ENHANCING MATHEMATICAL ACHIEVEMENT AND PROBLEM SOLVING ABILITIES OF GRADE 10 STUDENTS BY POLYA'S FOUR STEPS AND SCHOENFELD'S BEHAVIOR CATEGORIES

การส่งเสริมผลสัมฤทธิ์และความสามารถในการแก้ปัญหาทางคณิตศาสตร์ของนักเรียนชั้นมัธยมศึกษาปีที่ 4 โดยเทคนิคการแก้ปัญหาสี่ขั้นตอนของโพลยาและหมวดหมู่ทางพฤติกรรมของชอนเฟลด์

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Abstract

The purposes of this classroom action research were to enhance students' mathematical achievement and to enhance students' problem solving abilities by using Polya's Four Steps and Schoenfeld's Behavior Categories. The participants were 32 grade 10 students in the first semester of academic year 2017 at a secondary school in Bangkok, Thailand. The topic used in this study was real number system. Apart from Polya's Four Steps and Schoenfeld's Behavior Categories, the researcher also applied group work activity in teaching problem solving. Instruments in this study were 8 lesson plans, mathematics achievement test, and problem solving test. Teaching and learning lasted 13 periods with 50 minutes in each period. There were three cycles of action plan. Each cycle was composed of planning, acting, observing, and reflecting. Data were collected from those instruments, including students' homework, worksheets, and teacher's reflection. Results showed that students' achievement scores and problem solving scores were high to the levels required by the researcher. From reflections, researcher adjusted teaching by frequently reminding students of resources, general strategies, content-specific strategies, to think about their thinking, control and checking problem solutions.

Keywords: Mathematical problem solving; Polya's Four Steps; Schoenfeld's Behavior Categories

บทคัดย่อ

งานวิจัยนี้มีจุดประสงค์เพื่อส่งเสริมผลสัมฤทธิ์ทางการเรียนคณิตศาสตร์ และความสามารถในการแก้ปัญหาโดยการใช้การแก้ปัญหาสี่ขั้นตอนของโพลยาและหมวดหมู่ทางพฤติกรรมของชอนเฟลด์ กลุ่มตัวอย่างคือ นักเรียนชั้นมัธยมศึกษาปีที่ 4 โรงเรียนมัธยมแห่งหนึ่งในจังหวัดกรุงเทพมหานคร จำนวน 32 คน ภาคเรียนที่ 1 ปีการศึกษา 2560 หัวข้อที่ใช้ในงานวิจัยนี้ คือระบบจำนวนจริง นอกจากการแก้ปัญหาสี่ขั้นตอนของโพลยาและหมวดหมู่ทางพฤติกรรมของชอนเฟลด์ ผู้วิจัยได้ประยุกต์กิจกรรมการทำงานกลุ่มในการสอนการแก้ปัญหา เครื่องมือที่ใช้ในงานวิจัยประกอบด้วย แผนจัดการเรียนรู้ 8 แผน แบบทดสอบผลสัมฤทธิ์ทางการเรียนคณิตศาสตร์แบบทดสอบการแก้ปัญหา การบ้านและใบงานของนักเรียน และบันทึกผลการจัดการเรียนการสอนของครู การเรียนการสอนใช้เวลา 13 คาบ คาบละ 50 นาที ใช้วงจรการปฏิบัติงาน 3 วงจร แต่ละวงจรประกอบด้วย การวางแผน การจัดกิจกรรม การสังเกต และการสะท้อนผล ผลการวิจัยพบว่า ผลสัมฤทธิ์ทางการเรียนคณิตศาสตร์ และการแก้ปัญหาของนักเรียน

สูงถึงระดับที่ผู้วิจัยต้องการ จากการสะท้อนผล ผู้วิจัยปรับการสอนโดยการย้ำเตือนในด้านความรู้ ทฤษฎี กลวิธีทั่วไป กลวิธีเฉพาะเนื้อหา ความตระหนักและความเข้าใจในกระบวนการคิดของตัวเอง การควบคุมและตรวจสอบคำตอบ

คำสำคัญ: การแก้ปัญหาทางคณิตศาสตร์ การแก้ปัญหาสี่ขั้นตอนของโพลยา หมวดหมู่ทางพฤติกรรมของชอนเฟลด์

1. INTRODUCTION

Problem solving means engaging in a task for which the solution method is not known in advance. It is an important process which students should know and develop in themselves. It also helps students approach diversity of ideas, thinking process and reasoning. Problem solving is not only a goal of learning mathematics, but also a major means of doing so [1]. Problem solving is the core matter of the 1980 yearbook of the National Council of Teachers of Mathematics (NCTM) [2]. It stated that “problem solving must be the focus of school mathematics” and problem solving was a position statement of NCTM in its agenda for actions.

From teaching experiences and teacher interview, the researcher found that students’ main problems were: 1) Solving equations and inequalities, 2) Simplifying algebraic expressions, and 3) Solving algebraic word problems. In solving mathematics problems, most students neither know how to start nor continue to finish a problem. They were not taught to use problem solving strategies and to control their thinking until the solutions were reached. Moreover, students’ mathematics achievement in Thailand was not satisfactory as shown in the mathematics test scores of Ordinary National Education Test (O-net) school year 2014, 2015 and 2016. The average scores in mathematics on Grade 12 were 21.74%, 26.59%, and 24.88% respectively. Solving equations and inequalities which are in real number system will be mainly used by students in their further study in various fields. So, the researcher was interested in solving this problem, solving mathematics problems in the real number system. As a mathematics teacher, it was appropriate to apply classroom action research. The researcher tried to find out effective ways to help students improve solving mathematics problems.

Among many studies and suggestions on mathematical problem solving, the outstanding and accepted ideas are from those of George P. Polya and of Alan H. Schoenfeld. Polya is well known for his classic book, *How to Solve It*. Many researchers in mathematics education adopted Polya’s suggestions in their studies. And the results showed increasing in students’ problem solving abilities. Polya identifies four basic steps in solving any mathematics problems. The steps are 1) Understanding the problem, 2) Devising a plan, 3) Carrying out the plan, and 4) Looking back [3].

In his research at University of California at Berkley about mathematics problem solving, Schoenfeld came to a very important conclusion about students’ behavior categories in solving mathematics problems. These categories helps students in step 2 and 3 of Polya’s suggestion, plan to do it and carry out the plan. Schoenfeld’s Behavior Categories is valuable to remind problem solvers to prepare resources and think about their thinking while solving a problem. The categories are the following: 1) Resources, 2) Heuristics, 3) Control, and 4) Beliefs systems [4]. So, the researcher decided to adopt both Polya Four Steps and Schoenfeld’s Behavior Categories to improve students’ achievement and problem solving abilities.

2. RESEARCH OBJECTIVES

The objectives were to enhance students' mathematical achievement and to enhance students' problem solving abilities of grade 10 learners who have been taught to solve mathematical problems by using Polya's Four Steps and Schoenfeld's Behavior Categories. The author aimed 70% of all students to pass mathematics achievement test, and problem solving test. The criterion for each student to pass was to gain at least 70% of total score of mathematics achievement test, and problem solving test.

3. RESEARCH METHODOLOGY

This research took model of the cyclical nature of the typical action research process [5]. Each cycle was composed of planning, acting, observing, and reflecting. Researcher intended to use this model with three cycles.

The participants in this study were 32 grade 10 students in the first semester of academic year 2017 at a secondary school in Bangkok, Thailand.

4. RESEARCH INSTRUMENTS

The instruments applied in this study were lesson plans, mathematics achievement test, problem solving test and teacher's reflection as follows:

1. Lesson Plans: All eight lesson plans were included with three reflections: reflection 1 (at the end of 4^{th} period), reflection 2 (at the end of 9^{th} period) and reflection 3 (at the end of 13^{th} period). Lesson plans consisting of eight lessons with 13 periods on real number system in grade 10 were integrated by using Polya's Four Steps and Schoenfeld's Behavior Categories to enhance students' mathematical achievement and to enhance students' problem solving abilities. These lesson plans were evaluated and improved by three expert teachers. Topics on real number system were Introduction to real numbers, Properties of real numbers, Solving polynomial equations by Remainder Theorem, Properties of inequalities, Interval notations, Solving linear inequalities, Absolute values and solving linear equations in the form of absolute values, and Solving inequalities in the form of absolute values. Teaching applied four steps with were: 1) Reviewing previous knowledge, 2) Introducing new content and examples, 3) Solving problems by using Polya's Four Steps (Understanding the problem, Devising a plan, Carrying out the plan, Looking back) and Schoenfeld's Behavior Categories (Resources, Heuristics, Control and Belief systems), and 4) Conclusion. At the end of each period, the researcher reflected on students' behaviors and results for improvement in the next period.

2. Mathematics achievement test: Mathematics achievement test constructed by the researcher was used to evaluate mathematical achievement. The test consisted of 20 items focusing on knowledge of the real number system. The items in the test were presented in multiple choice format (1-point per item). Each item had four alternatives with one correct answer. Total score of this test were 20 points. The test items aimed to measure the following behaviors: knowledge, comprehension, application, and analysis. The participants took the mathematics achievement test after finishing the final period (at the end of 13^{th} period). The test difficulty obtained from trying out with other groups that had studied this topic before. The test content validity was evaluated by three expert teachers.

3. Problem solving test I and II: Problem solving test I and II constructed by the researcher were used to evaluate mathematical problem solving abilities which measured from 1) step in solving problem and 2) applying Schoenfeld's Behavior Categories: resources, heuristics, and control. Problem solving test I consisted of five written items. Total score of this test were 20 points. The participants took this test at the end of 9^{th} period. Problem solving test II consisted of three written items. Total score of this test were 20 points. The participants took this test after finishing the final period (at the end of 13^{th} period). These two tests were evaluated and improved by three expert teachers.

4. Teacher's reflections: In this research, there were three reflections to improve problem solving teaching. At the same time, the researcher reflected on students' problem solving abilities from students' homework and worksheets. Comments on reflections were also given by expert teachers.

5. DATA COLLECTION

The data collection of this study was performed during 13 periods of instruction described in the previous section. The researcher collected data from mathematics achievement test, problem solving test I and II, students' homework and worksheets, and teacher's reflections. After finishing 9^{th} period, the participants took problem solving test I. Researchers observed and recorded the students' behavior in solving problems from students' homework and worksheets in each period. At the last period, the participants took problem solving test II and mathematics achievement test.

6. DATA ANALYSIS

Quantitative data: The data from instruments were used to evaluate effectiveness of students' mathematical achievement and students' problem solving abilities by using Polya's Four Steps and Schoenfeld's Behavior Categories. Instruments were mathematics achievement test and problem solving test I and II. The researcher expected that 70% of students passed mathematics achievement test and problem solving test I and II. The criterion for each student to pass was to gain at least 70% of total scores of mathematics achievement test and problem solving test I and II.

Qualitative data: The researcher analyzed qualitative data from students' homework and worksheets, and teachers' reflection. They were also used to consider students' mathematical problem solving abilities.

7. RESULTS

Results from quantitative data collection

Mathematics achievement test was managed at the final period. Results are shown in table 1.

Table 1 Results from mathematics achievement test

Item	N	Total Score	70 percent of total score	$\overline{X}$	SD	Number of passing students (percent)
Mathematics achievement test	32	20	14	16.84	1.78	31 (96.88)
Total	32	20	14	16.84	1.78	-

From table 1, mean score of mathematics achievement test was 16.84 (SD = 1.78). Thirty one students from thirty two students passed the test which was 96.88 percent. So, more than 70 percent of students passed mathematics achievement test as required by researcher.

Problem solving test I was managed at the end of 9^{th} period. Results of problem solving test I are shown in table 2.

Table 2 Results from problem solving test I

Item	N	Total Score	70 percent of total score	$\overline{X}$	SD	Number of passing students (percent)
Problem solving test I	32	20	14	15.5	1.83	28 (87.5)

From table 2, the findings showed mean score of problem solving test I which was 15.5 (SD = 1.83). Twenty eight students from thirty two students passed the test which was 87.5 percent. So, more than 70 percent of students passed problem solving test I as required by researcher.

For students' behavior in solving problems, the researcher observed and recorded from problem solving test I. Students' behaviors were divided according to steps of Polya. Results of students' problem solving behaviors from problem solving test I are shown in table 3.

Table 3 Mean scores in each step expressed by students in problem solving test I

Problem solving no.	Understanding the problem (1)	Devising a plan (1)	Carrying out the plan (1)	Looking back (1)	Total Score (4)	Mean Score	Percentage of Mean Score
1	0.91	0.84	0.81	0.31	4	2.88	71.88
2	0.91	0.81	0.72	0.41	4	2.84	71.09
3	0.91	0.78	0.84	0.28	4	2.81	70.31
4	0.91	0.94	0.84	0.31	4	3	75
5	0.88	0.84	0.91	0.34	4	2.97	74.22
Total	4.5	4.22	4.13	1.66	20	14.5	-
Average	0.9	0.84	0.83	0.33	4	2.9	72.5

Table 3 showed that there were five items with 20 total score. Nearly all students applied step 1, 2, and 3 but less than 50 percent of all students did not apply step 4 (Looking back).

The researcher also analyzed students' behavior categories. They are resources, heuristic, and control. Results are shown in table 4.

Table 4 Mean scores in each categories expressed by students in problem solving test I

Problem solving no.	Resources (1)	Heuristic (1)	Control (1)	Total Score (3)	Mean Score	Percentage of Mean Score
1	0.91	0.88	0.66	3	2.44	81.25
2	0.94	0.84	0.66	3	2.44	81.25
3	0.84	0.72	0.78	3	2.34	78.13
4	0.81	0.69	0.75	3	2.25	75
5	0.94	0.88	0.72	3	2.53	84.38
Total	4.44	4.00	3.56	15	12	-
Average	0.89	0.80	0.71	3	2.4	80

Table 4 showed that nearly all students applied resource, heuristic, and control in solving problems.

At the end of final period, the researcher managed problem solving test II. Results from problem solving test II are shown in table 5.

Table 5 Results from problem solving test II

Item	N	Total Score	70 percent of total score	$\overline{X}$	SD	Number of passing students (percent)
Problem solving test II	32	20	14	15.97	1.60	30 (93.75)

Table 5 showed mean score of problem solving test II which was 15.97 (SD = 1.60). Thirty students from thirty two students passed the test which was 93.75 percent. So, more than 70 percent of students passed this problem solving test as required by the researcher.

Table 6 Mean scores in each step expressed by students in problem solving test II

Problem solving no.	Understanding the problem (1)	Devising a plan (1)	Carrying out the plan (1)	Looking back (1)	Total Score (4)	Mean Score	Percentage of Mean Score
6	0.94	0.81	0.88	0.28	4	2.91	72.66
7	0.88	0.84	0.81	0.44	4	2.97	74.22
8	0.91	0.81	0.88	0.25	4	2.84	71.09
Total	2.72	2.47	2.56	0.97	12	8.72	-
Average	0.91	0.82	0.85	0.32	4.00	2.91	72.66

Table 6 showed that there were three items with 12 total score. Nearly all students applied step 1, 2, and 3 but less than 50 percent of all students did not apply step 4 (Looking back).

Table 7 Mean scores in each categories expressed by students in problem solving test II

Problem solving no.	Resources (1)	Heuristic (1)	Control (1)	Total Score (3)	Mean Score	Percentage of Mean Score
6	0.94	0.91	0.72	3	2.56	85.42
7	0.90	0.88	0.75	3	2.53	84.27
8	0.88	0.91	0.81	3	2.59	86.46
Total	2.72	2.69	2.28	9	7.68	-
Average	0.91	0.90	0.76	3	2.56	85.38

Table 7 showed that nearly all students applied resource, heuristic, and control in solving problems.

Results from qualitative data collection

Teacher's reflections

Reflection is a part of classroom action research cycle. In this study, the researcher applied three action research cycles. Reflection consisting of feedback after teaching and teaching implementation plan for the next cycle was included in each cycle as follows:

Reflection from cycle 1

Most students hardly expressed their opinions or solved the problem in classroom because they feared and were worried about wrong answers. But some students tried to give their opinions or solved the problem by explaining or presenting in classroom.

From the above observation in this cycle, the researcher planned to use group activity. Students participated in learning more by discussion in groups. Moreover, students could link between previous and current knowledge. They shared resources, heuristics to describe answers, and expressed their opinions with teacher.

Reflection from cycle 2

Students preferred learning and exchanging ideas with friends to listening to teacher. Students could plan to solve problems with diversity of methods based on the processes of Polya’s Four Steps and Schoenfeld’s Behavior Categories. Some students got wrong answers because of misunderstanding on the process of problem solving or not checking the answers.

In the next classes, the researcher encouraged students to apply thinking processes and giving reasons in problem solving to prevent them from getting wrong answers.

Reflection from cycle 3

Most students were enthusiastic and also interested in learning. Students expressed their opinions better than that of the previous periods and could associate other knowledge to solve or describe their answers which were related to resources and strategies in solving problems. Besides, students showed active roles in solving problems focusing on their plans and showing good control.

The researcher gave students more chance in active learning and thinking with open-ended questions. The researcher used questions to motivate thinking to solve problems. Students had an opportunity to exchange knowledge and to solve problems with their peers by using Polya’s Four Steps in discussion.

8. CONCLUSION AND DISCUSSION

The purposes of this research were to enhance students' mathematical achievement and to enhance students' problem solving abilities of grade 10 students who have been taught to solve mathematical problems by using Polya’s Four Steps and Schoenfeld’s Behavior Categories.

The results from this study revealed that both students’ mathematical achievement and students’ problem solving abilities were high and met the levels required by the researcher, more than 70% of the total scores of both tests. These were the results from applying Polya’s Four Steps and Schoenfeld’s Behavior Categories to help students solve mathematical problems. In part of Polya’s Four Steps, most students applied steps 1, 2, and 3 but less than 50% of students did not applied step 4 (Looking back). For Schoenfeld’s Behavior Categories, most students applied resources, heuristic and control in solving problem. As mentioned above, well-prepared lesson plans and steps in general teaching (reviewing previous knowledge, introducing of new content and examples, solving problems and conclusion) are another factor playing an important role in enhancing students’ mathematical achievement and students’ problem solving abilities. According to the results from reflections, the researcher applied group work, problem solving steps, strategies, categories, and exchanging ideas.

The results of this study is consistent with many studies in the past [6]. According to Yuwalee [7], the results demonstrated that these problem solving strategies help students improve their mathematical problem solving and be more effective problem solvers. Students gain systematic thinking and problem solving skills. Moreover, Sirirat [8] pointed out that collaboration helps students create an environment for learners and improve learner’s communication between peers and teacher which benefit to the learners. From this study, there are many details to be considered in helping students to be good problem solvers. They include 1) sequence of content presentation to make students understand the contents, 2) teaching problem solving, 3) problem selection which is appropriate to students, and 4) reflecting one’s teaching and then adjusting teaching strategies. The important point gained from this study is that teaching problem solving to students is a complex process and needs time and effort to help a student to be a successful problem solver.

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