A MATHEMATICAL MODEL OF SALINITY CONTROL IN THE CHAO PHRAYA RIVER WITH BARRAGE DAM USING EXPLICIT FINITE DIFFERENCE METHOD

PORNPON OTHATA

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN APPLIED MATHEMATICS DEPARTMENT OF MATHEMATICS SCHOOL OF SCIENCE

KING MONGKUT'S INSTITUTE OF TECHNOLOGY LADKRABANG

2021

KMITL-2021-SC-D-001-003

Thesis Title	A Mathematical Model of Salinity Control in the Chao Phraya River with Barrage Dam using Explicit Finite Difference Methods
Student Name	Mr.Pornpon Othata
Student ID	60605008
Degree	Doctor of Philosophy of Science (Applied Mathematics)
Department	Mathematics
Year	2021
Thesis Advisor	Asst.Prof.Dr.Nopparat Pochai

Abstract

Salinity relates to the amount of salt in rivers, such salt potentially taking many different forms. There are two main methods of defining the concentration of salt in water the total dissolved solid measurement (TDS) and the electrical conductivity measurement (EC). Salinity is measured by evaporating water to dryness and weighing the solid residue. The electrical conductivity measurement is measured by passing an electric current through water and measuring how readily the current it flows. The total amount of salt in water can affect the taste of water. Drinking water becomes significantly and increasingly unpalatable at salinity levels greater than about 1.0 g/L. In this research, two models the internal wave hydrodynamic model and the salinity dispersion model are proposed. First, a modified model of salinity control in a river with a barrage dam with generated salinity data was introduced. Second, a modified model of salinity control in a river with a barrage dam with field measurement raw salinity data was introduced. Finally, a modified model of salinity control in a river with a barrage dam with the salinity flow velocity by using the internal wave hydrodynamic model was introduced. The proposed model provides salinity control by the release fresh water from a barrage dam. An unconditionally stable explicit finite difference technique is used to approximate the salinity level under several conditions from the proposed model.

Keywords : Salinity, Water quality, Barrage dam, Sualyev method, Wave hydrodynamic model, One-dimensional salinity dispersion model, A modified Lax-diffusive method.

Acknowledgements

I would like to thank the many people who support me. My advisor Asst. Prof. Dr. Nopparat Pochai, for taking me on as a student, introducing me to mathematics and methodology knowledge and enduring my frequent tardiness. Asst. Prof. Dr. Nopparat Pochai, for give me a scholarship. My family for your encouragement and counseling in my studies. The thesis committee for providing suggestions on how to improve this thesis.

Pornpon Othata

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Research Motivation

1.1.1 Chao Phraya River

The Chao Phraya River is an important river in Thailand, formed by the merger of two main rivers from the north, the Ping and Nan rivers. By converging in front of the dam in the town of Pak Nam Pho Subdistrict, Mueang Nakhon Sawan District, Nakhon Sawan Province Which can clearly see the differences of both streams That is to say, the Nan River is quite red and the Ping River is quite green. When they meet, they gradually gather together to form a large river. Which can be considered that Nakhon Sawan is the origin city of the river, then flows southwards through Uthai Thani, Chai Nat, Sing Buri, Ang Thong, Phra Nakhon Si Ayutthaya, Pathum Thani, Nonthaburi and Bangkok. Before going to the Gulf of Thailand at Pak Nam. The Chao Phraya River as important as the main arteries of the central region Both in travel and way of life In addition to the construction of many bridges and waterfront There are also rivers, branches, natural canals and digging canal, which connects the Chao Phraya River with internal areas to be able to connect to each other with many rivers, branches and canals.

Figure 1.1: Chao Phraya River. Form : https://sites.google.com/site/sirindasss33/sthan-thi-thxng-theiyw/tn-maena-ceaphraya-pakna-pho

The Chao Phraya River is used as a transport route for excursions, tourist routes, cargo transportation, sports, water bathing, container washing. It is a recreation and lifestyle place for agriculture, animal husbandry and fishery, uses electricity and drainage industry, and most importantly, is a raw water source for tap water production for consumption in Bangkok and its vicinity.

1.1.2 Water production

Water production means the removal of surface water or raw water from natural water sources such as rivers, canals, reservoirs and the sea into the production process for the quality and quantity as per requirement such as tap water and pure water for use in consumer, agriculture and industry. Each type of production water can use different production technologies ”water production procedure”. Water supply is water that passes various processes much more than tap water can be provided to the people there are several production steps and requires a very high investment as the following production process

1) Pumping Water production starts with ”low pressure pumping stations” pumping raw water from natural sources to be transported into the production system. The raw water that can be used to produce tap water must be colorless, odorless, tasteless, and not contaminated with excess contaminants which has been analyzed and verified by scientists that can be used to produce tap water and must have a sufficient amount to continuously produce tap water.

2) Improvement of raw water quality raw water that has already been pumped will be mixed with chemicals such as alum and lime in order to improve the raw water quality alum solution helps to better precipitate and the lime solution will inhibit the growth of algae or algae in the water or sometimes chlorine is added to kill germs that may mix with water in this first layer.

3) Sedimentation, this process will release water mixed with alum and lime which causes the rotation to allow water and chemicals to combine to help with the coagulation of sediment better and will bring those water to a large sedimentation tank to cause stagnant water large sediment, heavy weight Will fall to the bottom of the tank and was thrown away the clear water above will flow along the water receiving trough to the next step.

4) Filtration, in the filtering process, coarse and fine sand is used for filtering very small sediments in the water and to have more clarity In this process, the water that is filtered is very clear, but the turbidity remains at approximately $0.2-2.0$ turbidity units and the filter sand will be washed regularly for effective filtering.

5) Disinfection, the filtered water will be clear. But there may be germs contaminated with water, so it must be disinfected using chlorine, chlorine, which can kill germs as well. The water that has been mixed with chlorine is called ”tap water”. It can be used for consumption and will be stored in a large tank called a clear water tank to manage the service further.

6) Water quality control, this step is an important step because the tap water that has already been produced must be analyzed and examined again by scientists and this inspection will be carried out regularly to get clean tap water that is safe for

consumption.

7) The distribution of the water supply that is produced must be delivered to the houses of the users by passing through the pipelines. Therefore, metering is necessary by sending from a tall tank that can service in the nearby area and in areas far away or very tall, water pressure compressors are necessary. So that the water supply can be serviced thoroughly.

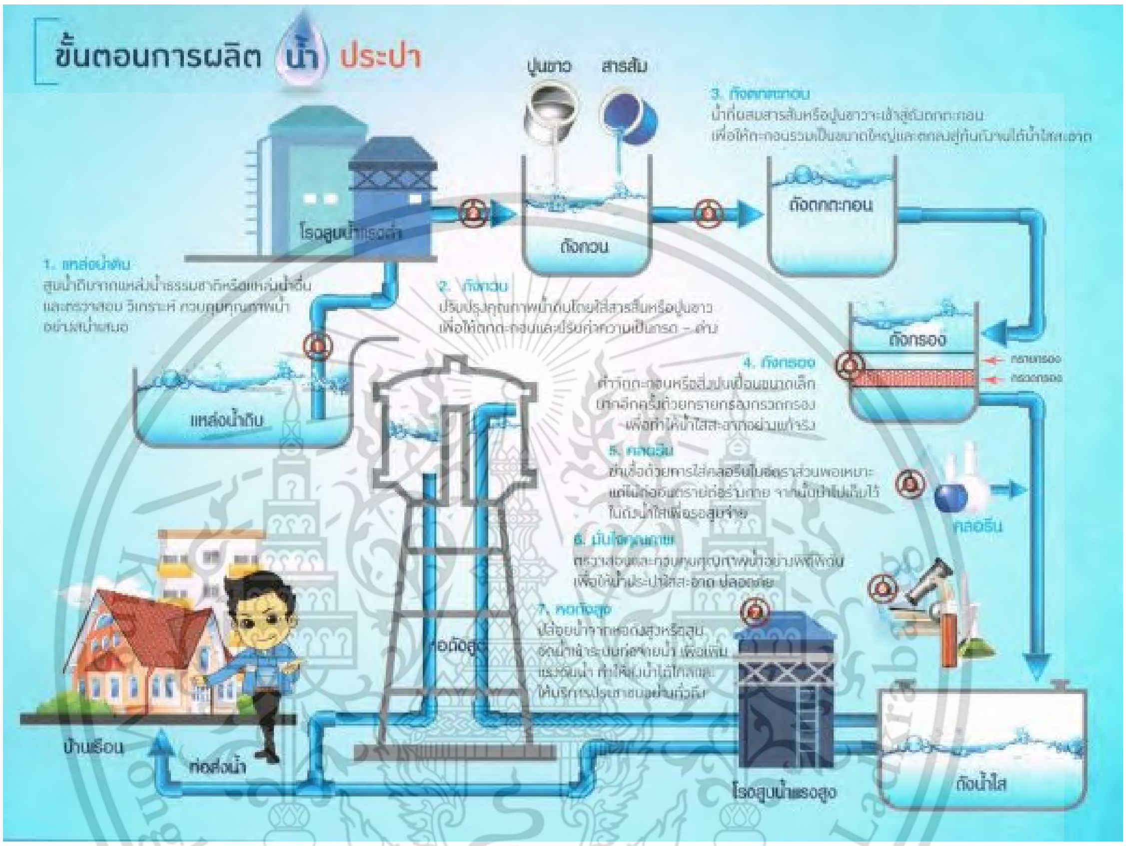


Figure 1.2: Water production process. From : http://www.pe.eng.ku.ac.th/files/semimar/2010/Group6/howto.html

Water supply systems will use surface water or raw water to produce water for use in water, which will be used for consumption, agriculture and certain industries that do not require high quality water. There are many factors that affect the quality of the water produced such as salinity of the water. It is a very important factor in the production because it can not be treated in the normal way. So, bring the water to the water treatment process, it is necessary to have a salinity standard.

The Waterworks Authority of Thailand has nine water quality monitoring stations located throughout the river. Each station has a distance from the estuary as shown in the Table 1.1.

Currently, the station used to pump raw water for use in the water supply process for consumption in Bangkok has a problem of salinity of water over than standard. That make an impact on the quality of water produced has a salinity up to

Figure 1.3: The Waterworks Authority of Thailand. From : https://www.pwa.co.th/province/branch/all

Table 1.1: Water quality monitoring stations in river, Thailand where S_7 is the main water supply pumping station for Bangkok.

Stations	Distance from the estuary
S_1	12
S_2	27
S_3	35
S_4	50
S_5	64
S_6	91
S_7	96
S_8	108

standard.

1.1.3 The effects of drinking water with salinity over standards

Today, there are research studies on the effects of drinking water with salinity over standards, such as [6], Salinity intrusion in coastal Bangladesh has serious population health implications, which are yet to be clearly understood. The study was undertaken through the Assessing Health, Livelihoods, Ecosystem Services and Poverty Alleviation in Populous Deltas project in coastal Bangladesh. Drinking water salinity and blood pressure measurements were carried out during the a household survey campaign. The study explored association among Socio-Ecological Systems (SESs), drinking water salinity and blood pressure. High blood pressure (prehyperten-

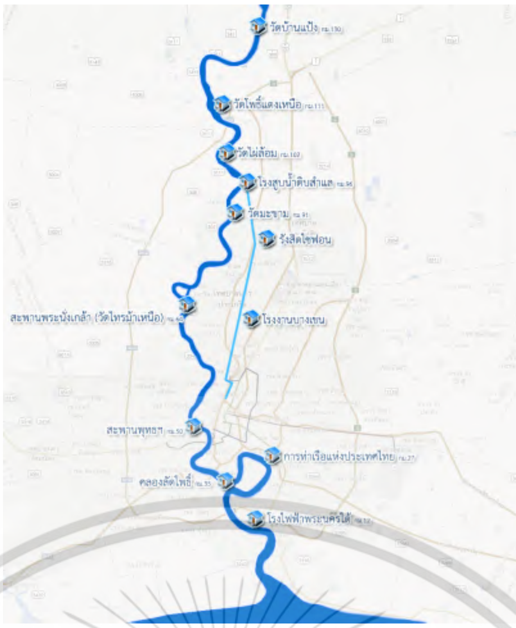


Figure 1.4: Map of salinity and water quality measurement stations in the Chao Phraya River. From : https://rwc.mwa.co.th/page/.

sion and hypertension) was found significantly associated with drinking water salinity. People exposed to slightly saline (1000-2000 mg/l) and moderately saline (2000 mg/l) concentration drinking water had respectively 17% and 42% higher chance of being hypertensive than those who consumed fresh water. Women had 31% higher chance of being hypertensive than men. Also, respondents of 35 years and above were about 2.4 times more likely to be hypertensive compared to below 35 years age group. For the 35 years and above age group, both prehypertension and hypertension were found higher than national rural statistics (50.1%) for saline water categories (53.8% for slightly and 62.5% for moderate saline). For moderate salinity exposure, hypertension prevalence was found respectively 21%, 60% and 48% higher than national statistics (23.6%) in consecutive survey rounds among the respondents. Though there was small seasonal variation in drinking water salinity, however blood pressure showed an increasing trend and maximum during the dry season. Mean salinity and associated hypertension prevalence were found higher for deep aquifer (21.6%) compared to shallow aquifer (20.8%).Localized increase in soil and groundwater salinity was predicted over the study area. Shallow aquifer salinity increase was projected based on modeled output of soil salinity. Rather than uniform increase, there were localized extreme values. Deep aquifer salinity was also predicted to exhibit increasing trend over the period. Study findings and recommendations are suggested for immediate and planned intervention.

There is also research on drinking salt water exceeding standards in children,such as [6], This study examines the impact of drinking water salinity on children?s education using a unique and rich dataset collected from eight southwest coastal districts

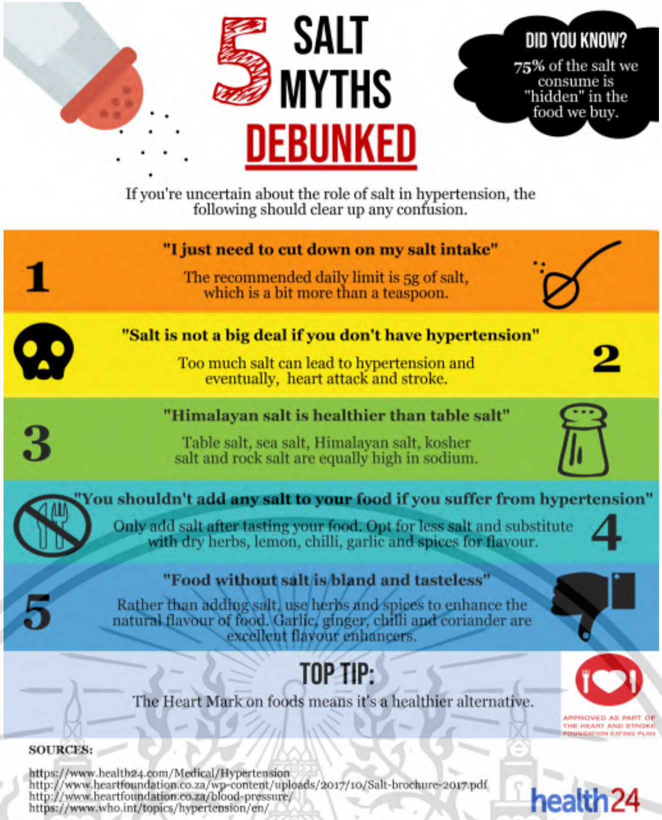


Figure 1.5: The dangers of drinking salinity water. From : https://www.health24.com/Medical/Hypertension/ About-hypertension/ salt-and-hypertension-5-common-myths-debunked-20190517-2

of Bangladesh. Salinity concentration in drinking water is measured at the household level using water samples from households primary source of drinking water during the summer, wet and dry season of 2014-2015. A third of the deep tube-well water samples was found to be slightly ($1,000 < TDS < 2,000$ mg/l) to moderately (TDS $\leq$ 2,000 mg/l) saline. Linking the child-level data on educational outcome to water salinity (i.e. TDS level), the study reveals a statistically significant negative effect of excessive salinity on grade advancement for 7-12 year old children. More specifically, exposure to excessive drinking water salinity ($TDS > 1,000$ mg/l) decreases the grade advancement likelihood of 7-12 year old children by 6.7 percentage points. The results remain robust to alternative model and econometric specifications. The adverse effect of salinity on grade advancement does not vary significantly across the gender of the child while poverty, as expected, exacerbates the effect. Impaired cognitive development due to early childhood exposure appears to be the most plausible channel through which the negative effects of excessive sodium consumption permeate to young children educational deficit. Additionally, poor health of the adults and elevated medical expenditure play a small yet significant mediating role.

1.1.4 Salinity water problems in Thailand.

In Thailand, the intrusion of saltwater is an important issue. This problem affects many things, such as agriculture, raising fish in floating cages and most importantly producing tap water. The salinity of the water affects the health of people in Bangkok and its vicinity, including people in the area near the estuary. This problem will intensify during at the dry season. By measuring the salinity as of December 14, 2019 and December 25, 2019, the salinity value is as appeared in the table 1.2.

Table 1.2: The salinity as of December 14, 2019 and December 25, 2019

Stations	Salinity level at 14/12/2019	Salinity level at 25/12/2019
S_1	24.74	23.78
S_2	19.64	18.72
S_3	14.6	17.34
S_4	8.82	11.72
S_5	3.99	5.79
S_6	0.25	1.73
S_7	0.2	0.79

Figure 1.6: Salinity water problems in Thailand. From : https:// www.facebook.com/Mono29News/posts/933285303740290/

The solution to the salinity intrusion of the Waterworks Authority of Thailand is to release the water from the dam at the beginning of the Chao Phraya River. In which the release of water from the dam to reduce salinity water can actually reduce the amount of salinity water. Because the fresh water that is discharged from the dam takes about 3 days to reach the pumping station, causing the water supply during that time to exceed the standard salinity. And most importantly, the release of water from the dam cannot be done all the time because fresh water is needed to be reserved for consumption. When the water is discharged from the dam for a while, the salinity water will rise again.

1.2 Literature Review

In [1], the finite difference method was utilized to explain water contamination models. In [2], the finite difference method was utilized to solve a hydrodynamic model with consistent coefficients in a closed uniform reservoir. In [3], a analytical solutio for a hydrodynamic model in a closed uniform reservoir was proposed. In [4], the Lax-Wendroff finite difference method was also proposed to approximate the salinity elevation and salinity flow velocity. In [?], the fourth-order method for a one-dimensional water quality model in a non-uniform flow stream was proposed. In [5], a non-dimensional form of a two-dimensional hydrodynamic model with generalized boundary condition and initial conditions for describing the elevation of water wave in an open uniform reservoir was proposed.

In this way, look into has been introduced on the expansion of salt water, for example, [7]. The outstanding numerical model of preservationist property was used for defining the diffusion of salinity water in a one-dimensional model [8].

$$A(x)\frac{\partial S}{\partial t} + Q(x,t)\frac{\partial S}{\partial x} = \frac{\partial}{\partial x}\left[A(x)D_x\frac{\partial S}{\partial x}\right], \tag{1.1}$$

where $A(x)$ is cross-sectional area of the river (m^2), $Q(x,t)$ is Flow rates (m^3/s), D_x is diffusion coefficient of water (m^2/s), S is salinity value (ppt), x is distance (m) and t is times (s)

In this research, a one-dimensional mathematical model of salinity measurement with non-uniform internal waves in a river is proposed. A one-dimensional hydrodynamic model of the internal salinity flow in a river is introduced. A modified model of salinity control in a river with a barrage dam is also proposed. A modified Lax-diffusive method is used to approximate the solution of the internal wave hydrodynamic model. An unconditionally stable explicit finite difference technique is used to approximate the saltiness level under a few conditions from the proposed model. The proposed computational method can be applied in reasonable situations for water supply forms.

1.3 Objectives of the study

1) We will introduce a field salinity measurement by monitoring stations in a river.

2) We will propose a mathematical model of salinity measurement in a river under assumption of uniform salinity flow.

3) We will propose a mathematical model of salinity measurement in a river under assumption of uniform salinity flow with a barrage dam.

4) We will propose a mathematical model of salinity control in a river under assumption of uniform salinity flow with a barrage dam.

5) We will propose a mathematical model of salinity measurement in a river under assumption of non-uniform salinity flow.

6) We will propose a mathematical model of salinity control in a river under assumption of non-uniform salinity flow with barrage dam.

1.4 Scopes of the study

1) Study the rate of salinity water diffusion rate to be constant.

2) Study the rate of salinity water as diffusion a interpolation function.

3) Study the rate of salinity water as diffusion in the Chao Phraya River.

4) Use the salinity of the water from the water supply as the criterion for comparing the accuracy.

1.5 Methodology

1) Study a field salinity measurement by monitoring stations in a river.

2) Generated a mathematical model of salinity measurement in a river under assumption of uniform salinity flow.

3) Generated a mathematical model of salinity measurement in a river under assumption of uniform salinity flow with a barrage dam.

4) Generated a mathematical model of salinity control in a river under assumption of uniform salinity flow with a barrage dam.

5) Define initial condition and boundary condition consistent with problem of a mathematical model of salinity in a river under assumption of uniform salinity flow.

6) Define a mathematical model of salinity measurement in a river under assumption of non-uniform salinity flow.

7) Define a mathematical model of salinity control in a river under assumption of non-uniform salinity flow with barrage dam.

8) Define initial condition and boundary condition consistent with problem of a mathematical model of salinity measurement in a river under assumption of non-uniform salinity flow.

9) Simulated the simulation of problem of salinity control in a river with a barrage dam.

10) Using numerical methods to approximate solution problem of salinity control in a river with a barrage dam.

1.6 Benefits of the study

1) We can measure salinity of water in the river.

2) We can forecast the level of salinity in the water along the river.

3) We can forecast the velocity of fresh water form barrage dam to diluted the salinity level of the water.

Chapter 2

Governing Equations

2.1 One-dimensional salinity dispersion model in a river with a barrage dam

In a salinity dispersion model, the governing equation is the dynamic one-dimensional advection-dispersion equation. A simplified representation, averaging the equation over the depths, as shown in [11]

$$\frac{\partial c}{\partial t} + u(x,t)\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2}, \tag{2.1}$$

for all $(x,t) \in \Omega = [0,L] \times [0,T]$, u is the salinity flow velocity, and D is given diffusion coefficient.

Expecting that the saltiness is weakened by the freshwater. These are then the saltiness shift in weather conditions level is decreased by the freshwater speed. The rate capacity of freshwater to weaken saltiness is accepted by $0 \leq k \leq 1$. The one-dimensional saltiness water contamination estimation model in a waterway with a torrent dam [11] can be expected by

$$\frac{\partial c}{\partial t} + (u_s - ku_w)\frac{\partial c}{\partial x} = D_s\frac{\partial^2 c}{\partial x^2}, \tag{2.2}$$

where $c(x,t)$ is the salinity concentration(kg/m^3), u_s is advective velocity of salinity water (m/s), k is water salinity removal efficiency rate, u_w is the fresh water flow velocity.

2.1.1 Initial conditions

The initial condition is defined by an interpolation function of measured raw salinity data, it is adjust on the length of the a stream from the estuary as far as possible of thought about territory. The initial condition is assumed by

$$c(x,0) = f(x), \tag{2.3}$$

for all $x \in [0,L]$, where $f(x)$ is a interpolation function of measured salinity data.

2.1.2 Left boundary condition

The left boundary condition is a interpolation function of estimated crude information, It depends on the saltiness of a stream at the main station that shut to the estuary. The boundary condition is assumed by

$$c(0,t) = g(t), \tag{2.4}$$

for all $t \in [0,T]$, where $g(t)$ is a given interpolation function by approximated salinity information at the first observing station.

2.1.3 Right boundary condition

The right boundary condition defined by the pace of progress of saltiness zone of the water. The condition can be assumed by

$$\frac{\partial c}{\partial x} = C_R, \tag{2.5}$$

for all $t \in [0,T]$, where C_R is an approximated pace of progress of saltiness around the last observing station.

2.2 One-dimensional salinity internal wave hydrodynamic model

The one-dimensional shallow water equations are obtained by integrating the Navier-Stokes equations over the flow depth under the assumptions as hydrostatic pressure distribution and small bottom slope. The hydrodynamic flows are high velocity and can be considered as advection dominated shallow salinity flows. Therefore, the eddy viscosity terms can be neglected. The governing equations on conversation and vector form can be written in the system of partial differential equations [10] as

$$\partial_x \begin{pmatrix} h \\ hu \end{pmatrix} + \partial_t \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix} = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix}, \tag{2.6}$$

where x is the longitudinal distance along a stream (m), t is time (s), $h(x,t)$ is the elevation of the salinity wave above the bottom (m/s), $z(x)$ is the function characterizing the bottom topography (m), and $u(x,t)$ is the velocity components (m/s), for all $(x,t) \in [0,L] \times [0,T]$. The initial conditions are given by

$$u(x,0) \;=\; u_1(x) \;\; for\ all \;\; 0 \le x \le L \tag{2.7}$$

and

$$h(x,0) \;=\; h_1(x) \;\; for\ all \;\; 0 \le x \le L \tag{2.8}$$

The boundary conditions are also given by

$$\frac{\partial u(0,t)}{\partial x} = f_1(t), t > 0, \tag{2.9}$$

$$\frac{\partial u(L,t)}{\partial x} = f_2(t), t > 0, \tag{2.10}$$

$$\frac{\partial h(0,t)}{\partial x} = g_1(t), t > 0, \tag{2.11}$$

$$\frac{\partial h(L,t)}{\partial x} = g_2(t), t > 0. \tag{2.12}$$

So as to be predictable with the physical marvel of a hydrodynamic from the left to one side. At t = 0, the dam breakdown and the stream issue comprises of a stun wave voyaging downstream and a rarefaction wave voyaging upstream.

Chapter 3

Numerical Techniques

We now discretize the domain by dividing the interval $[0, L]$ into M subintervals such that $M\Delta x = L$ and the time interval $[0, T]$ into N subintervals such that $N\Delta t = T$. The grid points (x_m, t_n) are defined by $x_m = m\Delta x$ for all $m = 1, 2, 3, ..., M$ and $t_n = n\Delta t$ for all $n = 1, 2, 3, ..., N$ in which M and N are positive integers. We can then approximate $c(x_m, t_n)$ by C_m^n , value of the difference approximation of $c(x, t)$ at point $x = m\Delta x$ and $t = n\Delta t$, where $0 \le m \le M$ and $0 \le n \le N$.

3.1 A modified Lax-diffusive method for the hydrodynamic model

The hydrodynamic model gives the speed field and height of the water. At that point the determined consequences of the model will be the contribution to the dispersion model which gives the contamination focus field. In this section, the modify method of a traditional Lax-diffusive method for the hydrodynamic model of [12] is proposed.

We will modify f^* from the traditional method of [12] to be the three points average. The discretization of Eq.(2.6) is base on a Lax-diffusive scheme. The semi-discrete scheme is applied to Eq.(2.6) and using a uniform spatial grid $(x_m, t_n) = (m\Delta x, n\Delta t)$, we can define

$$f_x = \frac{f_{m+1}^n - f_{m-1}^n}{2\Delta x}, \tag{3.1}$$

$$f_t = \frac{f_m^{n+1} - f_m^*}{\Delta t}, \tag{3.2}$$

where

$$f^* = \frac{f_{m+1}^n + f_m^n + f_{m-1}^n}{3}, \tag{3.3}$$

The partial derivative of h and u with respect to x and t are approximated by using Eqs.(3.1-3.3), respectively. We can see that Eq.(2.6) is written in a matrix form as

$$A_t + B_x + C = 0, \tag{3.4}$$

where

$$A = \begin{pmatrix} h \\ hu \end{pmatrix}, B = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix}, \tag{3.5}$$

It follows that Eq.(3.5) can be written by the uniform spatial grids as

$$A_m^n = \begin{pmatrix} h_m^n \\ h_m^n u_m^n \end{pmatrix}, B_m^n = \begin{pmatrix} h_m^n u_m^n \\ h_m^n (u_m^n)^2 + \frac{1}{2}g(h_m^n)^2 \end{pmatrix}, C_m^n = \begin{pmatrix} 0 \\ -gh_m^n \partial_x z \end{pmatrix}, \tag{3.6}$$

Substituting the finite difference approximations of Eqs.(3.1-3.3) into Eq.(3.4), we obtain that

$$A_m^{n+1} = \frac{\Delta t}{2\Delta x}\left(B_{m-1}^n - B_{m+1}^n\right) + A^*, \tag{3.7}$$

where $A^* = \begin{pmatrix} h^* \\ (hu)^* \end{pmatrix}$. Substituting Eq.(3.6) into Eq.(3.7), we can see that

$$\begin{aligned}\begin{pmatrix} h_m^{n+1} \\ h_m^{n+1}u_m^{n+1} \end{pmatrix} &= \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{m-1}^n u_{m-1}^n - h_{m+1}^n u_{m+1}^n \\ h_{m-1}^n\left(u_{m-1}^n\right)^2 - h_{m+1}^n\left(u_{m+1}^n\right)^2 + \frac{1}{2}g\left(\left(h_{m-1}^n\right)^2 - \left(h_{m+1}^n\right)^2\right) \end{pmatrix} \\ &+ \begin{pmatrix} h_{m-1}^n + h_m^n + h_{m+1}^n \\ h_{m-1}^n u_{m-1}^n + h_m^n u_m^n + h_{m+1}^n u_{m+1}^n \end{pmatrix}, \end{aligned} \tag{3.8}$$

for all $1 \leq m \leq M$ and $0 \leq n \leq N-1$. For upper boundary, where $m = 0$, plug the known value of the left boundary by $u_{-1}^n = u_0^n$ and $h_{-1}^n = h_0^n$ into Eq.(3.8) in the right-hand side, we obtain

$$\begin{pmatrix} h_1^{n+1} \\ h_1^{n+1}u_1^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_0^n u_0^n - h_1^n u_1^n \\ h_0^n(u_0^n)^2 - h_1^n(u_1^n)^2 + \frac{1}{2}g\left((h_0^n)^2 - (h_1^n)^2\right) \end{pmatrix} + \begin{pmatrix} 2h_0^n + h_1^n \\ 2h_0^n u_0^n + h_1^n u_1^n \end{pmatrix}, \tag{3.9}$$

For lower boundary, where $m = M$, substituting the approximate unknown value of the right boundary by boundary conditions, we can let $u_{M+1}^n = u_M^n$ and $h_{M+1}^n = h_M^n$ by rearranging, we obtain

$$\begin{aligned}\begin{pmatrix} h_M^{n+1} \\ h_M^{n+1}u_M^{n+1} \end{pmatrix} &= \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{M-1}^n u_{M-1}^n - h_M^n u_M^n \\ h_{M-1}^n\left(u_{M-1}^n\right)^2 - h_M^n(u_M^n)^2 + \frac{1}{2}g\left(\left(h_{M-1}^n\right)^2 - (h_M^n)^2\right) \end{pmatrix} \\ &+ \begin{pmatrix} h_{M-1}^n + 2h_M^n \\ h_{M-1}^n u_{M-1}^n + 2h_M^n u_M^n \end{pmatrix}, \end{aligned} \tag{3.10}$$

The stability condition of the scheme needed CFL number as [12],

$$C_n = u_{\text{max}}\left(\frac{\Delta t}{\Delta x}\right) \leq 1. \tag{3.11}$$

3.2 Forward Time Central Space Finite Difference Scheme

Taking the forward time central space technique [11] into Eq.(2.6) , we get the following discretization:

$$c\left(x_i, t_n\right) \cong C_i^n, \tag{3.12}$$

$$\left.\frac{\partial c}{\partial t}\right|_{(x_i,t_n)} \cong \frac{C_i^{n+1} - C_i^n}{\Delta t}, \tag{3.13}$$

$$\left.\frac{\partial c}{\partial x}\right|_{(x_i,t_n)} \cong \frac{C_{i+1}^n - C_{i-1}^n}{2\Delta x}, \tag{3.14}$$

$$\left.\frac{\partial^2 c}{\partial x^2}\right|_{(x_i,t_n)} \cong \frac{C_{i+1}^n + C_{i-1}^n - 2C_i^n}{\left(\Delta x\right)^2}, \tag{3.15}$$

$$u_s(x_i, t_n) = u_{s_i}^n, \tag{3.16}$$

$$u_w(x_i, t_n) = u_{w_i}^n. \tag{3.17}$$

Substituting Eqs.(3.12-3.17) into Eq.(2.6), we get the finite difference equation,

$$\frac{C_i^{n+1}-C_i^n}{\Delta t}+(u_{s_i}^n-ku_{w_i}^n)\left(\frac{C_{i+1}^n-C_{i-1}^n}{2\Delta x}\right)=D_s\left(\frac{C_{i+1}^n+C_{i-1}^n-2C_i^n}{(\Delta x)^2}\right). \tag{3.18}$$

Then the explicit finite difference equation becomes

$$C_{i+1}^{n+1}=(\lambda+0.5r_i^n)C_{i-1}^n+(1-2\lambda)C_i^n+(\lambda-0.5r_i^n)C_{i+1}^n, \tag{3.19}$$

for all $i=1,2,3,...,M-1$, where $\lambda=\frac{D_s\Delta t}{(\Delta x)^2}$ and $r_i^n=\frac{(u_{s_i}^n-ku_{w_i}^n)\Delta t}{\Delta x}$. The forward time central space scheme is conditionally stable subject to constraints in Eq.(3.18). The stability requirements for the scheme is [11] $\lambda<\frac{1}{2}$ and $r_i^n<1$.

For the right boundary condition Eq.(2.11), the right boundary condition defined by the rate of change of salinity area of the water. The right boundary condition is assumed by

$$\frac{\partial c}{\partial x}\approx\frac{c(L_2,t)-c(L_1,t)}{L_2-L_1}, \tag{3.20}$$

for all $t\in[0,T]$, where L_1 and L_2 is the distance from the upstream to the point before the and after the water supply source respectively. If substituting the approximate unknown value of the right boundary, we obtain

$$C_{M+1}^n=\left(\frac{C_{M_2}^n-C_{M_1}^n}{L_2-L_1}\right)\Delta x+C_{M-1}^n. \tag{3.21}$$

The forward time central space scheme is conditionally stable subject to constraints in Eq.(3.18). The stability requirements for the scheme is [11]. It can be obtained that the strictly stability requirements are the main disadvantage of this scheme.

3.3 Saulyev explicit finite difference scheme for one-dimensional salinity dispersion model in a river with a barrage dam

The Saulyev scheme is unconditionally steble [11]. Obviously the nonstrictly dependability prerequisite of Saulyev scheme is the principle of preferred position and conservative to utilize. Taking Saulyev technique [11] into Eq. (2.6), it tends to be gotten the following discretization:

$$c(x_m,t_n)\cong C_m^n, \tag{3.22}$$

$$\left.\frac{\partial c}{\partial t}\right|_{(x_m,t_n)}\cong\frac{C_m^{n+1}-C_m^n}{\Delta t}, \tag{3.23}$$

$$\left.\frac{\partial c}{\partial x}\right|_{(x_m,t_n)}\cong\frac{C_{m+1}^n-C_{m-1}^{n+1}}{2\Delta x}, \tag{3.24}$$

$$\left.\frac{\partial^2 c}{\partial x^2}\right|_{(x_m,t_n)}\cong\frac{C_{m+1}^n-C_m^n-C_m^{n+1}+C_{m-1}^{n+1}}{(\Delta x)^2}, \tag{3.25}$$

$$u_{s_m}^n\cong u_m^n, \tag{3.26}$$

$$u_{w_m}^n=u_w(x_m,t_n). \tag{3.27}$$

Substituting Eqs.(3.22-3.25) into Eq.(2.6), we get the finite difference equation,

$$\frac{C_m^{n+1}-C_m^n}{\Delta t}+(u_{s_m}^n-ku_{w_m}^n)\left(\frac{C_{m+1}^n-C_{m-1}^{n+1}}{2\Delta x}\right)=D_s\left(\frac{C_{m+1}^n-C_m^n-C_m^{n+1}+C_{m-1}^{n+1}}{(\Delta x)^2}\right) \quad (3.28)$$

Then the explicit finite difference equation becomes

$$C_{m+1}^{n+1}=\left(\frac{1}{1+\lambda}\right)\left[\left(\lambda+\frac{1}{2}r_m^n\right)C_{m-1}^{n+1}+(1-\lambda)C_m^n+\left(\lambda-\frac{1}{2}r_m^n\right)C_{m+1}^n\right]. \quad (3.29)$$

for all $i=1,2,3,...,M-1$, where $\lambda=\frac{D_s\Delta t}{(\Delta x)^2}$ and $r_m^n=\frac{(u_{s_m}^n-ku_{w_m}^n)\Delta t}{\Delta x}$. For $i=M$ the right boundary condition Eq.(2.9), if substituting the approximate unknown value of the right boundary, we obtain

$$C_{M+1}^n=\left(\frac{C_{M_2}^n-C_{M_1}^n}{L_2-L_1}\right)\Delta x+C_{M-1}^n. \quad (3.30)$$

Using Taylor series expansions on the approximation, [9] has shown that the truncation error is $O\left\{(\Delta x)^2+(\Delta t)^2+(\Delta t/\Delta x)^2\right\}$. The Saulyev technique is an unconditionally stable method [11]. It follows that the utilization of the explicit Saulyev finite difference technique is prudent calculation execution.

3.4 Lagrange interpolating polynomial

The problem of evaluating the first degree polynomials that passes through the (x_0,y_0) separate points (x_0,y_0) and (x_1,y_1) is the same as estimating a f function for which $f(x_0)=y_0$ and $f(x_1)=y_1$ by means of a first degree polynomial interpolation at the specified points with the values of f. Using this polynomial for approximation is called polynomial interpolation within the interval provided by the endpoints. Define the functions

$$L_0(x)=\frac{x-x_1}{x_0-x_1},\ \ L_1(x)=\frac{x-x_0}{x_1-x_0}, \quad (3.31)$$

The linear Lagrange interpolating polynomial through (x_0,y_0) and (x_1,y_1) is

$$\begin{aligned} P_n(x) &= L_0(x)f(x_0)+L_1(x)f(x_1) \\ &= \frac{x-x_1}{x_0-x_1}f(x_0)+\frac{x-x_0}{x_1-x_0}f(x_1). \end{aligned} \quad (3.32)$$

Note that

$$L_0(x_0)=1,\ \ L_0(x_1)=0,\ \ L_1(x_0)=0,\ \ L_1(x_1)=1, \quad (3.33)$$

which implies that

$$\begin{aligned} P(x_0) &= 1\cdot f(x_0)+0\cdot f(x_1)=f(x_0)=y_0, \\ P(x_1) &= 0\cdot f(x_0)+1\cdot f(x_1)=f(x_1)=y_1. \end{aligned} \quad (3.34)$$

Then, P is the most unique polynomial of degree that passes through (x_0,y_0) and (x_1,y_1). In this case, we construct first, for every $k=0,1,2,...,n$, a function $L_{n,k}(x)=1$

with the property that $L_{n,k}(x_i) = 0$ when $i \neq k$ and $L_{n,k}(x) = 1$. To satisfy $L_{n,k}(x_i) = 0$ for each $i \neq k$,it is required that the numerator of $L_{n,k}(x)$ includes the term $(x-x_0)(x-x_1)...(x-x_{k-1})(x-x_{k+1})...(x-x_n)$.

To satisfy $L_{n,k}(x) = 1$, The $L_{n,k}(x)$ denominator must be the same term but must be valued at $x = x_k$. Thus,

$$L_{n,k}(x) = \frac{(x-x_0)\dots(x-x_{k-1})(x-x_{k+1})\dots(x-x_n)}{(x_k-x_0)\dots(x_k-x_{k-1})(x_k-x_{k+1})\dots(x_k-x_n)} \quad (3.35)$$

Theorem 3.4.1. *If $x_0, x_1, ..., x_n$ are $n+1$ distinct numbers and f is a function whose values are given at these numbers, then a unique polynomial $P(x)$ of degree at most n exists with $f(x_k) = P(x_k)$, for each $k = 0, 1, ..., n$.*

This polynomial is given by

$$\begin{aligned} P(x) &= f(x_0)L_{n,0}(x) + \dots + f(x_n)L_{n,n}(x) \\ &= \sum_{k=0}^{n} f(x_k)L_{n,k}(x), \end{aligned} \quad (3.36)$$

where, for each $k = 0, 1, ..., n$,

$$L_{n,k}(x) = \prod_{i=0, i\neq k}^{n} \frac{(x-x_i)}{(x_k-x_i)}, \quad (3.37)$$

We can write $L_{n,k}(x)$ simply as $L_k(x)$ when there is no doubt as to the degree.

The approximation error of Lagrange interpolation polynomial is $\left|P(x) - \tilde{f}(x)\right|$, where $\tilde{f}(x)$ is the polynomial interpolating.

Chapter 4

Numerical models of salinity controlling

4.1 Numerical simulation with generated salinity data

4.1.1 Simulation 1 : salinity control in an ideal case.

We consider a segment of a river with 108 km of length as shown in Table 1.1 Assuming that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30% and the given simulated station any time is 1 day. It is estimated into 100 time steps. This means that 1 time step is referred to 14.4 min. Their physical parameters and give spacing are shown in Table 4.1. In [6], the theoretical solution is given by

$$c(x,t) = \frac{1}{\sqrt{4t+1}} \exp\left[-\frac{(x-1-(u_s-ku_w)t)^2}{D(4t+1)}\right]. \tag{4.1}$$

Actually when using the FTCS scheme Eq.(3.19) and the Sualyev technique Eq. (3.29) when their physical parameter are following by Table 4.1, we get the approximated solution $c(x,t)$. The theoretical solution is illustrated by a surface of solution in Fig 4.1 The FTCS approximated solution is illustrated by Fig 4.2 The Sualyev approximated solution is also illustrated by Fig 4.3 The maximum absolute error of both finite difference approximations are compared in Table 4.2

Table 4.1: Physical parameters of simulation 1.

D_s (m^2/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (s)
0.1	0.065	0.25	0.3	108	100

Table 4.2: The maximum absolute error defined by $err_{\max} = \max|\tilde{c}(x_i,T) - c(x_i,T)|$ for all $i = 0,1,..,N$ where $T = 10, 20, 30\ and\ 40$.

	err_{max}	
t	FTCS	Saulyev
10	5.9442×10^{-4}	5.1141×10^{-4}
20	0.0044	0.0042
30	0.0083	0.0082
40	0.0107	0.0107

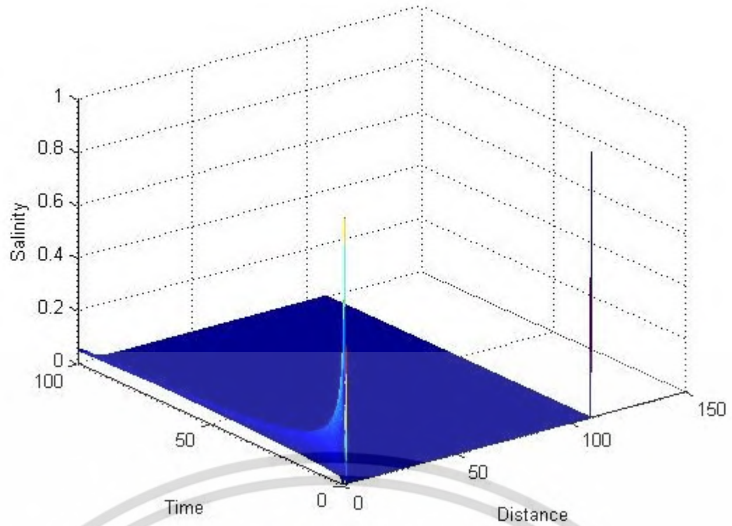


Figure 4.1: The exact solution of simulation 1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 100$.

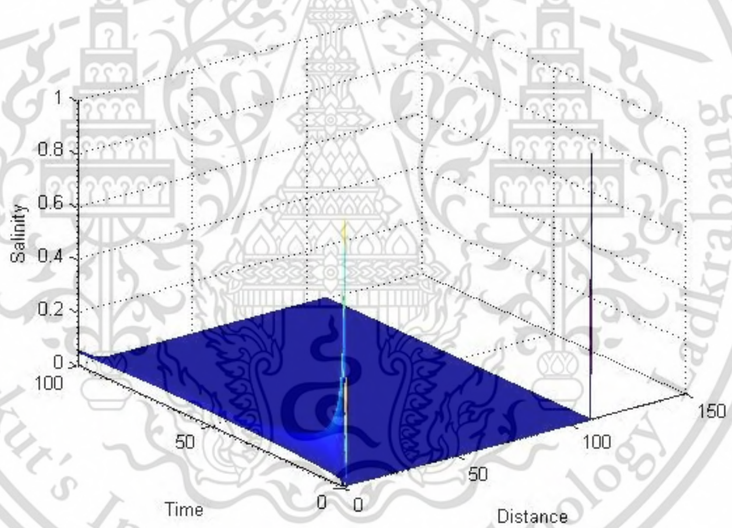


Figure 4.2: The FTCS solution of simulation 1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 100$.

4.1.2 Simulation 2 : the salinity is diluted by releasing the fresh water form a barrage dam with difference flow velocities.

We consider a segment of a river with 108 km of length as shown in Table 1.1. Assuming that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30% and the given simulated station any time is 10 days. It is estimated into 1000 time steps. This means that 1 time step is referred to 14.4 min. Their physical parameters and give spacing are shown in Table 4.3. In this simulation Sualyev technique is used to approximate the

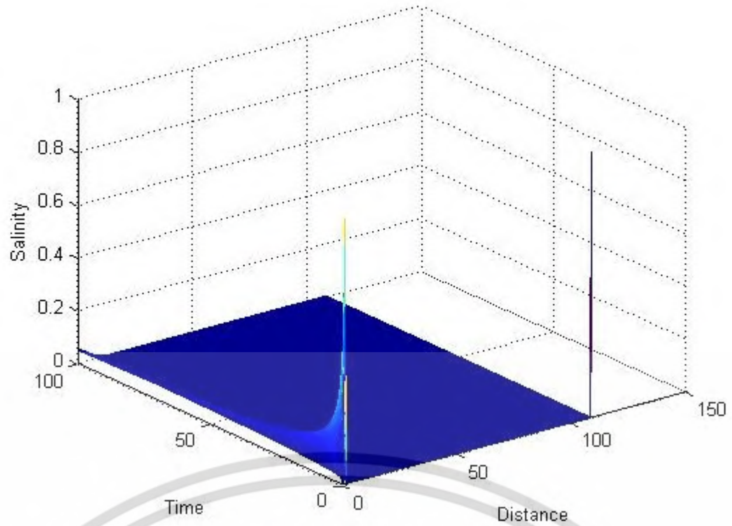


Figure 4.3: The Sualyev solution of simulation 1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 100$.

solution due to the technique will always gives stable solutions as shown in Table 4.4. According to the good agreement approximated solutions of the Sualyev method, the method Eq.(3.29) is chosen to approximate the solution of the simulation. The several fresh water flow velocities $u_w = 0.20, 0.25, 0.30$ m/s from the barrage dam are simulate until the salinity level at the controlled monitoring station S_7 becomes standardized level as shown in Fig 4.4-4.5.

Table 4.3: Physical parameters of simulation 2.

D_s (m^2/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (s)
0.1	0.065	0.3	0.3	108	100
0.1	0.065	0.25	0.3	108	100
0.1	0.065	0.2	0.3	108	100

4.1.3 Simulation 3 : the salinity is diluted by releasing the fresh water form a barrage dam and change flow velocities after the salinity come to standard.

We consider a segment of a river with 108 km of length as shown in Table 1.1. Assuming that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30% and the given simulated station any time is 10 days. It is estimated into 1000 time steps. This means that 1 time step is referred to 14.4 min. Their physical parameters and give spacing are shown in Table 4.5. Assuming that there are 8 monitoring station along a considered river segment as shown in Table 1.1 The controlled monitoring station is the station

Table 4.4: Convergence of FTCS method and Sualyev method for some grid spacing.

T	Δx	Δt	FTCS	Saulyev
100	0.10	0.04	Stable	Stable
		0.05	Stable	Stable
		0.06	Untable	Stable
		0.07	Untable	Stable
100	0.05	0.04	Untable	Stable
		0.05	Untable	Stable
		0.06	Untable	Stable
		0.07	Untable	Stable
100	0.025	0.04	Untable	Stable
		0.05	Untable	Stable
		0.06	Untable	Stable
		0.07	Untable	Stable

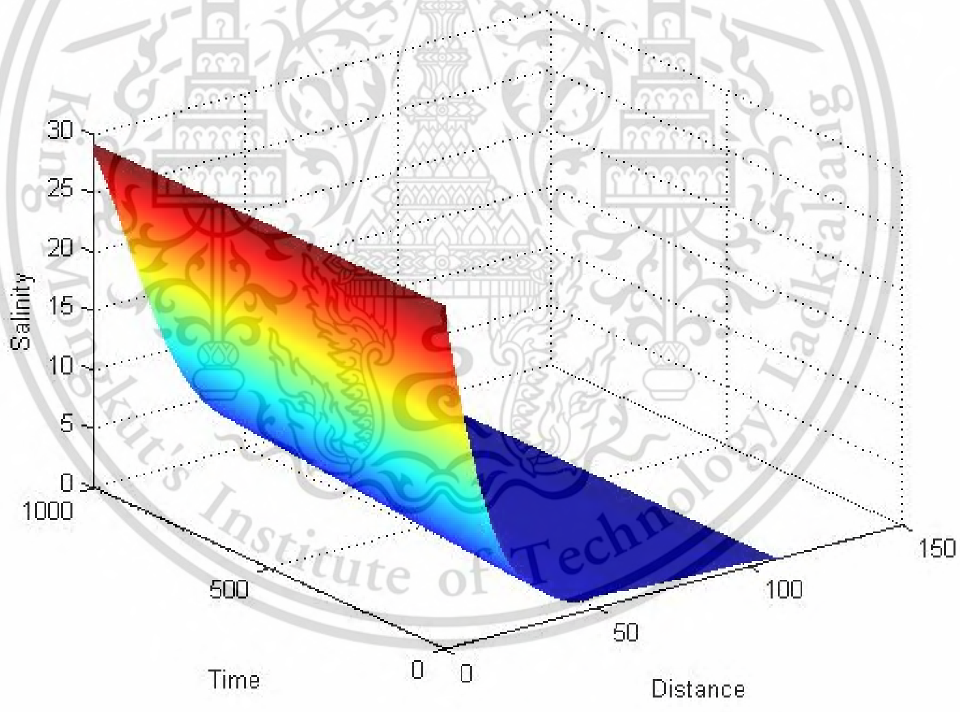


Figure 4.4: Approximated salinity along the river by using the Sualyev technique where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 1000$.

S_7. We need to control the salinity level at the station S_7 to be under the salinity standard level $C_{ST} = 0.3\ kg/m^3$. The salinity is controlled by a process as follows:

1) If the salinity level at the station S_7 $c(96,t) > C_{ST}$, then the fresh water will be released in a high speed from the barrage dam by controlled flow velocity.

2) If the salinity level at the station S_7 $c(96,t) < C_{ST}$, then the fresh water will

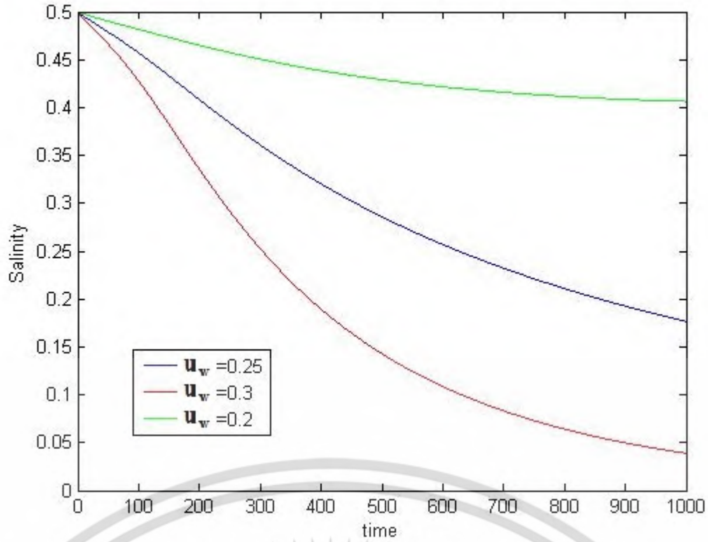


Figure 4.5: Approximated salinity at $c(96, t)$ by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$ when $u_w = 0.3, 0.25$ and 0.2.

be release in a low speed level from the barrage dam.

We can obtain the approximated salinity level along the considered river segment as shown in Fig 4.6 and Table 4.6 The salinity level at the several monitoring stations S_1, S_5 and S_7 as shown in Fig 4.7, 4.8 and 4.9 respectively.

Table 4.5: Physical parameters of simulation 3.

$c(x,t)$ at S_7	D (m^2/s)	u_s (m/s)	u_w (m/s)
$> C_{ST}$	0.1	0.065	0.25
$< C_{ST}$	0.1	0.065	0.205
K	T	L (km)	c(0,t)
0.3	1000	108	g(t)
0.3	1000	108	g(t)

4.1.4 Simmulation 4 : diluted the salinity water by releasing fresh water before salinity water arrives at the pumping station

We consider a segment of a river with 108 km of length as shown in Table 1.1 Assuming that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30% and the given simulated station any time is 10 days. It is estimated into 1000 time steps. This means that 1 time step is referred to 14.4 min. Their physical psrameters and give spacing are shown in Table 4.7 Assuming that there are 8 monitoring station along a considered river segment as shown in Table 1.1 The controlled monitoring station is the station S_7. We need to control the salinity level at the station S_7 before salinity level at

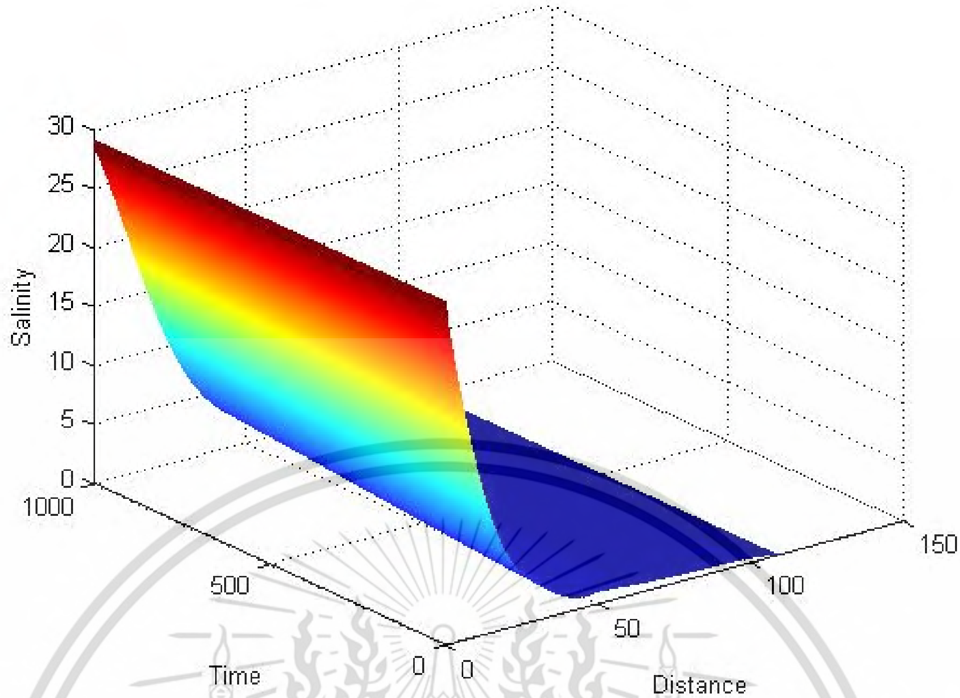


Figure 4.6: The numerical solution of simulation 3 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 1000$.

Table 4.6: Approximated salinity $c(x,t)$ of simulation 3 for all monitoring stations.

t	S_1	S_2	S_3	S_4
1	12.1040	4.0187	2.0224	1.0978
5000	11.3456	3.6020	1.9844	1.0058
10000	11.3391	3.6002	2.0397	1.0128
15000	13.7382	4.4013	2.4788	1.1159
20000	15.5617	5.3633	3.0060	1.2576
t	S_5	S_6	S_7	S_8
1	0.7955	0.5316	0.4995	0.1444
5000	0.7667	0.4807	0.3844	0.0027
10000	0.7476	0.4098	0.2993	0.0023
15000	0.7704	0.3996	0.2987	0.0032
20000	0.8025	0.3961	0.2983	0.0034

the station S_7 to be over the salinity standard level $C_{ST} = 0.05\ kg/m^3$ about 3 days. In this simulation Sualyev technique is used to approximate the solution due to the technique will always gives stable solutions. The salinity is controlled by a process as follows:

1) If the salinity level at the station S_5 $c(91,t) < C_{ST}$, then the fresh water is released in a normal speed level is from the barrage dam.

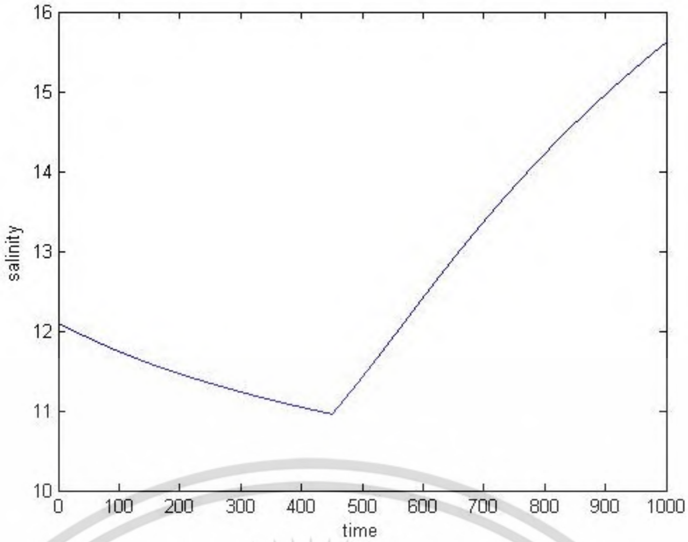


Figure 4.7: Approximated salinity of simulation 3 at station S_1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$.

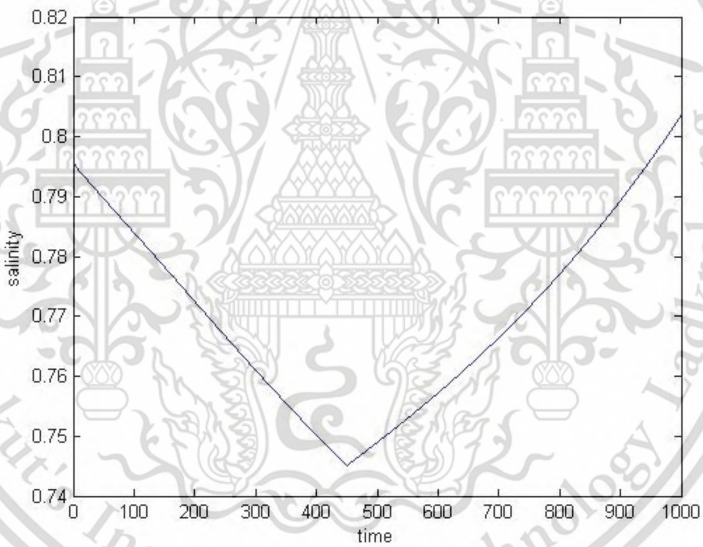


Figure 4.8: Approximated salinity of simulation 3 at station S_5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$.

2) If the salinity level at the station S_5 $c(91, t) > C_{ST}$, then the fresh water will be released in a high speed from the barrage dam which is used control is the salinity.

We can obtain the approximated salinity level along the considered river segment as shown in Table 4.8 and Fig 4.10 The salinity level at the several monitoring stations S_1, S_5 and S_7 as shown in Fig 4.11, 4.12 and 4.13 respectively.

4.1.5 Discussion of numerical simulation with generated salinity data

In simulation 1, we get good agreement approximated solutions of the FTCS and Sualyev finite difference techniques. The maximum error are less than 1%. In

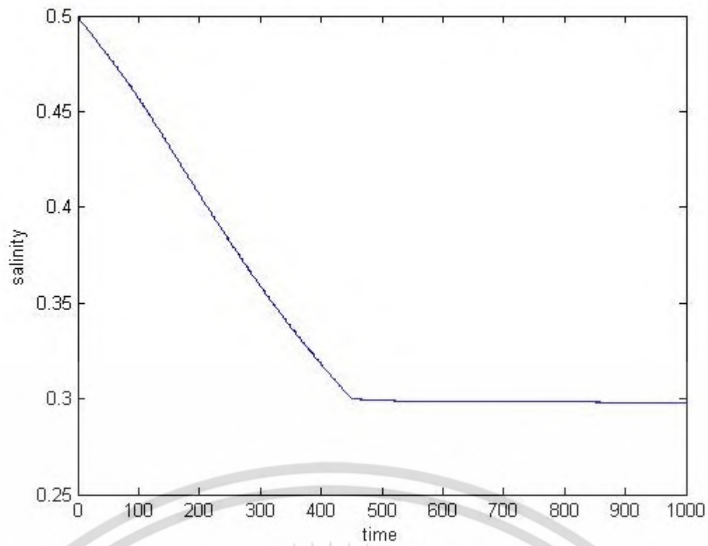


Figure 4.9: Approximated salinity of simulation 3 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$.

Table 4.7: Physical parameters of simulation 4.

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)
$< C_{ST}$	0.1	0.065	0
$> C_{ST}$	0.1	0.065	0.25
K	T	L (km)	c(0,t)
0.3	1000	108	g(t)
0.3	1000	108	g(t)

Table 4.8: Approximated salinity $c(x,t)$ of simulation 4 for all monitoring stations.

t	S_1	S_2	S_3	S_4
1	12.1040	4.0187	2.0224	1.0978
5000	20.3804	7.4884	4.0812	1.4095
10000	17.8501	7.0540	3.9099	1.4447
15000	16.1112	6.5658	3.7400	1.4458
20000	14.8449	6.1007	3.5647	1.4298
t	S_5	S_6	S_7	S_8
1	0.7957	0.4010	0.2544	0.1395
5000	0.9276	0.4791	0.3919	0.0040
10000	0.8945	0.4144	0.2993	0.0020
15000	0.8669	0.3456	0.2327	0.0015
20000	0.8380	0.2888	0.1868	0.0011

simulation 2, we can obtain that the Sualyev technique is better than the FTCS technique due to the limitation of stability conditions. The Sualyev technique gives stable

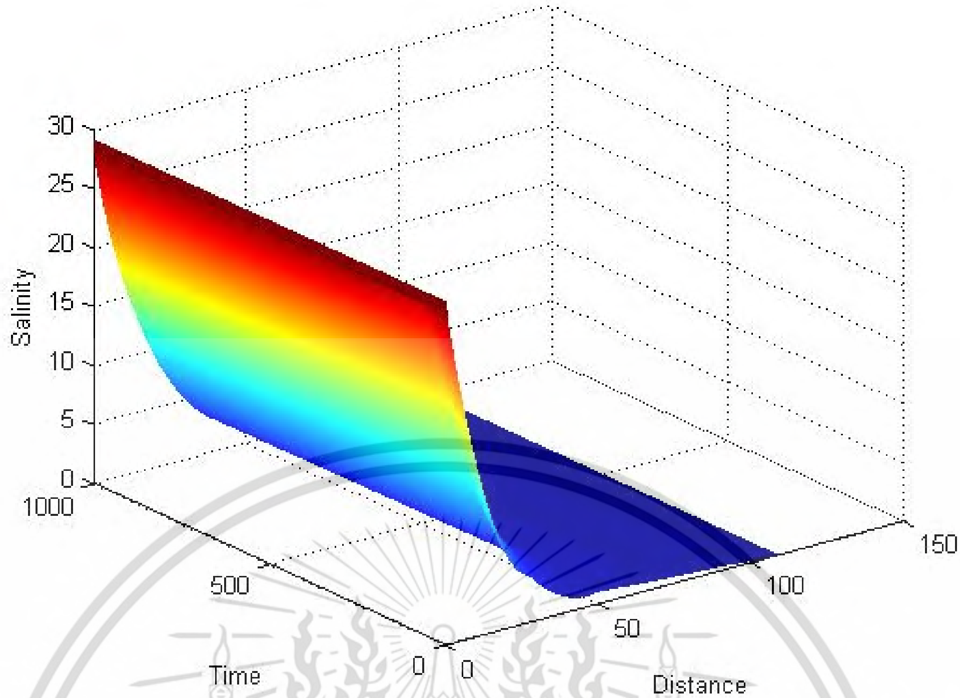


Figure 4.10: The numerical solution of simulation 4 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 1000$.

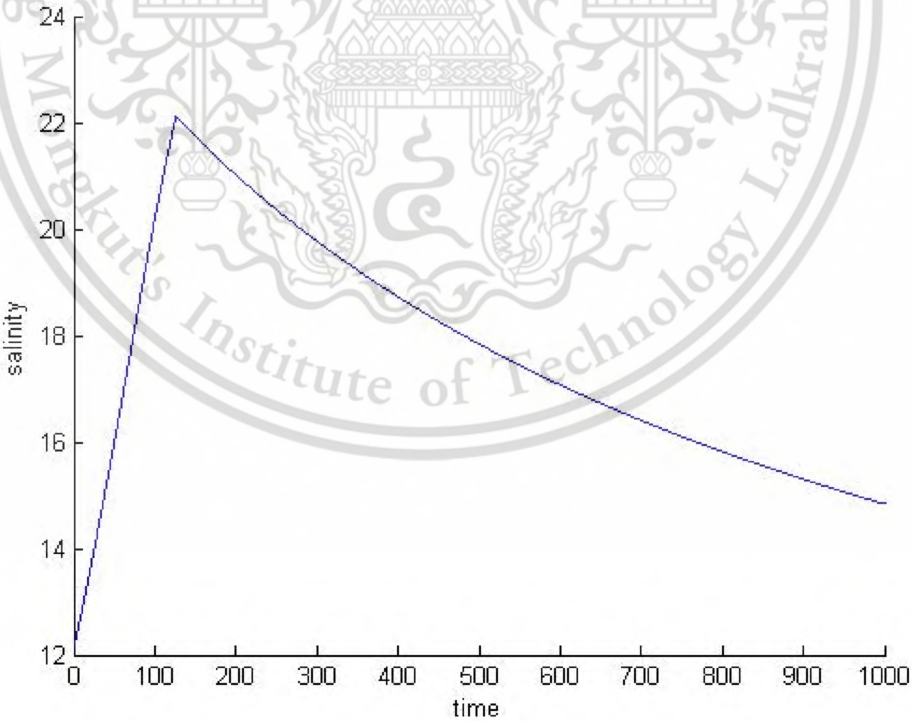


Figure 4.11: Approximated salinity of simulation 4 at station S_1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$.

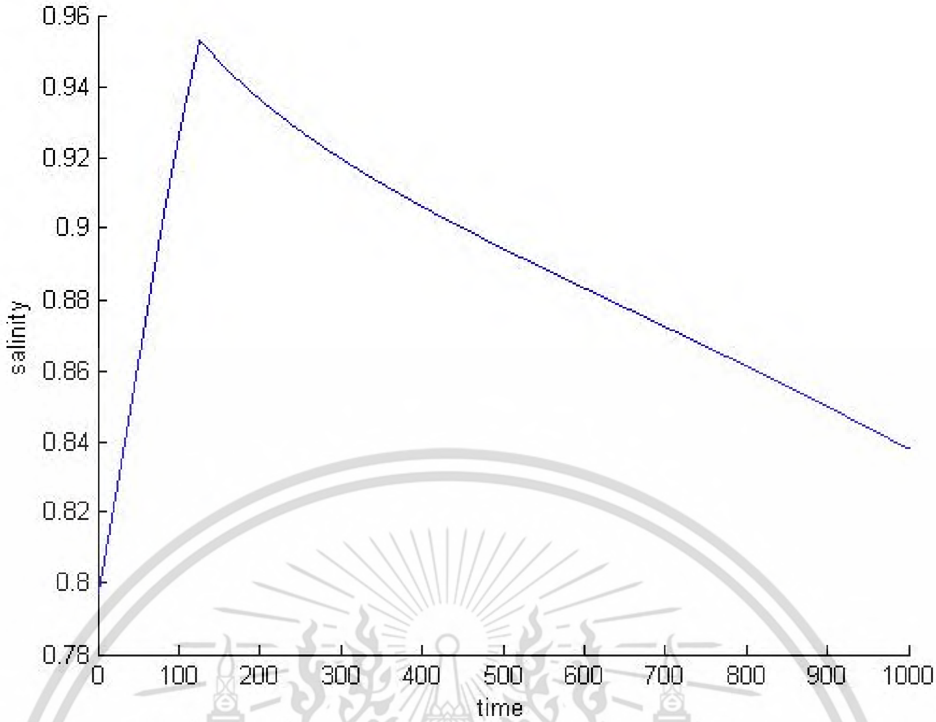


Figure 4.12: Approximated salinity of simulation 4 at station S_5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$.

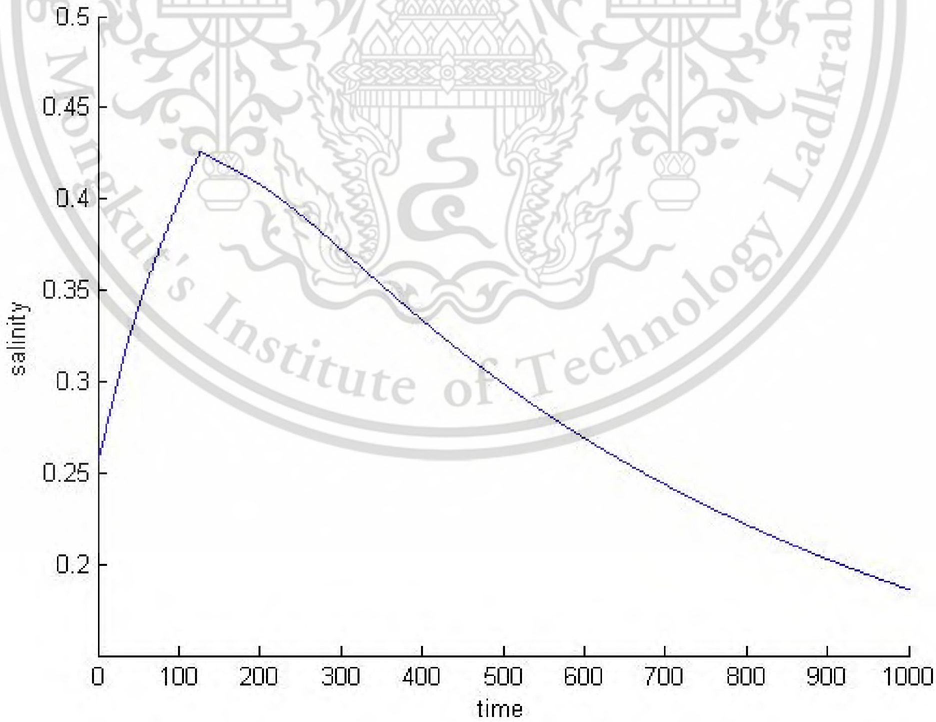


Figure 4.13: Approximated salinity of simulation 4 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$.

approximated solution. Otherwise, the FTCS is limited by its stability conditions. These are then the Sualyev technique is preferred to use in other realistic simulations. We can see that the salinity level will be reduced when the fresh water flow velocity is increasing as shown in Fig 4.5. In simulation 3, a salinity control process is simulated. The salinity is reduced the salinity level come to standard after that we can decreased the the fresh water flow velocity to maintain the salinity level in standard level as shown in fig 4.9. In simulation 4, a salinity control process is simulated. The salinity is reduced before the salinity level going to be touch the salinity standard level. The proposed process can reduce the salinity level when the fresh water is released from the barrage dam at least amount as shown in Fig 4.11-4.13.

4.2 Numerical simulation with field measurement raw salinity data

4.2.1 Simulation 1 : interpolation for the initial condition and left boundary conditions.

The observation stations with 90 km along the river and data on salinity level are considered, as appeared in Table 1.1 and Table 1.2 respectively. We simulated the boundary and initial conditions by using Eq.(3.31-3.32). The comparison of interpolation of boundary and initial condition with the measuring salinity data as appeared in Fig 4.14-4.15 respectively.

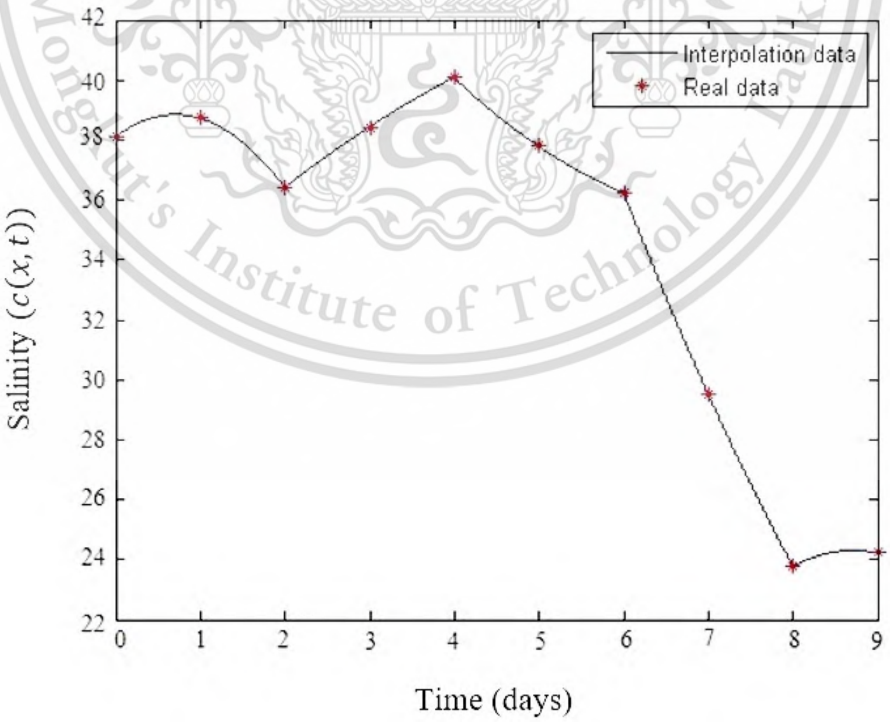


Figure 4.14: The comparison of interpolation in initial condition with the measuring salinity data

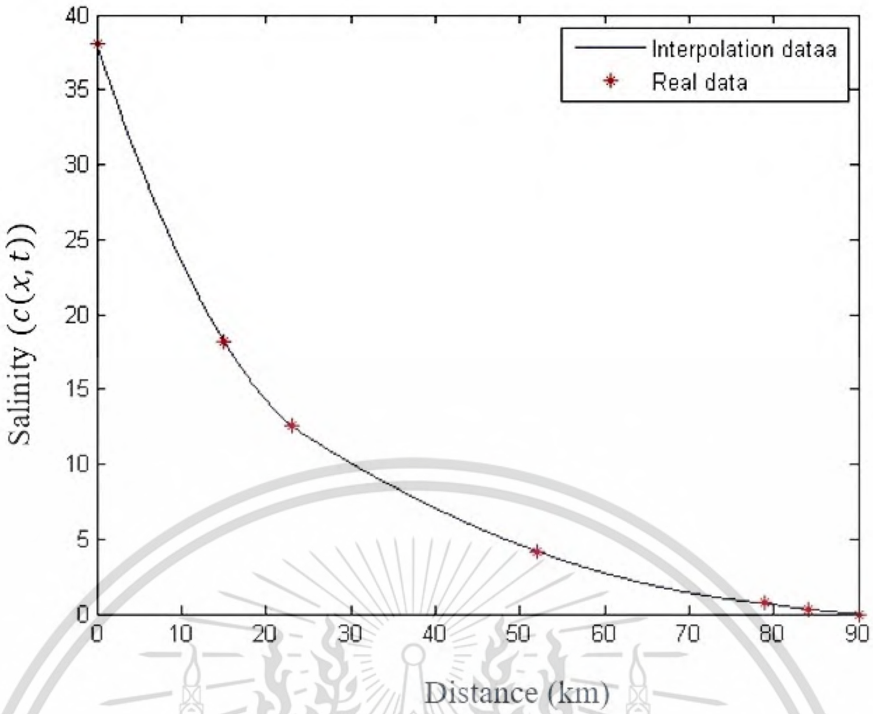


Figure 4.15: The comparison of interpolation in left boundary condition condition with the measuring salinity data

4.2.2 Simulation 2 : the spread of salinity water into rivers.

We will to find the approximate solution of Eq.(3.14) for all observation stations with 90 km along the river, as appeared in Table 1.1 Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\ m/s$, the efficiency of eliminating salinity of fresh water discharge is 30%, and time of simulation is 9 days. The physical parameters are appeared in Table 4.9 The approximate solution of salinity along the river and the salinity at observation station S_6 compare with the real data as appeared in Fig 4.16-4.17 respectively.

Table 4.9: The parameters of physical of simulation 2.

D_s (m^2/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (days)
0.1	0.06	0.3	0.3	90	9

4.2.3 Simulation 3 : release fresh water from the barrage dam to dilute the salinity.

We will to find the approximate solution of Eq.(3.14) for all observation stations with 90 km along the river, as appeared in Table 1.1 Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\ m/s$, the efficiency of eliminating salinity of fresh water discharge is 30%, and time of sim-

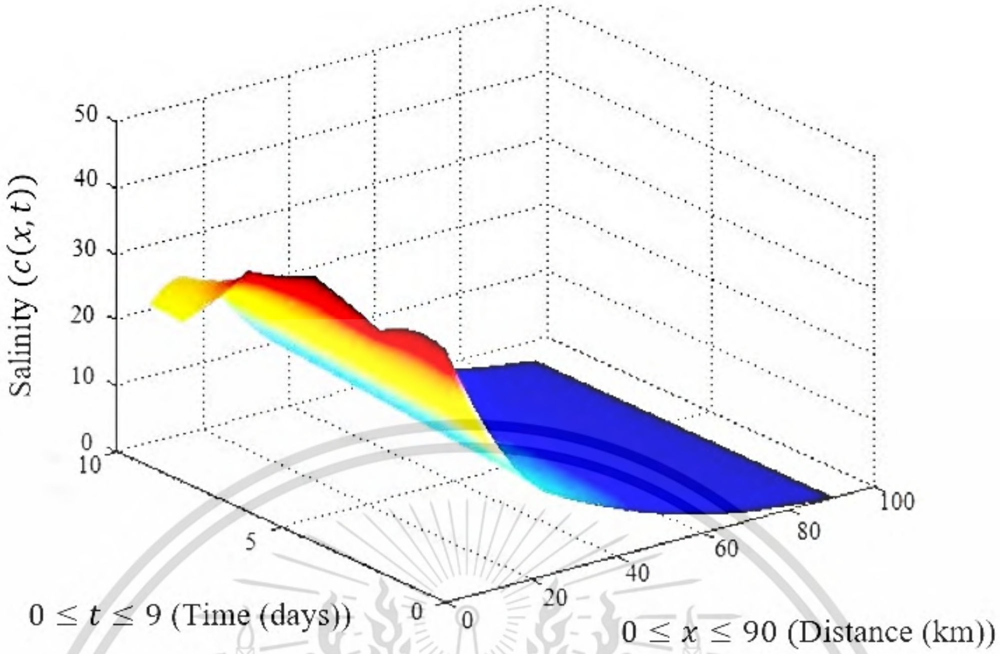


Figure 4.16: The approximated salinity level of simulation 2 where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 90$ and $0 \le t \le 9$.

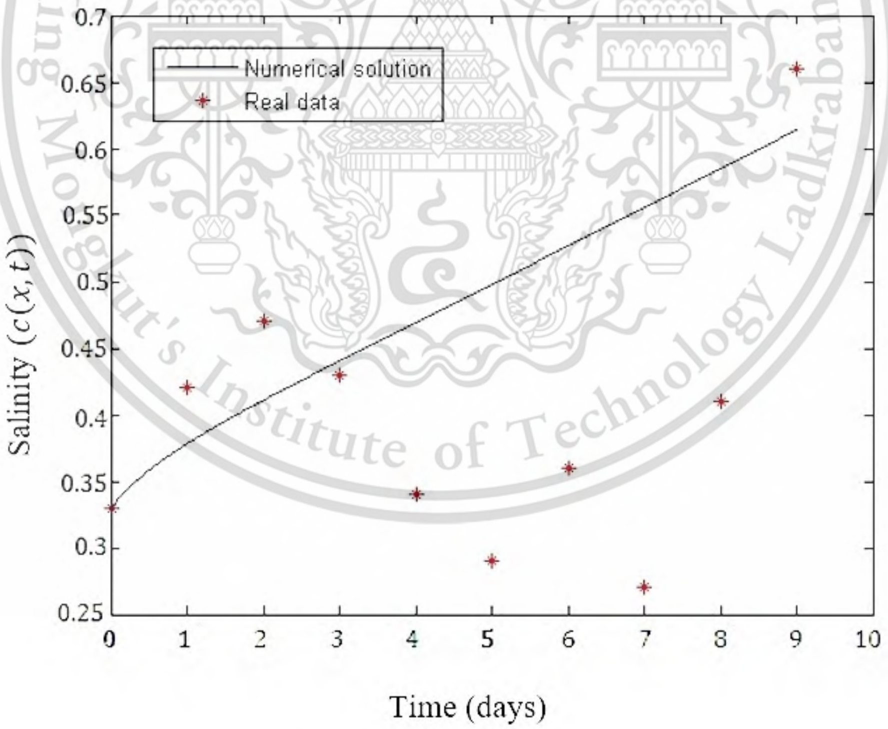


Figure 4.17: The approximated salinity level at station S_6 of simulation 2 where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 90$ and $0 \le t \le 9$.

ulation 9 days. The physical parameters are appeared in Table 4.10 The approximate solution of salinity along the river and the salinity level at the observation station S_6

as appeared in Fig 4.18-4.19 respectively.

Table 4.10: The parameters of physical of simulation 3.

D_s (m^2/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (days)
0.1	0.06	0.3	0.3	90	9

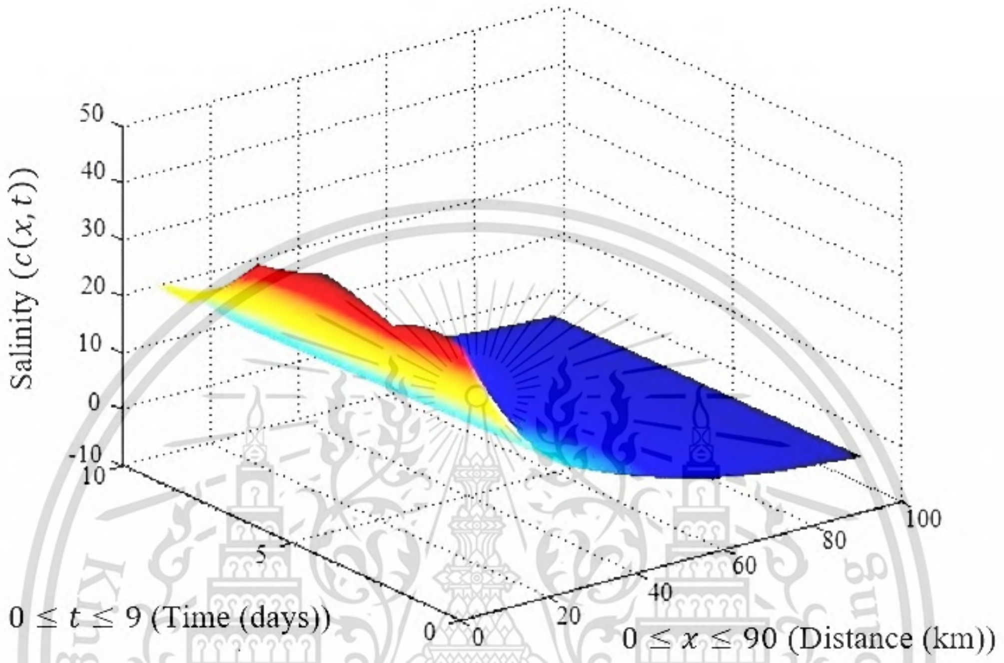


Figure 4.18: The approximated salinity level of simulation 3 where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 90$ and $0 \le t \le 9$.

4.2.4 Simulation 4 : maintaining a constant level of salinity to the standard by reducing the speed of water discharge from the barrage dam.

We will to find the approximate solution of Eq.(3.14) for all observation stations with 90 km along the river, as appeared in Table 1.1 Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\ m/s$, the efficiency of eliminating salinity of fresh water discharge is 30%, and time of simulation is 9 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.72\ kg/m^3$ by the controlled release of water from barrage dams with the following process:

1) Release at high speed when the salinity level $c(84,t) > C_{ST}$ at the station S_6.

2) Release at low speed when the salinity level $c(84,t) < C_{ST}$ at the station S_6.

Their parameters of physical are appeared in Table 4.11.

The approximate salinity level for all observation station can be obtained, as appeared in Fig 4.20 and Table 4.12 The saltiness level at the various observation stations S_6, as appeared in Fig 4.21

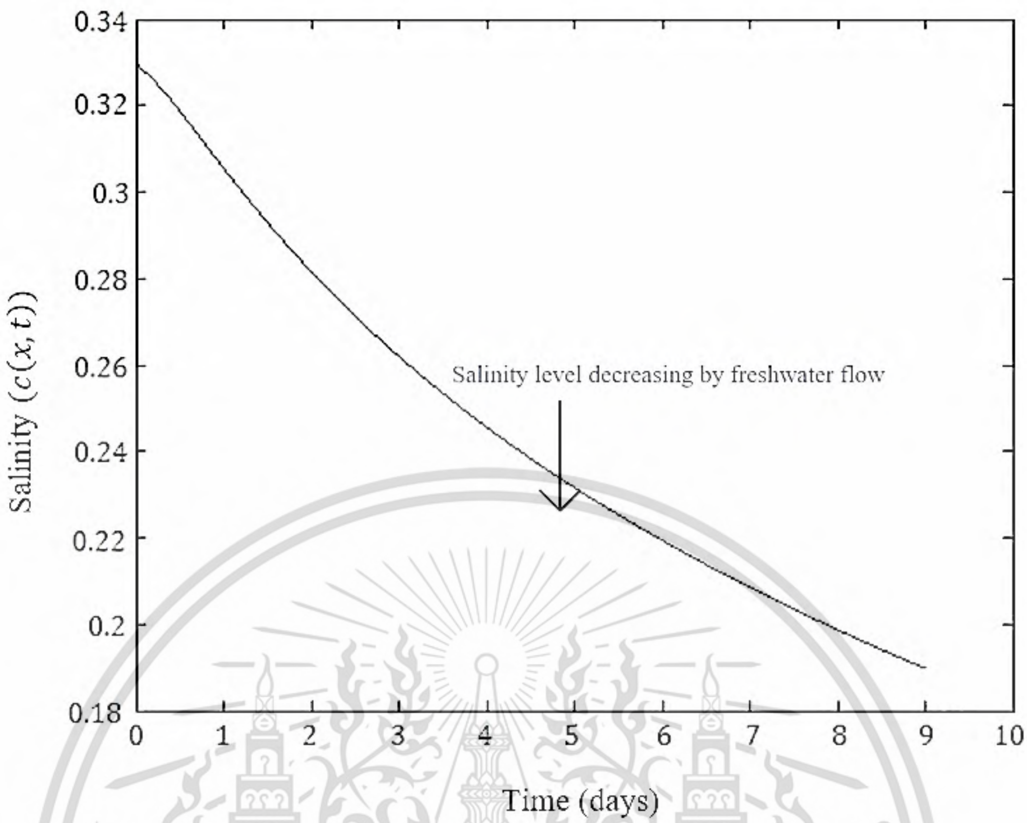


Figure 4.19: The approximated salinity level at station S_6 of simulation 3 where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 9]$.

Table 4.11: The parameters of physical of simulation 4.

$c(x,t)$ at S_6	D (m^2/s)	u_s (m/s)	u_w (m/s)
$> C_{ST}$	0.1	0.06	0.23
$< C_{ST}$	0.1	0.06	0.205
K	T (days)	L (km)	c(0,t)
0.3	9	90	g(t)
0.3	9	90	g(t)

Table 4.12: The approximated salinity level for all observation stations of simulation 4.

t	S_2	S_3	S_4	S_5	S_6
1	18.2300	12.5500	4.2000	0.7200	0.3300
5000	17.6659	12.4890	3.9232	0.5884	0.2765
10000	17.1508	12.1402	3.6688	0.5191	0.2385
15000	16.7113	11.7453	3.4336	0.4653	0.2113
20000	16.3177	11.5042	3.2701	0.4392	0.2025

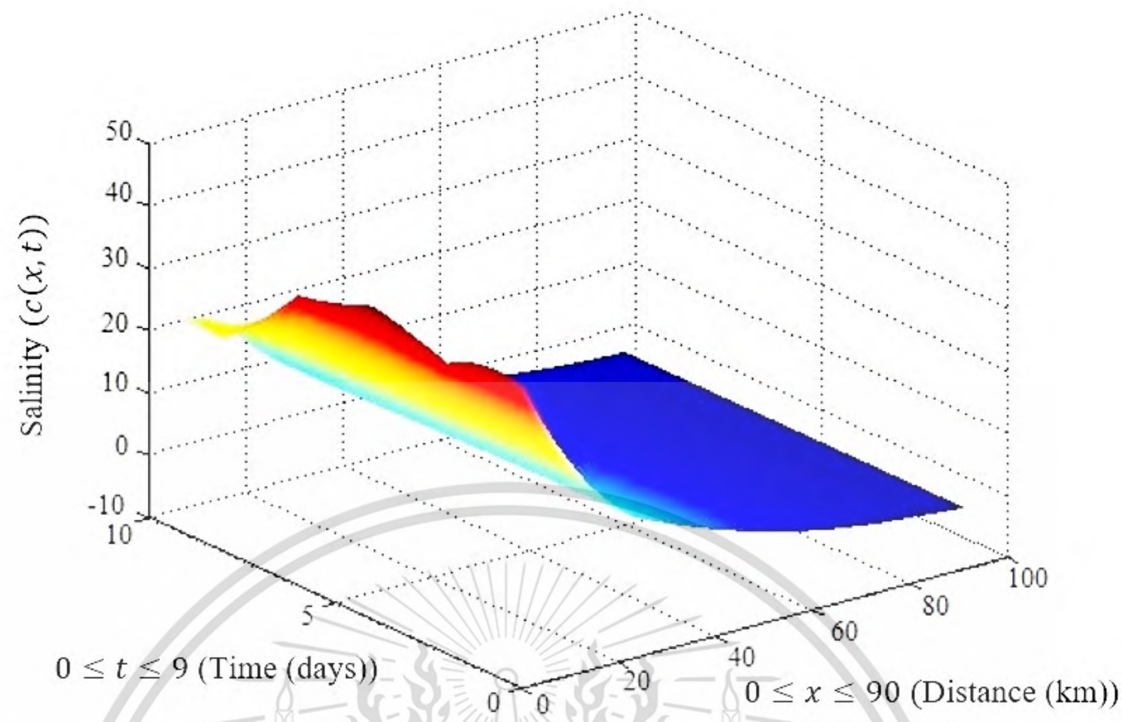


Figure 4.20: The approximated salinity level of simulation 4 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 90$ and $0 \le t \le 9$.

4.2.5 Simulation 5 : reduce the level of salinity before the salinity exceeds the standard.

We will to find the approximate solution of Eq.(3.14) for all observation stations with 90 km along the river, as appeared in Table 1.1 Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\ m/s$, the efficiency of eliminating salinity of fresh water discharge is 30%, and time of simulation is 9 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.72\ kg/m^3$ about 3 days by the controlled release of water from barrage dams with the following process:

1) Release at normal speed when the salinity level $c(79,t) < C_{ST}$ at the station S_5.

2) Release at high speed when the salinity level $c(79,t) > C_{ST}$ at the station S_5 Their parameters of physical are appeared in Table 4.13.

The approximate salinity level for all observation station can be obtained, as appeared in Fig 4.22 and Table 4.14 The saltiness level at the various observation station S_5 and S_6, as appeared in Fig 4.23 and 4.24.

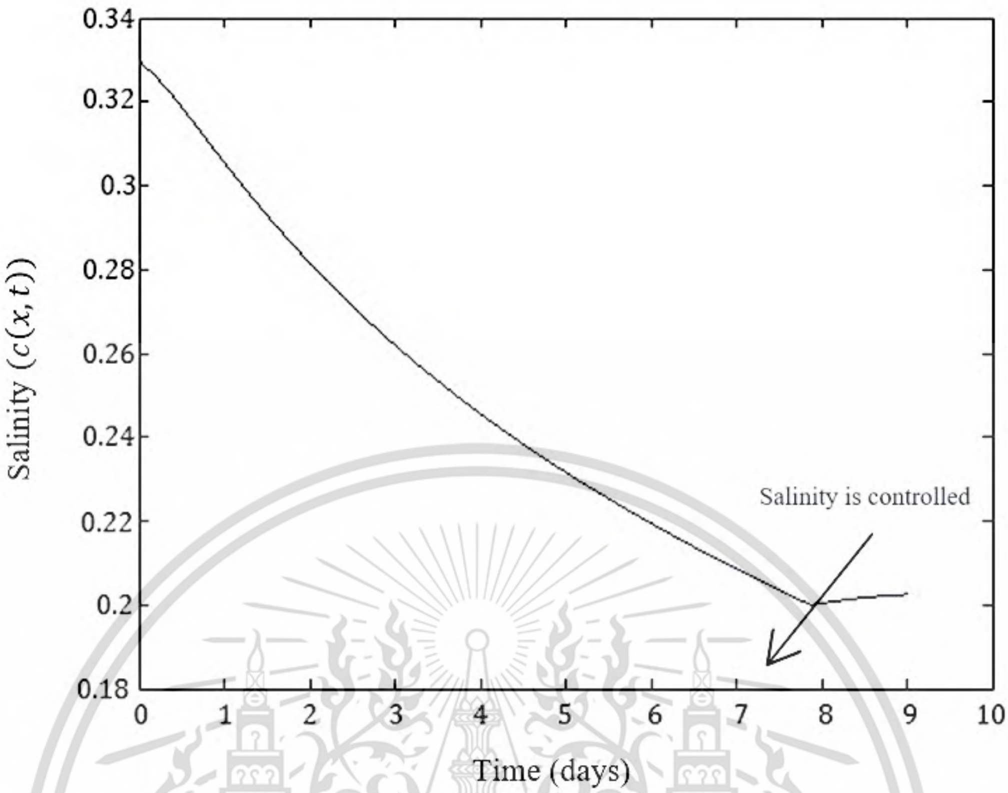


Figure 4.21: The approximated salinitylevel at station S_6 of simulation 4 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 9]$.

Table 4.13: The parameters of physical of simulation 5.

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)
$< C_{ST}$	0.1	0.06	0
$> C_{ST}$	0.1	0.06	0.22
K	T (days)	L (km)	c(0,t)
0.3	9	90	g(t)
0.3	9	90	g(t)

Table 4.14: The approximated salinity level of simulation 5 for all observation stations.

t	S_2	S_3	S_4	S_5	S_6
1	18.2300	12.5500	4.2000	0.7200	0.1500
5000	19.4352	13.5555	4.3278	0.6805	0.3248
10000	19.5986	13.7948	4.2646	0.6784	0.3252
15000	18.7978	13.3153	3.9964	0.5969	0.2759
20000	17.8724	12.8036	3.7464	0.5321	0.2426

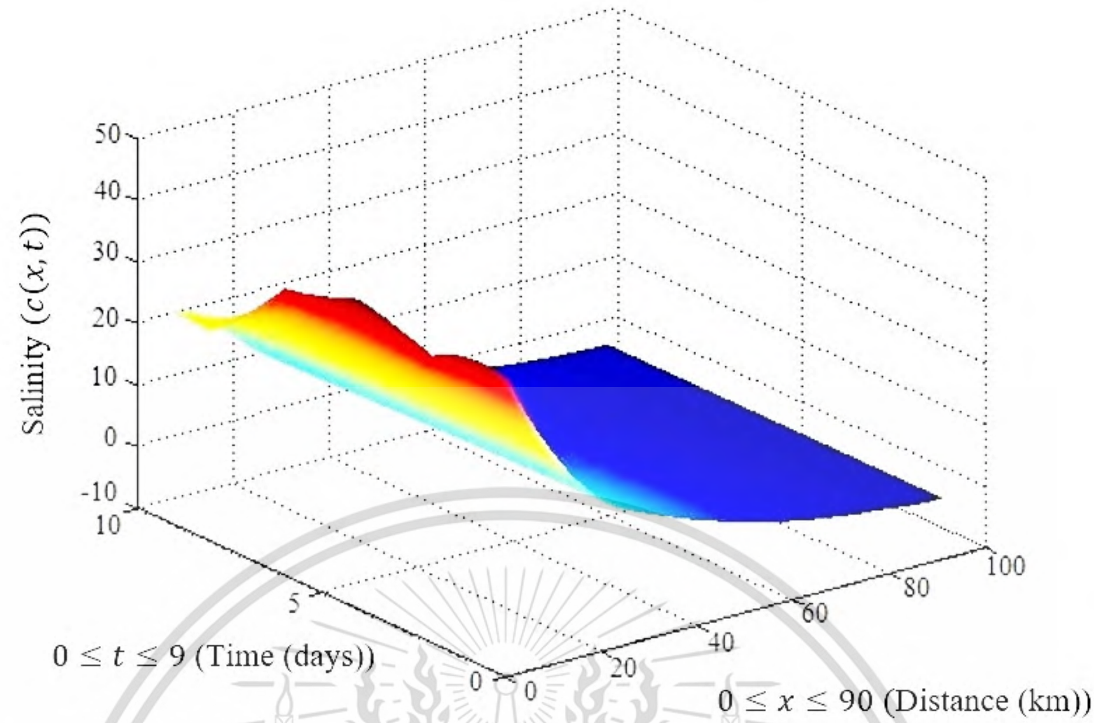


Figure 4.22: The approximated salinity level of simulation 5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 90$ and $0 \leq t \leq 9$.

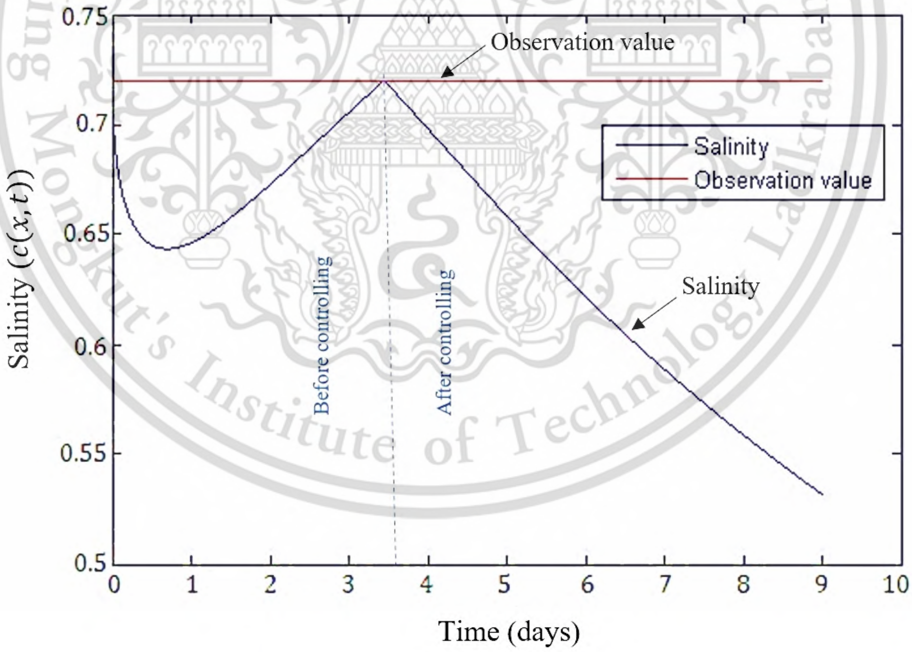


Figure 4.23: The approximated salinity level at stations S_5 of simulation 5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 9$.

4.2.6 Simulation 6 : reduce the salinity before the salinity exceeds the standard and reduce the emission from the dam when the salinity is low.

We will to find the approximate solution of Eq.(3.14) for all observation stations with 90 km along the river as, appeared in Table 1.1 Provided that saltiness water

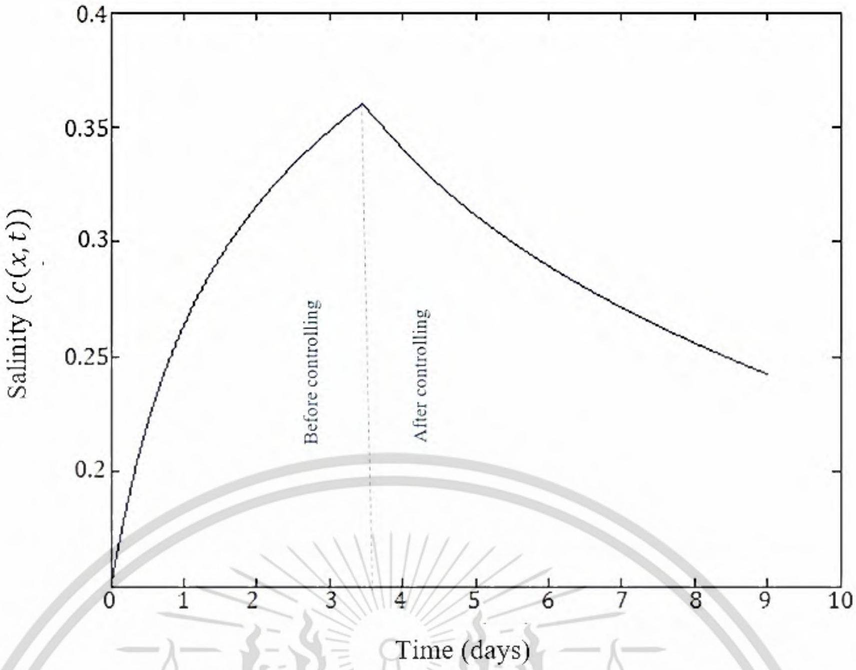


Figure 4.24: The approximated salinity level of simulation 5 at station S_6 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 9$.

coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\ m/s$, the efficiency of eliminating salinity of fresh water discharge is 30%, the fresh water dilution efficiency is 30%, and time of simulation is 9 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.72$ kg/m^3 about 3 days by the controlled release of water from barrage dams with the following process:

1) Release at normal speed when the salinity level $c(79,t) < C_{ST}$ at the station S_5.

2) Release at high speed when the salinity level $c(79,t) > C_{ST}$ at the station S_5 and change normal speed when S_5 $c(79,t) < C_{ST}$.

Their parameters of physical are appeared in Table 4.15

The approximate salinity level for all observation station can be obtained, as appeared in Fig 4.25 and Table 4.16. The S_5 and S_6 level at the various observation station S_5 and S_6, as appeared in Fig 4.26 and 4.27.

4.2.7 Discussion of numerical simulation with real salinity data

In simulation 1, approximate solutions of the initial and boundary conditions are obtained using the interpolation method. In simulation 2, the calculated solutions can be obtained by demonstrating the salinity level along the river with a maximum error of less than 30%, as appeared in Fig 4.16 and Fig 4.17. In simulation 3, the salinity value will decrease as the velocity of fresh water flow increases, as appeared

Table 4.15: The parameters of physical of simulation 6.

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)
$< C_{ST}$	0.1	0.06	0
$> C_{ST}$	0.1	0.06	0.25
K	T (days)	L (km)	c(0,t)
0.3	9	90	g(t)
0.3	9	90	g(t)

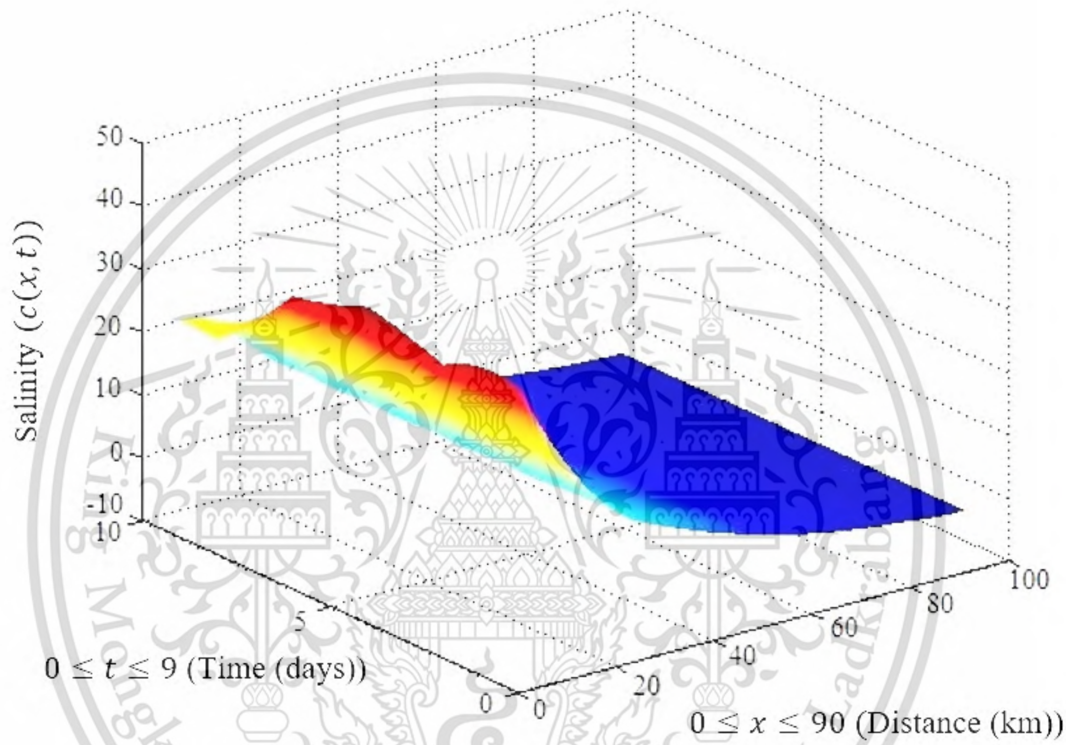


Figure 4.25: The approximated salinity level of simulation 6 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 90$ and $0 \le t \le 9$.

Table 4.16: The approximated salinity level for all observation station of simulation 6.

t	S_2	S_3	S_4	S_5	S_6
1	18.2300	12.5500	4.2000	0.7200	0.1500
5000	19.2985	13.4818	4.3009	0.6688	0.3107
10000	19.3886	13.6694	4.2232	0.6614	0.3138
15000	19.4798	13.7761	4.1554	0.6549	0.3110
20000	19.4353	13.9705	4.1413	0.6656	0.3215

in Fig 4.18 and Fig 4.19. In simulation 4, the salinity control process is simulated. If the salinity level becomes normal after that, the speed of fresh water flow should be lowered to preserve salinity at the normal level, as appeared in Fig 4.20 and Fig

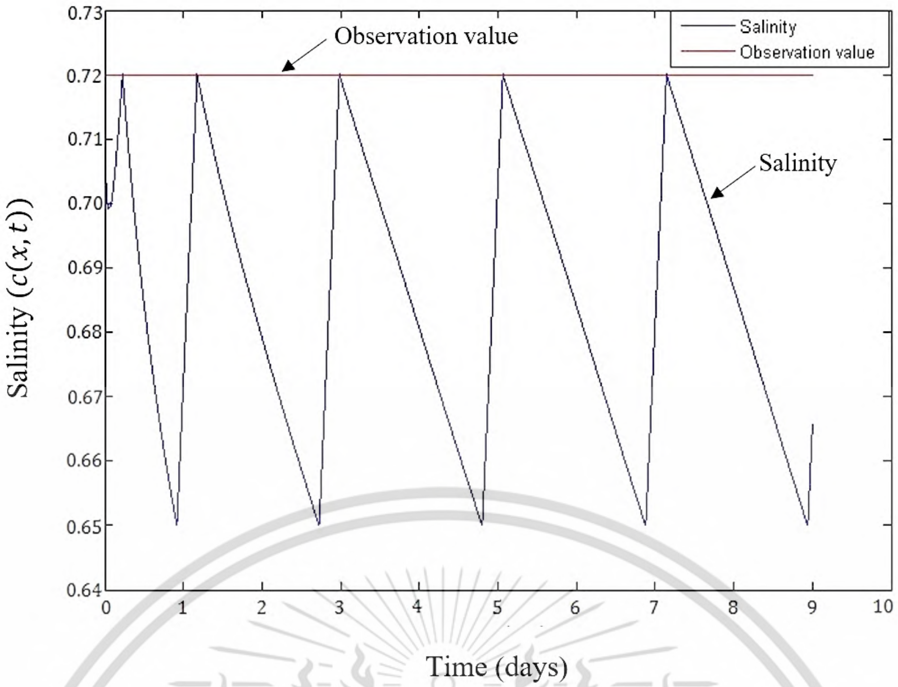


Figure 4.26: The estimated salinity level at station S_5 of simulation 6 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 9$.

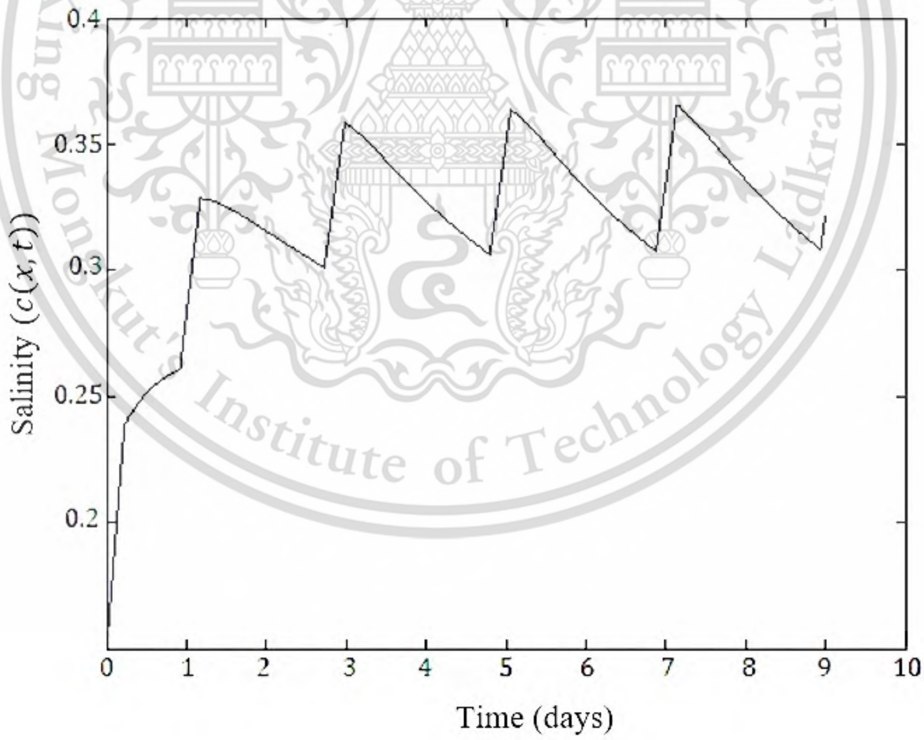


Figure 4.27: The estimated salinity level of simulation 6 at station S_6 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 9$.

4.21. In simulation 5, the salinity control mechanism is simulated. Salinity reduces until the saltiness level reaches the standard. The suggested process could reduce saltiness by at least releasing fresh water from the dam, as appeared in Fig 4.22, Fig 4.23 and Fig 4.24. In simulation 6, the salinity control mechanism is simulated. The salinity decreases to less than the standard value and increases as the velocity of fresh water decreases alternately. The suggested technique is to reduce salinity by at least an amount and fresh water is released when salinity levels become standard as appeared in Fig 4.25, Fig 4.26 and Fig 4.27.

4.3 Numerical models of salinity controlling with internal wave factor

4.3.1 Simulation 1 : the salinity flow velocity and the salinity elevation in a river with effect of internal wave.

The observation stations with 90 km along the river are considered, as appeared in Table 1.1 We find the approximate solution of a one-dimensional salinity internal wave hydrodynamic model Eq.(2.6). the efficiency of eliminating salinity of fresh water discharge is 30%, and time of simulation is 10 days. The boundary and initial condition functions are shown in Tables 4.17 and 4.18, respectively.

Table 4.17: Parameters of physical of simulation 1.

D_s (m^2/s)	u_w (m/s)	K	L(km)	T(days)
0.1	0.25	0.3	108	10

Table 4.18: The boundary and initial condition function of simulation 1.

Parameters	Given functions
$u_1(x)$	0.06
$h_1(x)$	$0.05 + 0.1sin(m\Delta t)$
$f_1(t)$	0
$f_2(t)$	0
$g_1(t)$	$0.05 + 0.1sin(m\Delta t)$
$g_2(t)$	0

The salinity diffusion coefficient is 0.1 m^2/s. By using a modified Lax-diffusive method for the hydrodynamic model, Eqs.(3.8-3.10), we obtain a graph of elevation of the water above the bottom and, by using the hydrodynamic model, we obtain a graph of the salinity flow, shown in Fig 4.28 and 4.29, respectively. We can see that this simulation is to approximate the salinity flow velocity from the estuary.

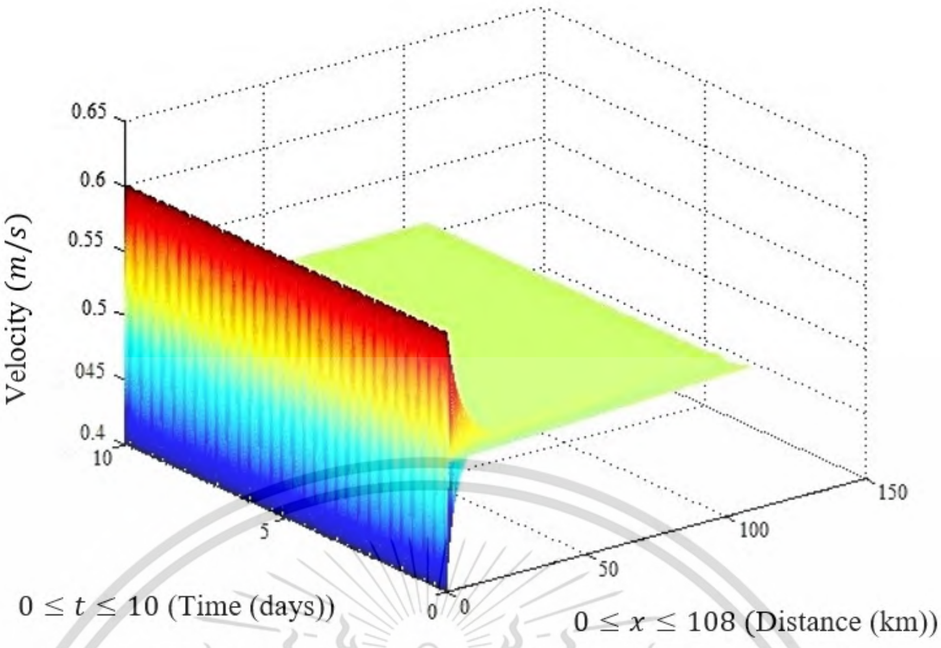


Figure 4.28: The salinity flow velocity for $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 10$.

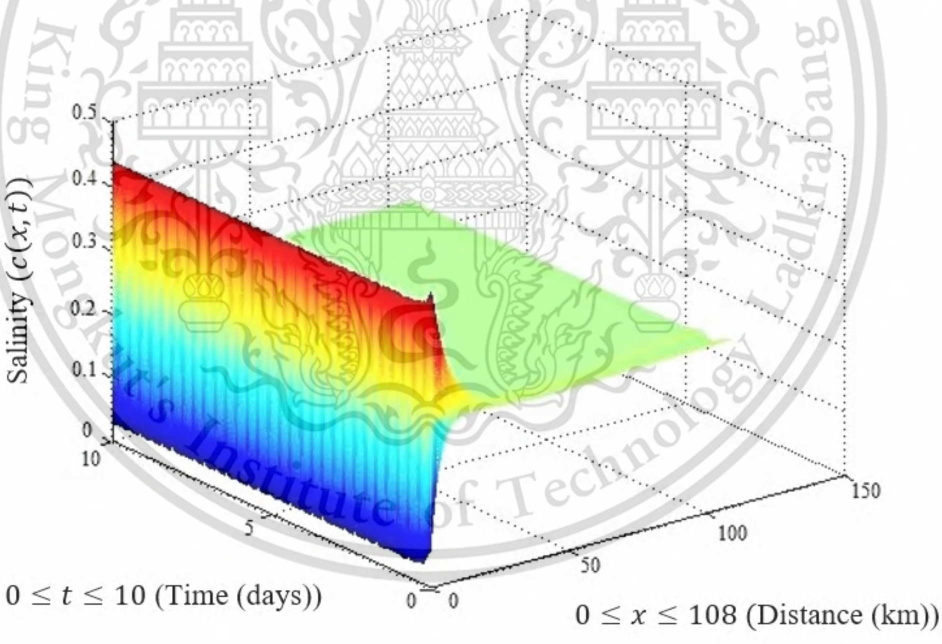


Figure 4.29: The salinity flow velocity for $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 10$.

4.3.2 Simulation 2 : release fresh water from the barrage dam to dilute the salinity without the salinity internal wave factor.

We will to find the approximate solution of Eq.(3.14) for all observation stations with 108 km along the river, as appeared in Table 1.1 Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.065\ m/s$, the efficiency of eliminating salinity of fresh water discharge is 30%, and time of simulation 10 days. The parameters of physical are appeared in Table 4.19.

Table 4.19: Parameters of physical of simulation 2.

D_s (m^2/s)	u_w	u_w (m/s)	K	L (km)	T (days)
0.1	0.065	0.25	0.3	108	10

We get the approximated solution $c(x,t)$ by using Sualyev scheme are appeared in Table 4.20 The level of salinity at the operated S_7 observation station will be standardized, as shown in Fig 4.30 and 4.31, respectively. We can see that this simulation is to reduce the saltiness level which releases fresh water from the dam.

Table 4.20: The estimated salinity level of simulation 2 for all observation stations.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	11.7025	3.7114	2.0435	1.0136
10000	11.2247	3.5663	2.0230	1.0088
15000	10.8874	3.4530	1.9999	0.9991
20000	10.6373	3.3562	1.9699	0.9854
t	S_5	S_6	S_7	S_8
1	0.8025	0.5325	0.5053	0.1618
5000	0.7730	0.4865	0.3955	0.0148
10000	0.7458	0.4081	0.2975	0.0061
15000	0.7221	0.3370	0.2304	0.0029
20000	0.6975	0.2798	0.1836	0.0015

4.3.3 Simulation 3 : release fresh water from the barrage dam to dilute the salinity with the salinity internal wave factor.

We will to find the approximate solution of Eq.(3.14) for all observation stations with 108 km along the river, as appeared in Table 1.1 Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow from simulation 1, the efficiency of eliminating salinity of fresh water discharge is 30%, and time of simulation 10 days. The parameters of physical are appeared in Table 4.21

We get the approximated solution $c(x,t)$ by using Sualyev scheme are appeared in Table 4.22 The saltiness concentration level at the station S_7 becomes a standard level. We can see that this simulation is to reduce the salinity level which releases fresh water from the dam. A comparison of the simulated salinity levels at the controlled observation station as shown in Fig 4.32-4.33.

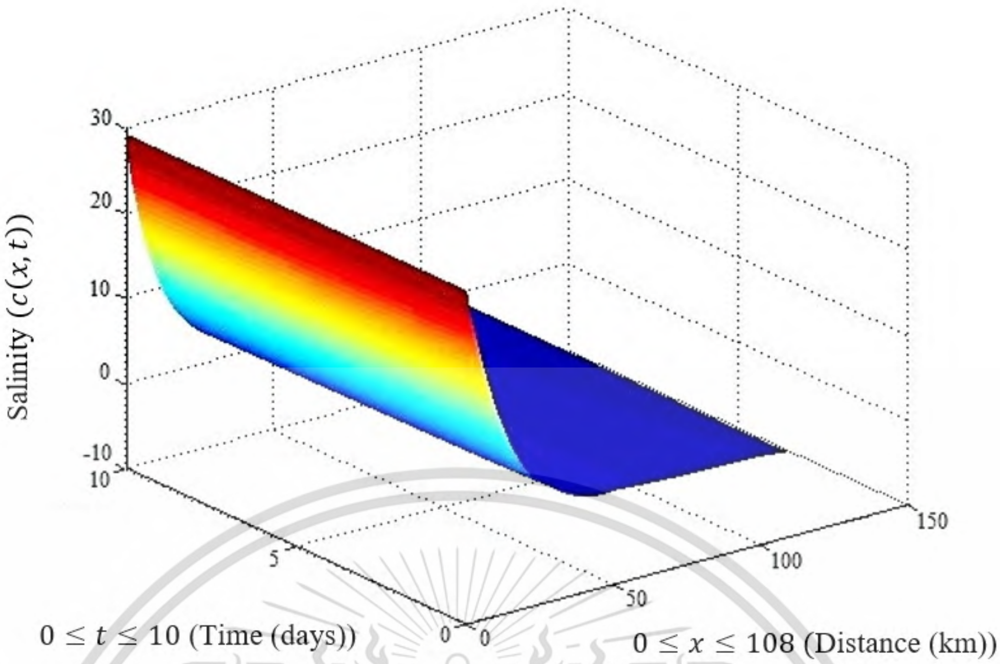


Figure 4.30: The estimated salinity level of simulation 2 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 10$.

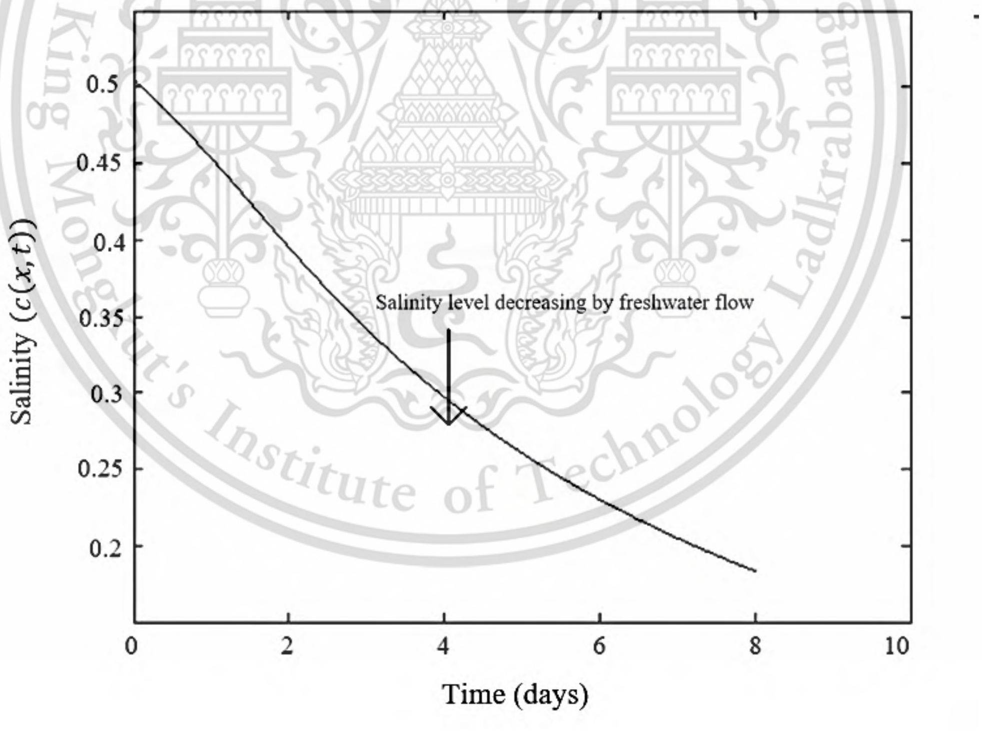


Figure 4.31: The estimated salinity level of simulation 2 at station S_7 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $0 \leq t \leq 10$.

Since, the water fresh water flow velocity is 0.25 m/s can diluted the salinity water in time and not waste too much water from the barrage dam and the saltiness flow velocity that flows into the river in nature is not constant at all times, the com-

Table 4.21: Parameters of physical of simulation 3.

D_s (m^2/s)	u_w (m/s)	K	L (km)	T (days)
0.1	0.2	0.3	108	10
0.1	0.25	0.3	108	10
0.1	0.3	0.3	108	10

Table 4.22: The estimated salinity level of simulation 3 for all observation stations.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	8.6881	2.7478	1.5514	0.9373
10000	6.0982	1.9581	1.2625	0.8286
15000	4.4861	1.5021	1.0642	0.7351
20000	3.4869	1.2107	0.9131	0.6494
t	S_5	S_6	S_7	S_8
1	0.8025	0.5325	0.5053	0.1618
5000	0.7122	0.4204	0.2960	0.0040
10000	0.6393	0.2369	0.1302	0.0022
15000	0.5627	0.1206	0.0574	0.0017
20000	0.4604	0.0604	0.0262	0.0010

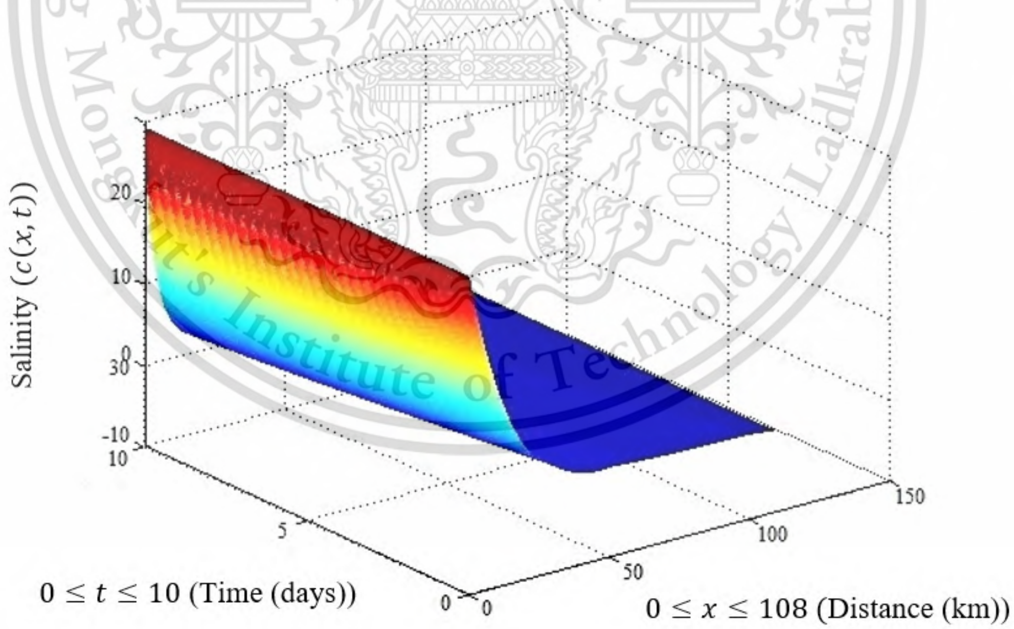


Figure 4.32: The estimated salinity level of simulation 3 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 10$.

parison of The salinity concentration level at the controlled observation station S_7 of the simulation 2 and 3 is shown in Fig 4.34. Therefore, in the following simulations, it

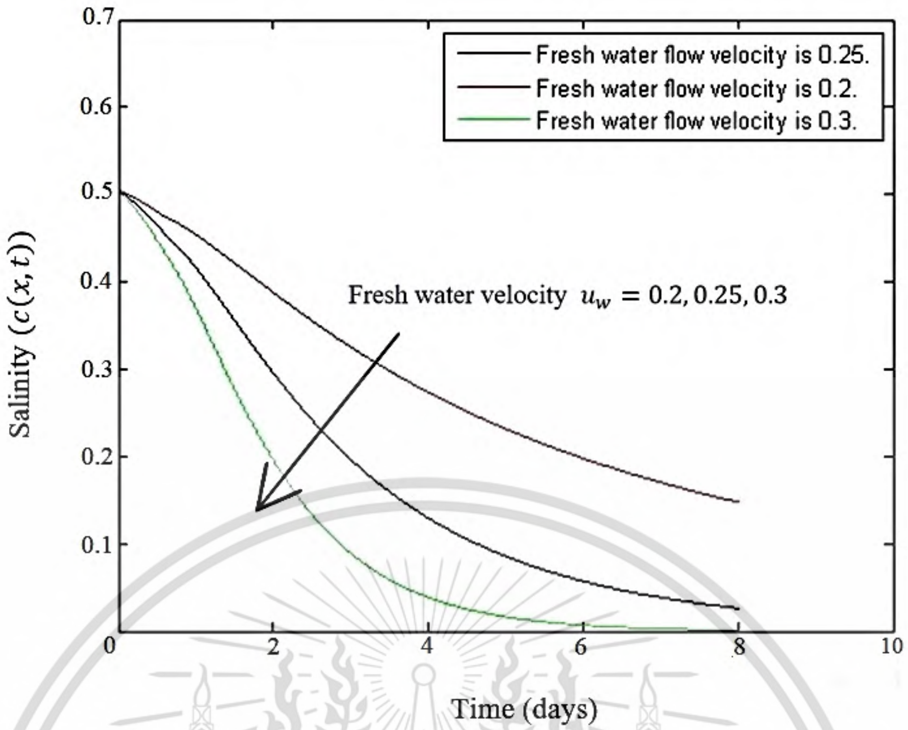


Figure 4.33: The estimated salinity level of simulation 3 at station S_7 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $0 \leq t \leq 10$.

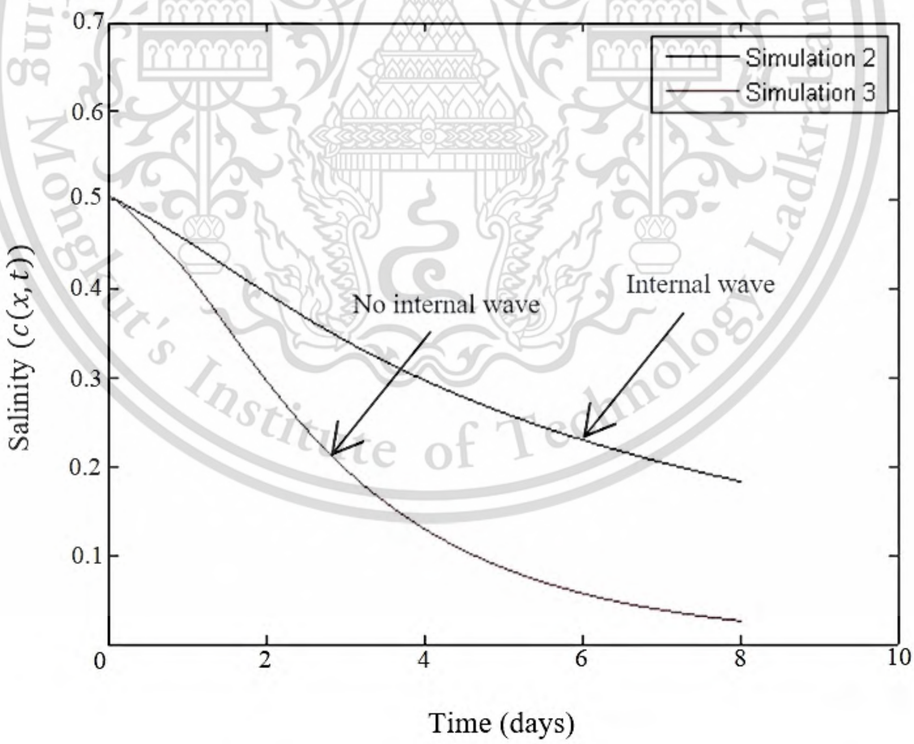


Figure 4.34: The comparison of The saltiness concentration level at station S_7 of simulation 2 and 3 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 10$.

will be using the saltiness flow velocity obtained from simulation 1.

4.3.4 Simulation 4 : maintaining a constant level of salinity to the standard by reducing the speed of water discharge from the barrage dam.

We will to find the approximate solution of Eq.(3.14) for all observation stations with 108 km along the river, as appeared in Table 1.1 Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow from the simulation 1, the efficiency of eliminating saltiness of fresh water discharge is 30%, and time of simulation is 10 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.3\ kg/m^3$ by the controlled release of water from barrage dams with the following process:

1) Release at high speed when the salinity level $c(96,t) > C_{ST}$ at the station S_6.

2) Release at low speed when the salinity level $c(96,t) < C_{ST}$ at the station S_6.

Their parameters of physical are appeared in Table 4.23.

The approximate salinity level for all observation station can be obtained, as appeared in Table 4.24 and Fig 4.35 The saltiness level at the various observation stations S_7, as appeared in Fig 4.36 We can see that the technique is to reduce the saltiness level which releases fresh water from the dam and use the amount of fresh water not too much.

Table 4.23: Parameters of physical of simulation 4.

$c(x,t)$ at S_7	D (m^2/s)	u_w (m/s)	K
$> C_{ST}$	0.1	0.25	0.3
$< C_{ST}$	0.1	0.15	0.3
	T (days)	L (km)	c(0,t)
	10	108	g(t)
	10	108	g(t)

4.3.5 Simulation 5 : reduce the level of salinity before the salinity exceeds the standard.

We will to find the approximate solution of Eq.(3.14) for all observation stations with 108 km along the river, as appeared in Table 1.1 Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow from the simulation 1, the efficiency of eliminating salinity of fresh water discharge is 30%, and time of simulation is 10 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.3\ kg/m^3$ about 3 days by the controlled release of water from barrage dams with the following process:

1) Release at normal speed when the salinity level $c(79,t) < C_{ST}$ at the station S_5.

Table 4.24: The estimated salinity level of simulation 4 for all observation stations.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	8.7906	2.7751	1.5642	0.9396
10000	11.5783	3.3642	1.9124	0.9836
15000	13.9586	4.1634	2.3118	1.0693
20000	15.6397	5.1400	2.8139	1.1899
t	S_5	S_6	S_7	S_8
1	0.8025	0.5325	0.5053	0.1618
5000	0.7140	0.4230	0.2997	0.0112
10000	0.7322	0.3990	0.2946	0.0126
15000	0.7514	0.3897	0.2931	0.0149
20000	0.7776	0.3852	0.2910	0.0153

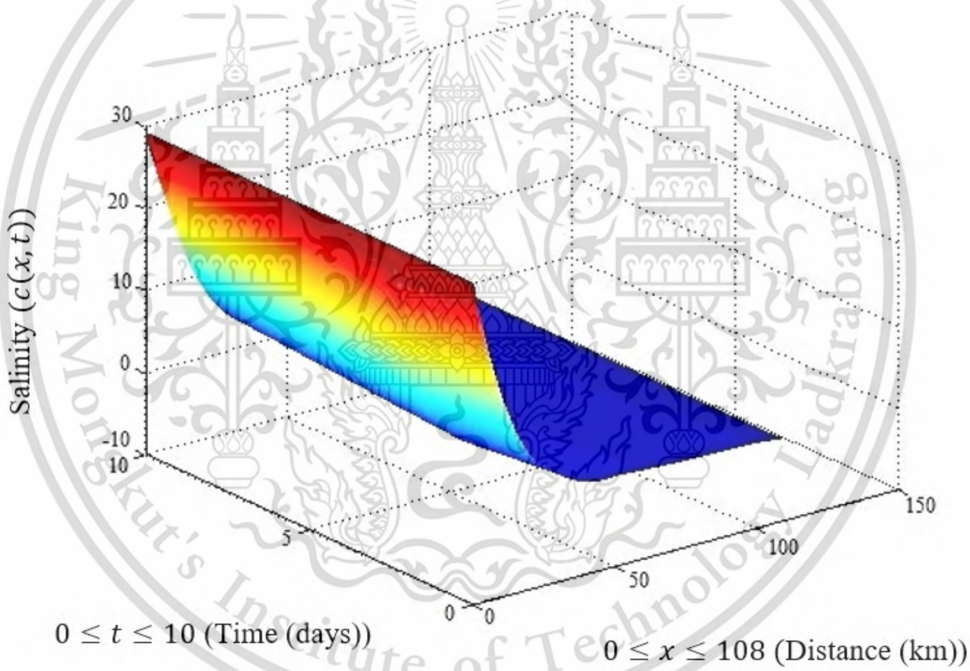


Figure 4.35: The estimated salinity level of simulation 4 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 108$ and $0 \le t \le 10$.

2) Release at high speed when the salinity level $c(79, t) > C_{ST}$ at the station S_5 Their parameters of physical are appeared in Table 4.25.

The approximate salinity level for all observation station can be obtained, as appeared in Table 4.26 and Fig 4.37 The saltiness level at the various observation stations S_7, as appeared in Fig 4.38 We can see that the technique is to reduce the saltiness level which releases fresh water from the dam before the salinity is higher than standard.

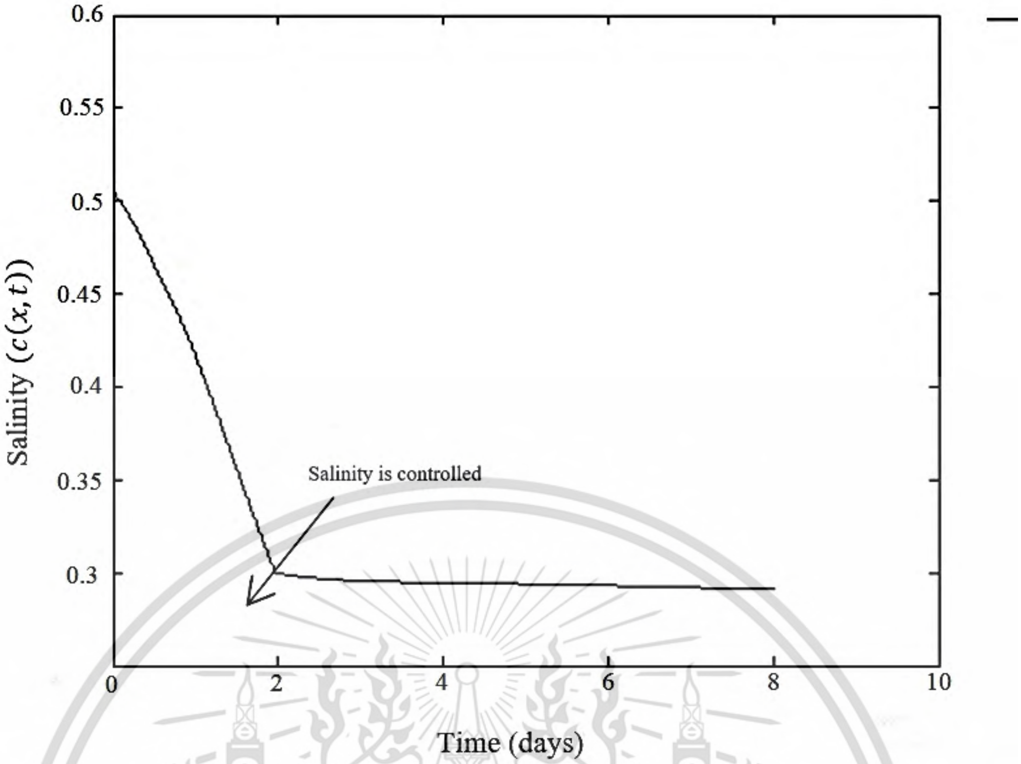


Figure 4.36: The estimated salinity level of simulation 4 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 10$.

Table 4.25: Parameters of physical of simulation 5.

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)
$< C_{ST}$	0.1	0.065	0
$> C_{ST}$	0.1	0.065	0.25
K	T (days)	L (km)	c(0,t)
0.3	10	108	g(t)
0.3	10	108	g(t)

4.3.6 Discussion of numerical simulation with effect of internal wave

In simulation 1, the approximate solutions of salinity flow velocity are obtained by using the modified Lax-diffusive system for the hydrodynamic model, as shown in Figs 4.28-4.29. In simulation 2, we can obtain salinity of water at the pumping station without the salinity flow velocity form simulation 1 by using the Sualyev technique. We can see that this simulation can decrease the salinity level with salinity flow velocity is constant by releasing fresh water from the barrage dam, as shown in Figs 4.30-4.31. In simulation 3, we can obtain salinity of water at the pumping station with the salinity flow velocity from simulation 1 by using the Sualyev technique. We can see that this simulation can decrease the salinity level with the affect of salinity internal wave by releasing fresh water from the barrage dam, as shown in Figs 4.32-4.33 and the

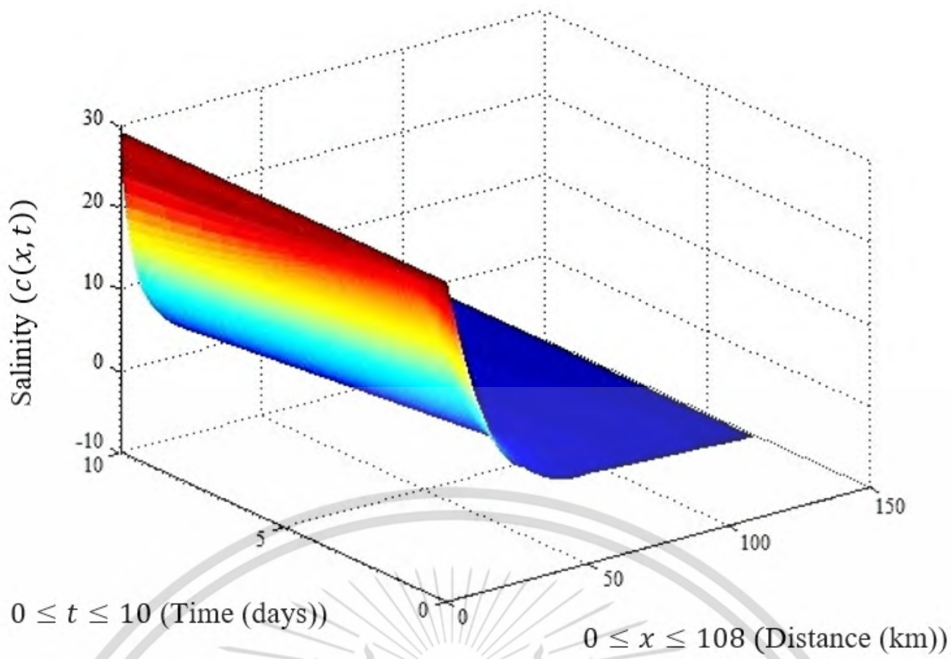


Figure 4.37: The estimated salinity level of simulation 5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 108$ and $0 \le t \le 10$.

Table 4.26: The estimated salinity level of simulation 5 for all observation stations.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	19.5112	6.9051	3.7499	1.3210
10000	13.4092	4.6944	2.6005	1.1293
15000	9.3605	3.2451	1.9022	0.9671
20000	6.6951	2.3258	1.4632	0.8235
t	S_5	S_6	S_7	S_8
1	0.8033	0.4052	0.2641	0.1378
5000	0.9057	0.4607	0.3732	0.0097
10000	0.7853	0.3005	0.1826	0.0041
15000	0.6692	0.1595	0.0806	0.0032
20000	0.5456	0.0808	0.0365	0.0019

comparison of the degree of salinity at a S_7 monitored observation station, as shown in Figure 4.34. In simulation 4, a salinity control process is simulated with the salinity flow velocity from Simulation 1, as shown in Figs 4.30-4.31. The salinity is reduced to the standard level after this, as shown in Figs 4.35-4.36, we will lower the fresh water flow rate to preserve the normal salinity level. In simulation 5, a process of with the salinity flow velocity from simulation 1 was proposed. The salinity is that until the normal salinity amount is reached. The suggested process may reduce the salinity

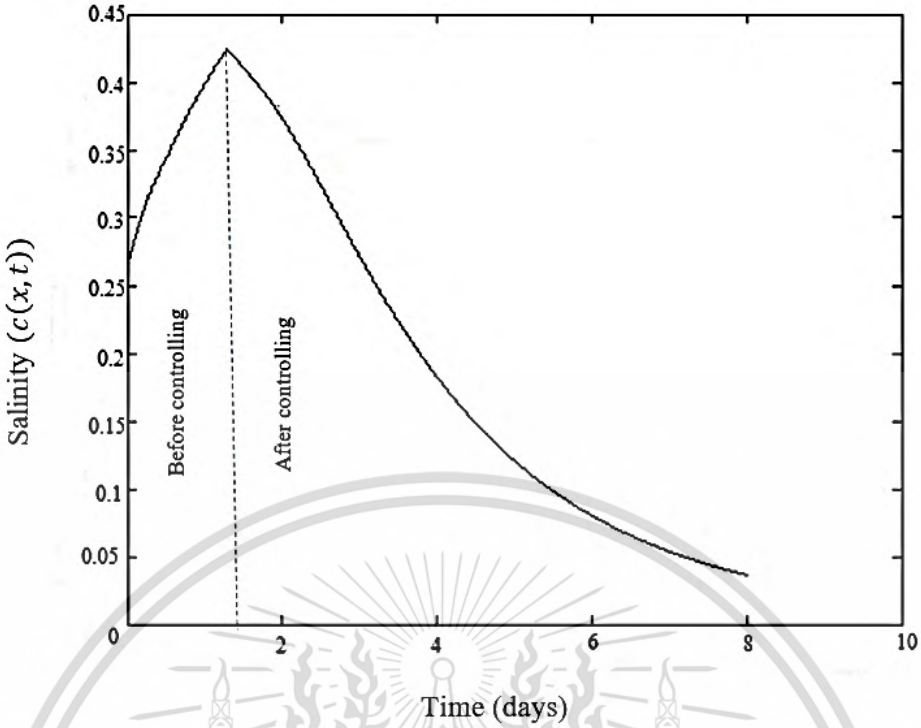


Figure 4.38: The estimated salinity level of simulation 5 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 10$.

level at least, as shown in Figs 4.37-4.38, when the barrage dam releases fresh water.

Chapter 5

Conclusion

We have proposed a one-dimensional mathematical model of salinity measurement in a river with a barrage dam. The propose model is concerned about salinity advection to a river and the fresh water flow from the barrage dam affects. The traditional forward time central space finite difference method is used to compare with the proposed Sualyev technique. the proposed Sualyev technique gives a stable solution in any grid spacing. The technique also gives accurately approximated solutions. The realistic problem is also simulated. The propose simulation can be use to several realistic salinity measurement. In the salinity control aspect, the proposed process can be reduce the salinity level before the level going to be over the standard. The proposed numerical simulation can be apply in practical salinity control in a river with barrage dam.

We also proposed a mathematical model for salinity water measurement in one-dimension with concerns the saltiness advection to the river and the effect of salinity flow velocity from an estuary into the river and the dam's release of fresh water. There are several scenarios in which the effects of internal salinity waves are simulated. The proposed simulations can be use in practical salinity measurement in similar topographical rivers. In the salinity control aspect, the proposed process may decrease the salinity level until the level reaches the normal line and the quantity of fresh water may not be used too much. Suggested simulations can be used in realistic salinity management situations in rivers with barrage dam systems along with savings in the volume of fresh water used. The result of all simulation as shown in Tables 5.1, 5.2 and 5.3 respectively.

Table 5.1: The result of numerical simulation with generated salinity data model.

Simulation	Result
1. Salinity control in an ideal case.	Exact solution of model was calculated.
2. The salinity is diluted by releasing the fresh water form a barrage dam with difference flow velocities.	Sualyev is chosen to approximate the solution of the simulation.
3. The salinity is diluted by releasing the fresh water form a barrage dam and change flow velocities.	The salinity level decreased to less than standard value for acceptable level with quantity of released fresh water is high.
4. Diluted the salinity water by releasing fresh water before salinity water arrives at the pumping station	The salinity level decreased to less than standard value for acceptable level with quantity of released fresh water is quite high.

Table 5.2: The result of numerical simulation with field measurement raw salinity data model.

Simulation	Result
1. Interpolation for the initial condition and left boundary conditions.	The initial condition and left boundary conditions of the model was approximated.
2. The spread of salinity water into rivers.	The model can be approximating the spread of salinity water into rivers.
3. Release fresh water from the barrage dam to dilute the salinity.	The salinity level decreased to less than standard value with quantity of released fresh water is high.
4. Maintaining a constant level of salinity to the standard by reducing the speed of water discharge from the barrage dam.	The salinity level decreased to less than standard value for acceptable level with quantity of released fresh water is quite high.
5. Reduce the level of salinity before the salinity exceeds the standard.	The salinity level is not more than and is much less than the standard with quantity of released fresh water is high.
6. Reduce the salinity before the salinity exceeds the standard and reduce the emission from the dam when the salinity is low.	The salinity level is not more than and is much less than the standard with quantity of released fresh water is not too high.

Table 5.3: The result of numerical models of salinity controlling with internal wave factor model.

Simulation	Result
1. The salinity flow velocity and the salinity elevation in a river with effect of internal wave.	The salinity flow velocity in the river was approximated.
2. Release fresh water from the barrage dam to dilute the salinity without the salinity internal wave factor.	The salinity level was controlled without the salinity internal wave factor.
3. Release fresh water from the barrage dam to dilute the salinity with the salinity internal wave factor.	The salinity level was controlled with the salinity internal wave factor.
4. Maintaining a constant level of salinity to the standard by reducing the speed of water discharge from the barrage dam.	The salinity level decreased to less than standard value for acceptable level with quantity of released fresh water is quite high.
5. Reduce the level of salinity before the salinity exceeds the standard.	The salinity level is not more than and is much less than the standard with quantity of released fresh water is high.

In the future, this research could improve the accuracy of the model if parameters such as the diffusion velocity of salt water were measured, and could be applied to the invasion problem of the salinity of other rivers. It can also be applied to other river chemical diffusion problems.

References

[1] Tabuenca, P., Vila, J., Cardona, J., "Samartin, A., Finite element simulation of dispersion in the bay of santander," Advanced in Engineering Software, vol. 28, pp313–332, 1997

[2] Pochai, N., Tangmanee, S., Crane, L.J., Miller, J.J.H., "A water quality computation in the uniform channel," Journal of Interdisciplinary Mathematics, vol. 11, pp803–814, 2008

[3] Pochai, N., "A numerical computation of non-dimensional form of stream water quality model with hydrodynamic advection-dispersion-reaction equations," Journal of Nonlinear Analysis: Hybrid Systems, vol. 13, pp666–673, 2009

[4] Pochai, N., "A numerical computation of non-dimensional form of a nonlinear hydrodynamic model in a uniform reservoir," Journal of Nonlinear Analysis: Hybrid Systems, vol. 3, pp463–466, 2009

[5] Thongtha, K., Kasemsuwan, J., "Analytical solution to a hydrodynamic model in an open uniform reservoir," Advances in Difference Equations, vol. 2017, 2017

[6] Nahian, M.A., Ahmed, A., Lazar, A.N., Hutton, C.W., Salehin, M., "Streatfield, P.K., Drinking water salinity associated health crisis in coastal bangladesh," Elementa: Science of the Anthropocene, vol. 6, 2017

[7] Leewatchanakul, K., "Salinity intrusion in chao phraya river," PhD thesis, Chulalongkorn University, 1988

[8] Chapra, S.C., "Surface Water-Quality Modeling," McGraw-Hill, vol. 6., 2018

[9] Pochai, N., "A numerical treatment of nondimensional form of water quality model in a nonuniform flow stream using saulyev scheme," Mathematical Problems in Engineering 2011, vol. 2011, pp1–16, 2011

[10] Subklay, K., Pochai, N., "Numerical simulations of a water quality model in a flooding stream due to dam-break problem using implicit and explicit methods," Journal of Interdisciplinary Mathematics, vol. 20, pp461–495, 2017

[11] Othata, P., Pochai, N., "A one-dimensional mathematical simulation to salinity control in a river with a barrage dam using an unconditionally stable explicit finite difference method," Advances in Difference Equations, vol. 203(2019), 2019

[12] Akbari, G. and Firoozi, B., "Implicit and Explicit Numerical Solution of Saint-Venent Equations for Simulation Flood wave in Natural Rivers," 5th National Congress on Civil Engineering, Ferdowsi University of Mashhad, Iran, 2010

[13] Samalerk, P. and Pochai, N., "Numerical Simulation of a One-Dimensional Water-Quality Model in a Stream Using a Saulyev Technique with Quadratic Interpolated Initial-Boundary Conditions," Abstract and Applied Analysis, vol. 2018, 2018

[14] Kraychang, W. and Pochai, N., "Implicit Finite Difference Simulation of Water Pollution Control in a Connected Reservoir System," IAENG International Journal of Applied Mathematics, vol. 46, no.4, pp47–57, 2016

[15] Salah, F. and H. Elhafian, M., "Numerical Solution for Heat Transfer of Non – Newtonian Second – Grade Fluid Flow over Stretching Sheet via Successive Linearization Method," IAENG International Journal of Applied Mathematics, vol. 49, no.4, pp505-512, 2019.

[16] Christodoulides, P., Grimshaw, R. and Demetriades, C., "Three-fluid System Short-wave Instability and Gap-solitons," IAENG International Journal of Applied Mathematics, vol. 41, no.3, pp235-240, 2011

[17] Klankaew, P. and Pochai, N., "Numerical Simulation of a Nonlinear Thin Fluid Film Flow Velocity Model of a Third Grade Fluid on a Moving Belt using Finite Difference Method with Newton Iterative Scheme," Proceedings of The International Multi Conference of Engineers and Computer Scientists 2019, pp318-321, 2019

[18] Timpitak, W. and Pochai, N., "Numerical Simulations to a One-dimensional Groundwater Pollution Measurement Model Through Heterogeneous Soil," IAENG International Journal of Applied Mathematics, vol. 50, no.3, pp558-565, 2020

[19] Kraychang, W. and Pochai, N., "A Simple Numerical Model for Water Quality Assessment with Constant Absorption around Nok Phrao Island of Trang River," Engineering Letters, vol. 28, no.3, pp912-922, 2020

[20] Konglok, S.A. and Pochai, N., "Numerical Computations of Three-dimensional Air-Quality Model with Variations on Atmospheric Stability Classes and Wind Velocities using Fractional Step Method," IAENG International Journal of Applied Mathematics, vol. 46, no.1, pp112-120, 2016

[21] Suebyat, W. and Pochai, N., "A Numerical Simulation of a Three-dimensional Air Quality Model in an Area Under a Bangkok Sky Train Platform Using an Explicit Finite Difference Scheme," IAENG International Journal of Applied Mathematics, vol. 40, no.4, pp471-476, 2016

Appendix

Appendix A

Othata and Pochai *Advances in Difference Equations* (2019) 2019:203
https://doi.org/10.1186/s13662-019-2106-4

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

A one-dimensional mathematical simulation to salinity control in a river with a barrage dam using an unconditionally stable explicit finite difference method

Pornpon Othata[1,2*] and Nopparat Pochai[1,2*]

*Correspondence:
pornpon.othata@gmail.com;
nop_math@yahoo.com
[1] Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand
[2] Centre of Excellence in Mathematics, CHE, Bangkok, Thailand

Abstract

Salinity refers to the amount of salt in rivers, where the salt can be in many different forms. There are two main methods of defining the concentration of salt in water such as the total dissolved solid measurement (TDS) and the electrical conductivity measurement (EC). The salinity is measured by evaporating water to dryness and weighing the solid residue. The electrical conductivity measurement is measured by passing an electric current through the water and measuring how readily the current flows. The total amount of salt in the water can affect the taste of water. The World Health Organization's guideline on water palatability is that water with a salinity level of less than about 0.50–0.60 g/L is generally considered to be of a standard level. The drinking-water becomes significantly and increasingly unpalatable at salinity levels greater than about 1.0 g/L. In this research, a one-dimensional mathematical model of salinity measurement in a river is proposed. A modified model of salinity control in a river with a barrage dam is also introduced. An unconditionally stable explicit finite difference technique is used to approximate the salinity level under several conditions from the proposed model. The proposed computational technique gives good agreement results in realistic scenarios for water supply processes.

MSC: 65N06; 65M06; 92F99

Keywords: Salinity; Water quality; Barrage dam; River; Saulyev method

1 Introduction

Water production means the removal of surface water or raw water from natural water sources such as rivers, canals, reservoirs, and the sea into the production process for the quality and quantity as per requirement such as tap water and pure water for use in consumption, agriculture, and industry. Each type of production water can use different production technologies.

Water supply systems will use surface water or raw water to produce water, which will be used for consumption, agriculture, and certain industries that do not require high quality water. There are many factors that affect the quality of the water produced such as salinity of the water. It is a very important factor in the production because it cannot be treated in

Table 1 Water quality monitoring stations in river, Thailand where S_7 is the main water supply pumping station for Bangkok

Stations	Distance from the estuary
S_1	12
S_2	27
S_3	35
S_4	50
S_5	64
S_6	91
S_7	96
S_8	108

the normal way. So, for bringing the water to the water treatment process, it is necessary to have a salinity standard.

The Waterworks Authority of Thailand has eight water quality monitoring stations located throughout the river. Each station has a distance from the estuary as shown in Table 1. Currently, the station used to pump raw water for use in the water supply process for consumption in Bangkok has a problem of salinity of water over the standard. That makes an impact on the quality of water produced has a salinity up to standard.

In [1] and [2], the finite element method was used to solve the water pollution models. In [3], the finite difference method was used to solve the hydrodynamic model with the constant coefficients in the closed uniform reservoir. In [4], an analytical solution to the hydrodynamic model in a closed uniform reservoir was proposed. In [5], the Lax–Wendroff finite difference method was also proposed to approximate the water elevation and water flow velocity. In [6], the fourth-order method for a one-dimensional water quality model in a nonuniform flow stream was proposed. In [7], a nondimensional form of a two-dimensional hydrodynamic model with generalized boundary condition and initial conditions for describing the elevation of water wave in an open uniform reservoir was proposed.

Today, there are research studies on the effects of drinking water with salinity over standards, such as [8, 9], and [10]. We will see that the water is too salty to the standards that affect the body. Therefore, research has been presented on the increase of salt water, such as [11] and [12]. The well-known mathematical model uses the conservative property for defining the diffusion of salinity water in a one-dimensional equation [13]

$$A\frac{\partial S}{\partial t} + Q\frac{\partial S}{\partial x} = \frac{\partial}{\partial x}\left[AD_x\frac{\partial S}{\partial x}\right], \tag{1}$$

where A is a cross-sectional area of the river (m^2), Q is flow rate (m^3/s), D_x is diffusion coefficient of water (m^2/s), S is salinity value (ppt), x is distance (m), and t is time (s).

In this research, a one-dimensional mathematical model of salinity measurement in a river is proposed. A modified model of salinity control in a river with a barrage dam is also introduced. An unconditionally stable explicit finite difference technique is used to approximate the salinity level under several conditions from the proposed model. The proposed computational technique can be applied in realistic scenarios for water supply processes.

2 Governing equations

2.1 Salinity water pollution measurement model

In a stream water quality model, the governing equation is the dynamic one-dimensional advection-dispersion equation. A simplified representation, averaging the equation over the depths, is shown in [6]:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2} \tag{2}$$

for all $(x, t) \in \Omega = [0, L] \times [0, T]$, u is the flow velocity and D is a given diffusion coefficient.

Assume that the salinity is diluted by the freshwater, then the salinity advection level is reduced by the freshwater velocity. The percentage ability of freshwater to dilute salinity is assumed to be $0 \le k \le 1$. The one-dimensional salinity water pollution measurement model in a river can be given as follows:

$$\frac{\partial c}{\partial t} + (u_s - ku_w)\frac{\partial c}{\partial x} = D_s\frac{\partial^2 c}{\partial x^2}, \tag{3}$$

where $c(x, t)$ is the salinity concentration (kg/m^3), u_s is advective velocity of salinity water (m/s), k is water salinity removal efficiency rate, u_w is the fresh water flow velocity.

2.2 Initial conditions

The initial condition is defined by an interpolation function of measured raw salinity data. It is aligned on the length of the river from the estuary to the end of the considered area. The initial condition is assumed to be

$$c(x, 0) = f(x) \tag{4}$$

for all $x \in [0, L]$, where $f(x)$ is an interpolation function of measured salinity data.

2.3 Boundary condition

2.3.1 Left boundary condition

The left boundary condition is an interpolation function of measured raw data. It is based on the salinity of a river at the first station close to the estuary. The boundary condition is assumed to be

$$c(0, t) = g(t) \tag{5}$$

for all $t \in [0, T]$, where $g(t)$ is a given interpolation function by measured salinity data at the first monitoring station.

2.3.2 Right boundary condition

The right boundary condition is defined by the rate of change of salinity area of the water. The condition can be given as follows:

$$\frac{\partial c}{\partial x} = C_R \tag{6}$$

for all $t \in [0, T]$, where C_R is an approximated rate of change of salinity around the last monitoring station.

3 Explicit finite difference method for a one-dimensional salinity water pollution measurement model

We now discretize the domain by dividing the interval $[0, L]$ into M subintervals such that $M\Delta x = L$ and the time interval $[0, T]$ into N subintervals such that $N\Delta t = T$. The grid points (x_i, t_n) are defined by $x_i = i\Delta x$ for all $i = 1, 2, 3, \ldots, M$ and $t_n = n\Delta t$ for all $n = 1, 2, 3, \ldots, N$, in which M and N are positive integers. We can then approximate $c(x_i, t_n)$ by C_i^n, value of the difference approximation of $c(x, t)$ at point $x = i\Delta x$ and $t = n\Delta t$, where $0 \le i \le M$ and $0 \le n \le N$. We will employ the forward time central space finite difference scheme (FTCS) and the Saulyev method into Eq. (2).

3.1 Forward time central space finite difference scheme

Taking the forward time central space technique [4] into Eq. (2), we get the following discretization:

$$c(x_i, t_n) \cong C_i^n, \tag{7}$$

$$\frac{\partial c}{\partial t}|_{(x_i,t_n)} \cong \frac{C_i^{n+1} - C_i^n}{\Delta t}, \tag{8}$$

$$\frac{\partial c}{\partial x}|_{(x_i,t_n)} \cong \frac{C_{i+1}^n - C_{i-1}^n}{2\Delta x}, \tag{9}$$

$$\frac{\partial^2 c}{\partial x^2}|_{(x_i,t_n)} \cong \frac{C_{i+1}^n + C_{i-1}^n - 2C_i^n}{(\Delta x)^2}, \tag{10}$$

$$u_s(x_i, t_n) = u_{s_i}^n, \tag{11}$$

$$u_w(x_i, t_n) = u_{w_i}^n. \tag{12}$$

Substituting Eqs. (7–12) into Eq. (2), we get the finite difference equation:

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} + \left(u_{s_i}^n - ku_{w_i}^n\right)\left(\frac{C_{i+1}^n - C_{i-1}^n}{2\Delta x}\right) = D_s\left(\frac{C_{i+1}^n + C_{i-1}^n - 2C_i^n}{(\Delta x)^2}\right). \tag{13}$$

Then the explicit finite difference equation becomes

$$C_{i+1}^{n+1} = \left(\lambda + 0.5r_i^n\right)C_{i-1}^n + (1 - 2\lambda)C_i^n + \left(\lambda - 0.5r_i^n\right)C_{i+1}^n \tag{14}$$

for all $i = 1, 2, 3, \ldots, M - 1$, where $\lambda = \frac{D_s\Delta t}{(\Delta x)^2}$ and $r_i^n = \frac{(u_{s_i}^n - ku_{w_i}^n)\Delta t}{\Delta x}$. The forward time central space scheme is conditionally stable subject to constraints in Eq. (13). The stability requirements for the scheme are [6], $0 < \lambda < \frac{1}{2}$, and $0 < r_i^n < 1$.

3.1.1 Right boundary condition approximation

For the right boundary condition Eq. (6), the right boundary condition is defined by the rate of change of salinity area of the water. The right boundary condition is assumed to be

$$\frac{\partial c}{\partial x} \approx \frac{c(L_2, t) - c(L_1, t)}{L_2 - L_1} \tag{15}$$

for all $t \in [0, T]$, where L_1 and L_2 are the distance from the upstream to the point before and after the water supply source, respectively. If we substitute the approximate unknown value of the right boundary, we obtain

$$C_{M+1}^n = \left(\frac{C_{M_2}^n - C_{M_1}^n}{L_2 - L_1}\right)\Delta x + C_{M-1}^n. \tag{16}$$

The forward time central space scheme is conditionally stable subject to constraints in Eq. (13). The stability requirements for the scheme are [6]. It can be obtained that the strict stability requirements are the main disadvantage of this scheme.

3.2 Saulyev explicit finite difference scheme

The Saulyev scheme is unconditionally stable [3]. It is clear that the non-strict stability requirement of the Saulyev scheme is the main advantage and economical to use. Taking Saulyev technique [3] into Eq. (2), the following discretization can be obtained:

$$c(x_i, t_n) \cong C_i^n, \tag{17}$$

$$\frac{\partial c}{\partial t}\Big|_{(x_i,t_n)} \cong \frac{C_i^{n+1} - C_i^n}{\Delta t}, \tag{18}$$

$$\frac{\partial c}{\partial x}\Big|_{(x_i,t_n)} \cong \frac{C_{i+1}^n - C_{i-1}^{n+1}}{2\Delta x}, \tag{19}$$

$$\frac{\partial^2 c}{\partial x^2}\Big|_{(x_i,t_n)} \cong \frac{C_{i+1}^n - C_i^n - C_i^{n+1} + C_{i-1}^{n+1}}{(\Delta x)^2}. \tag{20}$$

Substituting Eqs. (17–20) into Eq. (2), we get the finite difference equation

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} + \left(u_{s_i}^n - ku_{w_i}^n\right)\left(\frac{C_{i+1}^n - C_{i-1}^{n+1}}{2\Delta x}\right) = D_s\left(\frac{C_{i+1}^n - C_i^n - C_i^{n+1} + C_{i-1}^{n+1}}{(\Delta x)^2}\right). \tag{21}$$

Then the explicit finite difference equation becomes

$$C_{i+1}^{n+1} = \left(\frac{1}{1+\lambda}\right)\left[\left(\lambda + \frac{1}{2}r_i^n\right)C_{i-1}^{n+1} + (1-\lambda)C_i^n + \left(\lambda - \frac{1}{2}r_i^n\right)C_{i+1}^n\right] \tag{22}$$

for all $i = 1, 2, 3, \ldots, M-1$, where $\lambda = \frac{D_s \Delta t}{(\Delta x)^2}$ and $r_i^n = \frac{(u_{s_i}^n - ku_{w_i}^n)\Delta t}{\Delta x}$. For $i = M$, the right boundary condition Eq. (5), if substituting the approximate unknown value of the right boundary, we obtain $C_{M+1}^n = (\frac{C_{M_2}^n - C_{M_1}^n}{L_2 - L_1})\Delta x + C_{M-1}^n$.

Using Taylor series expansions on the approximation, [14] has shown that the truncation error is $O\{(\Delta x)^2 + (\Delta t)^2 + (\Delta t/\Delta x)^2\}$.

The Saulyev method is an unconditionally stable method [15]. It follows that the application of the explicit Saulyev finite difference technique is economical in terms of computation implementation.

4 Numerical simulations

4.1 Simulation 1: salinity control in an ideal case

We consider a segment of a river with 108 km of length as shown in Table 1. Assume that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30%, and the given simulated station at any

Table 2 Physical parameters of simulation 1

D_s (m²/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (s)
0.1	0.065	0.25	0.3	108	100

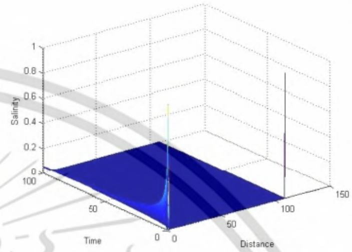


Figure 1 The exact solution of simulation 1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 108$ and $0 \le t \le 1000$

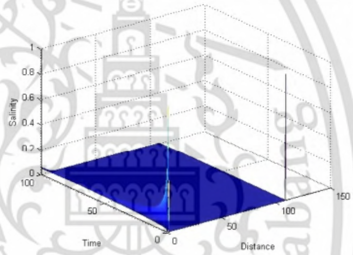


Figure 2 The FTCS solution of simulation 1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 108$ and $0 \le t \le 1000$

Table 3 The maximum absolute error defined by $\mathbf{err}_{max} = \max|\bar{c}(x_i, T) - c(x_i, T)|$ for all $i = 0, 1, \ldots, N$, where $T = 10, 20, 30$, and 40

T	FTCS	Saulyev
	$\mathbf{err}_{max}$	$\mathbf{err}_{max}$
10	5.9442×10^{-4}	5.1141×10^{-4}
20	0.0044	0.0042
30	0.0083	0.0082
40	0.0107	0.0107

time is 100. Their physical parameters and given spacing are shown in Table 2. In [11], the theoretical solution is given by

$$c(x,t) = \frac{1}{\sqrt{4t+1}} \exp\left[-\frac{(x - 1 - (u_s - ku_w)t)^2}{D(4t+1)}\right]. \quad (23)$$

Actually, when using the FTCS scheme Eq. (14) and the Saulyev technique Eq. (23), when their physical parameters are as given in Table 2, we get the approximated solution $c(x,t)$. The theoretical solution is illustrated by a surface of solution in Fig. 1. The FTCS approximated solution is illustrated by Fig. 2. The Saulyev approximated solution is also illustrated by Fig. 3. The maximum absolute error of both finite difference approximations is compared in Table 3.

Figure 3 The Saulyev solution of simulation 1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 1000$

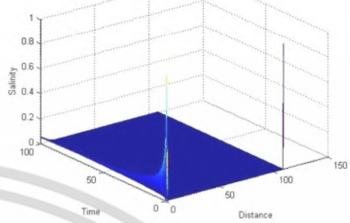


Table 4 Physical parameters of simulation 2

D_s (m²/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (s)
0.1	0.065	0.3	0.3	108	100
0.1	0.065	0.25	0.3	108	100
0.1	0.065	0.2	0.3	108	100

Table 5 Convergence of FTCS method and Saulyev method for some grid spacing

T	Δx	Δt	FTCS	Saulyev
100	0.10	0.04	Stable	Stable
		0.05	Stable	Stable
		0.06	Untable	Stable
		0.07	Untable	Stable
100	0.05	0.04	Untable	Stable
		0.05	Untable	Stable
		0.06	Untable	Stable
		0.07	Untable	Stable
100	0.025	0.04	Untable	Stable
		0.05	Untable	Stable
		0.06	Untable	Stable
		0.07	Untable	Stable

4.2 Simulation 2: the salinity is diluted by releasing the fresh water from a barrage dam with different flow velocities.

We consider a segment of a river with 108 km of length as shown in Table 1. Assuming that the salinity diffusion coefficient is 0.1 m²/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30%, and the given simulated station at any time is 1000. Their physical parameters and given spacing are shown in Table 4. In this simulation, the Saulyev technique is used to approximate the solution since the technique will always give stable solutions as shown in Table 5. According to the good agreement of approximated solutions of the Saulyev method, the method in Eq. (23) is chosen to approximate the solution of the simulation. The several fresh water flow velocities $u_w = 0.20, 0.25, 0.30$ m/s from the barrage dam are simulated until the salinity level at the controlled monitoring station S_7 becomes standardized level as shown in Figs. 4–5.

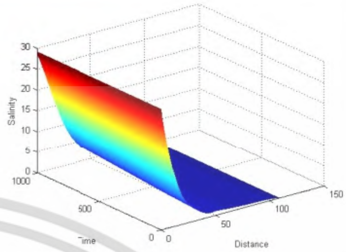


Figure 4 Approximated salinity along the river by using the Saulyev technique, where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 1000$

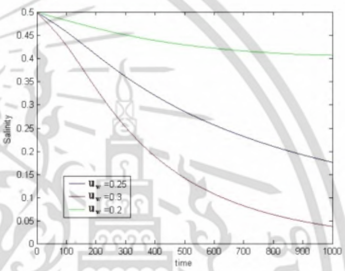


Figure 5 Approximated salinity at $c(96, t)$ by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$ when $u_w = 0.3, 0.25$ and 0.2

Table 6 Physical parameters of simulation 3

$c(x,t)$ at S_7	D (m^2/s)	u_s (m/s)	u_w (m/s)	K	T	L (km)	$c(0,t)$
$> C_{ST}$	0.1	0.065	0.25	0.3	1000	108	$g(t)$
$< C_{ST}$	0.1	0.065	0.205	0.3	1000	108	$g(t)$

4.3 Simulation 3: the salinity is diluted by releasing the fresh water from a barrage dam and changing flow velocities after the salinity comes to standard

We consider a segment of a river with 108 km of length as shown in Table 1. Assuming that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30%, and the given simulated station at any time is 1000. Their physical parameters and given spacing are shown in Table 6. Assume that there are eight monitoring stations along a considered river segment as shown in Table 1. The controlled monitoring station is station S_7. We need to control the salinity level at station S_7 to be under the salinity standard level $C_{ST} = 0.3\ kg/m^3$. The salinity is controlled by a process as follows:

(1) If the salinity level at station S_7 $c(96, t) > C_{ST}$, then the fresh water will be released at a high speed from the barrage dam by controlled flow velocity.

(2) If the salinity level at station S_7 $c(96, t) < C_{ST}$, then the fresh water will be released at a low speed level from the barrage dam.

We can obtain the approximated salinity level along the considered river segment as shown in Fig. 6 and Table 7. The salinity level at several monitoring stations S_1, S_5, and S_7 is shown in Figs. 7, 8, and 9, respectively.

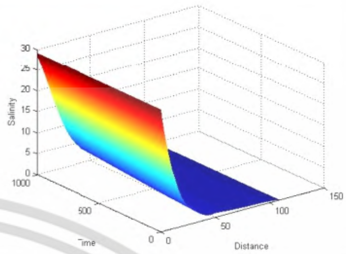


Figure 6 The numerical solution of simulation 2 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 108$ and $0 \le t \le 1000$

Table 7 Approximated salinity $c(x,t)$ of simulation 2 for all monitoring stations

t	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
1	12.1040	4.0187	2.0224	1.0978	0.7955	0.5316	0.4995	0.1444
5000	11.3456	3.6020	1.9844	1.0058	0.7667	0.4807	0.3844	0.0027
10,000	11.3391	3.6002	2.0397	1.0128	0.7476	0.4098	0.2993	0.0023
15,000	13.7382	4.4013	2.4788	1.1159	0.7704	0.3996	0.2987	0.0032
20,000	15.5617	5.3633	3.0060	1.2576	0.8025	0.3961	0.2983	0.0034

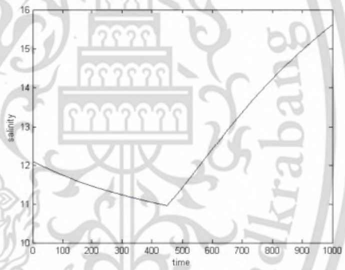


Figure 7 Approximated salinity of simulation 2 at station S_1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le t \le 1000$

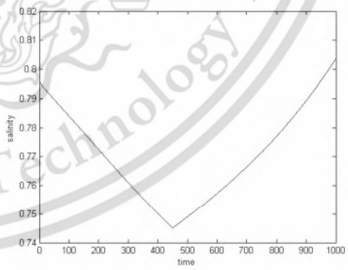


Figure 8 Approximated salinity of simulation at station S_5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le t \le 1000$

4.4 Simulation 4: diluting the salinity of water by releasing fresh water before salinity water arrives at the pumping station

We consider a segment of a river with 108 km of length as shown in Table 1. Assume that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30%, and the given simulated station at any time is

Figure 9 Approximated salinity of simulation at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le t \le 1000$

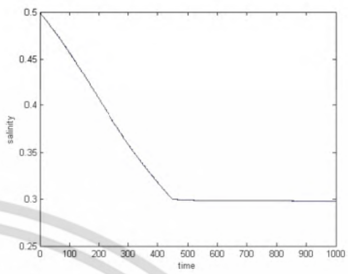
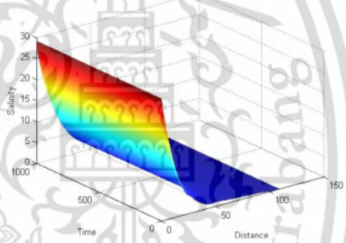


Table 8 Physical parameters of simulation 2

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)	K	T	L (km)	$c(0,t)$
$< C_{ST}$	0.1	0.065	0	0.3	1000	108	$g(t)$
$> C_{ST}$	0.1	0.065	0.25	0.3	1000	108	$g(t)$

Figure 10 The numerical solution of simulation 3 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 108$ and $0 \le t \le 1000$



1000. Their physical parameters and given spacing are shown in Table 8. Assume that there are eight monitoring stations along the considered river segment as shown in Table 1. The controlled monitoring station is station S_7. We need to control the salinity level at station S_7 before salinity level at station S_7 is over the salinity standard level $C_{ST} = 0.05\ \mathrm{kg/m^3}$ for about three days. In this simulation, the Saulyev technique is used to approximate the solution since the technique will always give stable solutions. The salinity is controlled by a process as follows:

(1) If the salinity level at station S_5 $c(91,t) < C_{ST}$, then the fresh water is released at a normal speed level from the barrage dam.

(2) If the salinity level a station S_5 $c(91,t) > C_{ST}$, then the fresh water will be released at a high speed from the barrage dam which is used to control the salinity.

We can obtain the approximated salinity level along the considered river segment as shown in Table 9 and Fig. 10. The salinity level at several monitoring stations S_1, S_5, and S_7 is shown in Figs. 11, 12, and 13, respectively.

5 Discussion

In simulation 1, we get good agreement between approximated solutions of the FTCS and the Saulyev finite difference techniques. The maximum error is less than 1%. In simulation 2, we can obtain that the Saulyev technique is better than the FTCS technique due to

Table 9 Approximated salinity $c(x,t)$ of simulation 3 for all monitoring stations

t	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
1	12.1040	4.0187	2.0224	1.0978	0.7957	0.4010	0.2544	0.1395
5000	20.3804	7.4884	4.0812	1.4095	0.9276	0.4791	0.3919	0.0040
10,000	17.8501	7.0540	3.9099	1.4447	0.8945	0.4144	0.2993	0.0020
15,000	16.1112	6.5658	3.7400	1.4458	0.8669	0.3456	0.2327	0.0015
20,000	14.8449	6.1007	3.5647	1.4298	0.8380	0.2888	0.1868	0.0011

Figure 11 Approximated salinity of simulation 3 at station S_1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le t \le 1000$

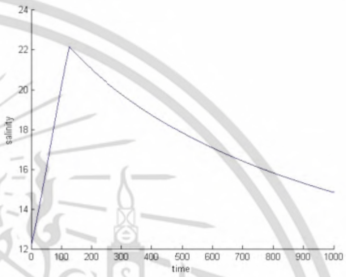


Figure 12 Approximated salinity of simulation 3 at station S_5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le t \le 1000$

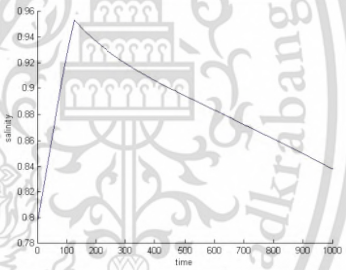
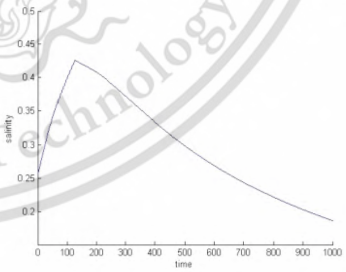


Figure 13 Approximated salinity of simulation 3 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le t \le 1000$



the limitation of stability conditions. The Saulyev technique gives a stable approximated solution. Otherwise, the FTCS is limited by its stability conditions. Thus the Saulyev technique is preferred in other realistic simulations. We can see that the salinity level will be reduced when the fresh water flow velocity is increasing as shown in Fig. 5. In simulation 3, a salinity control process is simulated. The salinity is reduced, the salinity level comes to standard, after that we can decrease the fresh water flow velocity to maintain the salinity level at the standard level as shown in Fig. 9. In simulation 4, a salinity control process is simulated. The salinity is reduced before the salinity level touches the standard salinity level. The proposed process can reduce the salinity level when the fresh water is released from the barrage dam at least amount as shown in Figs. 11–13.

6 Conclusion

We have proposed a one-dimensional mathematical model of salinity measurement in a river with a barrage dam. The proposed model deals with salinity advection to a river and the fresh water flow from the barrage dam effects. The traditional forward time central space finite difference method is compared with the proposed Saulyev technique. The proposed Saulyev technique gives a stable solution in any grid spacing. The technique also gives accurately approximated solutions. The realistic problem is also simulated. The proposed simulation can be used in several realistic salinity measurements. In the salinity control aspect, the proposed process can reduce the salinity level before the level is over the standard. The proposed numerical simulation can be applied in practical salinity control in a river with a barrage dam.

Funding
This paper is supported by the Centre of Excellence in Mathematics, the Commission on Higher Education, Thailand. The authors greatly appreciate valuable comments received from the referees.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. The authors read and approved the final manuscript.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 30 January 2019 Accepted: 17 April 2019 Published online: 24 May 2019

References

1. Pochai, N., Tangmanee, S., Crane, L.J., Miller, J.J.H.: A mathematical model of water pollution control using the finite element method. In: Proceedings in Applied Mathematics and Mechanics, 27–31 March 2006, Berlin (2006)
2. Tabuenca, P., Vila, J., Cardona, J., Samartin, A.: Finite element simulation of dispersion in the Bay of Santander. Adv. Eng. Softw. **28**, 313–332 (1997)
3. Pochai, N., Tangmanee, S., Crane, L.J., Miller, J.J.H.: A water quality computation in the uniform channel. J. Interdiscip. Math. **11**, 803–814 (2008)
4. Pochai, N.: A numerical computation of non-dimensional form of stream water quality model with hydrodynamic advection-dispersion-reaction equations. Nonlinear Anal. Hybrid Syst. **13**, 666–673 (2009)
5. Pochai, N.: A numerical computation of non-dimensional form of a nonlinear hydrodynamic model in a uniform reservoir. Nonlinear Anal. Hybrid Syst. **3**, 463–466 (2009)
6. Pochai, N.: Unconditional stable numerical techniques for a water-quality model in a non-uniform flow stream. Adv. Differ. Equ. **2017**, 286 (2017)
7. Thongtha, K., Kasemsuwan, J.: Analytical solution to a hydrodynamic model in an open uniform reservoir. Adv. Differ. Equ. **2017**, 149 (2017)
8. Nahian, M.A., Ahmed, A., Lazar, A.N., Hutton, C.W., Salehin, M., Streatfield, P.K.: Drinking water salinity associated health crisis in coastal Bangladesh. Elem. Sci. Ant. **6**, 2 (2018)
9. Suarez, D.L., Lebron, I.: Water quality criteria for irrigation with highly saline water. In: Towards the Rational Use of High Salinity Tolerant Plants, vol. 28, pp. 389–397 (1996)
10. Letey, J.: Relationship between salinity and efficient water use. Irrigation science. Irrig. Sci. **14**, 75–84 (1993)
11. Leewatchanakul, K.: Salinity intrusion in Chao Phraya river. PhD thesis, Chulalongkorn University (1988)
12. Grant, R.F.: Salinity water use and yield of maize: testing of the mathematical model ecosys. Plant Soil **172**, 389–397 (1995)
13. Chapra, S.C.: Surface Water-Quality Modeling. McGraw-Hill, New York (1997)
14. Pochai, N.: A numerical treatment of nondimensional form of water quality model in a nonuniform flow stream using Saulyev scheme. Math. Probl. Eng. **2011**, 1–16 (2011)
15. Samalerk, P., Pochai, N.: Numerical simulation of a one-dimensional water-quality model in a stream using a Saulyev technique with quadratic interpolated initial-boundary conditions (2018)

Appendix B

A Mathematical Model of Salinity Control in a River with an Effect of Internal Waves using Two Explicit Finite Difference Methods

Pornpon Othata, and Nopparat Pochai

Abstract—**Salinity is related to the quantity of salt in rivers, and that salt can take several different forms. Salinity is determined by water evaporation to dryness, and form residual measuring. The total salt content in water can in influence the taste of water. Drinking water at levels of salinity greater than around 1.0 g/L is drastically and rapidly unpalatable. In this research, two models, the internal wave hydrodynamic model and the salinity dispersion model, are proposed. The internal waves hydrodynamic model provides salinity internal wave and salinity flow velocity. The salinity dispersion model provides the salinity level. A modified salinity control model is also used in a river with a barrage dam. The suggested model provides salinity control releasing fresh water from a barrage dam where the salinity level is not higher than the standard and does not waste too much fresh water. An explicit finite difference method which is unconditionally stable is used for approximating the degree of salinity from the proposed model under many conditions.**

Index Terms—**Salinity Control, Internal Waves, Explicit Finite Difference Methods, Hydrodynamic model, Salinity dispersion model**

I. Introduction

Water creation implies the evacuation of surface water or crude water from characteristic water sources, for example, streams, channels, stores, and the ocean, in the generation procedure, meeting the quality and amount prerequisites for tap water and unadulterated water for buyer use or use in horticulture and industry. Each sort of creation water can utilize diverse generation advances.

Surface water or crude water is available in the water which will be utilized for utilization, horticulture, and certain enterprises that don't require top notch water. There are numerous elements that influence the nature of the water delivered, for example, saltiness of the water. It is a significant factor under way, on the grounds that it can't be dealt with customary ways. Thus, when carrying water into the treatment procedure, it is important to have a saltiness standard.

Thailand's Waterworks Authority has nine water quality control stations all over a river. Each station is within reaching distance of the estuary as seen in Table 1.

At present, the pumping station has an excess of salinity, which affects the quality of tap water in Bangkok. This impacts the quality of water produced.

P. Othata is a PhD candidate of Department of Applied Mathematics, Faculty of Science, King Mongkuts Institute of Technology Ladkrabang, Bangkok, 10520, Thailand (e-mail: pornpon.othata@gmail.com).

N. Pochai is an Assistant Professor of Department of Applied Mathematics, Faculty of Science, King Mongkuts Institute of Technology Ladkrabang, Bangkok, 10520, Thailand (corresponding author to provide phone: 662-329-8400; fax: 662-329-8400; e-mail: nop_math@yahoo.com).

TABLE I
DISTANCE FROM THE ESTUARY OF EACH OBSERVATION STATIONS

Stations	Distance from the estuary(km)
S_1	12
S_2	27
S_3	35
S_4	50
S_5	64
S_6	91
S_7	96
S_8	108

In [1], the finite difference method was utilized to explain water contamination models. In [17] and [18], the three dimension advection diffusion model was introduced. In [13], the implicit finite difference method was to solve the water pollution model. In [5], [15] and [16], the fluid flow model was proposed. In [14], the numerical method was used to solve the fluid flow problem. In [2], a hydrodynamic model with constant coefficients in a closed uniform reservoir was solved by using the finite difference method. In [3], an theoretical solution for a hydrodynamic model in a closed uniform reservoir was suggested. In [4], The Lax-Wendroff scheme was also proposed for approximating salinity elevation and salinity flow rate. Research reports on the effect of salt drinking water on standards have been undertaken, such as [6]. The water was excessively salty, up to levels that influence the body. In this way, the expansion of salinity water in Chao Phraya River has been introduced in [7]. In [8], the one-dimensional salinity water model was proposed

$$A(x)\frac{\partial S}{\partial t} + Q(x,t)\frac{\partial S}{\partial x} = \frac{\partial}{\partial x}\left[A(x)D_x\frac{\partial S}{\partial x}\right], \qquad (1)$$

where $A(x)$ is the river cross-sectional area (m^2), $Q(x,t)$ is flow rates (m^3/s), The coefficient of water diffusion is D_x (m^2/s), S is water salinity level (ppt), x is the length of river (m) and t is times (s)

To solve this problem, a one-dimensional salinity management model with non-uniform internal waves in a river is proposed. A one-dimensional hydrodynamic model of the internal saltiness flow in a river is introduced. A saltiness control in a river with a barrage dam model is also proposed. A modified Lax-diffusive method is used to approximate the solution of the internal wave hydrodynamic model. the Sualyev scheme is used to estimate the saltiness level. The suggested evaluation method for water supply forms can be used in reasonable circumstances.

Engineering Letters, 29:2, EL_29_2_37

II. Salinity measurement model in a River with the effect of internal wave

2.1 One-dimensional salinity internal wave hydrodynamic model

Under the assumptions, by combining the Navier-Stokes equations over the flow direction as a hydrostatic pressure distribution and a heavy downward slope, one-dimensional shallow water equations are obtained. The hydrodynamic flows are swift and can be regarded as shallow salinity flows driven by advection. Hence the concept of eddy viscosity may be overlooked. The system of partial differential equations of governing equation and vector form equations [10] can be written as

$$\partial_x \begin{pmatrix} h \\ hu \end{pmatrix} + \partial_t \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix} = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix}, \quad (2)$$

where x is the horizontal difference across a river(m), t is time(s), $h(x,t)$ is the salinity wave lifting above the bottom(m/s), $z(x)$ is the feature that characterizes the underneath geography(m), and $u(x,t)$ is component of velocity(m/s), for all $(x,t) \in [0,L] \times [0,T]$. We can defined the initial conditions by

$$u(x,0) = u_1(x) \quad for\ all\ x \in [0,L] \quad (3)$$

and

$$h(x,0) = h_1(x) \quad for\ all\ x \in [0,L] \quad (4)$$

The boundary conditions shall also be determined by

$$\frac{\partial u(0,t)}{\partial x} = f_1(t), t > 0, \quad (5)$$

$$\frac{\partial u(L,t)}{\partial x} = f_2(t), t > 0, \quad (6)$$

$$h(0,t) = g_1(t), t > 0, \quad (7)$$

$$\frac{\partial h(L,t)}{\partial x} = g_2(t), t > 0. \quad (8)$$

So as to be predictable with the physical marvel of a hydrodynamic from the left to one side at $t = 0$.

2.2 One-dimensional salinity water measurement model

The fluid one-dimensional advection-dispersion equation is the governing equation in the salinity dispersion model. Simpler representation, The equation is measured over the waters, as appeared in [11]

$$\frac{\partial c}{\partial t} + u(x,t)\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2}, \quad (9)$$

where $(x,t) \in [0,L] \times [0,T]$, u is the speed of the salinity flow and D is coefficient of diffusion.

Expecting that the saltiness is weakened by the fresh water. These are then the saltiness shift in weather conditions level is decreased by the fresh water speed. The rate of fresh water efficiency to weaken saltiness is accepted by $k \in [0,1]$. In [11], the model for salinity problem was proposed

$$\frac{\partial c}{\partial t} + (u_s - ku_w)\frac{\partial c}{\partial x} = D_s\frac{\partial^2 c}{\partial x^2}, \quad (10)$$

where $c(x,t)$ is the saltiness level of water(kg/m^3), k is the efficiency rate of water salinity removal, u_s is the salinity water advective velocity (m/s) and u_w is the velocity of fresh water flow

2.2.1 Initial conditions: A Lagrange interpolating polynomials of the salinity data of salinity data is defined as the initial condition, along the river form the estuary is taken into consideration using salinity data, defined by

$$c(x,0) = f(x), \quad (11)$$

where $x \in [0,L]$ and $f(x)$ is a measured salinity data interpolation function.

2.2.2 Left boundary condition: The left boundary condition is a Lagrange interpolating polynomials of estimated crude information, it depends on the saltiness of a stream at the main station that shut to the estuary, defined by

$$c(0,t) = g(t), \quad (12)$$

where $t \in [0,T]$ and $g(t)$ is a given Lagrange interpolating polynomials at the first observation station via calculated salinity data.

2.2.3 Right boundary condition: The right boundary condition were determined by the change in salinity at the last station, defined by

$$\frac{\partial c}{\partial x} = C_R, \quad (13)$$

where $t \in [0,T]$ and C_R is an estimated the rate of change of the saltiness level at the last observation station.

III. Explicit finite difference scheme for one-dimensional salinity internal wave hydrodynamic model

We can discriminate against the domain by splitting the interval $[0,L]$ into M sub intervals such that $M\Delta x = L$ and the time interval $[0,T]$ into N subintervals such that $N\Delta t = T$. The grid points (x_m, t_n) are defined by $x_m = m\Delta x$ for all $m = 1,2,3,...,M$ and $t_n = n\Delta t$ for all $n = 1,2,3,...,N$ in which M and N are positive integers. We can then estimate $c(x_m,t_n)$ by C_m^n, value of the difference approximation of $c(x,t)$ at point $x = m\Delta x$ and $t = n\Delta t$, where $m \in [0,M]$ and $n \in [0,N]$.

3.1 A modified Lax-diffusive method for the hydrodynamic model

The hydrodynamic model gives the speed field and the height of the water at that point the model's calculated effect would be the contribution to the dispersion model that provides the target area of pollution. This section suggests the alteration approach of a standard Lax-diffusive method for the hydrodynamic model of [12].

We'll adjust f^* from the standard formula of [12] to be the sum of three points. The semi-discrete scheme is applied to Eq.(2) and usage of an existing spatial grid $(x_m, t_n) = (m\Delta x, n\Delta t)$, we can define

$$f_x = \frac{f_{m+1}^n - f_{m-1}^n}{2\Delta x}, \quad (14)$$

$$f_t = \frac{f_m^{n+1} - f_m^*}{\Delta t} \quad (15)$$

where

$$f^* = \frac{f_{m+1}^n + f_m^n + f_{m-1}^n}{3} \quad (16)$$

Engineering Letters, 29:2, EL_29_2_37

The partial derivative of h and u is estimated by using Eqs.(14-16), respectively, with respect to x and t. We will see that Eq.(2) is in a form of a matrix as

$$A_t + B_x + C = 0, \tag{17}$$

where

$$A = \begin{pmatrix} h \\ hu \end{pmatrix}, B = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix} \tag{18}$$

It implies that the standardized spatial grids will write the Eq.(18) as

$$A_m^n = \begin{pmatrix} h_m^n \\ h_m^n u_m^n \end{pmatrix}, B = \begin{pmatrix} h_m^n u_m^n \\ h_m^n (u_m^n)^2 + \frac{1}{2}g(h_m^n)^2 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 \\ -gh_m^n \partial_x z \end{pmatrix}, \tag{19}$$

Substituting Eqs.(14-15) and Eq.(16) into Eq.(17), we obtain that

$$A_m^{n+1} = \frac{\Delta t}{2\Delta x}\left(B_{m-1}^n - B_{m+1}^n\right) + A^*, \tag{20}$$

where $A^* = \begin{pmatrix} h^* \\ (hu)^* \end{pmatrix}$. Substituting Eq.(19) into Eq.(20), we can see that

$$\begin{pmatrix} h_m^{n+1} \\ h_m^{n+1} u_m^{n+1} \end{pmatrix} = \begin{pmatrix} h_{m-1}^n + h_m^n + h_{m+1}^n \\ h_{m-1}^n u_{m-1}^n + h_m^n u_m^n + h_{m+1}^n u_{m+1}^n \end{pmatrix}$$

$$+ \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{m-1}^n u_{m-1}^n - h_{m+1}^n u_{m+1}^n \\ h_{m-1}^n (u_{m-1}^n)^2 - h_{m+1}^n (u_{m+1}^n)^2 \\ +\frac{1}{2}g\left((h_{m-1}^n)^2 - (h_{m+1}^n)^2\right) \end{pmatrix}, \tag{21}$$

for all $m \in [1, M]$ and $n \in [0, N-1]$. For upper boundary, where $m = 0$, replaced by $u_{-1}^n = u_0^n$ and $h_{-1}^n = h_0^n$ into Eq.(21), we get

$$\begin{pmatrix} h_1^{n+1} \\ h_1^{n+1} u_1^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_0^n u_0^n - h_1^n u_1^n \\ h_0^n (u_0^n)^2 - h_1^n (u_1^n)^2 \\ +\frac{1}{2}g\left((h_0^n)^2 - (h_1^n)^2\right) \end{pmatrix}$$

$$+ \begin{pmatrix} 2h_0^n + h_1^n \\ 2h_0^n u_0^n + h_1^n u_1^n \end{pmatrix}, \tag{22}$$

For lower boundary, where $m = M$, substituted by boundary conditions for the estimated unknown value of the right boundary, we can let $u_{M+1}^n = u_M^n$ and $h_{M+1}^n = h_M^n$ by rearranging, we get

$$\begin{pmatrix} h_M^{n+1} \\ h_M^{n+1} u_M^{n+1} \end{pmatrix}$$

$$= \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{M-1}^n u_{M-1}^n - h_M^n u_M^n \\ h_{M-1}^n (u_{M-1}^n)^2 - h_M^n (u_M^n)^2 \\ +\frac{1}{2}g\left((h_{M-1}^n)^2 - (h_M^n)^2\right) \end{pmatrix}$$

$$+ \begin{pmatrix} h_{M-1}^n + 2h_M^n \\ h_{M-1}^n u_{M-1}^n + 2h_M^n u_M^n \end{pmatrix}, \tag{23}$$

The scheme's stability state required CFL number as the [12],

$$C_n = u_{\max}\left(\frac{\Delta t}{\Delta x}\right) \le 1. \tag{24}$$

3.2 Saulyev method for the salinity dispersion model

The Saulyev scheme is unconditionally stable [2]. Obviously the non strictly dependability prerequisite of Saulyev scheme is the principle of preferred position and conservative to utilize. Taking the Saulyev scheme into Eq.(10), it appears to have the following discretization:

$$c(x_m, t_n) \cong C_m^n, \tag{25}$$

$$\left.\frac{\partial c}{\partial t}\right|_{(x_m,t_n)} \cong \frac{C_m^{n+1} - C_m^n}{\Delta t}, \tag{26}$$

$$\left.\frac{\partial c}{\partial x}\right|_{(x_m,t_n)} \cong \frac{C_{m+1}^n - C_{m-1}^{n+1}}{2\Delta x}, \tag{27}$$

$$\left.\frac{\partial^2 c}{\partial x^2}\right|_{(x_m,t_n)} \cong \frac{C_{m+1}^n - C_m^n - C_m^{n+1} + C_{m-1}^{n+1}}{(\Delta x)^2}, \tag{28}$$

$$u_{s_m}^n \cong u_m^n, \tag{29}$$

$$u_{w_m}^n = u_w(x_m, t_n). \tag{30}$$

Substituting Eqs.(25-30) into Eq.(10), We obtain the equation of finite difference,

$$\frac{C_m^{n+1} - C_m^n}{\Delta t} + (u_{s_m}^n - ku_{w_m}^n)\left(\frac{C_{m+1}^n - C_{m-1}^{n+1}}{2\Delta x}\right)$$

$$= D_s\left(\frac{C_{m+1}^n - C_m^n - C_m^{n+1} + C_{m-1}^{n+1}}{(\Delta x)^2}\right). \tag{31}$$

The explicit equation of finite difference then becomes

$$C_{m+1}^{n+1} = \left(\frac{1}{1+\lambda}\right)[\left(\lambda + \frac{1}{2}r_m^n\right)C_{m-1}^{n+1} + (1-\lambda)C_m^n$$

$$+ \left(\lambda - \frac{1}{2}r_m^n\right)C_{m+1}^n]. \tag{32}$$

where $i = 1, 2, 3, ..., M-1$, $\lambda = \frac{D_s \Delta t}{(\Delta x)^2}$ and $r_m^n = \frac{(u_{s_m}^n - ku_{w_m}^n)\Delta t}{\Delta x}$. For $i = M$, replaced the unknown value in Eq.(5), we obtain

$$C_{M+1}^n = \left(\frac{C_{M_2}^n - C_{M_1}^n}{L_2 - L_1}\right)\Delta x + C_{M-1}^n. \tag{33}$$

The truncation error of Sualyev scheme is $O\left\{(\Delta x)^2 + (\Delta t)^2 + (\Delta t/\Delta x)^2\right\}$.

IV. Numerical Simulations

4.1 Simulation 1 : the salinity flow velocity and the salinity elevation in a river with effect of internal wave.

The observation stations with 90 km along the river are considered, as appeared in Table 1. We find the approximate solution of a one-dimensional salinity internal wave hydrodynamic model Eq.(2). the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation is 10 days. The boundary and initial condition functions are seen in Tables 2 and 3, respectively.

TABLE II
Parameters of physical of simulation 1.

D_s (m^2/s)	u_w (m/s)	K	L(km)	T(days)
0.1	0.25	0.3	108	10

The salinity diffusion coefficient is 0.1 m^2/s. By using a modified Lax-diffusive method for the hydrodynamic model,

TABLE III
THE BOUNDARY AND INITIAL CONDITION FUNCTION OF SIMULATION 1.

Parameters	Given functions
$u_1(x)$	0.06
$h_1(x)$	$0.05 + 0.1sin(m\Delta t)$
$f_1(t)$	0
$f_2(t)$	0
$g_1(t)$	$0.05 + 0.1sin(m\Delta t)$
$g_2(t)$	0

Eqs.(21-23), we obtain a graph of elevation of the water above the bottom and, by using the hydrodynamic model, we obtain a graph of the salinity flow, seen in Fig 1 and 2, respectively. We can see that this simulation is to approximate the salinity flow velocity from the estuary.

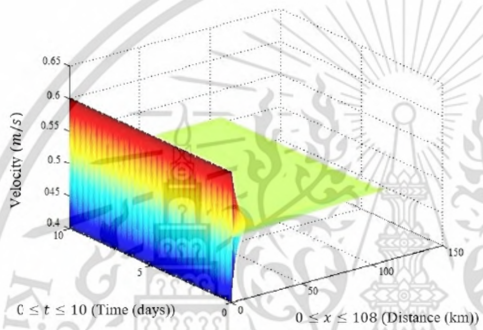


Fig. 1. The salinity flow velocity for $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

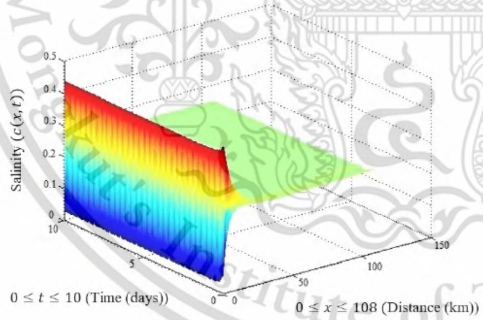


Fig. 2. The salinity flow velocity for $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

4.2 Simulation 2 : release fresh water from the barrage dam to dilute the salinity without the salinity internal wave factor.

We will to find the approximate solution of Eq.(10) for all observation stations with 108 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.065$ m/s, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation 10 days. The parameters of physical are appeared in Table 4.

We get the approximated solution $c(x, t)$ by using Sualyev

TABLE IV
PARAMETERS OF PHYSICAL OF SIMULATION 2.

D_s (m^2/s)	u_w	u_w (m/s)	K	L (km)	T (days)
0.1	0.065	0.25	0.3	108	10

scheme are appeared in Table 5. The level of salinity at the operated S_7 observation station will be standardized, as seen in Figs 3 and 4, respectively. We can see that this simulation is to reduce the saltiness level which releases fresh water from the dam.

TABLE V
THE ESTIMATED SALINITY LEVEL OF SIMULATION 2 FOR ALL OBSERVATION STATIONS.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	11.7025	3.7114	2.0435	1.0136
10000	11.2247	3.5663	2.0230	1.0088
15000	10.8874	3.4530	1.9999	0.9991
20000	10.6373	3.3562	1.9699	0.9854
t	S_5	S_6	S_7	S_8
1	0.8025	0.5325	0.5053	0.1618
5000	0.7730	0.4865	0.3955	0.0148
10000	0.7458	0.4081	0.2975	0.0061
15000	0.7221	0.3370	0.2304	0.0029
20000	0.6975	0.2798	0.1836	0.0015

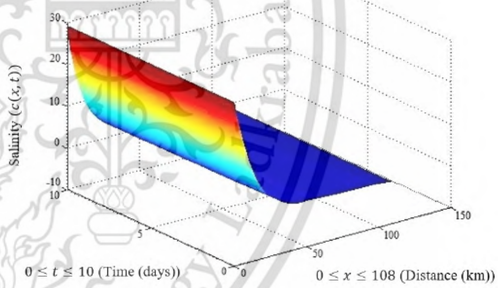


Fig. 3. The estimated salinity level of simulation 2 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

4.3 Simulation 3 : release fresh water from the barrage dam to dilute the salinity with the salinity internal wave factor.

We will to find the approximate solution of Eq.(10) for all observation stations with 108 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow from simulation 1, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation 10 days. The parameters of physical are appeared in Table 6.

We get the approximated solution $c(x, t)$ by using Sualyev scheme are appeared in Table 7. The saltiness concentration level at the station S_7 becomes a standard level. We can see that this simulation is to reduce the salinity level which

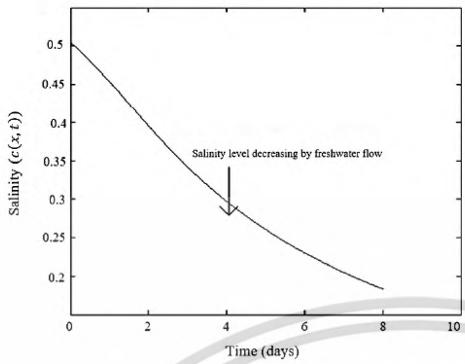


Fig. 4. The estimated salinity level of simulation 2 at station S_7 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $t \in [0, 10]$.

TABLE VI
PARAMETERS OF PHYSICAL OF SIMULATION 3.

D_s (m^2/s)	u_w (m/s)	K	L (km)	T (days)
0.1	0.2	0.3	108	10
0.1	0.25	0.3	108	10
0.1	0.3	0.3	108	10

releases fresh water from the dam. A comparison of the simulated salinity levels at the controlled observation station as seen in Figures 5-6.

TABLE VII
THE ESTIMATED SALINITY LEVEL OF SIMULATION 3 FOR ALL OBSERVATION STATIONS.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	8.6881	2.7478	1.5514	0.9373
10000	6.0982	1.9581	1.2625	0.8286
15000	4.4861	1.5021	1.0642	0.7351
20000	3.4869	1.2107	0.9131	0.6494
t	S_5	S_6	S_7	S_8
1	0.8025	0.5325	0.5053	0.1618
5000	0.7122	0.4204	0.2960	0.0040
10000	0.6393	0.2369	0.1302	0.0022
15000	0.5627	0.1206	0.0574	0.0017
20000	0.4604	0.0604	0.0262	0.0010

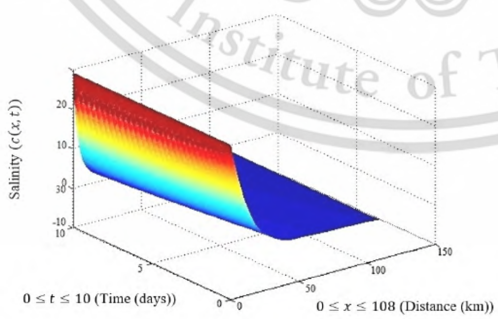


Fig. 5. The estimated salinity level of simulation 3 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

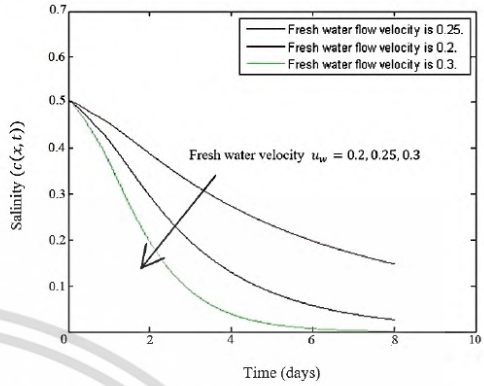


Fig. 6. The estimated salinity level of simulation 3 at station S_7 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $t \in [0, 10]$.

Since, the water fresh water flow velocity is 0.25 m/s can diluted the salinity water in time and not waste too much water from the barrage dam and the saltiness flow velocity that flows into the river in nature is not constant at all times, the comparison of The salinity concentration level at the controlled observation station S_7 of the simulation 2 and 3 is seen in Fig 7. Therefore, in the following simulations, it will

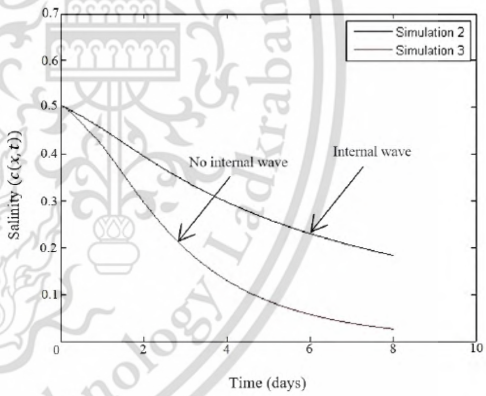


Fig. 7. The comparison of The saltiness concentration level at station S_7 of simulation 2 and 3 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

be using the saltiness flow velocity obtained from simulation 1.

4.4 Simulation 4 : maintaining a constant level of salinity to the standard by reducing the speed of water discharge from the barrage dam.

We will to find the approximate solution of Eq.(10) for all observation stations with 108 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow from the simulation 1, the efficiency of eliminating saltiness of fresh water discharge is $k = 30\%$, and time of simulation is 10 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.3$ kg/m^3 by the controlled release of water from barrage dams with the following process:

1) Release at high speed when the salinity level $c(96,t) > C_{ST}$ at the station S_6.

2) Release at low speed when the salinity level $c(96,t) < C_{ST}$ at the station S_6.

Their parameters of physical are appeared in Table 8.

The approximate salinity level for all observation station can be obtained, as appeared in Table 9 and Fig 8. The saltiness level at the various observation stations S_7, as appeared in Fig 9. We can see that the technique is to reduce the saltiness level which releases fresh water from the dam and use the amount of fresh water not too much.

TABLE VIII
PARAMETERS OF PHYSICAL OF SIMULATION 4.

$c(x,t)$ at S_7	D (m^2/s)	u_w (m/s)	K
$> C_{ST}$	0.1	0.25	0.3
$< C_{ST}$	0.1	0.15	0.3
	T (days)	L (km)	c(0,t)
	10	108	g(t)
	10	108	g(t)

TABLE IX
THE ESTIMATED SALINITY LEVEL OF SIMULATION 4 FOR ALL OBSERVATION STATIONS.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	8.7906	2.7751	1.5642	0.9396
10000	11.5783	3.3642	1.9124	0.9836
15000	13.9586	4.1634	2.3118	1.0693
20000	15.6397	5.1400	2.8139	1.1899
t	S_5	S_6	S_7	S_8
1	0.8025	0.5325	0.5053	0.1618
5000	0.7140	0.4230	0.2997	0.0112
10000	0.7322	0.3990	0.2946	0.0126
15000	0.7514	0.3897	0.2931	0.0149
20000	0.7776	0.3852	0.2910	0.0153

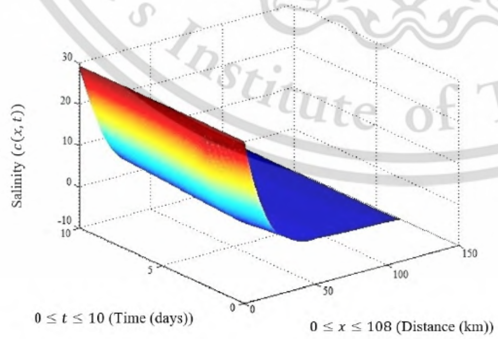


Fig. 8. The estimated salinity level of simulation 4 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

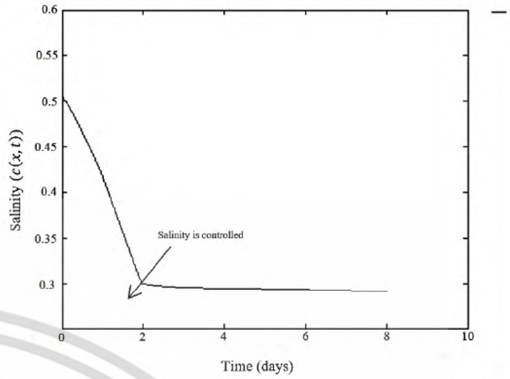


Fig. 9. The estimated salinity level of simulation 4 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 10]$.

4.5 Simulation 5 : reduce the level of salinity before the salinity exceeds the standard.

We will to find the approximate solution of Eq.(10) for all observation stations with 108 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow from the simulation 1, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation is 10 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.3$ kg/m^3 about 3 days by the controlled release of water from barrage dams with the following process:

1) Release at normal speed when the salinity level $c(79,t) < C_{ST}$ at the station S_5.

2) Release at high speed when the salinity level $c(79,t) > C_{ST}$ at the station S_5

Their parameters of physical are appeared in Table 10.

The approximate salinity level for all observation station can be obtained, as appeared in Table 11 and Fig 10. The saltiness level at the various observation stations S_7, as appeared in Fig 11. We can see that the technique is to reduce the saltiness level which releases fresh water from the dam before the salinity is higher than standard.

TABLE X
PARAMETERS OF PHYSICAL OF SIMULATION 5.

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)
$< C_{ST}$	0.1	0.065	0
$> C_{ST}$	0.1	0.065	0.25
K	T (days)	L (km)	c(0,t)
0.3	10	108	g(t)
0.3	10	108	g(t)

V. DISCUSSION

In simulation 1, the approximate solutions of salinity flow velocity are obtained by using the modified Lax-diffusive system for the hydrodynamic model, as seen in Figs 1-2. In simulation 2, we can obtain salinity of water at the pumping station without the salinity flow velocity form simulation 1 by using the Sualyev technique. We can see that this

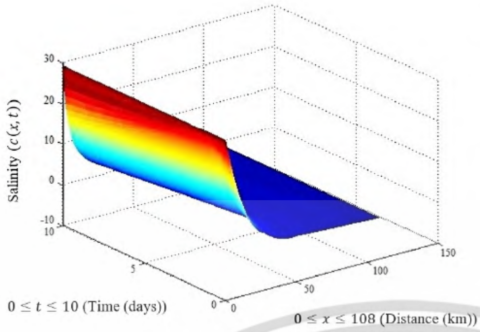


Fig. 10. The estimated salinity level of simulation 5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

TABLE XI
THE ESTIMATED SALINITY LEVEL OF SIMULATION 5 FOR ALL OBSERVATION STATIONS.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	19.5112	6.9051	3.7499	1.3210
10000	13.4092	4.6944	2.6005	1.1293
15000	9.3605	3.2451	1.9022	0.9671
20000	6.6951	2.3258	1.4632	0.8235
t	S_5	S_6	S_7	S_8
1	0.8033	0.4052	0.2641	0.1378
5000	0.9057	0.4607	0.3732	0.0097
10000	0.7853	0.3005	0.1826	0.0041
15000	0.6692	0.1595	0.0806	0.0032
20000	0.5456	0.0808	0.0365	0.0019

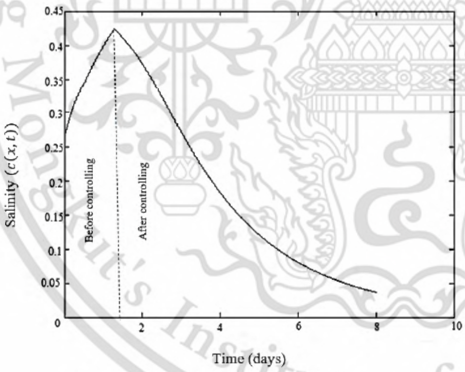


Fig. 11. The estimated salinity level of simulation 5 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 10]$.

simulation can decrease the salinity level with salinity flow velocity is constant by releasing fresh water from the barrage dam, as seen in Figs 3-4. In simulation 3, we can obtain salinity of water at the pumping station with the salinity flow velocity from simulation 1 by using the Sualyev technique. We can see that this simulation can decrease the salinity level with the affect of salinity internal wave by releasing fresh water from the barrage dam, as seen in Figs 5-6 and the comparison of the degree of salinity at a S_7 monitored observation station, as seen in Figure 7. In simulation 4, a salinity control process is simulated with the salinity flow velocity from Simulation 1, as seen in Figs 3-4. The salinity is reduced to the standard level after this, as seen in Figs 8-9, we'll lower the fresh water flow rate to preserve the normal salinity level. In simulation 5, a process of with the salinity flow velocity from simulation 1 was proposed. The salinity is that until the normal salinity amount is reached. The suggested process may reduce the salinity level at least, as seen in Figs 10-11, when the barrage dam releases fresh water.

VI. CONCLUSION

We also suggested a mathematical model for saltiness water measurement in one-dimension. The proposed model concerns the saltiness advection to the river and the effect of salinity flow velocity from an estuary into the river and the dam's release of fresh water. There are several scenarios in which the effects of internal salinity waves are simulated. The suggested simulations can be use in practical salinity measurement in similar topographical rivers. In the salinity control aspect, the suggested process may decrease the salinity level until the level reaches the normal line and the quantity of fresh water may not be used too much. Suggested simulations can be used in realistic salinity management situations in rivers with barrage dam systems along with savings in the volume of fresh water used.

ACKNOWLEDGEMENT

This paper is being sponsored by the Centre of Excellence in Mathematics, the Commission on Higher Education, Thailand. We are very grateful to experts for their appropriate and constructive suggestions to improve this paper.

REFERENCES

[1] Tabuenca, P., Vila, J., Cardona, J., "Samartin, A., Finite element simulation of dispersion in the bay of santander," Advanced in Engineering Software, vol. 28, pp313–332, 1997

[2] Pochai, N., Tangmanee, S., Crane, L.J., Miller, J.J.H., "A water quality computation in the uniform channel," Journal of Interdisciplinary Mathematics, vol. 11, pp803–814, 2008

[3] Pochai, N., "A numerical computation of non-dimensional form of stream water quality model with hydrodynamic advection-dispersion-reaction equations," Journal of Nonlinear Analysis: Hybrid Systems, vol. 13, pp666–673, 2009

[4] Pochai, N., "A numerical computation of non-dimensional form of a nonlinear hydrodynamic model in a uniform reservoir," Journal of Nonlinear Analysis: Hybrid Systems, vol. 3, pp463–466, 2009

[5] Thongtha, K., Kasemsuwan, J., "Analytical solution to a hydrodynamic model in an open uniform reservoir," Advances in Difference Equations, vol. 2017, 2017

[6] Nahian, M.A., Ahmed, A., Lazar, A.N., Hutton, C.W., Salehin, M., "Streatfield, P.K., Drinking water salinity associated health crisis in coastal bangladesh," Elementa: Science of the Anthropocene, vol. 6, 2017

[7] Leewatchanakul, K., "Salinity intrusion in chao phraya river," PhD thesis, Chulalongkorn University, 1988

[8] Chapra, S.C., "Surface Water-Quality Modeling," McGraw-Hill, vol. 6, 2018

[9] Pochai, N., "A numerical treatment of nondimensional form of water quality model in a nonuniform flow stream using saulyev scheme," Mathematical Problems in Engineering 2011, vol. 2011, pp1–16, 2011

[10] Subklay, K., Pochai, N., "Numerical simulations of a water quality model in a flooding stream due to dam-break problem using implicit and explicit methods," Journal of Interdisciplinary Mathematics, vol. 20, pp461–495, 2017

[11] Othata, P., Pochai, N., "A one-dimensional mathematical simulation to salinity control in a river with a barrage dam using an unconditionally stable explicit finite difference method," Advances in Difference Equations, vol. 2019, 2019

[12] Akbari, G. and Firoozi, B., "Implicit and Explicit Numerical Solution of Saint-Venent Equations for Simulation Flood wave in Natural Rivers," 5th National Congress on Civil Engineering, Ferdowsi University of Mashhad, Iran, 2010
[13] Kraychang, W. and Pochai, N., "Implicit Finite Difference Simulation of Water Pollution Control in a Connected Reservoir System," IAENG International Journal of Applied Mathematics, vol. 46, no.4, pp47–57, 2016
[14] Salah, F. and H. Elhafian, M., "Numerical Solution for Heat Transfer of Non –Newtonian Second – Grade Fluid Flow over Stretching Sheet via Successive Linearization Method," IAENG International Journal of Applied Mathematics, vol. 49, no.4, pp505-512, 2019.
[15] Christodoulides, P., Grimshaw, R. and Demetriades, C., "Three-fluid System Short-wave Instability and Gap-solitons," IAENG International Journal of Applied Mathematics, vol. 41, no.3, pp235-240, 2011
[16] Kraychang, W. and Pochai, N., "A Simple Numerical Model for Water Quality Assessment with Constant Absorption around Nok Phrao Island of Trang River," Engineering Letters, vol. 28, no.3, pp912-922, 2020
[17] Konglok, S.A. and Pochai, N., "Numerical Computations of Three-dimensional Air-Quality Model with Variations on Atmospheric Stability Classes and Wind Velocities using Fractional Step Method," IAENG International Journal of Applied Mathematics, vol. 46, no.1, pp112-120, 2016
[18] Suebyat, W. and Pochai, N., "A Numerical Simulation of a Three-dimensional Air Quality Model in an Area Under a Bangkok Sky Train Platform Using an Explicit Finite Difference Scheme," IAENG International Journal of Applied Mathematics, vol. 40, no.4, pp471-476, 2016

N. Pochai is a researcher of Centre of Excellence in Mathematics, CHE, Si Ayutthaya Road, Bangkok 10400, Thailand.

P. Othata is an assistant researcher of Centre of Excellence in Mathematics, CHE, Si Ayutthaya Road, Bangkok 10400, Thailand.

Appendix C

Engineering Letters, 29:2, EL_29_2_02

Irrigation Water Management Strategies for Salinity Control in the Chao Phraya River Using Sualyev Finite Difference Method With Lagrange Interpolation Technique

Pornpon Othata, and Nopparat Pochai

***Abstract*—The problem of salinity in tap water is a very important problem. Drinking water that is higher than the World Health Organization designation can greatly affect people's health. In this research, a salinity management model is also applied in a river with a dam with an interpolation process for initial and boundary conditions. An unconditionally stable explicit finite difference scheme with the Lagrange interpolating polynomials for the initial and boundary conditions is utilized to estimate the saltiness level with a few conditions of a proposed model. The suggested computational technique allows for a clear consensus on the effects of realistic implementations for water supply processes.**

***Index Terms*—Salinity, Barrage dam, Water quality, Interpolation method, Sualyev method**

I. Introduction

To supply tap water, water supply frameworks may utilize surface water or crude water. The salinity of the water is an important aspect impacting the quality of the water. As it can not be processed in the traditional way, this is a very critical factor in production. As such, it is required to have a salinity level to bring the water to the treatment process.

Thailand's Waterworks Authority has seven observation stations for water quality located along a river. Distance from the estuary of each station as appeared in Table 1.

TABLE I
DISTANCE FROM THE ESTUARY OF EACH OBSERVATION STATIONS

Stations	Distance
S_1	0
S_2	12
S_3	23
S_4	52
S_5	79
S_6	84
S_7	90

In Bangkok, the water supply handle for use includes a water saltiness issue above the norm. It has a salinity up to standard that impacts the quality of water generated. By measuring the salinity as of December 14, 2019 and December 25, 2019, the salinity value is as appeared in the table 2.

P. Othata is a PhD candidate of Department of Applied Mathematics, Faculty of Science, King Mongkuts Institute of Technology Ladkrabang, Bangkok, 10520, Thailand (e-mail: pornpon.othata@gmail.com).

N. Pochai is an Assistant Professor of Department of Applied Mathematics, Faculty of Science, King Mongkuts Institute of Technology Ladkrabang, Bangkok, 10520, Thailand (corresponding author to provide phone: 662-329-8400; fax: 662-329-8400; e-mail: nop_math@yahoo.com).

TABLE II
THE SALINITY AS OF DECEMBER 14, 2019 AND DECEMBER 25, 2019

Stations	Salinity level at 14/12/2019	Salinity level at 25/12/2019
S_1	24.74	23.78
S_2	19.64	18.72
S_3	14.6	17.34
S_4	8.82	11.72
S_5	3.99	5.79
S_6	0.25	1.73
S_7	0.2	0.79

At present, the pumping station has an excess of salinity, which affects the quality of tap water in Bangkok. This impacts the quality of water produced.

In [1], the finite difference method was utilized to explain water contamination models. In [13], the interpolation method was utilized to initial and boundary conditions. In [20] and [21], the three dimension advection diffusion model was proposed. In [18], the water pollution model was introduced. In [14], the implicit finite difference method was to solve the water pollution model. In [16], [17] and [19], the fluid flow model was proposed. In [15], the numerical method was used to solve the fluid flow problem. Research reports on the effect of salt drinking water on standards have been undertaken, such as [6]. The water was excessively salty, up to levels that influence the body. In this way, look into has been introduced on the expansion of salt water, for example, [7]. In [8], the one-dimensional salinity water was proposed,

$$A(x)\frac{\partial S}{\partial t} + Q(x,t)\frac{\partial S}{\partial x} = \frac{\partial}{\partial x}\left[A(x)D_x\frac{\partial S}{\partial x}\right], \qquad (1)$$

where $A(x)$ is the river cross-sectional area (m^2), $Q(x,t)$ is flow rates (m^3/s), The coefficient of water diffusion is D_x (m^2/s), S is water salinity level (ppt), x is the length of river (m) and t is times (s)

To solve this problem, a salinity management model is also introduced in a river with a barrage dam with an interpolation process suggested for the initial and boundary conditions. Under a few conditions from the suggested model, the Sualyev scheme is used to estimate the saltiness level. The suggested evaluation method for water supply forms can be used in reasonable circumstances.

II. Governing Equations

2.1 One-dimensional salinity water measurement model

The fluid one-dimensional advection-dispersion equation is the governing equation in the salinity dispersion model.

Simpler representation, The equation is measured over the waters, as appeared in [11]

$$\frac{\partial c}{\partial t} + u(x,t)\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2}, \tag{2}$$

where $(x,t) \in [0,L] \times [0,T]$, u is the speed of the salinity flow and D is coefficient of diffusion.

Expecting that the saltiness is weakened by the fresh water. These are then the saltiness shift in weather conditions level is decreased by the fresh water speed. The rate of fresh water efficiency to weaken saltiness is accepted by $k \in [0,1]$. In [11], the model for salinity problem was proposed

$$\frac{\partial c}{\partial t} + (u_s - ku_w)\frac{\partial c}{\partial x} = D_s\frac{\partial^2 c}{\partial x^2}, \tag{3}$$

where $c(x,t)$ is the salinity level of water(kg/m^3), k is the efficiency rate of water salinity removal, u_s is the salinity water advective velocity (m/s) and u_w is the velocity of fresh water flow

2.2 Initial conditions

A Lagrange interpolating polynomials of the salinity data of Chao Phraya River is defined as the initial condition, along 90 km of the river form the estuary is taken into consideration using salinity data, defined by

$$c(x,0) = f(x), \tag{4}$$

where $x \in [0,L]$ and $f(x)$ is a measured salinity data interpolation function.

2.3 Left boundary condition

The left boundary condition is a Lagrange interpolating polynomials of the raw data measured, dependent on the river's salinity at the first station, defined by

$$c(0,t) = g(t), \tag{5}$$

where $t \in [0,T]$ and $g(t)$ is a given Lagrange interpolating polynomials at the first observation station via calculated salinity data.

2.4 Right boundary condition

The right boundary condition were determined by the change in salinity at the last station, defined by

$$\frac{\partial c}{\partial x} = C_R, \tag{6}$$

where $t \in [0,T]$ and C_R is an estimated the rate of change of the water salinity level at the last station.

III. NUMERICAL TECHNIQUE

3.1 Lagrange interpolating polynomial

The problem of evaluating the first degree polynomials that passes through the (x_0, y_0) separate points (x_0, y_0) and (x_1, y_1) is the same as estimating a f function for which $f(x_0) = y0$ and $f(x_1) = y1$ by means of a first degree polynomial interpolation at the specified points with the values of f. Using this polynomial for approximation is called polynomial interpolation within the interval provided by the endpoints. Define the functions

$$L_0(x) = \frac{x - x_1}{x_0 - x_1}, \quad L_1(x) = \frac{x - x_0}{x_1 - x_0}, \tag{7}$$

The linear Lagrange interpolating polynomial through (x_0, y_0) and (x_1, y_1) is

$$\begin{aligned} P_n(x) &= L_0(x)f(x_0) + L_1(x)f(x_1) \\ &= \frac{x - x_1}{x_0 - x_1}f(x_0) + \frac{x - x_0}{x_1 - x_0}f(x_1). \end{aligned} \tag{8}$$

Note that

$$L_0(x_0) = 1, \quad L_0(x_1) = 0, \quad L_1(x_0) = 0, \quad L_1(x_1) = 1, \tag{9}$$

which implies that

$$\begin{aligned} P(x_0) &= 1 \cdot f(x_0) + 0 \cdot f(x_1) = f(x_0) = y_0, \\ P(x_1) &= 0 \cdot f(x_0) + 1 \cdot f(x_1) = f(x_1) = y_1. \end{aligned} \tag{10}$$

Then, P is the most unique polynomial of degree that passes through (x_0, y_0) and (x_1, y_1). In this case, we construct first, for every $k = 0, 1, 2, ..., n$, a function $L_{n,k}(x) = 1$ with the property that $L_{n,k}(x_i) = 0$ when $i \neq k$ and $L_{n,k}(x) = 1$. To satisfy $L_{n,k}(x_i) = 0$ for each $i \neq k$,it is required that the numerator of $L_{n,k}(x)$ includes the term $(x - x_0)(x - x_1)...(x - x_{k-1})(x - x_{k+1})...(x - x_n)$.

To satisfy $L_{n,k}(x) = 1$, The $L_{n,k}(x)$ denominator must be the same term but must be valued at $x = x_k$. Thus,

$$\begin{aligned} & L_{n,k}(x) \\ &= \frac{(x - x_0)...(x - x_{k-1})(x - x_{k+1})...(x - x_n)}{(x_k - x_0)...(x_k - x_{k-1})(x_k - x_{k+1})...(x_k - x_n)} \end{aligned} \tag{11}$$

Theorem 1. *If $x_0, x_1, ..., x_n$ are $n+1$ distinct numbers and f is a function whose values are given at these numbers, then a unique polynomial $P(x)$ of degree at most n exists with $f(x_k) = P(x_k)$, for each $k = 0, 1, ..., n$.*

This polynomial is given by

$$\begin{aligned} P(x) &= f(x_0)L_{n,0}(x) + ... + f(x_n)L_{n,n}(x) \\ &= \sum_{k=0}^{n} f(x_k)L_{n,k}(x), \end{aligned} \tag{12}$$

where, for each $k = 0, 1, ..., n$,

$$L_{n,k}(x) = \prod_{i=0, i\neq k}^{n} \frac{(x - x_i)}{(x_k - x_i)}, \tag{13}$$

We can write $L_{n,k}(x)$ simply as $L_k(x)$ when there is no doubt as to the degree.

The approximation error of Lagrange interpolation polynomial is $\left|P(x) - \tilde{f}(x)\right|$, where $\tilde{f}(x)$ is the polynomial interpolating.

3.2 Saulyev scheme

The Saulyev scheme is unconditionally stable [2]. Obviously the non strictly dependability prerequisite of Saulyev scheme is the principle of preferred position and conservative to utilize. Taking the Saulyev scheme into Eq.(3), it appears to have the following discretization:

$$c(x_m,t_n) \cong C_m^n, \quad (14)$$

$$\left.\frac{\partial c}{\partial t}\right|_{(x_m,t_n)} \cong \frac{C_m^{n+1}-C_m^n}{\Delta t}, \quad (15)$$

$$\left.\frac{\partial c}{\partial x}\right|_{(x_m,t_n)} \cong \frac{C_{m+1}^n-C_{m-1}^{n+1}}{2\Delta x}, \quad (16)$$

$$\left.\frac{\partial^2 c}{\partial x^2}\right|_{(x_m,t_n)} \cong \frac{C_{m+1}^n-C_m^n-C_m^{n+1}+C_{m-1}^{n+1}}{(\Delta x)^2}, \quad (17)$$

$$u_{s_m}^n \cong u_m^n, \quad (18)$$

$$u_{w_m}^n = u_w(x_m,t_n). \quad (19)$$

Substituting Eqs.(14-19) into Eq.(3), We obtain the equation of finite difference,

$$\frac{C_m^{n+1}-C_m^n}{\Delta t}+(u_{s_m}^n-ku_{w_m}^n)\left(\frac{C_{m+1}^n-C_{m-1}^{n+1}}{2\Delta x}\right) = D_s\left(\frac{C_{m+1}^n-C_m^n-C_m^{n+1}+C_{m-1}^{n+1}}{(\Delta x)^2}\right). \quad (20)$$

The explicit equation of finite difference then becomes

$$C_{m+1}^{n+1} = \left(\frac{1}{1+\lambda}\right)\left[\left(\lambda+\frac{1}{2}r_m^n\right)C_{m-1}^{n+1}+(1-\lambda)C_m^n + \left(\lambda-\frac{1}{2}r_m^n\right)C_{m+1}^n\right]. \quad (21)$$

where $i = 1,2,3,\dots,M-1$, $\lambda = \frac{D_s\Delta t}{(\Delta x)^2}$ and $r_m^n = \frac{(u_{s_m}^n-ku_{w_m}^n)\Delta t}{\Delta x}$. For $i=M$, replaced the unknown value in Eq.(5), we obtain

$$C_{M+1}^n = \left(\frac{C_{M_2}^n-C_{M_1}^n}{L_2-L_1}\right)\Delta x + C_{M-1}^n. \quad (22)$$

The truncation error of Sualyev scheme is $O\left\{(\Delta x)^2+(\Delta t)^2+(\Delta t/\Delta x)^2\right\}$.

IV. Numerical Simulations

4.1. Simulation 1 : Interpolation for the initial condition and left boundary conditions.

The observation stations with 90 km along the river and data on salinity level are considered, as appeared in Table 1 and Table 2 respectively. We simulated the boundary and initial conditions by using Eq.(7-8). The comparison of interpolation of boundary and initial condition with the measuring salinity data as appeared in Fig 1-2 respectively.

4.2. Simulation 2 : the spread of salinity water into rivers.

We will to find the approximate solution of Eq.(3) for all observation stations with 90 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\ m/s$, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation is 9 days. The physical parameters are appeared in Table 3. The approximate solution of salinity along the river and the salinity at observation station S_6 compare with the real data as appeared in Fig 3-4 respectively.

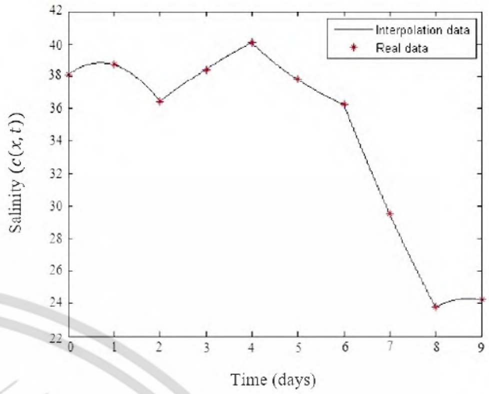


Fig. 1. The comparison of interpolation in initial condition with the measuring salinity data

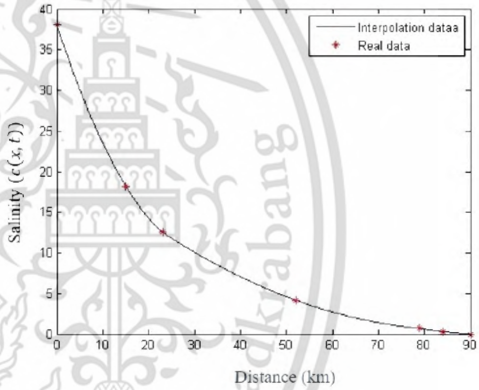


Fig. 2. The comparison of interpolation in left boundary condition condition with the measuring salinity data

TABLE III
The parameters of physical of simulation 2.

D_s (m^2/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (days)
0.1	0.06	0.3	0.3	90	9

4.3. Simulation 3 : release fresh water from the barrage dam to dilute the salinity.

We will to find the approximate solution of Eq.(3) for all observation stations with 90 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\ m/s$, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation 9 days. The physical parameters are appeared in Table 4. The approximate solution of salinity along the river and the salinity level at the observation station S_6 as appeared in Fig 5-6 respectively.

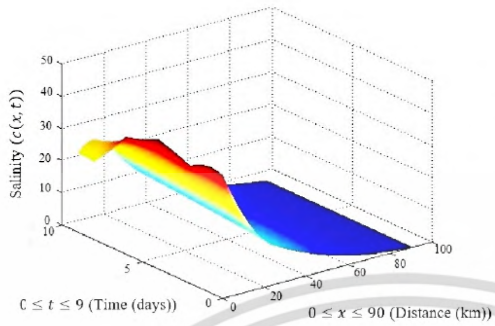


Fig. 3. The estimated saltiness level of simulation 2 where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 90]$ and $t \in [0, 9]$.

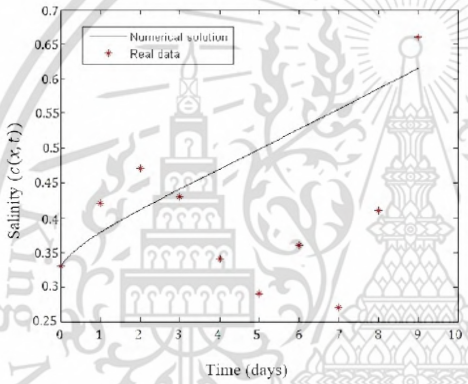


Fig. 4. The estimated saltiness level at station S_6 of simulation 2 where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 90]$ and $t \in [0, 9]$.

TABLE IV
THE PARAMETERS OF PHYSICAL OF SIMULATION 3.

D_s (m^2/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (days)
0.1	0.06	0.3	0.3	90	9

4.4. Simulation 4 : Maintaining a constant level of salinity to the standard by reducing the speed of water discharge from the barrage dam.

We will to find the approximate solution of Eq.(3) for all observation stations with 90 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\, m/s$, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation is 9 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.72\, kg/m^3$ by the controlled release of water from barrage dams with the following process:

1) Release at high speed when the salinity level $c(84, t) > C_{ST}$ at the station S_6.

2) Release at low speed when the salinity level $c(84, t) < C_{ST}$ at the station S_6.

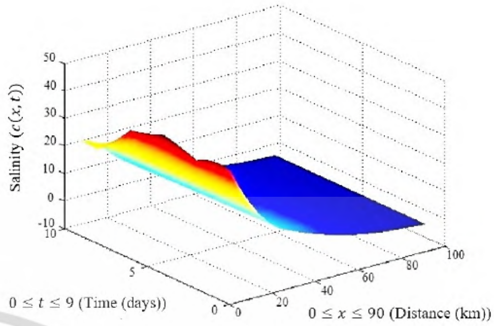


Fig. 5. The estimated saltiness level of simulation 3 where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 90]$ and $t \in [0, 9]$.

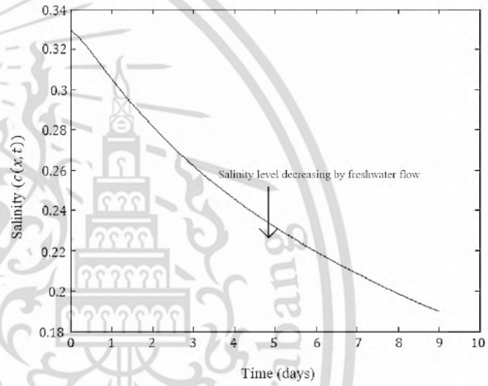


Fig. 6. The estimated saltiness level at station S_6 of simulation 3 where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 9]$.

Their parameters of physical are appeared in Table 5.

The approximate salinity level for all observation station can be obtained, as appeared in Fig 7 and Table 6. The saltiness level at the various observation stations S_6, as appeared in Fig 8.

TABLE V
THE PARAMETERS OF PHYSICAL OF SIMULATION 4.

$c(x,t)$ at S_6	D (m^2/s)	u_s (m/s)	u_w (m/s)
$> C_{ST}$	0.1	0.06	0.23
$< C_{ST}$	0.1	0.06	0.205
K	T (days)	L (km)	c(0,t)
0.3	9	90	g(t)
0.3	9	90	g(t)

TABLE VI
THE ESTIMATED SALTINESS LEVEL FOR ALL OBSERVATION STATIONS OF SIMULATION 4.

t	S_2	S_3	S_4	S_5	S_6
1	18.2300	12.5500	4.2000	0.7200	0.3300
5000	17.6659	12.4890	3.9232	0.5884	0.2765
10000	17.1508	12.1402	3.6688	0.5191	0.2385
15000	16.7113	11.7453	3.4336	0.4653	0.2113
20000	16.3177	11.5042	3.2701	0.4392	0.2025

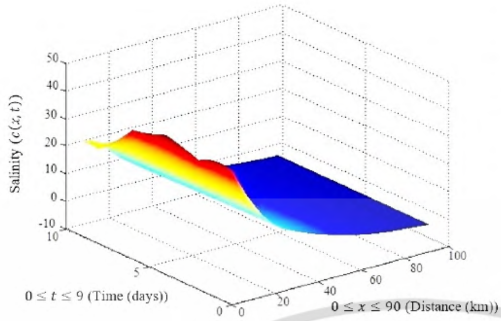


Fig. 7. The estimated saltiness level of simulation 4 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 90]$ and $t \in [0, 9]$.

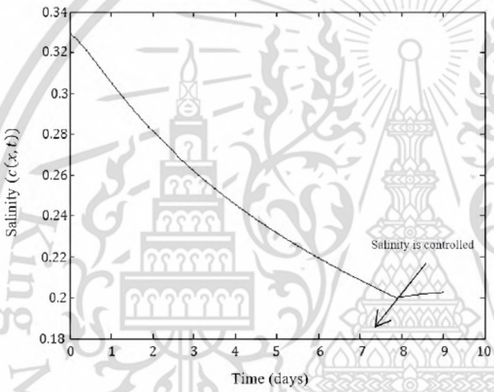


Fig. 8. The estimated saltiness level at station S_6 of simulation 4 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 9]$.

4.5. Simulation 5 : reduce the level of salinity before the salinity exceeds the standard.

We will to find the approximate solution of Eq.(3) for all observation stations with 90 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\ m/s$, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation is 9 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.72\ kg/m^3$ about 3 days by the controlled release of water from barrage dams with the following process:

1) Release at normal speed when the salinity level $c(79, t) < C_{ST}$ at the station S_5.

2) Release at high speed when the salinity level $c(79, t) > C_{ST}$ at the station S_5

Their parameters of physical are appeared in Table 7.

The approximate salinity level for all observation station can be obtained, as appeared in Fig 9 and Table 8. The saltiness level at the various observation station S_5 and S_6, as appeared in Fig 10 and 11.

TABLE VII
THE PARAMETERS OF PHYSICAL OF SIMULATION 5.

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)
$< C_{ST}$	0.1	0.06	0
$> C_{ST}$	0.1	0.06	0.22
K	T (days)	L (km)	c(0,t)
0.3	9	90	g(t)
0.3	9	90	g(t)

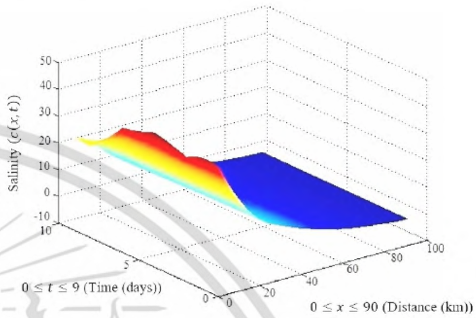


Fig. 9. The estimated saltiness level of simulation 5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 90]$ and $t \in [0, 9]$.

TABLE VIII
THE ESTIMATED SALTINESS LEVEL OF SIMULATION 5 FOR ALL OBSERVATION STATIONS.

t	S_2	S_3	S_4	S_5	S_6
1	18.2300	12.5500	4.2000	0.7200	0.1500
5000	19.4352	13.5555	4.3278	0.6805	0.3248
10000	19.5986	13.7948	4.2646	0.6784	0.3252
15000	18.7978	13.3153	3.9964	0.5969	0.2759
20000	17.8724	12.8036	3.7464	0.5321	0.2426

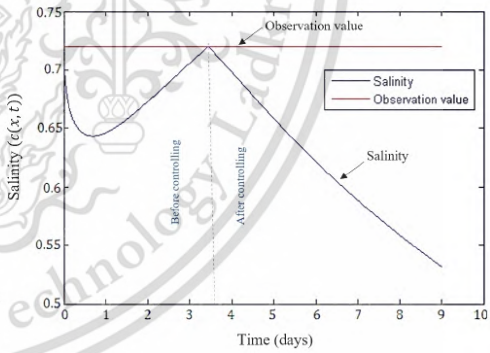


Fig. 10. The estimated saltiness level at stations S_5 of simulation 5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 9]$.

4.6. Simulation 6 : reduce the salinity before the salinity exceeds the standard and reduce the emission from the dam when the salinity is low.

We will to find the approximate solution of Eq.(3) for all observation stations with 90 km along the river as, appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.06\ m/s$, the efficiency of eliminating salinity of

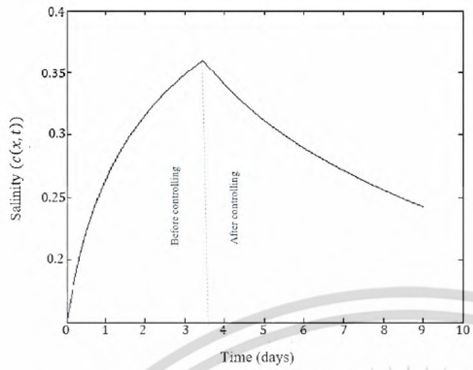


Fig. 11. The estimated saltiness level of simulation simulation 5 at station S_6 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 9]$.

fresh water discharge is $k = 30\%$, the fresh water dilution efficiency is 30%, and time of simulation is 9 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.72\ kg/m^3$ about 3 days by the controlled release of water from barrage dams with the following process:

1) Release at normal speed when the salinity level $c(79, t) < C_{ST}$ at the station S_5.

2) Release at high speed when the salinity level $c(79, t) > C_{ST}$ at the station S_5 and change normal speed when S_5 $c(79, t) < C_{ST}$.

Their parameters of physical are appeared in Table 9.

The approximate salinity level for all observation station can be obtained, as appeared in Fig 12 and Table 10. The S_5 and S_6 level at the various observation station S_5 and S_6, as appeared in Fig 13 and 14.

TABLE IX
THE PARAMETERS OF PHYSICAL OF SIMULATION 6.

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)
$< C_{ST}$	0.1	0.06	0
$> C_{ST}$	0.1	0.06	0.25
K	T (days)	L (km)	c(0,t)
0.3	9	90	g(t)
0.3	9	90	g(t)

TABLE X
THE ESTIMATED SALTINESS LEVEL FOR ALL OBSERVATION STATION OF SIMULATION 6.

t	S_2	S_3	S_4	S_5	S_6
1	18.2300	12.5500	4.2000	0.7200	0.1500
5000	19.2985	13.4818	4.3009	0.6688	0.3107
10000	19.3886	13.6694	4.2232	0.6614	0.3138
15000	19.4798	13.7761	4.1554	0.6549	0.3110
20000	19.4353	13.9705	4.1413	0.6656	0.3215

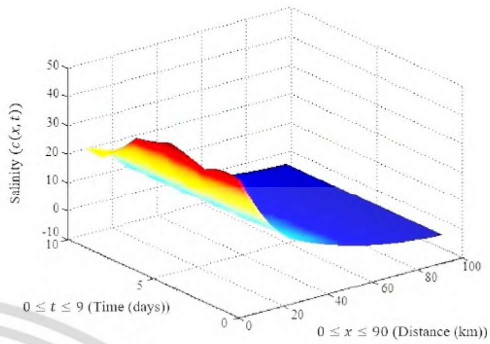


Fig. 12. The estimated saltiness level of simulation 6 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 90]$ and $t \in [0, 9]$.

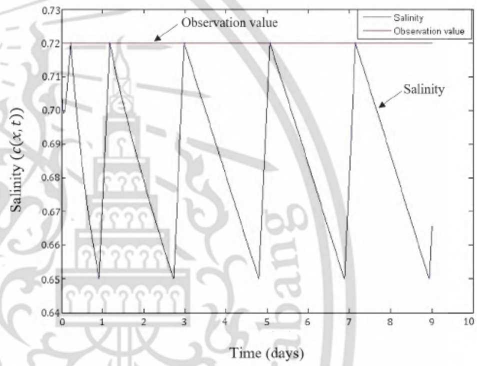


Fig. 13. The estimated salinity level at station S_5 of simulation 6 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 9]$.

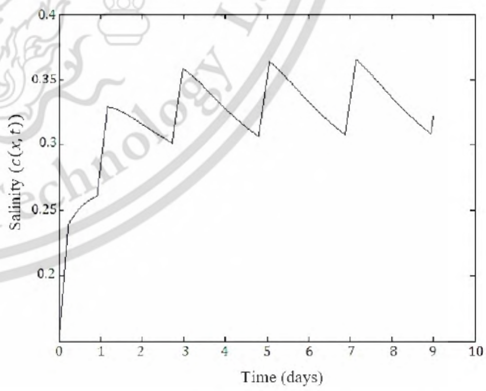


Fig. 14. The estimated salinity level of simulation simulation 6 at station S_6 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 9]$.

V. DISCUSSION

In simulation 1, approximate solutions of the initial and boundary conditions are obtained using the interpolation method. In simulation 2, the calculated solutions can be obtained by demonstrating the salinity level along the river with a maximum error of less than 30%, as appeared in Fig 3 and Fig 4. In simulation 3, the salinity value will decrease as the velocity of fresh water flow increases, as appeared in Fig 5 and Fig 6. The salinity control process is simulated in simulation 4. If the salinity level becomes normal after that, the speed of fresh water flow should be lowered to preserve salinity at the normal level, as appeared in Fig 7 and Fig 8. The salinity control mechanism is simulated in simulation 5. Salinity reduces until the saltiness level reaches the standard. The suggested process could reduce saltiness by at least releasing fresh water from the dam, as appeared in Fig 9, Fig 10 and Fig 11. The salinity control mechanism is simulated in simulation 6. The salinity decreases to less than the standard value and increases as the velocity of fresh water decreases alternately. The suggested technique is to reduce salinity by at least an amount and fresh water is released when salinity levels become standard as appeared in Fig 12, Fig 13 and Fig 14.

VI. Conclusion

We also suggested a mathematical model for saltiness water measurement in one-dimension. The proposed model concerns the salinity advection to the river and the effect of the dam's release of fresh water. Also, some practical problems are being simulated. For many practical salinity measurements, the proposed simulation can be used. The proposed process may decrease the level of salinity in the salinity control aspect until the level reaches the requirement. The proposed numerical simulation can be extended to the practical management of salinity. From each simulation, we can determine that the efficiency is appeared in Table 11.

TABLE XI
The efficiency of the simulation.

	Salinity level
Simulation 3	The salinity level decreased to much less than the standard value.
Simulation 4	The salinity level decreased to less than standard value for acceptable level.
Simulation 5	Salinity level is not more than and is much less than the standard.
Simulation 6	Salinity level not more than and less than standard value for acceptable level.
	Quantity of released fresh water
Simulation 3	High
Simulation 4	Quite high
Simulation 5	High
Simulation 6	Not too high

As a result, we can apply it to the salinity water problem in different rivers to reduce the impact of people on drinking water with salinity over the standard and help save fresh water used to reduce salinity levels in rivers.

Acknowledgement

This paper is being sponsored by the Centre of Excellence in Mathematics, the Commission on Higher Education, Thailand. We are very grateful to experts for their appropriate and constructive suggestions to improve this paper.

References

[1] Tabuenca, P., Vila, J., Cardona, J., "Samartin, A., Finite element simulation of dispersion in the bay of santander," Advanced in Engineering Software, vol. 28, pp313–332, 1997
[2] Pochai, N., Tangmanee, S., Crane, L.J., Miller, J.J.H., "A water quality computation in the uniform channel," Journal of Interdisciplinary Mathematics, vol. 11, pp803–814, 2008
[3] Pochai, N., "A numerical computation of non-dimensional form of stream water quality model with hydrodynamic advection-dispersion-reaction equations," Journal of Nonlinear Analysis: Hybrid Systems, vol. 13, pp666–673, 2009
[4] Pochai, N., "A numerical computation of non-dimensional form of a nonlinear hydrodynamic model in a uniform reservoir," Journal of Nonlinear Analysis: Hybrid Systems, vol. 3, pp463–466, 2009
[5] Thongtha, K., Kasemsuwan, J., "Analytical solution to a hydrodynamic model in an open uniform reservoir," Advances in Difference Equations, vol. 2017, 2017
[6] Nahian, M.A., Ahmed, A., Lazar, A.N., Hutton, C.W., Salehin, M., "Streatfield, P.K., Drinking water salinity associated health crisis in coastal bangladesh," Elementa: Science of the Anthropocene, vol. 6, 2017
[7] Leewatchanakul, K., "Salinity intrusion in chao phraya river," PhD thesis, Chulalongkorn University, 1988
[8] Chapra, S.C., "Surface Water-Quality Modeling," McGraw-Hill, vol. 6., 2018
[9] Pochai, N., "A numerical treatment of nondimensional form of water quality model in a nonuniform flow stream using saulyev scheme," Mathematical Problems in Engineering 2011, vol. 2011, pp1–16, 2011
[10] Subklay, K., Pochai, N., "Numerical simulations of a water quality model in a flooding stream due to dam-break problem using implicit and explicit methods," Journal of Interdisciplinary Mathematics, vol. 20, pp461–495, 2017
[11] Othata, P., Pochai, N., "A one-dimensional mathematical simulation to salinity control in a river with a barrage dam using an unconditionally stable explicit finite difference method," Advances in Difference Equations, vol. 2019, 2019
[12] Akbari, G. and Firoozi, B., "Implicit and Explicit Numerical Solution of Saint-Venent Equations for Simulation Flood wave in Natural Rivers," 5th National Congress on Civil Engineering, Ferdowsi University of Mashhad, Iran, 2010
[13] Samalerk, P. and Pochai, N., "Numerical Simulation of a One-Dimensional Water-Quality Model in a Stream Using a Saulyev Technique with Quadratic Interpolated Initial-Boundary Conditions," Abstract and Applied Analysis, vol. 2018, 2018
[14] Kraychang, W. and Pochai, N., "Implicit Finite Difference Simulation of Water Pollution Control in a Connected Reservoir System," IAENG International Journal of Applied Mathematics, vol. 46, no.4, pp47–57, 2016
[15] Salah, F. and H. Elhafian, M., "Numerical Solution for Heat Transfer of Non –Newtonian Second – Grade Fluid Flow over Stretching Sheet via Successive Linearization Method," IAENG International Journal of Applied Mathematics, vol. 49, no.4, pp505-512, 2019.
[16] Christodoulides, P., Grimshaw, R. and Demetriades, C., "Three-fluid System Short-wave Instability and Gap-solitons," IAENG International Journal of Applied Mathematics, vol. 41, no.3, pp235-240, 2011
[17] Klankaew, P. and Pochai, N., "Numerical Simulation of a Nonlinear Thin Fluid Film Flow Velocity Model of a Third Grade Fluid on a Moving Belt using Finite Difference Method with Newton Iterative Scheme," Proceedings of The International MultiConference of Engineers and Computer Scientists 2019, pp318-321, 2019
[18] Timpitak, W. and Pochai, N., "Numerical Simulations to a One-dimensional Groundwater Pollution Measurement Model Through Heterogeneous Soil," IAENG International Journal of Applied Mathematics, vol. 50, no.3, pp558-565, 2020
[19] Kraychang, W. and Pochai, N., "A Simple Numerical Model for Water Quality Assessment with Constant Absorption around Nok Phrao Island of Trang River," Engineering Letters, vol. 28, no.3, pp912-922, 2020
[20] Konglok, S.A. and Pochai, N., "Numerical Computations of Three-dimensional Air-Quality Model with Variations on Atmospheric Stability Classes and Wind Velocities using Fractional Step Method," IAENG International Journal of Applied Mathematics, vol. 46, no.1, pp112-120, 2016
[21] Suebyat, W. and Pochai, N., "A Numerical Simulation of a Three-dimensional Air Quality Model in an Area Under a Bangkok Sky Train Platform Using an Explicit Finite Difference Scheme," IAENG International Journal of Applied Mathematics, vol. 40, no.4, pp471-476, 2016

N. Pochai is a researcher of Centre of Excellence in Mathematics, CHE, Si Ayutthaya Road, Bangkok 10400, Thailand.

P. Othata is an assistant researcher of Centre of Excellence in Mathematics, CHE, Si Ayutthaya Road, Bangkok 10400, Thailand.

Author Biography

Name	Mr.Pornpon Othata
Date of Birth	24 August 1992
Address	5/230, Bangkok Province, Nong Chok District, 10530
Education	2015 Bachelor of Science in Applied Mathematics GPA 3.06
	King Mongkut's Institute of Technology Ladkrabang
	2017 Master of Science in Applied Mathematics GPA 3.56
	King Mongkut's Institute of Technology Ladkrabang
Scholarship(s)	Assistant Researcher by Centre of Excellence in Mathematics(CEM) Mahidol university.

Academic Publication(s)

Pornpon Othata and Nopparat Pochai, "A one-dimensional mathematical simulation to salinity control in a river with a barrage dam using an unconditionally stable explicit finite difference method", Advances in Difference Equations, 203(2019), 2019.

Pornpon Othata and Nopparat Pochai, "A mathematical model of salinity control in a river with an effect of internal waves using two explicit finite difference methods", Engineering letters, 2021. vol. 29, no. 2, pp689-696, 2021.

Pornpon Othata and Nopparat Pochai, "Irrigation water management strategies for salinity control in the Chao Phraya River using Sualyev finite difference method with Lagrange interpolation technique", Engineering letters, vol. 29, no. 2, pp332-338, 2021.