

TRACK MIS-REGISTRATION MITIGATION METHODS BASED ON READBACK  
SIGNALS IN BIT PATTERNED MEDIA RECORDING SYSTEM



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## บทคัดย่อ

วิทยานิพนธ์ฉบับนี้ได้นำเสนอวิธีการลดผลกระทบของการอ่านนอกแทร็กข้อมูลบนพื้นฐานของสัญญาณอ่านกลับในระบบการบันทึกข้อมูลเชิงแม่เหล็กแบบบิตแพตเทิร์นมีเดีย (bit patterned media recording : BPMP) การอ่านนอกแทร็ก (track mis-registration : TMR) หรือ ออฟแทร็ก (off-track) เป็นอีกหนึ่งปัญหาที่มีผลกระทบอย่างมากในระบบการบันทึกข้อมูลเชิงแม่เหล็กที่มีความหนาแน่นเชิงพื้นที่สูง เช่น ในระบบการบันทึกข้อมูลแบบ BPMP ซึ่งระบบการบันทึกข้อมูลเชิงแม่เหล็กแบบ BPMP ที่ได้รับผลกระทบจากการอ่านนอกแทร็กจะทำให้ประสิทธิภาพของระบบลดลงอย่างมาก การอ่านนอกแทร็กนั้นเกิดจากความคลาดเคลื่อนหัวอ่านที่ทำการอ่านไม่ตรงกับตำแหน่งกึ่งกลางแทร็ก โดยทั่วไปแล้วการนอกแทร็กสามารถตรวจสอบและแก้ไขโดยระบบเซอร์โว อย่างไรก็ตามในระบบเซอร์โวนั้นยากที่จะควบคุมการทำงานของหัวอ่านในขณะที่ TMR เกิดขึ้นเกินขอบเขต นอกจากนั้นแล้วระบบเซอร์โวจะต้องมีการสูญเสียพื้นที่เพื่อเก็บข้อมูลการทำงานของระบบเซอร์โวเพื่อตรวจสอบค่าการเกิด TMR ในระบบด้วย ดังนั้นในวิทยานิพนธ์ฉบับนี้ได้นำเสนอวิธีการลดผลกระทบของ TMR สำหรับระบบ BPMP บนพื้นฐานของสัญญาณอ่านกลับโดยได้ทำการประมาณค่าของ TMR ที่เกิดขึ้นในระบบจากสัญญาณอ่านกลับ อันดับแรกวิทยานิพนธ์ฉบับนี้ได้นำเสนอการออกแบบ คู่ของอีควอไลเซอร์และทาร์เก็ตแบบสองมิติที่เหมาะสมกับช่องสัญญาณของ BPMP สำหรับ TMR ในแต่ละระดับ จากนั้นทำการประมาณค่า TMR จากสัญญาณอ่านกลับที่มีความสัมพันธ์ระหว่างสัญญาณอ่านกลับและ TMR สุดท้ายของอีควอไลเซอร์และทาร์เก็ตแบบสองมิติที่เหมาะสมกับ TMR แต่ละระดับที่ออกแบบไว้แล้วจะถูกเลือกใช้เพื่อช่วยในการลดผลกระทบของ TMR ที่เกิดขึ้นในระบบ ผลการทดลองพบว่าวิธีที่นำเสนอขึ้นสามารถให้ค่าประสิทธิภาพที่สูงกว่าระบบทั่วไป โดยเฉพาะอย่างยิ่งเมื่อระบบได้รับผลกระทบจาก TMR ในระดับที่สูง

<b>Thesis</b>	Track Mis-registration Mitigation Methods based on Readback Signals in Bit Patterned Media Recording System
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## ABSTRACT

This thesis presents regarding Track Mis-Registration (TMR) mitigation methods based on readback signals for Bit patterned media recording (BPMR) system. TMR or off-track situation is one of the most significant problems in the extremely high-density magnetic recording system such as bit-patterned media recording (BPMR) system, since a track pitch becomes narrower thus the performance in magnetic recording system was intruded from high inter-track interference (ITI) and TMR effects. The TMR effect is occurred due to the misalignment between the center of the read head and that of the main data track. Typically, the TMR can be detected and handled by a servo system; however, it requires some special data to be inserted in the tracks so as to estimate the amount of head offset. Nonetheless, this thesis proposes a TMR mitigation method for BPMR systems based on the readback signals. Firstly, this thesis designs several pairs of the two dimensional (2D) asymmetric target and its corresponding 2D equalizers that match the BPMR channel for each TMR level. Then, we estimated TMR levels based on the read back signal in BPMR using the relationship between the readback signal and TMR levels. Lastly, a pair of the 2D asymmetric target and its corresponding 2D equalizer that is best fit to the estimated TMR level will be used to alleviate the TMR effect in the readback signal. Simulation results indicate that the proposed system can effectively estimate the TMR level and perform better than the conventional system without a TMR mitigation method, especially when the TMR level is high.

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# Chapter 1

## Introduction

### 1.1 Statement of the problem

As the internet and digital network communication are rapidly growing; therefore, the new technologies are developed to support for using in currently, such as social media data storage, big data storage, cloud application storage, and etc. These technologies are tasked based on the data storage system. Thus, the data storage system is important to propel these new technologies. Hard disk drive is one of the best ways to store the data because it is appropriate when compromising data and price. The high areal density (AD) magnetic recording technologies in hard disk drives are developed parallel with growth of new technologies, it is goal of magnetic recording system to develop in hard disk drives.

Bit patterned media recording (BPMR) system is a promising technology to enhance the areal density of hard disk drives beyond the limit imposed by the current perpendicular recording technology, and it can achieve an areal density up to 4 terabits per square inch (Tb/in<sup>2</sup>) [1], while, the perpendicular technology has super-paramagnetic limit effected and cannot store data more than 1 Tb/in<sup>2</sup> [2]. In the enhancing the areal density increasing in hard disk drives can be made by decreasing the distance of the island's data (bit island). However, while the bit islands are closed, it has been shown that inter-symbol interference (ISI) and inter-track interference (ITI) (or called two dimensional (2D) interference) and media noise can generate a significant impact on the error performance of BPMR systems [3]. These cases are the challenging problems in BPMR system. Other than, one of the major problems that increase ITI in BPMR is a track mis-registration (TMR) [4,5] due to the readhead sensing the magnetic islands in the adjacent track. Therefore, TMR problem in BPMR systems, will be mainly considered in this research.

## 1.2 Significance of the problem

A TMR effect is occurred due to the misalignment between the center of the readhead and that of the main data track [6,7] as illustrated in Figure 1.1.

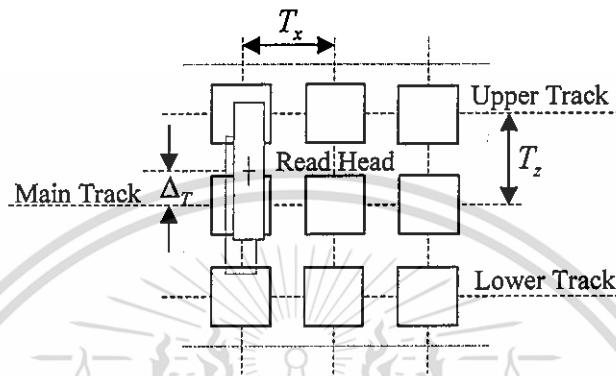


Figure 1.1 The TMR,  $\Delta_T$ , in a BPMR system.

Typically, the TMR can be detected and handled by a servo system; however, it requires some special data to be inserted in the tracks so as to estimate the amount of head offset and it is difficult to control the readhead when TMR is occurred beyond its limit [8-10]. To improve the system performance under the 2D interference effect, the previous work [11] has used iterative decision feedback detection (IDFD) and 2D generalized partial response equalization, which is optimized in minimum mean square error (MMSE), followed by 1D Viterbi algorithm (2D-GPR/1D-VA). Simulation result suggest that 2D-GPR/1D-VA performs better than IDFD.

Then, W. Chang and J.R. Cruz consider a multi-track detection technique, where the detection on the center track is aided by the information obtained from the detection of the sidetracks. This technique works with any equalizer capable of equalizing the channel to a 2D target. They apply this method to joint-track equalized and 2D equalized BPMR channels. Their simulation results show that the proposed technique provides a significance performance improvement [12]. To investigate the effect of ITI and TMR on the performance of the channel, Navabi et al. [13] have developed a 2D pulse response simulator for BPMR. Simulation results using these pulse response suggest that equalizer and detector optimized for zero TMR may not perform well in the presence of TMR. To mitigate the effects of TMR,

they proposed a modified trellis for Viterbi algorithm (VA). The modified Viterbi algorithm (MVA) takes into account the ITI while computing branch metrics. Simulation results show that the MVA can improve the bit error rate (BER) in the presence of TMR.

Another research that focus on how to mitigate the TMR effect, L. M. M. Myint and P. Supnithi [14] proposed an off-track detection based on the readback signals and improve the bit error performance by using an asymmetric target depending on the detected off-track direction. Specifically, they investigated the effects of off-track events on the target-shaping equalizer coefficients when the generalized partial-response target (GPR) is fixed. For a 3x3 channel matrix of bit patterned media recording system, the asymmetric targets offer the gain of about 1 to 2 dB at bit error rate (BER) =  $10^{-4}$  for the TMR levels of 20% to 25%.

From, these significances of 2D equalization and detection can mitigate ITI and TMR effects in BPMR system [11-14]. The BPMR channels can provide the different of readback signal when each TMR level occurred in the system [13,14]. Therefore, this thesis proposes the TMR prediction and mitigation methods based on the readback signal in BPMR system.

### 1.3 Goal and objectives

1. To study the TMR behavior and its effect on BPMR system.
2. To study the performance of BPMR systems which was from TMR effect.
3. To apply and develop a TMR mitigation method using the multiple readhead based on readback signals in BPMR system.
4. To improve the BER performance of BPMR system using the proposed TMR mitigation methods.

### 1.4 Hypothesis to be tested

From our primary study in a the magnetic recording system, we found that TMR effect is one of the most significant problems in the extremely high-density magnetic recording system such as BPMR system. The performance in BPMR system is severely decreased from TMR effect. Therefore, this research hypothesis is to find the

relationship between the readback signals and TMR levels in BPMR system, which can be used for estimating a TMR levels. Then, a pair of 2D target and its suitable corresponding 2D equalizer for each estimated TMR level will be selected to correct the TMR effect in the data detection process. Finally, the proposed method can yield a good performance if compared to the conventional recording system.

## 1.5 Conceptual framework

In this work, we focus on the discrete BPMR channel model and begin with studying the relationship among the statistical information of the readback signals, signal-to noise ratios (SNRs), and various TMR levels. Hence, these relationships will be employed to formulate the mathematical equations for estimating the SNR and the TMR level. We firstly estimate the SNR using the peak amplitude of the readback signal from one data sector. Then, this estimated SNR level will be utilized to estimate the TMR level according to the readback signal energy [14,15]. It should be pointed out that this estimation method can be adapted in designing a 2D equalizer and a 2D detector in BPMR systems so as to cope with this TMR problem [16,17]. Next, to mitigate the TMR effect from a pair of 2D target and its suitable corresponding 2D equalizer for the estimated TMR will be chose to correct the TMR in the data detection process. The target and its corresponding equalizer used in this work are priority designed for each TMR level based on minimizing a mean-square error (MSE) [18,19]. Finally, Viterbi detector used to estimate the recorded bits in the system. The simulation results are compared between the proposed method and the conventional method in term of BER performance [20]. These methods are significances to improve performance with TMR effects. Finally, an Iterative TMR mitigation method based on readback signal for BPMR and utilization of multiple readheads for TMR prediction and correction in BPMR are also proposed in this research.

## 1.6 Scope or limitation of the study

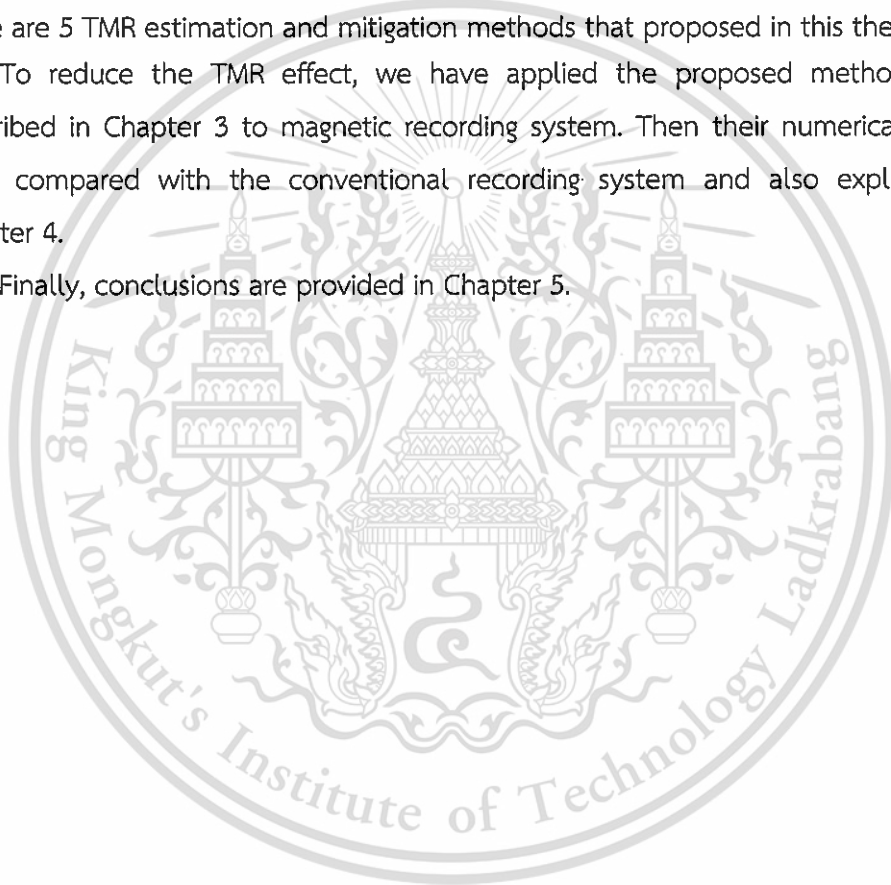
The scope of this research is to study the effect of TMR levels in BPMR system and proposed TMR estimation and mitigation methods based on readback signal. Finally, the simulation results are tested by MATLAB programing and the performances are shown in bit error rate (BER) for comparison of the methods. This thesis separate to 5 Chapters from this detail.

In Chapter 2, we provide an overview of the backgrounds on BPMR discrete channel model systems, the TMR effect in BPMR system, target and its corresponding equalizer are designed for each TMR level based on minimizing a mean-square error (MSE), at the end of this Chapter, we discuss the TMR detection base on readback signal in BPMR.

In Chapter 3, we proposed TMR mitigation method based on readback signal in BPMR system. This work proposes the novel TMR estimation and mitigation methods, which is based only on the readback signal. To model TMR, we use the 2D BPMR pulse response embed TMR effected which we explained in Chapter 2. Moreover, there are 5 TMR estimation and mitigation methods that proposed in this thesis.

To reduce the TMR effect, we have applied the proposed method which described in Chapter 3 to magnetic recording system. Then their numerical results were compared with the conventional recording system and also explained in Chapter 4.

Finally, conclusions are provided in Chapter 5.



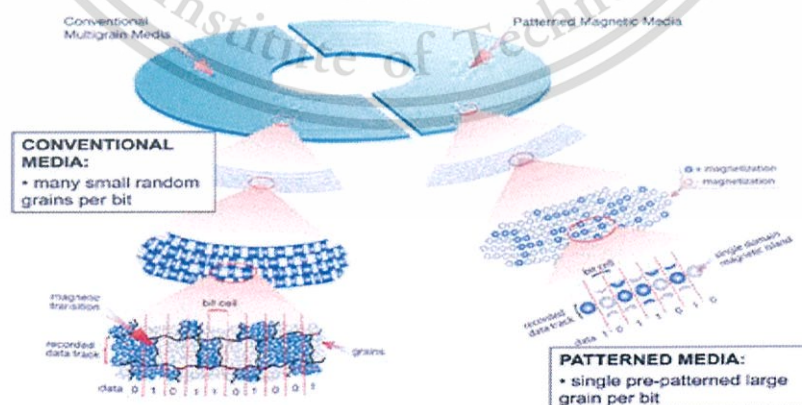
## Chapter 2

# Theories and Principles

In Chapter 2, this thesis would like to describe the overall system in BPMPR system. This Chapter considers a discrete channel model system which corrupted by additive white Gaussian noise (AWGN), two-dimensional (2D) interference, it is consisting of inter-symbol interference (ISI) and inter-track interference (ITI), media noise, and track mis-registration (TMR) effects. At the end of this Chapter, it also explain the target and its corresponding equalizer designed based on minimizing a mean-square error (MSE).

### 2.1 Bit patterned media recording system

While the data storage technology in hard disk drive has been expanding to rapidly growth, the aerial density (AD) in the magnetic recording system is one of the goal for developments. The perpendicular magnetic recording (PMR) systems are expected to reach storage density limits in the near future [1,2]. The high AD can be taken by reducing size of media's grain so, while the grain is smaller over the limited, the PMR system will not stable, this event is occurred from the thermal power of the outside to change the magnetization direction. Therefore, bit "0" change to "1" or inverts direction so, this is a limited effected in the high AD of PMR systems.



**Figure 2.1** Comparison of a simple media model between conventional media and patterned media [21].

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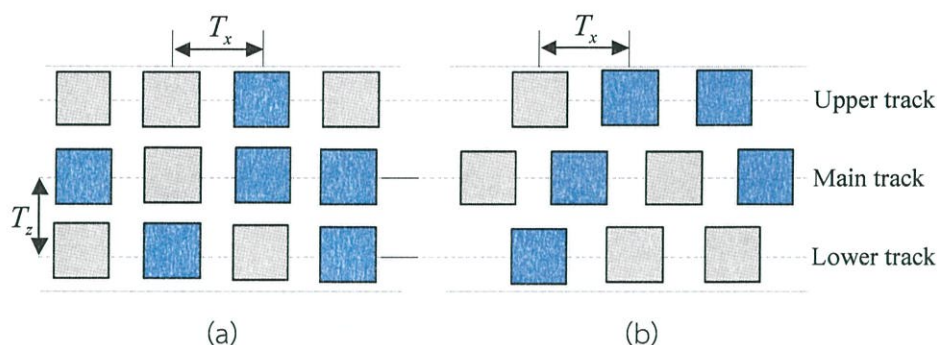


Figure 2.2 Grid of magnetic islands: (a) square (b) hexagonal.

Thus, BPMP system is a promising technology that can circumvent the super paramagnetic limit and offer higher density. In BPMP system, bits are stored in single domain magnetic islands and the regions between bits or magnetic islands are made of non-magnetic material. A schematic of BPMP based on the spinning disk is shown in Figure 2.1. Since isolated nano-scale magnetic islands, or nano-scale magnetic islands surrounded by non-magnetic material, are single domain or act as single grain magnetic islands, using many small magnetic grains per bit is not an issue any more. At the extreme, a magnetic island can be made of a single grain. Therefore, the use of single domain magnetic islands not only handles the super-paramagnetic limit, but also reduces or eliminates transition noise. Moreover, the locations of the islands are predetermined by patterning the media, which eliminates nonlinear bit shift and gives us some advantages in recovering the data. In addition in BPMP systems, by patterning the media, novel patterns for servo systems can be implemented which simplify tracking [14, 15] as well.

Typically, the BPMP system is divided into two categories, spinning disk system and probe based system [22]. In this work, it consider only signal processing system in spinning disk system of BPMP system while the behavior of recording system in square BPMP is similar to data commonly used (such perpendicular and longitudinal). Figure 2.2 demonstrates between square and hexagonal grid of magnetic islands while the performance of systems depend on these arrangements. The track pitch's adjacent spaced of hexagonal grid pattern is more than track pitch of the rectangular grid so, the performance of hexagonal grid pattern is better than performance of rectangular grid pattern [23]. However, this thesis focuses only the square of BPMP because it easy

to understand the behavior of ISI, ITI, media noise and TMR effects. The 2D pulse response in BPMR will be explained in section 2.2 and 2.3.

## 2.2 2D pulse response modeling in BPMR system

In this research, it assume perpendicular recording magnetic media and magneto-resistive (MR) readheads. With MR readheads, readback voltage is proportional to the signal flux injected into the MR element at the air bearing surface (ABS) and can be expressed as below [24]

$$V_{MR}(x, y) = C\phi(x, z), \quad (2.1)$$

where  $C$  is a constant,  $V_{MR}$  is the readback voltage,  $\phi$  is the signal flux,  $x$  corresponds to the along-track direction and  $z$  corresponds to the cross-track direction. Geometry of an MR readhead and a magnetic island is shown in Figure 2.3. Where,  $a$  is the length of a square island,  $\delta$  is the height of an island,  $d$  is the fly height,  $g$  is the length of the gap MR,  $t$  is the thickness of the MR element and  $W$  is the width of the MR element. Using reciprocity principle, the signal flux into the MR element at the air-bearing surface (ABS) can be expressed as follows [24]

$$\phi_{(x,y)} = \mu_0 \int_{-\alpha}^{\alpha} \int_d^{d+\delta} \int_{-\alpha}^{\alpha} \frac{H_y(x', y', z')}{i} M_y(x'-x, y', z'-z) dx' dy' dz', \quad (2.2)$$

when  $\mu_0$  is permeability of free space,  $i$  is imaginary coil,  $H_y$  is the MR readhead field generated by the imaginary coil,  $M_y(x, y, z)$  is media magnetization,  $y$  is the direction perpendicular to the recording medium. So, the magnetization in perpendicular and the integration of the response can be shaped in form of magnetic field potential ( $\psi$ ) as

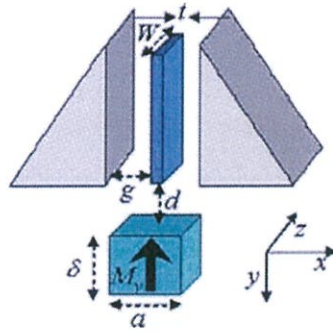


Figure 2.3 Geometry of an MR readhead and a magnetic island [24].

$$\phi_{(x,y)} = \frac{\mu_0}{i} \int_{-\alpha-d}^{\alpha} \int_{-d}^{d+\delta} \int_{-\alpha}^{\alpha} \frac{\partial \psi(x', y', z')}{\partial y'} M_y(x'-x, y', z'-z) dx' dy' dz', \quad (2.3)$$

or

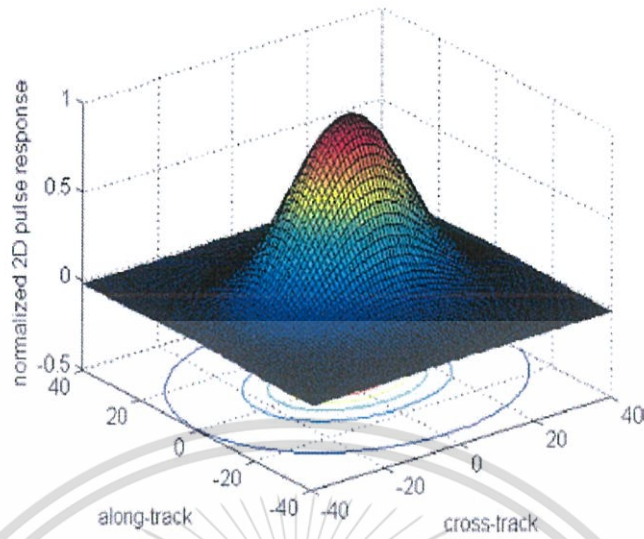
$$\phi_{(x,y)} = \frac{\mu_0}{i} \int_{-\alpha}^{\alpha} \int_{-d}^{d+\delta} \int_{-\alpha}^{\alpha} \psi(x', y', z') \left[ \frac{\partial M_y(x'-x, y', z')}{\partial y'} \right] dx' dy' dz'. \quad (2.4)$$

Assuming that there is no space between the soft under layer (SUL) and the media and the perpendicular magnetization is uniform along the thickness of an island, the pulse response can be expressed as follows

$$V(x, z) = C \int_{x-\alpha/2}^{x+\alpha/2} \int_{z-\alpha/2}^{z+\alpha/2} M(x'-x, y', z'-z) \{ \psi(x', d, z') - \psi(x', d+2\delta, z') \} dx' dz' \quad (2.5)$$

$$M = (x, y, z) = \begin{cases} M(\text{media-magnetization}), & x, z \in \text{island} \\ 0, & \text{else} \end{cases}$$

and with this method, the 2D pulse response for any geometry of a MR readhead and an island can be obtained.



**Figure 2.4** Numerical 2D pulse response for an island of length of 11 nm, height of 10 nm and fly height of 10 nm [24].

In the Figure 2.4 shows the simulated numerical 2D pulse response of a square island of length 11 nm ( $a = 11$  nm), thickness of 10 nm ( $\delta = 10$  nm), and fly height of 10 nm ( $d = 10$  nm).

### 2.3 2D Gaussian pulse response in BPMR system

In 2008, S. Nabavi proposed the responding of BPMR pulse in 2D pulse responding discrete model [25]. This model is estimated from calculation of 2D pulse responding of BPMR and readback signal was occurred from linear superposition. In other words, the readback signal from the readhead includes 2D interference, it can disturb and reduce the performance of BPMR system [11]. The 2D Gaussian pulse response can be explained from the following equation with a track pitch,  $T_z$  and a bit period,  $T_x$  to define as

$$P(nT_z, mT_x). \quad (2.6)$$

Furthermore, it consider the 2D Gaussian pulse response of the form [26] to define as

$$P(z, x) = A \exp \left\{ -\frac{1}{2c^2} \left[ \left( \frac{x}{PW_x} \right)^2 + \left( \frac{z}{PW_z} \right)^2 \right] \right\}, \quad (2.7)$$

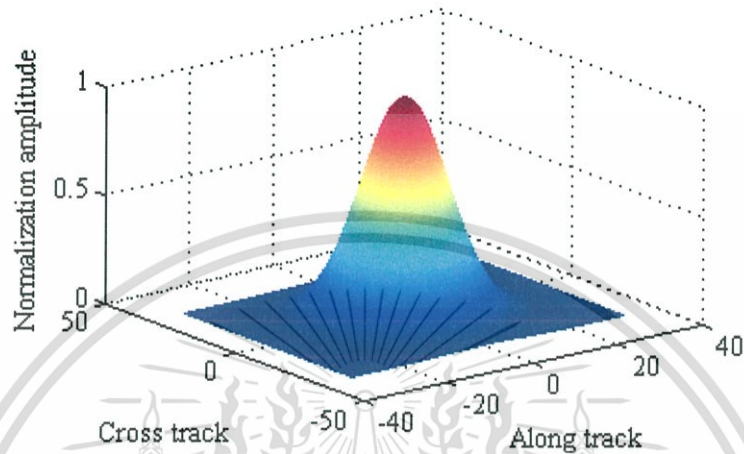


Figure 2.5 2D Gaussian pulse response from equation (2.7) with  $PW_x = 19.4$  nm and  $PW_z = 24.8$  nm.

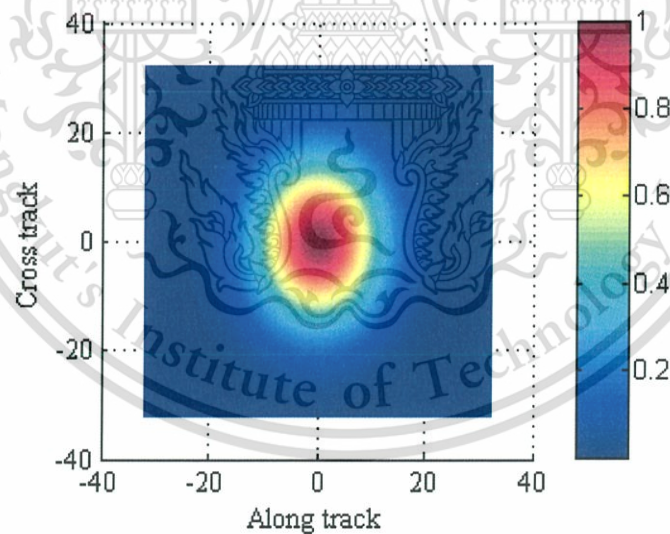


Figure 2.6 The contouring of 2D Gaussian pulse response.

where  $A = 1$  is assumed to be the peak amplitude of the pulse response,  $PW_x$  is the  $PW_{50}$  of the along-track pulse,  $PW_z$  is the  $PW_{50}$  of the across-track pulse,  $PW_{50}$  is the pulse width at half its maximum,  $c = 1/2.3548$  is a constant to account for the relationship between  $PW_{50}$  and the standard deviation of a Gaussian. Figure 2.5 and 2.6

show the 2D Gaussians pulse response in the tow view and the contouring view, respectively, which calculate from equation (2.7).

## 2.4 2D interference in BPMR system

In BPMR, the 2D interference, ISI and ITI become a major impairment due to the small track pitch. The 2D interference is occurred from the side track in along and cross track direction. So, pulse respond of upper, center, and lower tracks are disturbed or interference with adjacent tracks. The ITI is one of the biggest challenges in all high density magnetic recording systems [3,26], which can degrade the performance of the BPMR system. Therefore, to have a realistic BPMR system, ITI needs to be considered in the modeling of the channels. Also, to improve the performance of the BPMR channels, ITI needs to be considered when designing equalizers and detectors. Therefore, 2D interference is decreased the performance in BPMR especially ITI is the high effect more than ISI as Figure 2.7 and Figure 2.8.

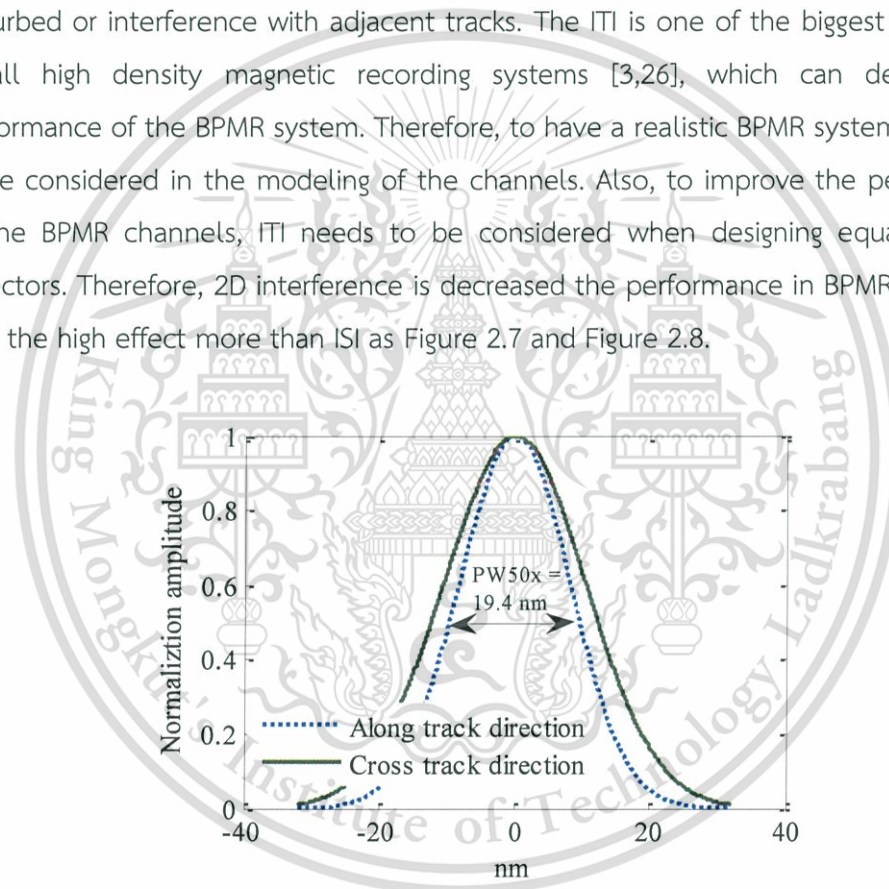


Figure 2.7 Comparison between the values of  $PW_{50}$  of Gaussians pulse response in along track direction.

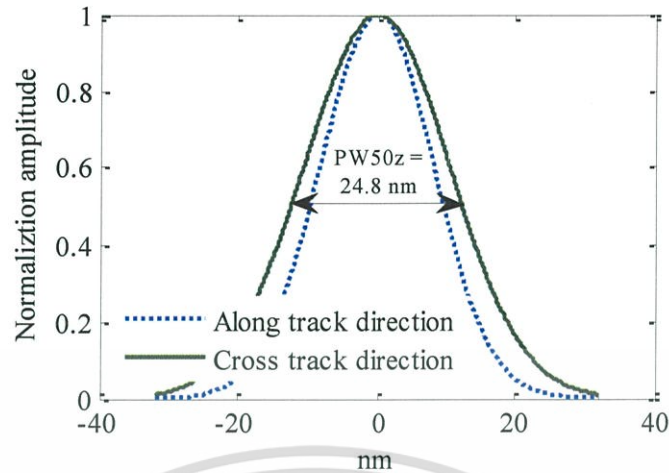


Figure 2.8 Comparison between values of  $PW_{50}$  of Gaussians pulse response in cross track direction.

The 2D pulse responding was coverage in cross track direction more than along track direction as Figure 2.7 and the sampling from 2D pulse responding show that value of cross track more than a long track also. It is confirm ITI is severe than ISI. In this pulse responding sampling where  $PW_x = 19.4$  nm and  $PW_z = 24.8$  nm of BPMR channel model from equation (2.7) can offer the coefficient of BPMR in along direction and cross track direction of multi areal density ( $2.0$  Tb/in<sup>2</sup>,  $2.5$  Tb/in<sup>2</sup> and  $3.0$  Tb/in<sup>2</sup>) as Table 2.1.

Table 2.1 The coefficients of BPMR channel of several AD.

AD (Tb/in <sup>2</sup> )	$T_x = T_z$ (nm)	2D channel coefficients		
2 Tb/in <sup>2</sup>	18 nm	0.0213	0.2321	0.0213
		0.0919	1.0000	0.0919
		0.0213	0.2321	0.0213
2.5 Tb/in <sup>2</sup>	16 nm	0.0478	0.3154	0.0478
		0.1517	1.0000	0.1517
		0.0478	0.3154	0.0478
3 Tb/in <sup>2</sup>	14.5 nm	0.0824	0.3876	0.0824
		0.2125	1.0000	0.2125
		0.824	0.3876	0.0824

## 2.5 Media noise

Media noise in BPMR is different from that in the conventional magnetic recording systems. In conventional magnetic recording systems, the dominant media noise is transition noise; however, because of the nano-scale single domain magnetic islands, transition noise is not an issue in BPMR systems. In BPMR, media noise is mainly due to fabrication imperfections [27,28]. Since such fabrication imperfections are slowly-varying, the media noise in adjacent islands might be correlated. Media noise changes the shape of the pulse response and the track profile, and therefore can degrade the performance of the channel. BPMR channels need to be investigated with this new form of media noise.

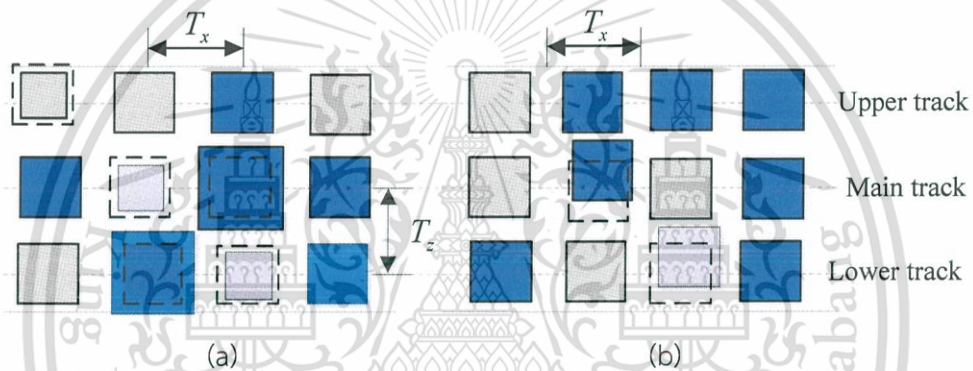


Figure 2.9 Fluctuation characteristic: (a) size fluctuation (b) position fluctuation.

The causes of media noise occur from many sources [25] such as location fluctuation of each island due to island should have track pitch or bit period less than 25 nm. Therefore, it is difficult to define all islands have similarly shape and size. Size fluctuation, height fluctuation, shape fluctuation, and saturation magnetization. Using the 2D pulse response expression as

$$P(z, x) = (A + \Delta_A) \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x + \Delta_x}{c(W_x + \Delta_{W_x})} \right)^2 + \left( \frac{z + \Delta_z}{c(W_z + \Delta_{W_z})} \right)^2 \right] \right\}, \quad (2.8)$$

where, any  $(z, x)$ , the location fluctuations can be modeled by introducing randomness to the  $z, x$  in the expression of the 2D Gaussian pulse response. At,  $\Delta_A$  is variations in the island size,  $A$  is the change the amplitude,  $W_x$  is the along-track  $PW_{50}$ , and  $W_z$  is the cross-track  $PW_{50}$  of the pulse response. As can be seen,  $A, W_x$  and  $W_z$  are almost linear

functions of the island size; thus, size fluctuations can be modeled by amplitude, along-track  $PW_{50}$ , and cross-track  $PW_{50}$  fluctuations. Other sources of media noise such as magnetic islands height and magnetization fluctuations can also be modeled by these fluctuations. Therefore the 2D Gaussian pulse response including the media noise effect can be expressed as equation 2.8. When the media noise occur in the system, the performance is high decreased. Thus, the receiver will be manage with this problem [25]. The media noise can be modeled and show the fluctuation characteristic on size fluctuation and position fluctuation show in Figure 2.9 (a) and Figure 2.9 (b), respectively.

## 2.6 Track mis-registration (TMR)

Track mis-registration (TMR) is the critical issue of BPMR system, which occurs when the center of read head is not aligned with that of the main track [6] as Figure 2.10 show that appropriate position of readhead to read on the media, where  $\Delta_r$  is offset of readhead or the distance between center of the head to the middle of the track, Iceland i.e. the center of readhead should be at center of main track and moving parallel follow a long track for achieving the best quality of readback signal. However, in the practice, the readhead have event to move out of the appropriate position by the readhead maybe move to upward offset or downward offset as Figure 2.11. So, in Figure 2.11 call this event as track mis-registration or readhead offset  $\Delta_r$ .

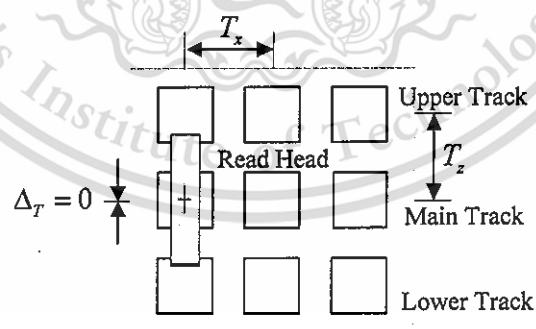


Figure 2.10 The MR position without TMR.

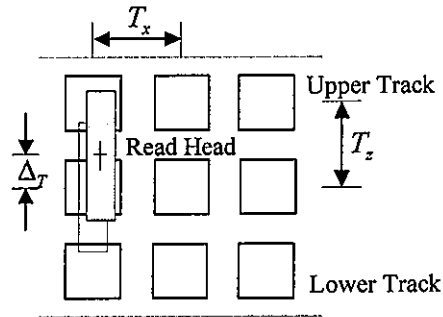


Figure 2.11 The MR position with TMR.

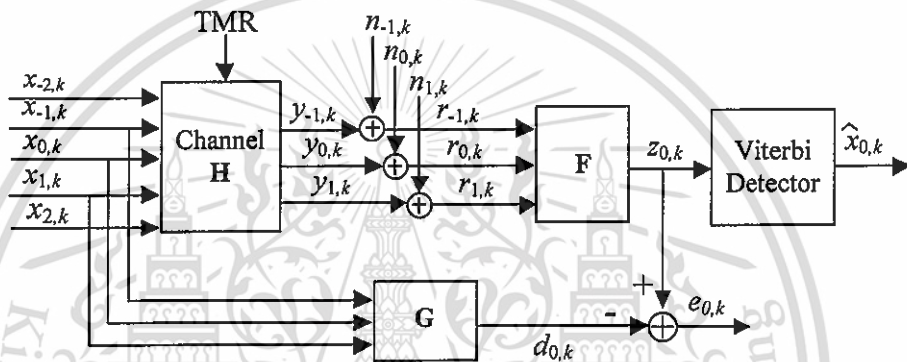


Figure 2.12 Block diagram of bit patterned media recording system.

To consider a discrete BPMR system, which disturbed with TMR effect as shown in Figure 2.12. A binary input data sequence  $x_{l,k} \in \{\pm 1\}$  with bit period  $T_x$ , where  $l = 0$  is the main track, and  $l = -1$  is the upper track, and  $l = 1$  is the lower track, is sent to the BPMR channel corrupted by TMR and electronics noise modeled as an additive white Gaussian noise (AWGN). Then, the readback signal of the  $k^{\text{th}}$  data bit on the main track can be expressed as

$$\begin{aligned} r_{l,k} &= x_{l,k} \otimes h_{l,k} + w_{l,k} \\ &= \sum_n \sum_m h_{-m,n} x_{-m,k-n} + n_{0,k}, \end{aligned} \quad (2.9)$$

where  $x_{0,k}$ 's are the recorded bits,  $h_{m,n}$ 's are the 2D channel coefficients,  $m$  and  $n$  represent the time indices of the bit island in the across- and the along-track directions, and  $n_{0,k}$  is an AWGN with zero mean and variance  $\sigma^2$ . Practically, the BPMR channel coefficients with TMR effected  $h_{m,n}$  can be generated by sampling a 2D Gaussian pulse response at the integer multiples of the bit period  $T_x$  and the track

period  $T_z$  according to

$$h_{m,n} = P(nT_z + \Delta_T, mT_x), \quad (2.10)$$

where  $P(z,x)$  is the 2D Gaussian pulse response,  $z$  and  $x$  are the time indices in the across- and the along-track directions,  $\{m,n\} \in (-L, \dots, 0, \dots, L)$ ,  $2L+1$  is the length of  $P(z,x)$ ,  $L$  is an integer, and  $\Delta_T$  is the head offset or the distance between the center of the readhead and that of the main track as depicted in Figure 2.11. Generally,  $L$  should be large enough to ensure that the tail amplitude of  $P(z,x)$  is enough small, where this paper considers  $L = 1$  for simplicity. In this paper, the TMR level is defined as

$$\text{TMR (\%)} = \frac{\Delta_T}{T_z} \times 100, \quad (2.11)$$

where the sign of  $\Delta_T$  is assumed to be positive for the upward offset as shown in Figure 2.11. Furthermore, it consider the 2D Gaussian pulse response of the form [25]

$$P(z,x) = A \exp \left\{ -\frac{1}{2c^2} \left[ \left( \frac{x}{PW_x} \right)^2 + \left( \frac{z + \Delta_T}{PW_z} \right)^2 \right] \right\}, \quad (2.12)$$

where  $A = 1$  is assumed to be the peak amplitude of the pulse response,  $PW_x$  is the  $PW_{50}$  of the along-track pulse,  $PW_z$  is the  $PW_{50}$  of the across-track pulse,  $PW_{50}$  is the pulse width at half its maximum,  $c = 1/2.3548$  is a constant to account for the relationship between  $PW_{50}$  and the standard deviation of a Gaussian pulse.

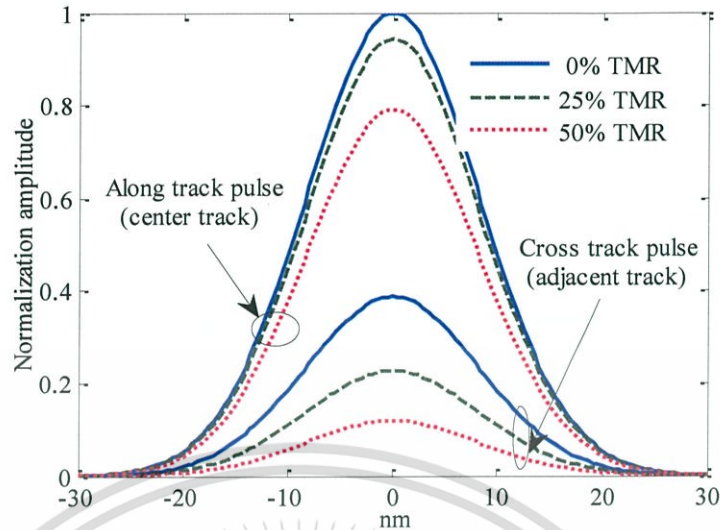


Figure 2.13 2D pulse response in along track and cross track with TMR.

In the simulation system under the TMR condition, it consider the  $AD = 3.0 \text{ Tb/in}^2$ , which correspond with  $T_x = T_z = 14.5 \text{ nm}$ ,  $PW_x = 19.8$  and  $PW = 24.8$ . It is seen that when the system is affected by TMR at each level of the readback signal will deviate from the real signal. The peak amplitude is reduced when the system is affected by a higher TMR levels as Figure 2.13. The system performance of BPMR by inclusion TMR affected with each levels show TMR is the major effect to decrease the performance of system, where x axis is signal to noise ratios (SNR) in decibel (dB) and y axis is BER. SNR can be defined from

$$20 \log_{10}(V_p / \sigma), \quad (2.13)$$

Where  $V = 1$ , is the peak amplitude and  $\sigma$  is standard deviation of AWGN as described in BPMR channel model. Assume that if the readback signal is affected with 2 levels (e.g., low and high levels) of AWGN noise severity. The readback signal which affected with the lower AWGN noise (high SNR) yields the lower peak amplitude as show in Figure 2.14 (b), while the read back signal which affected with high AWGN noise (low SNR) yields the higher peak amplitude as show in Figure 2.14 (a). The peak amplitude of the readback signals at SNR = 10 dB and 25 dB are Figure 2.14 (a), and 2.14 (b), respectively. It confirm as, while the SNR is low the peak amplitudes are high. On the other hand, It show as, while the SNR is high the peak amplitudes are low. These case show as the AWGN noise is depend on SNR levels, so AWGN noise is another one noise

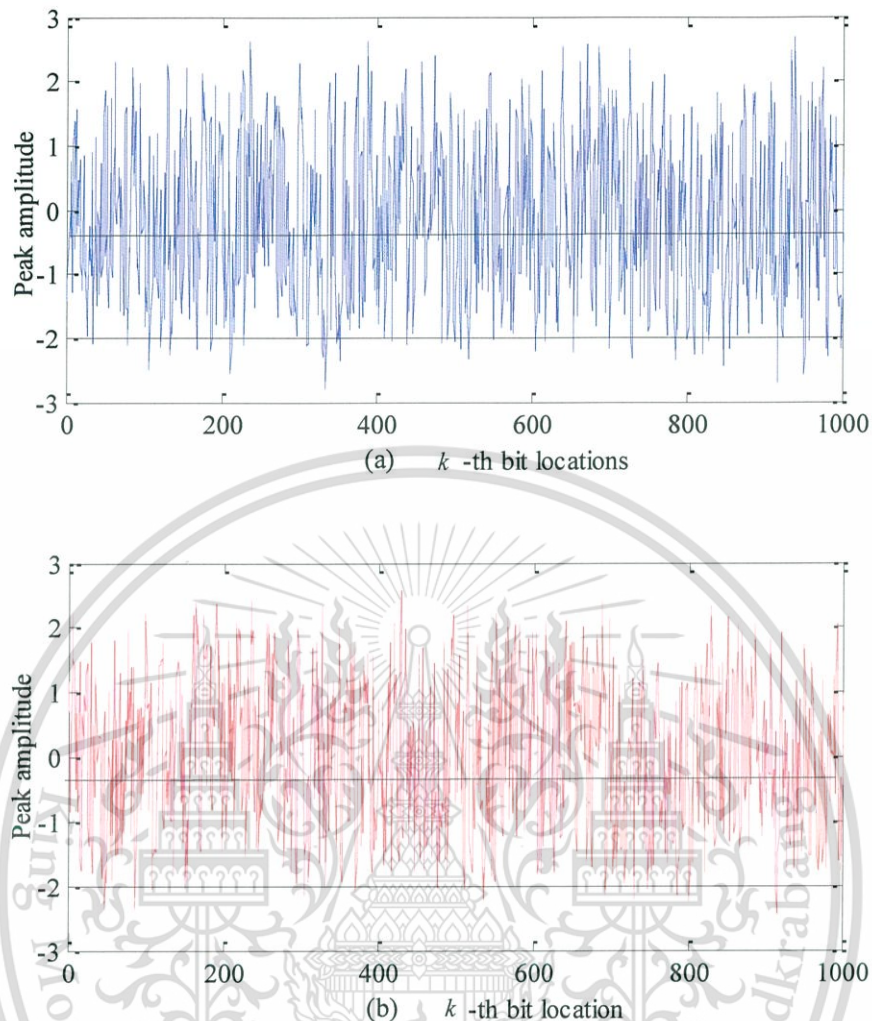
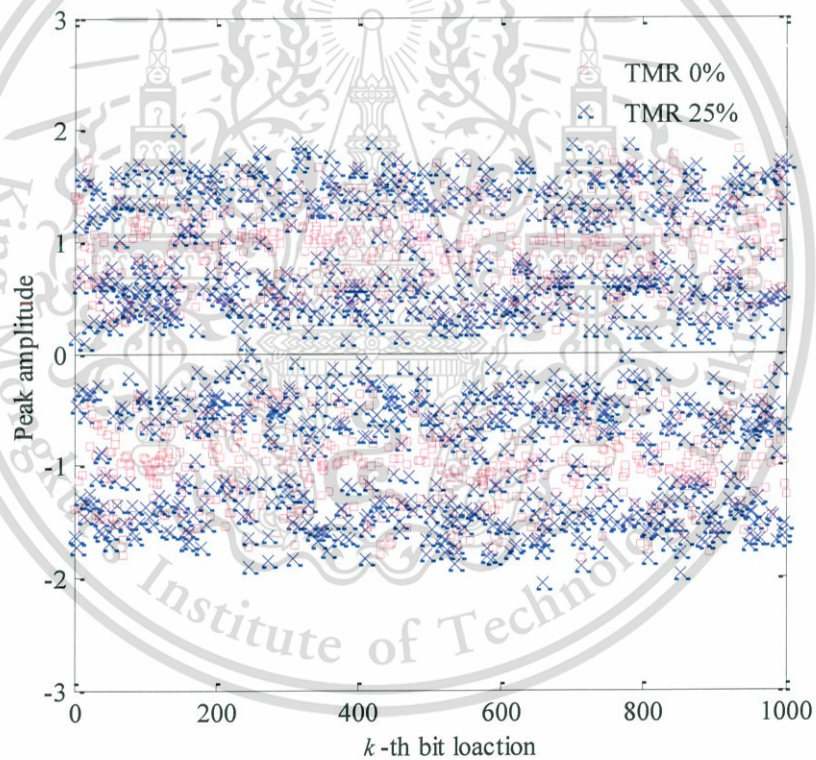


Figure 2.14 The example for the peak amplitude of the readback signal at  
(a) SNR = 10 dB and (b) SNR = 25 dB.

to decrease the performance in the system. Therefore, BER performance is compared by SNR levels. Normally, in the BPMR system without TMR effect. The coefficient from sampling of 2D Gaussian pulse response is symmetric channel. On the other hand, in the BPMR system under TMR effect. The coefficients of BPMR channel is asymmetric channel as show in table 2.2. When the TMR occurs in the system, this case observe that the peak amplitude of the readback signal can either be increased or decreased as show in Figure 2.15, the red boxes are the peak amplitude of readback signal without TMR (TMR = 0%), while the blue crosses are the peak amplitude of readback signal with TMR (TMR = 25%), The peak amplitude will be increased when the data bit of the adjacent track is similar.

**Table 2.2** The coefficients of BPMR with each TMR levels.

TMR (%)	2D channel coefficients			TMR (%)	2D channel coefficients		
0%	0.0824	<b>0.3876</b>	0.0824	15%	0.1094	<b>0.5151</b>	0.1094
	0.2125	1.0000	0.2125		0.2125	1.0000	0.2125
	0.0824	<b>0.3876</b>	0.0824		0.0620	<b>0.2917</b>	0.0620
5%	0.0905	<b>0.4261</b>	0.0905	20%	0.1203	<b>0.5663</b>	0.1203
	0.2125	1.0000	0.2125		0.2125	1.0000	0.2125
	0.0749	<b>0.3525</b>	0.0749		0.0564	<b>0.2653</b>	0.0564
10%	0.0996	<b>0.4685</b>	0.0996	25%	0.1323	<b>0.6226</b>	0.1323
	0.2125	1.0000	0.2125		0.2125	1.0000	0.2125
	0.0681	<b>0.3207</b>	0.0681		0.513	<b>0.2413</b>	0.0513

**Figure 2.15** The example for the peak amplitude of the readback signal in the case of the system under the absence/presence of TMR at SNR = 20 dB.

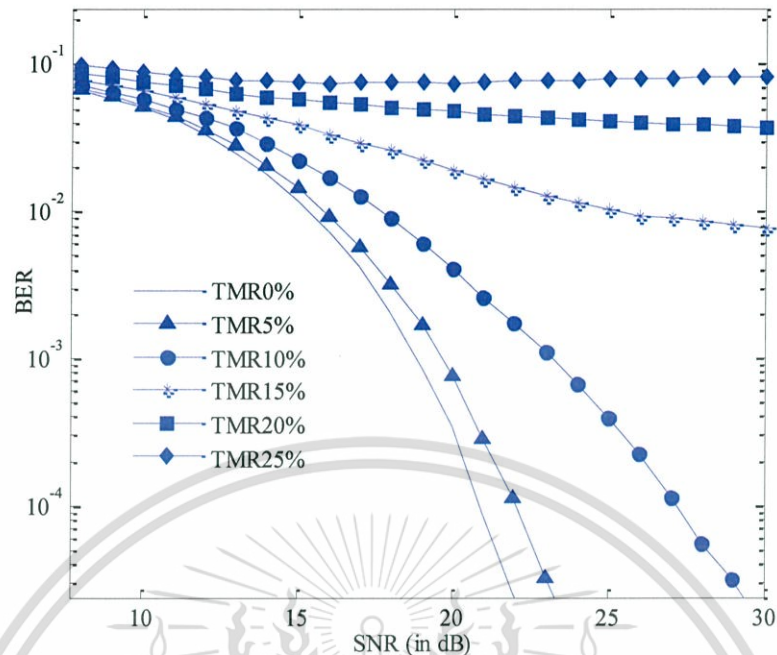


Figure 2.16 The BER performance of BPMR system with several TMR.

with the data bit of the main track, while the peak amplitude will be decreased if those data bits are difference. Assume that the data of main track is similar to the data of upper track; the peak amplitude becomes higher and higher according to the increasing amount of TMR. It is possible that the peak amplitude will be decreased in case the data bit are difference.

Figure 2.16 show the BER performance of the BPMR system, which effects from TMR effect for each levels. Here, the conventional 2D equalizer and 2D Viterbi detector were used for this simulation. The both of them are designed based on a monic constraint [26], where the 3x7-tap equalizer and the 3x3-tap target are designed at the SNR required to achieve BER of  $10^{-4}$  and compares the performance of different schemes at TMR level of 0%, 5%, 10%, 15%, 20%, and 25%. It is clear that the performance of the high TMR effected is unacceptable if compared to the TMR with low effected. It is because the TMR level are very severe in the BPMR system, especially at high ADs. Therefore, the conventional 2D equalizer is not appropriate system for processing in ultra-high BPMR system.

## 2.7 Target and equalizer design

The conventional block diagram of a BPMR system under TMR effects in this work is shown as Figure 2.12. This thesis consider that the readhead which detects the signals from the current bit island and the adjacent islands on its track as well as on the two adjacent tracks; hence the discrete BPM channel from (2.12) is represented by 3x3 channel response matrix, i.e.,

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{-1} \\ \mathbf{h}_0 \\ \mathbf{h}_1 \end{bmatrix} = \begin{bmatrix} h_{-1,-1} & h_{-1,0} & h_{-1,1} \\ h_{0,-1} & h_{0,0} & h_{0,1} \\ h_{1,-1} & h_{1,0} & h_{1,1} \end{bmatrix}. \quad (2.14)$$

The target and its corresponding equalizer used in this work are designed for each TMR level based on minimizing a mean-squared error (MSE) [19] according to

$$E\{\varepsilon_{i,k}^2\} = E\{(z_{i,k} - d_{i,k})^2\}, \quad (2.15)$$

where  $E\{\cdot\}$  is the expectation operator,  $\varepsilon_{i,k}$  is the error signal between the equalizer output  $z_{i,k}$  and the desired signal  $d_{i,k}$ . After expanding the right hand side in (2.15), it becomes

$$\begin{aligned} E\{\varepsilon_{i,k}^2\} &= E\{[(r_{i,k} \otimes f_{i,k}) - (x_{i,k} \otimes g_{i,k})]^2\} \\ &= f_{i,k} \otimes R_{i,k}^r \otimes f_{i,k} - 2f_{i,k} \otimes R_{i,k}^{r,x} \otimes g_{i,k} + g_{i,k} \otimes R_{i,k}^x \otimes g_{i,k}, \end{aligned} \quad (2.16)$$

where  $\otimes$  is the 2D convolution operator,  $R_{i,k}^r = E\{r_{i,j}r_{i-l,j-k}\}$  and  $R_{i,k}^x = E\{x_{i,j}x_{i-l,j-k}\}$  are the auto-correlations of the readback signals and the recorded bits from all three tracks, respectively, and  $R_{i,k}^{r,x} = E\{r_{i,j}x_{i-l,j-k}\}$  is the cross-correlation between the readback signals and the recorded bits.

To compute the solution of MSE in (2.16), it is convenient to represent the matrices in the vector forms [19]. To do so, this work let F be 3x(2N+1) equalizer matrix of the form

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_{-1} \\ \mathbf{f}_0 \\ \mathbf{f}_1 \end{bmatrix} = \begin{bmatrix} f_{-1,-N} \cdots f_{-1,0} \cdots f_{-1,N} \\ f_{0,-N} \cdots f_{0,0} \cdots f_{0,N} \\ f_{1,-N} \cdots f_{1,0} \cdots f_{1,N} \end{bmatrix}, \quad (2.17)$$

when  $f_{l,k}$ 's are the equalizer coefficients,  $l \in \{0, \pm 1\}$  is the track location,  $k \in \{-N, \dots, 0, \dots, N\}$  and  $2N + 1$  is the equalizer length. Similarly, let  $\mathbf{G}$  be a  $3 \times 3$  target matrix of the form

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_{-1} \\ \mathbf{g}_0 \\ \mathbf{g}_1 \end{bmatrix} = \begin{bmatrix} g_{-1,-1} & g_{-1,0} & g_{-1,1} \\ g_{0,-1} & g_{0,0} & g_{0,1} \\ g_{1,-1} & g_{1,0} & g_{1,1} \end{bmatrix}, \quad (2.18)$$

when  $g_{l,k}$ 's are the target coefficients,  $l \in \{0, \pm 1\}$  is the track location,  $k \in \{-1, 0, 1\}$ . In the general, the matrices  $\mathbf{F}$  and  $\mathbf{G}$  can be rearranged into the column vectors as  $\mathbf{f} = [f_{-1} \ f_0 \ f_1]^T$  and  $\mathbf{g} = [g_{-1} \ g_0 \ g_1]^T$ , respectively, where the component vectors are defined in (2.17) and (2.18), and  $[\cdot]^T$  is a transpose operator. Using those matrices, the MSE in (2.16) can be rewritten as

$$E\{\varepsilon_{l,k}^2\} = \mathbf{f}^T \mathbf{R}_f \mathbf{f} - 2\mathbf{f}^T \mathbf{R}_{rx} \mathbf{g} + \mathbf{g}^T \mathbf{R}_g \mathbf{g}, \quad (2.19)$$

where  $\mathbf{R}_f = [r_k r_k^T]$  is a  $3(2N+1) \times 3(2N+1)$  auto-correlation matrix of  $R_{l,k}^f$ ,  $r_k$  is the readback signal vector,  $\mathbf{R}_{rx} = [r_k x_k^T]$  is a  $3(2N+1) \times 9$  cross-correlation matrix of  $R_{l,k}^x$ ,  $x_k$  is the recorded bits vector, and  $\mathbf{R}_g = [x_k x_k^T]$  is  $9 \times 9$  auto-correlation of  $R_{l,k}^x$ . Because this work focus only on detecting the data on the main track (i.e.,  $l = 0$ ), the MSE in (2.15) can then be computed by

$$E\{\varepsilon_{0,k}^2\} = E\{(z_{0,k} - d_{0,k})^2\}. \quad (2.20)$$

Hence, in this case, the readback signals vector and the recorded bit vector will be given by  $r_k = [r_{1,k+N} \ r_{1,k+N-1} \ \cdots \ r_{0,0} \ \cdots \ r_{-1,k-N+1} \ r_{-1,k-N}]^T$  and  $x_k = [x_{1,k+1} x_{1,k} \ \cdots \ x_{0,0} \ \cdots \ x_{-1,k} x_{-1,k-1}]^T$ , respectively. During the minimization process of the MSE in (2.20), this work impose a constraint of  $\mathbf{e}^T \mathbf{g} = 1$  to avoid reaching trivial solution of  $\mathbf{f} = \mathbf{g} = 0$ , where

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$\mathbf{e} = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]^T$  is a column vector with 9 entries. Accordingly,  $\mathbf{f}$  and  $\mathbf{g}$  are chosen such that

$$E\{\mathbf{e}_{0,k}^2\} = \mathbf{f}^T \mathbf{R}_r \mathbf{f} - 2\mathbf{f}^T \mathbf{R}_{rx} \mathbf{g} + \mathbf{g}^T \mathbf{R}_x \mathbf{g} - 2\lambda^T (\mathbf{e}^T \mathbf{g} - 1), \quad (2.21)$$

is minimized, where  $\lambda$  is a Lagrange multiplier [22,23]. Then, the minimization process gives

$$\lambda = \frac{1}{\mathbf{e}^T (\mathbf{R}_x - \mathbf{R}_{rx}^T \mathbf{R}_r^{-1} \mathbf{R}_{rx})^{-1} \mathbf{e}} \quad (2.22)$$

$$\mathbf{g} = \lambda (\mathbf{R}_x - \mathbf{R}_{rx}^T \mathbf{R}_r^{-1} \mathbf{R}_{rx})^{-1} \mathbf{e} \quad (2.23)$$

$$\mathbf{f} = \mathbf{R}_r^{-1} \mathbf{R}_{rx} \mathbf{g}. \quad (2.24)$$

Note that if the target  $\mathbf{g}$  is given, one can still employ [19] to obtain the equalizer  $\mathbf{f}$  that minimizes the MSE in (2.21). In a conventional receiver that does not employ a TMR mitigation method, the readback signal  $r_{l,k}$  for  $l \in \{0, \pm 1\}$  is fed to a 2D equalizer followed by a 2D Viterbi detector to determine the most likely input sequence on the main track, i.e.,  $\hat{x}_{0,k}$ . Note that this paper does not take media noise into account and considers only the system that recovers the recorded data on the main track, as similar to [29]. Hence, three adjacent readback signals  $\{r_{-1,k}, r_{0,k}, r_{1,k}\}$  at the input of a 2D equalizer,  $\mathbf{F}$ , are required to generate a single output  $\{z_{0,k}\}$ , whereas three input data sequences  $\{x_{-1,k}, x_{0,k}, x_{1,k}\}$  are sent to a 2D target,  $\mathbf{G}$ , to output the desired data sequence  $\{d_{0,k}\}$ .

At the receiver from Figure 2.12, the readback signals  $r_{l,k}$  are equalized by the 2D equalizers to obtain the equalized signals,  $z_{l,k}$ . Next, these data sequences are fed to Viterbi detector to produce the estimated of data tracks. Finally, the system calculate the BER performance are calculate for providing with conventional system performed without the TMR estimation/mitigation schemes.

## 2.8 Summary

In this Chapter, this thesis describes the overall system in BPMR system corrupted with AWGN, ISI, ITI, media noise, this thesis also describes the BPMR channel modeling which corrupted with TMR effect. Then, this channel model will be used for entering the performance of the proposed methods in next Chapters. Moreover, at the end of this Chapter, we explain the 2D equalizer design with MMSE method for mitigation these effects. This thesis also explain how to design the 2D equalizer and 2D target based on the MMSE method, which will be used to mitigate the TMR effect as discussed in next Chapter.



## Chapter 3

### TMR Mitigation Methods

As discussed in Chapter 1, TMR effect is a major obstacle for increasing an areal density in ultra-high density magnetic recording systems such as BPMR system [3,30]. The TMR effect is occurred due to the misalignment between the center of the readhead and that of the main data track as illustrated in Figure 3.1. Because of TMR, the readback signal may experience even more severe inter track interference,

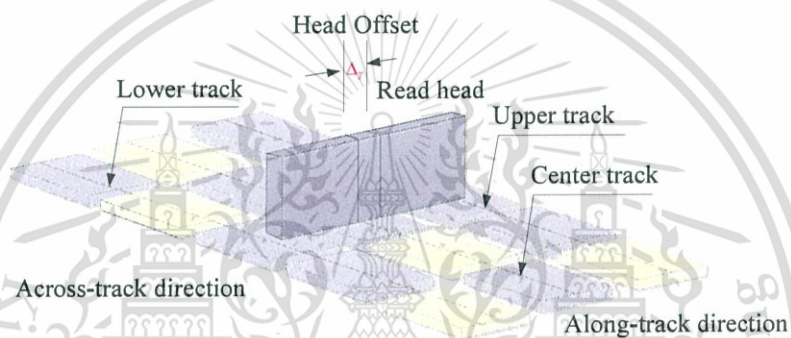


Figure 3.1 Illustration of TMR or head offset,  $\Delta$ , in a BPMR system.

which will further degrade the performance of data recovery process in BPMR systems [4,31]. Thus, this thesis proposes a novel TMR estimation and mitigation methods, which is based only on the readback signal. To model TMR, we use the two-dimensional (2D) BPMR system under TMR effect which explained in Chapter 2. This work, we propose 5 methods for estimation and mitigation TMR effect described as following.

#### 3.1 Simple TMR estimation methods

Track mis-registration (TMR) effect is a major obstacle for increasing an areal density (AD) in ultra-high density magnetic recording systems such as BPMR [3]. To handle the TMR effect, a servo system provides an inherent sector-level latency of the detection of servo bursts before any head adjustment can be made. This servo burst field has the information that can be used to estimate the amount of readhead offset. However, it is generally difficult to predict the TMR quantity to the next servo sector, when TMR is compensated by only burst signals, especially when TMR goes beyond the limit [7].

In practice, several TMR estimation methods based on the readback signal have been proposed in the literature [14,32]. Nonetheless, these methods require the knowledge of some recorded data and employ high complexity processing (e.g., calculating many correlation functions) for estimating TMR levels. Thus, this thesis proposes a novel TMR estimation method, which is based only on the readback signal.

This thesis begins with studying the relationship among the statistical information of the readback signal, signal to noise ratios (SNRs), and various TMR levels. Hence, these relationships will be employed to generate the mathematical equations to use estimating the SNR and the TMR level. In this work, we first estimate the SNR using the peak amplitude of the readback signal from one data sector. Then, this equation will be used for estimating the SNR and TMR levels, respectively.

### 3.1.1 Channel model

This work focuses on a discrete-time BPMR channel model with multi-track processing [9] as depicted in Figure 3.2. The readback signal corrupted with TMR effect can be calculated from (2.12) in Chapter 2. In the overall system of this proposed method as depicted in Figure 3.2, the recorded bits,  $x_{-1,k}$ ,  $x_{0,k}$  and  $x_{1,k}$  from upper, the main, and lower tracks, respectively, are sent to the BPMR channel corrupted by TMR and AWGN. At the read side, the readback signal is used to determine its peak amplitude,  $A_{\text{peak}}$ , and the average energy,  $E_{\text{ener}}$ , simultaneously. Then, the estimated SNR,  $\hat{\text{SNR}}$ , is obtained based on  $A_{\text{peak}}$ . Finally, we compute the estimated TMR,  $\hat{\text{TMR}}$ , using  $E_{\text{ener}}$  and  $\hat{\text{SNR}}$ .

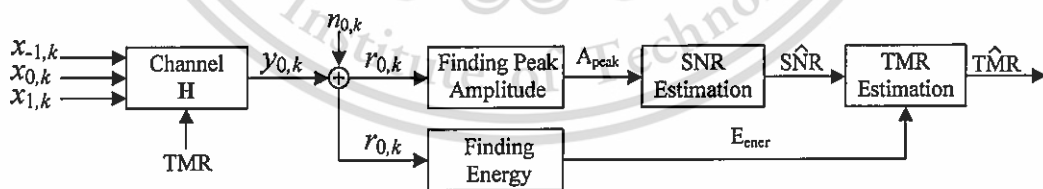
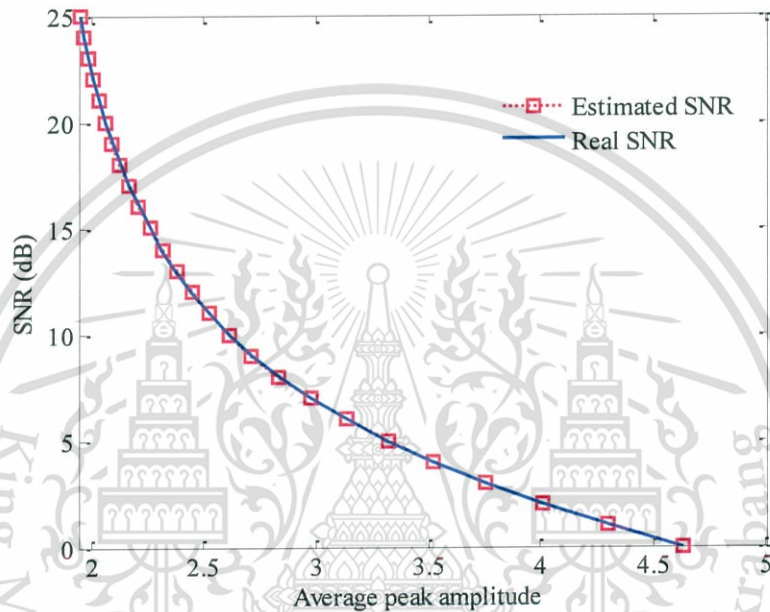


Figure 3.2 Block diagram of BPMR system with proposed SNR and TMR estimations.

### 3.1.2 SNR estimation method

In this work, we collect 100 samples of the readback signals at each SNR level from 0 to 25 decibel (dB), where each readback signal is affected by a random TMR level from 0% to 25%. For each SNR level, we average the peak amplitude of all readback signals. Using these two data sets of the SNRs and the average peak amplitudes, we propose to employ a curve fitting technique to obtain a polynomial



**Figure 3.3** The relationship between the SNR and the averaged peak amplitude of the readback signals at  $AD = 2 \text{ Tb/in}^2$ .

function to approximate the SNR based on the known average peak amplitude. The polynomial function using the curve fitting technique is

$$\hat{SNR} = a_0 + a_1y + a_2y^2 + \dots + a_Ny^N, \quad (3.1)$$

where  $\hat{SNR}$  is the estimated SNR,  $a_i$  is the  $i^{\text{th}}$  coefficient of the polynomial equation in (3.1),  $i \in \{0, 1, \dots, N\}$ . Based on extensive heuristic search, we found that  $N = 5$  is sufficient for our model because a higher order does not provide any performance gain on the accuracy of SNR estimation. In Figure 3.3, we compare the real SNR value and the estimated SNR using (3.1). The x-axis is the average peak amplitude of the readback signals and the y-axis is the SNR levels. The result is similar so that we use the curve fitting for estimated SNR level.

### 3.1.3 TMR estimation method

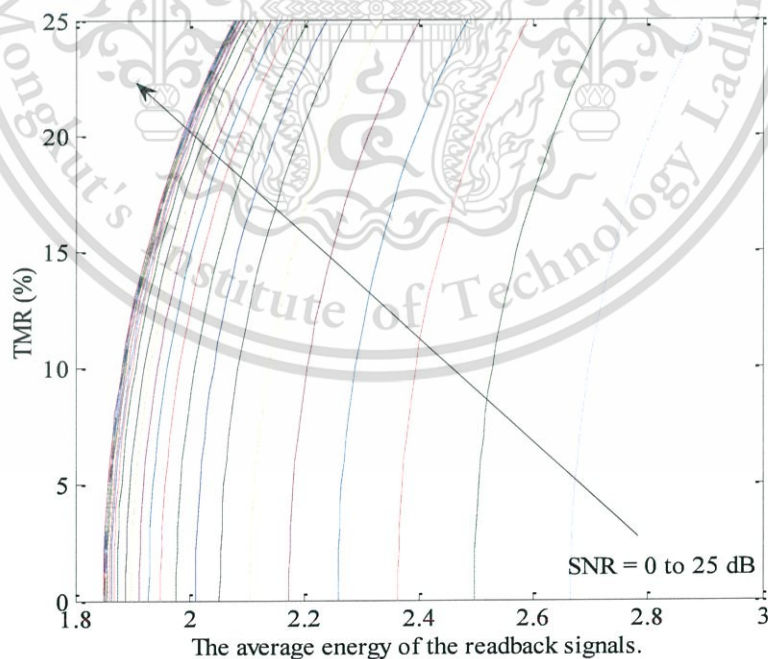
To estimate the TMR level, we compute the energy of the readback signal at SNR = 20 dB for each TMR level (i.e., 0, 5, 10, 15, 20, and 25%) according to

$$E_{\text{ener}} = \frac{1}{M} \sum_{k=1}^M r_{0,k}^2, \quad (3.2)$$

where  $M$  is the length of the readback signal, which is equal to 32768 bits as a 4k data sectors [33]. Next, we also use the curve fitting technique so as to obtain a polynomial function that provides the best fit line for the relation between the TMR and the average energy of all readback signals. This polynomial function can be utilized to approximate the TMR level according to

$$\hat{\text{TMR}} = b_0 + b_1 u + b_2 u^2 + \dots + b_F u^F, \quad (3.3)$$

where  $\hat{\text{TMR}}$  is the estimated TMR,  $b_i$  is  $i^{\text{th}}$  coefficient of the polynomial equation,  $i \in \{0, 1, \dots, F\}$ ,  $F$  is a degree of the polynomial equation (here we use  $F = 5$  for simplicity),  $u$  is the average signal energy, which is equal to  $E_{\text{ener}}$  in Figure 3.4.



**Figure 3.4** The relationship between the TMR level and the averaged energy of the readback signals at AD = 2.0 Tb/in<sup>2</sup>.

Figure 3.4 depicts the relationship between the TMR level and the average energy by plotting the average energy of all readback signals as a function of TMR levels. We repeat this step by varying the SNR level from 0 to 25 dB to obtain each curve in Figure 3.4. The dotted line is the SNR 0 dB and the next lines are the SNR level to increase by 1 dB up to SNR 25 dB. It is apparent that if we know the estimated SNR ( $\hat{SNR}$ ) and  $E_{ener}$ , we can easily compute the estimated  $\hat{TMR}$  from (3.3).

#### 3.1.4 Summary

In this topic, we propose the TMR estimation method by using only the readback signal for a BPMR system corrupted by AWGN and, inter-symbol interference and inter-track interference. Firstly, we estimate the SNR based on the peak amplitude of the readback signal. Then, the estimated TMR level is determined based on the readback signal energy and the estimated SNR level. However, in this method only estimated TMR levels without mitigation performance process. It should be pointed out that this estimation method can be adapted in designing a 2D equalizer and a 2D detector in BPMR systems so as to cope with this TMR problem following as next method.

### 3.2 A TMR mitigation method

Based on the first method for mitigation TMR effect, this topic also proposes a novel method to mitigate the TMR effect based on the readback signal. To do so, we study the statistical relationship among the readback signal, a SNR, and TMR amount for various cases similar to the first method. Next we formulate a mathematical equation so as to estimate the SNR and the TMR level based on the readback signal. Specifically, the SNR is estimated from the average peak readback amplitude, whereas the TMR level is computed from the estimated SNR and the energy of the readback signal. Then, the estimated TMR level will be utilized to choose the target and its corresponding equalizer from a look-up table that are suitable for the channel with TMR so as to facilitate the data detection process. Note that each pair of the target and its equalizer is designed for each TMR level and is stored in the look-up table.

### 3.2.1 Channel model

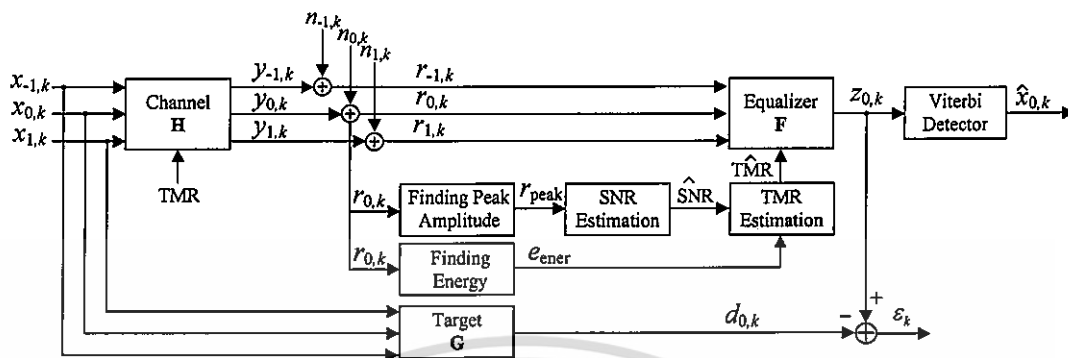


Figure 3.5 A BPMR channel model with the proposed method.

This work focuses on a discrete-time BPMR channel model with multi-track processing [14] as depicted in Figure 3.5. The readback signal corrupted TMR effect can calculate from (2.12) in Chapter 2 similar to previous work. In this overall proposed method show in Figure 3.5, we propose a novel method to subside the TMR effect in a BPMR channel, based on the readback signal. Specifically, we first estimate a TMR level with an aid of the estimated SNR and the average energy of the readback signal. Hence, the target and its corresponding equalizer suitable for the channel with TMR are selected according to the estimated TMR level so as to ease the data recovery process. The details of the proposed method can be explained as follows.

#### 3.2.2 SNR estimation scheme

In this work, the SNR is defined as [29]  $\text{SNR} = 20\log_{10}(V_p/\sigma)$  from (2.13) in Chapter 2. In decibel (dB), where  $V_p$  is the peak amplitude of the readback signal, which is assumed to be 1, and  $\sigma$  is a standard deviation of AWGN. This SNR will be estimated before predicting the amount of TMR. Here, we proposed to estimate the SNR from the peak amplitude of the readback signal. To do so, we collect a large number of sample (e.g., 1000 samples) of readback signal at each SNR ranged from 0 to 25 dB, where each readback signal is affected by a uniformly distributed random TMR level ranged form 0% to 25%. Then, for each SNR, the average value of peak amplitude of all readback signals,  $r_{\text{peak}}$  is computed, regardless of TMR levels. In Figure 3.6 illustrates the relationship between the SNR and the averaged peak amplitude of the readback signals at areal density of 2.0 Tb/in<sup>2</sup>, where we found that the SNR can be possibly estimated. To do so, we apply a least-squares (LS) fitting

technique to fit all data points to an  $M$ -degree polynomial equation according to

$$\hat{\text{SNR}} = a_0 + a_1 r_{\text{peak}} + a_2 r_{\text{peak}}^2 + \dots + a_M r_{\text{peak}}^M, \quad (3.4)$$

where  $\hat{\text{SNR}}$  is the estimated SNR,  $a_i$  is the  $i^{\text{th}}$  coefficient of the polynomial equation in (3.4),  $i \in \{0, 1, \dots, M\}$ . Based on extensive heuristic search, we found that  $M = 5$  is sufficient for our channel model at the AD up to  $3.0 \text{ Tb/in}^2$  because a higher order does not provide any benefit on the accuracy of SNR estimation. As depicted in Figure 3.6, it is apparent that the estimated SNR effectively coincides with the actual SNR.

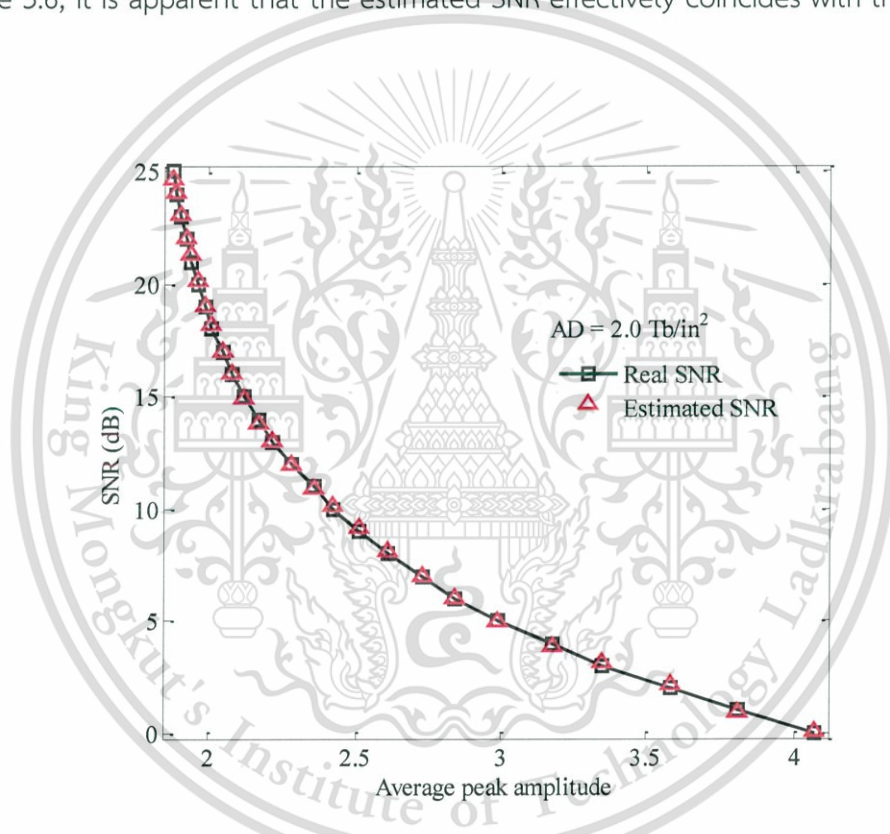


Figure 3.6 The relationship between the SNR and the averaged peak amplitude of the readback signals at  $\text{AD} = 2.0 \text{ Tb/in}^2$ .

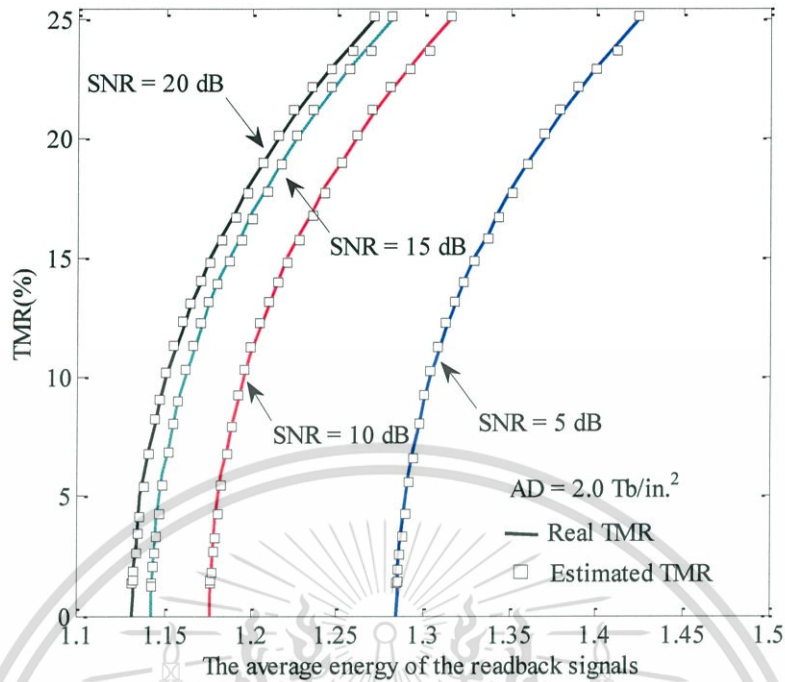


Figure 3.7 The relationship between the TMR level and the averaged energy of the readback signals at  $AD = 2.0 \text{ Tb/in.}^2$ .

### 3.2.3 TMR estimation scheme

For each SNR, we also propose to employ the energy of the readback signal to estimate the TMR level. To do so, we compute the averaged energy of the readback signal,  $e_{\text{ener}}$ , for each SNR and TMR level according to

$$E_r = \frac{1}{S} \sum_{k=1}^S r_{0,k}^2, \quad (3.5)$$

where  $S$  is the length of the readback signal samples (i.e.,  $S = 32768$  bits for a 4K-data sector) [33]. Next, the estimated TMR level is obtained based on a polynomial LS fitting technique, i.e.,

$$\hat{\text{TMR}} = b_0 + b_1 E_r + b_2 E_r^2 + \dots + b_Q E_r^Q, \quad (3.6)$$

where  $\hat{\text{TMR}}$  is the estimated TMR,  $b_i$  and  $Q$  are the  $i^{\text{th}}$  coefficient and a degree of the polynomial equation in (3.6), respectively, and  $i \in \{0, 1, \dots, Q\}$ . Similarly, we perform an extensive simulation search to find a suitable  $Q$ , where  $Q = 5$  provides

the best fit between the actual and the estimated TMR levels. Figure 3.7 shows the estimated TMR level as a function of the averaged energy of the readback signals at 2.0 Tb/in<sup>2</sup> for each SNR. Clearly, the TMR level can be effectively estimated from (3.6) based on  $\hat{S}\hat{N}R$  and  $E_r$ .

### 3.2.4 Equalizer and target design

The target and its corresponding equalizer used in this work are designed for each TMR level based on minimizing a mean square error (MSE) [19] according from (2.15) in Chapter 2. In this proposed method, we use the fixed 3x3 asymmetric general partial response GPR target  $G_{\text{asym}}$  which is similar to the channel matrix, i.e., for each TMR levels (0%, 5%, 10%, 15%, 20%, 25%) for designing the 2-D equalizer 3x7.

$$\begin{aligned}
 G_{\text{TMR } 0\%} &= \begin{bmatrix} 0.0824 & 0.3876 & 0.0824 \\ 0.2125 & 1.0000 & 0.2125 \\ 0.0824 & 0.3876 & 0.0824 \end{bmatrix}, & G_{\text{TMR } 5\%} &= \begin{bmatrix} 0.0905 & 0.4261 & 0.0905 \\ 0.2125 & 1.0000 & 0.2125 \\ 0.0749 & 0.3525 & 0.0749 \end{bmatrix}, \\
 G_{\text{TMR } 10\%} &= \begin{bmatrix} 0.0996 & 0.4685 & 0.0996 \\ 0.2125 & 1.0000 & 0.2125 \\ 0.0681 & 0.3207 & 0.0681 \end{bmatrix}, & G_{\text{TMR } 15\%} &= \begin{bmatrix} 0.1094 & 0.5151 & 0.1094 \\ 0.2125 & 1.0000 & 0.2125 \\ 0.0620 & 0.2917 & 0.0620 \end{bmatrix}, \\
 G_{\text{TMR } 20\%} &= \begin{bmatrix} 0.1203 & 0.5663 & 0.1203 \\ 0.2125 & 1.0000 & 0.2125 \\ 0.0564 & 0.2653 & 0.0564 \end{bmatrix}, & G_{\text{TMR } 25\%} &= \begin{bmatrix} 0.1323 & 0.6226 & 0.1323 \\ 0.2125 & 1.0000 & 0.2125 \\ 0.0513 & 0.2413 & 0.0513 \end{bmatrix},
 \end{aligned}$$

### 3.2.5 TMR mitigation methods

Here, this work proposes two methods to mitigate the TMR effect based on the used 2D targets (i.e., symmetric or asymmetric). Then, the performance of the proposed methods will be compared with that of a conventional receiver, which employs fixed 2D target and equalizer designed for 0% TMR level. Without TMR and media noise, the channel response in (2.10) will normally be symmetric, and we found that the 2D target  $\mathbf{G}$  obtained from this design is also symmetric because the target coefficients  $g_{-1}$  and  $g_1$  in (2.18) are almost equal.

For the first proposed method (denoted as the symmetric system), the symmetric 2D target as used in the conventional receiver is employed, but the equalizer is selected according to the estimated TMR level. To do so, we need to design the equalizers suitable for each TMR level based on (2.24), where the target is fixed, and store them in the look-up table. On the other hand, the second proposed method (denoted as the asymmetric system) utilizes the 2D target and its corresponding equalizer specially designed for each TMR level according to (2.22) - (2.24), where we refer to this 2D target as the asymmetric target because the target coefficients  $g_{-1}$  and  $g_1$  in (2.18) are not equal. Thus, each pair of the target and equalizer associated with a TMR level will be kept in the look-up table. Table 3.1 shows an example of the coefficients of the asymmetric 2D target for some TMR levels at the ADs of 2.0 and 3.0 Tb/in<sup>2</sup>, which are used in this TMR mitigation method.

**Table 3.1** The coefficients of the 2D target and its 2D equalizer for some TMR level.

TMR	2D target coefficients			2D equalizer coefficients		
0%	0.0692	0.3255	0.0692	-0.0107	0.0203	-0.0059
	0.1785	0.8398	0.1785	0.0197	0.8056	0.0108
	0.0692	0.3255	0.0692	0.0065	-0.0119	0.0211
10%	0.0829	0.3902	0.0829	-0.0075	0.0185	-0.0066
	0.1770	0.8328	0.1770	0.0124	0.7968	0.0137
	0.0567	0.2671	0.0567	-0.0069	0.0194	-0.0062
20%	0.0977	0.4598	0.0977	-0.0041	0.0155	-0.0107
	0.1725	0.8120	0.1725	0.0121	0.7808	0.0191
	0.0458	0.2154	0.0458	-0.0048	0.0164	-0.0114

It should be pointed that the 2D Viterbi detector is designed for the asymmetric 2D target [2] as illustrated in table 3.1. The three bits from the three adjacent tracks (upper, main, and lower tracks) are sent by the readhead so that each symbol represents these three bits resulting 8 combinations in total. The asymmetric 2D target has the current and 2 previous symbols (i.e., 2 memory taps) giving  $8^2 = 64$  states. Therefore, its trellis has 64 states and 8 outgoing branches each states.

### 3.2.6 Summary

The TMR estimation method by using only the readback signal for a BPMR system corrupted by AWGN, inter-symbol interference, and inter-track interference. We also propose a method for equalizer designing based on the estimated TMR level. Specially, we first estimate the SNR based on the peak amplitude of the readback signal. Then, the estimated TMR level is determined based on the readback signal energy and the estimated SNR level. At the read channel, the readback signal is equalized with the matched equalizer, which is designed based the TMR level. However, the TMR estimation of this method depends mainly on that of the SNR estimation. Thus, this method has several equations to calculate TMR levels, it depend on each SNR level also. Therefore, we will propose TMR estimation method without SNR estimation in the next section.

### 3.3 MSE Technique for TMR estimation

In this topic, we present to estimate the TMR levels from the readback signals but only calculate mean-square error (MSE) between the equalizer's outputs and the fixed target's outputs. Then, we mitigate the TMR effect by adjusting the equalizer's coefficient.

### 3.3.1 BPMR channel model

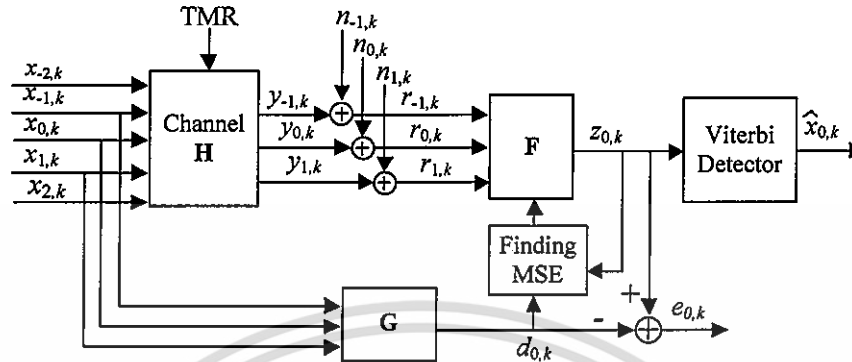


Figure 3.8 Block diagram of BPMR system with the proposed TMR mitigation approach.

This work focuses on a discrete-time BPMR channel model with multi-track processing [14] as depicted in Figure 3.8. The readback signal corrupted TMR effect can be calculated from (2.12) in Chapter 2 similar to previous work.

### 3.3.2 Proposed TMR mitigation approach

Firstly, we have designed the 2D equalizer coefficients with the fixed 2D target in each TMR levels e.g., at 0%, 5%, 10%, 15%, 20%, and 25%. These 2D equalizer coefficients will be kept in look-up table which storage in an alternate data memory.

Then, the MSE are determined using the equalizer's outputs,  $z_{0,k}$  and the fixed target's output,  $d_{0,k}$  for each TMR levels as shown in Figure 3.8. It is importance to note that the 2-D equalizer coefficients used in this process was designed without corrupting of TMR effect (TMR = 0%) and signal-to-noise ratio (SNR) is fixed at 20 decibel (dB). The MSE values for each TMR level can then be calculated as following equation,

$$\text{MSE} = \frac{1}{S} \sum_{k=1}^S (z_{0,k} - d_{0,k})^2, \quad (3.7)$$

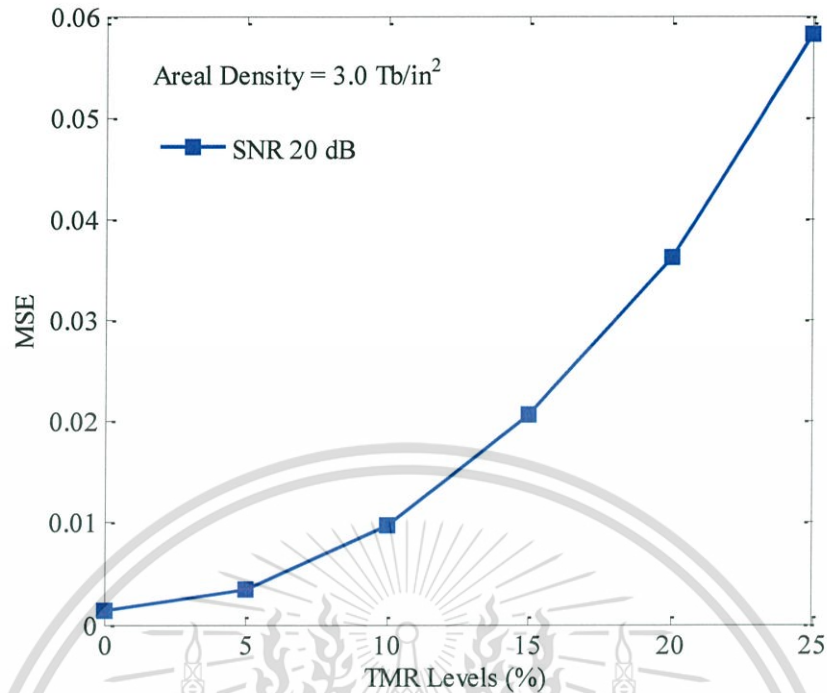


Figure 3.9 The relationship between the TMR levels (%) and the MSE of the equalizer's outputs and the fixed target's output.

where  $S$  is the length of the equalizer's outputs or target's outputs. Therefore, there will be six MSE values as illustrated in Figure 3.9, which will be next used for creating the estimated TMR equation. In this work, we use a curve fitting technique [14] so as to obtain a polynomial function that provides the best fit line from Figure 3.9 to the MSE values. This polynomial function can be utilized to approximate the TMR level according to following equation,

$$\hat{TMR} = b_0 + b_1 MSE + b_2 MSE^2 + \dots + b_N MSE^N, \quad (3.8)$$

where  $\hat{TMR}$  is the estimated TMR,  $b_i$  and  $N$  are the coefficient and a degree of the polynomial equation in (3.8), respectively, and  $i \in \{0, 1, \dots, N\}$ , where  $N = 2$  provides the best fit between the actual and the estimated TMR levels.

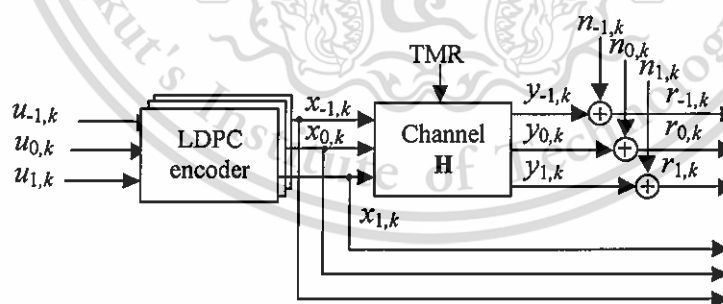
### 3.3.3 Summary

This topic presents the TMR estimation method by using MSE to find different values between the equalizer's outputs and fixed target's output for a BPMR system corrupted by AWGN and 2D interference. Then, these different values are utilized for selecting the properly 2D equalizer coefficients. However, this method simulation in Viterbi detector and we would like to adapt this method for iterative TMR mitigation method for increasing performance as following in next topic.

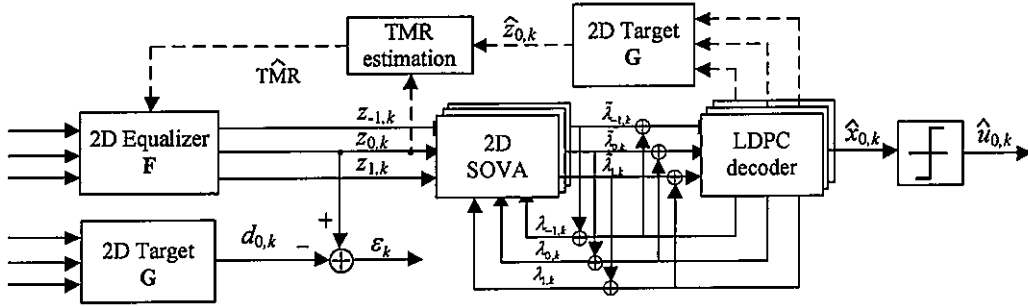
## 3.4 An iterative TMR mitigation method

To solve the TMR problem, we propose an iterative TMR mitigation method to alleviate the TMR effect based on the readback signals. We first design several pairs of the 2D asymmetric target and its corresponding 2D equalizer that match the BPMR channel for each TMR level. Next, we utilize the three adjacent data tracks obtained from the output of the low-density parity-check (LDPC) decoders to estimate the TMR level. Hence, a pair of the 2D target and its corresponding 2D equalizer that is best fit to the estimated TMR level will be employed to cope with the TMR-affected readback signal during the next global iteration.

### 3.4.1 BPMR Channel Model



(a) The sender.



(b) The receiver.

Figure 3.10 Block diagram of a coded BPMP channel model with the proposed iterative TMR mitigation method: (a) The sender and (b) The receiver.

To consider a discrete-time BPMP channel model [4], [16] in Figure 3.10. An input data sequence of  $u_{i,k} \in \{0,1\}$  length 3640 b with a bit period  $T_x$  is encoded by a rate-8/9 LDPC code to obtain a sequence  $x_{i,k} \in \{0,1\}$  of length 4095 bit, where the parity-check matrix has 3 ones in each column and 27 ones in each row. The readback signal of the  $k^{\text{th}}$  data bit of the  $l^{\text{th}}$  track can be expressed as (2.9) where the values of  $x_{l,k}$  are the recorded bits,  $l \in \{0, -1, +1\}$  represents the main, upper, and lower track, respectively, the values of  $h_{m,n}$  are the 2D channel coefficients,  $m$  and  $n$  represent the time indices of the bit island in the across-track and the along-track directions, respectively, and  $n_{i,k}$  is an additive white Gaussian noise with zero mean and variance  $\sigma^2$ . In practice,  $h_{m,n}$  can be obtained by sampling the isolated island pulse response at the integer multiples of the bit period  $T_x$  and the track pitch  $T_z$  according [25] in (2.10) for calculation the 2D Gaussian pulse response with media noise given by equation (2.8). In addition, the read-head offset or the TMR is assumed to be positive for the upward offset, as shown in Figure 3.1.

At the receiver, the readback data sequences  $\{r_{i,k}\}$  are equalized by a 2D equalizer to obtain the sequences  $\{z_{i,k}\}$  and sent to three 2D soft-output Viterbi algorithm (2D SOVA) detectors to produce the three-track soft information  $\tilde{\lambda}_{i,k}$  before passing them to the LDPC decoders to output the three-track log-likelihood ratio (LLR),  $\lambda_{i,k}$  of the input bits associated with the main and the two adjacent tracks. Note that the LDPC decoder is implemented based on the message-passing algorithm [34] with  $N_{\text{LDPC}}$  internal iterations. Then, the LLR sequences are fed back to each corresponding 2D-SOVA detector for the next turbo iteration,  $N_T$ .

At each global iteration,  $N_G$ , the three-track LLR sequences are mapped to  $\hat{x}_{i,k} \in \{\pm 1\}$ , and sent to the 2D asymmetric targets to produce all possible target outputs  $\{\hat{z}_{o,k}\}$ . Thus, the TMR estimation block computes the MSE between each sequence  $\hat{z}_{o,k}$  and the equalizer output  $z_{o,k}$  from the previous global iteration, and then outputs the estimated TMR,  $\hat{TMR}$ , corresponding to the 2D target used to generate the sequence  $\hat{z}_{o,k}$  that provides a minimum MSE (MMSE). Therefore, a new set of the 2D target and its corresponding 2D equalizer will be used for the next global iteration. In particular, the original readback data sequences  $\{r_{i,k}\}$  will be re-equalized by the new 2D equalizer before sending them to the 2D SOVA detectors implemented based on the new 2D target. Note that only one 2D equalizer is used to equalize three input sequences and then output three equalized sequences in the iteration process, whereas all 2D SOVA detectors employ the same 2D target. In addition, this paper focuses only on detecting the data on the main track, similar to [14].

### 3.4.2 Equalizer and target design

We design a pair of 2D target and its corresponding 2D equalizer for the TMR levels of 0%, 5%, 10%, 15%, 20%, and 25%, based on an MMSE approach [19,24], and employ them in the simulation. In practice, the target and its corresponding equalizer can be obtained by minimizing as (2.15). In the third proposed method, we use the optimized 3x3 GPR target  $G$  which is similar to the channel matrix, i.e.,

$$G = \begin{bmatrix} 0.0824 & 0.3876 & 0.0824 \\ 0.2125 & 1.0000 & 0.2125 \\ 0.0824 & 0.3876 & 0.0824 \end{bmatrix},$$

for each TMR levels (0%, 5%, 10%, 15%, 20%, 25%) and the size of the 2D equalizer is 3x7.

### 3.4.3 TMR estimation

The estimated TMR ( $\hat{TMR}$ ) is computed by the TMR estimation block. In particular, the LLR output of the LDPC decoders at the last turbo iteration ( $N_T = 3$ ) will be mapped to the estimated recorded sequence  $\hat{x}_{i,k}$  before convolving with each predesigned 2D target to obtain six data sequences,  $\{\hat{z}_{o,k}\}$ . Then, we calculate the MSE between each sequence  $\hat{z}_{o,k}$  and the equalizer output  $z_{o,k}$  according to

$$\text{MSE} = \frac{1}{S} \sum_{k=1}^S (z_{0,k} - \hat{z}_{0,k})^2. \quad (3.9)$$

Where  $S$  is the length of the equalizer output. Eventually, the 2D target used to generate  $\hat{z}_{0,k}$  that yields a minimum MSE and its corresponding 2D equalizer will be employed to process the original readback sequences  $\{r_{i,k}\}$  during the next global iteration.

#### 3.4.4 Summary

In this topic, we present the iterative TMR mitigation method for the BPMR system based on the readback signals. In particular, the three adjacent data tracks (i.e., the main and the two adjacent tracks) from the output of the LDPC decoders are used to estimate the TMR level. Then, the system performance can be improved using a new pair of the 2D target and its corresponding 2D equalizer, that is, the best fit to the estimated TMR level for the next global iteration.

### 3.5 TMR prediction and collection using multiple readheads

In our previous works [35,36] the TMR mitigation methods were introduced based on the readback signals with a single reader. Specifically, the TMR was predicted by the estimated signal-to-noise ratio (SNR) and the average energy of the readback signal [35]. Thus, the accuracy of the TMR estimation from this method depends mainly on that of the SNR estimation. An iterative method was employed to estimate the TMR level in a coded BPMR system [36]. Although this method is independent of the SNR level, three detectors are required to perform the TMR estimation. Nonetheless, these methods cannot accurately predict a TMR level, especially when the system operates in low TMR regime.

Recently, multiple readheads have been an interesting challenge for next-generation magnetic recording systems, and many researchers utilize a readhead array to estimate and mitigate inter-track interference (ITI) in magnetic recording systems [37-39]. Moreover, a practical multi-head multi-track (MHMT) detector [37] that provides a low-complexity approach to adaptively estimate the time-varying ITI was presented. In general, the MHMT detector can handle the ITI effect better than a single-head single-track detector. Thus, the reduced state sequence estimation algorithm [38] to significantly decrease the complexity of the MHMT detector was discussed. G. Mathew [39] et al., also show the advantages of using multiple

readheads in magnetic recording systems. Therefore, this paper proposes to employ multiple readheads to predict and correct the TMR in a BPMR system as illustrated in Figure 3.11. Specifically, we first predict a TMR level based on the signal energy ratio between the upper and lower tracks. Then, the estimated TMR level is used to select a pair of 2D target and its corresponding 2D equalizer from a look-up table that is suitable for the channel with TMR so as to facilitate the data detection process.

### 3.5.1 Channel model

To consider a multi-track multi-head BPMR system as shown in Figure 3.12, the readback signal  $r_{l,k}$ , of the  $k^{\text{th}}$  data bit of the  $l^{\text{th}}$  track can be expressed in (2.9) where  $x_{l,k}$ 's are the recorded bits,  $l \in \{0, -1, +1\}$  represent the main, the upper, and the lower track, respectively,  $h_{mn}$ 's are the 2D channel coefficients,  $m$  and  $n$  represent the time indices of the bit island in the across- and along-track directions, and  $n_{l,k}$  is

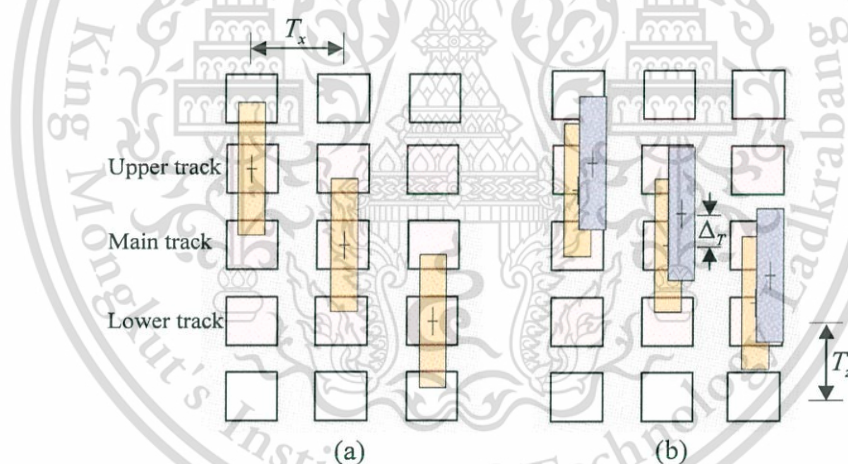


Figure 3.11 An array of three readheads for (a) the conventional scheme and (b) the proposed scheme in the BPMR system with TMR,  $\Delta_r$ .

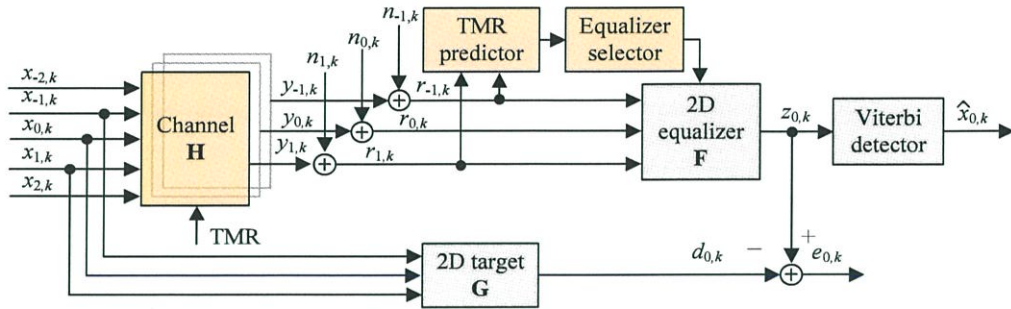


Figure 3.12 A multi-track multi-head BPMP channel model with the proposed TMR prediction and correction method.

an AWGN with zero mean and variance  $\sigma^2$ . In practice, the channel coefficients,  $h_{mn}$ , can be obtained by sampling the 2D Gaussian pulse response at the integer multiples of the track pitch,  $T_z$ , and the bit pitch,  $T_x$  according to [25,36] where  $A = 1$  is assumed to be the peak amplitude of the pulse response,  $PW_x$  is the  $PW_{50}$  of the along-track pulse,  $PW_z$  is the  $PW_{50}$  of the across-track pulse,  $PW_{50}$  is the pulse width at half its maximum,  $c = 1/2.3548$  is a constant to account for the relationship between  $PW_{50}$  and the standard deviation of a Gaussian, and  $\Delta_r$  is the head offset as shown in Figure 3.11. Here, we define the TMR level as

At a receiver, the readback signals are read by three readheads and are equalized by a 2D equalizer followed by the 2D Viterbi detector to determine the most likely data sequence on the main track,  $\hat{x}_{0,k}$ . Note that this paper focuses only on detecting the data on the main track. In addition, each pair of 2D target and 2D equalizer is designed based on a minimum mean-squared error (MMSE) approach [14,25] for each TMR level and is then stored in the look-up table. In a conventional system, each readhead will be positioned at the center of each track as shown in Figure 3.11 (a). In contrast, the proposed scheme employs an array of three readheads, where the upper and lower heads are moved closer to the center head (e.g., by 25% of  $T_z$ ) as depicted in Figure 3.11(b).

We propose to utilize three readheads to read the data on three adjacent tracks, and process them so as to predict and correct the TMR effect in the BPMP system.

### 3.5.2 TMR predictor

For each TMR level, we compute the energy ratio of the two readback signals associated with the upper and lower tracks (i.e.,  $r_{-1,k}$  and  $r_{1,k}$ ) according to

$$E_{ratio} = \sum_{k=1}^S (r_{-1,k})^2 / \sum_{k=1}^S (r_{1,k})^2, \quad (3.10)$$

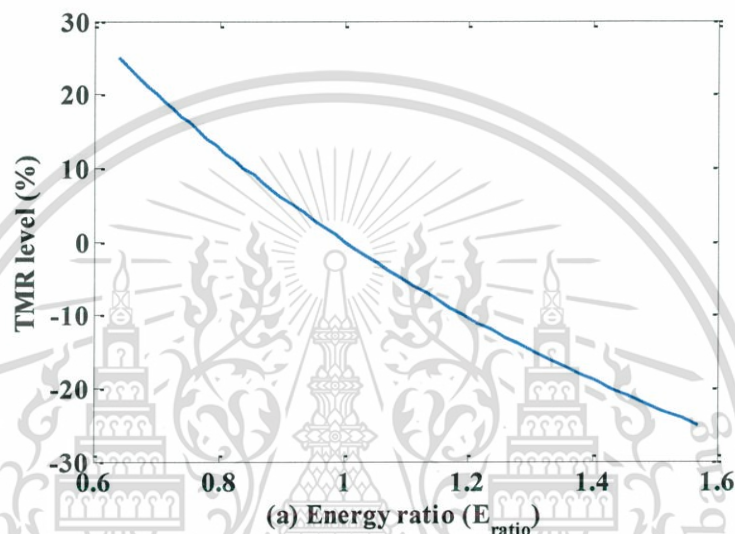


Figure 3.13 The relationship between the TMR levels versus the energy ratios at AD = 3.0 Tb/in<sup>2</sup>.

where  $S$  is the length of the readback signal samples (i.e.,  $S = 32,768$ ) bits for a 4K-data sector [23]). Next, the estimated TMR level is obtained based on a polynomial least-squares fitting technique, i.e.,

$$\hat{TMR} = b_0 + b_1 E_{ratio} + b_2 E_{ratio}^2 + \dots + b_M E_{ratio}^M, \quad (3.11)$$

where  $\hat{TMR}$  is the estimated TMR,  $b_i$  and  $M$  are the  $i^{\text{th}}$  coefficient and a degree of the polynomial equation in (3.11), respectively, and  $i \in \{0, 1, \dots, M\}$ . Then, we perform an extensive simulation search to find a suitable  $M$ , where  $M = 3$  provides the best fit between the actual and the estimated TMR levels. Figure 3.13 plots the estimated TMR level as a function of the energy ratios at 3.0 Tb/in<sup>2</sup>. It is apparent that the TMR level can be effectively estimated from (3.11) based on  $E_{ratio}$ .

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### 3.5.3 Equalizer selector

The 2D target and its corresponding 2D equalizer are designed for each TMR level based on MMSE approach, which minimizes a mean-squared error (MSE) [19] according in (2.20). In the this proposed method, we use the fixed 3x3 GPR target  $G$  which is close to the channel matrix, i.e.,

$$G = \begin{bmatrix} 0.0824 & 0.3876 & 0.0824 \\ 0.2125 & 1.0000 & 0.2125 \\ 0.0824 & 0.3876 & 0.0824 \end{bmatrix},$$

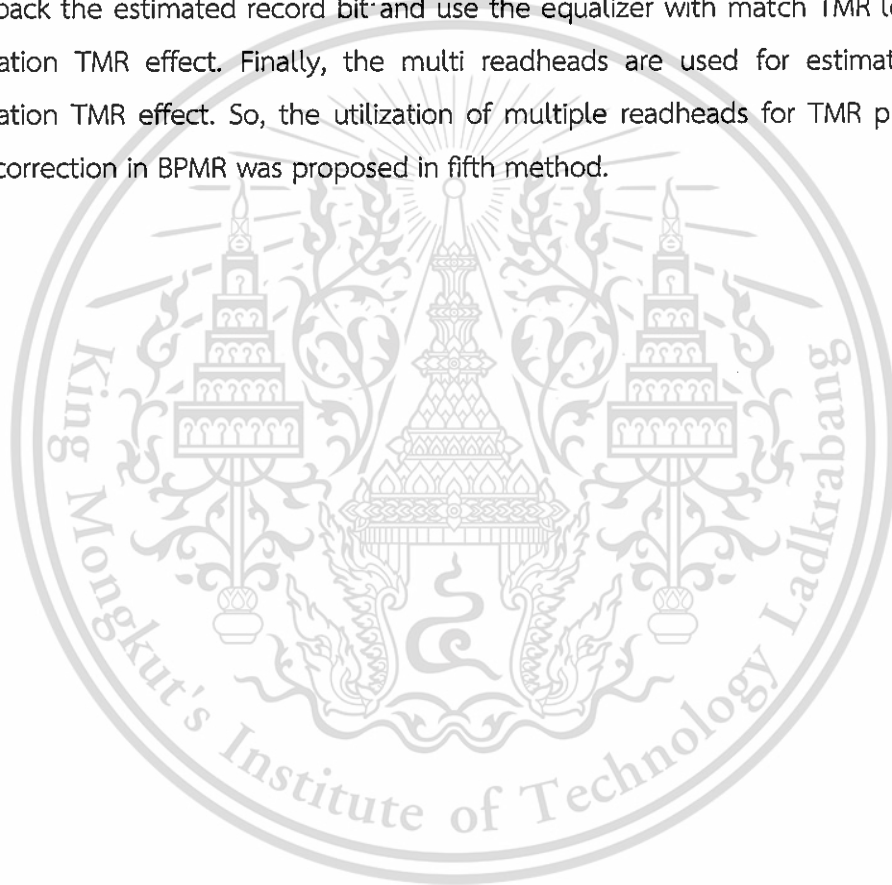
for each TMR levels (0%, 5%, 10%, 15%, 20%, 25%) and the size of the 2D equalizer is 3x7. Given the estimated TMR level, the equalizer selector will choose a pair of 2D target and 2D equalizer from the look-up table to alleviate the TMR effect embedded in the readback signal before sending it to the 2D Viterbi detector. Notice that this paper assumes that the 2D target coefficients are the 2D channel coefficients when the system has 0% TMR, similar to previous work [36]. Hence, we fix this 2D target and use it to design all 2D equalizers for each TMR level i.e., at -25%, -20%, ..., -5%, 0% 5%, ..., and 25%, based on the MMSE approach, which will be all stored in the look-up table.

### 3.5.4 Summary

This paper proposes the TMR prediction and correction method for an ultra-high density multi-track multi-head BPMR system. The energy ratio between the readback signals of the upper and lower readheads is used to predict the TMR level. Then, the TMR effect is corrected by using the 2D equalizer that is the best fit to the estimated TMR level.

### 3.6 Conclusion

In this Chapter proposed 5 methods for estimation and mitigation TMR effects based on readback signal in BPMR system. The first method proposed only TMR estimation without mitigation TMR effect. Then, the method to mitigate TMR levels base on equalizer designed with match TMR levels was proposed in second method. Next, the third method proposed estimation TMR levels from MSE values of output's equalizer and output's target. The fourth method proposed TMR estimation from feedback the estimated record bit and use the equalizer with match TMR levels for mitigation TMR effect. Finally, the multi readheads are used for estimation and mitigation TMR effect. So, the utilization of multiple readheads for TMR prediction and correction in BPMR was proposed in fifth method.



## Chapter 4

# The Results of Mitigation TMR Methods

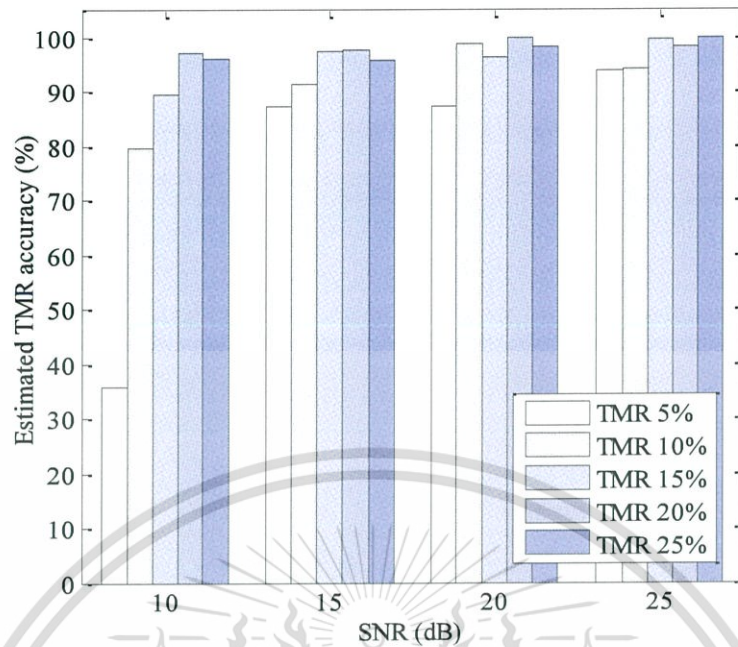
From previous in Chapter 3, this thesis proposed five methods for estimation and mitigation TMR effect based on the readback signal in BPMR system. In this chapter will be discussed the simulation results in the system from five methods, follow as the simple TMR estimation methods, a TMR mitigation method, MSE technique for TMR estimation, an iterative TMR mitigation method, TMR prediction and collection using multiple readheads, respectively. The simulation results of this thesis consider in BPMR system with several ADs of 2.0, 2.5 and 3.0 Tb/in<sup>2</sup>, where both the bit period and the track pitch are  $T_x = T_z = 18, 16,$  and  $14.5$  nm, respectively. Moreover, this work considers the 2D Gaussian pulse response with the along-track  $PW_{50}$  of 19.4 nm and the across-track  $PW_{50}$  of 24.8 nm as similar to [20]. In simulation, the SNR is defined as (2.13) where  $\sigma$  is a standard deviation of AWGN. In addition, the accuracy of TMR estimation is measured from

$$\text{accuracy (\%)} = 100 - \frac{|\hat{\text{TMR}} - \text{TMR}|}{\text{TMR}} \times 100, \quad (4.1)$$

for the simulation result, where  $\hat{\text{TMR}}$  is the estimated TMR obtained from the proposed method, and TMR is the actual TMR embedded in the readback signal. Finally, the conclusion will be discussed in the end of this chapter.

### 4.1 Simple TMR estimation methods results

The simple TMR estimation method studies the performance of the proposed methods in the BPMR system shown in block diagram in Figure 3.2 from chapter 3 at ADs of 2.0, 2.5, and 3.0 Tb/in<sup>2</sup>, where  $\hat{\text{TMR}}$  is the estimated TMR obtained from the proposed method, and TMR is the actual TMR embedded in the readback signal. Figure 4.1 demonstrates the percentage of the estimation accuracy of the proposed method in the BPMR system. Clearly, the proposed method can provide an accuracy of more than 80% when the TMR level is greater than 10%. In addition, the accuracy percentage approaches to nearly 100% when SNR is very high. For example, at high SNR (e.g., 20 dB and 25 dB) of each TMR level (from 5% to 25%), this work can achieve the accuracy of about 95%. Consequently, the proposed estimation method



**Figure 4.1** The relationship between the SNR levels and the percentage of the estimation accuracy of the proposed method at  $AD = 2.0 \text{ Tb/in}^2$ .

can be effectively employed to estimate the TMR level, especially when SNR and TMR are high. However, the low SNR levels provide an accuracy of less than 40%, in particularly, when the TMR level is low because the effect of TMR is dominant by high AWGN noise. Therefore, the accuracy of the proposed method is not well.

#### 4.1.1 Summary

The first method proposes a method to estimate a TMR amount based only on a readback signal. Firstly, the signal-to-noise ratio (SNR) is estimated based on the peak amplitude of the readback signal. Then, it determines a TMR level using the average signal energy and the estimated SNR. Simulation results show that the proposed method can predict the TMR level embedded in the readback signal with 95% accuracy, especially when TMR and SNR level are high.

## 4.2 A TMR mitigation method results

The second method studies the performance of the proposed methods (both the symmetric and the asymmetric systems) in the BPMR system shown in Figure 3.5 from Chapter 3 at ADs of 2.0, 2.5, and 3.0  $\text{Tb/in}^2$ , where both the bit period and the track pitch are  $T_x = T_z = 18$  and 14.5 nm, respectively. Additionally, this paper considers the 2D Gaussian pulse response with the along-track  $PW_{50}$  of 19.4 nm and the across-track  $PW_{50}$  of 24.8 nm. Each bit-error rate (BER) point is computed using as

many 4K-data sectors as required to collect a minimum number of 500 error bits. Furthermore, the accuracy of TMR estimation is measured from (4.1).

The accuracy of TMR estimation is similar to Figure 4.1, it demonstrates the TMR estimation accuracy (in percentage) of the proposed method in BPMP system at AD of 2 Tb/in<sup>2</sup>. Clearly, the proposed method can provide a good estimation of TMR level, especially when TMR is large and SNR is high. For example, it is possible to achieve 95% accuracy of TMR estimation when SNR is greater than 15 dB and TMR is larger than 15%. Therefore, it can be concluded that the proposed method can be effectively used to estimate the actual TMR embedded in the readback signal, especially when SNR and TMR are high. Moreover, this work found that the estimation accuracy is less than 40% when TMR and SNR are small. This might be because the effect of TMR ranged from 0% to 5% is very similar and AWGN is dominant because of low SNR.

The second method also compares the BER performances of the proposed methods and the conventional system for various ADs and various TMR effect levels. The curve labeled “Conv-TMR 0%” represents the conventional system without TMR effect, which yields the best performance, and the curves labeled “Conv-TMR 5%”, “Conv-TMR 10%” and “Conv-TMR 20%” represent the conventional system with TMR effect at 5%, 10% and 20%, respectively. Moreover, the curve labeled “Proposed-Sym-TMR X%” represents the proposed method with a symmetric target at the TMR level of X% whereas the curve labeled “Proposed-Asym-TMR X%” denotes the proposed method with an asymmetric target at the TMR level of X%. At AD = 2.0 Tb/in<sup>2</sup>, the results show that the proposed methods (symmetric and asymmetric systems) yield slightly better performance than the conventional system for all TMR levels, as illustrated in Figure 4.2 and Figure 4.3.

However, this method can obtain a higher performance gap when the AD is increased, e.g., at AD = 3.0 Tb/in<sup>2</sup>. Specifically, when AD increases, not only the ITI effect is more severe, but also the TMR can easily occur in the system even though the read head slightly moves away from the main track. Fortunately, the proposed method can handle the severe TMR effect. It can see that the symmetric system performs slightly better than the conventional system at 5% TMR level and offers the performance gain about 4 dB at BER 10<sup>-4</sup> and 10% of the TMR level: however, both the conventional and the symmetric systems cannot provide satisfactory

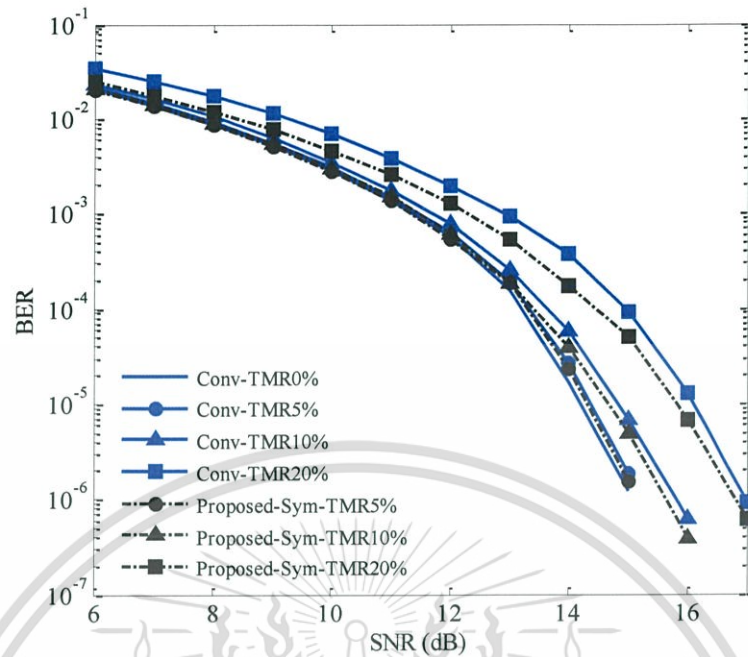


Figure 4.2 Performance comparison of various systems at  $AD = 2.0 \text{ Tb/in}^2$  with various TMR levels.

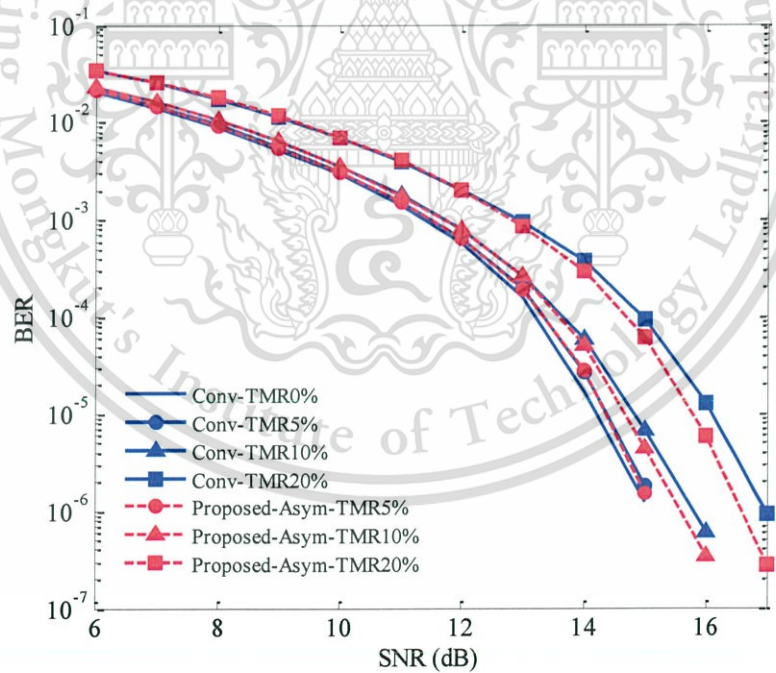


Figure 4.3 Performance comparison of various systems at  $AD = 2.0 \text{ Tb/in}^2$  with various TMR levels.

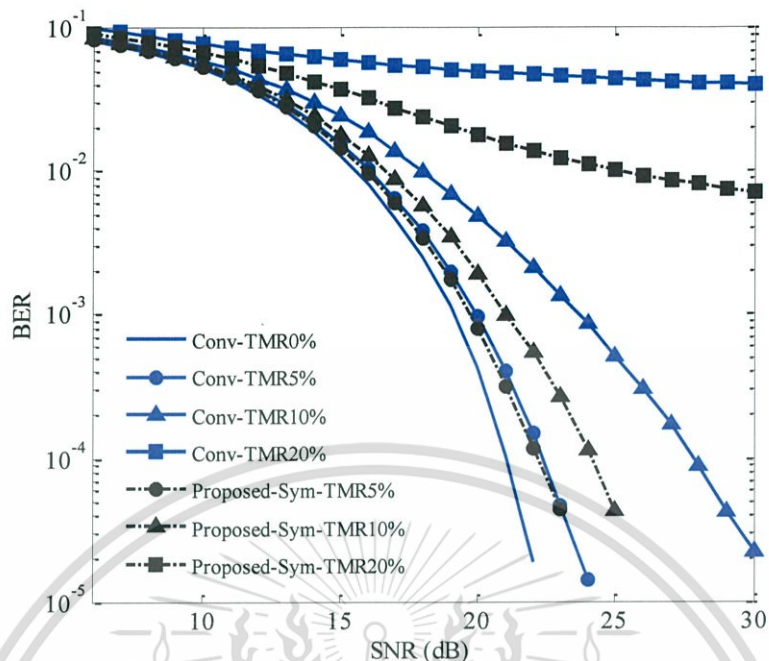


Figure 4.4 The performance between conventional and asymmetric systems for each TMR level at  $AD = 3.0 \text{ Tb/in}^2$ .

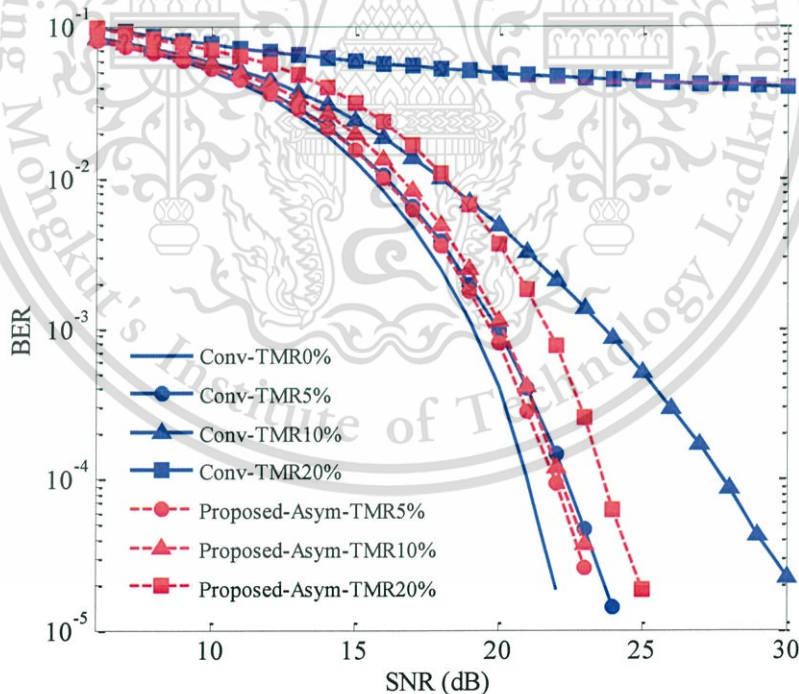


Figure 4.5 The performance between conventional and a symmetric systems for each TMR level at  $AD = 3.0 \text{ Tb/in}^2$ .

performance for high TMR levels, as demonstrated in Figure 4.4. On the other hand, the asymmetric system is superior to the conventional especially when TMR is high. Specifically, it can provide a performance gain over the conventional system about 0.5 and 6 dB at BER  $10^{-4}$  for the TMR level of 5% and 10%, respectively, as shown in Figure 4.5. Therefore, it is of importance to notice that the proposed TMR estimation method can be also well performed when it encounters with some media noise, e.g., position jitter (not shown here). Consequently, it can be implied that the proposed methods perform better than the conventional receiver because the detection process utilizes the 2D target and its corresponding equalizer that match with the BPMR channel with TMR. Nevertheless, the proposed methods require some extra memory to store the 2D target and the equalizer that are suitable for each TMR level.

#### 4.2.1 Summary

This work proposes a novel method to alleviate the TMR effect, based on the readback signal. Specifically, the readback signal is directly used to estimate a TMR level and is then further processed by the suitable target and equalizer designed for such a TMR level. Simulation results indicate that the proposed method can sufficiently estimate the TMR level and then yields a better performance than the conventional receiver (without a TMR mitigation method), especially when an areal density is high and/or a TMR level is large.

### 4.3 MSE Technique for TMR estimation results

The third method proposes TMR mitigation approach in the BPMR system at an areal density (AD) of  $3.0 \text{ Tb/in}^2$ , where  $\hat{\text{TMR}}$  is the estimated TMR obtained from the proposed method, and TMR is the actual TMR embedded in the equalizer's outputs.

The result in Figure 4.6 show as the TMR estimation accuracy (in percentage) of the proposed method in BPMR system at AD of  $3.0 \text{ Tb/in}^2$ . Clearly, the proposed method can achieve a good estimation of TMR level, especially when TMR large and SNR are high. For example, the proposed method provide the accuracy percentage more than 80% of TMR estimation when SNR is greater than 15 dB and TMR is larger than 15%. Therefore, it can be concluded that the proposed method can be effectively used to estimate the actual TMR from the MSE of the equalizer's outputs

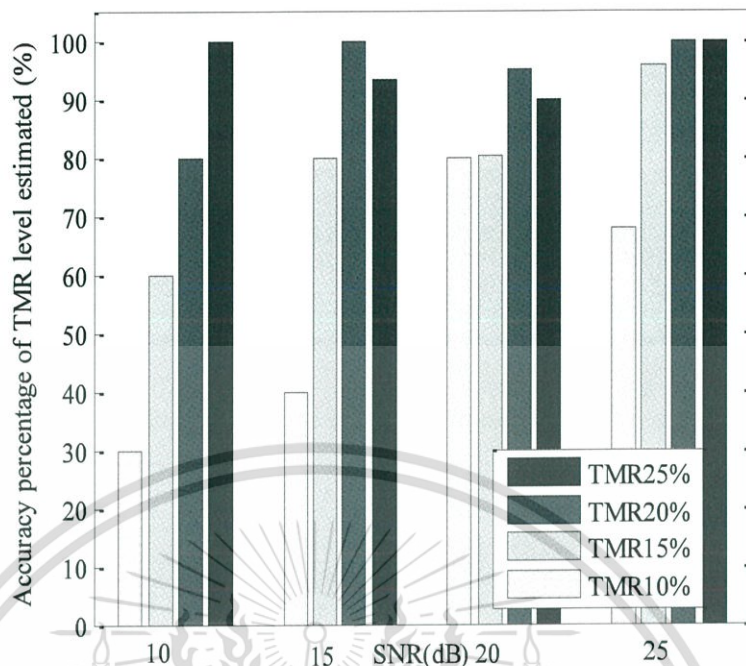


Figure 4.6 The relationship between the SNR levels and the percentage of the estimation accuracy of the proposed method.

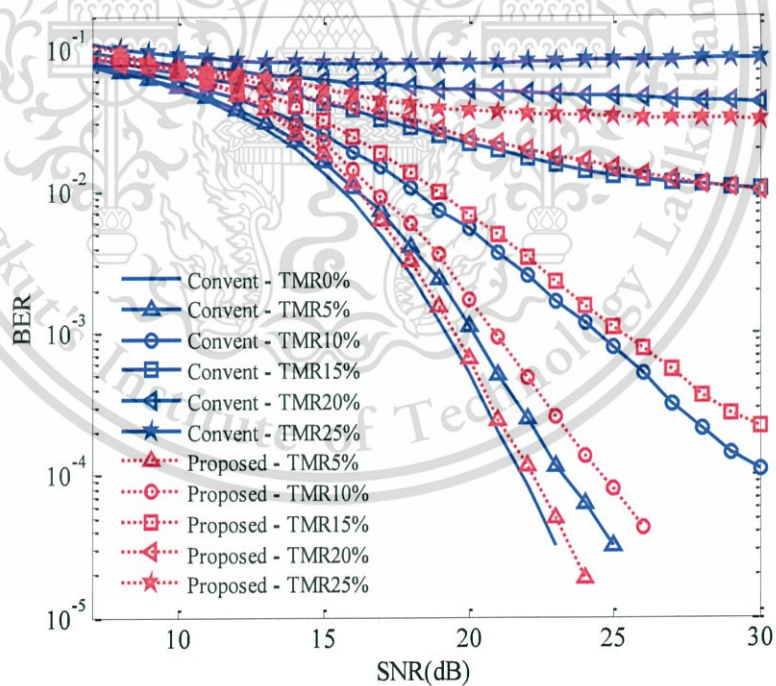


Figure 4.7 The BER performance comparison between the proposed TMR mitigation system and the conventional system.

and fixed target's output, especially when SNR and TMR are high. However, this methods estimate the low TMR level are not well. The estimation accuracy is still less than 40% when TMR and SNR are small. This might be because the effect of TMR ranged from 0% to 5% is very similar and AWGN is dominant because of low SNR.

This work also compares the BER performance of the proposed systems with the conventional system as illustrated in Figure 4.7, where the conventional system performs without any TMR mitigation method. The labeled as "Convent-TMR PT%" represents the conventional system. The PT% is the TMR effect PT%. The one labeled as "Proposed-TMR PT%" represent the proposed TMR mitigation approach. Clearly, the proposed system performs better than the conventional system for all TMR levels, especially when the TMR is high. It is very importance to note that the proposed method in this work, it does not consider the position jitter noise.

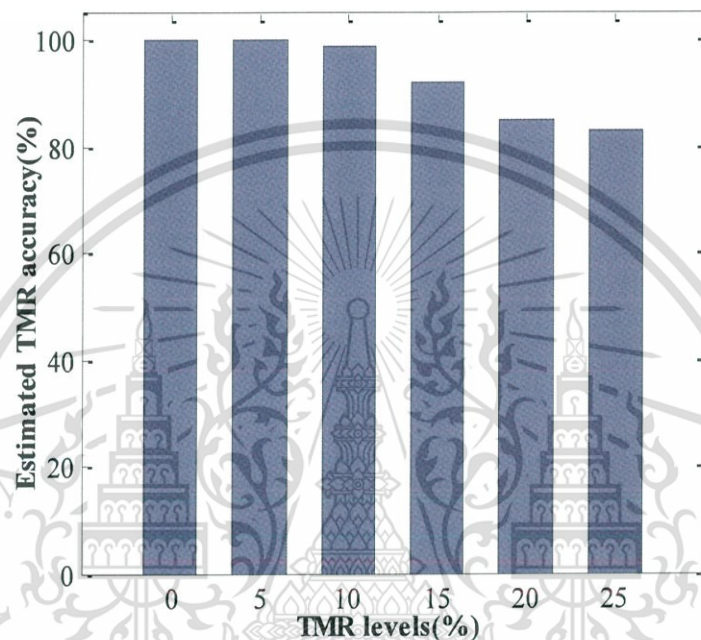
#### 4.3.1 Summary

This third method proposes a simple TMR mitigation approach based on the readback signal in bit-patterned media recording (BPMR) systems. It begins with calculating the mean-square error (MSE) between the equalizer's outputs and the fixed target's outputs for each TMR levels, which next will be collected in the look-up table. Then, the TMR levels are estimated utilizing these data from the look-up table before selecting the properly equalizer coefficients which matches with TMR level. In simulation results, it found that the proposed TMR mitigation approach offers the performance gain better than the conventional system, especially when the TMR is high.

#### 4.4 An iterative TMR mitigation method results

This fourth method evaluates the performance between 1) the conventional system (i.e., the system in Figure 3.10, but without TMR mitigation) and 2) the proposed method in Figure 3.10, at the AD of 3.0 Tb/in<sup>2</sup> (i.e.,  $T_x = T_z = 14.5$  nm), where the 2D Gaussian pulse response with the along-track PW50 of 19.4 nm and the across-track PW50 of 24.8 nm is considered, similar to [20]. This work also defines the signal-to-noise ratio as equation (2.13), where  $V_p = 1$  is assumed to be the peak value of the readback signal, and  $R$  is a code rate. Each bit error rate (BER) point is computed using as many 3640 bit data sectors as needed to collect 500 erroneous bits. In simulation, the 2D target and its corresponding 2D equalizer designed for 0% TMR level will be used at the first global iteration (i.e.,  $NG = 1$ ), and employ  $N_{LDPC} = 3$ ,  $NT = 3$ , and  $NG = 2$ .

This method first investigate the accuracy of our TMR estimation method. To do so, this method sends 5000 data sectors (each with random TMR ranged from 0% to 25%) into the system in Figure 3.10, and check how many times the TMR estimation block can output a correct  $\hat{TMR}$ . Simulation results indicate that, for the TMR levels of 0%, 5%, 10%, 15%, 20%, and 25%, our method yields the accuracies of 100%, 100%, 99%, 92%, 85%, and 83%, respectively as Figure 4.8. Clearly, the proposed



**Figure 4.8** The relationship between the TMR levels and the percentage of the estimation accuracy of the proposed method.

method can correctly estimate the TMR level almost 100% when the TMR is  $< 10\%$ , which is better than the method in [14]. Next, this method compares the BER performance of different systems for various TMR levels at the AD of  $3.0 \text{ Tb/in}^2$  in Figure 4.9. The curve labeled Convent-TMR X% and Proposed-TMR X% represent the conventional system and the proposed system with the TMR effect of X%, respectively. It is clear that the proposed system performs similar to the conventional system when no TMR effect is present in the system. However, at  $\text{BER} = 10^{-4}$ , the proposed system can provide the performance gain of  $> 1.5 \text{ dB}$  over the conventional system, when the TMR effect is  $> 5\%$ . This method also verifies that the proposed TMR mitigation scheme can perform well in the presence of media noise, e.g., position jitter, as shown in Figure. 4.10, where it fixes a TMR level at 10%

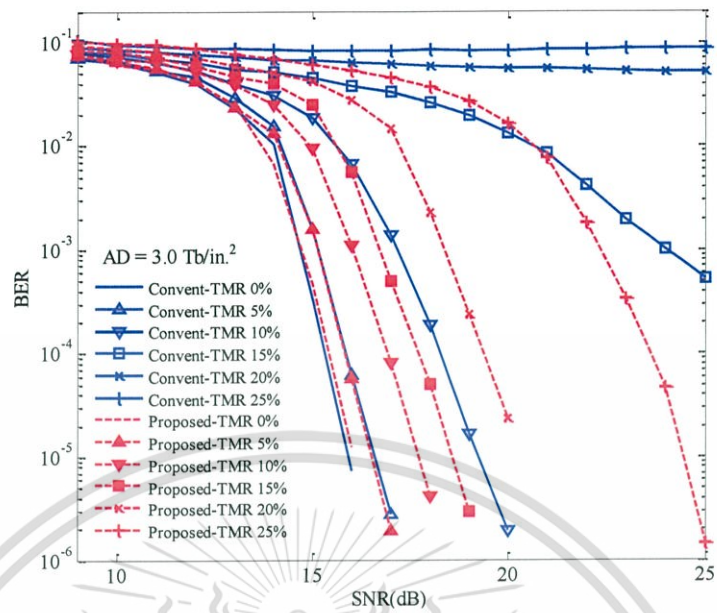


Figure 4.9 BER performance of different schemes for various TMR levels at the  $AD = 3.0 \text{ T/in}^2$ .

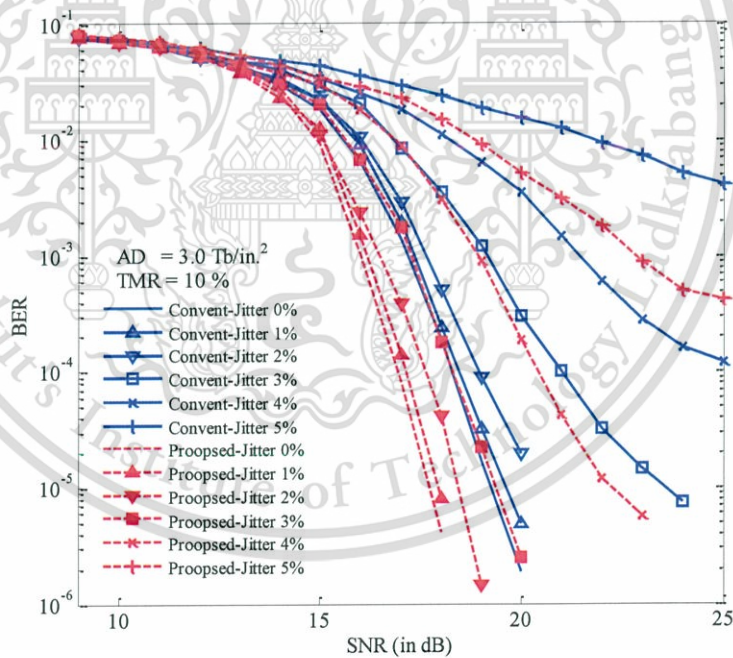


Figure 4.10 BER performance of different schemes for various position jitter amounts at the  $TMR = 10\%$  and the  $AD = 3.0 \text{ T/in}^2$ .

and vary the position jitter amount,  $\sigma_j/T_x$ , from 0% to 5%. The curve labeled Convent–Jitter X% and Proposed–Jitter X% represent the conventional system and the proposed system with the position jitter amount of X%, respectively. Again, the proposed system outperforms the conventional system for all position jitter amounts, especially when the position jitter is large. For instance, at BER =  $10^{-4}$ , it yields the performance improvement of > 4.5 dB over the conventional system when the position jitter amount is fixed at 4%. The reason that the proposed system performs well is because our TMR mitigation method can effectively estimate the TMR embedded in the readback signal. Using this information to select and use a new set of the 2D target and its corresponding 2D equalizer that matches the actual TMR to reprocess the readback signal will offer some performance improvement.

Therefore, it is worth employing the proposed TMR mitigation method in the extremely high-density BPMR system, in particular, when the TMR effect is dominant. However, it should be noted that the performance gain obtained from the proposed method needs to be balanced against the increased complexity.

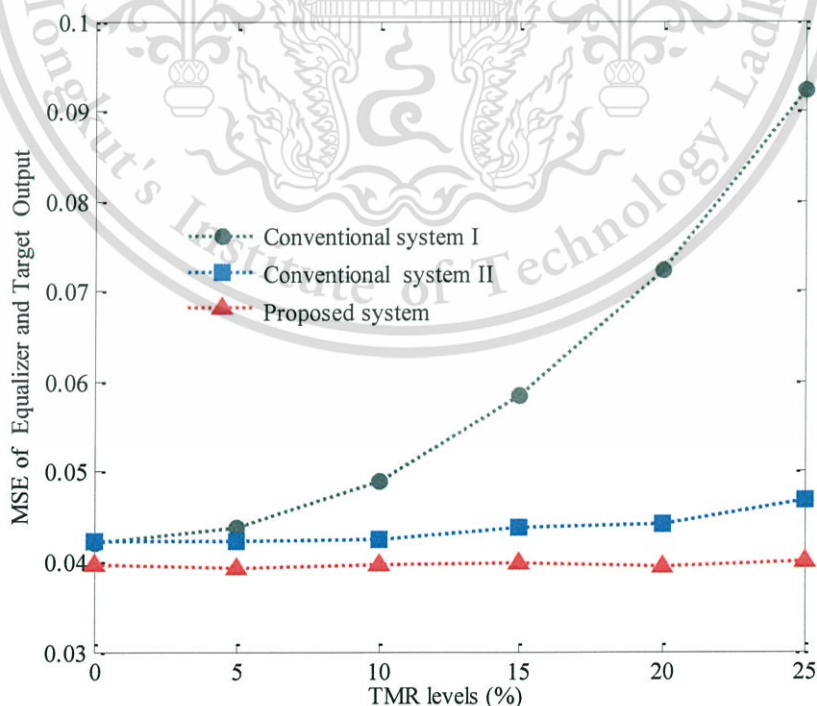
#### 4.4.1 Summary

This method proposes an iterative TMR mitigation method for BPMR systems based on the readback signals. First, it designs several pairs of the 2D asymmetric target and its corresponding 2D equalizer that match the BPMR channel for each TMR level. Then, this work exploits the three adjacent data tracks obtained from the low-density parity-check decoders to estimate the TMR level. Last, a pair of the 2D asymmetric target and its corresponding 2D equalizer that is best fit to the estimated TMR level will be used to alleviate the TMR effect in the readback signal for the next global iteration. Simulation results indicate that the proposed system can effectively estimate the TMR level and performs better than the conventional system without a TMR mitigation method, especially when the TMR level is high and/or the position jitter is large.

#### 4.5 TMR prediction and collection using multiple readheads results

The fifth method evaluate the system performance at the AD of 3.0 Tb/in<sup>2</sup> (i.e.,  $T_x = T_z = 14.5$  nm) among 1) the conventional system I without TMR prediction and correction method; 2) the conventional system II, assuming that TMR is known and the receiver uses the 2D equalizer designed for that TMR; and 3) the proposed system, where the 2D Gaussian pulse response with the along-track PW50 of 19.4 nm and the across-track PW50 of 24.8 nm is considered, similar to previous work [36]. In simulation, the SNR is defined as (2.13). This method also measures the accuracy of TMR estimation from (4.1) where  $\hat{TMR}$  is the estimated TMR obtained from the proposed method, and TMR is the actual TMR embedded in the readback signal. Figure 10 shows the accuracy of TMR estimation in percentage at different SNRs. Clearly, the proposed method can predict the TMR well, especially at high TMR levels. In addition, it founds that the accuracy of our TMR estimation method is independent of SNRs.

From now on, this work assumes that the BPMR system experiences the TMR effect only in the upward direction (i.e.,  $\Delta_r$  is a positive value). Note that this paper will not consider a media noise in the system so as to make it easy to understand the behavior of the TMR effect. Figure 11 compares the performance of different systems at AD = 3.0 Tb/in<sup>2</sup> in terms of the MSE in dB according to



**Figure 4.11** Performance of MSE versus TMR of difference systems at AD = 3.0 Tb/in<sup>2</sup>.

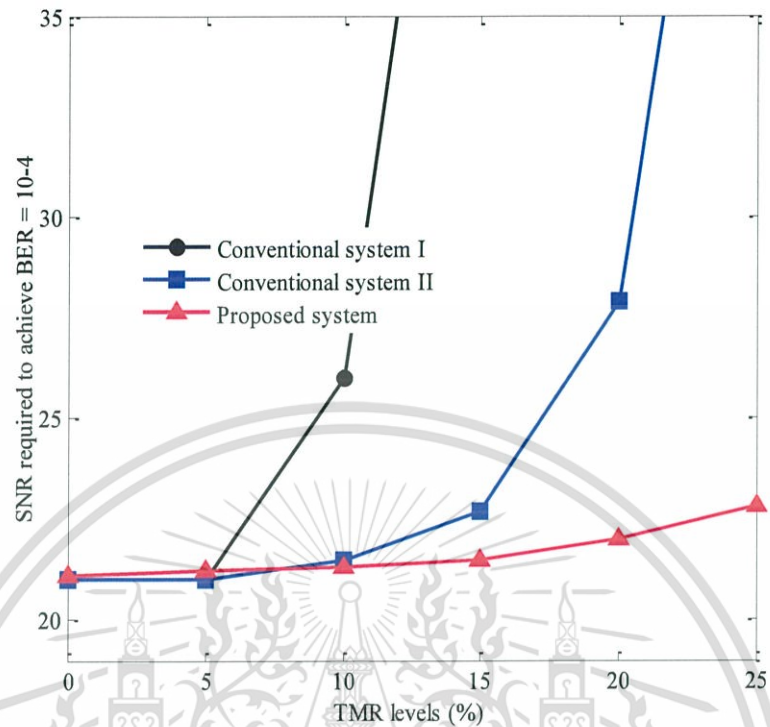


Figure 4.12 Performance comparison of difference systems at  $AD = 3.0 \text{ Tb/in}^2$ .

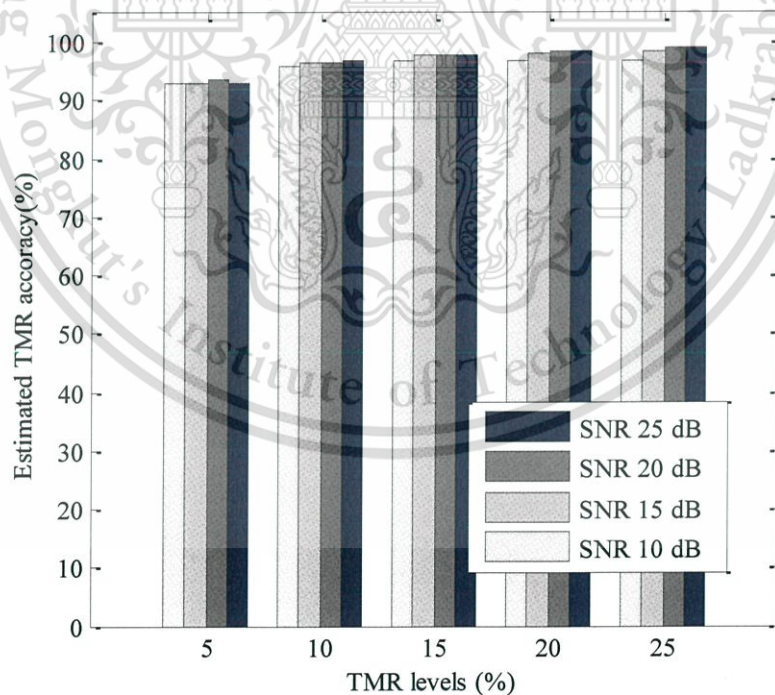


Figure 4.13 The relationship between the SNR levels and the percentage of the estimation accuracy of the proposed method at  $AD = 3.0 \text{ Tb/in}^2$ .

$$\text{MSE} = 10 \log_{10} \left( \sum_{k=1}^S \{z_{0,k} - d_{0,k}\}^2 \right). \quad (4.2)$$

Apparently, the proposed system yields the lowest MSE for all TMR levels. That means the proposed equalizer can perform with the proposed multi-track reading systems as Figure 11. Finally, Figure 4.12 compares the performance of different systems at  $AD = 3.0 \text{ Tb/in}^2$  by plotting the SNR required to achieve bit-error rate (BER) of  $10^{-4}$  as a function of TMR levels. It is clear that the proposed system is superior to other systems, especially when TMR is large. The reason might be because the proposed TMR estimation method can achieve a very high accuracy for all TMR levels as depicted in Figure 4.13. Therefore, it can be implied that the proposed method can help improve the quality of the readback signal before sending it to the 2D Viterbi detector, thus leading to a better BER performance. It can be concluded that by moving the upper and lower read heads closer to the center read head as shown in Figure 3.11(b) from Chapter 3, one can obtain the better readback signals with less ITI effect if compared to the conventional setting in Figure 3.11(a) from Chapter 3, thus leading to the improved system performance.

#### 4.5.1 Summary

This method proposes a utilization of multiple read heads to predict and correct a track mis-registration (TMR) in bit-patterned media recording (BPMR) based on the readback signals. This method propose to use the signal energy ratio between the upper and lower tracks from multiple read heads to estimate the TMR level. Then, a pair of two-dimensional (2D) target and its corresponding 2D equalizer associated with the estimated TMR will be chosen to correct the TMR in the data detection process. Numerical results show that the proposed system can achieve a very high accuracy of TMR prediction, thus performing better than the conventional system, especially when TMR is severe.

## 4.6 Discussion

This Chapter discuss the simulations result of all proposed methods for estimation and mitigation TMR effects based on readback signal in BPMP system. All method can achieve the high accuracy in percentage well. Especially while TMR are large they can achieve the performance gain more than the conventional methods. Next, this thesis would like to conclusion of all proposed methods in the Chapter 5.



## Chapter 5

# Conclusion and Recommendation

This thesis investigated signal processing for bit-patterned media recording (BPMR) channels with TMR effect. The contributions of this work and the conclusions are summarized in this chapter. Some suggestions for continuing this work and future work are provided at the end of this chapter. Based on the work carried out in this thesis, it conclude the following.

### 5.1 Simple TMR estimation methods

The first method proposed the TMR estimation method by using only the readback signal for a BPMR system corrupted by AWGN and, inter-symbol interference and inter-track interference. Specifically, This method first estimate the SNR based on the peak amplitude of the readback signal. Then, the estimated TMR level is determined based on the readback signal energy and the estimated SNR level. Experiment results indicate that the proposed method can effectively estimate the TMR level, especially when SNR and TMR are large. It should be pointed out that this estimation method can be adapted in designing a 2D equalizer and a 2D detector in BPMR systems so as to cope with this TMR problem.

This method can provide the high accuracy percentage of TMR estimation, specifically when the TMR are large. However, this method cannot estimate TMR in the low TMR levels. Moreover, in this method did not consider to mitigate TMR effect in the system. Therefore, a TMR mitigation method is proposed in the second method.

### 5.2 A TMR mitigation method

The second method proposed the TMR mitigation method for an ultra-high density BPMR system. The proposed method starts with estimating the SNR based on the average peak amplitude of the readback signals. Then, the estimated TMR level can be computed based on the estimated SNR and the average energy of the

readback signals. Once the TMR level is known, this work can choose the target and its corresponding equalizer that match.

In this method, the TMR estimation technique is similar to the first method and achieve the performance gain more than the conventional system (without TMR estimation). However, the TMR estimation method still depends on the SNR levels. To ignore the SNR estimation method and reduce the equations for estimating TMR levels. Therefore, the MSE technique for TMR estimation is proposed in the third method.

### 5.3 MSE technique for TMR estimation

The third method proposed the TMR estimation method by using MSE to find different values between the equalizer's outputs and fixed target's output for a BPMR system corrupted by AWGN and 2D interference. Then, these different values are utilized for selecting the properly 2D equalizer coefficients. The experiment results indicate that the proposed methods can effectively estimate the TMR levels and also shows that the proposed method can offer the BER performance gain over the conventional system at high AD and high TMR levels.

This TMR estimation method is independent of SNR estimation method, it can provide the high accuracy percentage of TMR estimation, especially when the TMR levels are large. However, the accuracy of TMR estimation is not well in the low TMR level. Therefore, for increasing the accuracy percentage in the low TMR levels, an iterative TMR mitigation method for improving the low TMR level estimation is proposed in the fourth method.

### 5.4 An iterative TMR mitigation method

The forth method proposed the iterative TMR mitigation method for the BPMR system based on the readback signals. In particular, the three adjacent data tracks (i.e., the main and the two adjacent tracks) from the output of the LDPC decoders are used to estimate the TMR level. Then, the system performance can be improved using a new pair of the 2D target and its corresponding 2D equalizer, that is, the best fit to the estimated TMR level for the next global iteration. As shown in simulation,

the proposed system is superior to the conventional system (without using a TMR mitigation method) in the presence of TMR both in the case of with and without media noise, in particular, when the TMR level is high and/or the position jitter is large.

This method provides the high accuracy percentage of TMR estimation in all TMR levels. However, this method uses several 2D-Sova detectors for feedback the estimated record bits of three tracks to estimate TMR levels, it has several detectors to provide the performance only the center track. Therefore, A TMR prediction and collection using multiple readheads in the fifth method is proposed for reducing the process feedback for TMR estimation.

### 5.5 TMR prediction and collection using multiple readheads

The fifth method proposed the TMR prediction and correction method for an ultra-high density multi-track multi-head BPMR system. The energy ratio between the readback signals of the upper and lower read heads is used to predict the TMR level. Then, the TMR effect is corrected by using the 2D equalizer that is the best fit to the estimated TMR level. Simulation results indicate that the proposed system can effectively estimate the all TMR level, and then performs better than the conventional system, especially when the system encounters high TMR amount.

From utilizing multi readheads in this method, it can provide the best performance more than all proposed methods in this thesis. However, it still achieve the best performance in the center track similar to the fourth method and this point is some significant for finding some technique to utilize three readheads for three tracks.

### 5.6 Suggestions for future work

Form the fifth method, this thesis use multi readheads for estimation and correction TMR in BPMR system, it can achieve the high accuracy percentage of TMR estimation and high performance gain more than all proposed methods in this thesis. However, this fifth method cannot improve the performance of both sidetracks. Therefore, this point is some significant for finding new methods to improve the performance of both sidetracks such as the equalizers or detectors designing.

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# Estimating Track Mis-Registration Based on Readback Signal in Bit-Patterned Media Recording Systems

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**Abstract**—Track mis-registration (TMR) is one of crucial factors on the performance of bit-patterned media recording (BPMR) systems. Because of the movement of a read head and the rotation of a magnetic disk, the center of the read head may move away from its center location of a data track, thus resulting in a TMR effect, which can deteriorate the overall system performance. In general, the TMR effect can be detected and handled by a servo system. However, the servo system usually requires inherent sector-level latency mechanisms for head adjustment. Therefore, this paper proposes a method to estimate a TMR amount based only on a readback signal. Firstly, the signal-to-noise ratio (SNR) is estimated based on the peak amplitude of the readback signal. Then, we determine a TMR level using the average signal energy and the estimated SNR. Simulation results show that the proposed method can predict the TMR level embedded in the readback signal with 95% accuracy, especially when TMR and SNR level are high.

**Keywords**—Bit-patterned media recording (BPMR); estimation method; signal-to-noise ratio (SNR); track mis-registration (TMR)

## I. INTRODUCTION

Track mis-registration (TMR) effect is a major obstacle for increasing an areal density (AD) in ultra-high density magnetic recording systems such as bit-patterned media recording (BPMR) [1], [2]. The TMR effect is occurred due to the misalignment between the center of the read head and that of the main data track as illustrated in Fig. 1. Because of TMR, the readback signal may experience even more severe inter-track interference, which will further degrade the performance of data recovery process in BPMR systems [3], [4].

To handle the TMR effect, a servo system provides an inherent sector-level latency of the detection of servo bursts before any head adjustment can be made. This servo burst field has the information that can be used to estimate the amount of read head offset. However, it is generally difficult to predict the TMR quantity to the next servo sector, when TMR is compensated by only burst signals, especially when TMR goes beyond the limit [5], [6], [7].

In practice, several TMR estimation methods based on the readback signal have been proposed in the literature [8], [9]. Nonetheless, these methods require the knowledge of some

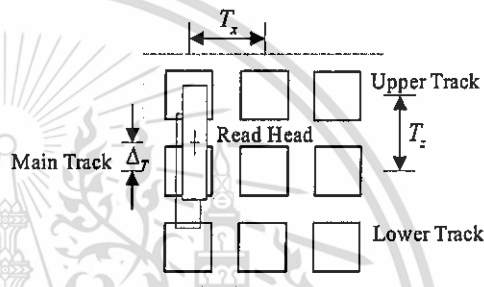


Fig. 1. TMR in a BPMR system, i.e.,  $\Delta_T$ .

recorded data and employ high complexity processing (e.g., calculating many correlation functions) for estimating TMR levels. Thus, this paper proposes a novel TMR estimation method, which is based only on the readback signal. Specifically, at high SNR, our method can provide up to 95% accuracy at high TMR level (i.e., 20%-25%).

This work begins with studying the relationship among the statistical information of the readback signal, signal-to-noise ratios (SNRs), and various TMR levels. Hence, these relationships will be employed to generate the mathematical equations to use estimating the SNR and the TMR level. In this work, we first estimate the SNR using the peak amplitude of the readback signal from one data sector. Then, this estimated SNR level will be utilized to estimate the TMR level according to the readback signal energy.

This paper is organized as follows. In Section II, the BPMR channel model is described. Section III explains the proposed TMR estimation method. The simulation results are given in Section IV. Finally, Section V concludes this paper.

## II. CHANNEL MODEL

In this paper, we focus on a discrete BPMR channel model [5], [6] as depicted in Fig. 2. The readback signal of the  $k^{\text{th}}$  data bit on the main track can be expressed as

$$r_{0,k} = \sum_n \sum_m h_{-m,n} x_{-m,k-n} + n_{0,k}, \quad (1)$$

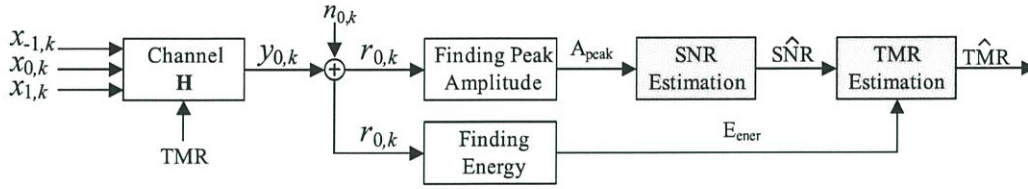


Fig. 2. Block diagram of BPMP system with proposed SNR and TMR estimations.

where  $x_{0,k}$ 's are the recorded bits on the main track,  $h_{m,n}$ 's are the 2D channel coefficients,  $m$  and  $n$  represent the time indices of the bit island in the across-track and the along-track directions, and  $n_{0,k}$  is an additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma^2$ . Practically, the channel response coefficients  $h_{m,n}$  of BPMP systems can be generated by sampling the 2D Gaussian pulse response at the integer multiples of the bit period,  $T_x$ , and the track period,  $T_z$ , i.e.,

$$h_{m,n} = P(nT_z + \Delta_T, mT_x), \quad (2)$$

where  $P(z, x)$  is the 2D Gaussian pulse response,  $z$  and  $x$  are the time indices in the across-track and the along-track directions,  $\{m, n\} \in (-L, \dots, 0, \dots, L)$ ,  $2L+1$  is the length of  $P(z, x)$ ,  $2L+1$  is the 2D channel length,  $L$  is an integer, and  $\Delta_T$  is the head offset or the distance between the center head and the center of the main track as shown in Fig. 1. Generally,  $L$  should be large enough to ensure that the tail amplitude of  $P(z, x)$  is small (here, we use  $L = 1$  for simplicity). In this paper, the sign of  $\Delta_T$  is assumed to be positive for the upward offset, where the TMR level is defined as

$$\text{TMR} (\%) = \frac{\Delta_T}{T_z} \times 100. \quad (3)$$

Furthermore, we consider the 2D Gaussian pulse response of the form [10]

$$P(z, x) = A \exp \left\{ -\frac{1}{2c^2} \left[ \left( \frac{x}{\text{PW}_x} \right)^2 + \left( \frac{z + \Delta_T}{\text{PW}_z} \right)^2 \right] \right\}, \quad (4)$$

where  $A = 1$  is assumed to be the peak amplitude of the pulse response,  $\text{PW}_x$  is the  $\text{PW}_{50}$  of the along-track pulse,  $\text{PW}_z$  is the  $\text{PW}_{50}$  of the across-track pulse,  $\text{PW}_{50}$  is the pulse width at half its maximum,  $c = 1/2.3548$  is a constant to account for the relationship between  $\text{PW}_{50}$  and the standard deviation of a Gaussian.

In Fig. 2, the recorded bits,  $x_{-1,k}$ ,  $x_{0,k}$ , and  $x_{1,k}$  from the upper, the main, and the lower tracks, respectively, are sent to the BPMP channel corrupted by TMR and AWGN. At the read side, the readback signal is used to determine its peak amplitude,  $A_{\text{peak}}$ , and the average energy,  $E_{\text{cner}}$ , simultaneously. Then, the estimated SNR,  $\hat{\text{SNR}}$ , is obtained based on  $A_{\text{peak}}$  (will be explained in Section III. A). Finally, we compute the estimated TMR,  $\hat{\text{TMR}}$ , using  $E_{\text{cner}}$  and  $\hat{\text{SNR}}$ .

### III. PROPOSED METHOD

#### A. SNR Estimation

In this work, we collect 100 samples of the readback signals at each SNR level from 0 to 25 decibel (dB), where

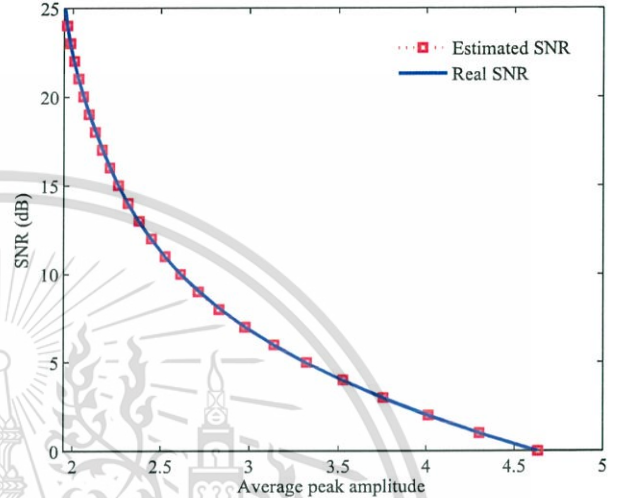


Fig. 3. The relationship between the SNR levels and the average peak amplitude of the readback signals.

each readback signal is affected by a random TMR level from 0% to 25%. For each SNR level, we average the peak amplitude of all readback signals. Using these two data sets of the SNRs and the average peak amplitudes, we propose to employ a curve fitting technique to obtain a polynomial function to approximate the SNR based on the known average peak amplitude. The polynomial function using the curve fitting technique is

$$\hat{\text{SNR}} = a_0 + a_1 y + a_2 y^2 + \dots + a_N y^N, \quad (5)$$

where  $\hat{\text{SNR}}$  is an estimated SNR,  $a_i$  is the  $i^{\text{th}}$  coefficient of the polynomial equation,  $i \in \{0, 1, \dots, N\}$ ,  $N$  is a degree of the polynomial equation,  $y$  is the peak amplitude of the readback signal. Based on extensive heuristic research, we found that  $N = 5$  is sufficient for our model because a higher order does not provide any performance gain on the accuracy of SNR estimation. In Fig. 3, we compare the real SNR value and the estimated SNR using (5). The x-axis is the average peak amplitude of the readback signals and the y-axis is the SNR levels. The result is similar so that we use the curve fitting for estimated SNR level.

#### B. TMR Estimation

To estimate the TMR level, we compute the energy of the readback signal at SNR = 0 dB for each TMR level (i.e., 0, 5,

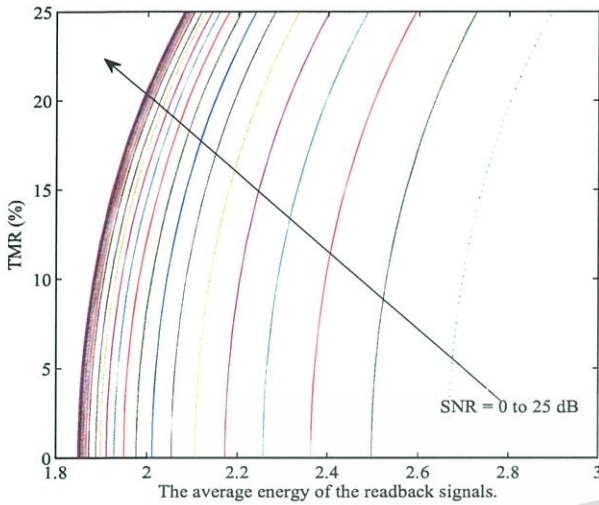


Fig. 4. The relationship between the TMR (%) levels and the average energy of the readback signals.

10, 15, 20, and 25%) according to

$$E_{\text{ener}} = \frac{1}{M} \sum_{k=1}^M r_{0,k}^2 \quad (6)$$

where  $M$  is the length of the readback signal, which is equal to 32768 bits as a 4k data sectors [11].

Next, we also use the curve fitting technique so as to obtain a polynomial function that provides the best fit line for the relation between the TMR and the average energy of all readback signals. This polynomial function can be utilized to approximate the TMR level according to

$$\hat{\text{TMR}} = b_0 + b_1 u + b_2 u^2 + \dots + b_F u^F, \quad (7)$$

where  $\hat{\text{TMR}}$  is an estimated TMR,  $b_i$  is the  $i^{\text{th}}$  coefficient of the polynomial equation,  $i \in \{0, 1, \dots, F\}$ ,  $F$  is a degree of the polynomial equation (here we use  $F = 5$  for simplicity),  $u$  is the average signal energy, which is equal to  $E_{\text{ener}}$  in Fig. 2.

Fig. 4 depicts the relationship between the TMR level and the average energy by plotting the average energy of all readback signals as a function of TMR levels. We repeat this step by varying the SNR level from 0 to 25 dB to obtain each curve in Fig. 4. The line dot is the SNR 0 dB and the next lines are the SNR level to increase by 1 dB up to SNR 25 dB. It is apparent that if we know the estimated SNR ( $\hat{\text{SNR}}$ ) and the average energy ( $E_{\text{ener}}$ ), we can easily compute the estimated TMR ( $\hat{\text{TMR}}$ ) from (7).

#### IV. SIMULATION RESULTS

We test the proposed method in the BPMR system given in Fig. 2 at an areal density (AD) of 2 Tb/in<sup>2</sup>, where both the bit period and the track pitch are  $T_x = T_z = 18$  nm, the along-track  $\text{PW}_{50}$  is 19.4 nm, and the across-track  $\text{PW}_{50}$  is 24.8 nm, as similar to [12]. The SNR is defined as

$$\text{SNR} = 20 \log_{10} \left( \frac{1}{\sigma} \right) \quad (\text{in dB}), \quad (8)$$

where  $\sigma$  is a standard deviation of AWGN.

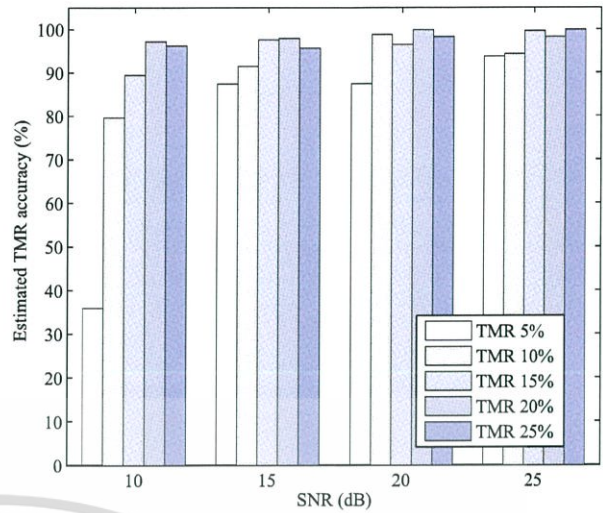


Fig. 5. The relationship between the SNR levels and the percentage of the estimation accuracy of the proposed method.

The performance of the proposed method is measured by the accuracy, which is defined as

$$\text{accuracy}(\%) = 100 - \frac{|\hat{\text{TMR}} - \text{TMR}|}{\text{TMR}} \times 100, \quad (9)$$

where  $\hat{\text{TMR}}$  is the estimated TMR, which is obtained from the proposed method, and  $\text{TMR}$  is the actual TMR level embedded in the readback signal.

Fig. 5 demonstrates the percentage of the estimation accuracy of the proposed method in the BPMR system. Clearly, the proposed method can provide an accuracy of more than 80% when the TMR level is greater than 10%. In addition, the accuracy percentage approaches to nearly 100% when SNR is very high. For example, at high SNR (e.g., 20 dB and 25 dB) of each TMR level (from 5% to 25%), we can achieve the accuracy of about 95%. Consequently, the proposed estimation method can be effectively employed to estimate the TMR level, especially when SNR and TMR are high. However, the low SNR levels provide an accuracy of less than 40%, in particular, when the TMR level is low because the effect of TMR is dominant by high AWGN noise. Therefore, the accuracy of the proposed method is not well.

#### V. CONCLUSION

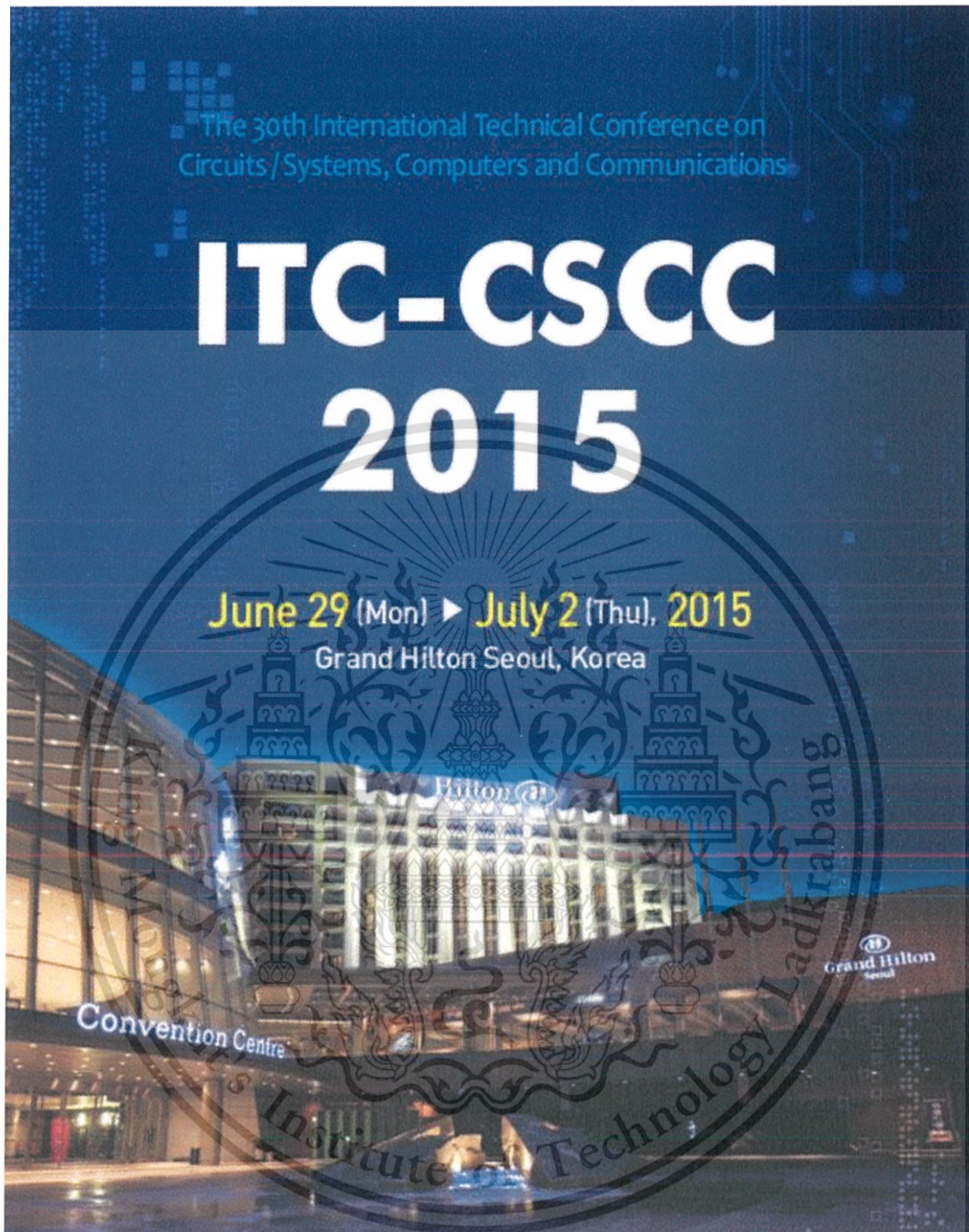
This paper proposes the TMR estimation method by using only the readback signal for a BPMR system corrupted by AWGN and, inter-symbol interference and inter-track interference. Specifically, we first estimate the SNR based on the peak amplitude of the readback signal. Then, the estimated TMR level is determined based on the readback signal energy and the estimated SNR level. Experiment results indicate that the proposed method can effectively estimate the TMR level, especially when SNR and TMR are large. It should be pointed out that this estimation method can be adapted in designing a two-dimensional (2D) equalizer and a 2D detector in BPMR systems so as to cope with this TMR problem.

## ACKNOWLEDGMENT

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# A Simple TMR Mitigation Approach for Bit Patterned Media Recording Based on Readback Signals

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## Abstract

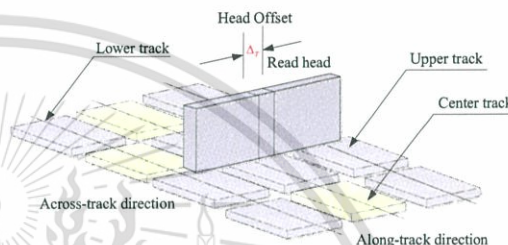
Track mis-registration (TMR) effect is the important issue in the ultra-high magnetic recording systems. Practically, this effect can be handled by the servo system. However, it needs some recording areas on the recorded media for storage the redundancy bits. Therefore, this paper proposes a simple TMR mitigation approach based on the readback signal in bit-patterned media recording (BPMR) systems. We begin with calculating the mean-square error (MSE) between the equalizer's outputs and the fixed target's outputs for each TMR levels, which next will be collected in the look-up table. Then, the TMR levels are estimated utilizing these data from the look-up table before selecting the properly equalizer coefficients which matches with TMR level. In simulation results, we found that the proposed TMR mitigation approach offers the performance gain better than the conventional system, especially when the TMR is high.

**Keywords:** bit-patterned media recording; track mis-registration; mean square error.

## 1. Introduction

Perpendicular magnetic recording (PMR) systems are expected to reach storage density limits in the near future. Bit-patterned media recording (BPMR) has been widely recognized as one of a promising technology that can overcome the superparamagnetic limitation [1]. We believe that this technology can be extended the storage density of magnetic storage systems beyond 1 Tera-bit per inches square (Tb/in<sup>2</sup>).

Track mis-registration (TMR) effect is one of the seriously problem in magnetic recording systems, especially when the areal density (AD) is high since the separation between tracks in BPMR has to be reduced to achieve higher densities [2-3]. Therefore, this results in a cause of easier TMR effect. TMR effect occurred when the center of read head misaligns the center of the read track data sequence as shown Fig. 1.



**Fig. 1:** Illustration of TMR or head offset,  $\Delta_T$ , in a BPMR system.

In the real system, the TMR effect can be controlled by the servo system [4]. However, the servo system needs some recording areas on the recorded media for storage the redundancy bits.

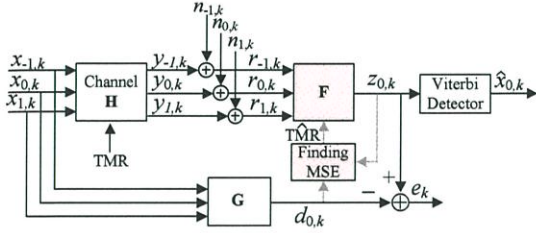
Alternatively, several TMR estimation methods based on the readback signal have been proposed in the literature [5-6]. Nonetheless, these methods required the knowledge of some recorded data and employed high complexity processing e.g., calculating many correlation functions for estimating TMR levels [5], finding both of readback signal energy and averaged peak amplitude [6].

In this paper; therefore, we propose to estimate the TMR levels from the readback signals but only calculate mean-square error (MSE) between the equalizer's outputs and the fixed target's outputs. Then, we mitigate the TMR effect by adjusting the equalizer's coefficient. Simulation results show that the proposed system performs better than the conventional system, especially when the TMR is high.

This paper is organized as follows. In Section 2, the BPMR channel model is described. Section 3, we explain the proposed TMR estimation/mitigation approach. The simulation results are given in Section 4. Finally, Section 5 concludes this paper.

## 2. BPMR Channel Model

In the channel model, we investigate on a discrete BPMR channel model [5-6] and the readback signal,  $r_{l,k}$ , from the  $k$ -th data bit on the  $l$ -th track can be determined by



**Fig. 2:** Block diagram of BPMR system with the proposed TMR mitigation approach.

$$r_{l,k} = \sum_n \sum_m h_{m,n} x_{l-m,k-n} + n_{l,k}, \quad (1)$$

where  $x_{l,k}$ 's are recorded bit,  $n_{l,k}$ 's are electronics noises modeled as an additive white Gaussian noise (AWGN), and  $h_{m,n}$ 's are the channel coefficients which can be obtained by sampling the isolated island pulse response at integer multiples of the bit period  $T_x$  and the track pitch  $T_z$ , i.e.,

$$h_{m,n} = P(mT_x, nT_z + \Delta_T), \quad (2)$$

where  $P(x,z)$  is the 2-D Gaussian pulse response,  $x$  and  $z$  are the time indices in along-track and across-track direction, and  $\Delta_T$  is the head offset or the distance between the center head and the center of the main track [6]. In this paper, the sign of  $\Delta_T$  is assumed to be positive for the upward offset, where the TMR level is defined as

$$\text{TMR} (\%) = 100 \times (\Delta_T / T_z). \quad (3)$$

At the receiver, the readback sequences are equalized by a 2-D equalizer and are fed to a Viterbi detector [5] to produce the most likely of the estimated data input sequence.

### 3. Proposed TMR Mitigation Approach

Firstly, we have designed the 2-D equalizer coefficients with the fixed 2-D target in each TMR levels e.g., at 0%, 5%, 10%, 15%, 20%, and 25%. Therefore, we will obtain the six sets of 2-D equalizer coefficients. These 2-D equalizer coefficients will be kept in look-up table which storage in an alternate data memory.

Then, the MSE are determined using the equalizer's outputs,  $z_{0,k}$  and the fixed target's output,  $d_{0,k}$  for each TMR levels as shown in Fig. 2. It is importance to note that the 2-D equalizer coefficients used in this process was designed without corrupting of TMR effect (TMR = 0%) and signal-to-noise ratio (SNR) is fixed at 20 decibel (dB). The MSE values for each TMR level can then be calculated as following equation,

$$\text{MSE} = \frac{1}{S} \sum_{k=1}^S (z_{0,k} - d_{0,k})^2, \quad (4)$$

where  $S$  is the length of the equalizer's outputs or target's outputs. Therefore, there will be six MSE values as illustrated in Fig. 2, which will be next used for creating the estimated TMR equation.

In this work, we use a curve fitting technique [6] so as to obtain a polynomial function that provides the best fit line to the MSE values as illustrate in Fig. 3. This polynomial function can be utilized to approximate the TMR level according to following equation,

$$\hat{\text{TMR}} = b_0 + b_1 \text{MSE} + b_2 \text{MSE}^2 + \dots + b_N \text{MSE}^N, \quad (5)$$

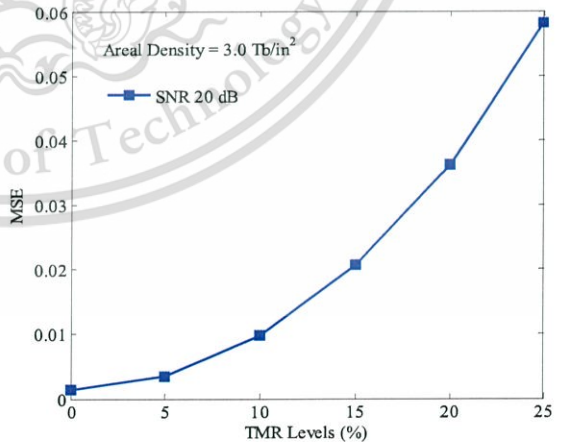
where  $\hat{\text{TMR}}$  is the estimated TMR,  $b_i$  and  $N$  are the coefficient and a degree of the polynomial equation in (5), respectively, and  $i \in \{0, 1, \dots, N\}$ , where  $N = 2$  provides the best fit between the actual and the estimated TMR levels.

### 4. Simulation Results

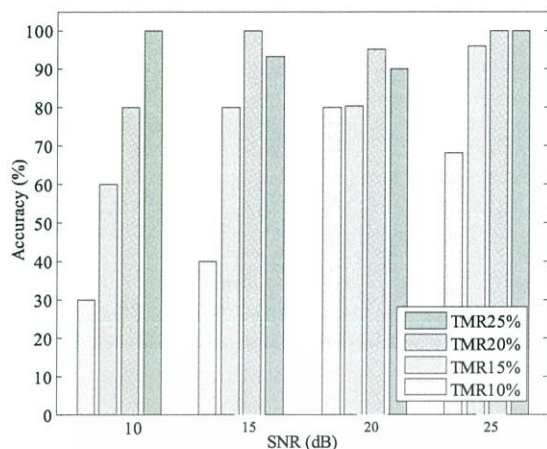
We test the proposed TMR mitigation approach in the BPMR system given in Fig. 2 at AD of 3.0 Tb/in<sup>2</sup>, where both the bit period and the track pitch are  $T_x = T_z = 14.5$  nm, the along-track  $\text{PW}_{50}$  is 19.4 nm, and the across-track  $\text{PW}_{50}$  is 24.8 nm, as similar to [5-6]. The SNR is defined as  $\text{SNR} = 20 \log_{10}(1/\sigma)$ , in decibel (dB), where  $\sigma$  is a standard deviation of AWGN. In addition, the accuracy of TMR estimation is measured by

$$\text{accuracy} (\%) = 100 - \frac{|\hat{\text{TMR}} - \text{TMR}|}{\text{TMR}} \times 100, \quad (6)$$

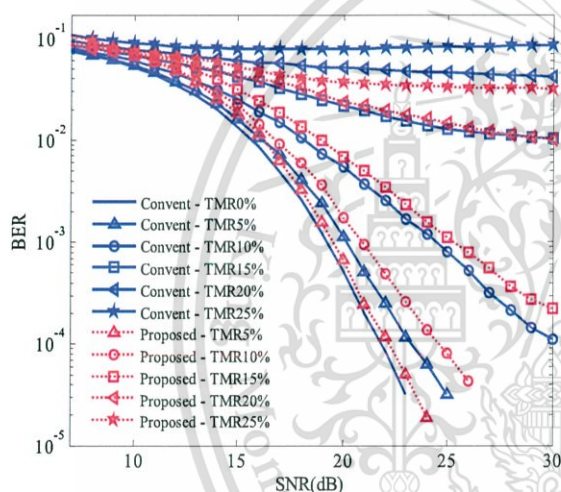
where  $\hat{\text{TMR}}$  is the estimated TMR obtained from the proposed method, and TMR is the actual TMR embedded in the equalizer's outputs.



**Fig. 3:** The relationship between the TMR levels (%) and the MSE of the equalizer's outputs and fixed target's output.



**Fig. 4:** The relationship between the SNR levels and the percentage of the estimation accuracy of the proposed method.



**Fig. 5:** The BER performance comparison between the proposed TMR mitigation approach and the conventional recording system.

Fig. 4 shows the TMR estimation accuracy (in percentage) of the proposed method in BPMR system at AD of 3.0 Tb/in<sup>2</sup>. Clearly, the proposed method can achieve a good estimation of TMR level, especially when TMR is large and SNRs are high. For example, the proposed method provide the accuracy percentage more than 80% of TMR estimation when SNR is greater than 15 dB and TMR is larger than 15%. Therefore, it can be concluded that the proposed method can be effectively used to estimate the actual TMR from the MSE of the equalizer's outputs and fixed target's output. However, at the low TMR level need to be considered since the accuracy percentage is low. The estimation accuracy is still less than 40%, when TMR and SNR are small. This might be because the effect of TMR at 0% to 5% is very similar level and AWGN is dominant because of low SNR.

We also compare the BER performance of the proposed systems with the conventional system as illustrated in Fig. 5, where the conventional system performs without any TMR mitigation method. The labeled as "Convent-TMR PT%" represents the conventional system. The PT % is the TMR effect PT%. The one labeled as "Proposed-TMR PT%" represent the proposed MTR mitigation approach. Clearly, the proposed system performs better than the conventional system for all TMR levels, especially when the TMR is high. It is very importance to note that the proposed method in this paper, we do not consider the position jitter noise.

## 5. Conclusion

This paper proposes the TMR estimation method by using MSE to find different values between the equalizer's outputs and fixed target's output for a BPMR system corrupted by AWGN and 2-D interference. Then, these different values are utilized for selecting the properly 2-D equalizer coefficients. The experiment results indicate that the proposed methods can effectively estimate the TMR levels and also shows that the proposed method can offer the BER performance gain over the conventional system at high AD and high TMR levels.

## Acknowledgement

This work was supported by College of Data Storage Innovation (D\*STAR), King Mongkut's Institute of Technology Ladkrabang Research Fund, and Nakhon Pathom Rajabhat University, Thailand.

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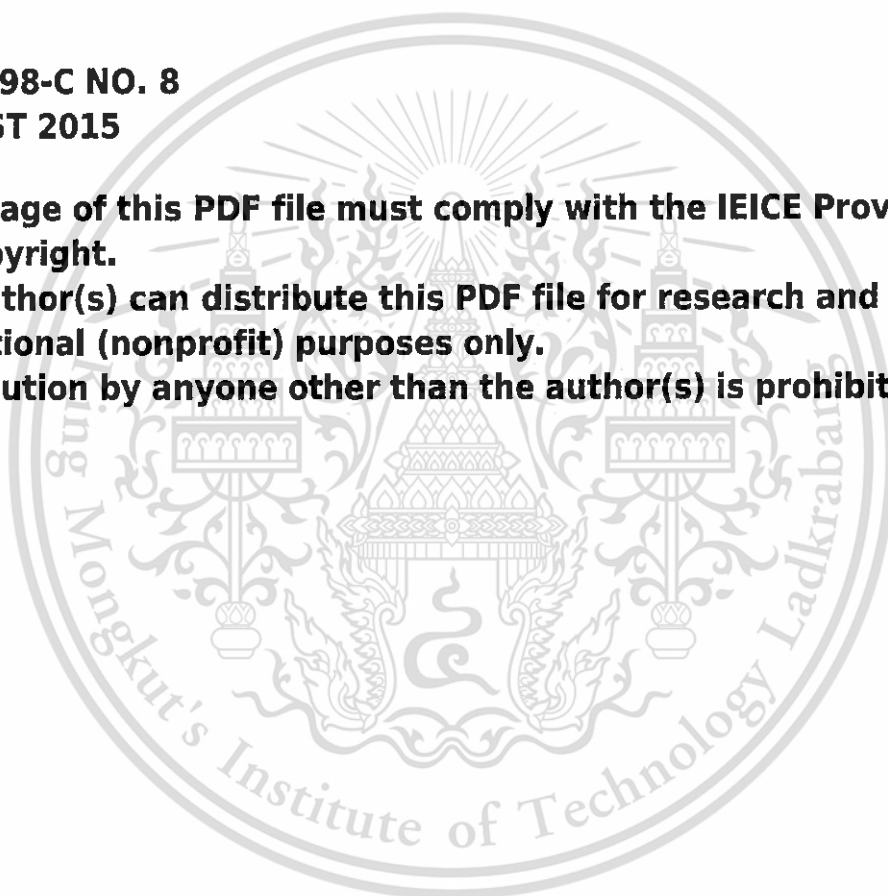
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## PAPER

# A TMR Mitigation Method Based on Readback Signal in Bit-Patterned Media Recording

Wiparat BUSYATRAS<sup>†</sup>, Nonmember, Chanon WARISARN<sup>†a)</sup>, Member, Lin M. M. MYINT<sup>††</sup>, Nonmember, and Piya KOVINTAVEWAT<sup>†††b)</sup>, Member

**SUMMARY** Track mis-registration (TMR) is one of the major problems in high-density magnetic recording systems such as bit-patterned media recording (BPMR). In general, TMR results from the misalignment between the center of the read head and that of the main track, which can deteriorate the system performance. Although TMR can be handled by a servo system, this paper proposes a novel method to alleviate the TMR effect, based on the readback signal. Specifically, the readback signal is directly used to estimate a TMR level and is then further processed by the suitable target and equalizer designed for such a TMR level. Simulation results indicate that the proposed method can sufficiently estimate the TMR level and then helps improve the system performance if compared to the conventional receiver that does not employ a TMR mitigation method, especially when an areal density is high and/or a TMR level is large.

**key words:** bit-patterned media recording, estimation method, signal-to-noise ratio, track mis-registration, two-dimensional equalization

## 1. Introduction

Bit-patterned media recording (BPMR) is one of the promising recording technologies for the next generation's hard disk drives, which can achieve an areal density (AD) up to 4 Tera-bits per square inch (Tb/in<sup>2</sup>) [1]. In BPMR, a data bit is stored in a single domain magnetic island, surrounded by non-magnetic material. To increase storage capacity, the spacing between the data bit islands in both the along- and the across-track directions must be decreased, thus leading to the increase of two-dimensional (2D) interference. In general, the 2D interference consisting of inter-symbol interference (ISI) and inter-track interference (ITI) [2] can considerably degrade the system performance if precautions are not taken.

In addition to the 2D interference, BPMR also encounters other challenging issues, including write synchronization error, media noise, and track mis-registration (TMR), which can further deteriorate the system performance. Therefore, a good read-channel design should provide robustness and reliability to tackle these issues. How-

ever, this paper focuses on how to mitigate the TMR effect, because it can significantly lead to performance degradation, especially in high-density BPMR systems.

Practically, TMR (or a head offset) is occurred when the center of the read head is not aligned with that of the main track [3], [4] as illustrated in Fig. 1. For example, the TMR could happen when the disk rotation speed is suddenly increased for high transfer rate and access time, while the read head moves to read data on the main track [5]. Generally, the TMR can yield a devastating impact on the data recovery process because it causes an unequal effect of the adjacent tracks on the main track, thus lowering the quality of the readback signal. Moreover, the TMR effect results in the mismatch between the readback signal and the design of the target and its corresponding equalizer, which in turn causes the detector to perform unreliably.

In general, the TMR can be controlled by a servo system [4], [5]. Specifically, a servo burst field has the information that can be used to estimate the amount of head offset, but it is difficult to estimate the behavior of the read head when the TMR occurred beyond the limit [5]. Alternatively, Myint and Supnithi [6] detected the presence of TMR from the readback signal based on the observation of 2D target-shaping equalizer coefficients, and then adjusted the 2D target and equalizer to be asymmetric so as to taken care of the TMR-affected readback signal. Nevertheless, we found that the method in [6] cannot accurately estimate the TMR level, and both the 2D target and equalizer are not efficiently matched with the BPMR channel with TMR, thus leading to the performance degradation at the data detection process.

To solve this problem, this paper proposes a novel

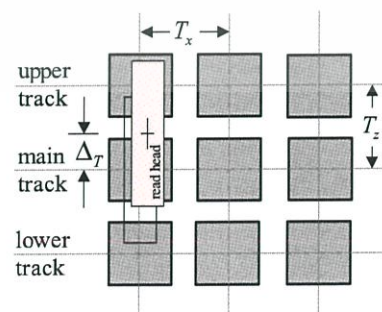


Fig. 1 The illustration of island structure configuration in a BPMR system with track mis-registration (TMR),  $\Delta_T$ .

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method to mitigate the TMR effect based on the readback signal. To do so, we study the statistical relationship among the readback signal, a signal-to-noise ratio (SNR), and TMR amount for various cases. Next, we formulate the mathematical equations so as to estimate the SNR and the TMR level based on the readback signal. Specifically, the SNR is estimated from the average peak readback amplitude, whereas the TMR level is computed from the estimated SNR and the energy of the readback signal. Then, the estimated TMR level will be utilized to choose the target and its corresponding equalizer from a look-up table that are suitable for the channel with TMR so as to facilitate the data detection process. Note that each pair of the target and its equalizer is designed for each TMR level and is stored in the look-up table.

The rest of this paper is organized as follows. Section 2 briefly describes a BPMP channel model, and Sect. 3 explains the proposed method. Simulation results are given in Sect. 4. Finally, Sect. 5 concludes this paper.

## 2. BPMP Channel Model

This work focuses on a discrete-time BPMP channel model with multi-track processing [7], as depicted in Fig. 2. A binary input data sequence  $x_{l,k} \in \{\pm 1\}$  with bit period  $T_x$ , where  $l = 0$  is the main track,  $l = -1$  is the upper track, and  $l = 1$  is the lower track, is sent to the BPMP channel corrupted by TMR and electronics noise modeled as an additive white Gaussian noise (AWGN). Then, the readback signal of the  $k$ th data bit on the  $l$ th track can be expressed as

$$\begin{aligned} r_{l,k} &= x_{l,k} \otimes h_{l,k} + w_{l,k} \\ &= \sum_m \sum_n h_{m,n} x_{l-m,k-n} + w_{l,k}, \end{aligned} \quad (1)$$

where  $x_{l,k}$ 's are the recorded bits,  $h_{m,n}$ 's are the 2D channel coefficients,  $m$  and  $n$  represent the time indices of the bit island in the across- and the along-track directions, and  $w_{l,k}$  is an AWGN with zero mean and variance  $\sigma^2$ .

Practically, the BPMP channel coefficients  $h_{m,n}$  can be generated by sampling a 2D Gaussian pulse response at the integer multiples of the bit period  $T_x$  and the track pitch  $T_z$  according to

$$h_{m,n} = P(mT_z + \Delta_T, nT_x) \quad (2)$$

where  $P(z, x)$  is the 2D Gaussian pulse response,  $z$  and  $x$  are the time indices in the across- and the along-track directions,  $\{m, n\} \in (-L, \dots, 0, \dots, L)$ ,  $2L + 1$  is the length of  $P(z, x)$ ,  $L$  is an integer, and  $\Delta_T$  is the head offset or the distance between the center of the read head and that of the main track as depicted in Fig. 1. Generally,  $L$  should be large enough to ensure that the tail amplitude of  $P(z, x)$  is small, where this paper considers  $L = 1$  for simplicity.

In this paper, the TMR level is defined as

$$\text{TMR (\%)} = \frac{\Delta_T}{T_z} \times 100, \quad (3)$$

where the sign of  $\Delta_T$  is assumed to be positive for the upward offset as shown in Fig. 1. Furthermore, we consider the 2D Gaussian pulse response of the form [2]

$$P(z, x) = A \exp \left\{ -\frac{1}{2c^2} \left[ \left( \frac{x}{PW_x} \right)^2 + \left( \frac{z + \Delta_T}{PW_z} \right)^2 \right] \right\}, \quad (4)$$

where  $A = 1$  is assumed to be the peak amplitude of the pulse response,  $PW_x$  is the  $PW_{50}$  of the along-track pulse,  $PW_z$  is the  $PW_{50}$  of the across-track pulse,  $PW_{50}$  is the pulse width at half its maximum, and  $c = 1/2.3548$  is a constant to account for the relationship between  $PW_{50}$  and the standard deviation of a Gaussian pulse.

In a conventional receiver that does not employ a TMR mitigation method, the readback signal  $r_{l,k}$  for  $l \in \{0, \pm 1\}$  is fed to a 2D equalizer followed by a 2D Viterbi detector to determine the most likely input sequence on the main track, i.e.,  $\hat{x}_{0,k}$ . Note that this paper does not take media noise into account and considers only the system that recovers the recorded data on the main track, as similar to [6]. Hence, three adjacent readback signals  $\{r_{-1,k}, r_{0,k}, r_{1,k}\}$  at the input of a 2D equalizer,  $F$ , are required to generate a single output  $\{z_{0,k}\}$ , whereas three input data sequences  $\{x_{-1,k}, x_{0,k}, x_{1,k}\}$  are sent to a 2D target,  $G$ , to output the desired data sequence  $\{d_{0,k}\}$ .

## 3. Proposed Method

We propose a novel method to subside the TMR effect in a BPMP channel, based on the readback signal. Specifically, we first estimate a TMR level with an aid of the estimated SNR and the average energy of the readback signal. Hence, the target and its corresponding equalizer suitable for the channel with TMR are selected according to the estimated TMR level so as to ease the data recovery process. The details of the proposed method can be explained as follows.

### 3.1 SNR Estimation

In this paper, the SNR is defined as [8]

$$\text{SNR} = 20 \log_{10}(V_p/\sigma), \quad (5)$$

in decibel (dB), where  $V_p$  is the peak amplitude of the readback signal, which is assumed to be 1, and  $\sigma$  is a standard deviation of AWGN. This SNR will be estimated before predicting the amount of the TMR. Here, we propose to estimate the SNR from the peak amplitude of the readback signal. To do so, we collect a large number of samples (e.g., 1000 samples) of the readback signals at each SNR ranged from 0 to 25 dB, where each readback signal is affected by a uniformly distributed random TMR level ranged from 0% to 25%. Then, for each SNR, the average value of the peak amplitude of all readback signals,  $r_{\text{peak}}$ , is computed, regardless of TMR levels.

Figure 3 illustrates the relationship between the SNR and the average peak amplitude of the readback signals at the AD of 2.0 Tb/in<sup>2</sup>, where we found that the SNR can be

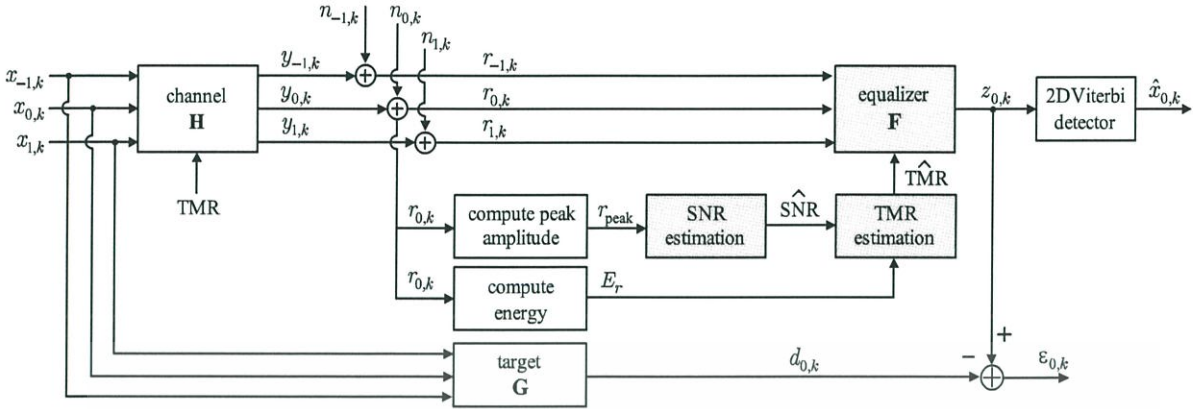


Fig. 2 A BPMR channel model with the proposed TMR mitigation method.

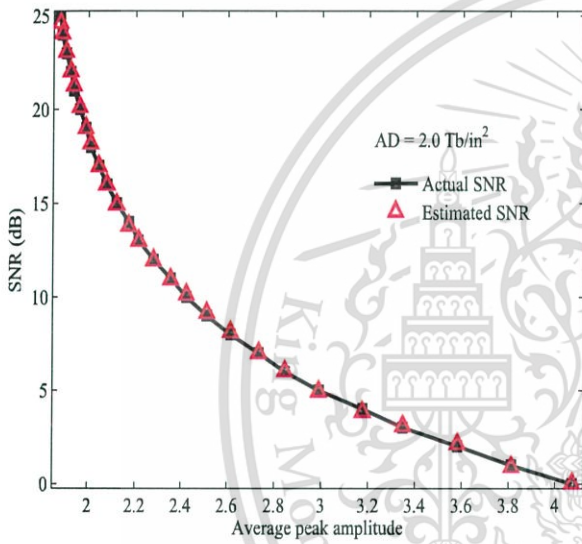


Fig. 3 The relationship between the SNR and the average peak amplitude of the readback signals at AD = 2.0 Tb/in<sup>2</sup>.

possibly approximated. To achieve this, we apply a least-squares (LS) fitting technique to fit all data points to an  $M$ -degree polynomial equation according to

$$\hat{\text{SNR}} = a_0 + a_1 r_{\text{peak}} + a_2 r_{\text{peak}}^2 + \dots + a_M r_{\text{peak}}^M \quad (6)$$

where  $\hat{\text{SNR}}$  is the estimated SNR,  $a_i$  is the  $i$ th coefficient of the polynomial equation in (6), and  $i \in \{0, 1, \dots, M\}$ . Based on extensive heuristic search, we found that  $M = 5$  is sufficient for our channel model at the AD up to 3.0 Tb/in<sup>2</sup> because a higher order does not provide any benefit on the accuracy of SNR estimation. As depicted in Fig. 3, it is apparent that the estimated SNR effectively coincides with the actual SNR.

### 3.2 TMR Estimation

For each SNR, we also propose to employ the energy of the readback signal to estimate the TMR level. To do so, we

compute the average energy of the readback signal,  $E_r$ , for each SNR and TMR level according to

$$E_r = \frac{1}{S} \sum_{k=1}^S r_{0,k}^2 \quad (7)$$

where  $S$  is the length of the readback signal samples (i.e.,  $S = 32768$  bits for a 4K-data sector [9]). Next, the estimated TMR level is obtained based on a polynomial LS fitting technique, i.e.,

$$\hat{\text{TMR}} = b_0 + b_1 E_r + b_2 E_r^2 + \dots + b_Q E_r^Q \quad (8)$$

where  $\hat{\text{TMR}}$  is the estimated TMR,  $b_i$  and  $Q$  are the  $i$ th coefficient and a degree of the polynomial equation in (8), respectively, and  $i \in \{0, 1, \dots, Q\}$ . Similarly, we perform an extensive simulation search to find a suitable  $Q$ , where  $Q = 5$  provides the best fit between the actual and the estimated TMR levels.

Figure 4 shows the estimated TMR level as a function of the average energy of the readback signal at the AD of 2.0 Tb/in<sup>2</sup> for various SNRs. Clearly, the TMR level can be effectively estimated from (8) based on  $\hat{\text{SNR}}$  and  $E_r$ .

### 3.3 Equalizer and Target Design

The target and its corresponding equalizer used in this work are designed for each TMR level based on minimizing a mean-squared error (MSE) [10] according to

$$E \{ \varepsilon_{l,k}^2 \} = E \{ (z_{l,k} - d_{l,k})^2 \}, \quad (9)$$

where  $E\{\cdot\}$  is an expectation operator, and  $\varepsilon_{l,k}$  is an error signal between the equalizer output  $z_{l,k}$  and the desired output  $d_{l,k}$ . By expanding the right-hand side in (9), we obtain

$$\begin{aligned} E \{ \varepsilon_{l,k}^2 \} &= E \{ [(r_{l,k} \otimes f_{l,k}) - (x_{l,k} \otimes g_{l,k})]^2 \} \\ &= f_{l,k} \otimes R_{l,k}^r \otimes f_{l,k} - 2 f_{l,k} \otimes R_{l,k}^r \otimes g_{l,k} \\ &\quad + g_{l,k} \otimes R_{l,k}^x \otimes g_{l,k}, \end{aligned} \quad (10)$$

where  $\otimes$  is the 2D convolution operator,  $R_{l,k}^r = E \{ r_{i,j} r_{i-l,j-k} \}$  and  $R_{l,k}^x = E \{ x_{i,j} x_{i-l,j-k} \}$  are the auto-correlations of the readback signals and the recorded bits from all three tracks, respectively, and  $R_{l,k}^{rx} = E \{ r_{i,j} x_{i-l,j-k} \}$  is the cross-correlation

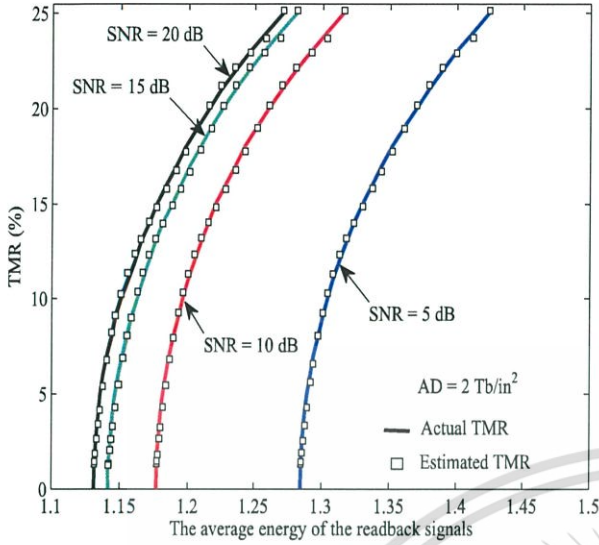


Fig. 4 The relationship between the TMR level and the average energy of the readback signals at AD = 2.0 Tb/in<sup>2</sup>.

between the readback signals and the recorded bits.

To find the solution of (10), it is convenient to represent the matrices in the vector forms [10]. To do so, we let  $\mathbf{F}$  be a  $3 \times (2N + 1)$  equalizer matrix of the form

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_{-1} \\ \mathbf{f}_0 \\ \mathbf{f}_1 \end{bmatrix} = \begin{bmatrix} f_{-1,-N} & f_{-1,0} & f_{-1,N} \\ f_{0,-N} & f_{0,0} & f_{0,N} \\ f_{1,-N} & f_{1,0} & f_{1,N} \end{bmatrix}, \quad (11)$$

where  $f_{l,k}$ 's are the equalizer coefficients,  $l \in \{0, \pm 1\}$  is the track location,  $k \in \{-N, \dots, 0, \dots, N\}$ , and  $2N + 1$  is the equalizer length. Similarly, let  $\mathbf{G}$  be a  $3 \times 3$  target matrix of the form

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_{-1} \\ \mathbf{g}_0 \\ \mathbf{g}_1 \end{bmatrix} = \begin{bmatrix} g_{-1,-1} & g_{-1,0} & g_{-1,1} \\ g_{0,-1} & g_{0,0} & g_{0,1} \\ g_{1,-1} & g_{1,0} & g_{1,1} \end{bmatrix}, \quad (12)$$

where  $g_{l,k}$ 's are the target coefficients,  $l \in \{0, \pm 1\}$  is the track location, and  $k \in \{-1, 0, 1\}$ .

In general, the matrices  $\mathbf{F}$  and  $\mathbf{G}$  can be rearranged into the column vectors as  $\mathbf{f} = [\mathbf{f}_{-1} \ \mathbf{f}_0 \ \mathbf{f}_1]^T$  and  $\mathbf{g} = [\mathbf{g}_{-1} \ \mathbf{g}_0 \ \mathbf{g}_1]^T$ , respectively, where the component vectors are defined in (11) and (12), and  $[\cdot]^T$  is a transpose operator. Using these matrices, the MSE in (10) can be rewritten as

$$E\{\varepsilon_{0,k}^2\} = \mathbf{f}^T \mathbf{R}_r \mathbf{f} - 2\mathbf{f}^T \mathbf{R}_{rx} \mathbf{g} + \mathbf{g}^T \mathbf{R}_x \mathbf{g}, \quad (13)$$

where  $\mathbf{R}_r = [\mathbf{r}_k \mathbf{r}_k^T]$  is a  $3(2N+1) \times 3(2N+1)$  auto-correlation matrix of  $R_{l,k}^r$ ,  $\mathbf{r}_k$  is the readback signal vector,  $\mathbf{R}_{rx} = [\mathbf{r}_k \mathbf{x}_k^T]$  is a  $3(2N+1) \times 9$  cross-correlation matrix of  $R_{l,k}^{rx}$ ,  $\mathbf{x}_k$  is the recorded bit vector, and  $\mathbf{R}_x = [\mathbf{x}_k \mathbf{x}_k^T]$  is a  $9 \times 9$  auto-correlation matrix of  $R_{l,k}^x$ .

Because we focus only on detecting the data on the main track (i.e.,  $l = 0$ ), the MSE in (9) can then be computed by

$$E\{\varepsilon_{0,k}^2\} = E\{(z_{0,k} - d_{0,k})^2\}. \quad (14)$$

Hence, in this case, the readback signal vector and the recorded bit vector will be given by  $\mathbf{r}_k = [r_{1,k+N} \ r_{1,k+N-1} \ \dots \ r_{0,0} \ \dots \ r_{-1,k-N+1} \ r_{-1,k-N}]^T$  and  $\mathbf{x}_k = [x_{1,k+1} \ x_{1,k} \ \dots \ x_{0,0} \ \dots \ x_{-1,k} \ x_{-1,k-1}]^T$ , respectively.

During the minimization process of the MSE in (13), we impose a constraint of  $\mathbf{e}^T \mathbf{g} = 1$  to avoid reaching trivial solutions of  $\mathbf{f} = \mathbf{g} = \mathbf{0}$ , where  $\mathbf{e} = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]^T$  is a column vector with 9 entries. Accordingly,  $\mathbf{f}$  and  $\mathbf{g}$  are chosen such that

$$E\{\varepsilon_{0,k}^2\} = \mathbf{f}^T \mathbf{R}_r \mathbf{f} - 2\mathbf{f}^T \mathbf{R}_{rx} \mathbf{g} + \mathbf{g}^T \mathbf{R}_x \mathbf{g} - 2\lambda^T (\mathbf{e}^T \mathbf{g} - 1) \quad (15)$$

is minimized, where  $\lambda$  is a Lagrange multiplier [10], [11]. Then, the minimization process gives

$$\lambda = \frac{1}{\mathbf{e}^T (\mathbf{R}_x - \mathbf{R}_{rx}^T \mathbf{R}_r^{-1} \mathbf{R}_{rx})^{-1} \mathbf{e}} \quad (16)$$

$$\mathbf{g} = \lambda (\mathbf{R}_x - \mathbf{R}_{rx}^T \mathbf{R}_r^{-1} \mathbf{R}_{rx})^{-1} \mathbf{e} \quad (17)$$

$$\mathbf{f} = \mathbf{R}_r^{-1} \mathbf{R}_{rx} \mathbf{g}. \quad (18)$$

Note that if the target  $\mathbf{g}$  is given, one can still employ (18) to obtain the equalizer  $\mathbf{f}$  that minimizes the MSE in (15).

### 3.4 TMR Mitigation Methods

In this work, we propose two methods to mitigate the TMR effect based on the structure of the 2D target (i.e., symmetric or asymmetric). Then, the performance of the proposed methods will be compared with that of a conventional receiver, which employs fixed 2D target and equalizer designed for 0% TMR level. Without TMR and media noise, the channel response in (2) will normally be symmetric, and we found that the 2D target  $\mathbf{G}$  obtained from this design is also *symmetric* because the target coefficients  $\mathbf{g}_{-1}$  and  $\mathbf{g}_1$  in (12) are almost equal.

For the first proposed method (denoted as the *symmetric* system), the symmetric 2D target as used in the conventional receiver is employed, but the equalizer is selected according to the estimated TMR level. To do so, we need to design the equalizers suitable for each TMR level based on (18), where the target is fixed, and store them in the look-up table. On the other hand, the second proposed method (denoted as the *asymmetric* system) utilizes the 2D target and its corresponding equalizer specially designed for each TMR level according to (16) – (18), where we refer to this 2D target as the *asymmetric* target because the target coefficients  $\mathbf{g}_{-1}$  and  $\mathbf{g}_1$  in (12) are not equal. Thus, each pair of the target and equalizer associated with a given TMR level will be kept in the look-up table. Table 1 shows an example of the coefficients of the asymmetric 2D targets for some TMR levels at the ADs of 2.0 and 3.0 Tb/in<sup>2</sup>, which are used in this paper.

It should be pointed that the 2D Viterbi detector is designed for the asymmetric 2D target [11] as illustrated in Table 1. The three bits from the three adjacent tracks (upper, main, and lower tracks) are sensed by the read head so that

**Table 1** The coefficients of the asymmetric 2D targets for some TMR levels at the ADs of 2.0 and 3.0 Tb/in<sup>2</sup>.

TMR (%)	2D target coefficients, G					
	2.0 Tb/in <sup>2</sup>			3.0 Tb/in <sup>2</sup>		
0%	0.0201	0.2187	0.0201	0.0692	0.3255	0.0692
	0.0866	0.9422	0.0866	0.1785	0.8398	0.1785
	0.0201	0.2187	0.0201	0.0692	0.3255	0.0692
5%	0.0232	0.2526	0.0232	0.0759	0.3571	0.0759
	0.0864	0.9402	0.0864	0.1781	0.8381	0.1781
	0.0173	0.1886	0.0173	0.0628	0.2955	0.0628
10%	0.0267	0.2904	0.0267	0.0829	0.3902	0.0829
	0.0859	0.9343	0.0859	0.1770	0.8238	0.1770
	0.0149	0.1619	0.0149	0.0567	0.2671	0.0567
20%	0.0348	0.3784	0.0348	0.0977	0.4598	0.0977
	0.0836	0.9090	0.0836	0.1725	0.8120	0.1725
	0.0108	0.1176	0.0108	0.0458	0.2154	0.0458

each symbol represents these three bits resulting 8 combinations in total. The asymmetric 2D target has the current and 2 previous symbols (i.e., 2 memory taps) giving  $8^2 = 64$  states. Therefore, its trellis has 64 states and 8 outgoing branches from each state.

#### 4. Simulation Results

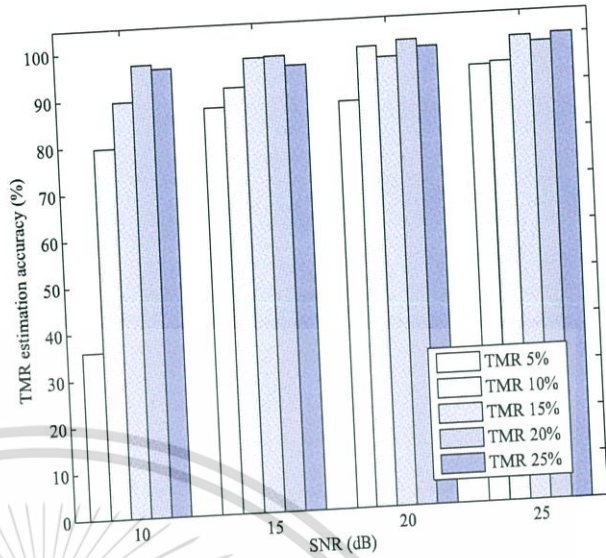
We study the performance of the proposed methods (both the symmetric and the asymmetric systems) in the BPMP system shown in Fig. 2 at the ADs of 2.0 and 3.0 Tb/in<sup>2</sup>, where both the bit period and the track pitch are  $T_x = T_z = 18$  and 14.5 nm, respectively. Additionally, this paper considers the 2D Gaussian pulse response with the along-track  $PW_{50}$  of 19.4 nm and the across-track  $PW_{50}$  of 24.8 nm, similar to [8]. Each bit-error rate (BER) point is computed using as many 4K-data sectors as required to collect a minimum number of 500 error bits. Furthermore, the accuracy of TMR estimation is measured by

$$\text{Accuracy}(\%) = 100 - \frac{|\hat{\text{TMR}} - \text{TMR}|}{\text{TMR}} \times 100, \quad (19)$$

where  $\hat{\text{TMR}}$  is the estimated TMR obtained from the proposed method, and TMR is the actual TMR embedded in the readback signal.

Figure 5 demonstrates the TMR estimation accuracy (in percentage) of the proposed method in BPMP system at the AD of 2.0 Tb/in<sup>2</sup>. Clearly, the proposed method can provide a good estimation of TMR level, especially when TMR is large and SNR is high. For example, it is possible to achieve 95% accuracy of TMR estimation when SNR is greater than 15 dB and TMR is larger than 15%. Therefore, it can be implied that the proposed method can be effectively used to estimate the actual TMR embedded in the readback signal, especially when SNR and TMR are high. Moreover, we found that the estimation accuracy is less than 40% when TMR and SNR are small. This might be because the effect of TMR ranged from 0% to 5% is very similar and AWGN is dominated because of low SNR.

We also compare the BER performance of the proposed systems with the conventional system for various ADs and


**Fig. 5** The relationship between the SNR levels and the percentage of the estimation accuracy of the proposed method at AD = 2.0 Tb/in<sup>2</sup>.

various TMR effect levels. The curve labeled “Conv-TMR 0%” represents the conventional system without TMR effect, which yields the best performance, and the curves labeled “Conv-TMR 5%”, “Conv-TMR 10%”, and “Conv-TMR 20%” represent the conventional system with TMR effect at 5%, 10%, and 20%, respectively. Moreover, the curve labeled “Proposed-Sym-TMR X%” represents the proposed method with a symmetric target at the TMR level of X%, whereas the curve labeled “Proposed-Asym-TMR X%” denotes the proposed method with an asymmetric target at the TMR level of X%.

At AD = 2.0 Tb/in<sup>2</sup>, the results show that the proposed methods (symmetric and asymmetric systems) yield slightly better performance than the conventional system for all TMR levels, as illustrated in Fig. 6 and Fig. 7. However, we can obtain a higher performance gap when the AD is increased, e.g., at AD = 3.0 Tb/in<sup>2</sup>. Specifically, when AD increases, not only the ITI effect is more severe, but also the TMR can easily occur in the system even though the read head slightly moves away from the main track. Fortunately, the proposed method can handle the severe TMR effect. We can see that the symmetric system performs slightly better than the conventional system at 5% TMR level and offers the performance gain about 4 dB at BER = 10<sup>-4</sup> and 10% of the TMR level; however, both the conventional and the symmetric systems cannot provide satisfactory performance for high TMR levels, as demonstrated in Fig. 8. On the other hand, the asymmetric system is superior to the conventional system, especially when TMR is high. Specifically, it can provide a performance gain over the conventional system about 0.5 and 6 dB at BER = 10<sup>-4</sup> for the TMR level of 5% and 10%, respectively, as shown in Fig. 9. Therefore, it is of importance to notice that the proposed TMR estimation method can be also well performed when it encounters with some media noise, e.g., position jitter (not shown here).

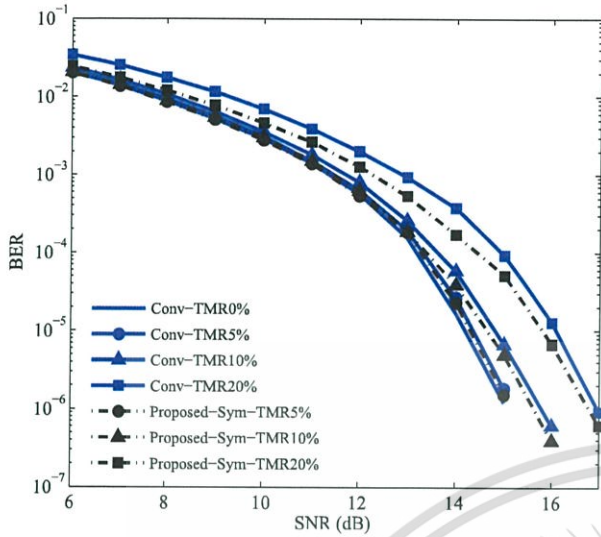


Fig. 6 BER performance between the conventional and the symmetric systems at  $AD = 2.0 \text{ Tb/in}^2$  for various TMR levels.

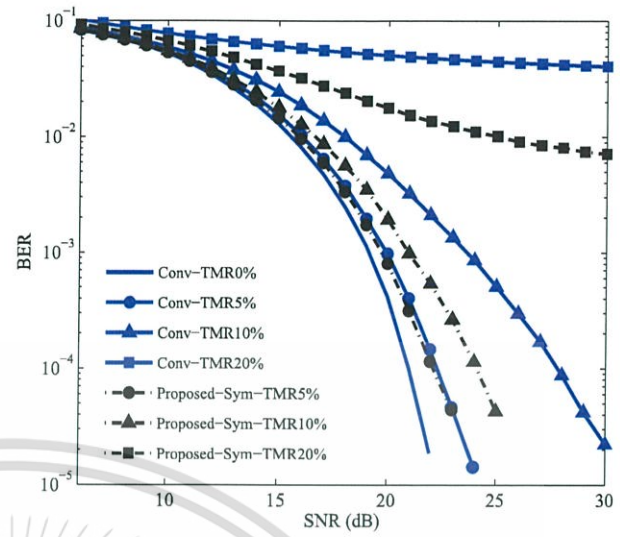


Fig. 8 BER performance between the conventional and the symmetric systems at  $AD = 3.0 \text{ Tb/in}^2$  for various TMR levels.

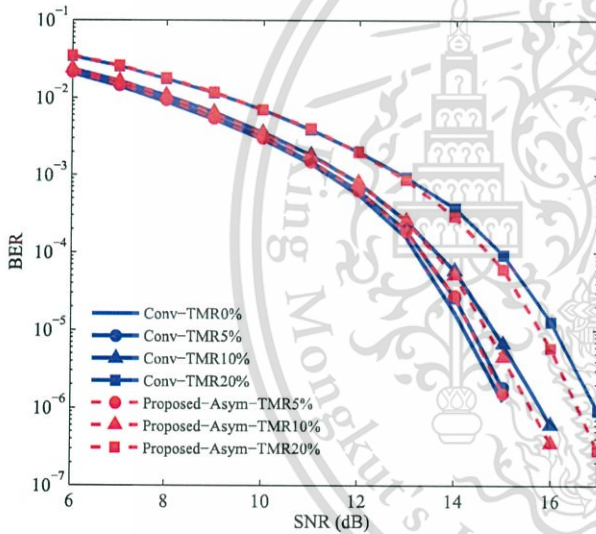


Fig. 7 BER performance between the conventional and the asymmetric systems at  $AD = 2.0 \text{ Tb/in}^2$  for various TMR levels.

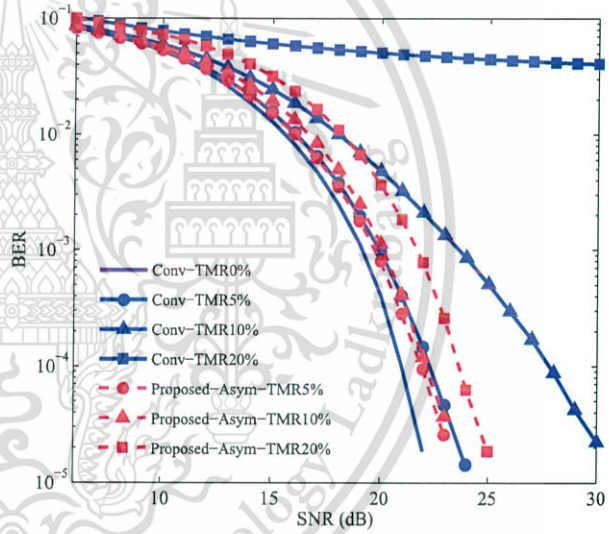


Fig. 9 BER performance between the conventional and the asymmetric systems at  $AD = 3.0 \text{ Tb/in}^2$  for various TMR levels.

Consequently, it can be implied that the proposed methods perform better than the conventional receiver because the detection process utilizes the 2D target and its corresponding equalizer that match with the BPMR channel with TMR. Nevertheless, the proposed methods require some extra memory to store the 2D target and the equalizer that are suitable for each TMR level.

### 5. Conclusion

This paper proposes the TMR mitigation method for a high-density BPMR system. It starts with estimating the SNR based on the average peak amplitude of the readback signals. Then, the estimated TMR level can be calculated based

on the estimated SNR and the average energy of the readback signals. Once the TMR level is known, we can choose the target and its corresponding equalizer that match with the BPMR channel with TMR in the data detection process so as to obtain a good system performance. Simulation results indicate that the proposed system can effectively estimate the TMR level, and it performs better than the conventional system, especially when the SNR and TMR are large.

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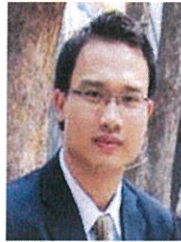
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# An Iterative TMR Mitigation Method Based on Readback Signal for Bit-Patterned Media Recording

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Off-track condition or track misregistration (TMR) is one of the most significant problems in the extremely high-density bit-patterned media recording (BPMR) system, since a track pitch becomes narrower. Typically, the TMR can be detected and handled by a servo system; however, it requires some special data to be inserted in the tracks so as to estimate the amount of head offset. Nonetheless, this paper proposes an iterative TMR mitigation method for BPMR systems based on the readback signals. First, we design several pairs of the 2-D asymmetric target and its corresponding 2-D equalizer that match the BPMR channel for each TMR level. Then, we exploit the three adjacent data tracks obtained from the low-density parity-check decoders to estimate the TMR level. Last, a pair of the 2-D asymmetric target and its corresponding 2-D equalizer that is best fit to the estimated TMR level will be used to alleviate the TMR effect in the readback signal for the next global iteration. Simulation results indicate that the proposed system can effectively estimate the TMR level and performs better than the conventional system without a TMR mitigation method, especially when the TMR level is high and/or the position jitter is large.

**Index Terms**—2-D equalization, bit-patterned media recording (BPMR), intertrack interference, track misregistration (TMR).

## I. INTRODUCTION

TRACK misregistration (TMR) is one of the major problems in the extremely high-density bit-patterned media recording (BPMR) system [1], [2] because a track pitch or a spacing between the adjacent tracks becomes narrower. This can easily cause the read head to position away from the center of the main track, as illustrated in Fig. 1. For example, this could happen when the disk rotation speed is suddenly increased for high transfer rate and access time, while the read head moves to read the data on the main track [3]. In general, the TMR can cause a devastating impact on the data recovery process, because it will enhance the interference from the adjacent tracks, i.e., intertrack interference, thus degrading the quality of the readback signal. Moreover, the TMR effect may lead to the mismatch between the readback signal and the design of the target and its corresponding equalizer, which in turn causes the detector to perform unreliably.

Although the servo system can be used to handle the TMR effect [3], [4], it requires some special data to be inserted in the tracks so as to estimate the amount of head offset. However, it is normally difficult to approximate the behavior of the read-head movement when the TMR occurs beyond its limitation [3], [5]. Alternatively, Myint and Supnithi [6] introduced the TMR detection method from the readback signals, based on the observation of the 2-D equalizer coefficients, and then adjusted the 2-D target and the 2-D equalizer to be asymmetric so as to handle the TMR-affected readback signal. Nevertheless, we found that this method can only perform

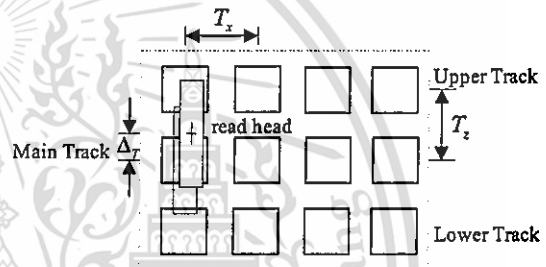


Fig. 1. TMR or a head offset,  $\Delta_T$ , in BPMR systems.

well when the TMR is not high (e.g.,  $\leq 10\%$ ) and an areal density (AD) is moderate (e.g.,  $\leq 2 \text{ Tb/in}^2$ ).

To solve the TMR problem, we propose an iterative TMR mitigation method to alleviate the TMR effect based on the readback signals. We first design several pairs of the 2-D asymmetric target and its corresponding 2-D equalizer that match the BPMR channel for each TMR level. Next, we utilize the three adjacent data tracks obtained from the output of the low-density parity-check (LDPC) decoders to estimate the TMR level. Hence, a pair of the 2-D target and its corresponding 2-D equalizer that is best fit to the estimated TMR level will be employed to cope with the TMR-affected readback signal during the next global iteration.

The rest of this paper is organized as follows. Section II describes the BPMR channel model, and Section III explains the proposed TMR mitigation method. Simulation results are given in Section IV. Finally, the conclusion is drawn in Section V.

## II. BPMR CHANNEL MODEL

Consider a discrete-time BPMR channel model [7], [8] in Fig. 2. An input data sequence  $u_{l,k} \in \{0, 1\}$  of length 3640 b with a bit period  $T_x$  is encoded by a rate-8/9 LDPC code [9] to obtain a sequence  $x_{l,k} \in \{\pm 1\}$  of length

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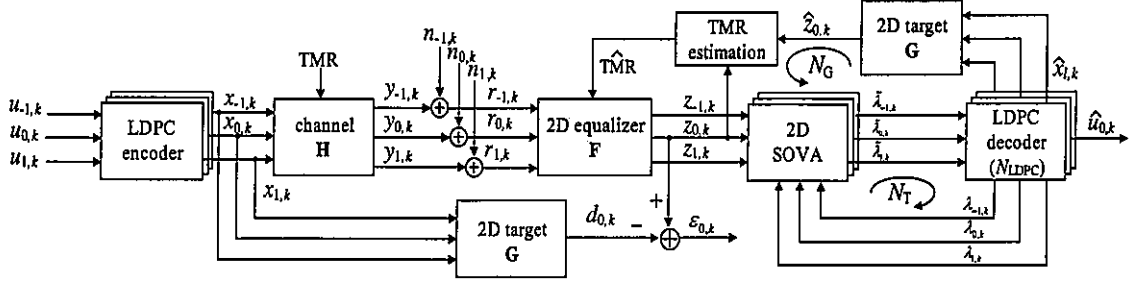


Fig. 2. BPMR channel model with the proposed TMR mitigation method.

4095 b, where the parity-check matrix has 3 ones in each column and 27 ones in each row. The readback signal of the  $k$ th data bit of the  $l$ th track can be expressed as

$$r_{l,k} = \sum_n \sum_m h_{m,n} x_{l-m,k-n} + n_{l,k} \quad (1)$$

where the values of  $x_{l,k}$  are the recorded bits,  $l \in \{0, -1, +1\}$  represents the main, upper, and lower track, respectively, the values of  $h_{m,n}$  are the 2-D channel coefficients,  $m$  and  $n$  represent the time indices of the bit island in the across-track and the along-track directions, respectively, and  $n_{l,k}$  is an additive white Gaussian noise with zero mean and variance  $\sigma^2$ .

In practice,  $h_{m,n}$  can be obtained by sampling the isolated island pulse response at the integer multiples of the bit period  $T_x$  and the track pitch  $T_z$  according to [10]

$$h_{m,n} = P(mT_z + \Delta_T, nT_x) \quad (2)$$

where  $P(z, x)$  is the 2-D Gaussian pulse response given by

$$P(z, x) = A \exp \left\{ \frac{-1}{2c^2} \left[ \left( \frac{z + \Delta_z}{PW_z} \right)^2 + \left( \frac{x + \Delta_x}{PW_x} \right)^2 \right] \right\} \quad (3)$$

$\{m, n\} \in (-L, \dots, 0, \dots, L)$ ,  $2L+1$  is the length of  $P(z, x)$ ,  $L$  is an integer,  $z$  and  $x$  are the time indices in the across-track and the along-track directions, respectively,  $A = 1$  is supposed to be the peak amplitude of  $P(z, x)$ ,  $c = 1/2.3548$  is a constant to account for the relationship between  $PW_{50}$  and the standard deviation of the Gaussian pulse [1],  $PW_{50}$  is the pulse width at half of its peak value,  $\Delta_x$  is the along-track location fluctuation (or position jitter [11]),  $\Delta_z$  is the across-track location fluctuation,  $PW_x$  is the  $PW_{50}$  of the along-track pulse, and  $PW_z$  is the  $PW_{50}$  of the across-track pulse. Here, we assume that  $\Delta_x$  and  $\Delta_z$  are modeled as a truncated Gaussian probability distribution function with zero mean and variance  $\sigma_j^2$ , where  $\sigma_j$  is specified as the percentage of  $T_x$ . In addition, the read-head offset or the TMR,  $\Delta_T$ , is assumed to be positive for the upward offset, as shown in Fig. 1, and the TMR level is defined as

$$\text{TMR}(\%) = \frac{\Delta_T}{T_z} \times 100. \quad (4)$$

At the receiver, the readback data sequences  $\{r_{l,k}\}$  are equalized by a 2-D equalizer to obtain the sequences  $\{z_{l,k}\}$ , and are sent to three 2-D soft-output Viterbi algorithm (2-D-SOVA) detectors to produce the three-track soft information  $\tilde{\lambda}_{l,k}$  before passing them to the LDPC decoders to output the

three-track log-likelihood ratio (LLR),  $\lambda_{l,k}$ , of the input bits associated with the main and the two adjacent tracks. Note that the LDPC decoder is implemented based on the message-passing algorithm [9] with  $N_{\text{LDPC}}$  internal iterations. Then, the LLR sequences are fed back to each corresponding 2-D-SOVA detector for the next turbo iteration,  $N_T$ .

At each global iteration,  $N_G$ , the three-track LLR sequences are mapped to  $\hat{x}_{l,k} \in \{\pm 1\}$ , and are sent to the 2-D asymmetric targets to produce all possible target outputs  $\{\hat{z}_{0,k}\}$ . Thus, the TMR estimation block computes the mean-squared error (MSE) between each sequence  $\hat{z}_{0,k}$  and the equalizer output  $z_{0,k}$  from the previous global iteration, and then outputs the estimated TMR,  $\hat{\text{TMR}}$ , corresponding to the 2-D target used to generate the sequence  $\hat{z}_{0,k}$  that provides a minimum MSE (MMSE). Therefore, a new set of the 2-D target and its corresponding 2-D equalizer will be used for the next global iteration. In particular, the original readback data sequences  $\{r_{l,k}\}$  will be reequalized by the new 2-D equalizer before sending them to the 2-D-SOVA detectors implemented based on the new 2-D target. Note that only one 2-D equalizer is used to equalize three input sequences and then output three equalized sequences in the iteration process, whereas all 2-D-SOVA detectors employ the same 2-D target. In addition, this paper focuses only on detecting the data on the main track, similar to [6].

### III. PROPOSED METHOD

#### A. Equalizer and Target Design

We design a pair of 2-D target and its corresponding 2-D equalizer for the TMR levels of 0%, 5%, 10%, 15%, 20%, and 25%, based on an MMSE approach [10]–[12], and employ them in the simulation. In practice, the target and its corresponding equalizer can be obtained by minimizing

$$E\{\varepsilon_{l,k}^2\} = E\{(z_{l,k} - d_{l,k})^2\} \quad (5)$$

where  $E\{\cdot\}$  is the expectation operator and  $\varepsilon_{l,k}$  is an error, which is the difference between the equalizer output  $z_{l,k}$  and the desired sequence  $d_{l,k}$ . Because we consider only the data on the main track, (5) can then be computed by

$$\begin{aligned} E\{\varepsilon_{l,k}^2\} &= E\{(z_{0,k} - d_{0,k})^2\} \\ &= E\{[(r_{l,k} \otimes f_{l,k}) - (x_{l,k} \otimes g_{l,k})]^2\} \end{aligned} \quad (6)$$

where the values of  $f_{l,k}$  are the equalizer coefficients, the values of  $g_{l,k}$  are the target coefficients, and  $\otimes$  is the

2-D convolution operator. After expanding the right-hand side in (6), we obtain

$$E\{e_{i,k}^2\} = f_{l,k} \otimes R_{l,k}^r \otimes f_{l,k} - 2f_{l,k} \otimes R_{l,k}^{rx} \otimes g_{l,k} + g_{l,k} \otimes R_{l,k}^{rx} \otimes g_{l,k} \quad (7)$$

where  $R_{l,k}^r = E\{r_{i,j}r_{i-l,j-k}\}$  and  $R_{l,k}^x = E\{x_{i,j}x_{i-l,j-k}\}$  are the autocorrelations of the readback signals and the recorded bits from all three tracks, respectively, and  $R_{l,k}^{rx} = E\{r_{i,j}x_{i-l,j-k}\}$  is the cross correlation between the readback signals and the recorded bits.

To compute the solution of MSE in (7), it is convenient to rewrite it in a matrix form [12]. Let  $\mathbf{F}$  be a  $3 \times (2N+1)$  equalizer matrix according to

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_{-1} \\ \mathbf{f}_0 \\ \mathbf{f}_1 \end{bmatrix} = \begin{bmatrix} f_{-1,-N} & \dots & f_{-1,0} & \dots & f_{-1,N} \\ f_{0,-N} & \dots & f_{0,0} & \dots & f_{0,N} \\ f_{1,-N} & \dots & f_{1,0} & \dots & f_{1,N} \end{bmatrix} \quad (8)$$

and let  $\mathbf{G}$  be a  $3 \times 3$  target matrix of the form

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_{-1} \\ \mathbf{g}_0 \\ \mathbf{g}_1 \end{bmatrix} = \begin{bmatrix} g_{-1,-1} & g_{-1,0} & g_{-1,1} \\ g_{0,-1} & g_{0,0} & g_{0,1} \\ g_{1,-1} & g_{1,0} & g_{1,1} \end{bmatrix}. \quad (9)$$

In general, the matrices  $\mathbf{F}$  and  $\mathbf{G}$  can be rearranged into the column vectors as  $\mathbf{f} = [\mathbf{f}_{-1} \mathbf{f}_0 \mathbf{f}_1]^T$  and  $\mathbf{g} = [\mathbf{g}_{-1} \mathbf{g}_0 \mathbf{g}_1]^T$ , respectively, where the component vectors are defined in (8) and (9), and  $[\cdot]^T$  is the matrix transpose operator. Using these matrices, the MSE in (7) can be rewritten as

$$E\{e_{i,k}^2\} = \mathbf{f}^T \mathbf{R}_r \mathbf{f} - 2\mathbf{f}^T \mathbf{R}_{rx} \mathbf{g} + \mathbf{g}^T \mathbf{R}_x \mathbf{g} \quad (10)$$

where  $\mathbf{R}_r = [r_k r_k^T]$  is a  $3(2N+1) \times 3(2N+1)$  autocorrelation matrix of  $R_{l,k}^r$ ,  $\mathbf{R}_{rx} = [r_k x_k^T]$  is a  $3(2N+1) \times 9$  cross-correlation matrix of  $R_{l,k}^{rx}$ ,  $\mathbf{R}_x = [x_k x_k^T]$  is a  $9 \times 9$  autocorrelation matrix of  $R_{l,k}^x$ ,  $r_k = [r_{1,k+N} \ r_{1,k+N-1} \ \dots \ r_{0,0} \ \dots \ r_{-1,k-N+1} \ r_{-1,k-N}]^T$  is a column vector of the readback signal, and  $x_k = [x_{1,k+1} \ x_{1,k} \ \dots \ x_{0,0} \ \dots \ x_{-1,k} \ x_{-1,k-1}]^T$  is a column vector of the recorded bits.

For a given 2-D target  $\mathbf{g}$  of each TMR level, the 2-D equalizer  $\mathbf{f}$  can be obtained by [12]

$$\mathbf{f} = \mathbf{R}_r^{-1} \mathbf{R}_{rx} \mathbf{g}. \quad (11)$$

Note that this paper assumes that the 2-D target coefficients at each TMR level are obtained by sampling the 2-D Gaussian pulse in (3) at the integer multiples of  $T_x$  and  $T_z$ . An example of the coefficients of the 2-D target and its corresponding 2-D equalizer is illustrated in Table I, where only three equalizer taps, i.e.,  $\{f_{-1,i}, f_{0,i}, f_{1,i}\}$ , where  $i \in \{-1, 0, 1\}$ , are shown.

### B. TMR Estimation

The estimated TMR ( $\hat{\text{TMR}}$ ) is computed by the TMR estimation block. In particular, the LLR output of the LDPC decoders at the last turbo iteration ( $N_T = 3$ ) will be mapped to the estimated recorded sequence  $\hat{x}_{l,k}$ , before convolving with each predesigned 2-D target to obtain six data sequences,  $\{\hat{z}_{0,k}\}$ . Then, we calculate the MSE between each sequence  $\hat{z}_{0,k}$  and the equalizer output  $z_{0,k}$  according to

TABLE I

EXAMPLE OF THE COEFFICIENTS OF THE 2-D TARGET AND ITS CORRESPONDING 2-D EQUALIZER FOR SOME TMR LEVELS

TMR	2D target coefficients			2D equalizer coefficients		
0%	0.0692	0.3255	0.0692	-0.0107	0.0203	-0.0059
	0.1785	0.8398	0.1785	0.0197	0.8056	0.0100
	0.0692	0.3255	0.0692	0.0065	-0.0119	0.0211
5%	0.0759	0.3571	0.0759	-0.0044	0.0172	-0.0085
	0.1781	0.8381	0.1781	0.0107	0.8035	0.0139
	0.0628	0.2955	0.0628	-0.0039	0.0166	-0.0029
10%	0.0829	0.3902	0.0829	-0.0075	0.0185	-0.0066
	0.1770	0.8238	0.1770	0.0124	0.7868	0.0137
	0.0567	0.2671	0.0567	-0.0069	0.0194	-0.0062
15%	0.0902	0.4245	0.0902	-0.0097	0.0169	-0.0086
	0.1751	0.8241	0.1751	0.0153	0.7939	0.0103
	0.0511	0.2404	0.0511	-0.0055	0.0154	-0.0066
20%	0.0977	0.4598	0.0977	-0.0041	0.0155	-0.0107
	0.1725	0.8120	0.1725	0.0121	0.7808	0.0191
	0.0458	0.2154	0.0458	-0.0048	0.0164	-0.0114

$\text{MSE} = 1/S \sum_{k=1}^S (z_{0,k} - \hat{z}_{0,k})^2$ , where  $S$  is the length of the equalizer output. Eventually, the 2-D target used to generate  $\hat{z}_{0,k}$  that yields a minimum MSE and its corresponding 2-D equalizer will be employed to process the original readback sequences  $\{r_{l,k}\}$  during the next global iteration.

## IV. SIMULATION RESULTS

We evaluate the performance between 1) the conventional system (i.e., the system in Fig. 2, but without TMR mitigation) and 2) the proposed method in Fig. 2, at the AD of 3 Tb/in<sup>2</sup> (i.e.,  $T_x = T_z = 14.5$  nm), where the 2-D Gaussian pulse response with the along-track  $\text{PW}_{50}$  of 19.4 nm and the across-track  $\text{PW}_{50}$  of 24.8 nm is considered, similar to [10]. We also define the signal-to-noise ratio as  $\text{SNR} = 10 \log_{10}(V_p/(R\sigma^2))$ , where  $V_p = 1$  is assumed to be the peak value of the readback signal, and  $R$  is a code rate. Each bit error rate (BER) point is computed using as many 3640 b data sectors as needed to collect 500 erroneous bits. In simulation, the 2-D target and its corresponding 2-D equalizer designed for 0% TMR level will be used at the first global iteration (i.e.,  $N_G = 1$ ), and we employ  $N_{\text{LDPC}} = 3$ ,  $N_T = 3$ , and  $N_G = 2$ .

We first investigate the accuracy of our TMR estimation method. To do so, we send 5000 data sectors (each with random TMR ranged from 0% to 25%) into the system in Fig. 2, and check how many times the TMR estimation block can output a correct TMR. Simulation results indicate that, for the TMR levels of 0%, 5%, 10%, 15%, 20%, and 25%, our method yields the accuracies of 100%, 100%, 99%, 92%, 85%, and 83%, respectively. Clearly, the proposed method can correctly estimate the TMR level almost 100% when the TMR is <10%, which is better than the method in [6].

Next, we compare the BER performance of different systems for various TMR levels at the AD of 3 Tb/in<sup>2</sup> in Fig. 3. The curve labeled Convent-TMR  $x\%$  and Proposed-TMR  $x\%$  represent the conventional system and the proposed system with the TMR effect of  $x\%$ , respectively. It is clear that the proposed system performs similar to the conventional system when no TMR effect is present in the system. However, at  $\text{BER} = 10^{-4}$ , the proposed system can provide the perfor-

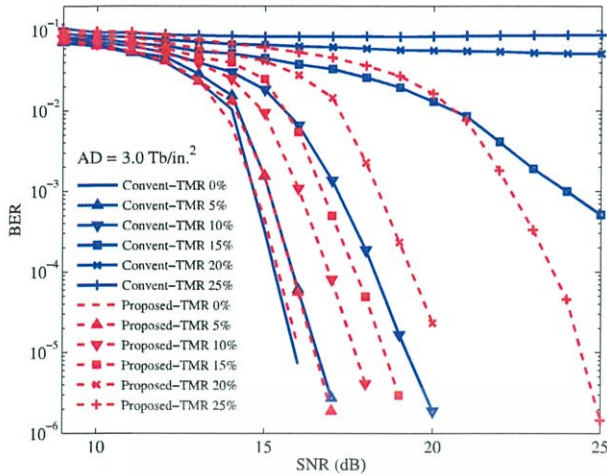


Fig. 3. Performance comparison for various TMR levels at AD of 3 Tb/in<sup>2</sup>.

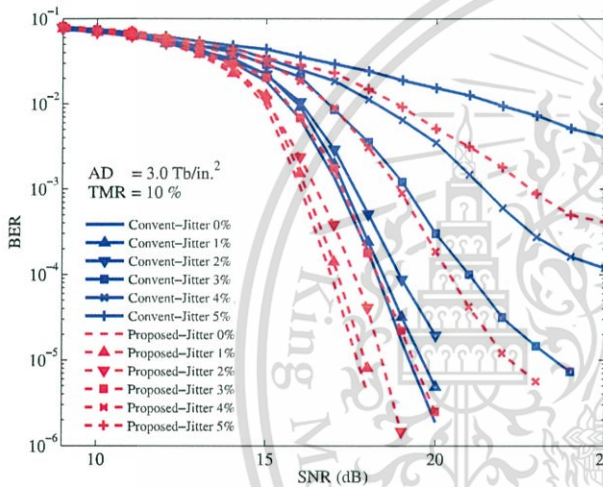


Fig. 4. BER performance of different schemes for several position jitter amounts at the TMR level of 10% and the AD of 3 Tb/in<sup>2</sup>.

performance gain of  $>1.5$  dB over the conventional system, when the TMR effect is  $>5\%$ .

We also verify that the proposed TMR mitigation scheme can perform well in the presence of media noise, e.g., position jitter, as shown in Fig. 4, where we fix a TMR level at 10% and vary the position jitter amount,  $\sigma_j/T_x$ , from 0% to 5%. The curve labeled Convent-Jitter  $x\%$  and Proposed-Jitter  $x\%$  represent the conventional system and the proposed system with the position jitter amount of  $x\%$ , respectively. Again, the proposed system outperforms the conventional system for all position jitter amounts, especially when the position jitter is large. For instance, at  $\text{BER} = 10^{-4}$ , it yields the performance improvement of  $>4.5$  dB over the conventional system when the position jitter amount is fixed at 4%.

The reason that the proposed system performs well is because our TMR mitigation method can effectively estimate the TMR embedded in the readback signal. Using this information to select and use a new set of the 2-D target and its corresponding 2-D equalizer that matches the actual TMR to reprocess the readback signal will offer some

performance improvement. Therefore, it is worth employing the proposed TMR mitigation method in the extremely high-density BPMR system, in particular, when the TMR effect is dominant. However, it should be noted that the performance gain obtained from the proposed method needs to be balanced against the increased complexity.

## V. CONCLUSION

In this paper, we proposed the iterative TMR mitigation method for the BPMR system based on the readback signals. In particular, the three adjacent data tracks (i.e., the main and the two adjacent tracks) from the output of the LDPC decoders are used to estimate the TMR level. Then, the system performance can be improved using a new pair of the 2-D target and its corresponding 2-D equalizer, that is, the best fit to the estimated TMR level for the next global iteration. As shown in simulation, the proposed system is superior to the conventional system (without using a TMR mitigation method) in the presence of TMR both in the case of with and without media noise, in particular, when the TMR level is high and/or the position jitter is large.

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## Utilization of multiple read heads for TMR prediction and correction in bit-patterned media recording

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## Utilization of multiple read heads for TMR prediction and correction in bit-patterned media recording

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This paper proposes a utilization of multiple read heads to predict and correct a track mis-registration (TMR) in bit-patterned media recording (BPMR) based on the readback signals. We propose to use the signal energy ratio between the upper and lower tracks from multiple read heads to estimate the TMR level. Then, a pair of two-dimensional (2D) target and its corresponding 2D equalizer associated with the estimated TMR will be chosen to correct the TMR in the data detection process. Numerical results show that the proposed system can achieve a very high accuracy of TMR prediction, thus performing better than the conventional system, especially when TMR is severe. © 2016 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>). [<http://dx.doi.org/10.1063/1.4972801>]

### I. INTRODUCTION

When the center of the read head is not aligned with that of the target track, it causes track mis-registration (TMR) or an off-track situation, which is one of major problems in bit-patterned media recording (BPMR), especially at high areal density (AD). Practically, the TMR can lead to a devastating impact on the data recovery process because of an unequal effect of adjacent tracks on the target track. In general, the servo mechanism can handle the TMR effect; however, it is difficult to control the read head when TMR occurs beyond its limit.<sup>1,2</sup>

In our previous works,<sup>3,4</sup> the TMR mitigation methods were introduced based on the readback signals with a single reader. Specifically, the TMR was predicted by the estimated signal-to-noise ratio (SNR) and the average energy of the readback signal.<sup>3</sup> Thus, the accuracy of the TMR estimation from this method depends mainly on that of the SNR estimation. An iterative method was employed to estimate the TMR level in a coded BPMR system.<sup>4</sup> Although this method is independent of the SNR levels, three detectors are required to perform the TMR estimation. Nonetheless, we found that these methods cannot accurately predict the TMR levels, especially when the system operates in a low TMR regime.

Recently, multiple read heads have been an interesting challenge for next-generation magnetic recording systems, and many researchers utilize a read head array to estimate and mitigate inter-track interference (ITI) in magnetic recording systems.<sup>5-7</sup> Moreover, a practical multi-head multi-track (MHMT) detector<sup>5</sup> that provides a low-complexity approach to adaptively estimate the time-varying ITI was presented. In general, the MHMT detector can handle the ITI effect better than a single-head single-track detector. Thus, the reduced state sequence estimation algorithm<sup>6</sup> to significantly decrease the complexity of the MHMT detector was discussed. G. Mathew<sup>7</sup> et. al., also show the advantages of using multiple read heads in magnetic recording systems. Therefore, this paper proposes to employ multiple read heads to predict and correct the TMR in a BPMR system as illustrated in Fig. 1. Specifically, we first predict a TMR level based on the signal energy ratio between the upper and



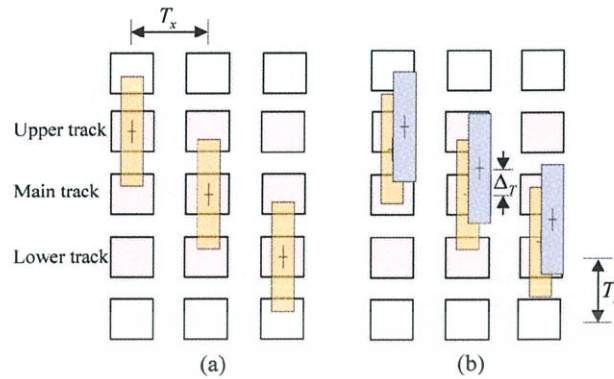


FIG. 1. An array of three read heads for (a) the conventional scheme and (b) the proposed scheme in the BPMR system with TMR,  $\Delta_T$ .

lower tracks. Then, the estimated TMR level is used to select a pair of two-dimensional (2D) target and its corresponding 2D equalizer from a look-up table that is suitable for the channel with TMR so as to facilitate the data detection process.

The rest of this paper is organized as follows. Section II briefly describes a BPMR channel model, and Section III explains the proposed method. Simulation results are given in Section IV. Finally, Section V concludes this paper.

**II. CHANNEL MODEL**

Consider a multi-track multi-head BPMR system as shown in Fig. 2. The readback signal,  $r_{l,k}$ , of the  $k$ -th data bit of the  $l$ -th track can be expressed as

$$r_{l,k} = \sum_n \sum_m h_{m,n} x_{l-m,k-n} + n_{l,k}, \tag{1}$$

where  $x_{l,k}$ 's are the recorded bits,  $l \in \{0, -1, +1\}$  represent the main, the upper, and the lower track, respectively,  $h_{m,n}$ 's are the 2D channel coefficients,  $m$  and  $n$  represent the time indices of the bit island in the across- and along-track directions, and  $n_{l,k}$  is an additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma^2$ . In practice, the channel coefficients,  $h_{m,n}$ , can be obtained by sampling the 2D Gaussian pulse response at the integer multiples of the track pitch,  $T_z$ , and the bit pitch,  $T_x$ , according to<sup>4,8</sup>

$$h_{m,n} = A \exp \left\{ -\frac{1}{2c^2} \left[ \left( \frac{mT_x}{PW_x} \right)^2 + \left( \frac{nT_z + \Delta_T}{PW_z} \right)^2 \right] \right\}, \tag{2}$$

where  $A = 1$  is assumed to be the peak amplitude of the pulse response,  $PW_x$  is the  $PW_{50}$  of the along-track pulse,  $PW_z$  is the  $PW_{50}$  of the across-track pulse,  $PW_{50}$  is the pulse width at half its maximum,  $c = 1/2.3548$  is a constant to account for the relationship between  $PW_{50}$  and the standard

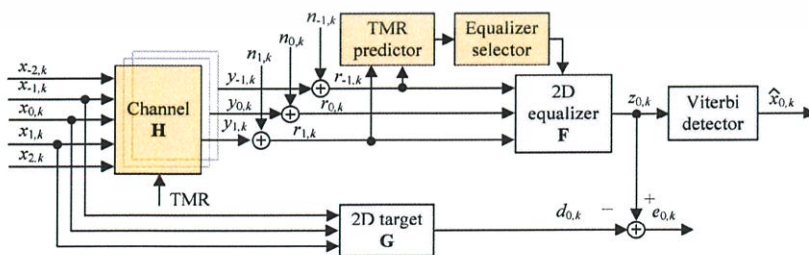


FIG. 2. A multi-track multi-head BPMR channel model with the proposed TMR prediction and correction method.

deviation of a Gaussian, and  $\Delta_T$  is the head offset as shown in Fig. 1. Here, we define the TMR level as

$$\text{TMR (\%)} = (\Delta_T / T_z) \times 100. \quad (3)$$

At the receiver, the readback signals are read by three read heads and are equalized by a 2D equalizer followed by the 2D Viterbi detector to determine the most likely data sequence on the main track,  $\hat{x}_{0,k}$ . Note that this paper focuses only on detecting the data on the main track. In addition, each pair of 2D target and 2D equalizer is designed based on a minimum mean-squared error (MMSE) approach<sup>9,10</sup> for each TMR level and is then stored in the look-up table. In a conventional system, each read head will be positioned at the center of each track as shown in Fig. 1(a). In contrast, the proposed scheme employs an array of three read heads, where the upper and lower heads are moved closer to the center head (e.g., by 25% of  $T_z$ ) as depicted in Fig. 1(b).

### III. PROPOSED METHOD

We propose to utilize three read heads to read the data on three adjacent tracks, and process them so as to predict and correct the TMR effect in the BPMR system.

#### A. TMR predictor

For each TMR level, we compute the energy ratio of the two readback signals associated with the upper and lower tracks (i.e.,  $r_{-1,k}$  and  $r_{1,k}$ ) according to

$$E_{ratio} = \sum_{k=1}^S (r_{-1,k})^2 / \sum_{k=1}^S (r_{1,k})^2, \quad (4)$$

where  $S$  is the length of the readback signal samples (i.e.,  $S = 32,768$  bits for a 4K-data sector<sup>11</sup>). Next, the estimated TMR level is obtained based on a polynomial least-squares fitting technique, i.e.,

$$\text{TMR} = b_0 + b_1 E_{ratio} + b_2 E_{ratio}^2 + \dots + b_M E_{ratio}^M, \quad (5)$$

where TMR is the estimated TMR,  $b_i$  and  $M$  are the  $i^{\text{th}}$  coefficient and a degree of the polynomial equation in (5), respectively, and  $i \in \{0, 1, \dots, M\}$ . Then, we perform an extensive simulation search to find a suitable  $M$ , where  $M = 3$  provides the best fit between the actual and the estimated TMR levels. Fig. 3(a) plots the estimated TMR level as a function of the energy ratios at 3.0 Tb/in<sup>2</sup>. It is apparent that the TMR level can be effectively estimated from (5) based on  $E_{ratio}$ .

#### B. Equalizer selector

The 2D target and its corresponding 2D equalizer are designed for each TMR level based on an MMSE approach, which minimizes a mean-squared error (MSE)<sup>12</sup> according to

$$E \{ e_{l,k}^2 \} = E \{ (z_{l,k} - d_{l,k})^2 \}, \quad (6)$$

where  $E\{\cdot\}$  is the expectation, and  $e_{l,k}$  is an error signal between the equalizer output  $z_{l,k}$  and the desired signal  $d_{l,k}$ .

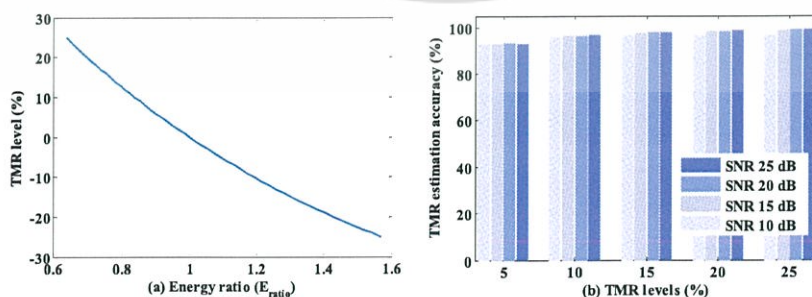


FIG. 3. (a) The relationship between the TMR levels versus the energy ratios and (b) the accuracy of TMR estimation depends the different SNRs, both of them were considered at AD = 3.0 Tb/in<sup>2</sup>.

Given the estimated TMR level, the equalizer selector will choose a pair of 2D target and 2D equalizer from the look-up table to alleviate the TMR effect embedded in the readback signal before sending it to the 2D Viterbi detector. Notice that this paper assumes that the 2D target coefficients are the 2D channel coefficients when the system has 0% TMR, similar to previous work.<sup>4</sup> Hence, we fix this 2D target and use it to design all 2D equalizers for each TMR level i.e., at -25%, -20%, ..., -5%, 0% 5%, ..., and 25%, based on the MMSE approach, which will be all stored in the look-up table.

#### IV. RESULTS AND DISCUSSION

We evaluate the system performance at the AD of 3.0 Tb/in<sup>2</sup> (i.e.,  $T_x = T_z = 14.5$  nm) among 1) the conventional system I without TMR prediction and correction method; 2) the conventional system II, assuming that TMR is known and the receiver uses the 2D equalizer designed for that TMR; and 3) the proposed system, where the 2D Gaussian pulse response with the along-track  $PW_{50}$  of 19.4 nm and the across-track  $PW_{50}$  of 24.8 nm is considered, similar to previous work.<sup>4</sup> In simulation, the SNR is defined as

$$\text{SNR} = 20 \log_{10} (1/\sigma) \text{ in decibel (dB)}, \quad (7)$$

where  $\sigma$  is a standard deviation of AWGN. We also measure the accuracy of TMR estimation by

$$\text{accuracy (\%)} = 100 - \frac{|\hat{\text{TMR}} - \text{TMR}|}{\text{TMR}} \times 100, \quad (8)$$

where  $\hat{\text{TMR}}$  is the estimated TMR obtained from the proposed method, and TMR is the actual TMR embedded in the readback signal. Fig. 3(b) shows the accuracy of TMR estimation in percentage at different SNRs. Clearly, the proposed method can predict the TMR well, especially at high TMR levels. In addition, we found that the accuracy of our TMR estimation method is independent of SNRs.

From now on, we assume that the BPMR system experiences the TMR effect only in the upward direction (i.e.,  $\Delta_T$  is a positive value). Note that this paper will not consider a media noise in the system so as to make it easy to understand the behavior of the TMR effect. Fig. 4(a) compares the performance of different systems at AD = 3.0 Tb/in<sup>2</sup> in terms of the MSE in dB according to

$$\text{MSE} = 10 \log_{10} \left( \sum_{k=1}^S |z_{0,k} - d_{0,k}|^2 \right). \quad (9)$$

Apparently, the proposed system yields the lowest MSE for all TMR levels. That means the proposed equalizer can perform with the proposed multi-track reading systems.

Finally, Fig. 4(b) compares the performance of different systems at AD = 3.0 Tb/in<sup>2</sup> by plotting the SNR required to achieve bit-error rate (BER) of  $10^{-4}$  as a function of TMR levels. It is clear that the proposed system is superior to other systems, especially when TMR is large. The reason might be because the proposed TMR estimation method can achieve a very high accuracy for all TMR levels as depicted in Fig. 3(b). Therefore, it can be implied that the proposed method can help improve the quality of the readback signal before sending it to the 2D Viterbi detector, thus leading to a better

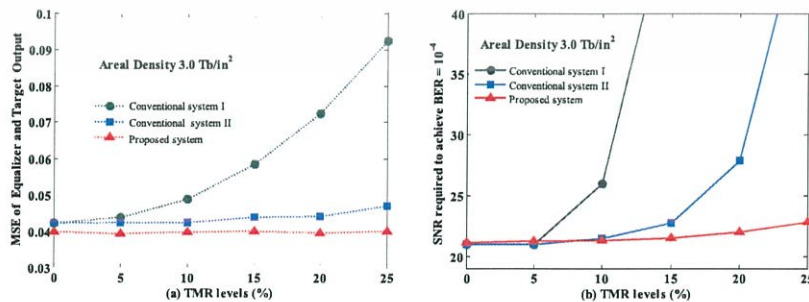


FIG. 4. (a) MSE and (b) BER performance comparison of different systems at AD of 3.0 Tb/in<sup>2</sup>.

BER performance. It can be concluded that by moving the upper and lower read heads closer to the center read head as shown in Fig. 1(b), one can obtain the better readback signals with less ITI effect if compared to the conventional setting in Fig. 1(a), thus leading to the improved system performance.

## V. CONCLUSION

This paper proposes the TMR prediction and correction method for a multi-track multi-head BPMR system. The energy ratio between the readback signals of the upper and lower tracks is used to predict the TMR level. Then, the TMR effect is corrected by using the 2D target and its corresponding 2D equalizer that are the best suit for the estimated TMR level. Simulation results indicate that the proposed system can effectively estimate the TMR level, thus performing better than the conventional systems, especially when the TMR is severe.

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### International Journal Publications:

- [1] W. Busyatras, C. Warisarn, L. M. M. Myint, and P. Kovintavewat, "A TMR mitigation method based on readback signal in bit-patterned media recording," *IEICE Trans. Electron.*, vol. E98-C, no. 8, pp. 892–898, Aug. 2015.
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### International Conference Proceedings:

- [1] W. Busyatras , A. Arrayangkool, C. Warisarn, Lin M. M. Myint, P. Supnithi and P. Kovintavewat, "Estimating Track Mis-Registration Based on Readback Signal in Bit-Patterned Media Recording Systems" in *Proc. of ITC-CSCC*, pp. 881-884, Phuket, Thailand, July 1- 4, 2014.
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