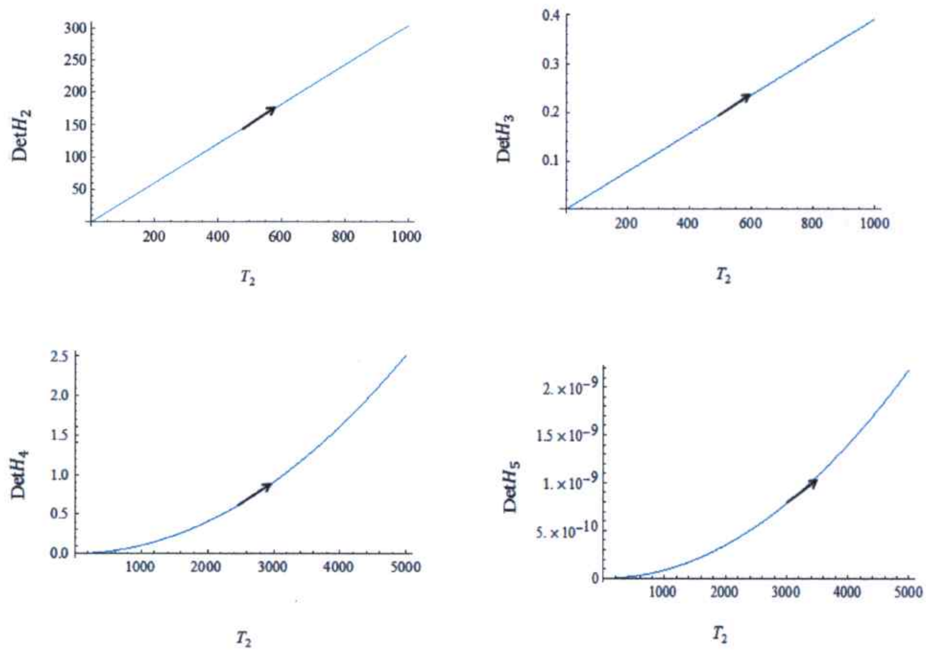


Figure 6.3. The parameter spaces for endemic disease equilibrium state, which satisfies the Routh-Hurwitz conditions, plotted onto
$(T_2, \det H_2)$, $(T_2, \det H_3)$, $(T_2, \det H_4)$ and $(T_2, \det H_5)$, respectively.
The values of parameter are follows:

$\omega_{c1} = 1/(17/2)$, $\omega_{c2} = 1/(19/2)$, $\eta_d = 1/(365*74.6)$ day^{-1}, $N_{tc} = 6000$, $N_{va1} = 5000$, $N_{vb1} = 2500$, $\alpha_{ac} = 0.2$, $\alpha_{bc} = 0.0714$, $\alpha_{va1} = 0.00000000576$, $\alpha_{vb1} = 0.00000435$, $\gamma_a = 0.08$, $\gamma_b = 0.047$ and $N_t = 100{,}000$.

From the above figure 6.3, the Routh-Hurwitz conditions are satisfied for $E_0 > 1$.

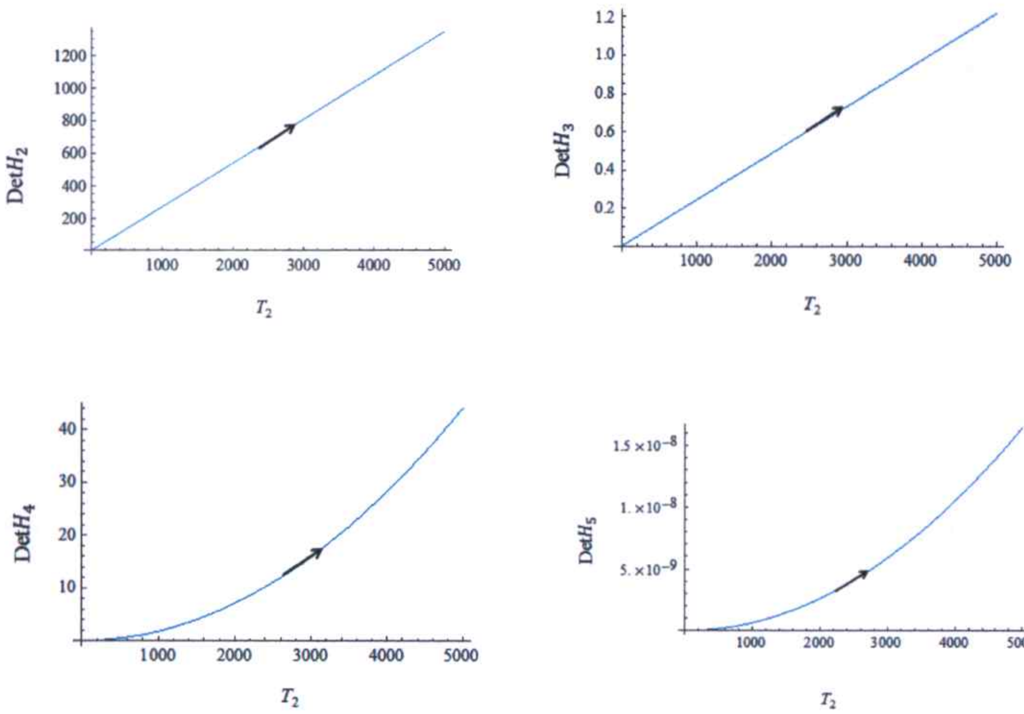


Figure 6.4. The parameter spaces for endemic disease equilibrium state, which satisfies the Routh - Hurwitz conditions, plotted onto $(T_2, \det H_2)$, $(T_2, \det H_3)$, $(T_2, \det H_4)$ and $(T_2, \det H_5)$, respectively.
The values of parameter are follows:

$\omega_{a1} = 1/(19/2)$, $\omega_{a2} = 1/(21/2)$, $\eta_d = 1/(365 * 74.6)$ day^{-1}, $N_{ta} = 4000$, $N_{va2} = 7000$, $N_{vb2} = 4300$, $\alpha_{aa} = 0.1667$, $\alpha_{ba} = 0.125$, $\alpha_{va2} = 0.00000000176$, $\alpha_{vb2} = 0.000000835$, $\gamma_a = 0.07$, $\gamma_b = 0.027$, and $N_t = 100000$.

From the above figures 6.4, the Routh-Hurwitz conditions are satisfies for $E_0 > 1$.

6.5 Numerical results

We consider the numerical solutions for dengue virus transmission. The main effect of introducing an age structure into the model is to change the definition of the basic reproductive rate. The parameters in this study are determined by the real life observations. The values of the parameters are as follows: $\eta_d = 1/(365*74.6)$ day^{-1}, corresponding to life expectancy of 74.6 years for human; $\omega_{c1} = 1/(8.5)$ and $\omega_{c2} = 1/(9.5)$ corresponding to the 8.5 days and 9.5 days of recovering due to biting of *Aedes aegypti* and *Aedes albopictus*, respectively. The death rate of mosquitoes are 1 /28 per day and 1/35 per day satisfies to the life time of 28 days for *Aedes aegypti* and the life time of 35 days for *Aedes albopictus* ,respectively $\omega_{a1} = 1/(9.5)$ and $\omega_{a2} = 1/(10.5)$ corresponding to the 9.5 days and 10.5 days of recovering of adult human population due to biting of *Aedes aegypti* and *Aedes albopictus*, respectively. The other parameters are arbitrary chosen. Numerical solutions of (6.21) – (6.30) are shown in the following figures.

Case 6.5.1, in children, we consider the locally asymptotically stable of disease free equilibrium point, when $\beta = 0$:

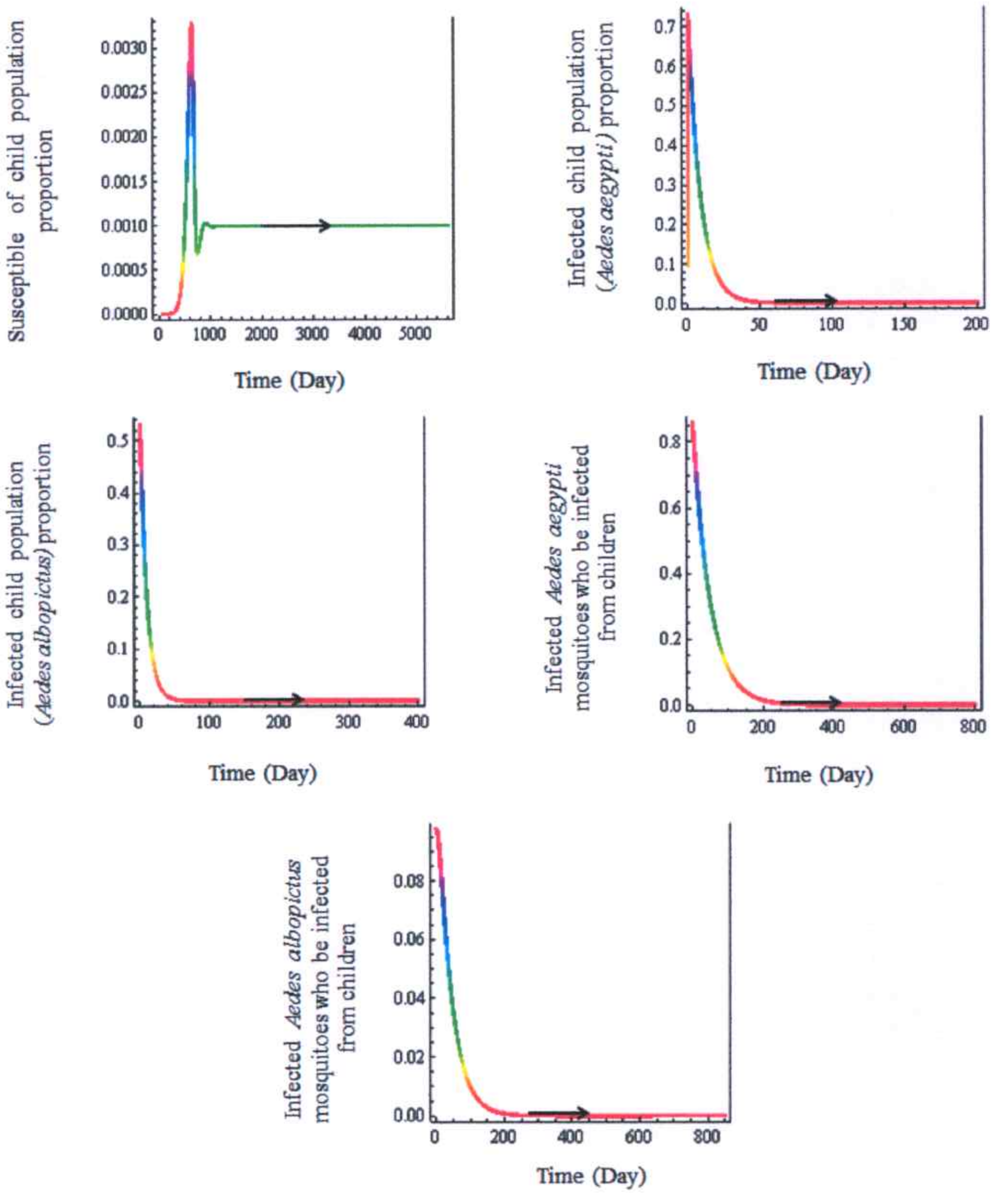


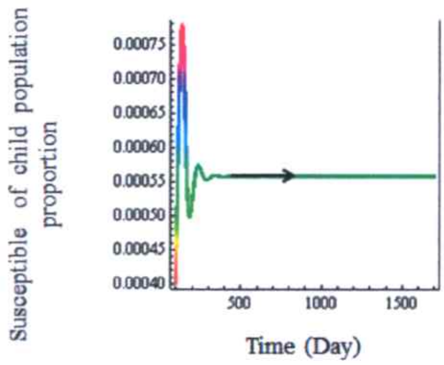
Figure 6.5. Time series solutions of $S_c, I_{c1}, I_{c2}, I_{va1}$ and I_{vb1} respectively.

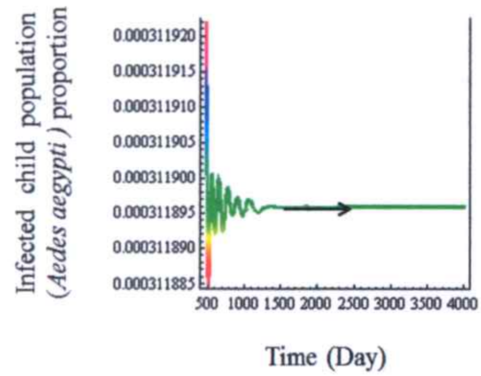
For $E_0 < 1$,and $E_{0c} = 0.000023944$ with following parameters:

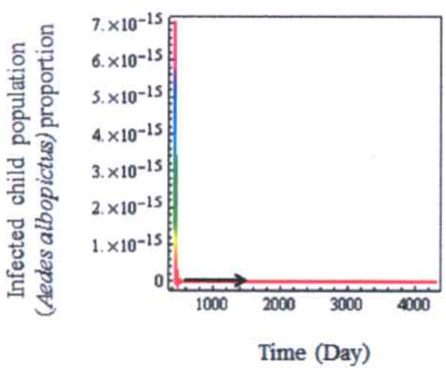
$\eta_{va1} = 1/49$, $\eta_{vb1} = 1/39$, $N_{tc} = 71000$, $N_{va1} = 5800$, $N_{vb1} = 10000$, $\alpha_{ac} = 0.0239$, $\alpha_{bc} = 0.0333$, $\alpha_{va1} = 0.00000000000347$, $\alpha_{vb1} = 0.0000000675$, $\gamma_a = 0.07$, $\gamma_b = 0.027$,and $N_t = 100{,}000$.

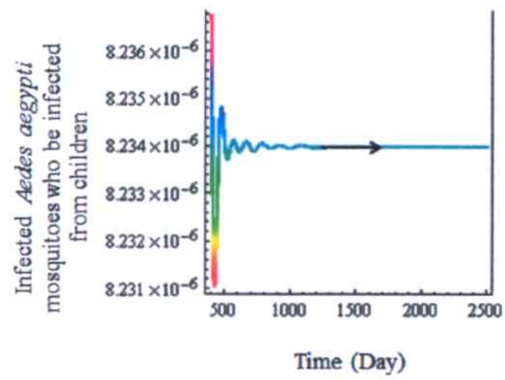
The proportions of populations $(S'_c, I'_{c1}, I'_{c2}, I'_{va1}, I'_{vb1})$ approach to the disease free equilibrium point $(1,0,0,0,0)$.

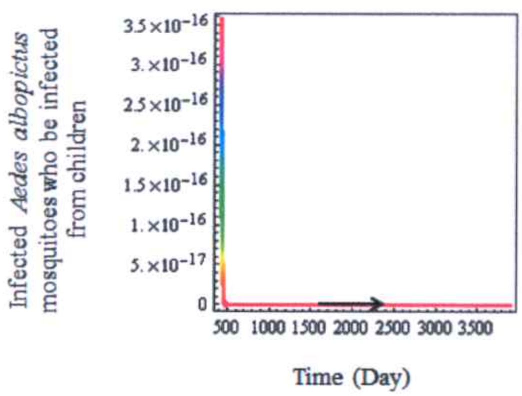
Case 6.5.2, in children, we consider the locally asymptotically stable of endemic equilibrium point, when $\beta = 0$:











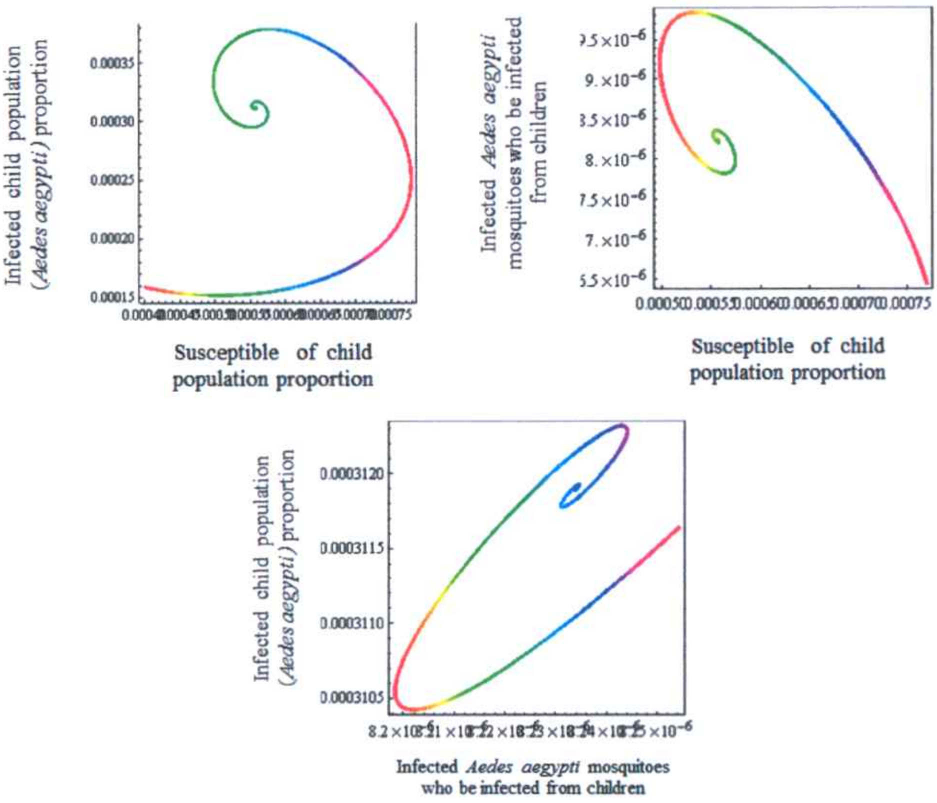


Figure 6.6. 6.6a)Time series solutions of S_c, I_{c1} , I_{c2}, I_{va1}, I_{vb1}. Values of parameters in the model are following:

$\eta_{va1} = 1/7$, $\eta_{vb1} = 1/14$, $N_{tc} = 50000$, $N_{va1} = 32000$, $N_{vb1} = 17000$, $\alpha_{ac} = 0.2$, $\alpha_{bc} = 0.125$, $\alpha_{va1} = 0.0000000058$, $\alpha_{vb1} = 0.00000000465$, $\gamma_a = 0.026$, $\gamma_b = 0.009$, and $N_t = 100{,}000$, when $E_{0c} = 174.473$

6.6b) Numerical solutions projected onto $(S_c^{'}, I_{c1}^{'})$, $(S_c^{'}, I_{va1}^{'})$, $(I_{va1}^{'}, I_{c1}^{'})$.

The solutions oscillate to the endemic equilibrium point $(S_c^*, I_{c1}^*, I_{c2}^*, I_{va1}^*, I_{vb1}^*)$ where $S_c^* = 0.000556913$, $I_{c1}^* = 0.0003$, $I_{c2}^* = 1.6622\times10^{-14}$, $I_{va1}^* = 8.23484\times10^{-6}$ and $I_{vb1}^* = 7.38289\times10^{-17}$,respectively.

Case 6.6.1, in adult, we consider the locally asymptotically stable of disease free equilibrium point, when $\beta = 0$:

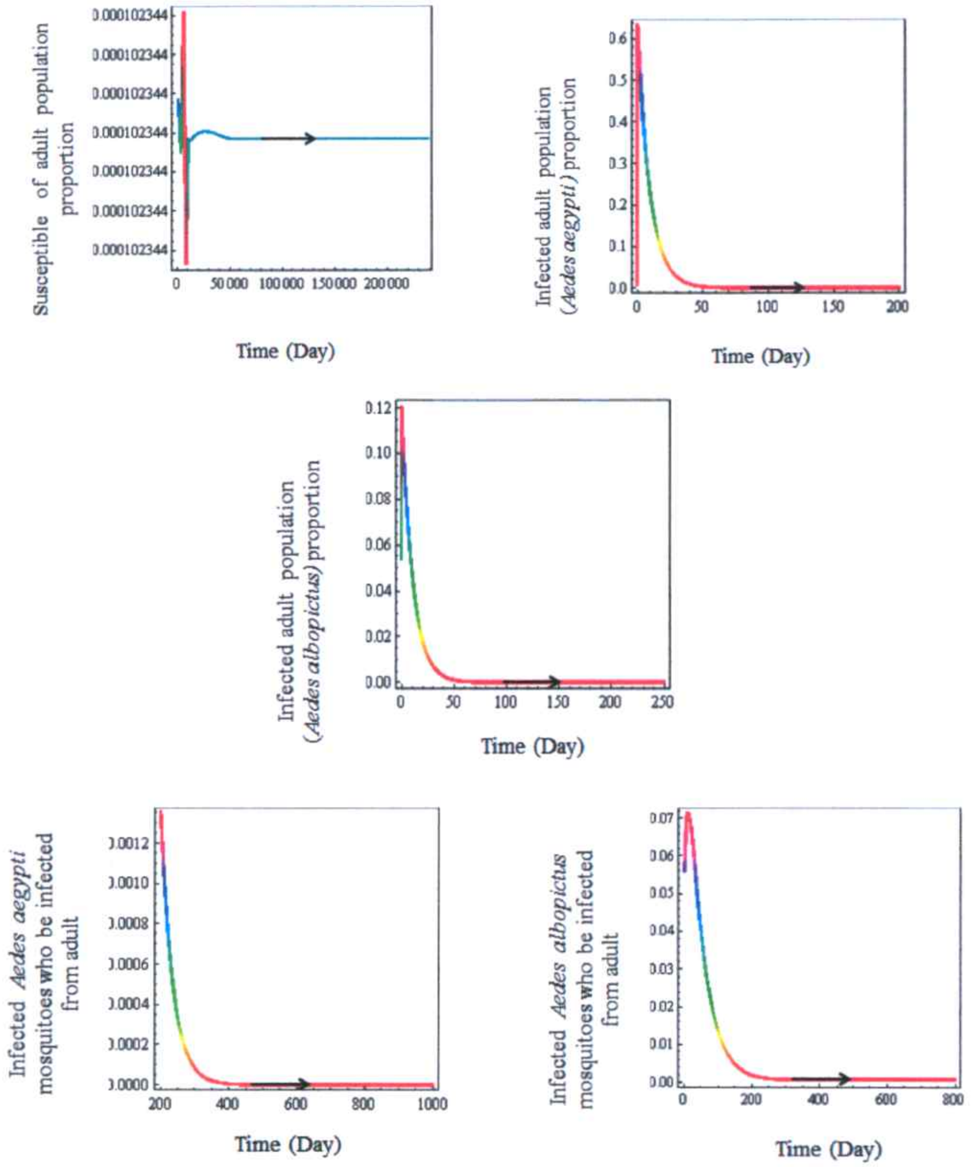
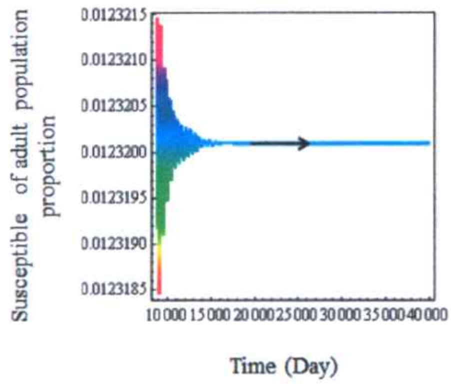


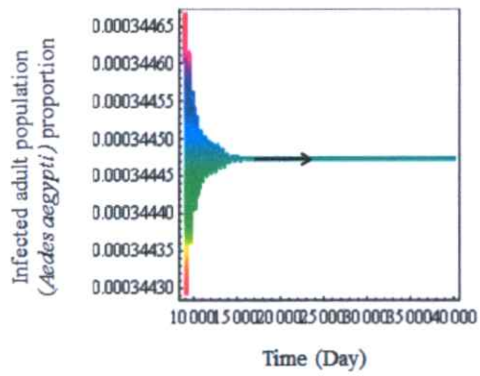
Figure 6.7. Time series solution of $S_a, I_{a1}, I_{a2}, I_{va2}, I_{vb2}$, respectively. For $E_0 < 1$, and $E_{0a} = 0.0307919$ with parameters are following :

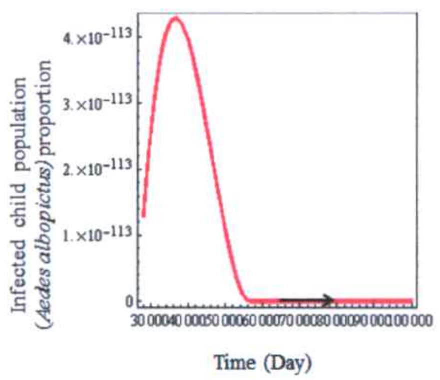
$\eta_{va2} = 1/36$, $\eta_{vb2} = 1/46$, $N_{ta} = 61000$, $N_{va2} = 4800$, $N_{vb2} = 10000$, $\alpha_{aa} = 0.03225$, $\alpha_{ba} = 0.02941$, $\alpha_{va2} = 0.000000000076$, $\alpha_{vb2} = 0.000000000664$, $\gamma_a = 0.04$, $\gamma_b = 0.06$, and $N_t = 100{,}000$.

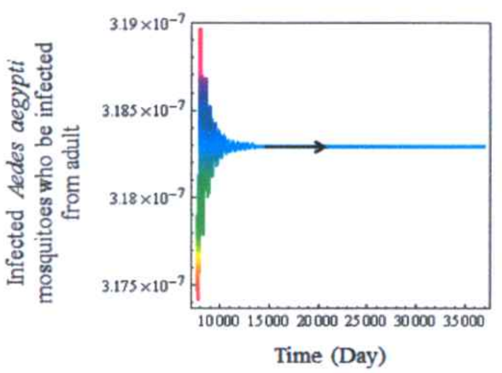
The proportions of populations $(S'_a, I'_{a1}, I'_{a2}, I'_{va2}, I'_{vb2})$ approach to the disease free equilibrium state $(1, 0, 0, 0, 0)$.

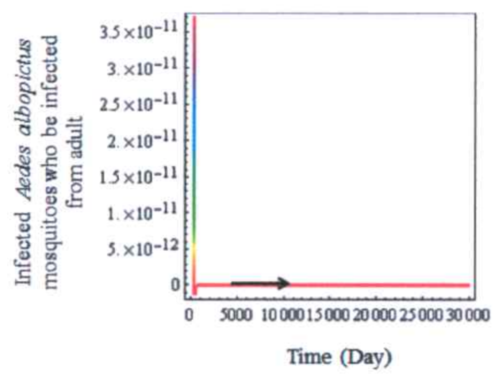
Case 6.6.2, in adult, we consider the locally asymptotically stable of endemic equilibrium point, when $\beta = 0$:











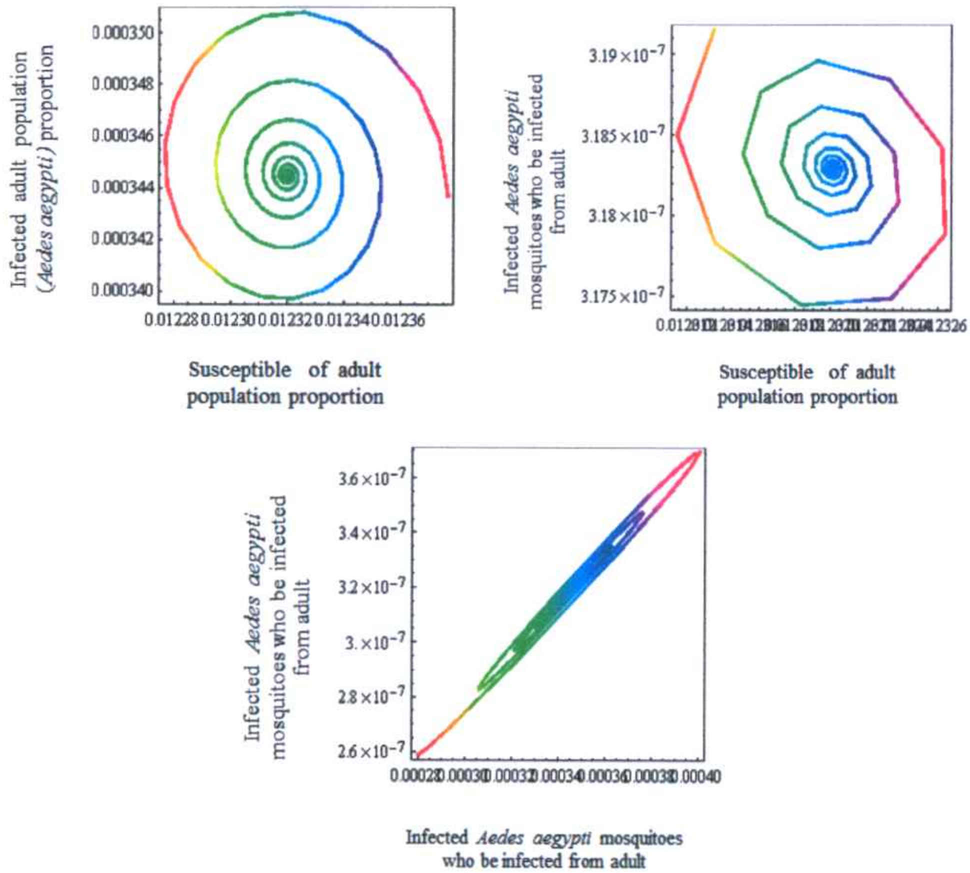


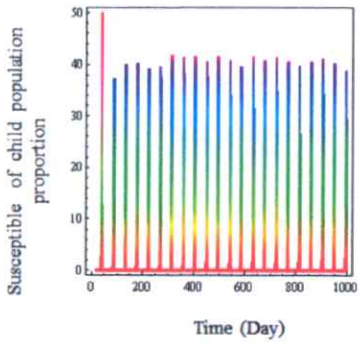
Figure 6.8. 6.8 a)Time series solutions of S_a, I_{a1} , I_{a2}, I_{va2}, I_{vb2} . Values of parameters in the model are following:

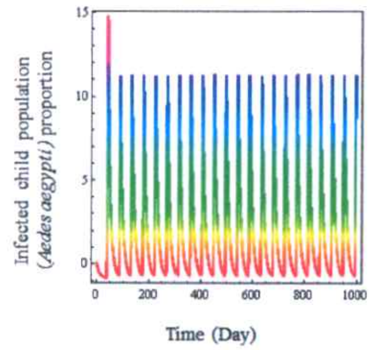
η_{va2} =1/7 , η_{vb2} =1/13 , N_{ta} =30000 , N_{va2} =37000 , N_{vb2} =19000 , α_{aa} =0.25 , α_{ba} = 0.1428 , α_{va2} = 0.0000000044 , α_{vb2} = 0.000000000335 , γ_a =0.02 , γ_b =0.07 ,and N_t = 100,000 , where E_{0a} =21.7785 in adult.

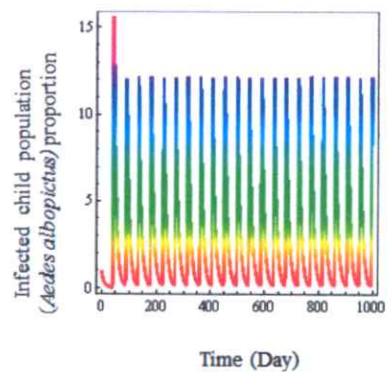
6.8 b) Numerical solutions projected onto $(S_a^{'}, I_{a1}^{'})$, $(S_a^{'}, I_{va2}^{'})$, $(I_{va2}^{'}, I_{a1}^{'})$.

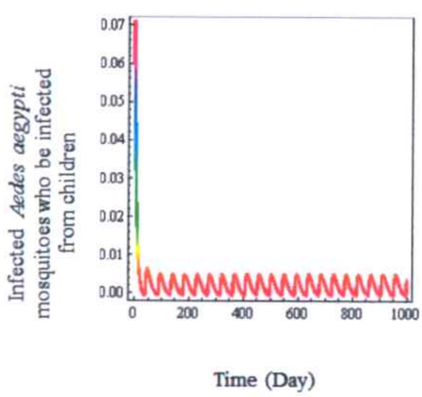
The solutions oscillate to the endemic equilibrium point $(S_a^*, I_{a1}^*, I_{a2}^*, I_{va2}^*, I_{vb2}^*)$ where S_c^* = 0.0123201 , I_{c1}^* = 0.000344474 , I_{c2}^* = 6.9961×10^{-14} , I_{va1}^* = 3.18294×10^{-7} and I_{vb1}^* = 1.40069×10^{-16} ,respectively.

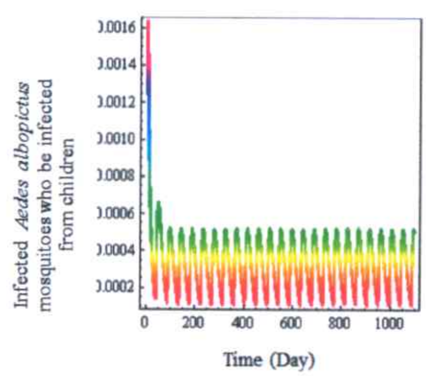
Case 6.7, in children, when $\beta \neq 0$:

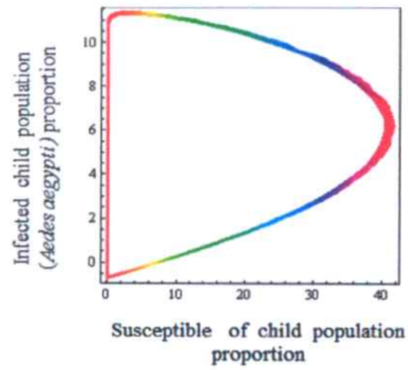


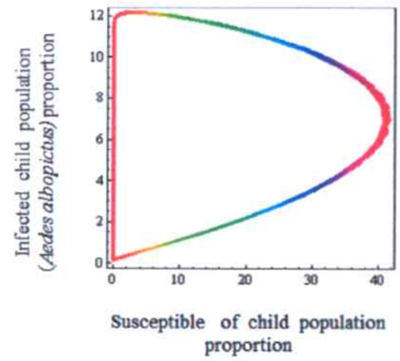


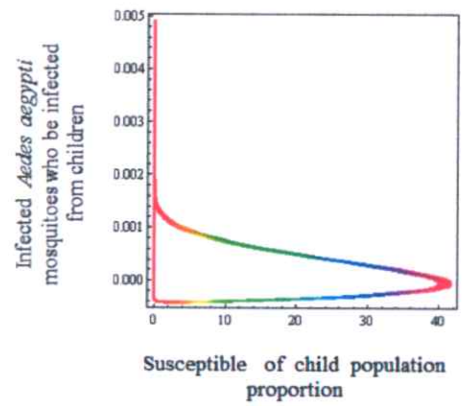


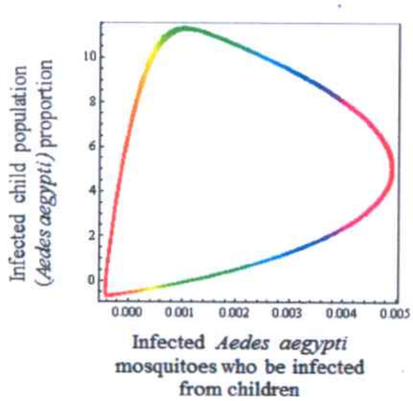


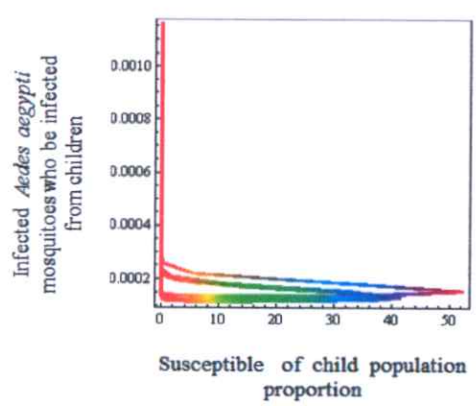



Infected child population (Aedes aegypti) proportion
Susceptible of child population proportion


Infected child population (Aedes albopictus) proportion
Susceptible of child population proportion


Infected Aedes aegypti mosquitoes who be infected from children
Susceptible of child population proportion


Infected child population (Aedes aegypti) proportion
Infected Aedes aegypti mosquitoes who be infected from children


Infected Aedes aegypti mosquitoes who be infected from children
Susceptible of child population proportion

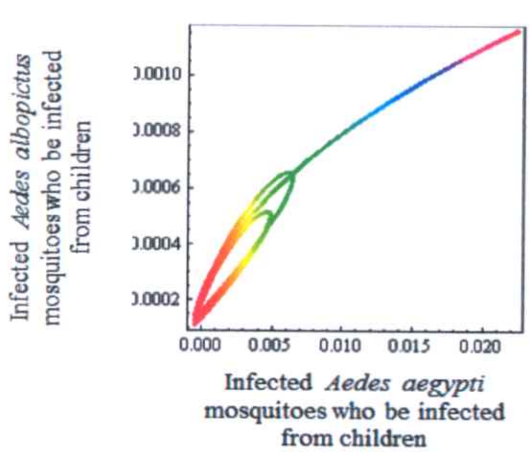
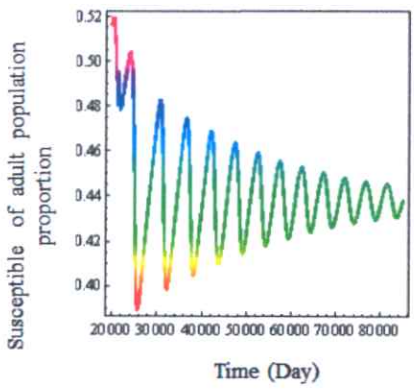
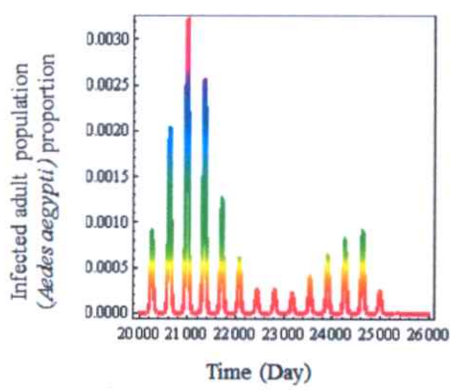

Infected Aedes albopictus mosquitoes who be infected from children
Infected Aedes aegypti mosquitoes who be infected from children

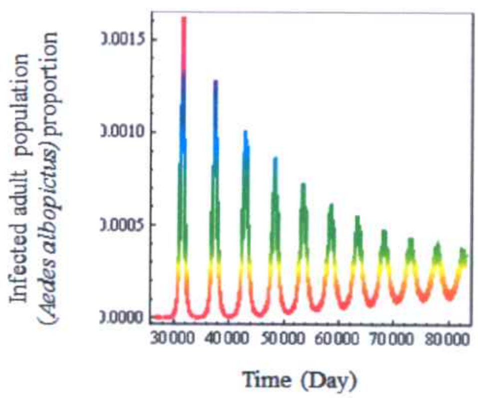
Figure 6.9. 6.9a) Time series solutions of $S_c, I_{c1}, I_{c2}, I_{va1}, I_{vb1}$. Values of parameters in the model are following :
$N_{tc} = 50000$, $N_{va1} = 32000$, $N_{vb1} = 17000$, $\alpha_{ac} = 0.2$, $\alpha_{bc} = 0.125$, $\alpha_{va1} = 0.0000000028$,
$\alpha_{vb1} = 0.000000000165$, $\gamma_a = 0.005$, $\gamma_b = 0.004$,and $N_t = 100{,}000$,and $E_{0c} = 22.8627$.

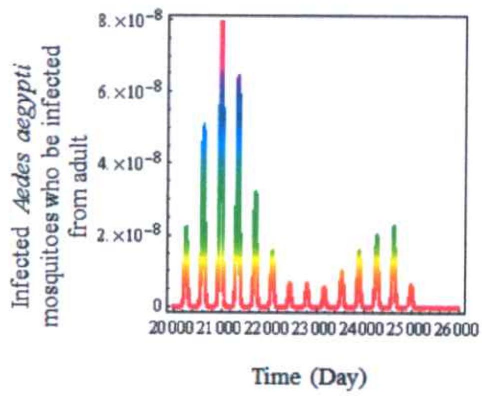
6.9b) Numerical solutions projected onto
$(S_c^{'}, I_{c1}^{'})$, $(S_c^{'}, I_{c2}^{'})$, $(S_c^{'}, I_{va1}^{'})$, $(S_c^{'}, I_{vb1}^{'})$, $(I_{va1}^{'}, I_{c1}^{'})$, $(I_{va1}^{'}, I_{vb1}^{'})$. Limit cycles are produced in this case.

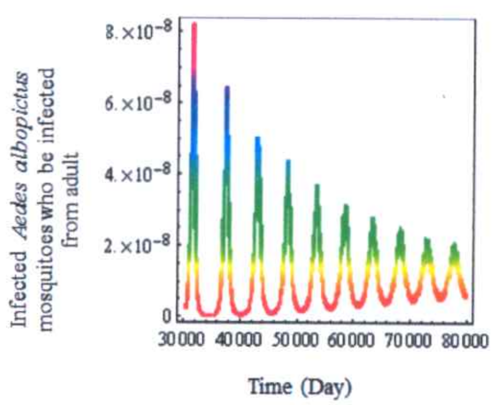
Case 6.8, in adult, when $\beta \neq 0$:











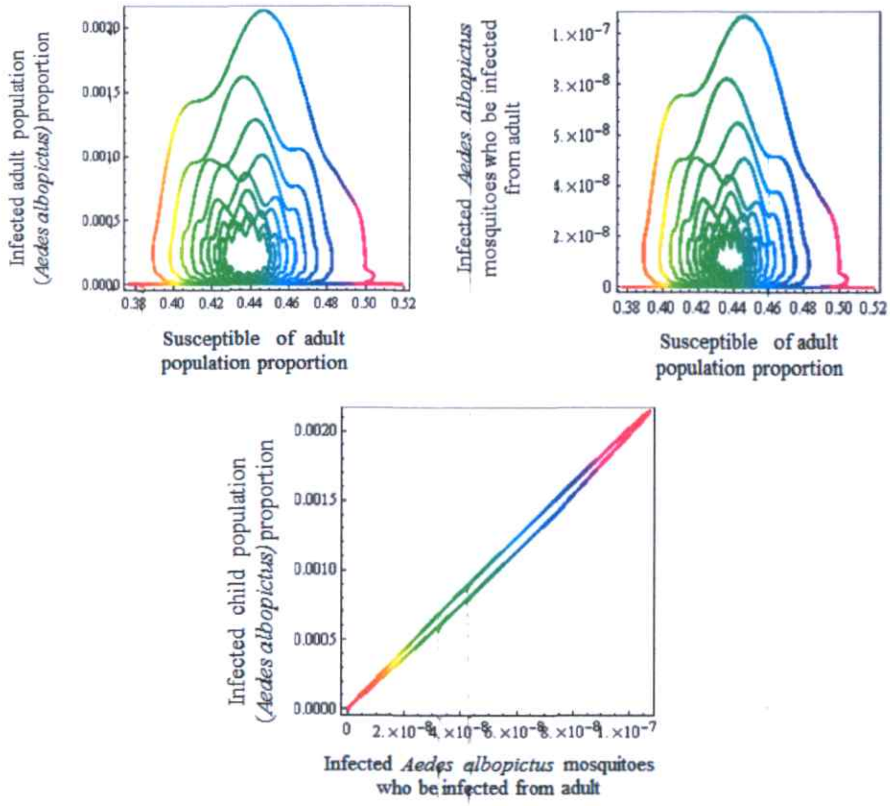


Figure 6.10. 6.10a) Time series solutions of $S_a, I_{a1}, I_{a2}, I_{va2}, I_{vb2}$. Values of parameters in the model are following:

$N_{ta}=50000$, $N_{va2}=34000$, $N_{vb2}=30000$, $\alpha_{aa}=0.25$, $\alpha_{ba}=0.1428$, $\alpha_{va2}=0.0000000075$, $\alpha_{vb2}=0.000000000625$, $\gamma_a=0.02$, $\gamma_b=0.07$, and $N_t=100{,}000$, when $E_{0a}=9.26764$.

6.10b) Numerical solutions projected onto $(S_a^{'}, I_{a2}^{'})$, $(S_a^{'}, I_{vb2}^{'})$, $(I_{vb2}^{'}, I_{a2}^{'})$. The solutions oscillate to the endemic equilibrium point $(S_a^*, I_{a1}^*, I_{a2}^*, I_{va2}^*, I_{vb2}^*)$ are limit cycles.

We present the numerical results from solving equation (6.21) – (6.30). Here , we will discuss the all of figures.

In figure 6.5, we present the time series solutions of $S_c, I_{c1}, I_{c2}, I_{va1}$ and I_{vb1} respectively. For $E_0 < 1$,and $E_{0c} = 0.000023944$ with following parameters:

$\eta_{va1} = 1/49$, $\eta_{vb1} = 1/39$, $N_{tc} = 71000$, $N_{va1} = 5800$, $N_{vb1} = 10000$, $\alpha_{ac} = 0.0239$, $\alpha_{bc} = 0.0333$, $\alpha_{va1} = 0.00000000000347$, $\alpha_{vb1} = 0.0000000675$, $\gamma_a = 0.07$, $\gamma_b = 0.027$,and $N_t = 100{,}000$.

The proportions of populations $(S_c', I_{c1}', I_{c2}', I_{va1}', I_{vb1}')$ approach to the disease free equilibrium point $(1, 0, 0, 0, 0)$.

In figure 6.6, we present the time series solutions of $S_c, I_{c1}, I_{c2}, I_{va1}, I_{vb1}$. Values of parameters in the model are following: $\eta_{va1} = 1/7$, $\eta_{vb1} = 1/14$, $N_{tc} = 50000$, $N_{va1} = 32000$, $N_{vb1} = 17000$, $\alpha_{ac} = 0.2$, $\alpha_{bc} = 0.125$, $\alpha_{va1} = 0.0000000058$, $\alpha_{vb1} = 0.00000000465$, $\gamma_a = 0.026$, $\gamma_b = 0.009$,and $N_t = 100{,}000$, when $E_{0c} = 174.473$.

The Numerical solutions projected onto (S_c', I_{c1}') , (S_c', I_{va1}') , (I_{va1}', I_{c1}') . The solutions oscillate to the endemic equilibrium point $(S_c^*, I_{c1}^*, I_{c2}^*, I_{va1}^*, I_{vb1}^*)$ where $S_c^* = 0.000556913$, $I_{c1}^* = 0.0003$, $I_{c2}^* = 1.6622 \times 10^{-14}$, $I_{va1}^* = 8.23484 \times 10^{-6}$ and $I_{vb1}^* = 7.38289 \times 10^{-17}$,respectively.

In figure 6.7, we present the time series solution of $S_a, I_{a1}, I_{a2}, I_{va2}, I_{vb2}$, respectively. For $E_0 < 1$, and $E_{0a} = 0.0307919$ with parameters are following :

$\eta_{va2} = 1/36$, $\eta_{vb2} = 1/46$, $N_{ta} = 61000$, $N_{va2} = 4800$, $N_{vb2} = 10000$, $\alpha_{aa} = 0.03225$, $\alpha_{ba} = 0.02941$, $\alpha_{va2} = 0.000000000076$, $\alpha_{vb2} = 0.000000000664$, $\gamma_a = 0.04$, $\gamma_b = 0.06$,and $N_t = 100{,}000$.

The proportions of populations $(S_a', I_{a1}', I_{a2}', I_{va2}', I_{vb2}')$ approach to the disease free equilibrium state $(1, 0, 0, 0, 0)$.

In figure 6.8, we present the time series solutions of $S_a, I_{a1}, I_{a2}, I_{va2}, I_{vb2}$. Values of parameters in the model are following: $\eta_{va2} = 1/7$, $\eta_{vb2} = 1/13$, $N_{ta} = 30000$, $N_{va2} = 37000$, $N_{vb2} = 19000$, $\alpha_{aa} = 0.25$, $\alpha_{ba} = 0.1428$, $\alpha_{va2} = 0.0000000044$, $\alpha_{vb2} = 0.000000000335$, $\gamma_a = 0.02$, $\gamma_b = 0.07$,and $N_t = 100{,}000$, where $E_{0a} = 21.7785$ in adult. The Numerical solutions projected onto (S_a', I_{a1}') , (S_a', I_{va2}') , (I_{va2}', I_{a1}') .The solutions oscillate to the endemic equilibrium point $(S_a^*, I_{a1}^*, I_{a2}^*, I_{va2}^*, I_{vb2}^*)$ where $S_c^* = 0.0123201$, $I_{c1}^* = 0.000344474$, $I_{c2}^* = 6.9961 \times 10^{-14}$, $I_{va1}^* = 3.18294 \times 10^{-7}$ and $I_{vb1}^* = 1.40069 \times 10^{-16}$, respectively.

In figure 6.9, we present the time series solutions of $S_c, I_{c1}, I_{c2}, I_{va1}, I_{vb1}$. Values of parameters in the model are following: $N_{tc} = 50000$, $N_{va1} = 32000$, $N_{vb1} = 17000$, $\alpha_{ac} = 0.2$, $\alpha_{bc} = 0.125$, $\alpha_{va1} = 0.0000000028$, $\alpha_{vb1} = 0.000000000165$, $\gamma_a = 0.005$, $\gamma_b = 0.004$,and $N_t = 100{,}000$,and $E_{0c} = 22.8627$.

The Numerical solutions projected onto $(S_c^{'}, I_{c1}^{'})$, $(S_c^{'}, I_{c2}^{'})$, $(S_c^{'}, I_{va1}^{'})$, $(S_c^{'}, I_{vb1}^{'})$, $(I_{va1}^{'}, I_{c1}^{'})$, $(I_{va1}^{'}, I_{vb1}^{'})$.Limit cycles are produced in this case.

In figure 6.10, we present the time series solutions of $S_a, I_{a1}, I_{a2}, I_{va2}, I_{vb2}$. Values of parameters in the model are following:

$N_{ta} = 50000$, $N_{va2} = 34000$, $N_{vb2} = 30000$, $\alpha_{aa} = 0.25$, $\alpha_{ba} = 0.1428$, $\alpha_{va2} = 0.0000000075$, $\alpha_{vb2} = 0.000000000625$, $\gamma_a = 0.02$, $\gamma_b = 0.07$, and $N_t = 100{,}000$, when $E_{0a} = 9.26764$.

The Numerical solutions projected onto $(S_a^{'}, I_{a2}^{'})$, $(S_a^{'}, I_{vb2}^{'})$, $(I_{vb2}^{'}, I_{a2}^{'})$.

The solutions oscillate to the endemic equilibrium point $(S_a^{*}, I_{a1}^{*}, I_{a2}^{*}, I_{va2}^{*}, I_{vb2}^{*})$ are limit cycles.

We will see the local stability can occur when there is no effect of sinusoidal variation $(\beta = 0)$ and limit cycle occurs when there is the effect of sinusoidal variation $\beta \neq 0$. Therefore sinusoidal variation is effect to the behaviors of the numerical solutions.

Chapter VII
Global Stability Analysis

7.1 Mathematical Model

We use susceptible –infected – recovered (SIR) for human and susceptible – infected (SI) for *Aedes* mosquito. We find the global stability of model 2.

7.1.1. The epidemic

The SIR model is given by the following system of ordinary differential equations:

$$\frac{dS}{dt} = \kappa_a N_h - \alpha_a (1 + \varepsilon_a Sin\delta t) I_{va} S - \omega_d S - \omega_h S - \alpha_b (1 + \varepsilon_b Sin\delta t) I_{vb} S \tag{7.1}$$

$$\frac{dI_a}{dt} = \alpha_a (1 + \varepsilon_a Sin\delta t) I_{va} S - \omega_d I_a - \omega_h I_a - \beta_{ha} I_a \tag{7.2}$$

$$\frac{dI_b}{dt} = \alpha_b (1 + \varepsilon_b Sin\delta t) I_{vb} S - \omega_d I_b - \omega_h I_b - \beta_{hb} I_b \tag{7.3}$$

$$\frac{dR}{dt} = -\omega_d R - \omega_h R - \beta_{ha} I_a - \beta_{hb} I_b \tag{7.4}$$

$$\frac{dS_{va}}{dt} = R_a - \alpha_{va} (1 + \varepsilon_{va} Sin\delta t) S_{va} I_a - \omega_{va} S_{va} \tag{7.5}$$

$$\frac{dI_{va}}{dt} = \alpha_{va} (1 + \varepsilon_{va} Sin\delta t) S_{va} I_a - \omega_{va} I_{va} \tag{7.6}$$

$$\frac{dS_{vb}}{dt} = R_b - \alpha_{vb} (1 + \varepsilon_{vb} Sin\delta t) S_{vb} I_b - \omega_{vb} S_{vb} \tag{7.7}$$

$$\frac{dI_{vb}}{dt} = \alpha_{vb} (1 + \varepsilon_{vb} Sin\delta t) S_{vb} I_b - \omega_{vb} I_{vb} \tag{7.8}$$

with the conditions $N_T = S(t) + I_a(t) + I_b(t) + R(t)$, $N_{va} = S_{va}(t) + I_{va}(t)$ and $N_{vb} = S_{vb}(t) + I_{vb}(t)$.

All parameter in this SIR model are non- negative. We will show that equations (5.1) – (5.8). The non – negative octant R_+^8 is positive invariant (where R_+^8 denotes the non – negative region). With respect to system (7.1) – (7.8), we have the following results.

Proposition 7.1. Let $(S(t), I_a(t), I_b(t), R(t), S_{va}(t), I_{va}(t), S_{vb}(t), I_{vb}(t))$ be the solution of (7.1) – (7.8) with the initial condition $(S(0), I_a(0), I_b(0), R(0), S_{va}(0), I_{va}(0), S_{vb}(0), I_{vb}(0))$ and the compact set

$$\Omega_a'' = \left\{ (S, I_a, I_b, R, S_{va}, I_{va}, S_{vb}, I_{vb}) \in R_+^8, W_1 \le N_T = \frac{\kappa_a N_h}{\omega_d + \omega_h}, W_2 \le N_{va} = \frac{R_a}{\omega_{va}}, W_3 \le N_{vb} = \frac{R_b}{\omega_{vb}} \right\}.$$

Then, under the flow described by (7.1) – (7.8), Ω_a'' is a positively invariant set that attracts all solutions in R_+^8.

Proof. We choose the Lyapunov function

$$W(t) = (W_1(t), W_2(t), W_3(t))$$

$$= (S + I_a + I_b + R\,,\, S_{va} + I_{va}\,, S_{vb} + I_{vb})$$

is positive definite on R_+^8 (by its time derivative satisfies) and we have

$$\frac{dW}{dt} = \left(\frac{dW_1}{dt}, \frac{dW_2}{dt}, \frac{dW_3}{dt} \right)$$

$$= \left(\frac{dS}{dt} + \frac{dI_a}{dt} + \frac{dI_b}{dt} + \frac{dR}{dt}, \frac{dS_{va}}{dt} + \frac{dI_{va}}{dt}, \frac{dS_{vb}}{dt} + \frac{dI_{vb}}{dt} \right)$$

$$= \left(\kappa_a N_h - (\omega_d + \omega_h)(S + I_a + I_b + R)\,,\, R_a - \omega_{va}(S_{va} + I_{va}), R_b - \omega_{vb}(S_{vb} + I_{vb}) \right)$$

$$= \left(\kappa_a N_h - (\omega_d + \omega_h) N_T\,,\, R_a - \omega_{va} N_{va}, R_b - \omega_{vb} N_{vb} \right).$$

We used the fact that $N_T = \frac{\kappa_a N_h}{\omega_d + \omega_h}$, $N_{va} = \frac{R_a}{\omega_{va}}$ and $N_{vb} = \frac{R_b}{\omega_{vb}}$.

With this in mind, it is not hard to prove that

$$\frac{dW_1}{dt} = \kappa_a N_h - (\omega_d + \omega_h) W_1 \le 0\,, \quad \text{for} \quad W_1 \ge \frac{\kappa_a N_h}{\omega_d + \omega_h} \tag{7.9}$$

$$\frac{dW_2}{dt} = R_a - \omega_{va} W_2 \le 0\,, \quad \text{for} \quad W_2 \ge \frac{R_a}{\omega_{va}} \tag{7.10}$$

$$\frac{dW_3}{dt} = R_b - \omega_{vb} W_3 \le 0\,, \quad \text{for} \quad W_3 \le \frac{R_b}{\omega_{vb}}. \tag{7.11}$$

From the above equation (7.9) – (7.11) one has that $\frac{dW}{dt} \le 0$ which implies that Ω_a'' is a positively invariant set. In other words, by solving (7.9) – (7.11), we obtain

$$0 \le (W_1(t), W_2(t), W_3(t)) \le ((\kappa_a N_h / (\omega_d + \omega_h)) + W_1(0) e^{-(\omega d + \omega h)t},$$

$$(R_a / \omega_{va}) + W_2(0) e^{-\omega va t}, (R_b / \omega_{vb}) + W(0) e^{-\omega vb t}),$$

where $W_1(0), W_2(0)$ and $W_3(0)$ are respectively, the initial conditions of $W_1(t)$, $W_2(t)$ and $W_3(t)$. Thus, as $t \to \infty$,
$0 \le (W_1(t), W_2(t), W_3(t)) \le (\kappa_a N_h / \omega_d + \omega_h, R_a / \omega_{va}, R_b / \omega_{vb}) = (N_T, N_{va}, N_{vb})$ and one can conclude that Ω_a'' is attractive set.

7.1.2. Equilibrium points

From equations (7.1) – (7.9), we set the right hand side of all equations to zero. We obtain two equilibrium points:

i) If $R_0 \le 1$, the only equilibrium is the disease free equilibrium

$$J_1(S, I_a, I_b, R, S_{va}, I_{va}, S_{vb}, I_{vb})$$

$$= J_1(\frac{\kappa_a N_h}{\omega_d + \omega_h}, 0, 0, 0, \frac{R_a}{\omega_{va}}, I_{va}, \frac{R_b}{\omega_{vb}}, 0) \in \Omega_a''.$$

ii) If $R_0 > 1$, there is the endemic equilibrium state

$$J_2(S^*, I_a^*, I_b^*, R^*, S_{va}^*, I_{va}^*, S_{vb}^*, I_{vb}^*) \in \Omega_a$$

and which satisfies $S^*, I_a^*, I_b^*, R^*, S_{va}^*, I_{va}^*, S_{vb}^*, I_{vb}^* > 0$, with

$$S^* = \frac{(\omega_{va}(I_a^* \gamma_{aa} + \omega_{va})\omega_{vb}(N_h \gamma_{bb} \kappa_a + (\beta_{hb} + \omega_d + \omega_h)\omega_{vb}))}{(\gamma_{bb}\omega_{va}^2(\gamma_{hb} R_b + (\omega_d + \omega_h)\omega_{vb}) + I_a^* \gamma_{aa}\gamma_{bb}(\gamma_{hb} R_b \omega_{va} + \gamma_{ha} R_a \omega_{vb} + (\omega_d + \omega_h)\omega_{va}\omega_{vb}))}, \tag{7.12}$$

$$\begin{aligned} I_b^* = {} & (N_h \gamma_{bb} \gamma_{hb} R_b \kappa_a \omega_{va}(I_a^* \gamma_{aa} + \omega_{va}) - (\beta_{hb} + \omega_d + \omega_h)((\omega_d + \omega_h)\omega_{va}{}^2 + I_a^* \gamma_{aa}(\gamma_{ha} R_a + \\ & (\omega_d + \omega_h)\omega_{va}))\omega_{vb}{}^2) / (\gamma_{bb}(\beta_{hb} + \omega_d + \omega_h)(\omega_{va}{}^2(\gamma_{hb} R_b + (\omega_d + \omega_h)\omega_{vb}) + I_a^* \gamma_{aa}(\gamma_{hb} R_b \omega_{va} +, \\ & \gamma_{ha} R_a \omega_{vb} + (\omega_d + \omega_h)\omega_{va}\omega_{vb}))) \end{aligned} \tag{7.13}$$

$$R^{*}=\{(\beta_{ha}(-\gamma_{bb}\gamma_{hb}R_b(\beta_{ha}+\omega_d+\omega_h)\omega_{va}-\gamma_{bb}(N_h\gamma_{aa}\gamma_{ha}R_a\kappa_a+(\omega_d+\omega_h)$$
$$(\beta_{ha}+\omega_d+\omega_h)\omega_{va}^{2})\omega_{vb}+\gamma_{aa}\gamma_{ha}R_a(\beta_{hb}+\omega_d+\omega_h)\omega_{vb}^{2}))/(\gamma_{aa}(\beta_{ha}+\omega_d+\omega_h)$$
$$(\gamma_{hb}R_b\omega_{va}+\gamma_{ha}R_a\omega_{vb}+(\omega_d+\omega_h)\omega_{va}\omega_{vb})+(\beta_{hb}(N_h\gamma_{bb}\gamma_{hb}R_b\kappa_a\omega_{va}(I_a^{*}\gamma_{aa}+\omega_{va}), \tag{7.14}$$
$$-(\beta_{hb}+\omega_d+\omega_h)((\omega_d+\omega_h)\omega_{va}^{2}+I_a^{*}\gamma_{AAaa}(\gamma_{ha}R_a+(\omega_d+\omega_h)\omega_{va}))\omega_{vb}^{2})/$$
$$((\beta_{hb}+\omega_d+\omega_h)(\omega_{va}^{2}(\gamma_{hb}R_b+(\omega_d+\omega_h)\omega_{vb})+I_a^{*}\gamma_{aa}(\gamma_{hb}R_b\omega_{va}+\gamma_{ha}R_a\omega_{vb}$$
$$+(\omega_d+\omega_h)\omega_{va}\omega_{vb}))\}/\{\gamma_{bb}(\omega_d+\omega_h)\}$$

$$S_{va}^{*}=\frac{R_a}{I_a^{*}\gamma_{aa}+\omega_{va}}, \tag{7.15}$$

$$I_{va}^{*}=\frac{I_a^{*}\gamma_{aa}R_a}{I_a^{*}\gamma_{aa}\omega_{va}+\omega_{va}^{2}}, \tag{7.16}$$

$$S_{vb}^{*}=((\beta_{hb}+\omega_d+\omega_h)(\omega_{va}^{2}(\gamma_{hb}R_b+(\omega_d+\omega_h)\omega_{vb})+I_a^{*}\gamma_{aa}(\gamma_{hb}R_b\omega_{va}+\gamma_{ha}R_a\omega_{vb}$$
$$(\omega_d+\omega_h)\omega_{va}\omega_{vb})))/(\gamma_{hb}\omega_{va}(I_a^{*}\gamma_{aa}+\omega_{va})(N_h\gamma_{bb}\kappa_a+(\beta_{hb}+\omega_d+\omega_h)\omega_{vb}))^{,} \tag{7.17}$$

$$I_{vb}^{*}=(N_h\gamma_{bb}\gamma_{hb}R_b\kappa_a\omega_{va}(I_a^{*}\gamma_{aa}+\omega_{va})-(\beta_{hb}+\omega_d+\omega_h)((\omega_d+\omega_h)\omega_{va}^{2}+I_a^{*}\gamma_{aa}(\gamma_{ha}R_a+$$
$$(\omega_d+\omega_h)\omega_{va}))\omega_{vb}^{2})/(\gamma_{hb}\omega_{va}(I_a^{*}\gamma_{aa}+\omega_{va})\omega_{vb}(N_h\gamma_{bb}\kappa_a+(\beta_{hb}+\omega_d+\omega_h)\omega_{vb})), \tag{7.18}$$

$$I_a^{*}=(-\gamma_{bb}\gamma_{hb}R_b(\beta_{ha}+\omega_d+\omega_h)\omega_{va}^{2}-\gamma_{bb}(-N_h\gamma_{aa}\gamma_{ha}R_a\kappa_a+(\omega_d+\omega_h)$$
$$(\beta_{ha}+\omega_d+\omega_h)\omega_{va}^{2})\omega_{vb}+\gamma_{aa}\gamma_{ha}R_a(\beta_{hb}+\omega_d+\omega_h)\omega_{vb}^{2})/(\gamma_{aa}\gamma_{bb}(\beta_{ha}+\omega_d+\omega_h), \tag{7.19}$$
$$(\gamma_{hb}R_b\omega_{va}+\gamma_{ha}R_a\omega_{vb}+(\omega_d+\omega_h)\omega_{va})\omega_{vb}))$$

and

$$\gamma_{ha}=\alpha_a(1+\varepsilon_a(Sin\delta t)),\qquad \gamma_{hb}=\alpha_b(1+\varepsilon_b(Sin\delta t))$$

$$\gamma_{aa}=\alpha_{va}(1+\varepsilon_{va}(Sin\delta t)),\qquad \gamma_{bb}=\alpha_{vb}(1+\varepsilon_{vb}(Sin\delta t)).$$

The basic reproduction number for system (7.1) – (7.9), we obtain

$$R_0=\frac{\gamma_{aa}\gamma_{ha}R_a\omega_{va}(N_h\gamma_{bb}\kappa_a+(\beta_{hb}+\omega_d+\omega_h)\omega_{vb})}{\gamma_{bb}(\beta_{ha}+\omega_d+\omega_h)\omega_{va}^{2}(\gamma_{hb}R_b+(\omega_d+\omega_h)\omega_{vb})}.$$

7.2. Global stability of the disease free equilibrium

We study the global behavior of the disease free equilibrium state for the equations (7.1) – (7.9).

Theorem 1. Assume that

$$\begin{cases} (\omega_{va})(t) &= \alpha_a (1+\varepsilon_a Sim\delta t) S^*, \\ (\omega_{vb})(t) &= \alpha_b (1+\varepsilon_b Sim\delta t) S^*, \\ (\omega_d+\omega_h)(t) &= \alpha_{va}(1+\varepsilon_{va} Sim\delta t) S_{va}^*, \alpha_{vb}(1+\varepsilon_{vb} Sim\delta t) S_{vb}^*, \end{cases} \tag{7.20}$$

when $R_0 \le 1$, then the disease free equilibrium J_1 is globally asymptotically stable on Ω_a''.

Proof. Consider the Lyapunov function on Ω_a.

$$\psi(t) = (S - S^* \ln S) + I_a + I_b + R + (S_{va} - S_{va}^* \ln S_{va}) + I_{va} + (S_{vb} - S_{vb}^* \ln S_{vb}) + I_{vb} \tag{7.21}$$

The derivative with respect to time yields

$$\frac{d\psi(t)}{dt} = \frac{dS}{dt}(1-\frac{S^*}{S}) + \frac{dI_a}{dt} + \frac{dI_b}{dt} + \frac{dR}{dt} + \frac{dS_{va}}{dt}(1-\frac{S_{va}^*}{S_{va}}) + \frac{dI_{va}}{dt} + \frac{dS_{vb}}{dt}(1-\frac{dS_{vb}^*}{S_{vb}}) + \frac{dI_{vb}}{dt} \tag{7.22}$$

$$\begin{aligned} \frac{d\psi(t)}{dt} = & (\kappa_a N_h - \alpha_a(1+\varepsilon_a Sin\delta t) I_{va} S - \omega_d S - \omega_h S - \alpha_b(1+\varepsilon_b Sin\delta t) I_{vb} S)(1-\frac{S^*}{S}) \\ & +(\alpha_a(1+\varepsilon_a Sin\delta t) I_{va} S - \omega_d I_a - \omega_h I_a - \beta_{ha} I_a) \\ & +(\alpha_b(1+\varepsilon_b Sin\delta t) I_{vb} S - \omega_d I_b - \omega_h I_b - \beta_{hb} I_b) \\ & +(-\omega_d R - \omega_h R - \beta_{ha} I_a - \beta_{hb} I_b) \\ & +(R_a - \alpha_{va}(1+\varepsilon_{va} Sin\delta t) S_{va} I_a - \omega_{va} S_{va})(1-\frac{S_{va}^*}{S_{va}}) \\ & +(\alpha_{va}(1+\varepsilon_{va} Sin\delta t) S_{va} I_a - \omega_{va} I_{va}) \\ & +(R_b - \alpha_{vb}(1+\varepsilon_{vb} Sin\delta t) S_{vb} I_b - \omega_{vb} S_{vb})(1-\frac{S_{vb}^*}{S_{vb}}) \\ & +(\alpha_{vb}(1+\varepsilon_{vb} Sin\delta t) S_{vb} I_b - \omega_{vb} I_{vb}) \end{aligned} \tag{7.23}$$

$$\begin{aligned} \frac{d\psi(t)}{dt} = & \kappa_a N_h (1-\frac{S^*}{S}) + R_a(1-\frac{S_{va}^*}{S_{va}}) + R_b(1-\frac{S_{vb}^*}{S_{vb}}) \\ & + I_{va}(\alpha_a(1+\varepsilon_a Sin\delta t) S^* - \omega_{va}) + I_{vb}(\alpha_b(1+\varepsilon_b Sin\delta t) S^* - \omega_{vb}) \\ & + I_a(\alpha_{va}(1+\varepsilon_{va} Sin\delta t) S_{va}^* - (\omega_d+\omega_h)) + I_b(\alpha_{vb}(1+\varepsilon_{vb} Sin\delta t) S_{vb}^* - (\omega_d+\omega_h)) \\ & + \omega_h S^*(1-\frac{S}{S^*}) + \omega_d S^*(1-\frac{S}{S^*}) + \omega_{va} S_{va}^*(1-\frac{S_{va}}{S_{va}^*}) + \omega_{vb} S_{vb}^*(1-\frac{S_{vb}}{S_{vb}^*}) - \omega_d R - \omega_h R. \end{aligned} \tag{7.24}$$

We define

$$(\omega_{va})(t) = \alpha_a(1+\varepsilon_a Sim\delta t)S^*,$$

$$(\omega_{vb})(t) = \alpha_b(1+\varepsilon_b Sim\delta t)S^*,$$

$(\omega_d+\omega_h)(t) = \alpha_{va}(1+\varepsilon_{va} Sim\delta t)S^*_{va}$ and

$$(\omega_d+\omega_h)(t) = \alpha_{vb}(1+\varepsilon_{vb} Sim\delta t)S^*_{vb}.$$

Note that on Ω''_a, we have $S^* = \dfrac{\kappa_a N_h}{\omega_d+\omega_h}$, $S^*_{va} = \dfrac{R_a}{\omega_{va}}$ and $S^*_{vb} = \dfrac{R_b}{\omega_{vb}}$. With above this is equation, [21] becomes

$$\begin{aligned}\frac{d\psi(t)}{dt} &= \kappa_a N_h(1-\frac{S^*}{S})+R_a(1-\frac{S^*_{va}}{S_{va}})+R_b(1-\frac{S^*_{vb}}{S_{vb}}) \\ &\quad +\omega_h S^*(1-\frac{S}{S^*})+\omega_d S^*(1-\frac{S}{S^*})+\omega_{va}S^*_{va}(1-\frac{S_{va}}{S^*_{va}})+\omega_{vb}S^*_{vb}(1-\frac{S_{vb}}{S^*_{vb}})-\omega_d R-\omega_h R\end{aligned} \quad (7.25)$$

$$\frac{d\psi(t)}{dt} = \kappa_a N_h(2-\frac{S^*}{S}-\frac{S}{S^*})+R_a(2-\frac{S^*_{va}}{S_{va}}-\frac{S_{va}}{S^*_{va}})+R_b(2-\frac{S^*_{vb}}{S_{vb}}-\frac{S_{vb}}{S^*_{vb}})-\omega_d R-\omega_h R \quad (7.26)$$

$$\frac{d\psi(t)}{dt} = -\kappa_a N_h\frac{(S^*-S)^2}{S^*S}-R_a\frac{(S^*_{va}-S_{va})^2}{S^*_{va}S_{va}}-R_b\frac{(S^*_{vb}-S_{vb})^2}{S^*_{vb}S_{vb}}-\omega_d R-\omega_h R. \quad (7.27)$$

We can see that all of terms in (7.27) are always non-positive. By using the LaSelle's extension to Lyapunov's theorem and we have $\dfrac{d\psi(t)}{dt}\le 0$. The limit set of each solution is contained in the largest invariant set for which $S=S^*, S_{va}=S^*_{va}, S_{vb}=S^*_{vb}$ and $R=0$ which is the singleton $\{J_1\}$. Then, we use the LaSelle's invariant principle implies that the disease free equilibrium J_1 is globally asymptotically stable on Ω''_a.

Next, we consider the global property of the endemic equilibrium of (7.1) – (7.9).

Theorem 7.2. If $R_0>1$, there is the endemic equilibrium state

$$J_2(S^*, I^*_a, I^*_h, R^*, S^*_{va}, I^*_{va}, S^*_{vb}, I^*_{vb}) \in \Omega''_a$$

exists and is globally asymptotically stable on Ω''_a if

$$\begin{cases} \omega_{va}(t) = (\alpha_a(1+\varepsilon_a Sin\delta t)N_h) \\ \omega_{vb}(t) = (\alpha_b(1+\varepsilon_b Sin\delta t)N_h) \\ (\omega_d+\omega_h+\alpha_{ha})(t) = Q\alpha_{va}(1+\varepsilon_{va}Sin\delta t)S_{va}^* \\ (\mu_d+\mu_h+\alpha_{hb})(t) = Q\alpha_{vb}(1+\varepsilon_{vb}Sin\delta t)S_{vb}^* \end{cases} \tag{7.28}$$

where

$$A_1(t) = \alpha_a(1+\varepsilon_a Sin\delta\, t)$$

$$A_2(t) = \alpha_b(1+\varepsilon_b Sin\delta t).$$

Proof. The Lyapunov function is in the form

$$\begin{aligned} \psi(t) = {} & (S-S^*\ln S)+I_a+I_b+\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)(S_{va}-S_{va}^*\ln S_{va})+\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)I_{va} \\ & +\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)(S_{vb}-S_{vb}^*\ln S_{vb})+\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)I_{vb} \end{aligned} \tag{7.29}$$

Its derivative along the trajectories of (7.1) – (7.8),

$$\begin{aligned} \frac{d\psi(t)}{dt} = {} & \frac{dS}{dt}(1-\frac{S^*}{S})+\frac{dI_a}{dt}+\frac{dI_b}{dt}+\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)\frac{dS_{va}}{dt}(1-\frac{S_{va}^*}{S_{va}})+\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)\frac{dI_{va}}{dt} \\ & +\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)\frac{dS_{vb}}{dt}(1-\frac{dS_{vb}^*}{S_{vb}})+\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)\frac{dI_{vb}}{dt} \end{aligned} \tag{7.30}$$

$$\begin{aligned} \frac{d\psi(t)}{dt} = {} & (\kappa N_h-\alpha_a(1+\varepsilon_a Sin\delta\, t)I_{va}S-\omega_dS-\omega_hS-\alpha_b(1+\varepsilon_b Sin\delta t)I_{vb}S)(1-\frac{S^*}{S}) \\ & +(\alpha_a(1+\varepsilon_a Sin\delta t)I_{va}S-\omega_dI_a-\omega_hI_a-\beta_{ha}I_a) \\ & +(\alpha_b(1+\varepsilon_b Sin\delta t)I_{vb}S-\omega_dI_b-\omega_hI_b-\beta_{hb}I_b) \\ & +\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)(Q_a-\alpha_{va}(1+\lambda_{va}Sin\delta t)S_{va}I_a-\omega_{va}S_{va})(1-\frac{S_{va}^*}{S_{va}}) \\ & +\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)(\alpha_{va}(1+\varepsilon_{va}Sin\delta t)S_{va}I_a-\omega_{va}I_{va}) \\ & +\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)(Q_b-\alpha_{vb}(1+\varepsilon_{vb}Sin\delta t)S_{vb}I_b-\omega_{vb}S_{vb})(1-\frac{S_{vb}^*}{S_{vb}}) \\ & +\left(\frac{\omega_d+\omega_h+\beta_{ha}+\beta_{hb}}{A_1S_{va}^*+A_2S_{vb}^*}\right)(\alpha_{vb}(1+\varepsilon_{vb}Sin\delta t)S_{vb}I_b-\omega_{vb}I_{vb}). \end{aligned} \tag{7.31}$$

Since we assume that total number of populations are constants, so we have $\kappa N_h = N_T(\omega_d + \omega_h)$, $Q_a = N_{va}\omega_{va} = \omega_{va}(S_{va} + I_{va})$ and $Q_b = N_{vb}\omega_{vb} = \omega_{vb}(S_{vb} + I_{vb})$. Then above equation become

$$\begin{aligned}\frac{d\psi(t)}{dt} &= (\omega_d + \omega_h)N_T(1 - \frac{S^*}{S}) + Q\omega_{va}N_{va}(1 - \frac{S^*_{va}}{S_{va}}) + Q\omega_{vb}N_{vb}(1 - \frac{S^*_{vb}}{S_{vb}})\\ &+ I_{va}(\alpha_a(1 + \varepsilon_a Sin\delta)N_h) - Q\omega_{va}\frac{S_{va}}{S^*_{va}})\\ &+ I_{vb}(\alpha_b(1 + \varepsilon_b Sin\delta)N_h) - Q\omega_{vb}\frac{S_{vb}}{S^*_{vb}})\\ &+ I_a(Q\alpha_{va}(1 + \varepsilon_{va} Sin\delta)S^*_{va} - (\omega_d + \omega_h + \beta_{ha}))\\ &+ I_b(Q\alpha_{vb}(1 + \varepsilon_{vb} Sin\delta)S^*_{vb} - (\omega_d + \omega_h + \beta_{hb}))\end{aligned} \tag{7.32}$$

when $Q = (\dfrac{\omega_d + \omega_h + \beta_{ha} + \beta_{hb}}{A_1 S^*_{va} + A_2 S^*_{vb}})$.

Substituting conditions of (7.28) into (7.32), we have

$$\begin{aligned}\frac{d\psi(t)}{dt} &= (\omega_d + \omega_h)N_T(1 - \frac{S^*}{S}) + Q\omega_{va}S_{va}(1 - \frac{S^*_{va}}{S_{va}}) + Q_1(1 - \frac{S^*_{va}}{S_{va}} - \frac{S_{va}}{S^*_{va}})\\ &+ QS_{vb}\omega_{vb}(1 - \frac{S^*_{vb}}{S_{vb}}) + Q_2(1 - \frac{S^*_{vb}}{S_{vb}} - \frac{S_{vb}}{S^*_{vb}}) - (\omega_d + \omega_h + \beta_{ha})I_a - (\omega_d + \omega_h + \beta_{hb})I_b\\ &+ I_{va}(\alpha_a(1 + \varepsilon_a Sin\delta)N_h) + I_{vb}(\alpha_b(1 + \varepsilon_b Sin\delta)N_h)\\ &+ I_a Q\alpha_{va}(1 + \varepsilon_{va} Sin\delta)S^*_{va} + I_b Q\alpha_{vb}(1 + \varepsilon_{vb} Sin\delta)S^*_{vb}\ .\end{aligned} \tag{7.33}$$

Noting that Ω''_a, we have $Q_1 = I_{va}Q\omega_{va}$ and $Q_2 = I_{vb}Q\omega_{vb}$. The above equation (7.33) become.

$$\begin{aligned}\frac{d\psi(t)}{dt} &= (\omega_d + \omega_h)N_T(1 - \frac{S^*}{S}) + Q\omega_{va}(t)S_{va}(1 - \frac{S^*_{va}}{S_{va}}) + Q_1(1 - \frac{S^*_{va}}{S_{va}} - \frac{S_{va}}{S^*_{va}})\\ &+ QS_{vb}\omega_{vb}(t)(1 - \frac{S^*_{vb}}{S_{vb}}) + Q_2(1 - \frac{S^*_{vb}}{S_{vb}} - \frac{S_{vb}}{S^*_{vb}})\\ &+ I_{va}(\omega_a(1 + \varepsilon_a Sin\delta)N_h) - \omega_{va}(t))\\ &+ I_{vb}(\alpha_b(1 + \varepsilon_b Sin\delta)N_h) - \omega_{vb}(t))\\ &+ I_a Q\alpha_{va}(1 + \varepsilon_{va} Sin\delta)S^*_{va} - (\omega_d + \omega_h + \beta_{ha})(t)I_a\\ &+ I_b Q\alpha_{vb}(1 + \varepsilon_{vb} Sin\delta)S^*_{vb} - (\omega_d + \omega_h + \beta_{hb})(t)I_b\end{aligned} \tag{7.34}$$

$$\begin{aligned}\frac{d\psi(t)}{dt} &= (\omega_d + \omega_h)N_T(2 - \frac{S^*}{S} - \frac{S}{S^*}) + Q\mu_{va}(t)S_{va}(2 - \frac{S^*_{va}}{S_{va}} - \frac{S_{va}}{S^*_{va}}) + Q_1(2 - \frac{S^*_{va}}{S_{va}} - \frac{S_{va}}{S^*_{va}})\\ &+ QS_{vb}\omega_{vb}(t)(2 - \frac{S^*_{vb}}{S_{vb}} - \frac{S_{vb}}{S^*_{vb}}) + Q_2(2 - \frac{S^*_{vb}}{S_{vb}} - \frac{S_{vb}}{S^*_{vb}})\end{aligned} \tag{7.35}$$

$$\frac{d\psi(t)}{dt} = -(\omega_d + \omega_h) N_T \frac{(S-S^*)^2}{SS^*} - Q\,\omega_{va}(t) S_{va} \frac{(S_{va}-S_{va}^*)^2}{S_{va} S_{va}^*} - Q_1 \frac{(S_{va}-S_{va}^*)^2}{S_{va} S_{va}^*}$$
$$- Q S_{vb}\, \omega_{vb}(t) \frac{(S_{vb}-S_{vb}^*)^2}{S_{vb} S_{vb}^*} - Q_2 \frac{(S_{vb}-S_{vb}^*)^2}{S_{vb} S_{vb}^*} \quad . \tag{7.36}$$

We use the LaSalle's invariant principle to show that $\frac{d\psi(t)}{dt} \le 0$ for all $(S^*, I_a^*, I_b^*, R^*, S_{va}^*, I_{va}^*, S_{vb}^*, I_{vb}^*) \in \Omega_a$, and the strict equality $\frac{d\psi(t)}{dt} = 0$ holds only for $S = S^*, I_a = I_a^*, I_b = I_b^*, R = R^*, S_{va} = S_{va}^*, I_{va} = I_{va}^*, S_{vb} = S_{vb}^*$ and $I_{vb} = I_{vb}^*$. Then, the equilibrium state J_2 is the only invariant set of the equations (7.1) – (7.8) contained entirely in
$\{(S^*, I_a^*, I_b^*, R^*, S_{va}^*, I_{va}^*, S_{vb}^*, I_{vb}^*), S = S^*, I_a = I_a^*, I_b = I_b^*, R = R^*, S_{va} = S_{va}^*, I_{va} = I_{va}^*, S_{vb} = S_{vb}^*$ and $I_{vb} = I_{vb}^*\}$ and hence the asymptotic stability theorem, the positive endemic equilibrium state J_2 is global asymptotic stability in Ω_a''.

Chapter VIII
Conclusions and Suggestions

In this research, we present a non – linear ODE system that include seasonality into the transmission model of dengue virus by two species of mosquitoes (*Aedes aegypti* and *Aedes albopictus*). In formulating our model, our model simulates seasonality, it is set to the θ - periodic seasonality function. We explore the biological consequences of combining SIR (Susceptible – Infected – Recover) system for dengue virus transmission in the human population with a corresponding non – autonomous, multistage SI (Susceptible – Infected) system for vectors (*Aedes aegypti* and *Aedes albopictus*). We assume that the human and mosquito populations remain constant. We derive a reproductive number and establish conditions for existence of an endemic equilibrium point for this research. We then incorporate the spatial element into the ODE system to obtain a non – linear system of coupled reaction – diffusion equation.

We prove that if the initial conditions are positive, the size of the total human population is constant. We then do variable substitutions to obtain a simplified, proportion – based system. Next, we set $\theta = \frac{2\pi}{365}$ day, one reproductive cycle, taking place over the course of one year, is fixed. Under this and other assumption, we are able to derive conditions for the existence of a threshold parameter, the basic reproductive ratio, R_0 denoting the expected number of secondary cases produced by a typically infective individual. We analyze the reported data from the Bureau of Epidemiology, Ministry of Public Health, Thailand during year 2003 – 2011 in figure 4.1 and formulated the system of differential equations of dengue virus transmission by biting of two species of mosquitoes (*Aedes aegypti* and *Aedes albopictus*). We use the standard dynamical analysis to determine the conditions of parameters for local asymptotically stability. Numerical solutions are shown to confirm the analytical results. We constructed the Lyapunov functions with the conditions of parameters for global asymptotically stability.

For each model, there exists a threshold parameter, the basic reproductive number for disease free and endemic equilibrium point. The disease free equilibrium point exists and is locally asymptotical stable if the basic reproductive number is less than one and become unstable when the basic reproductive number is more than one. We used numerical simulations to confirm these results.

The basic reproductive numbers are the geometric mean of these quantities (R_{01} – R_{04} and E_0)

In chapter III, we study the SIR transmission of dengue disease with the different contact rates, we have

$$R_{01} = \frac{\dfrac{\dfrac{\lambda^2 C_{hs\to v} C_{v\to hs} N_{vs} N_{Ts}}{(N_{Ts}+d)^2} + \dfrac{\lambda^2 C_{hw\to v} C_{v\to hw} N_{vw} N_{Tw}}{(N_{Tw}+d)^2}}{(\beta+\delta d+\delta h)} + \dfrac{\lambda^2 C_{hr\to v} C_{v\to hr} N_{vr} N_{Tr}}{(N_{Tr}+d)^2}}{\delta v}.$$

In chapter IV, We develop a mathematical model considering the transmission of dengue virus from *Aedes aegypti* and *Aedes albopictus* mosquitoes, we have

$$R_{02} = \frac{2(N_{va}\chi_{va}(N_t\,\chi_{vb}(\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\eta_h+\eta_d)+\omega_a\omega_{va}\rho_5)\chi_a+2\chi_{vb}\,\rho_4\,(2(\eta_h+\eta_d)+N_{vb}(2+\omega_b\,\omega_{vb})\chi_b)}{N_{va}\chi_{va}(2N_t\chi_{va}(2+\omega_a\,\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\eta_h+\eta_d)+2(2+\omega_a\,\omega_{va})\rho_5\chi_a+2N_{vb}\chi_{vb}\omega_b\,\omega_{vb}\rho_4\chi_b}$$

For the mathematical model in chapter V, the transmission of dengue disease with the different incubation rate for each season into SEIR model, we have

$$R_{03} = [(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{Tr} N_{Vr}\alpha_{hr}\beta_{vr}}{(N_{Tr}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)})$$

$$+(\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{Tw} N_{Vw}\alpha_{hw}\beta_{vw}}{(N_{Tw}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)})$$

$$+(\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{Ts} N_{Vs}\alpha_{hs}\beta_{vs}}{(N_{Ts}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)})].$$

In chapter VI, the SIR and SI models which provide suitable for the states of children and adult in two species are used in this study.(*Aedes aegypti* and *Aedes albopictus*), we have

$$E_0 = \max\ \{ \frac{2N_{vb1}\gamma_b\alpha_{bc}\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{v_a}\gamma_{vb}+N_{va1}\alpha_{ac}\alpha_{va1}(2(N_{tc}\alpha_{vb1}\eta_d+(\omega_{c2}+\eta_d)\eta_{v_b})(2+\gamma_a\gamma_{va})+2N_{tc}\alpha_{vb1}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{v_a}(2\eta_d+N_{vb1}\alpha_{bc}(2+\gamma_b\gamma_{vb}))+2N_{va1}\alpha_{ac}\alpha_{va1}(\alpha_a(\omega_{c2}+\eta_d)\eta_{v_b}\gamma_{va}+N_{tc}\alpha_{vb1}\,\eta_d(\gamma_a\gamma_{va}+(\gamma_a+\gamma_{va})\gamma_{vb})))},$$

$$\frac{2N_{vb2}\gamma_b\alpha_{ba}\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{v_a}\gamma_{vb}+N_{va2}\alpha_{aa}\alpha_{va2}(2(N_{ta}\alpha_{vb2}\eta_d+(\omega_{a2}+\eta_d)\eta_{v_b})(2+\gamma_a\gamma_{va})+2N_{ta}\alpha_{vb2}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{v_a}(2\eta_d+N_{vb2}\alpha_{ba}(2+\gamma_b\gamma_{vb}))+2N_{va2}\alpha_{aa}\alpha_{va2}(\gamma_a(\omega_{a2}+\eta_d)\eta_{v_b}\gamma_{va}+N_{ta}\alpha_{vb2}\,\eta_d(\gamma_a\gamma_{va}+(\gamma_a+\gamma_{va})\gamma_{vb})))}\}$$

For the mathematical model in the last chapter, we presented a mathematical study on global stalility of our model using the Lyapunov function, we have

$$R_{04} = \frac{\gamma_{aa}\gamma_{ha}R_a\omega_{va}(N_h\gamma_{bb}\kappa_a + (\beta_{hb}+\omega_d+\omega_h)\omega_{vb})}{\gamma_{bb}(\beta_{ha}+\omega_d+\omega_h)\omega_{va}{}^2(\gamma_{hb}R_b+(\omega_d+\omega_h)\omega_{vb})}$$

For the transmission model, we formulate the model for describing the transmission of dengue disease:

chapter III describes the transmission of dengue disease with the different contact rates of three seasons in Thailand.

chapter IV describes the transmission of dengue disease when human are infected from *Aedes aegypti* and *Aedes albopictus* mosquitoes

For $\theta = 0$, the threshold parameter for this model is

$$R_{02} = \frac{2(N_{va}\chi_{va}(N_t\,\chi_{vb}(\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\eta_h+\eta_d) + \omega_a\omega_{va}\rho_5)\chi_a + 2\chi_{vb}\rho_4(2(\eta_h+\eta_d)+N_{vb}(2+\omega_b\,\omega_{vb})\chi_b)}{N_{va}\chi_{va}(2N_t\chi_{va}(2+\omega_a\,\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\eta_h+\eta_d)+2(2+\omega_a\,\omega_{va})\rho_5\chi_a + 2N_{vb}\chi_{vb}\omega_b\,\omega_{vb}\rho_4\chi_b}.$$

For $\theta \neq 0$ the threshold parameter for this model is

$$\begin{aligned}R_{02} = {} & [2(N_{va}\chi_{va}(N_t\,\chi_{vb}(\omega_a\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\eta_h+\eta_d) + \omega_a\omega_{va}\rho_5)\chi_a Cos(2\theta t)+\chi_{vb}(2\,\rho_4(2+\omega_b\,\omega_{vb})\chi_b\,\rho_4(\eta_h+\eta_d)+N_{vb}(2+\omega_b\,\omega_{vb})\chi_b) \\ & +4(\rho_4(\omega_{vb}(\eta_h+\eta_d)+N_{vb}(\omega_b+\omega_{vb})\,\chi_b)Sin(\theta t)+N_t\,N_{va}\chi_{va}\rho_4\omega_a\omega_{va}\omega_{vb}(\eta_h+\eta_d)\chi_a\;Sin(3\theta t)]\;/[(2N_{vb}\chi_{vb}\rho_b\,\omega_{vb}\rho_4\chi_b Cos(2\theta t)+ \\ & N_{va}\chi_{va}\chi_a(2N_t\chi_{vb}(2+\omega_a\,\omega_{va}+(\omega_a+\omega_{va})\omega_{vb})(\eta_h+\eta_d)+2(2+\omega_a\,\omega_{va})\rho_5+(N_t\chi_{vb}(4(\omega_a+\omega_{va})\omega_{vb})(\eta_h+\eta_d)+ \\ & 4((\omega_a+\omega_{va})\rho_5)Sin(\theta t)]\end{aligned}$$

Analysis of this model reveals the existence of two equilibrium points. The quantity $\widetilde{R}_0 = \sqrt{R_0}$ is called the basic reproductive number of disease, it represents the average number of secondary cases that one case can produce if he/she introduced into a susceptible population. We consider human and vector (*Aedes aegypti* and *Aedes albopictus*) populations, when values of $\theta = 0$ and $\theta \neq 0$. If $\theta = 0$, we use Routh-Hurwitz criteria to determine the locally asymptotically stable of equilibrium points. If $\theta \neq 0$, we considered time series of human and vector (*Aedes aegypti* and *Aedes albopictus*) populations, when the transmission probability from *Aedes aegypti* to human and the transmission probability from *Aedes albopictus* to human are different.

For the disease free equilibrium point $(\theta = 0)$, $P_0 = (1,0,0,0,0)$,we represents the state in which dengue disease is not endemic in human and it is locally asymptotically stable for $R_0 < 1$. Figure 4.1. shows the proportions of human and vectors (*Aedes aegypti* and *Aedes albopictus*). The proportions of all population $(S', I'_a, I'_b, I'_{va}, I'_{vb})$ approach to the disease free state $(1,0,0,0,0)$ when $R_0 < 1$.

For the endemic equilibrium point $P_1 = (S^*, I^*_a, I^*_b, I^*_{va}, I^*_{vb})$, when $\theta = 0$ the endemic equilibrium point locally asymptotically stable for $R_0 > 1$. Figure 4.2 shows the proportions of human and vectors for $(S^*, I^*_a, I^*_b, I^*_{va}, I^*_{vb})$,when $R_0 > 1$.

For the mathematical model in chapter V, the transmission of dengue disease is analyzed by using the standard dynamical modeling method. We have shown that several standard theorems in mathematical epidemiology can be extended to this kind of SEIR model. If basic reproductive number less than one, the disease-free equilibrium state is local stable. If basic reproductive number more than one, then an endemic equilibrium is local stable. The results of this study are used for finding the condition of parameters to be the way for controlling this disease.

In chapter VI, Several investigations have been conducted using the SIR and SI models.

The basic reproductive number of equations (6.21) - (6.30) is defined as follows [55]:

$$E_0 = \max\ \left\{ \frac{2N_{vb1}\gamma_b\alpha_{bc}\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{v_a}\gamma_{vb} + N_{va1}\alpha_{ac}\alpha_{va1}(2(N_{tc}\alpha_{vb1}\eta_d + (\omega_{c2}+\eta_d)\eta_{v_b})(2+\gamma_a\gamma_{va}) + 2N_{tc}\alpha_{vb1}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{v_a}(2\eta_d + N_{vb1}\alpha_{bc}(2+\gamma_b\gamma_{vb})) + 2N_{va1}\alpha_{ac}\alpha_{va1}(\alpha_a(\omega_{c2}+\eta_d)\eta_{v_b}\gamma_{va} + N_{tc}\alpha_{vb1}\eta_d(\gamma_a\gamma_{va} + (\gamma_a+\gamma_{va})\gamma_{vb})))}, \right.$$

$$\left. \frac{2N_{vb2}\gamma_b\alpha_{ba}\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{v_a}\gamma_{vb} + N_{va2}\alpha_{aa}\alpha_{va2}(2(N_{ta}\alpha_{vb2}\eta_d + (\omega_{a2}+\eta_d)\eta_{v_b})(2+\gamma_a\gamma_{va}) + 2N_{ta}\alpha_{vb2}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{v_a}(2\eta_d + N_{vb2}\alpha_{ba}(2+\gamma_b\gamma_{vb})) + 2N_{va2}\alpha_{aa}\alpha_{va2}(\gamma_a(\omega_{a2}+\eta_d)\eta_{v_b}\gamma_{va} + N_{ta}\alpha_{vb2}\eta_d(\gamma_a\gamma_{va} + (\gamma_a+\gamma_{va})\gamma_{vb})))} \right\}$$

describes the number of infectious human produced from primary infection of children and adult. Using the initial values and parameter values from data, the obtained result of threshold parameter value E_0 for E_{0c} and E_{0a} can be rewritten in mathematical form as follows:

$$E_{0c} = \frac{2N_{vb1}\gamma_b\alpha_{bc}\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{v_a}\gamma_{vb} + N_{va1}\alpha_{ac}\alpha_{va1}(2(N_{tc}\alpha_{vb1}\eta_d + (\omega_{c2}+\eta_d)\eta_{v_b})(2+\gamma_a\gamma_{va}) + 2N_{tc}\alpha_{vb1}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb1}(\omega_{c1}+\eta_d)\eta_{v_a}(2\eta_d + N_{vb1}\alpha_{bc}(2+\gamma_b\gamma_{vb})) + 2N_{va1}\alpha_{ac}\alpha_{va1}(\gamma_a(\omega_{c2}+v_d)\eta_{v_b}\gamma_{va} + N_{tc}\alpha_{vb1}\eta_d(\gamma_a\gamma_{va} + (\gamma_a+\gamma_{va})\gamma_{vb})))}$$

in children.

$$E_{0a} = \frac{2N_{vb2}\gamma_b\alpha_{ba}\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{v_a}\gamma_{vb} + N_{va2}\alpha_{aa}\alpha_{va2}(2(N_{ta}\alpha_{vb2}\eta_d + (\omega_{a2}+\eta_d)\eta_{v_b})(2+\gamma_a\gamma_{va}) + 2N_{ta}\alpha_{vb2}\eta_d(\gamma_a+\gamma_{va}))}{(2\alpha_{vb2}(\omega_{a1}+\eta_d)\eta_{v_a}(2\eta_d + N_{vb2}\alpha_{ba}(2+\gamma_b\gamma_{vb})) + 2N_{va2}\alpha_{aa}\alpha_{va2}(\gamma_a(\omega_{a2}+\eta_d)\eta_{v_b}\gamma_{va} + N_{ta}\alpha_{vb2}\eta_d(\gamma_a\gamma_{va} + (\gamma_a+\gamma_{va})\gamma_{vb})))}$$

in adult.

The reproductive rates are depend on the number of infected mosquitoes I_{va1}, I_{vb1} in children and I_{va2}, I_{vb2} in adult.

Table 8.1. Determination of the values E_0 of infected mosquitoes

I_{va1} value	I_{vb1} value	I_{va2} value	I_{vb2} value	E_0 value
-	-	2.3860X10-18	6.44744X10-7	0.00310
2.5687X10-26	0.00006	-	-	0.00732
0	0.00036	-	-	0.06795
-	-	4.9155X10-23	0.0007175	0.68842
-	-	7.3376X10-14	1.07169X10-8	3.61291
8.234X10-6	7.982X10-17	-	-	38.6066
3.16331X10-5	1.118X10-6	-	-	80.3505
-	-	9.933X10-8	2.822X10-17	89.3077

From the above table (table 8.1), we will see that if the number of infected mosquitoes are increased, the basic reproductive rate is also increased.

Moreover, we have considered the effect of sinusoidal variation (β), we will see that if $(\beta = 0)$, then the solutions oscillation to the steady state. The limit cycle occurs for $(\beta \neq 0)$. Thus the limit cycles occurs while there is the seasonal variation of mosquitoes (*Aedes aegypti* and *Aedes albopictus*). It can be seen that the dynamical behavior of the endemic state changes while there is the influence of season.

The basic reproductive number of disease is defined by $\widetilde{E}_0 = \sqrt{E_0}$. This value is the threshold condition for the existence of the endemic state. When $E_0 \leq 1$, the solutions oscillate to disease free equilibrium state, whereas when $E_0 > 1$, the solutions oscillate to the endemic state. The behaviors of the proportion of susceptible, infective human into two classes (a child class and an adult class) and infective vectors of the two species (*Aedes aegypti* and *Aedes albopictus*) are initially positive numbers. If this can be seen as follow; the infective human are introduced into the susceptible is bitten during each period, by the fraction $\frac{bN_h}{(N_h + m)}(\frac{1}{\eta_v})$ (that the biting rate b of mosquitoes is the average number of bites per mosquito per day, η_v is the per capita mortality rate of mosquito, of these bites becomes new infective in the human population, N_h is the human population and N_v is the vector population and m is the number of alternative hosts available as blood sources) [13,14,15,45,48,49]. The parameters α_{ac}, α_{bc}, α_{aa}, α_{ba}, α_{ac}, α_{va1}, α_{va2}, α_{vb1} and α_{vb2} are effects to the basic reproductive number of this disease as we see in (6.21) - (6.30). If the basic reproductive number is less than equal to one, then the infective replaces less than one, then disease dies. On the other hand, if this number is greater than one and when the susceptible fraction gets large enough to birth of new susceptible, then there are secondary infections and endemic equilibrium state occur. As we can see in this study, the seasonal parameters such as γ_{va} and γ_{vb} which are the measure of influence on the transmission process reflect the environment.

For the mathematical model in the last chapter, the basic reproductive number of equations (7.1) - (7.8) is defined as follows [55]. Then, we define $R_0 = \sqrt{R_0}$ as the basic reproductive number of disease. Also, it represents the average number of secondary cases produced from susceptible population. We consider human and vector (*Aedes aegypti* and *Aedes albopictus*) populations. It depends on the transmission rate of dengue virus. The global stability of transmission of dengue disease in human and vector (*Aedes aegypti* and *Aedes albopictus*) by using Lyapunov functions. If basic productive number less than one, then each solution converges to the disease – free equilibrium state and it is globally asymptotically stable, in the feasible region and the disease dies out of population. If basic reproductive more than one, then there is the unique endemic equilibrium state is globally asymptotically stable in the interior of feasible region and the disease is present. If the disease is present in the population,then it will persist.

For the future, other reseachers will develop an SIR model for the transmission of dengue virus by considering the side effect of the susceptible and infected populations. The life cycle of mosquito and the influence of environment should be furture considered.

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Appendix

Appendix A.1

Theoretical Background

The models are characterized by quantities that interact. The systems of equations need to be analyzed. Below is the brief introduction to the analysis of nonlinear systems of ordinary differential equations (ODEs) [49,50].

Suppose we consider a system of two autonomous first – order equations

$$\frac{dx}{dt} = f(x,y) \tag{A.1}$$

$$\frac{dy}{dt} = g(x,y) \tag{A.2}$$

The system has autonomous equations because f and g are independent of t. A solution of (x_0, y_0) is a pair of two functions $x(t)$ and $y(t)$ that satisfy the equation plus any initial conditions, this system is defined by $f(x_0,y_0) = g(x_0,y_0) = 0$. It is rare that an analytic formula for the solution of (A.1) - (A.2) can be constructed so we usually construct a numerical solution or study the qualitative behavior of the solution.

If f and g are nonlinear functions. A solution (x_0, y_0) of this system is defined by $f(x_0,y_0) = g(x_0,y_0) = 0$ and (x_0, y_0) is called a steady state solution, equilibrium point at the origin, singular point or critical points.

The critical points are classified based on the vector field around the point. The Jacobian of the vector quantity is defined by

$$J\,(x_0, y_0) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix}_{(x_0, y_0)}$$

Let $\alpha = a_{11} + a_{22}$, $\omega = a_{11}a_{22} - a_{12}a_{21}$ and $\eta = \alpha^2 - 4\omega$, then the characteristic equation is $\lambda^2 - \alpha\lambda + \omega = 0$ and the eigenvalues are $\lambda_{1,2} = \frac{\alpha \pm \sqrt{\eta}}{2}$.

Theorem A.1 An equilibrium or critical point [45,49,50] $x=(x_0,y_0)\in R$ of dynamic system is stable if all eigenvalues of the equilibrium points of system have negative real parts. The behavior of the equilibrium points can be determined form the characteristics of eigenvalues of Jacobrian matrix J.

Let λ_1 and λ_2 be eigenvalues of the Jacobrian matrix with a_1 and a_2 as the corresponding eigenvectors. Then

Case I

a) If both eigenvalues are real and positive $(\lambda_1>\lambda_2>0)$, then we have an unstable node. The phase orbits (trajectories) near (x_0,y_0) are a parabola shaped family of curve that are tangent which is the eigenvector associated with the smaller of the two eigenvalues.

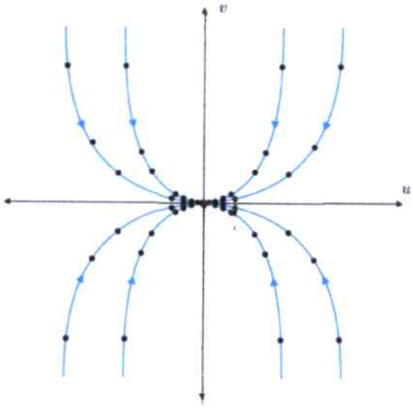

Figure A.1 An stable two –tangent node (Sink).

Theorem A.1 An equilibrium or critical point [45,49,50] $x=(x_0, y_0) \in R$ of dynamic system is stable if all eigenvalues of the equilibrium points of system have negative real parts. The behavior of the equilibrium points can be determined form the characteristics of eigenvalues of Jacobrian matrix J.

Let λ_1 and λ_2 be eigenvalues of the Jacobrian matrix with a_1 and a_2 as the corresponding eigenvectors. Then

Case I

a) If both eigenvalues are real and positive $(\lambda_1 > \lambda_2 > 0)$, then we have an unstable node. The phase orbits (trajectories) near (x_0, y_0) are a parabola shaped family of curve that are tangent which is the eigenvector associated with the smaller of the two eigenvalues.

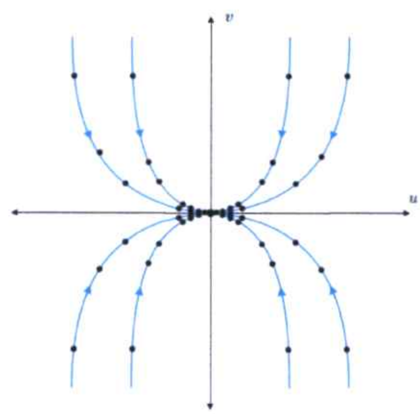

Figure A.1 An stable two –tangent node (Sink).

b) If both eigenvalues are real and negative ($\lambda_1 < \lambda_2 < 0$, that is $|\lambda_2| > |\lambda_1|$) we have a stable node. The phase orbits are a family of parabola shaped curves that are tangent.

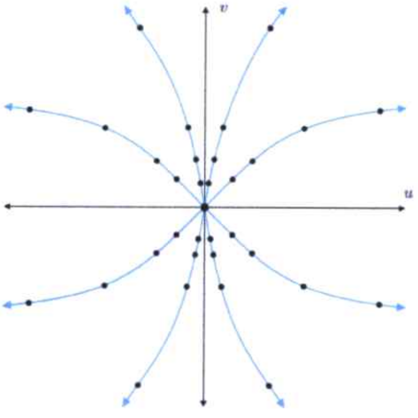

Figure A.2 An unstable two –tangent node (Source).

c) if the eigenvalues coincide $\lambda_1 = \lambda_2$ then we have an inflected node: stable if $\lambda_1 = \lambda_2 < 0$ and unstable if $\lambda_1 = \lambda_2 > 0$

c.1) $\lambda_1 = \lambda_2 > 0$

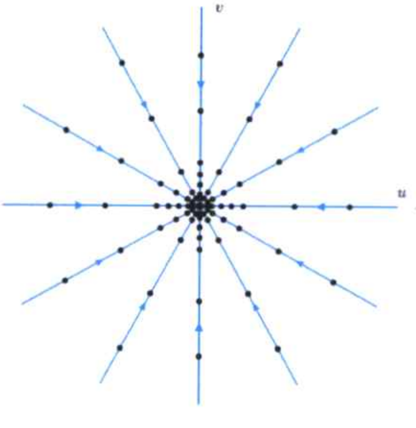

Figure A.3 Stable Star.

c.2) $\lambda_1 = \lambda_2 < 0$

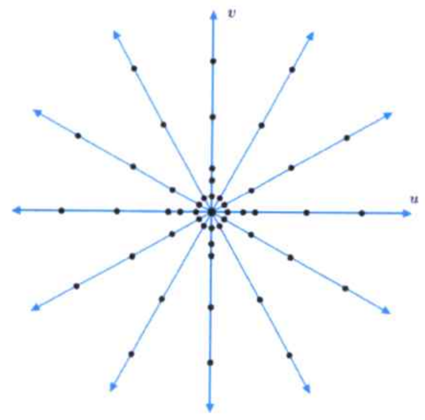

Figure A.4. Unstable Star.

c.3) $0 < |\lambda| < 1$, $|\lambda| = 1$, $|\lambda| > 1$

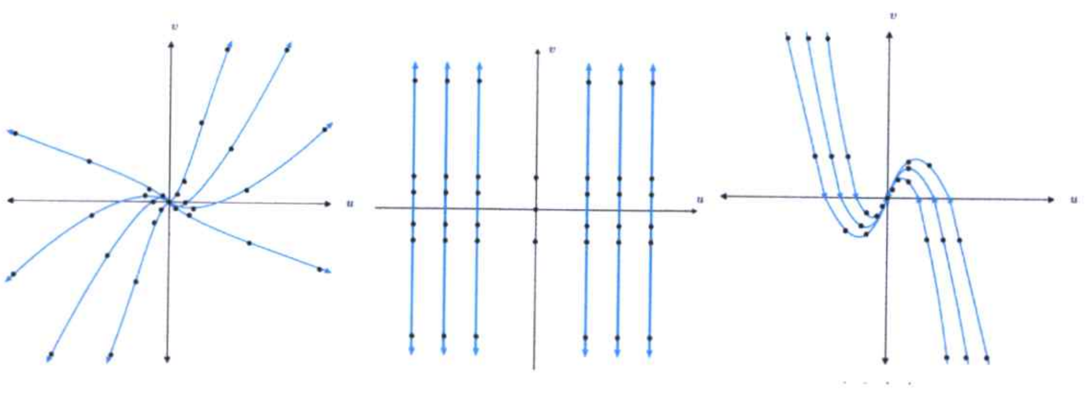

Figure A.5. $0 < |\lambda| < 1$, $|\lambda| = 1$, $|\lambda| > 1$

d) If both eigenvalues are real and one is positive $\lambda_2 > 0$ and the other is negative $\lambda_1 < 0$ then we have a *saddle.* The phase orbits are a hyperbola shaped family of curves that are unstable in the *line* direction and stable in the *line* direction.

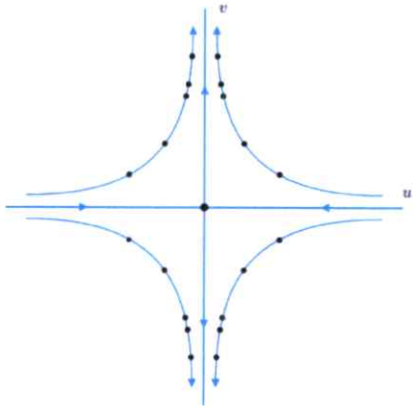

Figure A.6. A saddle node.

Case II

Let $\alpha = tr(J)$ and $\omega = \det(J)$, where J is the jacobian matrix evaluated at a fixed point of the two – dimensional dynamical system, $x' = f(x,y)$ and $y' = g(x,y)$, of two autonomous differential equations. Then the eigenvalues of J, λ_1 and λ_2, satisfy the quadratic equation $\lambda^2 - \alpha\lambda + \omega = 0$, and the fixed point is locally asymptotically stable if $\mathrm{Re}(\lambda_1)<0$ and $\mathrm{Re}(\lambda_2)<0$ or, equivalently, if $tr(J) < 0$ and $\det(J) > 0$.

a) If $\omega<0$, (so the eigenvalues are real, one positive and one negative) then the fixed point is a saddle.

b) If $\omega>0$, $\alpha <0$, and $\alpha^2 - 4\omega >= 0$ (so the eigenvalues are real and negative) then the fixed point is a sink node.

c) If $\omega>0$ and $\alpha >0$, and $\alpha^2 - 4\omega >= 0$ (so the eigenvalues are real and positive) then the fixed point is a source node.

d) If $\omega>0$ and $\alpha = 0$ (so the eigenvalues are complex with zero real parts) then the fixed point is a center.

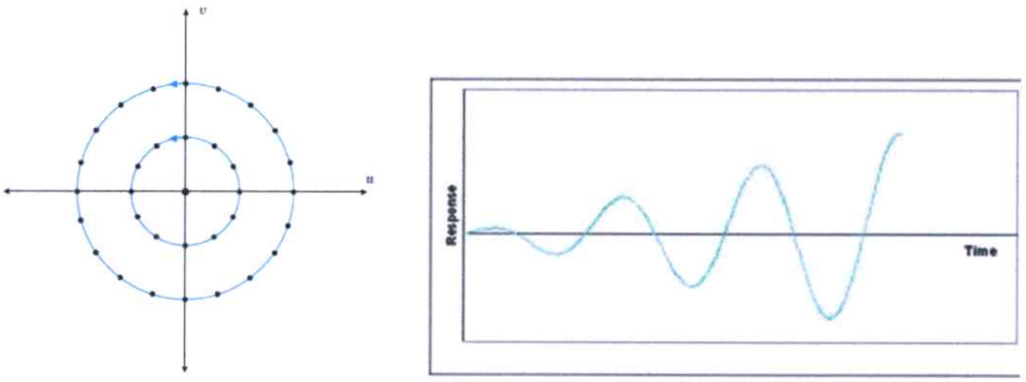

Figure A.7. A center.

e) If $\alpha<0$ and $\alpha^2-4\omega<0$,(so the eigenvalues are complex with negative real parts) then the fixed point is a spiral sink.

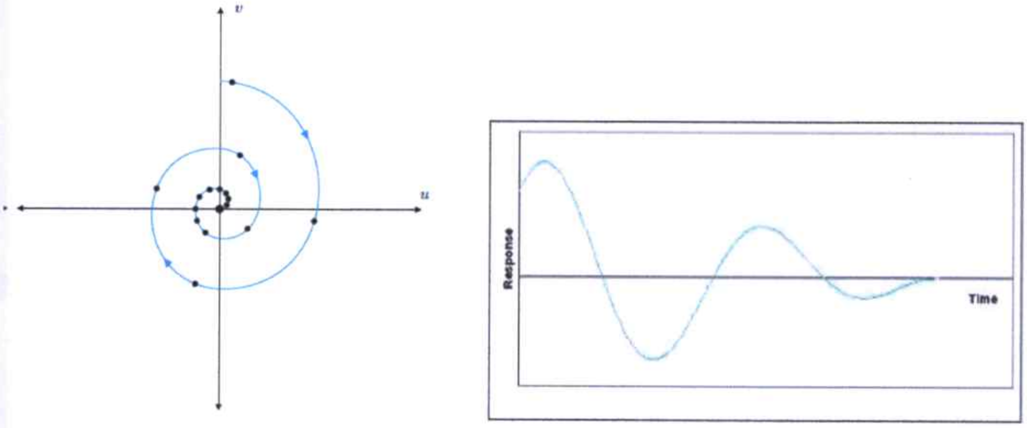

Figure A.8. A spiral sink.

f) If $\alpha>0$ and $\alpha^2-4\omega<0$,(so the eigenvalues are complex with positive real parts) then the fixed point is a spiral source.

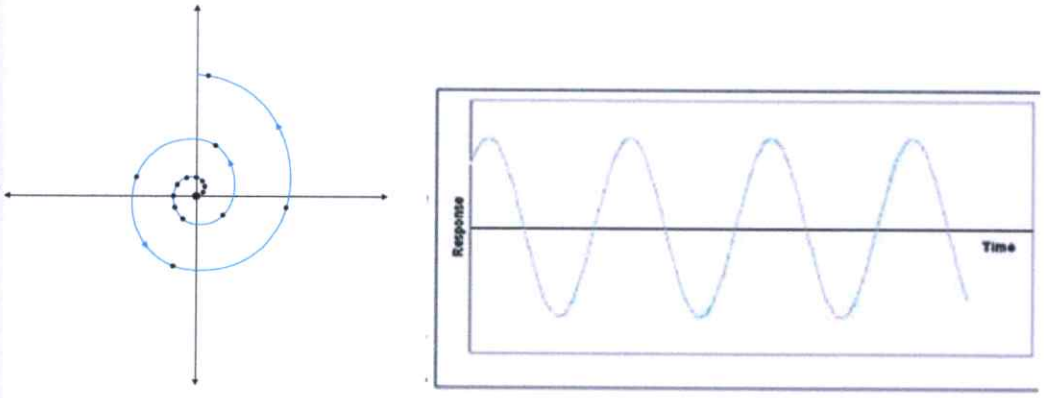

Figure A.9. A spiral source.

Next we consider stability of steady state for n – variables system:

$$\frac{dx_i}{dt} = f_i(x_1, x_2, x_3, ..., x_n) \qquad \text{with} \quad i = 1, 2, 3, \dots, n$$

The equilibrium point $\overline{P}$ is obtained by solving $F(P) = 0$.The next step, The equilibrium point would be to determine stability properties of this steady solution. By the steady state us leads to consider the Jacobian matrix:

$$J = \frac{\partial}{\partial x} F(\overline{P}) \ ,$$

$$J(\overline{P}) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} .$$

This J is now a nxn matrix. The eigenvalues are obtained by solving the following equation: $\det(J - \lambda I) = 0$. The characteristic equation is a polynomial of degree n :

$$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + ... + a_n = 0. \qquad \text{(A.3)}$$

The stability of the equilibrium point can be determinant without actually solving for these eigenvalues by checking certain criteria, by using Routh – Hurwitz.

Theorem A.2 (Routh – Hurwitz Criteria)[46,49] to determine local asymptotic stability of an equilibrium for characteristic equation (2), where the coefficients a_i are real constants, $i = 1,2,3,...,n$, define the n Routh – Hurwitz matices using the coefficients a_i of the characteristic polynomial:

$$H_1 = (a_1),$$

$$H_2 = \begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix},$$

$$H_3 = \begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix}, \ldots,$$

$$H_j = \begin{pmatrix} a_1 & 1 & 0 & 0 & \ldots & 0 \\ a_3 & a_2 & a_1 & 1 & \ldots & 0 \\ a_5 & a_4 & a_3 & a_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{2j-1} & a_{2j-2} & a_{2j-3} & a_{2j-4} & & a_j \end{pmatrix}, \ldots,$$

$$H_N = \begin{pmatrix} a_1 & 1 & 0 & \ldots & 0 \\ a_3 & a_2 & a_1 & \ldots & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & a_N \end{pmatrix}$$

where the (l, m) element in the matrix H_j is

a_{2l-m} for $0 < 2l - m < N$

1 for $2l = m$

0 for $2l < m$ or $2l > N + m$

Then, all eigenvalues have negative real parts (steady stable) if and only if the determinants of all Routh – Hurwitz matrixes are positive:

$\det(H_j) > 0$ when $j = 1, 2, 3, \ldots$

Show conditions of Routh – Hurwitz criteria for $j = 2,3,4$ and 5 as follows:

When $j = 2$, the Routh – Hurwitz criteria simplify to

a) $\det H_1 = \det(a_1) = a_1 > 0$

b) $\det H_2 = \det\begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix} = \det\begin{pmatrix} a_1 & 1 \\ 0 & a_2 \end{pmatrix} = a_1 a_2 > 0$.

For characteristic polynomial of $j = 2$, the Routh – Hurwitz criteria are H $a_1 > 0$ and H $a_2 > 0$.

When $j = 3$, the Routh – Hurwitz criteria simplify to

a) $\det H_1 = \det(a_1) = a_1 > 0$

b) $\det H_2 = \det\begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix} = a_1 a_2 - a_3 > 0$

c) $\det H_3 = \det\begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix} = \det\begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ 0 & 0 & a_3 \end{pmatrix} = a_3\ (a_1 a_2 - a_3) > 0$

If coefficients a_4 and a_5 in 3rd order characteristic polynomial equation equal to zero.

For characteristic polynomial of $j = 3$, the Routh – Hurwitz criteria are $a_1 > 0$, $a_1 a_2 - a_3 > 0$ and $a_3 > 0$.

When $j = 4$, the Routh – Hurwitz criteria simplify to

a) $\det H_1 = \det(a_1) = a_1 > 0$

b) $\det H_2 = \det\begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix} = a_1 a_2 - a_3 > 0$

c) $\det H_3 = \det\begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix} = \det\begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{pmatrix} = a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0$

d)

$$\det H_4 = \det\begin{pmatrix} a_1 & 1 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 \\ a_5 & a_4 & a_3 & a_2 \\ a_7 & a_6 & a_5 & a_4 \end{pmatrix} = \det\begin{pmatrix} a_1 & 1 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 \\ 0 & a_4 & a_3 & a_2 \\ 0 & 0 & 0 & a_4 \end{pmatrix} = a_1\, a_2\, a_3\, a_4 - a_3^2 a_4 - a_1^2 a_4^2 > 0$$

If coefficients a_5, a_6 and a_7 in 4th order characteristic polynomial equation equal to zero.

For characteristic polynomial of $j = 5$, the Routh – Hurwitz criteria are $a_1 > 0$, $a_1 a_2 - a_3 > 0$, $a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0$ and $a_1\, a_2\, a_3\, a_4 - a_3^2 a_4 - a_1^2 a_4^2 > 0$.

When $j = 5$, the Routh – Hurwitz criteria simplify to

a) $\det H_1 = \det(a_1) = a_1 > 0$

b) $\det H_2 = \det\begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix} = a_1a_2 - a_3 > 0$

c) $\det H_3 = \det\begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix} = \det\begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix} = a_1a_2a_3 + a_1a_5 - a_3^2 - a_1^2a_4 > 0$

d) $\det H_4 = \det\begin{pmatrix} a_1 & 1 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 \\ a_5 & a_4 & a_3 & a_2 \\ a_7 & a_6 & a_5 & a_4 \end{pmatrix} = \det\begin{pmatrix} a_1 & 1 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 \\ a_5 & a_4 & a_3 & a_2 \\ 0 & 0 & a_5 & a_4 \end{pmatrix}$

$$= a_1a_2a_3a_4 - a_3^2a_4 - a_1^2a_4^2 - a_1a_2^2a_5 - a_5^2 + a_2a_3a_5 + 2a_1a_4a_5 > 0$$

e) $\det H_5 = \det\begin{pmatrix} a_1 & 1 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 \\ a_7 & a_6 & a_5 & a_4 & a_3 \\ a_9 & a_8 & a_7 & a_6 & a_5 \end{pmatrix} = \det\begin{pmatrix} a_1 & 1 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 \\ 0 & 0 & a_5 & a_4 & a_3 \\ 0 & 0 & 0 & 0 & a_5 \end{pmatrix}$

$$= a_1a_2a_3a_4a_5 - a_3^2a_4a_5 - a_1^2a_4^2a_5 - a_1a_2^2a_5^2 - a_5^3 + a_2a_3a_5^2 + 2a_1a_4a_5^2 > 0$$

If coefficients a_6, a_7, a_8 and a_9 in 5th order characteristic polynomial equation equal to zero.

For characteristic polynomial of $j = 4$, the Routh – Hurwitz criteria are

$$a_1 > 0\ ,\ a_1a_2 - a_3 > 0,\ a_1a_2a_3 + a_1a_5 - a_3^2 - a_1^2a_4 > 0,$$

$$a_1a_2a_3a_4 - a_3^2a_4 - a_1^2a_4^2 - a_1a_2^2a_5 - a_5^2 + a_2a_3a_5 + 2a_1a_4a_5 > 0 \text{ and}$$

$$a_1a_2a_3a_4a_5 - a_3^2a_4a_5 - a_1^2a_4^2a_5 - a_1a_2^2a_5^2 - a_5^3 + a_2a_3a_5^2 + 2a_1a_4a_5^2 > 0\ .$$

Theorem A.3 (*Hopf Bifurcations*)[45,49] A *Hopf bifurcation* occurs when a periodic solution or limit cycles, surrounding an equilibrium point, arises or goes away as a parameter μ varies. When a stable limit cycle surrounds an unstable equilibrium point, the bifurcation is called a *supercritical Hopf bifurcation*. If the limit cycle is unstable and surrounds a stable equilibrium point, then the bifurcation is called a *subcritical Hopf bifurcation*. Consider the system:

Lyapunov Stability Theorem

Theorem A.4 (Stability) [44,45,47,48,51]. Let $x=0$ be an equilibrium point $\dot{x}=f(x)$ and $f:D\subset R^n$. Let $V:D\rightarrow R$ be a continuously differentiable function such that:

1. $V(0)=0$ and $V(x)=0$
2. $\dot{V}(x)>0$, $in\ D-\{0\}$
3. If $\dot{V}(x)\leq 0$, $in\ D-\{0\}$,then $x=0$ is "stable".

Theorem A.5 (Asymptotically Stable) [44,45,47,48,51]. Let $x=0$ be an equilibrium point $\dot{x}=f(x)$ and $f:D\subset R^n$. Let $V:D\rightarrow R$ be a continuously differentiable function such that:

1. $V(0)=0$ and $V(x)=0$
2. $\dot{V}(x)>0$, $in\ D-\{0\}$
3. If $\dot{V}(x)<0$, $in\ D-\{0\}$, then $x=0$ is "asymptotically stable".

Theorem A.6 (Globally Asymptotically Stable) [44,45,47,48,51] Let $x=0$ be an equilibrium point $\dot{x}=f(x)$ and $f:D\subset R^n$. Let $V:D\rightarrow R$ be a continuously differentiable function such that:

1. $V(0)=0$
2. $\dot{V}(x)>0$, $in\ D-\{0\}$
3. $\dot{V}(x)$ is "radially unbounded"
4. If $\dot{V}(x)<0$, $in\ D-\{0\}$, then $x=0$ is " globally asymptotically stable".

Theorem A.7 (LaSalle's theorem) [35,36] Let $f:D\subset R^n$ be a compact invariant set with respect to $\dot{x}=f(x)$. Let $V(x)$ be a continuously differentiable function defined over R^n such that $\dot{V}(x)\leq 0$ in D .

Let P be a set of all points in D where $\dot{V}(x)=0$ and E be the largest invariant set in P . Then every solution starting in D approaches E as $t\rightarrow\infty$.

Moreover, if the set P contains only one point $x=0$, then $x=0$ this point is asymptotically stable in D .

Theorem A.8 (Krazovskii - LaSalle's theorem) Let $x=0$ be an equilibrium point for $\dot{x}=f(x)$ and $D \subset R^n$. Let $V: D \subset R$ be a continuously differentiable positive definite function over R^n such that $\dot{V}(x) \leq 0$ in D . Let $P=\{x \in D: \dot{V}(x)=0\}$ and suppose that no other solution can stay in P , other than $x=0$. Then $x=0$ is asymptotically stable.

A2. Accepted Papers for Publication and Presented

For publication:

1.R.Sungchasit,P.Pungsumpun and I.M.Tang. **Transmission Model of Dengue Virus *Aedes aegypti And Aedes albopictus***. Far East Journal of Mathematical Sciences (FJMS), Volum83, Number1, 85-112, 2013.

2. R.Sungchasit,P.Pungsumpun and I.M.Tang. **SIR Transmission Model of Dengue Virus Taking Into Account Two Species of Mosquitoes and an Age Structure in the Human Population**. American Journal of Applied Sciences, 12(6): 426 – 443, 2015.

For presentation:

1. R. Sungchasit, P.Pongsumpun. **Transmission model of dengue disease with the different contact of three season in Thailand.** Proceedings of the 38th Congress on Science and Technology of Thailand. Chiang Mai, Thailand,2012.

2. R. Sungchasit, P. Pongsumpun. **Dengue transmission model with the different incubation rate for each season**, 1st Mae Fah Luang University International Conference 2012.

3. P. Pongsumpun, R. Sungchasit. **Mathematical model of dengue disease with the different seasons.** The Fourth TKU – KMITL Joint Symposium on Mathematics and Applied Mathematics (MAM 2014). Department of Mathematics, School of Science, Tokai University.

4. R.Sungchasit, P. Pongsumpun. **Effect of season on the transmission model of dengue disease.** The 1 st International Conference on Interdisciplinary Development Reseach 17 – 18 September 2015 (IDR 2015).

5.R.Sungchasit, P. Pongsumpun. **Analyzing of model for Dengue with its characteristics.** The 1 st International Conference on Interdisciplinary Development Reseach 17 – 18 September 2015 (IDR 2015).

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TRANSMISSION MODEL OF DENGUE VIRUS BY *Aedes aegypti* AND *Aedes albopictus*

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Abstract

Mathematical models are used for describing many diseases. Dengue disease is occurred by biting of infected *Aedes aegypti* and *Aedes albopictus* mosquitoes. Dengue outbreak is found during the rainy season. Each *Aedes* mosquito has the different dengue outbreaks and

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they depend on the temperature and areas. The standard dynamical modeling method is used in this study. The SIR (susceptible-infected-recovered) model is modified to describe the transmission of dengue virus by two species of vectors. The transmission of dengue virus is varied with time. The dynamical analysis method is used for analyzing this model. We confirm these results by using numerical results.

Introduction

Dengue is the disease transmitted by arthropods of the genus *Aedes*, is universally in different parts of the world. There are two species of dengue vectors such as *Aedes aegypti* and *Aedes albopictus*. *Aedes aegypti* is usually found in urban areas. *Aedes albopictus* is usually found in the suburban/rural areas.

Dengue disease has the different three forms: Dengue Fever (DF), Dengue Haemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS). It has emerged as an international public health problem, which is now an endemic in more than 100 countries. It affects about 40% of the world population (2.5 billion people) living in tropical and subtropical regions [1]. There are four serotypes of dengue virus, namely DEN-1, DEN-2, DEN-3 and DEN-4. Dengue disease cannot be directly transmitted between people. The infected female *Aedes* mosquito is the primary vector of this disease. The development of virus and mosquitoes is affected by climatic factors. The effect of extrinsic incubation period of dengue virus caused the seasonality transmission of this disease [2, 3]. When a vector bites someone who is infected with dengue virus, the virus is transferred to mosquito and it becomes infected mosquito. After the infected vector bites the susceptible human, the virus moves into the human bloodstream and it spreads throughout the body. Symptoms appear about eight to ten days after the biting of an infected mosquito. Symptoms are flu-like illness and can include high fever, nausea, vomiting, body aches, and headache. The moisture content, temperature, season and rainfall influenced the mosquito development. Dengue infection is endemic in Thailand. From data of dengue cases in 1999-2011, we can see that most dengue patients are occurred in rainy season.

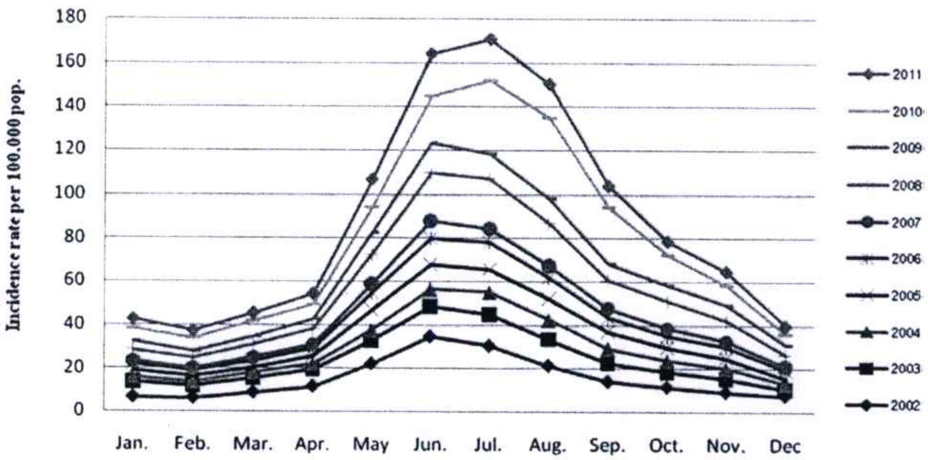


Figure 1. Reported cases of dengue disease per 100,000 population in Thailand during 2002 and 2011 (month-by-month).

The *Aedes aegypti* has spread throughout the tropics, especially in urban areas. *Aedes aegypti*, a day bitting mosquito, is highly anthropohilic and often resides near human dwellings [4]. *Aedes aegypti* mosquitoes are known to rest in secluded and unreachable places such as inside wardrobes and under beds [5, 6]. Insecticide-related factors (type, dosage, droplet size) and environmental factors (air temperature, wind direction, speed) also influenced the efficacy of ultra-low-volume insecticide application [6-8].

Aedes albopictus is commonly known as the Asian Tiger mosquito. It has been confirmed as the primary dengue vector for several outbreaks [9, 10]. *Aedes albopictus* is relatively exophilic, feeding on various kinds of hosts finding and feeding on human hosts at every gonotrophic cycle is not easy for *Aedes albopictus*. It is likely that the number of bites given to human per cycle of bites per gonotrophic cycle of *Aedes albopictus* is lower than that of *Aedes aegypti*, although the exact numbers are unknown [11, 12]. Vector mortality is important for entomological parameter that influences to the efficiency of viral transmission in mosquito-borne disease [13, 14]. Thus, we believe that it is important to consider the number of female *Aedes albopictus* per person that caused a large dengue outbreak in the past.

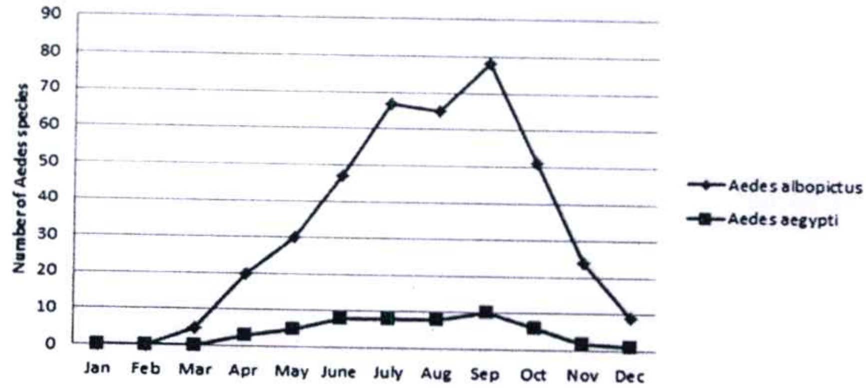


Figure 2. Monthly distribution of *Aedes aegypti* and *Aedes albopictus* from January to December 2010 [15].

From data of dengue vectors such as *Aedes aegypti* and *Aedes albopictus* from January to December, we can see that most mosquitoes are in June to October and during the rainy season.

This may provide some perspectives towards reducing the dengue epidemic potential in Thailand but also in other temperate regions where *Aedes aegypti* and *Aedes albopictus* are the vectors for dengue disease. Therefore T_0 is defined as the expected number of secondary cases produced by index case in a population during the entire period of infectiousness. When $T_0 > 1$, transmission is maintained and spread between the people; conversely, $T_0 < 1$ indicates that transmission declines and ceases.

Esteva and Vargas developed a model for dengue disease transmission and included the dynamics of the *Aedes aegypti* mosquito into standard SIR (susceptible-infected-recovered) epidemic model [16]. In 2010, Kongnuy and Pongsumpun [17] considered the transmission of dengue disease with the effect of season. Erickson et al. [18] considered a dengue model with a dynamic movement of *Aedes albopictus* vector population. Pongsumpun and Kongnuy [19] considered the Lyapunov function of dengue model infant with maternal antibody.

In this paper, we develop a mathematical model for dengue transmission dynamics on the reported number of dengue diseases. We used SIR model for analyzing and finding the method to decrease the outbreak of this disease. The purpose of this study is to use mathematical model to study the behavior for the transmission of dengue disease in population containing two species of dengue vectors such as *Aedes aegypti* and *Aedes albopictus*. In the next section, we formulate the model. In Section 3, we analyze the model. The equilibrium states of this dynamical system and their stability are obtained. In the last section, we give the numerical results and conclusion.

Methods

Mathematical model

To create our model, we define an SIR and SI equation into both human and mosquito population, respectively. Then we modify it by incorporating the different behaviors of *Aedes aegypti* and *Aedes albopictus*.

The considered population in this study are human and mosquito (*Aedes aegypti* and *Aedes albopictus*), we assume that human and mosquito population have constant sizes. The human population is divided into susceptible, infected and recovered classes. The mosquito population is divided into susceptible and infected classes because the mosquito never recovers from infection.

Let

$S'(t)$ be the number of susceptible human population at time t,

$I'_a(t)$ be the number of infected human population who be infected with *Aedes aegypti* at time t,

$I'_b(t)$ be the number of infected human population who be infected with *Aedes albopictus* at time t,

$R'(t)$ be the number of recovered human population at time t,

$S'_{va}(t)$ be the number of susceptible *Aedes aegypti* population at time t,

$I'_{va}(t)$ be the number of infected *Aedes aegypti* population at time t,

$S'_{vb}(t)$ be the number of susceptible *Aedes albopictus* population at time t,

$I'_{vb}(t)$ be the number of infected mosquito population *Aedes albopictus* at time t.

The dynamical change of human population is given by

$$\frac{d}{dt}S'(t) = \delta N_t - \lambda_a(1 + \varepsilon_a \operatorname{Sin}\gamma t)I'_{va}(t)S'(t) - \eta_d S'(t) - \eta_h S'(t)$$
$$- \lambda_b(1 + \varepsilon_b \operatorname{Sin}\gamma t)I'_{vb}(t)S'(t), \tag{1}$$

$$\frac{d}{dt}I'_a(t) = \lambda_a(1 + \varepsilon_a \operatorname{Sin}\gamma t)I'_{va}(t)S'(t) - \eta_d I'_a(t) - \eta_h I'_a(t) - \beta_{ha} I'_a(t), \tag{2}$$

$$\frac{d}{dt}I'_b(t) = \lambda_b(1 + \varepsilon_b \operatorname{Sin}\gamma t)I'_{vb}(t)S'(t) - \eta_d I'_b(t) - \eta_h I'_b(t) - \beta_{hb} I'_b(t), \tag{3}$$

$$\frac{d}{dt}R'(t) = -\eta_d R'(t) - \eta_h R'(t) - \beta_{ha} I'_a(t) - \beta_{hb} I'_b(t). \tag{4}$$

The dynamical change of mosquito population is given by

$$\frac{d}{dt}S'_{va}(t) = K_a - \alpha_{va}(1 + \varepsilon_{va} \operatorname{Sin}\gamma t)S'_{va}(t)I'_a(t) - \eta_{va} S'_{va}(t), \tag{5}$$

$$\frac{d}{dt}I'_{va}(t) = \alpha_{va}(1 + \varepsilon_{va} \operatorname{Sin}\gamma t)S'_{va}(t)I'_a(t) - \eta_{va} I'_{va}(t), \tag{6}$$

$$\frac{d}{dt}S'_{vb}(t) = K_b - \alpha_{vb}(1 + \varepsilon_{vb} \operatorname{Sin}\gamma t)S'_{vb}(t)I'_b(t) - \eta_{vb} S'_{vb}(t), \tag{7}$$

$$\frac{d}{dt}I'_{vb}(t) = \alpha_{vb}(1 + \varepsilon_{vb} \operatorname{Sin}\gamma t)S'_{vb}(t)I'_{vb}(t) - \eta_{vb} I'_{vb}(t). \tag{8}$$

We define

δ is birth rate of human population,

N_t is the total human population,

λ_a is the biting rate of *Aedes aegypti* population,

λ_b is the biting rate of *Aedes albopictus* population,

ε_a is a measure of influence on the transmission process from human population to *Aedes aegypti*,

ε_b is a measure of influence on the transmission process from human population to *Aedes albopictus*,

ε_{va} is a measure of influence on the transmission process from *Aedes aegypti* to human population,

ε_{vb} is a measure of influence on the transmission process from *Aedes albopictus* to human population,

β_{ha} is the recovery rate of human population who be infected with *Aedes aegypti*,

β_{hb} is the recovery rate of human population who be infected with *Aedes albopictus*,

η_h is the natural death rate of human population,

η_d is the death rate of human population due to the disease,

η_{va} is the death rate of *Aedes aegypti* population,

η_{vb} is the death rate of *Aedes albopictus* population,

α_{va} is the transmission probability of dengue disease from vector (*Aedes aegypti*) to human population,

α_{vb} is the transmission probability of dengue disease from vector (*Aedes albopictus*) to human population,

K_a is the constant recruitment rate of vector population (*Aedes aegypti*),

K_b is the constant recruitment rate of vector population (*Aedes albopictus*).

Analysis of the transmission models

We suppose that $N_t = S'(t) + I'_a(t) + I'_b(t) + R'(t)$, $N_{va} = S'(t) + I'_{va}(t)$ and $N_{vb} = S'(t) + I'_{vb}(t)$. We normalize equations by defining new variables

$$S = \frac{S'(t)}{N_t}, \quad I_a = \frac{I'_a(t)}{N_t}, \quad I_b = \frac{I'_b(t)}{N_t}, \quad R = \frac{R'(t)}{N_t},$$

$$S_{va} = \frac{S'_{va}(t)}{N_{va}}, \quad I_{va} = \frac{I'_{va}(t)}{N_{va}}, \quad S_{vb} = \frac{S'_{vb}(t)}{N_{vb}}, \quad I_{vb} = \frac{I'_{vb}(t)}{N_{vb}},$$

where N_{va} is the total vector population (*Aedes aegypti*),

N_{vb} is the total vector population (*Aedes albopictus*).

The total human and vector populations have constant sizes, thus rates of change for total human and vector populations equal to zero

$$\frac{d}{dt}N_t = 0, \ \frac{d}{dt}N_{va} = 0 \ \text{ and } \ \frac{d}{dt}N_{vb} = 0.$$

Thus, the birth and death rates are equivalent for human populations $(\delta = \eta_d + \eta_h)$; the total vector populations equal to $\frac{K_a}{\eta_{va}}$ for *Aedes aegypti* and the total vector populations equal to $\frac{K_b}{\eta_{vb}}$ for *Aedes albopictus*. Then the reduced equations become

$$\frac{d}{dt}S(t) = (\eta_d + \eta_h) - [\eta_d + \eta_h + \lambda_a(1 + \varepsilon_a \operatorname{Sin}\gamma t)I_{va}(t)N_{va} + \lambda_b(1 + \varepsilon_b \operatorname{Sin}\gamma t)I_{vb}(t)N_{vb}]S(t), \tag{9}$$

$$\frac{d}{dt}I_a(t) = \lambda_a(1 + \varepsilon_a \operatorname{Sin}\gamma t)I_{va}(t)N_{va}S(t) - (\eta_d + \eta_h + \beta_{ha})I_a(t), \tag{10}$$

$$\frac{d}{dt}I_b(t) = \lambda_b(1 + \varepsilon_b \operatorname{Sin}\gamma t)I_{vb}(t)N_{vb}S(t) - (\eta_d + \eta_h + \beta_{hb})I_b(t), \tag{11}$$

$$\frac{d}{dt}I_{va}(t) = \alpha_{va}(1 + \varepsilon_{va} \operatorname{Sin}\gamma t)S_{va}(t)I_a(t)N_t - \eta_{va}I_{va}(t), \tag{12}$$

$$\frac{d}{dt}I_{vb}(t) = \alpha_{vb}(1 + \varepsilon_{vb} \operatorname{Sin}\gamma t)S_{vb}(t)I_b(t)N_t - \eta_{vb}I_{vb}(t) \tag{13}$$

with conditions $S + I_a + I_b + R = 1$, $S_{va} + I_{va} = 1$ and $S_{vb} + I_{vb} = 1$.

A. Equilibrium points

In this paper, equations (9)-(13) have two equilibrium points: the disease free equilibrium and endemic equilibrium points. Two equilibrium points are found by setting the right hand sides of (9)-(13) to zero. The results are as follows:

(i) The disease free equilibrium point

$$E_0 = (1, 0, 0, 0, 0).$$

(ii) The endemic disease equilibrium point

$$E_1 = (S^*,\ I_a^*,\ I_b^*,\ I_{va}^*,\ I_{vb}^*),$$

where

$$S^* = \frac{\tau_1}{\tau_2}, \tag{14}$$

$$I_a^* = \frac{I_{va}^* N_{va} \lambda_a (1 + \varepsilon_a\, Sin\, \gamma t)(\tau_1)}{(\beta_{ha} + \eta_h + \eta_d)(\tau_2)(\tau_3)}, \tag{15}$$

$$I_b^* = \frac{I_{vb}^* N_{vb} \lambda_b (1 + \varepsilon_b\, Sin\, \gamma t)(\tau_1)}{(\beta_{hb} + \eta_d + \eta_h)(\tau_2)(\tau_3)}, \tag{16}$$

$$\begin{aligned} I_{va}^* = {} & [2(N_{va}\alpha_{va}(2 + \varepsilon_a\varepsilon_{va})\tau_5\lambda_a + \tau_6(-2\tau_4 \\ & + N_t N_{va}\alpha_{va}(2 + \varepsilon_a\varepsilon_{va} + (\varepsilon_a + \varepsilon_{va})\varepsilon_{vb})\lambda_a) - N_{vb}\alpha_{vb}(2 + \varepsilon_b\varepsilon_{vb})\lambda_b\tau_4) \\ & -2(N_{va}\alpha_{va}(N_t\alpha_{vb}(\varepsilon_a\varepsilon_{va} + (\varepsilon_a + \varepsilon_{va})\varepsilon_{vb})(\eta_h + \eta_d) + \varepsilon_a\varepsilon_{va}\tau_5)\lambda_a \\ & - N_{vb}\alpha_{vb}\varepsilon_b\varepsilon_{vb}\lambda_b\tau_4)Cos(2\gamma t) + (4N_{va}\alpha_{va}(\varepsilon_a + \varepsilon_{va})\tau_5\lambda_a \\ & + \tau_6(-4\varepsilon_{vb}\tau_4 + N_t N_{va}\alpha_{va}(4(\varepsilon_a + \varepsilon_{va}) + (4 + 3\varepsilon_a\varepsilon_{va})\varepsilon_{vb})\lambda_a) \\ & - 4N_{vb}\alpha_{vb}(\varepsilon_b + \varepsilon_{vb})\tau_4\lambda_6)Sin(\gamma t) - N_t N_{va}\alpha_{va}\varepsilon_a\varepsilon_{va}\varepsilon_{vb}\tau_6\lambda_a Sin(3\gamma t)]/ \\ & [2N_{va}\lambda_a(1 + \varepsilon_a Sin(\gamma t))(N_t\alpha_{va}\alpha_{vb}(2 + \varepsilon_{va}\varepsilon_{vb})(\eta_h + \eta_d) \\ & + 2\alpha_{vb}\tau_4 + 2\alpha_{va}\tau_5 - N_t\alpha_{va}\varepsilon_{va}\varepsilon_{vb}\tau_6 Cos(\gamma t) \\ & + (N_t\alpha_{va}(\varepsilon_{va} + \varepsilon_{vb})\tau_6 + \alpha_{vb}\varepsilon_{vb}\tau_4 + \alpha_{va}\varepsilon_{va}\tau_5)Sin(\gamma t))], \end{aligned} \tag{17}$$

$$I_{vb}^* = \frac{\begin{aligned}(-(\tau_5)(\eta_d + \eta_h + I_{va}^* N_{va}\lambda_a) + N_t N_{vb}\lambda_b\tau_6 \\ + Sin(\gamma t)(-I_{va}^* N_{va}\lambda_a\varepsilon_a\tau_5 + N_t N_{vb}(\varepsilon_b + \varepsilon_{vb})\lambda_b\tau_6 \\ + N_t N_{vb}\varepsilon_b\varepsilon_{vb}\lambda_b\tau_6 Sin(\gamma t)))\end{aligned}}{(N_{vb}\lambda_b(1 + \varepsilon_b Sin(\gamma t))(N_t\tau_6 + \tau_5 + N_t\varepsilon_{vb}\tau_6 Sin(\gamma t)))} \tag{18}$$

and

$$\tau_1 = N_t\alpha_{vb}(\eta_h + \eta_d) + (\beta_{hb} + \eta_h + \eta_d)\eta_{vb} + N_t\alpha_{vb}\varepsilon_{vb}(\eta_h + \eta_d)Sin(\gamma t),$$

$\tau_2 = (\eta_h + \eta_d + I^*_{va} N_{va} \lambda_a + N_{vb} \lambda_b + (I^*_{va} N_{va} \lambda_a \varepsilon_a N_{va} \lambda_a + N_{vb} \varepsilon_b \lambda_b) Sin(\gamma t)),$

$\tau_3 = N_t \alpha_{vb} (1 + \varepsilon_{vb} Sin(\gamma t)),$

$\tau_4 = (\beta_{ha} + \eta_h + \eta_d) \eta_{va},$

$\tau_5 = (\beta_{hb} + \eta_h + \eta_d) \eta_{vb},$

$\tau_6 = \alpha_{vb} (\eta_h + \eta_d).$

B. Local stability

The local stability of each equilibrium point is determined from linearizing equations (9)-(13) about equilibrium point examining the eigenvalues of the resulting Jacobian matrix. We now consider the eigenvalues of the Jacobian matrix at each equilibrium point. If all eigenvalues have negative real parts, then that equilibrium point is locally asymptotically stable. The standard dynamical modeling method is used in this study.

Proposition 1. *If* $T_0^* < 1$ *and* $\gamma = 0$, *then the equilibrium* E_0 *is locally asymptotically stable.*

Proof. For the disease free equilibrium point $E_0 = (1, 0, 0, 0, 0)$.

From equations (9)-(13), the Jacobian matrix evaluated at $E_0 = (1, 0, 0, 0, 0)$ is given by

$$\begin{bmatrix} -(\eta_d + \eta_h) & 0 & 0 & -\lambda_a(1 + \varepsilon_a Sin(\gamma t)) N_{va} & -\lambda_b(1 + \varepsilon_b Sin(\gamma t)) N_{vb} \\ 0 & -(\eta_d + \eta_h + \beta_{ha}) & 0 & \lambda_a(1 + \varepsilon_a Sin(\gamma t)) N_{va} & 0 \\ 0 & 0 & -(\eta_d + \eta_h + \beta_{hb}) & 0 & \lambda_b(1 + \varepsilon_b Sin(\gamma t)) N_{vb} \\ 0 & \alpha_{va}(1 + \varepsilon_{va} Sin(\gamma t)) N_t & 0 & -\eta_{va} & 0 \\ 0 & 0 & \alpha_{vb}(1 + \varepsilon_{vb} Sin(\gamma t)) N_t & 0 & -\eta_{vb} \end{bmatrix}.$$

The eigenvalues are solutions of the characteristic equation, $\det(J - \lambda I_5) = 0$.

For $\gamma = 0$, we obtain the following characteristic equation:

$$(\lambda + \eta_h + \eta_d)(\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4) = 0, \tag{19}$$

where

$$a_1 = \beta_{ha} + \beta_{hb} + 2\eta_h + 2\eta_d + \eta_{va} + \eta_{vb}, \tag{20}$$

$$\begin{aligned} a_2 &= (\eta_h + \eta_d)(\eta_h + \eta_d + 2\eta_{va}) + (2(\eta_h + \eta_d) + \eta_{va})\eta_{vb} \\ &\quad + \beta_{hb}(\eta_h + \eta_d + \eta_{va} + \eta_{vb}) + \beta_{ha}(\beta_{hb} + \eta_h + \eta_d + \eta_{va} + \eta_{vb}) \\ &\quad - N_t(N_{va}\alpha_{va}\lambda_a + N_{vb}\alpha_{vb}\lambda_b), \end{aligned} \tag{21}$$

$$\begin{aligned} a_3 &= (\beta_{ha} + \eta_h + \eta_d)(\beta_{hb} + \eta_h + \eta_d)\eta_{va} \\ &\quad + ((\beta_{ha} + \eta_h + \eta_d)((\beta_{hb} + \eta_h + \eta_d) + (\beta_{ha} + \beta_{hb} + 2(\eta_h + \eta_d))\eta_{va})\eta_{vb} \\ &\quad - N_t(N_{va}\alpha_{va}(\beta_{hb} + \eta_h + \eta_d + \eta_{vb})\lambda_a \\ &\quad + N_{vb}\alpha_{vb}(\beta_{ha} + \eta_h + \eta_d + \eta_{va})\lambda_b)), \end{aligned} \tag{22}$$

$$\begin{aligned} a_4 &= ((\beta_{ha} + \eta_h + \eta_d)\eta_{va} - N_t N_{va}\alpha_{va}\lambda_a)((\beta_{hb} + \eta_h + \eta_d)\eta_{vb} \\ &\quad - N_t N_{vb}\alpha_{vb}\lambda_b). \end{aligned} \tag{23}$$

There are five eigenvalues corresponding to (19). We can see that one eigenvalue is $\lambda = -\eta_h - \eta_d$, if $T_0^* < 1$. Next, we check the sign of other four eigenvalues are obtained by solving the equation $(\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4) = 0$. The remaining four eigenvalues have negative real parts if they satisfy Routh-Hurwitz criteria (20)-(23), each equilibrium point is locally asymptotically stable if the following conditions are satisfied:

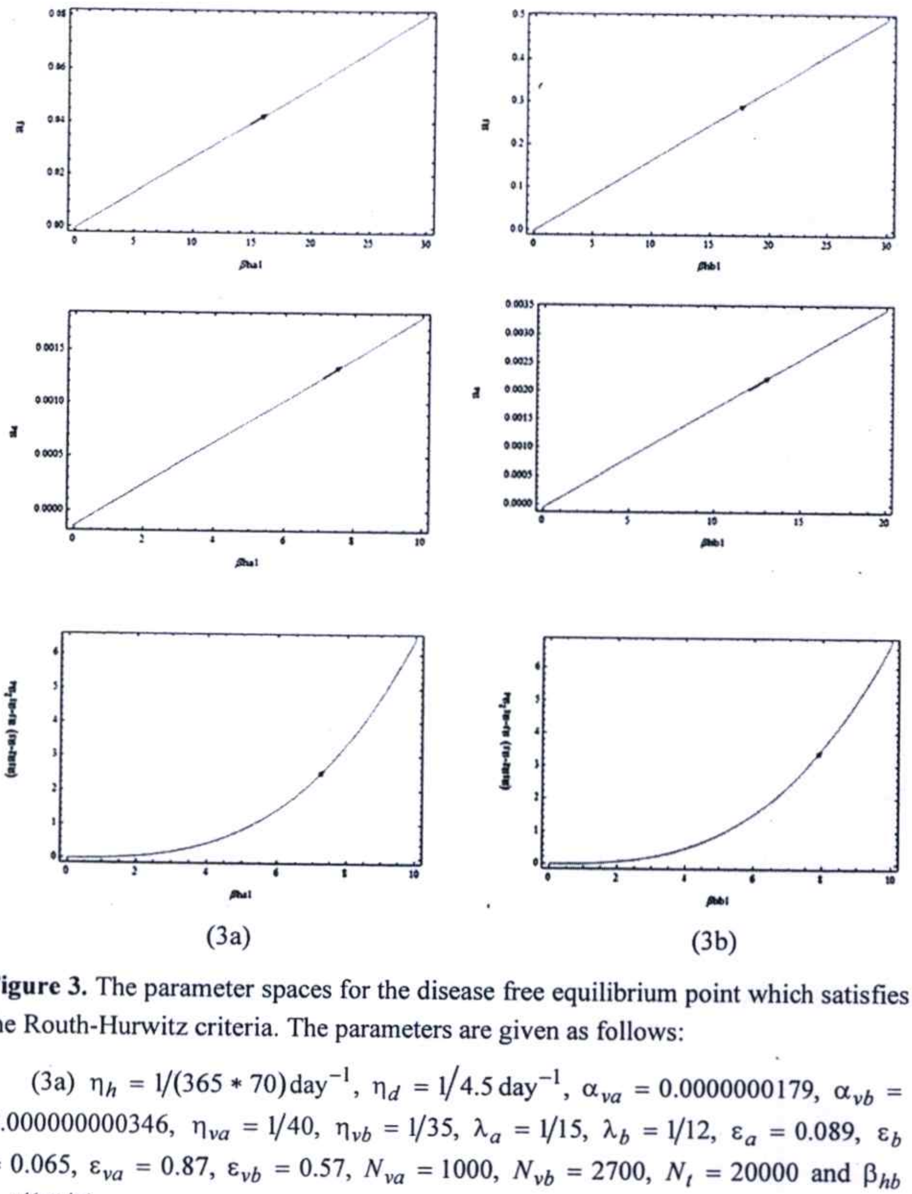
$$a_1 > 0, \tag{24}$$

$$a_3 > 0, \tag{25}$$

$$a_4 > 0, \tag{26}$$

$$a_1 a_2 a_3 > a_3^2 + a_1^2 a_4. \tag{27}$$

After checking the locally asymptotically stable of E_0, we can see that a_1 is always positive. For the equations given by (25)-(27), we show these conditions by using the following figure:



(3a) (3b)

Figure 3. The parameter spaces for the disease free equilibrium point which satisfies the Routh-Hurwitz criteria. The parameters are given as follows:

(3a) $\eta_h = 1/(365*70)\,\text{day}^{-1}$, $\eta_d = 1/4.5\,\text{day}^{-1}$, $\alpha_{va} = 0.0000000179$, $\alpha_{vb} = 0.000000000346$, $\eta_{va} = 1/40$, $\eta_{vb} = 1/35$, $\lambda_a = 1/15$, $\lambda_b = 1/12$, $\varepsilon_a = 0.089$, $\varepsilon_b = 0.065$, $\varepsilon_{va} = 0.87$, $\varepsilon_{vb} = 0.57$, $N_{va} = 1000$, $N_{vb} = 2700$, $N_t = 20000$ and $\beta_{hb} = 1/(19/2)$.

(3b) $\eta_h = 1/(365*70)\,\text{day}^{-1}$, $\eta_d = 1/4.5\,\text{day}^{-1}$, $\alpha_{va} = 0.0000000179$, $\alpha_{vb} = 0.000000000346$, $\eta_{va} = 1/40$, $\eta_{vb} = 1/35$, $\lambda_a = 1/15$, $\lambda_b = 1/12$, $\varepsilon_a = 0.089$, $\varepsilon_b = 0.065$, $\varepsilon_{va} = 0.87$, $\varepsilon_{vb} = 0.57$, $N_{va} = 1000$, $N_{vb} = 2700$, $N_t = 20000$ and $\beta_{ha} = 1/(17/2)$.

From the above figures, the Routh-Hurwitz conditions are satisfied for $T_0^* < 1$.

C. Disease endemic equilibrium point

For the endemic disease equilibrium point, E_1, we now consider eigenvalues of the Jacobian matrix at each equilibrium point. If all eigenvalues have negative real parts, then the local stability analysis of each equilibrium state is given in the following proposition.

Proposition 2. *If $\widetilde{T}_0 > 1$ and $\gamma = 0$, then the equilibrium point E_1 is locally asymptotically stable.*

Proof. For the endemic disease equilibrium point $E_1 = (S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$.

From equations (9)-(13), the Jacobian matrix evaluated at $E_1 = (S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ is given by

$$\begin{bmatrix} -[\eta_d + \eta_h + \lambda_a(1 + \varepsilon_a \, Sin\,\gamma t) I_{va}^* N_{va} + \lambda_b(1 + \varepsilon_b \, Sin\,\gamma t) I_{vb}^* N_{vb}] & 0 \\ \lambda_a(1 + \varepsilon_a \, Sin\,\gamma t) I_{va}^* N_{va} & -(\eta_d + \eta_h + \beta_{ha}) \\ \lambda_b(1 + \varepsilon_b \, Sin\,\gamma t) I_{vb}^* N_{vb} & 0 \\ 0 & \alpha_{va}(1 + \varepsilon_{va} \, Sin\,\gamma t)(1 - I_{va}^*) N_t \\ 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 0 & -\lambda_a(1 + \varepsilon_a \, Sin\,\gamma t) N_{va} S^* & -\lambda_b(1 + \varepsilon_b \, Sin\,\gamma t) I_{vb}^* N_{vb} S^* \\ 0 & \lambda_a(1 + \varepsilon_a \, Sin\,\gamma t) N_{va} S^* & 0 \\ -(\eta_d + \eta_h + \beta_{hb}) & 0 & \lambda_b(1 + \varepsilon_b \, Sin\,\gamma t) N_{vb} S^* \\ 0 & -\eta_{va} & 0 \\ \alpha_{vb}(1 + \varepsilon_{vb} \, Sin\,\gamma t)(1 - I_{vb}^*) N_t & 0 & -\eta_{vb} \end{matrix} \Bigg].$$

The eigenvalues are solutions of the characteristic equation, $\det(J - \lambda I_5) = 0$. We consider two cases:

$\gamma = 0$ and $\gamma \neq 0$. For $\gamma = 0$, we obtain the following characteristic equation:

$$(\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5) = 0,$$

where S^*, I_a^*, I_b^*, I_{va}^* and I_{vb}^* are given by equations (14)-(18). The characteristic equations for the Jacobian matrix evaluated at the equilibrium point are given by (9)-(13). It can be seen that eigenvalues, obtained by solving $(\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5) = 0$, where

$$a_1 = \frac{\gamma_{11}}{\gamma_5} + \frac{1}{\gamma_9}((\beta_{ha} + \beta_{hb} + 3\eta_h + 3\eta_d + \eta_{va} + \eta_{vb})\gamma_9$$

$$+ N_{vb}\lambda_b(-\gamma_{10}\tau_5 + N_t N_{vb}\lambda_b\tau_6)),$$

$$a_2 = \frac{\gamma_{11}(\beta_{ha} + \beta_{hb} + 2\eta_h + 2\eta_d + \eta_{va} + \eta_{vb} + N_t\alpha_{va}\gamma_{12})}{\gamma_5}$$

$$+ \frac{1}{\gamma_9}(\gamma_9(3(\eta_h + \eta_d)(\eta_h + \eta_d + \eta_{va}) + (3(\eta_h + \eta_d) + \eta_{va})\eta_{vb}$$

$$+ \beta_{hb}(2\eta_h + 2\eta_d + \eta_{va} + \eta_{vb}) + \beta_{ha}(\beta_{hb} + 2\eta_h + 2\eta_d + \eta_{va} + \eta_{vb})$$

$$- N_t(N_{va}\alpha_{va}\lambda_a + N_{vb}\alpha_{vb}\lambda_b)\gamma_{12})$$

$$+ N_{vb}\lambda_b(\beta_{ha} + \beta_{hb} + 2\eta_h + 2\eta_d + \eta_{va} + \eta_{vb} + N_t\alpha_{va}\gamma_{12})$$

$$(-\gamma_{10}\tau_5 + N_t N_{vb}\lambda_b\tau_6)),$$

$$a_3 = \frac{1}{\gamma_5\gamma_9}(\gamma_{11}\gamma_9((\eta_h + \eta_d)(\eta_h + \eta_d + 2\eta_{va}) + (2(\eta_h + \eta_d) + \eta_{va})\eta_{vb}$$

$$+ \beta_{hb}(\eta_h + \eta_d + \eta_{va} + \eta_{vb}) + \beta_{ha}(\beta_{hb} + \eta_h + \eta_d + \eta_{va} + \eta_{vb})$$

$$+ N_t(\alpha_{va}(\beta_{hb} + 2(\eta_h + \eta_d) + \eta_{vb}) - N_{vb}\alpha_{vb}\lambda_b)\gamma_{12})$$

$$+ N_t N_{vb}(\alpha_{va} + \alpha_{vb})\lambda_b\gamma_{12}(-\gamma_{10}\tau_5 + N_t N_{vb}\lambda_b\tau_6))$$

$$+ \gamma_5(\gamma_9(\beta_{hb}(\eta_h + \eta_d)(\eta_h + \eta_d + 2\eta_{va}) + \beta_{hb}(2(\eta_h + \eta_d) + \eta_{va})\eta_{vb})$$

$$+ (\eta_h + \eta_d)((\eta_h + \eta_d)(\eta_h + \eta_d + 3\eta_{va}) + 3\tau_5)$$

$$+ \beta_{ha}((\eta_h + \eta_d)(\eta_h + \eta_d + 2\eta_{va}) + (2(\eta_h + \eta_d) + \eta_{va})\eta_{vb}$$

$$+ \beta_{hb}(\eta_h + \eta_d + \eta_{va} + \eta_{vb})) - N_t(N_{va}\alpha_{va}(\beta_{hb} + 2(\eta_h + \eta_d) + \eta_{vb})\lambda_a$$

$$+ N_{vb}\alpha_{vb}(\beta_{ha} + 2(\eta_h + \eta_d) + \eta_{va})\lambda_b)\gamma_{12})$$

$$+ N_{vb}\lambda_b((\eta_h + \eta_d)(\eta_h + \eta_d + 2\eta_{va}) + 2(\eta_h + \eta_d) + \eta_{va})\eta_{vb}$$

$$+ \beta_{hb}(\eta_h + \eta_d + \eta_{va} + \eta_{vb}) + \beta_{ha}(\beta_{hb} + \eta_h + \eta_d + \eta_{va} + \eta_{vb})$$
$$+ N_t(\alpha_{vb}(\beta_{ha}2(\eta_h + \eta_d) + \eta_{va}) - N_{va}\alpha_{va}\lambda_a)\gamma_{12})$$
$$(-\gamma_{10}\tau_5 + N_t N_{vb}\lambda_b\tau_6))),$$

$$a_4 = \frac{1}{\gamma_5\gamma_9}(\gamma_{11}\gamma_9((\beta_{hb} + \eta_h + \eta_d)\tau_4 + (\beta_{ha} + \eta_h + \eta_d)(\beta_{hb} + \eta_h + \eta_d)$$
$$+ (\beta_{ha} + \beta_{hb} + 2(\eta_h + \eta_d))\eta_{va}) + \eta_{vb} + N_t\gamma_{12}(\alpha_{va}(\eta_h + \eta_d))\eta_{vb}$$
$$- N_{vb}\alpha_{vb}(\beta_{ha} + \eta_h + \eta_d + \eta_{va})\lambda_b - N_t N_{vb}\alpha_{va}\alpha_{vb}\lambda_b\gamma_{12}))$$
$$+ N_t N_{vb}\lambda_b\gamma_{12}(\alpha_{vb}(\beta_{ha} + \eta_h + \eta_d + \eta_{va}) + \alpha_{va}(\beta_{hb} + \eta_h + \eta_d + \eta_{vb})$$
$$+ N_t\alpha_{va}\alpha_{vb}\gamma_{12})(-\gamma_{10}\tau_5 + N_t N_{vb}\lambda_b\tau_6))$$
$$+ \gamma_5(\gamma_9((\eta_h + \eta_d)(\beta_{hb} + \eta_h + \eta_d)\tau_4$$
$$+ ((\eta_h + \eta_d)(\beta_{ha} + \eta_h + \eta_d)(\beta_{hb} + \eta_h + \eta_d)$$
$$+ (\beta_{ha}(\beta_{hb} + 2(\eta_h + \eta_d)) + (\eta_h + \eta_d)(2\beta_{hb} + 3(\eta_h + \eta_d)))\eta_{va})\eta_{vb}$$
$$+ N_t\gamma_{12}(-N_{va}\alpha_{va}(\beta_{hb}(\eta_h + \eta_d + \eta_{vb}) + (\eta_h + \eta_d)(\eta_h + \eta_d + 2\eta_{vb}))\lambda_a$$
$$- N_{vb}\alpha_{vb}(\beta_{ha}(\eta_h + \eta_d + \eta_{va}) + (\eta_h + \eta_d)(\eta_h + \eta_d + 2\eta_{va})\lambda_b$$
$$+ N_t N_{va}N_{vb}\alpha_{va}\alpha_{vb}\lambda_a\lambda_b\gamma_{12}))$$
$$+ N_{vb}\lambda_b((\beta_{hb} + \eta_h + \eta_d)\tau_4 + ((\beta_{ha} + \eta_h + \eta_d)(\beta_{hb} + \eta_h + \eta_d)$$
$$+ (\beta_{ha} + \beta_{hb} + 2(\eta_h + \eta_d))\eta_{va})\eta_{vb} + N_t\gamma_{12}(\tau_6(\beta_{ha} + \eta_h + \eta_d)$$
$$+ \alpha_{vb}(\beta_{ha} + 2(\eta_h + \eta_d))\eta_{va} - N_{va}\alpha_{va}(\beta_{hb} + \eta_h + \eta_d + \eta_{vb})\lambda_a$$
$$- N_t N_{va}\alpha_{va}\alpha_{vb}\lambda_a\gamma_{12}))(-\gamma_{10}\tau_5 + N_t N_{vb}\lambda_b\tau_6))),$$

$$a_5 = \frac{1}{\gamma_5\gamma_9}(\gamma_5(\tau_4 - N_t N_{va}\alpha_{va}\lambda_a\gamma_{12})(\eta_h + \eta_d)\gamma_9(\tau_5 - N_t N_{vb}\alpha_{vb}\lambda_b\gamma_{12})$$
$$+ N_{vb}\lambda_b(\tau_5 + N_t\alpha_{vb}(\eta_h + \eta_d)\gamma_{12}(-\gamma_{10}\tau_5 + N_t N_{vb}\lambda_b\tau_6))$$
$$+ \gamma_{11}(\gamma_9(\tau_4 + N_t\alpha_{va}(\eta_h + \eta_d)\gamma_{12}(\tau_5 - N_t N_{vb}\alpha_{vb}\lambda_b\gamma_{12})$$
$$+ N_t N_{vb}\lambda_b\gamma_{12}(\alpha_{vb}\tau_4 + \alpha_{va}\tau_5 + N_t\alpha_{va}\alpha_{vb}(\eta_h + \eta_d)\gamma_{12}$$
$$(-\gamma_{10}\tau_5 + N_t N_{vb}\lambda_b\tau_6))).$$

These five eigenvalues have negative real parts if they satisfy Routh-Hurwitz criteria:

$$a_1 > 0, \tag{28}$$

$$a_2 > 0, \tag{29}$$

$$a_3 > 0, \tag{30}$$

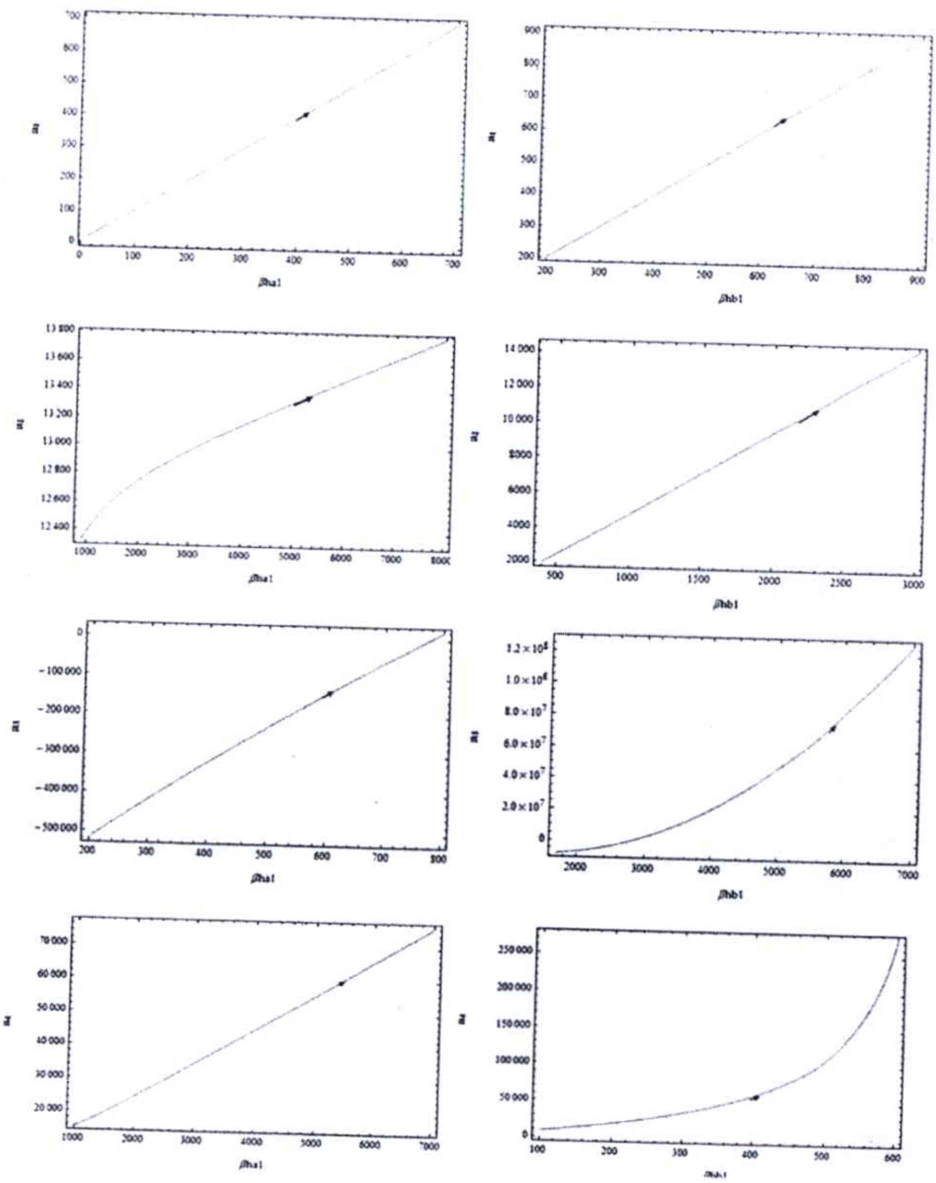
$$a_4 > 0, \tag{31}$$

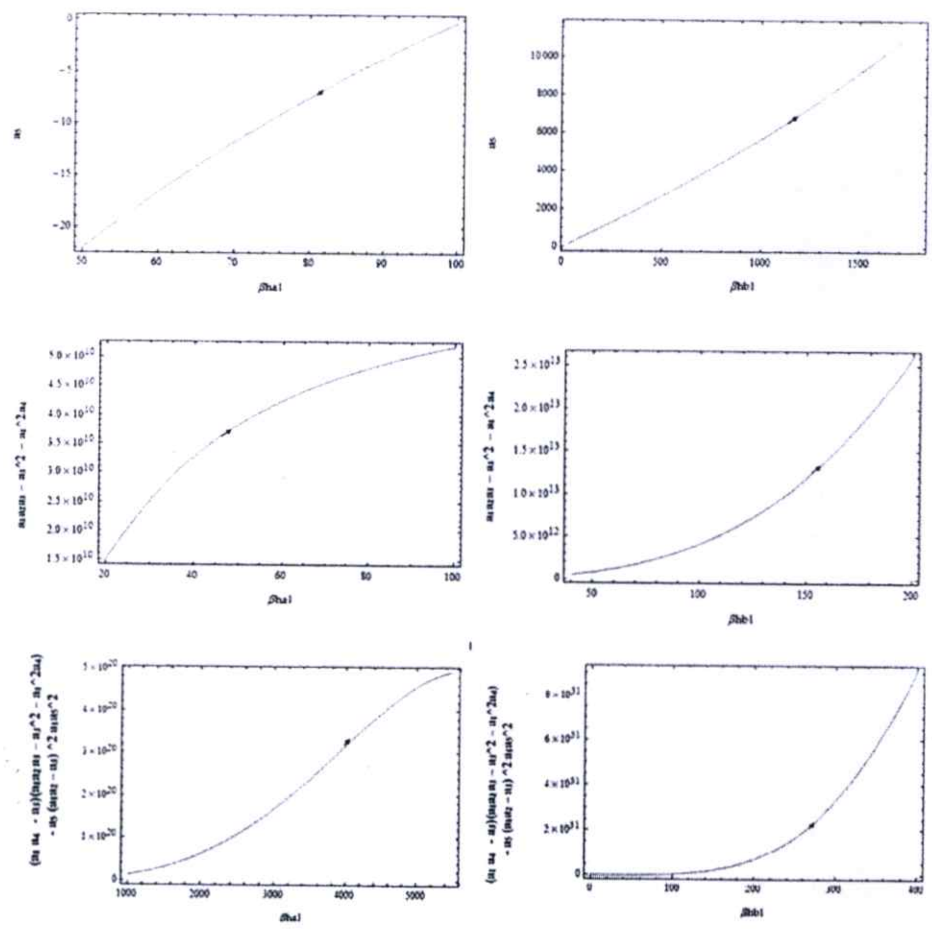
$$a_5 > 0, \tag{32}$$

$$a_1 a_2 a_3 > a_3^2 + a_1^2 a_4, \tag{33}$$

$$(a_1 a_4 - a_5)(a_1 a_2 a_3 - a_3^2 - a_1^2 a_4) - a_5(a_1 a_2 - a_3)^2 - a_1 a_5^2 > 0. \tag{34}$$

We check the stability of endemic equilibrium point by using conditions (28)-(34). We show the above conditions by Figure 4:


βha1
βhb1



(4a) (4b)

Figure 4. The parameter spaces for endemic disease equilibrium point which satisfies the Routh-Hurwitz criteria with the value of parameters: respectively.

(4a) $\eta_h = 1/(365 * 70)\text{day}^{-1}$, $\eta_d = 1/4.5\,\text{day}^{-1}$, $\alpha_{va} = 0.00000079$, $\alpha_{vb} = 0.0000000046$, $\eta_{va} = 1/12$, $\eta_{vb} = 1/18$, $\lambda_a = 1/5$, $\lambda_b = 1/8$, $\varepsilon_a = 0.07$, $\varepsilon_b = 0.04$, $\varepsilon_{va} = 0.6$, $\varepsilon_{vb} = 0.03$, $N_{va} = 8000$, $N_{vb} = 3000$, $N_t = 20000$ and $\beta_{hb} = 1/(19/2)$.

(4b) $\eta_h = 1/(365 * 70)\text{day}^{-1}$, $\eta_d = 1/4.5\,\text{day}^{-1}$, $\alpha_{va} = 0.00000079$, $\alpha_{vb} = 0.0000000046$, $\eta_{va} = 1/12$, $\eta_{vb} = 1/18$, $\lambda_a = 1/5$, $\lambda_b = 1/8$, $\varepsilon_a = 0.07$, $\varepsilon_b = 0.04$, $\varepsilon_{va} = 0.6$, $\varepsilon_{vb} = 0.03$, $N_{va} = 8000$, $N_{vb} = 3000$, $N_t = 20000$ and $\beta_{ha} = 1/(17/2)$.

From the above figures, the Routh-Hurwitz conditions are satisfied for $T_0^* > 1$.

D. Numerical results

In this paper, the numerical results are used for analyzing behaviors of above seasonality transmission models. γ is the seasonality influence on the transmission process. We use γ as an index parameter. The values of the parameters used in this study are $\eta_h = 1/(365 * 70)$ per day corresponding to life expectancy of 70 years in human. $\eta_d = 1/4.5$ corresponding to death rate due to the disease of human. $\beta_{ha} = 1/(8.5)$ and $\beta_{hb} = 1/(9.5)$ corresponding to the recovery rate of human population due to biting of *Aedes aegypti* and *Aedes albopictus*, respectively. The transmission probability of *Aedes aegypti* (α_{va}) and *Aedes albopictus* (α_{vb}) are arbitrarily chosen. We assume that no alternative host. The other parameters are arbitrarily chosen. We present numerical solutions of (9)-(13) as follows:

First case, $\gamma = 0$:

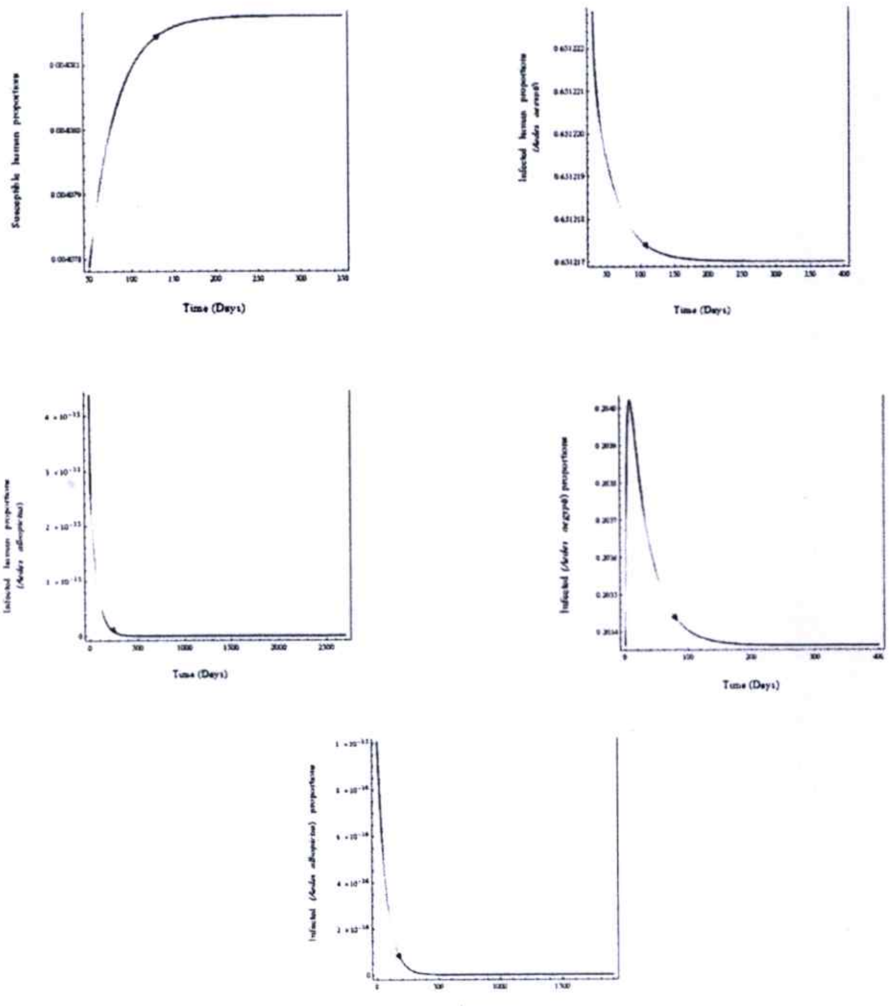


Figure 5. Numerical solution of (9)-(13) demonstrates the solution trajectories, projected onto S, I_a, I_b, I_{va} and I_{vb}, respectively. For $T_0^* < 1$, when $T_0 = 0.33878$ with $\eta_h = 1/(365 * 70)\,\text{day}^{-1}$, $\eta_d = 1/4.5\,\text{day}^{-1}$, $\alpha_{va} = 0.00000049$, $\alpha_{vb} = 0.00000026$, $\eta_{va} = 1/40$, $\eta_{vb} = 1/35$, $\lambda_a = 1/15$, $\lambda_b = 1/11$, $\varepsilon_a = 0.07$, $\varepsilon_b = 0.04$, $\varepsilon_{va} = 0.08$, $\varepsilon_{vb} = 0.03$, $N_{va} = 4000$, $N_{vb} = 2300$, $N_t = 20000$, $\beta_{ha} = 1/(17/2)$ and $\beta_{hb} = 1/(19/2)$. The fractions of populations $(S, I_a, I_b, I_{va}, I_{vb})$ approach to the disease free equilibrium point $(1, 0, 0, 0, 0)$.

Second case, $\gamma \neq 0$:

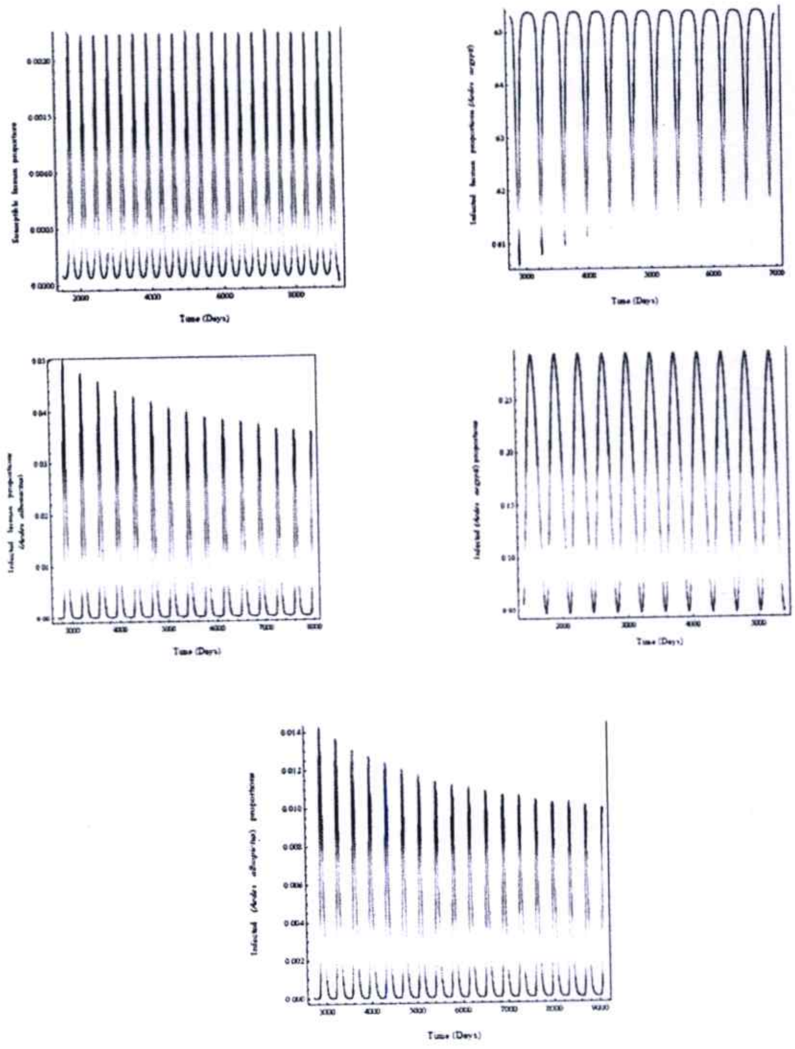


Figure 6. Numerical solutions of the times series of S^*, I_a^*, I_b^*, I_{va}^* and I_{vb}^* for $\widetilde{T}_0 > 1$, respectively. For $T_0^* > 1$, when $\gamma = \dfrac{2\pi}{365}$ with $\eta_h = 1/(365 * 70)\text{day}^{-1}$, $\eta_d = 1/4.5\,\text{day}^{-1}$, $\alpha_{va} = 0.00000059$, $\alpha_{vb} = 0.00000036$, $\eta_{va} = 1/10$, $\eta_{vb} = 1/16$, $\lambda_a = 1/4$, $\lambda_b = 1/6$, $\varepsilon_a = 0.7$, $\varepsilon_b = 0.4$, $\varepsilon_{va} = 0.8$, $\varepsilon_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\beta_{ha} = 1/(17/2)$ and $\beta_{hb} = 1/(19/2)$. The behaviors of $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ are limit cycles.

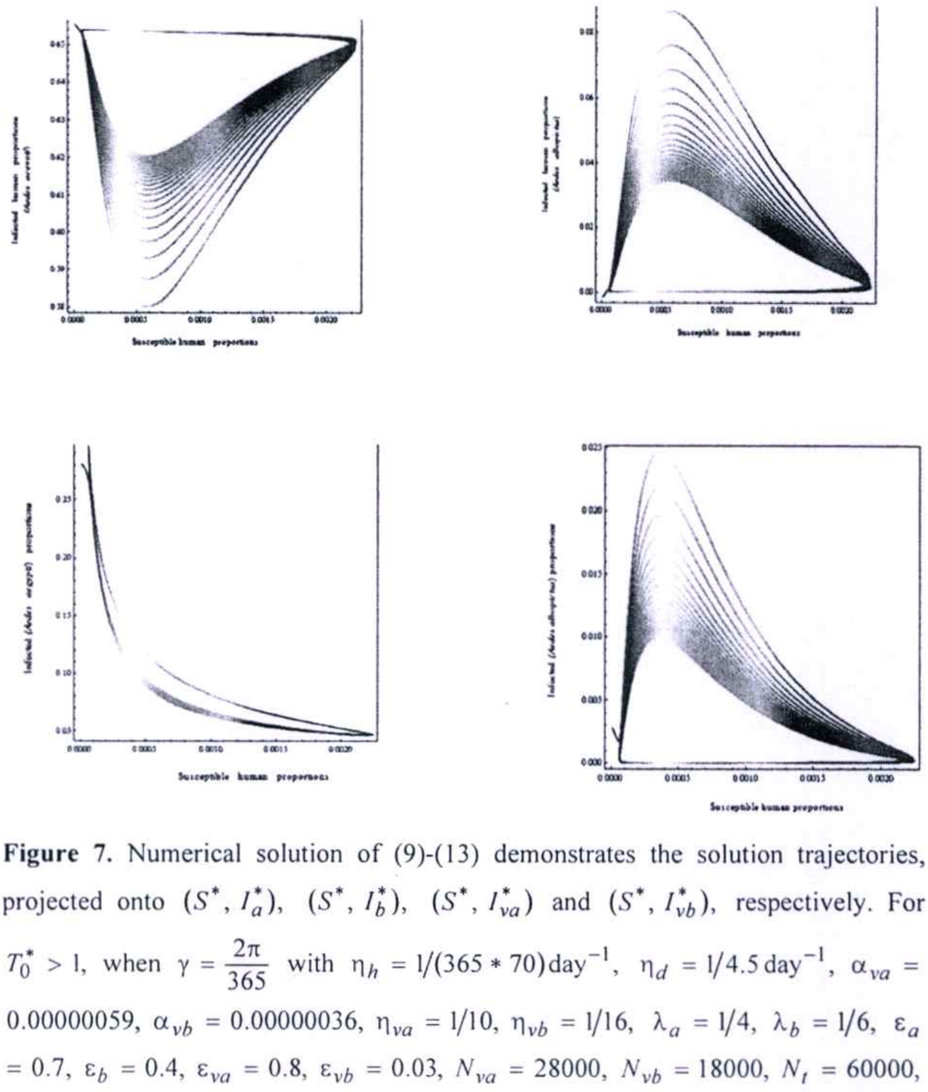


Figure 7. Numerical solution of (9)-(13) demonstrates the solution trajectories, projected onto (S^*, I_a^*), (S^*, I_b^*), (S^*, I_{va}^*) and (S^*, I_{vb}^*), respectively. For $T_0^* > 1$, when $\gamma = \frac{2\pi}{365}$ with $\eta_h = 1/(365 * 70)\,\text{day}^{-1}$, $\eta_d = 1/4.5\,\text{day}^{-1}$, $\alpha_{va} = 0.00000059$, $\alpha_{vb} = 0.00000036$, $\eta_{va} = 1/10$, $\eta_{vb} = 1/16$, $\lambda_a = 1/4$, $\lambda_b = 1/6$, $\varepsilon_a = 0.7$, $\varepsilon_b = 0.4$, $\varepsilon_{va} = 0.8$, $\varepsilon_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\eta_d = 1/4.5\,\text{day}^{-1}$, and $\beta_{hb} = 1/(19/2)$. The behaviors of $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ are limit cycles.

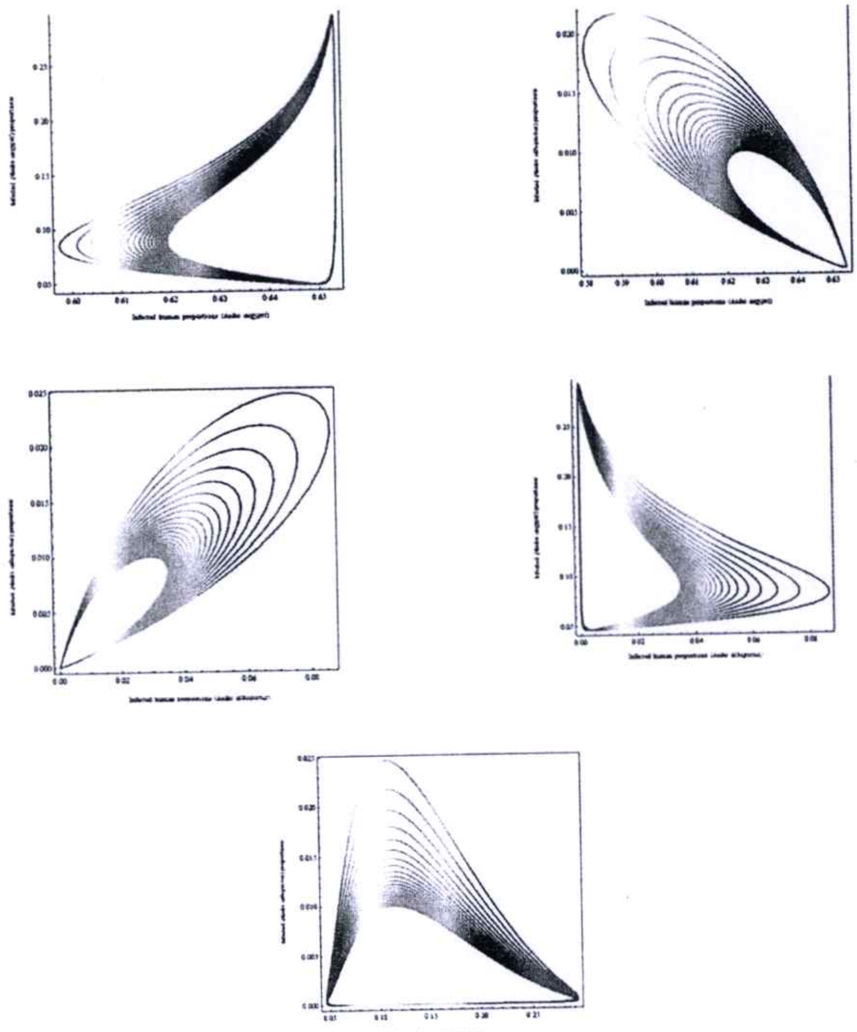


Figure 8. Numerical solution of (9)-(13) demonstrates the solution trajectories, projected onto (I_a^*, I_b^*), (I_a^*, I_{va}^*), (I_a^*, I_{vb}^*), (I_b^*, I_{vb}^*), (I_b^*, I_a^*), (I_b^*, I_{va}^*) and (I_{va}^*, I_{vb}^*), respectively. For $T_0^* > 1$, when $\gamma = \frac{2\pi}{365}$ with $\eta_h = 1/(365 * 70)\text{day}^{-1}$, $\eta_d = 1/4.5\,\text{day}^{-1}$, $\alpha_{va} = 0.00000059$, $\alpha_{vb} = 0.00000036$, $\eta_{va} = 1/10$, $\eta_{vb} = 1/16$, $\lambda_a = 1/4$, $\lambda_b = 1/6$, $\varepsilon_a = 0.7$, $\varepsilon_b = 0.4$, $\varepsilon_{va} = 0.8$, $\varepsilon_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\beta_{ha} = 1/(17/2)$ and $\beta_{hb} = 1/(19/2)$. The behaviors of $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ are limit cycles.

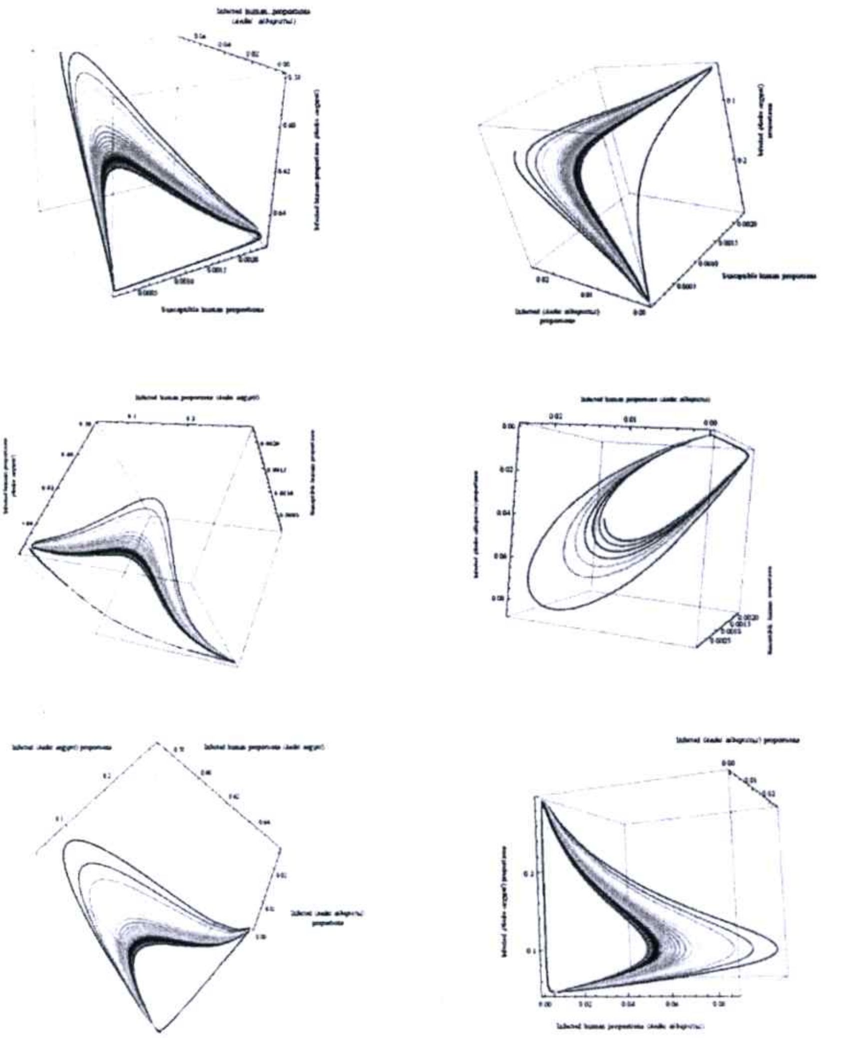


Figure 9. Numerical solution of (9)-(13) demonstrates the solution trajectories, projected onto (S^*, I_a^*, I_b^*), $(S^*, I_{va}^*, I_{vb}^*)$, (S^*, I_a^*, I_{va}^*), (S^*, I_b^*, I_{vb}^*), $(I_a^*, I_{va}^*, I_{vb}^*)$ and $(I_b^*, I_{vb}^*, I_{va}^*)$, respectively. For $T_0^* > 1$, when $\gamma = \frac{2\pi}{365}$ with $\eta_h = 1/(365 * 70)\text{day}^{-1}$, $\eta_d = 1/4.5\,\text{day}^{-1}$, $\alpha_{va} = 0.00000059$, $\alpha_{vb} = 0.00000036$, $\eta_{va} = 1/10$, $\eta_{vb} = 1/16$, $\lambda_a = 1/4$, $\lambda_b = 1/6$, $\varepsilon_a = 0.7$, $\varepsilon_b = 0.4$, $\varepsilon_{va} = 0.8$, $\varepsilon_{vb} = 0.03$, $N_{va} = 28000$, $N_{vb} = 18000$, $N_t = 60000$, $\beta_{ha} = 1/(17/2)$ and $\beta_{hb} = 1/(19/2)$. The behaviors of $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ are limit cycles.

Discussion and Conclusion

We develop a mathematical model considering the transmission of dengue virus from *Aedes aegypti* and *Aedes albopictus* mosquitoes.

For $\gamma = 0$, the threshold parameter for this model is

$$T_0 = \frac{\begin{array}{c} 2(N_{va}\alpha_{va}(N_t\alpha_{vb}(\varepsilon_a\varepsilon_{va} + (\varepsilon_a + \varepsilon_{va})\varepsilon_{vb})(\eta_h + \eta_d) + \varepsilon_a\varepsilon_{va}\tau_5)\lambda_a \\ + 2\alpha_{vb}\tau_4(2(\eta_h + \eta_d) + N_{vb}(2 + \varepsilon_b\varepsilon_{vb})\lambda_b) \end{array}}{\begin{array}{c} N_{va}\alpha_{va}(2N_t\alpha_{va}(2 + \varepsilon_a\varepsilon_{va} + (\varepsilon_a + \varepsilon_{va})\varepsilon_{vb})(\eta_h + \eta_d) \\ + 2(2 + \varepsilon_a\varepsilon_{va})\tau_5\lambda_a + 2N_{vb}\alpha_{vb}\varepsilon_b\varepsilon_{vb}\tau_4\lambda_b \end{array}}.$$

For $\gamma \neq 0$, the threshold parameter for this model is

$$\begin{aligned} T_0 = {} & [2(N_{va}\alpha_{va}(N_t\alpha_{vb}(\varepsilon_a\varepsilon_{va} + (\varepsilon_a + \varepsilon_{va})\varepsilon_{vb})(\eta_h + \eta_d) \\ & + \varepsilon_a\varepsilon_{va}\tau_5)\lambda_a Cos(2\gamma t) + \alpha_{vb}(2\tau_4(2 + \varepsilon_b\varepsilon_{vb})\lambda_b\tau_4(\eta_h + \eta_d) \\ & + N_{vb}(2 + \varepsilon_b\varepsilon_{vb})\lambda_b) + 4(\tau_4(\varepsilon_{vb}(\eta_h + \eta_d) + N_{vb}(\varepsilon_b + \varepsilon_{vb})\lambda_b)Sin(\gamma t) \\ & + N_t N_{va}\alpha_{va}\varepsilon_a\varepsilon_{va}\varepsilon_{vb}(\eta_h + \eta_d)\lambda_a Sin(3\gamma t)]/ \\ & [2(N_{vb}\alpha_{vb}\varepsilon_b\varepsilon_{vb}\tau_4\lambda_b Cos(2\gamma t) + N_{va}\alpha_{va}\lambda_a(2N_t\alpha_{vb}(2 + \varepsilon_a\varepsilon_{va} \\ & + (\varepsilon_a + \varepsilon_{va})\varepsilon_{vb})(\eta_h + \eta_d) + 2(2 + \varepsilon_a\varepsilon_{va})\tau_5 \\ & + (N_t\alpha_{vb}(4(\varepsilon_a + \varepsilon_{va})\varepsilon_{vb})(\eta_h + \eta_d) + 4((\varepsilon_a + \varepsilon_{va})\tau_5)Sin(\gamma t)]. \end{aligned}$$

Analysis of this model reveals the existence of two equilibrium points. One is the disease free equilibrium point and it is locally asymptotically stable if and only if $T_0 < 1$. The another one is an endemic equilibrium point. This equilibrium point is locally asymptotically stable if and only if $T_0 > 1$. The quantity $\widetilde{T_0} > \sqrt{T_0}$ is called the basic reproductive number of disease, it represents the average number of secondary cases that one case can produce if introduced into a susceptible population. We consider human and vector (*Aedes aegypti*, *Aedes albopictus*) populations, when values of $\gamma = 0$ and $\gamma \neq 0$. If $\gamma = 0$, we used Routh-Hurwitz criteria to determine the locally asymptotically stable of equilibrium points. If $\gamma \neq 0$, we consider time series of human and vector (*Aedes aegypti*, *Aedes albopictus*) populations, when the transmission probability from *Aedes aegypti* to human and the transmission probability from *Aedes albopictus* to human are different.

The disease free equilibrium point $E_0 = (1, 0, 0, 0, 0, 0)$, represents the state in which dengue disease is not endemic in human and it is locally asymptotically stable for $T_0 < 1$. Figure 3 shows the proportions of population human and vectors (*Aedes aegypti*, *Aedes albopictus*). The proportions of all population $(S, I_a, I_b, I_{va}, I_{vb})$ approach to the disease free state (1, 0, 0, 0, 0) when $T_0^* < 1$.

For the endemic equilibrium point $E_1 = (S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$, when $\gamma = 0$ the endemic equilibrium point is locally asymptotically stable for $T_0 > 1$. Figure 4 shows the proportions of human and vectors for $(S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$, when $T_0 > 1$.

When $\gamma \neq 0$, the endemic equilibrium point $E_1 = (S^*, I_a^*, I_b^*, I_{va}^*, I_{vb}^*)$ is locally asymptotically stable for $T_0 > 1$. These behaviors correspond to diagram as shown in Figure 4 to Figure 9. We can see that limit cycle occurs in this case. The fraction of *Aedes aegypti* population oscillates between 0.2961 and 0.04587. The fraction of *Aedes albopictus* population oscillates between 0.01026 and 0.00002.

We can see that when $T_0 < 1$ and $\gamma = 0$, E_0 will be locally asymptotically stable and for $T_0 > 1$, E_1 will be locally asymptotically stable. The locally asymptotically stable of all equilibrium points are determined by the threshold number T_0. To reduce transmission of this disease, we should control the above threshold number.

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Original Research Paper

SIR Transmission Model of Dengue Virus Taking Into Account Two Species of Mosquitoes and an Age Structure in the Human Population

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Abstract: Dengue is a vector-borne disease. It is transmitted to humans by the bites of the *Aedes aegypti* and *Aedes albopictus* mosquitoes. The human population is separated into two classes, a child class and an adult class, each class being described by a SIR model. The transmission rates of the two mosquito species are different and depend on what class the humans belong to. We develop a single model taking into account the presence of two type of mosquitoes and two age classes and apply it to dengue fever. The model shows how it is possible for the maximum level of infected human to be reached in a short time. The nature of stability of the equilibrium state and the trajectories of the individual classes in the model are determined by the values of the basic reproduction number by setting the values of the parameters in the model to different values which reflect the environment in which the epidemic is occurring in the model.

Keywords: *Aedes Aegypti*, *Aedes Albopictus*, Dengue Disease, Endemic Disease State, Equilibrium State, SIR Model

Introduction

Dengue fever is regarded as a serious infectious disease that risks about 2.5 billion people around the world, especially in tropical countries. It is a major epidemic disease occurred in Southeast Asia. Such epidemic arises due to climate change, knowledge of people and awareness on dengue fever so as to the dengue fever possibly become an endemic for a long time. Moreover, World Health Organization (WHO) (WHO, 2009) estimated 50 to 100 million cases worldwide. About 500,000 people are estimated to be infected by dengue hemorrhagic fever each year.

Dengue fever is caused by four serotypes and they are closely related as a family of dengue virus 1 (DEN 1), virus 2 (DEN2), virus 3 (DEN3) and virus 4 (DEN4). There are viruses carried by two kinds of mosquitoes such as Aedes aegypti and Aedes albopictus. This disease is transmitted to the human though biting of mosquitoes. Recently, It was detected in Asia. However, *Aedes aegypti* is still the principal vector of dengue fever transmission. Another interesting fact is the shift of patients' phenomena where dengue fever previously attacks children of primary school age, but now everybody is vulnerable to fever (Pongsumpun and Tang, 2001; Syafruddin and Noorani, 2012).

Dengue virus is transmitted between the human by biting of an infected Aedes mosquito. When a vector bites someone who be infected with dengue virus, the virus is transferred to mosquitoes and it become infected mosquito. After the infected vector bites the susceptible human, then the virus move into the human bloodstream and it spreads throughout the body. Usually, the mosquitoes bite susceptible people during the day time. Dengue fever is the most common disease in urban areas. The outbreaks commonly occur during the rainy season when the mosquitoes heavily breed in standing water. The dengue fever cases are increasing worldwide. The complications of the disease are leading cases of serious illness and most death in children (Kerpaninch *et al*., 2001; Kabilan *et al*., 2003; Malhotra *et al*., 2006; Wiwanitkit, 2006; Pongsumpum, 2011; Joshi *et al*., 2002; Koenraadt *et al*., 2007). One of the major public health problems in many tropical and subtropical regions where *Aedes aegypti* and *Aedes albopictus* are present.

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It is noted that *Aedes albopictus* was the principal vector in the 1940 s outbreaks in Japan (Hotta, 1998), whereas, *Aedes aegypti* is commonly the principal dengue vector in the tropical and subtropical regions. *Aedes aegypti* is highly domesticated and exhibits strong anthropophilia.

Traditional modeling in epidemiology focus on stability equilibria, since this characterizes if a disease will become endemic and this is a major concern for public health officers. The concept of a basic reproductive number (R_0) was introduced and became a modeling paradigm (Smith *et al.*, 2012) for a very recent review on the works by Ross and Macdonald from a medical modeling point of view. In a fairly large class of models, we can define R_0 unambiguously and it can be shown that if $R_0<1$, the disease is extinct while if $R_0>1$ it becomes endemic (Diekmann and Heesterbeek, 2000).

Hence, in this study, we analyze the SIR (Susceptible-Infected-Recovered) equations for human and SI (Susceptible-Infected) equations for mosquitoes. The model will apply empirically on data of dengue patients reports by Ministry of Public Health, Thailand (2002-2012) as shown in Fig. 1. The purpose of this paper is to study the age structural model of dengue disease incorporated the influence of *Aedes aegypti* and *Aedes albopictus.*

Mathematical Model

The SIR and SI simulates the spread of dengue virus between host and vector populations. The model is based on the Susceptible, Infected and Recovered (SIR) model of infected disease epidemiology, which was adopted by (Nuraini *et al.*, 2007; Yaacob, 2007). The age structure is introduced into a model, i.e., children and adults, then we modify it by incorporating the different behaviors of *Aedes aegypti* and *Aedes albopictus*. In Fig. 1, we show the age distribution of the incidence rates in one province in Thailand during 2002-2012 epidemic. As we see, most cases occur in children under the age of 15. However, a small number of cases do occur in older people. Similar distributions are seen in the other provinces in the country.

This model with age structure, the dynamics of each component of the human is given by:

$$\frac{dS_c}{dt} = P_c N_{tc} - \beta_{ac}(1+\alpha_a \sin\varepsilon t) I_{va1} S_c - \beta_{hc}(1+\alpha_h \sin\varepsilon t) I_{vh1} S_c - \mu_d S_c \tag{1a}$$

$$\frac{dI_{c1}}{dt} = \beta_{ac}(1+\alpha_a \sin\varepsilon t)\, I_{va1} S_c - \kappa_{c1} I_{c1} - \mu_d I_{c1} \tag{1b}$$

$$\frac{dI_{c2}}{dt} = \beta_{hc}(1+\alpha_h \sin\varepsilon t) I_{vh1} S_c - \kappa_{c2} I_{c2} - \mu_d I_{c2} \tag{1c}$$

$$\frac{dR_c}{dt} = \kappa_{c1} I_{c1} + \kappa_{c2} I_{c2} - \mu_d R_c \tag{1d}$$

$$\frac{dS_a}{dt} = P_a N_{ta} - \beta_{aa}(1+\alpha_a \sin\varepsilon t) I_{va2} S_a - \beta_{ha}(1+\alpha_h \sin\varepsilon t) I_{vh2} S_a - \mu_d S_a \tag{1e}$$

$$\frac{dI_{a1}}{dt} = \beta_{aa}(1+\alpha_a \sin\varepsilon t)\, I_{va2} S_a - \kappa_{a1} I_{a1} - \mu_d I_{a1} \tag{1f}$$

$$\frac{dI_{a2}}{dt} = \beta_{ha}(1+\alpha_h \sin\varepsilon t) I_{vh2} S_a - \kappa_{a2} I_{a2} - \mu_d I_{a2} \tag{1g}$$

$$\frac{dR_a}{dt} = \kappa_{a1} I_{a1} + \kappa_{a2} I_{a2} - \mu_d R_a \tag{1h}$$

where the variables and parameters are defined in Table 1.

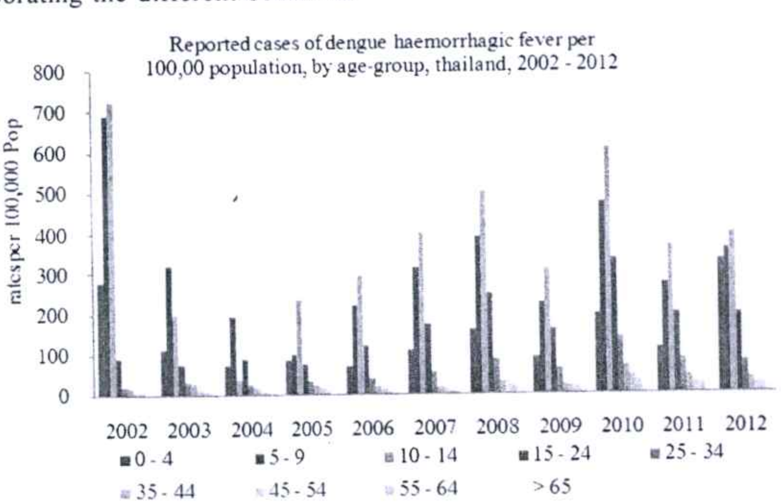

Fig. 1. Age distribution of the 2002-2012 Dengue fever incidence rates in Thailand

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Table 1. Parameters for equations (1a)-(1h) and their definitions

Variable/parameter	Definition
S_c, I_{c1}, I_{c2}, R_c	The numbers of susceptible children, infected from *Aedes aegypti* and *Aedes albopictus* in children and recovered children
S_a, I_{a1}, I_{a2}, R_a	The numbers of susceptible adult, infected from *Aedes aegypti* and *Aedes albopictus* in adult and recovered adult
N_t	The total population
N_{tc} and N_{ta}	The total population in children and the total population in adult they are constant variables
P_c and P_a	The birth rate of children and adult human
β_{ac}	The transmission probability of dengue virus from *Aedes aegypti* to children
β_{bc}	The transmission probability of dengue virus from *Aedes albopictus* to children
β_{aa}	The transmission probability of dengue virus from *Aedes aegypti* to adult
β_{ba}	The transmission probability of dengue virus from *Aedes albopictus* to adult
κ_{c1}	The rate at which the infected children from *Aedes aegypti* can recover
κ_{c2}	The rate at which the infected children from *Aedes albopictus* can recover
κ_{a1}	The rate at which the infected adult from *Aedes aegypti* can recover
κ_{a2}	The rate at which the infected adult from *Aedes albopictus* can recover
μ_d	The natural death rate of human
a_a	The measure of influence on the transmission process from *Aedes aegypti* mosquito to human
a_b	The measure of influence on the transmission process from *Aedes albopictus* to human
ρ_{va}	The measure of influence on the transmission process from *Aedes aegypti* to human
ρ_{vb}	The measure of influence on the transmission process from *Aedes albopictus* to human

It we add Equation (1a) – (1h) together, we get:

$$\frac{dN_t}{dt} = \frac{dN_{tc}}{dt} + \frac{dN_{ta}}{dt}$$
$$= (S_c + I_{c1} + I_{c2} + R_c) + (S_a + I_{a1} + I_{a2} + R_a)$$

The total children and adult populations are supposed to have constant sizes, i.e., $\frac{dN_{tc}}{dt} = 0$ and $\frac{dN_{ta}}{dt} = 0$, the birth rate would have to be equivalent to the death rate, $P_c = P_a = \mu_d$ in children and adult, respectively.
where, N_{tc} is the total number of children and is equivalent to $S_c + I_{c1} + I_{c2} + R_c$, N_{ta}, the total population in adult and is equal to $S_a + I_{a1} + I_{a2} + R_a$.

The dynamics of mosquitoes is described by:

$$\frac{dS_{va1}}{dt} = A_{v_a} - \lambda_{va1}(1 + \rho_{va}\sin\varepsilon t) I_{c1} S_{va1} - \mu_{v_a} S_{va1} \quad (2a)$$

$$\frac{dI_{va1}}{dt} = \lambda_{va1}(1 + \rho_{va}\sin\varepsilon t) I_{c1} S_{va1} - \mu_{v_a} I_{va1} \quad (2b)$$

$$\frac{dS_{va2}}{dt} = A_{v_a} - \lambda_{va2}(1 + \rho_{va}\sin\varepsilon t) I_{a1} S_{va2} - \mu_{v_a} S_{va2} \quad (2c)$$

$$\frac{dI_{va2}}{dt} = \lambda_{va2}(1 + \rho_{va}\sin\varepsilon t) I_{a1} S_{va2} - \mu_{v_a} I_{va2} \quad (2d)$$

$$\frac{dS_{vb1}}{dt} = A_{v_b} - \lambda_{vb1}(1 + \rho_{vb}\sin\varepsilon t) I_{c2} S_{vb1} - \mu_{v_b} S_{vb1} \quad (2e)$$

$$\frac{dI_{vb1}}{dt} = \lambda_{vb1}(1 + \rho_{vb}\sin\varepsilon t) I_{c2} S_{vb1} - \mu_{v_b} I_{vb1} \quad (2f)$$

$$\frac{dS_{vb2}}{dt} = A_{v_b} - \lambda_{vb2}(1 + \rho_{vb}\sin\varepsilon t) I_{a2} S_{vb2} - \mu_{v_b} S_{vb2} \quad (2g)$$

$$\frac{dI_{vb2}}{dt} = \lambda_{vb2}(1 + \rho_{vb}\sin\varepsilon t) I_{a2} S_{vb2} - \mu_{v_b} I_{vb2} \quad (2h)$$

where, the variable and parameters are given in Table 2.

If we add Equation (2a)-(2h) together, we get:

$$\frac{d(S_{va1} + I_{va1})}{dt} = A_{v_a} - \mu_{v_a} N_{va1} \quad (3a)$$

$$\frac{d(S_{va2} + I_{va2})}{dt} = A_{v_a} - \mu_{v_a} N_{va2} \quad (3b)$$

$$\frac{d(S_{vb1} + I_{vb1})}{dt} = A_{v_b} - \mu_{v_b} N_{vb1} \quad (3c)$$

$$\frac{d(S_{vb2} + I_{vb2})}{dt} = A_{v_b} - \mu_{v_b} N_{vb2} \quad (3d)$$

where, N_{va1} and N_{va2} are the numbers of *Aedes aegypti* in children and adult respectively, which is equal to $S_{va1}+I_{va1}$ and $S_{va2} + I_{va2}$. N_{vb1} and N_{vb2} are the numbers of *Aedes albopictus* in children and adult respectively, which is equal to $S_{vb1} + I_{vb1}$ and $S_{vb2} + I_{vb2}$. If the numbers of mosquitoes are also constant each other (3a)-(3d) gives $N_{va1} = A_{v_a} / \mu_{v_a}$, $N_{va2} = A_{v_a} / \mu_{v_a}$, $N_{vb1} = A_{v_b} / \mu_{v_b}$ and $N_{vb2} = A_{v_b} / \mu_{v_b}$.

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Table 2. Parameters for equations (2a)-(2g) and their definitions

Variable parameter	Definition
S_{va1} and I_{va1}	The number of susceptible and infected *Aedes aegypti* mosquitoes who be infected from children
μ_{v_a}	The death rate of *Aedes aegypti* mosquito
A_{v_a}	The carrying capacity of the environment for *Aedes aegypit*
λ_{va1}	The probability that a dengue virus transmitted to the *Aedes aegypti* from an infected children
S_{va2} and I_{va2}	The number of susceptible and infected *Aedes aegypti* mosquitoes who be infected from adult
μ_{v_a}	The death rate of *Aedes aegypti* mosquito
A_{v_a}	The carrying capacity of the environment for *Aedes aegypit* mosquito
λ_{va2}	The probability that a dengue virus transmitted to the *Aedes aegypti* from an infected adult human
S_{vb1} and I_{vb1}	The number of susceptible and infected *Aedes albopictus* mosquitoes who be infected from children
μ_{v_b}	The death rate of *Aedes albopictus* mosquito
A_{v_b}	The carrying capacity of the environment for *Aedes albopictus* mosquito
λ_{vb1}	The probability that a dengue virus transmitted to the *Aedes albopictus* from an infected human in children
S_{vb2} and I_{vb2}	The number of susceptible and infected *Aedes albopictus* mosquitoes who be infected from adult human
μ_{v_b}	The death rate of *Aedes albopictus* mosquito
A_{v_b}	The carrying capacity of the environment for *Aedes albopictus* mosquito
λ_{vb2}	The probability that a dengue virus transmitted to the *Aedes albopictus* from an infected adult

We normalize parameter (1a)-(1h) and (2a)-(2h) by writing $S'_c=\frac{S_c}{N_{tc}}$, $I'_{c1}=\frac{I_{c1}}{N_{tc}}$, $I'_{c2}=\frac{I_{c2}}{N_{tc}}$, $R'_c=\frac{R_c}{N_{tc}}$ in children and $S'_a=\frac{S_a}{N_{ta}}$, $I'_{a1}=\frac{I_{a1}}{N_{ta}}$, $I'_{a2}=\frac{I_{a2}}{N_{ta}}$, $R'_a=\frac{R_a}{N_{ta}}$ in adult. $S'_{va1}=\frac{S_{va1}}{N_{va1}}$, $I'_{va1}=\frac{I_{va1}}{N_{va1}}$, $S'_{va2}=\frac{S_{va2}}{N_{va2}}$, $I'_{va2}=\frac{I_{va2}}{N_{va2}}$, $S'_{vb1}=\frac{S_{vb1}}{N_{vb1}}$, $I'_{vb1}=\frac{I_{vb1}}{N_{vb1}}$, $S'_{vb2}=\frac{S_{vb2}}{N_{vb2}}$ and $I'_{vb2}=\frac{I_{vb2}}{N_{vb2}}$, then the reduced equations become:

$$\frac{d}{dt}S'_c=\mu_d-\beta_{ac}\ (1+\alpha_a\sin\varepsilon t)\ I'_{va1}N_{va1}S'_c-\beta_{bc}\ (1+\alpha_b\sin\varepsilon t)\ I'_{vb1}N_{vb1}S'_c-\mu_d S'_c \tag{4a}$$

$$\frac{d}{dt}I'_{c1}=\beta_{ac}(1+\alpha_a\sin\varepsilon t)\ I'_{va1}N_{va1}S'_c-\kappa_{c1}I'_{c1}-\mu_d I'_{c1} \tag{4b}$$

$$\frac{d}{dt}I'_{c2}=\beta_{bc}(1+\alpha_b\sin\varepsilon t)I'_{vb1}N_{vb1}S'_c-\kappa_{c2}I'_{c2}-\mu_d I'_{c2} \tag{4c}$$

$$\frac{d}{dt}S'_a=\mu_d-\beta_{aa}(1+\alpha_a\sin\varepsilon t)\ I'_{va2}N_{va2}S'_a-\beta_{ba}(1+\alpha_b\sin\varepsilon t)I'_{vb2}N_{vb2}S'_a-\mu_d S'_a \tag{4d}$$

$$\frac{d}{dt}I'_{a1}=\beta_{aa}(1+\alpha_a\sin\varepsilon t)\ I'_{va2}N_{va2}S'_a-\kappa_{a1}I'_{a1}-\mu_d I'_{a1} \tag{4e}$$

$$\frac{d}{dt}I'_{a2}=\beta_{ba}(1+\alpha_b\sin\varepsilon t)I'_{vb2}N_{vb2}S'_a-\kappa_{a2}I'_{a2}-\mu_d I'_{a2} \tag{4f}$$

$$\frac{d}{dt}I'_{va1}=\lambda_{va1}(1+\rho_{va}\sin\varepsilon t)I'_{c1}N_{tc}S'_{va1}-\mu_{v_a}I'_{va1} \tag{4g}$$

$$\frac{d}{dt}I'_{va2}=\lambda_{va2}(1+\rho_{va}\sin\varepsilon t)I'_{a1}N_{ta}S'_{va2}-\mu_{v_a}I'_{va2} \tag{4h}$$

$$\frac{d}{dt}I'_{vb1}=\lambda_{vb1}(1+\rho_{vb}\sin\varepsilon t)I'_{c2}N_{tc}S'_{vb1}-\mu_{v_b}I'_{vb1} \tag{4i}$$

$$\frac{d}{dt}I'_{vb2}=\lambda_{vb2}(1+\rho_{vb}\sin\varepsilon t)I'_{a2}N_{ta}S'_{vb2}-\mu_{v_b}I'_{vb2} \tag{4j}$$

Where:

$$S'_c+I'_{c1}+I'_{c2}+R'_c=1,\ S'_a+I'_{a1}+I'_{a2}+R'_a=1,$$
$$S_{va1}+I_{va1}=1,\ S'_{va2}+I'_{va2}=1,\ S'_{vb1}+I'_{vb1}=1\ and\ S'_{vb2}+I'_{vb2}=1$$

Mathematical Analysis

Equilibrium States

The equilibrium states $(S^*_c, I^*_{c1}, I^*_{c2}, S^*_a, I^*_{a1}, I^*_{a2}, I^*_{va1}, I^*_{vb1}, I^*_{va2}, I^*_{vb2})$ are obtained by setting the right hand side of (4a)-(4j) to zero. Therefore we obtain $(S^*_c, I^*_{c1}, I^*_{c2}, I^*_{va1}, I^*_{vb1})$ and $(S^*_a, I^*_{a1}, I^*_{a2}, I^*_{va2}, I^*_{vb2})$. Doing this, we get four equilibrium states.

A. The two group disease free equilibrium state:

$$E_0=(1,0,0,1,0,0,0,0,0,0)$$

A1.the disease free equilibrium state:

$$E_{oc}=(S^*_c, I^*_{c1}, I^*_{c2}, I^*_{va1}, I^*_{vb1})=(1,0,0,0,0)\ in\ children$$

A2.the disease free equilibrium state:

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$$E_{0a} = (S^{*}_{a}, I^{*}_{a1}, I^{*}_{a2}, I^{*}_{va2}, I^{*}_{vh2}) = (1,0,0,0,0) \text{ in adult}$$

B. The two group endemic equilibrium state:

$$\hat{E} = (S^{*}_{c}, I^{*}_{c1}, I^{*}_{c2}, I^{*}_{va1}, I^{*}_{vh1}, S^{*}_{a}, I^{*}_{a1}, I^{*}_{a2}, I^{*}_{va2}, I^{*}_{vh2})$$

B1.the endemic state:

$$E_{1c} = (S^{*}_{c}, I^{*}_{c1}, I^{*}_{c2}, I^{*}_{va1}, I^{*}_{vh1}) \text{ in children}$$

Where:

$$S^{*}_{c} = \frac{(N_{tc}\mu_d\lambda_{vh1} + (\kappa_{c2}+\mu_d)\mu_{vh1} + N_{tc}\mu_d\lambda_{vh1}\rho_{vh}\sin\varepsilon t)}{(N_{tc}\lambda_{vh1}(I^{*}_{va1}N_{va1}\beta_{ac} + N_{vh1}\beta_{hc} + \mu_d + (I^{*}_{va1}N_{va1}\alpha_a\beta_{ac} + N_{vh1}\alpha_h\beta_{hc})\sin\varepsilon t)(1+\rho_{vh}\sin\varepsilon t)))} \quad (5a)$$

$$I^{*}_{c1} = \frac{I^{*}_{va1}N_{va1}\beta_{ac}(1+\alpha_a\sin\varepsilon t)(N_{tc}\mu_d\lambda_{vh1} + (\kappa_{c2}+\mu_d)\mu_{v_h} + N_{tc}\mu_d\lambda_{vh1}\rho_{vh}\sin\varepsilon t))}{(N_{tc}\lambda_{vh1}(\kappa_{c1}+\mu_d)(I^{*}_{va1}N_{va1}\beta_{ac} + N_{vh1}\beta_{hc} + \mu_d + (I^{*}_{va1}N_{va1}\alpha_a\beta_{ac} + N_{vh1}\alpha_h\beta_{hc})\sin\varepsilon t)(1+\rho_{vh}\sin\varepsilon t))} \quad (5b)$$

$$I^{*}_{c2} = \frac{(I^{*}_{vh1}N_{vh1}\beta_{hc}(1+\alpha_h\sin\varepsilon t)(N_{tc}\lambda_{vh1}\mu_d + (\kappa_{c2}+\mu_d)\mu_{v_h} + N_{tc}\lambda_{vh1}\mu_d\rho_{vh}\sin\varepsilon t))}{(N_{tc}\lambda_{vh1}(\kappa_{c2}+\mu_d)(I^{*}_{va1}N_{va1}\beta_{ac} + I^{*}_{vh1}N_{vh1}\beta_{hc} + \mu_d + (I^{*}_{va1}N_{va1}\alpha_a\beta_{ac} + N_{vh1}\alpha_h\beta_{hc})\sin\varepsilon t)(1+\rho_{vh}\sin\varepsilon t))} \quad (5c)$$

$$\begin{aligned}
I^{*}_{va1} = [(&2(-2\lambda_{vh1}\mu_d(\kappa_{c1}+\mu_d)\mu_{v_a} \\
&+2N_{va1}\beta_{ac}\lambda_{va1}(\kappa_{c2}+\mu_d)\mu_{v_h} \\
&+N_{va1}\alpha_a\beta_{ac}\lambda_{va1}(\kappa_{c2}+\mu_d) \\
&\mu_{v_h}\rho_{va} - N_{vh1}\beta_{hc}\lambda_{vh1}(\kappa_{c1}+\mu_d)\mu_{v_a}(2+\alpha_h\rho_{vh}) \\
&+N_{tc}N_{va1}\beta_{ac}\lambda_{va1}\lambda_{vh1}\mu_d(2+\rho_{va}\rho_{vh}+\alpha_a(\rho_{va}+\rho_{vh}))) \\
&-2(N_{va1}\alpha_a\beta_{ac}\lambda_{va1}(\kappa_{c2}+\mu_d)\mu_{v_h}\rho_{va} \\
&-N_{vh1}\alpha_h\beta_{hc}\lambda_{vh1}(\kappa_{c1}+\mu_d)\mu_{v_a}\rho_{vh} \\
&+N_{tc}N_{va1}\beta_{ac}\lambda_{va1}\lambda_{vh1}\mu_d(\rho_{va}\rho_{vh}+\alpha_a(\rho_{va}+\rho_{vh})))\cos \\
&2\varepsilon t + (4(N_{va1}\beta_{ac}\lambda_{va1}(\kappa_{c2}+\mu_d)\mu_{v_h}(\alpha_a+\rho_{va}) \\
&-\lambda_{vh1}\mu_d(\kappa_{c1}+\mu_d)\mu_{v_a} - N_{vh1}\beta_{hc}\lambda_{vh1}(\kappa_{c1}+\mu_d)\mu_{v_a}(\alpha_h+\rho_{vh})) \\
&+N_{tc}N_{va1}\beta_{ac}\lambda_{va1}\lambda_{vh1}\mu_d(4(\rho_{va}+\rho_{vh}) \\
&+\alpha_a(4+3\rho_{va}\rho_{vh})))\sin\varepsilon t \\
&-N_{tc}N_{va1}\alpha_a\beta_{ac}\lambda_{va1}\lambda_{vh1}\mu_d\rho_{va}\rho_{vh}\sin\varepsilon t)] \\
/[(&2N_{va1}\beta_{ac}(1+\alpha_a\sin\varepsilon t)(2\lambda_{vh1}(\kappa_{c1}+\mu_d)\mu_{v_a} \\
&+2\lambda_{va1}(\kappa_{c2}+\mu_d)\mu_{v_h}(\kappa_{c2}+\mu_d)\mu_{v_h} \\
&+N_{tc}\mu_d\lambda_{va1}\lambda_{vh1}(2+\rho_{va}\rho_{vh}) - N_{tc}\mu_d\lambda_{va1}\lambda_{vh1}\rho_{va}\rho_{vh}\cos 2\varepsilon t \\
&+2(\lambda_{va1}(\kappa_{c2}+\mu_d)\mu_{v_h}\rho_{va} + \lambda_{vh1}(\kappa_{c1}+\mu_d)\mu_{v_a}\rho_{vh} \\
&+N_{tc}\mu_d\lambda_{va1}\lambda_{vh1}(\rho_{va}+\rho_{vh}))\sin\varepsilon t))]
\end{aligned} \quad (5d)$$

$$\begin{aligned}
I^{*}_{vh1} = [(&N_{tc}N_{vh1}\beta_{hc}\lambda_{vh1}\mu_d - (I^{*}_{va1}N_{va1}\beta_{ac}+\mu_d)(\kappa_{c2}+\mu_d)\mu_{v_h} \\
&+\sin\varepsilon t(-I^{*}_{va1}N_{va1}\alpha_a\beta_{ac}(\kappa_{c2}+\mu_d)\mu_{v_h} \\
&+N_{tc}N_{vh1}\beta_{hc}\lambda_{vh1}\mu_d(\alpha_h+\rho_{vh}) \\
&+N_{tc}N_{vh1}\alpha_h\beta_{hc}\lambda_{vh1}\mu_d\rho_{vh}\sin\varepsilon t))] \\
/[&N_{vh1}\beta_{hc}(1+\alpha_h\sin\varepsilon t)(N_{tc}\lambda_{vh1}\mu_d \\
&+(\kappa_{c2}+\mu_d)\mu_{v_h} + N_{tc}\lambda_{vh1}\mu_d\rho_{vh}\sin\varepsilon t))]
\end{aligned} \quad (5e)$$

B2. The endemic state:

$$E_{1a} = (S^{*}_{a}, I^{*}_{a1}, I^{*}_{a2}, I^{*}_{va2}, I^{*}_{vh2}) \text{ in adult}$$

Where:

$$S^{*}_{a} = \frac{(N_{ta}\mu_d\lambda_{vh2} + (\kappa_{a2}+\mu_d)\mu_{v_h} + N_{ta}\mu_d\lambda_{vh2}\rho_{vh}\sin\varepsilon t)}{(N_{ta}\lambda_{vh2}(I^{*}_{va2}N_{va2}\beta_{aa} + N_{vh2}\beta_{ha} + \mu_d + (I^{*}_{va2}N_{va2}\alpha_a\beta_{aa} + N_{vh2}\alpha_h\beta_{ha})\sin\varepsilon t)(1+\rho_{vh}\sin\varepsilon t)))} \quad (6a)$$

$$I^{*}_{a1} = \frac{I^{*}_{va2}N_{va2}\beta_{aa}(1+\alpha_a\sin\varepsilon t)(N_{ta}\mu_d\lambda_{vh2} + (\kappa_{a2}+\mu_d)\mu_{v_h} + N_{ta}\mu_d\lambda_{vh2}\rho_{vh}\sin\varepsilon t))}{(N_{ta}\lambda_{vh2}(\kappa_{a1}+\mu_d)(I^{*}_{va2}N_{va2}\beta_{aa} + N_{vh2}\beta_{ha} + \mu_d + (I^{*}_{va2}N_{va2}\alpha_a\beta_{aa} + N_{vh2}\alpha_h\beta_{ha})\sin\varepsilon t)(1+\rho_{vh}\sin\varepsilon t))} \quad (6b)$$

$$\begin{aligned}
I^{*}_{a2} = [(&I^{*}_{vh2}N_{vh2}\beta_{ha}(1+\alpha_h\sin\varepsilon t)(N_{ta}\lambda_{vh2}\mu_d \\
&+(\kappa_{a2}+\mu_d)\mu_{v_h} + N_{ta}\lambda_{vh2}\mu_d\rho_{vh}\sin\varepsilon t)) \\
/\,&N_{ta}\lambda_{vh2}(\kappa_{a2}+\mu_d)(I^{*}_{va2}N_{va2}\beta_{aa} \\
&+I^{*}_{vh2}N_{vh2}\beta_{ha} + \mu_d + (I^{*}_{va2}N_{va2}\alpha_a\beta_{aa} \\
&+N_{vh2}\alpha_h\beta_{ha})\sin\varepsilon t)(1+\rho_{vh}\sin\varepsilon t))]
\end{aligned} \quad (6c)$$

$$\begin{aligned}
I^{*}_{va2} = [(&2(-2\lambda_{vh2}\mu_d(\kappa_{a1}+\mu_d)\mu_{v_a} + 2N_{va2}\beta_{aa}\lambda_{va2}(\kappa_{a2}+\mu_d)\mu_{v_h} \\
&+N_{va2}\alpha_a\beta_{aa}\lambda_{va2}(\kappa_{a2}+\mu_d)\mu_{v_h}\rho_{va} \\
&-N_{vh2}\beta_{ha}\lambda_{vh2}(\kappa_{a1}+\mu_d)\mu_{v_a}(2+\alpha_h\rho_{vh}) \\
&+N_{ta}N_{va2}\beta_{aa}\lambda_{va2}\lambda_{vh2}\mu_d(2+\rho_{va}\rho_{vh}+\alpha_a(\rho_{va}+\rho_{vh}))) \\
&-2(N_{va2}\alpha_a\beta_{aa}\lambda_{va2}(\kappa_{a2}+\mu_d)\mu_{v_h}\rho_{va} \\
&-N_{vh2}\alpha_h\beta_{ha}\lambda_{vh2}(\kappa_{a1}+\mu_d)\mu_{v_a}\rho_{vh} \\
&+N_{ta}N_{va2}\beta_{aa}\lambda_{va2}\lambda_{vh2}\mu_d(\rho_{va}\rho_{vh}+\alpha_a(\rho_{va}+\rho_{vh})))\cos 2\varepsilon t \\
&+(4(N_{va2}\beta_{aa}\lambda_{va2}(\kappa_{a2}+\mu_d)\mu_{v_h}(\alpha_a+\rho_{va}) \\
&-\lambda_{vh2}\mu_d(\kappa_{a1}+\mu_d)\mu_{v_a} - N_{vh2}\beta_{ha}\lambda_{vh2}(\kappa_{a1}+\mu_d)\mu_{v_a}(\alpha_h+\rho_{vh})) \\
&+N_{ta}N_{va2}\beta_{aa}\lambda_{va2}\lambda_{vh2}\mu_d(4(\rho_{va}+\rho_{vh}) \\
&+\alpha_a(4+3\rho_{va}\rho_{vh})))\sin\varepsilon t \\
&-N_{ta}N_{va2}\alpha_a\beta_{aa}\lambda_{va2}\lambda_{vh2}\mu_d\rho_{va}\rho_{vh}\sin\varepsilon t)] \\
/[(&2N_{va2}\beta_{aa}(1+\alpha_a\sin\varepsilon t)(2\lambda_{vh2}(\kappa_{a1}+\mu_d)\mu_{v_a} \\
&+2\lambda_{va2}(\kappa_{a2}+\mu_d)\mu_{v_h}(\kappa_{a2}+\mu_d)\mu_{v_h} + N_{ta}\mu_d\lambda_{va2}\lambda_{vh2}(2+\rho_{va}\rho_{vh}) \\
&-N_{ta}\mu_d\lambda_{va2}\lambda_{vh2}\rho_{va}\rho_{vh}\cos 2\varepsilon t + 2(\lambda_{va2}(\kappa_{a2}+\mu_d)\mu_{v_h}\rho_{va} \\
&+\lambda_{vh2}(\kappa_{a1}+\mu_d)\mu_{v_a}\rho_{vh} + N_{ta}\mu_d\lambda_{va2}\lambda_{vh2}(\rho_{va}+\rho_{vh}))\sin\varepsilon t))]
\end{aligned} \quad (6d)$$

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$$\begin{aligned} I^{*}_{vb2} = {} & [(N_{ta} N_{vb2} \beta_{aa} \lambda_{vb2} \mu_d - (I^{*}_{va2} N_{va2} \beta_{aa} + \mu_d)(\kappa_{a2} + \mu_d)\mu_{v_b} \\ & + \sin\varepsilon t(-I^{*}_{va2} N_{va2} \alpha_a \beta_{aa} (\kappa_{a2} + \mu_d)\mu_{v_b} \\ & + N_{ta} N_{vb2} \beta_{ba} \lambda_{vb2} \mu_d (\alpha_b + \rho_{vb}) \\ & + N_{ta} N_{vb2} \alpha_b \beta_{ba} \lambda_{vb2} \mu_d \rho_{vb} \sin\varepsilon t))]/ \\ & [N_{vb2} \beta_{ba}(1 + \alpha_b \sin\varepsilon t)(N_{ta} \lambda_{vb2} \mu_d + (\kappa_{a2} + \mu_d)\mu_{v_b} \\ & + N_{ta} \lambda_{vb2} \mu_d \rho_{vb} \sin\varepsilon t))] \end{aligned} \qquad (6e)$$

Local Asymptotically Stability

The local stability of each equilibrium state is determined from Jacobian matrix of right hand side of the above set of differential equations evaluated at the equilibrium state.

Proposition A. If $S_0<1$, $S_{0c}<1$ and $S_{0a}<1$ when $\varepsilon = 0$, then the disease free equilibrium state E_{0c} in children and E_{0a} in adult are locally asymptotically stable, where:

$$S_0 = \max\left\{ \begin{aligned} & \frac{\begin{aligned} & 2N_{vb1}\alpha_b\beta_{bc}\lambda_{vb1}(\kappa_{c1}+\mu_d)\mu_{v_a}\rho_{vb} + N_{va1}\beta_{ac}\lambda_{va1}(2(N_{tc}\lambda_{vb1}\mu_d + (\kappa_{c2}+\mu_d)\mu_{v_b}) \\ & (2+\alpha_a\rho_{va}) + 2N_{tc}\lambda_{vb1}\mu_d(\alpha_a+\rho_{va})) \end{aligned}}{\begin{aligned} & (2\lambda_{vb1}(\kappa_{c1}+\mu_d)\mu_{v_a}(2\mu_d + N_{vb1}\beta_{bc}(2+\alpha_b\rho_{vb})) + 2N_{va1}\beta_{ac}\lambda_{va1}(\alpha_a(\kappa_{c2}+\mu_d)\mu_{v_b}\rho_{va} \\ & + N_{tc}\lambda_{vb1}\mu_d(\alpha_a\rho_{va} + (\alpha_a+\rho_{va})\rho_{vb}))) \end{aligned}}, \\ & \frac{\begin{aligned} & 2N_{vb2}\alpha_b\beta_{ba}\lambda_{vb2}(\kappa_{a1}+\mu_d)\mu_{v_a}\rho_{vb} + N_{va2}\beta_{aa}\lambda_{va2}(2(N_{ta}\lambda_{vb2}\mu_d + (\kappa_{a2}+\mu_d)\mu_{v_b}) \\ & (2+\alpha_a\rho_{va}) + 2N_{ta}\lambda_{vb2}\mu_d(\alpha_a+\rho_{va})) \end{aligned}}{\begin{aligned} & (2\lambda_{vb2}(\kappa_{a1}+\mu_d)\mu_{v_a}(2\mu_d + N_{vb2}\beta_{ba}(2+\alpha_b\rho_{vb})) + 2N_{va2}\beta_{aa}\lambda_{va2}(\alpha_a(\kappa_{a2}+\mu_d) \\ & \mu_{v_b}\rho_{va} + N_{ta}\lambda_{vb2}\mu_d(\alpha_a\rho_{va} + (\alpha_a+\rho_{va})\rho_{vb}))) \end{aligned}} \end{aligned} \right\} \qquad (6f)$$

$$S_{0c} = \frac{\begin{aligned} & 2N_{vb1}\alpha_b\beta_{bc}\lambda_{vb1}(\kappa_{c1}+\mu_d)\mu_{v_a}\rho_{vb} + N_{va1}\beta_{ac}\lambda_{va1}(2(N_{tc}\lambda_{vb1}\mu_d + (\kappa_{c2}+\mu_d)\mu_{v_b}) \\ & (2+\alpha_a\rho_{va}) + 2N_{tc}\lambda_{vb1}\mu_d(\alpha_a+\rho_{va})) \end{aligned}}{\begin{aligned} & (2\lambda_{vb1}(\kappa_{c1}+\mu_d)\mu_{v_a}(2\mu_d + N_{vb1}\beta_{bc}(2+\alpha_b\rho_{vb})) + 2N_{va1}\beta_{ac}\lambda_{va1}(\alpha_a(\kappa_{c2}+\mu_d)\mu_{v_b}\rho_{va} \\ & + N_{tc}\lambda_{vb1}\mu_d(\alpha_a\rho_{va} + (\alpha_a+\rho_{va})\rho_{vb}))) \end{aligned}} \qquad (6g)$$

in children

$$S_{0a} = \frac{\begin{aligned} & 2N_{vb2}\alpha_b\beta_{ba}\lambda_{vb2}(\kappa_{a1}+\mu_d)\mu_{v_a}\rho_{vb} + N_{va2}\beta_{aa}\lambda_{va2}(2(N_{ta}\lambda_{vb2}\mu_d + (\kappa_{a2} \\ & +\mu_d)\mu_{v_b})(2+\alpha_a\rho_{va}) + 2N_{ta}\lambda_{vb2}\mu_d(\alpha_a+\rho_{va})) \end{aligned}}{\begin{aligned} & (2\lambda_{vb2}(\kappa_{a1}+\mu_d)\mu_{v_a}(2\mu_d + N_{vb2}\beta_{ba}(2+\alpha_b\rho_{vb})) + 2N_{va2}\beta_{aa}\lambda_{va2}(\alpha_a(\kappa_{a2}+\mu_d)\mu_{v_b}\rho_{va} \\ & + N_{ta}\lambda_{vb2}\mu_d(\alpha_a\rho_{va} + (\alpha_a+\rho_{va})\rho_{vb}))) \end{aligned}} \text{ in adult} \qquad (6h)$$

Proof.

For the disease free equilibrium state in children $E_{0c} = (1, 0, 0, 0, 0)$ and in adult $E_{0a} = (1, 0, 0, 0, 0)$.

The system defined by Equation (4a) - (4j), the Jacobian matrix evaluated at E_{0c} and E_{0a} respectively, given by:

$$J_c = \begin{bmatrix} -(\mu_d)\ 0\ 0 - \beta_{ac}(1+\alpha_a \sin(\varepsilon t)) \\ N_{va1} - \beta_{bc}(1+\alpha_b \sin(\varepsilon t))N_{vb1} \\ 0 - (\kappa_{c1} + \mu_d)\ 0\ \beta_{ac}(1+\alpha_a \sin(\varepsilon t))N_{va1}\ 0 \\ 0\ 0\ -(\kappa_{c2} + \mu_d)\ 0\beta_{bc}(1+\alpha_b \sin(\varepsilon t))N_{vb1} \\ 0\ \lambda_{va1}(1+\rho_{va}\sin(\varepsilon t))\ 0 - \mu_{va1}\ \ 0 \\ 0\ 0\ \lambda_{vb1}(1+\rho_{vb}\sin(\varepsilon t))\ 0 - \mu_{vb1} \end{bmatrix} \qquad (7a)$$

$$J_a = \begin{bmatrix} -(\mu_d)\ 0\ 0 - \beta_{aa}(1+\alpha_a \sin(\varepsilon t))N_{va2} \\ -\beta_{ba}(1+\alpha_b \sin(\varepsilon t))N_{vb2} \\ 0 - (\kappa_{a1} + \mu_d)\ 0\ \beta_{aa}(1+\alpha_a \sin(\varepsilon t))N_{va2}\ 0 \\ 0\ 0 - (\kappa_{a2} + \mu_d)0\ \beta_{ba}(1+\alpha_b \sin(\varepsilon t))N_{vb2} \\ 0\ \lambda_{va2}(1+\rho_{va}\sin(\varepsilon t))\ 0 - \mu_{va2}\ 0 \\ 0\ 0\ \lambda_{vb2}(1+\rho_{vb}\sin(\varepsilon t))0 - \mu_{vb2} \end{bmatrix} \qquad (7b)$$

The eigenvalues are obtained by solving the characteristic equations, det $|\eta I_5 - J_i| = 0$. Where I_5 is the 5×5 identity matrix and $J_i(J_i;\ i = c,a)$ is the Jacobian matrix matrix for (7a) and (7b), respectively. To evaluate the determinant, we get the following characteristic equations:

$$(\eta + \mu_d)(\eta^4 + W_1\eta^3 + W_2\eta^2 + W_3\eta + W_4) = 0 \qquad (8a)$$

$$(\eta + \mu_d)(\eta^4 + F_1\eta^3 + F_2\eta^2 + F_3\eta + F_4) = 0 \qquad (8b)$$

Where:

$$W_1 = \kappa_{c1} + \kappa_{c2} + 2\mu_d + \mu_{v_a} + \mu_{v_b} \qquad (9a)$$

$$\begin{aligned} W_2 = {} & -N_{va1}\beta_{ac}\lambda_{va1} - N_{vb1}\beta_{bc}\lambda_{vb1}) \\ & + \mu_d^2 + 2\mu_d\mu_{v_a} + 2\mu_d\mu_{v_b} + \mu_{v_a}\mu_{v_b} \\ & + \kappa_{c2}(\mu_d + \mu_{v_a} + \mu_{v_b}) + \kappa_{c1}(\kappa_{c2} + \mu_d + \mu_{v_a} + \mu_{v_b}) \end{aligned} \qquad (9b)$$

$$\begin{aligned} W_3 = {} & (\kappa_{c1} + \mu_d)(\kappa_{c2} + \mu_d)\mu_{v_a} - N_{vb1}\beta_{bc}\lambda_{vb1}(\kappa_{c1} + \mu_d + \mu_{v_a}) \\ & + ((\kappa_{c1} + \mu_d)(\kappa_{c2} + \mu_d) + (\kappa_{c1} + \kappa_{c2} + 2\mu_d)\mu_{v_a}) \\ & \mu_{v_b} - N_{va1}\beta_{ac}\lambda_{va1}(\kappa_{c2} + \mu_d + \mu_{v_b}) + (\beta_{ac}\lambda_{va1}(\kappa_{c2} + \mu_d + \mu_{v_b})) \end{aligned} \qquad (9c)$$

$$\begin{aligned} W_4 = {} & (N_{va1}\beta_{ac}\lambda_{va1} - (\kappa_{c1} + \mu_d)\mu_{v_a}) \\ & (N_{vb1}\beta_{bc}\lambda_{vb1} + (\kappa_{c2} + \mu_d)\mu_{v_b}) \end{aligned} \qquad (9d)$$

$$F_1 = \kappa_{a1} + \kappa_{a2} + 2\mu_d + \mu_{v_a} + \mu_{v_b} \qquad (9e)$$

$$\begin{aligned} F_2 = {} & -N_{va2}\beta_{aa}\lambda_{va2} - N_{vb2}\beta_{ba}\lambda_{vb2}) + \mu_d^2 + 2\mu_d\mu_{v_a} + 2\mu_d\mu_{v_b} \\ & + \mu_{v_a}\mu_{v_b} + \kappa_{a2}(\mu_d + \mu_{v_a} + \mu_{v_b}) + \kappa_{a1}(\kappa_{a2} + \mu_d + \mu_{v_a} + \mu_{v_b}) \end{aligned} \qquad (9f)$$

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$$F_3=(\kappa_{a1}+\mu_d)(\kappa_{a2}+\mu_d)\mu_{v_a}-N_{vh2}\beta_{ha}\lambda_{vh2}(\kappa_{a1}+\mu_d+\mu_{v_a})$$
$$+((\kappa_{a1}+\mu_d)(\kappa_{a2}+\mu_d)+(\kappa_{a1}+\kappa_{a2}+2\mu_d)\mu_{v_a})\mu_{v_b} \quad (9g)$$
$$-N_{va2}\beta_{aa}\lambda_{va2}(\kappa_{a2}+\mu_d+\mu_{v_b})+(\beta_{aa}\lambda_{va2}(\kappa_{a2}+\mu_d+\mu_{v_b}))$$

$$F_4=(N_{va2}\beta_{aa}\lambda_{va2}-(\kappa_{a1}+\mu_d)\mu_{v_a})(N_{vh2}\beta_{ha}\lambda_{vh2}+(\kappa_{a2}+\mu_d)\mu_{v_b}) \quad (9h)$$

From the characteristic equation, Equation (8a)-(8b), we see that eigenvalues are $\eta_c=-\mu_d$ and $\eta_a=-\mu_d$, all of these eigenvalues are negative, for $S_0<1$. The sign of other four eigenvalues can be ascertained by solving equation $(\eta^4+W_1\eta^3+W_2\eta^2+W_3\eta+W_4)=0$ and $(\eta^4+F_1\eta^3+F_2\eta^2+F_3\eta+F_4)=0$. The remaining four eigenvalues have negative real parts if they satisfy Routh-Hurwitz criteria (10a) - (10d) (Esteva and Vargas, 1998), each equilibrium state is locally asymptotically stable if the following conditions are satisfied:

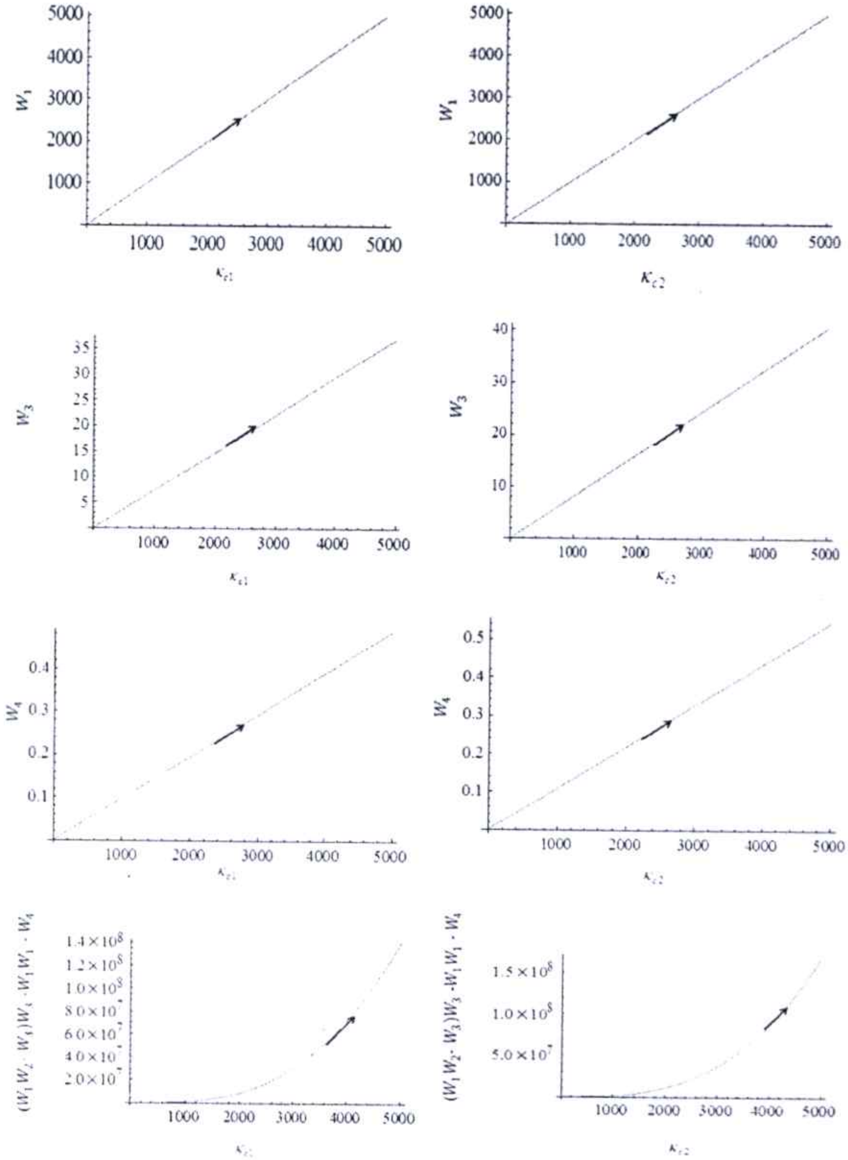
$$W_1 \text{ and } F_1>0 \quad (10a)$$

$$W_3 \text{ and } F_3>0 \quad (10b)$$

$$W_4 \text{ and } F_4>0 \quad (10c)$$

$$(W_1W_2-W_3)W_3+W_1^2W_4>0 \text{ and } (F_1F_2-F_3)F_3+F_1^2F_4>0 \quad (10d)$$

After we use *Mathematica* to show the conditions of locally asymptotically stable, we can see that W_1 and F_1 are always positive. For the equations given by (10b)-(10d), we show these conditions by using the following Fig. 2.



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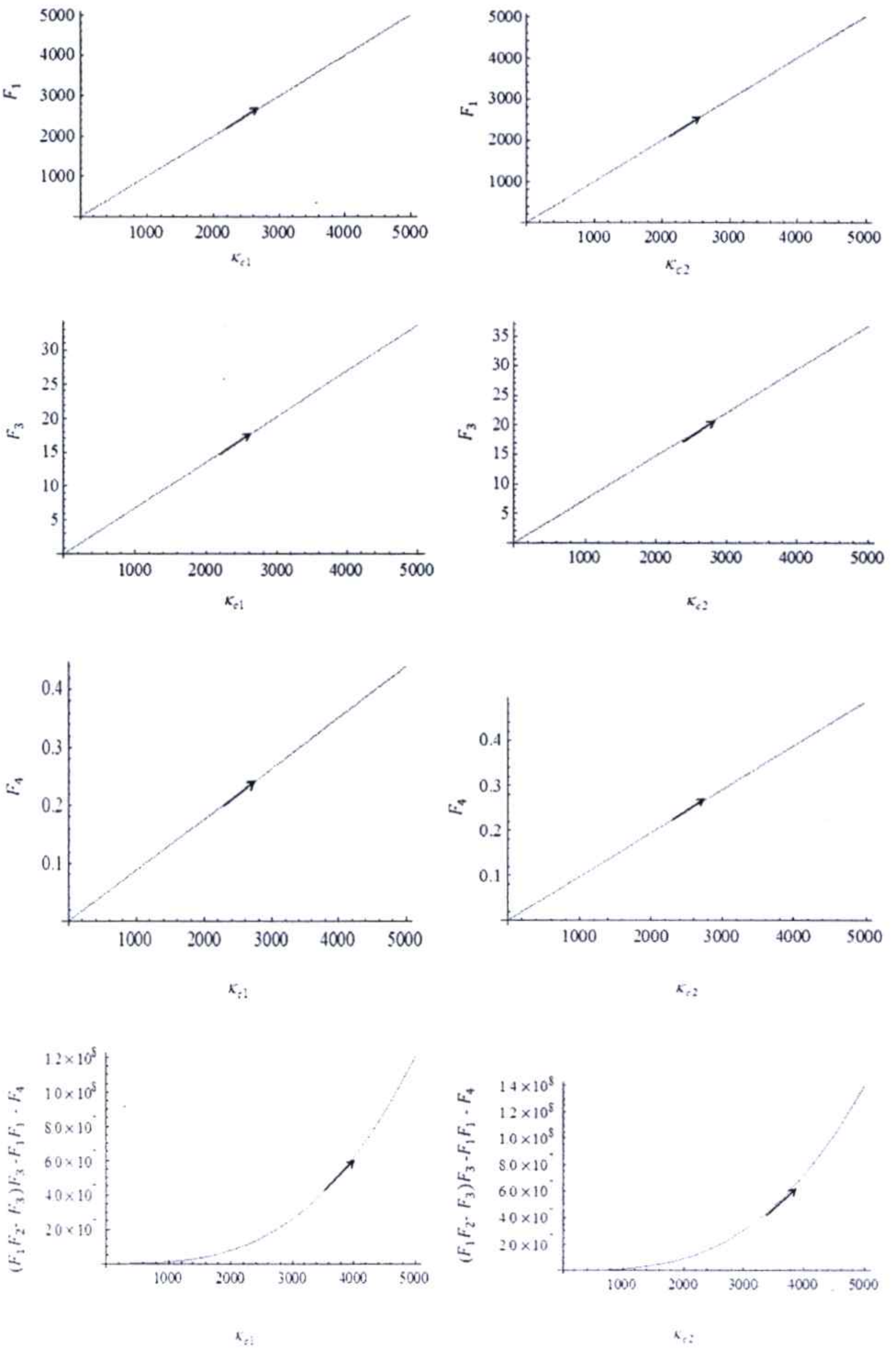

Fig. 2. The parameter spaces for the disease free equilibrium state, which satisfies the Routh-Hurwitz conditions, show onto (κ_{c1}, W_3), (κ_{c2}, W_3), (κ_{a1}, F_3), (κ_{a2}, F_3), (κ_{c1}, W_4), (κ_{c2}, W_4), (κ_{a1}, F_4), (κ_{a2}, F_4), $(\kappa_{c1}, ((W_1W_2 - W_3)W_3 - W_1^2W_4))$, $(\kappa_{c1}, ((W_1W_2 - W_3)W_3 - W_1^2W_4))$, $(\kappa_{a1}, ((F_1F_2 - F_3)F_3 - F_1^2F_4))$, $(\kappa_{a2}, ((F_1F_2 - F_3)F_3 - F_1^2F_4))$, respectively. The values of parameter are follows: $\kappa_{c1} = 1/(17/2)$, $\kappa_{c2} = 1/(19/2)$, $\mu_d = 1/(365*746)$ day^{-1}, $N_{tc} = 9000$, $N_{va1} = 4000$, $N_{vb1} = 5500$, $\beta_{ac} = 0.00769$, $\beta_{bc} = 0.000246$, $\lambda_{va1} = 0.00000576$, $\lambda_{vb1} = 0.00000335$, $\alpha_a = 0.07$, $\alpha_b = 0.067$ and $N_t = 100000$, $\kappa_{a1} = 1/(19/2)$, $\kappa_{a2} = 1/(21/2)$, $\mu_d = 1/(365*74.6)$ day^{-1}, $N_{ta} = 6000$, $N_{va2} = 3000$, $N_{vb2} = 4100$, $\beta_{aa} = 0.000045$, $\beta_{ba} = 0.000067$, $\lambda_{va2} = 0.0066$, $\lambda_{vb2} = 0.00235$, $\alpha_a = 0.07$, $\alpha_b = 0.067$ and $N_t = 100000$. From the above figures, the Routh-Hurwitz conditions are satisfied for $S_0 > 1$

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Endemic Disease State

Proposition B. If $S_0>1$, when $\varepsilon = 0$, then the equilibrium state $\hat{S}=(S_c^*, I_{c1}^*, I_{c2}^*, I_{va1}^*, I_{vb1}^*, S_a^*, I_{a1}^*, I_{a2}^*, I_{va2}^*, I_{vb2}^*)$ is locally asymptotically stable.

Proof.

For the endemic disease equilibrium state $E_{1c} = (S_c^*, I_{c1}^*, I_{c2}^*, I_{va1}^*, I_{vb1}^*)$ in children and $E_{1a} = (S_a^*, I_{a1}^*, I_{a2}^*, I_{va2}^*, I_{vb2}^*)$ in adult, we obtain the characteristic equation:

$$(\eta^5 + D_1\eta^4 + D_2\eta^3 + D_3\eta^2 + D_4\eta + D_5) = 0 \text{ in children} \quad (11a)$$

$$(\eta^5 + G_1\eta^4 + G_2\eta^3 + G_3\eta^2 + G_4\eta + G_5) = 0 \text{ in adult} \quad (11b)$$

Where:

$$D_1 = N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \kappa_{c1} + \kappa_{c2} + 3\mu_d + \mu_{v_a} + \mu_{v_b} \quad (12a)$$

$$\begin{aligned}
D_2 = {} & \kappa_{c1}\kappa_{c2} + 2\kappa_{c1}\mu_d + 2\kappa_{c2}\mu_d + 3\mu_d^2 + \kappa_{c1}\mu_{v_a} + \kappa_{c2}\mu_{va1} \\
& + 3\mu_d\mu_{v_a}(\kappa_{c1} + \kappa_{c2} + 3\mu_d + \mu_{va1})\mu_{v_b} + N_{va1}\beta_{ac} \\
& (-(\lambda_{va1}\mu_d / N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2\mu_d) + \theta_1(\kappa_{c1} + \kappa_{c2} + 2\mu_d \\
& + (\lambda_{va1}\mu_d / N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2\mu_d) + \mu_{v_a} + \mu_{v_b})) \\
& + N_{vh1}\beta_{hc}(-(\lambda_{vh1}\mu_d / N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2\mu_d) + \theta_2(\kappa_{c1} \\
& + \kappa_{c2} + 2\mu_d + (\lambda_{vh1}\mu_d / N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2\mu_d)\mu_{v_a} + \mu_{v_b}))
\end{aligned} \quad (12b)$$

$$\begin{aligned}
D_3 = {} & \frac{1}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d}(N_{va1}^2\beta_{ac}^2\theta_1^2(\mu_d^2 + 2\mu_d\mu_{va1} \\
& + 2\mu_d\mu_{v_b} + \mu_{v_a}\mu_{v_b} + \kappa_{c2}(\mu_d + \mu_{v_a} + \mu_{v_b}) + \kappa_{c1}(\kappa_{c2} + \mu_d \\
& + \mu_{v_a} + \mu_{v_b})) + N_{vh1}^2\beta_{hc}^2\theta_2^2 \\
& (\mu_d^2 + 2\mu_d\mu_{v_a} + 2\mu_d\mu_{v_b} \\
& + \mu_{v_a}\mu_{v_b} + \kappa_{c2}(\mu_d + \mu_{v_a} + \mu_{v_b}) + \kappa_{c1}(\kappa_{c2} + \mu_d + \mu_{v_a} \\
& + \mu_{v_b})) + \mu_d(\kappa_{c2}(\mu_d^2 + \mu_{v_a}\mu_{v_b} + 2\mu_d(\mu_{v_a} + \mu_{v_b})) \\
& + \mu_d(\mu_d^2 + 3\mu_{v_a}\mu_{v_b} + 3\mu_d(\mu_{v_a} + \mu_{v_b})) + \kappa_{c1}(\mu_d^2 + \mu_{v_a}\mu_{v_b} \\
& + 2\mu_d(\mu_{v_a} + \mu_{v_b}) + \kappa_{c2}(\mu_d + \mu_{v_a} + \mu_{v_b}))) + N_{vb1}\beta_{bc} \\
& (-\lambda_{vh1}\mu_d(\kappa_{c1} + 2\mu_d + \mu_{v_a}) + \theta_2(\mu_d(2\mu_d(\kappa_{c2} + \lambda_{vb1} \\
& + \mu_d) + (3\kappa_{c2} + \lambda_{vh1} + 5\mu_d)\mu_{v_a}) + (\kappa_{c2}(3\mu_d + \mu_{v_a}) \\
& + \mu_d(5\mu_d + 4\mu_{v_a}))\mu_{v_b} + \kappa_{c2}(2\mu_d + \mu_{v_a} + \mu_{v_b})))) \\
& + N_{va1}\beta_{ac}(-\lambda_{va1}\mu_d \\
& (\kappa_{c2} + 2\mu_d + \mu_{v_b}) + \theta_1(\kappa_{c2}\mu_d(\lambda_{va1} \\
& + 2\mu_d + 3\mu_{v_b}) + \kappa_{c2}(3\mu_d + \mu_{v_a})\mu_{v_b} \\
& + \mu_d(2\mu_d(\lambda_{va1} + \mu_d) \\
& + 5\mu_d\mu_{v_a} + (\lambda_{va1} + 5\mu_d + 4\mu_{v_a})\mu_{v_b}) + \kappa_{c1}(2\mu_d^2 + \mu_{v_a}\mu_{v_b} \\
& + 3\mu_d(\mu_{v_a} + \mu_{v_b}) + \kappa_{c2}(2\mu_d + \mu_{v_a} + \mu_{v_b}))) \\
& + N_{vh1}\beta_{hc}(-\theta_2\lambda_{va1}\mu_d + \theta_1(-\lambda_{vh1}\mu_d + \theta_2(\mu_d(\lambda_{va1} + \lambda_{vh1} \\
& + 2\mu_d + 4\mu_{v_a}) + 2(2\mu_d + \mu_{v_b})\mu_{v_b} \\
& + 2\kappa_{c2}(\mu_d + \mu_{v_a} + \mu_{v_b}) \\
& + 2\kappa_{c1}(\kappa_{c2} + \mu_d + \mu_{v_a} + \mu_{v_b}))))))
\end{aligned} \quad (12c)$$

$$\begin{aligned}
D_4 = {} & \kappa_{c1}\kappa_{c2}\mu_d\mu_{v_a} + \kappa_{c1}\mu_d^2\mu_{v_a} + \kappa_{c2}\mu_d^2\mu_{v_a} + \mu_d^3\mu_{v_a} + \kappa_{c1}\kappa_{c2}\mu_d\mu_{v_b} \\
& + \kappa_{c1}\mu_d^2\mu_{v_b} + \kappa_{c2}\mu_d^2\mu_{v_b} + \mu_d^3\mu_{v_b} + \kappa_{c1}\kappa_{c2}\mu_{v_a}\mu_{v_b} + 2\kappa_{c1}\mu_d\mu_{v_a}\mu_{v_b} \\
& + 2\kappa_{c2}\mu_d\mu_{v_a}\mu_{v_b} + 3\mu_d^3\mu_{v_a}\mu_{v_b} + \frac{1}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d} \\
& N_{vh1}\beta_{hc}((-1+\theta_2)\lambda_{vh1}\mu_d^2(\mu_d + 2\mu_{v_a}) + \theta_2(N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d) \\
& (\mu_d(\kappa_{c2} + \mu_d)\mu_{v_a} + (\mu_d(\kappa_{c2} + \mu_d) + (\kappa_{c2} + 2\mu_d)\mu_{v_a})\mu_{v_b}) \\
& + \kappa_{c1}((-1+\theta_2)\lambda_{vh1}\mu_d(\mu_d + 2\mu_{v_a}) + \theta_2(N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d)((\kappa_{c2} + \mu_d)\mu_{v_a} \\
& + (\kappa_{c2} + \mu_d + \mu_{v_a})\mu_{v_b}))) + N_{va1}\beta_{ac}(-\frac{\kappa_{c2}\lambda_{va1}\mu_d^2}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d} + \frac{\theta_1\kappa_{c2}\lambda_{va1}\mu_d^2}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d} \\
& - \frac{\lambda_{va1}\mu_d^3}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d} + \frac{\theta_1\lambda_{va1}\mu_d^3}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d} + \theta_1\kappa_{c1}\kappa_{c2}\mu_{v_a} + \theta_1\kappa_{c1}\mu_d\mu_{v_a} \\
& + \theta_1\kappa_{c2}\mu_d\mu_{v_a} + \theta_1\mu_d^2\mu_{v_a} + \theta_1\kappa_{c1}\kappa_{c2}\mu_{v_b} + \theta_1\kappa_{c1}\mu_d\mu_{v_b} + \theta_1\kappa_{c2}\mu_d\mu_{v_b} + \theta_1\mu_d^2\mu_{v_b} \\
& - \frac{\kappa_{c2}\lambda_{va1}\mu_d\mu_{v_b}}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d} + \frac{\theta_1\kappa_{c2}\lambda_{va1}\mu_d\mu_{v_b}}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d} - \frac{2\lambda_{va1}\mu_d^2\mu_{v_b}}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d} + \frac{2\theta_1\lambda_{va1}\mu_d^2\mu_{v_b}}{N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d} \\
& + \theta_1\kappa_{c1}\mu_{v_a}\mu_{v_b} + \theta_1\kappa_{c2}\mu_{v_a}\mu_{v_b} + 2\theta_1\mu_d\mu_{v_a}\mu_{v_b} + \frac{1}{(N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{hc}\theta_2 + \mu_d)^2}N_{vh1}\beta_{hc}\mu_d(N_{va1}\beta_{ac}\theta_1 \\
& (-\theta_1\lambda_{vh1}(\kappa_{c1} + \mu_d + \mu_{v_a}) + \theta_2((-1+\theta_1)\kappa_{c2}\lambda_{va1} - \lambda_{va1}(\mu_d + \mu_{v_b}) + \theta_1(\lambda_{vh1}(\kappa_{c1} + \mu_d + \mu_{v_a}) + \lambda_{va1}(\mu_d + \mu_{v_b})))) \\
& + N_{vh1}\beta_{hc}\theta_2(-\theta_1\lambda_{vh1}(\kappa_{c1} + \mu_d + \mu_{v_a}) + \theta_2((-1+\theta_1)\kappa_{c2}\lambda_{va1} - \lambda_{va1}(\mu_d + \mu_{vb1}) + \theta_1(\lambda_{vh1}(\kappa_{c1} + \mu_d + \mu_{v_a}) \\
& + \lambda_{va1}(\mu_d + \mu_{v_b})))) + \mu_d(-\lambda_{vh1}(-\lambda_{va1} + \theta_1(\kappa_{c1} + \lambda_{va1} + \mu_d + \mu_{v_a})) + \theta_2((-1+\theta_1)\kappa_{c2}\lambda_{va1} - \lambda_{va1}(\lambda_{vh1} + \mu_d + \mu_{v_b}) + \\
& \theta_1(\kappa_{c1}\lambda_{vh1} + \lambda_{vh1}(\mu_d + \mu_{v_a}) + \lambda_{va1}(\lambda_{vh1} + \mu_d + \mu_{v_b}))))))
\end{aligned} \quad (12d)$$

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$$\begin{aligned}
D_5 = {} & \frac{1}{(N_{va1}\beta_{ac}\theta_1 + N_{vh1}\beta_{bc}\theta_2 + \mu_d)^2}(N_{va1}^3\beta_{ac}^3\theta_1^3(\kappa_{c1}+\mu_d)(\kappa_{c2}+\mu_d)\mu_{va1}\mu_{v_b} \\
& +(N_{vh1}\beta_{hc}\theta_2+\mu_d)(\kappa_{c1}+\mu_d)\mu_{v_a}(N_{vh1}\beta_{bc}(-1+\theta_2)\lambda_{vh1}\mu_d^2+(N_{vh1}\beta_{hc}\theta_2+\mu_d)^2(\kappa_{c2}+\mu_d)\mu_{v_b}) \\
& +N_{va1}^2\beta_{ac}^2\theta_1(N_{vh1}\beta_{hc}\theta_1(-1+\theta_2)\lambda_{vh1}\mu_d(\kappa_{c1}+\mu_d)\mu_{v_a}+(N_{vh1}\beta_{bc}\theta_2+\mu_d)(\kappa_{c2}+\mu_d) \\
& ((-1+\theta_1)\lambda_{va1}\mu_d+3\theta_1(\kappa_{c1}+\mu_d)\mu_{v_a})\mu_{v_b})+N_{va1}\beta_{ac}(\mu_d^2(\kappa_{c2}+\mu_d)((-1+\theta_1)\lambda_{va1}\mu_d \\
& +3\theta_1(\kappa_{c1}+\mu_d)\mu_{v_a})\mu_{v_b})+N_{vh1}^2\beta_{bc}^2\theta_2(\theta_1(-1+\theta_2)\lambda_{vh1}\mu_d(\kappa_{c1}+\mu_d)\mu_{v_a}+\theta_2(\kappa_{c2}+\mu_d)((-1+\theta_1)\lambda_{va1}\mu_d+3\theta_1(\kappa_{c1}+\mu_d)\mu_{v_a})\mu_{v_b}) \\
& +N_{vh1}\beta_{hc}\mu_d((-1+\theta_2)\lambda_{vh1}\mu_d+2\theta_1(\kappa_{c1}+\mu_d)\mu_{v_a})+2\theta_2(\kappa_{c2}+\mu_d)((-1+\theta_1)\lambda_{va1}\mu_d+3\theta_1(\kappa_{c1}+\mu_d)\mu_{v_a})\mu_{v_b})))
\end{aligned} \tag{12e}$$

$$G_1 = N_{va2}\beta_{aa}\theta_3 + N_{vh2}\beta_{ha}\theta_4 + \kappa_{a1} + \kappa_{a2} + 3\mu_d + \mu_{v_a} + \mu_{v_b} \tag{12f}$$

$$\begin{aligned}
G_2 = {} & \kappa_{a1}\kappa_{a2}+2\kappa_{a1}\mu_d+2\kappa_{a2}\mu_d+3\mu_d^2+\kappa_{a1}\mu_{v_a}+\kappa_{a2}\mu_{v_a}+3\mu_d\mu_{v_a}+ \\
& (\kappa_{a1}+\kappa_{a2}+3\mu_d+\mu_{v_a})\mu_{v_b}+N_{va2}\beta_{aa}(-(\lambda_{va2}\mu_d / N_{va2}\beta_{aa}\theta_3+N_{vb2}\beta_{ba}\theta_4\mu_d) \\
& +\theta_3(\kappa_{a1}+\kappa_{a2}+2\mu_d+(\lambda_{va2}\mu_d / N_{va2}\beta_{aa}\theta_3+N_{vb2}\beta_{ba}\theta_4\mu_d)+\mu_{v_a}+\mu_{v_b}))+N_{vb2}\beta_{ba} \\
& (-(\lambda_{vh2}\mu_d / N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4\mu_d)+\theta_4(\kappa_{a1}+\kappa_{a2}+2\mu_d+(\lambda_{vb2}\mu_d / N_{va2}\beta_{aa}\theta_3+N_{vb2}\beta_{ba}\theta_4\mu_d)\mu_{v_a}+\mu_{v_b}))
\end{aligned} \tag{12g}$$

$$\begin{aligned}
G_3 = {} & \frac{1}{N_{va2}\beta_{aa}\theta_3+N_{vb2}\beta_{ha}\theta_4+\mu_d}(N_{va2}^2\beta_{aa}^2\theta_3^2(\mu_d^2+2\mu_d\mu_{v_a}+2\mu_d\mu_{v_b} \\
& +\mu_{va2}\mu_{v_b}+\kappa_{a2}(\mu_d+\mu_{v_a}+\mu_{v_b})+\kappa_{a1}(\kappa_{a2}+\mu_d+\mu_{v_a}+\mu_{v_b}))+N_{vh2}^2\beta_{ha}^2\theta_4^2(\mu_d^2+2\mu_d\mu_{v_a} \\
& +2\mu_d\mu_{v_b}+\mu_{v_a}\mu_{v_b}+\kappa_{a2}(\mu_d+\mu_{v_a}+\mu_{v_b})+\kappa_{a1}(\kappa_{a2}+\mu_d+\mu_{v_a}+\mu_{v_b}))+\mu_d(\kappa_{a2}(\mu_d^2+\mu_{v_a}\mu_{v_b} \\
& +2\mu_d(\mu_{v_a}+\mu_{v_b}))+\mu_d(\mu_d^2+3\mu_{v_a}\mu_{v_b}+3\mu_d(\mu_{v_a}+\mu_{v_b}))+\kappa_{a1}(\mu_d^2+\mu_{v_a}\mu_{v_b} \\
& +2\mu_d(\mu_{v_a}+\mu_{v_b})+\kappa_{a2}(\mu_d+\mu_{v_a}+\mu_{v_b})))+N_{vh2}\beta_{ha}(-\lambda_{vh2}\mu_d(\kappa_{a1}+2\mu_d+\mu_{v_a}) \\
& +\theta_4(\mu_d(2\mu_d(\kappa_{a2}+\lambda_{vh2}+\mu_d)+(3\kappa_{a2}+\lambda_{vh2}+5\mu_d)\mu_{v_a})+(\kappa_{a2}(3\mu_d+\mu_{v_a})+\mu_d(5\mu_d+4\mu_{v_a}))\mu_{v_b} \\
& +\kappa_{a2}(2\mu_d+\mu_{v_a}+\mu_{v_b}))))+N_{va2}\beta_{aa}(-\lambda_{va2}\mu_d(\kappa_{a2}+2\mu_d+\mu_{v_b})+\theta_3(\kappa_{a2}\mu_d(\lambda_{va2} \\
& +2\mu_d+3\mu_{v_a})+\kappa_{a2}(3\mu_d+\mu_{v_a})\mu_{vb2}+\mu_d(2\mu_d(\lambda_{va2}+\mu_d)+5\mu_d\mu_{v_a}+(\lambda_{va2}+5\mu_d+4\mu_{v_a})\mu_{v_b}) \\
& +\kappa_{a1}(2\mu_d^2+\mu_{v_a}\mu_{v_b}+3\mu_d(\mu_{v_a}+\mu_{v_b})+\kappa_{a2}(2\mu_d+\mu_{v_a}+\mu_{v_b})))+N_{vh2}\beta_{ha}(-\theta_4\lambda_{va2}\mu_d \\
& +\theta_3(-\lambda_{vh2}\mu_d+\theta_4(\mu_d(\lambda_{va2}+\lambda_{vh2}+2\mu_d+4\mu_{v_a})+2(2\mu_d+\mu_{v_b})\mu_{v_b}+2\kappa_{a2}(\mu_d+\mu_{v_a}+\mu_{v_b})+2\kappa_{a1}(\kappa_{a2}+\mu_d+\mu_{va2}+\mu_{v_b}))))))
\end{aligned} \tag{12h}$$

$$\begin{aligned}
G_4 = {} & \kappa_{a1}\kappa_{a2}\mu_d\mu_{v_a}+\kappa_{a1}\mu_d^2\mu_{v_a}+\kappa_{a2}\mu_d^2\mu_{v_a}+\mu_d^3\mu_{v_a}+\kappa_{a1}\kappa_{a2}\mu_d\mu_{v_b} \\
& +\kappa_{a1}\mu_d^2\mu_{v_b}+\kappa_{a2}\mu_d^2\mu_{v_b}+\mu_d^3\mu_{v_b}+\kappa_{a1}\kappa_{a2}\mu_{v_a}\mu_{v_b}+2\kappa_{a1}\mu_d\mu_{v_a}\mu_{v_b} \\
& +2\kappa_{a2}\mu_d\mu_{v_a}\mu_{v_b}+3\mu_d^3\mu_{v_a}\mu_{v_b}+\frac{1}{N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4+\mu_d}N_{vh2}\beta_{ha}((-1+\theta_4) \\
& \lambda_{vh2}\mu_d^2(\mu_d+2\mu_{v_a})+\theta_4(N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4+\mu_d)(\mu_d(\kappa_{a2}+\mu_d)\mu_{v_a}+(\mu_d(\kappa_{a2}+\mu_d) \\
& +(\kappa_{a2}+2\mu_d)\mu_{v_a})\mu_{v_b})+\kappa_{a1}((-1+\theta_4)\lambda_{vh2}\mu_d(\mu_d+2\mu_{v_a})+\theta_4(N_{va2}\beta_{aa}\theta_3 \\
& +N_{vh2}\beta_{ha}\theta_4+\mu_d)((\kappa_{a2}+\mu_d)\mu_{v_a}+(\kappa_{a2}+\mu_d+\mu_{v_a})\mu_{v_b})))+N_{va2}\beta_{aa} \\
& (-\frac{\kappa_{a2}\lambda_{va2}\mu_d^2}{N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4+\mu_d}+\frac{\theta_3\kappa_{a2}\lambda_{va2}\mu_d^2}{N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4+\mu_d}-\frac{\lambda_{va2}\mu_d^3}{N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4+\mu_d}+\frac{\theta_3\lambda_{va2}\mu_d^3}{N_{va2}\beta_{aa}\theta_3+N_{vb2}\beta_{ba}\theta_4+\mu_d} \\
& +\theta_3\kappa_{a1}\kappa_{a2}\mu_{v_a}+\theta_3\kappa_{a1}\mu_d\mu_{v_a}+\theta_3\kappa_{a2}\mu_d\mu_{v_a}+\theta_3\mu_d^2\mu_{v_a}+\theta_3\kappa_{a1}\kappa_{a2}\mu_{v_b}\theta_2\kappa_{a1}\mu_d\mu_{vh}+\theta_3\kappa_{a2}\mu_d\mu_{vh}+\theta_3\mu_d^2\mu_{v_b} \\
& -\frac{\kappa_{a2}\lambda_{va2}\mu_d\mu_{v_b}}{N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4+\mu_d}+\frac{\theta_3\kappa_{a2}\lambda_{va2}\mu_d\mu_{v_b}}{N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4+\mu_d}-\frac{2\lambda_{va2}\mu_d^2\mu_{v_b}}{N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4+\mu_d}+\frac{2\theta_3\lambda_{va2}\mu_d^2\mu_{v_b}}{N_{va2}\beta_{aa}\theta_3+N_{vb2}\beta_{ba}\theta_4+\mu_d} \\
& +\theta_3\kappa_{a1}\mu_{v_a}\mu_{v_b}+\theta_3\kappa_{a2}\mu_{v_a}\mu_{v_b}+2\theta_3\mu_d\mu_{v_a}\mu_{v_b}+\frac{1}{(N_{va2}\beta_{aa}\theta_3+N_{vh2}\beta_{ha}\theta_4+\mu_d)^2} \\
& N_{vh2}\beta_{ha}\mu_d(N_{va2}\beta_{aa}\theta_3(-\theta_3\lambda_{vh2}(\kappa_{a1}+\mu_d+\mu_{v_a})+\theta_4((-1+\theta_3)\kappa_{a2}\lambda_{va2}-\lambda_{va2}(\mu_d+\mu_{v_b}) \\
& +\theta_3(\lambda_{vh2}(\kappa_{a1}+\mu_d+\mu_{v_a})+\lambda_{va2}(\mu_d+\mu_{v_b}))))+N_{vh2}\beta_{ha}\theta_4(-\theta_3\lambda_{vh2}(\kappa_{a1}+\mu_d+\mu_{v_a})+\theta_4((-1+\theta_3)\kappa_{a2}\lambda_{va2} \\
& -\lambda_{va2}(\mu_d+\mu_{v_b})+\theta_3(\lambda_{vh2}(\kappa_{a1}+\mu_d+\mu_{v_a})+\lambda_{va2}(\mu_d+\mu_{v_b}))))+\mu_d(-\lambda_{vh2}(-\lambda_{va2}+\theta_3(\kappa_{a1}+\lambda_{va2} \\
& +\mu_d+\mu_{v_a}))+\theta_4((-1+\theta_3)\kappa_{a2}\lambda_{va2}-\lambda_{va2}(\lambda_{vh2}+\mu_d+\mu_{v_b})+\theta_3(\kappa_{a1}\lambda_{vh2}+\lambda_{vh2}(\mu_d+\mu_{v_a})+\lambda_{va2}(\lambda_{vh2}+\mu_d+\mu_{v_b}))))))
\end{aligned} \tag{12i}$$

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$$G_5 = \frac{1}{(N_{va2}\,\beta_{aa}\theta_3 + N_{vb2}\,\beta_{ba}\,\theta_4 + \mu_d)^2}(N_{vb2}^3\,\beta_{aa}^3\theta_3^3(\kappa_{a1} + \mu_d)(\kappa_{a2} + \mu_d)$$
$$\mu_{v_a}\mu_{v_b} + (N_{vb2}\,\beta_{ba}\,\theta_4 + \mu_d)(\kappa_{a1} + \mu_d)\,\mu_{v_a}\,(N_{vb2}\beta_{ba}(-1+\theta_4)\,\lambda_{vb2}\mu^2_d + (N_{vb2}\beta_{ba}\,\theta_4 + \mu_d)^2$$
$$(\kappa_{a2} + \mu_d)\,\mu_{v_b}) + N_{va2}^2\,\beta_{aa}^2\,\theta_3\,(N_{vb2}\,\beta_{ba}\,\theta_3(-1+\theta_4)\lambda_{vb2}\,\mu_d(\kappa_{a1} + \mu_d)\mu_{v_a} + (N_{vb2}\beta_{ba}\,\theta_4 + \mu_d)$$
$$(\kappa_{a2} + \mu_d)((-1+\theta_3)\,\lambda_{va2}\,\mu_d + 3\,\theta_3(\kappa_{a1} + \mu_d)\mu_{v_a})\,\mu_{v_b}) + N_{va2}\,\beta_{aa}(\mu^2_d(\kappa_{a2} + \mu_d)((-1+\theta_3)\lambda_{va2}\,\mu_d \tag{12 j}$$
$$+ 3\,\theta_3(\kappa_{a1} + \mu_d)\,\mu_{v_a})\,\mu_{v_b}) + N_{vb2}^2\,\beta_{ba}^2\,\theta_4(\theta_3(-1+\theta_4)\,\lambda_{vb2}\,\mu_d\,(\kappa_{a1} + \mu_d)\,\mu_{v_a} + \theta_4(\kappa_{a2} + \mu_d)$$
$$((-1+\theta_3)\,\lambda_{va2}\,\mu_d + 3\,\theta_3(\kappa_{a1} + \mu_d)\,\mu_{v_a})\mu_{v_b}) + N_{vb2}\,\beta_{ba}\mu_d((-1+\theta_4)\,\lambda_{vb2}\,\mu_d + 2\,\theta_3(\kappa_{a1} + \mu_d)\mu_{v_a})$$
$$+2\theta_4(\kappa_{a2} + \mu_d)((-1+\theta_3)\,\lambda_{va2}\mu_d + 3\theta_3\,(\kappa_{a1} + \mu_d)\,\mu_{v_a})\,\mu_{v_b})))$$

Where:

$$\theta_1 = \frac{\begin{array}{c}(N_{tc}\,N_{va1}\,\beta_{ac}\,\lambda_{va1}\,\lambda_{vb1}\,\mu_d - \lambda_{vb1}\,(N_{vb1}\,\beta_{bc} + \mu_d)\\ (\kappa_{c1} + \mu_d)\,\mu_{v_a} + N_{va1}\,\beta_{ac}\,\lambda_{va1}\,(\kappa_{c2} + \mu_d)\,\mu_{v_b})\end{array}}{\begin{array}{c}(N_{va1}\,\beta_{ac}\,(N_{tc}\,\lambda_{va1}\,\lambda_{vb1}\,\mu_d + \lambda_{vb1}\\ (\kappa_{c1} + \mu_d)\,\mu_{v_a} + \lambda_{va1}(\kappa_{c2} + \mu_d)\,\mu_{v_b}))\end{array}} \tag{13a}$$

$$\theta_2 = \frac{N_{tc}\,N_{vb1}\,\beta_{bc}\,\lambda_{vb1}\,\mu_d - (N_{va1}\,\beta_{ac}\,\theta_1 + \mu_d)(\kappa_{c2} + \mu_d)\mu_{v_b}}{N_{vb1}\,\beta_{bc}\,(N_{tc}\,\lambda_{vb1}\,\mu_d + (\kappa_{c2} + \mu_d)\mu_{v_b})} \tag{13b}$$

$$\theta_3 = \frac{\begin{array}{c}(N_{ta}\,N_{va2}\,\beta_{aa}\,\lambda_{va2}\,\lambda_{vb2}\,\mu_d - \lambda_{vb2}\,(N_{vb2}\,\beta_{ba} + \mu_d)\\ (\kappa_{a1} + \mu_d)\,\mu_{v_a} + N_{va2}\,\beta_{aa}\,\lambda_{va2}\,(\kappa_{a2} + \mu_d)\,\mu_{v_b})\end{array}}{\begin{array}{c}(N_{va2}\,\beta_{aa}\,(N_{ta}\,\lambda_{va2}\,\lambda_{vb2}\,\mu_d\\ + \lambda_{vb2}\,(\kappa_{a1} + \mu_d)\,\mu_{v_a} + \lambda_{va2}\,(\kappa_{a2} + \mu_d)\,\mu_{v_b}))\end{array}} \tag{13c}$$

$$\theta_4 = \frac{N_{ta}\,N_{vb2}\,\beta_{ba}\,\lambda_{vb2}\,\mu_d - (N_{va2}\,\beta_{aa}\,\theta_3 + \mu_d)(\kappa_{a2} + \mu_d)\mu_{v_b}}{N_{vb2}\,\beta_{ba}\,(N_{ta}\,\lambda_{vb2}\,\mu_d + (\kappa_{a2} + \mu_d)\,\mu_{v_b})} \tag{13d}$$

From the characteristic Equation (11a)-(11b), the eigenvalues are found by solving $(\eta^5 + D_1\eta^4 + D_2\eta^3 + D_3\eta^2 + D_4\eta + D_5) = 0$ in children and in adult $(\eta^5 + G_1\eta^4 + G_2\eta^3 + G_3\eta^2 + G_4\eta + G_5) = 0$, when $T_1 = D_1$ and G_1 for children and adult, $T_2 = D_2$ *and* G_2, for children and adult, $T_3 = D_3$ and G_3 for children and adult $T_4 = D_4$ and G_4 for children and adult, $T_5 = D_5$ and G_5 for children and adult. The five eigenvalues have negative real parts if they satisfy Routh-Hurwitz criteria (14b)-(14e) (Edelstein-Keshet, 1988), each equilibrium state is locally asymptotically stable, when it satisfies the following conditions:

$$\det H_1 = T_1 > 0 \tag{14a}$$

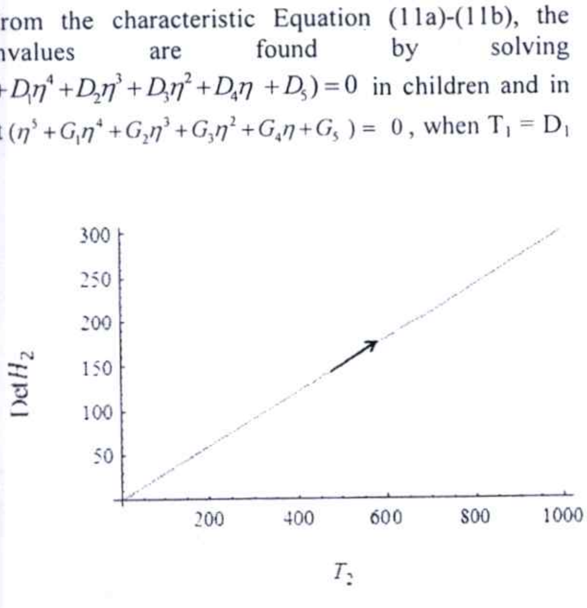
$$\det H_2 = T_1 T_2 - T_3 > 0 \tag{14b}$$

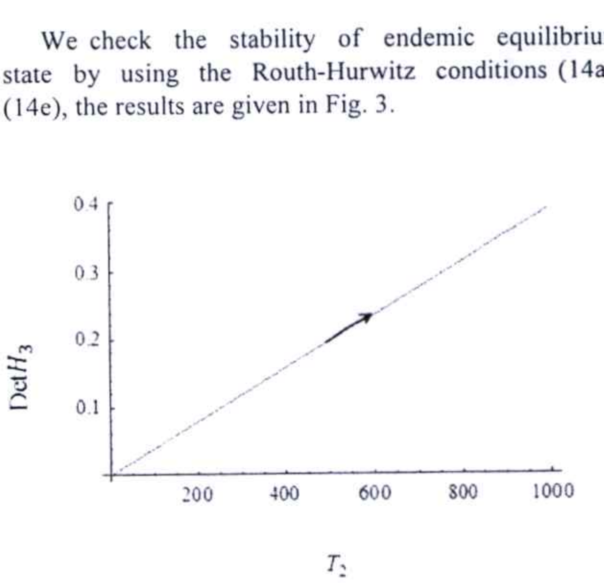
$$\det H_3 = T_1 T_2 T_3 - T_3^2 - T_1^2 T_4 > 0 \tag{14c}$$

$$\det H_4 = T_1 T_2 T_3 T_4 - T_3^2 T_4 - T_1^2 T_4^2 > 0 \tag{14d}$$

$$\det H_5 = T_1 T_2 T_3 T_4 T_5 T_3^2\, T_4 T_5 T_1^2 T_4^2 T_5 T_1 T_2^2 T_5^2 + T_2 T_3 T_5^2 + 2\, T_1 T_4 T_5^2 T_5^3 > 0 \tag{14e}$$

We check the stability of endemic equilibrium state by using the Routh-Hurwitz conditions (14a)-(14e), the results are given in Fig. 3.





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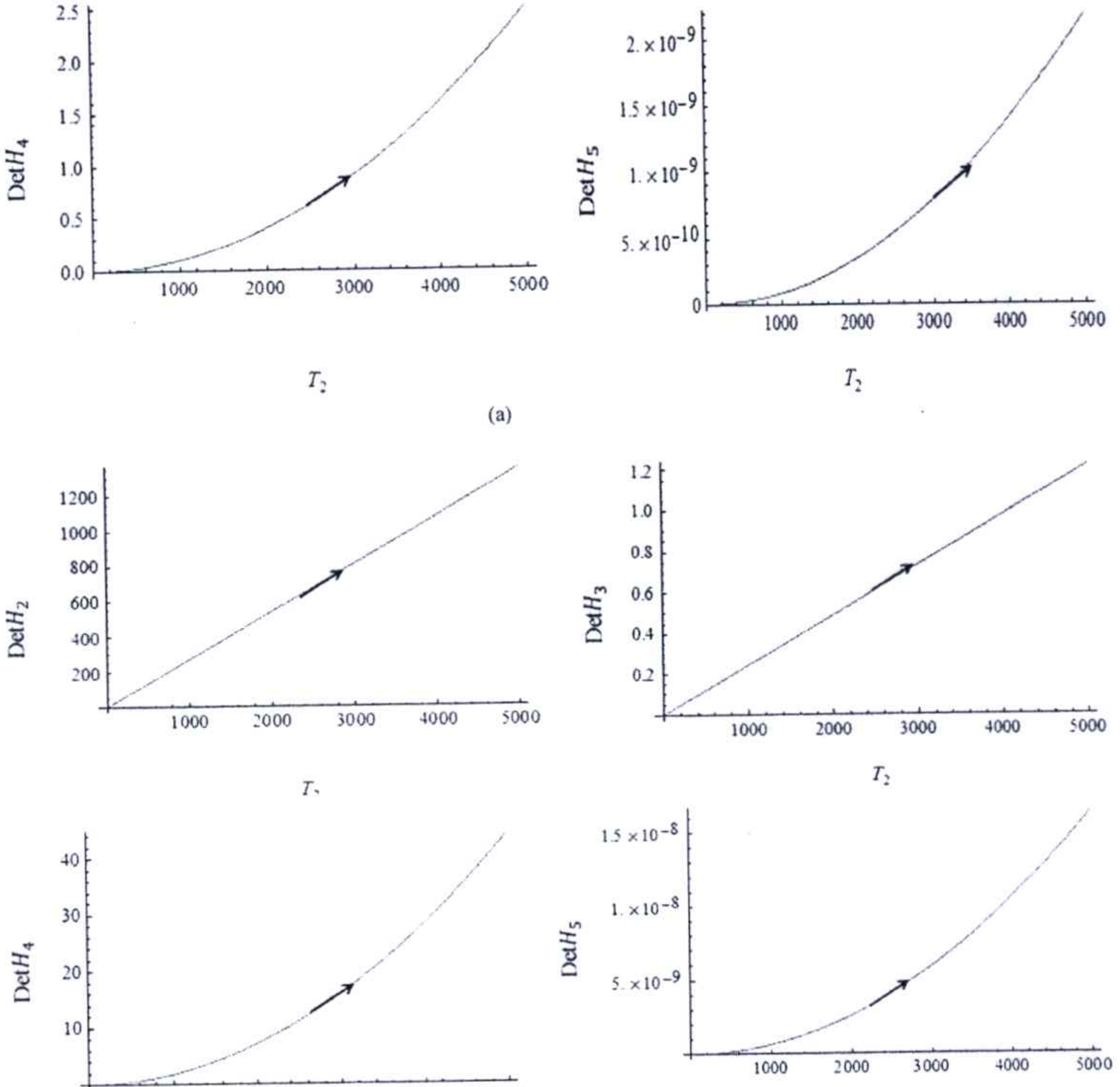

Fig. 3. The parameter spaces for endemic disease equilibrium state, which satisfies the Routh-Hurwitz conditions, plotted onto $(\kappa_{c1}, \det H_2)$, $(\kappa_{c1}, \det H_3)$, $(\kappa_{c1}, \det H_4)$, $(\kappa_{c1}, \det H_5)$, $(\kappa_{a1}, \det H_2)$, $(\kappa_{a1}, \det H_3)$, $(\kappa_{a1}, \det H_4)$ and $(\kappa_{a1}, \det H_5)$, respectively. The values of parameter are follows: (a) $\kappa_{c1} = 1/(17/2)$, $\kappa_{c2} = 1/(19/2)$, $\mu_d = 1/(365*74.6)$ day^{-1}, $N_{tc} = 6000$, $N_{va1} = 5000$, $N_{vb1} = 2500$, $\beta_{ac} = 0.2$, $\beta_{bc} = 0.0714$, $\lambda_{va1} = 0.00000000576$, $\lambda_{vb1} = 0.00000435$, $\alpha_a = 0.08$, $\alpha_b = 0.047$ and $N_t = 100{,}000$, (b) $\kappa_{a1} = 1/(19/2)$, $\kappa_{a2} = 1/(21/2)$, $\mu_d = 1/(365*746)$ day^{-1}, $N_{ta} = 4000$, $N_{va2} = 7000$, $N_{vb2} = 4300$, $\beta_{au} = 0.1667$, $\beta_{ha} = 0.125$, $\lambda_{va2} = 0.00000000176$, $\lambda_{vb2} = 0.000000835$, $\alpha_a = 0.07$, $\alpha_b = 0.027$ and $N_t = 100000$. From the above figures, the Routh-Hurwitz conditions are satisfies for $S_0 > 1$

Numerical Results

We consider the numerical solutions for dengue virus transmission. The main effect of introducing an age structure into the model is to change the definition of the basic reproductive rate. The parameters in this study are determined by the real life observations. The values of the parameters are as follows: $\mu_d = 1/(365*74.6)$ day^{-1}, corresponding to life expectancy of 74.6 years for human; $\kappa_{c1} = 1/(8.5)$ and $\kappa_{c2} = 1/(9.5)$ corresponding to the 8.5 days and 9.5 days of recovering due to biting of *Aedes aegypti* and *Aedes albopictus*, respectively. The death rate of mosquitoes are 1/28 per day and

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1/35 per day satisfies to the life time of 28 days for *Aedes aegypti* and the life time of 35 days for *Aedes albopictus*, respectively κ_{a1} = 1/(9.5) and κ_{a1} = 1/(10.5) corresponding to the 9.5 days and 10.5 days of recovering of adult human population due to biting of *Aedes aegypti* and *Aedes albopictus*, respectively. The other parameters are arbitrary chosen. Numerical solutions of (4a)-(4j) are shown in Fig. 4-9.

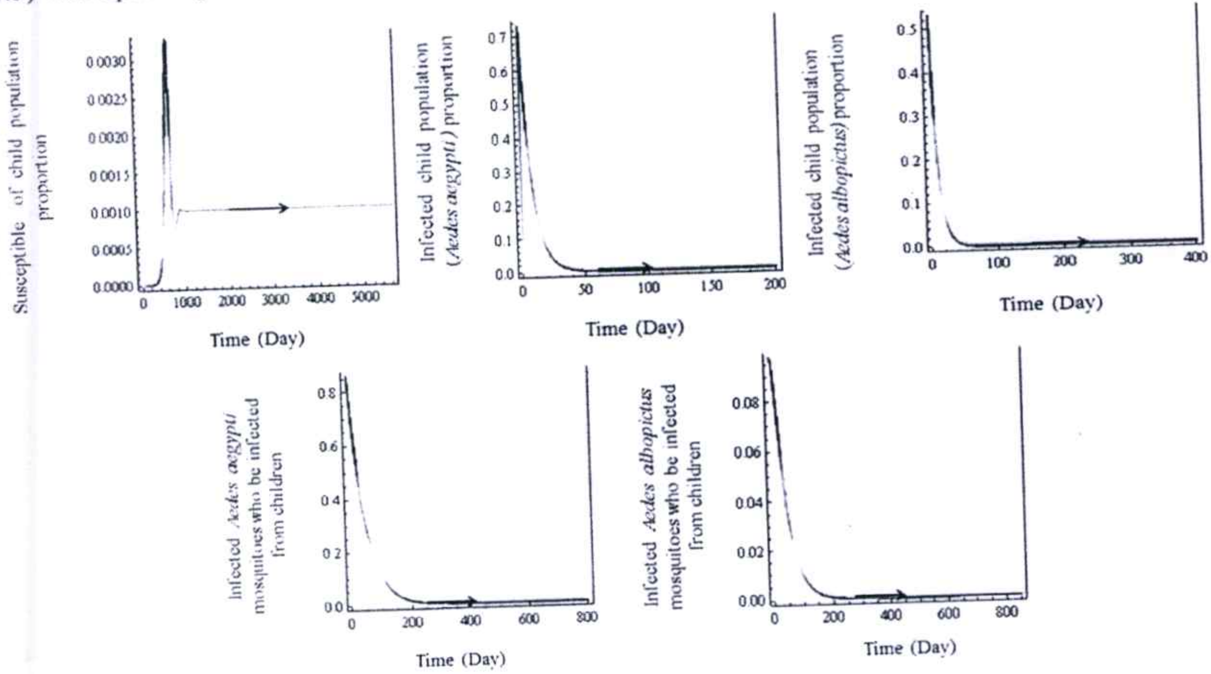
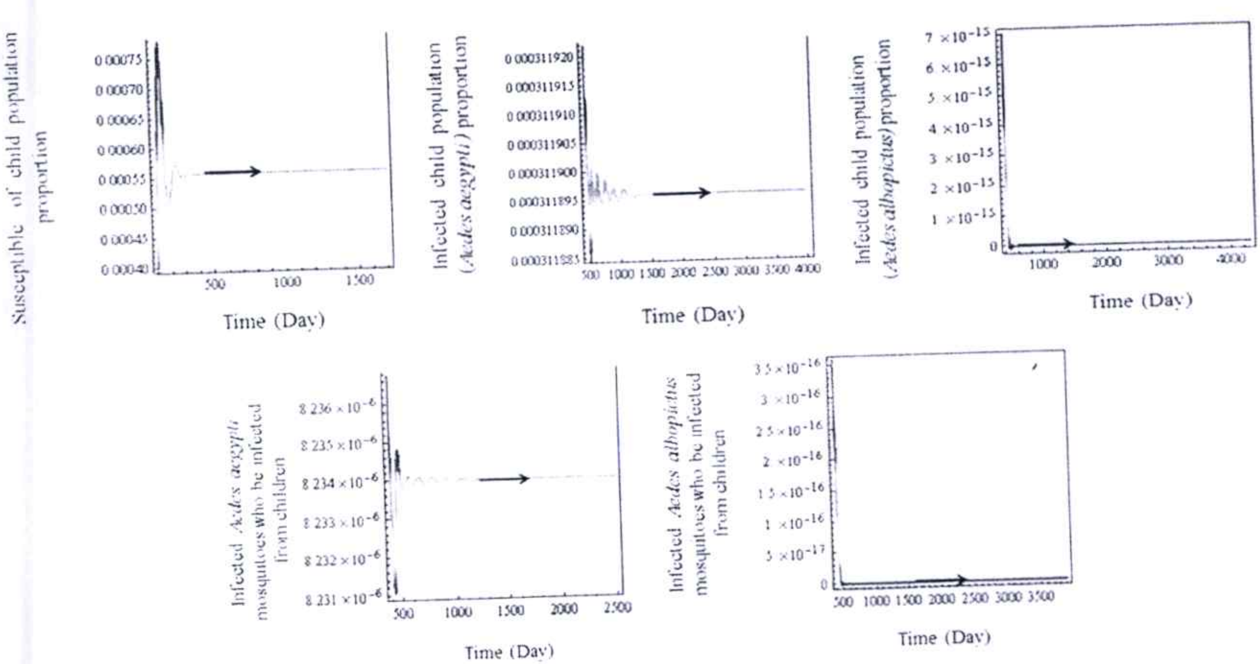


Fig. 4. Time series solutions of S_c, I_{c1}, I_{c2}, I_{va1} and I_{vb1} respectively. For $S_0 < 1$ and S_{0c} = 0.000023944 with parameters are following: $\mu_{va1} = 1/49$, $\mu_{vb1} = 1/39$, $N_{tc} = 71000$, $N_{va1} = 5800$, $N_{vb1} = 10000$, $\beta_{ac} = 0.0239$, $\beta_{bc} = 0.0333$, $\lambda_{va1} = 0.00000000000347$, $\lambda_{vb1} = 0.0000\,000675$, $\alpha_a = 0.07$, $\alpha_b = 0.027$ and $N_t = 100{,}000$. The proportions of populations (S'_c, I'_{c1}, I'_{c2}, I'_{va1}, I'_{vb1}) approach to the disease free equilibrium state (1,0,0,0,0)



(a)

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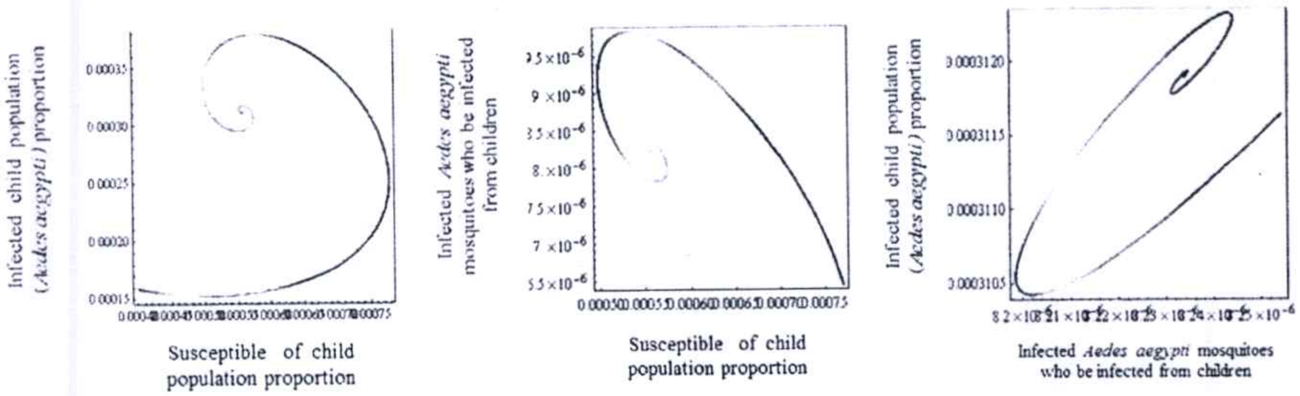

(b)

Fig. 5. (a) Time series solutions of $S_c, I_{c1}, I_{c2}, I_{va1}, I_{vb1}$. Values of parameters in the model are following: $\mu_{va1} = 1/7$, $\mu_{vb1} = 1/14$, $N_{tc} = 50000$, $N_{va1} = 32000$, $N_{vb1} = 17000$, $\beta_{ac} = 0.2$, $\beta_{bc} = 0.125$, $\lambda_{va1} = 0.0000000058$, $\lambda_{vb1} = 0.00000000465$, $\alpha_a = 0.026$, $\alpha_b = 0.009$ and $N_t = 100{,}000$, when $S_{0c} = 174.473$. (b) Numerical solutions projected onto (S_c', I_{c1}'), (S_c', I_{va1}'), (I_{va1}', I_{c1}'). The solutions oscillate to the endemic equilibrium state $(S_c^*, I_{c1}^*, I_{c2}^*, I_{va1}^*, I_{vb1}^*)$ where $S_c^* = 0.000556913$, $I_{c1}^* = 0.0003$, $I_{c2}^* = 1.6622 \times 10^{-14}$, $I_{va1}^* = 8.23484 \times 10^{-6}$ and $I_{vb1}^* = 7.38289 \times 10^{-17}$, respectively

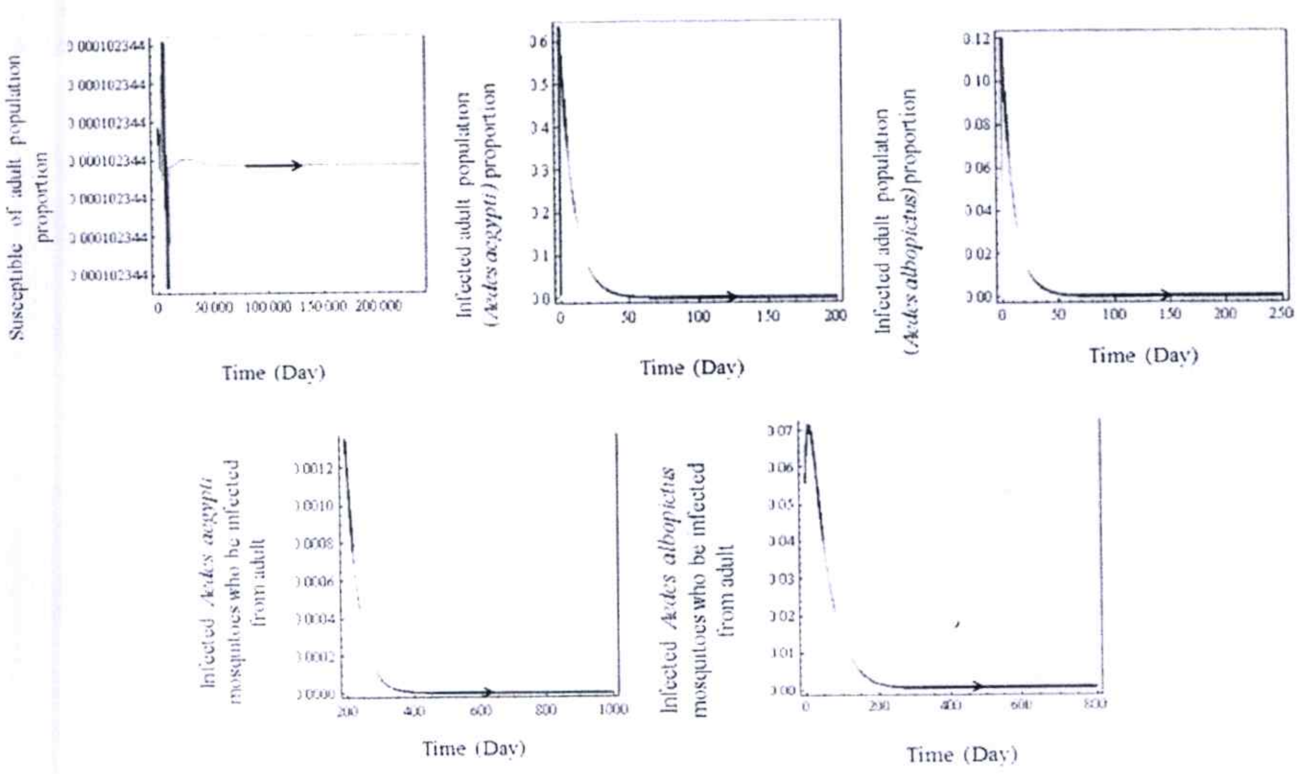

Fig. 6. Time series solution of S_a, I_{a1}, I_{a2}, I_{va2} and I_{vb2} respectively. For $S_0 < 1$ and $S_{0a} = 0.0307919$ with parameters are following: $\mu_{va2} = 1/36$, $\mu_{vb2} = 1/46$, $N_{ta} = 61000$, $N_{va2} = 4800$, $N_{vb2} = 10000$, $\beta_{aa} = 0.03225$, $\beta_{ba} = 0.02941$, $\lambda_{va2} = 0.000000000076$, $\lambda_{vb2} = 0.000000000664$, $\alpha_a = 0.04$, $\alpha_b = 0.06$ and $N_t = 100{,}000$. The proportions of populations $(S_a', I_{a1}', I_{a2}', I_{va2}', I_{vb2}')$ approach to the disease free equilibrium state (1,0,0,0,0)

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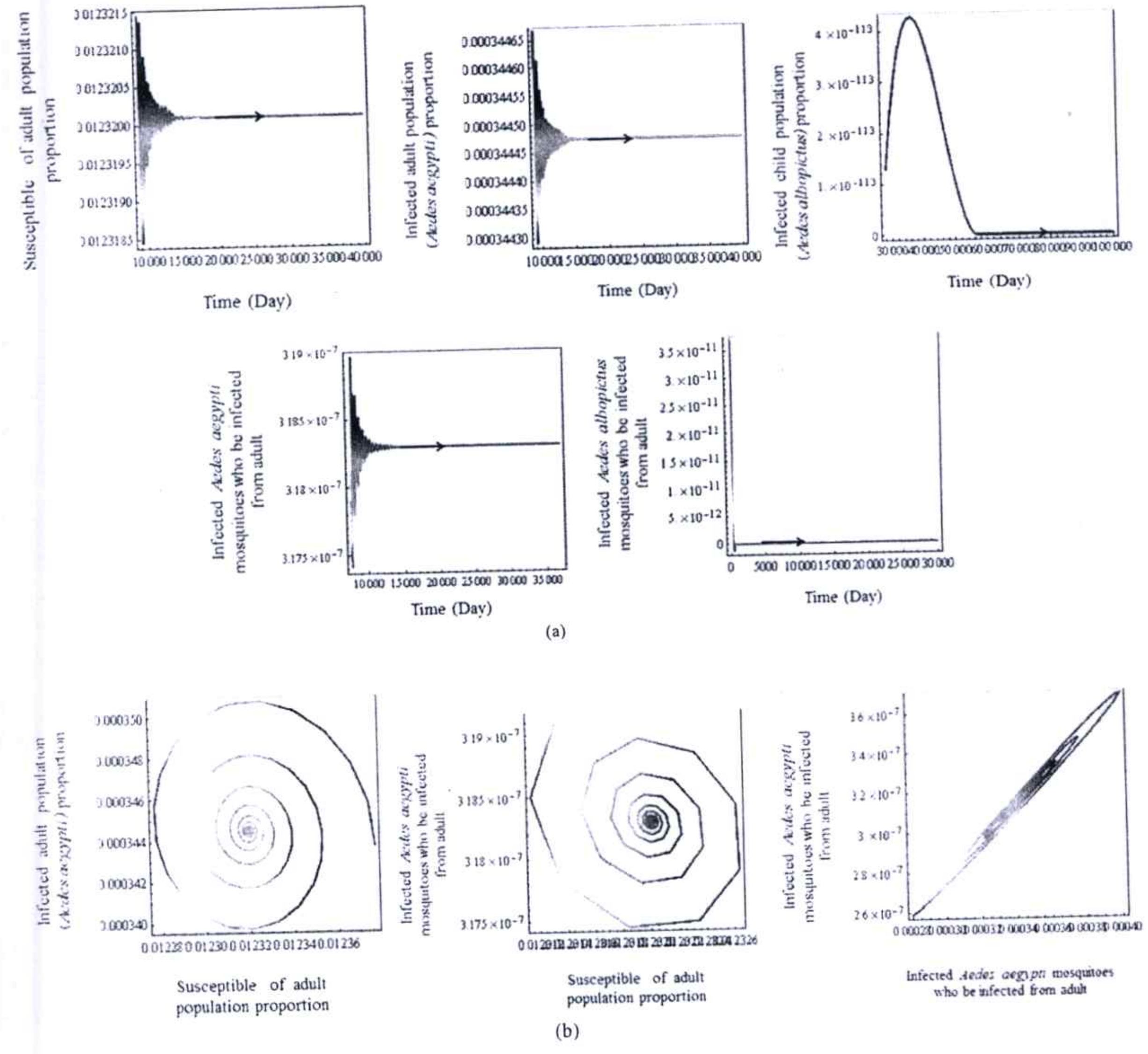

Fig. 7. (a) Time series solutions of $S_a, I_{a1}, I_{a2}, I_{va2}, I_{vb2}$. Values of parameters in the model are following: $\mu_{va2} = 1/7$, $\mu_{vb2} = 1/13$, $N_{ta} = 30000$, $N_{va2} = 37000$, $N_{vb2} = 19000$, $\beta_{au} = 0.25$, $\beta_{ba} = 0.1428$, $\lambda_{va2} = 0.0000000044$, $\lambda_{vb2} = 0.000000000335$, $\alpha_a = 0.02, \alpha_b = 0.07$ and $N_t = 100{,}000$, where S_{0a} = 21.7785 in adult. (b) Numerical solutions projected onto (S_a', I_{a1}'), (S_a', I_{va2}'), (I_{va2}', I_{a1}'). The solutions oscillate to the endemic equilibrium state $(S_a^*, I_{a1}^*, I_{a2}^*, I_{va2}^*, I_{vb2}^*)$ where $S_c^* = 0.0123201$, $I_{c1}^* = 0.000344474$, $I_{c2}^* = 6.9961\times10^{-14}$, $I_{va1}^* = 3.18294\times10^{-7}$ and $I_{vb1}^* = 1.40069\times10^{-16}$, respectively

Case A.1, in children, we consider the locally asymptotically stable of disease free equilibrium state, when ε = 0 as shown in Fig. 4.

Case A.2, in children, we consider the locally asymptotically stable of endemic equilibrium state, when ε = 0 as shown in Fig. 5.

Case B.1, in adult, we consider the locally asymptotically stable of disease free equilibrium state, when ε = 0 as shown in Fig. 6.

Case B.2, in adult, we consider the locally asymptotically stable of endemic equilibrium state, when ε = 0 as shown in Fig. 7.

Case C, in children, when ε ≠ 0 as shown in Fig. 8.

Case D, in adult, when ε ≠ 0 as shown in Fig. 9.

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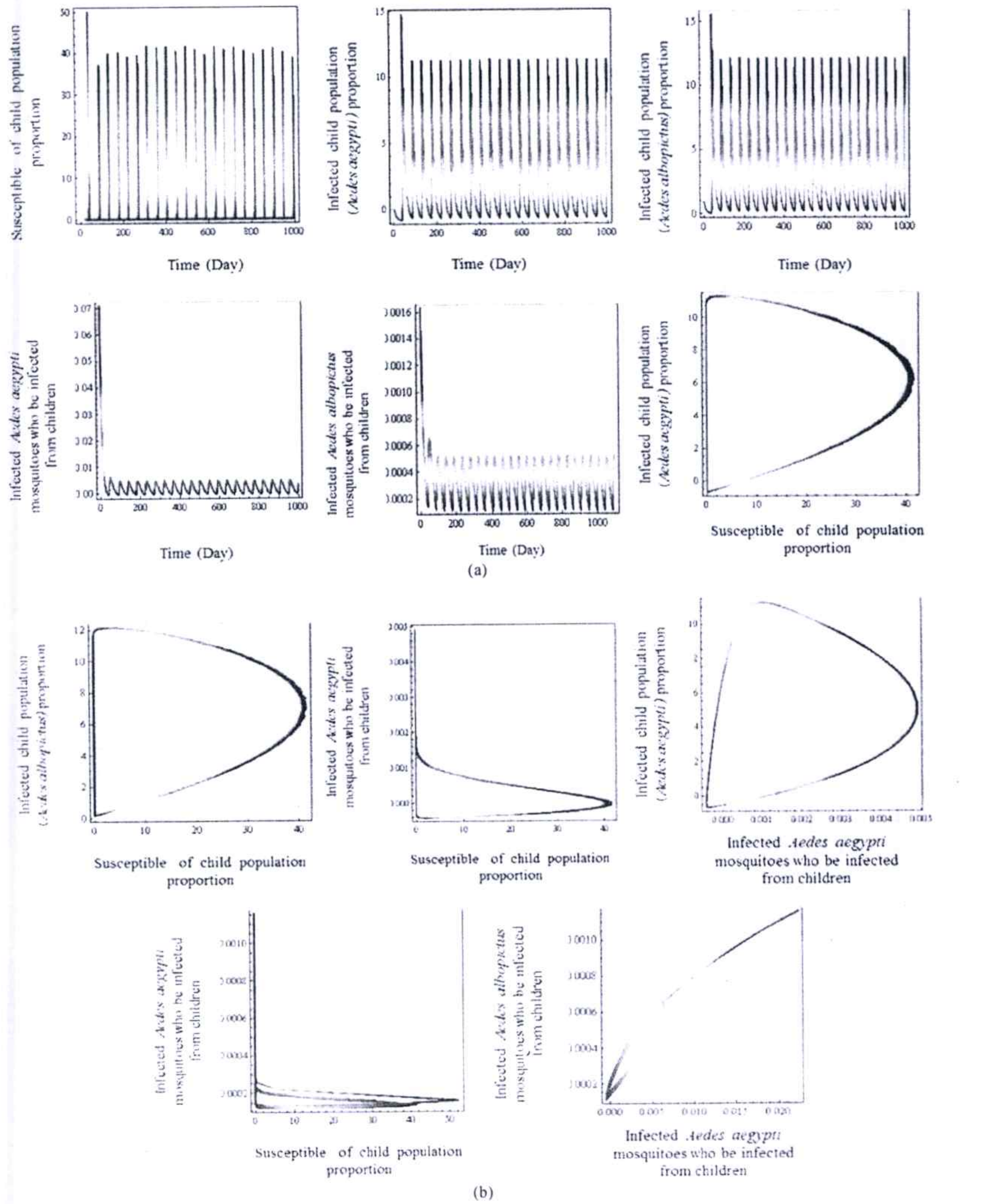

Fig. 8. (a) Time series solutions of $S_c, I_{c1}, I_{c2}, I_{va1}, I_{vb1}$. Values of parameters in the model are following: $N_{tc} = 50000$, $N_{va1} = 32000$, $N_{vb1} = 17000$, $\beta_{ac} = 0\ 2$, $\beta_{bc} = 0.125$, $\lambda_{va1} = 0.0000000028$, $\lambda_{vb1} = 0.000000000165$, $\alpha_a = 0.005$, $\alpha_b = 0.004$ and $N_t = 100{,}000$ and $S_{0c} = 22.8627$, (b) Numerical solutions projected onto (S_c', I_{c1}'), (S_c', I_{c2}'), (S_c', I_{va1}'), (S_c', I_{vb1}'), (I_{va1}', I_{c1}'), (I_{va1}', I_{vb1}'). Limit l cycles are produced in this case

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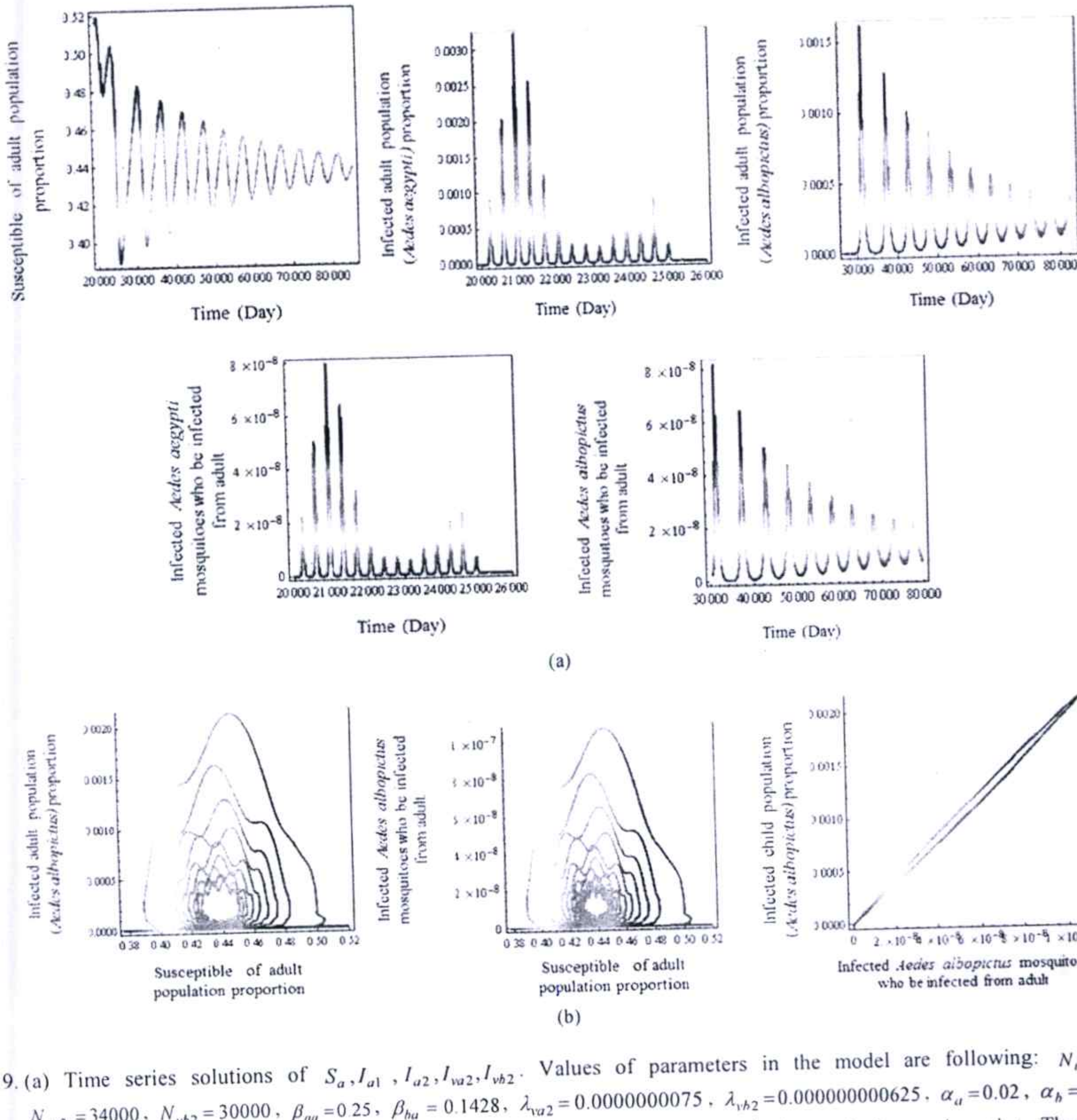

Fig. 9. (a) Time series solutions of $S_a, I_{a1}, I_{a2}, I_{va2}, I_{vb2}$. Values of parameters in the model are following: $N_{ta} = 50000$, $N_{va2} = 34000$, $N_{vb2} = 30000$, $\beta_{aa} = 0.25$, $\beta_{ha} = 0.1428$, $\lambda_{va2} = 0.0000000075$, $\lambda_{vb2} = 0.000000000625$, $\alpha_a = 0.02$, $\alpha_h = 0.07$ and $N_t = 100.000$, when $S_{0a} = 9.26764$. (b) Numerical solutions projected onto $(S_a^{'}, I_{a2}^{'})$, $(S_a^{'}, I_{vb2}^{'})$, $(I_{vb2}^{'}, I_{a2}^{'})$. The solutions oscillate to the endemic equilibrium state $(S_a^{*}, I_{a1}^{*}, I_{a2}^{*}, I_{va2}^{*}, I_{vb2}^{*})$ are limit cycles

Discussion

Several investigations have been conducted using the SIR and SI models. The SIR and SI models which provide suitable for the states of children and adult in two species are used in this study. (*Aedes aegypti* and *Aedes albopictus*).

The basic reproductive number of equations (4a)-(4j) is defined as follows (Chong *et al.*, 2013):

$$2N_{vh1}\alpha_h\beta_{hc}\lambda_{vh1}(\kappa_{c1} + \mu_d)\mu_{v_a}\rho_{vh}$$
$$+ N_{va1}\beta_{ac}\lambda_{va1}(2(N_{tc}\lambda_{vh1}\mu_d + (\kappa_{c2} + \mu_d)\mu_{v_b})$$

$$S_0 = \max \{ \frac{(2 + \alpha_a\rho_{va}) + 2N_{tc}\lambda_{vh1}\mu_d(\alpha_a + \rho_{va}))}{(2\lambda_{vh1}(\kappa_{c1} + \mu_d)\mu_{v_a}(2\mu_d + N_{vh1}\beta_{hc}},$$
$$(2 + \alpha_h\rho_{vh})) + 2N_{va1}\beta_{ac}\lambda_{va1}(\alpha_a(\kappa_{c2} + \mu_d)$$
$$\mu_{v_a}\rho_{va} + N_{tc}\lambda_{vh1}\mu_d(\alpha_a\rho_{va} + (\alpha_a + \rho_{va})\rho_{vh})))$$
$$2N_{vh2}\alpha_h\beta_{ha}\lambda_{vh2}(\kappa_{a1} + \mu_d)\mu_{v_a}\rho_{vh}$$
$$+ N_{va2}\beta_{aa}\lambda_{va2}(2(N_{ta}\lambda_{vh2}\mu_d + (\kappa_{a2} + \mu_d)\mu_{v_b})$$
$$\frac{(2 + \alpha_a\rho_{va}) + 2N_{ta}\lambda_{vh2}\mu_d(\alpha_a + \rho_{va}))}{(2\lambda_{vh2}(\kappa_{a1} + \mu_d)\mu_{v_a}(2\mu_d + N_{vh2}\beta_{ha}(2 + \alpha_h\rho_{vh}))} \}$$
$$+2N_{va2}\beta_{aa}\lambda_{va2}(\alpha_a(\kappa_{a2} + \mu_d)\mu_{v_b}\rho_{va}$$
$$+N_{ta}\lambda_{vh2}\mu_d(\alpha_a\rho_{va} + (\alpha_a + \rho_{va})\rho_{vb})))$$

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Table 3. Determination of the values S_0 of infected mosquitoes

I_{va1} value	I_{vb1} value	I_{va2} value	I_{va2} value	S_0 value
		2.3860×10-18	6.44744×10-7	0.00310
2.5687×10-26	0.000065	-	-	0.00732
0	0.00036	-	-	0.06795
-	-	4.9155×10-23	0.0007175	0.68842
-	-	7.3376×10-14	1.07169×10-8	3.61291
8.234×10-6	7.982×10-17	-	-	38.6066
3.16331×10-5	1.118×10-6	-	-	80.3505
-	-	9.933×10-8	2.822×10-17	89.3077

S_0 describes the number of infectious human produced from primary infection of children and adult. Using the initial values and parameter values from data, the obtained result of threshold parameter value S_0 for S_{0c} and S_{0a} can be rewritten in mathematical form as follows:

$$S_{0c} = \frac{\begin{array}{l} 2N_{vh1}\alpha_h\beta_{hc}\lambda_{vh1}(\kappa_{c1}+\mu_d)\mu_{v_a}\rho_{vh} \\ +N_{va1}\beta_{ac}\lambda_{va1}(2(N_{Ic}\lambda_{vh1}\mu_d+(\kappa_{c2}+\mu_d) \\ \mu_{v_b})(2+\alpha_a\rho_{va})+2N_{Ic}\lambda_{vh1}\mu_d(\alpha_a+\rho_{va})) \end{array}}{\begin{array}{l} (2\lambda_{vh1}(\kappa_{c1}+\mu_d)\mu_{v_a}(2\mu_d+N_{vh1}\beta_{hc} \\ (2+\alpha_h\rho_{vh}))+2N_{va1}\beta_{ac}\lambda_{va1}(\alpha_a(\kappa_{c2}+\mu_d) \\ \mu_{v_b}\rho_{va}+N_{Ic}\lambda_{vh1}\mu_d(\alpha_a\rho_{va}+(\alpha_a+\rho_{va})\rho_{vh}))) \end{array}} \text{ in children}$$

$$S_{0a} = \frac{\begin{array}{l} 2N_{vh2}\alpha_h\beta_{ha}\lambda_{vh2}(\kappa_{a1}+\mu_d)\mu_{v_a}\rho_{vh}+ \\ N_{va2}\beta_{aa}\lambda_{va2}(2(N_{Ia}\lambda_{vh2}\mu_d+(\kappa_{a2}+\mu_d)\mu_{v_b}) \\ (2+\alpha_a\rho_{va})+2N_{Ia}\lambda_{vh2}\mu_d(\alpha_a+\rho_{va})) \end{array}}{\begin{array}{l} (2\lambda_{vh2}(\kappa_{a1}+\mu_d)\mu_{v_a}(2\mu_d+N_{vh2}\beta_{ba} \\ (2+\alpha_h\rho_{vb}))+2N_{va2}\beta_{aa}\lambda_{va2}(\alpha_a(\kappa_{a2}+\mu_d) \\ \mu_{v_b}\rho_{va}+N_{Ia}\lambda_{vh2}\mu_d(\alpha_a\rho_{va}+(\alpha_a+\rho_{va})\rho_{vb}))) \end{array}} \text{ in adult}$$

The reproductive rate is depend on the number of infected mosquitoes I_{va1}, I_{vb1} in children and I_{va2}, I_{vb2} in adult.

From the above table (Table 3), we will see that if the number of infected mosquitoes is increased, the basic reproductive rate is also increased.

Moreover, we consider the effect of sinusoidal variation (ε), we will see that if $\varepsilon = 0$, then the solutions oscillation to the steady state. The limit cycle occurs for $\varepsilon \neq 0$. Thus the limit cycles occurs while there is the seasonal variation of mosquitoes (*Aedes aegypti* and *Aedes albopictus*). It can be seen that the dynamical behavior of the endemic state change while there is the influence of season.

Conclusion

The basic reproductive number of disease is defined by $\tilde{S}_0 = \sqrt{S_0}$. This value is the threshold condition for the existence of the endemic state. When $S_0 \leq 1$, the solutions oscillate to disease free equilibrium state, whereas $S_0 > 1$, the solutions oscillate to the endemic state. The behaviors of the proportion of susceptible, infective human into two classes (a child class and an adult class) and infective vectors of the two species (*Aedes aegypti* and *Aedes albopictus*) are initially positive. If this can be seen as follow; the infective human are introduced into the susceptible is bitten during each period, by the fraction $\frac{bN_h}{(N_h+m)}\left(\frac{1}{\mu_v}\right)$(that the biting rate b of mosquitoes is the average number of bites per mosquito per day, μ_v is the per capita mortality rate of mosquito, of these bites becomes new infective in the human population) (Esteva and Vargas, 1998). The parameters β_{ac}, β_{bc}, β_{aa}, β_{ba}, β_{ac}, λ_{va1}, λ_{va2}, λ_{vb1} and $\lambda_{vb2}$are effects to the basic reproductive number of this disease as we see in (6f)-(6h). If the basic reproductive number is less or equal than one, then the infective replaces less than one, then disease dies. On the other hand, if this number is greater than one and when the susceptible fraction get large enough to birth of new susceptible, then there are secondary infections and endemic equilibrium state is occurred. As we can see in this study, the seasonal parameters such as ε, ρ_{va} and ρ_{vb} which are the measure of influence on the transmission process reflect the environment.

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Author's Contributions

The work preseted here was carried out in collaboration between all authors. R. Sungchasit, P. Pongsumpun and I.M. Tang. Designed and analyzed the model. All authors have contributed to, seen and approved the manuscript.

Ethics

The authors confirm that this work is original and contains has not published elsewhere.

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TRANSMISSION MODEL OF DENGUE DISEASE WITH THE DIFFERENT CONTACT RATES OF THREE SEASONS IN THAILAND

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Abstract: A Mathematical model is used for describing the transmission of dengue disease. The person can infected this disease by biting of the infected *Aedes* mosquitoes. Dengue disease is separated into 4 serotypes: DEN-1,DEN-2, DEN-3 and DEN-4. In this study, we formulate the model with different contact rates in each season. The standard dynamical modeling method is used in this study. We derive stability conditions of parameters for disease free equilibrium state and disease endemic equilibrium state. The basic reproductive number of this disease is found. The results of this study will point the way for controlling this disease.

Introduction: Dengue Fever (DF), Dengue Haemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS) have emerged as an international public health problem, which is now endemic in more than 100 countries and affecting about 40% of the world population (2.5 billion people) living in tropical and subtropical regions[1]. There are four serotypes of dengue virus, namely DEN-1, DEN-2, DEN-3 and DEN-4. Dengue disease can not be directly transmitted between the people. The infected female *Aedes* mosquito is the primary vector for this disease. The development of the virus and the mosquito are affected by the climatic factors. The effect of extrinsic incubation period of dengue virus caused the seasonality transmission of this disease[2]. When a vector bites someone who be infected with dengue virus, the virus is transferred to that mosquito and it become infected mosquito. After the infected vector bites the susceptible human then the virus move into the human bloodstream , and it spread throughout the body. Symptoms appeared about eight to ten days after the bitting from an infected mosquito. Symptoms are flu-like illness and can include high fever, nausea, vomiting, body aches, and headache. The moisture content ,temperature, season and rainfall are influence to the mosquito development. Dengue infection is endemic in Thailand. From the data of Dengue cases in 1999 – 2010, we can see that most dengue patients are occurred in rainy season as shown in figure 1.

Esteva and Vargas developed a model for dengue disease transmission and included the dynamics of the *Aedes aegypti* mosquito into standard SIR (susceptible – infective-recovered) epidemic model[3]. In 2010, R.Kongnuy and P.Pongsumpun[4] considered the transmission of dengue disease with the effect of season. In this paper, we used SIR model for analyzing and finding the method to decrease the outbreak of this disease. We analyze the dengue model of seasonality compartment (rainy season, winter season and summer season).

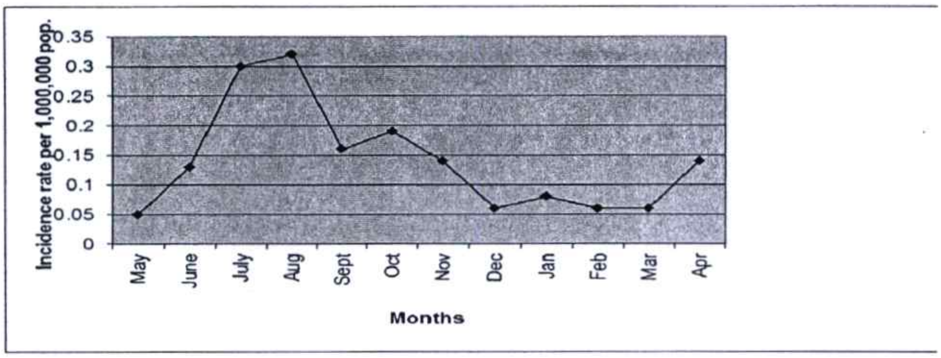

Figure 1: Reported cases of Dengue disease per 100,000 population in Thailand during May 2007 and April 2008.

Methodology: The mathematical modeling for dengue disease describes the relevance of human and mosquito population. In this study, we assume that the human and mosquito population have constant sizes. The human population is divided into susceptible , infected and recovered classes for the first model. The mosquito population is divided into susceptible and infected classes because the mosquito never recover from infection.

The dynamics of human population are given by

$$\frac{d}{dt}S_{hr} = P_r N_{Tr} - \delta_d S_{hr} - \delta_h S_{hr} - \lambda \frac{C_{v \to hr}}{N_{Tr}+d} I_{rr} S_{hr} \tag{1}$$

$$\frac{d}{dt}I_{hr} = \lambda \frac{C_{v \to hr}}{N_{Tr}+d} I_{rr} S_h - \delta_d I_{hr} - \delta_h I_{hr} - \beta I_{hr} \tag{2}$$

$$\frac{d}{dt}R_{hr} = \beta I_{hr} - \delta_d R_{hr} - \delta_h R_{hr} \tag{3}$$

$$\frac{d}{dt}S_{hw} = P_w N_{Tw} - \delta_d S_{hw} - \delta_h S_{hw} - \lambda \frac{C_{v \to hw}}{N_{Tw}+d} I_{rw} S_{hw} \tag{4}$$

$$\frac{d}{dt}I_{hw} = \lambda \frac{C_{v \to hw}}{N_{Tw}+d} I_{rw} S_h - \delta_d I_{hw} - \delta_h I_{hw} - \beta I_{hw} \tag{5}$$

$$\frac{d}{dt}R_{hw} = \beta I_{hw} - \delta_d R_{hw} - \delta_h R_{hw} \tag{6}$$

$$\frac{d}{dt}S_{hs} = P_s N_{Ts} - \delta_d S_{hs} - \delta_h S_{hs} - \lambda \frac{C_{v \to hs}}{N_{Ts}+d} I_{rs} S_{hs} \tag{7}$$

$$\frac{d}{dt}I_{hs} = \lambda \frac{C_{v \to hs}}{N_{Ts}+d} I_{rs} S_{hw} - \delta_d I_{hs} - \delta_h I_{hs} - \beta I_{hs} \tag{8}$$

$$\frac{d}{dt}R_{hs} = \beta I_{hs} - \delta_d R_{hs} - \delta_h R_{hs} \tag{9}$$

We define

S_{hr} is the number of susceptible human population in rainy season,

I_{hr} is the number of infectious human population in rainy season,

R_{hr} is the number of recovered human population in rainy season,

S_{hw} is the number of susceptible human population in winter season,

I_{hw} is the number of infectious human population in winter season,

R_{hw} is the number of recovered human population in winter season,

S_{hs} is the number of susceptible human population in summer season,

I_{hs} is the number of infectious human population in summer season,

R_{hs} is the number of recovered human population in summer season

The dynamics of the mosquito population can be described as the following equations :

$$\frac{d}{dt}S_{vr} = A_r - \lambda\frac{C_{hr\to v}}{N_{Tr}+d}I_{hr}S_{vr} - \delta_v S_{vr} \quad (10)$$

$$\frac{d}{dt}I_{vr} = \lambda\frac{C_{hr\to v}}{N_{Tr}+d}I_{hr}S_{vr} - \delta_v I_{vr} \quad (11)$$

$$\frac{d}{dt}S_{vw} = A_w - \lambda\frac{C_{hw\to v}}{N_{Tw}+d}I_{hw}S_{vw} - \delta_v S_{vw} \quad (12)$$

$$\frac{d}{dt}I_{vw} = \lambda\frac{C_{hr\to v}}{N_{Tw}+d}I_{hw}S_{vw} - \delta_v I_{vw} \quad (13)$$

$$\frac{d}{dt}S_{vs} = A_s - \lambda\frac{C_{hs\to v}}{N_{Ts}+d}I_{hs}S_{vs} - \delta_v S_{vs} \quad (14)$$

$$\frac{d}{dt}I_{vs} = \lambda\frac{C_{hs\to v}}{N_{Ts}+d}I_{hs}S_{vs} - \delta_v I_{vs} \quad (15)$$

We define

S_{vr} is the number of susceptible mosquito population in rainy season,

I_{vr} is the number of infectious mosquito population in rainy season,

S_{vw} is the number of susceptible mosquito population in winter season,

I_{vw} is the number of infectious mosquito population in winter season,

S_{vs} is the number of susceptible mosquito population in summer season,

I_{vs} is the number of infectious mosquito population in summer season.

Where the parameters are defined as follows :

N_{Tr} is the total human population in rainy season,

N_{Tw} is the total human population in winter season,

N_{Ts} is the total human population in summer season,

N_{Vr} is the total mosquito population in rainy season,

N_{Vw} is the total mosquito population in winter season,

N_{Vs} is the total mosquito population in summer season,

δ_h is the natural death rate of human population,

δ_d is the death rate of human population due to the disease,

δ_v is the death rate of mosquito population,

P is the birth rate of human population,

$C_{v\to hr}$ is the transmission probability of dengue disease from mosquito to human in rainy season,

$C_{v\to hw}$ is the transmission probability of dengue disease from mosquito to human in winter season ,

$C_{v\to hs}$ is the transmission probability of dengue disease from mosquito to human in summer season,

$C_{hr\to v}$ is the transmission probability of dengue disease from human to mosquito in rainy season,

$C_{hw\to v}$ is the transmission probability of dengue disease from human to mosquito in winter season,

$C_{hs\to v}$ is the transmission probability of dengue disease from human to mosquito in summer season,

β is the recovery rate of human population,

λ is the biting rate of mosquito population,

d is the number of other animals available as blood sources.

We suppose that $N_{Hr} = S_{hr} + I_{hr} + R_{hr}$, $N_{Hw} = S_{hw} + I_{hw} + R_{hw}$, $N_{Hs} = S_{hs} + I_{hs} + R_{hs}$, $N_{Vr} = S_{vr} + I_{vr}$, $N_{Vw} = S_{vw} + I_{vw}$ and $N_{Vs} = S_{vs} + I_{vs}$

We normalize equations (1) – (15) by defining new variables.

$$\overline{S_{hr}} = \frac{S_{hr}}{N_{Tr}},\ \overline{I_{hr}} = \frac{I_{hr}}{N_{Tr}},\ \overline{R_{hr}} = \frac{R_{hr}}{N_{Tr}},\ \overline{S_{hw}} = \frac{S_{hw}}{N_{Tw}},\ \overline{I_{hw}} = \frac{I_{hw}}{N_{Tw}},\ \overline{R_{hw}} = \frac{R_{hw}}{N_{Tw}},$$

$$\overline{S_{hs}} = \frac{S_{hs}}{N_{Ts}},\ \overline{I_{hs}} = \frac{I_{hs}}{N_{Ts}},\ \overline{R_{hs}} = \frac{R_{hs}}{N_{Ts}}$$

$$\overline{S_{vr}} = \frac{S_{vr}}{N_{Vr}},\ \overline{S_{vw}} = \frac{S_{vw}}{N_{Vw}},\ \overline{S_{vs}} = \frac{S_{vs}}{N_{Vs}},\ \overline{I_{vr}} = \frac{I_{vr}}{N_{Vr}},\ \overline{I_{vw}} = \frac{I_{vw}}{N_{Vw}},\ \overline{I_{vs}} = \frac{I_{vs}}{N_{Vs}}$$

The total human and mosquito populations have constant sizes , thus rates of change for total human and mosquito populations equal to zero. Thus , the birth and death rates are equivalent for human population , the total mosquito population equals to $\frac{A_r}{\delta_v}$ in rainy season , $\frac{A_w}{\delta_v}$ in winter season , $\frac{A_s}{\delta_v}$ in summer season. These give

$$\frac{d}{dt}\overline{S_{hr}} = (\delta_d + \delta_h) - (\delta_d + \delta_h + \lambda \frac{C_{v\to hr}}{N_{Tr}+d}\overline{I_{vr}}\, N_{Vr})\overline{S_{hr}} \tag{16}$$

$$\frac{d}{dt}\overline{I_{hr}} = \lambda \frac{C_{v\to hr}}{N_{Tr}+d}\overline{I_{vr}}\, N_{Vr}\overline{S_{hr}} - (\beta + \delta_d + \delta_h)\overline{I_{hr}} \tag{17}$$

$$\frac{d}{dt}\overline{S_{hw}} = (\delta_d + \delta_h) - (\delta_d + \delta_h + \lambda \frac{C_{v\to hw}}{N_{Tw}+d}\overline{I_{vw}}\, N_{Vw})\overline{S_{hw}} \tag{18}$$

$$\frac{d}{dt}\overline{I_{hw}} = \lambda \frac{C_{v\to hw}}{N_{Tw}+d}\overline{I_{vw}}\, N_{Vw}\overline{S_{hw}} - (\beta + \delta_d + \delta_h)\overline{I_{hw}} \tag{19}$$

$$\frac{d}{dt}\overline{S_{hs}} = (\delta_d + \delta_h) - (\delta_d + \delta_h + \lambda \frac{C_{v\to hs}}{N_{Ts}+d}\overline{I_{vs}}\, N_{Vs})\overline{S_{hs}} \tag{20}$$

$$\frac{d}{dt}\overline{I_{hs}} = \lambda \frac{C_{v\to hs}}{N_{Ts}+d}\overline{I_{vs}}\, N_{Vs}\overline{S_{hs}} - (\beta + \delta_d + \delta_h)\overline{I_{hs}} \tag{21}$$

$$\frac{d}{dt}\overline{I_{vr}} = \lambda \frac{C_{hr\to v}}{N_{Tr}+d}\overline{I_{hr}}\, N_{Tr}\overline{S_{vr}} - \delta_v \overline{I_{vr}} \tag{22}$$

$$\frac{d}{dt}\overline{I_{vw}} = \lambda \frac{C_{hw\to v}}{N_{Tw}+d}\overline{I_{hw}}\, N_{Tw}\overline{S_{vw}} - \delta_v \overline{I_{vw}} \tag{23}$$

$$\frac{d}{dt}\overline{I_{vs}} = \lambda \frac{C_{hs\to v}}{N_{Ts}+d}\overline{I_{hs}}\, N_{Ts}\overline{S_{vs}} - \delta_v \overline{I_{vs}} \tag{24}$$

R_{hr}, R_{hw}, R_{hs} and S_{vr}, S_{vw}, S_{vs} can be obtained from conditions $S_{hr} + I_{hr} + R_{hr} = 1$, $S_{hw} + I_{hw} + R_{hw} = 1$, $S_{hs} + I_{hs} + R_{hs} = 1$ and $S_{vr} + I_{vr} = 1$, $S_{vw} + I_{vw} = 1$, $S_{vs} + I_{vs} = 1$.

Analysis of the Mathematical Model

A. Equilibrium points

The equilibrium points are found by setting the right hand side of (16) – (24) equal to zero. This gives

1)The disease free equilibrium point $E_1 = (1,0,0,1,0,0,1,0,0)$

2)The endemic disease equilibrium point

$E_1 = (S^*_{hr}, I^*_{hr}, I^*_{vr}, S^*_{hw}, I^*_{hw}, I^*_{vw}, S^*_{hs}, I^*_{hs}, I^*_{vs})$,

where

$$S^*_{hr} = \frac{(\delta_d+\delta_h)}{(\delta_d+\delta_h)+\dfrac{\lambda C_{v\to hr}N_{Vr}}{(N_{Tr}+d)+\dfrac{(N_{Tr}+d)^2\delta_v}{\lambda C_{hr\to v}I^*_{hr}N_{Tr}}}}, \quad I^*_{vr} = \frac{1}{1+\dfrac{(N_{Tr}+d)\delta_v}{\lambda C_{hr\to v}I^*_{hr}N_{Tr}}},$$

$$I^*_{hr} = \frac{(\delta_d+\delta_h)(-(N_{Tr}+d)^2(\beta+\delta_d+\delta_h)\delta_v+N_{Tr}N_{Vr}\lambda^2 C_{v\to hr}C_{hr\to v})}{N_{Tr}(\beta+\delta_d+\delta_h)\lambda C_{hr\to v}((N_{Tr}+d)(\delta_d+\delta_h)+N_{Vr}\lambda C_{v\to hr})},$$

$$S^*_{hw} = \frac{(\delta_d+\delta_h)}{(\delta_d+\delta_h)+\dfrac{\lambda C_{v\to hw}N_{Vw}}{(N_{Tw}+d)+\dfrac{(N_{Tw}+d)^2\delta_v}{\lambda C_{hw\to v}I^*_{hw}N_{Tw}}}}, \quad I^*_{vw} = \frac{1}{1+\dfrac{(N_{Tw}+d)\delta_v}{\lambda C_{hw\to v}I^*_{hw}N_{Tw}}},$$

$$I^*_{hw} = \frac{(\delta_d+\delta_h)(-(N_{Tw}+d)^2(\beta+\delta_d+\delta_h)\delta_v+N_{Tw}N_{Vw}\lambda^2 C_{v\to hw}C_{hw\to v})}{N_{Tw}(\beta+\delta_d+\delta_h)\lambda C_{hw\to v}((N_{Tw}+d)(\delta_d+\delta_h)+N_{Vw}\lambda C_{v\to hw})},$$

$$S^*_{hs} = \frac{(\delta_d+\delta_h)}{(\delta_d+\delta_h)+\dfrac{\lambda C_{v\to hs}N_{Vs}}{(N_{Ts}+d)+\dfrac{(N_{Ts}+d)^2\delta_v}{\lambda C_{hs\to v}I^*_{hs}N_{Ts}}}}, \quad I^*_{vs} = \frac{1}{1+\dfrac{(N_{Ts}+d)\delta_v}{\lambda C_{hs\to v}I^*_{hs}N_{Ts}}},$$

$$I^*_{hs} = \frac{(\delta_d+\delta_h)(-(N_{Ts}+d)^2(\beta+\delta_d+\delta_h)\delta_v+N_{Ts}N_{Vs}\lambda^2 C_{v\to hs}C_{hs\to v})}{N_{Ts}(\beta+\delta_d+\delta_h)\lambda C_{hs\to v}((N_{Ts}+d)(\delta_d+\delta_h)+N_{Vs}\lambda C_{v\to hs})}$$

B. Local Stability

The local stability of each equilibrium point is determined from linearizing equation (16) – (24) about equilibrium point examining the eigenvalues of the resulting Jacobian matrix. We now consider the eigenvalues of the Jacobian matrix at each equilibrium point. From equation (16)-(24) ,we can write in the matrix form as follows:

$$J_{E_1} = \begin{pmatrix} -(\frac{\lambda C_{v\to hr}}{(N_{Tr}+d)} I^*_{vr} N_{Vr} + \delta_d + \delta_h) & 0 & -(\frac{\lambda C_{v\to hr}}{(N_{Tr}+d)} N_{Vr} S^*_{hr}) \\ \frac{\lambda C_{v\to hr}}{(N_{Tr}+d)} I^*_{vr} N_{Vr} & -(\beta+\delta_d+\delta_h) & (\frac{\lambda C_{v\to hr}}{(N_{Tr}+d)} N_{Tr} S^*_{hr}) \\ 0 & (\frac{\lambda C_{hr\to v}}{(N_{Tr}+d)} N_{Tr} S^*_{vr}) & -\delta_v \end{pmatrix}$$

,

$$J_{E_2} = \begin{pmatrix} -(\frac{\lambda C_{v\to hw}}{(N_{Tw}+d)} I^*_{vw} N_{Vw} + \delta_d + \delta_h) & 0 & -(\frac{\lambda C_{v\to hw}}{(N_{Tw}+d)} N_{Vw} S^*_{hw}) \\ \frac{\lambda C_{v\to hw}}{(N_{Tw}+d)} I^*_{vw} N_{Vw} & -(\beta+\delta_d+\delta_h) & (\frac{\lambda C_{v\to hw}}{(N_{Tw}+d)} N_{Tw} S^*_{hw}) \\ 0 & (\frac{\lambda C_{hw\to v}}{(N_{Tw}+d)} N_{Tw} S^*_{vw}) & -\delta_v \end{pmatrix}$$

$$J_{E_1} = \begin{pmatrix} -(\frac{\lambda C_{v\to hs}}{(N_{Ts}+d)} I^*_{vs} N_{Vs} + \delta_d + \delta_h) & 0 & -(\frac{\lambda C_{v\to hs}}{(N_{Ts}+d)} N_{Vs} S^*_{hs}) \\ \frac{\lambda C_{v\to hs}}{(N_{Ts}+d)} I^*_{vs} N_{Vs} & -(\beta+\delta_d+\delta_h) & (\frac{\lambda C_{v\to hs}}{(N_{Ts}+d)} N_{Ts} S^*_{hs}) \\ 0 & (\frac{\lambda C_{hs\to v}}{(N_{Ts}+d)} N_{Ts} S^*_{vs}) & -\delta_v \end{pmatrix}$$

The eigenvales (A) are the solution of the Characteristic equation[5]

$$\det(J - AI_3) = 0$$

where J is the Jacobian matrix evaluated at the equilibrium point . I is the identity matrix.

C. Disease free state

Equilibrium point of disease free state E_1 = (1,0,0,1,0,0,1,0,0) has eigenvalues as follows:

$$A_1 = -\delta_d - \delta_h,\quad A_2 = \frac{1}{2}\left(-\beta-\delta_d-\delta_h-\delta_v+\sqrt{\frac{(N_{Tr}+d)^2(\beta+\delta_d+\delta_h)^2+4N^2_{Tr}S_{vr}\lambda^2C_{v\to hr}C_{hr\to v}}{(N_{Tr}+d)}}\right)$$

$$A_3 = \frac{1}{2}\left(-\beta-\delta_d-\delta_h-\delta_v-\sqrt{\frac{(N_{Tr}+d)^2(\beta+\delta_d+\delta_h)^2+4N^2_{Tr}S_{vr}\lambda^2C_{v\to hr}C_{hr\to v}}{(N_{Tr}+d)}}\right)$$

$$A_4 = -\delta_d - \delta_h,\quad A_5 = \frac{1}{2}\left(-\beta-\delta_d-\delta_h-\delta_v+\sqrt{\frac{(N_{Tw}+d)^2(\beta+\delta_d+\delta_h)^2+4N^2_{Tw}S_{vw}\lambda^2C_{v\to hw}C_{hw\to v}}{(N_{Tw}+d)}}\right)$$

$$A_6 = \frac{1}{2}\left(-\beta-\delta_d-\delta_h-\delta_v+\sqrt{\frac{(N_{Tw}+d)^2(\beta+\delta_d+\delta_h)^2+4N^2_{Tw}S_{vw}\lambda^2C_{v\to hw}C_{hw\to v}}{(N_{Tw}+d)}}\right)$$

$$A_7 = -\delta_d - \delta_h,\quad A_8 = \frac{1}{2}\left(-\beta-\delta_d-\delta_h-\delta_v+\sqrt{\frac{(N_{Ts}+d)^2(\beta+\delta_d+\delta_h)^2+4N^2_{Ts}S_{vs}\lambda^2C_{v\to hs}C_{hs\to v}}{(N_{Ts}+d)}}\right)$$

$$A_9 = \frac{1}{2}\left(-\beta-\delta_d-\delta_h-\delta_v-\sqrt{\frac{(N_{Ts}+d)^2(\beta+\delta_d+\delta_h)^2+4N^2_{Ts}S_{vs}\lambda^2C_{v\to hs}C_{hs\to v}}{(N_{Ts}+d)}}\right)$$

From our evaluations, all eigenvalues have negative real parts when $R_0 < 1$. So this disease free equilibrium point is local stability when $R_0 < 1$. We define

$$R_0 = \frac{\dfrac{\dfrac{\lambda^2 C_{hs\to v}C_{v\to hs}N_{Vs}N_{Ts}}{(N_{Ts}+d)^2}+\dfrac{\lambda^2 C_{hw\to v}C_{v\to hw}N_{Vw}N_{Tw}}{(N_{Tw}+d)^2}}{(\beta+\delta d+\delta h)}+\dfrac{\lambda^2 C_{hr\to v}C_{v\to hr}N_{Vr}N_{Tr}}{(N_{Tr}+d)^2(\beta+\delta d+\delta h)}}{\delta v}.$$

D. Endemic disease state

The endemic disease equilibrium point

$E_1 = (S^*_{hr}, I^*_{hr}, I^*_{vr}, S^*_{hw}, I^*_{hw}, I^*_{vw}, S^*_{hs}, I^*_{hs}, I^*_{vs})$ has eigenvalues as follows:

$$A_1 = -\beta-\delta_d-\delta_h,\quad A_2 = -\delta_v$$

$$A_3 = \left[\delta_h + \frac{I^*_{hr}b_1}{b_7b_2} + \frac{b_7b_2^3\delta_d b_3 + b_2(I^*_{hr}b_2N_{Vr}\delta_d - b_7N^2_{Tr}S_{vr}b_3^2b_4)b_6 + I^*_{hr}N_{Vr}N_{Tr}(N_{vr}-N_{Tr})S_{vr}b_3b_4b_5}{b_2^2(b_7b_2b_3 + I^*_{hr}b_1)}\right] / \left(-1 + \frac{b_7b_4b_3b_6N^2_{Tr}S_{vr}}{b_2(b_7b_3b_2 + I^*_{hr}b_1)}\right)$$

$$A_4 = -\beta-\delta_d-\delta_h,\quad A_5 = -\delta_v$$

$$A_6 = \left[\delta_h + \frac{I^*_{hw}f_1}{f_7f_2} + \frac{f_7f_2^3\delta_d f_3 + f_2(I^*_{hw}f_2N_{Vw}\delta_d - b_7N^2_{Tw}S_{vw}f_3^2f_4)f_6 + I^*_{hw}N_{Vw}N_{Tw}(N_{vw}-N_{Tw})S_{vw}f_3f_4f_5}{f_2^2(f_7f_2f_3 + I^*_{hw}f_1)}\right] / \left(-1 + \frac{f_7f_4f_3f_6N^2_{Tw}S_{vw}}{f_2(f_7f_3f_2 + I^*_{hw}f_1)}\right)$$

$$A_7 = -\beta-\delta_d-\delta_h,\quad A_8 = -\delta_v$$

$$A_9 = \left[\delta_h + \frac{I^*_{hs}g_1}{g_7g_2} + \frac{g_7g_2^3\delta_d g_3 + g_2(I^*_{hs}g_2N_{Vs}\delta_d - g_7N^2_{Ts}S_{vs}g_3^2g_4)g_6 + I^*_{hs}N_{Vs}N_{Ts}(N_{vs}-N_{Ts})S_{vs}g_3g_4g_5}{g_2^2(g_7g_2g_3 + I^*_{hs}g_1)}\right] / \left(-1 + \frac{g_7g_4g_3g_6N^2_{Ts}S_{vs}}{g_2(g_7g_3g_2 + I^*_{hs}g_1)}\right)$$

After our calculations, we found that all eigenvalues have negative real parts when $R_0 > 1$. This means that the endemic disease state is local stability for $R_0 > 1$.

Results, Discussion and Conclusion: This paper studied the transmission model of dengue disease with different contact rates in each season. The standard dynamical modeling method is used in this study. We found two equilibrium states; disease free state and endemic disease state. The disease free state is local stability when $R_0 < 1$ .The endemic disease state is local stability when $R_0 > 1$. R_0 is defined as the basic reproductive number. The output of this study should introduce the alternative way for controlling the outbreak of dengue disease, ie., if we can control the parameters same as in the conditions of local stability for each equilibrium points, then we can reduce the transmission of this disease.

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Keywords: Dengue disease, local stability, mathematical model, season.

DENGUE TRANSMISSION MODEL WITH THE DIFFERENT INCUBATION RATE FOR EACH SEASON

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Abstract

A model is used for describing the transmission of dengue disease. This disease is occurred by biting of the infected *Aedes* mosquitoes. Dengue outbreak is found in the rainy, winter and summer seasons. Each season has the different dengue outbreaks and they are depend on the temperature of the environment. The standard dynamical modeling method is used in this study. The SEIR (S = susceptible , E= exposed , I = infected and R = recovered) model is used. We use conditions of parameters for determining the local stability of disease free equilibrium state and disease endemic equilibrium state. The basic reproductive number of disease is found. The disease free state is local stability when $R_0 < 1$ .The endemic disease state is local stability when $R_0 > 1$. The control of this disease is discussed in this paper.

Keywords: Dengue disease, local stability, transmission model ,SEIR model, season, incubation rate.

Introduction

Transmission of dengue virus may serve to retain viral pathogen in nature during inter-epidemic periods of the disease [2]. Dengue fever (DF) , Dengue Haemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS) are three types of dengue disease. There are four serotypes of dengue virus, namely DEN-1, DEN-2, DEN-3 and DEN-4. Dengue disease can not be directly transmitted between the people. Transmission is occurred by biting of the female *Aedes* mosquito. The development of virus and mosquito are affected by the climatic factors. When infected mosquito bites the human , thus the human are exposed and infected. Symptoms of dengue fever are depend on age. In older children, teenagers and adults, the most common symptoms of dengue disease are fever that comes on quickly and lasts two to seven days but this usually is not severe, muscle and joint pain, a red rash that starts on chest, back or stomach and spreads to your limbs and face, feeling sick, vomiting and diarrhea. The symptoms of dengue fever usually begin between five and eight days after each person be get bitten by an infected mosquito. Dengue fever is caused by a type of virus called a *flavivirus*, which is transmitted by infected female *Aedes* mosquitoes. We can catch the virus if we get be bitten by an infected mosquito. Mosquitoes become infected when they bite an infected person and are able to pass on the virus for the rest of their life. In Thailand, the annual estimations of dengue fever are depend on the season. The *Aedes aegypti* is the principal transmitter of Dengue fever in Thailand but it also transmits Chikungunya fever, yellow fever and Filariasis among other diseases. The Aedes aegypti prefers feed during

daylight hours. They adapt very easily to human surroundings and will lay their eggs where there is water, including plastic containers, bins, plant pots etc. Thailand's rainy season, starting from May through September, is also the high risk period for dengue fever, a potentially serious condition is the most prevalent in tropical countries. *Aedes aegypti* mosquitoes carry the virus that causes dengue fever, and they infect 50 million people a year, including 500,000 serious cases requiring hospitalization[8].The moisture content ,temperature,season and rainfall are influence to the mosquito development. Dengue infection is endemic in Thailand. From the data of Dengue cases in 1999 – 2010, we can see that most dengue patients are occurred in rainy season. We can see as shown in figure 1.
The purpose of this paper is to incorporate this feature into the SEIR model. Models keep track of an individual's infection-age for particular diseases, for instance tubercolosis[3]. Esteva and Vargas developed a model for dengue disease transmission and included the dynamics of the *Aedes aegypti mosquito into standard SIR* (susceptible – infective-recovered) epidemic model [5]. In this paper , we used SEIR model for analyzing and finding the method to decrease the outbreak of this disease. We analyze dengue model of seasonality compartment (rainy season, winter season and summer season).

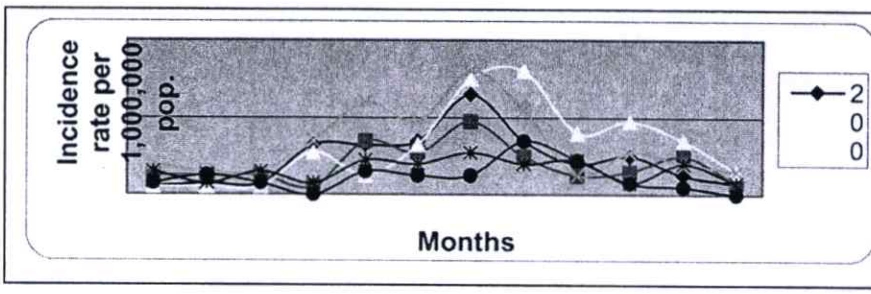

Figure 1 Reported cases of Dengue disease per 100,000 population in Thailand during year 2005 and 2010 .

Methodology

The mathematical modeling for dengue disease describes the relevance of human and mosquito population. In this study, we assume that the total human and mosquito population have constant sizes. The human population is divided into susceptible , exposed , infected and recovered classes for the first model. The mosquito population is divided into susceptible , exposed and infected classes because the mosquito never recover from infection. The model considers transmission of dengue virus in human and mosquito population by model :
The dynamics of human population are given by

$$\frac{d}{dt} S_{hr} = C_r N_{Tr} - \lambda_d S_{hr} - \lambda_h S_{hr} - \delta \frac{K_{v\to hr}}{N_{Tr}+g} I_{rr} S_{hr} \quad (1)$$

$$\frac{d}{dt} E_{hr} = \delta \frac{K_{v\to hr}}{N_{Tr}+g} I_{rr} S_{hr} - \lambda_d E_{hr} - \lambda_h E_{hr} - \alpha_{hr} E_{hr} \quad (2)$$

$$\frac{d}{dt} I_{hr} = \alpha_{hr} - \lambda_d I_{hr} - \lambda_h I_{hr} - \rho I_{hr} \quad (3)$$

$$\frac{d}{dt} R_{hr} = \rho I_{hr} - \lambda_d R_{hr} - \lambda_h R_{hr} \quad (4)$$

$$\frac{d}{dt} S_{hw} = C_w N_{Tw} - \lambda_d S_{hw} - \lambda_h S_{hw} - \delta \frac{K_{v\to hw}}{N_{Tw}+g} I_{rw} S_{hw} \quad (5)$$

$$\frac{d}{dt}E_{hw} = \delta\frac{K_{v\to hw}}{N_{Tw}+g}I_{rw}S_{hw} - \lambda_d E_{hw} - \lambda_h E_{hw} - \alpha_{hw}E_{hw} \quad (6)$$

$$\frac{d}{dt}I_{hw} = \alpha_{hw} - \lambda_d I_{hw} - \lambda_h I_{hw} - \rho I_{hw} \quad (7)$$

$$\frac{d}{dt}R_{hw} = \rho I_{hw} - \lambda_d R_{hw} - \lambda_h R_{hw} \quad (8)$$

$$\frac{d}{dt}S_{hs} = C_s N_{Ts} - \lambda_d S_{hs} - \lambda_h S_{hs} - \delta\frac{K_{v\to hs}}{N_{Ts}+g}I_{rs}S_{hs} \quad (9)$$

$$\frac{d}{dt}E_{hs} = \delta\frac{K_{v\to hs}}{N_{Ts}+g}I_{rs}S_{hs} - \lambda_d E_{hs} - \lambda_h E_{hs} - \alpha_{hs}E_{hs} \quad (10)$$

$$\frac{d}{dt}I_{hs} = \alpha_{hs} - \lambda_d I_{hs} - \lambda_h I_{hs} - \rho I_{hs} \quad (11)$$

$$\frac{d}{dt}R_{hs} = \rho I_{hs} - \lambda_d R_{hs} - \lambda_h R_{hs} \quad (12)$$

We define

S_{hr} is the number of susceptible human population in rainy season,
E_{hr} is the number of exposed human population in rainy season,
I_{hr} is the number of infectious human population in rainy season,
R_{hr} is the number of recovered human population in rainy season,
S_{hw} is the number of susceptible human population in winter season,
E_{hw} is the number of exposed human population in winter season,
I_{hw} is the number of infectious human population in winter season,
R_{hw} is the number of recovered human population in winter season,
S_{hs} is the number of susceptible human population in summer season,
E_{hs} is the number of exposed human population in summer season,
I_{hs} is the number of infectious human population in summer season,
R_{hs} is the number of recovered human population in summer season

The dynamics of the mosquito population are given by :

$$\frac{d}{dt}S_{vr} = Z_r - \delta\frac{K_{hr\to v}}{N_{Tr}+g}I_{hr}S_{vr} - \lambda_v S_{vr} \quad (13)$$

$$\frac{d}{dt}E_{vr} = \delta\frac{K_{hr\to v}}{N_{Tr}+g}I_{hr}S_{vr} - \lambda_v E_{vr} - \beta_{vr}E_{vr} \quad (14)$$

$$\frac{d}{dt}I_{vr} = \beta_{vr}E_{vr} - \lambda_v I_{vr} \quad (15)$$

$$\frac{d}{dt}S_{vw} = Z_w - \delta\frac{K_{hw\to v}}{N_{Tw}+g}I_{hw}S_{vw} - \lambda_v S_{vw} \quad (16)$$

$$\frac{d}{dt}E_{vw} = \delta\frac{K_{hw\to v}}{N_{Tw}+g}I_{hw}S_{vw} - \lambda_v E_{vw} - \beta_{vw}E_{vw} \quad (17)$$

$$\frac{d}{dt}I_{vw} = \beta_{vw}E_{vw} - \lambda_v I_{vw} \quad (18)$$

$$\frac{d}{dt}S_{vs} = Z_s - \delta\frac{K_{hs\to v}}{N_{Ts}+g}I_{hs}S_{vs} - \lambda_v S_{vs} \quad (19)$$

$$\frac{d}{dt}E_{vs} = \delta\frac{K_{hs\to v}}{N_{Ts}+g}I_{hs}S_{vs} - \lambda_v E_{vs} - \beta_{vs}E_{vs} \tag{20}$$

$$\frac{d}{dt}I_{vs} = \beta_{vs}E_{vs} - \lambda_v I_{vs} \tag{21}$$

We define

S_{vr} is the number of susceptible mosquito population in rainy season,

E_{vr} is the number of exposed mosquito population in rainy season,

I_{vr} is the number of infectious mosquito population in rainy season,

S_{vw} is the number of susceptible mosquito population in winter season,

E_{vw} is the number of exposed mosquito population in winter season,

I_{vw} is the number of infectious mosquito population in winter season,

S_{vs} is the number of susceptible mosquito population in summer season,

E_{vs} is the number of exposed mosquito population in summer season,

I_{vs} is the number of infectious mosquito population in summer season.

Where the parameters are defined as follows :

N_{Tr} is the total human population in rainy season,

N_{Tw} is the total human population in winter season,

N_{Ts} is the total human population in summer season,

N_{Vr} is the total mosquito population in rainy season,

N_{Vw} is the total mosquito population in winter season,

N_{Vs} is the total mosquito population in summer season,

λ_h is the natural death rate of human population,

λ_d is the death rate of human population due to the disease,

λ_v is the death rate of mosquito population,

K is the birth rate of human population,

$K_{v\to hr}$ is the transmission probability of dengue disease from mosquito to human in rainy season,

$K_{v\to hw}$ is the transmission probability of dengue disease from mosquito to human in winter season ,

$K_{v\to hs}$ is the transmission probability of dengue disease from mosquito to human in summer season,

$K_{hr\to v}$ is the transmission probability of dengue disease from human to mosquito in rainy season,

$K_{hw\to v}$ is the transmission probability of dengue disease from human to mosquito in winter season,

$K_{hs\to v}$ is the transmission probability of dengue disease from human to mosquito in summer season,

α_{hr} is the incubation rate of human population in rainy season,

α_{hw} is the incubation rate of human population in winter season,

α_{hs} is the incubation rate of human population in summer season,

α_{vr} is the incubation rate of mosquito population in rainy season

α_{vw} is the incubation rate of mosquito population in winter season

α_{vs} is the incubation rate of mosquito population in summer season,

ρ is the recovery rate of human population,

δ is the biting rate of mosquito population,

g is the number of other animals available as blood sources.

We suppose that $N_{Hr} = S_{hr} + E_{hr} + I_{hr} + R_{hr}$, $N_{Hw} = S_{hw} + E_{hw} + I_{hw} + R_{hw}$, $N_{Hs} = S_{hs} + E_{hs} + I_{hs} + R_{hs}$, $N_{Vr} = S_{vr} + E_{vr} + I_{vr}$, $N_{Vw} = S_{vw} + E_{vw} + I_{vw}$ and $N_{Vs} = S_{vs} + E_{vs} + I_{vs}$

we assume the total human and mosquito populations have constant sizes $\frac{dN_{hr}}{dt} = 0$, $\frac{dN_{vr}}{dt} = 0$ in rainy season, $\frac{dN_{hw}}{dt} = 0$, $\frac{dN_{vw}}{dt} = 0$ in winter season and $\frac{dN_{hs}}{dt} = 0$, $\frac{dN_{vs}}{dt} = 0$ in summer season.

$$\overline{S_{hr}} = \frac{S_{hr}}{N_{Tr}},\ \overline{E_{hr}} = \frac{E_{hr}}{N_{Tr}},\ \overline{I_{hr}} = \frac{I_{hr}}{N_{Tr}},\ \overline{R_{hr}} = \frac{R_{hr}}{N_{Tr}},$$

$$\overline{S_{hw}} = \frac{S_{hw}}{N_{Tw}},\ \overline{E_{hw}} = \frac{E_{hw}}{N_{Tw}},\ \overline{I_{hw}} = \frac{I_{hw}}{N_{Tw}},\ \overline{R_{hw}} = \frac{R_{hw}}{N_{Tw}},$$

$$\overline{S_{hs}} = \frac{S_{hs}}{N_{Ts}},\ \overline{E_{hs}} = \frac{E_{hs}}{N_{Ts}},\ \overline{I_{hs}} = \frac{I_{hs}}{N_{Ts}},\ \overline{R_{hs}} = \frac{R_{hs}}{N_{Ts}}$$

$$\overline{S_{vr}} = \frac{S_{vr}}{N_{Vr}},\ \overline{S_{vw}} = \frac{S_{vw}}{N_{Vw}},\ \overline{S_{vs}} = \frac{S_{vs}}{N_{Vs}},\ \overline{E_{vr}} = \frac{E_{vr}}{N_{Vr}},\ \overline{E_{vw}} = \frac{E_{vw}}{N_{Vw}},\ \overline{E_{vs}} = \frac{E_{vs}}{N_{Vs}}$$

$$\overline{I_{vr}} = \frac{I_{vr}}{N_{Vr}},\ \overline{I_{vw}} = \frac{I_{vw}}{N_{Vw}},\ \overline{I_{vs}} = \frac{I_{vs}}{N_{Vs}}$$

These give

$$\frac{d}{dt}\overline{S_{hr}} = (\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta\frac{K_{v\to hr}}{N_{Tr} + g}\overline{I_{vr}}N_{Vr})\overline{S_{hr}} \tag{22}$$

$$\frac{d}{dt}\overline{E_{hr}} = \delta\frac{K_{v\to hr}}{N_{Tr} + g}\overline{I_{vr}}N_{Vr}\overline{S_{hr}} - (\alpha_{hr} + \lambda_d + \lambda_h)\overline{E_{hr}} \tag{23}$$

$$\frac{d}{dt}\overline{I_{hr}} = \alpha_{hr}\overline{E_{hr}} - (\lambda_d + \lambda_h + \rho)\overline{I_{hr}} \tag{24}$$

$$\frac{d}{dt}\overline{S_{hw}} = (\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta\frac{K_{v\to hw}}{N_{Tw} + g}\overline{I_{vw}}N_{Vw})\overline{S_{hw}} \tag{25}$$

$$\frac{d}{dt}\overline{E_{hw}} = \delta\frac{K_{v\to hw}}{N_{Tw} + g}\overline{I_{vw}}N_{Vw}\overline{S_{hw}} - (\alpha_{hw} + \lambda_d + \lambda_h)\overline{E_{hw}} \tag{26}$$

$$\frac{d}{dt}\overline{I_{hw}} = \alpha_{hw}\overline{E_{hw}} - (\lambda_d + \lambda_h + \rho)\overline{I_{hw}} \tag{27}$$

$$\frac{d}{dt}\overline{S_{hs}} = (\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta\frac{K_{v\to hs}}{N_{Ts} + g}\overline{I_{vs}}N_{Vs})\overline{S_{hs}} \tag{28}$$

$$\frac{d}{dt}\overline{E_{hs}} = \delta\frac{K_{v\to hs}}{N_{Ts} + g}\overline{I_{vs}}N_{Vs}\overline{S_{hs}} - (\alpha_{hs} + \lambda_d + \lambda_h)\overline{E_{hs}} \tag{29}$$

$$\frac{d}{dt}\overline{I_{hs}} = \alpha_{hs}\overline{E_{hs}} - (\lambda_d + \lambda_h + \rho)\overline{I_{hs}} \tag{30}$$

$$\frac{d}{dt}\overline{E_{vr}} = \delta \frac{K_{hr\to v}}{N_{Tr}+g}\overline{I_{hr}}\, N_{Tr}\overline{Sv} - (\lambda_v + \beta_{vr})\overline{E_{vr}} \tag{31}$$

$$\frac{d}{dt}\overline{I_{vr}} = \beta_{vr}\overline{E_{vr}} - \lambda_v \overline{I_{vr}} \tag{32}$$

$$\frac{d}{dt}\overline{E_{vw}} = \delta \frac{K_{v\to hw}}{N_{Tw}+g}\overline{I_{hw}}\, N_{Tw}\overline{S_{vr}} - (\lambda_v + \beta_{vw})\overline{E_{vw}} \tag{33}$$

$$\frac{d}{dt}\overline{I_{vw}} = \beta_{vw}\overline{E_{vw}} - \lambda_v \overline{I_{vw}} \tag{34}$$

$$\frac{d}{dt}\overline{E_{vs}} = \delta \frac{K_{v\to hs}}{N_{Ts}+g}\overline{I_{hs}}\, N_{Ts}\overline{S_{vs}} - (\lambda_v + \beta_{vs})\overline{E_{vs}} \tag{35}$$

$$\frac{d}{dt}\overline{I_{vs}} = \beta_{vs}\overline{E_{vs}} - \lambda_v \overline{I_{vs}} \tag{36}$$

R_{hr}, R_{hw}, R_{hs} and S_{vr}, S_{vw}, S_{vs} can be obtained from conditions $S_{hr} + E_{hr} + I_{hr} + R_{hr} = 1$, $S_{hw} + E_{hw} + I_{hw} + R_{hw} = 1$, $S_{hs} + E_{hs} + I_{hs} + R_{hs} = 1$ and $S_{vr} + E_{vr} + I_{vr} = 1$, $S_{vw} + E_{vw} + I_{vw} = 1$, $S_{vs} + E_{vs} + I_{vs} = 1$.

Analysis of the Mathematical Model

The equilibrium points are found by setting the right hand side of (22) – (36) equal to zero. This gives

1)The disease free equilibrium point $M_1 = (1,0,0,0,0,1,0,0,0,0,1,0,0,0,0)$

2)The endemic disease equilibrium point

$M_2 = (S^*_{hr}, E^*_{hr}, I^*_{hr}, E^*_{vr}, I^*_{vr}, S^*_{hw}, E^*_{hw}, I^*_{hw}, E^*_{vw}, I^*_{vw}, S^*_{hs}, E^*_{hs}, I^*_{hs}, E^*_{vs}, I^*_{vs})$,

where

$$S^*_{hr} = \frac{(\lambda_d + \lambda_h)}{(\lambda_d + \lambda_h) + \dfrac{I^*_{hr} N_{Tr} N_{Vr} \beta_{vr} \delta K_{hr\to v} \delta K_{v\to hr}}{(N_{Tr}+g)(\beta_{vr}+\lambda_v)(I^*_{hr} N_{Tr} \delta K_{hr\to v} + (N_{Tr}+g)\lambda_v)}},$$

$$E^*_{hr} = \frac{I^*_{hr} N_{Vr} \beta_{vr} N_{Tr} \delta K_{hr\to v} \delta K_{v\to hr} (\lambda_d + \lambda_h)}{(N_{Tr}+g)(\alpha_{hr}+\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v)(I^*_{hr} N_{Tr}\delta K_{hr\to v} + (N_{Tr}+g)\lambda_v)(\lambda_d + \lambda_h + \dfrac{I^*_{hr} N_{Vr} \beta_{vr} N_{Tr} \delta K_{hr\to v} \delta K_{v\to hr}}{(N_{Tr}+g)(\beta_{vr}+\lambda_v)(I^*_{hr} N_{Tr}\delta K_{hr\to v} + (N_{Tr}+g)\lambda_v)}))$$

$$, I^*_{hr} = \frac{(\lambda_d+\lambda_h)(-(N_{Tr} N_{Vr} \alpha_{hr} \beta_{vr} \delta K_{hr\to v} \delta K_{v\to hr} + (N_{Tr}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)))}{(N_{Tr}\delta K_{hr\to v}(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)(N_{Vr}\beta_{vr}\delta K_{v\to hr} + (N_{Tr}+g)(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v)))},$$

$$E^*_{vr} = \frac{I^*_{hr} N_{Tr} \delta K_{hr\to v} \lambda_v}{(\beta_{vr}+\lambda_v)(I^*_{hr} N_{Tr} \delta K_{hr\to v} + (N_{Tr}+g)\lambda_v)}, I^*_{vr} = \frac{I^*_{hr} N_{Tr} \delta K_{hr\to v} \beta_{vr}}{(\beta_{vr}+\lambda_v)(I^*_{hr} N_{Tr}\delta K_{hr\to v} + (N_{Tr}+g)\lambda_v)}$$

$$S^*_{hw} = \frac{(\lambda_d + \lambda_h)}{(\lambda_d + \lambda_h) + \dfrac{I^*_{hw} N_{Tw} N_{Vw} \beta_{vw} \delta K_{hw\to v} \delta K_{v\to hw}}{(N_{Tw}+g)(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw} \delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)}},$$

$$E^*_{hw} = \frac{I^*_{hw} N_{Vw} \beta_{vw} N_{Tw} \delta K_{hw\to v} \delta K_{v\to hw} (\lambda_d + \lambda_h)}{(N_{Tw}+g)(\alpha_{hw}+\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw}\delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)(\lambda_d + \lambda_h + \dfrac{I^*_{hw} N_{Vw} \beta_{vw} N_{Tw} \delta K_{hw\to v} \delta K_{v\to hw}}{(N_{Tw}+g)(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw}\delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)}))$$

$$, I^*_{hw} = \frac{(\lambda_d+\lambda_h)(-(N_{Tw} N_{Vw} \alpha_{hw} \beta_{vw} \delta K_{hw\to v} \delta K_{v\to hw} + (N_{Tw}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)))}{(N_{Tw}\delta K_{hw\to v}(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)(N_{Vw}\beta_{vw}\delta K_{v\to hw} + (N_{Tw}+g)(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v)))},$$

$$E^*_{vw} = \frac{I^*_{hw} N_{Tw} \delta K_{hw\to v} \lambda_v}{(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw} \delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)}, I^*_{vw} = \frac{I^*_{hw} N_{Tw} \delta K_{hw\to v} \beta_{vw}}{(\beta_{vw}+\lambda_v)(I^*_{hw} N_{Tw}\delta K_{hw\to v} + (N_{Tw}+g)\lambda_v)}$$

$$S^{*}_{hs}=\frac{(\lambda_d+\lambda_h)}{(\lambda_d+\lambda_h)+\dfrac{I^{*}_{hs}N_{Ts}N_{Vs}\beta_{vs}\delta K_{hs\to v}\delta K_{v\to hs}}{(N_{Ts}+g)(\beta_{vs}+\lambda_v)(I^{*}_{hs}N_{Ts}\delta K_{hs\to v}+(N_{Ts}+g)\lambda_v)}},$$

$$E^{*}_{hs}=\frac{I^{*}_{hs}N_{Vs}\beta_{vs}N_{Ts}\delta K_{hs\to v}\delta K_{v\to hs}(\lambda_d+\lambda_h)}{(N_{Ts}+g)(\alpha_{hs}+\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v)(I^{*}_{hs}N_{Ts}\delta K_{hs\to v}+(N_{Ts}+g)\lambda_v)(\lambda_d+\lambda_h+\dfrac{I^{*}_{hs}N_{Vs}\beta_{vs}N_{Ts}\delta K_{hs\to v}\delta K_{v\to hs}}{(N_{Ts}+g)(\beta_{vs}+\lambda_v)(I^{*}_{hs}N_{Ts}\delta K_{hs\to v}+(N_{Ts}+g)\lambda_v)}))}$$

, $$I^{*}_{hs}=\frac{(\lambda_d+\lambda_h)(-(N_{Ts}N_{Vs}\alpha_{hs}\beta_{vs}\delta K_{hs\to v}\delta K_{v\to hs}+(N_{Ts}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)))}{(N_{Ts}\delta K_{hs\to v}(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)(N_{Vs}\beta_{vs}\delta K_{v\to hs}+(N_{Ts}+g)(\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v)))},$$

$$E^{*}_{vs}=\frac{I^{*}_{hs}N_{Ts}\delta K_{hs\to v}\lambda_v}{(\beta_{vs}+\lambda_v)(I^{*}_{hs}N_{Ts}\delta K_{hs\to v}+(N_{Ts}+g)\lambda_v)},\quad I^{*}_{vs}=\frac{I^{*}_{hs}N_{Ts}\delta K_{hs\to v}\beta_{vs}}{(\beta_{vs}+\lambda_v)(I^{*}_{hs}N_{Ts}\delta K_{hs\to v}+(N_{Ts}+g)\lambda_v)}$$

and $R_0>1$, $R_0=$

$$\left[\left(\frac{\delta^2 K_{hr\to v}K_{v\to hr}N_{tr}N_{vr}\alpha_{hr}\beta_{vr}}{(g+N_{tr})^2(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)}\right)+\left(\frac{\delta^2 K_{hw\to v}K_{v\to hw}N_{tw}N_{vw}\alpha_{hw}\beta_{vw}}{(g+N_{tw})^2(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)}\right)+\left(\frac{\delta^2 K_{hs\to v}K_{v\to hs}N_{ts}N_{vw}\alpha_{hs}\beta_{vs}}{(g+N_{ts})^2(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)}\right)\right]$$

B. Stability

The stability of each equilibrium point is determined from linearizing equation (13) – (21) about equilibrium point examining the eigenvalues of the resulting Jacobian matrix. We now consider the eigenvalues of the Jacobian matrix at each equilibrium point. From equation (13)-(21) ,we can write in the matrix form as follows:

$$J_{E_1}=\begin{pmatrix} -(\frac{\delta K_{v\to hr}}{(N_{Tr}+g)}I^{*}_{vr}N_{Vr}+\lambda_d+\lambda_h) & 0 & 0 & 0 & -(\frac{\delta K_{v\to hr}}{(N_{Tr}+g)}N_{Vr}S^{*}_{hr}) \\ \frac{\delta K_{v\to hr}}{(N_{Tr}+g)}I^{*}_{vr}N_{Vr} & -(\lambda_d+\lambda_h+\alpha_{hr}) & 0 & 0 & (\frac{\delta K_{v\to hr}}{(N_{Tr}+g)}N_{Vr}S^{*}_{hr}) \\ 0 & \alpha_{hr} & -(\lambda_d+\lambda_h+\rho) & 0 & 0 \\ 0 & 0 & (\frac{\delta K_{hr\to v}}{(N_{Tr}+g)}N_{Tr}S^{*}_{vr}) & -(\lambda_v+\beta_{Vr}) & 0 \\ 0 & 0 & 0 & \beta_{Vr} & -\lambda_v \end{pmatrix}$$

$$J_{E_2}=\begin{pmatrix} -(\frac{\delta K_{v\to hw}}{(N_{Tw}+g)}I^{*}_{vw}N_{Vw}+\lambda_d+\lambda_h) & 0 & 0 & 0 & -(\frac{\delta K_{v\to hw}}{(N_{Tw}+g)}N_{Vw}S^{*}_{hw}) \\ \frac{\delta K_{v\to hw}}{(N_{Tw}+g)}I^{*}_{vw}N_{Vw} & -(\lambda_d+\lambda_h+\alpha_{hw}) & 0 & 0 & (\frac{\delta K_{v\to hw}}{(N_{Tw}+g)}N_{Vw}S^{*}_{hw}) \\ 0 & \alpha_{hw} & -(\lambda_d+\lambda_h+\rho) & 0 & 0 \\ 0 & 0 & (\frac{\delta K_{hw\to v}}{(N_{Tw}+g)}N_{Tw}S^{*}_{vw}) & -(\lambda_v+\beta_{Vw}) & 0 \\ 0 & 0 & 0 & \beta_{Vw} & -\lambda_v \end{pmatrix}$$

$$J_{E_3}=\begin{pmatrix} -(\frac{\delta K_{v\to hs}}{(N_{Ts}+g)}I^{*}_{vs}N_{Vs}+\lambda_d+\lambda_h) & 0 & 0 & 0 & -(\frac{\delta K_{v\to hs}}{(N_{Ts}+g)}N_{Vs}S^{*}_{hs}) \\ \frac{\delta K_{v\to hs}}{(N_{Ts}+g)}I^{*}_{vs}N_{Vs} & -(\lambda_d+\lambda_h+\alpha_{hs}) & 0 & 0 & (\frac{\delta K_{v\to hs}}{(N_{Ts}+g)}N_{Vs}S^{*}_{hs}) \\ 0 & \alpha_{hs} & -(\lambda_d+\lambda_h+\rho) & 0 & 0 \\ 0 & 0 & (\frac{\delta K_{hs\to v}}{(N_{Ts}+g)}N_{Ts}S^{*}_{vs}) & -(\lambda_v+\beta_{Vs}) & 0 \\ 0 & 0 & 0 & \beta_{Vs} & -\lambda_v \end{pmatrix},$$

The eigenvalues (A) are the solution of the Characteristic equation

$$\det(J - BI_5) = 0$$

where J is the Jacobian matrix evaluated at the equilibrium point . I is the identity matrix.

C. Disease free state

Equilibrium point of disease free state $E_{1,2,3}$ = (1,0,0,0,0,1,0,0,0,0,1,0,0,0,0) has eigenvalues as follows:

$$(-B-\lambda_d-\lambda_h)\left(-\frac{\delta^2 K_{hr\to v}K_{v\to hr}N_{Tr}N_{Vr}S_{Vr}\alpha_{hr}\beta_{vr}}{(N_{Tr}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hr}-\lambda_d-\lambda_h)(-B-\beta_{vr}-\lambda_v)\right)$$

The eigenvalues are

$B=-\lambda_d-\lambda_h$ and the remaining 4 eigenvalues are the solutions of

$$\left(-\frac{\delta^2 K_{hr\to v}K_{v\to hr}N_{Tr}N_{Vr}S_{Vr}\alpha_{hr}\beta_{vr}}{(N_{Tr}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hr}-\lambda_d-\lambda_h)(-B-\beta_{vr}-\lambda_v)\right)=0$$

$$(-B-\lambda_d-\lambda_h)\left(-\frac{\delta^2 K_{hw\to v}K_{v\to hw}N_{Tw}N_{Vw}S_{Vw}\alpha_{hw}\beta_{vw}}{(N_{Tw}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hw}-\lambda_d-\lambda_h)(-B-\beta_{vw}-\lambda_v)\right)$$

The eigenvalues are

$B=-\lambda_d-\lambda_h$ and the remaining 4 eigenvalues are the solutions of

$$\left(-\frac{\delta^2 K_{hw\to v}K_{v\to hw}N_{Tw}N_{Vw}S_{Vw}\alpha_{hw}\beta_{vw}}{(N_{Tw}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hw}-\lambda_d-\lambda_h)(-B-\beta_{vw}-\lambda_v)\right)=0$$

$$(-B-\lambda_d-\lambda_h)\left(-\frac{\delta^2 K_{hs\to v}K_{v\to hs}N_{Ts}N_{Vs}S_{Vs}\alpha_{hs}\beta_{vs}}{(N_{Ts}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hs}-\lambda_d-\lambda_h)(-B-\beta_{vs}-\lambda_v)\right)$$

The eigenvalues are

$B=-\lambda_d-\lambda_h$ and the remaining 4 eigenvalues are the solutions of

$$\left(-\frac{\delta^2 K_{hs\to v}K_{v\to hs}N_{Ts}N_{Vs}S_{Vs}\alpha_{hs}\beta_{vs}}{(N_{Ts}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hs}-\lambda_d-\lambda_h)(-B-\beta_{vs}-\lambda_v)\right)=0$$

or $\lambda^4+A_3\lambda^3+A_2\lambda^2+A_1\lambda+A=0$

where

$$A_3=\rho+\alpha_{hr}+\beta_{vr}+2\lambda_d+2+2\lambda_v$$

$$A_2=(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v+\lambda_v^2+\beta_{vr}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hr}(\rho+\beta_{vr}+\lambda_d+\lambda_h+2\lambda_v)$$

$$A_1=\beta_{vr}(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)+(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vr}(\rho+2\lambda_d+2\lambda_h)+\alpha_{hr}(\beta_{vr}+2(\rho+\lambda_d+\lambda_h)))\lambda_v+(\rho+\alpha_{hr}+2\lambda_d+2\lambda_h)\lambda_v^2$$

$$A=\frac{1}{(N_{Tr}+g)^2}\left(-\delta^2 K_{hr\to v}K_{v\to hr}N_{Tr}N_{Vr}S_{Vr}\alpha_{hr}\beta_{vr}+(N_{Tr}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)\right)$$

$$A_3=\rho+\alpha_{hw}+\beta_{vw}+2\lambda_d+2+2\lambda_v$$

$$A_2=(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v+\lambda_v^2+\beta_{vw}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hw}(\rho+\beta_{vw}+\lambda_d+\lambda_h+2\lambda_v)$$

$$A_1=\beta_{vw}(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)+(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vw}(\rho+2\lambda_d+2\lambda_h)+\alpha_{hw}(\beta_{vw}+2(\rho+\lambda_d+\lambda_h)))\lambda_v+(\rho+\alpha_{hw}+2\lambda_d+2\lambda_h)\lambda_v^2$$

$$A=\frac{1}{(N_{Tw}+g)^2}\left(-\delta^2 K_{hw\to v}K_{v\to hw}N_{Tr}N_{Vw}S_{Vw}\alpha_{hw}\beta_{vw}+(N_{Tw}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)\right)$$

$$A_3=\rho+\alpha_{hs}+\beta_{vs}+2\lambda_d+2+2\lambda_v$$

$$A_2=(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v+\lambda_v^2+\beta_{vs}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hs}(\rho+\beta_{vs}+\lambda_d+\lambda_h+2\lambda_v)$$

$$A_1=\beta_{vs}(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)+(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vs}(\rho+2\lambda_d+2\lambda_h)+\alpha_{hs}(\beta_{vs}+2(\rho+\lambda_d+\lambda_h)))\lambda_v+(\rho+\alpha_{hs}+2\lambda_d+2\lambda_h)\lambda_v^2$$

$$A=\frac{1}{(N_{Ts}+g)^2}\left(-\delta^2 K_{hs\to v}K_{v\to hs}N_{Ts}N_{Vs}S_{Vs}\alpha_{hs}\beta_{vs}+(N_{Ts}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)\right)$$

Equilibrium point is local stability if eigenvalues have negative real parts. when they are according to the Routh – Hurwitz criteria:

$$A_3 \quad > 0,$$

$$A_1 \quad > 0,$$

$$A \quad > 0,$$

$$A_3A_2A_1 - A_1^2 - A_3^2 A \quad > 0$$

Consider condition of Routh – Hurwitz criteria as show with parameter above. Condition in above is always true . So this disease free state is local stability for $R_0 < 1$ when R_0 =

$$\left[\left(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr}}{(g + N_{tr})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h)\lambda_v(\beta_{vr} + \lambda_v)}\right) + \left(\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{tw} N_{vw} \alpha_{hw} \beta_{vw}}{(g + N_{tw})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hw} + \lambda_d + \lambda_h)\lambda_v(\beta_{vw} + \lambda_v)}\right) + \left(\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{ts} N_{vw} \alpha_{hs} \beta_{vs}}{(g + N_{ts})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hs} + \lambda_d + \lambda_h)\lambda_v(\beta_{vs} + \lambda_v)}\right)\right]$$

D. Endemic disease state

The endemic disease equilibrium point $E_1 = (S^*_{hr}, E^*_{hr}, I^*_{hr}, E^*_{vr}, I^*_{vr}, S^*_{hw}, E^*_{hw}, I^*_{hw}, E^*_{vw}, I^*_{vw}, S^*_{hs}, E^*_{hs}, I^*_{hs}, E^*_{vs}, I^*_{vs})$ has characteristic equation as follows:

$$\lambda^5 + A_4\lambda^4 + A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A = 0$$

Where

$$A4 = \rho + \alpha_{hr} + 3\lambda_d + 3\lambda_h + 2\lambda_v + \beta_{vr}\left(1 + \frac{\delta^2 i_{hr} K_{hr\to v} K_{v\to hr} N_{Tr} N_{vr}}{(g + N_{tr})(\beta_{vr} + \lambda_v)(\delta i_{hr} K_{hr\to v} N_{Tr} + (g + N_{tr})\lambda_v)}\right)$$

$$A3 = -\frac{1}{(g + N_{tr})(\beta_{vr} + \lambda_v)(\delta i_{hr} K_{hr\to v} N_{Tr} + (g + N_{tr})\lambda_v)} \Big(-(g + N_{tr})^2 \lambda_v (\beta_{vr} + \lambda_v)\big((\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h) + 2(\rho + 3\lambda_d + 3\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho + 3\lambda_d + 3\lambda_h + \lambda_v) + \alpha_{hr}(\rho + \beta_{vr} + 2\lambda_d + 2\lambda_h + 2\lambda_v)\big) - \delta i_{hr} K_{hr\to v} N_{Tr} \big(\delta K_{v\to hr} N_{vr} \beta_{vr}(\rho + \alpha_{hr} + \beta_{vr} + 2\lambda_d + 2\lambda_h + 2\lambda_v) + (g + N_{tr})(\beta_{vr} + \lambda_v)((\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h) + 2(\rho + 3\lambda_d + 3\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho + 3\lambda_d + 3\lambda_h + \lambda_v) + \alpha_{hr}(\rho + \beta_{vr} + 2\lambda_d + 2\lambda_h + 2\lambda_v))\big)\Big)$$

$$A2 = -\frac{1}{(g + N_{tr})(\beta_{vr} + \lambda_v)(\delta i_{hr} K_{hr\to v} N_{Tr} + (g + N_{tr})\lambda_v)} \Big(-(g + N_{tr})^2 \lambda_v(\beta_{vr} + \lambda_v)\big((\lambda_d + \lambda_h)^2(\rho + \lambda_d + \lambda_h) + 2(\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h)\lambda_v + (\rho + 3\lambda_d + 3\lambda_h)\lambda_v^2 + \beta_{vr}((\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h) + (\rho + 3\lambda_d + 3\lambda_h)\lambda_v) + \alpha_{hr}((\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + 2(\rho + 2\lambda_d + 2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho + 2\lambda_d + 2\lambda_h + \lambda_v))\big) - \delta i_{hr} K_{hr\to v} N_{Tr}\big(\delta K_{v\to hr} N_{vr}\beta_{vr}((\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + 2(\rho + 2\lambda_d + 2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho + 2\lambda_d + 2\lambda_h + \lambda_v) + \alpha_{hr}(\rho + \beta_{vr} + \lambda_d + \lambda_h + 2\lambda_v)) + (g + N_{tr})(\beta_{vr} + \lambda_v)((\lambda_d + \lambda_h)^2(\rho + \lambda_d + \lambda_h) + 2(\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h)\lambda_v + (\rho + 3\lambda_d + 3\lambda_h)\lambda_v^2 + \beta_{vr}((\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h) + (\rho + 3\lambda_d + 3\lambda_h)\lambda_v) + \alpha_{hr}((\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + 2(\rho + 2\lambda_d + 2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho + 2\lambda_d + 2\lambda_h + \lambda_v)))\big)\Big)$$

$$A1 = -\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr}}{(g + N_{tr})^2} + \rho\alpha_{hr}\beta_{vr}\lambda_d + \alpha_{hr}\beta_{vr}\lambda_d^2 + \rho\alpha_{hr}\beta_{vr}\lambda_h + 2\alpha_{hr}\beta_{vr}\lambda_d\lambda_h + \alpha_{hr}\beta_{vr}\lambda_h^2 - \frac{1}{(g + N_{tr})^2}\beta_{vr}\left(-(g + N_{tr})^2(\lambda_d + \lambda_h)^2(\rho + \lambda_d + \lambda_h) - \delta^2 i_{hr} K_{hr\to v} K_{v\to hr} N_{Tr} N_{vr}(\rho + \alpha_{hr} + 2\lambda_d + 2\lambda_h)\right) + (\alpha_{hr}(2(\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + \beta_{vr}(\rho + 2\lambda_d + 2\lambda_h)) + (\lambda_d + \lambda_h)(2(\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + \beta_{vr}(2\rho + 3\lambda_d + 3\lambda_h)))\lambda_v + (\alpha_{hr}(\rho + 2\lambda_d + 2\lambda_h) + (\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h))\lambda_v^2 + \frac{\delta^2 i_{hr} K_{hr\to v} K_{v\to hr} N_{Tr} N_{vr} \beta_{vr}^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h)}{(g + N_{tr})(-\delta i_{hr} K_{hr\to v} N_{Tr} + (g + N_{tr})\beta_{vr})(\beta_{vr} + \lambda_v)} + \left(\delta^2 i_{hr} K_{hr\to v} K_{v\to hr} N_{Tr} N_{vr} \beta_{vr}\left((g + N_{tr})^2 \beta_{vr}(\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h) + \delta^2 i_{hr}^2 K_{hr\to v}^2 N_{Tr}^2(\rho + \alpha_{hr} + 2\lambda_d + 2\lambda_h) + \delta i_{hr} K_{hr\to v}(g + N_{tr}) N_{Tr}(2(\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + \beta_{vr}(\rho + 2\lambda_d + 2\lambda_h) + \alpha_{hr}(\beta_{vr} + 2(\rho + \lambda_d + \lambda_h)))\right)\right) / \left((g + N_{tr})^2(-\delta i_{hr} K_{hr\to v} N_{Tr} + (g + N_{tr})\beta_{vr})(\delta i_{hr} K_{hr\to v} N_{Tr} + (g + N_{tr})\lambda_v)\right) + \left(\delta^2 i_{hr} K_{hr\to v}^2 K_{v\to hr} N_{tr} N_{Tr} N_{vr} \alpha_{hr} \beta_{vr}(\delta K_{v\to hr} N_{vr}\beta_{vr} + (g + N_{tr})(\lambda_d + \lambda_h)(\beta_{vr} + \lambda_v))\right) / \left((g + N_{tr})^2((g + N_{tr})^2(\lambda_d + \lambda_h)\lambda_v(\beta_{vr} + \lambda_v) + \delta i_{hr} K_{hr\to v} N_{Tr}(\delta K_{v\to hr} N_{vr}\beta_{vr} + (g + N_{tr})(\lambda_d + \lambda_h)(\beta_{vr} + \lambda_v)))\right)$$

$$A = \Big(\lambda_v\big(\delta^2 i_{hr}^2 K_{hr\to v}^2 N_{Tr}^2(\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h)(\delta K_{v\to hr} N_{vr}\beta_{vr} + (g + N_{tr})(\lambda_d + \lambda_h)(\beta_{vr} + \lambda_v))^2 + (g + N_{tr})^2(\lambda_d + \lambda_h)^2\lambda_v(\beta_{vr} + \lambda_v)(-\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr}\alpha_{hr}\beta_{vr} + (g + N_{tr})^2(\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h)\lambda_v(\beta_{vr} + \lambda_v)) + \delta i_{hr} K_{hr\to v}(g + N_{tr}) N_{Tr}(\lambda_d + \lambda_h)(\beta_{vr} + \lambda_v)(-\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr}\alpha_{hr}\beta_{vr}(\lambda_d + \lambda_h) + 2(g + N_{tr})(\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h)\lambda_v(\delta K_{v\to hr} N_{vr}\beta_{vr} + (g + N_{tr})(\lambda_d + \lambda_h)(\beta_{vr} + \lambda_v)))\big)\Big) / \Big((g + N_{tr})(\delta i_{hr} K_{hr\to v} N_{Tr} + (g + N_{tr})\lambda_v)((g + N_{tr})^2(\lambda_d + \lambda_h)\lambda_v(\beta_{vr} + \lambda_v) + \delta i_{hr} K_{hr\to v} N_{Tr}(\delta K_{v\to hr} N_{vr}\beta_{vr} + (g + N_{tr})(\lambda_d + \lambda_h)(\beta_{vr} + \lambda_v)))\Big)$$

$$\lambda 4 = \rho + \alpha_{hw} + 3\lambda_d + 3\lambda_h + 2\lambda_v + \beta_{vw}\left(1 + \frac{\delta^2 i_{hw} K_{hw\to v} K_{v\to hw} N_{hw} N_{vw}}{(g + N_{hw})(\beta_{vw} + \lambda_v)(\delta i_{hw} K_{hw\to v} N_{hw} + (g + N_{hw})\lambda_v)}\right)$$

$$\lambda 3 = \frac{1}{(g + N_{hw})(\beta_{vw} + \lambda_v)(\delta i_{hw} K_{hw\to v} N_{hw} + (g + N_{hw})\lambda_v)}\Big((g + N_{hw})^2 \lambda_v (\beta_{vw} + \lambda_v)\big((\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h) + 2(\rho + 3\lambda_d + 3\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho + 3\lambda_d + 3\lambda_h + \lambda_v) + \alpha_{hw}(\rho + \beta_{vw} + 2\lambda_d + 2\lambda_h + 2\lambda_v)\big) + \delta i_{hw} K_{hw\to v} N_{hw}\big(\delta K_{v\to hw} N_{vw} \beta_{vw}(\rho + \alpha_{hw} + \beta_{vw} + 2\lambda_d + 2\lambda_h + 2\lambda_v) + (g + N_{hw})(\beta_{vw} + \lambda_v)\big((\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h) + 2(\rho + 3\lambda_d + 3\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho + 3\lambda_d + 3\lambda_h + \lambda_v) + \alpha_{hw}(\rho + \beta_{vw} + 2\lambda_d + 2\lambda_h + 2\lambda_v)\big)\big)\Big)$$

$$\lambda 2 = \frac{1}{(g + N_{hw})(\beta_{vw} + \lambda_v)(\delta i_{hw} K_{hw\to v} N_{hw} + (g + N_{hw})\lambda_v)}\Big((g + N_{hw})^2 \lambda_v (\beta_{vw} + \lambda_v)\big((\lambda_d + \lambda_h)^2(\rho + \lambda_d + \lambda_h) + 2(\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h)\lambda_v + (\rho + 3\lambda_d + 3\lambda_h)\lambda_v^2 + \beta_{vw}((\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h) + (\rho + 3\lambda_d + 3\lambda_h)\lambda_v) + \alpha_{hw}((\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + 2(\rho + 2\lambda_d + 2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho + 2\lambda_d + 2\lambda_h + \lambda_v))\big) + \delta i_{hw} K_{hw\to v} N_{hw}\big(\delta K_{v\to hw} N_{vw} \beta_{vw}((\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + 2(\rho + 2\lambda_d + 2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho + 2\lambda_d + 2\lambda_h + \lambda_v) + \alpha_{hw}(\rho + \beta_{vw} + \lambda_d + \lambda_h + 2\lambda_v)) + (g + N_{hw})(\beta_{vw} + \lambda_v)((\lambda_d + \lambda_h)^2(\rho + \lambda_d + \lambda_h) + 2(\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h)\lambda_v + (\rho + 3\lambda_d + 3\lambda_h)\lambda_v^2 + \beta_{vw}((\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h) + (\rho + 3\lambda_d + 3\lambda_h)\lambda_v) + \alpha_{hw}((\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + 2(\rho + 2\lambda_d + 2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho + 2\lambda_d + 2\lambda_h + \lambda_v)))\big)\Big)$$

$$\lambda 1 = -\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{hw} N_{vw} \alpha_{hw} \beta_{vw}}{(g + N_{hw})^2} + \rho\,\alpha_{hw} \beta_{vw} \lambda_d + \alpha_{hw} \beta_{vw} \lambda_d^2 + \rho\,\alpha_{hw} \beta_{vw} \lambda_h + 2\alpha_{hw} \beta_{vw} \lambda_d \lambda_h + \alpha_{hw} \beta_{vw} \lambda_h^2 - \frac{\beta_{vw}\left(-(g + N_{hw})^2(\lambda_d + \lambda_h)^2(\rho + \lambda_d + \lambda_h) - \delta^2 i_{hw} K_{hw\to v} K_{v\to hw} N_{hw} N_{vw}(\rho + \alpha_{hw} + 2\lambda_d + 2\lambda_h)\right)}{(g + N_{hw})^2} + (\alpha_{hw}(2(\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + \beta_{vw}(\rho + 2\lambda_d + 2\lambda_h)) + (\lambda_d + \lambda_h)(2(\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + \beta_{vw}(2\rho + 3\lambda_d + 3\lambda_h)))\lambda_v + (\alpha_{hw}(\rho + 2\lambda_d + 2\lambda_h) + (\lambda_d + \lambda_h)(2\rho + 3\lambda_d + 3\lambda_h))\lambda_v^2 + \frac{\delta^2 i_{hw} K_{hw\to v} K_{v\to hw} N_{hw} N_{vw} \beta_{vw}^2(\rho + \lambda_d + \lambda_h)(\alpha_{hw} + \lambda_d + \lambda_h)}{(g + N_{hw})(-\delta i_{hw} K_{hw\to v} N_{hw} + (g + N_{hw})\beta_{vw})(\beta_{vw} - \lambda_v)} + \Big(\delta^2 i_{hw} K_{hw\to v} K_{v\to hw} N_{hw} N_{vw} \beta_{vw}\big((g + N_{hw})^2 \beta_{vw}(\rho + \lambda_d + \lambda_h)(\alpha_{hw} + \lambda_d + \lambda_h) + \delta^2 i_{hw}^2 K_{hw\to v}^2 N_{hw}^2(\rho + \alpha_{hw} + 2\lambda_d + 2\lambda_h) - \delta i_{hw} K_{hw\to v}(g + N_{hw}) N_{hw}(2(\lambda_d + \lambda_h)(\rho + \lambda_d + \lambda_h) + \beta_{vw}(\rho + 2\lambda_d + 2\lambda_h) + \alpha_{hw}(\beta_{vw} + 2(\rho + \lambda_d + \lambda_h)))\big)\Big) \Big/ \Big((g + N_{hw})^2(-\delta i_{hw} K_{hw\to v} N_{hw} + (g + N_{hw})\beta_{vw})(\delta i_{hw} K_{hw\to v} N_{hw} + (g + N_{hw})\lambda_v)\Big) + \frac{\delta^2 i_{hw} K_{hw\to v}^2 K_{v\to hw} N_{hw} N_{hw} N_{vw} \alpha_{hw} \beta_{vw}(\delta K_{v\to hw} N_{vw} \beta_{vw} + (g + N_{hw})(\lambda_d + \lambda_h)(\beta_{vw} + \lambda_v))}{(g + N_{hw})^2((g + N_{hw})^2(\lambda_d + \lambda_h)\lambda_v(\beta_{vw} + \lambda_v) + \delta i_{hw} K_{hw\to v} N_{hw}(\delta K_{v\to hw} N_{vw} \beta_{vw} + (g + N_{hw})(\lambda_d + \lambda_h)(\beta_{vw} + \lambda_v)))}$$

The eigenvalues of endemic disease state have negative real parts ,when they are according to the Routh – Hurwitz criteria:

$$A_i > 0 \quad (i = 1, 2, 3, 4), \tag{37}$$

$$A > 0, \tag{38}$$

$$A_4 A_3 A_2 - A_2^2 - A_4^2 A_1 > 0, \tag{39}$$

$$(A_4 A_1 - A)(A_4 A_3 A_2 - A_2^2 - A_4^2 A_1) - A(A_4 A_3 - A_2)^2 - A_4^2 > 0 \tag{40}$$

Consider condition of Routh – Hurwitz criteria as show with parameter above. Condition in above is always true. We can represent conditions (39)-(40) with the figure 2 as follows:

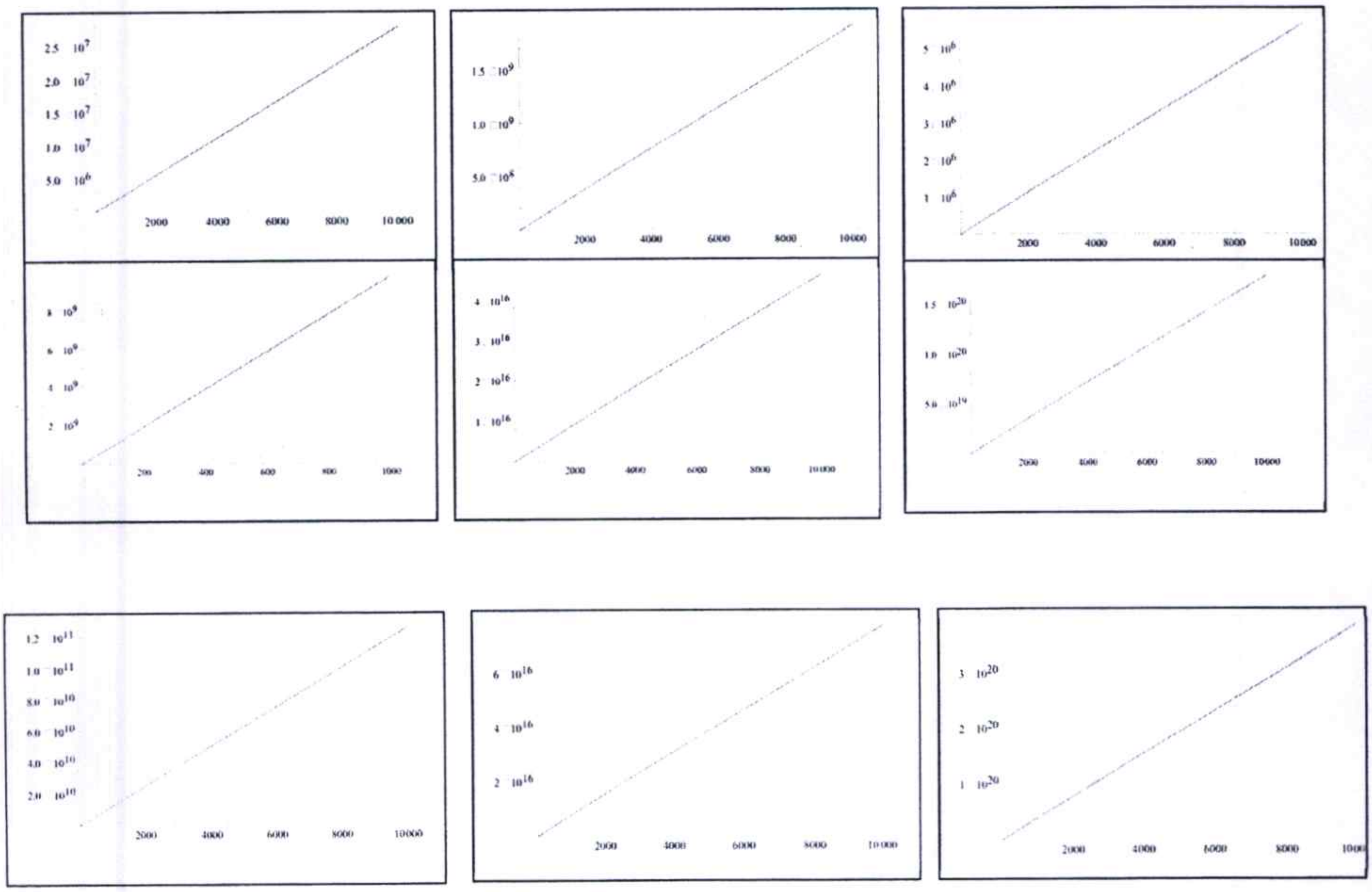

Figure 2 The condition of Routh – Hurwitz criteria(39) – (40) in endemic disease state for rainy season, winter season and summer season, respectively.

We found that all eigenvalues have negative real parts when $R_0 > 1$. This means that the endemic disease state is local stability [3] for $R_0 > 1$ when

$$R_0 = \left[\left(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} a_{hr} \beta_{vr}}{(g+N_{tr})^2 (\rho+\lambda_d+\lambda_h)(a_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)}\right)\cdot\left(\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{tw} N_{vw} a_{hw} \beta_{vw}}{(g+N_{tw})^2 (\rho+\lambda_d+\lambda_h)(a_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)}\right)\cdot\left(\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{ts} N_{vs} a_{hs} \beta_{vs}}{(g+N_{ts})^2 (\rho+\lambda_d+\lambda_h)(a_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)}\right)\right]$$

Results, Discussion and Conclusion

The transmission of dengue disease is analyzed by using the standard dynamical modeling method. We have shown that several standard theorems in mathematical epidemiology can be extended to this kind of SEIR model. R_0 can be expressed as

$$R_0 = \left[\left(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr}}{(g + N_{tr})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h) \lambda_v (\beta_{vr} + \lambda_v)}\right) \cdot \left(\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{tw} N_{vw} \alpha_{hw} \beta_{vw}}{(g + N_{tw})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hw} + \lambda_d + \lambda_h) \lambda_v (\beta_{vw} + \lambda_v)}\right) \cdot \left(\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{ts} N_{vw} \alpha_{hs} \beta_{vs}}{(g + N_{ts})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hs} + \lambda_d + \lambda_h) \lambda_v (\beta_{vs} + \lambda_v)}\right)\right]$$

is the basic reproductive number of the disease. If $R_0 < 1$, the disease-free equilibrium state is local stable . If $R_0 > 1$, then an endemic equilibrium is local stable. The results of this study are used for finding the condition of parameters to be the way for controlling this disease.

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Mathematical model of dengue disease with the different seasons

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Abstract

In this paper, the transmission of dengue disease in the mathematical model is analyzed. This disease is occurred by biting of the infected *Aedes* mosquitoes. Dengue outbreak is found in the rainy, winter and summer seasons. Each season has the different dengue outbreaks and they are depending on the environment temperature. The standard dynamical modeling method is used in this study. The SIR (S = susceptible , I = infected and R = recovered) model with season is studied. Conditions of parameters for determining the local stability of disease free equilibrium and endemic equilibrium states are found. The basic reproductive number (R_0) is calculated, giving a numerical measure of this particular epidemic's infectiousness. The disease free state is local stability when $R_0 < 1$ .The endemic disease state is local stability when $R_0 > 1$. The control of this disease is discussed in this paper.

Keywords : Dengue disease, transmission model ,SIR model, season.

Introduction

Dengue is a mosquito–borne viral disease found in tropical and subtropical regions around the world. Dengue disease can not be directly transmitted between the people. Transmission is occurred by biting of the female *Aedes* mosquito. The development of virus and mosquito are affected by the climatic factors. When infected mosquito bites the human , thus the human are exposed and infected. Dengue fever (DF), Dengue Haemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS) are three types of dengue disease. There are four serotypes of dengue virus, namely DEN-1, DEN-2, DEN-3 and DEN-4. Infection from one serotype grants life-long immunity to that strain, and also appears to temporarily grant the host a degree of cross – protection. Dengue fever is caused by a type of virus called a *flavivirus*, which is transmitted by infected female *Aedes* mosquitoes. We can catch the virus if we get be bitten by an infected mosquito. Mosquito become infected when it bites an infected person and pass on the virus for the rest of its life. *Aedes aegypti* mosquitoes carry the virus that causes dengue fever and they infect 50 million people a year, including 500,000 serious cases requiring hospitalization [8]. In older children, teenagers and adults, the most common symptoms of dengue disease are fever that comes on quickly and lasts two to seven days but this usually is not severe, muscle and joint pain, a red rash that starts on chest, back or stomach and spreads to your limbs and face, feeling sick, vomiting and diarrhea. The symptoms of dengue fever usually begin between five and eight days after each person be get bitten by an infected mosquito. In Thailand, the annual estimations of dengue fever cases are depend on the season. *Aedes aegypti* is the principal transmitter of Dengue fever in Thailand but it also transmits Chikungunya fever, yellow fever and Filariasis among other diseases. *Aedes aegypti* prefers feed during daylight hours. They adapt very easily to human surroundings and lay their eggs where there is water, including plastic containers, bins, plant pots, etc. Thailand's rainy season, starting from May through September, is also the high risk period for dengue fever, a potentially serious condition is the most prevalent in tropical countries. The moisture content ,temperature ,season and rainfall are influence to the mosquito development. Dengue infection is endemic in Thailand. From the data of Dengue cases in 1992 – 2011, we can see that most dengue patients are occurred in rainy season. We can see in figure 1.

The purpose of this paper is to incorporate this feature into the SEIR model. Models keep track of an individual's infection-age for particular diseases, for instance tubercolosis[3]. Esteva and Vargas developed a model for dengue disease transmission and included the dynamics of the *Aedes aegypti* mosquito into standard SIR (susceptible – infective-recovered) epidemic model [5].

In this paper, we used SIR model for analyzing and finding the method to decrease the outbreak of this disease. We analyze dengue model of seasonality compartment (rainy season, winter season and summer season).

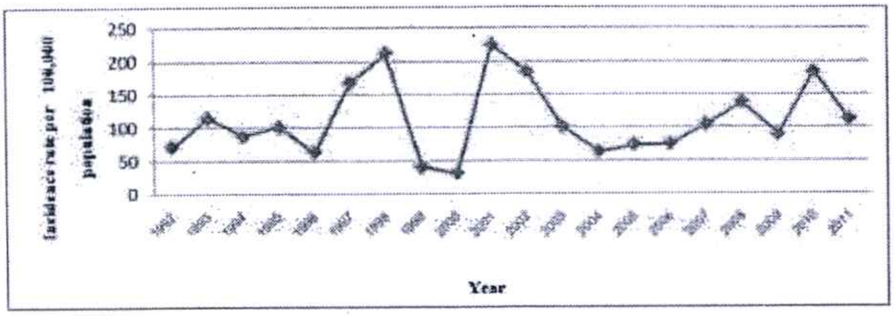

Figure 1: Reported cases of Dengue disease per 100,000 population in Thailand during year 1992 and 2011 .

Methodology

The mathematical modeling for dengue disease describes the relevance of human and mosquito population. In this study, we assume that the total human and mosquito population have. constant sizes. The human population is divided into susceptible, infected and recovered classes for the first model. The mosquito population is divided into susceptible and infected classes because the mosquito never recover from infection. The model considers transmission of dengue virus in human and mosquito population:

The dynamics of human population are given by

$$\frac{d}{dt}S_{hr} = H_r N_{Tr} - \tau_d S_{hr} - \tau_h S_{hr} - \rho \frac{d_{v \to hr}}{N_{Tr} + a} I_{vr} S_{hr} \tag{1}$$

$$\frac{d}{dt}I_{hr} = \rho \frac{d_{v \to hr}}{N_{Tr} + a} I_{vr} S_{hr} - \tau_d I_{hr} - \tau_h I_{hr} - \theta I_{hr} \tag{2}$$

$$\frac{d}{dt}R_{hr} = \theta I_{hr} - \tau_d I_{hr} - \tau_h I_{hr} \tag{3}$$

$$\frac{d}{dt}S_{hw} = H_w N_{Tw} - \tau_d S_{hw} - \tau_h S_{hw} - \rho \frac{d_{v \to hw}}{N_{Tw} + a} I_{vw} S_{hw} \tag{4}$$

$$\frac{d}{dt}I_{hw} = \rho \frac{d_{v \to hw}}{N_{Tw} + a} I_{vw} S_{hw} - \tau_d I_{hw} - \tau_h I_{hw} - \theta I_{hw} \tag{5}$$

$$\frac{d}{dt}R_{hw} = \theta I_{hw} - \tau_d I_{hw} - \tau_h I_{hw} \tag{6}$$

$$\frac{d}{dt}S_{hs} = H_s N_{Ts} - \tau_d S_{hs} - \tau_h S_{hs} - \rho \frac{d_{v \to hs}}{N_{Ts} + a} I_{vs} S_{hs} \tag{7}$$

$$\frac{d}{dt}I_{hs} = \rho \frac{d_{v \to hs}}{N_{Ts} + a} I_{vs} S_{hs} - \tau_d I_{hs} - \tau_h I_{hs} - \theta I_{hs} \tag{8}$$

$$\frac{d}{dt}R_{hs} = \theta I_{hs} - \tau_d I_{hs} - \tau_h I_{hs} \tag{9}$$

We define

S_{hr} is the number of susceptible human population in rainy season,

I_{hr} is the number of infectious human population in rainy season,

R_{hr} is the number of recovered human population in rainy season,

S_{hw} is the number of susceptible human population in winter season,

I_{hw} is the number of infectious human population in winter season,

R_{hw} is the number of recovered human population in winter season,

S_{hs} is the number of susceptible human population in summer season,

I_{hs} is the number of infectious human population in summer season,

R_{hs} is the number of recovered human population in summer season

The dynamics of the mosquito population are given by:

$$\frac{d}{dt}S_{vr} = P_r - \tau_{vr}S_{vr} - \rho\frac{d_{hr\to v}}{N_{Tr}+a}I_{hr}S_{vr} \tag{10}$$

$$\frac{d}{dt}I_{vr} = \rho\frac{d_{hr\to v}}{N_{Tr}+a}I_{hr}S_{vr} - \tau_{vr}I_{vr} \tag{11}$$

$$\frac{d}{dt}S_{vr} = P_w - \tau_{vw}S_{vw} - \rho\frac{d_{hw\to v}}{N_{Tw}+a}I_{vw}S_{vw} \tag{12}$$

$$\frac{d}{dt}I_{vw} = \rho\frac{d_{hw\to v}}{N_{Tw}+a}I_{hw}S_{vw} - \tau_{vw}I_{vw} \tag{13}$$

$$\frac{d}{dt}S_{vs} = P_s - \tau_{vs}S_{vs} - \rho\frac{d_{hs\to v}}{N_{Ts}+a}I_{hs}S_{vs} \tag{14}$$

$$\frac{d}{dt}I_{vs} = \rho\frac{d_{hs\to v}}{N_{Ts}+a}I_{hs}S_{vs} - \tau_{vs}I_{vs} \tag{15}$$

We define

S_{vr} is the number of susceptible mosquito population in rainy season,

I_{vr} is the number of infectious mosquito population in rainy season,

S_{vw} is the number of susceptible mosquito population in winter season,

I_{vw} is the number of infectious mosquito population in winter season,

S_{vs} is the number of susceptible mosquito population in summer season,

I_{vs} is the number of infectious mosquito population in summer season,

where the parameters are defined as follows :

N_{Tr} is the total human population in rainy season,

N_{Tw} is the total human population in winter season,

N_{Ts} is the total human population in summer season,

N_{vr} is the total mosquito population in rainy season,

N_{vw} is the total mosquito population in winter season,

N_{vs} is the total mosquito population in summer season,

τ_h is the natural death rate of human population,

τ_d is the death rate of human population due to the disease,

τ_{vr} is the death rate of mosquito population in rainy season,

τ_{vw} is the death rate of mosquito population in winter season,

τ_{vs} is the death rate of mosquito population in summer season,

H_r is the birth rate of human population in rainy season,

H_w is the birth rate of human population in winter season,

H_s is the birth rate of human population in summer season,

$d_{v\to hr}$ is the transmission probability of dengue disease from mosquito to human in rainy season,

$d_{v\to hw}$ is the transmission probability of dengue disease from mosquito to human in winter season ,

$d_{v\to hs}$ is the transmission probability of dengue disease from mosquito to human in summer season,

$d_{hr\to v}$ is the transmission probability of dengue disease from human to mosquito in rainy season,

$d_{hw\to v}$ is the transmission probability of dengue disease from human to mosquito in winter season,

$d_{hs\to v}$ is the transmission probability of dengue disease from human to mosquito in summer season,

θ is the recovery rate of human population,

ρ is the biting rate of mosquito population,

a is the number of other animals available as blood sources.

We suppose that $N_{hr}=S_{hr}+I_{hr}+R_{hr}$, $N_{hw}=S_{hw}+I_{hw}+R_{hw}$, $N_{hs}=S_{hs}+I_{hs}+R_{hs}$, $N_{vr}=S_{vr}+I_{vr}$, $N_{vw}=S_{vw}+I_{vw}$,and $N_{vs}=S_{vs}+I_{vs}$

we assume the total human and mosquito populations have constant sizes $\frac{d}{dt}N_{hr}=0$, $\frac{d}{dt}N_{vr}=0$ in rainy season , $\frac{d}{dt}N_{hw}=0$, $\frac{d}{dt}N_{vw}=0$ in winter season and $\frac{d}{dt}N_{hs}=0$, $\frac{d}{dt}N_{vs}=0$ in summer season.

$$\bar{S}_{hr}=\frac{S_{hr}}{N_{Tr}},\ \bar{I}_{hr}=\frac{I_{hr}}{N_{Tr}},\ \bar{R}_{hr}=\frac{R_{hr}}{N_{Tr}},\ \bar{S}_{hw}=\frac{S_{hw}}{N_{Tw}},\ \bar{I}_{hw}=\frac{I_{hw}}{N_{Tw}},\ \bar{R}_{hw}=\frac{R_{hw}}{N_{Tw}},$$

$$\bar{S}_{hs}=\frac{S_{hs}}{N_{Ts}},\ \bar{I}_{hs}=\frac{I_{hs}}{N_{Ts}},\ \bar{R}_{hs}=\frac{R_{hs}}{N_{Ts}}$$

$$\bar{S}_{vr}=\frac{S_{vr}}{N_{vs}},\ \bar{I}_{vr}=\frac{I_{vr}}{N_{vr}},\ \bar{S}_{vw}=\frac{S_{vw}}{N_{vw}},\ \bar{I}_{vw}=\frac{I_{vw}}{N_{vw}}\ \bar{S}_{vs}=\frac{S_{vs}}{N_{vs}},\ \bar{I}_{vs}=\frac{I_{vs}}{N_{vs}}$$

The total human and mosquito populations have constant sizes. Thus rates of change for human and mosquito populations equal to zero. Thus , the birth and death rates are equivalent for human populations , the total mosquito equal to $\frac{P_r}{\tau_{vr}}$ in rainy season , the total mosquito equal to $\frac{P_w}{\tau_{vw}}$ in winter season and the total mosquito equal to $\frac{P_s}{\tau_{vs}}$ in summer season.

The reduced equation become :

$$\frac{d}{dt}\bar{S}_{hr}=(\tau_h+\tau_d)-(\tau_d+\tau_h+\rho\frac{d_{v\to hr}}{N_{Tr}+a}\bar{I}_{vr}N_{vr})\bar{S}_{hr} \quad (16)$$

$$\frac{d}{dt}\bar{I}_{hr}=(\rho\frac{d_{v\to hr}}{N_{Tr}+a}\bar{I}_{vr}N_{vr}\bar{S}_{hr})-(\tau_d+\tau_h+\theta)\bar{I}_{hr} \quad (17)$$

$$\frac{d}{dt}\bar{I}_{vr}=\rho\frac{d_{hr\to v}}{N_{Tr}+a}\bar{I}_{hr}N_{Tr}\bar{S}_{vr}-\tau_{vr}\bar{I}_{vr} \quad (18)$$

$$\frac{d}{dt}\bar{S}_{hw}=(\tau_h+\tau_d)-(\tau_d+\tau_h+\rho\frac{d_{v\to hw}}{N_{Tw}+a}\bar{I}_{vw}N_{vw})\bar{S}_{hw} \quad (19)$$

$$\frac{d}{dt}\bar{I}_{hw}=\rho\frac{d_{v\to hw}}{N_{Tw}+a}\bar{I}_{vw}N_{Tw}\bar{S}_{hw}-(\tau_d+\tau_h+\theta)\bar{I}_{hw} \quad (20)$$

$$\frac{d}{dt}\bar{I}_{vw}=\rho\frac{d_{hw\to v}}{N_{Tw}+a}\bar{I}_{hw}N_{Tw}\bar{S}_{vw}-\tau_{vw}\bar{I}_{vw} \quad (21)$$

$$\frac{d}{dt}\bar{S}_{hs} = (\tau_h + \tau_d) - (\tau_d + \tau_h + \rho \frac{d_{v \to hs}}{N_{Ts} + a} \bar{I}_{vs} N_{vs}) \bar{S}_{hs} \quad (22)$$

$$\frac{d}{dt}\bar{I}_{hs} = \rho \frac{d_{v \to hs}}{N_{Ts} + a} \bar{I}_{vs} N_{vs} \bar{S}_{hs} - (\tau_d + \tau_h + \theta) \bar{I}_{hs} \quad (23)$$

$$\frac{d}{dt}\bar{I}_{vs} = \rho \frac{d_{hs \to v}}{N_{Ts} + a} \bar{I}_{hs} N_{Ts} \bar{S}_{vs} - \tau_{vs} \bar{I}_{vs} \quad (24)$$

R_{hr} , R_{hw} , R_{hs} and S_{vr} , S_{vw} , S_{vs} can be obtained from conditions $S_{hr} + I_{hr} + R_{hr} = 1$, $S_{hw} + I_{hw} + R_{hw} = 1$, $S_{hs} + I_{hs} + R_{hs} = 1$, $S_{vr} + I_{vr} = 1$, $S_{vw} + I_{vw} = 1$, and $S_{vs} + I_{vs} = 1$.

Analysis of the Mathematical Model

A. Equilibrium Points

The equilibrium points are found by setting the right hand side of (16) – (24) equal to zero. This gives

1)The disease free equilibrium point in rainy season, in winter season and in summer season , respectively:

$$E_{r0} = (1,0,0)$$

$$E_{w0} = (1,0,0)$$

$$E_{s0} = (1,0,0)$$

2)The endemic disease equilibrium point in rainy season, in winter season and in summer season.

$$E_{r1} = (S^*_{hr}, I^*_{hr}, I^*_{vr}),\ E_{w1} = (S^*_{hw}, I^*_{hw}, I^*_{vw}) \text{ and } E_{s1} = (S^*_{hs}, I^*_{hs}, I^*_{vs})$$

where

$$S^*_{hr} = \frac{1}{1 + \frac{I^*_{hr} N_{Tr} N_{vr} \rho^2 d_{hr \to v} d_{v \to hr}}{(N_{Tr} + a)(\tau_h + \tau_d)(I^*_{hr} N_{Tr} \rho d_{hr \to v} + (N_{Tr} + a)\tau_{vr})}},$$

$$I^*_{hr} = \frac{(\tau_h + \tau_d)(N_{Tr} N_{vr} \rho d_{hr \to v} \rho d_{v \to hr} - (N_{Tr} + a)^2 (\theta + \tau_h + \tau_d)\tau_{vr})}{N_{Tr} \rho d_{hr \to v} (\theta + \tau_h + \tau_d) N_{Tr} \rho d_{v \to hr} + (N_{Tr} + a)(\tau_h + \tau_d))},\quad I^*_{vr} = \frac{1}{1 + \frac{(N_{Tr} + a)\tau_{vr}}{I^*_{hr} N_{Tr} \rho d_{hr \to v}}}$$

$$S^*_{hw} = \frac{1}{1 + \frac{I^*_{hw} N_{Tw} N_{vw} \rho^2 d_{hw \to v} d_{v \to hw}}{(N_{Tw} + a)(\tau_h + \tau_d)(I^*_{hw} N_{Tw} \rho d_{hw \to v} + (N_{Tw} + a)\tau_{vw})}},$$

$$I^*_{hw} = \frac{(\tau_h + \tau_d)(N_{Tw} N_{vw} \rho d_{hw \to v} \rho d_{v \to hw} - (N_{Tw} + a)^2 (\theta + \tau_h + \tau_d)\tau_{vw})}{N_{Tw} \rho d_{hw \to v} (\theta + \tau_h + \tau_d) N_{Tw} \rho d_{v \to hw} + (N_{Tw} + a)(\tau_h + \tau_d))},\quad I^*_{vw} = \frac{1}{1 + \frac{(N_{Tw} + a)\tau_{vw}}{I^*_{hw} N_{Tw} \rho d_{hw \to v}}}$$

$$S^*_{hs} = \frac{1}{1 + \frac{I^*_{hs} N_{Ts} N_{vs} \rho^2 d_{hs \to v} d_{v \to hs}}{(N_{Ts} + a)(\tau_h + \tau_d)(I^*_{hs} N_{Ts} \rho d_{hs \to v} + (N_{Ts} + a)\tau_{vs})}},$$

$$I_{hs}^{*}=\frac{(\tau_h+\tau_d)(N_{Ts}N_{vs}\rho d_{hs\to v}\rho d_{v\to hs}-(N_{Ts}+a)^2(\theta+\tau_h+\tau_d)\tau_{vs})}{N_{Ts}\rho d_{hs\to v}(\theta+\tau_h+\tau_d)N_{Ts}\rho d_{v\to hs}+(N_{Ts}+a)(\tau_h+\tau_d))},\ I_{vs}^{*}=\frac{1}{1+\dfrac{(N_{Ts}+a)\tau_{vs}}{I_{hs}^{*}N_{Ts}\rho d_{hs\to v}}}$$

B. Local Stability

The local stability of each equilibrium point is determined from linearizing equation (16) – (24) about equilibrium point examining the eigenvalues of the resulting Jacobian matrix. We now consider the eigenvalues of the Jacobian matrix at each equilibrium point.

From equation (16)-(24) ,The Jacobian matrix evaluated at E_{r0}, E_{w0}, E_{s0} is the 3x3 matrix given by

$$\begin{bmatrix} -(\tau_d+\tau_h) & 0 & -(\rho\frac{d_{v\to hr}}{N_{Tr}+a}N_{vr}) \\ 0 & -(\tau_d+\tau_h+\theta) & (\rho\frac{d_{v\to hr}}{N_{Tr}+a}N_{vr}) \\ 0 & \rho\frac{d_{hr\to v}}{N_{Tr}+a}N_{Tr} & -\tau_{vr} \end{bmatrix} \text{ for rainy season,}$$

$$\begin{bmatrix} -(\tau_d+\tau_h) & 0 & -(\rho\frac{d_{v\to hw}}{N_{Tw}+a}N_{vw}) \\ 0 & -(\tau_d+\tau_h+\theta) & (\rho\frac{d_{v\to hw}}{N_{Tw}+a}N_{vw}) \\ 0 & \rho\frac{d_{hw\to v}}{N_{Tw}+a}N_{Tw} & -\tau_{vw} \end{bmatrix} \text{ for winter season,}$$

$$\begin{bmatrix} -(\tau_d+\tau_h) & 0 & -(\rho\frac{d_{v\to hs}}{N_{Ts}+a}N_{vs}) \\ 0 & -(\tau_d+\tau_h+\theta) & (\rho\frac{d_{v\to hs}}{N_{Tr}+a}N_{vs}) \\ 0 & \rho\frac{d_{hs\to v}}{N_{Ts}+a}N_{Ts} & -\tau_{vs} \end{bmatrix} \text{ for summer season.}$$

The eigenvalues are the solution of the characteristic equation , $\det(J-\lambda I_3)=0$. To evaluate the determinant, where J is the Jacobian matrix evaluated at the equilibrium point . I_3 is the identity matrix.

$(-\lambda-\tau_h-\tau_d)(\lambda^2+r_1\lambda+r_2) = 0$, $(-\lambda-\tau_h-\tau_d)(\lambda^2+w_1\lambda+w_2) = 0$, $(-\lambda-\tau_h-\tau_d)(\lambda^2+s_1\lambda+s_2) = 0$

Where

$$r_1=\theta+\tau_h+\tau_d+\tau_{vr}\ ,\ r_2=-\frac{\rho d_{hr\to v}d_{v\to hr}N_{Tr}N_{vr}}{(N_{Tr}+a)^2}+(\theta+\tau_h+\tau_d)\tau_{vr}$$

$$w_1=\theta+\tau_h+\tau_d+\tau_{wr}\ ,\ w_2=-\frac{\rho d_{hw\to v}d_{v\to hw}N_{Tw}N_{vw}}{(N_{Tw}+a)^2}+(\theta+\tau_h+\tau_d)\tau_{vw}\ ,$$

$$s_1=\theta+\tau_h+\tau_d+\tau_{vs}\ ,\ s_2=-\frac{\rho d_{hs\to v}d_{v\to hs}N_{Ts}N_{vs}}{(N_{Ts}+a)^2}+(\theta+\tau_h+\tau_d)\tau_{vs}$$

From the above characteristic equation , we can see that one of the eigenvalues are $-\tau_h-\tau_d$ in rainy season, $-\tau_h-\tau_d$ in winter season and $-\tau_h-\tau_d$ in summer season.

From $(\lambda^2+r_1\lambda+r_2)$, $(\lambda^2+w_1\lambda+w_2)$ and $(\lambda^2+s_1\lambda+s_2)$ equal to zero.

The two conditions of Routh-Hurwitz criteria for local stability in second order characteristic equation polynomial equation are

1) $r_1>0$, $r_2>0$ in rainy season
2) $w_1>0, w_2>0$ in winter season

3) $s_1 > 0$, $s_2 > 0$ in summer season

After we check the stability of equilibrium point, we can see r_1, r_2, w_1, w_2 and s_1, s_2 are positive when $R_{0r} = \dfrac{(N_{Tr}+a)^2(\theta+\tau_h+\tau_d)\tau_{vr}}{\rho^2 N_{Tr} N_{vr} d_{hr\to v} d_{v\to hr}}$ <1 in rainy season, $R_{0w} = \dfrac{(N_{Tw}+a)^2(\theta+\tau_h+\tau_d)\tau_{vw}}{\rho^2 N_{Tw} N_{vw} d_{hw\to v} d_{v\to hw}}$ <1 in winter season and $R_{0s} = \dfrac{(N_{Ts}+a)^2(\theta+\tau_h+\tau_d)\tau_{vs}}{\rho^2 N_{Ts} N_{vs} d_{hs\to v} d_{v\to hs}}$ <1 in summer season. Moreover, we found that the disease free equilibrium point is local stable.

C. Disease Endemic Equilibrium Point

The stability of the endemic disease equilibrium point, E_{r1}, E_{w1}, E_{s1} is determinant by looking at the eigenvalues of the Jacobian matrix, given follows:

$$\begin{bmatrix} -(\tau_d+\tau_h+\rho\frac{d_{v\to hr}}{N_{Tr}+a}I^*_{vr}N_{vr}) & 0 & -(\rho\frac{d_{v\to hr}}{N_{Tr}+a}N_{vr}S^*_{hr}) \\ (\rho\frac{d_{v\to hr}}{N_{Tr}+a}I^*_{vr}N_{vr}) & -(\tau_d+\tau_h+\theta) & (\rho\frac{d_{v\to hr}}{N_{Tr}+a}N_{vr}S^*_{hr}) \\ 0 & \rho\frac{d_{hr\to v}}{N_{Tr}+a}N_{Tr}S^*_{vr} & -\tau_{vr} \end{bmatrix} \text{ for rainy season,}$$

$$\begin{bmatrix} -(\tau_d+\tau_h+\rho\frac{d_{v\to hw}}{N_{Tw}+a}I^*_{vw}N_{vw}) & 0 & -(\rho\frac{d_{v\to hw}}{N_{Tw}+a}N_{vw}S^*_{hw}) \\ (\rho\frac{d_{v\to hw}}{N_{Tw}+a}I^*_{vw}N_{vw}) & -(\tau_d+\tau_h+\theta) & (\rho\frac{d_{v\to hw}}{N_{Tw}+a}N_{vw}S^*_{hw}) \\ 0 & \rho\frac{d_{hw\to v}}{N_{Tw}+a}N_{Tw}S^*_{vw} & -\tau_{vw} \end{bmatrix} \text{ for winter season,}$$

$$\begin{bmatrix} -(\tau_d+\tau_h+\rho\frac{d_{v\to hs}}{N_{Ts}+a}I^*_{vs}N_{vs}) & 0 & -(\rho\frac{d_{v\to hs}}{N_{Ts}+a}N_{vs}S^*_{hs}) \\ (\rho\frac{d_{v\to hs}}{N_{Ts}+a}I^*_{vs}N_{vs}) & -(\tau_d+\tau_h+\theta) & (\rho\frac{d_{v\to hs}}{N_{Tr}+a}N_{vs}S^*_{hs}) \\ 0 & \rho\frac{d_{hs\to v}}{N_{Ts}+a}N_{Ts}S^*_{vs} & -\tau_{vs} \end{bmatrix} \text{ for summer season.}$$

where $S^*_{hr}, I^*_{hr}, I^*_{vr}, S^*_{hw}, I^*_{hw}, I^*_{vw}, S^*_{hs}, I^*_{hs}$ and I^*_{vs} are given by equation the above. The characteristic equation for the Jacobian matrix evaluated at the second equilibrium point, given by (16) – (24), is

$\lambda^3 + r_{11}\lambda^2 + r_{22}\lambda + r_{33} = 0$ in rainy season, $\lambda^3 + w_{11}\lambda^2 + w_{22}\lambda + w_{33} = 0$ in winter season and $\lambda^3 + s_{11}\lambda^2 + s_{22}\lambda + s_{33} = 0$ in summer season.

where

$$r_{11} = (N_{Tr}\rho^2 d_{hr\to v} d_{v\to hr}(\tau_h+\tau_d)(\rho d_{v\to hr}N_{vr} + a(\theta+2\tau_h+2\tau_d) + N_{Tr}(\theta+2\tau_h+2\tau_d)) + (N_{Tr}+a)(-\rho d'_{v\to hr}(N_{Tr}+a) (\tau_h+\tau_d)(\theta+\tau_h+\tau_d) + \rho d_{v\to hr}(a(\theta+\tau_h+\tau_d)(\theta+2\tau_h+2\tau_d) + N_{Tr}(\theta^2 + (\tau_h+\tau_d)(3\theta + d_{hr\to v} + 2\tau_h + 2\tau_d))))\tau_{vr} + (N_{Tr}+a)^2 \rho d_{v\to hr}(\theta+\tau_h+\tau_d)\tau_{vr}^2 / ((N_{Tr}+a)\rho d_{v\to hr}(N_{Tr}\rho d_{hr\to v}(\tau_h+\tau_d) + (N_{Tr}+a)(\theta+\tau_h+\tau_d)\tau_{vr})),$$

$$r_{22} = ((N_{Tr}+a)\rho^2 d_{hr\to v} d_{v\to hr}(N_{Tr}\rho d_{hr\to v}(\tau_h+\tau_d) + (N_{Tr}+a)(\theta+\tau_h+\tau_d)\tau_{vr})\,((\tau_h+\tau_d)(\theta+\tau_h+\tau_d) + (\theta+2\tau_h+2\tau_d)\tau_{vr}) - \rho d_{v\to hr}(-N_{Tr}N_{vr}\rho^2 d_{hr\to v} d_{v\to hr}(\tau_h+\tau_d)(\theta+\tau_h+\tau_d) + \rho d_{hr\to v}(\tau_h+\tau_d)(a^2(\theta+\tau_h+\tau_d)^2 + N_{Tr}^2(\theta+\tau_h+\tau_d)(\theta+\rho d_{hr\to v}+\tau_h+\tau_d) + N_{Tr}(-N_{vr}\rho^2 d_{hr\to v} d_{v\to hr} + a\rho d_{hr\to v}(\theta+\tau_h+\tau_d) + 2a(\theta+\tau_h+\tau_d)^2))\tau_{vr} + (N_{Tr}+a)^2(\theta+\tau_h+\tau_d)(\rho d_{hr\to v}(\tau_h+\tau_d) + + \rho d_{hr\to v}(\theta+\tau_h+\tau_d))\tau_{vr}^2)) / ((N_{Tr}+a)\rho^2 d_{hr\to v} d_{v\to hr}(N_{Tr}\rho d_{hr\to v}(\tau_h+\tau_d) + (N_{Tr}+a)(\theta+\tau_h+\tau_d)\tau_{vr})),$$

$$r_{33} = -((\tau_h+\tau_d)(\theta+\tau_h+\tau_d)\tau_{vr})\,(N_{Tr}\rho d_{hr\to v}(-(N_{Tr}+a)\rho^2 d_{hr\to v} d_{v\to hr}(\tau_h+\tau_d) + \rho d_{v\to hr}(-N_{vr}\rho^2 d_{hr\to v} d_{v\to hr} + \rho d_{hr\to v}(N_{Tr}+a)(\tau_h+\tau_d))) + (N_{Tr}+a)^2 (\rho d_{v\to hr}(\rho d_{hr\to v} + \rho d_{hr\to v}) - \rho^2 d_{hr\to v} d_{v\to hr})(\theta+\tau_h+\tau_d)\tau_{vr})) / (N_{Tr}+a)\rho^2 d_{hr\to v} d_{v\to hr}(N_{Tr}\rho d_{hr\to v}(\tau_h+\tau_d) + (N_{Tr}+a)(\theta+\tau_h+\tau_d)\tau_{vr})),$$

$$w_{11} = (N_{Tw}\rho^2 d_{hw\to v} d_{v\to hw}(\tau_h+\tau_d)(\rho d_{v\to hw}N_{vw} + a(\theta+2\tau_h+2\tau_d) + N_{Tw}(\theta+2\tau_h+2\tau_d)) + (N_{Tw}+a)(-\rho d_{v\to hw}(N_{Tw}+a)$$
$$(\tau_h+\tau_d)(\theta+\tau_h+\tau_d) + \rho d_{v\to hw}(a(\theta+\tau_h+\tau_d)(\theta+2\tau_h+2\tau_d) + N_{Tw}(\theta^2+(\tau_h+\tau_d)(3\theta+d_{hw\to v}+2\tau_h+2\tau_d))))\tau_{vw} +$$
$$(N_{Tw}+a)^2\rho d_{v\to hw}(\theta+\tau_h+\tau_d)\tau_{vw}^2 / ((N_{Tw}+a)\rho d_{v\to hw}(N_{Tw}\rho d_{hw\to v}(\tau_h+\tau_d) + (N_{Tw}+a)(\theta+\tau_h+\tau_d)\tau_{vw})),$$

$$w_{22} = ((N_{Tw}+a)\rho^2 d_{hw\to v} d_{v\to hw}(N_{Tw}\rho d_{hw\to v}(\tau_h+\tau_d) + (N_{Tw}+a)(\theta+\tau_h+\tau_d)\tau_{vw})((\tau_h+\tau_d)(\theta+\tau_h+\tau_d) + (\theta+2\tau_h+2\tau_d)\tau_{vw}) -$$
$$\rho d_{v\to hw}(-N_{Tw}N_{vw}\rho^2 d_{hw\to v}d_{v\to hw}(\tau_h+\tau_d)(\theta+\tau_h+\tau_d) + \rho d_{hw\to v}(\tau_h+\tau_d)(a^2(\theta+\tau_h+\tau_d)^2 + N_{Tw}^2(\theta+\tau_h+\tau_d)(\theta+\rho d_{hw\to v}+\tau_h+\tau_d)$$
$$+ N_{Tw}(-N_{vw}\rho^2 d_{hw\to v}d_{v\to hw} + a\rho d_{hw\to v}(\theta+\tau_h+\tau_d) + 2a(\theta+\tau_h+\tau_d)^2))\tau_{vw} + (N_{Tw}+a)^2(\theta+\tau_h+\tau_d)(\rho d_{hw\to v}(\tau_h+\tau_d)+$$
$$+ \rho d_{hw\to v}(\theta+\tau_h+\tau_d))\tau_{vw}^2))/((N_{Tw}+a)\rho^2 d_{hw\to v}d_{v\to hw}(N_{Tw}\rho d_{hw\to v}(\tau_h+\tau_d) + (N_{Tw}+a)(\theta+\tau_h+\tau_d)\tau_{vw})),$$

$$w_{33} = -((\tau_h+\tau_d)(\theta+\tau_h+\tau_d)\tau_{vw})(N_{Tw}\rho d_{hw\to v}(-(N_{Tr}+a)\rho^2 d_{hw\to v}d_{v\to hw}(\tau_h+\tau_d) + \rho d_{v\to hw}(-N_{vw}\rho^2 d_{hw\to v}d_{v\to hw} + \rho d_{hw\to v}(N_{Tw}+a)(\tau_h+\tau_d))) + (N_{Tw}+a)^2$$
$$(\rho d_{v\to hw}(\rho d_{hw\to v} + \rho d_{hw\to v}) - \rho^2 d_{hw\to v}d_{v\to hw})(\theta+\tau_h+\tau_d)\tau_{vw}))/(N_{Tw}+a)\rho^2 d_{hw\to v}d_{v\to hw}(N_{Tw}\rho d_{hw\to v}(\tau_h+\tau_d) + (N_{Tw}+a)(\theta+\tau_h+\tau_d)\tau_{vw})),$$

$$s_{11} = (N_{Ts}\rho^2 d_{hs\to v} d_{v\to hs}(\tau_h+\tau_d)(\rho d_{v\to hs}N_{vs} + a(\theta+2\tau_h+2\tau_d) + N_{Ts}(\theta+2\tau_h+2\tau_d)) + (N_{Ts}+a)(-\rho d_{v\to hs}(N_{Ts}+a)$$
$$(\tau_h+\tau_d)(\theta+\tau_h+\tau_d) + \rho d_{v\to hs}(a(\theta+\tau_h+\tau_d)(\theta+2\tau_h+2\tau_d) + N_{Ts}(\theta^2+(\tau_h+\tau_d)(3\theta+d_{hs\to v}+2\tau_h+2\tau_d))))\tau_{vs} +$$
$$(N_{Ts}+a)^2\rho d_{v\to hs}(\theta+\tau_h+\tau_d)\tau_{vs}^2 / ((N_{Ts}+a)\rho d_{v\to hs}(N_{Ts}\rho d_{hs\to v}(\tau_h+\tau_d) + (N_{Ts}+a)(\theta+\tau_h+\tau_d)\tau_{vs})),$$

$$s_{22} = ((N_{Ts}+a)\rho^2 d_{hs\to v} d_{v\to hs}(N_{Ts}\rho d_{hs\to v}(\tau_h+\tau_d) + (N_{Ts}+a)(\theta+\tau_h+\tau_d)\tau_{vs})((\tau_h+\tau_d)(\theta+\tau_h+\tau_d) + (\theta+2\tau_h+2\tau_d)\tau_{vs}) -$$
$$\rho d_{v\to hs}(-N_{Ts}N_{vs}\rho^2 d_{hs\to v}d_{v\to hs}(\tau_h+\tau_d)(\theta+\tau_h+\tau_d) + \rho d_{hs\to v}(\tau_h+\tau_d)(a^2(\theta+\tau_h+\tau_d)^2 + N_{Ts}^2(\theta+\tau_h+\tau_d)(\theta+\rho d_{hs\to v}+\tau_h+\tau_d)$$
$$+ N_{Ts}(-N_{vs}\rho^2 d_{hs\to v}d_{v\to hs} + a\rho d_{hs\to v}(\theta+\tau_h+\tau_d) + 2a(\theta+\tau_h+\tau_d)^2))\tau_{vs} + (N_{Ts}+a)^2(\theta+\tau_h+\tau_d)(\rho d_{hs\to v}(\tau_h+\tau_d)+$$
$$+ \rho d_{hs\to v}(\theta+\tau_h+\tau_d))\tau_{vs}^2))/((N_{Ts}+a)\rho^2 d_{hs\to v}d_{v\to hs}(N_{Ts}\rho d_{hs\to v}(\tau_h+\tau_d) + (N_{Ts}+a)(\theta+\tau_h+\tau_d)\tau_{vs})),$$

$$s_{33} = -((\tau_h+\tau_d)(\theta+\tau_h+\tau_d)\tau_{vs})(N_{Ts}\rho d_{hs\to v}(-(N_{Ts}+a)\rho^2 d_{hs\to v}d_{v\to hs}(\tau_h+\tau_d) + \rho d_{v\to hs}(-N_{vs}\rho^2 d_{hs\to v}d_{v\to hs} + \rho d_{hs\to v}(N_{Ts}+a)(\tau_h+\tau_d))) + (N_{Ts}+a)^2$$
$$(\rho d_{v\to hs}(\rho d_{hs\to v} + \rho d_{hs\to v}) - \rho^2 d_{hs\to v}d_{v\to hs})(\theta+\tau_h+\tau_d)\tau_{vs}))/(N_{Ts}+a)\rho^2 d_{hs\to v}d_{v\to hs}(N_{Ts}\rho d_{hs\to v}(\tau_h+\tau_d) + (N_{Ts}+a)(\theta+\tau_h+\tau_d)\tau_{vs}))$$

By using Routh-Hurwitz criteria, each equilibrium point is local stability when the following conditions are satisfied,

1) r_{11} , w_{11} , $s_{11} > 0$,
2) r_{33} , w_{33} , $s_{33} > 0$
3) $r_{11}r_{22} > r_{33}$, $w_{11}w_{22} > w_{33}$, $s_{11}s_{22} > s_{33}$

We can see r_{11} , w_{11} and s_{11} are always positive. then we consider the next conditions. Consider r_{33} , w_{33} and s_{33} are always positive , and $r_{11}r_{22} - r_{33} > 0$, $w_{11}w_{22} - w_{33} > 0$, $s_{11}s_{22} - s_{33} > 0$ are positive when $R_0 < 1$

Discussion and Conclusion

Mathematical model of dengue disease is analyzed by using the standard dynamical modeling method. We have shown that the outbreak of this disease can be reduced by the basic reproductive number (R_0) and we defined R_0 for each season as follows:

$$R_{0r} = \frac{(N_{Tr}+a)^2(\theta+\tau_h+\tau_d)\tau_{vr}}{\rho^2 N_{Tr}N_{vr}d_{hr\to v}d_{v\to hr}}, \quad R_{0w} = \frac{(N_{Tw}+a)^2(\theta+\tau_h+\tau_d)\tau_{vw}}{\rho^2 N_{Tw}N_{vw}d_{hw\to v}d_{v\to hw}} \text{ and}$$

$$R_{0s} = \frac{(N_{Ts}+a)^2(\theta+\tau_h+\tau_d)\tau_{vs}}{\rho^2 N_{Ts}N_{vs}d_{hs\to v}d_{v\to hs}}$$ for rainy ,winter and summer seasons, respectively.

If R_{0r}, R_{0w} and $R_{0s} < 1$ then the disease-free equilibrium state is local stable. If R_{0r}, R_{0w} and $R_{0s} > 1$ then the endemic equilibrium state is local stable. The results of this study are used for finding the condition of parameters to be the way for reducing the disease outbreak in each season.

Acknowledgment

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Effect of season on the transmission model of dengue disease

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ABSTRACT

The transmission of dengue disease in the mathematical model is analyzed. We used SIRS model to describe the transmission of dengue disease in mosquito due to the different of dengue transmission rate in each season. *Aedes aegypti* and *Aedes albopictus* are primary vectors for the disease. The human population is separated into three population such as human in winter season, human in summer season and vector population. The infection rate of dengue disease depends on difference of the season and the number of the mosquito infected in season. We use standard dynamic analysis method for analyzing mathematical model. The conditions for the disease free equilibrium state and the endemic state are determined from the value of the reproductive number for the disease.

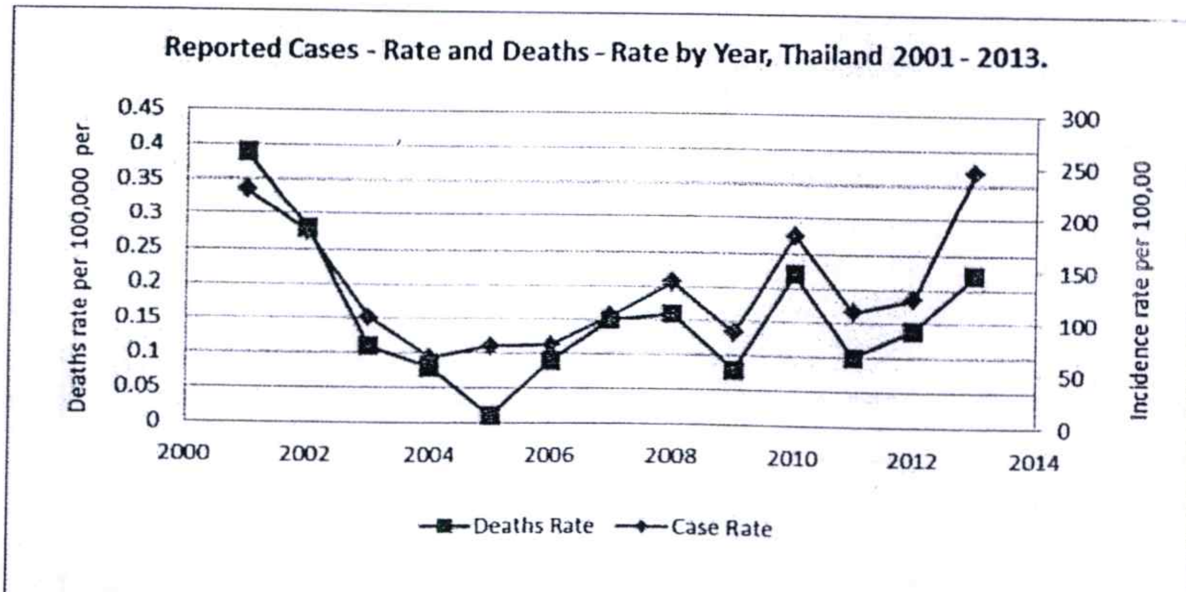
Keywords : Dengue, equilibrium state, mathematical model, SIRS model.

1. Introduction

Dengue fever is transmitted to human by bitting of the female mosquitoes infected with a dengue virus. When the female mosquitoes bites the human with dengue virus in their blood, thus the human are exposed and infected. It can't be spread directly from human to another human. In a human, the virus incubates for 3 to 14 days before symptoms appear, with an average symptom onset at 4 to 7 days. The exactly symptoms depend on age. In older children, teenage and adults, the most common symptoms of dengue disease are a fever that comes on quickly and lasts two to seven days, a headache, muscle and joint pain (dengue fever is also known as "breakbone fever"), a red rash that starts on chest, pain behind eyes and feeling sick and vomiting. Dengue fever (DF), Dengue Haemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS) are three types of dengue disease. There are four serotypes of dengue virus, namely DEN-1, DEN-2, DEN-3 and DEN-4[1,2,3]. Infection from one serotype grants life-long immunity to that strain, and also appears to temporarily grant the host a degree of cross – protection. Dengue fever is caused by a type of virus called a *flavivirus*, which is transmitted by infected female *Aedes* mosquitoes. *Aedes* mosquitoes are commonly found in tropical and subtropical countries. Each year, an estimated 100 million cases of dengue fever occur worldwide. Most of these cases are in tropical areas of the world, with the greatest risk occurring in the Indian subcontinent, Southeast Asia, Southern China, Taiwan, Mexico, Africa and India. There are about 2.5 billion people—nearly 40 percent of the world's population live in areas where the disease can be acquired from local mosquitoes. According to the WHO, many dengue cases were reported in Southeast Asian and South Asian countries during the first to eight months of 2010, with 60,000 cases recorded in Indonesia, 58,000 in Thailand and 27,000 in Sri Lanka[7,8,9,10,11].

In recent decades, mathematical models were developed to investigate the infectious epidemiology. Most of the models incorporate several factors of the disease to predict the possible magnitude of the outbreaks. The moisture content, temperature, season and rainfall are influence to the mosquito development. Dengue infection is endemic in Thailand. From the data of Dengue cases rate and deaths rate in 2001 – 2013, we can see that most dengue patients are occurred in rainy season. We can see as shown in figure 1.

Figure1 : The reports cases rate and deaths rate of dengue disease in Thailand by year during 2001 – 2013.[4]



Esteva and Vargas developed a model for dengue disease transmission and included the dynamics of the *Aedes aegypti* mosquito into standard SIR (susceptible – infective-recovered) epidemic model [5]. This disease is occurred by biting of infected *Aedes aegypti* and *Aedes albopictus* mosquitoes. Dengue outbreak is found during the rainy season. Each Aedes mosquito has the different dengue outbreaks and they are depend on the temperature and areas[3]. Dengue outbreak is found in the rainy, winter and summer seasons. Each season has the different dengue outbreaks and they are depend on the temperature of the environment[6]. In this paper, we develop the transmission of dengue disease by formulating the mathematical models. We used SIRS model for analyzing and finding the method to decrease the outbreak of this disease. We analyze dengue model of seasonality compartment (rainy season, winter season and summer season).

2. Methodology

The model consists of the standard SIRS model, where S, I ,R denote the number of susceptible, infectious, and recovered hosts, respectively. The mathematical modeling for dengue disease describes the relevance of human and mosquito population. In this study, we assume that the total human and mosquito population have constant sizes. Hence, the set of ordinary differential equations (ODEs) representing the *SIRS* model is given by:

The dynamics of human population are given by

$$\frac{d\bar{S}_r}{dt} = RN_T - \tau\frac{b_{v\to hr}}{N_{Tr}+g}\bar{I}_v\bar{S}_r - \eta_d\bar{S}_r + \theta\bar{R}_r \tag{1a}$$

$$\frac{d\bar{I}_r}{dt} = \tau\frac{b_{v\to hr}}{N_{Tr}+g}\bar{I}_v\bar{S}_r - \eta_d\bar{I}_r - \gamma\bar{I}_r \tag{1b}$$

$$\frac{d\bar{R}_r}{dt} = \gamma\bar{I}_r - \eta_d\bar{R}_r - \theta\bar{R}_r \tag{1c}$$

$$\frac{d\bar{S}_w}{dt} = WN_T - \tau\frac{b_{v\to hw}}{N_{Tw}+g}\bar{I}_v\bar{S}_w - \eta_d\bar{S}_w + \theta\bar{R}_w \tag{1d}$$

$$\frac{d\bar{I}_w}{dt} = \tau\frac{b_{v\to hw}}{N_{Tw}+g}\bar{I}_v\bar{S}_w - \eta_d\bar{I}_w - \gamma\bar{I}_w \tag{1e}$$

$$\frac{d\bar{R}_w}{dt} = \gamma\bar{I}_w - \eta_d\bar{R}_w - \theta\bar{R}_w \tag{1f}$$

$$\frac{d\bar{S}_s}{dt} = SN_T - \tau\frac{b_{v\to hs}}{N_{Ts}+g}\bar{I}_v\bar{S}_s - \eta_d\bar{S}_s + \theta\bar{R}_s \tag{1g}$$

$$\frac{d\bar{I}_s}{dt} = \tau\frac{b_{v\to hs}}{N_{Ts}+g}\bar{I}_v\bar{S}_s - \eta_d\bar{I}_s - \gamma\bar{I}_s \tag{1h}$$

$$\frac{d\bar{R}_s}{dt} = \gamma\bar{I}_s - \eta_d\bar{R}_s - \theta\bar{R}_s \tag{1i}$$

The variables are defined as follows: $\bar{S}_r$ is the number of susceptible human population in rainy season, $\bar{I}_r$ is the number of infectious human population in rainy season, $\bar{R}_r$ is the number of recovered human population in rainy season, $\bar{S}_w$ is the number of susceptible human population in winter season, $\bar{I}_w$ is the number of infectious human population in winter season, $\bar{R}_w$ is the number of recovered human population in winter season, $\bar{S}_s$ is the number of susceptible human population in summer season, $\bar{I}_s$ is the number of infectious human population in summer season, $\bar{R}_s$ is the number of recovered human population in summer season.

The dynamics of the mosquito population are given by:

$$\frac{d\bar{S}_v}{dt} = V - \left(\tau\left(\frac{b_{hr\to v}}{N_{Tr}+g}\bar{I}_r\right)+\tau\left(\frac{b_{hw\to v}}{N_{Tw}+g}\bar{I}_w\right)+\tau\left(\frac{b_{hs\to v}}{N_{Ts}+g}\bar{I}_s\right)\right)\bar{S}_v - \eta_d\bar{S}_v \quad (2a)$$

$$\frac{d\bar{I}_v}{dt} = \left(\tau\left(\frac{b_{hr\to v}}{N_{Tr}+g}\bar{I}_r\right)+\tau\left(\frac{b_{hw\to v}}{N_{Tw}+g}\bar{I}_w\right)+\tau\left(\frac{b_{hs\to v}}{N_{Ts}+g}\bar{I}_s\right)\right)\bar{S}_v - \eta_d\bar{I}_v \quad (2b)$$

We define

Table1 : Definitions of variables and parameters for our model.

variable/parameter	definition
$\bar{S}_v$	the number of susceptible mosquito population
$\bar{I}_v$	the number of infectious mosquito population
N_T	the total human population
N_{Tr}	the total human population in rainy season
N_{Tw}	the total human population in winter season
N_{Ts}	the total human population in summer season
N_v	the total mosquito population
η_d	the natural death rate of human population
R	the birth rate of human population in rainy season
W	the birth rate of human population in winter season
S	the birth rate of human population in summer season
$b_{v\to hr}$	the transmission probability of dengue disease from mosquito to human in rainy season
$b_{v\to hw}$	the transmission probability of dengue disease from mosquito to human in winter season
$b_{v\to hs}$	the transmission probability of dengue disease from mosquito to human in summer season
$b_{v\to hs}$	the transmission probability of dengue disease from mosquito to human in summer season
$b_{hr\to v}$	the transmission probability of dengue disease from human to mosquito in rainy season
$b_{hw\to v}$	the transmission probability of dengue disease from human to mosquito in winter season
$b_{hs\to v}$	the transmission probability of dengue disease from human to mosquito in summer season
γ	the recovery rate of human population
τ	the biting rate of mosquito population
θ	the infection rate of human population
g	the number of other animals available as blood sources

We suppose that $N_r=\bar{S}_r+\bar{I}_r+\bar{R}_r$, $N_w=\bar{S}_w+\bar{I}_w+\bar{R}_w$, $N_s=\bar{S}_s+\bar{I}_s+\bar{R}_s$ and $N_v=\bar{S}_v+\bar{I}_v$.

We assume the total human and mosquito populations have constant sizes $\frac{dN_{Tr}}{dt}=0$ in rainy season, $\frac{dN_{Tw}}{dt}=0$ in winter season, $\frac{dN_{Ts}}{dt}=0$ in summer season and $\frac{dN_v}{dt}=0$.

$S_r=\frac{\bar{S}_r}{N_{Tr}}$, $I_r=\frac{\bar{I}_r}{N_{Tr}}$, $R_r=\frac{\bar{R}_r}{N_{Tr}}$, $S_w=\frac{\bar{S}_w}{N_{Tw}}$, $I_w=\frac{\bar{I}_w}{N_{Tw}}$, $R_w=\frac{\bar{R}_w}{N_{Tw}}$, $S_s=\frac{\bar{S}_s}{N_{Ts}}$, $I_s=\frac{\bar{I}_s}{N_{Ts}}$, $R_s=\frac{\bar{R}_s}{N_{Ts}}$,

$S_v=\frac{\bar{S}_v}{N_v}$, $I_v=\frac{\bar{I}_v}{N_{vr}}$

The total human and mosquito populations have constant sizes. Thus rates of change for human and mosquito populations equal to zero. Thus, the birth and death rates are equivalent for human populations , the total mosquito equal to $\frac{V}{\eta_v}$.

The reduced equations become:

$$\frac{dS_r}{dt} = R - \tau\frac{b_{v\to hr}}{N_{Tr}+g} I_v \left(\frac{V}{\eta_v}\right) S_r - \eta_d S_r + \theta(1 - S_r - I_r) \tag{3a}$$

$$\frac{dI_r}{dt} = \tau\frac{b_{v\to hr}}{N_{Tr}+g} I_v \left(\frac{V}{\eta_v}\right) S_r - \eta_d I_r - \gamma I_r \tag{3b}$$

$$\frac{dS_w}{dt} = W - \tau\frac{b_{v\to hw}}{N_{Tw}+g} I_v \left(\frac{V}{\eta_v}\right) S_w - \eta_d S_w + \theta(1 - S_w - I_w) \tag{3c}$$

$$\frac{dI_w}{dt} = \tau\frac{b_{v\to hw}}{N_{Tw}+g} I_v \left(\frac{V}{\eta_v}\right) S_w - \eta_d I_w - \gamma I_w \tag{3d}$$

$$\frac{dS_s}{dt} = S - \tau\frac{b_{v\to hs}}{N_{Ts}+g} I_v \left(\frac{V}{\eta_v}\right) S_s - \eta_d S_s + \theta(1 - S_s - I_s) \tag{3e}$$

$$\frac{dI_s}{dt} = \tau\frac{b_{v\to hs}}{N_{Ts}+g} I_v \left(\frac{V}{\eta_v}\right) S_s - \eta_d I_s - \gamma I_s \tag{3f}$$

$$\frac{dI_v}{dt} = \left(\tau\left(\frac{b_{hr\to v}}{N_{Tr}+g} I_r N_{Tr}\right) + \tau\left(\frac{b_{hw\to v}}{N_{Tw}+g} I_w N_{Tw}\right) + \tau\left(\frac{b_{hs\to v}}{N_{Ts}+g} I_s N_{Ts}\right)\right) S_v - \eta_d I_v \tag{3g}$$

from conditions $S_r + I_r + R_r = 1$, $S_w + I_w + R_w = 1$, $S_s + I_s + R_s = 1$ and $S_v + I_v = 1$.

3. Analysis of the Mathematical Model

From the above equations, we find two equilibrium states; the disease free equilibrium state $A_0 = (1,0,1,0,1,0,0)$ and the endemic disease equilibrium state $A_1 = (S'_r, I'_r, S'_w, I'_w, S'_s, I'_s, I'_v)$ where

$$S'_r = \frac{(\gamma+\eta_d)\eta_v(R+\theta)}{(\gamma+\eta_d)\eta_v(\eta_d+\theta) + I'_v V(\gamma+\eta_d+\theta)\tau\varpi_1}$$

$$I'_r = \frac{I'_v V(R+\theta)\tau\varpi_1}{(\gamma+\eta_d)\eta_v(\eta_d+\theta) + I'_v V(\gamma+\eta_d+\theta)\tau\varpi_1}$$

$$S'_w = \frac{(\gamma+\eta_d)\eta_v(W+\theta)}{(\gamma+\eta_d)\eta_v(\eta_d+\theta) + I'_v V(\gamma+\eta_d+\theta)\tau\varpi_2}$$

$$I'_w = \frac{I'_v V(W+\theta)\tau\varpi_2}{(\gamma+\eta_d)\eta_v(\eta_d+\theta) + I'_v V(\gamma+\eta_d+\theta)\tau\varpi_2}$$

$$S'_s = \frac{(\gamma+\eta_d)\eta_v(S+\theta)}{(\gamma+\eta_d)\eta_v(\eta_d+\theta) + I'_v V(\gamma+\eta_d+\theta)\tau\varpi_3}$$

$$I'_s = \frac{I'_v V(R+\theta)\tau\varpi_3}{(\gamma+\eta_d)\eta_v(\eta_d+\theta) + I'_v V(\gamma+\eta_d+\theta)\tau\varpi_3}$$

and

$$I'_v = 1 - \frac{\eta_v}{\eta_v I'_r N_{Tr}\eta_1\tau + \eta_v I'_w N_{Tw}\eta_2\tau + \eta_v I'_s N_{Ts}\eta_3\tau}$$

with

$$\varpi_1 = \frac{b_{v\to hr}}{N_{Tr}+g}, \qquad \varpi_2 = \frac{b_{v\to hw}}{N_{Tw}+g}, \qquad \varpi_3 = \frac{b_{v\to hs}}{N_{Ts}+g},$$

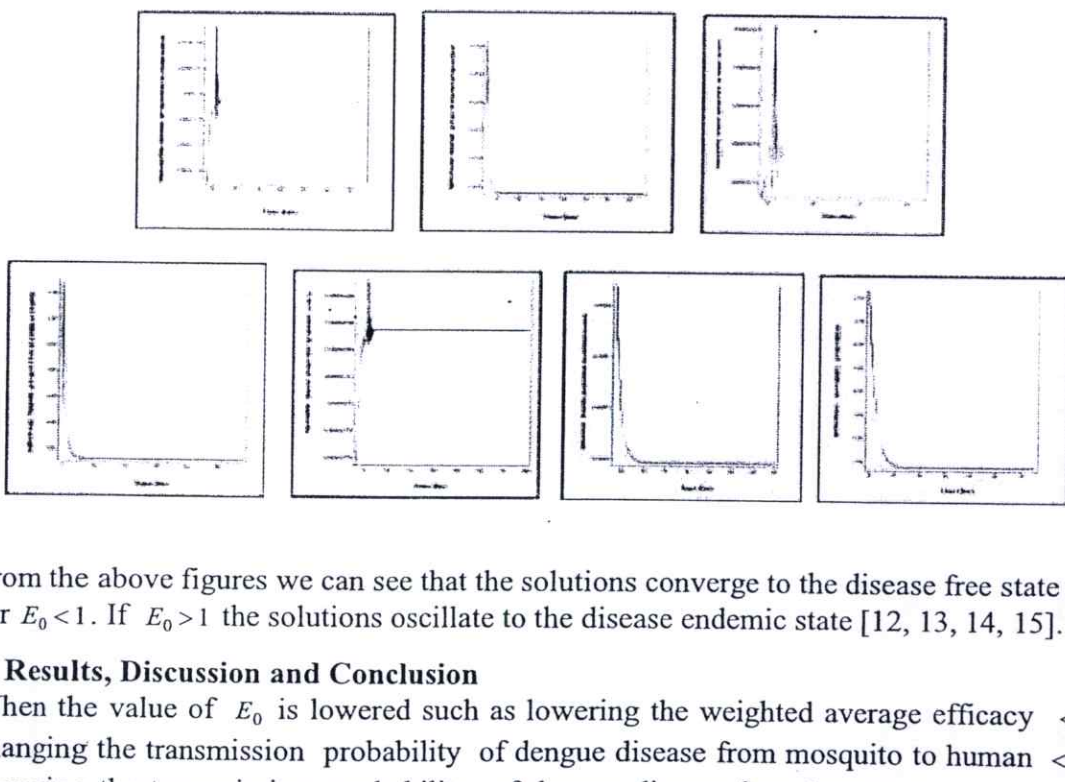
$$\eta_1 = \frac{b_{hr\to v}}{N_{Tr}+g}, \qquad \eta_2 = \frac{b_{hw\to v}}{N_{Tw}+g}, \qquad \eta_3 = \frac{b_{hs\to v}}{N_{Ts}+g}$$

The reproductive number, E_0 is defined by $E_0 = \frac{\sum_{i=1}^{\alpha} \eta_i \varpi_i N_{T_i}}{N_T}$. The parameters are defined as follows: $\eta_d = 1/(365*74.61)$, $\gamma = 1/4.5$, $\theta = 1/((180+365)/2)$. WE suppose that $\gamma_1 = <N_T><\eta_i><\varpi_i>\tau^2$. Thus, we obtain $E_0 = \frac{3.961x10^{25}\gamma_1}{8.8083x10^{22}}$. $<N_T>$ the weighted average defined as $<N_T> = N_{Tr} + N_{Tw} + N_{Ts}$, $<\eta_i>$ when $i=1,2,3$ and $<\varpi_i>$ when $i=1,2,3$. .Which is η_i the efficiency of the transmission of the dengue virus to the mosquito from human in each season (rainy, winter, summer).

4. Numerical Simulation

In this paper, we are interested in transmission of dengue disease with the season. The values of the parameter used in this study are as follows: $\eta_d = 1/(365*74.61)$ per day corresponds to a life expectancy of 74.61 years in human. The other parameters are arbitrarily chosen. We presented the numerical solutions of (3a) – (3g) for endemic equilibrium state on the follows figures.

Figure 2 : Numerical solutions of (3a)-(3g), demonstrate the times series of each human population, for $E_0 > 1$ in rainy season, in winter and in summer. $N_{tr} = 190{,}000$, $N_{tw} = 95{,}000$, $N_{ts} = 30{,}000$, $N_v = 20{,}000$, $\eta_d = 1/(365*74.61)$, $\gamma = 1/4.5$, $\tau = 35$, $\eta_v = 1/4$, $\theta = 1/((180+365)/2)$, $g = 3{,}800$ and $R_0 = 2.13674$.

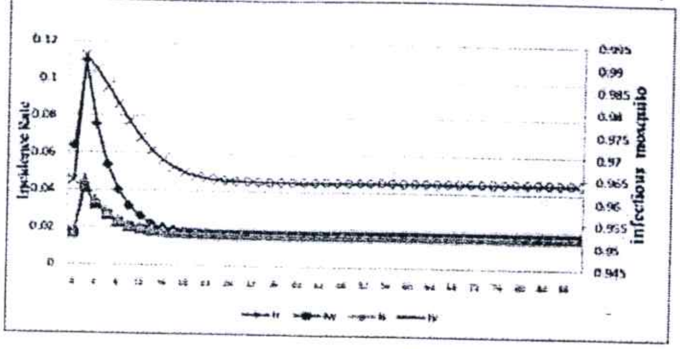


From the above figures we can see that the solutions converge to the disease free state for $E_0 < 1$. If $E_0 > 1$ the solutions oscillate to the disease endemic state [12, 13, 14, 15].

5. Results, Discussion and Conclusion

When the value of E_0 is lowered such as lowering the weighted average efficacy $<\varpi_i>$or changing the transmission probability of dengue disease from mosquito to human $<\eta_i>$ or changing the transmission probability of dengue disease from human to mosquito in the difference season. The incidence rate at the equilibrium state A_0 , can be determined by setting the time rate of change of the different season (by using equations 3a – 3g) and the equivalent equations for the infected and recovered to zero.

Figure 3 : The value of solutions of each human population, for in infective rainy season, winter season and summer season. $\eta_d = 1/(365*74.61)$, $\gamma = 1/4.5$, $\eta_v = 1/4$, $\theta = 1/((180+365)/2)$.



From fig.3, the variables are $\bar{I}_r$, $\bar{I}_w$, $\bar{I}_s$ and $\bar{I}_v$. The number of infectious mosquito population affect the value of $\bar{I}_r$, $\bar{I}_w$ and $\bar{I}_s$. When are the number of infectious mosquito population increase will do the number of infectious population of different season.

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Analyzing of model for Dengue with its characteristics

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ABSTRACT

Model of dengue epidemics are parametrized on disease incidence data and therefore high – quality data are essential. Dengue disease is occurred by biting of the infected *Aedes* mosquitoes. When infected mosquito bites the human, thus the human are exposed and infected. Dengue fever (DF), Dengue Haemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS) are three types of dengue disease. dengue virus is separated into DEN-1, DEN-2, DEN-3 and DEN-4. The Symptom of each dengue case is depending on type of dengue disease (DF,DHF,DSS) that each person is infected. In this model, we studied the yearly distribution of dengue cases between 2000 and 2012.The curves are fitted with data. Mathematical model is formulated by type of dengue disease. Numerical simulations of the model are shown by solving a system of differential equations. It shows that the local stability of endemic and disease free equilibrium states are depend on the basic reproductive rate of the disease.

Keywords; DF,DHF,DSS, transmission model, SIR model, *Aedes aegypti* .

1. Introduction

Transmission of dengue discase is occurred by biting of the female *Aedes* mosquito. The development of virus and mosquito are affected by the climatic factors. When infected mosquito bites the human, thus the human are exposed and then infected. Dengue fever (DF), Dengue Haemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS) are three types of dengue disease. There are four serotypes of dengue virus, namely DEN-1, DEN-2, DEN-3 and DEN-4. Infection from one serotype gives life-long immunity to that strain, and also appears temporarily immune with the other strains. Dengue fever is caused by a type of virus called a *flavivirus*, which is transmitted by infected female *Aedes* mosquitoes. We can catch the virus if we get be bitten by an infected mosquito. Mosquito become infected when it bites an infected person and pass on the virus for the rest of its life. *Aedes aegypti* mosquitoes carry the virus that causes dengue disease. There are 50 million cases a year, including 500,000 serious cases requiring hospitalization[1,2,3,4].

Dengue fever (DF), a viral mosquito – borne infection is a major international public health concern with about 3 billion people at risk of acquiring the infection [5]. It is estimated that every year, there are 70–500 million dengue infections, 36 million cases of DF and 2.1 million cases of DHF/DSS. There are more than 20,000 deaths per year [5,6]. Infection by dengue virus causes a wide range of clinical manifestations and its classification into DF and DHF/DSS are given according to World Health Organization (WHO) guidelines. DF is an acute febrile viral disease frequently presenting with headaches, bone or joint and muscular pains, rash and leukopenia as symptoms. DHF is characterized by four major clinical manifestations: high fever, haemorrhagic phenomena, often with hepatomegaly and, in severe cases, signs of circulatory failure. Infected patients may develop hypovolaemic shock resulting from plasma leakage. This is designated DSS and can be fatal [7].If DF cases are often benign or asymptomatic, DHF cases may evolve towards a group of symptoms with haemorrhagic fever leading to shock or DSS. In Thailand, the annual estimations of dengue fever cases are depend on the season. *Aedes aegypti* is the principal transmitter of Dengue fever in Thailand but it also transmits Chikungunya fever, yellow fever and Filariasis among other diseases. *Aedes aegypti* prefers feed during daylight hours. They adapt very easily to human surroundings and lay their eggs where there is water, including plastic containers, bins, plant pots, etc.This disease is occurred by biting of infected *Aedes aegypti* and *Aedes albopictus* mosquitoes. Dengue outbreak is found during the rainy season. Each *Aedes* mosquito has the different dengue outbreaks and they are depend on the temperature and areas [7]. The symptom of each dengue patient is depend on the type of dengue disease that each person is infected[8]. Dengue infection is endemic in Thailand. From the data of Dengue cases in 2000 – 2012, we can see that most dengue patients are occurred in rainy season [9]. Models keep track of an individual's infection-age for particular diseases, for instance tubercolosis[10]. Esteva and Vargas developed a model for dengue disease transmission and included the dynamics of the *Aedes aegypti* mosquito into standard SIR (susceptible – infective-recovered) epidemic model [11] and each season has the different dengue outbreaks and they are depend on the temperature of the environment. The standard dynamical modeling method is used in this study. The SEIR (S = susceptible , E= exposed , I = infected and R = recovered) model is used[12].

In this paper, we study the yearly distribution of dengue cases (DF,DHF,DSS) in Thailand (2000 -2012) by finding the analytical approximated equations. We used SIR model for analyzing and finding the method to decrease the outbreak of this disease.

2. Methodology

We analyze the data of dengue cases in Thailand [13] , between 1992 and 2012 by fitting curves.

We denote x_i as the i[th] year and $f(x_i)$ as the number of dengue cases (DF, DHF,DSS) of i[th] year .Here, we use the polynomial curve fitting to find the best analytical approximated equations.

Consider the general form of Polynomial order j

$$f(x_i) = a + b_1x_i + b_2x_i^2 + b_3x_i^3 + ... + b_jx_i^j = a + \sum_{k=1}^{j} b_kx_i^k \quad (1)$$

where a, $b_1, b_2, \ldots, b_j$ are some coefficients; i = 1,2,3,…,n and n is the total data sets. Error-Least squares approach is the method to find the general form of the Error -Least squares approach. We define

$$err = \sum_{i=1}^{n} (d_i)^2 \quad (2)$$

where $d_i = y_i - f(x_i)$; i = 1,2,3,…,n; y_i is the real data of dengue cases in each year and $f(x_i)$ is the approximated number of dengue cases from fitting curves.

Thus,

$$err = \sum_{i=1}^{n} (y_i - f(x_i))^2 \quad (3)$$

then we obtain;

$$err = \sum_{i=1}^{n} (y_i - (a + \sum_{k=1}^{j} b_kx_i^k))^2 \quad (4)$$

Coefficient of determination is the proportion of variability in a data set. The value of R^2 is always positive and it is between 0 and 1. If this value is closed to 1, then the model is appropriated. We define

$$R^2 = \frac{\sum_{i=1}^{n} (f(x_i) - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \quad (5)$$

where $\bar{y}$ is the average of real data. From the data of dengue cases (DF, DHF,DSS), we found the polynomial equations and coefficient of determination (R^2) as follows:

Table 1 : Polynomial equations and R^2 of DF cases by year (2000 – 2012)

Polynomial equations	**R^2**
$0.2497x^2 - 0.2908x + 27.197$	0.3871
$0.0033x^3 + 0.1797x^2 + 0.1159x + 26.637$	0.3872
$-0.0776x^4 + 2.1759x^3 - 19.895x^2 + 68.243x - 36.679$	0.6209
$0.0208x^5 - 0..8073x^4 + 11.453x^3 - 71.702x^2 + 188.68x + 121.74$	0.7743
$-0.0011.x^6 + 0.068x^5 - 1.15714x^4 + 17.452x^3 - 95.067x^2 + 229.68x - 145.46$	0.778

Table 2 : Polynomial equations and R^2 of DHF cases by year (2000 – 2012)

Polynomial equations	**R^2**
$0.468x^2 - 9.0354x + 105.69$	0.0834
$0.1337x^3 - 2.3393x^2 + 7.2739x + 83.235$	0.1031
$-0.1925x^4 + 5.5232x^3 - 52.138x^2 + 176.27x - 73.831$	0.4993
$0.056x^5 - 2.1508x^4 + 30.422x^3 - 191.18x^2 + 499.5x + 302.12$	0.8036
$-0.0115x^6 + 0.5402x^5 - 10.001x^4 + 92.048x^3 - 431.21x^2 + 920.72x + 545.86$	0.9117

Table 3 : Polynomial equations and R^2 of DSS cases by year (2000 – 2012)

Polynomial equations	R^2
$0.0213x^2 - 0.3056x + 3.1742$	0.0402
$0.0032x^3 - 0.046x^2 + 0.0851x + 2.6362$	0.0495
$-0.0077x^4 + 0.22x^3 - 2.0495x^2 + 6.8846x - 3.6831$	0.5755
$0.0013x^5 - 0.053x^4 + 0.7956x^3 - 5.2638x^2 + 14.357x - 8.9605$	0.7089
$-0.0002x^6 + 0.0106x^5 - 0.2032x^4 + 1.9749x^3 - 9.8569x^2 + 22.417x - 13.625$	0.7414

Table 4 : Polynomial equations and R^2 of DF deaths by year (2000 – 2012)

Polynomial equations	R^2
$-0.000004x^2 + 0.0006x - 0.0013$	0.0013
$-0.0000008x^3 + 0.0002x^2 - 0.0006x 0.0004$	0.4
$-0.0000002x^4 - 0.000006x^3 + 0.0007x^2 - 0.0025x + 0.0023$	0.4712
$0.00000009x^5 - 0.000003x^4 + 0.0004x^3 - 0.002x^2 + 0.0041x - 0.0026$	0.6631
$-0.000000002x^6 + 0.0000002x^5 - 0.000005x^4 + 0.0005x^3 - 0.0026x^2 + 0.0052x - 0.0032$	0.6642

Table 5 : Polynomial equations and R^2 of DHF deaths by year (2000 – 2012)

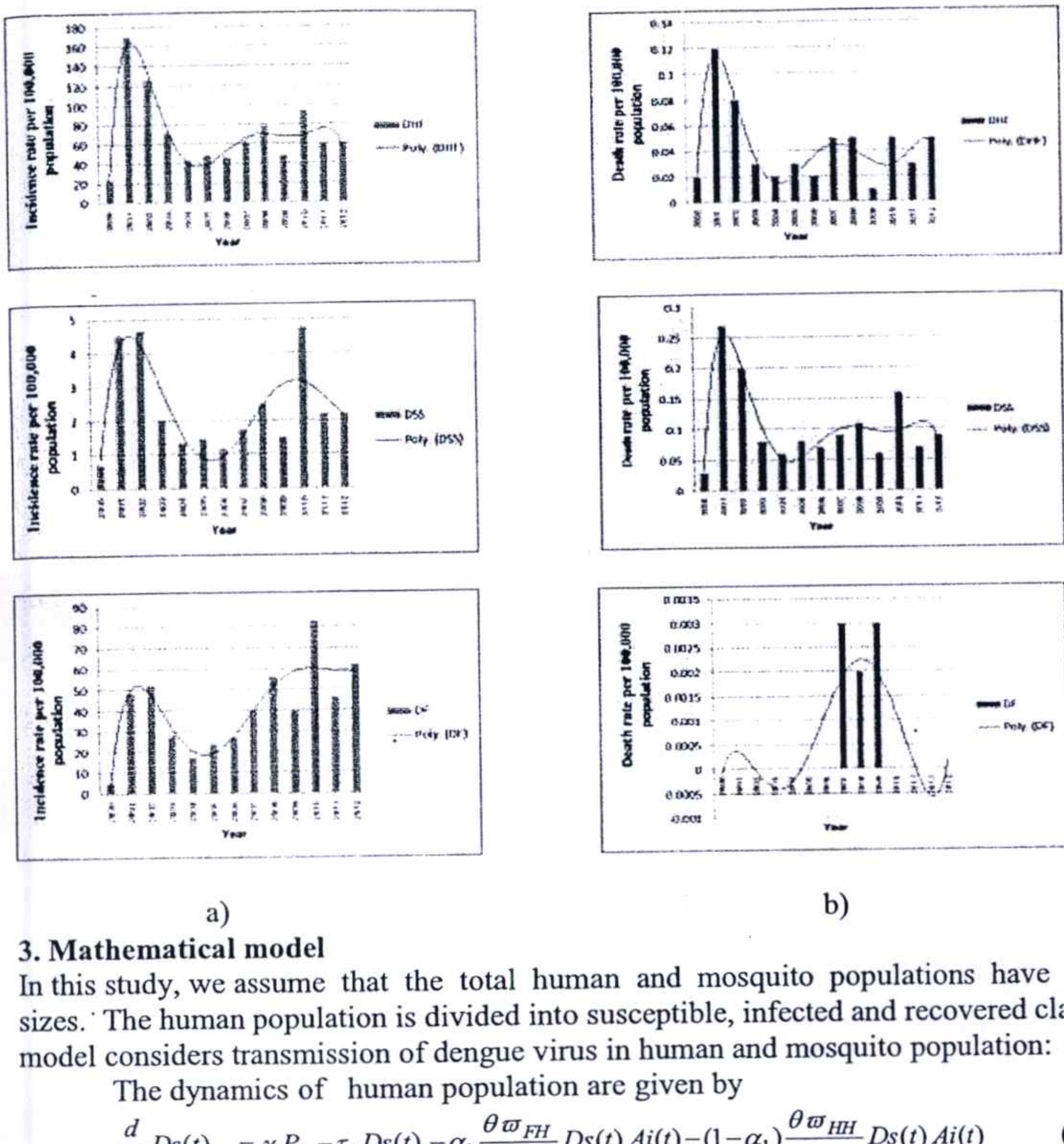
Polynomial equations	R^2
$0.0006x^2 - 0.0106x + 0.0788$	0.1412
$0.0001x^3 - 0.0017x^2 + 0.0029x + 0.0602$	0.1649
$-0.000009x^4 + 0.0025x^3 - 0.024x^2 + 0.0784x - 0.01$	0.3036
$0.000005x^5 - 0.0018x^4 + 0.024x^3 - 0.1441x^2 + 0.3578x - 0.2073$	0.7019
$-0.000001x^6 + 0.0005x^5 - 0.0088x^4 + 0.0798x^3 - 0.3595x^2 + 0.7356x - 0.4259$	0.8544

Table 6 : Polynomial equations and R^2 of DSS deaths by year (2000 – 2012)

Polynomial equations	R^2
$0.0008x^2 - 0.0147x + 0.1606$	0.0796
$0.0002x^3 - 0.0033x^2 + 0.0091x + 0.1278$	0.0944
$-0.0003x^4 + 0.0084x^3 - 0.0788x^2 + 0.2653x - 0.1103$	0.415
$0.000009x^5 - 0.0036x^4 + 0.05x^3 - 0.3112x^2 + 0.8056x - 0.4918$	0.7144
$-0.000002x^6 + 0.001x^5 - 0.0176x^4 + 0.1605x^3 - 0..7418x^2 + 1.5612x - 0.9291$	0.837

From the above tables, the appropriated equations are 6 th order polynomial equations because R^2 converges to 1.

Figure 1 : The real data and the corresponding fitted curves of dengue disease (DHF,DSS ,DF) in Thailand (a)Cases and (b) Deaths by Year).



a) b)

3. Mathematical model

In this study, we assume that the total human and mosquito populations have constant sizes. The human population is divided into susceptible, infected and recovered classes. The model considers transmission of dengue virus in human and mosquito population:

The dynamics of human population are given by

$$\frac{d}{dt}Ds(t) = \gamma P_h - \tau_h Ds(t) - \alpha_1 \frac{\theta \varpi_{FH}}{P_h + c} Ds(t)Ai(t) - (1-\alpha_1)\frac{\theta \varpi_{HH}}{P_h + c} Ds(t)Ai(t) \quad (6.1)$$

$$\frac{d}{dt}Fi(t) = \alpha_1 \frac{\theta \varpi_{FH}}{P_h + c} Ds(t)Ai(t) - \tau_h Fi(t) - \delta_1 Fi(t) \quad (6.2)$$

$$\frac{d}{dt}Hi(t) = (1-\alpha_1)\frac{\theta \varpi_{HH}}{P_h + c} Ds(t)Ai(t) - \tau_h Hi(t) - \beta Hi(t) - (1-\beta)Hi(t) \quad (6.3)$$

$$\frac{d}{dt}HFi(t) = \beta Hi(t) - \tau_h HFi(t) - \delta_2 HFi(t) \quad (6.4)$$

$$\frac{d}{dt}Si(t) = (1-\beta)Hi(t) - \tau_d Si(t) - \tau_h Si(t) - \delta_3 Si(t) \quad (6.5)$$

$$\frac{d}{dt}Dr(t) = \delta_1 Fi(t) + \delta_2 Hi(t) + \delta_3 Si(t) - \tau_h Dr(t) \quad (6.6)$$

We define

$Ds(t)$ is the number of susceptible human population at time t,

$Fi(t)$ is the number of DF infectious human at time t,

$Hi(t)$ is the number of infectious human who be suspected with DHF infection,

$HFi(t)$ is the number of DHF infectious human at time t,

$Si(t)$ is the number of DSS infectious human at time t,

$Dr(t)$ is the number of recovered human population at time t.

The dynamics of the mosquito population are given by :

$$\frac{d}{dt}As(t) = P_v - \frac{\theta\varpi_{FV}}{P_h+c}Fi(t)As(t) + \frac{\theta\varpi_{HV}}{P_h+c}Hi(t)As(t) - \tau_v As(t) \quad (7.1)$$

$$\frac{d}{dt}Ai(t) = \frac{\theta\varpi_{FV}}{P_h+c}Fi(t)As(t) + \frac{\theta\varpi_{HV}}{P_h+c}Hi(t)As(t) - \tau_v Ai(t) \quad (7.2)$$

We define

$As(t)$ is the number of susceptible mosquito population at time t,

$Ai(t)$ is the number of infectious mosquito population at time t.

The parameters of our equations are defined as follows:

P_h is the total human population,

P_v is the constant recruitment rate of mosquito population,

γ is the birth rate of human population,

θ is the biting rate of mosquito population,

α_1 is the probability of infection with DF,

$1-\alpha_1$ is the probability of infection with DHF,

β is the probability of patient with type DHF plasma leakage is not in shock,

$1-\beta$ is the probability of patient with type DHF plasma leakage in shock,

c is the number of other animals available as blood sources,

τ_h is the natural death rate of human population,

τ_d is the death rate of human population due to the disease,

τ_v is the death rate of mosquito population,

ϖ_{FH} is the transmission probability of DF from mosquito to human ,

ϖ_{HH} is the transmission probability of DHF from mosquito to human,

ϖ_{FV} is the transmission probability of DF from human to mosquito,

ϖ_{HV} is the transmission probability of DHF from human to mosquito,

δ_1 is the recovery rate of human population from DF infection,

δ_2 is the recovery rate of human population from DHF infection,

δ_3 is the recovery rate of human population from DSS infection.

We suppose that $P_h = Ds+Fi+Hi+HFi+Si+Dr$ and $N_v = As+Ai$,we assume the total human and mosquito populations have constant sizes $\frac{d}{dt}P_h=0$ and $\frac{d}{dt}P_v=0$

4. Analysis of the mathematical model

Equilibrium Points:

The equilibrium points $(Ds^*, Fi^*, Hi^*, HFi^*, Si^*, Dr^*, As^*, Ai^*)$ are found by setting the right hand side of (1.1) – (1.6) and (2.1) – (2.2) equal to zero. The system has two possible equilibrium points: disease free equilibrium point and endemic disease equilibrium point. This gives

1) Disease free equilibrium point: $S_0 = (\frac{P_h\gamma}{\tau_h}, 0,0,0,0,0, \frac{P_v}{\tau_v}, 0)$

2) Endemic disease equilibrium point: $S_1 = (Ds^*, Fi^*, Hi^*, HFi^*, Si^*, Dr^*, As^*, Ai^*)$ where

$$Ds^* = \frac{P_h \gamma}{Ai(\alpha_1(\varepsilon_1 - \varepsilon_2) + \varepsilon_2) + \tau_h} \tag{8.1}$$

$$Fi^* = \frac{Ai\, P_h \alpha_1 \gamma \varepsilon_1}{(Ai(\alpha_1(\varepsilon_1 - \varepsilon_2) + \varepsilon_2) + \tau_h)(\delta_1 + \tau_h)} \tag{8.2}$$

$$Hi^* = \frac{Ai\, P_h(-1+\alpha_1)\gamma \varepsilon_2}{(1+\tau_h)(Ai(\alpha_1(\varepsilon_1 - \varepsilon_2) + \varepsilon_2) + \tau_h)} \tag{8.3}$$

$$HFi^* = \frac{Ai\, P_h(-1+\alpha_1)\,\beta \gamma \varepsilon_2}{(1+\tau_h)(Ai(\alpha_1(\varepsilon_1 - \varepsilon_2) + \varepsilon_2) + \tau_h)(\delta_2 + \tau_h)} \tag{8.4}$$

$$Si^* = \frac{Ai\, P_h(-1+\alpha_1)(-1+\beta)\gamma \varepsilon_2}{(1+\tau_h)(Ai(\alpha_1(\varepsilon_1 - \varepsilon_2) + \varepsilon_2) + \tau_h)(\delta_2 + \tau_h + \tau_d)} \tag{8.5}$$

$$Dr^* = \frac{Ai\, P_h \gamma \left(-\frac{\alpha_1 \varepsilon_1 \delta_1}{\delta_1 + \tau_h} - \frac{(-1+\alpha_1)\beta \varepsilon_2 \delta_2}{(1+\tau_h)(\delta_2 + \tau_h)} - \frac{(-1+\alpha_1)\beta \varepsilon_2 \delta_3}{(1+\tau_h)(\delta_3 + \tau_h + \tau_d)} \right)}{\tau_h (Ai(\alpha_1(\varepsilon_1 - \varepsilon_2) + \varepsilon_2) + \tau_h)} \tag{8.6}$$

$$As^* = \frac{P_v}{\frac{Ai\, P_h \gamma (\alpha_1 \varepsilon_1 \varepsilon_3 + (-1+\alpha_1)\varepsilon_2 \varepsilon_4 \delta_1 + (\alpha_1 \varepsilon_1 \varepsilon_3 + (-1+\alpha_1)\varepsilon_2 \varepsilon_4)\tau_h)}{(1+\tau_h)(Ai(\alpha_1(\varepsilon_1 - \varepsilon_2) + \varepsilon_2) + \tau_h)(\delta_1 + \tau_h)} + \tau_v} \tag{8.7}$$

$$Ai^* = \frac{P_v P_h \gamma (\alpha_1 \varepsilon_1 \varepsilon_3 + (-1+\alpha_1)\varepsilon_2 \varepsilon_4 \delta_1 + (\alpha_1 \varepsilon_1 \varepsilon_3 + (-1+\alpha_1)\varepsilon_2 \varepsilon_4)\tau_h) - \tau_h(1+\tau_h)(\delta_1 + \tau_h)\tau_v^2)}{(\tau_v (P_h \gamma (\alpha_1 \varepsilon_1 \varepsilon_3 + (-1+\alpha_1)\varepsilon_2 \varepsilon_4 \delta_1 + (\alpha_1 \varepsilon_1 \varepsilon_3 + (-1+\alpha_1)\varepsilon_2 \varepsilon_4)\tau_h) + (\alpha_1(\varepsilon_1 - \varepsilon_2) + \varepsilon_2)(1+\tau_h)(\delta_1 + \tau_h)\tau_v))} \tag{8.8}$$

with $\varepsilon_1 = \frac{(\theta \varpi_{FH})}{(P_h + c)}$, $\varepsilon_2 = \frac{(\theta \varpi_{HH})}{(P_h + c)}$, $\varepsilon_3 = \frac{(\theta \varpi_{FV})}{(P_h + c)}$, $\varepsilon_4 = \frac{(\theta \varpi_{HV})}{(P_h + c)}$

and

$$D_0 = \frac{P_v P_h \gamma \alpha_1 (\varepsilon_1 \varepsilon_3 (1+\tau_h) + \varepsilon_2 \varepsilon_4 (\delta_1 + \tau_h))}{(\delta_1 + \tau_h)(P_v P_h \gamma \varepsilon_2 \varepsilon_4 + \tau_h (1+\tau_h)\tau_v^2)}$$

It can be seen from the above equations that the steady state solution is positive for $D_0 > 1$ .The local stability of each equilibrium point is determined by the sign of eigenvalues for each steady state. If all eigenvalues have negative real parts, then that steady state is local stability [9]. The eigenvalues are obtained by solving the following characteristic equation

$$\det(J - \psi I_8) = 0$$

where I_8 the identity matrix dimension 8x8 .

or the disease free steady state $S_0 = (\frac{P_h \gamma}{\tau_h}, 0,0,0,0,0, \frac{P_v}{\tau_v}, 0)$, the *Jacobian* matrix is given by

$$\begin{bmatrix}
(-(\tau_h)-(\alpha_1\varepsilon_1 Ai)-((1-\alpha_1)(\varepsilon_2)Ai))-\psi & 0 & 0 & 0 & 0 & 0 & 0 & -(\alpha_1\varepsilon_1 Ds)-((1-\alpha_1)(\varepsilon_2)Ds) \\
(\alpha_1\varepsilon_1 Ai) & (-(\tau_h)-(\delta_1))-\psi & 0 & 0 & 0 & 0 & 0 & (\alpha_1\varepsilon_1 Ds) \\
((1-\alpha)(\varepsilon_2)Ai)) & 0 & (-(\tau_h)-(\beta)-(1-\beta))-\psi & 0 & 0 & 0 & 0 & ((1-\alpha_1)(\varepsilon_2)Ds) \\
) & 0 & \beta & (-(\tau_h)-(\delta_2))-\psi & 0 & 0 & 0 & 0 \\
\text{ɔ} & 0 & (1-\beta) & 0 & (-(\tau_h)-(\tau_d)-(\delta_3))-\psi & 0 & 0 & 0 \\
0 & \delta_1 & 0 & \delta_2 & \delta_3 & -(\tau_h)-\psi & 0 & 0 \\
0 & -(\varepsilon_3 As) & -(\varepsilon_4 As) & 0 & 0 & 0 & (-(\varepsilon_3 Fi)-(\varepsilon_4 Hi)-(\tau_v))-\psi & 0 \\
0 & (\varepsilon_3 As) & (\varepsilon_4 As) & 0 & 0 & 0 & (\varepsilon_3 Fi)+(\varepsilon_4 Hi) & (-\tau_v)-\psi
\end{bmatrix}$$

where

$$\psi = -\tau_h \,,\ \psi = -\tau_h, \psi = -\tau_h - \delta_2, \psi = -\tau_h - \delta_3 - \tau_d, \psi = -\tau_v,$$

Tl.e characteristic equation is defined by $\psi^3 + A_1 \psi^2 + A_2 \psi + A_3$.

wherr

$\delta_1 + 2\tau_h + \tau_v$,

$$A_2 = (1+\tau_h)+(\delta_1+\tau_h)-\frac{P_h P_v \gamma(\alpha_1\varepsilon_1\varepsilon_3+\varepsilon_2\,\varepsilon_4-\alpha_1\varepsilon_2\,\varepsilon_4)}{\tau_h\tau_v}+(1+\delta_1+2\tau_h)\tau_v\,,$$

$$A_3 = \frac{1}{\tau_h\tau_v}(-P_h P_v \gamma(\alpha_1\varepsilon_1\varepsilon_3-(-1+\alpha_1)\,\varepsilon_2\,\varepsilon_4\,\delta_1+(\alpha_1\varepsilon_1\varepsilon_3+\varepsilon_2\,\varepsilon_4-\alpha_1\,\varepsilon_2\,\varepsilon_4)\tau_h)+\tau_h(1+\tau_h)(\delta_1+\tau_h)\tau_v^2$$

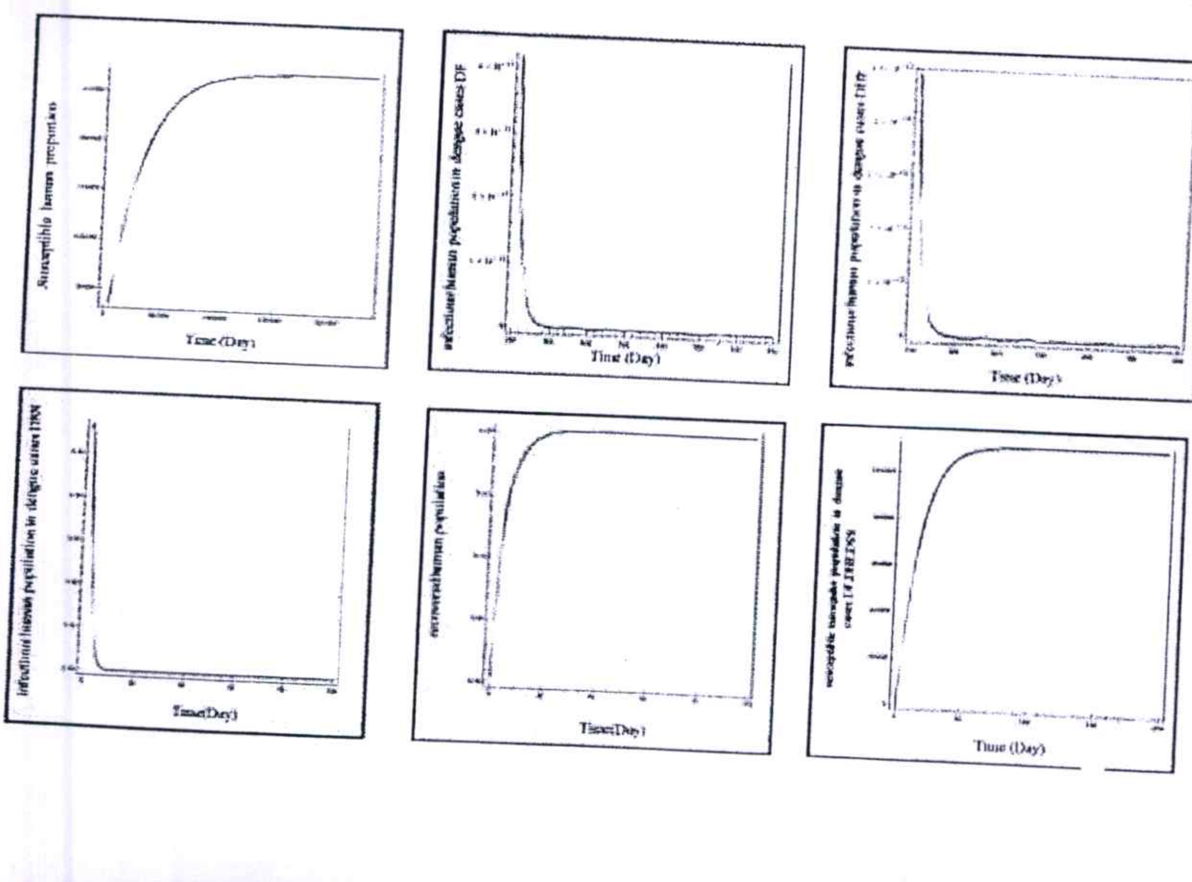
We check the sign of eigenvalues by using Routh Hurwitz criteria can impact eigenvalues with negative real parts. If the characteristic equation satisfy Routh Hurwitz criteria, we can say that the steady state is local stability. Thus, this disease free state will be local stability when $D_0 < 1$.On similarly method, we found that the disease endemic state will be local stability when $D_0 > 1$.

5. Numerical results

The parameters are given as follows: $\tau_h = 1/(365*74.6)$ corresponds to the life expectancy of 74.6 years for human, $\delta_1 = 1/5$ corresponds to the 5 days at which quarantine human change to be recovered human for DF, $\delta_2 = 1/6$ corresponds to the 6 days at which quarantine human change to be recovered human for DHF, $\delta_3 = 1/3$ corresponds to the 3 days at which quarantine human change to be recovered human for DSS. The other parameters are arbitrary chosen

Cases1. $D_0 < 1$;

Figure 2 : Times series solutions of susceptible, DF infectious human, DHF infectious human, DSS infectious human, susceptible mosquito population, infectious mosquito population, respectively.

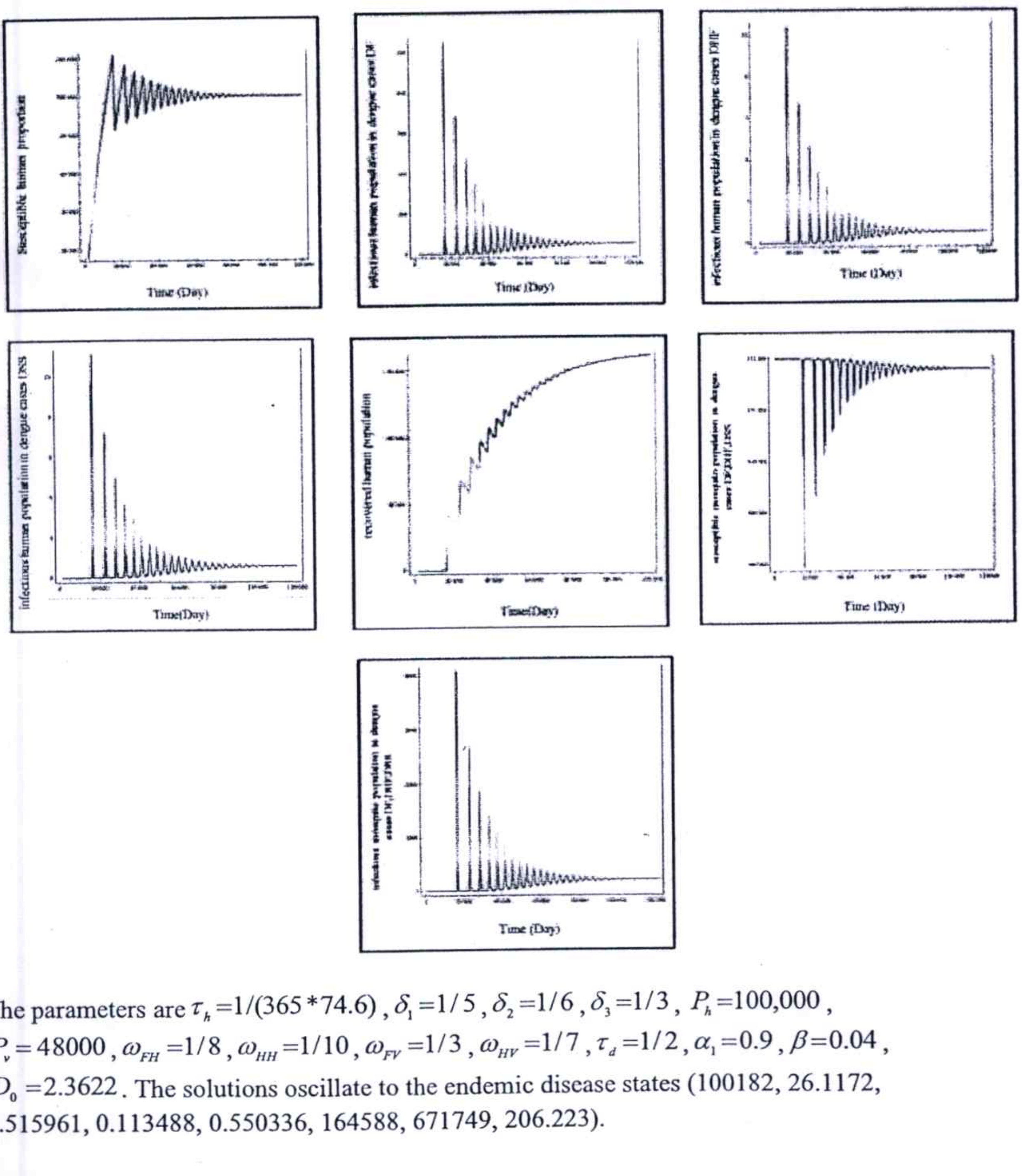


The parameters are

τ_h =1/(365 *74.6) , δ_1 =1/5 , δ_2 =1/6 , δ_3 =1/3 , P_h =100,000 , P_v =8000 , ω_{FH} =1/8 , ω_{HH} =1/10 , ω_{FV} =1/3 , ω_{HV} =1/7 , τ_d =1/2 , α_1 =0.7 , β=0.007 , D_0 =0.319108 .The solutions converge to the disease free states (272290, 0, 0, 0, 0, 0, 98000, 0)

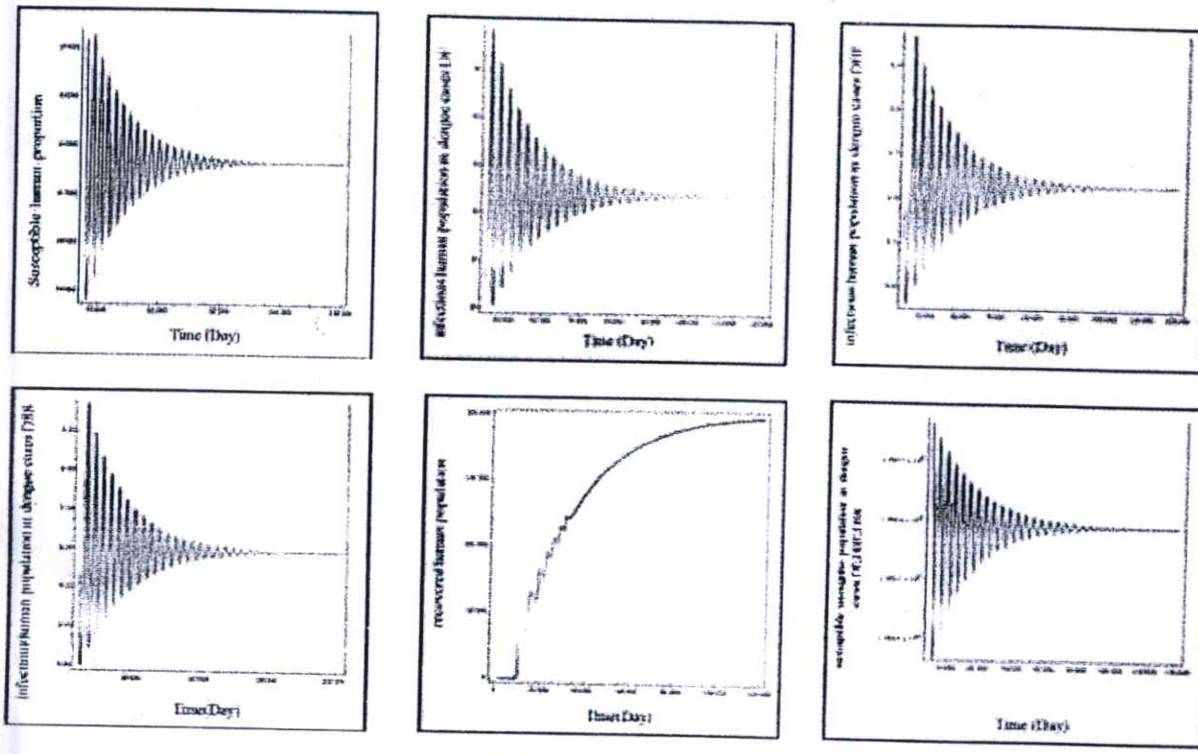
Cases2. $D_0 > 1$;

Figure 3 : Times series solutions of susceptible, DF infectious human, DHF infectious human, DSS infectious human, susceptible mosquito population, infectious mosquito population, respectively.



The parameters are τ_h =1/(365 *74.6) , δ_1 =1/5 , δ_2 =1/6 , δ_3 =1/3 , P_h =100,000 , P_v = 48000 , ω_{FH} =1/8 , ω_{HH} =1/10 , ω_{FV} =1/3 , ω_{HV} =1/7 , τ_d =1/2 , α_1 =0.9 , β=0.04 , D_0 =2.3622 . The solutions oscillate to the endemic disease states (100182, 26.1172, 0.515961, 0.113488, 0.550336, 164588, 671749, 206.223).

Figure 4 : Times series solutions of susceptible , DF infectious human, DHF infectious human, DSS infectious human, susceptible mosquito population, infectious mosquito population, respectively.



The parameters are $\tau_h = 1/(365*74.6)$, $\delta_1 = 1/5$, $\delta_2 = 1/6$, $\delta_3 = 1/3$, $P_h = 100{,}000$, $P_v = 78000$, $\omega_{FH} = 1/8$, $\omega_{HH} = 1/10$, $\omega_{FV} = 1/3$, $\omega_{HV} = 1/7$, $\tau_d = 1/2$, $\alpha_1 = 0.7$, $\beta = 0.007$, $D_0 = 3.43912$. The solutions oscillate to the endemic disease states (61654.8, 31.9637, 0.631462, 0.138894, 0.673532, 201432, 0.000001091, 410.1).

When value of probability of infection with DF (α_1) and probability of patient with type DHF plasma leakage is not in shock (β) ,as a results Fig 3 converge to endemic faster than Fig 4.

6. Discussion and co. .lusion

In this study, we constr.icted the mathematical model of dengue cases (DF,DHF,DSS) and analyzed the results by using standard dynamical modeling method. The basic reproductive number is defined by D_0 .From Fig 3 ,parameter values are $\alpha_1 = 0.9$, $\beta=0.04$ and Fig 4, parameter values are $\alpha_1 = 0.7$, $\beta=0.007$,so that the convergence of the graph are different. The others parameter were, too. Therefore, we know that, when values α_1 (the probability of infection with DF) and β (the probability of patient with type DHF plasma leakage is not in shock), the convergence to endemic state are different. The time series parameter inference shows different dynamic behaviours, depending on the data collection to be described via the modeling approaches. The output of this model should introduce the way for reducing the transmission of this disease.

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