A MATHEMATICAL MODEL OF AIRBORNE INFECTION MEASUREMENT AN OUT - PATIENT ROOM WITH A VENTILATION SYSTEM

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Abstract

At present, the epidemic of infectious disease has spread through many countries. Many of the world's countries have huge issues that directly impact people's livelihoods, such as economic development and different modes of travel, etc. In this research, we develop and demonstrate a simple mathematical model that predicts the risk of airborne infectious diseases, such as tuberculosis or COVID-19, under unsteady -state conditions, by analyzing exhaled air by infectors and susceptible to hospital outpatients. The proposed model provides the concentration of exhaled air and the risk probability of airborne transmission in a realistic situation inside an outpatient room with a ventilation system. The proposed model can be used to provide a better design for limiting the capacity of patients in each outpatient room. This means that the risk of airborne infectious diseases is controlled by the proposed model.

Keywords : carbon dioxide, epidemic, outpatient room, ventilation system, airborne infection, exhaled air.

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Introduction

1.1 Inception and Importance

At present, the epidemic of infectious diseases has spread to many countries. Many countries have huge issues that directly impact people's livelihoods, such as economic development and different modes of travel, etc. In this research, we developed and demonstrated a simple mathematical model that predicts the risk of airborne infectious diseases, such as tuberculosis or COVID19, under unsteady state conditions, by analyzing exhaled air by infectors and susceptible hospital outpatients. The proposed model provides the concentration of exhaled air and the risk probability of airborne transmission in a realistic situation inside an outpatient room with a ventilation system. The proposed model could be used to provide a better design for limiting the capacity of patients in each outpatient room. This means that the risk of airborne infectious diseases is controlled by the proposed model.

The patient room contributes to increased ventilation depending on the human respiratory system. And physical activity that increases the respiratory rate to a greater frequency. When the exhaled air concentration increases in a room with an infected person, the probability of Weak individuals are also increasingly exposed to airborne infection as the exhaled air from an infected person often contains airborne infectious particles within the sediment nucleus, which may remain in the air. It is prolonged and when inhaled can result in a new infection of the vulnerable . The progression of infection to many diseases depends on a number of factors, including the state of the system. Immunity in the body of the vulnerable and the severity of the infectious strain. Dispersant behaviors such as talking, coughing, sneezing and singing can affect the production of more respiratory particles, when infectious airborne particles are inhaled by susceptible individuals. Only a fraction of the infected particles breathed in can reach a targeted infection point in the respiratory system. This is because infectious particles of different sizes have different deposition fractions in different anatomical regions of the airway. Critical size from 1 micro meter to 5 micro meter has a high probability of reaching an infection risk. 5 micro meter will be in the upper respiratory tract. This means that all airborne infectious particles cannot be reached or stored at the target infected area. Therefore, when assessing the risk of airborne infectious diseases, the remnants of the respiratory accumulation of airborne infectious particles must also be taken into account.

In this research, we have developed a mathematical model that is close to real-world situations to predict the risk of airborne infectious disease under unstable

conditions. By examining the exhaled air by an infected person in a ventilated outpatient room. We begin by explaining the amount of air generated in an infected room that is inhaled by a susceptible person with a high likelihood of contracting airborne infectious disease, as shown in the section below.

1.2 Objectives of the Study

1) Developed mathematical models to manage ventilation systems in hospitals under unsteady state conditions by monitoriing exhaled air by patients in an outpatient room.
2) To predict the likelihood of carbon dioxide concentrations within an outpatient room that is a marker of the patient's exhaled air.
3) To manage the population who come to the outpatient room with the least risk of airborne infection.

1.3 Scope of the Study

We perform an approximation of the range of polynomial functions and find the ordinary differential equations solution of a mathematical model by using numerical technique: The Quadratic Lagrange Interpolating Polynomial and The Runge–Kutta fourth-order method.

1.4 Benefits of the Study

This mathematical model can be applied to other types of air borne infectious diseases in the future and to be developed in the air system with various facilities such as rooms. hotels, or even in educational facilities.

1.5 Research Methodology

1) Study the basic knowledge of common derivative equations.
2) Study the numerical regulation that can find the inequality of the equation.
3) to research Main Factors Variables affecting air borne infections in both static and unstable conditions under
4) limited areas. Research similar research
5) Meet the objectives and comply with the assumptions set forth. Apply real situations to mathematical models created to study the risk trends set.
6) Analyze data from mathematical models.
7) Report on the results and development of mathematical models
8) Summary and evaluation under the limited terms of work scope

Table 1.1: The research schedule

Activity	Time frame (month of year)									
	2019		2020						2021	
	8-10	11-12	1-2	3-4	5-6	7-8	9-10	11-12	1-3	4-5
Step 1	⟷									
Step 2		⟷	⟷							
Step 3			⟷	⟷						
Step 4				⟷	⟷	⟷				
Step 5						⟷	⟷			
Step 6							⟷	⟷		
Step 7								⟷	⟷	
Step 8										⟷

Chapter 2

Preliminaries

This chapter introduces the basic knowledge that must be known about the theories and documents that are related to research, including numerical methodology, in order to use it as a basis for further work.

2.1 Airborne transmission in hospitals

The spread of hospital airborne infections is a major medical and public health concern around the world. In Thailand, tuberculosis and COVID-19 are examples of airborne pathogens and are now a major problem as the incidence of disease increases and drug resistance is also found in many species.In addition, emerging diseases of the respiratory system, including SARS, Avian Influenza, A H5N1, may be transmitted via air or by aerosol. In some situations, prevention of the spread of airborne infection in a medical facility is a must.In 1930, William F. Wells, a sanitary engineer at Harvard University, conducted a study with medical student Richard Reiley and reported that the droplet nuclei, which contained microorganisms, were responsible for diffusion. Airborne dispersion as particles less than or equal to 5 microns can remain suspended in the air for a long time without falling to the ground and remain in the air very far from their origin, which is consistent with the equation used to calculate the velocity at 1 - 100 microns of particles will fall to the ground, proposed by Lewis Strokes.In this equation, particles of 1 - 5 microns in still air have a drop rate of 1 yard per hour, however, with strong air currents, the airborne flotation of such particles is longer as they are inhaled. 1 - 5 microns in size pass the cilia and mucosal defenses in the upper respiratory tract. It accumulates in the alveoli (alveoli). If the pathogen is still alive on the droplet nuclei, it can cause disease.

Education of airborne pathogen transmission have been widely found after the incidence of disease has increased. Including the incidence of severe acute respiratory disease caused by SARS Corona virus, which is evidence that the disease is transmitted by air. In some circumstances, at present there are 3 groups of pathogens that spread through the air:

1) **Obligated airborne transmission**

   Pathogens are primarily spread by air. Disease with evidence that This spread is tuberculosis. In addition, Measles, Aspergillus spp. And Rhizopus spp may be in this group. This is because there is strong evidence that airborne spread is the main channel.

2) **Preferential airborne transmission**

Pathogens in this group can spread in a number of ways, but if spread through Air or aerosol forms and accumulates in the tip of the lungs, causing pathogens to spread throughout the body and complete the disease (full - blown disease). Pathogens in this group include Varicella - Zoster, Smallpox, fungi. Acremonium spp. For influenza and avian influenza A H5N1, there is evidence that it is likely to be in this group, especially avian influenza. Influenza A H5N1 found that the receptor was at the tip of the lungs and the viral division in That position as well Pathogens in this group if spread by other means. The severity of the disease is reduced.

3) **Opportunistically airborne transmission**

These pathogens naturally spread by other means. But in some circumstances, such as aerosol and inhalation of pathogens into the distal part of the lungs, it can cause disease.Potential pathogens in this category are SARS Corona Virus and Viral hemorrhagic fever, Ebola. , Lassa, Marburg, Hanta

In addition, diseases that spread by air may be divided into 2 groups according to the risk of exposure as follows.

- The first group is a group of diseases in which normal people who do not have immunity to that disease are at risk of infection, including tuberculosis, Measles virus, Varicella - Zoster virus, Smallpox, SARS corona virus, Influenza A H5N1 and Viral hemorrhagic fever. Preventing the spread of these diseases requires prevention at the source (patient), that is, the patient must be in a separate room for the prevention of airborne transmission and the person entering it. In the same room, the patient must wear a mask that blocks particles less than 5 microns in size, which is known as Airborne precautions. administrative control, environmental control and respiratory protection control etc.

- The second group is a group of diseases that usually cause diseases only to those who are immunocompromised: fungi such as Aspergillus spp., Rlizopuss spp., Acremonium spp., Fusarium spp. This means that the transplant patients must minimize the contamination of the air in their separate rooms by filtering the air to the patient's room. With a HEPA filter and the air pressure in the patient room must be positive (positive air pressure) compared to the ambient air to prevent the outside air, which may be contaminated with pathogens, from entering the room. Sick through the holes in the room.

However, in addition to these two groups of patients, which can be identified and measures can be taken to control the spread or reduce the infection, there are still a number of cases that we do not yet know can be transmitted by air. Or at risk of infection that spreads through the air? Including patients who have undergone surgery

and nebulization Examination of internal organs in critical patients or even patients waiting for examination If there is a cough, it can spread the infection by air to other people, including personnel. In a hospital as a standard is essential.

2.2 Air quality and Ventilation systems in hospitals

Air quality control for the prevention of airborne contamination within health-care facilities has a number of guidelines, calculations, designs and requirements depending on the area to be controlled.

1) **Aeration of fresh air from the outside**
   Increasing the rate of aeration with fresh air from the outside will reduce the concentration of airborne contaminants inside the room. However, the position to receive fresh air from outside must also be suitable. This is to ensure that the outside air that will enter the room is free of any contamination, but due to the climate in Thailand is hot - humid. In order to determine the appropriate external aeration rate, it must be taken with great care to control the effects of temperature and relative humidity of the indoor air conditioning system.

2) **Pressure control between areas**
   The air flow direction is proportional to the air pressure within each area. Determining the difference in air pressure for each area that requires proper air quality control is a matter of consideration.

3) **Air flow direction control**
   In the event of airborne contamination, position the air from the area surrounding the room to allow air to pass through the healthcare personnel before exiting the room on the headboard wall. This will greatly reduce the risk of airborne infection if there are multiple beds within the controlled area, the location and area required for the medical service must be taken into account.

Control and prevention of airborne infection through ventilation methods will It is the ventilation from the room or area where there is a sick person or the air source And bring the air from the source or spread out to the outside, which can be classified in 2 ways as follows

- Natural ventilation
  It is the architectural design of the nursing home area to facilitate the flow of air, the outside air will flow into the area and carry Contamination of the area to the outside The use of this method for ventilation must take into account the direction of the flow of outside air into the area during each season. Location of use within the area Operational positions of medical personnel As Thailand is in the equatorial tropical-humid region, there is a general flow of air between

the rainy and winter seasons. In the opposite direction Therefore, it is difficult to control the direction of air flow through the hospital area.

Figure 2.1*:* The patient room opens a window to allow natural air flow.

- Mechanical Ventilation

  It is the use of mechanical tools to ventilate the area In general, different types of ventilation fans and duct systems are used to carry air into or out of the ventilation area. Of the air inside the area at any time, independent of the season However, there are disadvantages when compared to natural ventilation: higher installation and operating costs.

Figure 2.2*:* Examples of different types of fans used for ventilation.

2.3 Probability Distributions for Discrete Random Variables

2.3.1 Poisson distribution

Most Poison distributions are associated with rare events, such as the number of times the radioactive leakage from a nuclear power plant has occurred each month, etc. We will shorten the time period so that the event is less likely to occur in accordance with the nature of the Poisson experiment as defined below.

Definition 2.1. The Poisson Experiment is an experiment that looks like this.

1) The number of success occurrences in a particular period of time. Or a specific area is independent of the number of success occurrences at another time or area.
2) Probability of one success in a very short time period. Or a very small area will vary depending on the time or the size of that area. And is independent of the number of successful results occurring outside the time period. Or outside the said area.
3) Probability of more than one success in a very short period of time. Or a very small area with very little value that it must not be taken into account Or can be cut off.

Definition 2.2. We call the random variable X, which represents the number of successes achieved over a given period of time or area in the Poisson experiment as the poison random variable.

Definition 2.3. If X is a random variable representing the number of successes that have occurred during a given period or region. In the Poisson experiment and the mean of the successes over the period or area is λ , the probability mass function or the probability distribution of X is

$$Pr\left(X=x\right)=f\left(x\right)=P\left(x;\lambda\right)=\frac{e^{-\lambda}\lambda^{x}}{x!} \qquad \text{where } x=0,1,2,3,...$$

Definition 2.4. If X is a Poisson random variable, then the mean and variance of X are

$$\mu=E\left(X\right)=\lambda \text{ and } \sigma^{2}=Var\left(X\right)=\lambda$$

Example 2.5. According to a bank customer survey, the average number of customers is 3 in 4 minutes.Find the probability that 10 customers use the service in an 8-minute period.

Let X represent the number of customers who arrived in the service within 4 minutes. Therefore, X is 0, 1, 2, ...

$$Pr\left(X=x\right)=f\left(x\right)=P\left(x;\lambda\right)=\frac{e^{-\lambda}\lambda^{x}}{x!} \qquad \text{where } x=0,1,2,3,...$$

The problem wants to find the probability that 10 customers come to use the service in an 8-minute period, but the problem requires that the average number of customers in 4 minutes is equal to 3 people, therefore, in an 8-minute period, there will be an average of 6 customers.

Substitute the formula

$$Pr\left(X=10\right)=P\left(10;6\right)=\frac{e^{-6}\left(6\right)^{10}}{10!}=0.0413 \qquad \text{where } x=0,1,2,3,...$$

That is, the probability of getting 10 customers at this bank in 8 minutes is 0.0413.

2.3.2 Exponential distribution

An exponential distribution is a distribution that is related to the Poisson distribution. Since the exponential distribution is the distribution of the amount of time waiting until the first interest event occurs, the Poisson distribution's number of times has been distributed, for example: The number of customers who come to use the service in 1 hour is distributed in Poisson. The time between customers who come to use the service will have exponential distribution. The count of consecutive times continues until the Poisson event occurs. If X is this consecutive count of time, X will be a random variable with exponential distribution. Which has a probability function as follows:

The random variable X represents the time waiting for the first event, or the time between two consecutive events, has a rate of incidence per unit time of λ , and then X is exponential distribution.

$$f\left(x\right)=\lambda e^{-\lambda x}\ ;\ 0<x<\infty,\ \lambda>0 \tag{2.1}$$

The expected values and the variance are as follows.

$$E\left(X\right)=\frac{1}{\lambda} \quad \text{and} \quad V\left(X\right)=\frac{1}{\lambda^2}$$

The probability calculation is based on the cumulative probability.

$$F\left(X\right)=1-e^{-\lambda x}\ ;\ x>0 \tag{2.2}$$

2.4 Numerical Methods

Numerical methods are widely used to solve engineering problems because some computational equations are complex and complex, requiring advanced mathematics to solve. Therefore, knowledge and understanding of numerical methodology is necessary and much studied in order to be used in solving these problems. However, the solutions obtained from numerical methods are approximate only. In this section, we will show two topics, the solution estimation of ordinary differential equations (ODEs) by using the Runge Kutta 4th order method (RK4) and the quadratic Lagrange interpolating polynomial.

2.4.1 The Runge-Kutta 4th order method

The Runge-Kuta 4th order method (RK4) is the most commonly used method for solving ordinary differential equations (ODEs) because of its high accuracy and precision. Let an initial value problem be specified as follows:

$$\frac{dC}{dt} = f(t, C) \ , \qquad C(t_0) = C_0. \tag{2.3}$$

Here C is an unknown function (scalar or vector) of time t and $\frac{dC}{dt}$ is the rate at which C changes. It is a function of t and of C itself.At the initial time t_0 the corresponding C value is C_0.The function f and the initial condition t_0, C_0 are given [13].

Now let's pick a step size $h > 0$ and define, for $t_{n+1} = t_n + h$ where $n = 0, 1, 2, 3, \ldots$,

$$C_{n+1} = C_n + \frac{1}{6}h\left(k_1 + 2k_2 + 2k_3 + k_4\right) \ , \qquad t_{n+1} = t_n + h \tag{2.4}$$

where

$$\begin{aligned} k_1 &= f(t_n, C_n), \\ k_2 &= f\left(t_n + \tfrac{h}{2}, C_n + h\tfrac{k_1}{2}\right), \\ k_3 &= f\left(t_n + \tfrac{h}{2}, C_n + h\tfrac{k_2}{2}\right), \\ k_4 &= f\left(t_n + h, C_n + hk_3\right). \end{aligned}$$

Here C_{n+1} is the RK4 approximation of $C\left(t_{n+1}\right)$ obtained by the current value $C\left(t_{n+1}\right)$ plus the weighted average of four increments, where each increment is the item of the size of the interval, h, and an approximated slope specified by function f on the right-hand side of the equation.

- the term k_1 is the slope at the starting of the interval, using C (Euler's method)
- the term k_2 is the slope at the midpoint of the interval, using C and k_1.
- the term k_3 is again the slope at the midpoint, but now using C and k_2
- the term k_4 is the slope at the end of the interval, using C and k_3.

In averaging the four slopes, a greater weight is given to the slopes at the mid-point. If f is independent of C, so that the differential equation is equivalent to a simple integral, then RK4 is Simpson's rule.

The local truncation error is on the order of $O(h^5)$, while the total accumulated error is on the order of $O(h^4)$.

Example 2.6. Solve the differential equation using the Runge-Kuta 4th order method

$$y' = f(x,y) = -2x^3 + 12x^2 - 20x + 8.5\ , \quad 0 \le x \le 4$$

When step size is $h = 0.5$ and the initial condition is $x = 0,\ y = 1$

From the RK4 method $$y_{n+1} = y_n + \left[\frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\right] h$$

where

$$\begin{aligned} k_1 &= f(x_n, y_n) \\ &= f(0,1) \\ &= -2(0)^3 + 12(0)^2 - 20(0) + 8.5 \\ &= 8.5 \end{aligned}$$

$$\begin{aligned} k_2 &= f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}(k_1)\right) \\ &= f\left(0 + \frac{0.5}{2}, 1 + \frac{0.5}{2}(8.5)\right) \\ &= f(0.25, 3.125) \\ &= -2(0.25)^3 + 12(0.25)^2 - 20(0.25) + 8.5 \\ &= 4.21875 \end{aligned}$$

$$\begin{aligned} k_3 &= f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}(k_2)\right) \\ &= f\left(0.25, 1 + \frac{1}{2}(0.5)(4.21875)\right) \\ &= f(0.25, 2.05469) \\ &= 4.21875 \end{aligned}$$

$$\begin{aligned} k_4 &= f(x_n + h, y_n + hk_3) \\ &= -2(0.5)^3 + 12(0.5)^2 - 20(0.5) + 8.5 \\ &= 1.25 \end{aligned}$$

$$y_{n+1} = y_n + \left[\frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\right] h$$

we have

$$\begin{aligned} y(0.5) &= 1 + \left[\frac{1}{6}(8.5 + 2(4.21875) + 2(4.21875) + 1.25)\right] 0.5 \\ &= 3.21875 \end{aligned}$$

Continue to repeat this pattern until we get the solution of $y(4)$

2.4.2 Lagrange interpolation

Lagrange polynomials are the most commonly used interpolation polynomials, as well as Newtonian interpolation polynomials because polynomials are easy to understand. In addition, the Lagrange interpolation polynomial was also used to form the Newton-Coates integral formula for use in numerical integration and to construct an element interpolation Functions.

The Lagrange interpolation formula writes the interpolating polynomial for $\{(x_i, y_i)\}_{i=1}^{n}$ as a linear combination.The interpolation polynomial is calculated as:

$$L(x) = \sum_{i=1}^{n} y_i l_i(x). \tag{2.5}$$

In this paper we looked specifically at the version where $n = 3$ so we obtain the following **The Quadratic Lagrange interpolating polynomial** for $\{(x_1, y_1), (x_2, y_2), (x_3, y_3)\}$:

$$l_1 = \left(\frac{x - x_2}{x_1 - x_2}\right)\left(\frac{x - x_3}{x_1 - x_3}\right),$$

$$l_2 = \left(\frac{x - x_1}{x_2 - x_1}\right)\left(\frac{x - x_3}{x_2 - x_3}\right),$$

$$l_3 = \left(\frac{x - x_1}{x_3 - x_1}\right)\left(\frac{x - x_2}{x_3 - x_2}\right),$$

then

$$L(x) = y_1 l_1 + y_2 l_2 + y_3 l_3.$$

Example 2.7. Define the data set as in the following table.

x	2	2.5	4
$f(x)$	0.5	0.4	0.26

Find the Quadratic Lagrange interpolating polynomial of the above data set, and find the approximate value of $f(3)$.

We will find the Quadratic Lagrange interpolating polynomial, we need to find l_1, l_2, l_2 by

$$l_1 = \left(\frac{x-x_2}{x_1-x_2}\right)\left(\frac{x-x_3}{x_1-x_3}\right) = \left(\frac{x-2.5}{2-2.5}\right)\left(\frac{x-4}{2-4}\right) = x^2 - 6.5x + 10$$

$$l_2 = \left(\frac{x-x_1}{x_2-x_1}\right)\left(\frac{x-x_3}{x_2-x_3}\right) = \left(\frac{x-2}{2-2.5}\right)\left(\frac{x-5}{2.5-4}\right) = \frac{-4x^2+24x-32}{3}$$

$$l_3 = \left(\frac{x-x_1}{x_3-x_1}\right)\left(\frac{x-x_2}{x_3-x_2}\right) = \left(\frac{x-2}{4-2}\right)\left(\frac{x-2.5}{4-2.5}\right) = \frac{x^2-4.5x+5}{3}$$

we have

$$\begin{aligned} L(x) &= f(x_1)l_1 + f(x_2)l_2 + f(x_3)l_3 \\ &= 0.5\left(x^2 - 6.5x + 10\right) + 0.4\left(\frac{-4x^2+24x-32}{3}\right) + 0.25\left(\frac{x^2-4.5x+5}{3}\right) \\ &= 0.05x^2 - 0.452x + 1.15 \end{aligned}$$

thus, $f(3) \approx L(3) = 0.325$

Chapter 3

A mathematical model for carbon dioxide concentration measurement

In this chapter, we created mathematical models to predict the concentration of carbon dioxide in hospitals. We specifically target outpatient rooms because they are the areas with the most patients entering the hospital, as well as the ventilated rooms, which are the main parameters that affect carbon dioxide concentration.

3.1 A mathematical model for carbon dioxide concentration measurement

In general, the increased concentration of carbon dioxide in the room is due to the rate of exhaled air production of the population and the ventilation imported from the outside, that is, the air from the environment. Since the exhaled air from an infected person contains infectious particles in the air, we consider carbon dioxide as a substitute for the exhaled air. The exhaled air contains approximately 40,000 ppm of carbon dioxide, compared to approximately 400 ppm of CO_2 in the environmental air. This made us hypothesize that in a closed area, such as an outpatient room in a hospital, the environmental carbon dioxide concentration of approximately 400 ppm was added to the total number of people in the outpatient room. This means that the concentration of exhaled air may contain infectious airborne particles from the presence of airborne pathogens released by infected persons. We think the outpatient population equally contributes to the generation of carbon dioxide, a key marker of exhaled air.

The basic equation for the exhaled carbon dioxide air accumulation rate in the outpatient room is equal to the rate of exhaled air generated by all residents, plus the external intake rate, minus the environmental carbon dioxide. The air exhaled by the rate of ventilation from the outpatient room. It is illustrated in the following equation:

$$V\frac{dC}{dt} = npC_a + D_{in}C_e - D_{out}C \tag{3.1}$$

where

V (m^3) is the volume of oupatient room.

C_e (ppm) is the environmental carbon dioxide concentration.

D_{in} (L/s) is the inlet ventilation rate.

D_{out} (L/s) is the outlet ventilation rate.

n (peoples) is a number of people in an outpatient room.

C (ppm) is the carbon dioxide concentration indoor exhaled air.

p (L/s) is the breathing rate for each person in the room.

C_a (ppm) is the carbon dioxide fraction contained in breathed air.

t (minutes) is times.

3.2 Numerical Simulation of Carbon Dioxide Concentration in an Outpatient Room

A mathematical model of the cumulative concentration of carbon dioxide in an outpatient room.

3.2.1 Static number of population.

In all hospitals, it is imperative to control the air flow and ventilation with the design of the hospital ventilation system.Especially in the outpatient rooms, where there is a high number of people entering the service, it can lead to chances of airborne infection if not properly managed and controlled. We therefore used a mathematical model designed to predict the concentration of carbon dioxide in an outpatient room under unstable conditions using numerical methods to estimate the solution of the differential equation.

We simulated the events to use mathematical models to predict carbon dioxide concentrations when the population was constant inside an outpatient room under the following conditions:

1) The volume of the outpatient room $V = 125 \ \ m^3$.
2) Concentration of carbon dioxide in the ambient state, $C_e = 400 \ \ ppm$.
3) Inlet ventilation rate $D_{in} = 5 \ \ L/s$.
4) Outlet ventilation rate $D_{out} = 5 \ \ L/s$.
5) Number of people who come to use the service $n = 50$ peoples.
6) The exhalation rate of each person, $p = 0.12 \ \ L/s$.
7) The fraction of the concentration of carbon dioxide contained in the exhaled exhalation, $C_a = 0.04 \ \ ppm$.

When we simulate an event under the limited conditions above, we will predict the cumulative concentration of carbon dioxide in the outpatient room using mathematical models.

We substitute the parameters into Equation (3.1)

$$\begin{aligned} 125\frac{dC}{dt} &= 50\,(0.12)\,(0.04) + 5\,(400) - 5C \\ \frac{dC}{dt} &= \frac{50\,(0.12)\,(0.04) + 5\,(400) - 5C}{125} \\ \frac{dC}{dt} &= \frac{2000.24 - 5C}{125} \end{aligned}$$

Thus, we have

$$f\,(t_n, C_n) = \frac{2000.24 - 5C}{125} \tag{3.2}$$

When step size is $h = 0.1$ and the initial condition is $t = 0,\ C = 400,\ 0 \le t \le 160$
We would like to estimate the solution of Equation (3.2) by the RK4 method. ,

$$C_{n+1} = C_n + \frac{1}{6}h\,(k_1 + 2k_2 + 2k_3 + k_4)\,,$$

$$t_{n+1} = t_n + h$$

where

$$\begin{aligned} k_1 &= f(t_n, C_n) \\ &= f\,(0, 400) \\ &= \frac{2000.24 - 5\,(400)}{125} \\ &= 0.0019 \end{aligned}$$

$$\begin{aligned} k_2 &= f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}(k_1)\right) \\ &= f\left(0 + \frac{0.1}{2}, 400 + \frac{0.1}{2}\,(0.0019)\right) \\ &= f\,(0.0500, 400.0001) \\ &= \frac{2000.24 - 5\,(400.0001)}{125} \\ &= 0.0019 \end{aligned}$$

$$\begin{aligned} k_3 &= f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\,(k_2)\right) \\ &= f\left(0 + \frac{0.1}{2}, 400 + \frac{0.1}{2}\,(0.0019)\right) \\ &= f\,(0.0500, 400.0001) \\ &= \frac{2000.24 - 5\,(400.0001)}{125} \\ &= 0.0019 \end{aligned}$$

$$
\begin{aligned}
k_4 = \; & f\left(x_n + h, y_n + hk_3\right) \\
= \; & f\left(0 + 0.1, 400 + 0.1\left(0.0019\right)\right) \\
= \; & f\left(0.1, 400.0002\right) \\
= \; & \frac{2000.24 - 5\left(400.0002\right)}{125} \\
= \; & 0.0019
\end{aligned}
$$

$$
C_{n+1} = \; C_n + \left[\frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)\right] h
$$

we have

$$
\begin{aligned}
C\left(0.1\right) = \; & 400 + \left[\frac{1}{6}\left(0.0019 + 2\left(0.0019\right) + 2\left(0.0019\right) + 0.0019\right)\right] 0.1 \\
= \; & 3.21875
\end{aligned}
$$

We use the numerical method RK4 to estimate the model solution that is generated under uncertain conditions.

Table 3.1: Carbon dioxide concentrations distributed in outpatient rooms with fixed numbers of people.

Time (min)	0	40	80	120	140	160
CO_2 (L/s)	400	783.6511	860.5789	876.1226	878.4323	879.2331

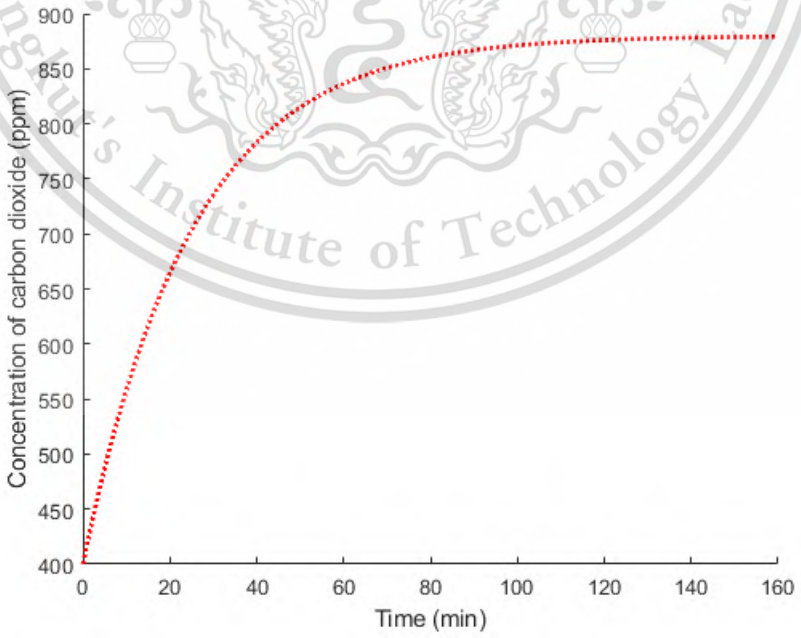

Figure 3.1: Graphs of carbon dioxide concentrations distributed in outpatient rooms with fixed numbers of people, where $D_{in} = D_{out} = 5\ ppm$.

From the graph above, we can observe the trend of carbon dioxide concentration within a constant population of 50 people. It can be seen that over time, the carbon dioxide concentration will increase steadily until a certain period of time, the graph will gradually stabilize. We simulated various events to compare the carbon dioxide concentration, divided into the following cases.

Case I : The population is n = 50 people, and the inlet ventilation rate equal to the outlet ventilation rate.

Table 3.2*:* Comparing the cumulative concentration of carbon dioxide in an outpatient room where the inlet ventilation rate equal to the outlet ventilation rate.

Time (min)	0	40	80	120	140	160
$CO_2; D = 3$ (L/s)	400	894.4563	1083.3564	1155.2341	1172.3452	1183.3201
$CO_2; D = 4$ (L/s)	400	833.2034	954.6732	987.3211	993.4322	996.5431
$CO_2; D = 5$ (L/s)	400	783.6511	860.5789	876.1226	878.4323	879.2331

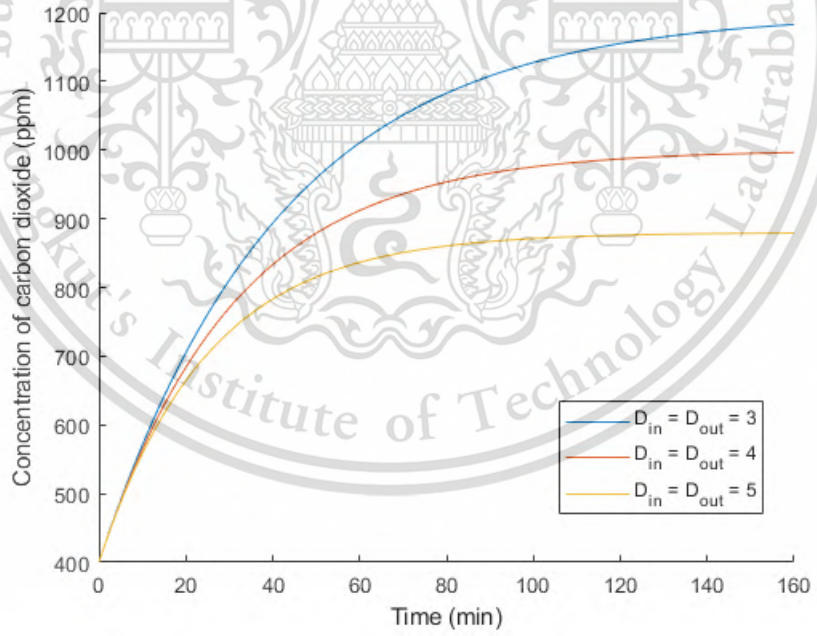

Figure 3.2*:* Graphs of comparing the cumulative concentration of carbon dioxide in an outpatient room where the inlet ventilation rate equal to the outlet ventilation rate.

Case II : The population is n = 50 people, and the inlet ventilation rate changes while the outlet ventilation rate static.

Table 3.3*:* Comparing the cumulative concentration of carbon dioxide in an outpatient room where the inlet ventilation rate changes while the outlet ventilation rate static.

Time (min)	0	40	80	120	140	160
$CO_2; D_{out} = 4$	400	761.3211	861.1298	889.4577	894.8924	897.2218
$CO_2; D_{out} = 5$	400	655.9853	707.2267	717.8995	719.5457	719.7869
$CO_2; D_{out} = 6$	400	571.3211	596.8771	599.6701	600.1124	600.6788

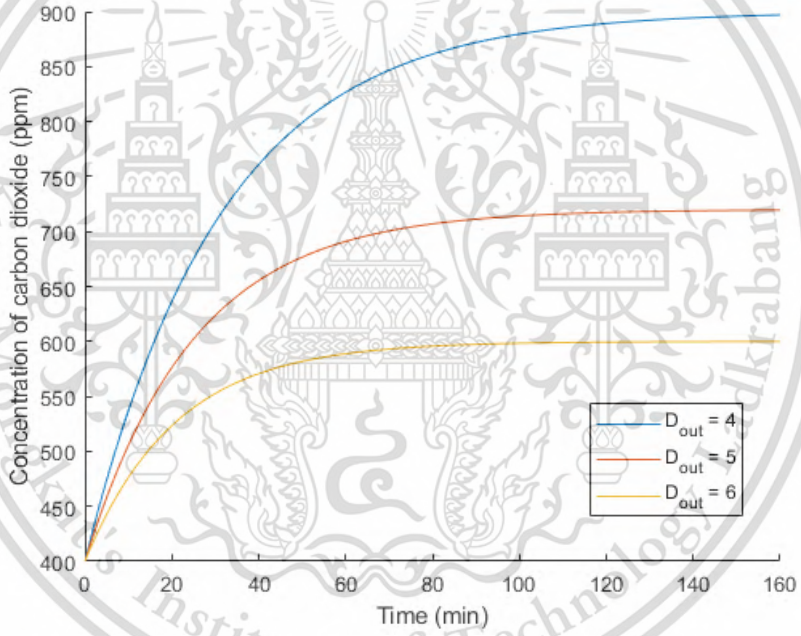

Figure 3.3*:* Graphs of comparing the cumulative concentration of carbon dioxide in an outpatient room where the inlet ventilation rate changes while the outlet ventilation rate static.

Case III : The population is n = 50 people, and the outlet ventilation rate changes while the inlet ventilation rate static.

Table 3.4*:* Comparing the cumulative concentration of carbon dioxide in an outpatient room where the outlet ventilation rate changes while the inlet ventilation rate static.

Time (min)	0	40	80	120	140	160
$CO_2; D_{in} = 4$	400	976.2201	1197.5661	1281.9023	1301.7611	1313.5490
$CO_2; D_{in} = 5$	400	1058.7522	1310.0923	1407.2311	1430.8011	1444.9821
$CO_2; D_{in} = 6$	400	1141.3088	1424.3708	1533.3229	1558.4490	1574.1189

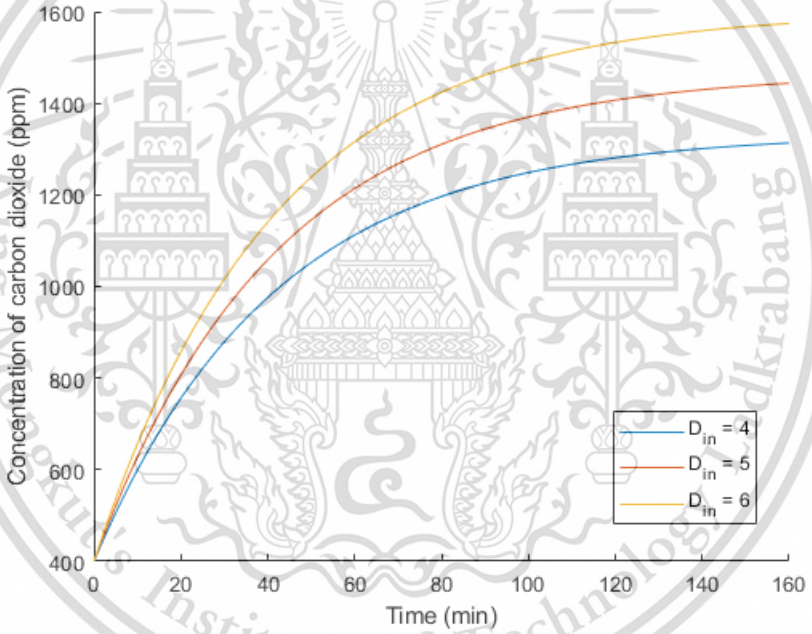

Figure 3.4*:* Graphs of comparing the cumulative concentration of carbon dioxide in an outpatient room where the outlet ventilation rate changes while the inlet ventilation rate static.

The mathematical model shown in the graph above shows that, over time, the cumulative concentration of carbon dioxide in the outpatient room increases or decreases depending on the intake of air and the ventilation rate. Obviously, as the inlet vantilation rate increases, the cumulative concentration of carbon dioxide increases. On the other hand, as the outlet ventilation rate increases, the total concentration of carbon dioxide will decrease. There is another important factor that affects the change in the cumulative carbon dioxide concentration. That is, the population. Next, we will look at the population's increase and decrease over time, so we need to simulate events to reflect the current situation.

3.2.2 Dynamic number of population

In the following simulations, we will apply the mathematical model to the real situation using the data table ,we simulated as in the following table.

Table 3.5*:* The information of the number of people every 10 minutes.

Time (min)	0	10	20	30	40	50	60	...	160
Number of people (person)	0	55	60	50	49	53	50	...	48

From the data table we assumed, it can be seen that, over time, the number of people visiting the outpatient room will change over time. So we use the quadratic Lagrange interpolating polynomial. To find the representative function of the data table, displayed in the following graph.

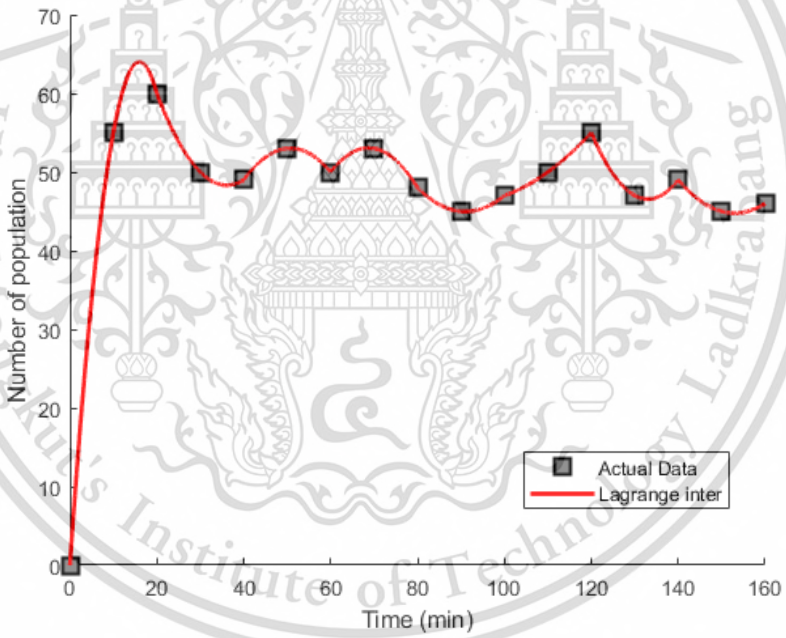

Figure 3.5*:* Comparative graph of approximation during the Lagrange second order polynomial function.

We estimate the solution of the mathematical model equations to be more consistent with real life, we construct a function to represent the data and use the RK4 numerical methodology to predict the carbon dioxide concentration, as shown in the following cases.

Case I : The population is changing over time n(t) people, and the inlet ventilation rate equal to the outlet ventilation rate.

Table 3.6*:* Comparing the cumulative concentration of carbon dioxide in an outpatient room where the outlet ventilation rate changes while the inlet ventilation rate static.

Time (min)	0	40	80	120	140	160
$CO_2; D = 3$	400	921.0167	1086.8901	1163.1001	1165.9001	1155.7811
$CO_2; D = 4$	400	855.3452	952.7811	994.7811	985.0991	971.8710
$CO_2; D = 5$	400	800.3711	856.6533	883.5622	871.9024	856.0621

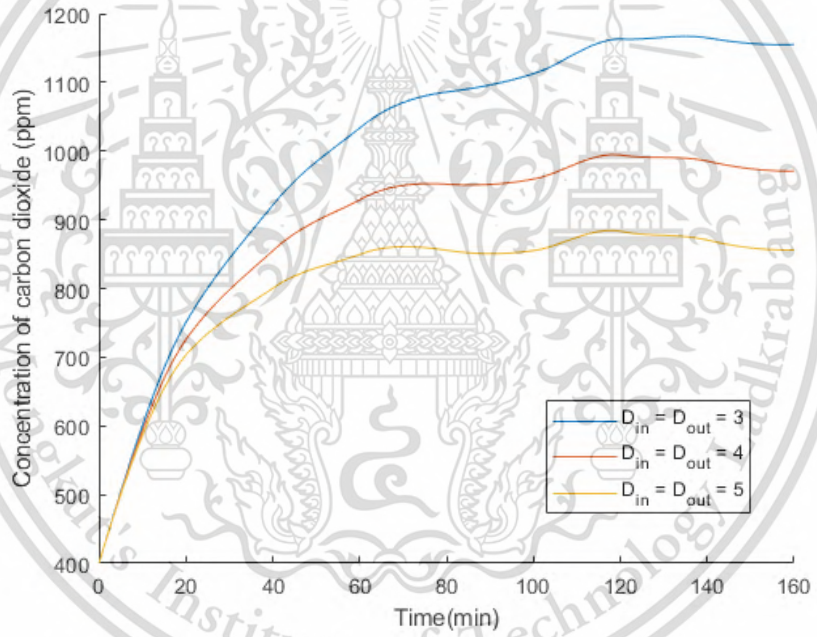

Figure 3.6*:* Graphs of comparing the cumulative concentration of carbon dioxide in an outpatient room with the number of population changing over time where the inlet ventilation rate equal to the outlet ventilation rate.

Case II : The population is changing over time n(t) people, and the inlet ventilation rate changes while the outlet ventilation rate static.

Table 3.7*:* Comparing the cumulative concentration of carbon dioxide in an outpatient room with the number of population changing over time where the inlet ventilation rate changes while the outlet ventilation rate static.

Time (min)	0	40	80	120	140	160
$CO_2; D_{out} = 4$	400	783.4589	860.7642	896.9012	887.2566	872.2278
$CO_2; D_{out} = 5$	400	673.6332	702.4320	725.8902	711.0567	696.6772
$CO_2; D_{out} = 6$	400	584.6722	589.7781	607.9002	592.5529	579.7811

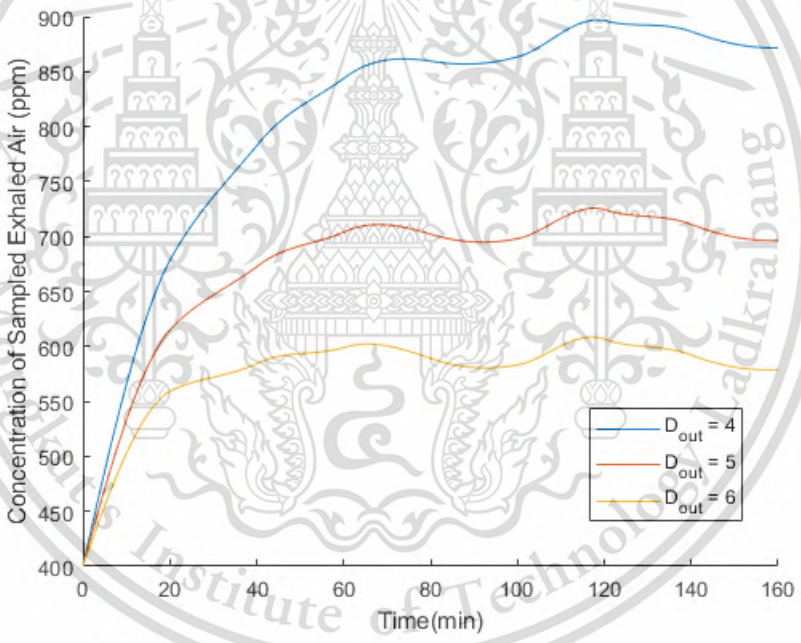

Figure 3.7*:* Graphs of comparing the cumulative concentration of carbon dioxide in an outpatient room with the number of population changing over time where the inlet ventilation rate changes while the outlet ventilation rate static.

Case III : The population is changing over time n(t) people, and the outlet ventilation rate changes while the inlet ventilation rate static.

Table 3.8: Comparing the cumulative concentration of carbon dioxide in an outpatient room with the number of population changing over time where the outlet ventilation rate changes while the inlet ventilation rate static.

Time (min)	0	40	80	120	140	160
$CO_2; D_{in} = 4$	400	1003.9956	1200.8921	1289.0921	1294.8921	1286.5671
$CO_2; D_{in} = 5$	400	1086.3189	1314.4672	1415.7622	1422.6523	1416.7844
$CO_2; D_{in} = 6$	400	1168.5421	1427.0986	1541.8713	1551.7845	1547.7884

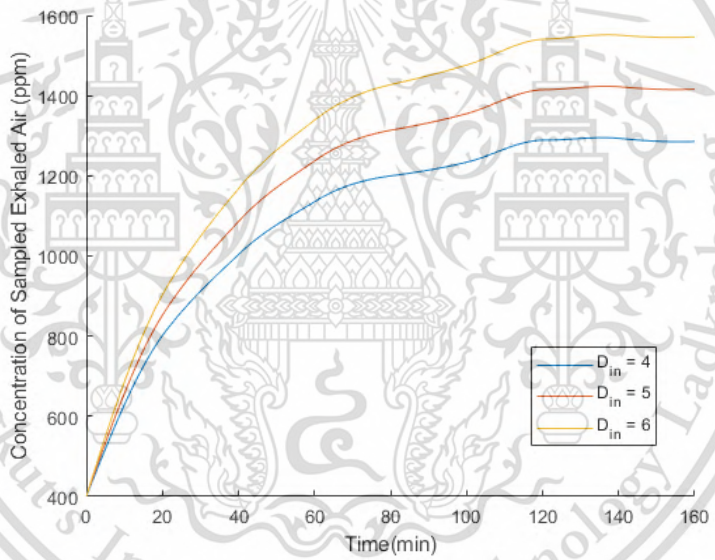

Figure 3.8: Graphs of comparing the cumulative concentration of carbon dioxide in an outpatient room with the number of population changing over time where the outlet ventilation rate changes while the inlet ventilation rate static.

The mathematical model shown in the graph above shows that, over time, the cumulative concentration of carbon dioxide in the outpatient room increases or decreases depending on the intake of air and the ventilation rate. Obviously, as the inlet vantilation rate increases, the cumulative concentration of carbon dioxide increases. On the other hand, as the outlet ventilation rate increases, the total concentration of carbon dioxide will decrease. There is another important factor that affects the change in the cumulative carbon dioxide concentration. That is, the population. Next, we will look at the population's increase and decrease over time, so we need to simulate events to reflect the current situation.

Chapter 4

Mathematical models for predicting the likelihood of airborne infection

In this chapter, we will develop a mathematical model in Chapter 3 to be a mathematical model used to predict the concentration accumulation of carbon dioxide in the closed area under the various constraints applied to airborne infectious diseases.

4.1 Model development

Airborne infection has a number of important infectious factors, and we consider carbon dioxide concentrations as the main marker of airborne infection. We have developed a more appropriate mathematical model where we only consider air that is vulnerable to airborne contamination.Although this mathematical model is widely used in the analysis of airborne infectious disease outbreaks, for example, it analyzes the probability of TB infection for people with low immunity at risk of infection. When they come into direct contact with an infected person In the meantime This mathematical model is limited, it can only predict the risk of infectious disease as well as constant and consistent ventilation conditions.But in reality, airborne infections contain more or less infectious particles, depending on the individual's immunity as well as the severity of the strain released by the pathogen. This makes us unable to accurately measure or determine the number of infectious particles. Infected airborne particles disperse and float infinitely in space. When a vulnerable person is exposed to airborne infectious particles. They have a hundred percent chance of being infected, or perhaps not at all, depending on the infection of the infected particles and the immune response in the person's body that is weak.The probability of infection is higher, when the inhaled infectious particle travels to the point of infection or the particle concentration is above the threshold, causing the infection. This has led to the development of mathematical models that predict the likelihood of a vulnerable person's risk of infection under unstable conditions. Therefore, when considering the transmission of tuberculosis, we take into account the duration of exposure and the proximity to it. This means that if the vulnerable person is closer to the infected person, the infection will occur in a shorter period of time than people who are far away.

To develop this mathematical model, we made three hypotheses. First, the ventilation rate in each area should not be the same in all areas in the outpatient room.Second, the exhaled air of an infected person carries infectious particles in the air, allowing vulnerable people to become infected and become infected with airborne pathogens.And finally, the probability of individual infection is independent of

each other. This is because the air in the room is uneven and the concentration of infectious particles in the air is high when it is released from the source, that is, it is caused by the infected person being released, making the area higher than the rest of the area.As mentioned earlier, the air exhaled by an infected person carries airborne infectious particles that cause infectious diseases. We have the ability to monitor carbon dioxide concentrations to mark the air generated in the outpatient room. Therefore, the concept of air emission rate was used to develop mathematical models that predict the risk of airborne pathogen transmission under unstable conditions.

We determine the concentration of exhaled air in an outpatient room with volume $V\ \ (m^3)$ with the total population n (people) active and C_a is the fraction of the CO_2 in the exhaled air. While the ventilation rate is $D_{out}\ \ (L/s)$ and the cumulative concentration of exhaled air inside the outpatient room is $C\ \ (ppm)$. We noticed that infectious particles in the air were generated from exhaled air in the outpatient room, but because this area was also populated by the infected, it was not possible to remove the infection. Thus, the exhalation air accumulation rate in the outpatient room is equal to the exhalation air delivery rate in the outpatient chamber minus the exhaust air ventilation rate, as shown in this differential equation:

$$V\frac{dC}{dt} = npC_a - D_{out}C\ , \quad n \geq 2 \tag{4.1}$$

Where n is the total number of the population. (Infected person and susceptible person) and p is the rate of exhalation (L / s).

The results of simulations of the carbon dioxide concentrations of airborne infection.

We have developed a mathematical model that is flexible and suitable for predicting the risk of airborne infectious disease.We consider the specific concentrations of carbon dioxide produced by the breath of an infected person that is released as a carrier of the infection. Using numerical methodology to estimate the solution of the simulated equation, that is, the differential equation.

We continue to define parameters similar to Chapter 3: predicting the CO concentration in the outpatient room with the constant population entering the hospital in the outpatient area. This will be demonstrated under the following conditions.

1) We simulate the volume of the ventilated outpatient room, $V = 125\ \ m^3$.

2) Concentration of carbon dioxide in the ambient state, $C_e = 400\ \ ppm$.

3) Outlet ventilation rate $D_{out} = 5\ \ L/s$.

4) Number of people who come to use the service $n = 50$ peoples.

5) The exhalation rate of each person, $p = 0.12\ \ L/s$.

6) The fraction of the concentration of carbon dioxide contained in the exhaled exhalation, $C_a = 0.04 \ \ ppm$.

When we simulate an event under the limitation conditions above. Next, we will predict the cumulative concentration of carbon dioxide in the outpatient room using mathematical models.

We substitute the parameters into Equation (4.1)

$$\begin{aligned} 125\frac{dC}{dt} &= 50\,(0.12)\,(0.04) - 5C \\ \frac{dC}{dt} &= \frac{50\,(0.12)\,(0.04) - 5C}{125} \\ \frac{dC}{dt} &= \frac{0.24 - 5C}{125} \end{aligned}$$

Thus, we have

$$f\,(t_n, C_n) = \frac{0.24 - 5C}{125} \tag{4.2}$$

We determined that the cumulative carbon dioxide concentration in the outpatient room was 0 ppm, since we hypothesized that at the start time in the outpatient room there were no incoming populations, there was no carrier carbon dioxide concentration. In the period of infection from 0 to 160 minutes and Step size h = 0.1, we would like to estimate the solution of equation (4.2) by the RK4 method. as shown in the figure below.

Table 4.1: The cumulative concentration of carbon dioxide in an outpatient room where the outlet ventilation rate $D_{out} = 5\ (ppm)$.

Time (min)	0	40	80	120	140	160
CO_2 (ppm)	400	464.3221	477.1507	479.6161	480.8285	480.9015

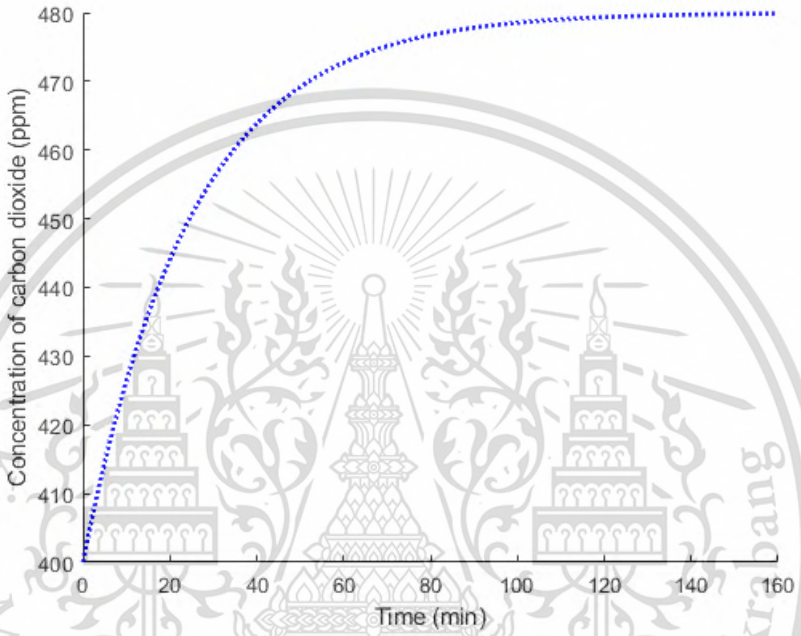

Figure 4.1: Graphs of the cumulative concentration of carbon dioxide in an outpatient room where the outlet ventilation rate $D_{out} = 5\ (ppm)$.

4.1.1 The average volume fraction of exhaled air

From the above differential equation of the exhaled air. We can determine the mean volume fraction of exhaled air in an outpatient room under unstable conditions.We assume that the exhalation air intensity begins to rise from 0 to $C\,(T)\ ppm$ and a person enters the service from 0 to T minutes. Taking into account that the exhaled air volume fraction is formed by the exhaled air concentration of a sample in air divided by the exhaled carbon dioxide fraction C_a, the following equation is given:

$$f\,(t_i) = \frac{C\,(t_i)}{Ca} \qquad \text{where} \quad i = 1, 2, 3, \ldots \tag{4.3}$$

Led to the calculation of the average volume fraction of the exhaled air $\bar{f}$ represents the volume fraction of exhaled air considering time from 0 to T:

$$\bar{f} = \frac{\sum_{i=1}^{n} f(t_i)}{n} \qquad \text{where} \quad i = 1, 2, 3, \ldots \tag{4.4}$$

As previously mentioned, the likelihood that airborne infectious particles released by an infected person will cause infection for susceptible individuals or those with low

immunity is very high, If they reach the point of infection that is higher than the threshold. However, some infectious particles can get trapped in the upper respiratory tract or spread to other parts of the body. There is a possibility that these infections will not affect airborne infections. We define β as the rate of total airborne infectious particles emitted by the infected person (particles / s) and μ is the mortality rate of airborne infectious particles that cannot be transmitted to the infected target (particles / s) .Therefore, the survival rate of airborne infectious particles is equal to $\beta - \mu$ (particles / s) , as shown in the figure below.

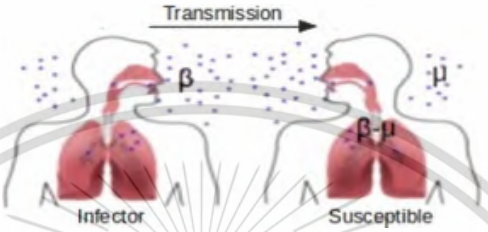

Figure 4.2: The movement of infectious particles in the air.

4.1.2 The average concentration of airborne infectious disease particles

The concentration of airborne particles that caused the infection is equal to the fraction of the air produced by the infected person multiplied by the average concentration of airborne infectious particles released by the infected person when the particle travels to the point of infection. Get as the following equation :

$$N(t_i) = \left(\frac{I(t_i)\, f(t_i)}{n(t_i)}\right)\left(\frac{\beta - \mu}{p}\right) \quad I(t_i) \geq 1 \text{ and } \beta - \mu \geq 1 \tag{4.5}$$

Where $I(t_i)$ represents the number of infected people who come to the outpatient room.

The equation for **the average concentration of airborne infectious particles** is obtained:

$$\bar{N} = \frac{\sum_{i=1}^{n} N(t_i)}{n} \quad \text{where } i = 1, 2, 3, \ldots \tag{4.6}$$

This is because some infectious particles cannot reach the point of infection and will accumulate in other parts of the body. We define θ as airborne particulate matter that can successfully pass through the infected area, thus ensuring that the average number of airborne infectious particles that a vulnerable person receives is equal to the product of the respiratory volume of the vulnerable multiplied by the fraction of the accumulation of airborne infectious particles and the mean concentrations of airborne infectious particles released by the infected person is given in the following equation.

Let $\bar{\lambda}$ is the average number of airborne infectious particles that a vulnerable person receives.

$$\bar{\lambda} = pt\theta\bar{N} \quad , t > 0 \tag{4.7}$$

Where t is the time the infection has passed to the point of infection.

4.2 The probability of airborne transmission risk for susceptible individuals

By assuming that airborne transmission is based on the Poisson distribution, we express the probability of transmission of airborne infectious disease as follows:

$$P(T \leq t \mid I, D_{out}, V, p, \theta, \mu, \beta) = 1 - e^{-\bar{\lambda}} \tag{4.8}$$

Where $P(T \leq t \mid I, D_{out}, V, p, \theta, \mu, \beta)$ represents the probability of airborne transmission risk for susceptible individuals, and $T \leq t$ is random variables showing infection risk for susceptible individuals up to time considered in an outpatient room.

Therefore, to predict the risk of transmission of airborne diseases such as tuberculosis or coronavirus under unstable conditions in our research, we examined the concentrations of air generated by infected people in the outpatient room and the exhaled air accumulation rate to calculate the fraction of the mean volume of direct exhaled air. And the exponent of this equation is equal to the average number of airborne infectious particles that are inhaled by a vulnerable individual in the air to induce the infection.

4.3 Simulated results of infection probability of airborne infectious disease.

Airborne infection is caused by a variety of factors described in Chapter 2, leading to the development of a mathematical model for predicting the transmission risk of airborne infectious disease. In this section, we will simulate events and analyze the likelihood of people who use the ambulatory room to determine the probability of airborne infection. We have classified it into 2 types as follows.

4.3.1 Static number of population

We looked at the risk of airborne infectious disease transmission in an outpatient facility in the event of a constant traffic population and set the parameters and conditions similar to the preceding section.In the next step, the results of the simulation can be explained in the following cases.

Simulation I : The population is 50 people and simulates the ventilation rate change.

Table 4.2*:* Comparison of the concentration of carbon dioxide when the ventilation system changes.

Time (min)	0	40	80	120	140	160
$CO_2; D_{out} = 3$	0	494.0456	683.5534	755.9021	772.6722	783.3351
$CO_2; D_{out} = 4$	0	433.8921	554.9823	587.0943	593.5662	596.7711
$CO_2; D_{out} = 5$	0	383.5671	460.6781	476.0934	478.6781	479.5611

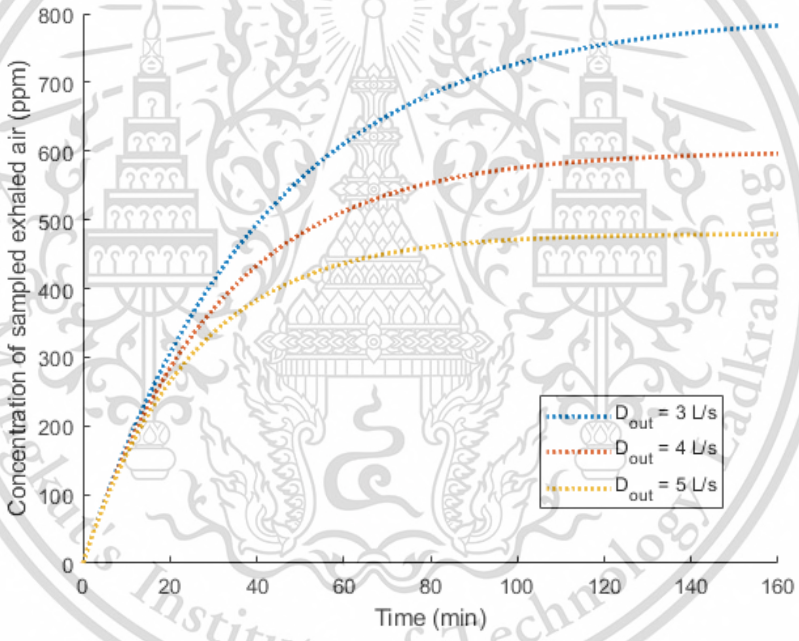

Figure 4.3*:* Comparison of the concentration of carbon dioxide when the ventilation system changes.

Table 4.3: Comparison of the probability of infection of airborne diseases when the ventilation system changes.

Time (min)	0	20	40	80	100	120	140	160
$Prob; D_{out} = 3$	0	0.1574	0.2901	0.4961	0.5754	0.6423	0.6986	0.7460
$Prob; D_{out} = 4$	0	0.1297	0.2426	0.4263	0.5007	0.5655	0.6218	0.6709
$Prob; D_{out} = 5$	0	0.1099	0.2077	0.3722	0.4412	0.5026	0.5572	0.60558

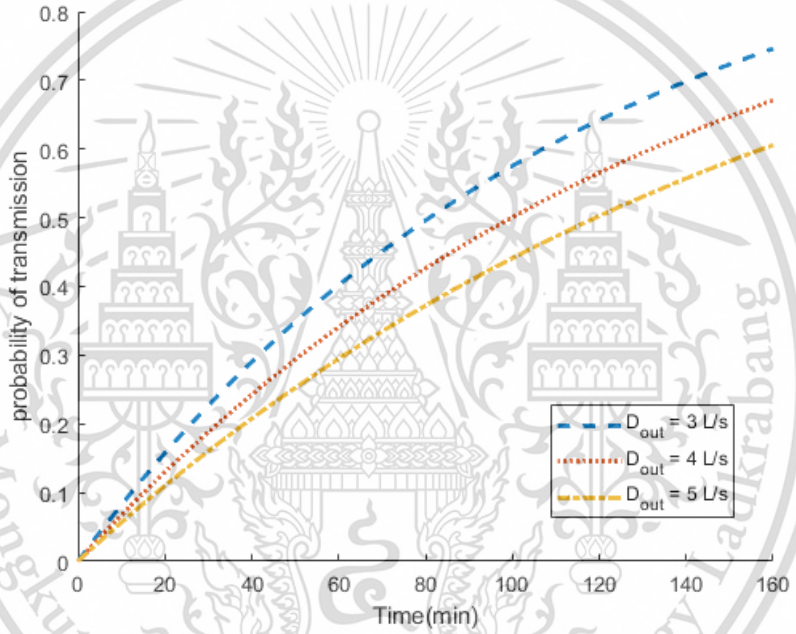

Figure 4.4: Comparison of the probability of infection of airborne diseases when the ventilation system changes.

4.3.2 Dynamic number of population

In the following simulations, we will apply the mathematical model to the real situation using the data table ,we simulated as in the following table.

Table 4.4: The information of the number of people every 10 minutes.

Time (min)	0	10	20	30	40	50	60	...	160
Number of people (person)	0	15	60	70	79	63	60	...	26

From the data table we assumed, it can be seen that over time, the number of people visiting the outpatient room will change over time. So we use the quadratic lagrange interpolating polynomial. To find the representative function of the data table, displayed in the following graph.

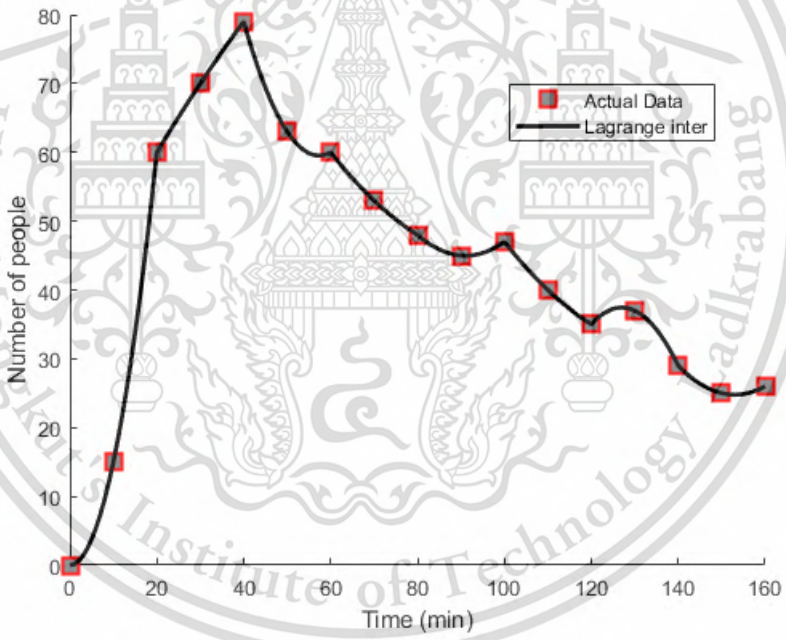

Figure 4.5: Comparative graph of approximation during the Lagrange second order polynomial function.

Simulation II : The population depends on the time being considered and simulates the ventilation rate change.

Table 4.5*:* Comparison of the concentration of carbon dioxide when the ventilation system changes.

Time (min)	0	40	80	120	140	160
$CO_2; D_{out} = 3$	0	381.6721	503.5512	429.4311	385.7611	318.5260
$CO_2; D_{out} = 4$	0	276.6435	263.8681	200.4636	176.2331	137.9926
$CO_2; D_{out} = 5$	0	211.5571	171.5509	128.7080	113.8208	86.6917

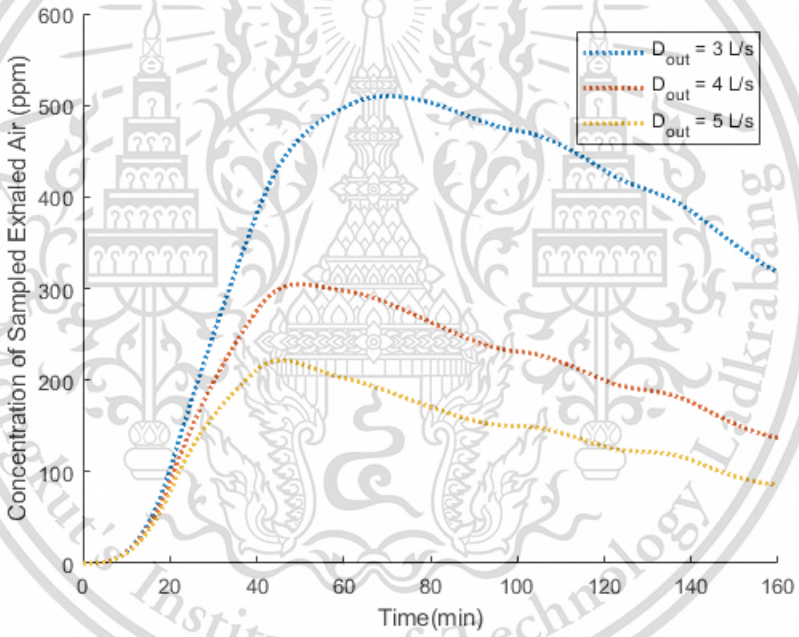

Figure 4.6*:* Comparison of the concentration of carbon dioxide when the ventilation system changes.

Table 4.6: Comparison of the Probability of infection of airborne diseases when the ventilation system changes.

Time (min)	0	20	40	80	100	120	140	160
$Prob; D_{out} = 3$	0	0.1267	0.2374	0.4184	0.4921	0.5565	0.6127	0.6618
$Prob; D_{out} = 4$	0	0.0711	0.1372	0.2556	0.3085	0.3577	0.4034	0.4458
$Prob; D_{out} = 5$	0	0.0491	0.0959	0.1825	0.2227	0.2609	0.2972	0.3317

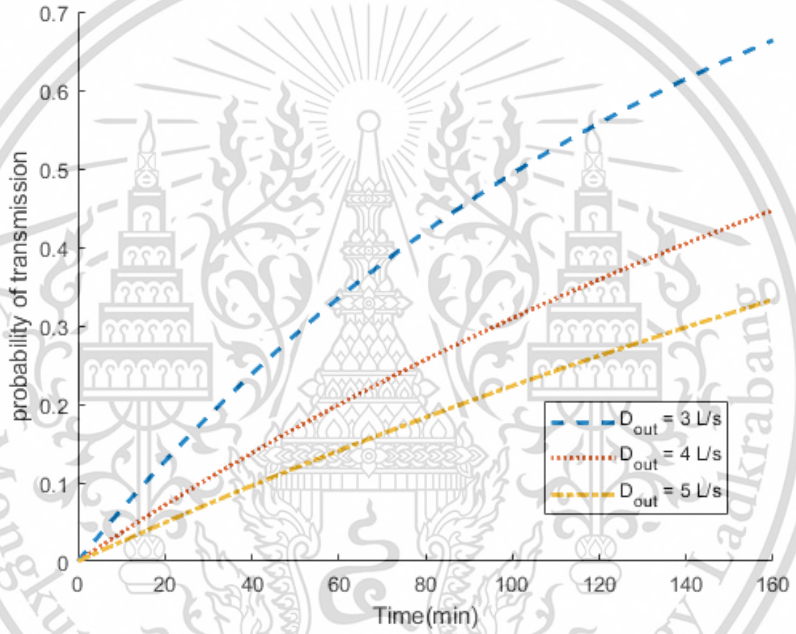

Figure 4.7: Comparison of the Probability of infection of airborne diseases when the ventilation system changes.

The probability of transmission of infectious diseases by air increases as the ventilation rate decreases. This suggests that in an atmosphere dense with concentrations of carbon dioxide greatly increases the risk of infection. However, we need to manage the ventilation system according to the room volume and the population that use the service. Next, we will apply the mathematical model derived from studying the probability of airborne infectious disease transmission to the outpatient room under acceptable risk.

4.4 Applying a model of the probability of spreading infectious diseases by air to control the population that comes to use the service accordingly.

We can calculate the probability of airborne infection using the concentration of carbon dioxide released from the breath of an infected person. Resulting in simulation results according to the previous topic leading to application to actual use in hospitals, especially outpatient rooms, by considering the acceptable probabilities.We simulated as in the following table.

Table 4.7*:* The information of the number of people every 10 minutes.

Time (min)	0	10	20	30	40	50	60	...	160
Number of people (person)	0	15	60	70	79	63	60	...	26

From the data table we assumed, it can be seen that over time, the number of people visiting the outpatient room will change over time.

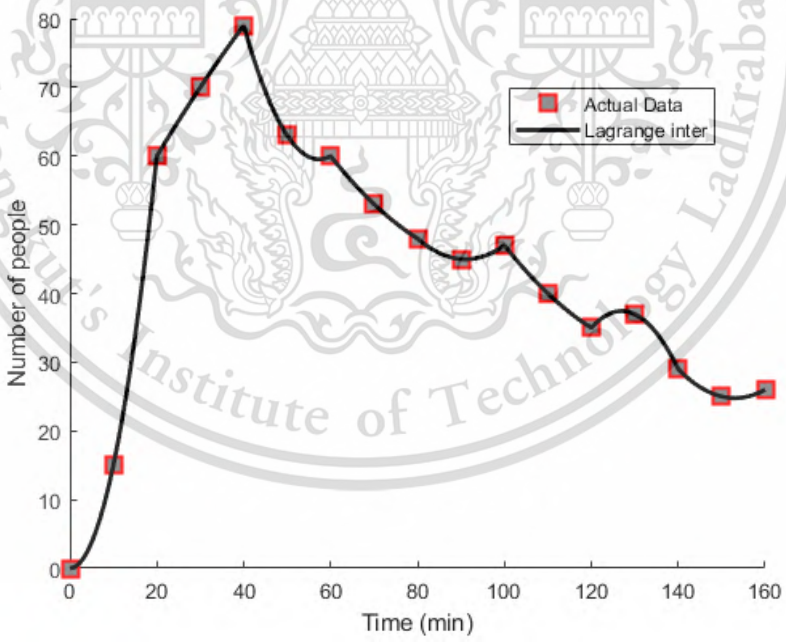

Figure 4.8*:* Comparative graph of approximation during the Lagrange second order polynomial function.

We simulate a graph showing the generally acceptable probabilities we consider a probability value 50 percent or 0.5 under restricted conditions and a ventilation system $D_{out} = 5/ppm$. The following graph is concerned.

Table 4.8: The probability of spreading an infectious disease by air considering the probability of 0.5

Time (min)	0	20	40	80	100	120	140	160
$Prob; D_{out} = 3$	0	0.1267	0.2374	0.4184	0.4921	0.5565	0.6127	0.6618

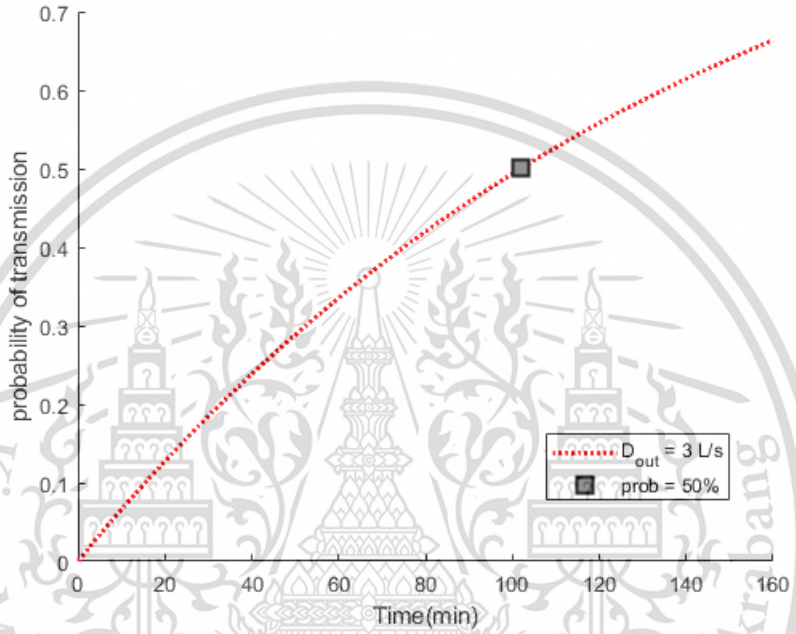

Figure 4.9: The graph shows the probability of spreading an infectious disease by air considering the probability of 0.5.

From the graph above, we can see that under various constraints, we can control the number of populations entering the outpatient room under the probability that 0.5 will be able to receive 21 people within a period of 102. minutes, depending on the acceptable probabilities.

Chapter 5

Discussion and Conclusion

In this chapter, we will discuss and summarize the results of airborne infectious disease simulations using mathematical models to predict the spread of airborne infectious diseases and to find ways to reduce the risk of infection. By controlling the ventilation system and the number of people entering the service in the outpatient room.

5.1 Discussion

In an unstable state, we have developed a mathematical model that accurately predicts the risk of airborne infectious disease transmission using carbon dioxide concentrations as a marker of exhaled air in an outpatient room.In addition, we investigated the application of the model to different factors and conditions resulting from the likelihood of airborne transmission risk. Therefore, we created a mathematical model to determine the risk of airborne infection under unstable conditions. Since it best reflects the real world, we show that normal breathing, including human talk, can contain air containing infectious particles in the air that can cause infectious disease in the outpatient room. The risk of air transmission probability increases significantly as the number of infected people and airborne particles increases.

5.2 Conclusion

We studied a mathematical model of carbon dioxide concentration in an outpatient room in which the carbon dioxide contributing factor was composed. The influx of outside air and the breath of the population who come to use the outpatient room is therefore the cumulative concentration of carbon dioxide in the outpatient room. Next, we developed a mathematical model to determine the probability of airborne infection under various conditions and considered the amount of carbon dioxide released by the infected person, $C_E = 0ppm$. Since external sources have no effect on airborne infections, a mathematical model of the air concentrations that are true carriers of airborne infections is therefore obtained. From the simulation results, it was concluded that the ventilation rate is an important variable that increases or decreases the likelihood of infection. That is, when the ventilation rate is low, the carbon dioxide concentration will increase. As air increases, the concentration of carbon dioxide will be reduced.

5.3 Suggestions

This research has developed a mathematical model that is flexible and can be used in real life. This research can be applied to areas other than outpatient rooms such as classrooms, buses or other densely populated areas and to change the numerical methodology to suit the developed model. Other variables that cause airborne infections may also need to be studied.

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