A MATHEMATICAL MODEL OF THE RISK OF AIRBORNE TRANSMISSION ASSESSMENT IN A CLASSROOM TAKING INTO ACCOUNT THE VENTILATION SYSTEM AND FACE MASK EFFICIENCY

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Thesis Title	A Mathematical Model of the Risk of Airborne Transmission Assessment in a Classroom Taking into Account the Ventilation System and Face Mask Efficiency
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Abstract

TB, COVID-19, MERS, and SARS are all serious infectious diseases that are transmitted by the air or aerosol via coughing, spitting, sneezing, speaking, or wounds. The more and longer a person interacts with others, the greater the risk of COVID-19 spreading. As a result, we must be aware of this treatment and management. As a result, effective air quality management is essential for controlling and reducing potentially polluted air, such as CO_2 levels. They investigated the protective effectiveness of face masks against airborne transmission of infectious SARS-CoV-2 droplets and aerosols in response to the World Health Organization's recommendation to wear face masks to prevent the spread of COVID-19. Using nine different forms of mask efficiency, this research provides a mathematical model for calculating the chance of airborne transmission in a classroom. The fourth order Runge-Kutta approach is used to approximate the model solution. The proposed strategy strikes a balance between the number of students allowed to stay in the classroom and the effectiveness of nine different masks. We can see how utilizing nine different masks and a well-ventilated system in the classroom can help to reduce the risk of airborne infection.

Keywords: Airborne transmission, Classroom, Mathematical model, Risk, Ventilation system

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Table of Contents

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Table of Contents

King Mongkut's Institute of Technology Ladkrabang

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

List of Tables

Table **Page**

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List of Figures

Figure **Page**

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Abbreviations / Symbols

Symbols	Description
C	The concentration of carbon dioxide in the air on a bus (ppm)
D	A diffusion coefficient of carbon dioxide (m^2/s)
n	The number of people
p	The breathing rate (L/s)
t	The breathing rate (min)
C_a	The carbon dioxide fraction contained in breathed air (ppm)
Q	The ventilation rate (L/s)
V	The volume of air in the class room (m^3)
I	The number of infectors in the class room
β	The total airborne infectious particle generation rate released by an infector (particles/s)
μ	The mortality rate of generated airborne infectious particles by the infector that do not reach the alveolar (particles/s)
pt	The volume of breathed air by susceptibles
θ	A respiratory deposition fraction of airborne infectious particles
$f(x,t)$	The volume fraction of exhaled air
$N(x,t)$	The concentration of airborne infectious particles
$\lambda(x,t)$	The number of airborne infectious particles
$P(x,t)$	The probability of airborne infectious diseases risk for susceptible individuals

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Chapter 1

Introduction

1.1 Inception and Significance of Research

Every day, the number of airborne infections is increasing day by day. Tuberculosis (TB), the coronavirus disease that began in 2019 (COVID-19), Middle East Respiratory Syndrome (MERS), and severe acute respiratory syndrome (SARS) are dangerous contagious diseases. It is spread from person to person through air or droplets in various forms such as coughing, saliva, sneezing, talking, or through wounds. On average, people infected with the virus show symptoms in 5–6 days. In [1], new research by the US NIH found evidence of the COVID-19 virus. Spread through the air (Airborne) through small aerosols (Aerosol) is the main channel. By the virus that causes COVID-19 can be transmitted through the air through large aerosols (Droplet) and small aerosols (Aerosol), but the spread of air in the international way. Refers to only small aerosols. Which is less than 5 micrometers in size, small enough to travel through the air over long distances and stay in the air for a long time. Therefore, coviral disease is a major public health problem in Thailand. In [2], a new procedure was developed to study the distribution of epidemics for predicting the possibility of airborne infectious diseases in high-density urban areas. It can analyze the chance of spread in sub-transportation, and it can also help understand dispersion of airborne diseases in public transportation in China. In [3], from a study in US hospitals COVID-19 spread through the air from a study in a hospital in Boston. Between September 2020 and April 2021, the hospital had 803 beds, 28% of which were double rooms. A total of 11,290 patients slept in double rooms. Of those, 25 people who slept in twin rooms had no symptoms of coronavirus. The first day gave a negative result. Repeat the test in 3 days. Positive result means that infected with covid-19 virus out of hospitals, there were 31 patients sleeping in the same room with the infected, the mean age was 64 years. It turned out that 12 out of 31 people were infected from patients sleeping in the same room, representing 39 percent. Most were infected within 5 days, so stay at home or not? Go to various places should wear masks.

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In [4], the researchers developed an infectious diseases model of SARS by using two methods for estimating both small-scale SARS outbreak parameter at the Amoy Gardens, Hong Kong and large-scale outbreak parameter in the entire Hong Kong Special Administrative Region Super-spreading events for infectious diseases occur when some infected individuals infect more than the average number of secondary cases. Several super-spreading individuals have been identified for the 2003 outbreak of severe acute respiratory syndrome (SARS). In [5], airflow and the airborne spread of infectious agents from an indoor environment was focused on. From this, it was confirmed that infected individuals and susceptible individuals should use masks, and also should use personalized ventilation for a short-range airborne route.

1.2 Research Motivation

Because airborne infectious disease is a formidable disease. Difficult to control and spread the risk of transmission - simply touching or being in a vulnerable area can increase your risk of airborne infection. Likewise, the number of airborne infections in the population is increasing. As a result, population control is difficult and vulnerable to disease transmission. Therefore, a mathematical aggregate and study of Runge-kutta method was performed to reduce the success and transmission of airborne pathogens and an academic study affecting airborne pathogens and transmission. Air to control and reduce infection.

1.3 Literature Review

In [6], this paper discusses a comprehensive review of various workable technologies and methods to combat airborne viruses, e.g., COVID-19, in ventilation systems and enclosed spaces. These technologies and methods include an increase in ventilation, high-efficiency air filtration, ionization of the air, environmental condition control, ultraviolet germicidal irradiation, nonthermal plasma and reactive oxygen species, filter coatings, chemical disinfectants, and heat inactivation. Research gaps have been identified and discussed, and

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recommendations for applying such technologies and methods have also been provided in this article.

In [7], With great interest they read the recently published letter by Julian and colleagues highlighting the possibility of influenza airborne transmission as a result of using nebulizers. In the context of COVID-19, the use of nebulizers and other breathing aid devices is routine in the medical settings. However, high amounts of aerosols generated by these devices can expose healthcare workers and other patients to infectious particles thus increasing their risk of morbidity and mortality. Air samples have been shown to contain viable SARS-CoV-2 collected up to four meters away from COVID-19 patients in hospital rooms and isolation care units, and it was found that the viral load was significantly higher in patients fitted with a nasal cannula. The risk of airborne transmission of infectious agents can be mitigated by ventilation strategies.4Since the majority of SARS-CoV-2 transmission occurs indoors, efficient ventilation systems that can dilute the viral concentration in the air is highly desirable.

In [8], SARS-CoV-2 can spread by close contact through large droplet spray and indirect contact via contaminated objects. There is mounting evidence that it can also be transmitted by inhalation of infected saliva aerosol particles. These particles are generated when breathing, talking, laughing, coughing or sneezing. It can be assumed that aerosol particle concentrations should be kept low in order to minimize the potential risk of airborne virus transmission. This paper presents measurements of aerosol particle concentrations in a gym, where saliva aerosol production is pronounced. 35 test persons performed physical exercise and aerosol particle concentrations, CO_2 concentrations, air temperature and relative humidity were obtained in the room of 886 m^3. A separate test was used to discriminate between human endogenous and exogenous aerosol particles. Aerosol particle removal by mechanical ventilation and mobile air cleaning units was measured. The gym test showed that ventilation with air-change rate ACH = 2.2 h^{-1}, i.e. 4.5 times the minimum of the Dutch Building Code, was insufficient to stop the significant aerosol concentration rise over 30 min. Air cleaning alone with ACH = 1.39 h^{-1} had a similar effect as ventilation alone. Simplified mathematical models were engaged to provide further insight into ventilation, air cleaning and deposition. It was shown that combining the above-mentioned ventilation and air cleaning can reduce aerosol particle concentrations with 80 to 90%, depending on aerosol size. This combination of existing

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ventilation supplemented with air cleaning is energy efficient and can also be applied for other indoor environments.

In [9], The success of preventive measures of COVID-19 depends on a correct understanding of the transmission routes of the virus. Researchers had identified SARS-CoV-2 in the air sampling of the COVID-19 isolation ward and found that viruses in the aerosol can survive and remain infectious for quite a while. The culprit of multiple infection events in public transportation, apartments, shopping malls, restaurants, choirs, and other places appeared to be airborne transmission. As research showed, the suspension time of aerosol increased tenfold in a poorly ventilated space. With more and more academic evidence, World Health Organization (WHO) and the Centers for Disease Control and Prevention (US-CDC) have changed their positions recently, confirming that airborne transmission is an essential route of COVID-19 transmission. To reduce the air concentration of the virus and the viral exposure dose of people indoors, researchers also started to emphasize improving the ventilation and exhaust system as a core preventive suggestion, rather than maintaining safety social distance.

In [10], The science around the use of masks by the public to impede COVID-19 transmission is advancing rapidly. In this narrative review, they develop an analytical framework to examine mask usage, synthesizing the relevant literature to inform multiple areas: population impact, transmission characteristics, source control, wearer protection, sociological considerations, and implementation considerations. A primary route of transmission of COVID-19 is via respiratory particles, and it is known to be transmissible from presymptomatic, paucisymptomatic, and asymptomatic individuals. Reducing disease spread requires two things: limiting contacts of infected individuals via physical distancing and other measures and reducing the transmission probability per contact. The preponderance of evidence indicates that mask wearing reduces transmissibility per contact by reducing transmission of infected respiratory particles in both laboratory and clinical contexts. Public mask wearing is most effective at reducing spread of the virus when compliance is high.

In [11], Airborne transmission by droplets and aerosols is important for the spread of viruses. Face masks are a well-established preventive measure, but their effectiveness for mitigating severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) transmission is still under debate. They show that variations in mask efficacy can be explained by different regimes

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of virus abundance and are related to population average infection probability and reproduction number. For SARS-CoV-2, the viral load of infectious individuals can vary by orders of magnitude. They find that most environments and contacts are under conditions of low virus abundance (virus-limited), where surgical masks are effective at preventing virus spread. More-advanced masks and other protective equipment are required in potentially virus-rich indoor environments, including medical centers and hospitals. Masks are particularly effective in combination with other preventive measures like ventilation and distancing.

In [12], Prior to the coronavirus disease 2019 (COVID-19) pandemic, the efficacy of community mask wearing to reduce the spread of respiratory infections was controversial because there were no solid relevant data to support their use. During the pandemic, the scientific evidence has increased. Compelling data now demonstrate that community mask wearing is an effective nonpharmacologic intervention to reduce the spread of this infection, especially as source control to prevent spread from infected persons, but also as protection to reduce wearers' exposure to infection.

In [13], Face mask use by the general public for limiting the spread of the COVID-19 pandemic is controversial, though increasingly recommended, and the potential of this intervention is not well understood. They develop a compartmental model for assessing the communitywide impact of mask use by the general, asymptomatic public, a portion of which may be asymptomatically infectious. Model simulations, using data relevant to COVID-19 dynamics in the US states of New York and Washington, suggest that broad adoption of even relatively ineffective face masks may meaningfully reduce community transmission of COVID-19 and decrease peak hospitalizations and deaths. Moreover, mask use decreases the effective transmission rate in nearly linear proportion to the product of mask effectiveness , while the impact on epidemiologic outcomes (death, hospitalizations) is highly nonlinear, indicating masks could synergize with other nonpharmaceutical measures. Notably, masks are found to be useful with respect to both preventing illness in healthy persons and preventing asymptomatic transmission. Hypothetical mask adoption scenarios, for Washington and New York state, suggest that immediate near universal (80%) adoption of moderately (50%) effective masks could prevent on the order of 45% of projected deaths over two months in New York, while decreasing the peak daily death rate by 58%, absent other changes in

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epidemic dynamics. Even very weak masks (20% effective) can still be useful if the underlying transmission rate is relatively low or decreasing: In Washington, where baseline transmission is much less intense, 80% adoption of such masks could reduce mortality by 65% (and peak deaths 69%), compared to 9% mortality reduction in New York (peak death reduction 18%). Our results suggest use of face masks by the general public is potentially of high value in curtailing community transmission and the burden of the pandemic. The community-wide benefits are likely to be greatest when face masks are used in conjunction with other non-pharmaceutical practices.

In [14], the objectives of this article are to: (a) provide a comprehensive review of the airborne transmission characteristics of SARS-CoV-2 in enclosed spaces and a theoretical basis for HVAC operation guideline revision; (b) investigate HVAC-related guidelines to clarify the operational variations of HVAC systems during the pandemic; (c) analyses how operational variations of HVAC systems affect energy consumption; and (d) identify the innovations and research trends concerning future HVAC systems. Furthermore, this paper compares the energy consumption of HVAC system operation during the normal times versus pandemic period, based on a case study in China, providing a reference for other countries around the world. Results of this paper offer comprehensive insights into how to keep indoor environments safe while maintaining energy-efficient operation of HVAC systems.

1.4 Research Objectives

1. Capable of population control and estimating appropriate ventilation.

2. Study the mathematical model and determine the relevant parameters.

3. Determine the possibility that the virus will spread through the air.

4. Find the probability of preventing the spread of infection.

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1.5 Research Methodology

Step 1: Study fundamentals of mathematical model.

Step 2: Research the numerical technique, the numerical Runge-Kutta method, the variables operator, and other topics covered in related articles.

Step 3: Analyze mathematical models that have been tested.

Step 4: Test the values of the variables and use the values of the variables in the analysis.

Step 5: Optimize the value of the variable and use the variable's value to analyze the data.

Step 6: Analyze and verify the results of the applied variables.

Step 7: The data from the masks were analyzed, and the relevant values were used in the model to determine the risk and probabilities.

Step 8: Discuss and reach a conclusion about the outcome.

Step 9: Compose the thesis.

1.6 Scope of the Study

1. The carbon dioxide fraction in breathed air is stated.

2. Room air volume is given.

3. The breathing rate for each person in the room is given.

4. The ventilation rate is given.

5. The time spent in the room is given.

6. Types of favorite masks used in daily life are given in nine types.

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1.7 Benefits of the Study

1. Capable of analyzing risks and probabilities in order to reduce infection.

2. Plan or manage population numbers in order to reduce infection risks.

3. Plan the purchase of ventilation fans to suit your needs.

4. Capable of reducing or increasing the distribution of mask quantity based on type.

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Chapter 2

Basic knowledge

2.1 Initial Value Problem

The object of approximation method is to obtain approximations to the well-posed initial-value problem,

$$\frac{dy}{dt} = f(t, y),\ a \le t \le b,\ y(a) = \alpha. \tag{2.1}$$

A continuous approximation to the solution $y(t)$ will not be obtained instead, approximations to y will be generated at various values, called mesh points, in the interval $[a,\ b]$. Once the approximate solution is obtained at the points, the approximate solution at other points in the interval can be found by interpolation.

We first make the stipulation that the mesh points are equally distributed throughout the interval $[a,\ b]$. This condition is ensured by choosing a positive integer N and selecting the mesh points. For each $i = 0,\ 1,\ 2, \ldots,\ N,$

$$t_i = a + ih. \tag{2.2}$$

where the common distance between the points $h = \frac{b-a}{N}$ is called the step size,

$$a = t_0 \text{ and } b = t_N. \tag{2.3}$$

So that $[a,b] = [a,t_1] \cup [t_1,t_2] \cup [t_2,t_3] \cup \ldots \cup [t_{n-1},b]$, it is illustrated as listed below,

i	0	1	2	3	...	N
t_i	t_0	t_1	t_2	t_3	...	t_N
y_i	y_0	y_1	y_2	y_3	...	y_N

Note that $$y_i = y(t_i). \tag{2.4}$$

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2.2 A Fourth-Order Runge-Kutta Method

Runge-kutta methods have the high-order lacal truncation error of the Taylor methods but eliminate the need to compute and evaluate the derivatives of $f(t, y)$.

2.2.1 Runge-Kutta Order Four

Runge-Kutta methods of order three are not generally used. The most common Runge-Kutta method in use is of order four in difference-equation form, is given by the following.

$$y_{i+1} = y_i + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4], \tag{2.5}$$

where

$$k_1 = f(t_i, y_i), \tag{2.6}$$

$$k_2 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right), \tag{2.7}$$

$$k_3 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right), \tag{2.8}$$

$$k_4 = f\left(t_i + h, y_i + k_3 h\right), \tag{2.9}$$

for each $i = 0, 1, \ldots, N-1$. This method has local truncation error $O\left(h^4\right)$, provided the solution $y(t)$ has five continuous derivatives. We introduce the notation k_1, k_2, k_3, k_4 into the method is to eliminate the need for successive nesting in the second variable of $f(t, y)$.

2.2.2 Computational Comparisons

The main computational effort in applying the Runge-Kutta methods is the evaluation of f. In the second-order methods, the local truncation error is $O\left(h^2\right)$, and the cost is two function evaluations per step. The Runge-Kutta method of order four requires 4 evaluations per step, and the local truncation error is $O\left(h^4\right)$. In [15], they have established the relationship between the number of evaluations per step and the order of the local truncation error shown in Table 2.1. This table indicates why the methods of order less than five with smaller step size are used in preference to the higher-order methods using a larger step size.

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Table 2.1: the relationship between the number of evaluations per step and the order of the local truncation error

Evaluations per step	Best possible local truncation error
2	$O\left(h^2\right)$
3	$O\left(h^3\right)$
4	$O\left(h^4\right)$
$5 \le n \le 7$	$O\left(h^{n-1}\right)$
$8 \le n \le 9$	$O\left(h^{n-2}\right)$
$10 \le n$	$O\left(h^{n-3}\right)$

One measure of comparing the lower-order Runge-Kutta methods is described as follows: The Runge-Kutta method of order four requires four evaluations per step, whereas Euler's method requires only one evaluation. Hence if the Runge-Kutta method of order four is to be superior it should give more accurate answers than Euler's method with one-fourth the step size. Similarly, if the Runge-Kutta method of order four is to be superior to the second-order Runge-Kutta methods, which require two evaluations per step, it should give more accuracy with step size h than a second-order method with step size $h/2$.

2.3 Mask Filtration Studies

Public health experts recommend wearing face masks as tools to protect others from breathing potentially infectious particles. At the request of University of North Carolina (UNC) Hospitals, EPA scientists are working to understand the effectiveness of masks to protect the wearer against the virus through a series of projects in collaboration with UNC researchers [13]. Researchers tested how well different masks and modifications filter out airborne salt particles, which are the same size as the smallest SARS-CoV-2 particles, but are not harmful. Members of the research team wore the face coverings to do the testing themselves [16].

In one study, the researchers sought to determine whether alternatives to high-efficiency N95 masks reserved for health care workers could offer similar protection for hospital personnel in the event of shortages. They tested the filtration ability of expired N95 masks,

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N95 masks that had been sterilized for reuse, and dozens of other face mask alternatives. In another study, the researchers examined the filtration ability of a variety of medical procedure masks, cloth masks and coverings recommended for the public. They tested masks made from cotton, nylon, and other materials and in different styles, including masks with ear loops and ties. The effectiveness of face mask disinfection for 9 consumer-grade masks is shown in Table 2.2.

Table 2.2: Effectiveness of face mask disinfection [16]

Masks	Consumer-Grade Masks	Fitted Filtration Efficiency (FFE)
M1	2-layer woven nylon mask	0.4400
M2	cotton bandana	0.5010
M3	single layer woven polyester gaiter	0.6220
M4	single layer woven polyester/nylon mask with ties	0.6070
M5	non-woven polypropylene mask with fixed ear loops	0.7140
M6	3-layer knitted cotton mask with ear loops	0.7350
M7	N95 respirator	0.0160
M8	surgical mask with ties	0.2850
M9	procedure mask with ear loops	0.6150

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Nine types of consumer-grade masks

Figure 2.1: 2-layer woven nylon mask [16]

Figure 2.2: cotton bandana [16]

Figure 2.3: single layer woven polyester gaiter [16]

Figure 2.4: single layer woven polyester/nylon mask with ties [16]

Figure 2.5: non-woven polypropylene mask with fixed ear loops [16]

Figure 2.6: 3-layer knitted cotton mask with ear loops [16]

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Figure 2.7: procedure mask with ear loops [16]

Figure 2.8: N95 respirator [17]

Figure 2.9: surgical mask with ties [18]

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Chapter 3

Governing equation and Initial condition

3.1 Governing Equation

3.1.1 Mass balance equation

The basis for the description of the relationship between the mass or concentration of a gaseous substance in space as a function of time is the mass balance equation. Thus, the generalized tracer mass balance equation can be presented as the following first-order differential equation [19]:

$$V\frac{dC}{dt}=F-QC, \tag{3.1}$$

where C is the indoor exhaled air concentration (ppm), V is the volume of the classroom (m^3), and F is an emission of tracer gas into space by a tracer gas source (mass per time unit). Furthermore, QC is the transport of tracer gas from the room air to the outside (mass per time unit).

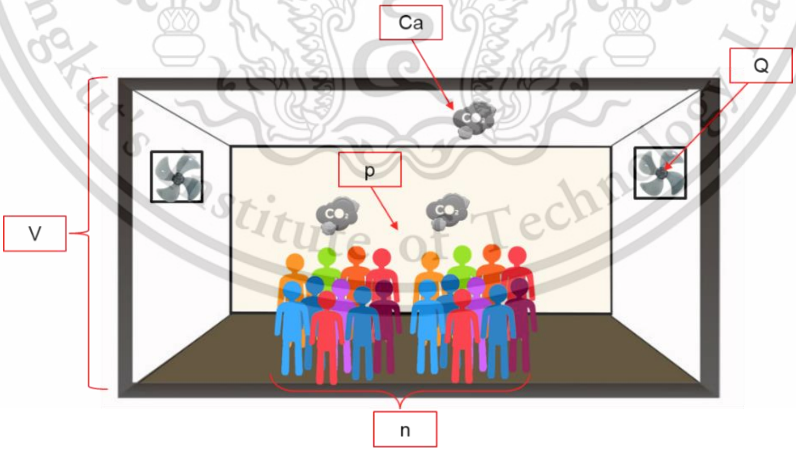

Figure 3.1: The sampled of variables

The basic equation for the exhaled air accumulation rate in an atmospheric carbon dioxide (CO_2) space, which is then occupied, is equal to the rate of exhaled air produced by the

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occupants plus the ambient rate of CO_2, minus the exhaled air eliminated by the rate of ventilation. Moreover, if we consider the term F of the mass balance equation in (3.1), we find the rate of exhaled air generated by occupants is the production rate of tracer by all sources within the enclosure, i.e., n are the number of people (person), p is the breathing rate for each person in the room (L/s), and C_a is the CO_2 fraction containing inbreathed air.

3.1.2 An exhaled air concentration model

The fundamental equation for exhaled air accumulation rate in the room with environmental CO_2 can be formulated as the following:

$$V\frac{dC}{dt} = npC_a - QC. \tag{3.2}$$

We assume that people in the room will contribute equally to the generation of CO_2 as a marker of exhaled air. At the start of the day, environmental CO_2 concentration is C_a (ppm); it becomes occupied by n. This implies that the level of exhaled air concentration that might contain airborne infectious particles given the presence of infectors will start to increase in the room depending on the ventilation rate Q (L/s) and n.

3.1.3 An exhaled air concentration model with a ventilation system

We consider the equation for exhaled air accumulation rate in the room with environmental CO_2 in (3.2), and we divide the ventilation rate into inlet ventilation rate and outlet ventilation rate, which can be written as:

$$V\frac{dC}{dt} = npC_a - Q_{out}C, \tag{3.3}$$

where Q_{out} are the inlet ventilation rate and outlet ventilation rate, respectively. After dividing both sides of (3.3) by the volume, we obtain an ordinary differential equation which describes the concentration change of the indoor exhaled air per time unit:

$$\frac{dC}{dt} = \frac{npC_a - Q_{out}C}{V}. \tag{3.4}$$

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In this research, we are interested in the amount of air pollution that leads to tuberculosis. Using the same initial equation as the above equation, the equation is used to describe the CO_2 concentration in the classroom.

3.1.4 A volume fraction of exhaled air concentration model

Taking into account that the volume fraction of exhaled air, f, is given by the sampled exhaled air concentration $C(t)$ in the space divided by carbon dioxide fraction in breathed air, C_a, we get

$$f = \frac{C(t)}{C_a}, \tag{3.5}$$

where t is the elapsed time in the given space.

3.1.5 A concentration of airborne infectious particles model

In [20], as discussed earlier, the likelihood of airborne infectious particles released by infectors causing infection for susceptible individuals is extremely high if they reach the target infection site of the host at a threshold level. However, some infectious particles can be trapped in the upper respiratory tract or be reflected to other parts of the body, where the probability of causing infection is almost negligible. Let β be the total airborne infectious particles generation rate released by an infector (particles/s), μ be the mortality rate of generated airborne infectious particles by the infector that do not reach the alveolar (particles/s) and FFE is Fitted Filtration Efficiency. Hence, the survival rate of airborne infectious particles released by the infector that reach the target infection site of the susceptible individual to cause infection at threshold level is $\beta-(FFE)\mu$ particles/s as demonstrated.

The average concentration of airborne infectious particles, $\bar{N}$, that cause infection, is equal to the average volume fraction of rebreathed air by infectors $(I\bar{f}/n)$, multiplied by the average concentration of airborne infectious particles released by infectors in the space that reach the target infection site of the respiratory tract $(\beta-(FFE\cdot\mu))/p$:

$$N(t) = \frac{If(\beta-(FFE\cdot\mu))}{n(t)p},\ I \geq 1 \text{ and } \beta-(FFE\cdot\mu) \geq 1. \tag{3.6}$$

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3.1.6 A number of airborne infectious particles model

Since not all infectious particles can reach and deposit at the alveolar, let θ be a respiratory deposition fraction of airborne infectious particles that successfully reach and deposit at the target infection site of the host. Hence, the number of airborne infectious particles, $\lambda(t)$, breathed by a susceptible individual that causes infection, is equal to the product of the volume of breathed air by susceptible, pt, respiratory deposition fraction of airborne infectious particles, θ $(0 < \theta < 1)$, and the concentration of airborne infectious particles $N(t)$ released by infectors:

$$\lambda(t) = pt\theta N(t), \quad t > 0, \tag{3.7}$$

where t is the time spent in the space up to the point of infection.

3.1.7 A percentage of airborne infectious particles model

Computing an expected average number of airborne infectious particles in equation (3.7), the percentage of airborne infectious particles, $\gamma(t)$, that cause airborne infectious diseases in exhaled air can be estimated as

$$\gamma(t) = \frac{\lambda(t)}{C(t)} \times 100, \tag{3.8}$$

where $C(t)$ is the sampled exhaled air in the given space.

3.1.8 A probability of airborne infectious particles model

In [21], considering assumed that airborne transmission follows a Poisson distribution. In [21] and [22], we express airborne transmission probability as

$$P(t) = 1 - e^{-\lambda(t)}, \tag{3.9}$$

where $P(t)$ denotes the probability of airborne transmission risk for susceptible individuals are the random variables representing infection risk for susceptible individuals up to the time spent in the confined space given the presence of an infectious environment in the space.

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Equation in (3.9) predicts the risk of airborne infectious disease by introducing very important parameters, including particle production, survival, mortality rates and successful deposition fraction at the site of infection. The model is applicable in multiple infective environmental conditions obeying the boundary condition of threshold level of infectious particles to induce infection.

Hence, equation in (3.9), predicts the risk of airborne infectious disease transmission, such as covid19, under non-steady state conditions. An exponential term of this equation is equal to the number of airborne infectious particles inhaled by each susceptible individual in the space to induce infection and not quanta. Additionally, for a susceptible individual to be infected, this number should reach or exceed the threshold level depending on the virulence of the infecting pathogen strains and host immune response. We believe that this approach provides a better and more flexible mathematical model for epidemiologists and the whole community in understanding infectious disease transmission risks.

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Chapter 4

Numerical simulations

4.1 Three static cases of the number of students in a classroom

Time (min) There is a risk of airborne infection when employing nine different types of mask efficiency when the number of students in a class room remains constant. Three different scenarios will be developed with 50, 40, and 30 people in the room. Table 4.2 and Figure 4.1 illustrate the exhaled air in the classroom when the number of students is 50, 40, and 30, respectively. As a result, Table 4.4 and Figure 4.2 both show the volume fraction of exhaled air. Table 4.6 and Figure 4.3 show the concentration of airborne infectious particles with nine different types of mask efficiency. As a result, Table 4.8 and Figure 4.4 show the number of airborne infectious particles for nine different types of mask efficiency. Finally, Table 4.9 and Figure 4.5 show the probability of airborne transmission risk for susceptible individuals with nine different types of mask efficiency.

Table 4.1: Parameters of $C(t)$

Parameters	**Values**
V	$350\ (m^3)$
C_a	$0.04\ (ppm)$
p	$0.12\ (L/s)$
n	$30,\ 40, 50$ (person)
Q	$3\ (L/s)$
$C(0)$	$0\ (ppm)$
h	0.1 (min)
T	180 (min)

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Table 4.2: The exhaled air concentration in the room $C(t)$ when number of students are static

Time (min)	n		
	50	**40**	**30**
0	0.0000	0.0000	0.0000
10	2.3000	1.8400	1.3800
20	4.4111	3.5289	2.6467
30	6.3488	5.0790	3.8093
60	11.2580	9.0064	6.7548
90	15.0541	12.0433	9.0325
120	17.9895	14.3916	10.7937
180	22.0145	17.6116	13.2087

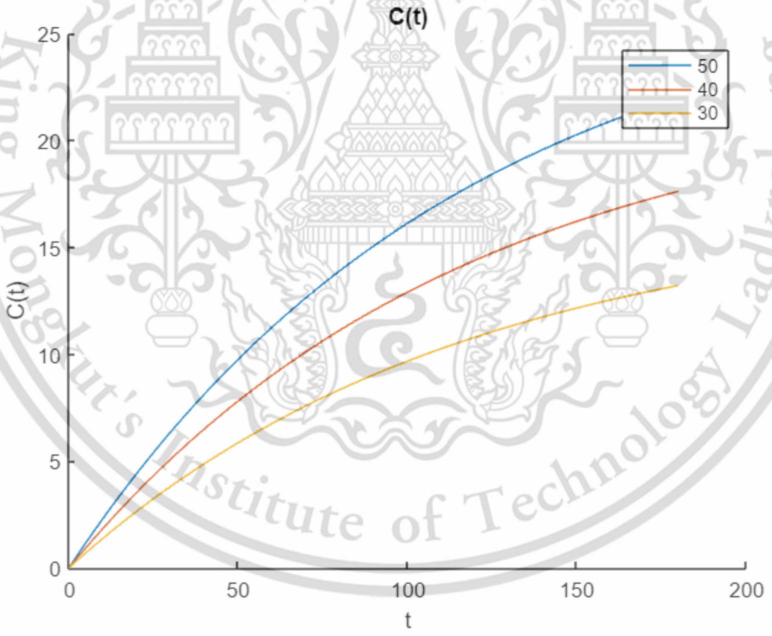

Figure 4.1: The sampled exhaled air in the room when number of are static

From Table 4.2 and Figure 4.1, it is seen that as the number of people increases from 30, 40, 50 the CO_2 concentration at time t increases and over time, the CO_2 concentration at time increases accordingly.

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Table 4.3: Parameters of $f(t)$

Parameters	Values
V	$350\ (m^3)$
C_a	$0.04\ (ppm)$
p	$0.12\ (L/s)$
n	30, 40, 50 (person)
Q	$3\ (L/s)$
$C(0)$	$0\ (ppm)$
h	0.1 (min)
t	180 (min)

Table 4.4: The volume fraction of exhaled air $f(t)$ when number of students are static

Time (min)	n		
	50	40	30
0	0.0000	0.0000	0.0000
10	57.5005	46.0004	34.5003
20	110.2777	88.2222	66.1666
30	158.7196	126.9757	95.2317
60	281.4507	225.1606	168.8704
90	376.3535	301.0828	225.8121
120	449.7379	359.7903	269.8427
180	550.361	440.289	273.5292

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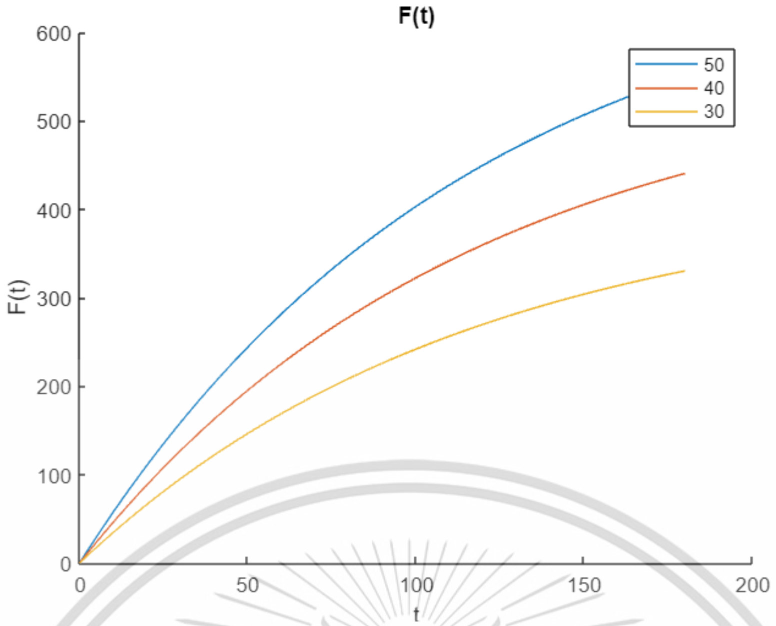

Figure 4.2: The volume fraction of exhaled air when number of students are static

From Table 4.4 and Figure 4.2, it can be seen that when the number of people increased from 30 40 50 people, the volume of exhaled air will increase as time t changes and will remain stable when the equilibrium is reached.

Table 4.5: Parameters of $N(t)$

Parameters	Values	Parameters	Values
β_{M1}	1.0000	μ_{M1}	0.5600
β_{M2}	1.0000	μ_{M2}	0.4990
β_{M3}	1.0000	μ_{M3}	0.3780
β_{M4}	1.0000	μ_{M4}	0.3930
β_{M5}	1.0000	μ_{M5}	0.2860
β_{M6}	1.0000	μ_{M6}	0.2650
β_{M7}	1.0000	μ_{M7}	0.9840
β_{M8}	1.0000	μ_{M8}	0.7150
β_{M9}	1.0000	μ_{M9}	0.3850
I	2.0000		

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Table 4.6: The concentration of airborne infectious particle $N(t)$ when number of students are static

Time (min)	Masks				
	M1	M2	M3	M4	M5
0	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0469	0.0533	0.0662	0.0646	0.0760
20	0.0899	0.1023	0.1270	0.1240	0.1458
30	0.1293	0.1473	0.1828	0.1784	0.2099
60	0.2293	0.2611	0.3242	0.3164	0.3721
90	0.3067	0.3492	0.4335	0.4230	0.4976
120	0.3665	0.4173	0.5180	0.5055	0.5947
180	0.4484	0.5106	0.6339	0.6186	0.7277
Time (min)	Masks				
	M6	M7	M8	M9	
0	0.0000	0.0000	0.0000	0.0000	
10	0.0783	0.0017	0.0303	0.0655	
20	0.1501	0.0033	0.0582	0.1256	
30	0.2160	0.0047	0.0838	0.1808	
60	0.3831	0.0083	0.1485	0.3205	
90	0.5123	0.0112	0.1986	0.4286	
120	0.6121	0.0133	0.2374	0.5122	
180	0.7491	0.0163	0.2905	0.6268	

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From Table 4.6, it can be seen that over time the value the concentration of infectious particles in the air will increase accordingly. The concentration of infectious particles in the air After 1 hour, the smallest values are 0.0083, 0.1485, 0.2293. After 2 hours, the smallest values are 0.0133, 0.2374, 0.3665 and after 3 hours, the smallest values are 0.0163, 0.2905, 0.4484. The masks were N95 respirator $(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$, respectively.

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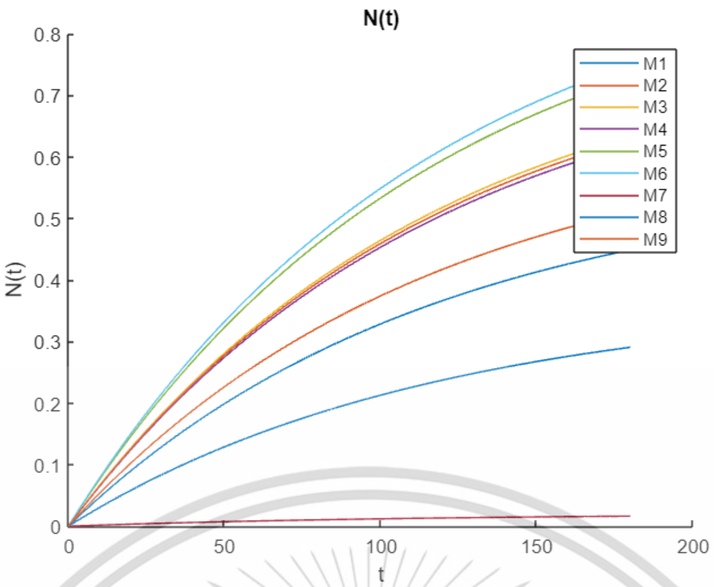

Figure 4.3: The concentration of airborne infectious particle when number of students are static

Figure 4.3, it can be seen that over time the value the concentration of infectious particles in the air will increase. Masks of Single layer woven polyester gaiter $(M3)$, Non-woven polypropylene mask with fixed ear loops $(M5)$ and 3-layer knitted cotton mask with ear loops $(M6)$ tend to increase. The concentration of infectious particles in the air At most it will go into a steady state. The least likely masks were N95 respirator$(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask$(M1)$, respectively.

Table 4.7: Parameters of $\lambda(t)$

Parameters	Values
p	$0.12\ (L/s)$
θ	1

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Table 4.8: The number of airborne infectious particles $\lambda(t)$ when number of students are static

Time (min)	Masks								
	M1	M2	M3	M4	M5	M6	M7	M8	M9
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0557	0.0634	0.0787	0.0768	0.0903	0.0930	0.0020	0.0361	0.0778
20	0.2146	0.2443	0.3033	0.2960	0.3482	0.3584	0.0078	0.1390	0.2999
30	0.4640	0.5284	0.6560	0.6401	0.7530	0.7751	0.0169	0.3006	0.6486
60	1.6484	1.8770	2.3303	2.2741	2.6749	2.7536	0.0599	1.0677	2.3040
90	3.3082	3.7669	4.6766	4.5639	5.3684	5.5262	0.1203	2.1428	4.4963
120	5.2725	6.0035	7.4534	7.2737	8.5559	8.8075	0.1917	3.4152	7.3696
180	9.6810	11.0231	13.6854	13.3554	15.7096	16.1716	0.3520	6.2706	13.5314

From Table 4.8, it can be seen that over time amount of dust in the air will increase accordingly amount of dust in the air After 1 hour, the smallest values were 0.0599, 1.0677, 1.6484. After 2 hours, the smallest values were 0.1917, 3.4152, 5.2725 and after 3 hours, the smallest values were 0.3520, 6.2706, 9.6810. The masks were N95 respirator $(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$, respectively.

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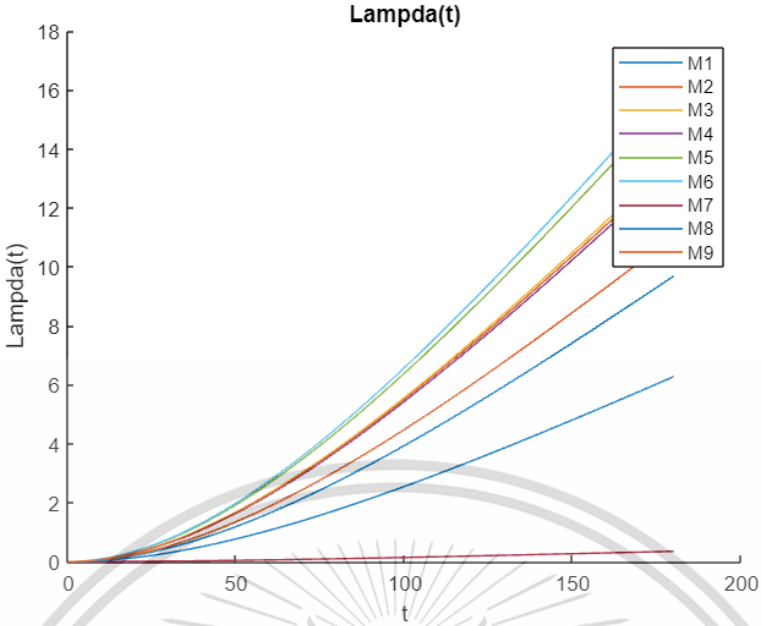

Figure 4.4: The number of airborne infectious particles when number of students are static

From Figure 4.4, it can be seen that over time the amount of particulate matter in the air will increase. Masks of Single layer woven polyester gaiter$(M3)$, Non-woven polypropylene mask with fixed ear loops$(M5)$ and 3-layer knitted cotton mask with ear loops $(M6)$ tend to increase. Air was the most, while the least likely masks were N95 respirator $(M7)$, Surgical mask with ties$(M8)$ and 2-layer woven nylon mask $(M1)$, respectively.

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Table 4.9: The probability of airborne transmission risk for susceptible individuals $p(t)$ when number of students are static

Time (min)	Masks				
	M1	M2	M3	M4	M5
0	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0541	0.0614	0.0757	0.0739	0.0864
20	0.1931	0.2168	0.2616	0.2562	0.2940
30	0.3713	0.4104	0.4811	0.4728	0.5290
60	0.8076	0.8469	0.9027	0.8971	0.9311
90	0.8469	0.9769	0.9907	0.9896	0.9953
120	0.9949	0.9975	0.9994	0.9993	0.9998
180	0.9999	0.9978	1.0000	1.0000	1.0000
Time (min)	Masks				
	M6	M7	M8	M9	
0	0.0000	0.0000	0.0000	0.0000	
10	0.0888	0.0020	0.0354	0.0748	
20	0.3012	0.0078	0.1298	0.2591	
30	0.5394	0.0167	0.2596	0.4772	
60	0.9363	0.0582	0.6562	0.9001	
90	0.9960	0.1133	0.8827	0.9902	
120	0.9999	0.1745	0.9671	0.9994	
180	1.0000	0.2967	0.9981	1.0000	

From Table 4.9, it can be seen that over time the probability of transmission risk will increase accordingly probability of transmission risk After 1 hour, the smallest values were 0.0582, 0.6562, 0.8076. After 2 hours, the smallest values were 0.1745, 0.9671, 0.9949 and after 3 hours, the smallest values were 0.2967, 0.9981, 0.9999. The masks were N95 respirator $(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$, respectively.

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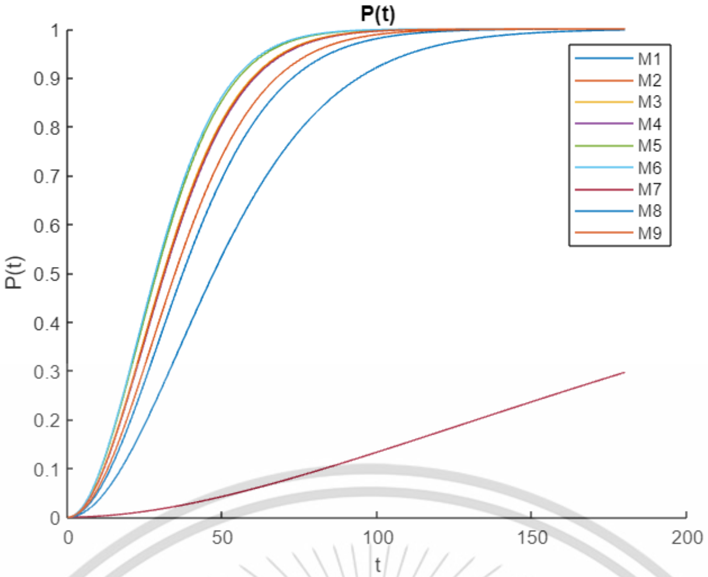

Figure 4.5: The probability of airborne transmission risk for susceptible individuals when number of students are static

Figure 4.5, over time the probability of transmission risk increases and N95 respirator $(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$ had the highest probability of transmission risk, indicating that the three masks had the best protection, respectively.

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4.2 Three examples of student numbers that are assumed to be functions in a classroom

There is a risk of airborne infection when employing nine different types of mask efficiency when the number of students in a class room is assumed to be function in a classroom. Three different scenarios will be developed with 50, 40, and 30 people in the room. Table 4.11 and Figure 4.6 illustrate the exhaled air in the classroom when the number of students is 50, 40, and 30, respectively. As a result, Table 4.13 and Figure 4.7 both show the volume fraction of exhaled air. Table 4.15 and Figure 4.8 show the concentration of airborne infectious particles with nine different types of mask efficiency. As a result, Table 4.17 and Figure 4.9 show the number of airborne infectious particles for nine different types of mask efficiency. Finally, Table 4.18 and Figure 4.10 show the probability of airborne transmission risk for susceptible individuals with nine different types of mask efficiency.

Table 4.10: Parameters of $C(t)$

Parameters	Values
V	$350\ (m^3)$
C_a	$0.04\ (ppm)$
p	$0.12\ (L/s)$
n	$50 + 10sin\ (0.1t)$ $50 + 20sin\ (0.1t)$ $50 + 30sin\ (0.1t)$
Q	$3\ (L/s)$
$C(0)$	$0\ (ppm)$
h	0.1 (min)
t	180 (min)

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Table 4.11: The exhaled air concentration in the room $C(t)$ when number of students are varied

Time (min)	n		
	$50 + 10sin\ (0.1t)$	$50 + 10sin\ (0.1t)$	$50 + 10sin\ (0.1t)$
0	0.0000	0.0000	0.0000
10	2.4821	2.6642	2.8462
20	5.1901	5.7678	6.3454
30	7.0864	7.8241	8.5618
60	11.2230	11.1880	11.1529
90	15.8495	16.6448	17.4402
120	18.1445	18.2995	18.4544
180	22.4003	22.7862	23.172

From Table 4.11, over time, the CO_2 concentration at time t increases accordingly. and when adding the number of people by time is the number of people as a function that depends on time, so the CO_2 concentration at time t increases or decreases according to the function value n.

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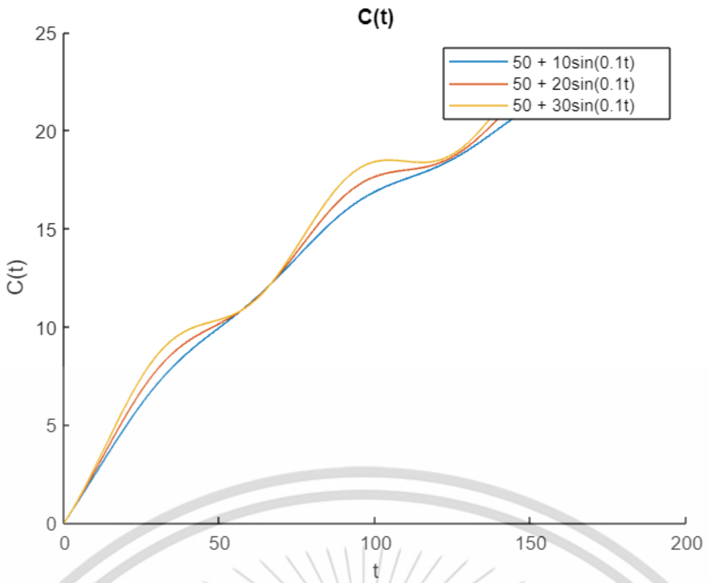

Figure 4.6: The sampled exhaled air in the room when number of are varied

From Figure 4.6, it can be seen that when increasing the number of people from $50+10sin(0.1t)$, $50+20sin(0.1t)$, $50+30sin(0.1t)$. The CO_2 concentration at time t increases as the time changes. and will increase or decrease according to the function n.

Table 4.12: Parameters of $f(t)$

Parameters	Values
V	$350\ (m^3)$
C_a	$0.04\ (ppm)$
p	$0.12\ (L/s)$
n	$50 + 10sin\ (0.1t)$ $50 + 20sin\ (0.1t)$ $50 + 30sin\ (0.1t)$
Q	$3\ (L/s)$
$C(0)$	$0\ (ppm)$
h	$0.1\ (\text{min})$
t	$180\ (\text{min})$

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Table 4.13: The volume fraction of exhaled air $f(t)$ when number of students are varied

Time (min)	n		
	$50 + 10sin\ (0.1t)$	$50 + 20sin\ (0.1t)$	$50 + 30sin\ (0.1t)$
0	0.0000	0.0000	0.0000
10	62.0523	66.6041	71.1559
20	123.9117	137.5458	151.1799
30	177.1611	195.6026	214.0441
60	280.5748	279.6988	278.8228
90	396.2369	416.1204	436.0038
120	453.6122	571.2071	461.3609
180	560.0080	569.6540	579.3010

From Table 4.13, it can be seen that over time the value fraction of exhaled air will increase accordingly and when increasing the number of people over time, i.e., the number of people is a function that increases with time, the volume of exhaled air will increase or decrease with the function value n.

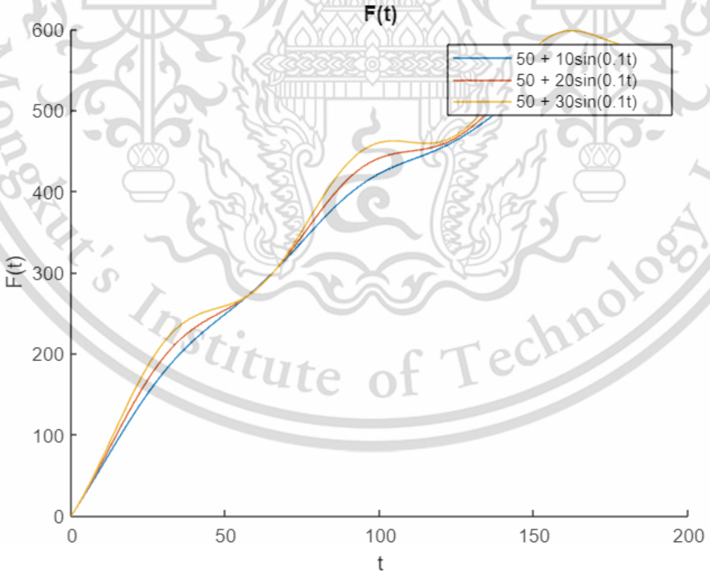

Figure 4.7: The volume fraction of exhaled air when number of students are varied

Figure 4.7, it can be seen that when increasing the number of people from $50+10sin(0.1t)$, $50+20sin(0.1t)$, $50+30sin(0.1t)$ the volume of exhaled air will increase or decrease with the function n.

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Table 4.14: Parameters of $N(t)$

Parameters	Values	Parameters	Values
β_{M1}	1.0000	μ_{M1}	0.5600
β_{M2}	1.0000	μ_{M2}	0.4990
β_{M3}	1.0000	μ_{M3}	0.3780
β_{M4}	1.0000	μ_{M4}	0.3930
β_{M5}	1.0000	μ_{M5}	0.2860
β_{M6}	1.0000	μ_{M6}	0.2650
β_{M7}	1.0000	μ_{M7}	0.9840
β_{M8}	1.0000	μ_{M8}	0.7150
β_{M9}	1.0000	μ_{M9}	0.3850
I	2.0000		

Table 4.15: The concentration of airborne infectious particle $N(t)$ when number of students are varied

Time (min)	Masks				
	M1	M2	M3	M4	M5
0	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0386	0.0440	0.0546	0.0533	0.0627
20	0.0796	0.0906	0.1125	0.1134	0.1291
30	0.1599	0.1821	0.2261	0.2206	0.2595
60	0.2748	0.3130	0.3885	0.3792	0.4460
90	0.2836	0.3229	0.4009	0.3912	0.4602
120	0.5586	0.6360	0.7896	0.7706	0.9064
180	0.8654	0.9853	1.2233	1.1938	1.4042
Time (min)	Masks				
	M6	M7	M8	M9	
0	0.0000	0.0000	0.0000	0.0000	
10	0.0645	0.0014	0.0250	0.0540	
20	0.1329	0.0029	0.0515	0.1112	
30	0.2671	0.0058	0.1036	0.2235	

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60	0.4591	0.0100	0.1780	0.3842	
90	0.4737	0.0103	0.1837	0.3964	
120	0.9331	0.0203	0.3618	0.7807	
180	1.4455	0.0315	0.5605	1.2095	

From Table 4.15, it can be seen that over time the value the concentration of infectious particles in the air will increase accordingly. The concentration of infectious particles in the air After 1 hour, the smallest values were 0.0100, 0.1780, 0.2748. After 2 hours, the smallest values were 0.0203, 0.3618, 0.5586 and after 3 hours, the smallest values were 0.0315, 0.5605, 0.8654. The masks were N95 respirator $(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$, respectively.

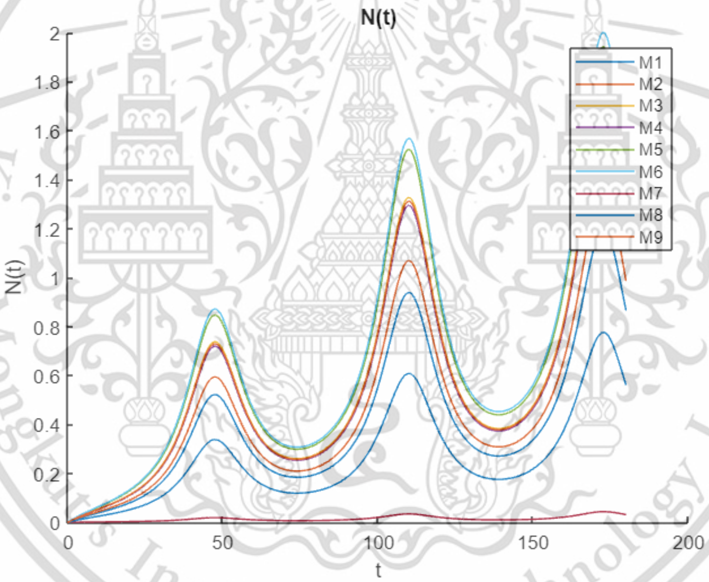

Figure 4.8: The concentration of airborne infectious particle when number of students are varied

Figure 4.8, it can be seen that over time the value the concentration of infectious particles in the air Increased or decreased by function value n. Single layer woven polyester gaiter $(M3)$, Non-woven polypropylene mask with fixed ear loops $(M5)$ and 3-layer knitted cotton mask with ear loops $(M6)$ masks tended to increase or decrease with function n. concentration of infectious particles in the air At most it will go into a steady state. The least likely masks were N95 respirator $(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$, respectively.

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Table 4.16: Parameters of $\lambda(t)$

Parameters	Values
p	$0.12\ (L/s)$
θ	1

Table 4.17: The number of airborne infectious particles $\lambda(t)$ when number of students are varied

Time (min)	Masks								
	M1	M2	M3	M4	M5	M6	M7	M8	M9
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0459	0.0522	0.0648	0.0633	0.0744	0.0766	0.0017	0.0297	0.0641
20	0.1900	0.2164	0.2686	0.2621	0.3084	0.3174	0.0069	0.1231	0.2656
30	0.5738	0.6533	0.8111	0.7916	0.9311	0.9585	0.0209	0.3717	0.8020
60	1.9756	2.2495	2.7928	2.7254	3.2059	3.3002	0.0718	1.2797	2.7614
90	3.0594	3.4835	4.3249	4.2206	4.9645	5.1106	0.1113	1.9816	4.2762
120	8.0367	9.1509	11.3610	11.0870	13.0414	13.4250	0.2922	5.2056	11.2331
180	18.6813	21.2712	26.4085	25.7716	30.3146	31.2062	0.6793	12.1004	26.1113

From Table 4.17, it can be seen that over time amount of dust in the air will increase accordingly amount of dust in the air After 1 hour, the smallest values were 0.0718, 1.2797,

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1.9756. After 2 hours, the smallest values were 0.2922, 5.2056, 8.0367 and after 3 hours, the smallest values were 0.6793, 12.1004, 18.6813. The masks were N95 respirator $(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$, respectively.

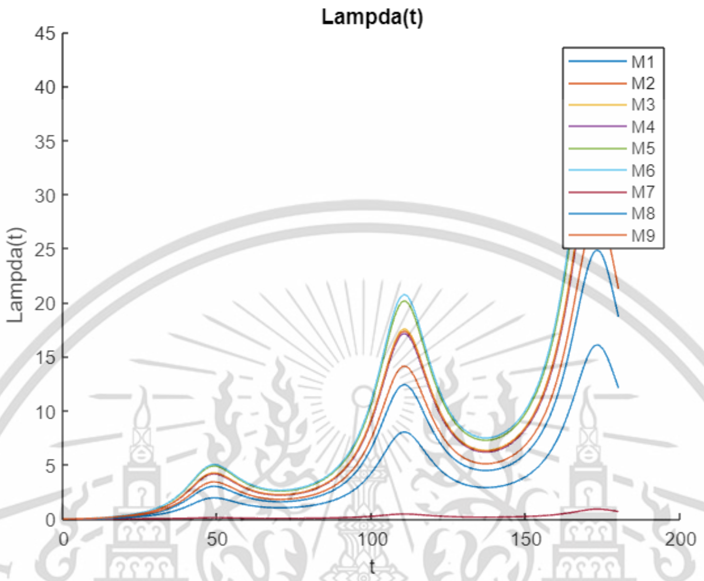

Figure 4.9: The number of airborne infectious particles when number of students are varied

Figure 4.9, it can be seen that over time amount of dust in the air the amount of dust increases or decreases with the n function. Single layer woven polyester gaiter $(M3)$, Non-woven polypropylene mask with fixed ear loops $(M5)$ and 3-layer knitted cotton mask with ear loops $(M6)$ masks tend to increase or decrease with function values. Airborne aerosol was the most, while the least likely masks were N95 respirator $(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$, respectively.

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Table 4.18: The probability of airborne transmission risk for susceptible individuals $p(t)$ when number of students are varied

Time (min)	Masks								
	M1	M2	M3	M4	M5	M6	M7	M8	M9
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0448	0.0509	0.0628	0.0613	0.0717	0.0738	0.0017	0.0293	0.0621
20	0.1731	0.1946	0.2356	0.2306	0.2653	0.2720	0.0069	0.1158	0.2333
30	0.4438	0.4797	0.5556	0.5469	0.6059	0.6165	0.0206	0.3104	0.5516
60	0.8613	0.8945	0.9388	0.9345	0.9595	0.9631	0.0693	0.7219	0.9368
90	0.9531	0.9693	0.9868	0.9853	0.9930	0.9940	0.1053	0.8622	0.9861
120	0.9997	0.9999	1.0000	1.0000	1.0000	1.0000	0.2534	0.9945	1.0000
180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.4930	1.0000	1.0000

From Table 4.18, it can be seen that over time the probability of transmission risk It increases or decreases with the function n the probability of transmission risk. After 1 hour, the smallest values were 0.0693, 0.7219, 0.8613. After 2 hours, the smallest values were 0.2534, 0.9945, 0.9997 and after 3 hours, the smallest values were 0.4930, 1.0000, 1.0000. The masks were N95 respirator $(M7)$, Surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$, respectively.

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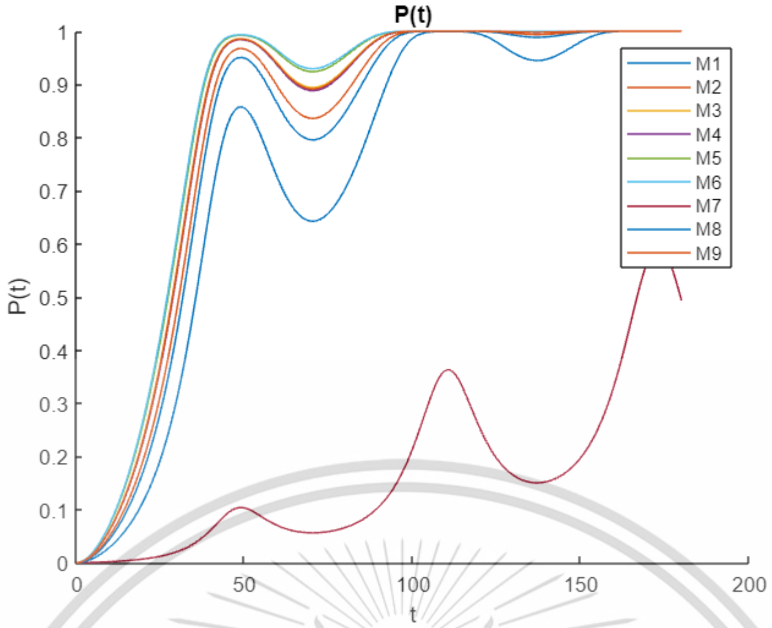

Figure 4.10: The probability of airborne transmission risk for susceptible individuals when number of students are varied

Figure 4.10, it can be seen that over time the probability of transmission risk N95 respirator$(M7)$, Surgical mask with ties $(M8)$ and 2 - layer woven nylon mask $(M1)$ masks tended to increase or decrease the likelihood of transmission risk. The best shows that these 3 types of masks can prevent the spread of infection the best, respectively.

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Chapter 5

Discussions and Conclusion

5.1 Discussions

This study suggests a mathematical model for measuring the risk of airborne transmission in a classroom with a ventilation system. The model solution is approximated using the fourth-order Runge-Kutta method. When n is added to the constant, it can be seen that the increase in $C(t)$ the oscillation increases accordingly. similar, when n is added to a constant, it can be seen that *P(t)* increases and oscillates more.

We can observe that nine different types of mask efficiency minimize the risk of airborne infection. Two scenarios are simulated, including three static cases of the number of students in a classroom and three examples of student numbers in a classroom that are supposed to be functions. As a result, the optimum protection is provided by the $M7$ mask, which is an N95 respirator. The $M5$ mask, which is made of non-woven polypropylene and has fixed ear loops, provides the least level of protection. However, if students remain in a room for an extended period of time, such as 2– 3 hours, they may be at risk of transmitting an airborne infection.

Hence, It can be seen that increasing the number of people function increases the values of the sampled exhaled air in the given space $C(t)$ .The change in n from the resulting $P(t)$ shows that over time the risk of infection increases. It depends on the number of people and the amount of time spent at that time. and type of mask we can see that Different masks have different efficacy. The most effective masks to prevent infection in 3 hours for number of students are static were N95 respirator $(M7)$ is 29.67%, surgical mask with ties $(M8)$ is 99.81% and 2-layer woven nylon mask $(M1)$ is 99.99%, and for number of students are varied were N95 respirator $(M7)$ is 49.30%, surgical mask with ties $(M8)$ and 2-layer woven nylon mask $(M1)$ is 100%, respectively.

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5.2 Conclusion

The more and longer a person interacts with others, the greater the risk of COVID-19 spreading. As a result, effective air produces better quality for controlling and reducing potentially polluted air, such as CO_2 levels. This research proposed a mathematical model for assessing the risk of the performance of different types of masks to the airborne transmission in a ventilated classroom. A mathematical model for assessing the risk of various masks in airborne transmission in a classroom with a ventilation system is proposed in this work. The fourth-order Runge-Kutta method is used to approximate the model solution. During the air quality control procedure, the suggested technique balances the number of students permitted to stay in the classroom with the efficiency of the air ventilation system and for assessing the risk of the performance of different types of masks to the airborne transmission.

This study develops a mathematical model for assessing the risk of airborne transmission in a classroom using nine distinct types of mask efficiency. The model solution is approximated using the fourth-order Runge-Kutta method. The proposed technique creates a balance between the number of students permitted to remain in the classroom and the efficacy of nine distinct masks. During the air quality control technique, the suggested technique balances the number of students allowed to stay in the classroom with the effectiveness of the air ventilation system, as well as the number of people differences, both static and variable. We can see how using nine different masks in the classroom and having a well-ventilated system can assist in limiting the risk of airborne infection.

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Appendix

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2022 SPUC National and International Conference

A MATHEMATICAL MODEL OF THE RISK ASSESSMENT FOR AIRBORNE TRANSMISSION IN A CLASSROOM WITH A SURGICAL MASK EFFICIENCY

Benjawan Janmanit *
Nopparat Pochai *

ABSTRACT

TB, COVID-19, MERS, and SARS are all severe infectious diseases that spread through the air or aerosol in a number of ways, such as coughing, spitting, sneezing, speaking, or wounds. This paper proposes a mathematical model for measuring the probability of airborne transmission in a classroom using surgical mask efficiency. To approximate the model solution, the fourth-order Runge-Kutta technique is utilized. The proposed method provides a balance between the number of students permitted to remain in the classroom and the efficacy of a surgical mask. We can see that using a surgical mask and a well-ventilated system can help to limit the risk of airborne infection in the classroom.

Keywords: Airborne transmission, Classroom, Mathematical model, Risk, Ventilation system, Surgical mask.

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Introduction

Tuberculosis (TB), Coronavirus Disease Starting in 2019 (COVID-19), Middle East Respiratory Syndrome (MERS), and Severe Acute Respiratory syndrome (SARS) are a hazardous communicable disease which are spread from person to person through the air or the aerosol in different ways, such as through coughing, spitting, sneezing, speaking, or through wounds. US scientists in the laboratory have shown that the virus can live in an aerosol and remain infectious for at least 3 hours. At present, we have an effective TB disinfectant. TB can be treated, but recovery takes a long time. If the treatment is not continued, or is incomplete, death may result. Therefore, TB is an important public health issue in Thailand (Doremalen, Bushmaker, Morris, Holbrook, Gamble, Williamson, Tamin, Harcourt, Thornburg, Gerber, Lloyd-Smith, Wit and Munster, 2020). A new procedure was developed to study the distribution of epidemics for predicting the possibility of airborne infectious diseases in high-density urban areas. It can analyze the chance of spread in sub-transportation, and it can also help understand dispersion of airborne diseases in public transportation in China (Shan, Zhou, Zhu, Zu, Zheng, Alexander and Peter, 2011). The researchers studied the behaviors of Korean TB infection. TB transmission dynamic was proposed by using mathematical TB model with exogenous reinfection. Then, the least squares method was used to approximate the considered parameters. From the results, the most significant factor was the case finding effort, which led to a decrease of active TB patients (Sara, Seoyoon, Junseong, Sanga, Yeon, and Sunmi, 2014). The researchers developed an infectious diseases model of SARS by using two methods for estimating both small-scale SARS outbreak parameter at the Amoy Gardens, Hong Kong and large-scale outbreak parameter in the entire Hong Kong Special Administrative Region (Mkhatshwa and Mummert, 2011). The inpatient nursing records from EMR of the University of Miyazaki Hospital were analyzed by using a text data mining technique. This result indicated that vocabulary related to appropriate treatment methods (Kushima, Araki, Suzuki, Araki, and Nikama, 2011). This research proposed a mathematical model for assessing the probability of airborne transmission in a classroom with surgical mask efficiency.

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Objectives

1. A mathematical model for assessing the risk of airborne transmission in a classroom with a surgical mask efficiency is proposed in this work.

2. The fourth-order Runge-Kutta method is used to approximate the model solution.

3. The suggested technique balances the number of students allowed to stay in the classroom with the effectiveness of a surgical mask.

Governing Equation

The basis for the description of the relationship between the mass or concentration of a gaseous substance in space as a function of time is the mass balance equation. Thus, the generalized tracer mass balance equation can be presented as the following first-order differential equation (Heidt and Werner,1986):

$$V\frac{dC}{dt} = F - QC, \tag{1}$$

where *C* is the indoor exhaled air concentration (ppm), *V* is the volume of the classroom (m^3), and *F* is an emission of tracer gas into space by a tracer gas source (mass per time unit). Furthermore, *QC* is the transport of tracer gas from the room air to the outside (mass per time unit). That show in figure 1.

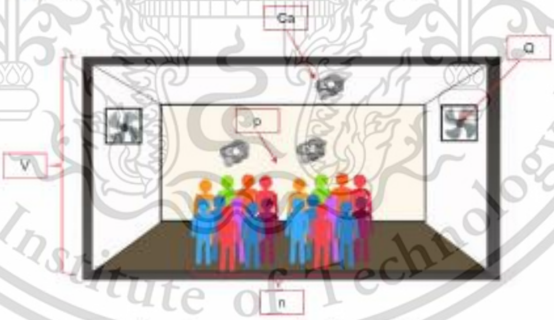

Fig. 1. The sampled of variables.

The basic equation for the exhaled air accumulation rate in an atmospheric carbon dioxide (CO2) space, which is then occupied, is equal to the rate of exhaled air produced by the occupants plus the ambient rate of CO2, minus the exhaled air eliminated by the rate of ventilation. Moreover, if we consider the term *F* of the mass balance equation in (1), we find the rate of exhaled air generated by occupants is the production rate of tracer by all sources within the enclosure, i.e., n are the

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number of people (person), p is the breathing rate for each person in the room (L/s), and C_a is the CO_2 fraction containing inbreathed air. Thus, the fundamental equation for exhaled air accumulation rate in the room with environmental CO_2 can be formulated as the following:

$$V\frac{dC}{dt} = npC_a - QC. \tag{2}$$

We assume that people in the room will contribute equally to the generation of CO_2 as a marker of exhaled air. At the start of the day, environmental CO_2 concentration is C_a (ppm); it becomes occupied by n. This implies that the level of exhaled air concentration that might contain airborne infectious particles given the presence of infectors will start to increase in the room depending on the ventilation rate Q (L/s) and n.

We consider the equation for exhaled air accumulation rate in the room with environmental CO_2 in (2), and we divide the ventilation rate into inlet ventilation rate and outlet ventilation rate, which can be written as:

$$V\frac{dC}{dt} = n(t)pC_a - Q_{out}C, \tag{3}$$

where Q_{out} are the inlet ventilation rate and outlet ventilation rate, respectively. After dividing both sides of (3) by the volume, we obtain an ordinary differential equation which describes the concentration change of the indoor exhaled air per time unit:

$$\frac{dC}{dt} = \frac{n(t)pC_a - Q_{out}C}{V}. \tag{4}$$

In this paper, we are interested in the amount of air pollution that leads to tuberculosis. Using the same initial equation as the above equation, the equation is used to describe the CO_2 concentration in the outpatient room of the hospital.

Taking into account that the volume fraction of exhaled air, f, is given by the sampled exhaled air concentration $C(t)$ in the space divided by carbon dioxide fraction in breathed air C_a, we get

$$f(t) = \frac{C(t)}{C_a}. \tag{5}$$

where t is the elapsed time in the given space.

As discussed earlier (Sze To and Chao, 2010), the likelihood of airborne infectious particles released by infectors causing infection for susceptible individuals is extremely high if they reach the target infection site of the host at a threshold

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level. However, some infectious particles can be trapped in the upper respiratory tract or be reflected to other parts of the body, where the probability of causing infection is almost negligible. Let β be the total airborne infectious particles generation rate released by an infector (particles/s), μ be the mortality rate of generated airborne infectious particles by the infector that do not reach the alveolar (particles/s) and FFE is Fitted Filtration Efficiency. Hence, the survival rate of airborne infectious particles released by the infector that reach the target infection site of the susceptible individual to cause infection at threshold level is $\beta - (FFE)\mu$ particles/s as demonstrated.

The average concentration of airborne infectious particles, $\bar{N}$, that cause infection, is equal to the average volume fraction of rebreathed air by infectors $(I\bar{f}/n)$, multiplied by the average concentration of airborne infectious particles released by infectors in the space that reach the target infection site of the respiratory tract $(\beta - (FFE \cdot \mu))/p$:

$$N(t) = \frac{If(\beta - (FFE \cdot \mu))}{n(t)p},\ I \geq 1 \text{ and } \beta - (FFE \cdot \mu) \geq 1. \tag{6}$$

Since not all infectious particles can reach and deposit at the alveolar, let θ be a respiratory deposition fraction of airborne infectious particles that successfully reach and deposit at the target infection site of the host. Hence, the average number of airborne infectious particles, $\lambda(t)$, breathed by a susceptible individual that causes infection, is equal to the product of the volume of breathed air by susceptible, (*pt*), respiratory deposition fraction of airborne infectious particles, $\theta(0 < \theta < 1)$, and the average concentration of airborne infectious particles ($\bar{N}$) released by infectors:

$$\lambda(t) = pt\theta N(t),\ t>0. \tag{7}$$

where *t* is the time spent in the space up to the point of infection.

Computing an expected average number of airborne infectious particles in equation (13), the percentage of airborne infectious particles, γ, that cause airborne infectious diseases in exhaled air can be estimated as

$$\gamma(t) = \frac{\lambda(t)}{C(t)} \times 100. \tag{8}$$

where C_T is the sampled exhaled air in the given space.

Considering (Wells, 1955) assumed that *TB* transmission follows a Poisson distribution. In (Wells, 1955; Rudnick and Milton, 2003), we express *TB* transmission probability as

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$$P(T \le t \mid I, Q, V, p, \theta, \mu, \beta) = 1 - e^{-\lambda(t)}. \tag{9}$$

where $P(T \le t \mid I, Q, V, p, \theta, \mu, \beta)$ denotes the probability of *TB* transmission risk for susceptible individuals and T ≤ t are the random variables representing infection risk for susceptible individuals up to the time spent in the confined space given the presence of an infectious environment in the space.

Eq. (9), predicts the risk of airborne infectious disease by introducing very important parameters, including particle production, survival, mortality rates and successful deposition fraction at the site of infection. The model is applicable in multiple infective environmental conditions obeying the boundary condition of threshold level of infectious particles to induce infection.

Hence, Eq. (9), predicts the risk of airborne infectious disease transmission, such as covid19, under non-steady state conditions. An exponential term of this equation is equal to the number of airborne infectious particles inhaled by each susceptible individual in the space to induce infection and not quanta. Additionally, for a susceptible individual to be infected, this number should reach or exceed the threshold level depending on the virulence of the infecting pathogen strains and host immune response. We believe that this approach provides a better and more flexible mathematical model for epidemiologists and the whole community in understanding infectious disease transmission risks.

Effectiveness of Face Masks Disinfection Method Against COVID

Public health experts recommend wearing face masks as tools to protect others from breathing potentially infectious particles. At the request of University of North Carolina (UNC) Hospitals, EPA scientists are working to understand the effectiveness of masks to protect the wearer against the virus through a series of projects in collaboration with UNC researchers. Researchers tested how well different masks and modifications filter out airborne salt particles, which are the same size as the smallest SARS-CoV-2 particles, but are not harmful. Members of the research team wore the face coverings to do the testing themselves (Environmental Protection Agency, 2021). The effectiveness of face mask disinfection surgical mask is shown in Table 1.

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Table 1. Effectiveness of face mask disinfection (Environmental Protection Agency, 2021)

Consumer-Grade Masks	Fitted Filtration Efficiency (FFE)
surgical mask with ties	0.7150

Numerical Simulation

There is a risk of airborne infection and surgical mask efficacy when the number of patients in an outpatient room remains stable. We will create three different scenarios in which there are 50, 40, and 30 individuals in the room. The effectiveness of face mask disinfection in Table 1 is used. Their concerned parameters are assumed as shown in Table 2. The fourth-order Runge-Kutta method is employed to approximate the solution of Eq. (4). The exhaled air in the classroom when the number of students is 50, 40, and 30, respectively, is shown in Table 3 and Fig. 2. Consequently, the volume fraction of exhaled air is also obtained in Table 4 and Fig. 3. Finally, the probability of airborne transmission risk for susceptible individuals when the number of students is 50, 40, and 30, respectively, is increasing, as shown in Table 5 and Fig. 4.

Table 2. Parameters of setting

Variable	Values
V	75 m^3
Ca	0.04 (ppm)
p	0.12 (L/s)
n(T)	30,40,50
Q	3 (L/s)
C (0)	0 (ppm)
h	0.1 (min)
t	180 (min)
β	1
μ	1
FFE	0.7150

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Table 3. Exhaled air in the room C(t)

Time (min)	N		
	50	40	30
0	0.0000	0.0000	0.0000
10	1.9781	1.5825	1.1868
20	3.3040	2.6432	1.9824
30	4.1928	3.3543	2.5157
60	5.4557	4.3646	3.2734
90	5.8361	4.6688	3.5016
120	5.9506	4.7605	3.5704
180	5.9955	4.7964	3.5973

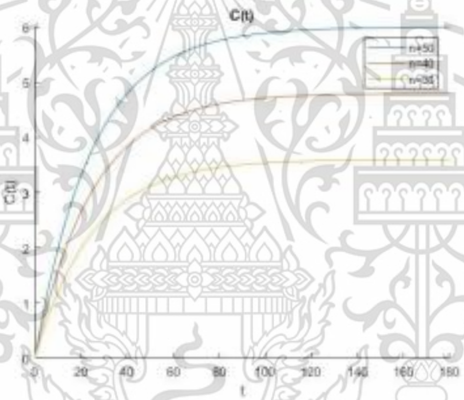

Fig. 2. Exhaled air in the room in 3 cases.

Table 4. The volume fraction of exhaled air f(t)

Time (min)	N		
	50	40	30
0	0.0000	0.0000	0.0000
10	49.4520	39.5616	29.6712
20	82.6007	66.0805	49.5604
30	104.8209	83.8567	62.8925
60	136.3923	109.1138	81.8354

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90	145.9014	116.7212	87.5409
120	148.7655	119.0124	89.2593
180	149.8880	119.9104	89.9328

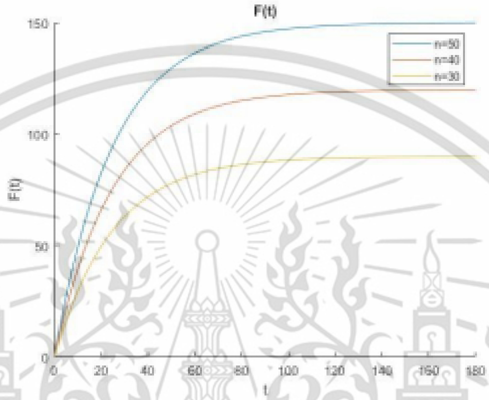

Fig. 3. The volume fraction of exhaled air in 3 cases.

Table 5. The probability of airborne transmission risk for susceptible individuals P(t)

Time	N		
(min)	50	40	30
0	0.0000	0.0000	0.0000
10	0.0305	0.0245	0.0184
20	0.0989	0.0876	0.0606
30	0.1800	0.1468	0.1123
60	0.4039	0.3390	0.2669
90	0.5643	0.4855	0.3925
120	0.6769	0.5949	0.4923
180	0.8187	0.7449	0.6411

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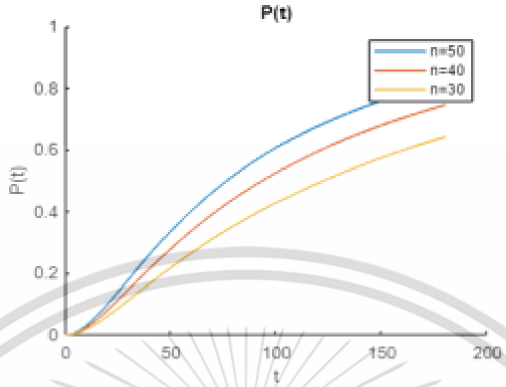

Fig. 4. The probability of airborne transmission risk for susceptible individuals in 3 cases.

Conclusion

The more and longer a person interacts with others, the greater the risk of COVID-19 spreading. As a result, effective air produces better quality for controlling and reducing potentially polluted air, such as CO2 levels. This research proposed a mathematical model for assessing the risk of airborne transmission in a ventilated classroom. A mathematical model for assessing the risk of airborne transmission in a classroom with a ventilation system is proposed in this work. The fourth-order Runge-Kutta method is used to approximate the model solution. The suggested technique balances the number of students allowed to stay in the classroom with the effectiveness of the air ventilation system during the air quality control technique and as the number of people increases, the value of carbon dioxide increases. As a result, the risk increases accordingly. We can see that the surgical mask and the sufficiently ventilated system can reduce the risk of airborne infection in a classroom.

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