รายงานการวิจัยฉบับสมบูรณ์

การคำนวณเชิงตัวเลขของตัวแบบการระบายน้ำในอ่างเก็บน้ำโดยใช้วิธีผลต่างจำกัด

A Numerical Computation of a Reservoir Drain Using Finite Difference Method

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ได้รับทุนสนับสนุนงานวิจัยจากเงินรายได้ ประจำปีงบประมาณ 2556
คณะวิทยาศาสตร์
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ขอขอบคุณคณะวิทยาศาสตร์ สถาบันเทคโนโลยีพระจอมเกล้าเจ้าคุณทหารลาดกระบัง ที่ให้การสนับสนุนทุนวิจัย ประเภทส่งเสริมนักวิจัยด้วยเงินรายได้คณะวิทยาศาสตร์ ประจำปีงบประมาณ 2556 จำนวน 50,000 บาท จนงานวิจัยนี้สำเร็จลุล่วงไปได้ด้วยดี

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จำนวนเงิน 50,000 บาท

ระยะเวลาทำการวิจัย 1 ปี ตั้งแต่ 1 ตุลาคม 2556 ถึง 30 กันยายน 2557

หน่วยงานและผู้ดำเนินการวิจัยพร้อมหน่วยงานที่สังกัด

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บทคัดย่อ

การจำลองแบบและการสร้างตัวแบบเพื่อการบริหารจัดการน้ำเป็นสิ่งที่มีความจำเป็น เนื่องด้วยหากสามารถจำลองแบบได้ จะทำให้คาดการณ์สถานการณ์น้ำที่ระบายออกและระดับน้ำที่กักเก็บ เพื่อนำมาประกอบกับข้อมูลทางอุตุนิยมวิทยาแล้วน้ำข้อมูลทั้งหลายมาประมวลผลเพื่อการบริหารจัดการน้ำอย่างมีประสิทธิภาพได้ โดยวิธีเชิงตัวเลขที่มีประสิทธิภาพสูงมีความจำเป็นในการหาผลเฉลยโดยประมาณของตัวแบบที่ให้ความแม่นยำสูง ดังนั้นงานวิจัยนี้มุ่งเน้นที่จะตอบโจทย์ของปัญหาดังกล่าวคือ สามารถหาวิธีเชิงตัวเลขที่เหมาะสม เพื่อหาผลเฉลยโดยประมาณที่มีความแม่นยำ สำหรับคำนวณหาความสัมพันธ์ของระดับน้ำกักเก็บเมื่อมีการระบายน้ำในช่วงเวลาต่างๆเพื่อเป็นข้อมูลประกอบการบริหารจัดการน้ำอย่างยั่งยืนต่อไป

Abstract

Simulation and modeling for water management is needed. It can be predict the water flow and water level at the reservoir. The simulated information can be attributed to the meteorology. The data will be processed for effective water management. The numerical methods are highly effective, it is necessary to find a solution by way of a high precision. This research aims to answer the above problem is to find a suitable numerical methods. It can be use to obtain the solutions that is estimated with precision for calculating the relationship between the water reservoir when the water at various times as a basis for sustainable water management.

สารบัญ

CHAPTER I
INTRODUCTION

1 A pond drainage problems

The purpose of a dam is to impound (store) water, wastewater or liquid borne materials for any of several reasons, e.g. flood control, human water supply, irrigation, livestock water supply, energy generation, containment of mine tailings, recreation or pollution control. Many dams fulfill a combination of the above functions.

Manmade dams may be classified according to the type of construction material used, the methods used in construction, the slope or cross-section of the dam, the way the dam resists the forces of the water pressure behind it, the means used for controlling seepage and, occasionally, according to the purpose of the dam.

The materials used for construction of dams include earth, rock, tailings from mining or milling, concrete, masonry, steel, timber, miscellaneous materials (such as plastic or rubber) and any combination of these materials.

Intentional release of water is confined to water releases through outlet works and spillways. A dam typically has a principal or mechanical spillway and a drawdown facility. Additionally, some dams are equipped with auxiliary spillways to manage extreme floods.

Outlet WorksIn addition to spillways that ensure that the reservoir does not overtop the dam, outlet works may be provided so that water can be drawn continuously, or as needed, from the reservoir. They also provide a way to draw down the reservoir for repair or safety concerns. Water withdrawn may be discharged into the river below the dam, run through generators to provide hydroelectric power, or used for irrigation. Dam outlets usually consist of pipes, box culverts or tunnels with intake inverts near minimum reservoir level. Such outlets are provided with gates or valves to regulate the flow rate.

The most common type of spillway is an ungated concrete chute. This chute may be located over the dam or through the abutment. To permit maximum use of storage volume, movable gates are sometimes installed above the crest to control discharge. Many smaller dams have a pipe and riser spillway, used to carry most flows, and a vegetated earth or rockcut spillway through an abutment to carry infrequent high flood flows. In dams such as those on the Mississippi River, flood discharges are of such magnitude that the spillway occupies the entire width of the dam and the overall structure appears as a succession of vertical piers supporting movable gates. High arch-type dams in rock canyons usually have downstream faces too steep for an overflow spillway. In Hoover Dam on the Colorado River, for example, a shaft spillway is used. In shaft spillways, a vertical shaft upstream from the dam drains water from the reservoir when the water level becomes high enough to enter the shaft or riser; the vertical shaft connects to a horizontal conduit through the dam or abutment into the river below.

2 Embankment dams

Embankment dams are the most common type of dam in use today. Materials used for embankment dams include natural soil or rock, or waste materials obtained from mining or milling operations. An embankment dam is termed an earthfill or rockfill dam depending on whether it is comprised of compacted earth or mostly compacted or dumped rock.

The ability of an embankment dam to resist the reservoir water pressure is primarily a result of the mass weight, type and strength of the materials from which the dam is made.

3 Concrete dams

Concrete dams may be categorized into gravity and arch dams according to the designs used to resist the stress due to reservoir water pressure. Typical concrete gravity dams are shown here and are the most common form of concrete dam. The mass weight of concrete and friction resist the reservoir water pressure.

A buttress dam is a specific type of gravity dam in which the large mass of concrete is reduced, and the forces are diverted to the dam foundation through vertical or sloping buttresses. Gravity dams are constructed of vertical blocks of concrete with flexible seals in the joints between the blocks.

Concrete arch dams are typically rather thin in cross-section. The reservoir water forces acting on an arch dam are carried laterally into the abutments. The shape of the arch may resemble a segment of a circle or an ellipse, and the arch may be curved in the vertical plane as well. Such dams are usually constructed of a series of thin vertical blocks that are keyed together; barriers to stop water from flowing are provided between blocks.

Variations of arch dams include multi-arch dams in which more than one curved section is used, and arch-gravity dams which combine some features of the two types of dams.

4 Concrete dams

Because the purpose of a dam is to retain water effectively and safely, the water retention ability of a dam is of prime importance. Water may pass from the reservoir to the downstream side of a dam by:

Passing through the main spillway or outlet works
Passing over an auxiliary spillway
Overtopping the dam
Seepage through the abutments
Seepage under the dam

Overtopping of an embankment dam is very undesirable because the embankment materials may be eroded away. Additionally, only a small number of concrete dams have been designed to be overtopped. Water normally passes through the main spillway or outlet works; it should pass over an auxiliary spillway only during periods of high reservoir levels and high water inflow. All embankment and most concrete dams have some seepage. However, it is important to control the seepage to prevent internal erosion and instability. Proper dam construction, and maintenance and monitoring of seepage provide this control.

5 Literature review

Dam reservoirs plays a vital function at various times and for different purposes such as supplying water for irrigation, hydropower, mitigating disastrous environmental effects and impacts, as well as ensuring flood mitigation and as an insurance during periods of drought etc. Information regarding reservoir inflow, is necessary in the analysis and design of several water resources projects such as dam construction, flood control and wastewater disposal.[ ]

In recent years, Artificial Neural Networks are being increasingly used to model hydrological processes due to their capability to represent any arbitrary nonlinear function given sufficientcomplexity of the trained networks. Some of the cited examples in the literatures are rainfall-runoff modeling, rainfall prediction, flood forecasting, water quality modeling, ground water modeling, development of water management policy, and reservoir operation studies.[ ]

Artificial Neural Network models were developed to forecast the monthly released water for Haditha Reservoir in west of Iraq by using the monthly inputs include the reservoir inflow , rainfall ,evaporation , storage , total demand and releases of previous time.It is better to have data record length as long as possible.This paper presents the results from study on the application of the feed-forward backpropagation neural networks to forecast the monthly reservoir release.[ ]

6 Objectives

In this research, we investigate a pond drains through a pipe. Under a number of simplifying assumption data. The problem is modeled in a form of ordinary differential equation that can be describes how depth of water level in a pond change with time.

CHAPTER II
MATHEMATICAL MODEL OF A POND DRAINAGE

In order to calculate the solutions of mathematical model of a pond drainage, we requires some basic knowledge of the the numerical methods. In this chapter, we will give the detail of some preliminaries of basic numerical techniques.

7 Fundamentals in flow analysis

The main distinction between a liquid and gas lies in their rate of change of *density*. However, one can be treated in the same way without taking into account the change of density, provided that the speed of flow is low as compared with the speed of sound propagating in the fluid. The fluid is called *incompressible* if the change of density is negligible [?].

7.1 Streamlines and streamfunction

Suppose that ink is injected into a gently moving fluid. We can observe a streak of ink, as shown by bold curve in Fig.1. The curve thus obtained is called *streakline.* In general, the ink at the point B travelled there, not along the streakline but along a different curve such as the one shown by the dotted line. This curve is called a *particle trajectory.*

We can consider yet another curve, which presents the flow pattern at the instant the ink reaches the point B. This curve is defined in such a way that the tangent to the curve and the flow velocity at the instant have the same direction at every point on the curve. The curve is called a *streamline.*

7.2 Steady flow and unsteady flow

If the streamline and the particle trajectory are coincide, only when the flow pattern does not change with time, the flow is called a *steady flow.* If flow pattern change with time, then it is called an *unsteady flow* or *transient flow.*

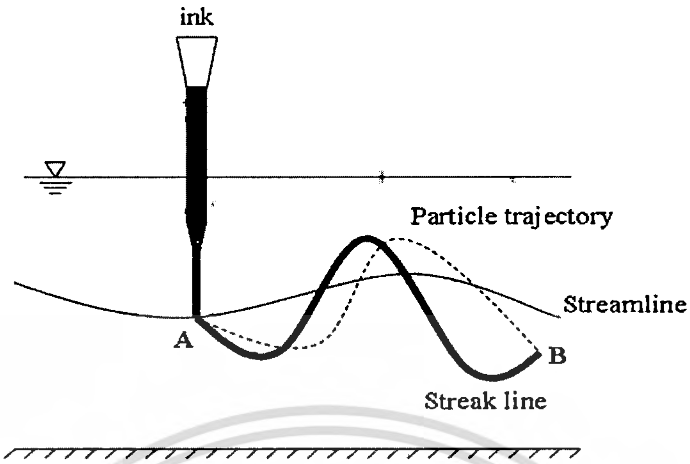

Figure 1: Movement of ink.

7.3 Flow velocity

We denote by u and v the x-component and y-component of flow velocity $(u, v) = u\hat{i} + v\hat{j}$ (m/sec). Since the flow vector with components u, v at every point on the streamline has the same direction as the tangent vector at the same point, we have

$$\frac{u}{v} = \frac{dx}{dy}, \tag{1}$$

$$\frac{dx}{u} = \frac{dy}{v}. \tag{2}$$

A smooth curve can be generally expressed by means of a function in two variables. We consider the function $\psi(x, y)$ such that the $\psi(x, y) = c$ where c is a constant, represents the streamlines. When ψ is a smooth function, the total derivatives along the line must be equal to zero. Namely,

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = 0. \tag{3}$$

The function ψ is called a *streamfunction*. As a possible consequence, it follows from Eq.(2) and Eq.(3) that

$$\frac{dx}{u} - \frac{dy}{v} = 0, \tag{4}$$

$$u \cdot dy - v \cdot dx = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = 0, \tag{5}$$

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x}. \tag{6}$$

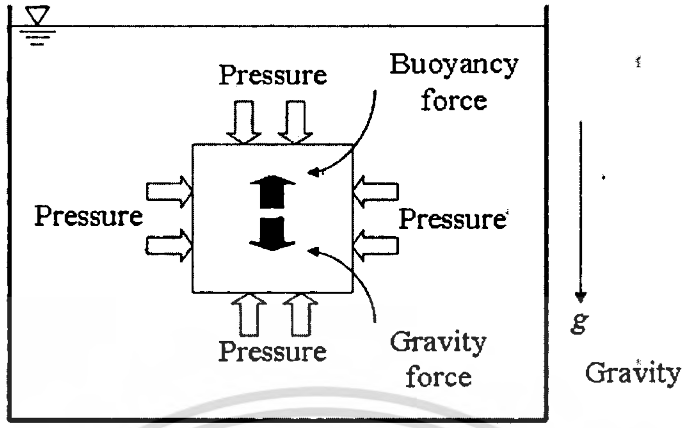

Figure 2: Forces acting in fluid at rest.

7.4 Viscosity and vorticity

Let us consider forces acting in a fluid that is stationary in a vessel. The *gravity forces* is acting in the isothermal fluid. Since fluid is stationary, a counterforce must be acting in a fluid. This force is called *pressure* which will be denoted by p.

The force per unit area (N/m^2) is called *stress.* Therefore, the pressure is a normal stress. We consider a imaginary cube in a fluid as shown in Fig.2. Pressure acting on vertical surfaces balance out. The difference between the pressure acting on top and bottom surface exerts an upward force, known as *buoyancy force*, which balances with the gravity force. As the result, the fluid remains stationary.

The moving viscous fluid encounters an internal frictional force in the direction of motion due to the *viscosity.* The force is expressed as *shearing stress*, which acts on the unit area of a surface in the tangential direction. Consider a fluid at rest between 2 parallel plates. If one plate begin to move at constant velocity, a motion is induced in the fluid. Through this process, the momentum is transported by the fluid due to its viscosity. For most types of fluid, the shearing stress τ(Pa) is proportional to the negative of the spatial variation of the flow velocity $\frac{du}{dy}$, which is known as *the rate of strain* such fluid is called *Newtonian fluid.* We have thus

$$\tau = -\mu\frac{du}{dy}, \tag{7}$$

where μ is the viscosity coefficient (Pa·sec). The quotient of viscosity divided by the density ρ(Kg/m^3) of the fluid is called *the kinematic viscosity* (m^2/sec), it is written as

$$\nu = \frac{\mu}{\rho}, \tag{8}$$

Real fluid have various degrees of viscosity. The fluid in which the shearing stress in presented is a *viscous fluid.* If the shearing stress is negligible small, then the fluid is considered *inviscid* and called *perfect* or *ideal fluid.*

7.5 Similarity of flows

For the physical similarity in flows of viscous fluid, the corresponding non-dimensional number is known as *Reynolds number*, which defined by

$$Re = \frac{UL}{\nu}, \tag{9}$$

where U is a characteristic velocity (m/sec) in the flow system, L is the characteristic length(m) and ν is the kinematic viscosity(m^2/sec). The flow fluctuates wildly as the Reynolds number Re become large, while the flow becomes gentle as Re become small. The flow is *larminar* at $Re \ll 1$. For larger Reynolds numbers $Re \approx 10^6$, the flow becomes *turbulent.* The Table 1 has shown their physical constants of water at 15^o C at atmospheric pressure and Table 2 summarizes important non-dimensional numbers.

8 A Mathematical Model of a Pond Drainage

A pound drains through a pipe. Under a number of simplifying assumptions, [1] and [2] are proposed the following differential equation describes how depth change with time,

$$\frac{dh}{dt} = -\frac{\pi d^2}{4A(h)}\sqrt{2g(h+e)}, \tag{10}$$

Table 1: Physical constants of air at 15^o C at atmospheric pressure.

Material constant	Water	Unit
Density ρ	1.225	kg/m^3
Viscosity μ	1.87×10^{-5}	Pa $\cdot$ sec
Kinematic viscosity ν	1.45×10^{-5}	m^2/sec
Heat conduction coefficient κ	2.51×10^{-2}	J/m·sec·K
Thermal conductivity λ	2.02×10^{-5}	m^2/sec

Table 2: Non-dimensional numbers

Numbers	Definition	Physical constants
Reynolds number	$Re = UL/\nu$	U Velocity (m/sec)
Péclet number	$Pe = UL/\lambda$	L Length (m)
Grashof number	$Gr = g\beta\Delta TL^3/\nu^2$	ν kinematic viscosity (m^2/sec)
Prandtl number	$Pr = \nu\lambda$	g Gravity m/sec^2
Schmidt number	$Sc = \nu/D$	β Thermal expansion coefficient $(1/K)$
Rayleigh number	$Ra = Gr \cdot Pr$	ΔT Temperature difference (K)
Froude number	$Fr = Re^2/Gr$	λ Thermal conductivity (m^2/sec)
		D diffusion coefficient (m^2/sec)

where h is the depth (m), t is a time increment (s), d is a pipe diameter, $A(h)$ is the pound surface area as a function of depth (m^2), g is a gravitational constant (9.81 m$^2/s$) and e a the depth of pipe outlet below the pound bottom (m).

In this consideration, the initial condition is assumed as

$$h(0) = h_0 \tag{11}$$

h_0 is a constant (m). The pound drainage system can be illustrated as Fig.3.

References

[1] Chapra, S.C., 1997, **Surface water-quality modeling**, McGraw-Hill, Singapore.

[2] Chapra, S.C. and Canale, R., 2009, **Numerical Methods for Engineers**, McGraw-Hill, Singapore.

CHAPTER III

Numerical Techniques

9 Cubic Spline Interpolation

The pound surface area data are collected from the field measurement. The function of pound surface area have to determine by using an accurate interpolation as a cubic spline technique. The problem will require that the interpolating function be continuously differentiable.

To achieve more accuracy from the interpolating function, higher-degree polynomial pieces must be used. The most common choice is cubic polynomials. These cubic polynomial pieces can be combined in different ways to produce the overall interpolating function. So, we will give the technique of cubic spline interpolation, that obtains the highest degree of smoothness from the piecewise interpolating function.

9.1 The cubic spline interpolant

Let f be a function defined on the interval $[a, b]$, and let

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b \tag{12}$$

be the $n+1$ distinct points at which f is to be interpolated. Recall that the x_i divide $[a, b]$ into n subintervals, referred to as a partition of $[a, b]$.

Definition A cubic spline interpolant of f relative to the partition

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b \tag{13}$$

is a function s that satisfies
(1) on each subinterval $[x_j, x_{j+1}], j = 0, 1, 2, \ldots, n-1, s$ coincides with the cubic polynomial

$$s(x) = s_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3 \tag{14}$$

(2) s interpolates f at $x_0, x_1, x_2, \ldots, x_n$
(3) s is continuous on $[a, b]$
(4) s' is continuous on $[a, b]$
(5) s'' is continuous on $[a, b]$.

The function s is composed of n different cubic polynomials, each with four coefficients, so there are a total of $4n$ unknowns. Interpolation provides $n + 1$ equations. We can write the equations which follow from the definition.

Interpolation:

$$s_j(x_j) = a_j = f(x_j), j = 0, 1, 2, \ldots, n \tag{15}$$

Continuity of spline:

$$a_{j+1} = a_j + b_j h_j + c_j h_j^2 + d_j h_j^3, j = 0, 1, 2, \ldots, n-2 \tag{16}$$

Continuity of spline derivative:

$$b_{j+1} = b_j + 2c_j h_j + 3d_j h_j^2, j = 0, 1, 2, \ldots, n-2 \tag{17}$$

Continuity of spline second derivative:

$$c_{j+1} = c_j + 3d_j h_j, j = 0, 1, 2, \ldots, n-2 \tag{18}$$

To simplify the equations, we have defined $h_j = x_{j+1} - x_j$. We are using $a_n = f(x_n) = f(b)$, which is a slight extension to the notation introduced in the definition of a cubic spline interpolation.

The interpolation conditions directly provide the values for the a_j, that is removing one-quarter of the unknowns. Next, we will solve the equation for the continuity of the spline second derivative for d_j,

$$d_j = \frac{c_{j+1} - c_j}{3h_j}. \tag{19}$$

Substituting this extension into the equations for the continuity of the spline and its first derivative gives

$$a_{j+1} = a_j + b_j h_j + c_j h_j^2 + \frac{c_{j+1} - c_j}{3} h_j^2, \tag{20}$$

$$= a_j + b_j h_j + \frac{c_{j+1} - c_j}{3} h_j^2 \tag{21}$$

and

$$b_{j+1} = b_j + 2c_jh_j + (c_{j+1} - c_j)h_j, \quad (22)$$
$$= b_j + (c_{j+1} + c_j)h_j. \quad (23)$$

Finally, we will solve Eq.(20) for b_j,

$$b_j = \frac{a_{j+1} - a_j}{h_j} - \frac{2c_j + c_{j+1}}{3}h_j. \quad (24)$$

and substituting the result into Eq.(22). After performing an algebraic manipulation and shifting the subscripts down by one, we can obtain that

$$h_{j-1}c_{j-1} + 2(h_{j-1} - h_j)c_j + h_jc_{j+1} = \frac{3}{h_j}(a_{j+1} - a_j) - \frac{3}{h_{j-1}}(a_j - a_{j-1}). \quad (25)$$

Equation (25) holds for $j = 1, 2, 3, \ldots, n-1$ and forms the basis a tridiagonal system of equations for determining the c_j. The equations for $j = 0$ and $j = n$ depend on the type of boundary conditions which are being applied.

9.2 Not-a-Knot boundary condition

These condition require that s''' be continuous $x = x_1$ and $x = x_{n-1}$. In terms of the spline coefficients, it follows that

$$d_0 = d_1, \quad (26)$$
$$d_{n-2} = d_{n-1}. \quad (27)$$

Using Eq.(19), and rearranging terms, these equations can be expressed in terms of c_j as

$$h_1c_0 - (h_0 + h_1)c_1 + h_0c_2 = 0, \quad (28)$$
$$h_{n-1}c_{n_2} - (h_{n-2} + h_{n-1})c_{n-1} + h_{n-2}c_n = 0. \quad (29)$$

Solve Eq.(28) for c_0 and Eq.(29) for c_n. This gives

$$c_0 = (1 + \frac{h_0}{h_1})c_1 - \frac{h_0}{h_1}c_2, \quad (30)$$
$$c_n = -\frac{h_{n-1}}{h_{n-2}}c_{n-2} + (1 + \frac{n_{n-1}}{h_{n-2}})c_{n-1}. \quad (31)$$

Now, substitute c_0 from Eq.(30) into Eq.(25), for $j = 1$, and group terms to obtain

$$(3h_0 + 2h_1 + \frac{h_0^2}{h_1})c_1 + (h_1 - \frac{h_0^2}{h_1})c_2 = \frac{3}{h_1}(a_2 - a_1) - \frac{3}{h_0}(a_1 - a_0). \tag{32}$$

Proceed in a similar manner with the expression for c_n from Eq.(31) substituted into Eq.(25) for $j = n - 1$ to produce

$$(h_{n-2} - \frac{h_{n-1}^2}{h_{n-2}})c_{n-2} + (3h_{n-1} + 2h_{n-2} + \frac{h_{n-1}^2}{h_{n-2}})c_{n-1} = \frac{3}{h_{n-1}}(a_n - a_{n-1}) - \frac{3}{h_{n-2}}(a_{n-1} - a_{n-2}). \tag{33}$$

Eq.(25), for $j = 2, 3, \ldots, n - 2$, together with Eq.(32) and Eq.(33) will construct a complete tridiagonal system for the coefficients $c_1, c_2, c_3, \ldots, c_{n-1}$.

10 Forth-Order Runge-Kutta Method

Consider, a first-order initial value problem,

$$y'(t) = f(t, y(t)), a \le t \le b \tag{34}$$

$$y(a) = \alpha. \tag{35}$$

Our objective is to determine a numerical approximation $w \approx y$, where $y(t)$ is the exact solution of Eqs.(34-35). We will determine values of w at the discrete set of points

$$a = t_0 < t_1 < t_2, \ldots, t_{N-1} < t_N = b, \tag{36}$$

and we will give w_i represents the approximation to $y_i = y(t_i)$. We will define the step size

$$h = (b - a)/N, \tag{37}$$

and then the t_i will be given by

$$t_i = a + ih, (i = 0, 1, 2, \ldots, N). \tag{38}$$

The fourth-order scheme updates the approximate solution at each time step according to the formula

$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \tag{39}$$

where

$$
\begin{aligned}
k_1 &= hf(t_i, w_i), && (40)\\
k_2 &= hf(t_i + \frac{h}{2}, w_i + \frac{k_1}{2}), && (41)\\
k_3 &= hf(t_i + \frac{h}{2}, w_i + \frac{k_2}{2}), && (42)\\
k_4 &= hf(t_i + h, w_i + k_3). && (43)
\end{aligned}
$$

CHAPTER IV
NUMERICAL EXPERIMENTS

10.1 Topography of a domain

As a field measurement, we assume pound surface area in each depth on Table 3.

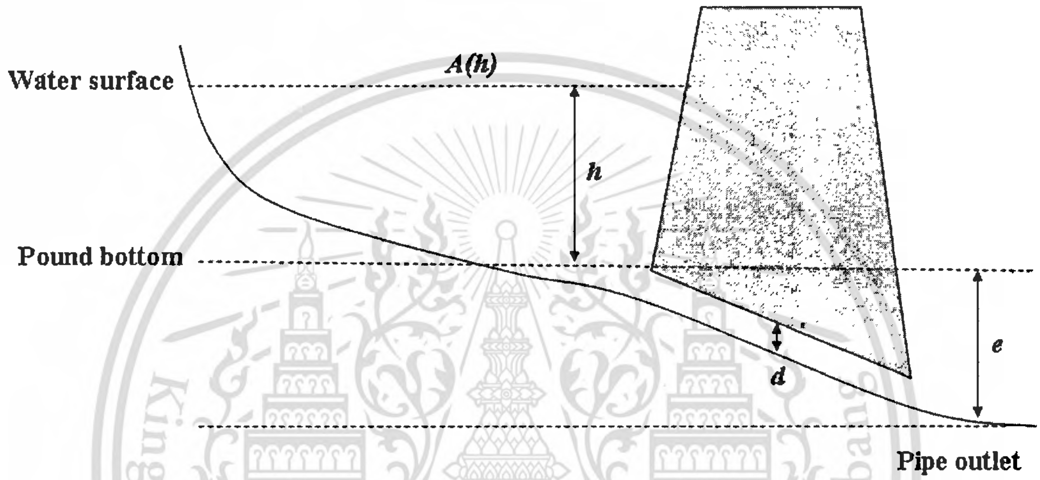

Figure 3: A pound drainage system

Physical parameter of a pound drainage system, that shown in Fig.(3), are assumed that the maximum of depth is 6 m, a pipe diameter is 0.2500 m and the depth of pipe outlet below the pound bottom is 1 m.

Table 3: Pound surface area in each depth (h) m.

Depth h (m)	6.00	5.00	4.00	3.00	2.00	1.00	0.00
Surface area $A(h)$	11700	9700	6700	4500	3200	1800	0

10.2 The initial conditions

The initial and boundary conditions are indicated in Fig.??. The initial condition, the atmosphere is assumed to be motionless. The boundary conditions for $t > 0$

are

$$
\begin{aligned}
C &= 0 \text{ at } 0 \leq x \leq 70, y = 34, && (44)\\
C &= 0 \text{ at } x = 0, 0 \leq y \leq 34, && (45)\\
\frac{\partial C}{\partial n} &= 0 \text{ at } 0 \leq x \leq 70, y = 0, && (46)\\
\frac{\partial C}{\partial n} &= 0 \text{ at } x = 70, 0 \leq y \leq 34. && (47)
\end{aligned}
$$

CHAPTER IV
DISCUSSION AND CONCLUSION

In this research, the pound surface area data are collected from the field measurement. So, we have to create the function of pound surface area. We also determine the function by using an accurate interpolation such as a cubic spline interpolation technique. The problem will require that the interpolating function be continuously differentiable.

To achieve more accuracy from the interpolating function, higher-degree polynomial pieces must be used. The most common choice is cubic polynomials. These cubic polynomial pieces can be combined in different ways to produce the overall interpolating function. So, we proposed the technique of cubic spline interpolation, that obtains the highest degree of smoothness from the piecewise interpolating function.

Consequently, we propose the Runge-Kutta method that can be obtained the approximate solutions of the initial value problem of the pond drainage problem.

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