A RISK MODEL OF AIRBORNE TRANSMISSION WITH VACCINE EFFICACY IN AN OUTPATIENT ROOM WITH A VENTILATION SYSTEM

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Thesis Title	A Risk Model of Airborne Transmission with Vaccine Efficacy in an Outpatient Room with a Ventilation System
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Abstract

TB, COVID-19, MERS, and SARS are all significant infectious diseases that are transmitted by the air or aerosol via coughing, spitting, sneezing, speaking, or wounds. Human breath emits a lot of carbon dioxide, which contributes a lot to airborne infections. We should also be informed about how patients in the outpatient room are managed. A risk model of airborne transmission in an outpatient room with a ventilation system is provided for variable patient quantities and vaccine efficacy. As can be seen, the probability of infection depends on the number of people present, the rate of ventilation, and the efficacy of each type of vaccination. The fourth-order Runge-Kutta method is used to approximate the model solution. Several scenarios for improving air quality are presented in the simulations. In the air quality management process, the proposed technique balances the number of people allowed to stay in the room with the capacity of the air ventilation system. We can see that the sufficiently ventilated system and the efficacy of each type of vaccination can reduce the risk of airborne infection in an outpatient room in a hospital. This research could be utilized to help control the risk of airborne infection to the desired level if there is a public vaccination database system.

Keywords: Breath, Carbon dioxide, Outpatient room, Ventilation, Mathematical model

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Abbreviations/Symbols

Symbols	Description
C	An outpatient room exhaled air concentration (ppm)
C_a	The carbon dioxide fraction contained in breathed air (ppm)
V	An outpatient room of volume (m^3)
Q	The ventilation rate (L/s)
p	The breathing rate (L/s)
$n(t)$	The total number of people
t	Time (min)
$f(t)$	The volume fraction of exhaled air
$N(t)$	The concentration of airborne infectious particles
$\lambda(t)$	The number of airborne infectious particles
pt	The volume of breathed air by susceptible
β	The total airborne infectious particles generation rate released by an infector
μ	The mortality rate of generated airborne infectious particles by the infector that do not reach the alveolar
θ	A respiratory deposition fraction of airborne infectious particles
I	The number of infectors in an outpatient room

Chapter 1
Introduction

1.1 Research motivation

Due to the current situation of the epidemic of infectious diseases related to the respiratory system, such as tuberculosis, COVID-19, the common cold, influenza, avian influenza, and SARS, etc., these diseases are still increasing.

As you know, the world is still concerned about the spread of the coronavirus disease 2019 (COVID-19) which directly affects human health. And can lead to acute respiratory failure and pneumonia. This virus is regarded as a global health disaster. Causing many deaths, the main route of transmission of the virus from person to person is through droplets from the respiratory system (coughing, sneezing, saliva, snot). When the body inhales these secretions, it can become infected. As a result, most countries have introduced "Social Distancing" measures to help control the spread and reduce the impact of the virus. In particular, people living in areas with poor air pollution may be at higher risk of contracting COVID-19.Air pollution It is also regarded as a major problem worldwide with the fifth-highest mortality rate among ischemic heart disease-related air pollution deaths 24%, chronic obstructive pulmonary disease 23%, and stroke 21%. And cardiovascular disease 19%.

In a 2003 study, researchers from the UCLA School of Public Health found that people exposed to high levels of air pollution were twice as likely to die from SARS. It is of the same family and is 80% similar to the novel coronavirus (COVID-19) (formally known as SARS-CoV-2), with the death rate of infected people in polluted areas as high as 8.9. % Compared to areas with less pollution, 4.08%.

Recognizing this problem, we conducted a modeling on the risk of infection in hospital outpatient rooms because hospitals are direct healthcare facilities. Therefore, we used a risk model for infection within the outpatient room to control the risk of not being too high. At the desired level and control the number of people who come to use the service in the outpatient room each day.

1.2 Literature review

In [1], they develop and demonstrate a flexible mathematical model that predicts the risk of airborne infectious diseases, such as tuberculosis, under steady

state and non-steady state conditions by monitoring exhaled air by infectors in a confined space. In the development of this model, we used the rebreathed air accumulation rate concept to directly determine the average volume fraction of exhaled air in a given space. From a biological point of view, exhaled air by infectors contains airborne infectious particles that cause airborne infectious diseases such as tuberculosis in confined spaces. Since not all infectious particles can reach the target infection site, they took into account that the infectious particles that commence the infection are determined by respiratory deposition fraction, which is the probability of each infectious particle reaching the target infection site of the respiratory tracts and causing infection. Furthermore, we compute the quantity of carbon dioxide as a marker of exhaled air, which can be inhaled in the room with high likelihood of causing airborne infectious disease given the presence of infectors. We demonstrated mathematically and schematically the correlation between TB transmission probability and airborne infectious particle generation rate, ventilation rate, average volume fraction of exhaled air, TB prevalence and duration of exposure to infectors in a confined space.

In [2], this research, a mathematical model for the risk analysis of airborne infectious disease in an outpatient room is proposed. Not only considering one type of person but also in this research, people are considered according to personal classifications. There are 4 types - patient, relative, worker, and outsider, staying in an outpatient room, which is in accordance with the real world. Air quality control manipulations are simulated using the inlet and outlet ventilation rates adjustment under the condition of a number of surrounding people with a personal classified factor. The fourth-order Runge-Kutta (RK4) is used to approximate the model solution. The proposed numerical model can be used to describe the dynamical dispersion of airborne infectious disease in an outpatient room. The results of the model are satisfactory, and it will be able to control airborne disease in more complicated structures.

In [3], US scientists in the laboratory have shown that the virus can live in an aerosol and remain infectious for at least 3 hours. Tuberculosis (commonly known as TB), this communicable disease is caused by Mycobacterium Tuberculosis, which most often affects the lungs. At present, we have an effective TB disinfectant. TB can be treated, but recovery takes a long time. If the treatment is not continued, or is

incomplete, death may result. Therefore, TB is an important public health issue in Thailand.

In [4], a new procedure was developed to study the distribution of epidemics for predicting the possibility of airborne infectious diseases in high-density urban areas. It can analyze the chance of spread in sub-transportation, and it can also help understand dispersion of airborne diseases in public transportation in China.

In [5], the researchers studied the behaviors of Korean TB infection. TB transmission dynamic was proposed by using mathematical TB model with exogenous reinfection. Then, the least squares method was used to approximate the considered parameters. From the results, the most significant factor was the case finding effort, which led to a decrease of active TB patients.

In [6], the researchers developed an infectious diseases model of SARS by using two methods for estimating both small-scale SARS outbreak parameter at the Amoy Gardens, Hong Kong, and large-scale outbreak parameter in the entire Hong Kong Special Administrative Region.

In [7], the inpatient nursing records from EMR of the University of Miyazaki Hospital were analyzed by using a text data mining technique. This result indicated that vocabulary related to appropriate treatment methods.

In [8], airflow and the airborne spread of infectious agents from an indoor environment was focused on. From this, it was confirmed that infected individuals and susceptible individuals should use masks, and also should use personalized ventilation for a short-range airborne route.

In [9], the researchers predicted an airborne disease transmission from infected patients in high-rise hospitals. This simulation could analyze the ventilation system by using multi-zone airflow simulation and tracer (CFD) simulation.

In [10], basic epidemiological and multivariate state-space models are proposed to predict optimal control measure strategies. This approach can be used for various diffusion diseases include Ebola, MERS.

In [11], [12], a vaccination strategy for the SEIR model was designed. It was oriented towards the measurement and used for the infectious population to epidemic models for designs the general time-varying, the vaccination control rule.

1.3 Objectives of the study

1) We will study the risk of airborne infection by modeling in a normal room.

2) We will introduce a risk of airborne infection through modeling in an outpatient room.

3) We will implement a method of controlling the risk of airborne infection in an outpatient room.

4) To introduce an effect of a risk of airborne infection while the number of people in an outpatient room is varied.

1.4 Research methodology

1) Their related research on modeling the risk of airborne infection in a normal room will be reviewed.

2) A risk of airborne infection through modeling in an outpatient room will be proposed.

3) Techniques for parameter setting will be proposed.

4) Initial condition of the model will be implemented.

5) Numerical methods will be selected to estimate the model solution.

6) Several different scenarios will be simulated.

7) Discuss and summarize the results of the experiment.

1.5 Scope(s) of the study

1) Only study in a ventilated outpatient room.

2) Examine the number of people in the outpatient department.

3) Examine each vaccine's efficacy in terms of lowering the risk of airborne infections.

4) The parameters for this study were the carbon dioxide fraction in breathed air, the volume of an outpatient room, the ventilation rate, the breathing rate, and the total number of people.

1.6 Benefits of the study

1) We can control the risk of infection in an outpatient room to the desired level.

2) We can assess the quality of the air in an outpatient room.

3) We can control the number of people in an outpatient room.

Chapter 2
Basic knowledge

2.1 History and mechanism of airborne transmission

In 1930 William F. Wells, a sanitation engineer from Harvard University. A study was conducted with medical student Richard Reiley and reported that the nebulizer core (droplet nuclei) that contain microorganisms It is what causes the spread of infection through the air. due to particles. Which has a size of ≤ 5 microns can be suspended in the air for a long time without falling to the ground. and will float in the air, going very far from the origin. This corresponds to the equation used to calculate the velocity at particle size 1-100 microns will fall to the ground, proposed by Lewis Strokes. From the above equation, it was found that the particle size 1-5 microns in the still air stream There will be a fall rate of 1 yard per hour. However, if there is a wind the air suspension force of such particles is longer when inhaled particles size 1-5 micron is able to pass through the cilia and mucosal defenses in the upper respiratory tract.

Collected in the air sacs of the lungs (alveoli). If the pathogen is still alive on the core of the aerosol, it will cause disease. The study of airborne pathogen transmission was extensive after the incidence of tuberculosis. Disease on the rise including the incidence of severe acute respiratory syndrome from SARS Corona virus. which has evidence that it is an airborne disease in some conditions causing the current division of pathogens Air diffusion designed into 3 groups: [27]

1.Obligated airborne transmission: Pathogens use the airborne transmission method as the main method. Diseases that have been shown to spread this way are tuberculosis, particularly pulmonary and laryngeal tuberculosis. Measles, Aspergillus spp. and Rhizopus spp. may be in this group. Because there is clear evidence that air diffusion is the main channel.

2. Preferential airborne transmission: Pathogens in this group can spread in many ways, but if spread through the air or in the form of aerosol and accumulate in the tip of the lungs. will cause the germs to spread throughout the body and have a full-blown disease. Pathogens in this group include Varicella-Zoster, Smallpox, Acremonium spp. for influenza and avian influenza (Influenza A H5N1), there is evidence that it may be

in this group. especially bird flu Influenza A H5N1 is found to have receptors located at the tip of the lungs. And there is a division of the virus in that location as well. Germs in this group if spread by other means. The severity of the disease will decrease.
3.Opportunistically airborne transmission: Germs in this group naturally spread. By other means, but in certain circumstances, for example, the germ takes the form of aerosol and is inhaled into the tip of the lungs can cause disease. Pathogens that are likely to be in this group include SARS Corona Virus and Viral hemorrhagic fever groups include Ebola, Lassa, Marburg, Hanta.

In addition, air-borne diseases may be divided into 2 groups according to the risk of the person exposed. That is to say

- The first group is a group of diseases in which normal people without immunity to that disease are at risk of contracting the disease. Diseases: Measles virus, Varicella-Zoster virus, Smallpox, SARS corona virus, Influenza A H5N1 and viral hemorrhagic fever. Prevention of the spread of these diseases. Prevention must be used at the source of the disease (patient), i.e. The patient must be placed in an isolated room to prevent airborne transmission. And those who will be in the same room as the patient must wear a mask that protects against particles < 5 microns in size. Known as Airborne precautions, the primary approach is administrative control, environmental control. and respiratory protection control.
- The second group is a group of diseases that often cause disease only in people with impaired immunity, namely Fungi such as Aspergillus spp., Rlizopuss spp., Acremonium spp., Fusarium spp. These diseases are necessary for specific risk groups, including organ transplant patients by having to do keep the air in these patients' isolated rooms to be contaminated with this pathogen to a minimum. The air that is supplied into the patient room is filtered with a HEPA filter, and the patient's room air pressure must be positive (positive air pressure) relative to the ambient outside air. To prevent the air from outside which may be contaminated with germs. not to enter the patient's room through the holes in the room.

2.1.1 Environmental controls

Prevention and control of airborne diseases the aim was to reduce the concentration of droplet nuclei in the air using engineering principles and this control architecture is divided into two levels: [28]

Primary environmental controls are control at the source of infection, possibly patient or laboratory specimens, by using local exhaust ventilation from hoods used in laboratories. or the patient's sputum storage room through dilution and elimination of germs. by the normal ventilation system of the building (General ventilation).

Secondary environmental controls are the control of the air around the disease source, such as the surrounding air-borne isolation room, by controlling the direction of the airflow, filtering with an air filter or using UVGI. Ultraviolet

germicidal irradiation) these environmental controls are discussed in detail in the section on Air Quality in Healthcare Facilities. and air conditioning system design standards and ventilation in hospitals.

2.1.2 Air quality and air conditioning system design standards and ventilation in hospitals

Air Quality Control

Air quality control to prevent airborne infections in healthcare facilities. There is a calculation method design and specifications many things depend on the area to be controlled. But can be divided into two main approaches as follows: [28]

1. Prevent airborne infection to the patient.
2. Prevent the spread of infection by air from the patient.

In details, the following will focus on the prevention of airborne transmission from patients. with various environmental control guidelines Preliminary as follows

1. Fresh air from outside: Increasing the aeration rate with fresh air from outside reduces the concentration of airborne contaminants in the room. The location for receiving fresh air from outside must also be set in an appropriate location. This ensures that the outside air to enter the room is free from contamination. Others, but because the weather in Thailand is hot - humid. Determination of aeration rate from outside, suitable. Therefore, it is necessary to be very careful to control the effects of temperature and relative humidity in the air conditioning system in the area.
2. Pressure control between areas: The direction of air flow is proportional to the air pressure within each area. Determining the difference in air pressure for each area to control air quality appropriately. therefore, it is necessary must be carefully considered.
3. Airflow direction control: In the event that the patient is spreading the virus through the air determine the location of air supply from the area around the room, so that

Air flows through the medical personnel before ventilating the room at the headboard wall. It can greatly reduce the risk of airborne infection if there are many beds in the controlled area.

4. Air filter: Currently, standards and guidelines from many agencies recommend the use of air filters. High efficiency (High - Efficiency Particulate Air Filter; HEPA Filter) or air filter panel Ultralow - Penetration Air Filter (ULPA) to filter airborne contaminants and germs. Because it is the device that has the ability to remove airborne contaminants with the highest efficiency today.

5. Using an ultraviolet lamp: Standards and guidelines recommend that ultraviolet lamps can be installed as an addition to the high-efficiency air filter but are not recommended as a replacement for the high-efficiency air filter. Because the use of ultraviolet lamps has several limitations that must be careful in choosing.

6. Temperature and relative humidity: Temperature and relative humidity control can help reduce the growth of airborne pathogens, as standards and guidelines from many agencies establish the range of temperature and relative humidity required. Control of different areas within the hospital but because Thailand has a climate of Hot and humid, the relative humidity control of the air in Thailand must be calculated and designed to suitable by professionals in this field directly Because the design of conventional air conditioning systems cannot control the relative humidity.

2.2 Prevention and control of air quality in hospitals

Preventing the spread of infection Means practice to prevent microorganisms. Livelihoods from infected patients or those who live but do not show symptoms (carrier or colonized) are transmitted to other patients, personnel, or relatives of patients. Preventing the spread of infection can be done in a number of ways, including separating the patient Hand hygiene, disinfection on medical equipment materials with appropriate methods, etc. In cases where the patient can spread the infection to others to personnel and relatives, it is necessary to properly isolate the patient. Because the separation of patients may cause inconvenience in nursing care. Causing increased time and cost. In some cases, it may cause psychological problems for patients, especially pediatric patients. Choosing the right separation method will solve the problem. And can effectively prevent the spread of infection in the hospital.

There are three methods of spreading the virus: [29]

1.Contact transmission

1.1 Direct contact transmission: Transmission of the disease through direct contact with a patient's infection, such as herpetic whitlow, or by touching scabies lesions, can also cause the person to contract the disease. Such incidents are rare, but when they come in contact, they are more prone to disease than indirect contact. It is effective in making the disease more prone to spread and most often it is an infection that occurs in careless personnel.

1.2 Indirect contact transmission: Transmission of the disease is due to indirect contact with the infected person not in direct contact with the source of the disease, such as being stung by a blood-stained needle. (If the patient's blood splashes directly into the wound of the infected person, it is considered direct contact transmission). The most important indirect contact transmission is transmission through the hands of personnel who do not clean their hands properly. appropriate after caring for a patient the contaminated germs on the personnel's hands are then transferred to other patients. Therefore, hand washing is one of the most important measures to prevent the spread of pathogens in hospitals.

2. Droplet transmission: the disease is spread with large droplets, the disease that spreads by this method, upper respiratory tract infections caused by viruses such as influenza 2 (avian influenza), severe acute respiratory syndrome (SARS), and streptococcal pharyngotonsillitis have been cured. Typically, droplets are larger than 5 microns and will not last long in the air. spread not far Protection is as simple as wearing a normal mask. May use the type used in the operating room (surgical mask) and do not approach the patient at a distance of less than 1 meter and keep the hands clean.

3. Airborne transmission: the spread of the disease with droplets smaller than 5 microns, which, with its small size, allow the infected droplets to spread far into the air. Diseases transmitted by this method are tuberculosis of the respiratory tract, measles, and chickenpox. Preventing the transmission of the above diseases can be done. By separating the patient into an isolated room with negative atmospheric pressure relative to the outside and those entering the isolation room should wear an N95 mask, a mask that can filter at least 95 percent of 0.3-micron particles.

2.3 Vaccines against COVID-19

There are four types: [30]

1. Genetic vaccines: including mRNA, this group of vaccines It uses a new technology to synthesize viral messenger RNA (mRNA) genetic material. Vaccines carry mRNA into cells and direct the cells to produce viral spike proteins. This protein stimulates the body's immune system to produce antibodies against the infection. Currently, available vaccines include Pfizer and Moderna vaccines. Currently, this vaccine is approximately 95% effective in preventing disease, preventing severe illness and preventing 100% of death. Two injections are administered intramuscularly 3 weeks apart, Moderna's vaccine should be administered intramuscularly, 4 weeks apart.

2. Recombinant viral vector vaccine: These vaccines use genetically engineered viruses, such as the Adenovirus, which are genetically modified, so they can't divide. And carry the genetic material of the COVID-19 virus. When injected with the virus, these vectors mimic natural infections. By stimulating the immune system to make antibodies against the COVID-19 virus. According to the genetic material inserted. However, even if it is a vaccine in which the adeno virus does not divide, It is also classified as a live virus when it enters the body. Therefore, it is not recommended for use in people with severely immunodeficiency. Until more clear information is available. Currently, four brands of this vaccine are widely used: Chimpanzee adenovirus by AstraZeneca, which is about 70-80% effective in preventing 100% of death; Human adenovirus type 5 by CanSinoBio is approximately 60% effective against symptoms; Human adenovirus type 26 by Johnson and Johnson is approximately 64-72% effective against symptoms. And Human adenovirus types 5 and26 by the Russian company Gamaleya, effective in preventing about 90% of symptoms.

3. Protein subunit vaccine: Vaccines produced by this technology The world has long been familiar with. Because it is used in the production of many vaccines, such as influenza vaccine. Hepatitis B vaccine, etc., is produced by producing viral proteins by cell culture, yeast, baculovirus, etc., and then mixed with immunosuppressants. When injected into the body, it stimulates the body to produce antibodies against the spike protein of the COVID-19 virus. The vaccine currently available is the Novavax brand vaccine, which is made from the baculovirus and uses Matrix M as an immune booster. It is effective in preventing symptoms about 60-90%, preventing 100% death.

4. Inactivated vaccine: This group of vaccines is produced by taking the COVID-19 virus. To raise a large number of and used to cause the germs to die. Vaccination stimulates the body to build immunity to all parts of the virus. As if directly infected with the

virus, but does not cause disease because the infection is dead, This technology is the method used for the hepatitis A vaccine. Injectable polio Therefore, it has been familiar with its effectiveness and safety for a long time. But because culturing the virus requires extreme caution. This made it slow and expensive to produce. The vaccine currently in use, the Sinovac vaccine, is 50-70% effective, prevents 100% of death.

Table 2.3.1: Comparison of the efficacy of the COVID-19 vaccine.

Vaccines	Vaccines Efficacy
Pfizer	95.0%
Moderna	94.5%
Novavax	89.3%
AstraZeneca	70.4%
J&J	66.0%
Sinovac	50.4%

This chapter is devoted to solving ordinary differential equations of the form [26]

$$\frac{dy}{dt} = f(t, y). \tag{2.0A}$$

In the chapter 1, we developed a numerical method to solve such an equation for the velocity of the free-falling bungee jumper. Recall that the method was of the general form

New value = old value + slope × step size

or, in mathematical terms,

$$y_{i+1} = y_i + \phi h, \tag{2.0B}$$

where the slope ϕ is called an *increment function*. According to this equation, the slope estimate of ϕ is used to extrapolate from an old value y_i to a new value y_{i+1} over a distance h. This formula can be applied step by step to trace out the trajectory of the solution into the future. Such approaches are called *one-step methods* because the value of the increment function is based on information at a single point i. They are also referred to as Runge-Kutta methods after the two applied mathematicians

who first discussed them in the early 1900s. Another class of methods called multistep methods use information from several previous points as the basis for extrapolating to a new value.

2.4 Runge-Kutta method

Runge-Kutta (RK) methods achieve the accuracy of a Taylor series approach without requiring the calculation of higher derivatives. [20]

$$y_{i+1} = y_i + \phi(x_i, y_i, h)h \ , \tag{2.1}$$

where $\phi(x_i, y_i, h)$ is called an increment function, which can be interpreted as a representative slope over the interval. The increment function can be written in general form as

$$\phi = a_1 k_1 + a_2 k_2 + ... + a_n k_n \ , \tag{2.2}$$

where the a's are constants and the k's are

$$k_1 = f(x_i, y_i) \ , \tag{2.3}$$

$$k_2 = f(x_i + p_1 h, y_i + q_{11} k_1 h) \ , \tag{2.4}$$

$$k_3 = f(x_i + p_2 h, y_i + q_{21} k_2 h) \ , \tag{2.5}$$

$$\vdots$$

$$k_n = f(x_i + p_{n-1} h, y_i + q_{n-1,1} k_1 h + q_{n-1,2} k_2 h + ... +, q_{n-1,n-1} k_{n-1} h) \ , \tag{2.6}$$

where the p's and q's are constants. Notice that the k's are recurrence relationships. That is, k_1 appears in the equation for k_2, which appears in the equation for k_3, and so forth. Because each k is a functional evaluation, this recurrence makes RK methods efficient for computer calculations.

Various types of Runge-Kutta methods can be devised by employing different numbers of terms in the increment function as specified by n. Note that the first-order RK method with $n = 1$ is, in fact, Euler's method. Once n is chosen, values for the a's, p's, and q's are evaluated by setting Eq. (2.1) equal to terms in a Taylor series expansion. Thus, at least for the lower-order versions, the number of terms, n, usually represents the order of the approach. For example, in the next section, second-order RK methods use an increment function with two terms $(n = 2)$. These second-order methods will be exact if the solution to the differential equation is quadratic. In addition, because terms with h^3 and higher are dropped during the derivation, the

local truncation error is $O(h^3)$ and the global error is $O(h^2)$. In subsequent sections, the third- and fourth-order RK methods ($n = 3$ and 4, respectively) are developed. For these cases, the global truncation errors are $O(h^3)$ and $O(h^4)$, respectively.

2.4.1 Fourth-order Runge-Kutta method

The most popular RK methods are fourth order. As with the second-order approaches, there are an infinite number of versions. The following is the most commonly used form, and we therefore call it the classical fourth-order RK method: [20]

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \ , \tag{2.7}$$

where

$$k_1 = f(x_i, y_i) \ , \tag{2.7.1}$$

$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h) \ , \tag{2.7.2}$$

$$k_3 = f(x_i + h, y_i - k_1 h + 2k_2 h) \ , \tag{2.7.3}$$

$$k_4 = f(x_i + h, y_i + k_3 h). \tag{2.7.4}$$

Various types of Runge-Kutta methods can be devised by employing different numbers of terms in the increment function as specified by n. Note that the first order RK method with $n = 1$ is, in fact, Euler's method. Once n is chosen, values for the $a's, p's$ and $q's$ are evaluated by setting Eq. (22.33) equal to terms in a Taylor series expansion. Thus, at least for the lower-order versions, the number of terms, n, usually represents the order of the approach. For example, in Section 22.4.1, second-order RK methods use an increment function with two terms $(n = 2)$. These second-order methods will be exact if the solution to the differential equation is quadratic. In addition, because terms with h^3 and higher are dropped during the derivation, the local truncation error is $O(h^3)$ and the global error is $O(h^2)$. In Section 22.4.2, the fourth-order RK method $(n = 4)$ is presented for which the global truncation errors is $O(h^4)$. [26]

Chapter 3
Governing equation

3.1 Governing equation

In general, the rate of exhaled air generation and ventilation per person determine the raised concentration of indoor carbon dioxide [16],[18], and [19]. Because an infected individual's exhaled air contains airborne infectious particles, carbon dioxide levels can be employed as an exhaled air surrogate [16],[17],[19],[14], and [20]. Exhaled air contains approximately 40,000 ppm of carbon dioxide, compared to 400 ppm of carbon dioxide in ambient air [16],[14], and [13].

3.1.1 A accumulation rate exhaled air concentration in a room with carbon dioxide environmental model

We assume that an indoor space, such as a room with a volume of V, begins the day with a carbon dioxide concentration of C_E of about 400 ppm and is occupied by a number of people, n. Given the presence of infectors, the concentration of exhaled air that may contain airborne contagious particles may tend to rise in the room, based on the rate of ventilation, Q, and the number of people in the room. We simply assume that persons in the room contribute substantially to the production of carbon dioxide, which serves as an exhaled air marker. The fundamental equation of the accumulation rate exhaled air concentration in a room with carbon dioxide environmental, is equal to the exhaled air rate generated by inhabitants plus the rate of carbon dioxide environmental, minus ventilation rate removes exhaled air: [1]

$$V\frac{dC}{dt} = npC_a + QC_E - QC \,, \tag{3.1}$$

where C is the concentration of indoor air exhaled (ppm),

p is the rate of breathing(L/s) for each person in the room,

C_a is the carbon dioxide fraction included in inbreathed air,

t is the duration time and T is the stationery simulation time.

If the value of Q assumed by Q_{in} and Q_{out}, then these values are named the inlet ventilation rate and the outlet ventilation respectively and in a simple scenario, a

number of people are unstable then a number of people depend on the time assumed by $n(t)$. In this study preferred to use Eq.(3.1) as follow:

$$V\frac{dC}{dt} = n(t)pC_a + Q_{in}C_E - Q_{out}C\ , \tag{3.2}$$

for all $0 \le t \le T$.

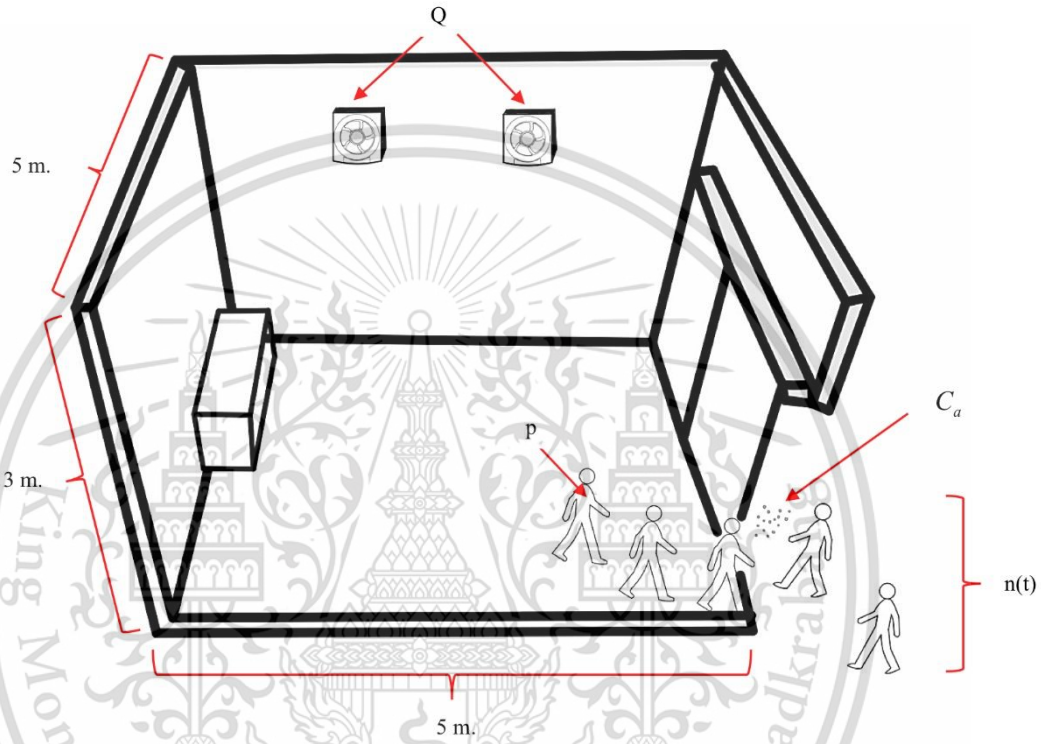

Fig.3.1 An outpatient room model

3.1.2 An exhaled air concentration in a room model

We obtain the concentration of sampled exhaled air, in the given space, which is the difference between indoor exhaled air concentration and outdoor exhaled air concentration (rebreathed air removed by ventilation) divided by the ventilation rate as show in Fig 3.1,

$$\frac{dC}{dt} = \frac{n(t)pC_a - QC}{V}. \tag{3.3}$$

3.1.3 The volume fraction of exhaled air

Taking into account that the volume fraction of exhaled air, f, is given by the sampled exhaled air concentration Eq.(3.3) in the space divided by carbon dioxide fraction in breathed air (C_a), we get

$$f(t) = \frac{C(t)}{C_a}\ , \tag{3.4}$$

for all $0 \le t \le T$.

3.1.4 The concentration of airborne infectious particles

However, some infectious particles can be trapped in the upper respiratory tract or be reflected to other parts of the body, where the probability of causing infection is almost negligible. let β be the total airborne infectious particles generation rate released by an infector (particles/s) and μ be the mortality rate of generated airborne infectious particles by the infector that do not reach the alveolar (particles/s). Hence, the survival rate of airborne infectious particles released by the infector that reach the target infection site of the susceptible individual to cause infection at threshold level is $(\beta - \mu)$ particles/s.

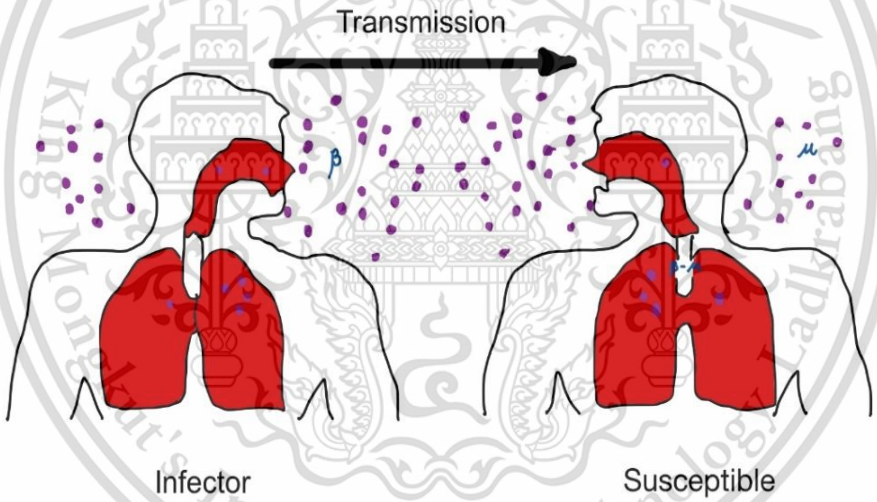

Fig.3.2 Movement of airborne infectious particles

The concentration of airborne infectious particles, N , that cause infection, is equal to the average volume fraction of rebreathed air by infectors $(If(t)/n(t))$, multiplied by the concentration of airborne infectious particles released by infectors in the space that reach the target infection site of the respiratory tract $(\beta - \mu)/p$:

$$N(t) = \frac{If(t)(\beta - \mu)}{n(t)p},\ I \ge 1 \quad \text{and} \quad (\beta - \mu) \ge 1, \tag{3.5}$$

for all $0 \le t \le T$.

3.1.5 The averaged number of airborne infectious particles

Since not all infectious particles can reach and deposit at the alveolar, let θ be a respiratory deposition fraction of airborne infectious particles that successfully reach and deposit at the target infection site of the host. Hence, the average number of airborne infectious particles, λ, breathed by a susceptible individual that causes infection, is equal to the product of the volume of breathed air by susceptible (pt), respiratory deposition fraction of airborne infectious particles, $\theta(0<\theta<1)$, and the concentration of airborne infectious particles $N(t)$ released by infectors

$$\lambda(t) = pt\theta N(t)\ ,\ t\ >\ 0\ , \tag{3.6}$$

where t is the time spent in the space up to the point of infection.

3.1.6 The risk of airborne infection in a room

Considering Wells (1955) assumed that TB transmission [1] follows a Poisson distribution (Wells, 1955; Rudnick and Milton, 2003), we express TB transmission probability as

$$P(T \le t \,|\, I, Q, V, p, \theta, \mu, \beta) = 1 - e^{-\lambda(t)}. \tag{3.7}$$

Eq. (3.7) predicts the risk of airborne infectious disease by introducing very important parameters, including particle production, survival, mortality rates and successful deposition fraction at the site of infection. The model is applicable in multiple infective environmental conditions obeying the boundary condition of threshold level of infectious particles to induce infection.

3.2 Parameter setting

There are several parameters that have to be given for the proposed models, as listed below.

C	An outpatient room exhaled air concentration (ppm),
C_a	The carbon dioxide fraction contained in breathed air (ppm),
V	An outpatient room of volume (m^3),
Q	The ventilation rate (L/s),
p	The breathing rate (L/s),
$n(t)$	The total number of people,
t	Duration (min),
$f(t)$	The volume fraction of exhaled air,

$N(t)$ The concentration of airborne infectious particles,

$\lambda(t)$ The number of airborne infectious particles,

pt The volume of breathed air by susceptible,

β The total airborne infectious particles generation rate released by an infector,

μ The mortality rate of generated airborne infectious particles by the infector that do not reach the alveolar,

θ A respiratory deposition fraction of airborne infectious particles,

I The number of infectors in an outpatient room.

3.3 Initial value problem and initial conditions

3.3.1 initial conditions

Initial Condition(s) are a condition, or set of conditions, on the solution that will allow us to determine which solution that we are after. Initial conditions are of the form,

$$C(t_0) = C_0 \ , \tag{3.8}$$

where C_0 is the given latent carbon dioxide concentration.

Chapter 4

Numerical simulation

In this research, we assume that people spend five hours on average in an outpatient room.

4.1 Simulation1: When the number of patients in an outpatient room remains consistent, there is a possibility of airborne infection and vaccination effectiveness. We will create 4 scenarios in which there are 30, 40, 50, and 60 people who are staying in the room. We assume that $C_0 = 10$ is the initial carbon dioxide concentrations.

Table 4.1.1: Physical parameters 1

V	C_E	C_a	p	Q	I	θ
75	0	0.04	0.12	1	2	0.25

Table 4.1.2: Compare the effectiveness of vaccines

Vaccine types	A (mRNA)	B (mRNA)	C (Protein -based)	D (Viral vector)	E (Viral vector)	F (Inactivated)
Vaccine efficacy (%)	95	94.5	89.3	70.4	66	50.4

Table 4.1.3: The probability of infection when the number of people in an outpatient room is constant, by considering the case of 30 people.

Probability of infection at n=30	Vaccine types					
	A	B	C	D	E	F
P(30)	0.1120	0.1224	0.2244	0.5049	0.5540	0.6921
P(60)	0.1486	0.1622	0.2912	0.6141	0.6650	0.7972
P(90)	0.1514	0.1652	0.2963	0.6217	0.6726	0.8038
P(120)	0.1393	0.1521	0.2746	0.5886	0.6395	0.7743
P(150)	0.1218	0.1331	0.2426	0.5364	0.5864	0.7242
P(180)	0.1036	0.1134	0.2087	0.4767	0.5247	0.6621
P(210)	0.0873	0.0956	0.1775	0.4176	0.4626	0.5958
P(240)	0.0737	0.0808	0.1511	0.3644	0.4058	0.5320
P(270)	0.0631	0.0692	0.1301	0.3200	0.3579	0.4760
P(300)	0.0552	0.0605	0.1144	0.2854	0.3202	0.4305

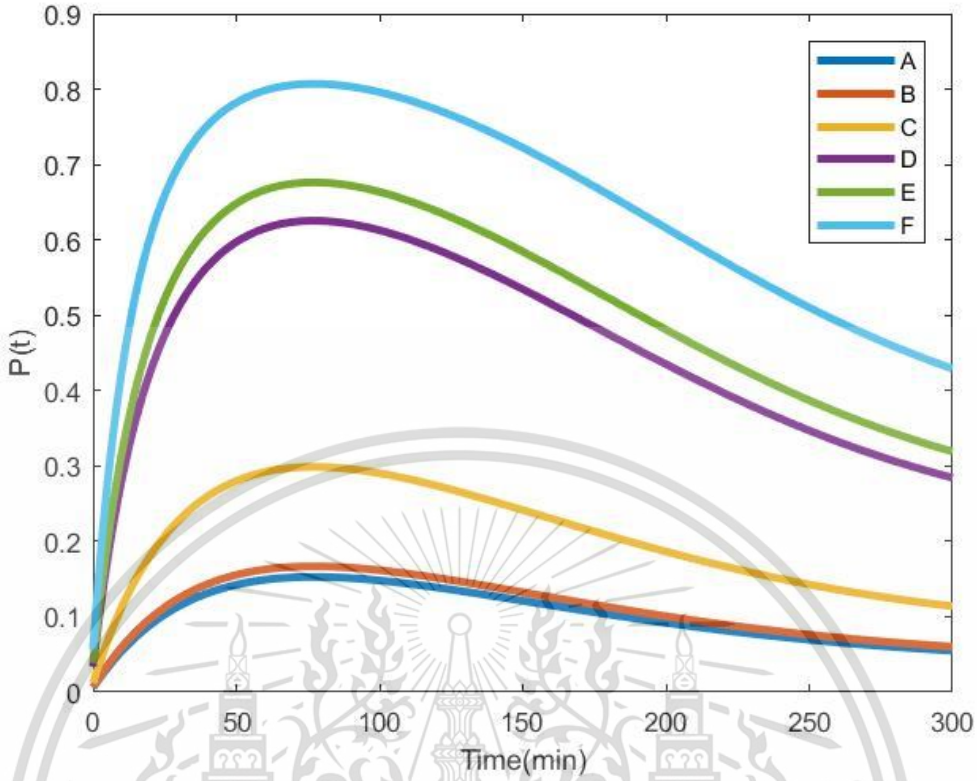

Fig.4.1 The probability of infection for case 30 people

Table 4.1.4: The probability of infection when the number of people in an outpatient room is constant, by considering the case of 40 people.

Probability of infection at n=40	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0854	0.0935	0.1739	0.4104	0.4550	0.5875
P(60)	0.1142	0.1249	0.2286	0.5123	0.5617	0.6998
P(90)	0.1170	0.1279	0.2338	0.5213	0.5710	0.7090
P(120)	0.1082	0.1184	0.2174	0.4923	0.5410	0.6789
P(150)	0.0952	0.1042	0.1928	0.4470	0.4936	0.6294
P(180)	0.0819	0.0897	0.1671	0.3970	0.4407	0.5716
P(210)	0.0700	0.0767	0.1439	0.3493	0.3896	0.5133
P(240)	0.0603	0.0661	0.1246	0.3081	0.3449	0.4605
P(270)	0.0529	0.0580	0.1098	0.2750	0.3089	0.4166
P(300)	0.0475	0.0522	0.0990	0.2505	0.2819	0.3832

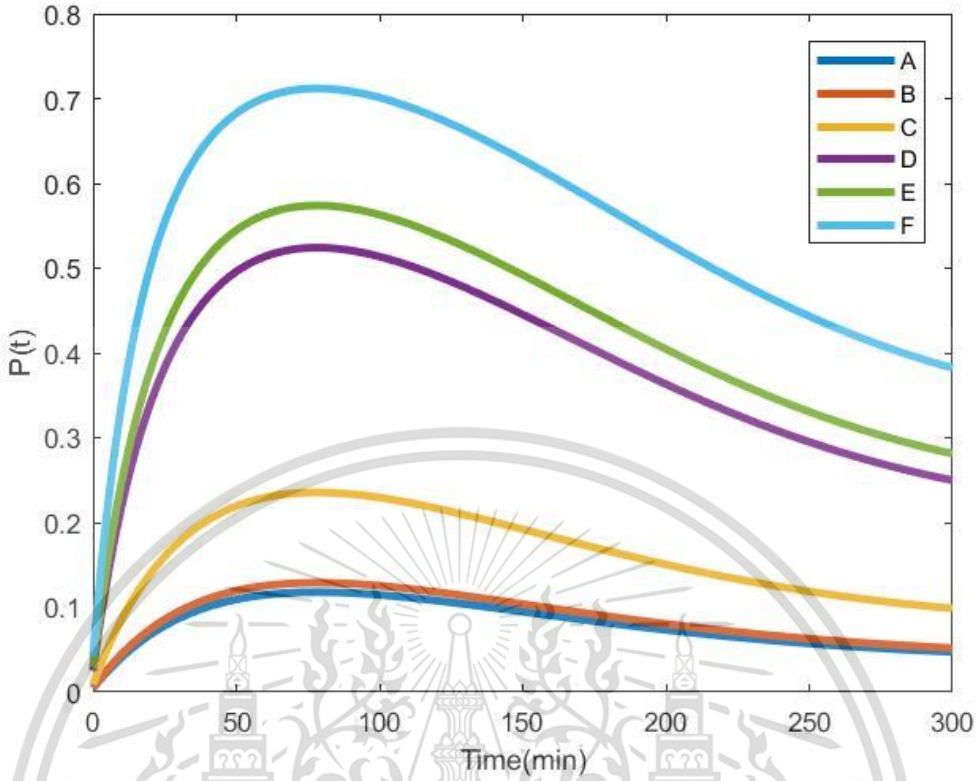

Fig.4.2 The probability of infection for case 40 people

Table 4.1.5: The probability of infection when the number of people in an outpatient room is constant, by considering the case of 50 people.

Probability of infection at n=50	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0691	0.0757	0.1420	0.3454	0.3853	0.5083
P(60)	0.0930	0.1018	0.1885	0.4388	0.4850	0.6202
P(90)	0.0957	0.1048	0.1937	0.4487	0.4954	0.6314
P(120)	0.0890	0.0974	0.1808	0.4241	0.4694	0.6033
P(150)	0.0789	0.0865	0.1613	0.3854	0.4283	0.5576
P(180)	0.0686	0.0752	0.1411	0.3435	0.3833	0.5060
P(210)	0.0595	0.0653	0.1230	0.3046	0.3411	0.4559
P(240)	0.0522	0.0573	0.1084	0.2719	0.3054	0.4124
P(270)	0.0467	0.0513	0.0973	0.2466	0.2777	0.3778
P(300)	0.0429	0.0471	0.0896	0.2287	0.2579	0.3529

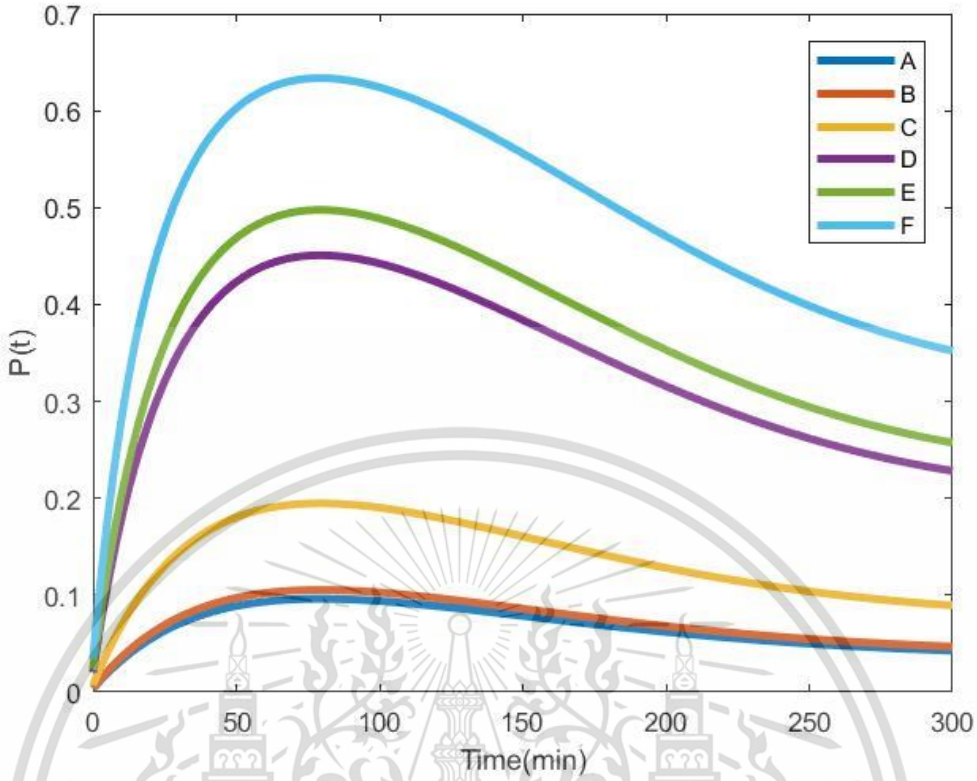

Fig.4.3 The probability of infection for case 50 people

Table 4.1.6: The probability of infection when the number of people in an outpatient room is constant, by considering the case of 60 people.

Probability of infection at n=60	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0580	0.0636	0.1201	0.2980	0.3340	0.4473
P(60)	0.0785	0.0860	0.1605	0.3838	0.4266	0.5557
P(90)	0.0812	0.0890	0.1658	0.3943	0.4378	0.5684
P(120)	0.0760	0.0832	0.1555	0.3735	0.4156	0.5433
P(150)	0.0679	0.0744	0.1397	0.3405	0.3801	0.5022
P(180)	0.0597	0.0654	0.1233	0.3052	0.3418	0.4567
P(210)	0.0524	0.0575	0.1089	0.2731	0.3067	0.4140
P(240)	0.0467	0.0513	0.0974	0.2467	0.2778	0.3780
P(270)	0.0426	0.0467	0.0889	0.2271	0.2561	0.3505
P(300)	0.0398	0.0437	0.0833	0.2139	0.2415	0.3318

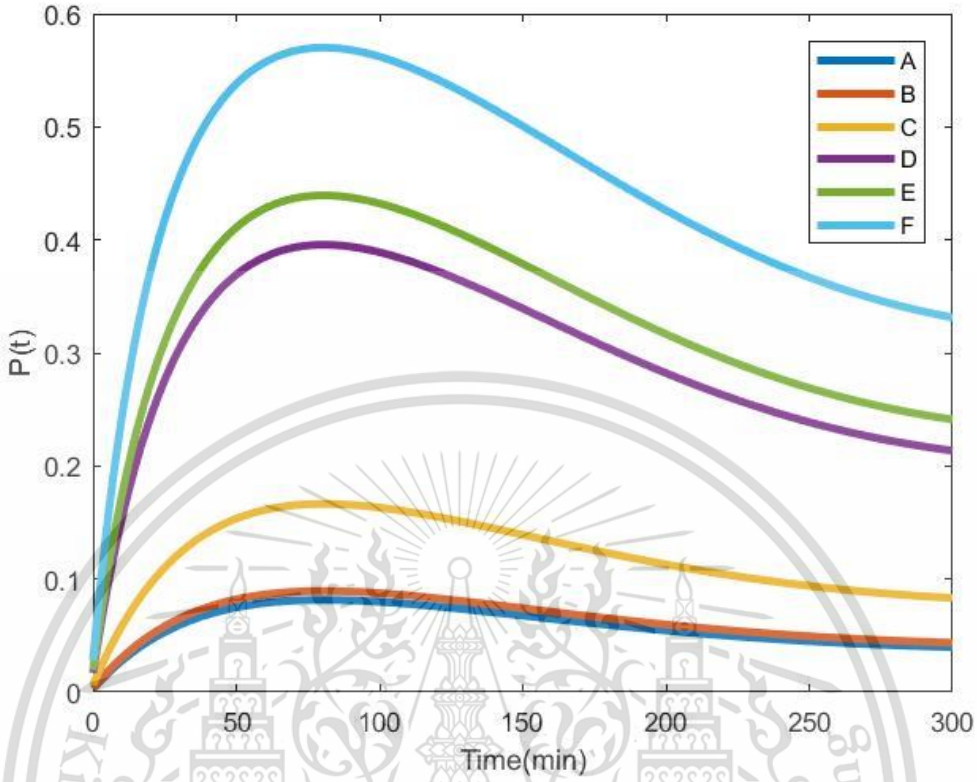

Fig.4.4 The probability of infection for case 60 people

4.2 Simulation2: When the air ventilation rate in an outpatient room is considered, there is a possibility of airborne infection and vaccination effectiveness. We will create 4 scenarios in which the ventilation rate inside the room is 1.5, 2.0, 2.5, and 3.0 and there are 30 people who are staying in the room.

We assume that $C_0 = 10$ is the initial carbon dioxide concentrations.

Table 4.2.1: Physical parameters 2

V	C_E	C_a	p	n	I	θ
75	0	0.04	0.12	30	2	0.25

Table 4.2.2: The probability of infection when the air ventilation rate inside an outpatient room is 1.5

Probability of infection at Q=1.5	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0933	0.1021	0.1891	0.4400	0.4862	0.6215
P(60)	0.1033	0.1130	0.2081	0.4755	0.5235	0.6608
P(90)	0.0879	0.0962	0.1787	0.4199	0.4650	0.5985
P(120)	0.0679	0.0744	0.1397	0.3405	0.3801	0.5022
P(150)	0.0507	0.0556	0.1053	0.2650	0.2978	0.4030
P(180)	0.03800	0.0417	0.0796	0.2050	0.2317	0.3192
P(210)	0.0296	0.0325	0.0623	0.1631	0.1850	0.2580
P(240)	0.0246	0.0270	0.0519	0.1370	0.1557	0.2187
P(270)	0.0219	0.0240	0.0463	0.1228	0.1397	0.1971
P(300)	0.0208	0.0229	0.0440	0.1170	0.1332	0.1882

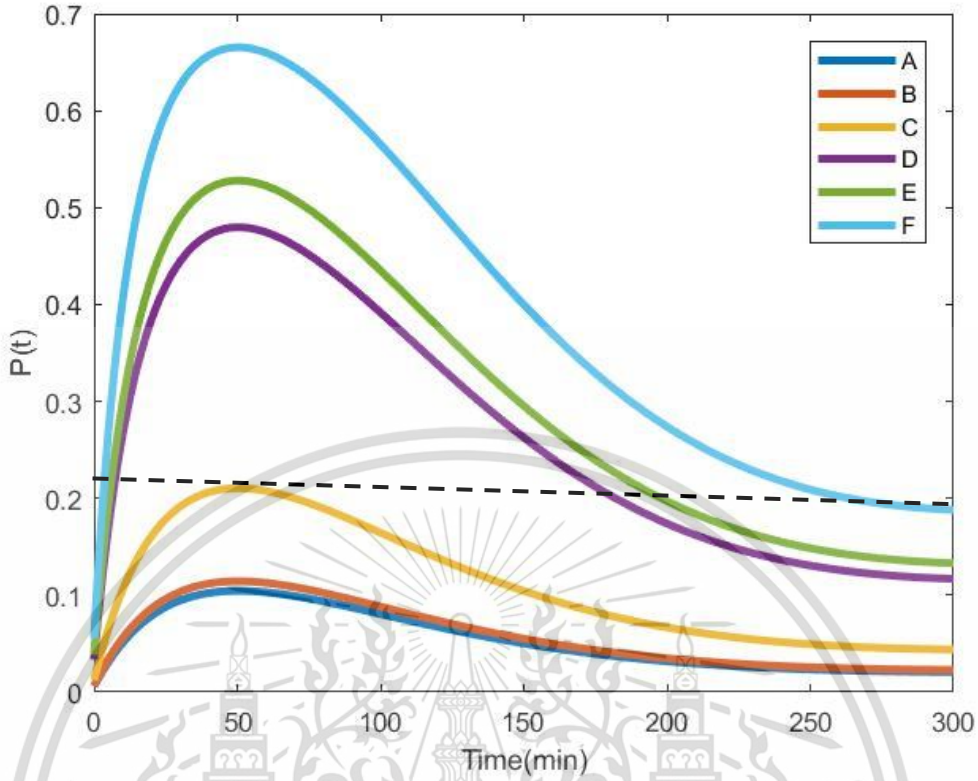

Fig.4.5 The probability of infection for the ventilation rate inside an outpatient room is 1.5

Table 4.2.3: The probability of infection when the air ventilation rate inside an outpatient room is 2.0

Probability of infection at Q=2.0	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0776	0.0850	0.1588	0.3801	0.4227	0.5513
P(60)	0.0713	0.0782	0.1464	0.3546	0.3953	0.5200
P(90)	0.0506	0.0555	0.1052	0.2646	0.2975	0.4026
P(120)	0.0333	0.0366	0.0699	0.1817	0.2057	0.2853
P(150)	0.0222	0.0244	0.0469	0.1245	0.1416	0.1997
P(180)	0.0161	0.0177	0.0342	0.0917	0.1046	0.1488
P(210)	0.0132	0.0146	0.0281	0.0759	0.0867	0.1239
P(240)	0.0123	0.0135	0.0261	0.0705	0.0806	0.1153
P(270)	0.0124	0.0136	0.0263	0.0710	0.0811	0.1161
P(300)	0.01300	0.0143	0.0276	0.0746	0.0852	0.1218

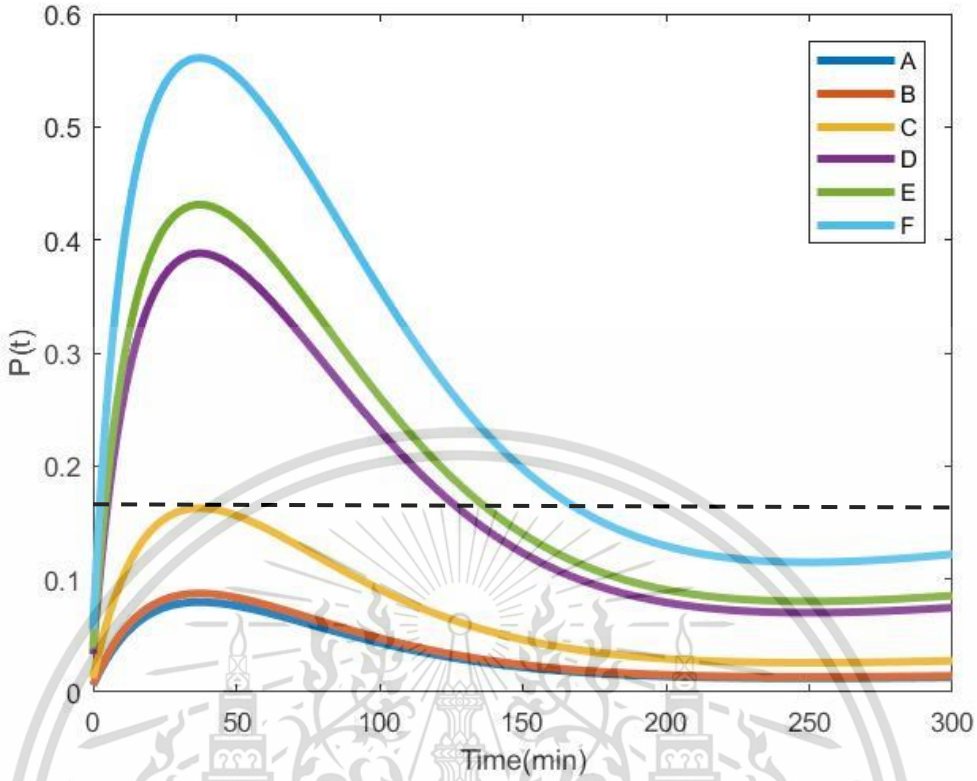

Fig.4.6 The probability of infection for the ventilation rate inside an outpatient room is 2.0

Table 4.2.4: The probability of infection when the air ventilation rate inside an outpatient room is 2.5

Probability of infection at Q=2.5	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0645	0.0707	0.1329	0.3260	0.3644	0.4838
P(60)	0.0491	0.0538	0.1021	0.2576	0.2897	0.3929
P(90)	0.0292	0.0321	0.0615	0.1610	0.1826	0.2549
P(120)	0.0169	0.0186	0.0359	0.0961	0.1096	0.1558
P(150)	0.0110	0.0120	0.0233	0.0631	0.0721	0.1035
P(180)	0.0086	0.0095	0.0183	0.0499	0.0572	0.0823
P(210)	0.0081	0.0089	0.0173	0.0470	0.0538	0.0776
P(240)	0.0084	0.0093	0.0180	0.0490	0.0560	0.0807
P(270)	0.0092	0.0101	0.0195	0.0530	0.0606	0.0872
P(300)	0.0100	0.0110	0.0213	0.0579	0.0663	0.0952

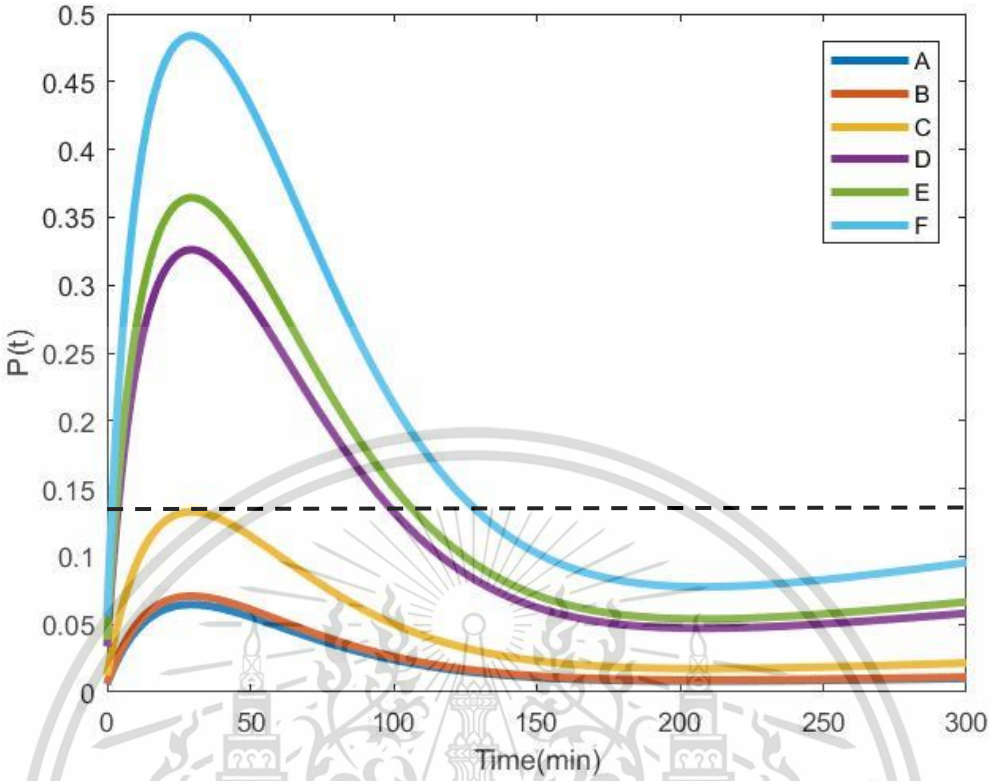

Fig.4.7 The probability of infection for the ventilation rate inside an outpatient room is 2.5

Table 4.2.5: The probability of infection when the air ventilation rate inside an outpatient room is 3.0

Probability of infection at Q=3.0	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0535	0.0587	0.1110	0.2779	0.3120	0.4205
P(60)	0.0337	0.0370	0.0708	0.1837	0.2080	0.2884
P(90)	0.0171	0.0188	0.0362	0.0970	0.1106	0.1572
P(120)	0.0092	0.0101	0.0196	0.0533	0.0610	0.0877
P(150)	0.0064	0.0070	0.0136	0.0372	0.0426	0.0615
P(180)	0.0058	0.0064	0.0123	0.0338	0.0387	0.0560
P(210)	0.0061	0.0067	0.0130	0.0356	0.0407	0.0589
P(240)	0.0067	0.0074	0.0144	0.0393	0.0450	0.0649
P(270)	0.0075	0.0083	0.0160	0.0436	0.0499	0.0720
P(300)	0.0083	0.0091	0.0177	0.0482	0.0552	0.0794

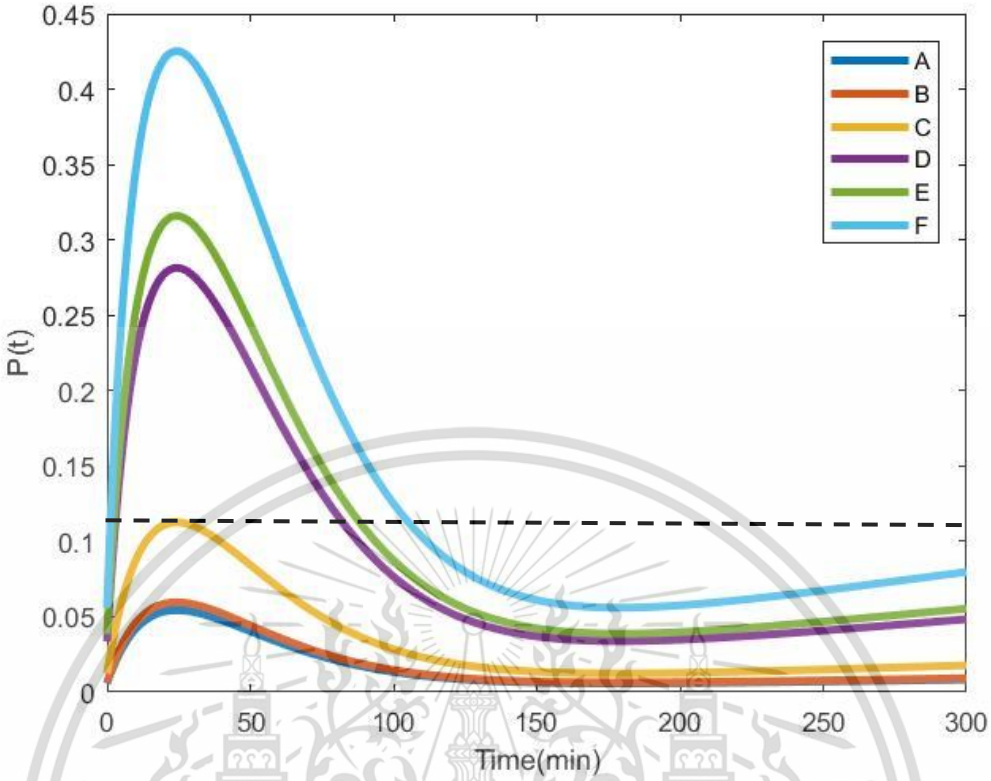

Fig.4.8 The probability of infection for the ventilation rate inside an outpatient room is 3.0

4.3 Simulation3: When the air ventilation rate and number of stayed people in an outpatient room are considered, there is a possibility of airborne infection and vaccination effectiveness. We will create 4 scenarios in which the ventilation rate inside the room is 1.5, 2.0, 2.5, and 3.0 and the number of people staying in the room is listed in Table 4.3.1.

We assume that $C_0 = 10$ is the initial carbon dioxide concentrations.

Table 4.3.1: Physical parameters 3

V	C_E	C_a	p	I	θ
75	0	0.04	0.12	2	0.25

Table 4.3.2: The number of people who stayed in an outpatient room at each period of time

t	0	30	60	90	120	150	180	210	240	270	300
n(t)	50	33	40	35	54	25	33	40	35	30	20

Table 4.3.3: The probability of infection when the air ventilation rate inside an outpatient room is 1.5 when the number of people in an outpatient room varies over time.

Probability of infection at Q=1.5	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0854	0.0935	0.1738	0.4104	0.4549	0.5874
P(60)	0.0808	0.0885	0.1649	0.3926	0.4360	0.5663
P(90)	0.0765	0.0838	0.1565	0.3755	0.4178	0.5457
P(120)	0.0407	0.0447	0.0852	0.2183	0.2464	0.3381
P(150)	0.0607	0.0665	0.1253	0.3096	0.3466	0.4625
P(180)	0.0388	0.0426	0.0812	0.2088	0.2358	0.3245
P(210)	0.0235	0.0259	0.0497	0.1315	0.1495	0.2105
P(240)	0.0228	0.0251	0.0483	0.1279	0.1454	0.2049
P(270)	0.0236	0.0259	0.0499	0.1319	0.1500	0.2111
P(300)	0.0292	0.0321	0.0615	0.1611	0.1827	0.2550

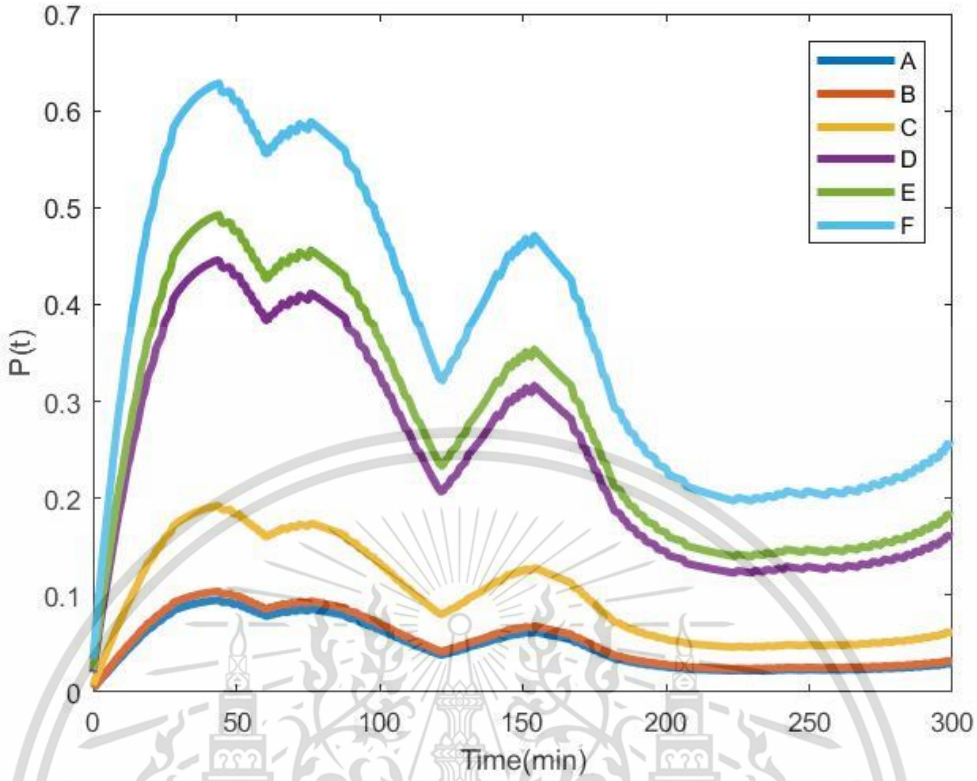

Fig.4.9 The probability of infection for the number of people in an outpatient room is not constant and the ventilation rate inside an outpatient room is 1.5

Table 4.3.4: The probability of infection when the air ventilation rate inside an outpatient room is 2.0 when the number of people in an outpatient room varies over time.

Probability of infection at Q=2.0	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0710	0.0778	0.1458	0.3533	0.3939	0.5183
P(60)	0.0556	0.0610	0.1152	0.2873	0.3223	0.4331
P(90)	0.0441	0.0484	0.0919	0.2341	0.2639	0.3604
P(120)	0.0202	0.0222	0.0427	0.1138	0.1295	0.1832
P(150)	0.0275	0.0302	0.0579	0.1522	0.1728	0.2417
P(180)	0.0164	0.0180	0.0348	0.0934	0.1065	0.1514
P(210)	0.0109	0.0120	0.0231	0.0627	0.0717	0.1029
P(240)	0.0121	0.0133	0.0258	0.0696	0.0796	0.1139
P(270)	0.0138	0.0151	0.0292	0.0787	0.0899	0.1284
P(300)	0.0179	0.0197	0.0379	0.1014	0.1156	0.1640

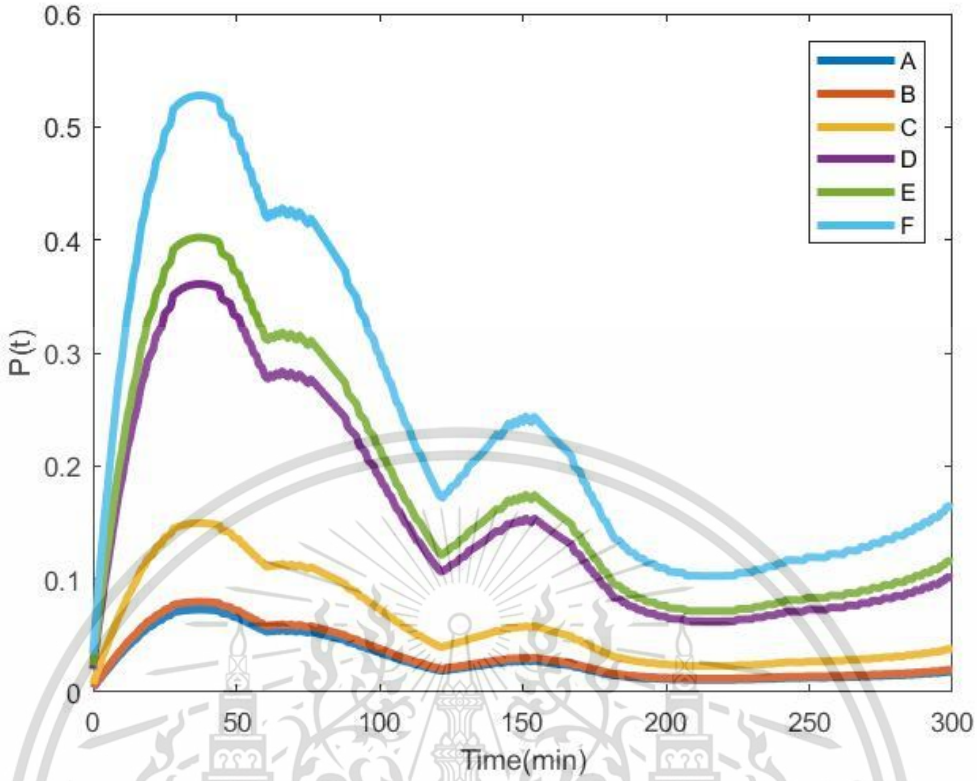

Fig.4.10 The probability of infection for the number of people in an outpatient room is not constant and the ventilation rate inside an outpatient room is 2.0

Table 4.3.5: The probability of infection when the air ventilation rate inside an outpatient room is 2.5 when the number of people in an outpatient room varies over time.

Probability of infection at Q=2.5	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0589	0.0646	0.1219	0.3021	0.3384	0.4527
P(60)	0.0382	0.0419	0.0800	0.2059	0.2327	0.3205
P(90)	0.0255	0.0280	0.0538	0.1420	0.1613	0.2263
P(120)	0.0105	0.0116	0.0224	0.0608	0.0696	0.0998
P(150)	0.0142	0.0156	0.0301	0.0810	0.0924	0.1319
P(180)	0.0087	0.0095	0.0184	0.0502	0.0574	0.0827
P(210)	0.0069	0.0075	0.0146	0.0400	0.0458	0.0661
P(240)	0.0086	0.0095	0.0184	0.0501	0.0573	0.0825
P(270)	0.0103	0.0113	0.0218	0.0592	0.0677	0.0972
P(300)	0.0134	0.0148	0.0285	0.077	0.0879	0.1256

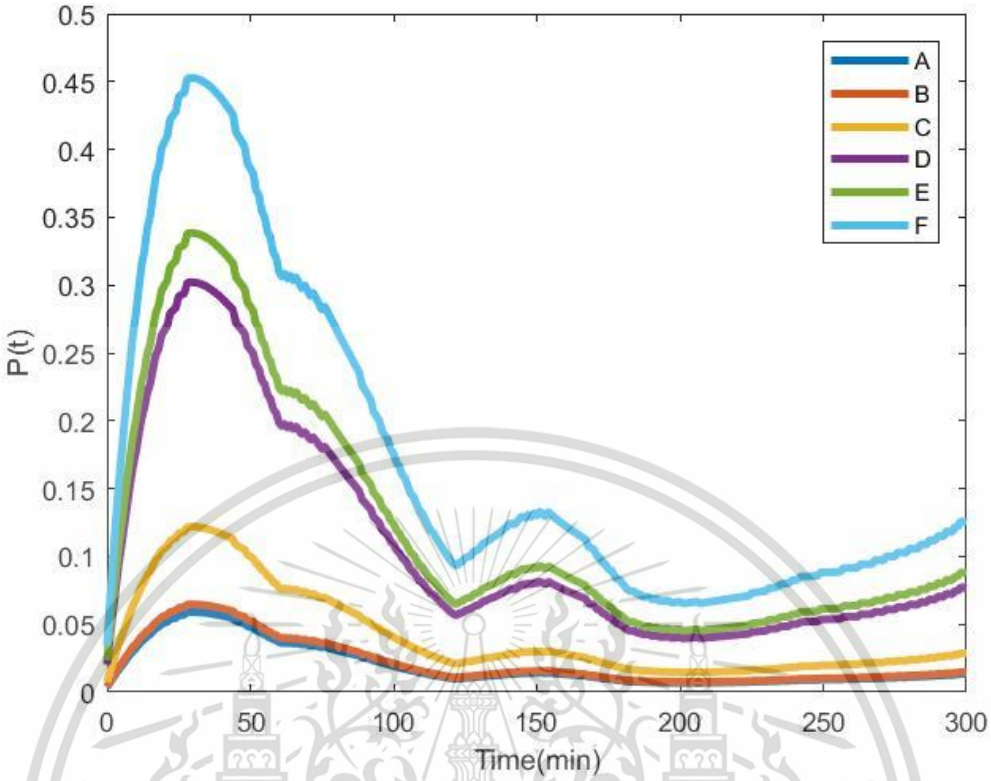

Fig.4.11 The probability of infection for the number of people in an outpatient room is not constant and the ventilation rate inside an outpatient room is 2.5

Table 4.3.6: The probability of infection when the air ventilation rate inside an outpatient room is 3.0 when the number of people in an outpatient room varies over time.

Probability of infection at Q=3.0	Vaccine types					
	A	B	C	D	E	F
P(30)	0.0489	0.0537	0.1018	0.2568	0.2889	0.3919
P(60)	0.0263	0.0288	0.0553	0.1457	0.1655	0.2319
P(90)	0.0150	0.0165	0.0319	0.0858	0.0979	0.1395
P(120)	0.0060	0.0066	0.0128	0.0350	0.0400	0.0579
P(150)	0.0086	0.0094	0.0183	0.0498	0.0570	0.0820
P(180)	0.0057	0.0062	0.0121	0.0332	0.0380	0.0549
P(210)	0.0053	0.0058	0.0112	0.0308	0.0353	0.0510
P(240)	0.0070	0.0077	0.0150	0.0408	0.0467	0.0674
P(270)	0.0084	0.0092	0.0178	0.0486	0.0556	0.0800
P(300)	0.0109	0.0119	0.0231	0.0626	0.0716	0.1027

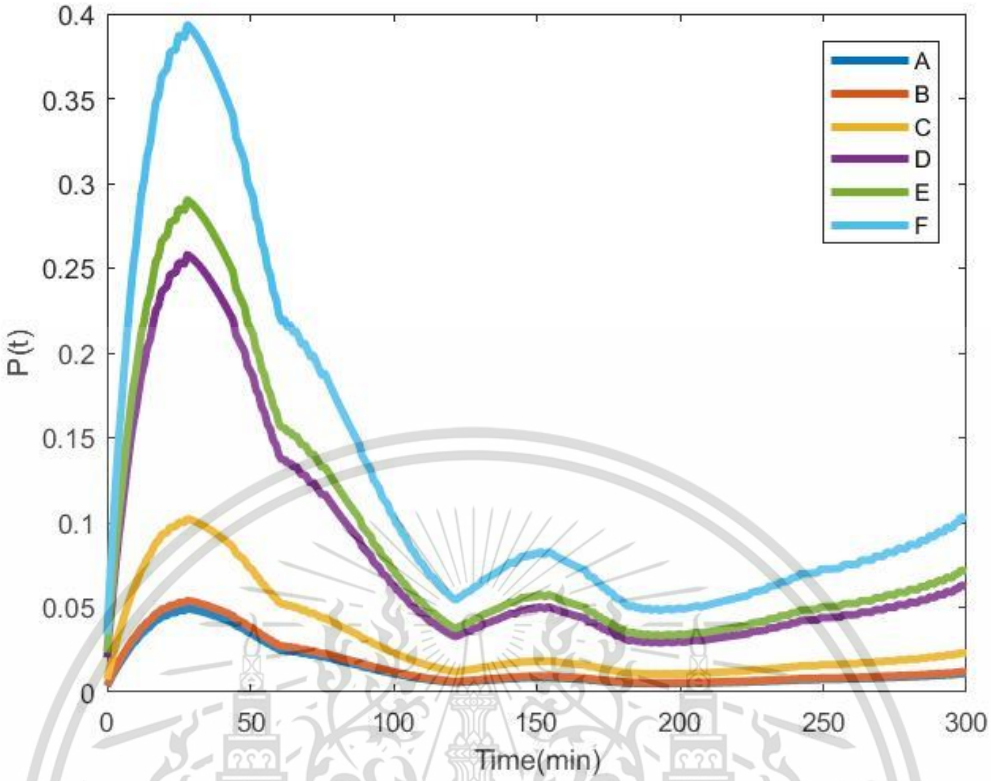

Fig.4.12 The probability of infection for the number of people in an outpatient room is not constant and the ventilation rate inside an outpatient room is 3.0

Chapter 5
Discussion and conclusion

5.1 Discussion

In simulation 1, with n=30 cases, we observe from Table 4.1.2 that the probability of infection is lowest when receiving types A and B vaccinations compared to the remainder. At the same time, as shown in Figure 3, the longer people stay in an outpatient room, the more probable it will be that normal people will get infected. And when the number of people in an outpatient room is increased statically to 40, 50, and 60, we found that the probability of infection was shown in Tables 4.1.4, 4.1.5 and 4.1.6, respectively. When the number of people in an outpatient room is fixed at 60, the probability of infection becomes the lowest.

Consider the rate of ventilation in an outpatient room in Simulation 2. We determined that when outpatient room ventilation rates were 1.5, 2.0, 2.5, and 3.0, the probability of infection was 1.5, as shown in Table 4.2.2, with types A and B vaccines having a very low probability of infection. At the same time, as shown in Figure 7, the longer people stay in an outpatient room, the more probable people are to become infected. Tables 4.2.3, 4.2.4, and 4.2.5 show the probability of infection when the ventilation rate in an outpatient room is increased to 2.0, 2.5, and 3.0, respectively.

Consider a situation where the number of people in the room changes from Table 4.3.2 and the ventilation rates are 1.5, 2.0, 2.5, and 3.0. At a ventilation rate of 1.5, the probability of infection varies with the number of people in an outpatient room at that time, as shown in Table 4.3.3. A and B are also less likely to be infected than the remaining four types, as shown in Figure 11. And when the ventilation rate was increased in an outpatient room to 2.0, 2.5, and 3.0, respectively, it was found that the probability of infection was as shown in Tables 4.3.4, 4.3.5 and 4.3.6. They will be less infected as shown in Figures 12, 13 and 14.

5.2 Conclusion

A risk model of airborne transmission in an outpatient room with a ventilation system is provided for variable patient quantities and vaccine efficacy. As can be seen, the probability of infection depends on the number of people present, the rate of

ventilation, and the efficacy of each type of vaccination. Using the RK4 method in the most optimum conditions, we show that the proposed strategy works for actual issues. Numerical models have been shown to provide beneficial results.

In the air quality management process, the suggested model balances the number of people permitted in an outpatient room with the ventilation system's ability to control the risk of infection within an outpatient room. We can see that the sufficiently ventilated system and the efficacy of each type of vaccination can reduce the risk of airborne infection in an outpatient room in a hospital. This research could be utilized to help control the risk of airborne infection to the desired level if there is a public vaccination database system.

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Appendix

Appendix A

The research paper

Engineering Letters, 30:2, EL_30_2_28

A Risk Model of Airborne Transmission and Vaccine Efficacy in an Outpatient Room with a Ventilation System

Watchareeporn Boonmeemapasuk, and Nopparat Pochai

Abstract— **Every day, a large number of patients visit the facility, creating a serious infectious transmission problem that might infect patients with respiratory infectious illnesses in outpatient rooms, putting their health at risk. TB, COVID-19, MERS, and SARS are all significant infectious diseases that are transmitted by the air or aerosol via coughing, spitting, sneezing, speaking, or wounds. COVID-19, tuberculosis, MERS, and SARS are all hazards, and the probability of a serious disease increasing the number of people admitted to the hospital. We should also be informed about how patients in the outpatient room are managed. When the number of patients in each room changes over time, it is challenging to measure and manage carbon dioxide in a hospital with a ventilation system. We should also be informed about the management of patients with these conditions. This research investigates the mathematical modeling of carbon dioxide concentration measurement and the risk assessment of airborne infection in an outpatient room with a ventilation system, while the number of patients in each room changes over time. As a result, efficient air quality monitoring, such as carbon dioxide (CO_2) concentrations, is required to monitor and decrease the possibility of contaminated air. It is indeed difficult to measure and manage carbon dioxide in a hospital with a ventilation system when the number of patients in each room changes over time. This research provided a risk model of airborne transmission and vaccination effectiveness in an outpatient room with a ventilation system. When the number of people and the rate of ventilation change, the model modifies the carbon dioxide concentration. To approximate the model solution, the fourth-order Runge-Kutta technique is used. In the presented simulations, there are several scenarios for improving air quality. The proposed approach balances the number of people allowed to stay in the room with the capacity of the air ventilation system in the air quality management process. As can be seen, the risk of infection is dependent on the number of people present, the rate of ventilation, and the efficacy of each type of vaccination. If there is a public vaccination database system, this research may be used to help control the risk of airborne infection to the desired level.**

Index Terms— **Risk, Airborne, Transmission, Outpatient Room, Ventilation, Vaccination**

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I. Introduction

Tuberculosis (TB), Coronavirus Disease Starting in 2019 (COVID-19), Middle East Respiratory Syndrome (MERS), and Severe Acute Respiratory Syndrome (SARS) are a hazardous communicable disease which are transfer from people to people through the air or the aerosol in different ways, such as through coughing, spitting, sneezing, speaking, or through wounds. In [1], In the laboratory, US scientists demonstrated that the virus can survive in an aerosol and remain infectious for at least 3 hours. Tuberculosis (often known as TB) is a communicable illness causcd by Mycobacterium Tuberculosis that mostly affects the lungs. At present, we have an effective TB disinfectant. TB can be treated, but recovery takes a long time. If the treatment is not continued, or is incomplete, death may result. Therefore, TB is an important public health issue in Thailand. In [2], a new procedure was developed to study the distribution of epidemics for predicting the possibility of airborne infectious diseases in high-density urban areas. It can analyze the chance of spread in sub-transportation, and it can also help understand dispersion of airborne diseases in public transportation in China. In [3], the researchers studied the behaviors of Korean TB infection. TB transmission dynamic was proposed by using mathematical TB model with exogenous reinfection. Then, the least squares method was used to approximate the considered parameters. From the results, the most significant factor was the case finding effort, which led to a decrease of active TB patients. In [4], the researchers developed an infectious diseases model of SARS by using two methods for estimating both small-scale SARS outbreak parameter at the Amoy Gardens, Hong Kong and large-scale outbreak parameter in the entire Hong Kong Special Administrative Region. In [5], the inpatient nursing records from EMR of the University of Miyazaki Hospital were analyzed by using a text data mining technique. This result indicated that vocabulary related to appropriate treatment methods. The focus of [6] was on airflow and the airborne dissemination of infectious pathogens from an indoor setting. From this, it was confirmed that infected individuals and susceptible individuals should use masks, and also should use personalized ventilation for a short-range airborne route. In [7], the researchers projected that diseased patient in high-rise hospitals would transmit airborne diseases. Using multi-zone airflow simulation and tracer (CFD) simulation, this simulation could examine the ventilation system. In [8], basic epidemiological and multivariate state-space models are proposed to predict

optimal control measure strategies. This approach can be used for various diffusion diseases include Ebola, MERS. In [9],[10], a vaccination strategy for the SEIR model was designed. It was oriented towards the measurement and used for the infectious population to epidemic models for designs the general time-varying, the vaccination control rule. In [11], a new discrete-time SEIR epidemic model was presented by using the Forward-Euler difference method. Besides, the numerical simulations were presented to compare the continuous-time epidemic and discrete-time system. In [12],[13],[14],[15], and [16], they proposed that infectious disease of airborne such as tuberculosis (TB) spread in several gathering locate areas with infectors and poor ventilation per person rates, [12],[13],[14],[15], and [16]. In [17],[18],[19], and [15], they proposed infectors could be dangerous if there no is a high concentration of indoor rebreathed air because it could contain infector-borne infectious particles, which could lead to the spread of airborne infectious diseases like tuberculosis. In [17], and [20], they proposed carbon dioxide be used as an indicator of air quality indoor, built on the notion that people release carbon dioxide at a rate dictated by their body weight and bodily movement, and that levels of carbon dioxide indoor are measured by fresh air clearance. In [17],[21], and [20], they propose carbon dioxide concentration in the air of approximately 400 ppm in a room, but when but people enter it, exhaled air concentration begins to rise, depending on the rate of ventilation per person, the length of the room, and the number of persons who are present in the room, because of their oxygen intake, respiratory quotient, and bodily movement, person in the room add to the rise in rebreathed air. In [15] and [13], they argued that when the concentration of exhaled air in a room grows in the presence of infectors, the risk of vulnerable persons developing infectious illnesses communicated by the air rises. This is due to the fact that contaminated people's exhaled air contains infectious airborne particles inside the nucleus with droplets that may remain airborne for lengthy periods of time and infect a vulnerable individual when inhaled. In [13],[22], and [23], they proposed the immune system's condition of the host, host physiology, and the virulence of the Mycobacterium tuberculosis (Mtb) infectious strain are all important factors in the advancement of infection to TB disease. The major method of transmission in [24] is droplet or airborne transfer, and the risk of infection is known to be substantially lower outside where ventilation is greater. As winter approaches in the northern hemisphere, opportunities for socializing and outdoor activity become more difficult, and fears of COVID-19 transmission increase. In [25], They proposed that about the efficacy of ventilation systems for human thermal comfort in terms of ceiling height, which contributes to green building architectures. Other advantages of ventilation, which we gain in high-ceilinged dwellings, cannot be overlooked. This would also assist to minimize moisture, smoke, odor, heat, dust, and germs. In this research, several numerical models of carbon dioxide concentration measurement in an outpatient room with an opened ventilation system is introduced. [29] describes a mathematical model that predicts the risk of airborne infectious diseases, such as TB, under steady-state and non-steady-state situations by monitoring infectors' breath in a confined area. This research used a risk model of airborne transmission and vaccination efficacy in an outpatient room with a ventilation system. [30] presents a logistic regression model for predicting high-risk patients' failure to complete their TB treatment regimen. In [31] and [32], air quality assessment models are considered.

This research used a risk model of airborne transmission and vaccination efficacy in an outpatient room with a ventilation system.

II. GOVERNING EQUATION

In general, the rate of exhaled air generation and ventilation per person [17], [19], and [20] determines the elevated concentration of indoor carbon dioxide. Carbon dioxide levels can be utilized as a substitute for exhaled air since ill people's exhaled air contains airborne infectious particles [17], [18], [15], and [26]. Exhaled air has around 40,000 parts per million of carbon dioxide, compared to 400 parts per million in ambient air [17], [15], and [14]. We suppose that an interior area, such as a room with a volume, has a carbon dioxide concentration of roughly 400 ppm at the start of the day and is occupied by n people. Based on the rate of ventilation and the number of people in the room, the concentration of exhaled air that may include airborne infectious particles may tend to grow in the room if infectors are present. We simply assume that everyone in the room contributes a significant amount of carbon dioxide to the atmosphere, which acts as a generator for exhaled air. Considering the volume proportion of exhaled air as a generator. The exhaled air rate generated by inhabitants plus the rate of carbon dioxide environmental, minus the ventilation rate that removes exhaled air, is equal to the exhaled air rate generated by inhabitants plus the rate of carbon dioxide environmental, minus the ventilation rate that removes exhaled air:

$$V\frac{dC}{dt} = npC_a + QC_E - QC \tag{1}$$

where C is the concentration of indoor air exhaled (ppm), p is the rate of breathing(L/s) for each person in the room and C_a is the carbon dioxide fraction included in inbreathed air. t is the duration time and T is the stationery simulation time. Initial condition $C(0) = C_0$ where C_0 is the latent carbon dioxide concentration.

If the value of Q assumed by Q_{in} and Q_{out}, then these values are named the inlet ventilation rate and the outlet ventilation respectively and in a simple scenario, a number of people are unstable then a number of people depend on the time assumed by $n(t)$. In this study preferred to use Eq.(1) as follow:

$$V\frac{dC}{dt} = n(t)pC_a + Q_{in}C_E - Q_{out}C\ , \tag{2}$$

for all $0 \le t \le T$.

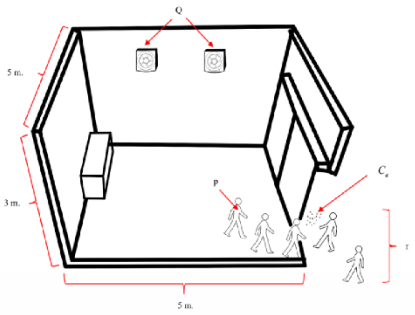

Fig.1 An outpatient room model

The concentration of sampled exhaled air in the given space is calculated by dividing the difference between indoor and outdoor exhaled air concentrations by the ventilation rate:

$$\frac{dC}{dt} = \frac{n(t)pC_a + QC_E - QC}{V} \quad (3)$$

Taking the volume fraction of exhaled air into consideration, f, is given by the sampled exhaled air concentration Eq.(3) in the space divided by carbon dioxide fraction in breathed air (C_a), we get

$$f(t) = \frac{C(t)}{C_a}, \quad (4)$$

for all $0 \le t \le T$.

Some infectious particles, on the other hand, can become lodged in the upper respiratory tract or be reflected into other parts of the body where the danger of infection is minimal. Let β represent the overall rate of airborne infectious particle generation emitted by an infector (particles/s) and μ represent the mortality rate of airborne infectious particle production by the infector that does not reach the alveoli (particles/s). As a result, the survival rate of infectious particles released by the infector and reaching the susceptible individual's target infection area to cause infection is $\beta - \mu$ particles/s.

The concentration of airborne infectious particles, N, that cause infection, is equal to the average volume fraction of rebreathed air by infectors $(If(t)/n(t))$, multiplied by the concentration of airborne infectious particles released by infectors in the space that reach the target infection site of the respiratory tract $(\beta-\mu)/p$:

$$N(t) = \frac{If(t)(\beta-\mu)}{n(t)p}, I \ge 1 \text{ and } (\beta-\mu) \ge 1 \quad (5)$$

for all $0 \le t \le T$.

Let θ be a respiratory deposition percentage of airborne infectious particles that successfully reach and deposit at the target infection site of the host, because not all infected particles can reach and deposit at the alveoli. As a result, the product of the volume of breathed air by susceptible (pt), respiratory deposition fraction of airborne infectious particles, $\theta(0<\theta<1)$, and the concentration of airborne infectious particles $N(t)$ released by infectors equals the average number of airborne infectious particles, λ, breathed by a susceptible individual that causes infection,

$$\lambda(t) = pt\theta N(t), t > 0 \quad (6)$$

where t is the time spent in the space up to the point of infection.

Considering Wells (1955) assumed that TB transmission follows a Poisson distribution [29], we express TB transmission probability as

$$P(T \le t | I, Q, V, p, \theta, \mu, \beta) = 1 - e^{-\lambda(t)} \quad (7)$$

III. NUMERICAL TECHNIQUES

A continuous approximation to the solution $C(t)$ will not be obtained; instead, approximations to C will be generated at various values, called mush points, in the interval $[0,T]$.

Interpolation can be used to find the approximate solution at additional points in the interval once the approximate solution at the points has been determined. We start by assuming that the grid points are spread evenly over the interval $[0,T]$. This requirement is met by picking a positive integer N and the grid points $t_i = a + ih$ for each $i = 0,1,2,...,N$. The common distance between the points $h = (T-0)/N = t_{i+1} + t_i$ is called the step size.

A. Fourth-order Runge-Kutta method

$$C \cong C_i \quad (8)$$

$$C_{i+1} = C_i + \frac{1}{6}[F_1 + 2F_2 + 2F_3 + F_4] \quad (9)$$

$$F_1 = hf(t_i, C_i) \quad (10)$$

$$F_2 = hf(t_i + \frac{h}{2}, C_i + \frac{F_1}{2}) \quad (11)$$

$$F_3 = hf(t_i + \frac{h}{2}, C_i + \frac{F_2}{2}) \quad (12)$$

$$F_4 = hf(t_i + h, C_i + F_3) \quad (13)$$

from Eq.(2), we get the classical fourth-order RK method

$$\frac{dC}{dt} = f(t_i, C_i) \quad (14)$$

$$f(t_i, C_i) = \frac{1}{V}(n(t)pC_a + Q_{in}C_E - Q_{out}C_i) \quad (15)$$

IV. Numerical Experiments and Results

*A. **Simulation1**: When the number of patients in an outpatient room remains consistent, there is a possibility of airborne infection and vaccination effectiveness. We will create 4 scenarios in which there are 60, 50, 40, and 30 people who are staying in the room.*

We assume that $C_0 = 10$ is the initial carbon dioxide concentrations.

TABLE I
PHYSICAL PARAMETERS.

V	C_E	C_a	P	Q	I	θ
75	0	0.04	0.12	1	2	0.25

TABLE II
COMPARE THE EFFECTIVENESS OF VACCINES.

vaccine types	A	B	C	D	E	F
vaccine efficacy (%)	95	94.5	89.3	70.4	66	50.4

TABLE III
THE PROBABILITY OF INFECTION WHEN THE NUMBER OF PEOPLE IN AN OUTPATIENT ROOM IS CONSTANT, BY CONSIDERING THE CASE OF 30 PEOPLE.

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0501	0.1120	0.1486	0.1514
B	0.0550	0.1224	0.1622	0.1652
C	0.1042	0.2244	0.2912	0.2963
D	0.2624	0.5049	0.6141	0.6217
E	0.2951	0.5540	0.6650	0.6726
F	0.3996	0.6921	0.7972	0.8038

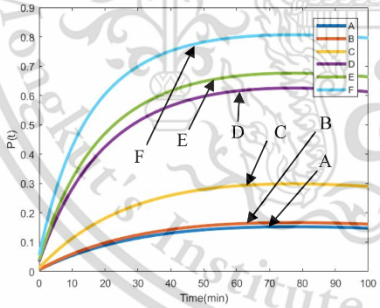

Fig.2 The probability of infection for case 30 people

TABLE IV
THE PROBABILITY OF INFECTION WHEN THE NUMBER OF PEOPLE IN AN OUTPATIENT ROOM IS CONSTANT, BY CONSIDERING THE CASE OF 40 PEOPLE.

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0379	0.0854	0.1142	0.1170
B	0.0416	0.0935	0.1249	0.1279
C	0.0793	0.1739	0.2286	0.2338
D	0.2042	0.4104	0.5123	0.5213
E	0.2308	0.4550	0.5617	0.5710
F	0.3181	0.5875	0.6998	0.7090

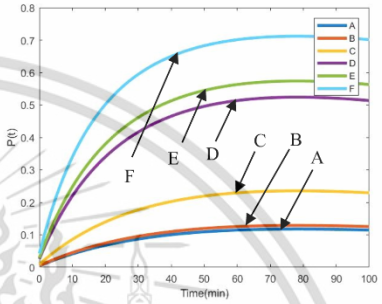

Fig.3 The probability of infection for case 40 people

TABLE V
THE PROBABILITY OF INFECTION WHEN THE NUMBER OF PEOPLE IN AN OUTPATIENT ROOM IS CONSTANT, BY CONSIDERING THE CASE OF 50 PEOPLE.

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0304	0.0691	0.0930	0.0957
B	0.0334	0.0757	0.1018	0.1048
C	0.0640	0.1420	0.1885	0.1937
D	0.1671	0.3454	0.4388	0.4487
E	0.1895	0.3853	0.4850	0.4954
F	0.2639	0.5083	0.6202	0.6314

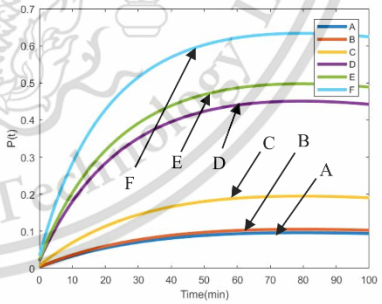

Fig.4 The probability of infection for case 50 people

TABLE VI
THE PROBABILITY OF INFECTION WHEN THE NUMBER OF PEOPLE IN AN OUTPATIENT ROOM IS CONSTANT, BY CONSIDERING THE CASE OF 60 PEOPLE.

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0254	0.0580	0.0785	0.0812
B	0.0279	0.0636	0.0860	0.0890
C	0.0536	0.1201	0.1605	0.1658
D	0.1414	0.2980	0.3838	0.3943
E	0.1607	0.3340	0.4266	0.4378
F	0.2255	0.4473	0.5557	0.5684

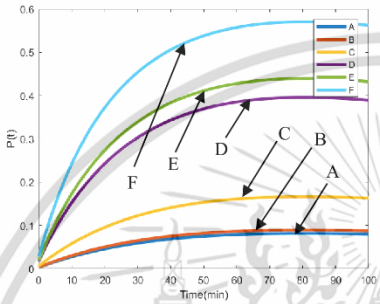

Fig.5 The probability of infection for case 60 people

*B. **Simulation2:** When the air ventilation rate in an outpatient room is considered, there is a possibility of airborne infection and vaccination effectiveness. We will create 4 scenarios in which the ventilation rate inside the room is 0.5, 1.5, 2.5, and 3.0 and there are 30 people who are staying in the room.*

We assume that $C_0 = 10$ is the initial carbon dioxide concentrations.

TABLE VII
PHYSICAL PARAMETERS.

V	C_E	C_a	p	n	I	θ
75	0	0.04	0.12	30	2	0.25

TABLE VIII
THE PROBABILITY OF INFECTION WHEN THE AIR VENTILATION RATE INSIDE AN OUTPATIENT ROOM IS 0.5

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0531	0.1341	0.2114	0.2551
B	0.0583	0.1465	0.2300	0.2767
C	0.1103	0.2652	0.3985	0.4675
D	0.2762	0.5736	0.7550	0.8250
E	0.3101	0.6243	0.8012	0.8650
F	0.4182	0.7603	0.9053	0.9461

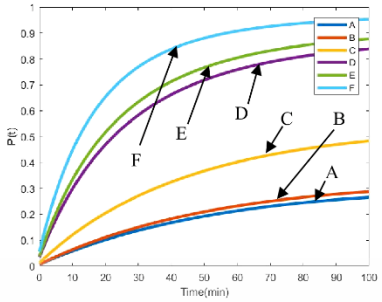

Fig.6 The probability of infection for the ventilation rate inside an outpatient room is 0.5

TABLE IX
THE PROBABILITY OF INFECTION WHEN THE AIR VENTILATION RATE INSIDE AN OUTPATIENT ROOM IS 1.5

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0473	0.0933	0.1033	0.0879
B	0.0519	0.1021	0.1130	0.0962
C	0.0984	0.1891	0.2081	0.1787
D	0.2493	0.4400	0.4755	0.4199
E	0.2806	0.4862	0.5235	0.4650
F	0.3566	0.6215	0.6608	0.5985

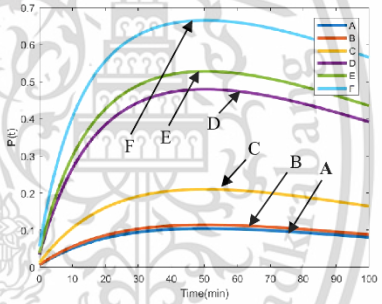

Fig.7 The probability of infection for the ventilation rate inside an outpatient room is 1.5

TABLE X
THE PROBABILITY OF INFECTION WHEN THE AIR VENTILATION RATE INSIDE AN OUTPATIENT ROOM IS 2.5

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0420	0.0645	0.0491	0.0292
B	0.0462	0.0707	0.0538	0.0321
C	0.0878	0.1329	0.1021	0.0615
D	0.2245	0.3260	0.2576	0.1610
E	0.2533	0.3644	0.2897	0.1826
F	0.3470	0.4838	0.3929	0.2549

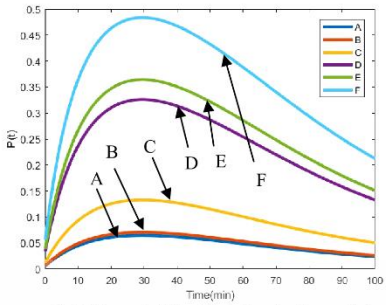

Fig.8 The probability of infection for the ventilation rate inside an outpatient room is 2.5

TABLE XI
THE PROBABILITY OF INFECTION WHEN THE AIR VENTILATION RATE INSIDE AN OUTPATIENT ROOM IS 3.0

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0446	0.0776	0.0713	0.0506
B	0.0489	0.0850	0.0782	0.0555
C	0.0930	0.1588	0.1464	0.1052
D	0.2366	0.3801	0.3546	0.2646
E	0.2667	0.4227	0.3953	0.2975
F	0.3639	0.5513	0.5200	0.4026

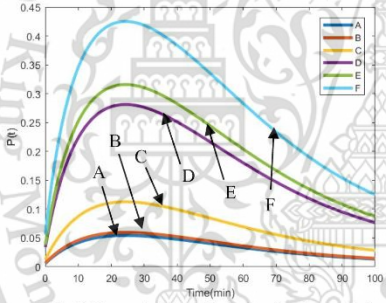

Fig.9 The probability of infection for the ventilation rate inside an outpatient room is 3.0

C. ***Simulation3:*** *When the air ventilation rate and number of stayed people in an outpatient room are considered, there is a possibility of airborne infection and vaccination effectiveness. We will create 4 scenarios in which the ventilation rate inside the room is 0.5, 1.5, 2.5, and 3.0 and the number of people staying in the room is listed in Table XIII.*

We assume that $C_0 = 10$ is the initial carbon dioxide concentrations.

TABLE XII
PHYSICAL PARAMETERS.

V	C_E	C_a	p	I	θ
75	0	0.04	0.12	2	0.25

TABLE XIII
THE NUMBER OF PEOPLE WHO STAYED IN AN OUTPATIENT ROOM AT EACH PERIOD OF TIME.

t	0	10	20	30	40	50	60	70	80	90	100
n(t)	50	33	40	35	54	25	33	40	34	42	40

TABLE XIV
THE PROBABILITY OF INFECTION WHEN THE AIR VENTILATION RATE INSIDE AN OUTPATIENT ROOM IS 0.5 WHEN THE NUMBER OF PEOPLE IN AN OUTPATIENT ROOM VARIES OVER TIME.

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0471	0.1195	0.2326	0.2036
B	0.0516	0.1306	0.2526	0.2215
C	0.0980	0.2383	0.4325	0.3856
D	0.2483	0.5291	0.7914	0.7401
E	0.2795	0.5790	0.8347	0.7873
F	0.3801	0.7169	0.9276	0.8954

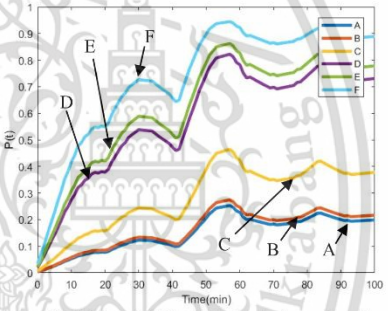

Fig.10 The probability of infection for the number of people in an outpatient room is not constant and the ventilation rate inside an outpatient room is 0.5

TABLE XV
THE PROBABILITY OF INFECTION WHEN THE AIR VENTILATION RATE INSIDE AN OUTPATIENT ROOM IS 1.5 WHEN THE NUMBER OF PEOPLE IN AN OUTPATIENT ROOM VARIES OVER TIME.

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0419	0.0829	0.1145	0.0689
B	0.0459	0.0908	0.1252	0.0756
C	0.0874	0.1691	0.2291	0.1417
D	0.2236	0.4010	0.5132	0.3448
E	0.2523	0.4449	0.5626	0.3847
F	0.3457	0.5763	0.7007	0.5076

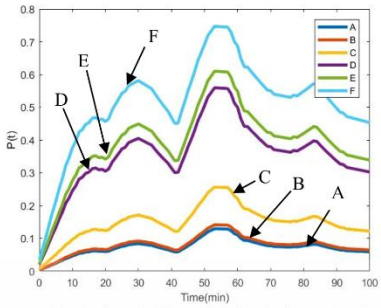

Fig.11 The probability of infection for the number of people in an outpatient room is not constant and the ventilation rate inside an outpatient room is 1.5

TABLE XVII
THE PROBABILITY OF INFECTION WHEN THE AIR VENTILATION RATE INSIDE AN OUTPATIENT ROOM IS 3.0 WHEN THE NUMBER OF PEOPLE IN AN OUTPATIENT ROOM VARIES OVER TIME.

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0351	0.0475	0.0376	0.0135
B	0.0385	0.0521	0.0413	0.0149
C	0.0736	0.0988	0.0788	0.0288
D	0.1906	0.2502	0.2032	0.0776
E	0.2157	0.2816	0.2296	0.0886
F	0.2984	0.3827	0.3165	0.1266

TABLE XVI
THE PROBABILITY OF INFECTION WHEN THE AIR VENTILATION RATE INSIDE AN OUTPATIENT ROOM IS 2.5 WHEN THE NUMBER OF PEOPLE IN AN OUTPATIENT ROOM VARIES OVER TIME.

vaccine types	probability of infection			
	P(10)	P(30)	P(60)	P(90)
A	0.0372	0.0572	0.0547	0.0230
B	0.0409	0.0628	0.0600	0.0253
C	0.0780	0.1185	0.1133	0.0485
D	0.2011	0.2945	0.2831	0.1286
E	0.2274	0.3302	0.3177	0.1463
F	0.3136	0.4427	0.4274	0.2060

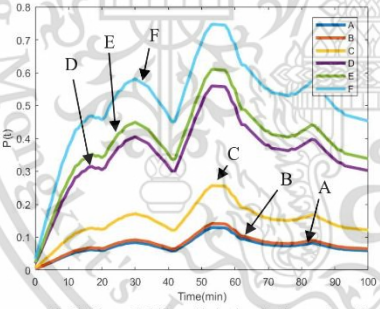

Fig.12 The probability of infection for the number of people in an outpatient room is not constant and the ventilation rate inside an outpatient room is 2.5

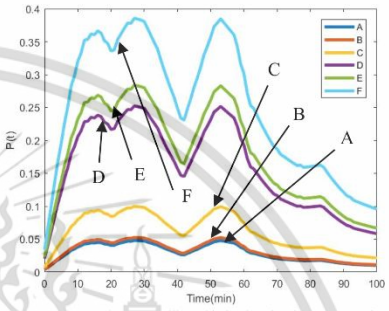

Fig.13 The probability of infection for the number of people in an outpatient room is not constant and the ventilation rate inside an outpatient room is 3.0

V. DISCUSSION

In simulation 1, with n=30 cases, we observe from Table II that the probability of infection is lowest when receiving types, A and B vaccinations compared to the remainder. At the same time, as shown in Figure 2, the longer people stay in an outpatient room, the more probable it will be that normal people will get infected. And when the number of people in an outpatient room is increased statically to 40, 50, and 60, we found that the probability of infection was shown in Tables IV, V and VI, respectively. When the number of people in an outpatient room is fixed at 60, the probability of infection becomes the lowest.

Consider the rate of ventilation in an outpatient room in Simulation 2. We determined that when outpatient room ventilation rates were 0.5, 1.5, 2.5, and 3.0, the probability of infection was 0.5, as shown in Table VIII, with types A and B vaccines having a very low probability of infection. At the same time, as shown in Figure 6, the longer people stay in an outpatient room, the more probable people are to become infected. Tables IX, X, and XI show the probability of infection when the ventilation rate in an outpatient room is increased to 1.5, 2.5, and 3.0, respectively.

Consider a situation where the number of people in the room changes from Table XIII and the ventilation rates are 0.5, 1.5, 2.5, and 3.0. At a ventilation rate of 0.5, the probability of infection varies with the number of people in an outpatient room at that time, as shown in Table XIV. A and B are also less likely to be infected than the remaining four types, as shown in Figure 10. And when the ventilation rate was increased in an outpatient room to 1.5, 2.5, and 3.0,

respectively, it was found that the probability of infection was as shown in Tables XV, XVI and XVII. They will be less infected as shown in Figures 11, 12 and 13.

VI. Conclusion

A risk model of airborne transmission in an outpatient room with a ventilation system is provided for variable patient quantities and vaccine efficacy. As can be seen, the likelihood of infection is affected by the number of people present, the rate of ventilation, and the effectiveness of each form of immunization. Using the RK4 method in the most optimum conditions, we show that the proposed strategy works for actual issues. Numerical models have been shown to provide beneficial results. In the air quality management process, the suggested model balances the number of people permitted in an outpatient room with the ventilation system's ability to control the risk of infection within an outpatient room. We can see that the sufficiently ventilated system and the efficacy of each type of vaccination can reduce the risk of airborne infection in an outpatient room in a hospital. This research could be utilized to help control the risk of airborne infection to the desired level if there is a public vaccination database system.

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Academic Publication(s)

W. Boonmeemapasuk and N. Pochai., "A Risk Model of Airborne Transmission and Vaccine Efficacy in an Outpatient Room with a Ventilation System", Engineering Letters, Vol 30, No 2(2022), pp.644-651.