รายงานวิจัย

The Analysis of Random Coefficient

Dynamic Regression Model

ชื่อผู้วิจัย

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ประจำปีงบประมาณ 2554

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Abstract

Random Coefficient Dynamic Regression (RCDR) model is to develop from Random Coefficient Autoregressive (RCA) model and Autoregressive (AR) model. The RCDR model are considered by adding exogenous variables. In this article, the concept of the Least Squares (LS) criterion is used to estimate the parameter on RCDR model. Simulation results have shown the proposed coefficient of AR model provided asymptotically unbiased estimates nearly for most six data generating models. The RCDR model is then applied to a series of daily observation of the exchange rate of Baht/GBP and Baht/EUR to illustrate the methodology. The prediction of LS criterion are used with those obtained on twenty hold out future values of withheld observations.

Keywords: Autoregressive; Least Squares criterion; Random Coefficient Dynamic Regression;

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Chapter 1

Introduction

A variety of fields in business, finance, economics and other areas are often collected in the forms of time series. The time series are an ordered sequence of observations taken at regular intervals of time such as daily exchange rates, weekly stock prices, monthly oil consumptions, and annual growth rates. Many economic time series exhibit nonstationarity and stochastic volatility which are important role in finance and business when the financial time series are often clear evidence of changing variance in time plot in data. There are literature based on the volatility, for instance, Thavaneswaran, Appadoo, and Peiris (2005) studied the problem of volatility forecasting for some financial time series model. Tsay (2005) defined the volatility as the conditional variance of the asset return and the characteristics can be seen commonly in underlying asset return. Roh (2007) studied the forecasting volatility of stock price index that can be enhanced by integrating financial time series model.

The model has been widely used to model volatility of financial and economic time series is the Autoregressive Conditional Heteroscedastic (ARCH) model were first introduced by Engle(1982) for modeling of the predictive variance for U.K. inflation rate. An ARCH(r) model assumes that

$$x_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 \tag{1.1}$$

where $\{\varepsilon_i\}$ is a sequence of independent and identically distributed(iid) random variables with mean zero and variance 1 , and ε_t is independent of $\{x_{t-k}, k \geq 1\}$ for all t, $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i > 0$.

Although the ARCH is simple, it often requires many parameters to adequately describe the volatility process. To obtain more flexibility, the ARCH model has been extended by Bollerslev(1986) who produced the Generalized ARCH (GARCH) model. The GARCH model is allowed the past data time series and the past volatility in this model.

The GARCH(p,q) model assume that

$$x_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{1.2}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$,for all i and $j > 0$, are constant, $\{\varepsilon_i\}$ is a sequence of independent and identically distributed(iid) random variables with mean zero and variance 1, and ε_t is independent of $\{x_{t-k}, k \geq 1\}$ for all t, when $\beta_j = 0$ the model reduces to ARCH(r).

There are other models which proposed the evolution of condition variance which is called the Conditional Heteroscadastic Autoregressive Moving Average (CHARMA) model by Tsay(1987). The CHARMA model is not similar the GARCH model, but these two models possess similar second-order condition properties. The CHARMA model is defined as

$$\phi(\mathbf{B})(x_t - \mu) = \theta(\mathbf{B})a_t \tag{1.3}$$

$$\delta_t(\mathbf{B})a_t = \omega_{0,t}[\hat{x}_{t-1}(1) - \mu] + \omega_t^*(\mathbf{B})(x_t - \mu) + \varepsilon_t \tag{1.4}$$

The equation (1.3) is called Observation Equation, where $\phi(\mathbf{B}) = 1 - \phi_1(\mathbf{B}) - \ldots - \phi_p\mathbf{B}^p$ and $\theta(\mathbf{B}) = 1 - \theta_1(\mathbf{B}) - \ldots - \theta_q\mathbf{B}^q$are constant coefficient polynomials in $\mathbf{B}$ of degree p and q, $\mathbf{B}$ is backshift operator such that $\mathbf{B}x_t = x_{t-1}$, and μ is a mean level of x_t.

In (1.4) is called Innovation Equation, where $\delta_t(\mathbf{B}) = 1 - \delta_{1,t}(\mathbf{B}) - \ldots - \delta_{r,t}\mathbf{B}^r$ and $\omega_t^*(\mathbf{B}) = 1 - \omega_{1,t}(\mathbf{B}) - \ldots - \omega_{s,t}\mathbf{B}^s$ are purely random coefficient polynomials in $\mathbf{B}$ of degree r and s, so the model is in form CHARMA(p,q,r,s).

The special case of CHARMA model was CHARMA(p,0,0,p) with $\omega_{0,t} = 0$ that reduces to the Random Coefficient Autoregressive(RCA) model which was studied by Nicholls and Quinn(1982).

The RCA model is used the conditional variance to evolve with previous observations in RCA(p). The RCA model is

$$x_t = \alpha + \sum_{i=1}^{p} \beta_{ti} x_{t-i} + \sigma \varepsilon_t, \qquad \underline{\beta}_t = \underline{\mu}_\beta + \Sigma_\beta^{1/2} \underline{u}_t \tag{1.5}$$

where $\underline{\beta}_t = (\beta_{t1}, \ldots, \beta_{tp})'$ and $\underline{\mu}_\beta = (\mu_{\beta 1}, \ldots, \mu_{\beta p})'$. The ε_t and $\underline{\mu}_\beta$ are sequences of independent of random vectors with mean zero and unit variance.

The above models consist of one variable but sometime it's not enough to forecast the future time series so the disadvantage can improve the model by adding exogenous variables which have some direct effect on the nonstationarity (or stationarity) of the process. Tong (1983) suggested the Threshold Autoregressive (TAR) model which is very useful for the parameters of the conditional expectation of the process, depended on the past or other explanatory variables.

One of the method is the estimation to practice, through Least Squares (LS) criterion that can be developed flexible statistics to a point estimation. Nicholls and Quinn (1982) studied the RCA model from univariate time series data that obtained the Least Squares estimates of the model parameter and showed that under suitable conditions the least squares estimates are strongly consistent. Chan (1993) considered the least squares estimator of the threshold autoregressive model and showed that the estimators are consistent.

Our research goal is to present the LS criterion to modeling nonstationary and stationary time series. We introduce the new class of Random Coefficient Dynamic Regression (RCDR) Model that are extensions of some of the models described above.

This article is organized as follow : Section 2 describes the RCDR model that developed from RCA model. Section 3 describes the parameter estimation procedure and how to implement the Least Squares (LS) criterion. Section 4 shows the results of simulation study and we discuss the results. We apply our proposed method to real data in Section 6. Finally, in Section 7, we indicate some directions for future research ideas.

Chapter 2

RCDR Model

The time series data can produce the time series dynamic modeling when the observations of time series data have correlated with exogenous variables, the dynamic modeling will help accurate forecasts in future time.

In this section, we will extend the RCA model in(1.5) by adding the exogenous variables of y_t denoted

$$\begin{aligned} x_t &= \alpha_t + \sum_{i=1}^{p} \beta_{ti} x_{t-i} + \sigma\varepsilon_t \\ \underline{\beta}_t &= \underline{\mu}_\beta + \Sigma_\beta^{1/2} \underline{u}_t \\ \alpha_t &= \sum_{j=1}^{q} \eta_j y_{t-j} + \varepsilon_t \end{aligned} \tag{2.1}$$

where $\underline{\beta}_t = (\beta_{t1}, \ldots, \beta_{tp})'$ and $\underline{\mu}_\beta = (\mu_{\beta 1}, \ldots, \mu_{\beta p})'$. The ε_t and $\underline{\mu}_\beta$ are sequences of independent of random vectors with mean zero and unit variance, so the model (2.1) is called Random Coefficient Dynamic Regression (RCDR) Model or RCDR(p,q) model.

We will consider the simplified case of RCDR model with $p = q = 1$,the RCDR$(1,1)$, and we will write in form (2.2) as

$$\begin{aligned} x_t &= \alpha_t + \beta_t x_{t-1} + \varepsilon_t \\ \beta_t &= \mu_\beta + \sigma_\beta u_t \\ \alpha_t &= \eta y_{t-1} + \varepsilon_t \end{aligned} \tag{2.2}$$

where β_t's are iid random variables with mean μ_β and variance σ_β^2, ε_t's are iid random variables with mean 0 and variance σ^2, and β_t's and ε_t's are independent.

The parameters of RCDR(1, 1) consist of the intercept term η, the mean μ_β and variance σ_β^2 of the coefficient β_t and the variance σ^2 of the ε_t, or define as $\theta = (\eta, \mu_\beta, \sigma_\beta^2, \sigma^2)'$. In the literature, there is the RCA(1) with the slight modifications to model setup that used the nature of problem at hand might motivate to assume the two random variables β_t and ϵ_t to be correlated, see Hwang and Basawa (1998).

Chapter 3

Parameter Estimation for RCDR$(1,1)$

To estimate parameter of RCDR(1,1) model, we propose the LS criterion to estimate parameter $\theta = (\eta, \mu_\beta, \sigma_\beta^2, \sigma^2)'$ by minimizing sum of residuals square in 2 steps.

The first step of this method is based on LS criterion. Let $u_t = x_t - \eta y_{t-1} - \mu_\beta x_{t-1}$, then it can be seen that

$$E(u_t) = 0 \quad \text{and} \quad E(u_t^2) = (1+\eta^2)\sigma^2 + \sigma_\beta^2 x_{t-1}^2$$

Given the sample $x_1, x_2, \ldots, x_n$, the $\hat{\eta}$ is to estimate by minimizing sum of square residuals by

$$\begin{aligned}
u_t &= x_t - \eta y_{t-1} - \mu_\beta x_{t-1} \\
\sum_{t=1}^{n}(u_t)^2 &= \sum_{t=1}^{n}(x_t - \eta y_{t-1} - \mu_\beta x_{t-1})^2 \\
\frac{\partial}{\partial \eta}\sum_{t=1}^{n}(u_t)^2 &= -2\sum_{t=1}^{n}(x_t - \eta y_{t-1} - \mu_\beta x_{t-1})y_{t-1} = 0 \\
\eta \sum_{t=1}^{n} y_{t-1}^2 &= \sum_{t=1}^{n} x_t y_{t-1} - \mu_\beta \sum_{t=1}^{n} x_{t-1}y_{t-1} \\
\hat{\eta} &= \frac{\sum_{t=1}^{n} x_t y_{t-1} - \mu_\beta \sum_{t=1}^{n} x_{t-1}y_{t-1}}{\sum_{t=1}^{n} y_{t-1}^2} \qquad (3.1)
\end{aligned}$$

The LS estimate of $\hat{\mu}_\beta$ is given by,

$$\begin{aligned}
u_t &= x_t - \eta y_{t-1} - \mu_\beta x_{t-1} \\
\sum_{t=1}^{n} (u_t)^2 &= \sum_{t=1}^{n} (x_t - \eta y_{t-1} - \mu_\beta x_{t-1})^2 \\
\frac{\partial}{\partial \mu_\beta} \sum_{t=1}^{n} (u_t)^2 &= -2 \sum_{t=1}^{n} (x_t - \eta y_{t-1} - \mu_\beta x_{t-1}) x_{t-1} = 0 \\
\mu_\beta \sum_{t=1}^{n} x_{t-1}^2 &= \sum_{t=1}^{n} x_t x_{t-1} - \hat{\eta} \sum_{t=1}^{n} x_{t-1} y_{t-1} \\
\hat{\mu}_\beta &= \frac{\sum_{t=1}^{n} x_t x_{t-1} - \hat{\eta} \sum_{t=1}^{n} x_{t-1} y_{t-1}}{\sum_{t=1}^{n} x_{t-1}^2} \qquad (3.2)
\end{aligned}$$

From (3.2), let us replace in (3.1) and $\hat{\eta}$ is obtain as,

$$\begin{aligned}
\hat{\eta} \sum_{t=1}^{n} y_{t-1}^2 &= \sum_{t=1}^{n} x_t y_{t-1} - \left[\frac{\sum_{t=1}^{n} x_t x_{t-1} - \hat{\eta} \sum_{t=1}^{n} x_{t-1} y_{t-1}}{\sum_{t=1}^{n} x_{t-1}^2} \right] \sum_{t=1}^{n} x_{t-1} y_{t-1} \\
\hat{\eta} \sum_{t=1}^{n} y_{t-1}^2 \sum_{t=1}^{n} x_{t-1}^2 &= \sum_{t=1}^{n} x_t y_{t-1} \sum_{t=1}^{n} x_{t-1}^2 - \sum_{t=1}^{n} x_t x_{t-1} \sum_{t=1}^{n} x_{t-1} y_{t-1} + \hat{\eta} \Big(\sum_{t=1}^{n} x_{t-1} y_{t-1}\Big)^2 \\
\hat{\eta} &= \frac{\sum_{t=1}^{n} x_t y_{t-1} \sum_{t=1}^{n} x_{t-1}^2 - \sum_{t=1}^{n} x_t x_{t-1} \sum_{t=1}^{n} x_{t-1} y_{t-1}}{\sum_{t=1}^{n} y_{t-1}^2 \sum_{t=1}^{n} x_{t-1}^2 - (\sum_{t=1}^{n} x_{t-1} y_{t-1})^2} \qquad (3.3)
\end{aligned}$$

Thus $\hat{\mu}_\beta$ is calculated by,

$$\hat{\mu}_\beta = \frac{\sum_{t=1}^{n} x_t x_{t-1} \sum_{t=1}^{n} y_{t-1}^2 - \sum_{t=1}^{n} x_t y_{t-1} \sum_{t=1}^{n} x_{t-1} y_{t-1}}{\sum_{t=1}^{n} x_{t-1}^2 \sum_{t=1}^{n} y_{t-1}^2 - (\sum_{t=1}^{n} x_{t-1} y_{t-1})^2} \qquad (3.4)$$

The second step is to form the residuals $\hat{u}_t = x_t - \hat{\eta} y_{t-1} - \hat{\mu}_\beta x_{t-1}$ and then regress $\hat{u}_t^2$ on x_{t-1}^2, which is equivalent to minimize
$\sum_{t=1}^{n} \left(\hat{u}_t^2 - (1-\hat{\eta}^2)\sigma^2 - \sigma_\beta^2 x_{t-1}^2\right)^2$, and let

$$\sum_{t=1}^{n} \tilde{u}_t^2 = \sum_{t=1}^{n} \left(\hat{u}_t^2 - (1-\hat{\eta}^2)\sigma^2 - \sigma_\beta^2 x_{t-1}^2\right)^2 \qquad (3.5)$$

Consequently the LS estimates of σ^2 and σ_β^2 are obtained by,

$$\begin{aligned}
\frac{\partial}{\partial \sigma^2} \sum_{t=1}^{n} \tilde{u}_t^2 &= -2 \sum_{t=1}^{n} \left(\hat{u}_t^2 - (1-\hat{\eta}^2)\sigma^2 - \sigma_\beta^2 x_{t-1}^2\right) (1-\hat{\eta}^2) = 0 \\
\sum_{t=1}^{n} \hat{u}_t^2 &= n(1-\hat{\eta}^2)\sigma^2 + \sigma_\beta^2 \sum_{t=1}^{n} x_{t-1}^2 \\
\hat{\sigma}^2 &= \frac{\sum_{t=1}^{n} \hat{u}_t^2 - \sigma_\beta^2 \sum_{t=1}^{n} x_{t-1}^2}{n(1-\hat{\eta}^2)} \qquad (3.6)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial}{\partial \sigma_\beta^2}\sum_{t=1}^{n}\tilde{u}_t^2 &= -2\sum_{t=1}^{n}\left(\hat{u}_t^2-(1-\hat{\eta}^2)\sigma^2-\sigma_\beta^2 x_{t-1}^2\right)x_{t-1}^2=0\\
\sum_{t=1}^{n}\hat{u}_t^2 x_{t-1}^2 &= (1-\hat{\eta}^2)\hat{\sigma}^2\sum_{t=1}^{n}x_{t-1}^2+\sigma_\beta^2\sum_{t=1}^{n}x_{t-1}^4\\
\sigma_\beta^2\sum_{t=1}^{n}x_{t-1}^4 &= \sum_{t=1}^{n}\hat{u}_t^2 x_{t-1}^2-(1-\hat{\eta}^2)\hat{\sigma}^2\sum_{t=1}^{n}x_{t-1}^2\\
\hat{\sigma}_\beta^2 &= \frac{\sum_{t=1}^{n}\hat{u}_t^2 x_{t-1}^2-(1-\hat{\eta}^2)\hat{\sigma}^2\sum_{t=1}^{n}x_{t-1}^2}{\sum_{t=1}^{n}x_{t-1}^4} \qquad (3.7)
\end{aligned}$$

The $\hat{\sigma}^2$ and $\hat{\sigma}_\beta^2$ can be written in general form as

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{n}x_{t-1}^4\sum_{t=1}^{n}\hat{u}_t^2-\sum_{t=1}^{n}x_{t-1}^2\sum_{t=1}^{n}\hat{u}_t^2x_{t-1}^2}{n(1-\hat{\eta}^2)\sum_{t=1}^{n}x_{t-1}^4+(\sum_{t=1}^{n}x_{t-1}^2)^2(1-\hat{\eta}^2)} \qquad (3.8)$$

$$\hat{\sigma}_\beta^2 = \frac{\sum_{t=1}^{n}x_{t-1}^2\sum_{t=1}^{n}\hat{u}_t^2+n\sum_{t=1}^{n}\hat{u}_t^2x_{t-1}^2}{n\sum_{t=1}^{n}x_{t-1}^4+(\sum_{t=1}^{n}x_{t-1}^2)^2} \qquad (3.9)$$

Hence, The LS estimator of RCDR model has the form

$$x_t=\hat{\eta}y_{t-1}+\hat{\mu}_\beta x_{t-1} \quad ,t=2,3,\ldots,n \qquad (3.10)$$

3.1 Efficiency of LS Estimator

Consider the RCDR model

$$x_t=\eta y_{t-1}+\mu_\beta x_{t-1}+\varepsilon_t \quad ,t=1,2,\ldots,n \qquad (3.11)$$

We can rewrite (3.11) in terms of the matrix form of a regression model as

$$\mathbf{Y}=\mathbf{X}\underline{\beta}+\underline{\varepsilon}$$

where

$$\mathbf{Y}=\begin{bmatrix}x_1\\ \vdots\\ x_n\end{bmatrix},\quad \mathbf{X}=\begin{bmatrix}y_0 & x_0\\ \vdots & \vdots\\ y_{n-1} & x_{n-1}\end{bmatrix},\quad \underline{\beta}=\begin{bmatrix}\eta\\ \mu_\beta\end{bmatrix},\text{and}\ \underline{\varepsilon}=\begin{bmatrix}\varepsilon_1\\ \vdots\\ \varepsilon_n\end{bmatrix}$$

Nicholls and Quinn (1982) show that under the normality of β_t and ε_t, if the process is second order stationary, then $\hat{\theta}$ is consistent for θ and moreover, if the forth moments of β_t and ε_t, then $\sqrt{n}(\hat{\theta}-\theta)$ has a limiting normal distribution. In this case, it is shown that if the η and μ_β consistent estimates for $\hat{\eta}$ and $\hat{\mu}_\beta$, then $\sqrt{n}(\hat{\eta}-\eta)$ and $\sqrt{n}(\hat{\mu}_\beta-\mu_\beta)$ has a limiting normal distribution.

Phillips(1987)showed that the sample moment of $\{y_t\}$ converges to random functions of Brownian motion or Wiener process :

$$\begin{aligned} n^{-3/2}\sum_{t=1}^{n} y_{t-1} &\xrightarrow{d} \sigma_\varepsilon \int_0^1 W_1(r)dr \\ n^{-2}\sum_{t=1}^{n} y_{t-1}^2 &\xrightarrow{d} \sigma_\varepsilon^2 \int_0^1 W_1(r)^2dr \\ n^{-1}\sum_{t=1}^{n} y_{t-1}\,\varepsilon_t &\xrightarrow{d} \sigma_\varepsilon^2 \int_0^1 W_1(r)dW_1(r) \end{aligned}$$

where $W_1(r)$ denotes a standard Brownian motion. The LS estimator, the estimator ($\underline{\hat{\beta}}$) is given by $\underline{\hat{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$. Then

$$\underline{\hat{\beta}} = \begin{bmatrix} \hat{\eta} \\ \hat{\mu}_\beta \end{bmatrix} = \begin{bmatrix} \sum_{t=1}^n y_{t-1}^2 & \sum_{t=1}^n x_{t-1}\, y_{t-1} \\ \sum_{t=1}^n x_{t-1}\, y_{t-1} & \sum_{t=1}^n x_{t-1}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{t=1}^n y_{t-1}x_t \\ \sum_{t=1}^n x_{t-1}x_t \end{bmatrix}$$

when following Dickey and Fuller(1979), we use $(\underline{\hat{\beta}}-\underline{\beta}) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\underline{\varepsilon}$, so we have

$$\begin{bmatrix} \hat{\eta}-\eta \\ \hat{\mu}_\beta-\mu_\beta \end{bmatrix} = \begin{bmatrix} \sum_{t=1}^n y_{t-1}^2 & \sum_{t=1}^n x_{t-1}\, y_{t-1} \\ \sum_{t=1}^n x_{t-1}\, y_{t-1} & \sum_{t=1}^n x_{t-1}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{t=1}^n y_{t-1}\varepsilon_t \\ \sum_{t=1}^n x_{t-1}\varepsilon_t \end{bmatrix}$$

then

$$\begin{bmatrix} n(\hat{\eta}-\eta) \\ n(\hat{\mu}_\beta-\mu_\beta) \end{bmatrix} = \begin{bmatrix} \frac{\sum_{t=1}^n y_{t-1}^2}{n^2} & \frac{\sum_{t=1}^n x_{t-1}\, y_{t-1}}{n^2} \\ \frac{\sum_{t=1}^n x_{t-1}\, y_{t-1}}{n^2} & \frac{\sum_{t=1}^n x_{t-1}^2}{n^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\sum_{t=1}^n y_{t-1}\varepsilon_t}{n} \\ \frac{\sum_{t=1}^n x_{t-1}\varepsilon_t}{n} \end{bmatrix}$$

$$\xrightarrow{d} \begin{bmatrix} \sigma_\varepsilon^2 \int_0^1 W_1(r)^2dr & \sigma_\varepsilon^2 \int_0^1 W_1(r)\, W_2(r)dr \\ \sigma_\varepsilon^2 \int_0^1 W_1(r)\, W_2(r)dr & \sigma_\varepsilon^2 \int_0^1 W_2(r)^2dr \end{bmatrix}^{-1} \begin{bmatrix} \sigma_\varepsilon^2 \int_0^1 W_1(r)dW_1(r) \\ \sigma_\varepsilon^2 \int_0^1 W_2(r)dW_2(r) \end{bmatrix}$$

It shows that

$$n(\hat{\eta}-\eta) \xrightarrow{d} \frac{\left[\int_0^1 W_2(r)^2dr \int_0^1 W_1(r)dW_1(r) - \int_0^1 W_1(r)\, W_2(r)dr \int_0^1 W_2(r)dW_2(r)\right]}{\left[\int_0^1 W_1(r)^2dr \int_0^1 W_2(r)^2dr - \left(\int_0^1 W_1(r)\, W_2(r)dr\right)^2\right]}$$

and

$$n(\hat{\mu}_\beta - \mu_\beta) \xrightarrow{d} \frac{\left[\int_0^1 W_1(r)^2 dr \ \int_0^1 W_2(r) dW_2(r) - \int_0^1 W_1(r)\ W_2(r) dr \ \int_0^1 W_1(r) dW_1(r)\right]}{\left[\int_0^1 W_1(r)^2 dr \ \int_0^1 W_2(r)^2 dr - \left(\int_0^1 W_1(r)\ W_2(r) dr\right)^2\right]}$$

The t statistics of η can be written as

$$t_\eta = \frac{\hat{\eta} - \eta}{S_{\hat{\eta}}}$$

where

$$S_{\hat{\eta}} = \left\{ \sigma_\varepsilon^2 \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \sum_{t=1}^n y_{t-1}^2 & \sum_{t=1}^n x_{t-1}\ y_{t-1} \\ \sum_{t=1}^n x_{t-1}\ y_{t-1} & \sum_{t=1}^n x_{t-1}^2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}^{1/2}$$

and

$$\sigma_\varepsilon^2 = \frac{\sum_{t=1}^n (x_t - \hat{\eta}\ y_{t-1} - \hat{\mu}_\beta x_{t-1})^2}{n-2}$$

Therefore

$$S_{\hat{\eta}}^2 = \sigma_\varepsilon^2 \left[\frac{\sum_{t=1}^n x_{t-1}^2}{\sum_{t=1}^n y_{t-1}^2 \ \sum_{t=1}^n x_{t-1}^2 - (\sum_{t=1}^n x_{t-1}\ y_{t-1})^2} \right]$$

hence,

$$n^2 S_{\hat{\eta}}^2 \xrightarrow{d} \left[\frac{\int_0^1 W_2(r)^2 dr}{\int_0^1 W_1(r)^2 dr \ \int_0^1 W_2(r)^2 dr - (\int_0^1 W_1(r)\ W_2(r) dr)^2} \right] \tag{3.12}$$

Rewriting t_η as

$$t_\eta = \frac{n(\hat{\eta} - \eta)}{[n^2 S_{\hat{\eta}}^2]^{1/2}} \tag{3.13}$$

and using (3.12) and (3.13), we have that

$$\begin{aligned} t_\eta &= \frac{n(\hat{\eta} - \eta)}{[n^2 S_{\hat{\eta}}^2]^{1/2}} \qquad (3.14) \\ &\xrightarrow{d} \frac{(\int_0^1 W_2(r)^2 dr)^{1/2} \int_0^1 W_1(r) dW_1(r)}{\left[\int_0^1 W_1(r)^2 dr \ \int_0^1 W_2(r)^2 dr - \left(\int_0^1 W_1(r)\ W_2(r) dr\right)^2\right]^{1/2}} \\ &\quad - \frac{(\int_0^1 W_2(r)^2 dr)^{-1/2} \int_0^1 W_1(r)\ W_2(r) dr \ \int_0^1 W_2(r) dW_2(r)}{\left[\int_0^1 W_1(r)^2 dr \ \int_0^1 W_2(r)^2 dr - \left(\int_0^1 W_1(r)\ W_2(r) dr\right)^2\right]^{1/2}} \end{aligned}$$

The t statistics of μ_β can be written as

$$t_{\mu_\beta} = \frac{\hat{\mu}_\beta - \mu_\beta}{S_{\hat{\mu}_\beta}}$$

where

$$S_{\hat{\mu}_\beta} = \left\{ \sigma_\varepsilon^2 \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \sum_{t=1}^{n} y_{t-1}^2 & \sum_{t=1}^{n} x_{t-1}\ y_{t-1} \\ \sum_{t=1}^{n} x_{t-1}\ y_{t-1} & \sum_{t=1}^{n} x_{t-1}^2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}^{1/2}$$

and

$$\sigma_\varepsilon^2 = \frac{\sum_{t=1}^{n} (x_t - \hat{\eta}\ y_{t-1} - \hat{\mu}_\beta x_{t-1})^2}{n-2}$$

Therefore

$$S_{\hat{\mu}_\beta}^2 = \sigma_\varepsilon^2 \left[\frac{\sum_{t=1}^{n} y_{t-1}^2}{\sum_{t=1}^{n} y_{t-1}^2\ \sum_{t=1}^{n} x_{t-1}^2 - (\sum_{t=1}^{n} x_{t-1}\ y_{t-1})^2} \right]$$

hence,

$$n^2 S_{\hat{\mu}_\beta}^2 \xrightarrow{d} \left[\frac{\int_0^1 W_1(r)^2 dr}{\int_0^1 W_1(r)^2 dr\ \int_0^1 W_2(r)^2 dr - (\int_0^1 W_1(r)\ W_2(r) dr)^2} \right] \tag{3.15}$$

Rewriting t_{μ_β} as

$$t_{\mu_\beta} = \frac{n(\hat{\mu}_\beta - \mu_\beta)}{[n^2 S_{\hat{\mu}_\beta}]^{1/2}} \tag{3.16}$$

and using (3.15) and (3.16), we have that

$$\begin{aligned} t_{\mu_\beta} &= \frac{n(\hat{\mu}_\beta - \mu_\beta)}{[n^2 S_{\hat{\mu}_\beta}^2]^{1/2}} \qquad (3.17) \\ &\xrightarrow{d} \frac{(\int_0^1 W_1(r)^2 dr)^{1/2} \int_0^1 W_2(r) dW_2(r)}{\left[\int_0^1 W_1(r)^2 dr\ \int_0^1 W_2(r)^2 dr - \left(\int_0^1 W_1(r)\ W_2(r) dr \right)^2 \right]^{1/2}} \\ &\quad - \frac{(\int_0^1 W_1(r)^2 dr)^{-1/2} \int_0^1 W_1(r)\ W_2(r) dr\ \int_0^1 W_1(r) dW_1(r)}{\left[\int_0^1 W_1(r)^2 dr\ \int_0^1 W_2(r)^2 dr - \left(\int_0^1 W_1(r)\ W_2(r) dr \right)^2 \right]^{1/2}} \end{aligned}$$

By Slutsky's Theorem, under $t \to z$ in distribution as $n \to \infty$, where z is a standard normal distribution. Therefore the $\hat{\eta}$ and $\hat{\mu}_\beta$ are consistent estimator as $n \to \infty$.

Chapter 4

A Simulation Study

The simulation study to estimate parameter $\theta = (\eta, \mu_\beta, \sigma_\beta^2, \sigma^2)'$ for the performance of LS criterion. At the beginning, we generate data $y_t, t = 1, 2, \ldots, n$ from the AR(1) model by taking $\eta = 0.1, 0.5$, and 0.9 following;

$$\alpha_t = y_t = \eta y_{t-1} + \varepsilon_t : AR(1) \tag{4.1}$$

To illustrate the implication of AR model, Figure 4.1 shows the 100 sample sizes for each 3 coefficients. It should be noted that $\eta = 0.1$ is strictly stationary, $\eta = 0.5$ is weakly stationary, and $\eta = 0.9$ is the random walk case.

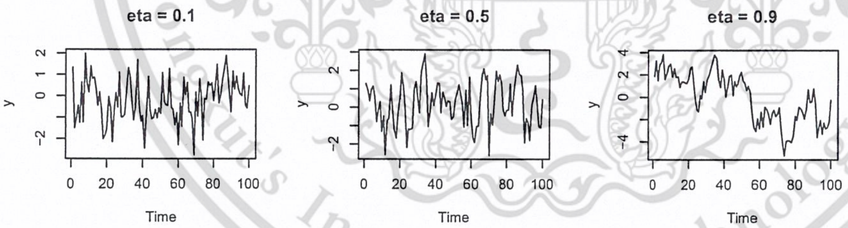

Figure 4.1: The time series plot for data generated from AR(1) of $y_t(\eta = 0.1, 0.5$, and 0.9)

Next, we consider the RCDR(1, 1) model where the AR(1) is generated from y_t. Therefore we obtain the x_t in term of RCDR(1, 1) written as

$$x_t = \eta y_{t-1} + \beta_t x_{t-1} + \varepsilon_t \tag{4.2}$$

In Figures 4.2-4.4, we present the data generating in 6 cases :

1. $\sigma^2 = 1$, $\mu_\beta = 0.5$ and $\sigma^2_\beta = 0.25$
2. $\sigma^2 = 1$, $\mu_\beta = 0.995$ and $\sigma^2_\beta = 0.01$
3. $\sigma^2 = 1$, $\mu_\beta = 0.1$ and $\sigma^2_\beta = 0.99$
4. $\sigma^2 = 1$, $\mu_\beta = -0.995$ and $\sigma^2_\beta = 0.01$
5. $\sigma^2 = 1$, $\mu_\beta = -0.1$ and $\sigma^2_\beta = 0.99$
6. $\sigma^2 = 1$, $\mu_\beta = 0$ and $\sigma^2_\beta = 1$

It should be noted that Case 2 is the random walk case and the Case 4 tends to around its mean value of 0. For Case 1, 3, 5,and 6, it can't define the character of the time series plot when x_t's depended on the η of y_t. We simulated data with the

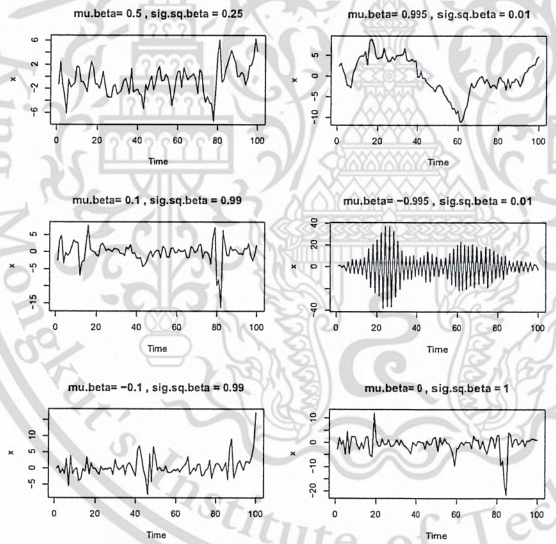

Figure 4.2: The time series plot for data generated from RCDR(1,1) of $x_t(\eta = 0.1)$

sample sizes $n = 100$ and 500, and repeated the data generation and model fitting 500 times. Tables 4.1- 4.6 show various Monte Carlo(MC) summary statistics of the estimates obtained by taking $\eta = 0.1, 0.5$ and 0.9 in 6 cases.

The third and the fourth columns of these tables represent the MC sample mean and standard deviation. The sample means for the lower and upper bounds of 95%

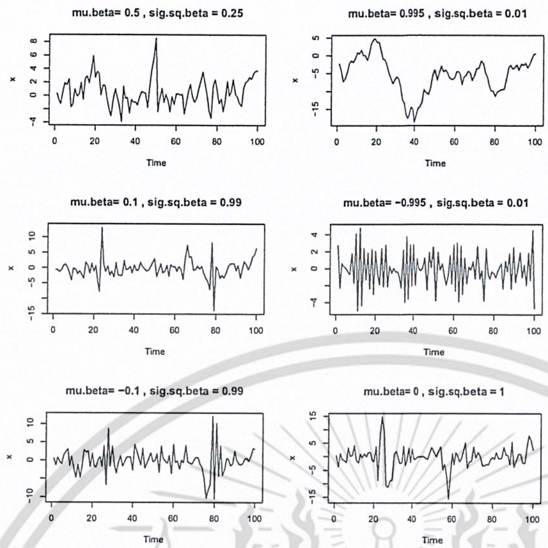

Figure 4.3: The time series plot for data generated from RCDR(1,1) of $x_t(\eta = 0.5)$

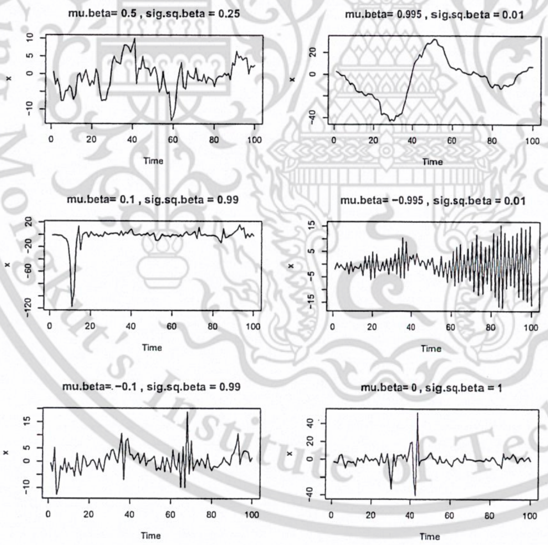

Figure 4.4: The time series plot for data generated from RCDR(1,1) of $x_t(\eta = 0.9)$

confidence interval are given in next two columns. The last two columns of these tables list the t-statistic, and p-values for hypothesis testing average of parameters.

By observing the p-values, the results appear following: from Tables 4.1- 4.6, the means of coefficient, η, under AR model provides asymptotically unbiased estimates nearly for most six generating models, especially for $\eta = 0.1$, and the average parameters seem to decrease with increasing sample sizes. However, the estimates of σ^2 is not good in these models, but the estimates of μ_β and σ_β^2 appear to be asymptotically unbiased when the data were generated in some cases.

The histogram of the coefficient estimates under AR model are presented in Figures 4.5-4.10. From the histogram is apparent that the distribution of coefficient estimates to be more normally distributed for larger sample sizes (especially for $\eta = 0.9$).

Table 4.1: The simulation of RCDR model for η = 0.1 and 100 sample sizes(500 replications)

Case	true	mean	s.d.	lci	uci	t-stat	p-value
Case 1	$\eta = 0.1$	0.107	0.212	0.088	0.125	0.745	0.456
	$\sigma^2 = 1$	2.311	0.743	2.246	2.376	39.425	0.000*
	$\mu_\beta = 0.5$	0.470	0.123	0.459	0.481	-5.301	0.000*
	$\sigma_\beta^2 = 0.25$	0.226	0.195	0.209	0.243	-2.680	0.000*
Case 2	$\eta = 0.1$	0.107	0.177	0.092	0.123	0.959	0.337
	$\sigma^2 = 1$	2.212	0.797	2.142	2.282	33.978	0.000*
	$\mu_\beta = 0.995$	0.973	0.031	0.970	0.976	-15.62	0.000*
	$\sigma_\beta^2 = 0.01$	0.009	0.011	0.008	0.010	-1.209	0.227
Case 3	$\eta = 0.1$	0.089	0.458	0.048	0.129	-0.525	0.599
	$\sigma^2 = 1$	12.082	39.054	8.561	15.514	6.345	0.000*
	$\mu_\beta = 0.1$	0.067	0.222	0.048	0.087	-3.234	0.001*
	$\sigma_\beta^2 = 0.99$	0.494	0.254	0.471	0.516	-43.52	0.000*
Case 4	$\eta = 0.1$	0.086	0.174	0.070	0.101	-1.772	0.076
	$\sigma^2 = 1$	2.318	0.745	2.252	2.383	39.55	0.000*
	$\mu_\beta = -0.995$	-0.971	0.033	-0.974	-0.968	15.88	0.000*
	$\sigma_\beta^2 = 0.01$	0.008	0.014	0.007	0.009	-2.131	0.033
Case 5	$\eta = 0.1$	0.048	0.048	0.006	0.091	-2.340	0.019
	$\sigma^2 = 1$	20.93	238.43	-0.017	41.881	1.869	0.062
	$\mu_\beta = -0.1$	-0.093	0.226	-0.113	-0.073	0.679	0.497
	$\sigma_\beta^2 = 0.99$	0.461	0.233	0.441	0.482	-50.67	0.000*
Case 6	$\eta = 0.1$	0.108	0.480	0.066	0.150	0.401	0.688
	$\sigma^2 = 1$	13.121	75.151	6.518	19.724	3.606	0.000*
	$\mu_\beta = 0$	0.003	0.214	-0.015	0.219	0.330	0.741
	$\sigma_\beta^2 = 1$	0.684	0.371	0.652	0.717	-18.99	0.000*

*indicates significance at 1% level

Table 4.2: The simulation of RCDR model for η = 0.1 and 500 sample sizes(500 replications)

Case	true	mean	s.d.	lci	uci	t-stat	p-value
Case 1	$\eta = 0.1$	0.105	0.096	0.097	0.114	1.306	0.192
	$\sigma^2 = 1$	2.163	0.357	2.132	2.195	72.84	0.000*
	$\mu_\beta = 0.5$	0.065	0.065	0.489	0.500	-1.634	0.102
	$\sigma_\beta^2 = 0.25$	0.216	0.096	0.208	0.225	-7.761	0.000*
Case 2	$\eta = 0.1$	0.105	0.129	0.094	0.117	0.995	0.318
	$\sigma^2 = 1$	2.161	1.923	1.992	2.330	13.501	0.000*
	$\mu_\beta = 0.995$	0.987	0.008	0.986	0.988	-20.00	0.000*
	$\sigma_\beta^2 = 0.01$	0.009	0.003	0.009	0.010	-0.275	0.783
Case 3	$\eta = 0.1$	0.128	0.297	0.102	0.154	2.133	0.033
	$\sigma^2 = 1$	23.34	163.5	8.479	37.72	3.056	0.002*
	$\mu_\beta = 0.1$	0.089	0.168	0.074	0.103	-1.439	0.150
	$\sigma_\beta^2 = 0.99$	0.532	0.196	0.515	0.549	-52.04	0.000*
Case 4	$\eta = 0.1$	0.093	0.103	0.084	0.102	-1.461	0.144
	$\sigma^2 = 1$	2.043	1.230	1.935	2.151	18.964	0.000*
	$\mu_\beta = -0.995$	-0.986	0.008	-0.987	-0.985	22.58	0.000*
	$\sigma_\beta^2 = 0.01$	0.009	0.003	0.009	0.010	-0.363	0.716
Case 5	$\eta = 0.1$	0.087	0.282	0.062	0.112	-1.010	0.312
	$\sigma^2 = 1$	13.90	33.11	10.99	16.81	8.71	0.000*
	$\mu_\beta = -0.1$	-0.083	0.178	-0.098	-0.067	2.119	0.034
	$\sigma_\beta^2 = 0.99$	0.504	0.147	0.491	0.517	-73.76	0.000*
Case 6	$\eta = 0.1$	0.108	0.298	0.081	0.134	0.601	0.548
	$\sigma^2 = 1$	15.61	105.74	6.320	24.902	3.089	0.002*
	$\mu_\beta = 0$	0.009	0.178	-0.005	0.025	1.233	0.218
	$\sigma_\beta^2 = 1$	0.494	0.145	0.481	0.507	-77.861	0.000*

*indicates significance at 1% level

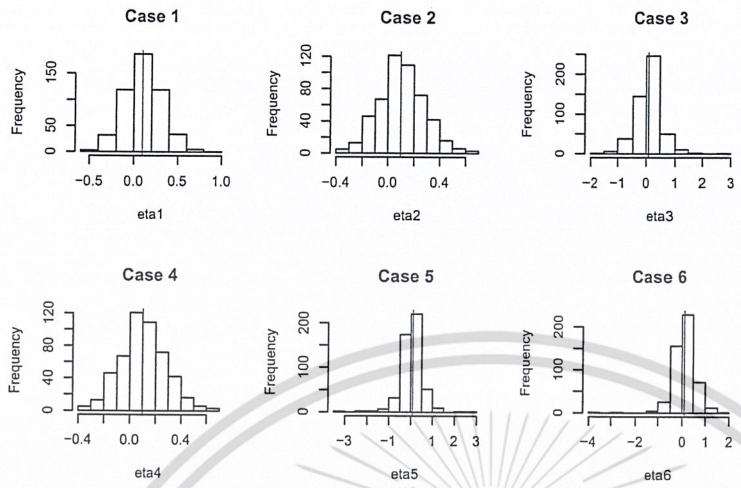

Figure 4.5: The histogram of $\eta = 0.1$ for $n = 100$ in 6 cases

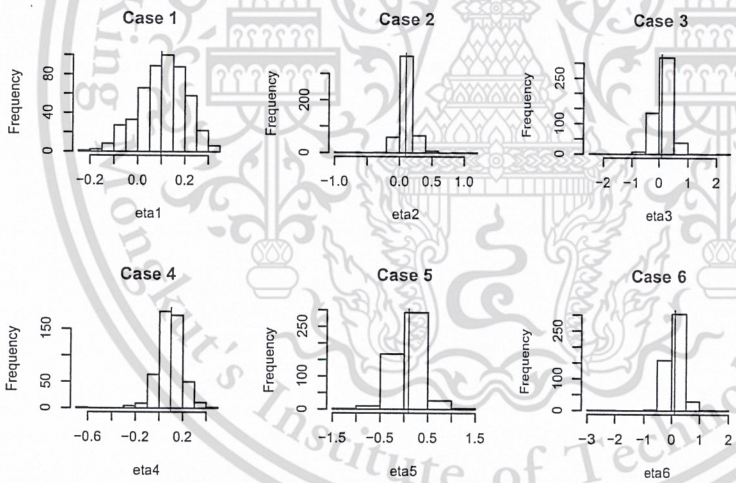

Figure 4.6: The histogram of $\eta = 0.1$ for $n = 500$ in 6 cases

Table 4.3: The simulation of RCDR model for η = 0.5 and 100 sample sizes(500 replications)

Case	true	mean	s.d.	lci	uci	t-stat	p-value
Case 1	$\eta = 0.5$	0.560	0.214	0.541	0.579	6.303	0.000*
	$\sigma^2 = 1$	11.975	3.312	11.68	12.26	74.08	0.000*
	$\mu_\beta = 0.5$	0.455	0.126	0.444	0.446	-7.881	0.000*
	$\sigma_\beta^2 = 0.25$	0.183	0.133	0.172	0.194	-11.21	0.000*
Case 2	$\eta = 0.5$	0.500	0.199	0.483	0.518	0.074	0.940
	$\sigma^2 = 1$	2.886	1.670	2.739	3.033	25.253	0.000*
	$\mu_\beta = 0.995$	0.978	0.024	0.976	0.981	-14.427	0.000*
	$\sigma_\beta^2 = 0.01$	0.010	0.007	0.009	0.010	0.414	0.678
Case 3	$\eta = 0.5$	0.523	0.476	0.482	0.565	1.124	0.261
	$\sigma^2 = 1$	320.2	1152.8	218.9	421.5	6.191	0.000*
	$\mu_\beta = 0.1$	0.077	0.237	0.056	0.098	-2.102	0.035
	$\sigma_\beta^2 = 0.99$	0.677	0.422	0.640	0.715	-16.51	0.000*
Case 4	$\eta = 0.5$	0.478	0.154	0.464	0.491	-3.148	0.001*
	$\sigma^2 = 1$	2.901	0.880	2.824	2.978	48.27	0.000*
	$\mu_\beta = -0.995$	-0.968	0.036	-0.971	-0.965	16.151	0.000*
	$\sigma_\beta^2 = 0.01$	0.011	0.018	0.010	0.013	2.229	0.026
Case 5	$\eta = 0.5$	0.503	0.478	0.461	0.545	0.142	0.887
	$\sigma^2 = 1$	12.01	21.34	10.14	13.89	11.53	0.000*
	$\mu_\beta = -0.1$	-0.097	0.213	-0.115	0-0.078	0.299	0.764
	$\sigma_\beta^2 = 0.99$	0.455	0.234	0.434	0.476	-50.86	0.000*
Case 6	$\eta = 0.5$	0.494	0.473	0.452	0.536	-0.261	0.793
	$\sigma^2 = 1$	13.283	29.72	10.67	15.89	9.239	0.000*
	$\mu_\beta = 0$	-0.009	0.212	-0.028	0.008	-1.049	0.294
	$\sigma_\beta^2 = 1$	0.490	0.254	0.468	0.512	-44.78	0.000*

*indicates significance at 1% level

Table 4.4: The simulation of RCDR model for η = 0.5 and 500 sample sizes(500 replications)

Case	true	mean	s.d.	lci	uci	t-stat	p-value
Case 1	$\eta = 0.5$	0.514	0.117	0.504	0.525	2.835	0.004
	$\sigma^2 = 1$	2.546	0.733	2.482	2.611	47.11	0.000*
	$\mu_\beta = 0.5$	0.488	0.068	0.482	0.494	-3.683	0.000*
	$\sigma^2_\beta = 0.25$	0.243	0.116	0.233	0.253	-1.248	0.212
Case 2	$\eta = 0.5$	0.522	0.225	0.502	0.542	2.216	0.027
	$\sigma^2 = 1$	3.787	23.81	1.695	5.880	2.617	0.009*
	$\mu_\beta = 0.995$	0.987	0.007	0.987	0.984	-21.52	0.000*
	$\sigma^2_\beta = 0.01$	0.010	0.003	0.009	0.010	1.531	0.126
Case 3	$\eta = 0.5$	0.504	0.331	0.475	0.533	0.314	0.753
	$\sigma^2 = 1$	41.20	142.9	28.64	53.77	6.287	0.000*
	$\mu_\beta = 0.1$	0.084	0.183	0.068	0.110	-1.932	0.053
	$\sigma^2_\beta = 0.99$	0.520	0.215	0.501	0.539	-48.72	0.000*
Case 4	$\eta = 0.5$	0.497	0.107	0.487	0.506	-0.608	0.542
	$\sigma^2 = 1$	3.362	6.231	2.814	3.909	8.477	0.000*
	$\mu_\beta = -0.995$	-0.986	0.010	-0.987	-0.985	18.71	0.000*
	$\sigma^2_\beta = 0.01$	0.010	0.003	0.009	0.010	0.552	0.580
Case 5	$\eta = 0.5$	0.458	0.293	0.432	0.484	-3.173	0.001*
	$\sigma^2 = 1$	21.31	99.33	12.58	30.04	4.573	0.000*
	$\mu_\beta = -0.1$	-0.081	0.166	-0.095	-0.066	2.519	0.012
	$\sigma^2_\beta = 0.99$	0.496	0.137	0.484	0.509	-80.60	0.000*
Case 6	$\eta = 0.5$	0.509	0.264	0.486	0.533	0.822	0.405
	$\sigma^2 = 1$	15.42	36.50	12.21	18.63	8.835	0.000*
	$\mu_\beta = 0$	-0.010	0.163	-0.024	0.004	-1.390	0.165
	$\sigma^2_\beta = 1$	0.491	0.141	0.478	0.503	-80.38	0.000*

*indicates significance at 1% level

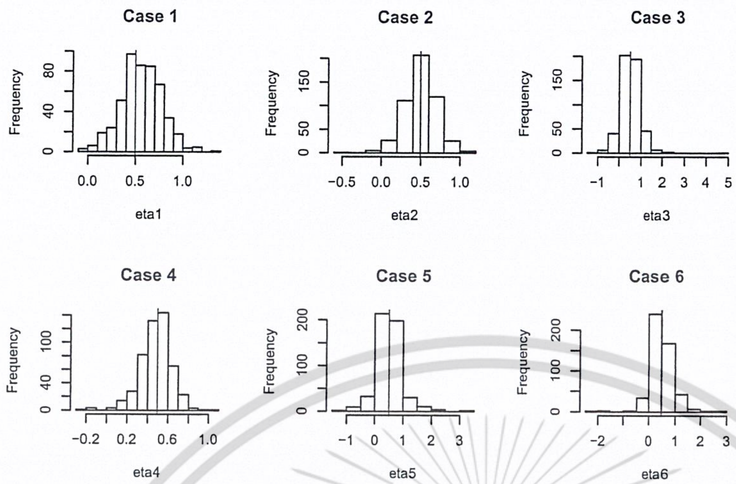

Figure 4.7: The histogram of $\eta = 0.5$ for $n = 100$ in 6 cases

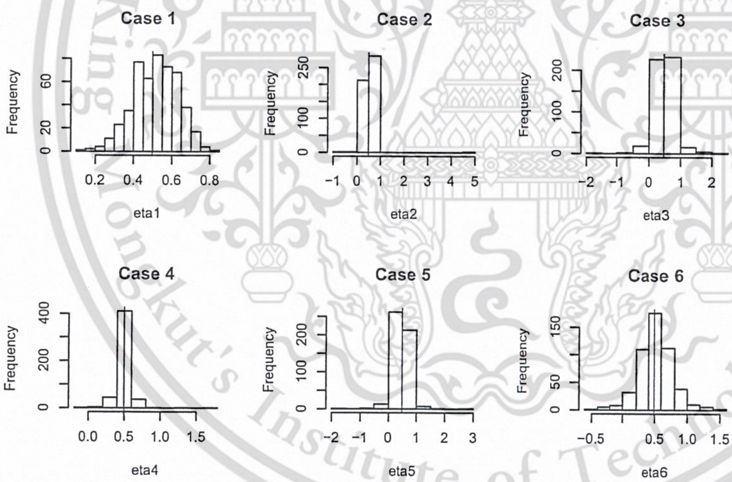

Figure 4.8: The histogram of $\eta = 0.5$ for $n = 500$ in 6 cases

Table 4.5: The simulation of RCDR model for η = 0.9 and 100 sample sizes(500 replications)

Case	true	mean	s.d.	lci	uci	t-stat	p-value
Case 1	$\eta = 0.9$	0.988	0.246	0.966	1.009	7.969	0.000*
	$\sigma^2 = 1$	9.071	11.157	8.090	10.05	16.17	0.000*
	$\mu_\beta = 0.5$	0.434	0.133	0.422	0.445	-11.03	0.000*
	$\sigma_\beta^2 = 0.25$	0.257	0.183	0.241	0.273	0.879	0.379
Case 2	$\eta = 0.9$	0.965	0.272	0.942	0.989	5.420	0.000*
	$\sigma^2 = 1$	30.15	160.9	16.00	44.29	4.049	0.000*
	$\mu_\beta = 0.995$	0.982	0.016	0.981	0.984	-16.56	0.000*
	$\sigma_\beta^2 = 0.01$	0.009	0.004	0.009	0.010	-1.293	0.196
Case 3	$\eta = 0.9$	0.929	0.480	0.886	0.971	1.354	0.176
	$\sigma^2 = 1$	66.48	285.3	41.41	91.55	5.131	0.000*
	$\mu_\beta = 0.1$	0.064	0.239	0.043	0.085	-3.301	0.001*
	$\sigma_\beta^2 = 0.99$	0.640	0.344	0.610	0.670	-22.67	0.000*
Case 4	$\eta = 0.9$	0.867	0.099	0.858	0.876	-7.321	0.000*
	$\sigma^2 = 1$	46.95	17.03	45.45	48.44	60.33	0.000*
	$\mu_\beta = -0.995$	-0.970	0.037	-0.973	-0.967	14.603	0.000*
	$\sigma_\beta^2 = 0.01$	0.011	0.016	0.009	0.012	1.577	0.15
Case 5	$\eta = 0.9$	0.885	0.436	0.846	0.923	-0.752	0.452
	$\sigma^2 = 1$	56.26	244.8	34.75	77.77	5.047	0.000*
	$\mu_\beta = -0.1$	-0.087	0.211	-0.106	-0.068	1.322	0.186
	$\sigma_\beta^2 = 0.99$	0.455	0.195	0.438	0.473	-60.96	0.000*
Case 6	$\eta = 0.9$	0.903	0.379	0.870	0.936	0.201	0.84
	$\sigma^2 = 1$	54.56	271.4	30.70	78.41	4.411	0.000*
	$\mu_\beta = 0$	-0.021	0.226	-0.041	-0.001	-2.142	0.032
	$\sigma_\beta^2 = 1$	0.475	0.227	0.455	0.495	-51.49	0.000*

*indicates significance at 1% level

Table 4.6: The simulation of RCDR model for η = 0.9 and 500 sample sizes(500 replications)

Case	true	mean	s.d.	lci	uci	t-stat	p-value
Case 1	$\eta = 0.9$	0.938	0.127	0.927	0.949	6.760	0.000*
	$\sigma^2 = 1$	25.41	18.00	23.83	26.99	30.32	0.000*
	$\mu_\beta = 0.5$	0.470	0.076	0.463	0.477	-8.514	0.000*
	$\sigma^2_\beta = 0.25$	0.215	0.100	0.206	0.224	-7.582	0.000*
Case 2	$\eta = 0.9$	0.940	0.269	0.917	0.964	3.379	0.000*
	$\sigma^2 = 1$	3.485	646.0	-53.27	60.24	0.086	0.931
	$\mu_\beta = 0.995$	0.988	0.006	0.988	0.989	-20.62	0.000*
	$\sigma^2_\beta = 0.01$	0.009	0.002	0.009	0.010	-0.115	0.908
Case 3	$\eta = 0.9$	0.924	0.281	0.900	0.949	1.967	0.049
	$\sigma^2 = 1$	116.75	824.6	44.29	189.2	3.138	0.001*
	$\mu_\beta = 0.1$	0.073	0.172	0.058	0.088	-3.399	0.000*
	$\sigma^2_\beta = 0.99$	0.532	0.186	0.515	0.548	-54.90	0.000*
Case 4	$\eta = 0.9$	0.891	0.052	0.887	0.896	-3.606	0.000*
	$\sigma^2 = 1$	19.99	12.60	18.89	21.10	33.69	0.000*
	$\mu_\beta = -0.995$	-0.986	0.009	-0.987	-0.985	19.517	0.000*
	$\sigma^2_\beta = 0.01$	0.009	0.003	0.009	0.010	-0.391	0.6954
Case 5	$\eta = 0.9$	0.879	0.257	0.856	0.901	-1.792	0.073
	$\sigma^2 = 1$	58.69	116.02	48.49	68.88	11.11	0.000*
	$\mu_\beta = -0.1$	-0.087	0.182	-0.103	-0.071	1.564	0.118
	$\sigma^2_\beta = 0.99$	0.523	0.156	0.509	0.536	-66.75	0.000*
Case 6	$\eta = 0.9$	0.907	0.237	0.886	0.928	0.711	0.477
	$\sigma^2 = 1$	123.03	241.37	101.82	144.24	11.305	0.000*
	$\mu_\beta = 0$	-0.005	0.179	-0.021	0.010	-0.698	0.485
	$\sigma^2_\beta = 1$	0.516	0.149	0.503	0.529	-72.437	0.000*

*indicates significance at 1% level

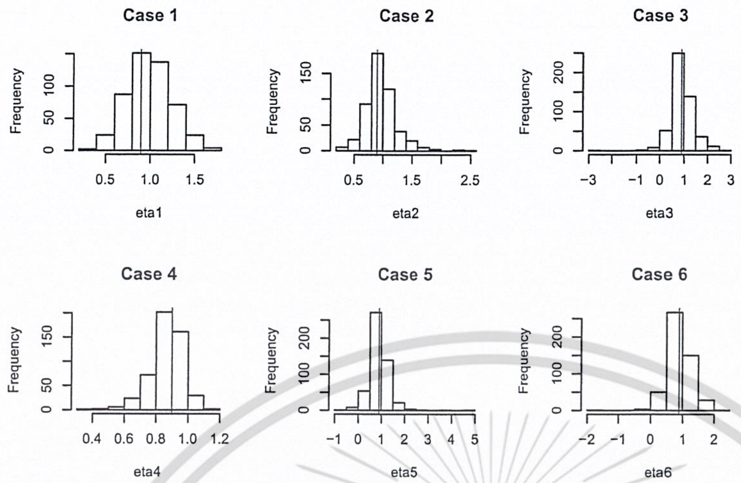

Figure 4.9: The histogram of $\eta = 0.9$ for $n = 100$ in 6 cases

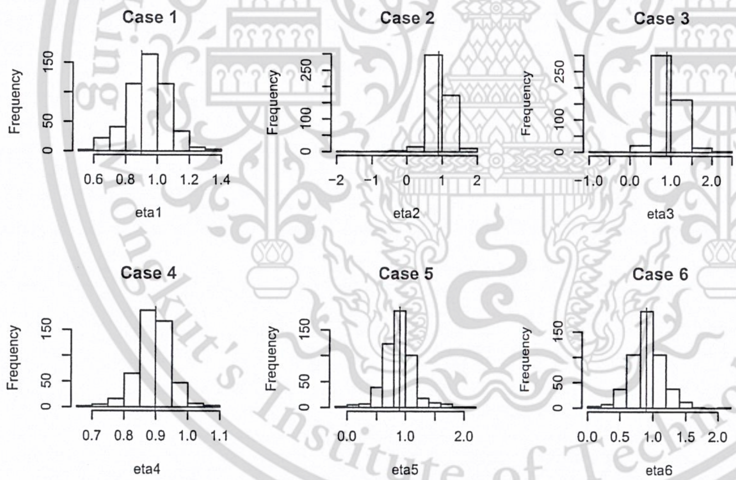

Figure 4.10: The histogram of $\eta = 0.9$ for $n = 500$ in 6 cases

Chapter 5

Applications for Real Data

In this section, we will consider the application of RCDR model using the Least Squares criterion that we developed in the previous section. The real data, we use the daily volume for Thai Baht to one pond (GBP) which is the official currency of the United Kingdom and Thai Baht to one euro (EUR) which is the official currency of the 27 member states of the European Union (EU) from May, 2005 to July, 2010 that consisted of 1,366 records. These data were obtained from `http://www.data360.org/ds_list.aspx`. The process for predictive future values following:

For starters, we considered the RCDR model following

$$\begin{aligned} x_t &= \eta y_{t-1} + \beta_t x_{t-1} + \varepsilon_t \\ \beta_t &= \mu_\beta + \sigma_\beta u_t \end{aligned}$$

where ε_t's are independently and identically distributed with mean 0 and variance σ^2.

Next step for parameter estimation, we fitted the RCDR model to obtain parameter estimator $\hat{\theta} = (\hat{\eta}, \hat{\mu}_\beta, \hat{\sigma}^2_\beta, \hat{\sigma}^2)'$.

Finally, we obtain forecast values of $x_{n+1}, \ldots, x_{n+m}$ using the LS criterion based on the RCDR model:

$$\hat{x}_{n+m} = \hat{\eta}\hat{y}_{n+m-1} + \hat{\mu}_\beta \hat{x}_{n+m-1} \tag{5.1}$$

We also computed the Mean Square Error (MSE) defined as follow:

$$\text{MSE} = \frac{1}{m}\sum_{k=1}^{m}(\hat{x}_{n+k} - x_{n+k})^2,$$

where $\hat{x}_{n+k}$ denotes predicted value of x_{n+k} obtained as the predictive distribution defined by equation 5.1.

Let y_t denote the daily volume for Thai Baht to one GBP and x_t denote the daily volume for Thai Baht to one EUR shown the time series plot in Figure 5.1. The

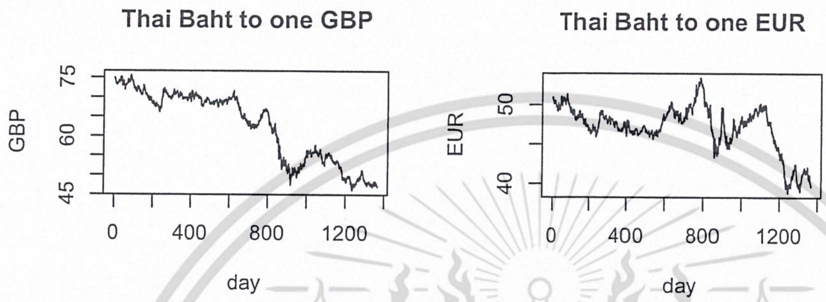

Figure 5.1: The time series plot for Thai Baht to one GBP and EUR

forecasting future values for next 20 days are shown in Table 5.1 . In comparison, we also proposed the different of true values, forecast values, and prediction interval in Figure 5.2.

It follows from the Figure 5.2 that the LS criterion for RCDR model provides predictive intervals that capture most of the true values under 95% predictive interval. The trend of forecasting future values looks decreasing as the actual values.

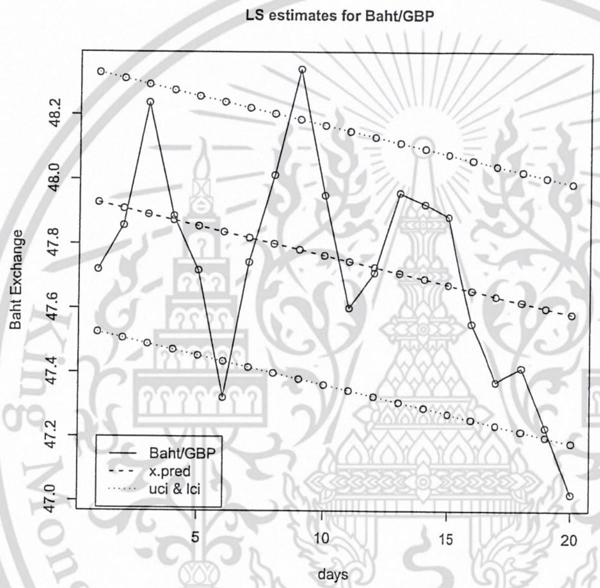

Figure 5.2: The actual exchange rate of Thai Baht to one GBP (Baht/GBP), forecast values (x.pred) and 95% prediction interval of future values (uci and lci) for LS criterion

Table 5.1: The actual Baht to 1 GBP, the forecast values (FV), 95% prediction interval of future values (95%PI), Mean Square Error (MSE)

m	Baht to 1 GBP	FV	95%PI
1	47.7208	47.9277	(47.5263,48.3291)
2	47.8574	47.9096	(47.5082,48.311)
3	48.2366	47.8915	(47.4899,48.293)
4	47.8866	47.8733	(47.4717,48.275)
5	47.7181	47.8552	(47.4534,48.257)
6	47.3218	47.8371	(47.4352,48.239)
7	47.7435	47.819	(47.417,48.221)
8	48.0134	47.8009	(47.3988,48.203)
9	48.3398	47.7828	(47.3806,48.185)
10	47.9509	47.7647	(47.3624,48.167)
11	47.6016	47.7466	(47.3442,48.149)
12	47.7119	47.7285	(47.326,48.131)
13	47.9587	47.7105	(47.3079,48.1131)
14	47.9235	47.6924	(47.2897,48.0951)
15	47.8849	47.6744	(47.2715,48.0772)
16	47.5529	47.6563	(47.2534,48.0592)
17	47.3716	47.6383	(47.2352,48.0413)
18	47.4157	47.6202	(47.2171,48.0234)
19	47.2292	47.6022	(47.199,48.0054)
20	47.0218	47.5842	(47.1809,47.9875)
MSE	-	0.0011	-

Chapter 6

Conclusion

In this section, we have developed the LS criterion with RCDR model. Through a Monte Carlo simulation study, we evaluated the performance of LS criterion method procedure and showed that the coefficient estimators ($\hat{\eta}(\cdot)$) work reasonably well for data of most to moderate sample sizes. The point estimates for parameters approach their corresponding true values as the sample sizes increase. For hypothesis testing based on the p-value, the estimator of σ^2 never support the null value, but μ_β and σ^2_β estimators can apply to estimate the null value in some cases.

For application in actual data, we are also interested the power of forecasting future values by considering the Mean Square Error (MSE). We can see that the LS criterion method performs significantly in terms of capturing future values within a 95% predictive interval.

As a part of further work, we will study the following aspects:

- Simultaneous estimation of the RCDR model using Maximum Likelihood (ML) method approach.

- Include the higher order RCDR models.

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