A NON-DIMENSIONAL MATHEMATICAL MODEL OF SHORELINE EVOLUTION WITH A GROIN STRUCTURE USING AN UNCONDITIONALLY STABLE EXPLICIT DIFFERENCE TECHNIQUE

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Abstract

Coastal erosion is a natural phenomenon that occurs when sediment transports away from the coast and is not countered by the formation of new material on the shoreline. A groin structure was created to prevent coastal erosion and floods. A qualitative analysis of the model coastal behavior in relation to the controlling process is required to research beach erosion and beach deposition. In this research, we provide a governing equation when a groin is introduced to a one-dimensional shoreline growth model. A non-dimensional shoreline evolution model and a groin structure model are also included in this model. The model now has the ability to manipulate physical parameters. When groin structural effects are present, the initial condition setting method and boundary condition approaches are also presented. To approximate the incremental model in each year, the traditional forward time centered space (FTCS) technique and the unconditionally stable Saulyev finite difference technique are used. The Saulyev finite difference technique can be very useful for computing a practical conceptual design of shoreline evolution since the number of grids has increased. The numerical models offered provide a viable simulation for evaluating long-term coastal development.

Keywords: Finite difference method, Groin structure, Mathematical model, Non-dimensional model, Shoreline evolution

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Mr. Surasak Manilam

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Abbreviations / Symbols

Symbols	Description
x	The alongshore coordinate (m)
t	Time (day)
$y(x,t)$	The shoreline positions and perpendicular to x-axis (m)
Q	The long-shore sand transport rate (m^3/day)
D_B	The average berm height (m)
D_C	The closure depth (m)
Q_0	The amplitude of the long-shore sand transport rate
ρ	The density of sea water (kg/m^3)
ρ_s	The density of the sediment (kg/m^3)
n	The porosity
K	The dimensionless coefficient which is a function of particle size
H	The wave height (m)
c_g	The wave group velocity
α_b	The impact angle between breaking wave crests angle with local shoreline (degree)
α_0	The angle between breaking wave crests and the x-axis (degree)
L	The length of alongshore (m)
τ	The time of simulation (day)
X	The variables of dimensionless (no units)
T	The variables of dimensionless (no units)
$Y(X,T)$	The variables of dimensionless (no units)
Y_*	The expected shoreline evolution (m)

Chapter 1
Introduction

1.1 Research Motivation

Erosion is a natural process that constantly changes the physical characteristics of coastal areas. It is influenced by the processes of waves, wind, currents, and the movement of sand sediments from one area to another. These influences result in coastlines being characterized by dynamics and susceptibility to change. A report by the Department of Marine and Coastal Resources found that Thailand has a coastline of approximately 3,148.32 kilometers. It is divided into the Gulf of Thailand coast covering 17 provinces, a distance of 2,055.18 kilometers and the Andaman Sea coast covering 6 provinces, a distance of 1,093.14 kilometers. At present, coastal areas are used for a variety of purposes including tourism activities, recreational, fisheries and aquaculture, habitat, industrial sites, ports and water transport, etc. During this decade, coastal erosion occurring along the coast has a tendency of increasing frequency and severity every year.

For example, at Pak Phanang District, Nakhon Si Thammarat Province, at Pranburi Beach, Prachuap Khiri Khan Province, Thailand and even abroad, such as at the village of Withernsea in the East Riding of Yorkshire, United Kingdom, this problem is also faced.

Figure 1.1 : Erosion at Pranburi beach, Thailand [1]

Figure 1.2 : Erosion at the village of Withernsea, United Kingdom [2]

Erosions have consequences in many areas, such as economically, socially and environmentally. To address the issue, both the public and private sectors have established two approaches to coastal protection: the non-structure approach and the structure approach. The non-structure approach on preserving and restoring resources near the coastline, such as mangrove planting and restoration, sand filling, the use of regulatory laws, including setback zone, etc.

Figure 1.3 : Mangrove planting [3]

These approaches are effective in reducing the long-term impact on people's lives and property. It is suitable for coastal areas with fewer dense communities and little erosion, but it has a limitation that it cannot mitigate the immediate impact. In some countries, to reduce this limitation, engineering structures or structure approaches may be included, such as breakwater, seawall, and groin, etc. These structures are effective to reduce the impact. They are created by waves and coastal erosion, where structures can dissipate the power of waves or help hold the shoreline.

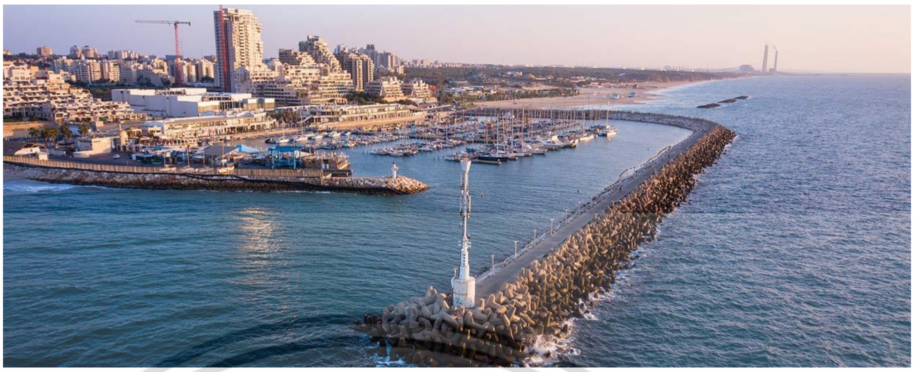

Figure 1.4 : Breakwater structure [4]

Figure 1.5 : Groin structure [5]

In this research, the focus is on the engineering structure of the groin system, which is a medium-sized artificial structure that is perpendicular to the coastline. It can be built as a standalone structure or as a set of groins. It is designed to trap the sand sediment that moves along the coastal currents causing the area to accumulate. Coastal evolution forecasting can be used for forecasting future trends in the rise and fall of landscapes and is therefore important because it can benefit management planning, sustainable solutions and can be used as part of an environmental impact assessment (EIA). With this motivation, the researchers manipulate a thesis titled: A Non-Dimensional Mathematical Model of Shoreline Evolution with a Groin Structure Using an Unconditionally Stable Explicit Finite Difference Technique.

1.2 Literature Review

Coastal erosion is a natural process that occurs when sediment movement away from the coast is not counterbalanced by fresh material growth along the shoreline. This is certainly a problem that is related to coastal erosion. A sea wall and groin were built to prevent coastal erosion and floods. The beach's future topography is analyzed using shoreline evolution research. Basic phrases that have a substantial influence on the coastal structure are erosion, accretion, and sea-level variations.

The partial differential equation represents several disciplines, including mass, heat, energy, velocity, and vorticity (see for example [6], [7]). In [8]–[13] these papers, the diffusion equation has been used to solve a variety of engineering problems, including pollutant and salinity transport in rivers and streams, and groundwater and contaminant dispersion in shallow lakes. In [14]–[16], they presented a case study of water level forecasting, a water quality evaluation based on probabilistic echo state networks, and the fluid dynamics of nonaqueous phase pollutants in groundwater.

Understanding the ideal shorelines' responses to the governing processes is important in the study of beach actions. A model for describing realistic situations involving general shoreline configuration settings and time-varying waves in more detail is proposed. As a result, numerical methods of shoreline evolution are preferred to analytical methods. In 1966, this paper [17] introduces modern logical design guidelines for groin structures. They are organized into three basic categories: Coastal processes, Functional design and Structural design. In [18] this paper expands on both the theoretical and practical concepts of mathematical models related to coastal behaviors: Theoretically, the influence of diffraction behind the groin is used to determine computer programming; Practically, the coastal constant in the theoretical model of the coast is expressed in terms of wave height and SVASEK's theoretical wave direction. In [19] this paper describes the development of the governing equations in general form and describes the assumptions and techniques used to obtain more than 25 analytical solutions. Solution for shoreline evolution with and without the influence of coastal structures. It covers situations involving beach filling of initial shape, sand mining, river discharge, groin and jetty and breakwater etc. The wind wave-driven longshore sediment transport rate and shoreline change are evaluated using a numerical model based on one-line theory in [20]. The model transforms waves from deep water into the surf zone and calculates their breaking characteristics. The model [21] provides complete, time-dependent simulations of shoreline evolution for coastlines driven by structures and a variety of boundary conditions that are both practical and reliable. This research looks at two case studies: New Jersey's Sea Isle City Beach and Egypt's Nile Delta Coast. The purpose of [22] is to quantify the changes in the shoreline along with the sand reclamation for the Sultan Mahmud Airport runway in Kuala Terengganu. Littoral Processes and Coastline Kinetics (LITPACK) numerical model is the numerical device employed to solve the shoreline problem. This research [23] describes a numerical modelling framework called GENESIS that is used to simulate long-term shoreline change caused by spatial and temporal variations in longshore sand transport at coastal engineering projects. The modelling system is managed via an organized and user-friendly interface, which reduces the need for the operator to get concerned with computer code specifics. The modelling-system application method is described from the viewpoint of engineers and planners involved in the evaluation of shore-protection projects. In [24]–[26], they are introducing a governing equation of a one-dimensional shoreline evolution model when a couple of groins is added, and the parameters that influence this model are described on a monthly basis over a period of one year. Consideration is given to the wave crest impact model for evaluating the impact of the wave crest at that stage.

Then, if a groin is added to a one-dimensional shoreline evolution model, we propose a governing equation for this research. It is provided with a non-dimensional shoreline evolution model with a groin construction model. The model can now be used to manipulate physical parameters.

1.3 Research Objectives

1. To introduce the setting of physical parameters.
2. To introduce the transformation of a one-dimensional model to a non-dimensional model and compare the efficiency of the calculation results.
3. To forecast coastal sand sediment erosion and deposition using long-term shoreline evolution models with groin structures.

4. To study the effects of shoreline evolution simulations if different scenarios are given.
5. To study and compare the efficiency of the calculation results between each of the finite difference methods.
6. Investigate methods and options for resolving coastal erosion issues.

1.4 Research Methodology

Step 1: Choose a thesis topic.

Step 2: Review and collect references related to shoreline evolution mathematical models.

Step 3: Improved the model of shoreline evolution.

Step 4: Research and improve the physical parameters that are relevant.

Step 5: Estimate the solution using appropriate numerical methods.

Step 6: Investigate and simulate shoreline evolution in various scenarios.

Step 7: Analyze and verify the results of the test.

Step 8: Present the test results in various formats, such as tables and graphs.

Step 9: Discuss and reach an agreement on the outcome.

Step 10: Suggest solutions for future problems.

Step 11: Compose the thesis.

1.5 Scopes of the Study

1. One-dimensional and non-dimensional shoreline evolution models are used as governing equations.
2. Two finite difference methods were used to approximate model solutions: the traditional forward time centered space (FTCS) method and the unconditionally Saulyev finite difference method.
3. The amplitude of the long-shore sand transport rate is given.
4. The average berm height is given.
5. The average closure depth is given.
6. The interpolation function of the measured evolutionary data is given.

1.6 Benefits of the Study

1. A non-dimensional model can be used to obtain an approximate solution to evolution.
2. We can predict the accumulation of sand sediment in the coastal area when a groin structure is used.
3. We can measure evolution over the long term.
4. We can offer solutions and possibilities for solving coastal erosion problems.
5. We have the ability to reduce erosion and increase coastal sand sediment.

Chapter 2

Preliminaries and Shoreline Evolution Model

This chapter introduces basic knowledge, concepts, and theories related to this research as well as numerical methods. To use as a basis for model development and further work.

2.1 Coastal Erosion Problem and Its Causes

Coastal erosion and deposition are among the problems of unbalanced coastal transformation. The impacts of erosion occur in many areas including environmental, economic, social and quality of life:

- *Environmental* – Coastal ecosystems (mangrove forests, seagrasses and coral reefs) are directly affected by the loss of existing shorelines or by sand sediment deposition. This has caused the degradation of seagrass and coral reef habitats. Affects ecosystem balance and decreases biodiversity.
- *Economic* - Businesses in the tourism sector are directly affected by the deteriorating coastline and the loss of beautiful coastlines. This affects the huge income generated by foreign tourists around the world. On the other hand, the state wastes resources, opportunities and large budgets to repair and prevent coastal erosion.
- *Social* – Coastal communities have to migrate from their old areas to new areas to avoid erosion that has resulted in the loss of their traditional livelihoods and the inability to carry on their jobs as usual.
- *Quality of life* – People and communities lose their land and asset. Anxiety arises in a new career which may affect family relationships and mental health.

There are two main causes of coastal erosion, which are:

1. It is caused by natural processes such as monsoons and storms that cause imbalance between the washed-out sand masses and the washed-out sand masses. At present, a natural process that has a huge impact is global climate change as sea level rise causes seawater to encroach on land resulting in severe coastal erosion and more frequent.

2. It is caused by human activities such as mangrove forest invasion, the negative impact of coastal engineering, the construction of dams and reservoirs at the upstream causes the flow of water to slow down and some sediment will be confined above the dam ang groundwater pumping contributes to soil subsidence, etc. The most impactful human action is the development of coastal areas for economic development. It has led to a large number of constructions in coastal areas such as the construction of waterways, the construction of industrial estates, the construction of deep-sea ports, etc. They do not take into account the actual site conditions such as corridors and directions of water, which would subsequently result in an imbalance in the coastal geology, resources and ecosystems in the area and ultimately lead to coastal erosion.

The causes of erosion are natural and artificial which detailed in the table below:

Table 2.1 : Causes of erosion are natural and artificial

Natural	Artificial
1. Action of breaking waves	1. Construction of unplanned structures
2. Effect of severe cyclonic storms	2. Reduction of sediment supply due to damming of rivers
3. Rise in sea level	3. Removal of sand from beaches
4. Deflation	4. Dredging of inlet channels
5. Tidal current	5. Unplanned reclamation

2.2 Protection from Coastal Erosion

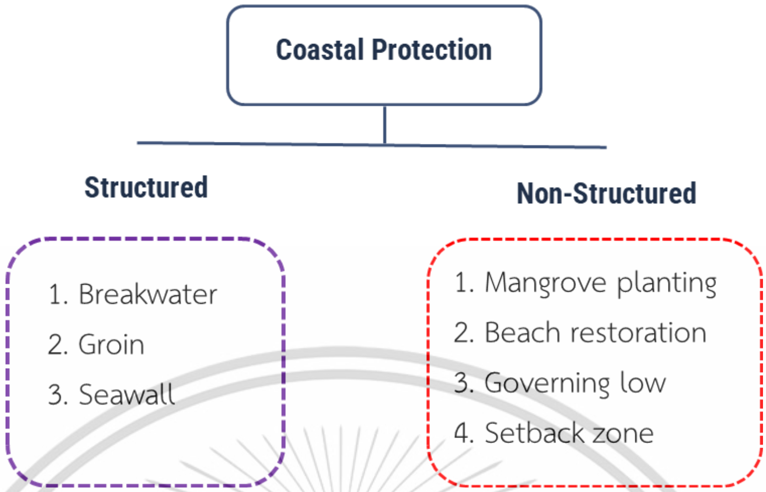

Figure 2.1 : Coastal protection

Coastal protection approaches are classified into two approaches: the non-structural approach and the non-structural approach, which the details are as follows:

2.2.1 The Non-Structural Approach

The non-structural approach aims to dissipate wave energy by mirroring natural forces and maintaining the natural topography of the coast. An example of this approach:

1) Mangroves Planting

Figure 2.2 : Mongrove planting [27]

2) Breach Restoration

Figure 2.3 : Breach restoration [28]

3) Governing Low

4) Setback Zone

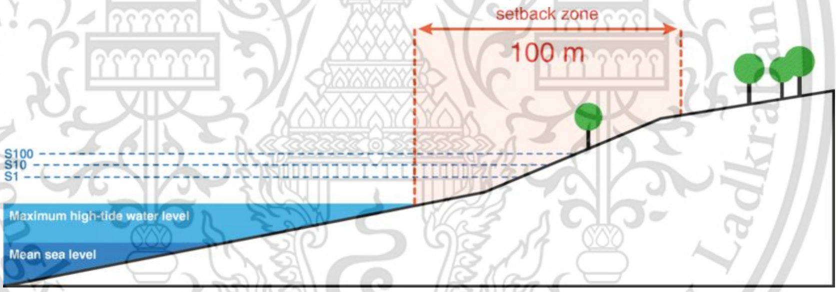

Figure 2.4 : Setback zone [29]

2.2.2 The Structural Approach

The structural approaches constructed on the beach or further offshore. These influence coastal processes to stop or reduce the rate of coastal erosion. An example of this approach:

1) Breakwater

Figure 2.5 : Breakwater [30]

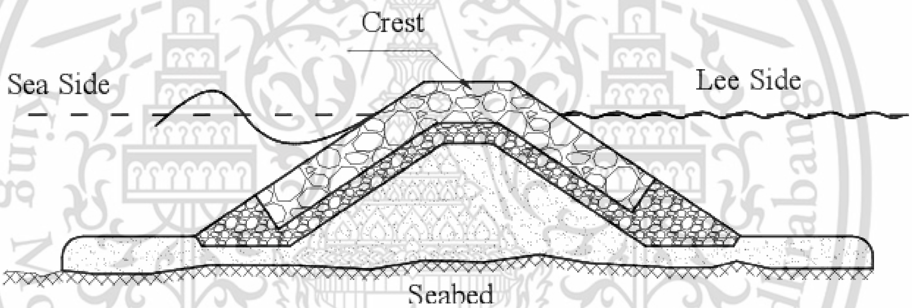

Figure 2.6 : Breakwater structure [31]

A breakwater is built out into the sea to create a safe harbor, marina, or anchorage for fishing vessels and protect the coast from waves. They are a structure that parallels or is perpendicular to the shore and serves as a wave absorber and reduces wave energy in its lee. it creates a salient or tombolo behind the structure that influences the longshore transport of sediment. Moreover, they become multipurpose artificial reefs where fish habitats develop and enhance surf breaking for water sports activities. These structures are appropriate for all coastlines. Their disadvantages are:

- They are relatively large structures and are difficult to build.
- They have to be specially designed.
- They are vulnerable to strong wave action.

2) Seawall

Figure 2.7 : Seawall [32]

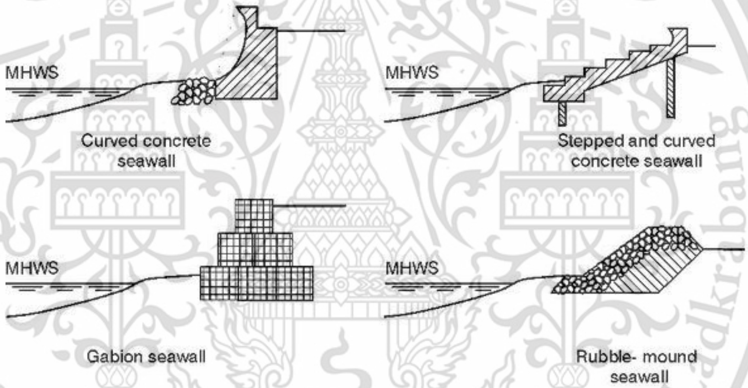

Figure 2.8 : Seawall structure [33]

A seawall is a large barrier built to protect coastal communities against flooding and mitigate the effects of erosion. Like the breakwater, they are a structure constructed parallel to the shoreline that shelters the shore from wave action. This structure has many different designs. It can be used to protect a cliff from wave attack and improve slope stability and it can also dissipate wave energy on sandy coasts. The disadvantages of this structure are:

- It creates wave reflections and promotes sediment transport offshore.
- It does not promote beach stability.
- It should be constructed along the whole coastline if not, erosion will occur on the adjacent coastline.

- It should be constructed along the whole coastline because otherwise the adjacent coastline will be eroded.

- It takes an expensive build budget.

3) Groin Structures or Groyne

Figure 2.9 : Groin [34]

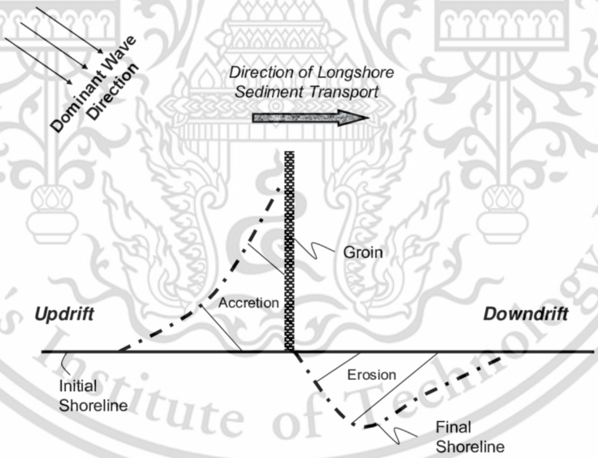

Figure 2.10 : Groin structures [35]

A groin is a medium-sized artificial structure built to the coastline from the shore into the sea to trap longshore sediment transport or control longshore currents. Unlike the breakwater, which generates calm water basins, groins are not constructed to create harbors and do not provide shelter to fishing boats, yachts, and vessels. It is built in series that work together to catch sediments in the surf zone brought by longshore drift. This type of structure is easy to construct from a variety of materials such as wood, rock, or bamboo and is normally used on sandy coasts. It has the following disadvantages:

- Requires regular maintenance.
- Induces local to scour at the base of structures.
- Typically, more than one structure is required.

The non-structural approach and the structural approach have both positive and negative aspects. Therefore, choosing a preventive approach is very important as it affects planning, implementation, and monitoring. In some countries, to minimize any side impacts and to maximize the effectiveness of prevention. So, they have adopted a combination of both approaches. To be a guideline for decision-making. Therefore, this research focuses on the groin structure introduced with the shoreline growth model. At present, coastal evolution analysis is used to predict future topographic trends.

2.3 The Governing Equation

The beach profile is supposed to travel landward and seaward while maintaining the same form in the one-line model, meaning that all bottom contours are parallel. As a result, specifying the horizontal location of the profile with respect to the baseline is sufficient under this assumption, and one contour line may be used to represent changes in the beach plan shape and volume as the beach reduces and accretes. Sand is carried alongshore between two well-defined limiting heights on the profile i.e., berm height and closure depth, according to the model's main assumption. If there is a variation in the alongshore sand transport rate at the lateral sides of the section and the related sand continuity, it contributes to the volume change.

At all times, the laws of mass conservation must be applied to the system. The following differential equation for shoreline evolution is produced using the given definitions,

$$\frac{\partial y}{\partial t} = \frac{1}{D_B + D_C}\left(-\frac{\partial Q}{\partial x}\right) , \quad (2.1)$$

where x is the alongshore coordinate (m),

y is the shoreline positions and perpendicular to x-axis (m),

t is time (day),

Q is the long-shore sand transport rate (m^3/day),

D_B is the average berm height (m),

D_C is the average closure depth (m).

In order to solve Eq. (2.1), an equation for the longshore sand transport rate Q must be specified. This quantity is thought to be created by a wave that strikes the coastline obliquely. In [36] provided a general expression for the long-shore sand transport rate,

$$Q = Q_0 \sin\left(2\alpha_b\right) , \tag{2.2}$$

where Q_0 is the amplitude of the long-shore sand transport rate.

The empirical predictive formula for the amplitude of the long-shore sand transport rate [37] is,

$$Q_0 = \frac{\rho}{16}\left(H_b^2 c_{gb}\right)\frac{K}{(\rho_s - \rho)(1-n)} , \tag{2.3}$$

where the subscript b represent the value at the point breaking,

ρ is the density of sea water (kg/m^3),

ρ_s is the density of the sediment (kg/m^3),

n is the porosity,

K is the dimensionless coefficient which is a function of particle size,

H is the wave height,

c_g is the wave group velocity.

The quantity α_b the impact angle between breaking wave crests angle with local shoreline, and may be written as,

$$\alpha_b = \alpha_0 - \tan^{-1}\left(\frac{\partial y}{\partial x}\right) , \tag{2.4}$$

where α_0 is the angle between breaking wave crests and the x-axis. For beaches with a slight slope, the breaking wave angle to the coastline is likely to be minimal. Assuming that,

$$\sin(2\alpha_b) \approx 2\alpha_b \ ,$$

and

$$\tan^{-1}\left(\frac{\partial y}{\partial x}\right) \approx \left(\frac{\partial y}{\partial x}\right) .$$

Substituting Eq.(2.4) into Eq.(2.2), and assuming the beach with mild slope yields,

$$Q = Q_0\left(2\alpha_0 - 2\frac{\partial y}{\partial x}\right) . \tag{2.5}$$

Substituting Eq.(2.5) into Eq.(2.1), and neglecting the sources or sinks along the coast. We obtain,

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} \ , \tag{2.6}$$

for all $(x,t) \in \Omega$ such that $\Omega = [0, L] \times [0, \tau]$ and $\tau > 0$, where $D = \dfrac{2Q_0}{D_B + D_C}$.

2.4 The Initial and Boundary Conditions

2.4.1 The Initial Condition

The initial condition is defined by the interpolation function of the measured evolutionary data. It rests along the length of the coast from its start to the end of the beach area being considered. The initial condition is assumed to be as follows,

$$y(x,0) = f(x) \ , \tag{2.7}$$

for all $x \in [0, L]$, where $f(x)$ is the interpolation function of the measured evolutionary data.

2.4.2 The Left Boundary Condition

The left boundary condition is defined by the interpolation function of the measured evolutionary data. It is shoreline evolution with the left-hand side groin system. The left boundary condition is assumed to be as follows,

$$y(0,t) = g(t), \tag{2.8}$$

for all $t \in [0,\tau]$, where $g(t)$ is a given interpolation function of the measured evolutionary data at the left-hand side groin system.

2.4.3 The Right Boundary Condition

The right boundary condition is defined by the interpolation function of the measured evolutionary data. It is shoreline evolution with the right-hand side groin system. The right boundary condition is assumed to be as follows,

$$y(L,t) = h(t), \tag{2.9}$$

for all $t \in [0,\tau]$, where $h(t)$ is a given interpolation function of the measured evolutionary data at the right-hand side groin system.

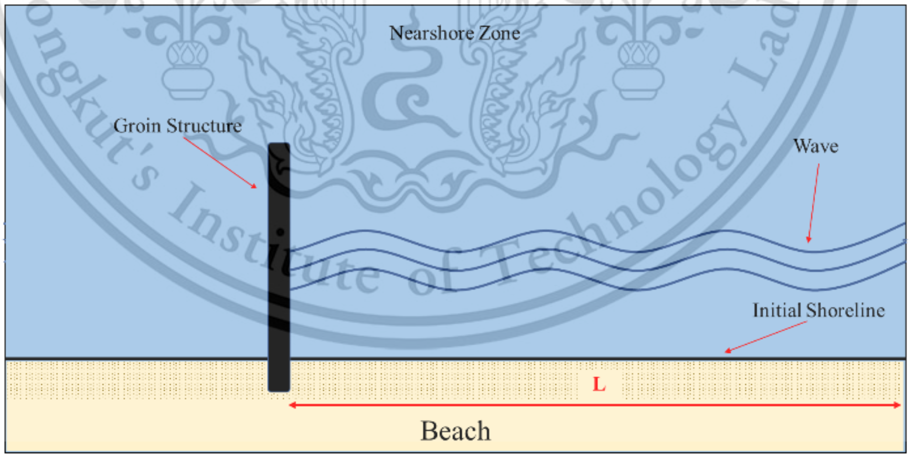

Figure 2.11 : Initial shoreline with configuration straight impermeable groins

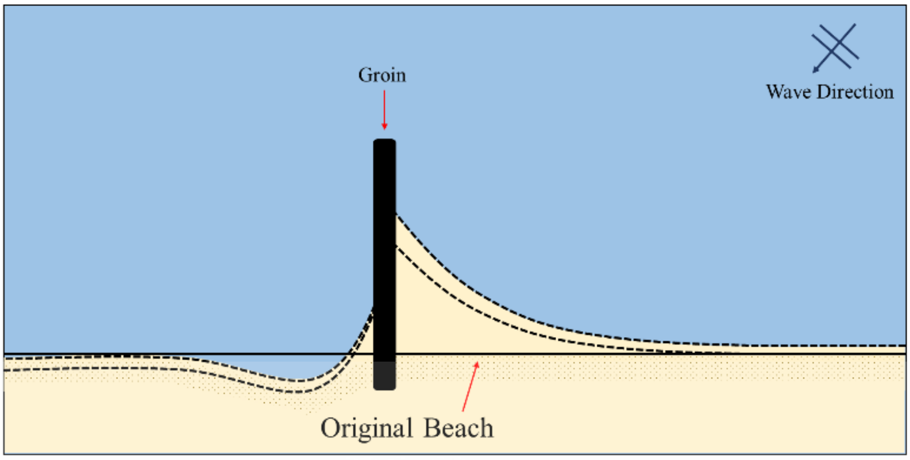

Figure 2.12 : Shoreline evolution with groins

2.5 Nondimensionalization

The partial or complete removal of physical dimensions from an equation involving physical values is known as nondimensionalization. This method can be used to simplify and parameterize problems using measured units. It has a lot in common with dimensional analysis. The phrase scaling is sometimes used to suggest that particular values are better assessed to a specific unit. This method is especially effective for systems that can be characterized using differential equations, as it saves time and money when it comes to measurement instruments, budgets, personnel, and software, among other things. To nondimensionalize a system of equations, one must do the following:

1. Make a list of all independent and dependent variables.
2. Substitute a quantity scaled relative to a to-be-determined characteristic unit of measure for each of them.
3. Remove the coefficient of the highest order polynomial or derivative term from the equation.
4. Carefully select each variable's characteristic unit specification such that the coefficients of as many terms as possible reach 1.
5. Rewrite the equations system in terms of their new dimensionless quantities.

2.6 Numerical Methods

Numerical methods are widely used in solving scientific, mathematical, and engineering problems because some equations are complex. Which requires advanced mathematics to solve problems. Therefore, an understanding of numerical methods is necessary and many studies are used to solve these problems. However, the solution given by the numerical method is only an approximation. In this section, we will consider the finite difference method mainly used for numerical methods.

2.6.1 Grid Spacing

We introduce the finite difference expression and describe the basic concepts and methods for approximating solutions. The typical grid point is given by $X_m = m\Delta X$ and $T_n = n\Delta T$, where $m = 0,1,2,\ldots,M$, $n = 0,1,2,\ldots,N$, ΔX and ΔT are the grid spacing or grid size or step size in the space and time coordinates, respectively, and positive integers M, N represent the number of subintervals, and approximate $U(X_m,T_n)$ by U_m^n, value of the difference approximation of $U(X_m,T_n)$. Consider a grid system as shown in Figure 2.13.

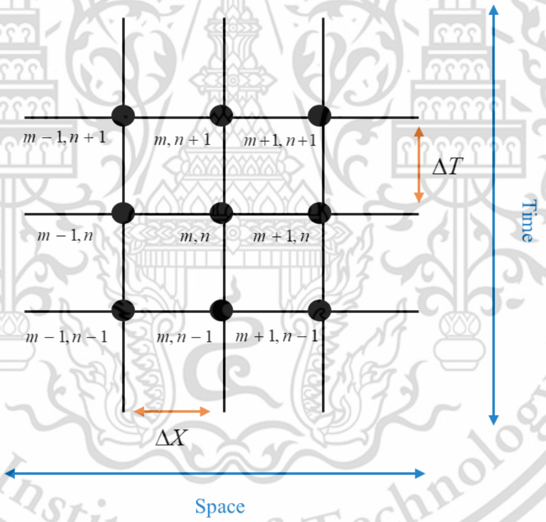

Figure 2.13 : Grid spacing

The Taylor series for a function $U(X,T)$ expanded about X_m at $(X_m + \Delta X)$ and $(X_m - \Delta X)$ are respectively. We obtain,

$$U(X+\Delta X,T) = U(X,T) + (\Delta X)\frac{\partial U(X,T)}{\partial X} + \frac{(\Delta X)^2}{2!}\frac{\partial^2 U(X,T)}{\partial^2 X} + \frac{(\Delta X)^3}{3!}\frac{\partial^3 U(X,T)}{\partial^3 X} + \ldots , \quad (2.10)$$

$$U(X-\Delta X,T) = U(X,T) - (\Delta X)\frac{\partial U(X,T)}{\partial X} + \frac{(\Delta X)^2}{2!}\frac{\partial^2 U(X,T)}{\partial^2 X} - \frac{(\Delta X)^3}{3!}\frac{\partial^3 U(X,T)}{\partial^3 X} + \ldots , \quad (2.11)$$

or corresponds to Eqs. (2.10) - (2.11) if note that ΔX equal to h and ΔT equal to k, we obtain,

$$U(X+h,T)=U(X,T)+h\frac{\partial U(X,T)}{\partial X}+\frac{h^2}{2!}\frac{\partial^2 U(X,T)}{\partial^2 X}+\frac{h^3}{3!}\frac{\partial^3 U(X,T)}{\partial^3 X}+\dots\ , \quad (2.12)$$

$$U(X-h,T)=U(X,T)-h\frac{\partial U(X,T)}{\partial X}+\frac{h^2}{2!}\frac{\partial^2 U(X,T)}{\partial^2 X}-\frac{h^3}{3!}\frac{\partial^3 U(X,T)}{\partial^3 X}+\dots\ . \quad (2.13)$$

Introducing U_m^n- notation in which the subscript denotes the X-position and the superscript denotes the T-position, the above expressions can be written as:

$$U_{m+1}^n=U_m^n+h\frac{\partial U_m^n}{\partial X}+\frac{h^2}{2!}\frac{\partial^2 U_m^n}{\partial X^2}+\frac{h^3}{3!}\frac{\partial^3 U_m^n}{\partial X^3}+\dots\ , \quad (2.14)$$

$$U_{m-1}^n=U_m^n-h\frac{\partial U_m^n}{\partial X}+\frac{h^2}{2!}\frac{\partial^2 U_m^n}{\partial X^2}-\frac{h^3}{3!}\frac{\partial^3 U_m^n}{\partial X^3}+\dots\ . \quad (2.15)$$

From Eq. (2.14),

$$h\frac{\partial U_m^n}{\partial X}=U_{m+1}^n-U_m^n-\frac{h^2}{2!}\frac{\partial^2 U_m^n}{\partial X^2}-\frac{h^3}{3!}\frac{\partial^3 U_m^n}{\partial X^3}-\dots\ , \quad (2.16)$$

$$\frac{\partial U_m^n}{\partial X}=\frac{U_{m+1}^n}{h}-\frac{U_m^n}{h}+\left(-\frac{h}{2!}\frac{\partial^2 U_m^n}{\partial X^2}-\frac{h^2}{3!}\frac{\partial^3 U_m^n}{\partial X^3}-\dots\right), \quad (2.17)$$

$$\frac{\partial U_m^n}{\partial X}=\frac{U_{m+1}^n-U_m^n}{h}+O(h)\ . \quad (2.18)$$

Similarly, from Eq. (2.15),

$$h\frac{\partial U_m^n}{\partial X}=U_m^n-U_{m-1}^n-\frac{h^2}{2!}\frac{\partial^2 U_m^n}{\partial X^2}-\frac{h^3}{3!}\frac{\partial^3 U_m^n}{\partial X^3}+\dots\ , \quad (2.19)$$

$$\frac{\partial U_m^n}{\partial X}=\frac{U_m^n}{h}-\frac{U_{m-1}^n}{h}+\left(-\frac{h}{2!}\frac{\partial^2 U_m^n}{\partial X^2}-\frac{h^2}{3!}\frac{\partial^3 U_m^n}{\partial X^3}+\dots\right), \quad (2.20)$$

$$\frac{\partial U_m^n}{\partial X}=\frac{U_m^n-U_{m-1}^n}{h}+O(h)\ . \quad (2.21)$$

Then, if the terms containing the second-order and higher derivative are truncation in these expressions. We obtain the forward difference and the backward difference approximations respectively for the first-order derivative, and the truncation error is $O(h)$ and $O(h)$ respectively.

If we take Eq. (2.14) subtract Eq. (2.15) and rearrange, we obtain the central difference:

$$2h\frac{\partial U_m^n}{\partial X} = U_{m+1}^n + U_m^n + \left(\frac{2h^3}{3!}\frac{\partial^3 U_m^n}{\partial X^3} + \dots\right), \tag{2.22}$$

$$\frac{\partial U_m^n}{\partial X} = \frac{U_{m+1}^n + U_m^n}{2h} + \left(\frac{h^2}{3!}\frac{\partial^3 U_m^n}{\partial X^3} + \dots\right), \tag{2.23}$$

$$\frac{\partial U_m^n}{\partial X} = \frac{U_{m+1}^n + U_m^n}{2h} + O\left(h^2\right), \tag{2.24}$$

and the truncation error is $O\left(h^2\right)$.

If we take Eq. (2.14) plus Eq. (2.15) and rearrange, we obtain the central difference for second-order derivative:

$$\frac{2h^2}{2!}\frac{\partial^2 U_m^n}{\partial X^2} = U_{m+1}^n + U_{m-1}^n - 2U_m^n + \left(-\frac{2h^4}{4!}\frac{\partial^4 U_m^n}{\partial X^4} - \dots\right), \tag{2.25}$$

$$\frac{\partial^2 U_m^n}{\partial X^2} = \frac{U_{m+1}^n - 2U_m^n + U_{m-1}^n}{h} + \left(-\frac{2h^2}{4!}\frac{\partial^4 U_m^n}{\partial X^4} - \dots\right), \tag{2.26}$$

$$\frac{\partial^2 U_m^n}{\partial X^2} = \frac{U_{m+1}^n - 2U_m^n + U_{m-1}^n}{h} + O\left(h^2\right), \tag{2.27}$$

and the truncation error is $O\left(h^2\right)$.

Similarly, for the T-derivatives, we obtain,

$$\frac{\partial U_m^n}{\partial T} = \frac{U_m^{n+1} - U_m^n}{k} + O(k), \tag{2.28}$$

$$\frac{\partial U_m^n}{\partial T} = \frac{U_m^n - U_m^{n-1}}{k} + O(k) \ , \tag{2.29}$$

$$\frac{\partial U_m^n}{\partial T} = \frac{U_m^{n+1} - U_m^{n-1}}{2k} + O(k^2) \ , \tag{2.30}$$

$$\frac{\partial^2 U_m^n}{\partial T^2} = \frac{U_m^{n+1} - 2U_m^n + U_m^{n-1}}{k^2} + O(k^2) \ . \tag{2.31}$$

Those equations (2.28) - (2.30) are called the forward difference, the backward difference and the central difference approximations, respectively for first-order derivative, and equation (2.31) is the central difference approximations for second-order derivative.

2.6.2 The Traditional Forward Time Centered Space (FTCS) Technique

The finite difference approximation for this technique becomes [24], [38]:

$$U(X_m, T_n) \cong U_m^n \ , \tag{2.32}$$

$$\frac{\partial U}{\partial T} \cong \frac{U_m^{n+1} - U_m^n}{\Delta T} \ , \tag{2.33}$$

$$\frac{\partial U}{\partial X} \cong \frac{U_{m+1}^n - U_{m-1}^n}{2\Delta X} \ , \tag{2.34}$$

$$\frac{\partial^2 U}{\partial X^2} \cong \frac{U_{m+1}^n - 2U_m^n + U_{m-1}^n}{(\Delta X)^2} \ . \tag{2.35}$$

2.6.3 An Unconditionally Saulyev Finite Difference (Saulyev) Technique

The finite difference approximation for this technique becomes [24], [38]:

$$U(X_m, T_n) \cong U_m^n \ , \tag{2.36}$$

$$\frac{\partial U}{\partial T} \cong \frac{U_m^{n+1} - U_m^n}{\Delta T} \ , \tag{2.37}$$

$$\frac{\partial U}{\partial X} \cong \frac{U_{m+1}^n - U_{m-1}^{n+1}}{2\Delta X} \ , \tag{2.38}$$

$$\frac{\partial^2 U}{\partial X^2} \cong \frac{U_{m+1}^{n} - U_{m}^{n} - U_{m}^{n+1} + U_{m-1}^{n+1}}{(\Delta X)^2} \quad . \tag{2.39}$$

2.7 Error Measurement

A simple measure used to measure the difference between actual and approximate values is the absolute error method. The absolute error formula be for each grid point (x_m, t_n) as follows:

$$E_m^n = \left| \tilde{y}_m^n - y_m^n \right| \ , \tag{2.40}$$

where $\tilde{y}_m^n$ is the analytical solution of shoreline evolution,
y_m^n is the approximate solution of shoreline evolution.

The calculated non-dimensional solutions have to be converted into the dimensional solutions which are used in Eq. (2.40).

Chapter 3

The Non-Dimensional Shoreline Evolution Model

This chapter introduces the development of a mathematical model for forecasting coastal evolution. The model is less complex than the old model but still capable of manipulating physical parameters. This newly developed model is called a non-dimensional shoreline evolution model. In addition, we will introduce the definitions of conditions according to this model and the numerical techniques.

3.1 The One-Dimensional Shoreline Evolution Model

The One-Dimensional Shoreline Evolution Model:

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} , \tag{3.1}$$

for all $(x,t) \in \Omega$ such that $\Omega = [0, L] \times [0, \tau]$ and $\tau > 0$, where $D = \frac{2Q_0}{D_B + D_C}$.

3.1.1 The Initial Condition

$$y(x,0) = f(x) , \tag{3.2}$$

for all $x \in [0, L]$, where $f(x)$ is the interpolation function of the measured evolutionary data.

3.1.2 The Left Boundary Condition

$$y(0,t) = g(t) , \tag{3.3}$$

for all $t \in [0, \tau]$, where $g(t)$ is a given interpolation function of the measured evolutionary data at the left-hand side groin system.

3.1.3 The Right Boundary Condition

$$y(L,t) = h(t) , \tag{3.4}$$

for all $t \in [0, \tau]$, where $h(t)$ is a given interpolation function of the measured evolutionary data at the right-hand side groin system.

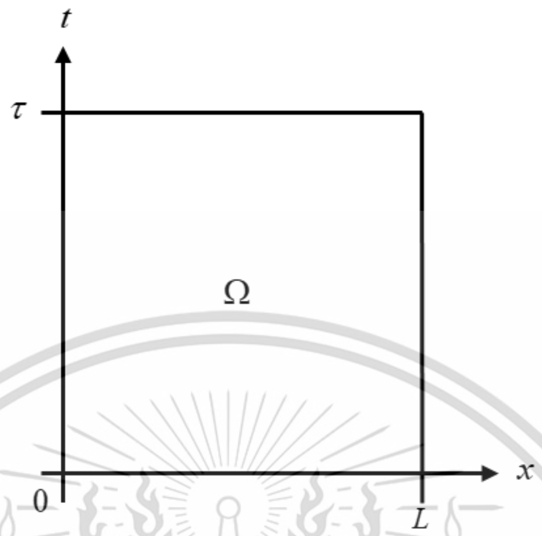

Figure 3.1 : Domain diagram of a one-dimensional model

For research, these equations (3.1)-(3.4) are called the one-dimensional model of shoreline evolution.

3.2 The Non-Dimensional Shoreline Evolution Model

To remove physical dimensions and simplify govern equations, which save time and cost in terms of measuring tools, budgets, people and software, etc. In this section, we will show how to develop a one-dimensional model into the term of a non-dimensional variable by using a nondimensionalization in Chapter 3 and a transformation technique [39]. We set $Y = \frac{y}{Y_*}$, $X = \frac{x}{L}$, and from the chain rule:

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial X}\frac{\partial X}{\partial x} , \tag{3.5}$$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial X}\left(\frac{1}{L}\right) , \tag{3.6}$$

and,

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) , \tag{3.7}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial X}\left(\frac{1}{L}\right)\right) , \tag{3.8}$$

$$= \frac{1}{L}\frac{\partial}{\partial x}\left(\frac{\partial y}{\partial X}\right) , \tag{3.9}$$

$$= \frac{1}{L}\frac{\partial}{\partial X}\left(\frac{\partial y}{\partial x}\right) , \tag{3.10}$$

$$= \frac{1}{L}\frac{\partial}{\partial X}\left(\frac{\partial y}{\partial X}\left(\frac{1}{L}\right)\right) , \tag{3.11}$$

$$= \frac{1}{L^2}\frac{\partial^2 y}{\partial X^2} . \tag{3.12}$$

Since $y = YY_*$.

Substituting these into the Eq.(3.1), we obtain,

$$\frac{\partial y}{\partial t} = D\left(\frac{1}{L^2}\frac{\partial^2 y}{\partial X^2}\right) , \tag{3.13}$$

$$\frac{\partial (YY_*)}{\partial t} = D\left(\frac{1}{L^2}\frac{\partial^2 (YY_*)}{\partial X^2}\right) , \tag{3.14}$$

$$Y_*\frac{\partial Y}{\partial t} = Y_*\frac{D}{L^2}\frac{\partial^2 Y}{\partial X^2} . \tag{3.15}$$

Cancelling Y_* leaves,

$$\frac{\partial Y}{\partial t} = \frac{D}{L^2}\frac{\partial^2 Y}{\partial X^2} , \tag{3.16}$$

writing $T = DL^{-2}t$ or $t = D^{-1}L^2T$ and applying the function of a function rule to the left side yields,

$$\frac{\partial Y}{\partial (D^{-1}L^2T)} = \frac{D}{L^2}\frac{\partial^2 Y}{\partial X^2} , \tag{3.17}$$

$$\frac{D}{L^2}\frac{\partial Y}{\partial T} = \frac{D}{L^2}\frac{\partial^2 Y}{\partial X^2} . \tag{3.18}$$

Cancelling $\frac{D}{L^2}$ leaves, we obtain the non-dimensional form:

$$\frac{\partial Y}{\partial T} = \frac{\partial^2 Y}{\partial X^2} \ , \tag{3.19}$$

for all $(X,T) \in \psi$ such that $\psi = [0,1] \times [0,\Gamma]$ and $\Gamma > 0$, where

$$X = \frac{x}{L} \ , \tag{3.20}$$

$$T = \frac{Dt}{L^2} \ , \tag{3.21}$$

$$Y = \frac{y}{Y_*} \ , \tag{3.22}$$

and the variables X , T and Y are dimensionless and have no units,

L is the length of alongshore (m),

Y_* is the expected shoreline evolution (m).

Equation (3.19) is similar to the heat equation which has a thermal conductivity coefficient of 1, so in order to solve a problem it is necessary to define the initial conditions and the boundary conditions.

3.3 The Initial and Boundary Conditions

The initial and boundary conditions for a non-dimensional model can be simply defined under the known initial and boundary conditions according to Eqs. (3.2) – (3.4) for the one-dimensional model, so we can define the necessary conditions as the follows:

3.3.1 The Initial Condition

$$Y(X,0) = F(X) \ , \tag{3.23}$$

for all $X \in [0,1]$, where $F(X) = \frac{f(x)}{Y_*}$.

3.3.2 The Left Boundary Condition

$$Y(0,T) = G(T) \ , \tag{3.24}$$

for all $T \in [0,\Gamma]$, where $G(T) = \dfrac{g(t)}{Y_*}$.

3.3.3 The Right Boundary Condition

$$Y(1,T) = H(T) \ , \tag{3.25}$$

for all $T \in [0,\Gamma]$, where $H(T) = \dfrac{h(t)}{Y_*}$.

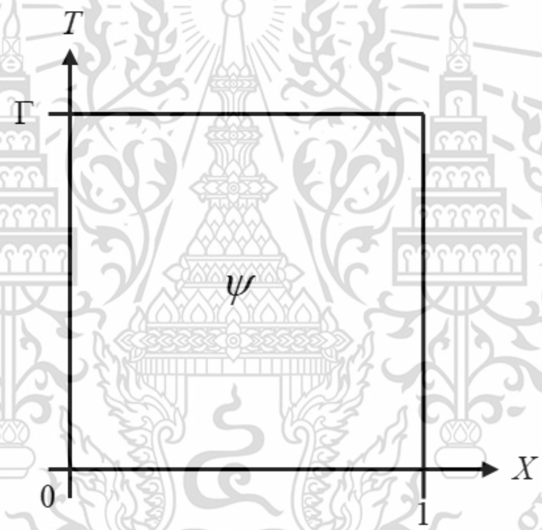

Figure 3.2 : Domain diagram of a non-dimensional model

For research, these equations (3.19) - (3.25) are called the non-dimensional model of shoreline evolution.

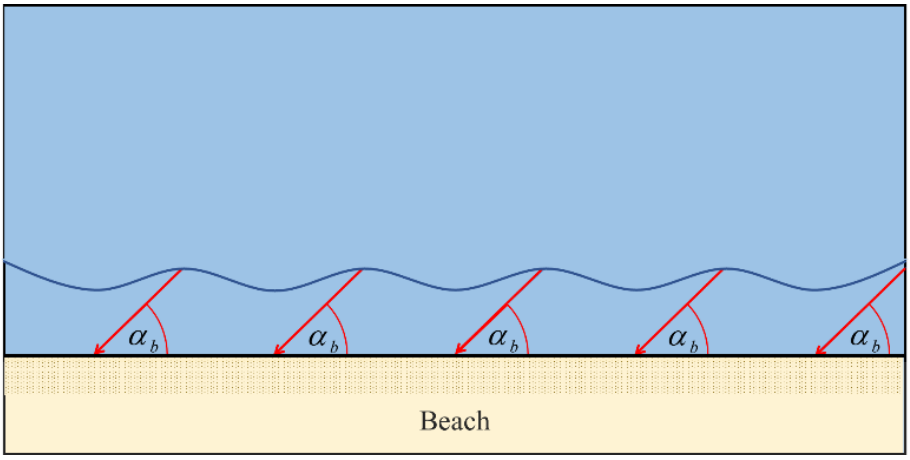

Figure 3.3 : Breaking wave crests impact angle

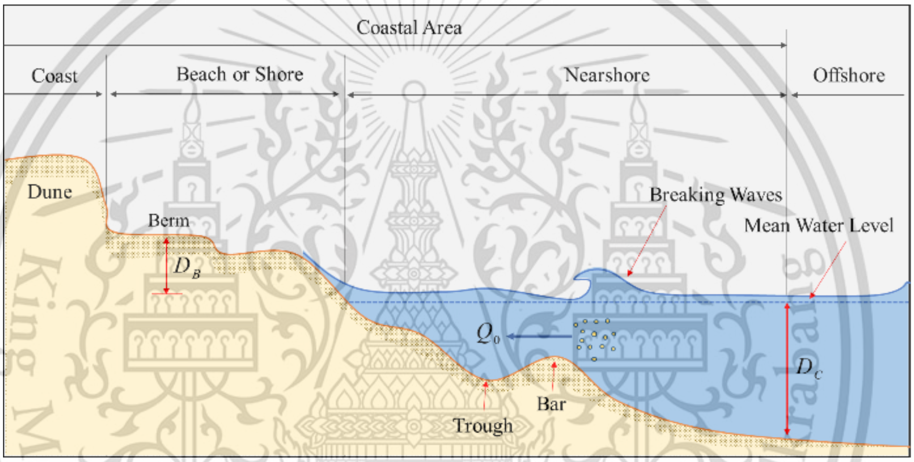

Figure 3.4 : Beach profile and shoreline physical parameters

3.4 Physical Parameters

Physical parameter of the model can be illustrated as show in Figure 3.3 - 3.4. that are listed below:

α_0 is the impact angle between breaking wave crests angle with the x-axis (degree),

Q_0 is the amplitude of the long-shore sand transport rate (m^3/day),

D_B is the average berm height (m),

D_C is the average closure depth (m),

L is the length of alongshore (m),

τ is time of simulation (day).

3.5 Numerical Techniques for the Non-Dimensional Shoreline Evolution Model

3.5.1 The Traditional Forward Time Centered Space (FTCS) Technique

A mesh grid-line is used to cover the problem domain. We now discretize the domain of Eq. (3.19) by dividing the interval $[0,1]$ into M subintervals such that $M\Delta X = 1$ and the time interval $[0,\Gamma]$ into N subintervals such that $N\Delta T = \Gamma$. We then approximate $Y(X_m, T_n)$ by Y_m^n , at the point $X_m = m\Delta X$ and $T_n = n\Delta T$, where $m = 0,1,\ldots,M$ and $n = 0,1,\ldots,N$ in which M and N are positive integers. Consider a grid-lines system as illustrated in Figure 3.5.

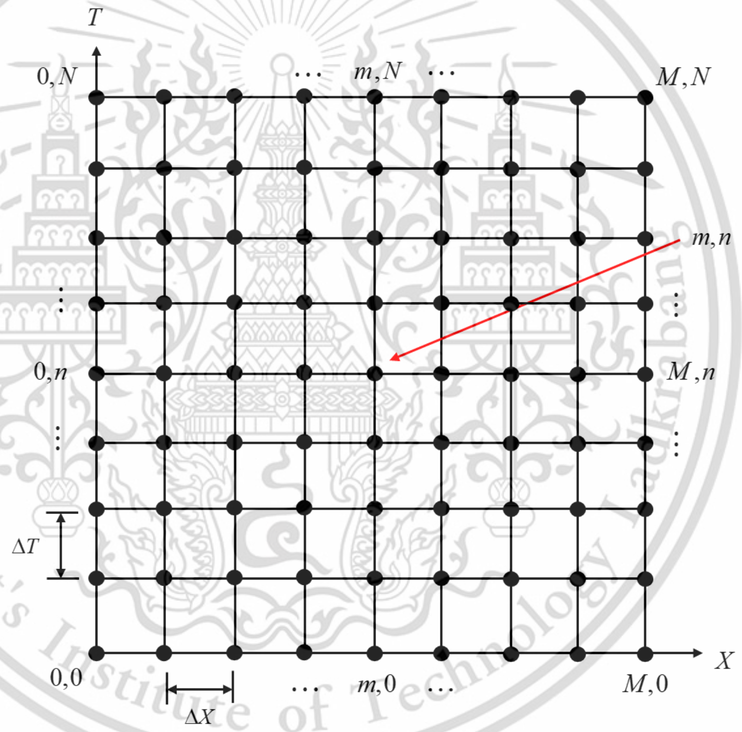

Figure 3.5 : A mesh grid-lines system where m and n are indexes indicating the position of points

The forward time centered space (FTCS) Technique is employed. Consequently, the finite difference approximation becomes:

$$Y(X_m, T_n) \cong Y_m^n \ , \tag{3.26}$$

$$\frac{\partial Y}{\partial T} \cong \frac{Y_m^{n+1} - Y_m^n}{\Delta T} \ , \tag{3.27}$$

$$\frac{\partial Y}{\partial X} \cong \frac{Y_{m+1}^{n} - Y_{m-1}^{n}}{2\Delta X} \quad , \tag{3.28}$$

$$\frac{\partial^2 Y}{\partial X^2} \cong \frac{Y_{m+1}^{n} - 2Y_{m}^{n} + Y_{m-1}^{n}}{\left(\Delta X\right)^2} \quad . \tag{3.29}$$

Substituting Eqs. (3.26) - (3.29) into Eq. (3.19), we obtain,

$$\frac{Y_{m}^{n+1} - Y_{m}^{n}}{\Delta T} \cong \frac{Y_{m+1}^{n} - 2Y_{m}^{n} + Y_{m-1}^{n}}{\left(\Delta X\right)^2} \quad , \tag{3.30}$$

for all $m = 1, 2, ..., M-1$ and $n = 0, 1, ..., N$.

Equation (3.30) can be written in an explicit form of finite difference as follows:

$$Y_{m}^{n+1} \cong \mu Y_{m+1}^{n} + \left(1 - 2\mu\right) Y_{m}^{n} + \mu Y_{m-1}^{n} \quad , \tag{3.31}$$

for all $m = 1, 2, ..., M-1$ and $n = 0, 1, ..., N$, where $\mu = \dfrac{\Delta T}{\left(\Delta X\right)^2}$.

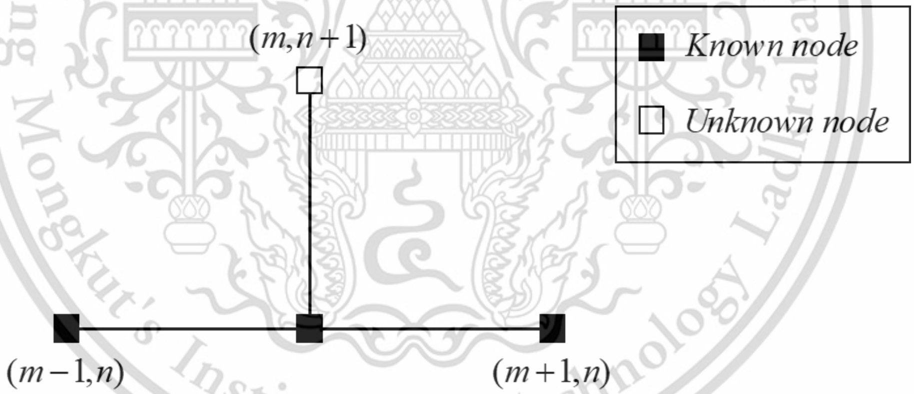

Figure 3.6 : Stencil diagram of the FTCS method

3.5.2 The Unconditionally Saulyev Finite Difference (Saulyev) Technique

A mesh grid-line is used to cover the problem domain. We now discretize the domain of Eq. (3.19) by dividing the interval $[0,1]$ into M subintervals such that $M\Delta X = 1$ and the time interval $[0,\Gamma]$ into N subintervals such that $N\Delta T = \Gamma$. We then approximate $Y\left(X_m, T_n\right)$ by Y_m^n , at the point $X_m = m\Delta X$ and $T_n = n\Delta T$, where $m = 0, 1, ..., M$ and $n = 0, 1, ..., N$ in which M and N are positive integers. Consider a grid-lines system as illustrated in Figure 3.5.

The Saulyev finite difference technique is employed. Consequently, the finite difference approximation becomes:

$$Y\left(X_m, T_n\right) \cong Y_m^n \ , \tag{3.32}$$

$$\frac{\partial Y}{\partial T} \cong \frac{Y_m^{n+1} - Y_m^n}{\Delta T} \ , \tag{3.33}$$

$$\frac{\partial Y}{\partial X} \cong \frac{Y_{m+1}^n - Y_{m-1}^{n+1}}{2\Delta X} \ , \tag{3.34}$$

$$\frac{\partial^2 Y}{\partial X^2} \cong \frac{Y_{m+1}^n - Y_m^n - Y_m^{n+1} + Y_{m-1}^{n+1}}{\left(\Delta X\right)^2} \ . \tag{3.35}$$

Substituting Eqs. (3.32) - (3.35) into Eq. (3.19), we obtain,

$$\frac{Y_m^{n+1} - Y_m^n}{\Delta T} \cong \frac{Y_{m+1}^n - Y_m^n - Y_m^{n+1} + Y_{m-1}^{n+1}}{\left(\Delta X\right)^2} \ , \tag{3.36}$$

for all $m = 1, 2, \ldots, M-1$ and $n = 0, 1, \ldots, N$.

Equation (3.36) can be written in an explicit form of finite difference as follows:

$$Y_m^{n+1} \cong \left(1+\mu\right)^{-1} \cdot \left(\mu Y_{m+1}^n + \left(1-\mu\right) Y_m^n + \mu Y_{m-1}^{n+1}\right) \ , \tag{3.37}$$

for all $m = 1, 2, \ldots, M-1$ and $n = 0, 1, \ldots, N$, where $\mu = \dfrac{\Delta T}{\left(\Delta X\right)^2}$.

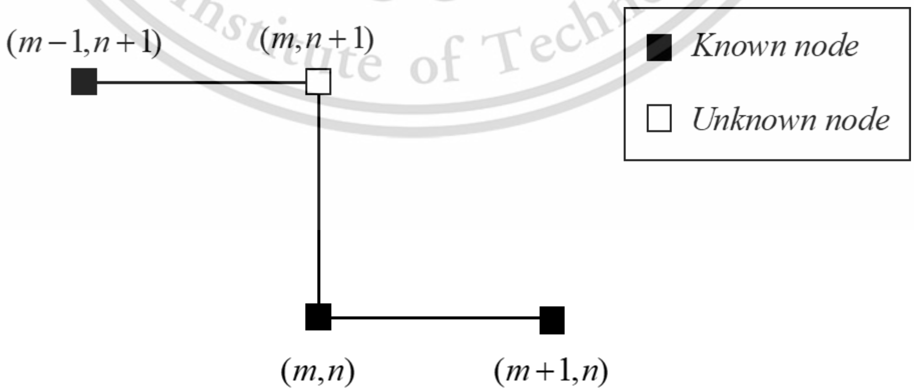

Figure 3.7 : Stencil diagram of the Saulyev method

Chapter 4

Numerical Experiments

In this chapter, we introduce predictions about sand sediment deposition on local shorelines with the groin structures. For prediction, we will set the physical parameters and apply two numerical techniques to the shoreline evolution model developed in Chapter 3.

4.1 Setting Physical Parameters for Numerical Experiments

In order to study the shoreline evolution in the long-term scale. Assume that the physical parameters of the simulation are illustrated in the table below. In Figures 4.1 – 4.2 illustrates the concept of an initial shoreline and an evolutionary shoreline.

Table 4.1 : The physical parameters

Parameter	Symbol (Unit)	Values
the length of considered shoreline	$L(m)$	5,000
the amplitude of the long-shore transport rate	$Q_0\left(m^3/day\right)$	7,500
the averaged berm height	$D_B(m)$	2
the averaged closure depth	$D_C(m)$	28
the breaking wave impact angle	$\alpha_0(degree)$	0.02
the expected shoreline evolution	$Y_*(m)$	20

We will employ the traditional forward time centered space (FTCS) technique Eq. (3.31), and the unconditionally Saulyev finite difference technique Eq. (3.37), to approximate the model solution.

The analytical solution of the simulation [40] is,

$$\tilde{y}(x,t)=\tan\alpha_0\sqrt{\frac{4Dt}{\pi}}\left(e^{-\frac{x^2}{4Dt}}-\frac{x\sqrt{\pi}}{2\sqrt{Dt}}erfc\left(\frac{x}{2\sqrt{Dt}}\right)\right), \tag{4.1}$$

where $erfc(z)$ is the complementary error function defined by:

$$erfc(z) = 1 - erf(z) \ , \tag{4.2}$$

which is $erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$.

Often the error function or the Gauss error function (efr) cannot be evaluated in closed form in terms of elementary functions, but by expanding into the Maclaurin series of the integrand (e^{-t^2}) and integrating. We obtain the error function in the form of the Maclaurin series:

$$erf(z) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{n!(2n+1)} \ , \tag{4.3}$$

or,

$$erf(z) = \frac{2}{\sqrt{\pi}} \left(z - \frac{z^3}{3} + \frac{z^5}{10} - \frac{z^7}{42} + \frac{z^9}{216} - \dots \right) \ , \tag{4.4}$$

which holds for every complex number z. And the formula for iterative calculation of this series can be rewritten as follows:

$$erf(z) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{z}{(2n+1)} \prod_{k=1}^{n} \frac{-z^2}{k} \ . \tag{4.5}$$

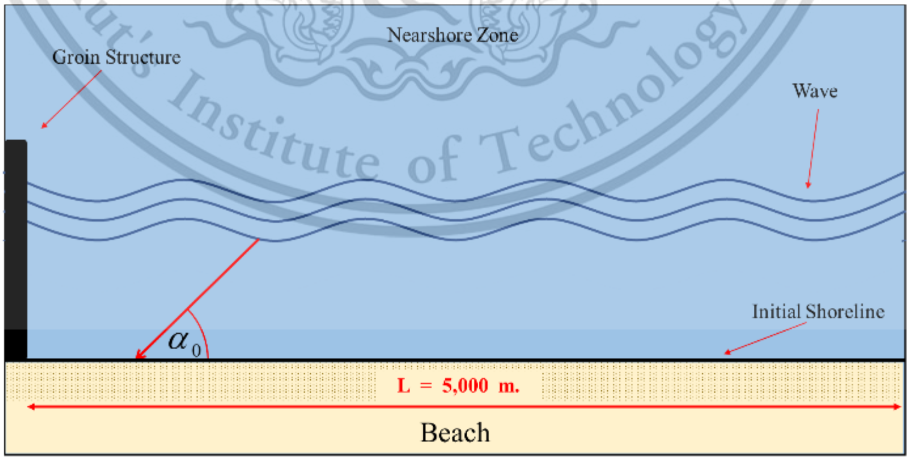

Figure 4.1 : The Initial shoreline

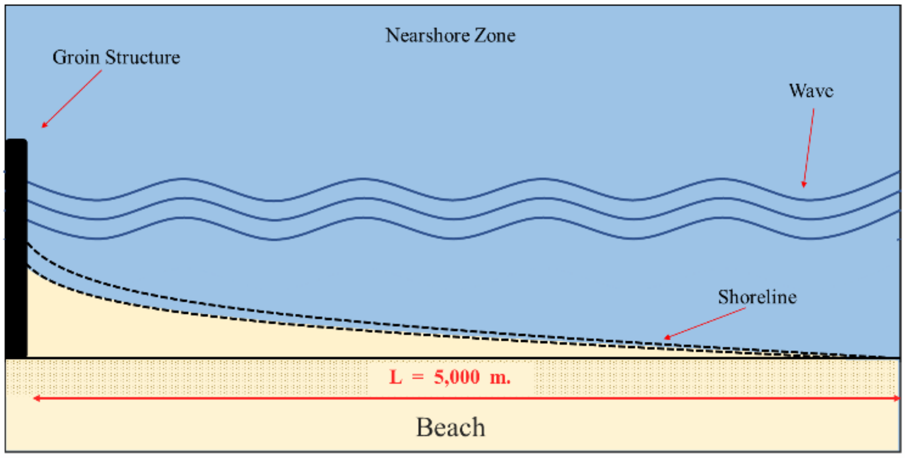

Figure 4.2 : The evolution from initial shoreline

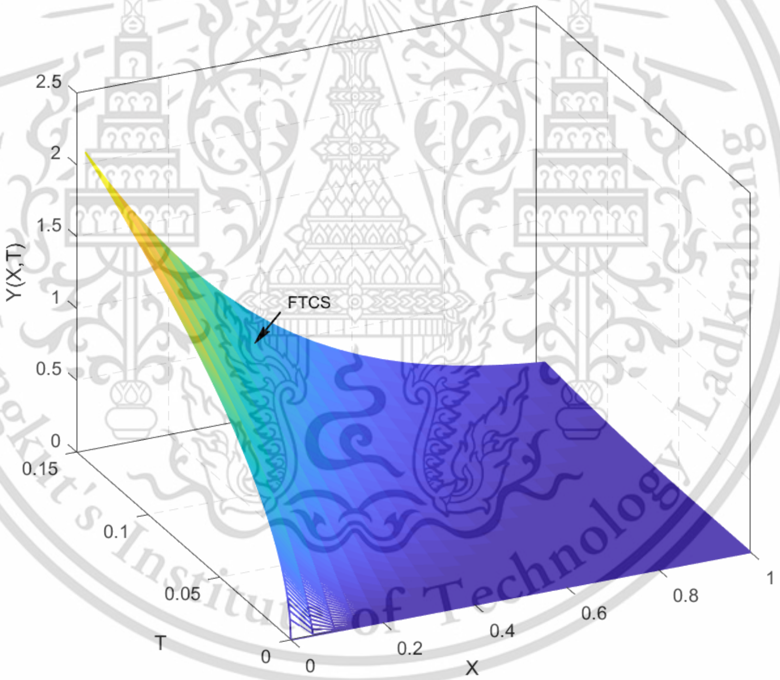

Figure 4.3 : Approximated shoreline evolution under the non-dimensional model in 20 years when the FTCS technique is used

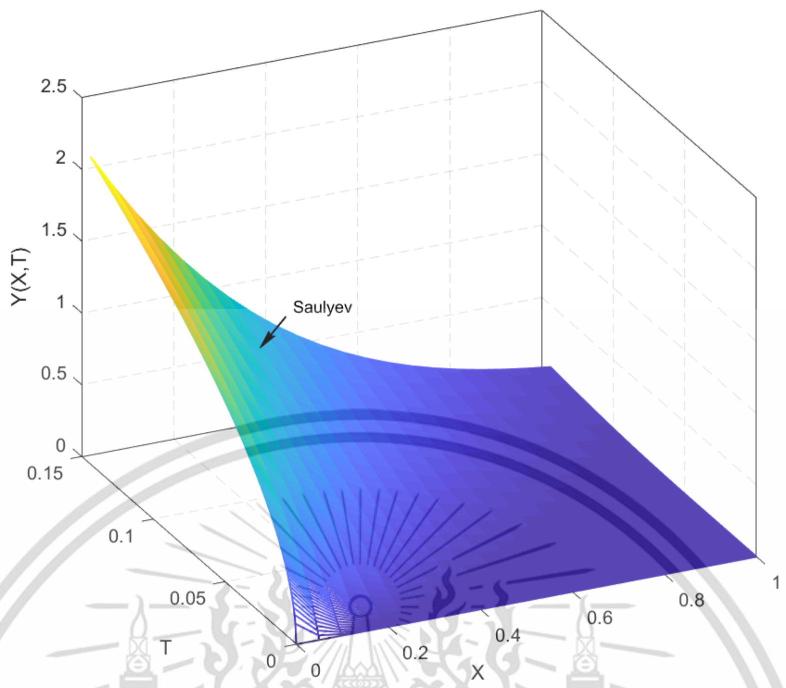

Figure 4.4 : Approximated shoreline evolution under the non-dimensional model in 20 years when the Saulyev technique is used

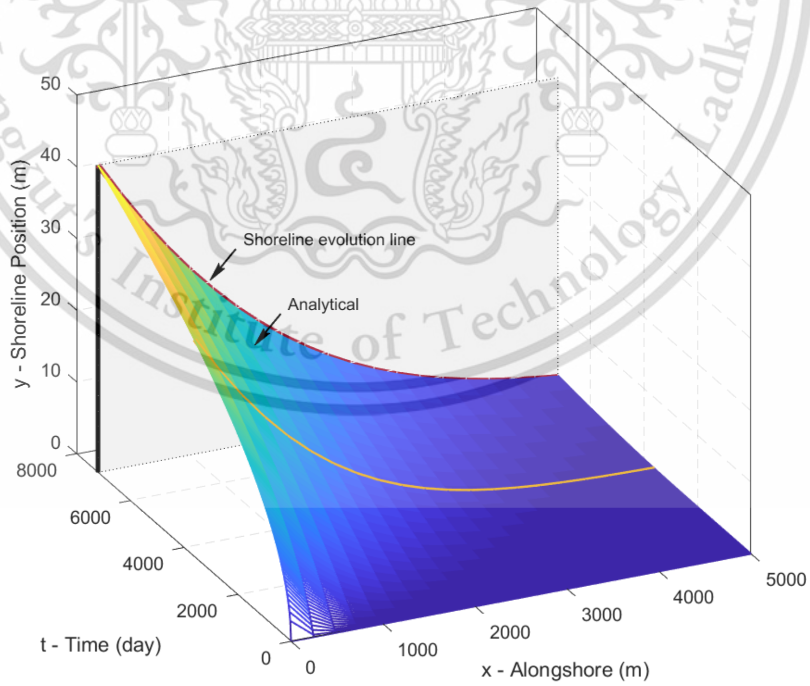

Figure 4.5 : Analytical shoreline evolution in 20 years

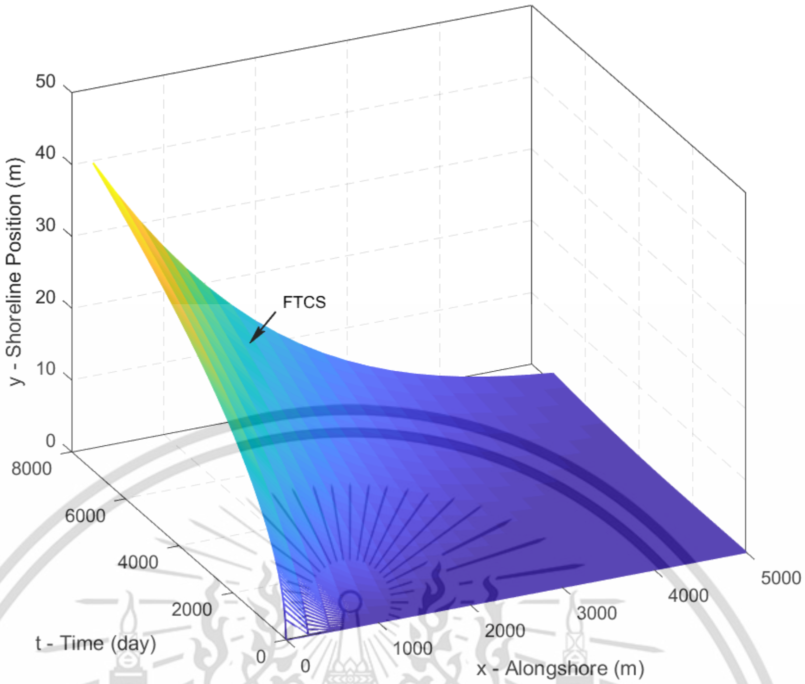

Figure 4.6 : Approximated shoreline evolution under the one-dimensional model in 20 years when the FTCS technique is used

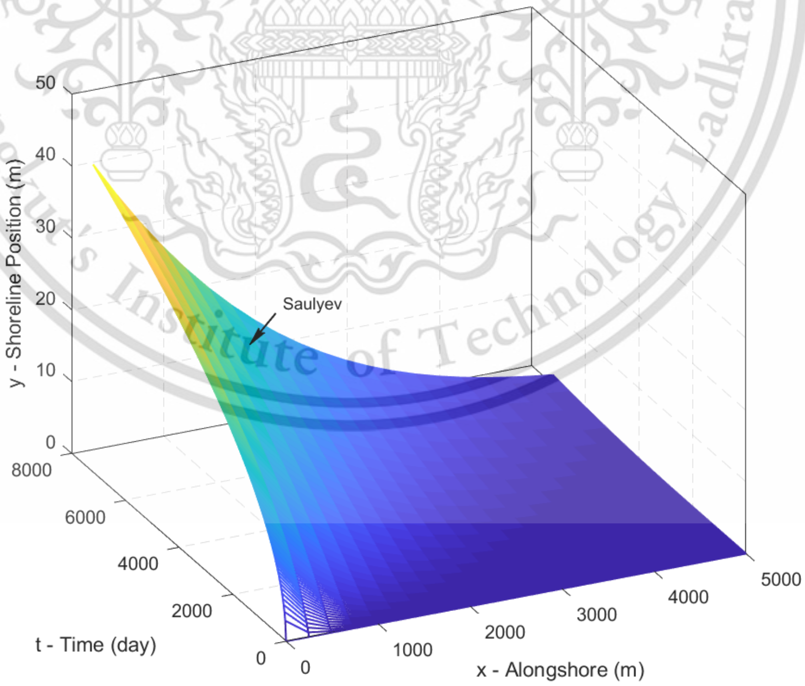

Figure 4.7 : Approximated shoreline evolution under the one-dimensional model in 20 years when the Saulyev technique is used

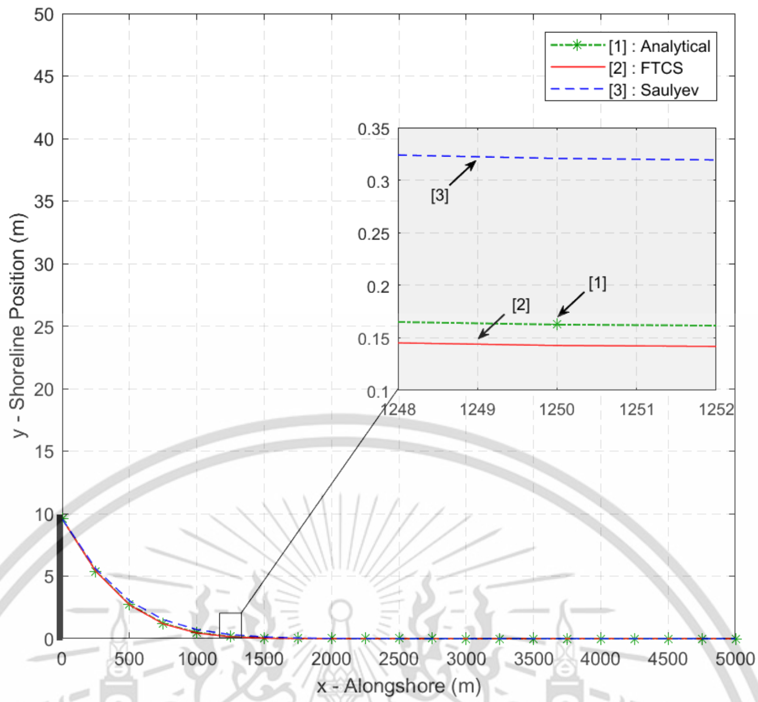

Figure 4.8 : Shoreline evolution in 1 year

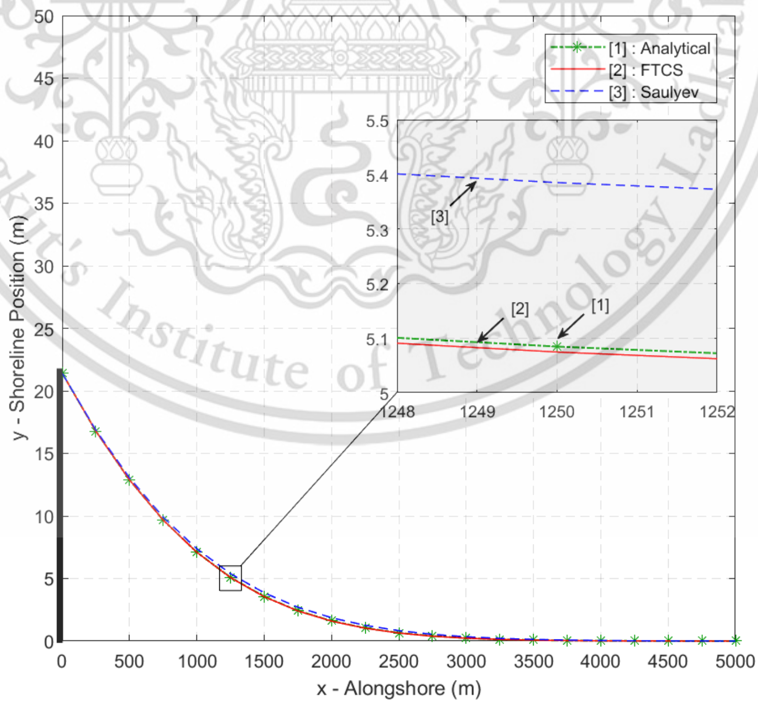

Figure 4.9 : Shoreline evolution in 5 years

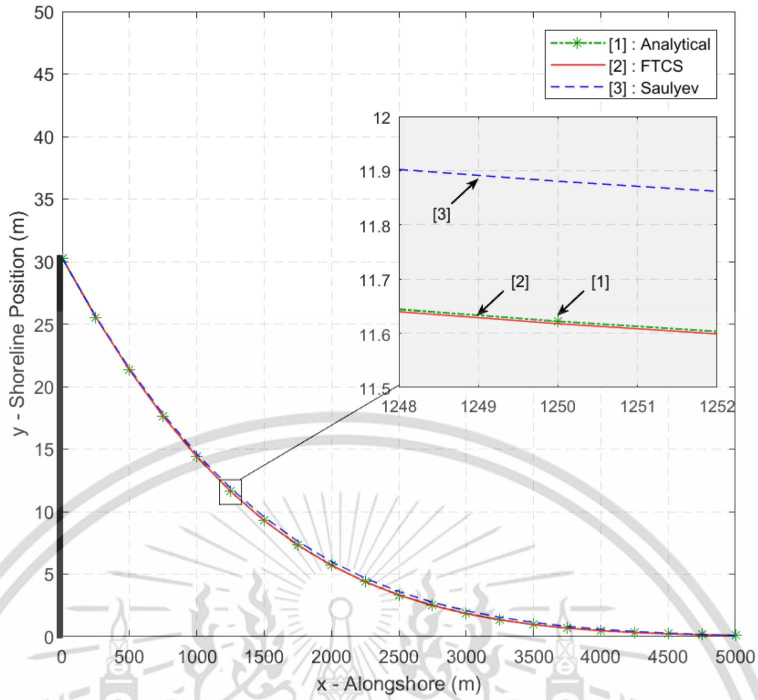

Figure 4.10 : Shoreline evolution in 10 years

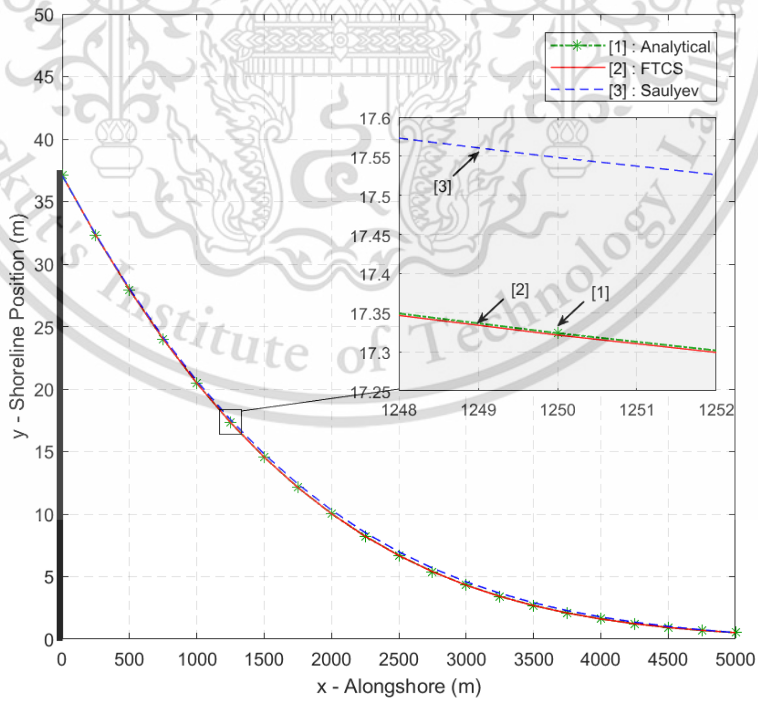

Figure 4.11 : Shoreline evolution in 15 years

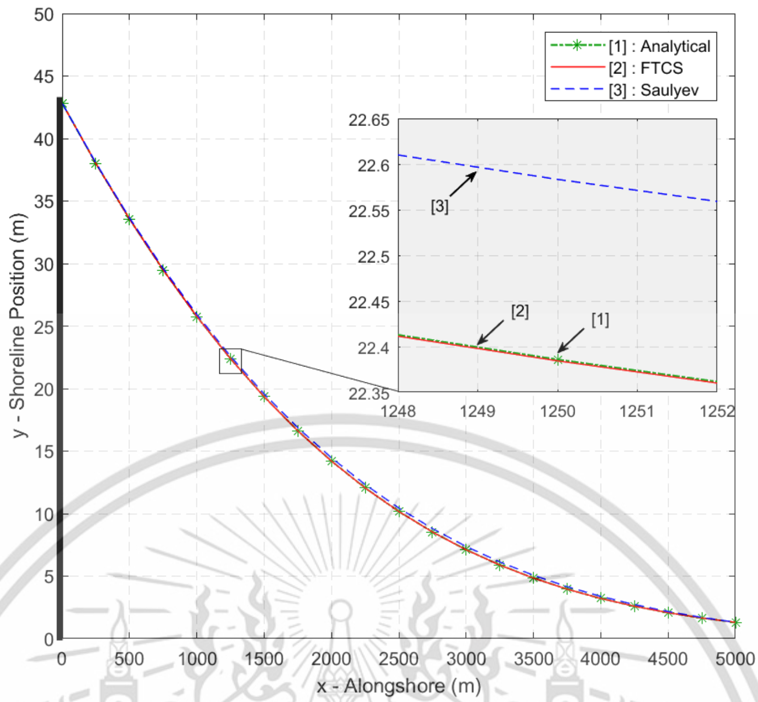

Figure 4.12 : Shoreline evolution in 20 years

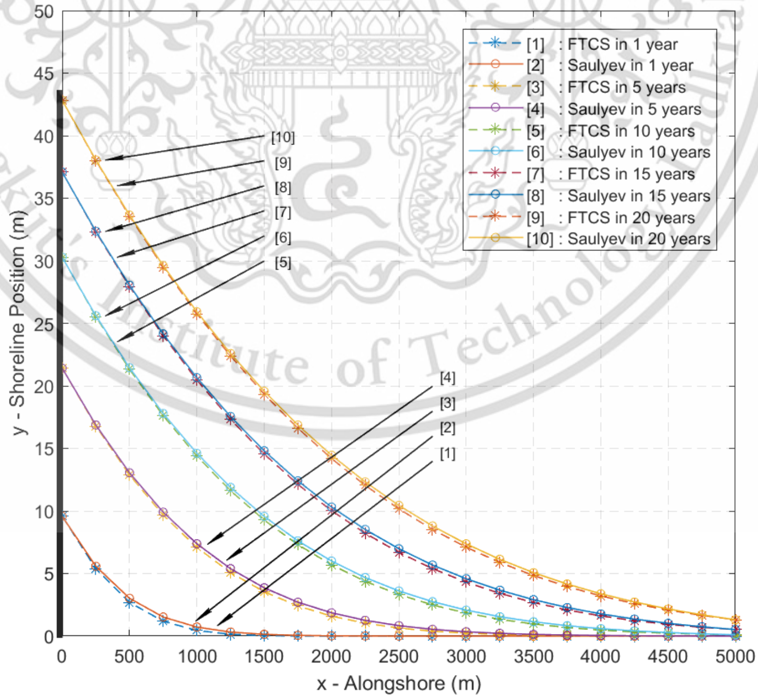

Figure 4.13 : Comparison of the approximate shoreline evolution results in 1, 5, 10, 15 and 20 years when the FTCS and Saulyev techniques are used

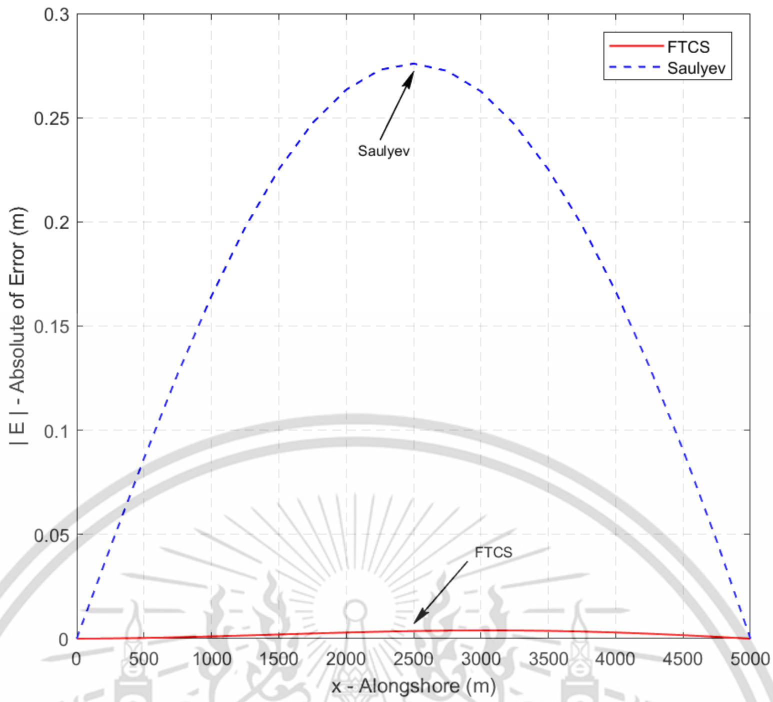

Figure 4.14 : Comparison of absolute error in 20 years between the FTCS and Saulyev techniques

The analytical solution is illustrated in Table 4.2 and the approximate solutions of both the numerical techniques are illustrated in Tables 4.3 - 4.4, respectively.

Table 4.2 : Analytical solution of shoreline evolution

Time (Years)	Distance (m)					
	0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.476	0.003	0.000	0.000	0.000
5	21.412	7.096	1.607	0.237	0.022	0.001
10	30.282	14.390	5.688	1.844	0.480	0.099
15	37.087	20.467	10.029	4.314	1.614	0.521
20	42.825	25.762	14.194	7.107	3.213	1.304

Table 4.3 : Approximated shoreline evolution along 20 years using the traditional forward time centered space (FTCS) technique

Time (Years)	Distance (m.)					
	0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.446	0.001	0.000	0.000	0.000
5	21.412	7.089	1.595	0.230	0.020	0.001
10	30.282	14.387	5.692	1.836	0.475	0.099
15	37.078	20.465	10.024	4.308	1.610	0.521
20	42.825	25.761	14.191	7.103	3.210	1.304

Table 4.4 : Approximated shoreline evolution along 20 years using the unconditionally Saulyev finite difference (Saulyev) technique

Time (Years)	Distance (m.)					
	0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.719	0.021	0.000	0.000	0.000
5	21.412	7.374	1.863	0.344	0.047	0.001
10	30.282	14.612	6.000	2.075	0.593	0.099
15	37.087	20.655	10.317	4.582	1.773	0.521
20	42.825	25.926	14.457	7.370	3.380	1.304

In order to be able to analyze the computational efficiency, the absolute error and comparisons of stability are illustrated in Tables 4.5 – 4.6, respectively.

Table 4.5 : Absolute error of shoreline evolution when using the FTCS and Saulyev techniques

Time (Years)	Distance (m.)	Absolute Error	
		FTCS	Saulyev
1	1000	0.2941×10^{-1}	0.24318
	2000	0.1385×10^{-2}	0.1846×10^{-1}
	3000	0.1283×10^{-5}	0.3314×10^{-3}
	4000	0.3588×10^{-14}	0.3425×10^{-5}
	5000	0.1233×10^{-31}	0.1233×10^{-31}
5	1000	0.7894×10^{-2}	0.2778
	2000	0.1187×10^{-1}	0.2560
	3000	0.6303×10^{-2}	0.1072
	4000	0.1533×10^{-2}	0.2482×10^{-1}
	5000	0.1822×10^{-16}	0.1822×10^{-16}
10	1000	0.3204×10^{-2}	0.2224
	2000	0.7287×10^{-2}	0.2997
	3000	0.7667×10^{-2}	0.2310
	4000	0.4505×10^{-2}	0.1134
	5000	0.4163×10^{-15}	0.4163×10^{-15}
15	1000	0.1791×10^{-2}	0.1888
	2000	0.4616×10^{-2}	0.2878
	3000	0.5833×10^{-2}	0.2644
	4000	0.4126×10^{-2}	0.1592
	5000	0.1110×10^{-15}	0.1110×10^{-15}
20	1000	0.1063×10^{-2}	0.1644
	2000	0.2933×10^{-2}	0.2634
	3000	0.3948×10^{-2}	0.2626
	4000	0.2950×10^{-2}	0.1667
	5000	0.2443×10^{-14}	0.2443×10^{-14}

Table 4.6 : Comparison of the stability in each grid spacing sizes

ΔT	Δt	ΔX	Δx	Stability	
				FTCS	Saulyev
0.0006	30	0.005	25	✗	✓
		0.010	50	✗	✓
		0.020	100	✗	✓
		0.050	250	✓	✓
		0.100	500	✓	✓
		0.200	1000	✓	✓
0.0003	15	0.005	25	✗	✓
		0.010	50	✗	✓
		0.020	100	✗	✓
		0.050	250	✓	✓
		0.100	500	✓	✓
		0.200	1000	✓	✓
0.0002	10	0.005	25	✗	✓
		0.010	50	✗	✓
		0.020	100	✓	✓
		0.050	250	✓	✓
		0.100	500	✓	✓
		0.200	1000	✓	✓
0.0001	5	0.005	25	✗	✓
		0.010	50	✗	✓
		0.020	100	✓	✓
		0.050	250	✓	✓
		0.100	500	✓	✓
		0.200	1000	✓	✓

Note that ✓ represents Stable and ✗ represents Unstable

Remark: In some cases of unstable solutions, their solutions are oscillated.

Chapter 5

Discussion and Conclusion

This chapter discusses and concludes the results of a shoreline evolution simulation using a non-dimensional shoreline evolution model to predict the accumulation of sand sediments on local shorelines with the groin structures in the long-term and to find a way to prevent erosion problems, etc.

5.1 Discussion

Using the traditional forward time centered space (FTCS) techniques and the Saulyev finite difference (Saulyev) techniques, the annual evolution of the shoreline can be determined.

In computational terms, when we compare the estimated shoreline evolution values under the non-dimensional model, the solutions obtained by the two numerical techniques are close (as shown in Figures 4.3 - 4.4). As a result, the shoreline evolution values obtained by converting back to the approximated solution under the one-dimensional model are also close (as shown in Figures 4.6 - 4.7). Therefore, these approximate solutions (as shown in Figures 4.6 - 4.7). are close to the analytical solution (as shown in Figures 4.5).

As demonstrated in Tables 4.2 – 4.4, and Figure 4.8, the distance from the furthest shoreline evolution after 1 year is 9.576 m. The smallest distance from the evolution of the shoreline is 0.000 m.

As demonstrated in Tables 4.2 – 4.4, and Figure 4.9, the distance from the furthest shoreline evolution after 5 years is 21.412 m. The smallest distance from the evolution of the shoreline is 0.001 m.

As demonstrated in Tables 4.2 – 4.4, and Figure 4.10, the distance from the furthest shoreline evolution after 10 years is 30.282 m. The smallest distance from the evolution of the shoreline is 0.099 m.

As demonstrated in Tables 4.2 – 4.4, and Figure 4.11, the distance from the furthest shoreline evolution after 15 years is 37.087 m. The smallest distance from the evolution of the shoreline is 0.521 m.

As demonstrated in Tables 4.2 – 4.4, and Figure 4.12, the distance from the furthest shoreline evolution after 20 years is 42.825 m. The smallest distance from the evolution of the shoreline is 1.304 m.

In Figure 4.13, it is clearly confirmed that the shoreline evolution values approximated by both numerical techniques are close and tend to increase over time.

When considering the absolute error as shown in Table 4.5 and Figure 4.14, the traditional forward time-centered space (FTCS) technique is more accurate than the Saulyev finite difference technique because this technique has a lower absolute error value. But it is worth noting that the traditional forward time-centered space (FTCS) technique cannot be handled in some cases the time increment is increased, on the other hand, the Saulyev finite difference techniques can still be used. The Saulyev finite difference technique can handle numerical solutions in almost every scenario as shown in Table 4.6, because the stability conditions are not constrained. As a result, the Saulyev finite difference technique can also be useful for computing a practical conceptual design of shoreline evolution when the number of grids is increased.

In terms of dimensions, we found that dimensional did not affect computational efficiency because the approximate solution obtained was still close to the analytical solution. But transforming the model to the form of non-dimensional variables is very useful as it simplifies the model and makes it more convenient to use numerical methods, saving time and cost.

5.2 Conclusion

The model includes a non-dimensional shoreline evolution model as well as a groin structure model. Physical parameters can now be manipulated by the model. The initial condition setting technique and boundary condition approaches are also presented where groin structural effects are involved. The forward time centered space (FTCS) technique and the unconditionally stable Saulyev finite difference strategies are employed to approximate the incremental model every year. The numerical models available give a realistic simulation for predicting long-term coastal evolution. Because the stability requirements are not limited, the Saulyev finite difference methods can handle numerical solutions in practically any circumstance. Since the number of grids has increased, the Saulyev finite difference technique can be highly beneficial for computing a practical conceptual design of shoreline evolution. The simulation described here can be used to forecast the success of a groin system on a local beach.

5.3 Remark

The focus of this research is on a non-dimensional shoreline evolution model with constants for the long-shore sand transport rate, average berm height, and average closure depth. If the parameters in the non-dimensional model are supposed to be functions, the numerical techniques described will run into the obstacle that the dimensionless-time independent variable T must depend on D. It is an ill-defined condition.

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Appendix

Appendix A

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A Non-Dimensional Mathematical Model of Shoreline Evolution with a Groin Structure Using an Unconditionally Stable Explicit Finite Difference Technique

Surasak Manilam, and Nopparat Pochai

Abstract—**Coastal erosion is a natural phenomenon that occurs when sediment transport away from the coast is not countered by the formation of new material on the shoreline. This is indeed a problem that is driving the erosion of coastal areas. A sea wall and a groin were created to prevent coastal erosion and floods. The future topography of the beach is being investigated using shoreline evolution analysis. Erosion, accretion, and sea level changes are basic stages that have a significant impact on the coastal structure. A qualitative analysis of the model coastal behavior in relation to the controlling process is required to research beach erosion and beach deposition. When stated in terms of non-dimensional variables, all are mathematically equivalent. In general, the models do not have to be dimensionally different. Those might just be modifications of the same problems. One can solve a wide range of models with a single solution to the related non-dimensional equation. In this research, we provide a governing equation when a groin is introduced to a one-dimensional shoreline growth model. A non-dimensional shoreline evolution model with a groin structure model is provided. The model now has the ability to manipulate physical parameters. When groin structural effects are present, the initial condition setting method and boundary condition approaches are also given. To approximate the incremental model in each year, the forward time-centered space technique and the unconditionally stable Saulyev finite difference methods are used. The Saulyev finite difference approach can handle numerical solutions in almost any scenario since the stability requirements are not restricted. The Saulyev finite difference technique can be very useful for computing a practical conceptual design of shoreline evolution since the number of grids has increased. The numerical models offered provide a viable simulation for evaluating long-term coastal development. The proposed modeling may be used to forecast the effectiveness of constructing a groin system on a local beach.**

Index Terms—**shoreline evolution, groin structure, non-dimension, mathematical model, finite difference method**

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INTRODUCTION

Coastal erosion is a natural process that occurs when sediment movement away from the coast is not counterbalanced by fresh material growth along the shoreline. This is certainly a problem that is related to coastal erosion. A sea wall and groin were built to prevent coastal erosion and floods. The beach's future topography is analyzed using shoreline evolution research. Basic phrases that have a substantial influence on the coastal structure are erosion, accretion, and sea-level variations.

The partial differential equation represents several disciplines, including mass, heat, energy, velocity, and vorticity (see for example [1-2]). In [3-8] these papers, the diffusion equation has been used to solve a variety of engineering problems, including pollutant and salinity transport in rivers and streams, and groundwater and contaminant dispersion in shallow lakes. In [9-11], they presented a case study of water level forecasting, a water quality evaluation based on probabilistic echo state networks, and the fluid dynamics of nonaqueous phase pollutants in groundwater.

Understanding the ideal shorelines' responses to the governing processes is important in the study of beach actions. A model for describing realistic situations involving general shoreline configuration settings and time-varying waves in more detail is proposed. As a result, numerical methods of shoreline evolution are preferred to analytical methods. In 1966, this paper [12] introduces modern logical design guidelines for groin structures. They are organized into three basic categories: Coastal processes, Functional design and Structural design. In [13] this paper expands on both the theoretical and practical concepts of mathematical models related to coastal behaviors: Theoretically, the influence of diffraction behind the groin is used to determine computer programming; Practically, the coastal constant in the theoretical model of the coast is expressed in terms of wave height and SVASEK's theoretical wave direction. In [14] this paper describes the development of the governing equations in general form and describes the assumptions and techniques used to obtain more than 25 analytical solutions. Solution for shoreline evolution with and without the influence of coastal structures. It covers situations involving beach filling of initial shape, sand mining, river discharge, groin and jetty and breakwater etc. The wind wave-driven longshore sediment transport rate and shoreline change are evaluated using a numerical model based on one-line theory in [15].

The model transforms waves from deep water into the surf zone and calculates their breaking characteristics. The model [16] provides complete, time-dependent simulations of shoreline evolution for coastlines driven by structures and a variety of boundary conditions that are both practical and reliable. This research looks at two case studies: New Jersey's Sea Isle City Beach and Egypt's Nile Delta Coast.
The purpose of [17] is to quantify the changes in the shoreline along with the sand reclamation for the Sultan Mahmud Airport runway in Kuala Terengganu. Littoral Processes and Coastline Kinetics (LITPACK) numerical model is the numerical device employed to solve the shoreline problem. This research [18] describes a numerical modelling framework called GENESIS that is used to simulate long-term shoreline change caused by spatial and temporal variations in longshore sand transport at coastal engineering projects. The modelling system is managed via an organized and user-friendly interface, which reduces the need for the operator to get concerned with computer code specifics. The modelling-system application method is described from the viewpoint of engineers and planners involved in the evaluation of shore-protection projects [19-21].

Then, if a groin is added to a one-dimensional shoreline evolution model, we propose a governing equation for this research. It is provided with a non-dimensional shoreline evolution model with a groin construction model. The model can now be used to manipulate physical parameters.

I. Governing Equation

A. Shoreline Evolution Model

The beach profile is supposed to travel landward and seaward while maintaining the same form in the one-line model, meaning that all bottom contours are parallel. As a result, specifying the horizontal location of the profile with respect to the baseline is sufficient under this assumption, and one contour line may be used to represent changes in the beach plan shape and volume as the beach reduces and accretes. Sand is carried alongshore between two well-defined limiting heights on the profile, according to the model's main assumption. If there is a variation in the alongshore sand transport rate at the lateral sides of the section and the related sand continuity, it contributes to the volume change.

At all times, the laws of mass conservation must be applied to the system. The following differential equation for shoreline evolution is produced using the given definitions:

$$\frac{\partial y}{\partial t}=\frac{1}{D_B+D_C}\left(-\frac{\partial Q}{\partial x}\right), \tag{1}$$

where x is the alongshore coordinate (m), y is the shoreline positions (m) and perpendicular to x-axis, t is time (day), Q is the long-shore sand transport rate (m^3/day), D_B is the average berm height (m) and D_C is the average closure depth (m).

In order to solve Eq.(1), an equation for the longshore sand transport rate Q must be specified. This quantity is thought to be created by a wave that strikes the coastline obliquely. [22] provided a general expression for the long-shore sand transport rate,

$$Q=Q_0\sin(2\alpha_b)\ , \tag{2}$$

where Q_0 is the amplitude of the long-shore sand transport rate. The empirical predictive formula for the amplitude of the long-shore sand transport rate is [23]:

$$Q_0=\frac{\rho}{16}\left(H_b^2c_{gb}\right)\frac{K}{(\rho_s-\rho)(1-n)}, \tag{3}$$

where the subscript b represent the value at the point breaking, ρ is the density of sea water (kg/m^3), ρ_s is the density of the sediment (kg/m^3), n is the porosity, K is the dimensionless coefficient which is a function of particle size, H is the wave height and c_g is the wave group velocity.

The quantity α_b the impact angle between breaking wave crests angle with local shoreline, and may be written as,

$$\alpha_b=\alpha_0-\tan^{-1}\left(\frac{\partial y}{\partial x}\right), \tag{4}$$

where α_0 is the angle between breaking wave crests and the x-axis. For beaches with a slight slope, the breaking wave angle to the coastline is likely to be minimal. Assuming that,

$$\sin(2\alpha_b)\approx 2\alpha_b\ ,$$

and

$$\tan^{-1}\left(\frac{\partial y}{\partial x}\right)\approx\left(\frac{\partial y}{\partial x}\right).$$

Substituting Eq.(4) into Eq.(2), and assuming the beach with mild slope yields,

$$Q=Q_0\left(2\alpha_0-2\frac{\partial y}{\partial x}\right), \tag{5}$$

Substituting Eq.(5) into Eq.(1), and neglecting the sources or sinks along the coast gives,

$$\frac{\partial y}{\partial t}=D\frac{\partial^2 y}{\partial x^2}\ , \tag{6}$$

for all $(x,t)\in[0,L]\times[0,\tau]$, where $D=\dfrac{2Q_0}{D_B+D_C}$.

B. The Initial and Boundary Conditions of the One-Dimensional Model

a) The Initial Condition

Groin system that is impermeable and straight. The shoreline of the initials is considered to be parallel to the x-axis.

Assume α_0 is the braking wave angle to the beach as show in Fig. 3. As a result, the rate of sand transport along the beach is homogeneous. As seen in Fig. 1, the groin is inserted instantly at $x = 0$. As a result, the initial conditions become,

$$y(x,0) = 0 \;, \tag{7}$$

for all $x \in [0, L]$.

b) The Left Boundary Condition

The left boundary condition is defined by the interpolation function of the measured evolutionary data. It is shoreline evolution with the left-hand side groin system. The boundary conditions are assumed to be as follows,

$$y(0,t) = g(t) \;, \tag{8}$$

for all $t \in [0, \tau]$, where $g(t)$ is a given interpolation function of the measured evolutionary data at the left-hand side groin system.

c) The Right Boundary Condition

The right boundary condition is defined by the interpolation function of the measured evolutionary data.
It is shoreline evolution with the right-hand side groin system. The boundary conditions are assumed to be as follows,

$$y(L,t) = h(t) \;, \tag{9}$$

for all $t \in [0, \tau]$, where $h(t)$ is a given interpolation function of the measured evolutionary data at the right-hand side groin system.

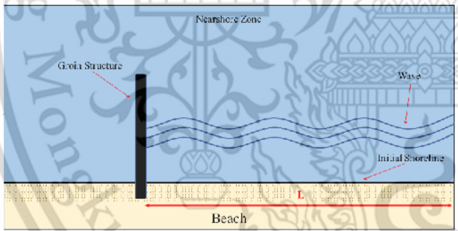

Fig. 1. Initial shoreline with configuration straight impermeable groins.

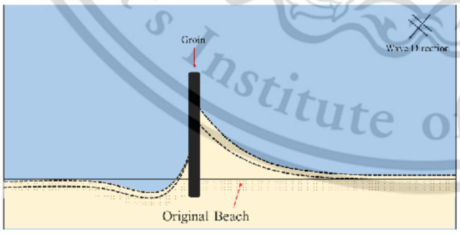

Fig. 2. Shoreline evolution with a groin structure.

C. Physical Parameters

Physical parameters of the model can be illustrated as shown in Figs. 3-4. that are listed below.

α_0 is the impact angle between breaking wave crests angle with the x-axis.

Q_0 is the amplitude of the long-shore sand transport rate (m^3/day).

D_B is the average berm height (m).

D_C is the average closure depth (m).

L is the length of alongshore (m).

τ is time of simulation (day).

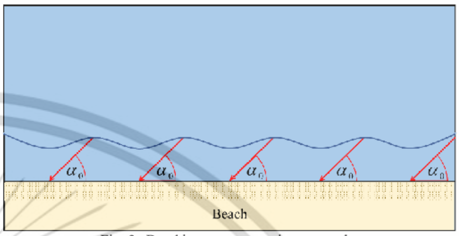

Fig. 3. Breaking wave crests impact angle.

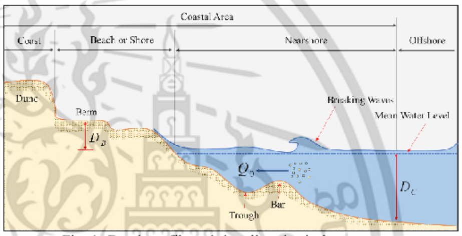

Fig. 4. Beach profile and shoreline physical parameters.

II. A NON-DIMENSIONAL SHORELINE EVOLUTION MODEL

A. A Non-Dimensional Shoreline Evolution Model

Taking non-dimensional technique [24] into Eq.(6), we obtain the following,

$$\frac{\partial Y}{\partial T} = \frac{\partial^2 Y}{\partial X^2} \;, \tag{10}$$

for all $(X,T) \in [0,1] \times [0,\Gamma]$, where L is the length of alongshore, Y_* is the expected shoreline evolution,

$Y = \frac{y}{Y_*}$, $X = \frac{x}{L}$, and $T = \frac{Dt}{L^2}$.

Equation (10) is similar to the one-dimensional heat equation which has a thermal conductivity coefficient of 1, so in order to solve a problem it is necessary to define the initial conditions and the boundary conditions.

B. The Initial and Boundary Conditions of the Non-Dimensional Model

The initial and boundary conditions for a non-dimensional model can be simply defined under the known initial and boundary conditions according to Eqs.(7)-(9) for a one-dimensional model, so we can define the necessary conditions as the follows. These means that the initial condition becomes,

$$Y(X,0) = F(X) \quad \text{for all } X \in [0,1] \;, \tag{11}$$

Boundary conditions are also defined by:

$$Y(0,T) = G(T) \qquad \text{for all } T \in [0,\Gamma], \tag{12}$$

and

$$Y(1,T) = H(T) \qquad \text{for all } T \in [0,\Gamma], \tag{13}$$

where $F(X) = 0$, $G(T) = \frac{g(t)}{Y_*}$ and $H(T) = \frac{h(t)}{Y_*}$.

III. Numerical Techniques

A. Grid Spacing

We now discretize the domain of Eq.(10) by dividing the interval $[0,1]$ into M subintervals such that $M\Delta X = 1$ and the time interval $[0,\Gamma]$ into N subintervals such that $N\Delta T = \Gamma$. We then approximate $Y(X_m, T_n)$ by Y_m^n, at the point $X_m = m\Delta X$ and $T_n = n\Delta T$, where $0 \le m \le M$ and $0 \le n \le N$ in which M and N are positive integers.

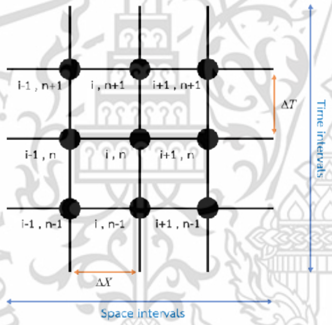

Fig. 5. Grid spacing.

B. The Traditional Forward Time Centered Space (FTCS) Techniques

The forward time centered space (FTCS) Technique is employed. Consequently, the finite difference approximation [19],[25] becomes,

$$Y(X_m, T_n) \cong Y_m^n, \tag{14}$$

$$\frac{\partial Y}{\partial T} \cong \frac{Y_m^{n+1} - Y_m^n}{\Delta T}, \tag{15}$$

$$\frac{\partial Y}{\partial X} \cong \frac{Y_{m+1}^n - Y_{m-1}^n}{2\Delta X}, \tag{16}$$

$$\frac{\partial^2 Y}{\partial X^2} \cong \frac{Y_{m+1}^n - 2Y_m^n + Y_{m-1}^n}{(\Delta X)^2}, \tag{17}$$

Substituting Eqs.(14)-(17) into Eq.(10), we obtain,

$$\frac{Y_m^{n+1} - Y_m^n}{\Delta T} \cong \frac{Y_{m+1}^n - 2Y_m^n + Y_{m-1}^n}{(\Delta X)^2}, \tag{18}$$

for $1 \le m \le M-1$ and $0 \le n \le N-1$. Equation (18) can be written in an explicit form of finite difference as follows,

$$Y_m^{n+1} \cong \mu Y_{m+1}^n + (1-2\mu) Y_m^n + \mu Y_{m-1}^n, \tag{19}$$

for $1 \le m \le M-1$ and $0 \le n \le N-1$. where $\mu = \frac{\Delta T}{(\Delta X)^2}$.

C. An Unconditionally Saulyev Finite Difference Techniques

The Saulyev finite difference Technique will be also employed. We can obtain that the finite difference approximatetion [19],[25] becomes,

$$Y(X_m, T_n) \cong Y_m^n, \tag{20}$$

$$\frac{\partial Y}{\partial T} \cong \frac{Y_m^{n+1} - Y_m^n}{\Delta T}, \tag{21}$$

$$\frac{\partial Y}{\partial X} \cong \frac{Y_{m+1}^n - Y_{m-1}^{n+1}}{2\Delta X}, \tag{22}$$

$$\frac{\partial^2 Y}{\partial X^2} \cong \frac{Y_{m+1}^n - Y_m^n - Y_m^{n+1} + Y_{m-1}^{n+1}}{(\Delta X)^2}, \tag{23}$$

Substituting Eqs.(20)-(23) into Eq.(10), we obtain,

$$\frac{Y_m^{n+1} - Y_m^n}{\Delta T} \cong \frac{Y_{m+1}^n - Y_m^n - Y_m^{n+1} + Y_{m-1}^{n+1}}{(\Delta X)^2}, \tag{24}$$

for $1 \le m \le M-1$ and $0 \le n \le N-1$. Equation (24) can be written in an explicit form of finite difference as follows,

$$Y_m^{n+1} \cong (1+\mu)^{-1} \cdot \left(\mu Y_{m+1}^n + (1-\mu) Y_m^n + \mu Y_{m-1}^{n+1}\right), \tag{25}$$

for $1 \le m \le M-1$ and $0 \le n \le N-1$. where $\mu = \frac{\Delta T}{(\Delta X)^2}$.

IV. Error measurement

A simple measure used to measure the difference between actual and approximate values is the absolute error method. The absolute error formula be as follows,

$$E_m^n = \left|\tilde{y}_m^n - y_m^n\right|, \tag{26}$$

where $\tilde{y}_m^n$ is the analytical solution of shoreline evolution and y_m^n is the approximate solution of shoreline evolution.

V. Numerical Experiment

In order to investigate the shoreline evolution in the long-term scale. Assuming that the length of considered shoreline L is 5000 m, the amplitude of the long-shore transport rate Q_0 is 7500 m^3/day, the averaged berm height D_B is 2 m, the averaged closure depth D_C is 28 m, the breaking wave impact angle α_0 is 0.02, and the expected shoreline evolution Y_* is 20 m. The simulation setting is illustrated in Fig. 6-7.

We will employ the traditional forward time centered space (FTCS) techniques Eq.(19), and the Saulyev finite difference techniques Eq.(25), to approximate the model solution.

The analytical solution of the simulation is [26]:

$$\tilde{y}(x,t) = \tan\alpha_0 \sqrt{\frac{4Dt}{\pi}} \left(e^{-\frac{x^2}{4Dt}} - \frac{x\sqrt{\pi}}{2\sqrt{Dt}} erfc\left(\frac{x}{2\sqrt{Dt}}\right) \right), \quad (27)$$

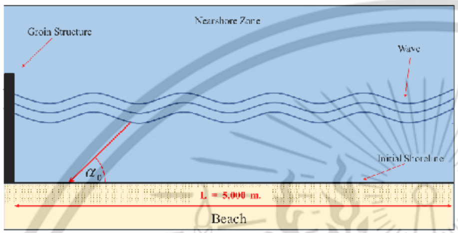

Fig. 6. Initial shoreline.

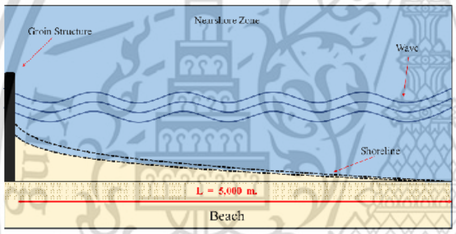

Fig. 7. The evolution from initial shoreline.

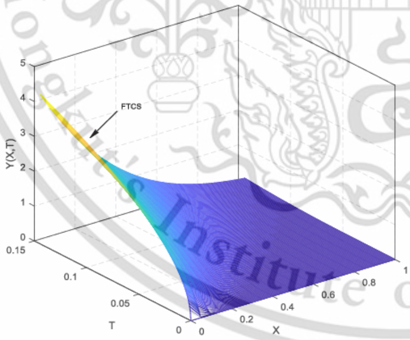

Fig. 8. Approximated shoreline evolution under the non-dimensional equations in 20 years when the FTCS technique is used.

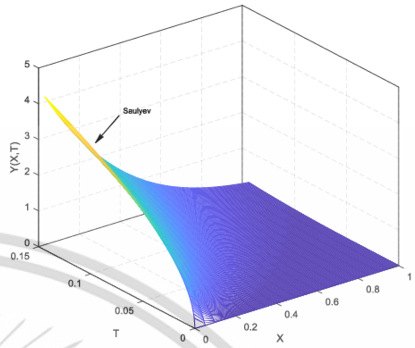

Fig. 9. Approximated shoreline evolution under the non-dimensional equations in 20 years when the Saulyev technique is used.

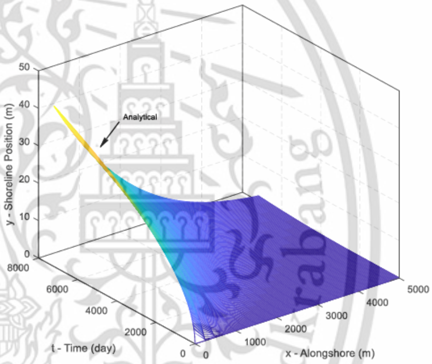

Fig. 10. Analytical shoreline evolution in 20 years.

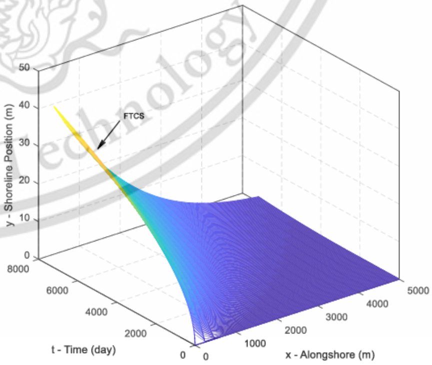

Fig. 11. Approximated shoreline evolution under the one-dimensional equations in 20 years when the FTCS technique is used.

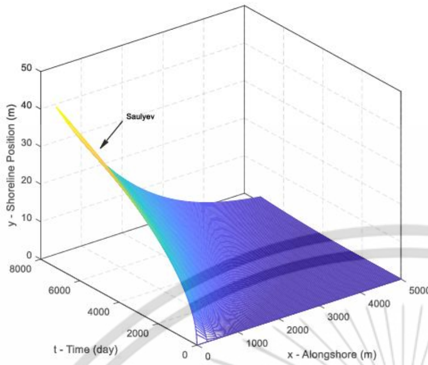

Fig. 12. Approximated shoreline evolution under the one-dimensional equations in 20 years when the Saulyev technique is used.

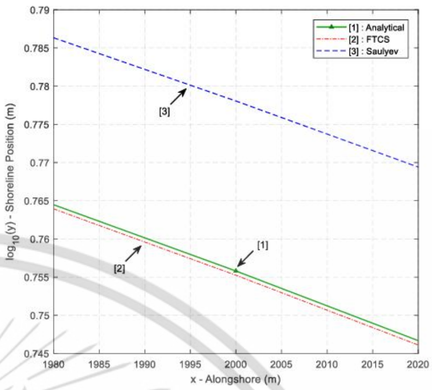

Fig. 15. Shoreline evolution in 10 years.

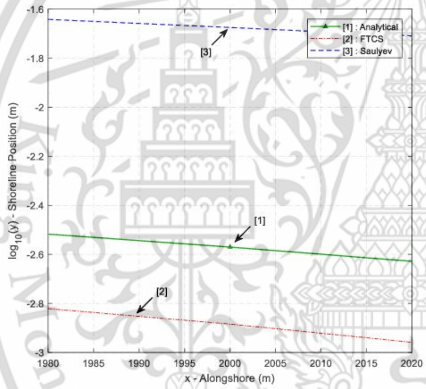

Fig. 13. Shoreline evolution in 1 year.

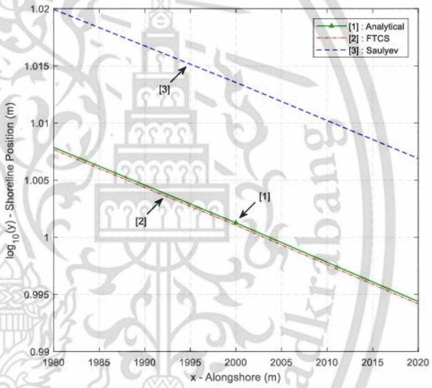

Fig. 16. Shoreline evolution in 15 years.

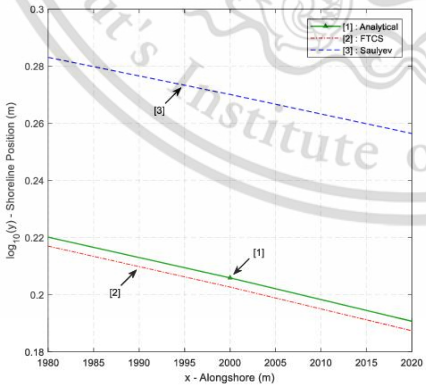

Fig. 14. Shoreline evolution in 5 years.

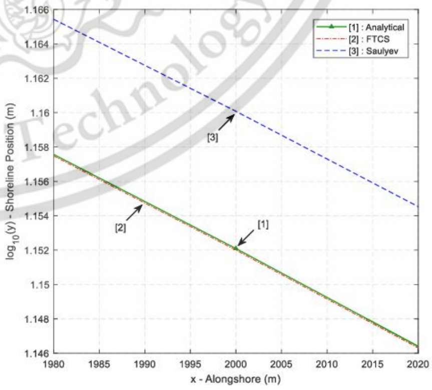

Fig. 17. Shoreline evolution in 20 years.

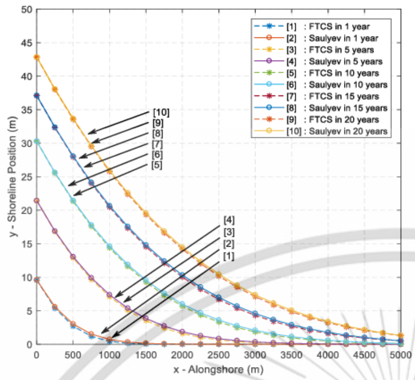

Fig. 18. Comparison of the approximate shoreline evolution results in 1, 5, 10, 15 and 20 years when the FTCS technique and the Saulyev technique are used.

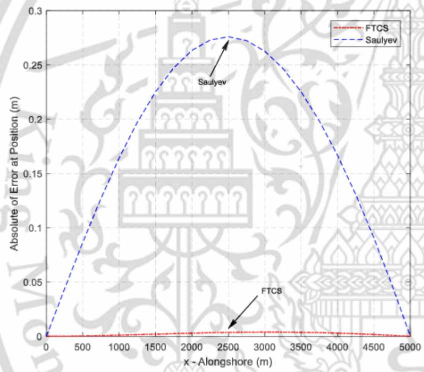

Fig. 19. Comparison of absolute error in 20 years between the FTCS technique and the Saulyev technique.

The analytical solution is illustrated in Table I and the approximate solutions of both the numerical techniques are illustrated in Tables II-III, respectively.

TABLE I
ANALYTICAL SOLUTION OF SHORELINE EVOLUTION

Time (Years)	Distance (m.) 0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.476	0.003	0.000	0.000	0.000
5	21.412	7.096	1.607	0.237	0.022	0.001
10	30.282	14.390	5.688	1.844	0.480	0.099
15	37.087	20.467	10.029	4.314	1.614	0.521
20	42.825	25.762	14.194	7.107	3.213	1.304

TABLE II
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE (FTCS) TECHNIQUE

Time (Years)	Distance (m.) 0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.446	0.001	0.000	0.000	0.000
5	21.412	7.089	1.595	0.230	0.020	0.001
10	30.282	14.387	5.692	1.836	0.475	0.099
15	37.078	20.465	10.024	4.308	1.610	0.521
20	42.825	25.761	14.191	7.103	3.210	1.304

TABLE III
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING AN UNCONDITIONALLY SAULYEV FINITE DIFFERENCE TECHNIQUE

Time (Years)	Distance (m.) 0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.719	0.021	0.000	0.000	0.000
5	21.412	7.374	1.863	0.344	0.047	0.001
10	30.282	14.612	6.000	2.075	0.593	0.099
15	37.087	20.655	10.317	4.582	1.773	0.521
20	42.825	25.926	14.457	7.370	3.380	1.304

In order to be able to analyze the computational efficiency, the absolute error and comparisons of stability are illustrated in Tables IV-V, respectively.

TABLE IV
ABSOLUTE ERROR OF SHORELINE EVOLUTION WHEN USING THE FTCS TECHNIQUES AND SAULYEV TECHNIQUES

Time (Years)	Distance (m.)	Absolute Error FTCS	Saulyev
1	1000	$0.2941\text{x}10^{-1}$	0.24318
	2000	$0.1385\text{x}10^{-2}$	$0.1846\text{x}10^{-1}$
	3000	$0.1283\text{x}10^{-5}$	$0.3314\text{x}10^{-3}$
	4000	$0.3588\text{x}10^{-14}$	$0.3425\text{x}10^{-5}$
	5000	$0.1233\text{x}10^{-31}$	$0.1233\text{x}10^{-31}$
5	1000	$0.7894\text{x}10^{-2}$	0.2778
	2000	$0.1187\text{x}10^{-1}$	0.2560
	3000	$0.6303\text{x}10^{-2}$	0.1072
	4000	$0.1533\text{x}10^{-2}$	$0.2482\text{x}10^{-1}$
	5000	$0.1822\text{x}10^{-16}$	$0.1822\text{x}10^{-16}$
10	1000	$0.3204\text{x}10^{-2}$	0.2224
	2000	$0.7287\text{x}10^{-2}$	0.2997
	3000	$0.7667\text{x}10^{-2}$	0.2310
	4000	$0.4505\text{x}10^{-2}$	0.1134
	5000	$0.4163\text{x}10^{-15}$	$0.4163\text{x}10^{-15}$
15	1000	$0.1791\text{x}10^{-2}$	0.1888
	2000	$0.4616\text{x}10^{-2}$	0.2878
	3000	$0.5833\text{x}10^{-2}$	0.2644
	4000	$0.4126\text{x}10^{-2}$	0.1592
	5000	$0.1110\text{x}10^{-15}$	$0.1110\text{x}10^{-15}$
20	1000	$0.1063\text{x}10^{-2}$	0.1644
	2000	$0.2933\text{x}10^{-2}$	0.2634
	3000	$0.3948\text{x}10^{-2}$	0.2626
	4000	$0.2950\text{x}10^{-2}$	0.1667
	5000	$0.2443\text{x}10^{-14}$	$0.2443\text{x}10^{-14}$

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TABLE V
COMPARISON OF THE STABILITY WHEN CHANGING GRID SPACING SIZES

ΔT	Δt	ΔX	Δx	Stability	
				FTCS	Saulyev
0.0006	30	0.005	25	U	S
		0.010	50	U	S
		0.020	100	U	S
		0.050	250	S	S
		0.100	500	S	S
		0.200	1000	S	S
0.0003	15	0.005	25	U	S
		0.010	50	U	S
		0.020	100	U	S
		0.050	250	S	S
		0.100	500	S	S
		0.200	1000	S	S
0.0002	10	0.005	25	U	S
		0.010	50	U	S
		0.020	100	S	S
		0.050	250	S	S
		0.100	500	S	S
		0.200	1000	S	S
0.0001	5	0.005	25	U	S
		0.010	50	U	S
		0.020	100	S	S
		0.050	250	S	S
		0.100	500	S	S
		0.200	1000	S	S

Note: S represents Stable, U represents Unstable.

VI. DISCUSSION

Using the traditional forward time centered space (FTCS) techniques and the Saulyev finite difference techniques, the annual evolution of the shoreline can be determined.

In the calculations, when we compared the approximated shoreline evolution solutions under the non-dimensionless model which occurred between the traditional forward times centered space (FTCS) technique and the Saulyev finite difference technique, they were found to be close together as shown in Figs. 8-9. For this reason, the approximated shoreline evolution solutions obtained when converted back to the solutions under the one-dimensional model are also close as shown in Figs. 11-12. Therefore, we can see that the approximate solutions as shown in Figs. 11-12 are close to the analytical solution as shown in Fig. 10.

As demonstrated in Tables II, III, and Fig. 13, the distance from the furthest shoreline evolution after 1 year is 9.576 m. The smallest distance from the evolution of the shoreline is 0.000 m.

As demonstrated in Tables II, III, and Fig. 14, the distance from the furthest shoreline evolution after 5 years is 21.412 m. The smallest distance from the evolution of the shoreline is 0.001 m.

As demonstrated in Tables II, III, and Fig. 15, the distance from the furthest shoreline evolution after 10 years is 30.282 m. The smallest distance from the evolution of the shoreline is 0.099 m.

As demonstrated in Tables II, III, and Fig. 16, the distance from the furthest shoreline evolution after 15 years is 37.087 m. The smallest distance from the evolution of the shoreline is 0.521 m.

As demonstrated in Tables II, III, and Fig. 17, the distance from the furthest shoreline evolution after 20 years is 42.825 m. The smallest distance from the evolution of the shoreline is 1.304 m.

In Fig.18, it is clearly confirmed that both numerical techniques produce shoreline evolution solutions close to each other even after 1, 5, 10, 15, and 20 years.

In terms of accuracy, the traditional forward time centered space (FTCS) technique is more accurate than the Saulyev finite difference techniques as shown in Table IV and Fig. 19, but the solution cannot be handled in some cases when the time increment is increased for the traditional forward time centered space (FTCS) techniques. The Saulyev finite difference techniques, on the other hand, can still be used. The Saulyev finite difference technique, on the other hand, can handle numerical solutions in almost every scenario as shown in Table V, because the stability conditions are not constrained. As a result, the Saulyev finite difference technique can also be useful for computing a practical conceptual design of shoreline evolution when the number of grids is increased.

VII. CONCLUSION

The model includes a non-dimensional shoreline evolution model as well as a groin structure model. Physical parameters can now be manipulated by the model. The initial condition setting technique and boundary condition approaches are also presented where groin structural effects are involved. The traditional forward time centered space (FTCS) technique and the unconditionally stable Saulyev finite difference strategies are employed to approximate the incremental model every year. The numerical models available give a realistic simulation for predicting long-term coastal evolution. Because the stability requirements are not limited, the Saulyev finite difference Techniques can handle numerical solutions in practically any circumstance. Since the number of grids has increased, the Saulyev finite difference technique can be highly beneficial for computing a practical conceptual design of shoreline evolution. The simulation described here can be used to forecast the success of a groin system on a local beach.

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Appendix B

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A Non-Dimensional Mathematical Model of Shoreline Evolution with a Groin Structure*

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Abstract

Beach erosion is a natural phenomenon that happens when sediment transport away from the coast is not countered by the creation of new shoreline volume. A sea wall and groin were created to prevent coastal erosion and sedimentation. Shoreline evolution modeling is used to forecast the actual topography of the shoreline. In this research, we present a governing equation when a groin is included into a one-dimensional shoreline development model. A non-dimensional shoreline evolution model and a groin structure model are included in the model. Each year, the forward time centered space technique is used to approximate the incremental model. In addition, a new MATLAB program is created to simulate the problem in several scenarios. From numerical experiments, although the model has been transformed into a non-dimensional form. We were also able to conclude that the approximate solution obtained was close to the analytical solution when using this numerical technique.

Keywords: Finite difference method, Groin structure, Mathematical model, Non-dimension, Shoreline evolution.

2020 MSC: Primary 65N06.

1 Introduction

Coastal erosion is a natural process that changes the physical characteristics of coastal areas that occur. It is influenced by the processes of waves, wind, currents, and the movement of sand sediments that are unbalanced by the growth of new materials along the shoreline. This research [1] describes a numerical modeling framework called GENESIS that is used to simulate long-term shoreline change caused by spatial and temporal variations in longshore sand transport at coastal engineering projects. The modeling system is managed

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via an organized and user-friendly interface, which reduces the need for the operator to get concerned with computer code specifics. The modeling-system application method is described from the viewpoint of engineers and planners involved in the evaluation of shore-protection projects [2], [3], [4].

Then if a groin is added to a one-dimensional shoreline evolution model, we propose a governing equation in this research. It provides a non-dimensional shoreline evolution model with a groin construction model. The model described here is a simplified version of the model presented in [5]. In this study, the technique of transformation of the initial and boundary conditions in dimensional form into non-dimensional form will also be proposed. The model can now be used to manipulate physical parameters.

2 Governing Equation

2.1 The Shoreline Evolution

The shoreline evolution is governed by a partial differential equation [4]:

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} , \tag{2.1}$$

for all $(x,t) \in [0, L] \times [0, \tau]$, where $D = \dfrac{2Q_0}{D_B + D_C}$, x is the alongshore coordinate (m), y is the shoreline positions (m), t is time (day), D_B is the average berm height (m), D_C is the average closure depth (m), Q_0 is the amplitude of the long-shore sand transport rate, L is the length of alongshore (m) and τ is time of simulation (day).

Therefore, to solve Eq. (2.1) it is necessary to define the initial and boundary conditions.

2.2 The Initial and Boundary Conditions of the One-Dimensional Model

The initial and boundary conditions are defined by the interpolation function of the measured evolutionary data.

The initial conditions are assumed to be as follows:

$$y(x,0) = f(x) . \tag{2.2}$$

The boundary conditions are assumed to be as follows:

$$y(0,t) = g(t) , \tag{2.3}$$

and

$$y(L,t) = h(t) , \tag{2.4}$$

for all $x \in [0, L]$, $t \in [0, \tau]$ and $\tau > 0$, where $f(x)$ is given interpolation function of the measured evolutionary data at the start time along shoreline and $g(t), h(t)$ are given interpolation function of the measured evolutionary data at the left-hand side and the right-hand side of groin system, respectively.

3 A Non-Dimensional Shoreline Evolution Model

3.1 The Non-Dimensional Shoreline Evolution Model

To remove physical dimensions and simplify govern equations, which save time and cost in terms of measuring tools, budgets, people and software, etc. Therefore, taking the non-dimensional technique [6] into Eq. (2.1), we obtain a non-dimensional equation of shoreline evolution as follows:

$$\frac{\partial Y}{\partial T} = \frac{\partial^2 Y}{\partial X^2}, \tag{3.1}$$

and the variables used for transformations as follows:

$$Y = \frac{y}{Y_*}, \tag{3.2}$$

$$X = \frac{x}{L}, \tag{3.3}$$

$$T = \frac{D\,t}{L^2}, \tag{3.4}$$

for all $(X,T) \in [0,1] \times [0,\Gamma]$, where the variable Y, X and T are dimensionless and no units, L is the length of alongshore and Y_* is the expected shoreline evolution.

Therefore, to solve Eq. (3.1) it is necessary to define the initial and boundary conditions.

3.2 The Initial and Boundary Conditions of the Non-Dimensional Model

The initial and boundary conditions for a non-dimensional model can be simply defined from known initial and boundary conditions of the one-dimensional model that are Eqs. (2.2) – (2.4). We can define the necessary condition as follows.
The initial condition becomes:

$$Y(X,0) = F(X) \tag{3.5}$$

The boundary conditions are also defined by:

$$Y(0,T) = G(T), \tag{3.6}$$

and

$$Y(1,T) = H(T), \tag{3.7}$$

for all $X \in [0,1]$, $T \in [0,\Gamma]$ and $\Gamma > 0$, where $F(X) = \dfrac{f(x)}{Y_*}$, $G(T) = \dfrac{g(t)}{Y_*}$ and $H(T) = \dfrac{h(t)}{Y_*}$.

4 Numerical Techniques

4.1 Grid Spacing

We now discretize the domain of Eq. (3.1) by dividing the space interval $\left[0,1\right]$ into M subintervals such that $M\Delta X = 1$ and the time interval $\left[0,\Gamma\right]$ into N subintervals such that $N\Delta T = \Gamma$. We then approximate $Y\left(X_m, T_n\right)$ or Y_m^n, at the point $X_m = m\,\Delta X$ and $T_n = n\,\Delta T$, where $m = 0, 1, 2, \ldots, M$ and $n = 0, 1, 2, \ldots, N$ in which M and N are positive integers.

4.2 The Traditional Forward Time Centered Space (FTCS) Techniques

The forward time centered space method is employed. Consequently, the finite difference approximation becomes [7],[8]:

$$Y\left(X_m, T_n\right) \cong Y_m^n, \tag{4.1}$$

$$\frac{\partial Y}{\partial T} \cong \frac{Y_m^{n+1} - Y_m^n}{\Delta T}, \tag{4.2}$$

$$\frac{\partial Y}{\partial X} \cong \frac{Y_{m+1}^n - Y_{m-1}^n}{2\,\Delta X}, \tag{4.3}$$

$$\frac{\partial^2 Y}{\partial X^2} \cong \frac{Y_{m+1}^n - 2\,Y_m^n + Y_{m-1}^n}{\left(\Delta X\right)^2}, \tag{4.4}$$

Substituting Eqs. (4.1) - (4.4) into Eq. (3.1), and rearrange the equation in an explicit form of finite difference as follows:

$$Y_m^{n+1} \cong \mu\,Y_{m+1}^n + \left(1 - 2\,\mu\right)Y_m^n + \mu\,Y_{m-1}^n, \tag{4.5}$$

for all $m = 1, 2, \ldots, M-1$ and $n = 0, 1, 2, \ldots, N-1$, where $\mu = \dfrac{\Delta T}{\left(\Delta X\right)^2}$.

5 Error Measurement

A simple measure used to measure the difference between actual and approximate values is the absolute error method. The absolute error formula be as follows:

$$E_m^n = \left|\tilde{y}_m^n - y_m^n\right|, \tag{5.1}$$

where $\tilde{y}_m^n$ is the analytical solution of shoreline evolution and y_m^n is the approximate solution of shoreline evolution.

6 Numerical Experiment

We assume that the length of considered shoreline L is 5,000 m, the amplitude of the long-shore transport rate Q_0 is 7,500 m^3/day, the averaged berm height D_B is 2 m, the averaged closure depth D_C is 28 m and the expected shoreline evolution Y_* is 20 m when studying long-term shoreline evolution. The idealized simulation setting is illustrated in Figure 1.

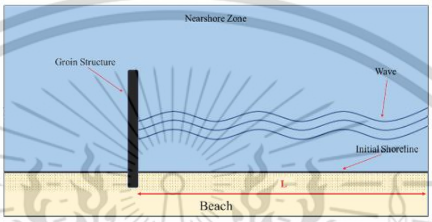

Figure 1: Initial shoreline.

We will employ the traditional forward time centered space (FTCS) techniques Eq. (4.5) to approximate the model solution. Additionally, we will use the absolute error formula as Eq. (5.1) to measure the error and efficiency of this numerical approximation technique. The analytical solution of the simulation is [5]:

$$\tilde{y}(x,t) = \tan\alpha_0 \sqrt{\frac{4Dt}{\pi}} \left\{ e^{-\frac{x^2}{4Dt}} - \frac{x\sqrt{\pi}}{2\sqrt{Dt}}\, erfc\left(\frac{x}{2\sqrt{Dt}}\right) \right\}, \tag{6.1}$$

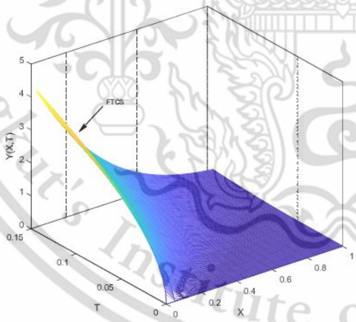

Figure 2: Approximated shoreline evolution under the non-dimensional equations in 20 years when the FTCS method is used.

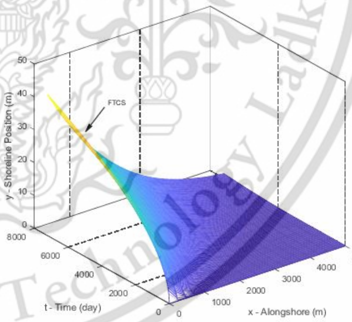

Figure 3: Approximated shoreline evolution under the one-dimensional equations in 20 years when the FTCS method is used.

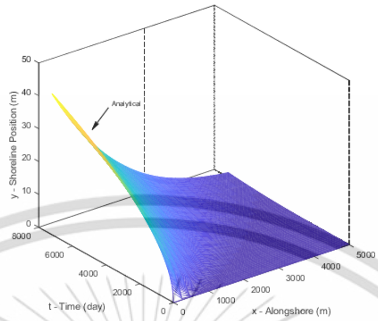

Figure 4: Analytical shoreline evolution in 20 years.

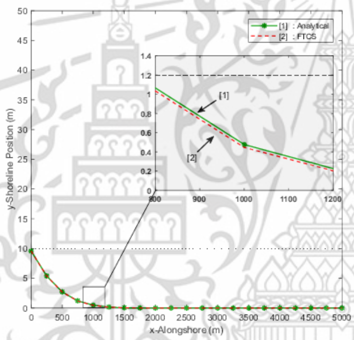

Figure 5: Shoreline evolution value in 1 year.

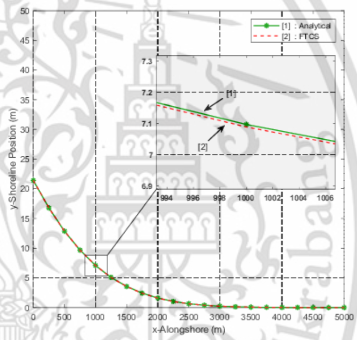

Figure 6: Shoreline evolution value in 5 years.

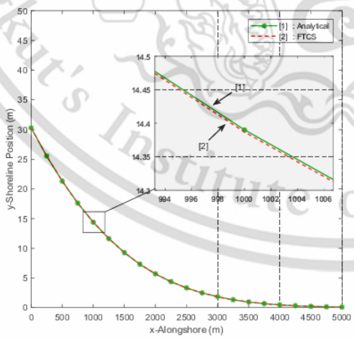

Figure 7: Shoreline evolution value in 10 years.

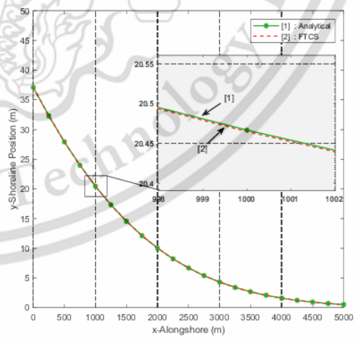

Figure 8: Shoreline evolution value in 15 years.

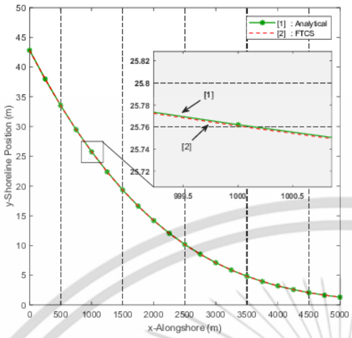

Figure 9: Shoreline evolution value in 20 years.

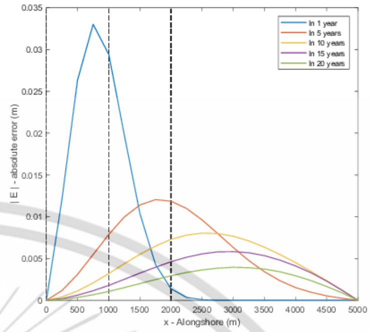

Figure 10: Comparison of absolute error in 1, 5, 10, 15 and 20 years.

The analytical solution is given in Table I and the approximated solutions of the traditional forward time centered space (FTCS) technique give approximated solutions in Table II.

Table I: Analytical solution of shoreline evolution.

Time	Distance evolution (m.)					
(Years)	0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.476	0.003	0.000	0.000	0.000
5	21.412	7.096	1.607	0.237	0.022	0.001
10	30.282	14.390	5.688	1.844	0.480	0.099
15	37.087	20.467	10.029	4.314	1.614	0.521
20	42.825	25.762	14.194	7.107	3.213	1.304

Table II: Approximated shoreline evolution along 20 years using FTCS techniques.

Time	Distance of evolution(m.)					
(Years)	0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.446	0.001	0.000	0.000	0.000
5	21.412	7.089	1.595	0.230	0.020	0.001
10	30.282	14.387	5.692	1.836	0.475	0.099
15	37.078	20.465	10.024	4.308	1.610	0.521
20	42.825	25.761	14.191	7.103	3.210	1.304

7 Discussion

Using the traditional forward time centered space (FTCS) techniques, the annual evolution of the shoreline can be determined.

In computational terms, when we convert the approximate solution under the non-dimensional equation (as Figure 2) into the approximate solution under the one-dimensional equation (as Figure 3), we can see that it is close to the analytical solutions (as Figure 4). Moreover, we can interpret that the model doesn't have to be dimensioned, which we can transformers to a non-dimensional form because it doesn't affect computational efficiency and it can benefit complex problems and reduce costs. For example, the cost in terms of time, people and budget include software.

In Tables I-II and Figures 5-9, it is confirmed that the approximate solution is close to the analytical solution. And, after 1, 5, 10, 15 and 20 years, the shoreline evolution value increases accordingly.

In terms of accuracy, the traditional forward time centered space (FTCS) techniques can handle the problem well if considering the absolute errors as shown in Figure 10 which the maximum value of the absolute error is no more than 1 m.

8 Conclusion

The model includes a one-dimensional model and a non-dimensional model as well as a groin structure model. Physical parameters can now be manipulated by the model. The initial condition setting technique and boundary condition approaches are also presented where groin structural effects are involved. The forward time centered space (FTCS) technique is employed to approximate the incremental model every year. From the computational results, we find that the approximate solution obtained is close to the analytical solution. Therefore, the non-dimensional models do not affect computational performance. In addition, it is also convenient to use. The suggested simulation might be used to predict how successful a groin system on a local beach would be.

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Academic Publication(s)

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2. S. Manilam and N. Pochai, "A Non-Dimensional Mathematical Model of Shoreline Evolution with a Groin Structure Using an Unconditionally Stable Explicit Finite Difference Technique," IAENG International Journal of Applied Mathematics (IJAM), vol. 52, no.2, pp473-481, 2022