A MATHEMATICAL MODEL OF THE RISK OF AIRBORNE INFECTION ASSESSMENT AMONG BUS PASSENGERS

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Abstract

Carbon dioxide emitted by human breath is a major contributor to airborne infections. Airborne infections can spread quickly, and breathing can expose us to airborne infections that are life-threatening. If there are passengers traveling by bus, there is a risk of infection. In this research, a mathematical model of carbon dioxide concentration measurement due to human breath is introduced. This research focuses on measuring the concentration of carbon dioxide due to bus passengers. An explicit finite difference technique is used to approximate the solution of the model. The model solution can be used to know how much the passengers allow for sitting on the bus while the level of carbon dioxide concentration is controlled. In addition, mathematical models were used to assess the risk of air infection among bus passengers equipped with ventilation systems to reduce the risk of air infection and improve ventilation. It was found in good agreement that the proposed air quality model allows us to know the balance between the number of passengers allowed to sit on the bus while controlling the risk of airborne infection as well as the concentration of carbon dioxide and the potential of the ventilation system.

Keywords: Bus, Carbon dioxide, Explicit finite difference technique, Mathematical model, Passengers breathing

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Miss Jenjira Sooknum

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Abbreviations/Symbols

Symbols	Description
C	The concentration of carbon dioxide in the air on a bus (ppm)
D	A diffusion coefficient of carbon dioxide (m^2 /s)
n	The number of people
p	The breathing rate (L/s)
t	Time (min)
C_a	The carbon dioxide fraction contained in breathed air (ppm)
Q	The ventilation rate (L/s)
V	The volume of air in the bus (m^3)
I	The number of infectors in the bus
β	The total airborne infectious particle generation rate released by an infector (particles/s)
μ	The mortality rate of generated airborne infectious particles by the infector that do not reach the alveolar (particles/s)
pt	The volume of breathed air by susceptibles
θ	A respiratory deposition fraction of airborne infectious particles
$f(x,t)$	The volume fraction of exhaled air
$N(x,t)$	The concentration of airborne infectious particles
$\lambda(x,t)$	The number of airborne infectious particles
$P(x,t)$	The probability of airborne infectious diseases risk for susceptible individuals

Chapter 1

Introduction

1.1 Research motivation

Most of the people in the metropolitan area travel by public transport, and one of them is by bus, with a large number of people using the bus, causing congestion inside the bus. If there is an epidemic of airborne infectious diseases such as tuberculosis, SARS and COVID-19, which are spread through respiratory droplets in various forms such as breathing, talking, coughing, and sneezing of infected people. If those aerosols are inhaled, infection and life threatening may occur. Airborne infections on a bus such as news in China indicates COVID-19 spreads 4.5 meters in the air for 30 minutes, which is a dangerous situation that we should be aware of.

Therefore, we are interested in studying and assess the risk of air infection among passengers on the bus, there is a ventilated system to control risks on the bus, such as controlling the number of passengers and improving ventilation systems.

1.2 Literature reviews

In [14], they analyzed three epidemiological models used to predict the transmission of airborne disease in confined spaces. Gammiatoni and Nucci's general formulation is shown to be the most suitable for modeling airborne transmission in ventilated spaces, and it is subsequently used in a parametric study to evaluate the effect of physical and environmental factors on the rate of disease transmission. In [5], they assessed the efficacy of recommended tuberculosis (TB) infection control measures by using a deterministic mathematical model for airborne contagion. The efficiency of these control measures decreases as the infectivity of the source case increases.

In [4], they measured carbon dioxide in classrooms under non-steady state conditions. Tuberculosis transmission criteria were assessed using a carbon dioxide-based risk equation. With a high smear positive rate, the proposal to achieve carbon dioxide levels of 1000 ppm through natural ventilation helps to meet WHO guidelines for providing children with healthy indoor environments and to control the TB epidemic in areas of high prevalence. In [10], they utilized a carbon dioxide decay technique measured ventilation in air change per hour (ACH) and used the Wells-Riley equation to estimate TB transmission risk in traditional homes. Low ventilation was found at baseline measurements when the windows were closed, thus there was a high risk of spreading TB. In [24], they reviews and critically evaluates the current ventilation strategies used in buildings to assess the state of the art and elaborates if there is room for further development, especially for high occupancy buildings, to reduce or eradicate the risk of pathogen transmission and adapt ventilation measures.

In [18], they developed a portable monitor to continuously sample carbon dioxide levels, which were combined with social contact diary records to estimate daily rebreathed liters. They then estimated the daily volumes of air rebreathed by adolescents living in a crowded township. They demonstrate the practical measurement of carbon dioxide levels to which individuals are exposed in a sequence of non-steady indoor environments. A novel metric of rebreathed air volume reflects social and environmental factors associated with airborne infection and can identify locations with high transmission potential. In [1], they propose that using a modified Well-Riley model for airborne disease transmission, they estimated the risk of tuberculosis transmission on 3 modes of public transit risk, using exhaled carbon dioxide as a natural tracer gas to evaluate air exchange. Given its poor ventilation and high respiratory contact rates, public transportation may play a critical role in sustaining tuberculosis transmission.

In [16], they considered, according to personal classifications. There are four types: patients, patient relatives, workers and outsiders, staying in an outpatient room. Air quality control manipulations are simulated using the inlet and outlet ventilation rates adjustment under the condition of a number of surrounding people. The fourth-order Runge-Kutta method was used to approximate the model solution. The proposed numerical model can be used to describe the dynamic dispersion of airborne infectious disease in an outpatient room. The results of the model will be able to control airborne disease in more complicated structures. In [22], they proposed a model sets the concentration of carbon dioxide at any point when the number of people and the rate of ventilation varies. The classical fourth-order Runge-Kutta method is employed to approximate the model solution. There are many cases of scenarios for improving air quality in the proposed simulations. In the air quality management process, the proposed model provides a balance between the number of persons allowed to stay in the room and the capacity of the air ventilation system. In [21], they develop and demonstrate a flexible mathematical model that predicts the risk of airborne infectious diseases, such as tuberculosis steady state and non-steady state conditions by monitoring exhaled air by infectors in a confined space. they demonstrated a mathematically and schematically the correlation between TB transmission probability and airborne infection particle generation rate, ventilation rate, average volume fraction of exhaled air, TB prevalence and duration of exposure to infectors in a confined space.

Therefore, if there is an epidemic of infectious diseases in the air or in a place where there are infected people. How do we know the risk of airborne infection at that location at that time? In this study, we studied a mathematical model to assess the risk of airborne infection among passengers on buses under closed conditions with ventilation system.

1.3 Objectives of the study

1) To determine the amount of carbon dioxide produced by human respiration or other polluting compounds.
2) To determine the quality of the air inside the bus.
3) To reduce the risk of airborne infection on board the bus.
4) To present a mathematical model of carbon dioxide or other toxic substance concentration measurement and methods for determining its parameters.

1.4 Research methodology

1) Choose a research topic.
2) Collect and review references related to mathematical of the concentration of carbon dioxide or other toxic substances.
3) Improve the carbon dioxide or other toxic substance concentration model.
4) To introduce an improved parameterization.

5) To estimate the solution using a numerical method.
6) Plot a graph of the concentration of carbon dioxide in the air inside the bus, the volume fraction of exhaled air, and the concentration of airborne infectious particles.
7) Plot a graph of the number of airborne infectious particles and the probability of airborne infectious disease risk for susceptible individuals.
8) Analyze the results of the experiment.
9) To simulate mathematical models.
10) Discuss and summarize the experiment's findings.
11) Outline potential solutions to future problems.

1.5 Scopes of the study

1) The rate of ventilation in a closed environment only on the bus.
2) The diffusion coefficient of carbon dioxide is given.
3) The number of people is given.
4) The carbon dioxide fraction in the breathed air is provided.
5) The rate of breathing is given.
6) The initial value of the concentration of carbon dioxide in the air on a bus is given.
7) The ventilation rate is specified.
8) The number of people changes every 10 minutes.
9) The Bangkok mass transit authority buses run in the suburbs, stopping every 10 minutes and traveling time is 1 hour.
10) Passengers are seated according to their position only.

1.6 Benefits of the study

1) Capable of limiting the number of bus passengers per round.
2) Capable of identifying the number of passengers who are at risk of COVID-19 or other airborne infectious diseases.
3) The ventilation system can be improved to reduce the risk of airborne infections.

Chapter 2

Basic knowledge

2.1 Bus structure

The dimensions of the car are 2.5 meters wide, 2.5 meters high and 12 meters long as shown in Figure 2.1.

Figure 2.1 A bus model.

The 48-seater air-conditioned bus is shown in the seat map in Figure 2.2 and has two exhaust fans at the front and rear of the bus as shown in Figure 2.3.

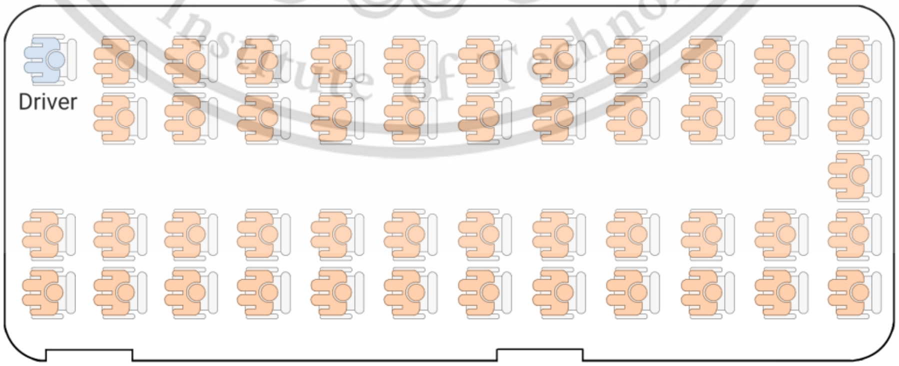


Figure 2.2 Seat map.

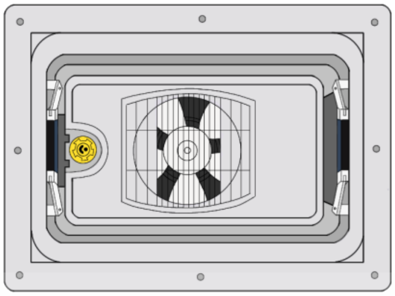


Figure 2.3 Exhaust fan.

2.2 Airborne transmission and airborne infections on a bus

2.2.1 Airborne transmission

Airborne transmission is the transmission of infectious diseases through dust particles that can stay in the air for a long time and can be moved to another location. There are many diseases that can be transmitted through the air, both in humans and animals. The germs that cause disease can be viruses, bacteria or fungi, and can be spread through breathing, speaking, coughing, sneezing, dusting, spraying, flushing the toilet bowl, and any activity that produces aerosol particles or droplets. Airborne infectious diseases generally do not count diseases caused by air pollutants such as volatile organic compounds, toxic gases or other particulate matter.

Traditionally, airborne transmission was thought to be different from aerosol transmission, which originated from a misunderstanding of the physical behavior of particles of various sizes. The respiratory secretion becomes large enough to quickly fall to the ground after being released. However, under the new definition, particles of any size that can be inhaled have the potential to be airborne infectious particles.

Each person can transmit aerosols and droplets of various sizes, quantities, and concentrations of germs. It depends on the difference of each person and the activities they do. Aerosols larger than 100 micrometers tend to fall rapidly to the ground at a distance of less than 2 meters from the point of origin, and may remain in the air for a period of time with pathogenic aerosol particles. The highest pathogenic concentrations are within a radius of 2 meters, but can travel to other locations and may accumulate to increase concentrations in other locations of that area as well.

This is because inhaled particles often come out of varying sizes at the same time and will either drift or fall to the ground depending on a number of environmental factors that do not depend only on their initial size. However, the idea has been used to prevent the spread of disease in hospitals for decades. New data on the indoor diffusion of respiratory secretions lead to the belief that 20 micrometer particles. In the first phase it was released to float along the air currents in the beginning, which may be air streams from coughing or sneezing from air conditioning, but when traveling further, it will gradually fall to the ground according to gravity. It has been found that particles of this size when entering the body are most strongly trapped in the mucous membrane of the nasopharynx and because this location is often the initial site of infection that causes COVID-19. That's why it's credible that particles of this size play a big part in causing the COVID-19 outbreak.

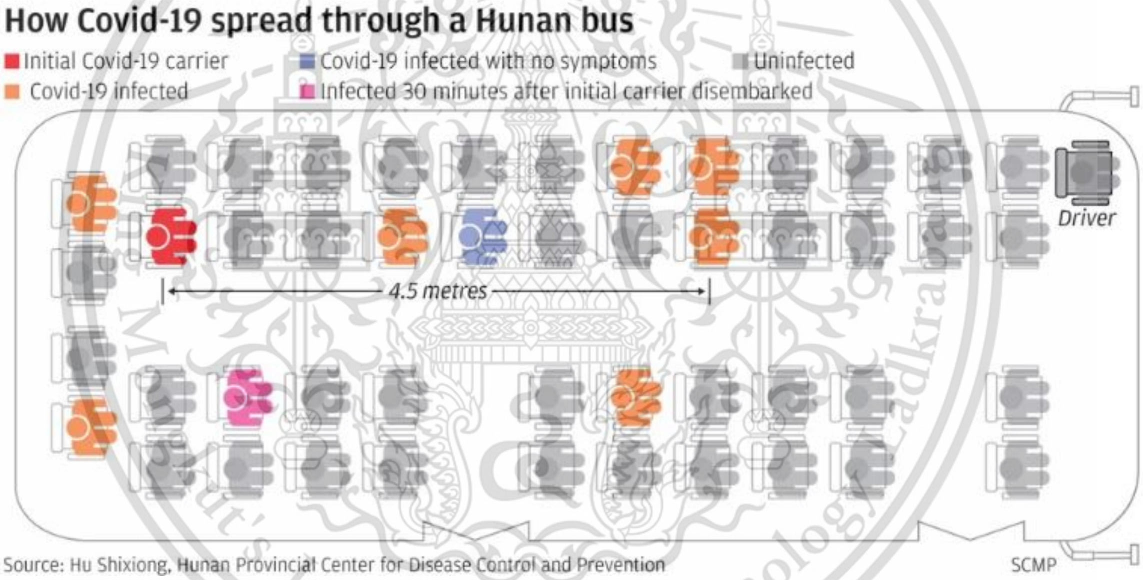
2.2.2 Airborne infections on a bus

Study results by a team of Chinese government epidemiologists. It was found that the new coronavirus has been in the air for at least 30 minutes and traveled 4.5 meters. Greater than the safe distance recommended by public health officials around the world to be 1-2 meters away from infected people.

The results of the study, compiled by Hubei researchers investigating cluster infection disease on January 22, 2020, were at the peak of the trip to celebrate Lunar New Year, a passenger suffering from illness and travel in a long-haul 48-seater bus, sitting in the second back row without mask, and most passengers or drivers do not wear masks. At the time, China had not declared the outbreak a national crisis, as China required all long-distance buses to be equipped with surveillance cameras, the resulting clips were very useful for researchers to study how the vehicle is spread. The bus with all windows closed, so it can be confirmed that in a closed environment with air conditioning. The new coronavirus will spread farther in a known safe distance, and there is also a risk that the virus may persist after the carrier has alighted the bus.

"The advice is to wear a mask all the time when in the bus", the epidemiologist said, adding that in this case, "A" had not interacted with anyone during the four-hour journey. The virus has spread from the carrier to seven other passengers and not the closest seats. Rather, it was a husband and wife who sat in the sixth row next to him, 4.5 meters away.

30 minutes after these passengers got off the car, another group of passengers came up. It turned out that one passenger in the front row was also infected. The researchers said the patient, who was not wearing a mask, was likely to inhale tiny aerosols released on the breath of an infected passenger. The reason for this is because in a closed area. The main air flow is from the hot air from the air conditioner. The weather is hot and the aerosols carry the virus go further.



How Covid-19 spread through a Hunan bus

Source: Hu Shixiong, Hunan Provincial Center for Disease Control and Prevention

Figure 2.4 Seat map on 48-seater bus showing the spread of covid-19 among carrier patients.

After getting off the carrier bus, the first person connects to the minibus and travels for an hour. As a result, two other passengers were infected, who were also sitting 4.5 meters away from the patient A. By the time this study was completed in mid-February. It was concluded that the carrier infecting at least 13 other people.

2.3 A one-dimensional advection-diffusion equation

Consider,

$$\frac{\partial u}{\partial t}+\gamma\frac{\partial u}{\partial x}=D\frac{\partial^2 u}{\partial x^2}\text{, for all }(x,t)\in\Omega\text{,} \tag{1}$$

where γ is a convection coefficient, D is a diffusion coefficient and $\Omega\in[0,L]\times[0,T]$.

2.3.1 The initial condition

$$u(x,0)=f(x)\text{, for all }0\le x\le L\text{,} \tag{2}$$

where $f(x)$ is a given function.

2.3.2 The boundary conditions

The left boundary condition (LBC):

$$u(0,t)=g_1(t)\text{, for all }t\in[0,T]\text{,} \tag{3}$$

where $g_1(t)$ is a given function.

The right boundary condition (RBC):

$$u(L,t)=g_2(t)\text{, for all }t\in[0,T]\text{,} \tag{4}$$

where $g_2(t)$ is a given function.

2.4 Numerical technique

A continuous approximation to the solution $u(x,t)$ will not be obtained; instead, approximations to u will be generated at various values, called mesh point, in the interval $[0,T]$.

Once the approximate solution at other points in the interval can be found by interpolation. We first make the stipulation that the mesh points are equally distributed throughout the interval $[0,T]$. This condition is ensured by choosing a positive integer M and selecting the mesh point $t_n=n\ell$, for each $n=0,1,2,\ldots,M$ where $\ell=(T-0)/M$ is called the time step. This condition is ensured by choosing a positive integer N and

$x_m = mh$, for each $m = 0,1,2,...,N$. The common distance between the points $h = (L-0)/N$ is called the step size.

2.4.1 The forward time centered space (FTCS) method

Use a forward time finite difference approximation:

$$\frac{\partial u}{\partial t} = \frac{u(x,t+\ell)-u(x,t)}{\ell} + O(\ell). \tag{5}$$

The forward difference is used to evaluate the time derivative at $t = t_n$.

Use a central difference in space approximation:

$$\frac{\partial u}{\partial x} = \frac{u(x+h,t)-u(x-h,t)}{2h} + O(h^2), \tag{6}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u(x-h,t)-2u(x,t)+u(x+h,t)}{h^2} + O(h^2). \tag{7}$$

The spatial derivative with the central difference operator and take all nodal value at time t_n.

Retaining in $\frac{\partial u}{\partial t}$, $\frac{\partial u}{\partial x}$ and $\frac{\partial^2 u}{\partial x^2}$ are replaced by (5), (6) and (7) in Eq. (1) and we will letting that

$$u \approx u_m^n, \tag{8}$$

$$u_m^n = u(x_m, t_n), \tag{9}$$

$$u_{m+1}^n = u(x_m + h, t_n), \tag{10}$$

$$u_m^{n+1} = u(x_m, t_n + \ell), \tag{11}$$

$$u_{m-1}^n = u(x_m - h, t_n). \tag{12}$$

From (1), we get the FTCS finite difference equation,

$$u_m^{n+1} = (\alpha + \frac{1}{2}r)u_{m-1}^n + (1-2\alpha)u_m^n + (\alpha - \frac{1}{2}r)u_{m+1}^n, \tag{13}$$

where $r = \gamma \frac{\ell}{h}$ is the convection number and $\alpha = D\frac{\ell}{h^2}$ is the diffusion number.

Chapter 3

Governing equation

3.1 Governing equation

In general, the rate of exhaled air generation and ventilation per person determine the raised concentration of indoor carbon dioxide [6], [8], and [9]. Because an infected individual's exhaled air contains airborne infectious particles, carbon dioxide levels can be employed as an exhaled air surrogate [4], [6], [7], [9], and [18]. The ambient carbon dioxide levels tend to average about 400 ppm [3], [4], and [6].

3.1.1 A one-dimensional exhaled air concentration measurement model: a bus with a constant number of passengers

We assume that a bus interior space, such as a bus with a volume of V. Given the presence of infectors, the concentration of exhaled air that may contain airborne contagious particles may tend to rise in the bus, based on the rate of ventilation Q, and the number of people in the bus.

We simply assume that persons in the bus contribute substantially to the production of carbon dioxide, which serves as an exhaled air marker. The fundamental equation of the accumulation rate exhaled air concentration in a bus with carbon dioxide is equal to the exhaled air rate generated by inhabitants plus the diffusion rate of carbon dioxide, minus ventilation rate removes exhaled air:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} + npC_a - QC, \text{ for all } (x,t) \in \Omega, \tag{14}$$

where $\Omega = \left[0, L\right] \times \left[0, T\right]$, C is the concentration of air inside the exhaled bus (ppm), p is the rate of breathing (L/s) for each person in the bus and C_a is the carbon dioxide fraction included in breathed air, t is the duration time, T is the stationary simulation time and L is the length of a considered bus. Initial condition is given by $C(x,0) = C_0$

where C_0 is the latent carbon dioxide concentration. The boundary conditions are given by $\frac{\partial C}{\partial x} = C_F$ where $x = 0$ and C_F is a given constant, and $\frac{\partial C}{\partial x} = C_B$ where $x = L$ and C_B is a given constant.

3.1.2 A one-dimensional exhaled air concentration measurement model: a bus with a variable number of passengers

In a simple scenario, a number of people are unstable then a number of people depend on the time assumed by $n(x,t)$. In this study preferred to use Eq. (14) as follow:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} + n(x,t)pC_a - QC\,, \tag{15}$$

for all $(x,t) \in \Omega$.

3.1.3 The volume fraction of exhaled air

We obtain the concentration of sampled exhaled air, $C(x,t)$, in the given space. By the volume fraction of exhaled air, $f(x,t)$, given by the concentration of sampled air divided by the carbon dioxide fraction included in breathed air, we get

$$f(x,t) = \frac{C(x,t)}{C_a}\,, \tag{16}$$

for all $(x,t) \in \Omega$.

3.1.4 The concentration of airborne infectious particles

The chances that airborne infectious particles released by infectors causing infection for susceptible individuals is extremely high if they reach the target infection site of the host at a threshold level [15]. However, some infectious particles can get trapped in the upper respiratory tract or spread to other parts of the body. Let β be the total airborne infectious particles generation rate released by an infector (particles/s) and μ be the mortality rate of generated airborne infectious particles by the infector that do not reach the alveolar (particles/s).

The concentration of airborne infectious particles, N, causing infection is equal to the volume fraction of rebreathed air by infectors, multiplied by the concentration of airborne infectious particles released by infectors in the space that reach the target infection site of the respiratory tract:

$$N(x,t)=\frac{If(x,t)(\beta-\mu)}{n(x,t)p},\ I\geq 1 \text{ and } \beta-\mu\geq 1, \tag{17}$$

where I is the number of infectors in the bus and for all $(x,t)\in\Omega$.

3.1.5 The number of airborne infectious particles

Since not all infectious particles can reach and deposit at the alveolar, let θ be a respiratory deposition fraction of airborne infectious particles that successfully reach and deposit at target infection site of the host. Hence, the number of airborne infectious particles, $\lambda(x,t)$, breathed by a susceptible individual that causes infection, is equal to the produce of the volume of breathed air by a susceptible, respiratory deposition fraction of airborne infectious particles, θ, which is greater than zero but less than 1, and the concentration of airborne infectious particles released by infectors:

$$\lambda(x,t)=pt\theta N(x,t), \tag{18}$$

where t is the time spent in the space up to the point of infection and for all $(x,t)\in\Omega$ **.**

3.1.6 The risk of airborne infection

The probability of airborne infection risk for susceptible individuals:

$$P(x,t)=1-e^{-\lambda(x,t)}, \tag{19}$$

where $(x,t)\in\Omega$.

3.2 Parameter setting

In our research, all parameters will be assumed by an assumption as show in this section.

C	The concentration of carbon dioxide due to passengers breathing in the air on a bus (ppm)
D	A diffusion coefficient of carbon dioxide (m^2/s)
$n(x,t)$	The number of passengers seated in the bus depends on the time
p	The breathing rate (L/s)
t	The time the bus takes to run (min)
C_a	The carbon dioxide fraction contained in breathed air (ppm)
Q	The ventilation rate (L/s)
V	The volume of air in the bus (m^3)
I	The number of infectors in the bus
α	The diffusion number
β	The total airborne infectious particle generation rate released by an infector (particles/s)
μ	The mortality rate of generated airborne infectious particles by the infector that do not reach the alveolar (particles/s)
θ	A respiratory deposition fraction of airborne infectious particles
$f(x,t)$	The volume fraction of exhaled air
$N(x,t)$	The concentration of airborne infectious particles
$\lambda(x,t)$	The number of airborne infectious particles
$P(x,t)$	The probability of airborne infectious diseases risk for susceptible individuals

3.3 Initial condition setting

$$C(x,0)=f(x)\,, \tag{20}$$

for all $(x,t)\in\Omega$ and $f(x)$ is a given function of the remained exhaled air concentration in an empty bus.

3.4 Boundary conditions setting

Assuming that there is no absorbance mechanism on the front and the back of the considered bus.

The left boundary condition (LBC):

$$\frac{\partial C}{\partial x}=C_F\,, \text{ for all } t>0 \text{ and } x=0\,. \tag{21}$$

The right boundary condition (RBC):

$$\frac{\partial C}{\partial x}=C_B\,, \text{ for all } t>0 \text{ and } x=L\,. \tag{22}$$

3.5 A forward-time centered-space finite difference method for A one-dimensional exhaled air concentration measurement model: a bus with a variable number of passengers

Retaining in $\dfrac{\partial u}{\partial t}$ and $\dfrac{\partial^2 u}{\partial x^2}$ are replaced by (5) and (7) in Eq. (15).

Letting that

$$C\approx C_m^n, \tag{23}$$

$$C_m^n=C(x_m,t_n)\,, \tag{24}$$

$$C_{m+1}^n=C(x_m+h,t_n)\,, \tag{25}$$

$$C_m^{n+1}=C(x_m,t_n+\ell)\,, \tag{26}$$

$$C_{m-1}^n=C(x_m-h,t_n)\,, \tag{27}$$

then

$$\frac{C_m^{n+1}-C_m^n}{\ell}=D\left(\frac{C_{m-1}^n-2C_m^n+C_{m+1}^n}{h^2}\right)+n(x,t)\,pC_a-QC_m^n\,,$$

$$C_m^{n+1} = \frac{D\ell}{h^2}\left(C_{m-1}^n - 2C_m^n + C_{m+1}^n\right) + n(x,t)pC_a\ell - QC_m^n\ell + C_m^n,$$

$$C_m^{n+1} = \alpha C_{m-1}^n - 2\alpha C_m^n + \alpha C_{m+1}^n + n(x,t)pC_a\ell - QC_m^n\ell + C_m^n,$$

from (15), we get the FTCS finite difference equation becomes

$$C_m^{n+1} = \alpha C_{m-1}^n - (2\alpha + Q\ell - 1)C_m^n + \alpha C_{m+1}^n + n(x,t)pC_a\ell, \tag{28}$$

where $\alpha = \frac{D\ell}{h^2}$ is the diffusion number and $p = 0.12$ (L/s) is the breathing rate. The stability condition of Eq. (28) is [26] $0 < \alpha \le \frac{1}{2}$.

Chapter 4

Numerical simulation

Assuming that the bus of volume is $V = 75$ (m^3), each person's breathing rate in the bus assumed by $p = 0.12$ (L/s), then the diffusion coefficient of carbon dioxide $D = 0.732$ (m^2/s), the carbon dioxide fraction included in breathed air $C_a = 0.04$ (ppm) and the rate of change of the carbon dioxide at the frontend and the backend in the bus is neglected which are $C_F = C_B = 0$.

4.1 Simulation 1: the probability of airborne infection risk for susceptible individuals on the bus.

Table 4.1 lists the number of passengers in each row. $C_0 = 0.1$ is the initial value of carbon dioxide in the air on a bus (ppm) and the bus uses the air vent rate is 75. We achieve the approximated solutions illustrated in Figure 4.2 to 4.4 by using the forward time centered space (FTCS) method (23) to (28) and (16) shown in Figure 4.5.

Table 4.1 The number of passengers on the bus in each row.

Time (min)	The number of passengers in each row												
	0	1	2	3	4	5	6	7	8	9	10	11	12
0-10	0	1	1	1	1	1	1	1	1	1	1	1	0
10-20	0	3	3	3	3	3	3	3	3	3	3	3	3
20-30	0	2	4	4	4	4	4	4	4	4	2	2	2
30-40	0	1	1	1	1	1	1	1	1	1	1	1	1
40-50	0	2	2	2	2	2	2	2	2	2	2	2	2
50-60	0	2	2	2	2	2	2	2	2	2	0	0	0

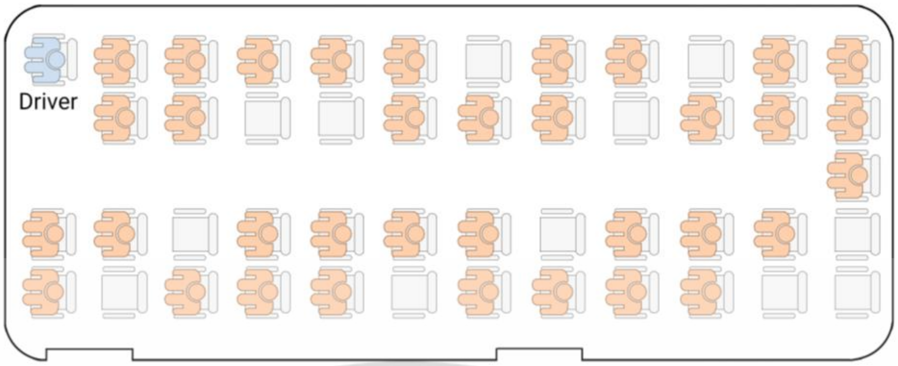


Figure 4.1 Seat map of the number of people seated on the bus for 10 to 20 minutes.

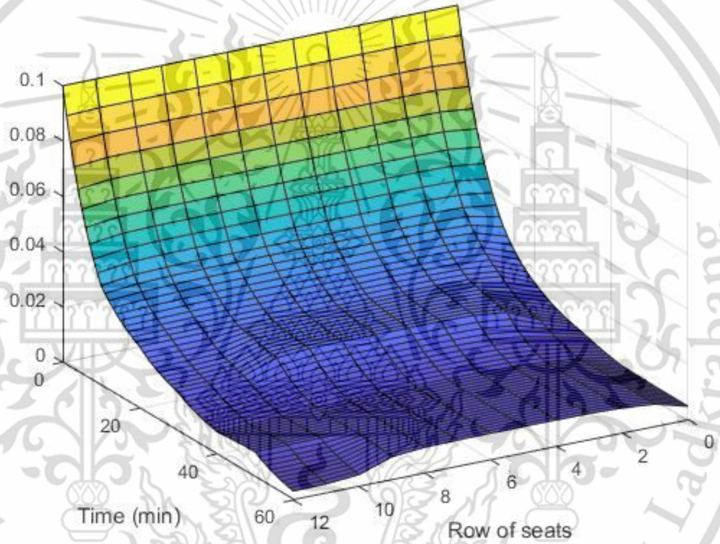


Figure 4.2 The approximated carbon dioxide concentration in the air on a bus with a ventilation system $\ell = 0.1 \ T = 60$.

We can see that the carbon dioxide concentration in the air decreases because there are fewer passengers in each row as passengers get off the bus.

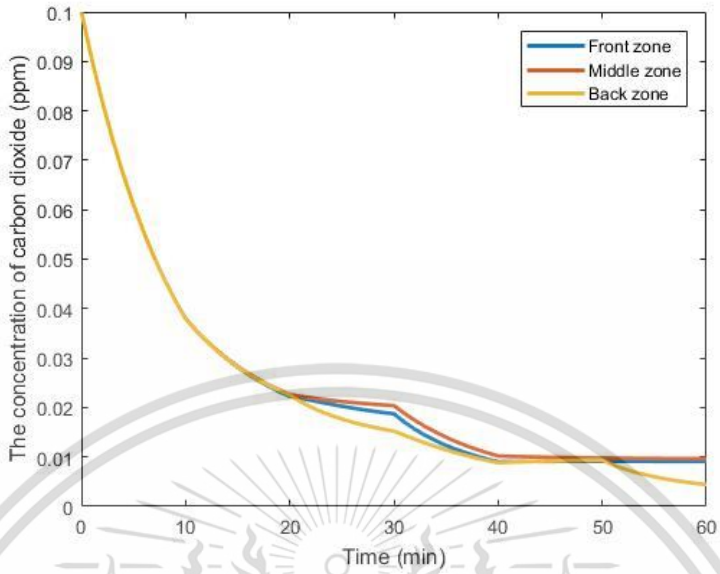


Figure 4.3 The concentration of carbon dioxide in the air in the front, middle and rear zones on a bus with a ventilation system $\ell = 0.1\ T = 60$.

We can see that the concentration of carbon dioxide in the middle zone is higher than the other zones.

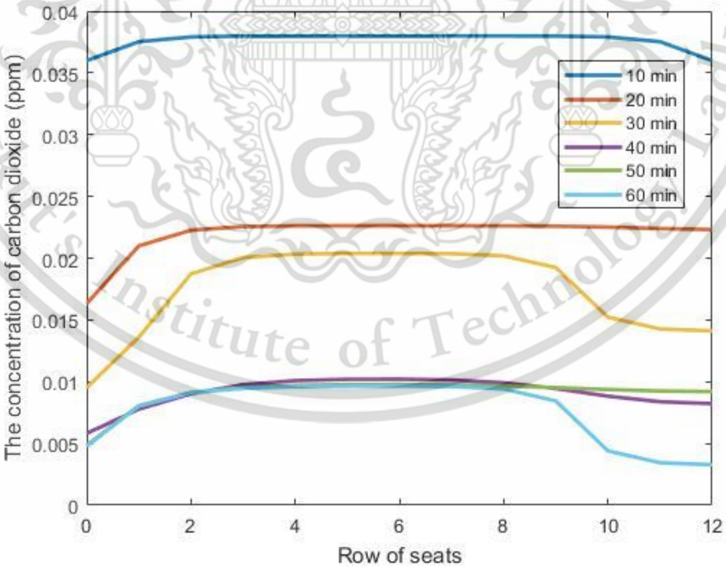


Figure 4.4 The concentration of carbon dioxide in the air on a bus every 10 minutes with a ventilation system $\ell = 0.1\ T = 60$.

We can see that in the case of a constant ventilation rate, the carbon dioxide concentration in the air decreases sharply with fewer passengers.

Table 4.2 The concentration of carbon dioxide in the air on a bus every 10 minutes.

	The concentration of carbon dioxide					
x	$t=10$	$t=20$	$t=30$	$t=40$	$t=50$	$t=60$
0	0.0360	0.0163	0.0095	0.0058	0.0048	0.0048
1	0.0375	0.0210	0.0136	0.0078	0.0081	0.0080
2	0.0379	0.0223	0.0187	0.0090	0.0091	0.0091
3	0.0380	0.0226	0.0200	0.0098	0.0095	0.0095
4	0.0380	0.0226	0.0203	0.0101	0.0097	0.0096
5	0.0380	0.0226	0.0204	0.0102	0.0098	0.0096
6	0.0380	0.0226	0.0204	0.0102	0.0098	0.0096
7	0.0380	0.0226	0.0204	0.0101	0.0097	0.0096
8	0.0380	0.0226	0.0202	0.0099	0.0097	0.0094
9	0.0380	0.0226	0.0192	0.0094	0.0095	0.0084
10	0.0379	0.0225	0.0152	0.0088	0.0094	0.0044
11	0.0375	0.0224	0.0143	0.0084	0.0092	0.0034
12	0.0360	0.0223	0.0141	0.0082	0.0092	0.0033

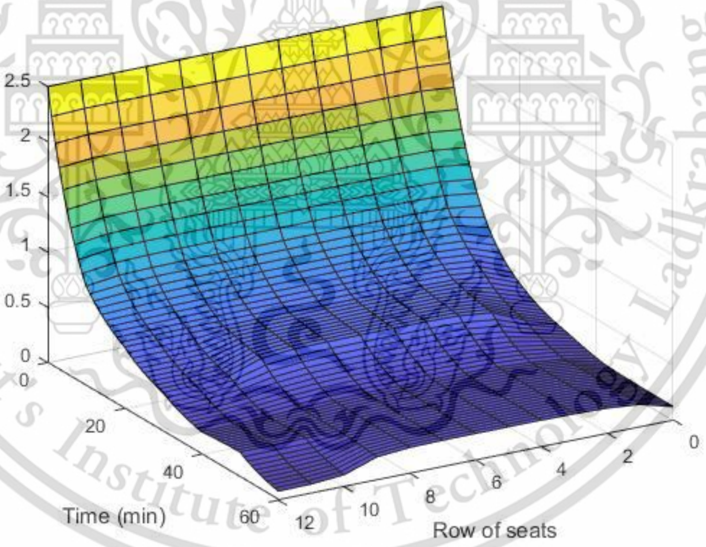


Figure 4.5 The volume fraction of exhaled air on a bus.

We can see that the passenger exhaled air volume fraction increases as the number of passengers increases.

Table 4.3 lists the physical parameters. We achieve the approximated solutions illustrated in Figure 4.6 to 4.10 by using (17) shown in Figure 4.6, (18) shown in Figure 4.7 and (19) shown in Figure 4.8 to 4.10.

Table 4.3 Physical parameters.

I	θ	β	μ
1	0.25	100	87

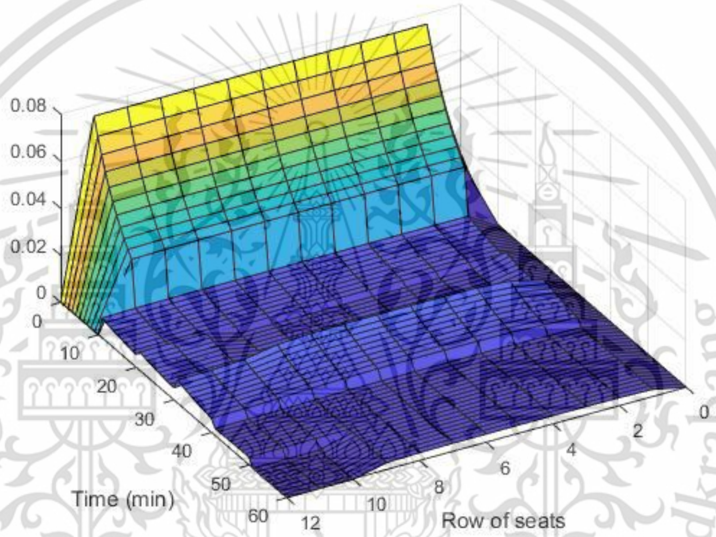


Figure 4.6 The concentration of airborne infectious particles.

We can see that the number of passengers at 0 to 10 minutes and at 30 to 40 minutes resulted in higher concentrations of airborne infectious particles than other times.

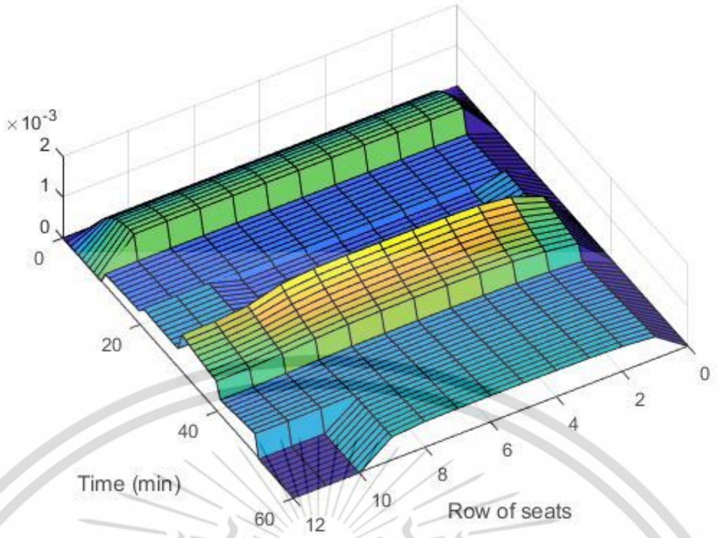


Figure 4.7 The number of airborne infectious particles.

We can see that the number of infectious particles in the air at 0 to 10 minutes and 30 to 40 minutes is high due to the low number of passengers.

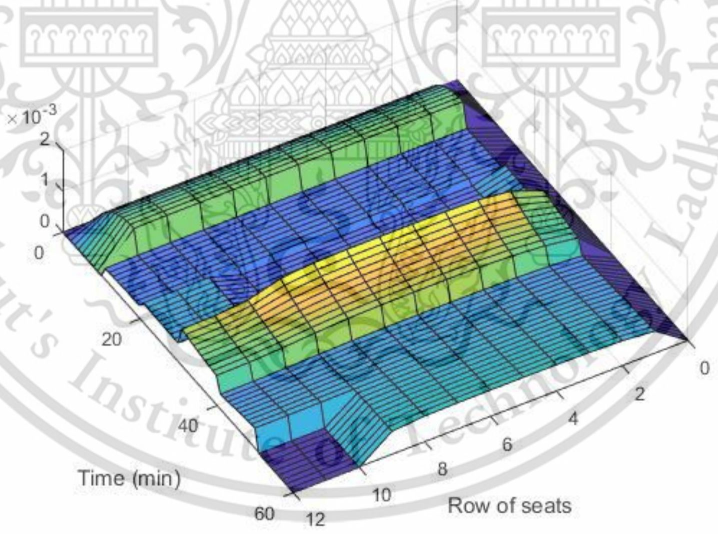


Figure 4.8 The probability of airborne infection risk for susceptible individuals.

We can see that passengers seated at 0 to 10 minutes and 30 to 40 minutes are at high risk of airborne infection.

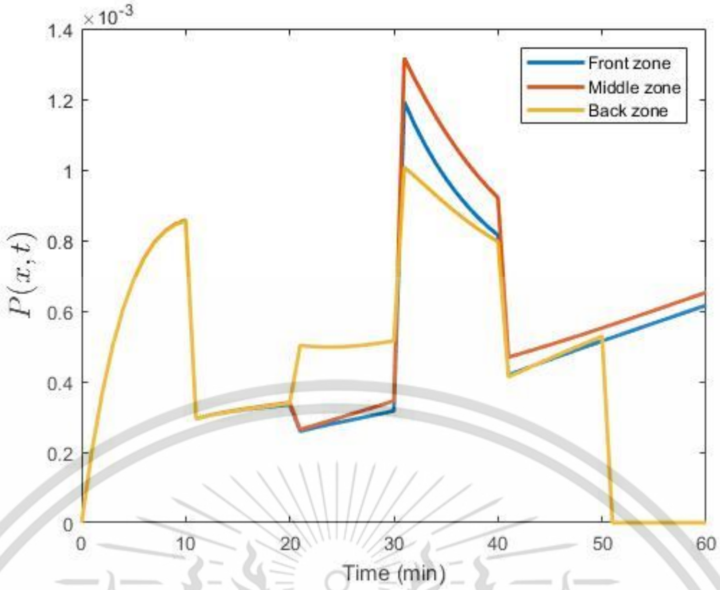


Figure 4.9 The probability of airborne infection risk for susceptible individuals in the front, middle and rear zones on a bus.

We can see that the passenger who take a seat on the middle zone have encounter higher risk than other zones.

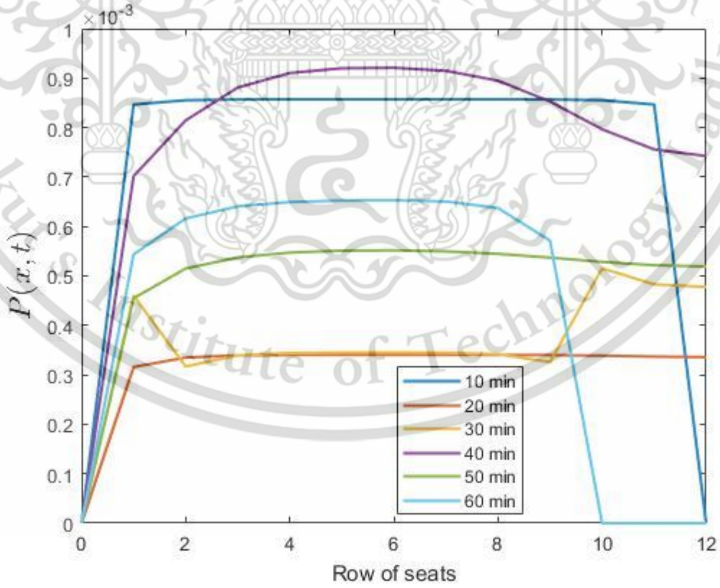


Figure 4.10 The probability of airborne infection risk for susceptible individuals every 10 minutes.

We can see that passengers seated at 10 minutes and 40 minutes are at high risk of airborne infection.

Table 4.4 The probability of airborne infection risk for susceptible individuals every 10 minutes.

	The probability of airborne infection risk for susceptible individuals ($\times 10^{-3}$)					
x	$t = 10$	$t = 20$	$t = 30$	$t = 40$	$t = 50$	$t = 60$
0	0	0	0	0	0	0
1	0.8466	0.3158	0.4602	0.7018	0.4550	0.5435
2	0.8552	0.3349	0.3169	0.8145	0.5149	0.6162
3	0.8569	0.3393	0.3392	0.8809	0.5375	0.6407
4	0.8571	0.3402	0.3441	0.9103	0.5472	0.6493
5	0.8571	0.3404	0.3450	0.9199	0.5511	0.6524
6	0.8571	0.3404	0.3451	0.9209	0.5517	0.6529
7	0.8571	0.3404	0.3447	0.9148	0.5496	0.6505
8	0.8571	0.3403	0.3417	0.8948	0.5447	0.6372
9	0.8569	0.3398	0.3257	0.8526	0.5370	0.5714
10	0.8552	0.3387	0.5153	0.7965	0.5283	0
11	0.8466	0.3369	0.4830	0.7559	0.5216	0
12	0	0.3357	0.4777	0.7425	0.5191	0

4.2 Simulation 2: the probability of airborne infection risk for susceptible individuals on the bus with reduced ventilation rates.

A number of passengers in each row as show in table 4.1. $C_0 = 0.1$ is the initial value of the concentration of carbon dioxide in the air on a bus (ppm) and the bus uses the air vent rate is 75, 37.5, 18.75, 0. We achieve the approximated solutions illustrated in Figure 4.12 by using the forward time centered space (FTCS) method (23) to (28) and (16) shown in Figure 4.13.

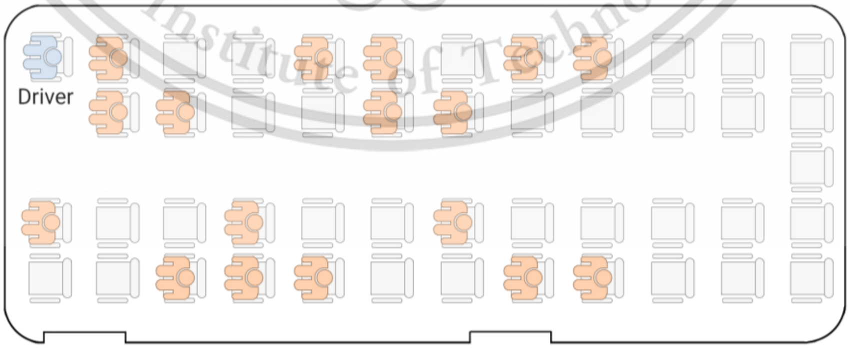


Figure 4.11 Seat map of the number of people seated on the bus for 50 to 60 minutes.

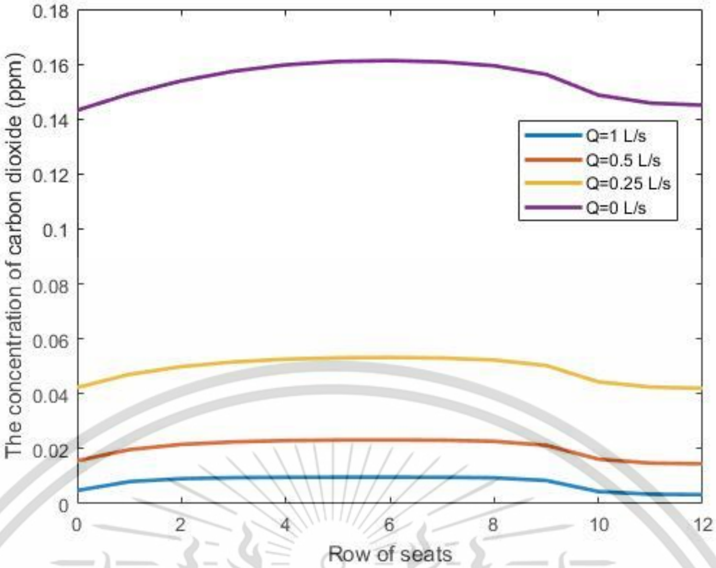


Figure 4.12 The approximated carbon dioxide concentration in the air on a bus at 60 min with a ventilation system $\ell = 0.1\ T = 60$.

We can see that the concentration of carbon dioxide in the air increases due to the reduced ventilation rate.

Table 4.5 The concentration of carbon dioxide in the air on a bus at 60 min.

	The concentration of carbon dioxide			
x	$Q=1$	$Q=0.5$	$Q=0.25$	$Q=0$
0	0.0048	0.0156	0.0423	0.1432
1	0.0080	0.0197	0.0471	0.1491
2	0.0091	0.0216	0.0499	0.1539
3	0.0095	0.0225	0.0516	0.1574
4	0.0096	0.0230	0.0526	0.1597
5	0.0096	0.0232	0.0531	0.1610
6	0.0096	0.0232	0.0532	0.1613
7	0.0096	0.0231	0.0530	0.1608
8	0.0094	0.0227	0.0523	0.1594
9	0.0084	0.0212	0.0503	0.1562
10	0.0044	0.0162	0.0443	0.1487
11	0.0034	0.0148	0.0424	0.1458
12	0.0033	0.0145	0.0420	0.1451

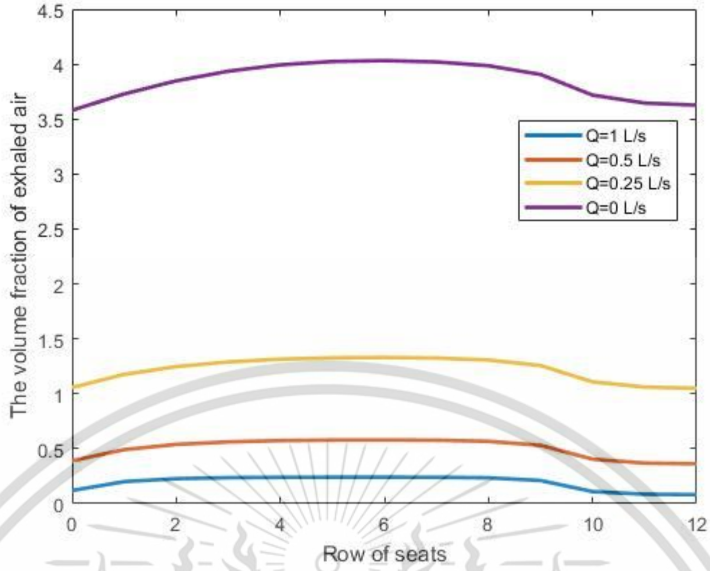


Figure 4.13 The volume fraction of exhaled air on a bus at 60 min.

We can see that at 60 minutes the fraction of exhaled air volume increases due to the decrease in ventilation rate.

A physical parameters as show in table 4.3. We achieve the approximated solutions illustrated in Figure 4.14 to 4.16 by using (17) shown in Figure 4.14, (18) shown in Figure 4.15 and (19) shown in Figure 4.16.

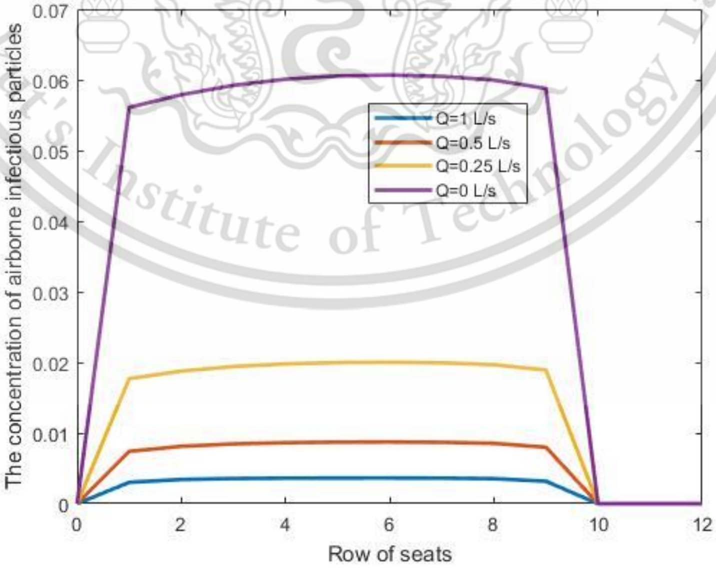


Figure 4.14 The concentration of airborne infectious particles on a bus at 60 min.

We can see that in 60 minutes the concentration of airborne infectious particles increases as the respiratory rate decreases.

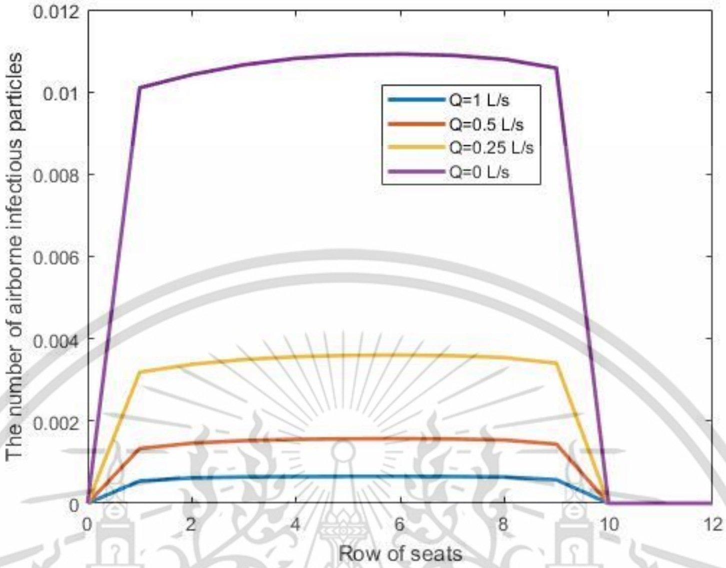


Figure 4.15 The number of airborne infectious particles on a bus at 60 min.

We can see that in 60 minutes the number of airborne infectious particles increases as the respiratory rate decreases.

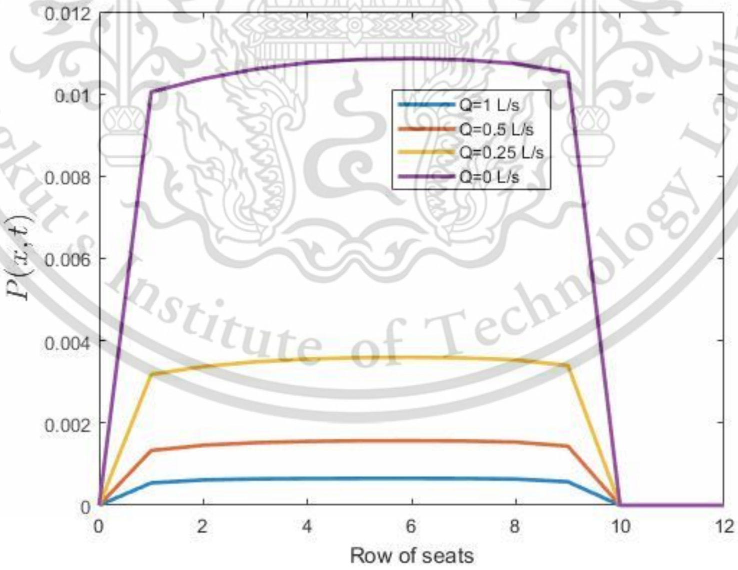


Figure 4.16 The probability of airborne infection risk for susceptible individuals on a bus at 60 min.

We can see that in 60 minutes at a ventilation rate of 1 (L/s), the passenger is at the lowest risk of airborne infection.

Table 4.6 The probability of airborne infection risk for susceptible individuals on a bus at 60 min.

The probability of airborne infection risk for susceptible individuals				
x	$Q=1$	$Q=0.5$	$Q=0.25$	$Q=0$
0	0	0	0	0
1	0.5435×10^{-3}	0.0013	0.0032	0.0100
2	0.6162×10^{-3}	0.0015	0.0034	0.0104
3	0.6407×10^{-3}	0.0015	0.0035	0.0106
4	0.6493×10^{-3}	0.0016	0.0036	0.0108
5	0.6524×10^{-3}	0.0016	0.0036	0.0108
6	0.6529×10^{-3}	0.0016	0.0036	0.0109
7	0.6505×10^{-3}	0.0016	0.0036	0.0108
8	0.6372×10^{-3}	0.0015	0.0035	0.0107
9	0.5714×10^{-3}	0.0014	0.0034	0.0105
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0

Chapter 5

Discussion and Conclusion

5.1 Discussion

In simulation 1, the number of passengers on the bus in each row is as shown in Table 4.1. When estimated using the forward time centered space (FTCS) method, the surface graph is obtained as shown in Figure 4.2. It can be seen that the concentration of carbon dioxide at the starting point decreases and increases gradually around about row 2 to row 10. Over a period of about 10 to 30 minutes, it gradually decreases again as the number of passengers has already disembarked the bus and fewer passengers sit in each row.

From Figure 4.3, we show the concentration of carbon dioxide in the front, middle and rear zones of the bus over a period of 60 minutes. It can be seen that the front zone and the middle zone have more passengers than the rear zone. Therefore, the concentration of carbon dioxide in the middle zone is higher than other zones. That is, each graph increases and decreases corresponding to the number of passengers. The concentration of carbon dioxide in the air on the bus every 10 minutes is shown in Figure 4.4. Since the ventilation rate is constant, it can be seen that during 0 to 20 minutes there are few bus passengers, thus reducing the concentration of carbon dioxide rapidly.

During 10 to 30 minutes, the number of bus passengers increased, thereby slowing the ventilation, resulting in a gradual decrease in the concentration of carbon dioxide. The approximate carbon dioxide concentration is shown in Table 4.2. From Figure 4.5, the volume fraction of exhaled air on a bus increases and decreases corresponding to the number of passengers. The physical parameters in Table 4.3 are estimated using (17) to (19) as shown in Figure 4.6 to 4.10, which we show that when the number of passengers seated in each row is low, the result is the concentration of airborne infectious particles, the number of airborne infectious particles and the probability of airborne infection risk for susceptible individuals increase.

From Figure 4.8, the front and the middle zones for 0 to 10 minutes and 30 to 40 minutes clearly show that the probability of airborne infection risk for susceptible individuals increases due to the small number of passengers seated in each row and the rear zone, the probability of airborne infection risk for susceptible individuals decreases due to the large number of passengers seated in each row and the probability of airborne infection risk for susceptible individuals is zero when no passengers are seated in each row.

From Figure 4.10, we show the probability of airborne infection risk for susceptible individuals every 10 minutes. At 10 minutes and 40 minutes, there are fewer passengers in each row, so there is a greater likelihood of infection. Where 20 minutes and 30 minutes are at low risk of infection due to the greater number of passengers in each row, the probability of airborne infection risk for susceptible individuals every 10 minutes is shown in Table 4.4.

In simulation 2, we estimated the concentration of carbon dioxide in the air on the bus with different ventilation rates by the number of passengers in each row as shown in Table 4.1. From Figure 4.12, we show the concentration of carbon dioxide at 60 min. It can be seen that as the ventilation rate decreases, the concentration of carbon dioxide increases. The approximate carbon dioxide concentration is shown in Table 4.5.

From Figure 4.13 to 4.16, we show the different ventilation rates on the bus at 60 minutes when the ventilation rate decreases, the volume fraction of exhaled air, the concentration of airborne infectious particles, the number of airborne infectious particles and the probability of airborne infection risk for susceptible individuals are increased. The probability of airborne infection risk for susceptible individuals is shown in Table 4.6.

5.2 Conclusion

A computational mathematical model is used to estimate the risk of airborne infection on a ventilated bus. As the number of passengers on the bus varies, we can see that the concentration of carbon dioxide, the volume fraction of exhaled air, the concentration of airborne infectious particles, the number of airborne infectious particles and the probability of airborne infection risk for susceptible individuals depend on the number of passengers in each cycle and the ventilation rate. We show that the proposed technique is applicable to real-world problems by using explicit finite difference techniques to approximate the solution of the model. It was found in good agreement that the proposed air quality model allows us to know the balance between the number of passengers allowed to sit on the bus while controlling the risk of airborne infection as well as the concentration of carbon dioxide and the potential of the ventilation system.

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Appendix

Appendix A

The research paper

Proceedings of the
26th Annual Meeting in Mathematics (AMM 2022)
School of Mathematics, Institute of Science
Suranaree University of Technology, Thailand

A Mathematical Model for Measuring Carbon Dioxide Concentration in a Bus Due to Passengers Breathing*

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Abstract

Human breath emits a significant concentration of carbon dioxide, which makes a significant contribution to airborne infections. Airborne diseases can spread easily, and breathing can expose us to possibly fatal airborne infections. There is a risk of infection for individuals going by bus. A mathematical model of carbon dioxide concentration measurement due to human breath is introduced in this study. The purpose of this research is to determine the concentration of carbon dioxide emitted by bus passengers. An explicit finite difference method is applied to approximate the model's solution. The model solution can be used to determine how much time passengers are willing to spend sitting on the bus while the carbon dioxide level is managed.

Keywords: Bus, Carbon dioxide, Explicit finite difference technique, Mathematical model, Passengers breathing.

2020 MSC: Primary 65M06.

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1 Introduction

In [1], they propose that using a modified Well-Riley model for airborne disease transmission, they estimated the risk of tuberculosis transmission on 3 modes of public transit risk, using exhaled carbon dioxide as a natural tracer gas to evaluate air exchange. Given its poor ventilation and high respiratory contact rates, public transportation may play a critical role in sustaining tuberculosis transmission. In [3], they measured carbon dioxide in classrooms under non-steady state conditions. Tuberculosis transmission criteria were assessed using a carbon dioxide-based risk equation. In [8], they considered, according to personal classifications. There are four types: patients, patient relatives, workers and outsiders, staying in an outpatient room. Air quality control manipulations are simulated using the inlet and outlet ventilation rates adjustment under the condition of a number of surrounding people. The fourth-order Runge-Kutta method was used to approximate the model solution. The proposed numerical model can be used to describe the dynamic dispersion of airborne infectious disease in an outpatient room. The results of the model will be able to control airborne disease in more complicated structures.

In [9], they developed a portable monitor to continuously sample carbon dioxide levels, which were combined with social contact diary records to estimate daily rebreathed liters. They then estimated the daily volumes of air rebreathed by adolescents living in a crowded township. They demonstrate the practical measurement of carbon dioxide levels to which individuals are exposed in a sequence of non-steady indoor environments. In [10], they develop and demonstrate a flexible mathematical model that predicts the risk of airborne infectious diseases, such as tuberculosis steady state and non-steady state conditions by monitoring exhaled air by infectors in a confined space. They demonstrated a mathematically and schematically the correlation between TB transmission probability and airborne infection particle generation rate, ventilation rate, average volume fraction of exhaled air, TB prevalence and duration of exposure to infectors in a confined space.

In [11], they proposed a model sets the concentration of carbon dioxide at any point when the number of people and the rate of ventilation varies. The classical fourth-order Runge-Kutta method is employed to approximate the model solution. There are many cases of scenarios for improving air quality in the proposed simulations. In the air quality management process, the proposed model provides a balance between the number of persons allowed to stay in the room and the capacity of the air ventilation system.

Therefore, if there is an epidemic of infectious diseases in the air or in a place where there are infected people. We therefore studied a mathematical model of measuring the concentration of carbon dioxide from human breath. We will know how much passengers are allowed to sit on the bus while controlling the concentration of carbon dioxide.

2 Governing Equation

In general, the human exhalation rate and ventilation are responsible for the increased concentrations of carbon dioxide [4], [6] and [7] because the exhaled air of an infected person contains infectious particles in the air. Carbon dioxide concentrations can therefore be used as exhalation agents [3], [4], [5], [7] and [9]. The ambient carbon dioxide levels tend to average about 400 ppm [2], [3] and [4].

We assume that a bus interior space, such as a bus with a volume of $V\,(m^3)$. Given the presence of infectors, the concentration of exhaled air that may contain airborne contagious particles may tend to rise in the bus, based on the rate of ventilation Q (L/s), and the number of people in the bus.

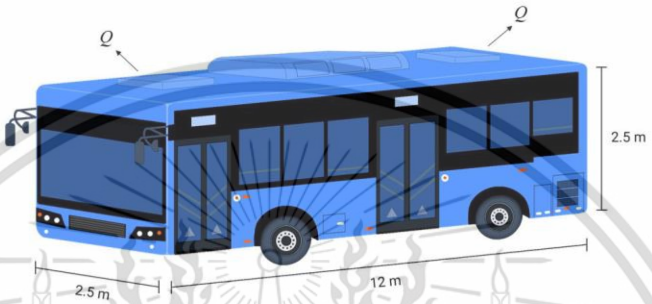

Figure 1: A bus model.

We simply assume that persons in the bus contribute substantially to the production of carbon dioxide, which serves as an exhaled air marker. The fundamental equation of the accumulation rate exhaled air concentration in a bus with carbon dioxide is equal to the exhaled air rate generated by inhabitants plus the diffusion rate of carbon dioxide, minus ventilation rate removes exhaled air:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} + npC_a - QC \qquad (1)$$

where C is the concentration of air inside the exhaled bus (ppm), p is the rate of breathing (L/s) for each person in the bus and C_a is the carbon dioxide fraction included in breathed air. t is the duration time and T is the stationery simulation time. Initial condition $C(x,0) = C_0$ where C_0 is the latent carbon dioxide concentration.

In a simple scenario, a number of people are unstable then a number of people depend on the time assumed by $n(x,t)$, then the diffusion coefficient of carbon dioxide D (m^2/s). In this study preferred to use (1) as follow:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} + n(x,t)pC_a - QC \qquad (2)$$

for all $0 \le t \le T$.

3 Numerical Techniques

A continuous approximation to the solution $C(x,t)$ will not be obtained; instead, approximations to C will be generated at various values, called mesh point, in the interval $[0,T]$.

Once the approximate solution at other points in the interval can be found by interpolation. We first make the stipulation that the mesh points are equally distributed throughout the interval $[0,T]$. This condition is ensured by choosing a positive integer M and selecting the mesh point $t_n = n\ell$, for each $n = 0,1,2,\ldots,M$ where $\ell = (T-0)/M$ is called the time step. This condition is ensured by choosing a positive integer N and $x_m = mh$, for each $m = 0,1,2,\ldots,N$. The common distance between the points $h = (L-0)/N$ is called the step size.

3.1 Initial Condition

$$C(x,0) = f(x) \tag{3}$$

for all $0 \le x \le L$.

3.2 Boundary Conditions

The left boundary condition (LBC):

$$C(0,t) = \frac{\partial C}{\partial x} = 0 \tag{4}$$

The right boundary condition (RBC):

$$C(L,t) = \frac{\partial C}{\partial x} = 0 \tag{5}$$

where t is greater than zero.

3.3 The Forward Time Centered Space (FTCS) Method

Forward time:

$$\frac{\partial C}{\partial t} = \frac{C(x,t+\ell) - C(x,t)}{\ell} + O(\ell) \tag{6}$$

Centered space:

$$\frac{\partial^2 C}{\partial x^2} = \frac{C(x-h,t) - 2C(x,t) + C(x+h,t)}{h^2} + O(h^2) \tag{7}$$

Letting that

$$C \approx C_m^n \tag{8}$$

$$C_m^n = C(x_m, t_n) \tag{9}$$

$$C_{m+1}^n = C(x_m + h, t_n) \tag{10}$$

$$C_m^{n+1} = C(x_m, t_n + \ell) \tag{11}$$

$$C_{m-1}^n = C(x_m - h, t_n) \tag{12}$$

from (2), we get the FTCS method

$$C_m^{n+1} = \lambda C_{m-1}^n - (2\lambda + Q\ell - 1)C_m^n + \lambda C_{m+1}^n + npC_a\ell \tag{13}$$

4 Numerical Experiments and Results

Assuming that the bus of volume $V = 75$ (m^3), each person's breathing rate in the bus assumed by $p = 0.12$ (L/s), then the diffusion coefficient of carbon dioxide $D = 0.732$ (m^2/s) and the carbon dioxide fraction included in breathed air $C_a = 0.04$.

4.1 Simulation 1: Concentration of Carbon Dioxide in The Air on a Bus

Table I lists the number of passengers in each row. $C_0 = 0.1$ is the initial value of the concentration of carbon dioxide in the air on a bus (ppm) and the bus uses the air vent rate is 75. We achieve the approximated solutions illustrated in Figure 3 to 5 by using the forward time centered space (FTCS) method (8) to (13).

Table I: The number of passengers on a bus in each row.

Time (min)	**The number of passengers in each row**												
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0-10	0	1	1	1	1	1	1	1	1	1	1	1	0
10-20	0	3	3	3	3	3	3	3	3	3	3	3	3
20-30	0	2	4	4	4	4	4	4	4	4	2	2	2
30-40	0	1	1	1	1	1	1	1	1	1	1	1	1
40-50	0	2	2	2	2	2	2	2	2	2	2	2	2
50-60	0	2	2	2	2	2	2	2	2	2	0	0	0

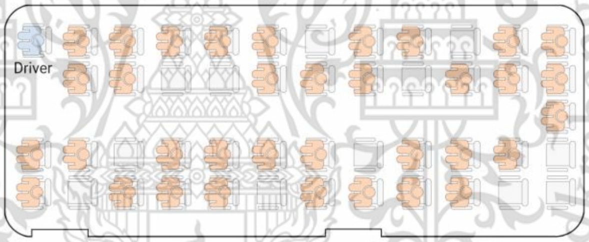

Figure 2: Seat map of the number of people seated on the bus for 10 to 20 minutes.

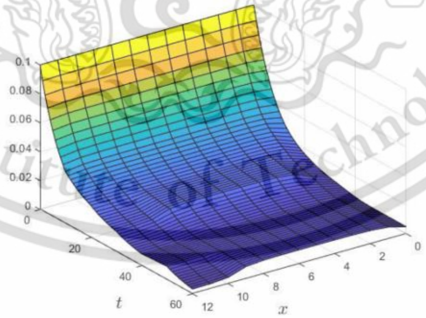

Figure 3: The approximated carbon dioxide concentration in the air on a bus with a ventilation system $\ell = 0.1$, $T = 60$.

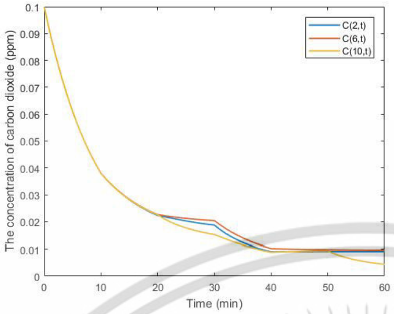

Figure 4: The concentration of carbon dioxide in the air in the front, middle and rear zones on a bus with a ventilation system $\ell = 0.1,\ T = 60$.

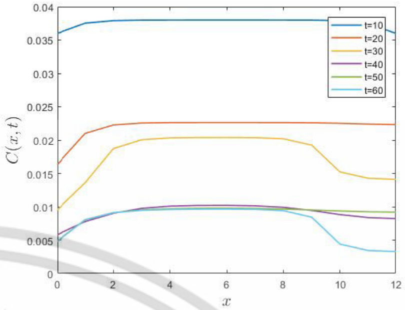

Figure 5: The concentration of carbon dioxide in the air on a bus every 10 minutes with a ventilation system $\ell = 0.1,\ T = 60$.

Table II: The concentration of carbon dioxide in the air on a bus every 10 minutes.

	The concentration of carbon dioxide					
x	$t = 10$	$t = 20$	$t = 30$	$t = 40$	$t = 50$	$t = 60$
0	0.0360	0.0163	0.0095	0.0058	0.0048	0.0048
1	0.0375	0.0210	0.0136	0.0078	0.0081	0.0080
2	0.0379	0.0223	0.0187	0.0090	0.0091	0.0091
3	0.0380	0.0226	0.0200	0.0098	0.0095	0.0095
4	0.0380	0.0226	0.0203	0.0101	0.0097	0.0096
5	0.0380	0.0226	0.0204	0.0102	0.0098	0.0096
6	0.0380	0.0226	0.0204	0.0102	0.0098	0.0096
7	0.0380	0.0226	0.0204	0.0101	0.0097	0.0096
8	0.0380	0.0226	0.0202	0.0099	0.0097	0.0094
9	0.0380	0.0226	0.0192	0.0094	0.0095	0.0084
10	0.0379	0.0225	0.0152	0.0088	0.0094	0.0044
11	0.0375	0.0224	0.0143	0.0084	0.0092	0.0034
12	0.0360	0.0223	0.0141	0.0082	0.0092	0.0033

4.2 Simulation 2: Concentration of Carbon Dioxide in The Air on a Bus with Reduced Ventilation Rates.

A number of passengers in each row as show in table I. $C_0 = 0.1$ is the initial value of the concentration of carbon dioxide in the air on a bus (ppm) and the bus uses the air vent rate is $75,\ 37.5,\ 18.75,\ 0$. We achieve the approximated solutions illustrated in Figure 7 by using the forward time centered space (FTCS) method (8) to (13).

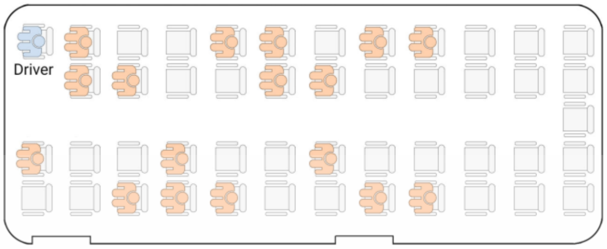

Figure 6: Seat map of the number of people seated on the bus for 50 to 60 minutes.

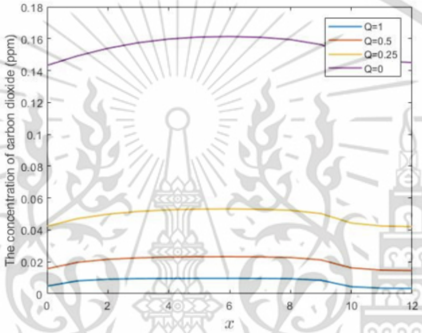

Figure 7: The approximated carbon dioxide concentration in the air on a bus at 60 min with a ventilation system $\ell = 0.1$, $T = 60$.

Table III: The concentration of carbon dioxide in the air on a bus at 60 min.

The concentration of carbon dioxide				
x	$Q = 1$	$Q = 0.5$	$Q = 0.25$	$Q = 0$
0	0.0048	0.0156	0.0423	0.1432
1	0.0080	0.0197	0.0471	0.1491
2	0.0091	0.0216	0.0499	0.1539
3	0.0095	0.0225	0.0516	0.1574
4	0.0096	0.0230	0.0526	0.1597
5	0.0096	0.0232	0.0531	0.1610
6	0.0096	0.0232	0.0532	0.1613
7	0.0096	0.0231	0.0530	0.1608
8	0.0094	0.0227	0.0523	0.1594
9	0.0084	0.0212	0.0503	0.1562
10	0.0044	0.0162	0.0443	0.1487
11	0.0034	0.0148	0.0424	0.1458
12	0.0033	0.0145	0.0420	0.1451

5 Discussion

In simulation 1, the number of passengers on the bus in each row is as shown in Table I. When estimated using the forward time centered space (FTCS) method, the surface graph is obtained as shown in Figure 3. It can be seen that the concentration of carbon dioxide at the starting point decreases and increases gradually around about row 2 to row 10. Over a period of about 10 to 30 minutes, it gradually decreases again as the number of passengers has already disembarked the bus and fewer passengers sit in each row. From Figure 4, we show the concentration of carbon dioxide in the front, middle and rear zones of the bus over a period of 60 minutes. It can be seen that the front zone and the middle zone have more passengers than the rear zone. Therefore, the concentration of carbon dioxide in the front zone and the middle zone is higher than the rear zone. That is, each graph increases and decreases corresponding to the number of passengers. The concentration of carbon dioxide in the air on the bus every 10 minutes is shown in Figure 5. Since the ventilation rate is constant, it can be seen that during 0 to 20 minutes there are few bus passengers, thus reducing the concentration of carbon dioxide rapidly. During 10 to 30 minutes, the number of bus passengers increased, thereby slowing the ventilation, resulting in a gradual decrease in the concentration of carbon dioxide. The approximate carbon dioxide concentration is shown in Table II.

In simulation 2, we estimated the concentration of carbon dioxide in the air on the bus with different ventilation rates by the number of passengers in each row as shown in Table I. From Figure 7, we show the concentration of carbon dioxide at 60 min. It can be seen that as the ventilation rate decreases, the concentration of carbon dioxide increases. The approximate carbon dioxide concentration is shown in Table III.

6 Conclusion

A computational mathematical model is used to estimate the concentration of carbon dioxide on a ventilated bus. As the number of passengers on the bus varies, we can see that the concentration of carbon dioxide depends on the number of passengers in each cycle and the ventilation rate. We show that the proposed technique is applicable to real-world problems by using explicit finite difference techniques to approximate the solution of the model. It was found in good agreement that the proposed air quality model allows us to know the balance between the number of passengers allowed to sit on the bus while controlling the concentration of carbon dioxide and the potential of the ventilation system.

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