STOCK SELECTIVE TECHNIQUE INTO PORTFOLIO BY FUZZY QUANTITATIVE ANALYSIS AND FUZZY MULTI-CRITERIA DECISION MAKING

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หัวข้อวิทยานิพนธ์	เทคนิคการเลือกหุ้นลงพอร์ตฟอลิโอโดยการวิเคราะห์เชิงปริมาณแบบฟัซซีและการตัดสินใจแบบฟัซซีหลายเกณฑ์
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บทคัดย่อ

งานวิจัยนี้นำเสนอเทคนิคการคัดเลือกหุ้นในตลาดหลักทรัพย์จากแต่ละกลุ่มหุ้นที่แบ่งกลุ่มตามประเภทของธุรกิจซึ่งหุ้นในแต่ละกลุ่มจะถูกคัดกรองและจัดอันดับตามค่าน้ำหนักของการลงทุนภายในกลุ่ม โดยใช้วิธีการวิเคราะห์เชิงปริมาณแบบฟัซซี และทำการประเมินกลุ่มหุ้นแต่ละกลุ่มโดยวิธีการตัดสินใจแบบฟัซซีหลายเกณฑ์ ในกระบวนการเปรียบเทียบระดับชั้นแบบฟัซซีจะคำนวณหาค่าน้ำหนักของเกณฑ์การตัดสินใจของผู้ตัดสินใจ และใช้เทคนิคสำหรับจัดระดับความชอบโดยคล้ายกับผลเฉลยในอุดมคติแบบฟัซซี และกำหนดค่าน้ำหนักของแต่ละกลุ่มหุ้นเพื่อใช้ในการจัดอันดับกลุ่มหุ้น ค่าน้ำหนักของกลุ่มหุ้นที่ได้และค่าน้ำหนักการลงทุนของหุ้นภายในแต่ละกลุ่มจะถูกนำมาใช้ประกอบการตัดสินใจในการลงทุน โดยค่าน้ำหนักทั้งสองจะถูกนำมาสรุปเป็นค่าน้ำหนักโดยรวมเพื่อใช้ในการจัดอันดับหุ้นทั้งหมด นักลงทุนสามารถเลือกหุ้นลงทุนได้โดยพิจารณาหุ้นที่มีค่าน้ำหนักโดยรวมสูงอยู่ในลำดับต้น นอกจากนี้ยังได้แสดงกรณีศึกษาโดยใช้เทคนิคที่ได้นำเสนออีกด้วย

คำสำคัญ : การเลือกหุ้น, การวิเคราะห์เชิงปริมาณ, การเปรียบเทียบระดับชั้นแบบฟัซซี, ตรรกศาสตร์ฟัซซี, หลายเกณฑ์การตัดสินใจ, เทคนิคสำหรับจัดระดับความชอบโดยคล้ายกับผลเฉลยในอุดมคติแบบฟัซซี

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Abstract

This research presents a stock selection approach assisted by fuzzy procedures. In this approach, stocks are classified into groups according to business types. Within each group, the stocks are screened and then ranked according to their investment weight obtained from fuzzy quantitative analysis. Groups were also ranked according to their group weight obtained from fuzzy analytic hierarchy process (FAHP) and technique for order preference by similarity to ideal solution method (TOPSIS). The overall weight for each stock was then derived from both of these weights and used for selecting a stock into the portfolio. As a demonstration, our analysis procedures were applied to a test set of data.

Keywords : Stock Selection, Quantitative Analysis, FAHP, Fuzzy logic, Multi Criteria Decision Making, TOPSIS

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Satit Yodmun

Table of Contents

Table of Contents (Continued)

List of Tables

List of Tables (Continued)

List of Tables (Continued)

List of Figure

List of Figure (Continued)

List of Figure (Continued)

Figure **Page**

Chapter 1
Introduction

1.1 Research Motivation

Presently, investors are more interested in investing in stocks and bonds than keeping their money in the bank because it yields a higher return. However, this higher return also comes with higher risk; investors may lose some of their investment, get a lower-than-expected return, or get a lower return than that from another type of investment. Therefore, they have to analyze a stock carefully before investing in it.

In addition to several established approaches to stock analysis—such as quantitative fundamental analysis, technical analysis, and stochastic analysis—new analytical tools have been developed and widely used including ones that are based on Brownian movement, fuzzy logic, and the analytic hierarchy process. The analytic hierarchy process (AHP) is a multi-criteria decision making approach and is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. It was developed by Saaty in the 1970s, to help one make decision when one is faced with the mixture of qualitative, quantitative, and sometimes conflicting factors that are taken into consideration. AHP has been very effective in making complicated, often irreversible decisions. It has been extensively studied and refined since then (e.g., [1–11] and references therein).

Fuzzy sets and fuzzy logic, especially, are of wide interest today. They are effective tools for modeling, in the absence of complete and precise information, complex business, finance, and management systems. The subjective judgement of experts who have used fuzzy logic techniques produces better results than the objective manipulation of inexact data. The concept of a fuzzy set is a reflection of reality reflection which serves as a point of departure for the development of theories which have the capability to model the pervasive imprecision and uncertainty of the real world. As applied to stock analysis (e.g., [12–15] and references therein), fuzzy logic uses integrated experiential knowledge of human experts to make better quantitative estimates, not possible with classical logic, based on robust mathematical principles.

By reason of vagueness of boundaries of stock data in future and the attendant imprecision, uncertainty, and preference of decision makers, therefore, fuzzy logic and AHP seem suitable for this problem. This paper proposes an approach to stock analysis based on calculated weights from fuzzy quantitative analysis and fuzzy multi-criteria decision making. The idea of using fuzzy quantitative analysis and fuzzy multi-criteria decision making to imply final investment weights for the stock selection into portfolio is different from the previous works. The practicality of the approach was demonstrated by an application to a test set of data.

1.2 Objectives of the study

To present a tactic of conveying the stock selection into portfolio by using two tactics, fuzzy quantitative analysis and fuzzy multi-criteria decision making.

1.3 Scope

In this research, we will only construct a tool for financial analysis only. We do not compare the tool with others. We demonstrate an application to a test set of data.

1.4 Benefits of the study

We obtain a tool for financial analysis that may be an alternative for investors.

Chapter 2
Theoretical Preliminary

In this chapter, we will introduce some fuzzy logic, financial ratio and fuzzy multi-criteria decision making.

2.1 Fuzzy logic and Inference[6]

2.1.1 Fuzzy set

Definition 2.1: Given a crisp set A of a universe $\mathcal{U}$, a fuzzy set $\tilde{A}$ on A is defined as

$$\tilde{A}=\{(x,u_{\tilde{A}}(x))\mid x\in A\} \text{ where } u_{\tilde{A}}(x)\in[0,1] \tag{2.1}$$

and $u_{\tilde{A}}$ is a membership function of $\tilde{A}$.

Example 2.1 Given $A=\{1,2,3,4,5\}$, defined a fuzzy set $\tilde{A}$ with the membership function $u_{\tilde{A}}(x)$ where $u_{\tilde{A}}(x)=\dfrac{1}{x}$.

Sine $u_{\tilde{A}}(x)=\dfrac{1}{x}$

x	1	2	3	4	5
$u_{\tilde{A}}(x)$	1	0.5	0.33	0.25	0.2

Hence $\tilde{A}=\{(1,1),(2,0.5),(3,0.33),(4,0.25),(5,0.2)\}$, $\tilde{A}$ can be expressed as the following figure:

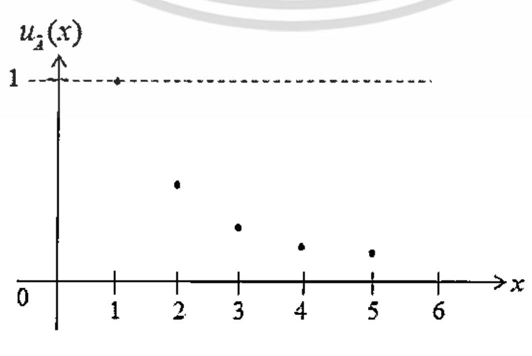

Figure 2.1 Membership function of fuzzy set $\tilde{A}$

Definition 2.2 Let $\tilde{A}$ be a fuzzy set under the membership function $u_{\tilde{A}}: A \rightarrow [0,1]$

$\tilde{A}$ is a normalized fuzzy set, i.e., $\exists x, u_{\tilde{A}}(x) = 1$.

$\tilde{A}$ is a nonnormalized fuzzy set, i.e., $\forall x, u_{\tilde{A}}(x) < 1$.

For nonnormalized fuzzy set $\tilde{A}$, we can make $\tilde{A}$ is a normailzed fuzzy set, defined as $\hat{u}_{\tilde{A}}(x) = \dfrac{u_{\tilde{A}}(x)}{\max u_{\tilde{A}}(y)}$.

2.1.2 Fuzzy number

Definition 2.3 Given a fuzzy set $\tilde{A}$, an α*-cut set* or *Level set* α, denoted by $[u_{\tilde{A}}]^{\alpha}$, for all $\alpha \in (0,1]$ is defined as

$$[u_{\tilde{A}}]^{\alpha} = \begin{cases} \{x \in A \mid u_{\tilde{A}}(x) \geq \alpha\} & ; 0 < \alpha \leq 1 \\ \overline{\{x \in A \mid u_{\tilde{A}}(x) > 0\}} & ; \alpha = 0 \end{cases} \tag{2.2}$$

Note: Closure B represented by the expression $\bar{B}$.

Given an $\mathbb{R}_F$ fuzzy number space, Definition 2.3 assures that every $u_{\tilde{A}} \in \mathbb{R}_F$ can be represented by a closed interval $[u_{\tilde{A}}]^{\alpha} = [\underline{u}(\alpha), \overline{u}(\alpha)]$ where $\underline{u}, \overline{u} : [0,1] \rightarrow \mathbb{R}$ are functions that satisfy the following conditions:

1) $\underline{u}$ is a bounded, left continuous, and non-decreasing function on $[0,1]$;
2) $\overline{u}$ is a bounded, right continuous, and non-increasing function on $[0,1]$;
3) $\underline{u}(\alpha) \leq \overline{u}(\alpha)$ for all $\alpha \in [0,1]$.

Definition 2.4 Let $\tilde{A}$ be a fuzzy set under the membership function $u_{\tilde{A}}: A \rightarrow [0,1]$, $\tilde{A}$ is called *Convex fuzzy set* if $u_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{u_{\tilde{A}}(x_1), u_{\tilde{A}}(x_2)\}$ for all $x_1, x_2 \in A$ and $\lambda \in [0,1]$.

Note: Since definition 2.4 can be expressed following figure.

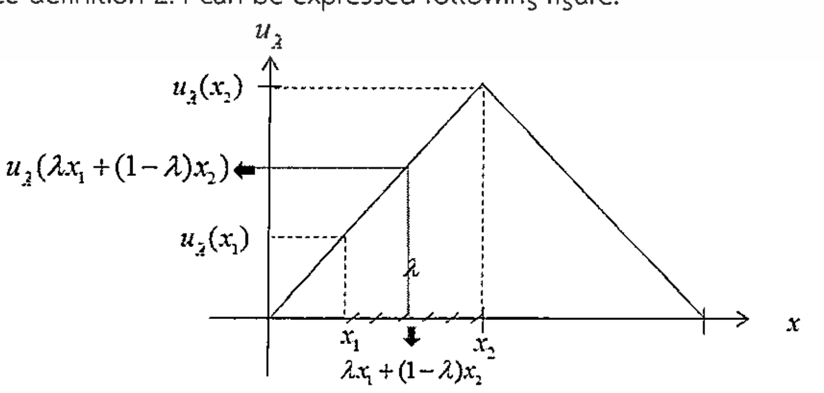

Figure 2.2 Shown convex fuzzy set $\tilde{A}$

Definition 2.5 Let $\tilde{A}$ be a fuzzy set under the membership function $u_{\tilde{A}}: A \rightarrow [0,1]$, $\tilde{A}$ is a fuzzy number if it satisfies the following conditions:

1) $\tilde{A}$ is a normal fuzzy set.
2) $\tilde{A}$ is a convex fuzzy set.
3) For every $\alpha \in [0,1]$, $[u_{\tilde{A}}]^{\alpha} = [a,b]$ for some closed interval $[a,b]$.

Definition 2.6 Given $a^L, a^{M_1}, a^{M_2}, a^U \in \tilde{A}$ where $a^L \le a^{M_1} \le a^{M_2} \le a^U$, a *trapezoidal fuzzy number* is a fuzzy number $\tilde{A}$ whose membership function $u_{\tilde{A}}(x)$ is defined by

$$u_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a^L}{a^{M_1}-a^L} & ; a_L \le x \le a^{M_1} \\ 1 & ; a^{M_1} \le x \le a^{M_2} \\ \dfrac{x-a^U}{a^{M_2}-a^U} & ; a^{M_2} \le x \le a^U \\ 0 & ; Otherwise \end{cases} . \qquad (2.3)$$

and represented by the expression $\tilde{A} = \langle a^L, a^{M_1}, a^{M_2}, a^U \rangle$.

Note: - A trapezoidal fuzzy number $\tilde{A} = \langle a^L, a^M, a^M, a^U \rangle$ is called a triangular fuzzy number and expressed as $\tilde{A} = \langle a^L, a^M, a^U \rangle$.
- For any real number a, $a = \langle a,a,a \rangle = \langle a,a,a,a \rangle$.

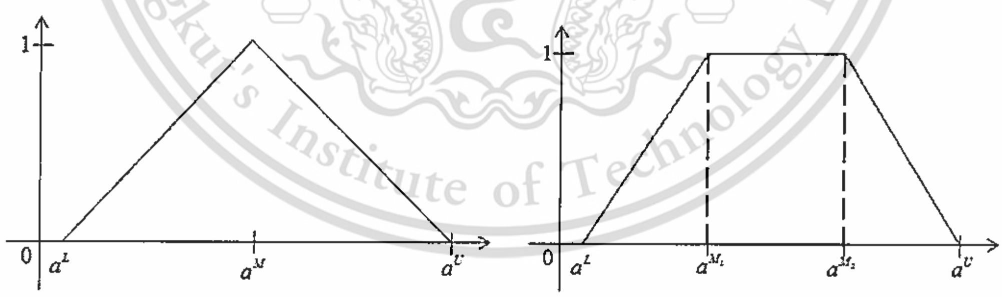

Figure 2.3 Triangular fuzzy number and trapezoidal fuzzy number

For convenience, $n,m \in \mathbb{N}\, ; I_n = \{1,2,...,n\}$, $I_m = \{1,2,...,m\}$ is defined for further use in this research.

Definition 2.7 $\tilde{A} = (\tilde{a}_{ij})_{m \times n}$ is a fuzzy matrix if $\tilde{a}_{ij}$ are fuzzy numbers for all $i \in I_m$ and $j \in I_n$.

2.1.3 Operation on fuzzy

Definition 2.8 Let $\tilde{A},\tilde{B}$ be a fuzzy set under membership function $u_{\tilde{A}}$ and $u_{\tilde{B}}$ respectively, the *intersection* of two fuzzy set $\tilde{A}$ and $\tilde{B}$ $\left(\tilde{A}\tilde{\cap}\tilde{B}\right)$ can be defined by the membership function $u_{\tilde{A}\tilde{\cap}\tilde{B}}(x)=\min\left\{u_{\tilde{A}}(x),u_{\tilde{B}}(x)\right\}$, for all $x\in\mathcal{U}$ or denoted by $u_{\tilde{A}\tilde{\cap}\tilde{B}}(x)=\wedge\left\{u_{\tilde{A}}(x),u_{\tilde{B}}(x)\right\}$.

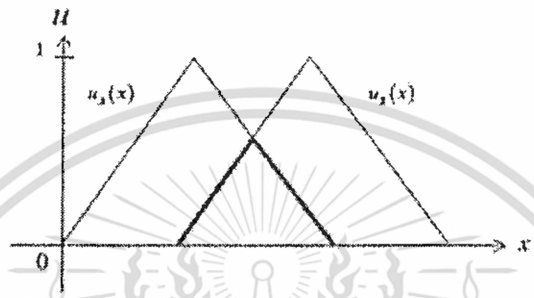

Figure 2.4 $\tilde{A}\tilde{\cap}\tilde{B}$

Definition 2.9 Let $\tilde{A},\tilde{B}$ be a fuzzy set under membership function $u_{\tilde{A}}$ and $u_{\tilde{B}}$ respectively, the *union* of two fuzzy set $\tilde{A}$ and $\tilde{B}$ $\left(\tilde{A}\tilde{\cup}\tilde{B}\right)$ can be defined by the membership function $u_{\tilde{A}\tilde{\cup}\tilde{B}}(x)=\max\left\{u_{\tilde{A}}(x),u_{\tilde{B}}(x)\right\}$, for all $x\in\mathcal{U}$ or denoted by $u_{\tilde{A}\tilde{\cup}\tilde{B}}(x)=\vee\left\{u_{\tilde{A}}(x),u_{\tilde{B}}(x)\right\}$.

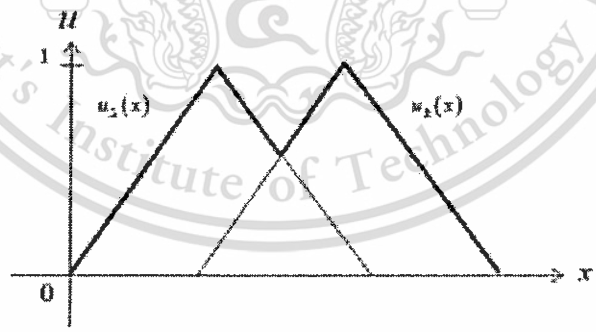

Figure 2.5 $\tilde{A}\tilde{\cup}\tilde{B}$

Definition 2.10 Given any two fuzzy numbers $\tilde{A}=\left\langle a^L,a^{M_1},a^{M_2},a^U\right\rangle$ and $\tilde{B}=\left\langle b^L,b^{M_1},b^{M_2},b^U\right\rangle$ and a real positive number $p\in\mathbb{R}^+$, operations $\oplus$, $\ominus$, $\otimes$, $\oslash$ between $\tilde{A}$ and $\tilde{B}$ and an operation $\odot$ between $\tilde{a}$ and p are defined as follows:

$$\tilde{A} \oplus \tilde{B} = \left\langle a^L + b^L, a^{M_1} + b^{M_1}, a^{M_2} + b^{M_2}, a^U + b^U \right\rangle \quad (2.4)$$

$$\tilde{A} \ominus \tilde{B} = \left\langle a^L - b^U, a^{M_1} - b^{M_2}, a^{M_2} - b^{M_1}, a^U - b^L \right\rangle \quad (2.5)$$

$$\tilde{A} \otimes \tilde{B} = \left\langle a^L b^L, a^{M_1} b^{M_1}, a^{M_2} b^{M_2}, a^U b^U \right\rangle \quad (2.6)$$

$$p \odot \tilde{A} = \left\langle pa^L, pa^{M_1}, pa^{M_2}, pa^U \right\rangle \quad (2.7)$$

$$\tilde{A} \oslash \tilde{B} = \left\langle a^L / b^U, a^{M_1} / b^{M_2}, a^{M_2} / b^{M_1}, a^U / b^L \right\rangle \quad (2.8)$$

Definition 2.11 Given two trapezoidal fuzzy numbers $\tilde{A} = \left\langle a^L, a^{M_1}, a^{M_2}, a^U \right\rangle$ and $\tilde{B} = \left\langle b^L, b^{M_1}, b^{M_2}, b^U \right\rangle$, the *distance between* $\tilde{A}$ and $\tilde{B}$ represented by the symbol $d\left(\tilde{A}, \tilde{B}\right)$ is defined as

$$d\left(\tilde{A}, \tilde{B}\right) = \sqrt{\frac{1}{4}\left[\left(a^L - b^L\right)^2 + \left(a^{M_1} - b^{M_1}\right)^2 + \left(a^{M_2} - b^{M_2}\right)^2 + \left(a^U - b^U\right)^2\right]} \quad (2.9)$$

Definition 2.12 $\tilde{\mathrm{M}} = \left(\tilde{M}_{ij}\right)_{n \times 1}$ is a fuzzy vector when all $\tilde{M}_i = \left\langle m_i^L, m_i^{M_1}, m_i^{M_2}, m_i^U \right\rangle, i \in I_n$ are trapezoidal fuzzy numbers. The *aggregation* of $\tilde{\mathrm{M}}$, represented by $\tilde{\mathrm{M}}_{agg}$, is defined as

$$\tilde{\mathrm{M}}_{agg} = \left\langle \min_{i=1}^{n}\{m_i^L\}, \frac{1}{n}\sum_{i=1}^{n} m_i^{M_1}, \frac{1}{n}\sum_{i=1}^{n} m_i^{M_2}, \max_{i=1}^{n}\{m_i^U\} \right\rangle. \quad (2.10)$$

2.1.4 Linguistic variable

Linguistic variables[7] are the input or output variables of the system whose values are words or sentences from a natural language, instead of numerical values. A linguistic variable is generally decomposed into a set of linguistic terms. For example speed is a linguistic variable then we have little faster(0-80km./h.), fast(80-120km./h) and ultrafast(120upkm/h.) are term of linguistic.

2.1.5 Fuzzy Inference systems

Fuzzy inference systems uses Mamdani Method, parallel If-Then rules form the deducing mechanism which indicates how to project input variables onto output space. A single fuzzy If-Then rule follows the form, in this consider fuzzy logic n sub conditions of rule $1,2,\ldots,q$.

Rule-1: if x_1 is $\tilde{A}_{11}$ and x_2 is $\tilde{A}_{12}$ ... and x_n is $\tilde{A}_{1n}$ then y is $\tilde{B}_1$,

Rule-2: if x_1 is $\tilde{A}_{21}$ and x_2 is $\tilde{A}_{22}$ ... and x_n is $\tilde{A}_{2n}$ then y is $\tilde{B}_2$,

⋮ ⋮

Rule-q: if x_1 is $\tilde{A}_{q1}$ and x_2 is $\tilde{A}_{q2}$ .... and x_n is $\tilde{A}_{qn}$ then y is $\tilde{B}_q$,

Fact: x_1 is $\tilde{A}_1$ and x_2 is $\tilde{A}_2$ ...and x_n is $\tilde{A}_n$

Conclusion : y is $\tilde{B}$

Fuzzy inference process consists of three steps:

1) Calculate the weights between Fact $\tilde{A}_i$ and conditions $\tilde{A}_{ki}$ of rule $k=1,2,\ldots,q$ and $i=1,2,\ldots,n$. Which is the minimum height of sub fuzzy fact and condition of rule, such that

$$w_k = \overset{n}{\underset{i=1}{\wedge}}\left(\underset{i}{\vee} u_{\tilde{A}_i \cap \tilde{A}_{ki}}(x_i)\right) = \overset{n}{\underset{i=1}{\wedge}}\left(\underset{i}{\vee}\left(u_{\tilde{A}_i}(x_i) \wedge u_{\tilde{A}_{ki}}(x_i)\right)\right) \tag{2.11}$$

For $k=1,2,\ldots,q$.

2) Truncate $\tilde{B}_k$ equal to w_k, with $\tilde{B}_k$ represented by $\tilde{B}_k$ and defiled membership function.

$$u_{\tilde{B}_k}(y) = w_k \wedge u_{\tilde{B}_k}(y) \tag{2.12}$$

3) Find fuzzy set $\tilde{B}$ from union of $\tilde{B}_k$, that is $\tilde{B} = \overset{m}{\underset{k=1}{\tilde{\cup}}} \tilde{B}_k$ and defiled membership function.

$$u_{\tilde{B}}(y) = \overset{m}{\underset{k=1}{\vee}} u_{\tilde{B}_k}(y) = \overset{m}{\underset{k=1}{\vee}}\left(w_k \wedge u_{\tilde{B}_k}(y)\right) \tag{2.13}$$

2.1.6 Defuzzification

In this research, performing defuzzification of the fuzzy output to a crisp output to a centroid method. A crisp z^{cg} is the average weight of the weight at each point z on domain B where $w_z = \dfrac{u_{\tilde{B}}(z)}{\int_B u_{\tilde{B}}(z)dz}$ for all $z \in B$,

i.e., the crisp output is $z^{cg} = \int_B z w_z dz = \dfrac{\int_B z u_{\tilde{B}}(z)dz}{\int_B u_{\tilde{B}}(z)dz}$.

2.1.7 m-partition of n-scale

Definition 2.13 Given n is positive integer , $\left\{\frac{0}{n-1}, \frac{1}{n-1}, \frac{2}{n-1}, \ldots, \frac{n-1}{n-1}\right\}$ is called n-scale set on $[0,1]$.

Definition 2.14 Given n-scale an m is number, m-partition of n-scale on $[0,1]$ denoted by $M_1, M_2, \ldots, M_m$ respectively, where

$$M_1 = [0, a_1],\ M_k = (a_{k-1}, a_k],\ M_m = (a_{m-1}, 1] \quad ;\ k = 2,3,\ldots,m-1 \text{ and } a_k = \frac{k}{n-1}.$$

Example 2.2 let n=4 and m=3.

We have n-scale set

$$\left\{\frac{0}{4-1}, \frac{1}{4-1}, \frac{2}{4-1}, \frac{3}{4-1}\right\} = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}.$$

And 3-partition is

$$M_1 = \left[0, \frac{1}{3}\right] \quad M_2 = \left(\frac{1}{3}, \frac{2}{3}\right] \quad M_3 = \left(\frac{2}{3}, 1\right].$$

2.2 Financial Ratios

The quantitative stock analysis[e.g.8,9,10,11] presented in this study is based on the following financial ratios: Price to Earnings Ratio or P/E Ratio; Price to Book Value Ratio or P/BV Ratio; and Price to Intrinsic Ratio or P/P_n Ratio, which are defined as follows.

Definition 2.15 Let n_1 , n_2 and n_3 be the number of common stock, preferred stock, and treasury stock respectively, P_t be current price per share, and E_r be r^{th}-quarter net profit, *price to earnings ratio or* P/E is defined as

$$P/E = \frac{P_t(n_1 + n_2 - n_3)}{E_r} \tag{2.14}$$

P/E denotes the stock price per 1 baht of net profit that the investor is willing to pay for.

Definition 2.16 Let n be the number of be the number of registered share, A_t and R_t be the asset and liability of the company respectively, and P_t be current price per share, *price to book value ratio or* P/BV is defined as

$$P/BV = \frac{P_t}{B_t} \tag{2.15}$$

where $B_t = \dfrac{A_t - R_t}{n}$.

P/BV denotes how many times the current stock price is compared to its account value.

Definition 2.17 Let r be the reference interest rate, D_k be the k^{th} year-end dividend per share, $k \in I_n$ and P_0 be the n^{th} -quarter historical price, *the current target price* P_n is defined as

$$P_n = P_0(1+r)^n - \sum_{k=1}^{n} D_k (1+r)^{n-k} \tag{2.16}$$

Definition 2.18 Let P_n be the current target price and P be the current stock price, *price per target price ratio* or P/P_n is defined as

$$P/P_n = \frac{P}{P_n} \tag{2.17}$$

P/P_n denotes how many times the current stock price is compared to the current target price.

2.3 Fuzzy Multi-Criteria Decision Making

We will introduce process of fuzzy multi-criteria decision making[12,13] consisting of fuzzy Analytic Hierarchy Process: AHP and Fuzzy Technique for Order Preference by Similarity to Ideal Solution Method: FTOPSIS .

2.3.1 Fuzzy analytic hierarchy process[14,15]

Definition 2.19 Let $A = (a_{ij})_{n\times n}$ be an $n \times n$ matrix where $a_{ij} > 0$ for all $i, j \in I_n$, A is *a consistency matrix* if there exist weight vectors $w = (w_i)_{n\times 1}$, $w_i > 0$, for all $i \in I_n$ where $\sum_{i=1}^{n} w_i = 1$ and $a_{ij} = w_i / w_j$ for all $i, j \in I_n$.

$$A = \begin{bmatrix} 1 & \frac{w_1}{w_2} & \frac{w_1}{w_3} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & 1 & \frac{w_2}{w_3} & \cdots & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & 1 & \cdots & \frac{w_3}{w_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & \cdots & 1 \end{bmatrix}.$$

Definition 2.20 Let $A=\left(a_{ij}\right)_{n\times n}$ be an $n\times n$ matrix where $a_{ij}>0$ for all $i,j\in I_n$, A is *a reciprocal matrix* if $a_{ji}=1/a_{ij}$ for all $i,j\in I_n$.

$$A=\begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{1n} & 1/a_{2n} & \cdots & 1 \end{bmatrix}$$

Definition 2.21 Let $\tilde{A}=\left(\tilde{a}_{ij}\right)_{n\times n}$ be an $n\times n$ fuzzy matrix where $\tilde{a}_{ij}>0$ are fuzzy numbers for all $i,j\in I_n$, $\tilde{A}$ is *a reciprocal fuzzy* matrix if $\tilde{a}_{ji}=1\oslash\tilde{a}_{ij}$ for all $i,j\in I_n$. In particular, if every member of $\tilde{A}=\left(\tilde{a}_{ij}\right)_{n\times n}$ is a triangular fuzzy number $\tilde{a}_{ij}=\left\langle a_{ij}^L,a_{ij}^M,a_{ij}^U\right\rangle$, $\tilde{A}$ is a reciprocal fuzzy matrix if $\tilde{a}_{ji}=\left\langle \frac{1}{a_{ij}^U},\frac{1}{a_{ij}^M},\frac{1}{a_{ij}^L}\right\rangle$ for all $i,j\in I_n$.

Definition 2.22 Let $\tilde{A}=\left(\tilde{a}_{ij}\right)_{n\times n}$ be an $n\times n$ fuzzy matrix where $\tilde{a}_{ij}=[\underline{a}_{ij}(\alpha),\overline{a}_{ij}(\alpha)]>0$ for all $i,j\in I_n$, $\tilde{A}$ is *a consistency fuzzy matrix* if there exist $a_{ij}^{\alpha}\in[\underline{a}_{ij}(\alpha),\overline{a}_{ij}(\alpha)]$ for all $i,j\in I_n$ and some $\alpha\in[0,1]$ with which $A=\left(a_{ij}^{\alpha}\right)_{n\times n}$ is a consistency matrix, i.e., there exist $w^{\alpha}=\left(w_i^{\alpha}\right)_{n\times 1}$, $w_i^{\alpha}>0$, for all $i\in I_n$ where $\sum_{i=1}^{n}w_i^{\alpha}=1$ and $a_{ij}^{\alpha}=w_i^{\alpha}/w_j^{\alpha}$ for all $i,j\in I_n$.

According to Definition 2.23, since $w_i^{\alpha}>0$ for all $i\in I_n$, there exist fuzzy vectors $\tilde{w}=(\tilde{w}_i)_{n\times 1}$ where $w_i^{\alpha}\in[\underline{w}_i(\alpha),\overline{w}_i(\alpha)]>0$ for all $i\in I_n$. These vectors are called fuzzy weight vectors.

It is clear that if $\tilde{A}$ is a fuzzy consistency matrix then it is a fuzzy reciprocal fuzzy matrix and $\tilde{A}$ is not a fuzzy consistency matrix if it is not a fuzzy reciprocal fuzzy matrix. Because of these reasons, construction of a fuzzy consistency matrix usually starts by first constructing a reciprocal fuzzy matrix $\tilde{A}$. J. Ramik[16,17] proposed a method for calculating fuzzy weight vector $\tilde{w}=(\tilde{w}_i)_{n\times 1}$ for a fuzzy reciprocal matrix $\tilde{A}=\left(\tilde{a}_{ij}\right)_{n\times n}$ where $\tilde{a}_{ij}=\left\langle a_{ij}^L,a_{ij}^M,a_{ij}^U\right\rangle$ for all $i,j\in I_n$ by using the method of geometric mean. $\tilde{w}_k=\left\langle w_k^L,w_k^M,w_k^U\right\rangle$ are defined for all $k\in I_n$ where

$$w_k^L = C_L \cdot \frac{\left(\prod_{j=1}^{n} a_{kj}^L\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}} \quad , \quad w_k^M = \frac{\left(\prod_{j=1}^{n} a_{kj}^M\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}} \quad , \quad w_k^U = C_U \cdot \frac{\left(\prod_{j=1}^{n} a_{kj}^U\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}} \qquad (2.18)$$

and

$$C_L = \min_{i \in I_n}\left\{\frac{\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}}{\left(\prod_{j=1}^{n} a_{ij}^L\right)^{1/n}}\right\} \text{ and } C_U = \max_{i \in I_n}\left\{\frac{\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}}{\left(\prod_{j=1}^{n} a_{ij}^U\right)^{1/n}}\right\}. \qquad (2.19)$$

In addition, J.Ramik[16,17] defined a consistency index for measuring the nearness of a fuzzy reciprocal matrix to the corresponding fuzzy consistency matrix as follows.

Definition 2.23 Let $\tilde{A} = \left(\tilde{a}_{ij}\right)_{n\times n}$ be a fuzzy reciprocal matrix, of which $\tilde{a}_{ij} = \left\langle a_{ij}^L, a_{ij}^M, a_{ij}^U\right\rangle$ are triangular fuzzy numbers, evaluated from a scale $S = \left[\frac{1}{\sigma}, \sigma\right]$ for some real number $\sigma > 1$, the consistency index of $\tilde{A}$ represented by the symbol $I_n^\sigma\left(\tilde{A}\right)$ is defined as

$$I_n^\sigma\left(\tilde{A}\right) = C_n^\sigma \bullet \max_{i,j}\left\{\max\left\{\left|\frac{w_i^L}{w_j^U} - a_{ij}^L\right|, \left|\frac{w_i^M}{w_j^M} - a_{ij}^M\right|, \left|\frac{w_i^U}{w_j^L} - a_{ij}^U\right|\right\}\right\} \qquad (2.20)$$

where $\tilde{w} = (\tilde{w}_i)_{n\times 1}$ are fuzzy weight vectors and $\tilde{w}_i = \left\langle w_i^L, w_i^M, w_i^U\right\rangle$ for all $i \in I_n$ as expressed in (2.18) and

$$C_n^\sigma = \begin{cases} \dfrac{1}{\max\left\{\sigma - \sigma^{\frac{2-2n}{n}}, \sigma^2\left(\left(\frac{2}{n}\right)^{\frac{2}{n-2}} - \left(\frac{2}{n}\right)^{\frac{n-2}{2}}\right)\right\}} & ; \sigma < \left(\frac{n}{2}\right)^{\frac{n}{n-2}} \\[2ex] \dfrac{1}{\max\left\{\sigma - \sigma^{\frac{2-2n}{n}}, \sigma^{\frac{2-2n}{n}} - \sigma\right\}} & ; \sigma \geq \left(\frac{n}{2}\right)^{\frac{n}{n-2}} \end{cases} \qquad (2.22)$$

If the consistency index $I_n^\sigma\left(\tilde{A}\right)=0$, the fuzzy reciprocal fuzzy matrix $\tilde{A}$ is absolutely consistent. The closer the value of $I_n^\sigma\left(\tilde{A}\right)$ to 0 is, the more consistent the matrix is. Generally, an acceptable value is $I_n^\sigma\left(\tilde{A}\right)<0.1$ or 10 %.

2.3.2 Fuzzy technique for order preference by similarity to ideal solutions method.

The other technique, FTOPSIS developed by C.T. Chan[18] and S. Balli, et al.[2], is a fuzzy technique for ranking preference levels by comparing the similarity of alternate choice to the ideal choice in order to find the best alternative. It covers diverse alternate choices, decision criteria, and decision makers.

Applying this technique to n_1 decision makers, n_2 decision criteria and n_3 alternative as alternate choices, the analysis steps are as follows.

Step2-1) Finding matrix for decision makers

Herein a reciprocal matrix is compares paired data that are metrics of real quantities such as price, weight, and preference. Here, these quantities are preferences. Levels of preferences are represented by numbers in a set $\Omega_n=\left\{\frac{1}{n},\frac{1}{n-1},\ldots,\frac{1}{3},\frac{1}{2},1,2,3,\ldots,n-1,n\right\}$. Generalizing this idea, the set of crisp preference values Ω_n is replaced by a set of fuzzy preference values $\tilde{\Omega}_n^\delta=\left\{\frac{1}{\tilde{n}_\delta},\frac{1}{\left(\tilde{n-1}\right)},\ldots,\frac{1}{\tilde{3}_\delta},\frac{1}{\tilde{2}_\delta},1,\tilde{2}_\delta,\tilde{3}_\delta,\ldots,\left(\tilde{n-1}\right)_\delta,\tilde{n}_\delta\right\}$ where $\tilde{k}_\delta=\langle k-\delta,k,k+\delta\rangle$ and $\frac{1}{\tilde{k}_\delta}=1\varnothing\tilde{k}_\delta=\left\langle\frac{1}{k+\delta},\frac{1}{k},\frac{1}{k-\delta}\right\rangle$ for all $k\in I_n$ and $0\le\delta\le1$.

In this step, a decision maker i , $i=1,\ldots,n_1$ is compared to another decision maker j in terms of their preference level based on a preference function $\varphi(i,j)$ defined as

$$\varphi(i,j)=\begin{cases}\tilde{c}_{ij} & ,\exists\tilde{c}_{ij}\in\tilde{\Omega}_n \ ; j>i\\ 1 & ; j=i.\\ 1\varnothing\varphi(j,i) & ; j<i\end{cases}$$

The decision maker's preference matrix $\tilde{D}=\left(\tilde{a}_{ij}\right)_{n_1\times n_1}$ is a reciprocal matrix where

$$\tilde{a}_{ij}=\begin{cases}\varphi(i,j) & ;i<j\\ 1 & ;i=j.\\ 1\varnothing\varphi(j,i) & ;i>j\end{cases}$$

Step2-2) Finding a fuzzy weight vector $\tilde{w}_d = (\tilde{w}_{d,k})_{n_1\times 1}$ for $\tilde{D} = \left(\tilde{a}_{ij}\right)_{n_1\times n_1}$

$\tilde{w}_{d,k} = \left\langle w_{d,k}^L, w_{d,k}^M, w_{d,k}^U \right\rangle$ is a fuzzy weight vector for all $k \in I_{n_1}$ where

$$w_{d,k}^L = C_L \cdot \frac{\left(\prod_{j=1}^{n_1} a_{kj}^L\right)^{1/n_1}}{\sum_{i=1}^{n_1}\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}} \quad , \quad w_{d,k}^M = \frac{\left(\prod_{j=1}^{n_1} a_{kj}^M\right)^{1/n_1}}{\sum_{i=1}^{n_1}\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}} \quad , \quad w_{d,k}^U = C_U \cdot \frac{\left(\prod_{j=1}^{n_1} a_{kj}^U\right)^{1/n_1}}{\sum_{i=1}^{n_1}\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}}$$

with

$$C_L = \min_{i\in I_{n_1}}\left\{\frac{\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}}{\left(\prod_{j=1}^{n_1} a_{ij}^L\right)^{1/n_1}}\right\} \text{ and } C_U = \max_{i\in I_{n_1}}\left\{\frac{\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}}{\left(\prod_{j=1}^{n_1} a_{ij}^U\right)^{1/n_1}}\right\}.$$

If its consistency index $I_{n_1}^{\sigma}\left(\tilde{D}\right)$ as defined in Definition 2.15 is less than 0.1, it is accepted as being valid. Otherwise, the decision maker's weight is re-evaluated by repeating Step2-1).

Step2-3) Decision makers $d_1, d_2, \ldots, d_{n_1}$ constructing decision criteria $c_1, c_2, \ldots, c_{n_2}$ for evaluating alternative $G_1, G_2, \ldots, G_{n_3}$ where $c_i, i = 1, \ldots, n_2$ is constructed from investment weight of n_3 individual groups given by decision makers in the term of following linguistic terms.

linguistic term	Fuzzy number
Very Low (VL)	$\langle 0,\ 0,\ 0.1, 0.2\rangle$
Low (L)	$\langle 0.1,\ 0.2,\ 0.3\rangle$
Medium Low (ML)	$\langle 0.2,\ 0.3,\ 0.4\rangle$
Medium (M)	$\langle 0.3,\ 0.4,\ 0.6, 0.7\rangle$
Medium High (MH)	$\langle 0.6,\ 0.7,\ 0.8\rangle$
High (H)	$\langle 0.7,\ 0.8,\ 0.9\rangle$
Very High (VH)	$\langle 0.8, 0.9\ , 1\ , 1\rangle$

The decision criteria constructed is in the form of a fuzzy matrix $\tilde{B}$ with members $b_{jik} = \left\langle b_{jik}^L, b_{jik}^{M_1}, b_{jik}^{M_2}, b_{jik}^U \right\rangle$, $j \in I_{n_3}, i \in I_{n_2}$ and $k \in I_{n_1}$, which are trapezoidal fuzzy numbers representing the linguistic terms of $c_1, c_2, \ldots, c_{n_2}$ shown in Table 2.1.

Table 2.1 Decision criteria for evaluating alternative $G_1, G_2, \ldots, G_{n_3}$

		d_1	d_2	$\ldots$	d_{n_1}
c_1	G_1	$\tilde{b}_{111}$	$\tilde{b}_{112}$	$\ldots$	$\tilde{b}_{11n_1}$
	G_2	$\tilde{b}_{211}$	$\tilde{b}_{212}$	$\ldots$	$\tilde{b}_{21n_1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	G_{n_3}	$\tilde{b}_{n_3 11}$	$\tilde{b}_{n_3 12}$	$\ldots$	$\tilde{b}_{n_3 1 n_1}$
c_2	G_1	$\tilde{b}_{121}$	$\tilde{b}_{122}$	$\ldots$	$\tilde{b}_{12n_1}$
	G_2	$\tilde{b}_{221}$	$\tilde{b}_{222}$	$\ldots$	$\tilde{b}_{22n_1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	G_{n_3}	$\tilde{b}_{n_3 21}$	$\tilde{b}_{n_3 22}$	$\ldots$	$\tilde{b}_{n_3 2 n_1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
c_{n_2}	G_1	$\tilde{b}_{1 n_2 1}$	$\tilde{b}_{1 n_2 2}$	$\ldots$	$\tilde{b}_{1 n_2 n_1}$
	G_2	$\tilde{b}_{2 n_2 1}$	$\tilde{b}_{2 n_2 2}$	$\ldots$	$\tilde{b}_{2 n_2 n_1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	G_{n_3}	$\tilde{b}_{n_3 n_2 1}$	$\tilde{b}_{n_3 n_2 2}$	$\ldots$	$\tilde{b}_{n_3 n_2 n_1}$

$= \tilde{B}$

Step2-4) Decision makers $d_1, d_2, \ldots, d_{n_1}$ evaluating decision criteria $c_1, c_2, \ldots, c_{n_2}$ constructing from the linguistic terms $\mathrm{VL, L, ML, M, MH, H, VH}$ as in step2-3). A fuzzy matrix $\tilde{C} = \left(\tilde{c}_{ij}\right)_{n_2 \times n_1}$ for evaluation is then obtained where $\tilde{c}_{ij} \in \{\mathrm{VL, L, ML, M, MH, H, VH}\}$ for all $i \in I_{n_2}$ and $j \in I_{n_1}$ as shown in Table 2.2.

Table 2.2 Evaluation of decision criteria $c_1, c_2, \ldots, c_{n_2}$

	d_1	d_2	$\ldots$	d_{n_1}
c_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\ldots$	$\tilde{c}_{1n_1}$
c_2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\ldots$	$\tilde{c}_{2n_1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
c_{n_2}	$\tilde{c}_{n_2 1}$	$\tilde{c}_{n_2 2}$	$\ldots$	$\tilde{c}_{n_2 n_1}$

$= \tilde{C}$

Step2-5) Calculating decision criteria based on decision makers' weights by multiplying the decision criterion of a decision maker in each column in Step2-4) (depicted in Table 2.2) with the corresponding decision maker's fuzzy weight vector $\tilde{w}_d = (\tilde{w}_{d,k})_{n \times 1}$ where $\tilde{w}_{d,k} = \left\langle w_{d,k}^L, w_{d,k}^M, w_{d,k}^U \right\rangle = \left\langle w_{d,k}^L, w_{d,k}^M, w_{d,k}^M, w_{d,k}^U \right\rangle$ calculated from Step2-2). Table 2.3 shows these multiplication results.

Table 2.3 Decision criteria based on weights of decision makers

	d_1	d_2	...	d_{n_1}
c_1	$\tilde{c}_{11} \otimes \tilde{w}_{d,1}$	$\tilde{c}_{12} \otimes \tilde{w}_{d,2}$	...	$\tilde{c}_{1n_1} \otimes \tilde{w}_{d,n_1}$
c_2	$\tilde{c}_{21} \otimes \tilde{w}_{d,1}$	$\tilde{c}_{22} \otimes \tilde{w}_{d,2}$	...	$\tilde{c}_{2n_1} \otimes \tilde{w}_{d,n_1}$
⋮	⋮	⋮	⋮	⋮
c_{n_2}	$\tilde{c}_{n_2 1} \otimes \tilde{w}_{d,1}$	$\tilde{c}_{n_2 2} \otimes \tilde{w}_{d,2}$	...	$\tilde{c}_{n_2 n_1} \otimes \tilde{w}_{d,n_1}$

$= \tilde{C}_w$

Next, multiplying the decision criterion for evaluating alternative in the column representing each decision maker constructed in Step2-3) with the corresponding decision maker's fuzzy weight vector $\tilde{w} = (\tilde{w}_{d,k})_{n \times 1}$ where $\tilde{w}_{d,k} = \left\langle w_{d,k}^L, w_{d,k}^M, w_{d,k}^U \right\rangle = \left\langle w_{d,k}^L, w_{d,k}^M, w_{d,k}^M, w_{d,k}^U \right\rangle$ calculated from Step2-2). The multiplication results are in Table 2.4.

Table 2.4 Decision criteria for evaluating alternative based on weights of decision makers.

		d_1	d_2	...	d_{n_1}
c_1	G_1	$\tilde{b}_{111} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{112} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{11n_1} \otimes \tilde{w}_{d,n_1}$
	G_2	$\tilde{b}_{211} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{212} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{21n_1} \otimes \tilde{w}_{d,n_1}$
	⋮	⋮	⋮	⋮	⋮
	G_{n_3}	$\tilde{b}_{n_3 11} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{n_3 12} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{n_3 1n_1} \otimes \tilde{w}_{d,n_1}$
c_2	G_1	$\tilde{b}_{121} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{122} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{12n_1} \otimes \tilde{w}_{d,n_1}$
	G_2	$\tilde{b}_{221} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{222} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{22n_1} \otimes \tilde{w}_{d,n_1}$
	⋮	⋮	⋮	⋮	⋮
	G_{n_3}	$\tilde{b}_{n_3 21} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{n_3 22} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{n_3 2n_1} \otimes \tilde{w}_{d,n_1}$
⋮	⋮	⋮	⋮	⋮	⋮
c_{n_2}	G_1	$\tilde{b}_{1n_2 1} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{1n_2 2} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{1n_2 n_1} \otimes \tilde{w}_{d,n_1}$
	G_2	$\tilde{b}_{2n_2 1} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{2n_2 2} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{2n_2 n_1} \otimes \tilde{w}_{d,n_1}$
	⋮	⋮	⋮	⋮	⋮
	G_{n_3}	$\tilde{b}_{n_3 n_2 1} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{n_3 n_2 2} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{n_3 n_2 n_1} \otimes \tilde{w}_{d,n_1}$

$= \tilde{B}_w$

Step2-6) Aggregating weights of decision criteria based on the decision makers' weights as follows.

$$\tilde{w}_{c,i} = \left\langle w_{c,i}^L, w_{c,i}^{M_1}, w_{c,i}^{M_2}, w_{c,i}^U \right\rangle$$

where $w_{c,i}^{L} = \min_{k=1}^{n_1}\{c_{w,ik}^{\ L}\}$, $w_{c,i}^{M_1} = \frac{1}{n_1}\sum_{k=1}^{n_1} c_{w,ik}^{\ M_1}$, $w_{c,i}^{M_2} = \frac{1}{n_1}\sum_{k=1}^{n_1} c_{w,ik}^{\ M_2}$, $w_{c,i}^{U} = \max_{k=1}^{n_1}\{c_{w,ik}^{\ U}\}$ for all $i \in I_{n_2}$, $\tilde{C}_w = \left(\tilde{c}_{w,jk}\right)_{n_2 \times n_1}$ and n_1 is the number of decision makers. Table 2.5 shows these aggregation results.

Table 2.5 Weights of decision criteria $c_1, c_2, \ldots, c_{n_2}$

	c_1	c_2	...	c_{n_2}
W_2	$\tilde{w}_{c,1}$	$\tilde{w}_{c,2}$	...	$\tilde{w}_{c,n_2}$

Next, aggregating alternative based on the decision makers' weights (Table 2.4) by the following equations:

$$\tilde{x}_{ji} = \left\langle x_{ji}^{L}, x_{ji}^{M_1}, x_{ji}^{M_2}, x_{ji}^{U} \right\rangle$$

where $x_{ji}^{L} = \min_{k=1}^{n_1}\{b_{w,jik}^{\ L}\}$, $x_{ji}^{M_1} = \frac{1}{n_1}\sum_{k=1}^{n_1} b_{w,jik}^{\ M_1}$, $x_{ji}^{M_2} = \frac{1}{n_1}\sum_{k=1}^{n_1} b_{w,jik}^{\ M_2}$, $x_{ji}^{U} = \max_{k=1}^{n_2}\{b_{w,jik}^{\ U}\}$ for all $j \in n_3, i \in n_2$, $\tilde{B}_w = \left(\tilde{b}_{w,jik}\right)_{n_3 n_2 \times n_1}$ and n_1 is the number of decision makers. These results are shown in Table 2.6.

Table 2.6 Evaluation matrix of alternative $G_1, G_2, \ldots, G_{n_3}$

	c_1	c_2	...	c_{n_2}
G_1	$\tilde{x}_{11}$	$\tilde{x}_{12}$	...	$\tilde{x}_{1n_2}$
G_2	$\tilde{x}_{21}$	$\tilde{x}_{22}$	...	$\tilde{x}_{2n_2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
G_{n_3}	$\tilde{x}_{n_3 1}$	$\tilde{x}_{n_3 2}$	...	$\tilde{x}_{n_3 n_2}$

$= \tilde{X}$

Step2-7) Constructing a decision matrix by normalizing the alternative' evaluation matrix $\tilde{X}$ (Table 2.6) as follows,

$$\hat{R} = \left(\tilde{r}_{ji}\right)_{n_3 \times n_2}, \quad \tilde{r}_{ji} = \left\langle \frac{x_{ji}^{L}}{x_i^{*}}, \frac{x_{ji}^{M_1}}{x_i^{*}}, \frac{x_{ji}^{M_2}}{x_i^{*}}, \frac{x_{ji}^{U}}{x_i^{*}} \right\rangle \text{ where } x_i^{*} = \max_{j}^{n_3}\left\{x_{ji}^{U}\right\}.$$

Then, multiplying the normalized matrix with the decision weights from Step 2-6),

$$\tilde{V} = \left(\tilde{v}_{ji}\right)_{n_3 \times n_2}$$

where $\tilde{v}_{ji} = \left\langle v_{ji}^{L}, v_{ji}^{M_1}, v_{ji}^{M_2}, v_{ji}^{U} \right\rangle$ and $\tilde{v}_{ji} = \tilde{r}_{ji} \otimes \tilde{w}_{c,i}$ when $j \in I_{n_3}, i \in I_{n_2}$.

Table 2.7 Alternative' evaluation matrix

	c_1	c_2	...	c_{n_2}
G_1	$\tilde{v}_{11}$	$\tilde{v}_{12}$	...	$\tilde{v}_{1n_2}$
G_2	$\tilde{v}_{21}$	$\tilde{v}_{22}$	...	$\tilde{v}_{2n_2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
G_{n_3}	$\tilde{v}_{n_3 1}$	$\tilde{v}_{n_3 2}$	...	$\tilde{v}_{n_3 n_2}$

$= \tilde{V}$

Step2-8) Defining positive ideal solution (G^*) and negative ideal solution (G^-) from Table 2.7 as $G^* = \left(\hat{v}_1^*, \hat{v}_2^*, \ldots, \hat{v}_{n_2}^*\right)$ and $G^- = \left(\hat{v}_1^-, \hat{v}_2^-, \ldots, \hat{v}_{n_2}^-\right)$, respectively, where $\hat{v}_i^* = \max_j^{n_3}\left\{v_{ji}^U\right\}$ and $\hat{v}_i^- = \min_j^{n_3}\left\{v_{ji}^L\right\}$; $j \in I_{n_3}$, $i \in I_{n_2}$, $\tilde{V} = \left(\tilde{v}_{ji}\right)_{n_3 \times n_2}$.

Step2-9) Calculating the distances between the alternative' evaluation results with the positive and negative ideal solutions, as defined by the following,

$$d_j^* = \sum_{i=1}^{n_2} d_v(\tilde{v}_{ji}, \hat{v}_i^*) \quad , j \in I_{n_3}$$

$$d_j^- = \sum_{i=1}^{n_2} d_v(\tilde{v}_{ji}, \hat{v}_i^-) \quad , j \in I_{n_3}$$

where $d_v(\tilde{v}_{ji}, \hat{v}_i^{*,-})$ are calculated in the same way as fuzzy numbers are calculated according to Definition 2.11 (depicted in Table 2.8).

Table 2.8 Distances between the alternative' evaluation results and positive and negative ideal solutions G^* and G^-

	c_1	c_2	...	c_{n_2}	$d_j^{*,-} = \sum_{i=1}^{n_2} d_v(\tilde{v}_{ji}, \hat{v}_j^{*,-})$
G_1	$d_v(\tilde{v}_{11}, \hat{v}_1^{*,-})$	$d_v(\tilde{v}_{12}, \hat{v}_2^{*,-})$	...	$d_v(\tilde{v}_{1n_2}, \hat{v}_{n_2}^{*,-})$	$d_1^{*,-}$
G_2	$d_v(\tilde{v}_{21}, \hat{v}_1^{*,-})$	$d_v(\tilde{v}_{22}, \hat{v}_2^{*,-})$	...	$d_v(\tilde{v}_{2n_2}, \hat{v}_{2n_2}^{*,-})$	$d_2^{*,-}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
G_{n_3}	$d_v(\tilde{v}_{n_3 1}, \hat{v}_1^{*,-})$	$d_v(\tilde{v}_{n_3 2}, \hat{v}_2^{*,-})$	...	$d_v(\tilde{v}_{n_3 n_2}, \hat{v}_{n_2}^{*,-})$	$d_{n_3}^{*,-}$

Step2-10) Calculating the nearness coefficients to the positive ideal solution CC_j, and ranking the alternative according to them. CC_j are defined as follows,

$$CC_j = \frac{d_j^-}{d_j^- + d_j^*} \quad , \; j \in I_{n_3} .$$

From the calculation, a set of investment weights for alternative, $W_3 = (w_1, w_2, ..., w_{n_3})$ where $w_1, w_2, ..., w_{n_3}$ are weights of alternative, is obtained. The alternative of which investment weight value is nearest to one (the closest to the positive ideal solution) is the best alternative.

Chapter 3

Stock Selection Procedure

This section presents the proposed stock selection procedure which is done in the following 3 Main steps:

Main step 1) Analysis of individual stocks within each industrial group

In this Main step, analysis of individual stocks within each industrial group from their financial ratios, using fuzzy logic principles to calculate the normalize investment weight for each individual stock;

Main step 2) Analysis of industrial groups

Since the investment weight for each individual stock can't be compared across industrial group. So, this research presents idea in analysis of industrial for ranking interestedness in investment follows industrial group. This Main step, present analysis of industrial groups (e.g. finance, communication, technology and property) using fuzzy multi-criteria decision-making principles to calculate the investment weight for each industrial group;

Main step 3) Analysis of individual stocks across all industrial groups

In this Main step, analysis of individual stocks across all industrial groups using the 2 types of weights from Main step 1) and 2) to calculate the final weight for ranking all individual stocks in the market.

3.1 Analysis of individual stocks within each industrial group

In this Main step, we apply the method of P.Bumlungpong, et al. [4] to analyze individual stocks within each industrial group. Price to Earnings Ratio (P/E Ratio), Price to Book Value Ratio (P/BV Ratio), and Price to Intrinsic Value Ratio (P/P_n Ratio) are used to calculate the investment weight for each individual stock within an industrial group based on quantitative fuzzy analysis under these assumptions:

1) A calculated investment weight of an individual stock can be compared only to another one in the same industrial group;

2) More recent data reflect current trend better than earlier ones;

3) Fuzzy rules are flexible and depend on expert information.

The specific steps of the fuzzy analysis are as follows.

Step1-1) Screening in only m individual stocks ($S_1, S_2, \ldots, S_m$) in the same industrial group of which sufficient financial data are provided for calculating P/E, P/BV, and P/P_n of n earlier years up to the present.

Step1-2) Calculating $(E/P)(S_k^i)$, $(P/BV)(S_k^i)$ and $(P/P_n)(S_k^i)$ for all $i \in I_n$ and $k \in I_m$ where S_k^i denotes the k^{th} stock in the i^{th} year.

Step1-3) Calculating the following weighted arithmetic mean: $(E/P)^w(S_k)$, $(P/BV)^w(S_k)$, and $(P/P_n)^w(S_k)$, $k \in I_m$ from the following equations:

$$(E/P)^w(S_k) = \sum_{i=1}^{n} w_i (E/P)(S_k^i),$$

$$(P/BV)^w(S_k) = \sum_{i=1}^{n} w_i (P/BV)(S_k^i)$$

and

$$(P/P_n)^w(S_k) = \sum_{i=1}^{n} w_i (P/P_n)(S_k^i)$$

where $w_i = \dfrac{2i}{n(n+1)}$, $i \in I_n$.

Step1-4) An expert constructing fuzzy sets in linguistic terms of the ranked financial ratios E/P, P/BV, and P/P_n and a fuzzy set W of the investment weights from $(E/P)^w(S_k)$, $(P/BV)^w(S_k)$ and $(P/P_n)^w(S_k)$, $k \in I_m$.

Step1-5) An expert constructing fuzzy rules for estimation based on the fuzzy sets constructed in Step 1-4). These fuzzy rules are in the form of an 'if-then' rule as follows:

Rule-1: if x_1 is $\tilde{A}_{11}$ and x_2 is $\tilde{A}_{12}$ ... and x_n is $\tilde{A}_{1n}$ then y is $\tilde{B}_1$,

Rule-2: if x_1 is $\tilde{A}_{21}$ and x_2 is $\tilde{A}_{22}$ ... and x_n is $\tilde{A}_{2n}$ then y is $\tilde{B}_2$,

$\vdots$ $\vdots$

Rule-q: if x_1 is $\tilde{A}_{q1}$ and x_2 is $\tilde{A}_{q2}$ .... and x_n is $\tilde{A}_{qn}$ then y is $\tilde{B}_q$,

where x_1, x_2, x_3 and y are fuzzy variables of E/P, P/BV, P/P_n and W respectively, and $\tilde{A}_{k1}$, $\tilde{A}_{k2}$, and $\tilde{A}_{k3}$, $k \in I_q$ are linguistic terms of E/P, P/BV, P/P_n, and W respectively, i.e., $E/P = \{\tilde{A}_{11}, \tilde{A}_{21}, \ldots, \tilde{A}_{q1}\}$, $P/BV = \{\tilde{A}_{12}, \tilde{A}_{22}, \ldots, \tilde{A}_{q2}\}$, $P/P_n = \{\tilde{A}_{13}, \tilde{A}_{23}, \ldots, \tilde{A}_{q3}\}$ and $W = \{\tilde{B}_1, \tilde{B}_2, \ldots, \tilde{B}_q\}$.

Step1-6) Importing the E/P, P/BV and P/P_n of the latest day and making estimation with Mamdani method using the fuzzy rules constructed in Step 1-5). Hence we obtain an output of a fuzzy set $\tilde{B}$ under the membership $u_{\tilde{B}}$ on B.

Step1-7) Performing defuzzification of the fuzzy output to a crisp output by a centroid method. A crisp z^{cg} is the average weight of the weight at each point z on domain B where

$$w_z = \frac{u_{\tilde{B}}(z)}{\int_B u_{\tilde{B}}(z)dz} \quad \text{for all} \quad z \in B,$$

i.e., the crisp output is $$z^{cg} = \int_B z w_z dz = \frac{\int_B z u_{\tilde{B}}(z)dz}{\int_B u_{\tilde{B}}(z)dz}.$$

It is the investment weight of each individual stock in a particular industrial group. These weights are then used to rank stocks in an industrial group.

3.2 Analysis of industrial groups

Industrial groups are ranked by weights calculated by the method of fuzzy multi-criteria decision-making consisting of AHP, fuzzy Analytic Hierarchy Process, and FTOPSIS, Fuzzy Technique for Order Preference by Similarity to Ideal Solution Method.

Applying this technique to n_1 decision makers, n_2 decision criteria and n_3 alternative as alternate choices, the analysis steps are as follows.

Step2-1) Finding matrix for decision makers

Herein a reciprocal matrix is compares paired data that are metrics of real quantities such as price, weight, and preference. Here, these quantities are preferences. Levels of preferences are represented by numbers in a set $\Omega_n = \left\{\frac{1}{n}, \frac{1}{n-1}, \ldots, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, \ldots, n-1, n\right\}$. Generalizing this idea, the set of crisp preference values Ω_n is replaced by a set of fuzzy preference values $\tilde{\Omega}_n^{\delta} = \left\{\frac{1}{\tilde{n}_\delta}, \frac{1}{(\widetilde{n-1})}, \ldots, \frac{1}{\tilde{3}_\delta}, \frac{1}{\tilde{2}_\delta}, 1, \tilde{2}_\delta, \tilde{3}_\delta, \ldots, (\widetilde{n-1})_\delta, \tilde{n}_\delta\right\}$ where $\tilde{k}_\delta = \langle k-\delta, k, k+\delta\rangle$ and $\frac{1}{\tilde{k}_\delta} = 1 \oslash \tilde{k}_\delta = \left\langle \frac{1}{k+\delta}, \frac{1}{k}, \frac{1}{k-\delta}\right\rangle$ for all $k \in I_n$ and $0 \le \delta \le 1$.

In this step, a decision maker i , $i = 1, \ldots, n_1$ is compared to another decision maker j in terms of their preference level based on a preference function $\varphi(i, j)$ defined as

$$\varphi(i,j) = \begin{cases} \tilde{c}_{ij} & , \exists \tilde{c}_{ij} \in \tilde{\Omega}_n \ ; \ j > i \\ 1 & ; \ j = i \\ 1 \oslash \varphi(j,i) & ; \ j < i \end{cases}$$

The decision maker's preference matrix $\tilde{D}=\left(\tilde{a}_{ij}\right)_{n_1\times n_1}$ is a reciprocal matrix where

$$\tilde{a}_{ij}=\begin{cases}\varphi(i,j) & ;i<j\\ 1 & ;i=j\\ 1\oslash\varphi(j,i) & ;i>j\end{cases}$$

Step2-2) Finding a fuzzy weight vector $\tilde{w}_d=(\tilde{w}_{d,k})_{n_1\times 1}$ for $\tilde{D}=\left(\tilde{a}_{ij}\right)_{n_1\times n_1}$

$\tilde{w}_{d,k}=\left\langle w_{d,k}^L, w_{d,k}^M, w_{d,k}^U\right\rangle$ is a fuzzy weight vector for all $k\in I_{n_1}$ where

$$w_{d,k}^L=C_L\cdot\frac{\left(\prod_{j=1}^{n_1}a_{kj}^L\right)^{1/n_1}}{\sum_{i=1}^{n_1}\left(\prod_{j=1}^{n_1}a_{ij}^M\right)^{1/n_1}},\quad w_{d,k}^M=\frac{\left(\prod_{j=1}^{n_1}a_{kj}^M\right)^{1/n_1}}{\sum_{i=1}^{n_1}\left(\prod_{j=1}^{n_1}a_{ij}^M\right)^{1/n_1}},\quad w_{d,k}^U=C_U\cdot\frac{\left(\prod_{j=1}^{n_1}a_{kj}^U\right)^{1/n_1}}{\sum_{i=1}^{n_1}\left(\prod_{j=1}^{n_1}a_{ij}^M\right)^{1/n_1}}$$

with $C_L=\min_{i\in I_{n_1}}\left\{\frac{\left(\prod_{j=1}^{n_1}a_{ij}^M\right)^{1/n_1}}{\left(\prod_{j=1}^{n_1}a_{ij}^L\right)^{1/n_1}}\right\}$ and $C_U=\max_{i\in I_{n_1}}\left\{\frac{\left(\prod_{j=1}^{n_1}a_{ij}^M\right)^{1/n_1}}{\left(\prod_{j=1}^{n_1}a_{ij}^U\right)^{1/n_1}}\right\}$.

If its consistency index $I_{n_1}^{\sigma}\left(\tilde{D}\right)$ as defined in Definition 2.15 is less than 0.1, it is accepted as being valid. Otherwise, the decision maker's weight is re-evaluated by repeating Step2-1).

Step2-3) Decision makers $d_1,d_2,\ldots,d_{n_1}$ constructing decision criteria $c_1,c_2,\ldots,c_{n_2}$ for evaluating industrial groups $G_1,G_2,\ldots,G_{n_3}$, where c_i, $i=1,\ldots,n_2$ is constructed from investment weight of n_3 individual groups given by decision makers in the term of following linguistic terms.

linguistic term	Fuzzy number
Very Low (VL)	$\langle 0,\ 0,\ 0.1, 0.2\rangle$
Low (L)	$\langle 0.1,\ 0.2,\ 0.3\rangle$
Medium Low (ML)	$\langle 0.2,\ 0.3,\ 0.4\rangle$
Medium (M)	$\langle 0.3,\ 0.4,\ 0.6, 0.7\rangle$
Medium High (MH)	$\langle 0.6,\ 0.7,\ 0.8\rangle$
High (H)	$\langle 0.7,\ 0.8,\ 0.9\rangle$
Very High (VH)	$\langle 0.8, 0.9, 1, 1\rangle$

The decision criteria constructed is in the form of a fuzzy matrix $\tilde{B}$ with members $b_{jik} = \langle b_{jik}^{L}, b_{jik}^{M_1}, b_{jik}^{M_2}, b_{jik}^{U} \rangle$, $j \in I_{n_3}, i \in I_{n_2}$ and $k \in I_{n_1}$, which are trapezoidal fuzzy numbers representing the linguistic terms of $c_1, c_2, ..., c_{n_2}$ shown in Table 3.1.

Table 3.1 Decision criteria for evaluating industrial groups $G_1, G_2, ..., G_{n_3}$

		d_1	d_2	...	d_{n_1}
c_1	G_1	$\tilde{b}_{111}$	$\tilde{b}_{112}$	...	$\tilde{b}_{11n_1}$
	G_2	$\tilde{b}_{211}$	$\tilde{b}_{212}$	...	$\tilde{b}_{21n_1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	G_{n_3}	$\tilde{b}_{n_311}$	$\tilde{b}_{n_312}$	...	$\tilde{b}_{n_31n_1}$
c_2	G_1	$\tilde{b}_{121}$	$\tilde{b}_{122}$	...	$\tilde{b}_{12n_1}$
	G_2	$\tilde{b}_{221}$	$\tilde{b}_{222}$	...	$\tilde{b}_{22n_1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	G_{n_3}	$\tilde{b}_{n_321}$	$\tilde{b}_{n_322}$	...	$\tilde{b}_{n_32n_1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
c_{n_2}	G_1	$\tilde{b}_{1n_21}$	$\tilde{b}_{1n_22}$	...	$\tilde{b}_{1n_2n_1}$
	G_2	$\tilde{b}_{2n_21}$	$\tilde{b}_{2n_22}$	...	$\tilde{b}_{2n_2n_1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	G_{n_3}	$\tilde{b}_{n_3n_21}$	$\tilde{b}_{n_3n_22}$	...	$\tilde{b}_{n_3n_2n_1}$

$= \tilde{B}$

Step2-4) Decision makers $d_1, d_2, ..., d_{n_1}$ evaluating decision criteria $c_1, c_2, ..., c_{n_2}$ constructing from the linguistic terms VL, L, ML, M, MH, H, VH as in step2-3). A fuzzy matrix $\tilde{C} = (\tilde{c}_{ij})_{n_2 \times n_1}$ for evaluation is then obtained where $\tilde{c}_{ij} \in \{VL, L, ML, M, MH, H, VH\}$ for all $i \in I_{n_2}$ and $j \in I_{n_1}$ as shown in Table 3.2.

Table 3.2 Evaluation of decision criteria $c_1, c_2, ..., c_{n_2}$

	d_1	d_2	...	d_{n_1}
c_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	...	$\tilde{c}_{1n_1}$
c_2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	...	$\tilde{c}_{2n_1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
c_{n_2}	$\tilde{c}_{n_21}$	$\tilde{c}_{n_22}$	...	$\tilde{c}_{n_2n_1}$

$= \tilde{C}$

Step2-5) Calculating decision criteria based on decision makers' weights by multiplying the decision criterion of a decision maker in each column in Step2-4) (depicted in Table 3.2) with the corresponding decision maker's fuzzy weight vector

$\tilde{w}_d = (\tilde{w}_{d,k})_{n\times 1}$ where $\tilde{w}_{d,k} = \langle w^L_{d,k}, w^M_{d,k}, w^U_{d,k} \rangle = \langle w^L_{d,k}, w^M_{d,k}, w^M_{d,k}, w^U_{d,k} \rangle$ calculated from Step2-2). Table 3.3 shows these multiplication results.

Table 3.3 Decision criteria based on weights of decision makers

	d_1	d_2	...	d_{n_1}
c_1	$\tilde{c}_{11} \otimes \tilde{w}_{d,1}$	$\tilde{c}_{12} \otimes \tilde{w}_{d,2}$	...	$\tilde{c}_{1n_1} \otimes \tilde{w}_{d,n_1}$
c_2	$\tilde{c}_{21} \otimes \tilde{w}_{d,1}$	$\tilde{c}_{22} \otimes \tilde{w}_{d,2}$	...	$\tilde{c}_{2n_1} \otimes \tilde{w}_{d,n_1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
c_{n_2}	$\tilde{c}_{n_2 1} \otimes \tilde{w}_{d,1}$	$\tilde{c}_{n_2 2} \otimes \tilde{w}_{d,2}$	...	$\tilde{c}_{n_2 n_1} \otimes \tilde{w}_{d,n_1}$

$= \tilde{C}_w$

Next, multiplying the decision criterion for evaluating industrial groups in the column representing each decision maker constructed in Step2-3) with the corresponding decision maker's fuzzy weight vector $\tilde{w} = (\tilde{w}_{d,k})_{n\times 1}$ where $\tilde{w}_{d,k} = \langle w^L_{d,k}, w^M_{d,k}, w^U_{d,k} \rangle = \langle w^L_{d,k}, w^M_{d,k}, w^M_{d,k}, w^U_{d,k} \rangle$ calculated from Step2-2). The multiplication results are in Table 3.4.

Table 3.4 Decision criteria for evaluating industrial groups based on weights of decision makers

		d_1	d_2	...	d_{n_1}
c_1	G_1	$\tilde{b}_{111} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{112} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{11n_1} \otimes \tilde{w}_{d,n_1}$
	G_2	$\tilde{b}_{211} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{212} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{21n_1} \otimes \tilde{w}_{d,n_1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	G_{n_3}	$\tilde{b}_{n_3 11} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{n_3 12} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{n_3 1n_1} \otimes \tilde{w}_{d,n_1}$
c_2	G_1	$\tilde{b}_{121} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{122} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{12n_1} \otimes \tilde{w}_{d,n_1}$
	G_2	$\tilde{b}_{221} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{222} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{22n_1} \otimes \tilde{w}_{d,n_1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	G_{n_3}	$\tilde{b}_{n_3 21} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{n_3 22} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{n_3 2n_1} \otimes \tilde{w}_{d,n_1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
c_{n_2}	G_1	$\tilde{b}_{1n_2 1} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{1n_2 2} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{1n_2 n_1} \otimes \tilde{w}_{d,n_1}$
	G_2	$\tilde{b}_{2n_2 1} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{2n_2 2} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{2n_2 n_1} \otimes \tilde{w}_{d,n_1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	G_{n_3}	$\tilde{b}_{n_3 n_2 1} \otimes \tilde{w}_{d,1}$	$\tilde{b}_{n_3 n_2 2} \otimes \tilde{w}_{d,2}$	...	$\tilde{b}_{n_3 n_2 n_1} \otimes \tilde{w}_{d,n_1}$

$= \tilde{B}_w$

Step2-6) Aggregating weights of decision criteria based on the decision makers' weights as follows.

$$\tilde{w}_{c,i} = \left\langle w_{c,i}^{L}, w_{c,i}^{M_1}, w_{c,i}^{M_2}, w_{c,i}^{U} \right\rangle$$

where $w_{c,i}^{L} = \min_{k=1}^{n_1}\{c_{w,ik}^{\ L}\}$, $w_{c,i}^{M_1} = \frac{1}{n_1}\sum_{k=1}^{n_1} c_{w,ik}^{\ M_1}$, $w_{c,i}^{M_2} = \frac{1}{n_1}\sum_{k=1}^{n_1} c_{w,ik}^{\ M_2}$, $w_{c,i}^{U} = \max_{k=1}^{n_1}\{c_{w,ik}^{\ U}\}$ for all $i \in I_{n_2}$, $\tilde{C}_w = \left(\tilde{c}_{w,jk}\right)_{n_2 \times n_1}$ and n_1 is the number of decision makers. Table 3.5 shows these aggregation results.

Table 3.5 Weights of decision criteria $c_1, c_2, \ldots, c_{n_2}$

	c_1	c_2	...	c_{n_2}
W	$\tilde{w}_{c,1}$	$\tilde{w}_{c,2}$	...	$\tilde{w}_{c,n_2}$

Next, aggregating industrial groups based on the decision makers' weights (Table 3.4) by the following equations:

$$\tilde{x}_{ji} = \left\langle x_{ji}^{L}, x_{ji}^{M_1}, x_{ji}^{M_2}, x_{ji}^{U} \right\rangle$$

where $x_{ji}^{L} = \min_{k=1}^{n_1}\{b_{w,jik}^{\ L}\}$, $x_{ji}^{M_1} = \frac{1}{n_1}\sum_{k=1}^{n_1} b_{w,jik}^{\ M_1}$, $x_{ji}^{M_2} = \frac{1}{n_1}\sum_{k=1}^{n_1} b_{w,jik}^{\ M_2}$, $x_{ji}^{U} = \max_{k=1}^{n_2}\{b_{w,jik}^{\ U}\}$ for all $j \in n_3, i \in n_2$, $\tilde{B}_w = \left(\tilde{b}_{w,jik}\right)_{n_3 n_2 \times n_1}$ and n_1 is the number of decision makers. These results are shown in Table 3.6.

Table 3.6 Evaluation matrix of industrial groups $G_1, G_2, \ldots, G_{n_3}$

	c_1	c_2	...	c_{n_2}
G_1	$\tilde{x}_{11}$	$\tilde{x}_{12}$	...	$\tilde{x}_{1n_2}$
G_2	$\tilde{x}_{21}$	$\tilde{x}_{22}$	...	$\tilde{x}_{2n_2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
G_{n_3}	$\tilde{x}_{n_3 1}$	$\tilde{x}_{n_3 2}$	...	$\tilde{x}_{n_3 n_2}$

$= \tilde{X}$

Step2-7) Constructing a decision matrix by normalizing the industrial groups' evaluation matrix $\tilde{X}$ (Table 3.6) as follows,

$$\hat{R} = \left(\tilde{r}_{ji}\right)_{n_3 \times n_2}, \quad \tilde{r}_{ji} = \left\langle \frac{x_{ji}^{L}}{x_i^*}, \frac{x_{ji}^{M_1}}{x_i^*}, \frac{x_{ji}^{M_2}}{x_i^*}, \frac{x_{ji}^{U}}{x_i^*} \right\rangle \text{ where } x_i^* = \max_{j}^{n_3}\left\{x_{ji}^{U}\right\}.$$

Then, multiplying the normalized matrix with the decision weights from Step2-6),

$$\tilde{V} = \left(\tilde{v}_{ji}\right)_{n_3 \times n_2} \quad \text{where } \tilde{v}_{ji} = \left\langle v_{ji}^{L}, v_{ji}^{M_1}, v_{ji}^{M_2}, v_{ji}^{U} \right\rangle$$

and $\tilde{v}_{ji} = \tilde{r}_{ji} \otimes \tilde{w}_{c,i}$ when $j \in I_{n_3}$, $i \in I_{n_2}$.

Table 3.7 Industrial groups' evaluation matrix

	c_1	c_2	...	c_{n_2}
G_1	$\tilde{v}_{11}$	$\tilde{v}_{12}$	...	$\tilde{v}_{1n_2}$
G_2	$\tilde{v}_{21}$	$\tilde{v}_{22}$	...	$\tilde{v}_{2n_2}$
⋮	⋮	⋮	⋮	⋮
G_{n_3}	$\tilde{v}_{n_3 1}$	$\tilde{v}_{n_3 2}$	...	$\tilde{v}_{n_3 n_2}$

$= \tilde{V}$

Step2-8) Defining positive ideal solution (G^*) and negative ideal solution (G^-) from Table 3.7 as $G^* = \left(\hat{v}_1^*, \hat{v}_2^*, ..., \hat{v}_{n_2}^*\right)$ and $G^- = \left(\hat{v}_1^-, \hat{v}_2^-, ..., \hat{v}_{n_2}^-\right)$, respectively, where $\hat{v}_i^* = \max_j^{n_3}\left\{v_{ji}^U\right\}$ and $\hat{v}_i^- = \min_j^{n_3}\left\{v_{ji}^L\right\}; j \in I_{n_3}, i \in I_{n_2}$, $\tilde{V} = \left(\tilde{v}_{ji}\right)_{n_3 \times n_2}$.

Step2-9) Calculating the distances between the industrial groups' evaluation results with the positive and negative ideal solutions, as defined by the following,

$$d_j^* = \sum_{i=1}^{n_2} d_v(\tilde{v}_{ji}, \hat{v}_i^*) \quad , j \in I_{n_3}$$

$$d_j^- = \sum_{i=1}^{n_2} d_v(\tilde{v}_{ji}, \hat{v}_i^-) \quad , j \in I_{n_3}$$

where $d_v(\tilde{v}_{ji}, \hat{v}_i^{*,-})$ are calculated in the same way as fuzzy numbers are calculated according to Definition 2.8 (depicted in Table 3.8).

Table 3.8 Distances between the industrial groups' evaluation results and positive and negative ideal solutions G^* and G^-

	c_1	c_2	...	c_{n_2}	$d_j^{*,-} = \sum_{i=1}^{n_2} d_v(\tilde{v}_{ji}, \hat{v}_j^{*,-})$
G_1	$d_v(\tilde{v}_{11}, \hat{v}_1^{*,-})$	$d_v(\tilde{v}_{12}, \hat{v}_2^{*,-})$	...	$d_v(\tilde{v}_{1n_2}, \hat{v}_{n_2}^{*,-})$	$d_1^{*,-}$
G_2	$d_v(\tilde{v}_{21}, \hat{v}_1^{*,-})$	$d_v(\tilde{v}_{22}, \hat{v}_2^{*,-})$	...	$d_v(\tilde{v}_{2n_2}, \hat{v}_{2n_2}^{*,-})$	$d_2^{*,-}$
⋮	⋮	⋮	⋮	⋮	⋮
G_{n_3}	$d_v(\tilde{v}_{n_3 1}, \hat{v}_1^{*,-})$	$d_v(\tilde{v}_{n_3 2}, \hat{v}_2^{*,-})$	...	$d_v(\tilde{v}_{n_3 n_2}, \hat{v}_{n_2}^{*,-})$	$d_{n_3}^{*,-}$

Step2-10) Calculating the nearness coefficients to the positive ideal solution, CC_j, and ranking the industrial groups according to them. CC_j are defined as follows,

$$CC_j = \frac{d_j^-}{d_j^- + d_j^*} \quad , j \in I_{n_3} .$$

From the calculation, a set of investment weights for industrial groups, $W_3 = (w_1, w_2, ..., w_{n_3})$ where $w_1, w_2, ..., w_{n_3}$ are weights of individual groups, is obtained. The industrial group of which investment weight value is nearest to one (the closest to the positive ideal solution) is the best industrial group.

3.3 Analysis of all stocks from different industrial groups

In this step, the two investment weights from Main Step 1) and Main Step 2) are used to calculate the integrated final investment weights for all of the stocks in the market, denoted as

$$W_{OA}(s_{ij}) \text{ where } W_{OA}(s_{ij}) = W_1(s_{ij}) \cdot W_2(G_j)$$

and $W_1(s_{ij})$ is the weight of the i^{th} stock from the j^{th} group from Main Step 1 and $W_2(G_j)$ is the weight of the j^{th} group from Main Step 2). These weights are then used to rank the stocks for making decisions and planning out strategies.

Chapter 4
Application of the analysis procedures to a demonstration case

As a demonstration of the applicability of our analysis procedures, a simulated case of stock selection into a portfolio for a given period of time is conducted. Suppose that the 6 industrial groups of investment interest were the following: agricultural and food industry (G_1) consumer product and service industry (G_2), financial industry (G_3), industrial product and technology industry (G_4), property and construction industry (G_5), and resource industry (G_6). Stocks from each individual industry were analyzed as follows.

Main step 1): Analysis of individual stocks within each industrial group

As an example, the analysis of the property and construction industry, G_5, is shown below.

In this group G_5, we use the past 5-year financial fact data of the companies from Stock Exchange of Thailand, 2010-2014, http://www.settrade.com.

Step1-1) Gathering the past 5-year financial data of the companies in this group and screening in stocks with complete data from 12 companies: CK, CNT, ITD, NWR, PREB, SEAFCO, STEC, STPI, SYNTEC, TRC , TTCL, and UNIQ.

Step1-2) Calculating the E/P, P/BV, and P/P_n values of each individual stock.

Step1-3) Calculating the following weighted arithmetic mean of E/P, P/BV, and P/P_n. Table 4.1, 4.2 and 4.3 show data of some stock(STPI), Table 4.4 show the weighted arithmetic mean of each individual stock in G_5.

<u>E/P</u>

We gather data of each individual stock for calculating E/P. We deal it following this flowchart shown in Figure 4.1.

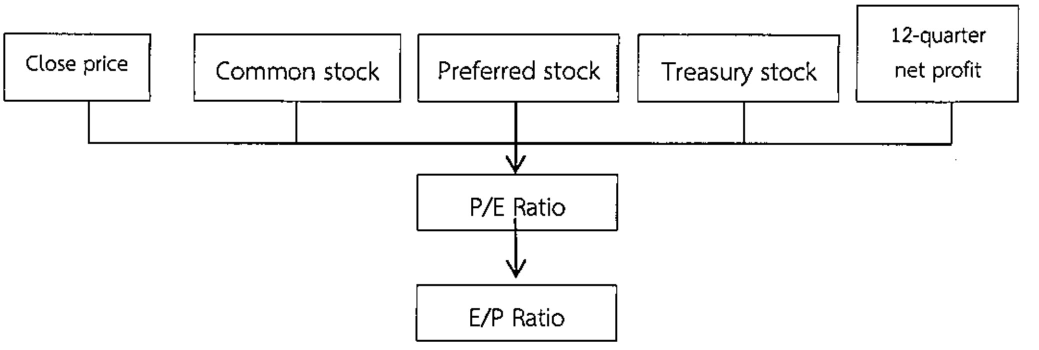

Figure 4.1 A flowchart representing a process for calculating E/P.

We calculate E/P of each individual stock by using Definition 2.15. The weighted arithmetic mean $(E/P)^w(S_k)$, $k \in I_m$ is computed from the formula:

$$(E/P)^w(S_k) = \sum_{i=1}^{n} w_i (E/P)(S_k^i)$$

where $w_i = \dfrac{2i}{n(n+1)}$, $i \in I_n$.

The weighted arithmetic mean E/P of some individual stock is shown in Table 4.1.

Table 4.1 E/P of STPI

STPI stock	14/10/2014	27/12/2013	28/12/1012	30/12/2011	30/12/2010
Closing price of common stock (baht)		15.7	62.75	28.75	27
Number of common stocks		369360995	368,492,092	367,873,233	367,546,097
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		1908520000	1089760000	399510000	2021430000
P/E	14.8500	3.0385	21.2183	26.4733	4.9093
E/P	0.0673	0.3291	0.0471	0.0378	0.2037
E/P (weighted average)	0.1383				
E/P (% weighted average)	13.83				

$\underline{P/BV}$

We gather data of each individual stock for calculating P/BV. We deal it following this flowchart shown in Figure 4.2.

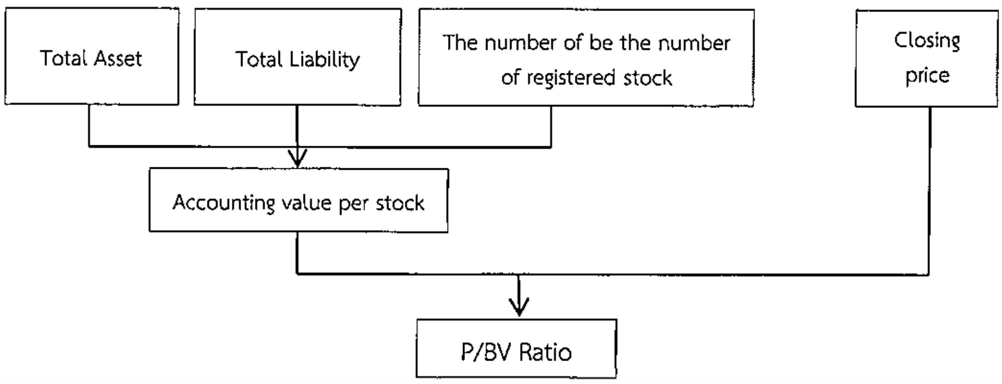

Figure 4.2 A flowchart representing a process for calculating P/BV.

We calculate P/BV by using Definition 2.16. The weighted arithmetic mean $(P/BV)^{w}(S_k)$, $k \in I_m$ is computed with the formula:

$$(P/BV)^{w}(S_k)=\sum_{i=1}^{n} w_i (P/BV)(S_k^i)$$

where $w_i=\dfrac{2i}{n(n+1)}$, $i \in I_n$. The weighted arithmetic mean P/BV of some individual stock is shown in Table 4.2.

Table 4.2 P/BV of STPI

STPI stock	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	15.7	62.75	28.75	27
Number of common stocks	1477443980	368,492,092	367,873,233	367,546,097
Number of preferred stocks	0	0	0	0
Total assets	10867008638	7347262706	3522893354	4259624240
Total liabilities	4956210154	2922198628	423972604	1021904292
Accounting value per share	4.000692117	12.00857271	8.42388212	8.809017357
P/BV	3.924320978	5.225433658	3.412915754	3.065041072
P/BV of 2014 (2nd quarter)	4.8			
P/BV (weighted average)	4.350963831			
P/BV (highest)	25.18861616			
P/BV (%)	17.27353263			

<u>P/P_n</u>

We gather data of each individual stock for calculating P/P_n. We deal it following this flowchart shown in Figure 4.3.

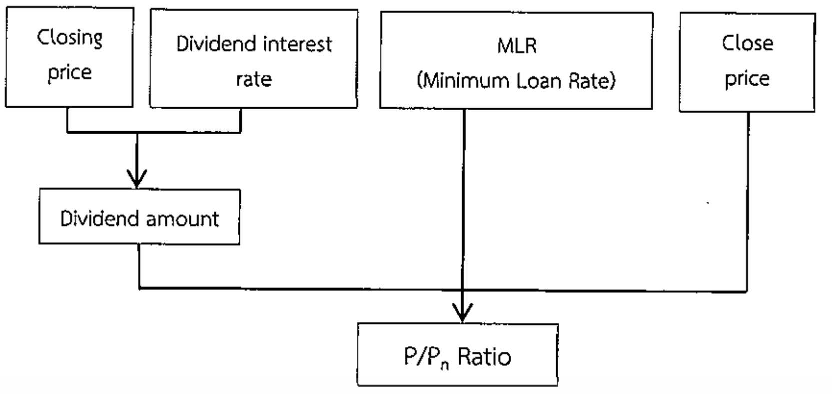

Figure 4.3 A flowchart representing a process for calculating P/P_n

In this research, we use MLR average from 2010-2014 for calculating P/P_n. MLR of each individual year from 2010-2014 is shown in Table 4.3

Table 4.3 MLR

	2010	2011	2012	2013	2014
BAY	6.5	7.625	7.375	6.875	7.125
BBL	6.125	7.25	7	7	6.75
BAY	6.125	7.25	7	6.8575	6.75
KBANK	6.12	7.25	7	6.75	6.75
CIMB	6.75	7.75	7.625	7.625	7.375
TMB	6.5	7.5	7.375	7.25	7.25
TNC	6.5	7.625	7.375	7.25	7.125
SCB	6.12	7.25	7	6.75	6.75
TISCO	6.5	7.625	7.375	7.125	7.125
UOB	6.85	7.875	7.625	7.25	7.25
Average	6.409	7.5	7.275	7.0733	7.025

We calculates P/P_n by using Definition 2.17 and Definition 2.18. The weighted arithmetic mean $(P/P_n)^w(S_k)$, $k \in I_m$ is computed from this formula:

$$(P/P_n)^w(S_k) = \sum_{i=1}^{n} w_i (P/P_n)(S_k^i)$$

where $w_i = \frac{2i}{n(n+1)}$, $i \in I_n$. The weighted arithmetic mean P/P_n of some individual stock is shown in Table 4.4.

Table 4.4 P/P_n of STPI

STPI stock	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	20.8	15.7	62.75	28.75	27
Dividend interest rate (%)	1.63	1.59	0.5	12.16	7.86
Dividend amount (baht)	0.339	0.2496	0.3138	3.496	2.1222
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment ($1+r$)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	29.3056				
Closing price to target price ratio	0.7098				

Therefore, financial ratio of stocks in G_5 is Table 4.4.

Table 4.5 E/P , P/BV , P/P_n of stocks in G_5

Financial Ratio	CK	CNT	ITD	NWR	PREB	SEAFCO	STEC	STPI	SYNTEC	TRC	TTCL	UNIQ
E/P (%)	10.86	7.86	0.89	6.14	10.5	6.38	5.59	13.83	3.26	7.98	4.2	7.66
P/BV(%)	8.71	9.1	7.35	4.73	8.19	7.19	16.06	17.27	3.8	8.99	16.19	8.3
P/P_n	2.43	1.12	0.94	2.38	2.94	0.97	1.67	0.71	1.86	0.83	2.87	2.4

Step1-4) An expert constructing a fuzzy set based on the latest 5-year financial data of which linguistic terms are represented by trapezoidal and triangular fuzzy numbers.

Values of E/P , P/BV, P/P_n were grouped into 3 levels: low(GL), medium(GM) , and high(GH), and so the fuzzy sets representing these levels were $\tilde{L}=\langle x^L, x^{M_1}, x^{M_2}, x^U\rangle$, $\tilde{M}=\langle y^L, y^{M_1}, y^{M_2}, y^U\rangle$, and $\tilde{H}=\langle z^L, z^{M_1}, z^{M_2}, z^U\rangle$.

The fuzzy sets of linguistic terms are as follows:

$$E/P \Rightarrow \tilde{LX}=\langle 0,0,1,3\rangle \quad , \quad \tilde{MX}=\langle 1,3,8,10\rangle \quad , \quad \tilde{HX}=\langle 8,10,100,100\rangle;$$

$$P/BV \Rightarrow \tilde{LY}=\langle 0,0,5,7\rangle \quad , \quad \tilde{MY}=\langle 5,7,10,16\rangle \quad , \quad \tilde{HY}=\langle 10,16,100,100\rangle;$$

$$P/P_n \Rightarrow \tilde{LZ}=\langle 0,0,1,1.1\rangle \quad , \quad \tilde{MZ}=\langle 1,1.1,1.9,2.3\rangle \quad , \quad \tilde{HZ}=\langle 1.9,2.3,100,100\rangle.$$

Let x, y, z and w be a fuzzy variable of E/P, P/BV and P/P_n , respectively.

Then the membership functions of each fuzzy sets of linguistic terms are as follows:

$$u_{\tilde{LX}}(x)=\begin{cases}1 & ;0\le x\le 1\\ -\dfrac{(x-3)}{2} & ;1\le x\le 3\end{cases}, \qquad u_{\tilde{MX}}(x)=\begin{cases}\dfrac{x-1}{2} & ;1\le x\le 3\\ 1 & ;3\le x\le 8\\ -\dfrac{(x-10)}{2} & ;8\le x\le 10\end{cases}$$

$$u_{\tilde{HX}}(x)=\begin{cases}\frac{x-8}{2} & ;8\le x\le 10\\ 1 & ;10\le x\le 100\end{cases}$$

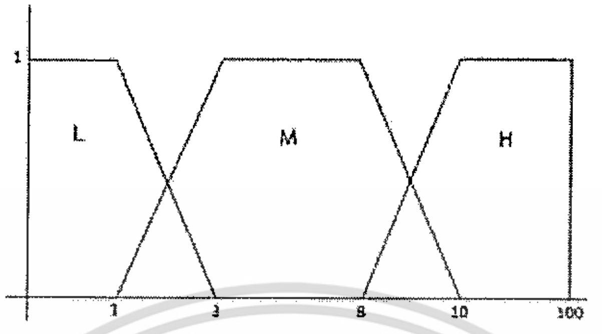

Figure 4.4 Fuzzy sets of linguistic terms of E/P

$$u_{\tilde{LY}}(y)=\begin{cases}1 & ;0\le y\le 5\\ -\frac{(y-7)}{2} & ;5\le y\le 7\end{cases},\quad u_{\tilde{MY}}(y)=\begin{cases}\frac{y-5}{2} & ;5\le y\le 7\\ 1 & ;7\le y\le 10\\ -\frac{(y-16)}{6} & ;10\le y\le 16\end{cases}$$

$$u_{\tilde{HY}}(y)=\begin{cases}\frac{y-10}{6} & ;10\le y\le 16\\ 1 & ;16\le y\le 100\end{cases}$$

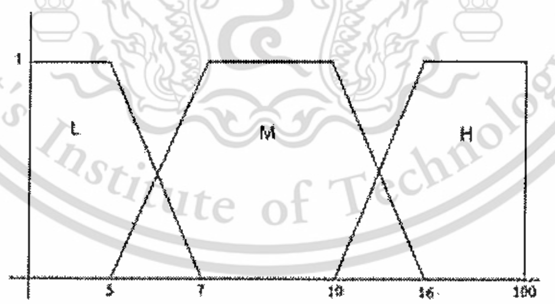

Figure 4.5 Fuzzy sets of linguistic terms of P/BV

$$u_{\tilde{LZ}}(z)=\begin{cases}1 & ;0\le z\le 1\\ -\frac{(z-1.1)}{0.1} & ;1\le z\le 1.1\end{cases},\quad u_{\tilde{MZ}}(z)=\begin{cases}\frac{z-1}{0.1} & ;1\le z\le 1.1\\ 1 & ;1.1\le z\le 1.9\\ -\frac{(z-2.3)}{0.4} & ;1.9\le z\le 2.3\end{cases}$$

$$u_{\tilde{H}Z}(z)=\begin{cases}\dfrac{z-1.9}{0.4} & ;1.9\le z\le 2.3\\ 1 & ;2.3\le z\le 100\end{cases}$$

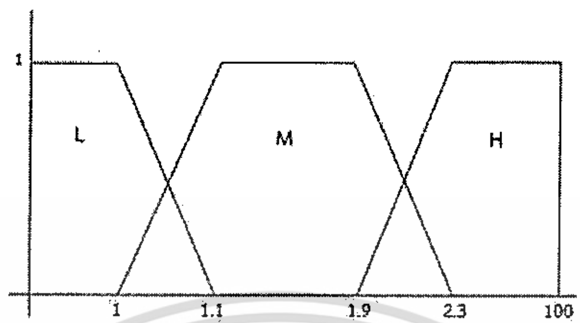

Figure 4.6 Fuzzy set of linguistic terms of P/P_n

Step1-5) An expert constructing fuzzy rules from the fuzzy sets constructed from Step 4 as follows:

In this research, we define fuzzy sets of linguistic terms of as follows:

$\tilde{L}=\langle 0,0,0.2,0.3\rangle$, $\tilde{RL}=\langle 0.2,0.3,0.4,0.5\rangle$, $\tilde{M}=\langle 0.4,0.5,0.6\rangle$

$\tilde{RH}=\langle 0.5,0.6,0.7,0.8\rangle$, $\tilde{H}=\langle 0.7,0.8,1,1\rangle$

which their membership functions are :

$$u_{\tilde{L}W}(w)=\begin{cases}1 & ;0\le w\le 0.2\\ -\dfrac{(w-0.3)}{0.1} & ;0.2\le w\le 0.3\end{cases}$$

$$u_{\tilde{RL}W}(w)=\begin{cases}\dfrac{w-0.2}{0.1} & ;0.2\le w\le 0.3\\ 1 & ;0.3\le w\le 0.4\\ -\dfrac{(w-0.5)}{0.1} & ;0.4\le w\le 0.5\end{cases}$$

$$u_{\tilde{M}W}(w)=\begin{cases}\dfrac{w-0.4}{0.1} & ;0.4\le w\le 0.5\\ -\dfrac{(w-0.6)}{0.1} & ;0.5\le w\le 0.6\end{cases}$$

$$u_{R\tilde{H}W}(w)=\begin{cases}\dfrac{w-0.5}{0.1} & ;0.5\le w\le 0.6\\ 1 & ;0.6\le w\le 0.7\\ -\dfrac{(w-0.8)}{0.1} & ;0.7\le w\le 0.8\end{cases}$$

$$u_{\tilde{H}W}(w)=\begin{cases}\dfrac{w-0.7}{0.1} & ;0.7\le w\le 0.8\\ 1 & ;0.8\le w\le 1\end{cases}$$

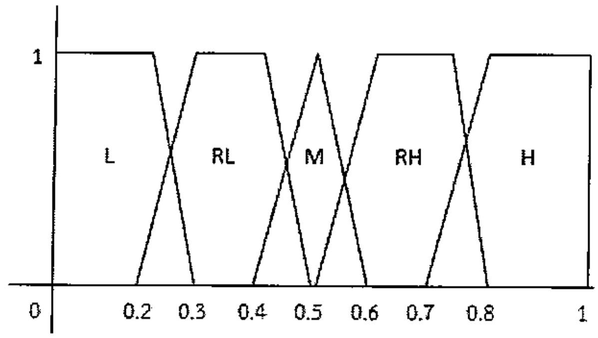

Figure 4.7 Fuzzy set of linguistic terms of W

We denote fuzzy set of linguistic terms of W by

$\tilde{LW}$ =: Low, $R\tilde{L}W$ =: Relatively low=, $\tilde{MW}$ =: Medium , $R\tilde{H}W$ =: Relatively high and $\tilde{HW}$ =: High .

We assume an expert determining fuzzy rules as follows:

Rule 1 If x is $\tilde{LX}$ and y is $\tilde{LY}$ and z is $\tilde{LZ}$ then w is $R\tilde{H}W$.

Rule 2 If x is $\tilde{LX}$ and y is $\tilde{LY}$ and z is $\tilde{MZ}$ then w is $\tilde{MW}$.

Rule 3 If x is $\tilde{LX}$ and y is $\tilde{LY}$ and z is $\tilde{HZ}$ then w is $R\tilde{L}W$.

Rule 4 If x is $\tilde{LX}$ and y is $\tilde{MY}$ and z is $\tilde{LZ}$ then w is $\tilde{MW}$.

Rule 5 If x is $\tilde{LX}$ and y is $\tilde{MY}$ and z is $\tilde{MZ}$ then w is $R\tilde{L}W$.

Rule 6 If x is $\tilde{LX}$ and y is $\tilde{MY}$ and z is $\tilde{HZ}$ then w is $\tilde{LW}$.

Rule 7 If x is $\tilde{LX}$ and y is $\tilde{HY}$ and z is $\tilde{LZ}$ then w is $R\tilde{L}W$.

Rule 8 If x is $\tilde{LX}$ and y is $\tilde{HY}$ and z is $\tilde{MZ}$ then w is $\tilde{LW}$.

Rule 9 If x is $\tilde{LX}$ and y is $\tilde{HY}$ and z is $\tilde{HZ}$ then w is $\tilde{LW}$.

Rule 10 If x is $\tilde{MX}$ and y is $\tilde{LY}$ and z is $\tilde{LZ}$ then w is $\tilde{HW}$.

Rule 11 If x is $\tilde{MX}$ and y is $\tilde{LY}$ and z is $\tilde{MZ}$ then w is $R\tilde{H}W$.

Rule 12 If x is $\tilde{MX}$ and y is $\tilde{LY}$ and z is $\tilde{HZ}$ then w is $\tilde{MW}$.

Rule 13 If x is $\tilde{MX}$ and y is $\tilde{MY}$ and z is $\tilde{LZ}$ then w is $R\tilde{H}W$.

Rule 14 If x is $\tilde{MX}$ and y is $\tilde{MY}$ and z is $\tilde{MZ}$ then w is $\tilde{MW}$.

Rule 15 If x is $\tilde{MX}$ and y is $\tilde{MY}$ and z is $\tilde{HZ}$ then w is $R\tilde{L}W$.

Rule 16 If x is $\tilde{MX}$ and y is $\tilde{HY}$ and z is $\tilde{LZ}$ then w is $\tilde{MW}$.

Rule 17 If x is $\tilde{MX}$ and y is $\tilde{HY}$ and z is $\tilde{MZ}$ then w is $R\tilde{L}W$.

Rule 18 If x is $\tilde{MX}$ and y is $\tilde{HY}$ and z is $\tilde{HZ}$ then w is $\tilde{LW}$.

Rule 19 If x is $\tilde{HX}$ and y is $\tilde{LY}$ and z is $\tilde{LZ}$ then w is $\tilde{HW}$.

Rule 20 If x is $\tilde{HX}$ and y is $\tilde{LY}$ and z is $\tilde{MZ}$ then w is $\tilde{HW}$.

Rule 21 If x is $\tilde{HX}$ and y is $\tilde{LY}$ and z is $\tilde{HZ}$ then w is $R\tilde{H}W$.

Rule 22 If x is $\tilde{HX}$ and y is $\tilde{MY}$ and z is $\tilde{LZ}$ then w is $\tilde{HW}$.

Rule 23 If x is $\tilde{HX}$ and y is $\tilde{MY}$ and z is $\tilde{MZ}$ then w is $R\tilde{H}W$.

Rule 24 If x is $\tilde{HX}$ and y is $\tilde{MY}$ and z is $\tilde{HZ}$ then w is $\tilde{MW}$.

Rule 25 If x is $\tilde{HX}$ and y is $\tilde{HY}$ and z is $\tilde{LZ}$ then w is $R\tilde{H}W$.

Rule 26 If x is $\tilde{HX}$ and y is $\tilde{HY}$ and z is $\tilde{MZ}$ then w is $\tilde{MW}$.

Rule 27 If x is $\tilde{HX}$ and y is $\tilde{HY}$ and z is $\tilde{HZ}$ then w is $R\tilde{L}W$.

Step1-6) Importing the values of current P/E , P/BV and P/P_n (inversing to E/P, BV/P and P_n/P), which, in this study, were the values of the 22nd of January 2015 shown in Table 4.5 below.

Table 4.6 Financial Ratios of the 22nd January 2015, http://www.settrade.com

Financial Ratio	CK	CNT	ITD	NWR	PREB	SEAFCO	STEC	STPI	SYNTEC	TRC	TTCL	UNIQ
E/P (%)	4.76	N.A	2.14	N.A	5.76	5.29	4.71	8.23	4.84	5.47	3.58	3.51
P/BV(%)	2.54	2.36	3.66	1.69	3.41	3.88	4.83	4.09	1.82	3.46	3.04	3.73
P/P_n	2.55	0.91	1.34	2.34	4.29	1.79	1.55	0.66	2.37	1.08	2.83	3.12

Note: The E/Ps of CNT and NWR were not applicable, meaning that they suffered a loss, so they were not included in further calculation.

Next, we only show detail of inference for STPI stock by using Mamdani method.
We input $E/P = 8.23$, $P/BV = 4.09$ and $P/P_n = 0.66$, i.e. input $x = 8.23$, $y = 4.09$ and $z = 0.66$.

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

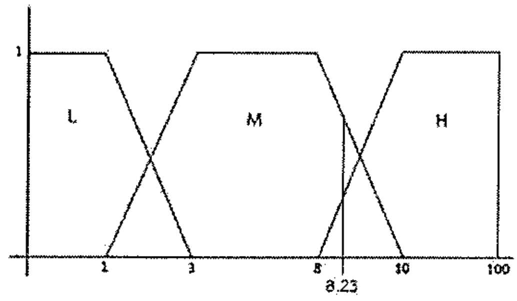

Figure 4.8 Intersection between input value and fuzzy set of linguistic terms of E/P,STPI.

For P/BV

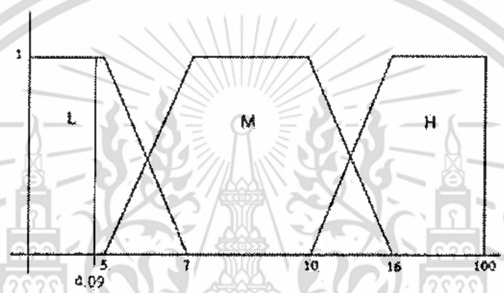

Figure 4.9 Intersection between input value and fuzzy set of linguistic terms of P/BV ,STPI.

For P/P_n

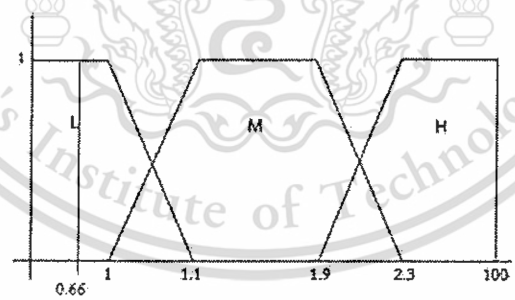

Figure 4.10 Intersection between input value and fuzzy set of linguistic terms of P/P_n,STPI.

From Figure 4.8-4.10, we have the concerned fuzzy rules as follows:

Rule 10 If x is $\tilde{MX}$ and y is $\tilde{LY}$ and z is $\tilde{LZ}$ then w is $\tilde{HW}$.

Rule 19 If x is $\tilde{HX}$ and y is $\tilde{LY}$ and z is $\tilde{LZ}$ then w is $\tilde{HW}$.

Since $u_{\tilde{MX}}(x) = -\dfrac{(x-10)}{2}$, $u_{\tilde{HX}}(x) = \dfrac{(x-8)}{2}$, $u_{\tilde{LY}}(y) = 1$ and $u_{\tilde{LZ}}(z) = 1$

we have,

$$h_{10} = u_{\tilde{MX}}(8.23) \wedge u_{\tilde{LY}}(4.09) \wedge u_{\tilde{LZ}}(0.66)$$
$$= 0.885 \wedge 1 \wedge 1 = 0.885$$
$$h_{19} = u_{\tilde{HX}}(8.23) \wedge u_{\tilde{LY}}(4.09) \wedge u_{\tilde{LZ}}(0.66)$$
$$= 0.115 \wedge 1 \wedge 1 = 0.115$$

Then $h = h_{10} \wedge h_{19} = 0.885 \wedge 0.115 = 0.115$

Since output of each rule is $\tilde{HW}$, so the union of truncated fuzzy outputs with $h = 0.115$ is equal to $\tilde{HW}$, that is, $\tilde{W}_{out}(w) = \tilde{HW}$.

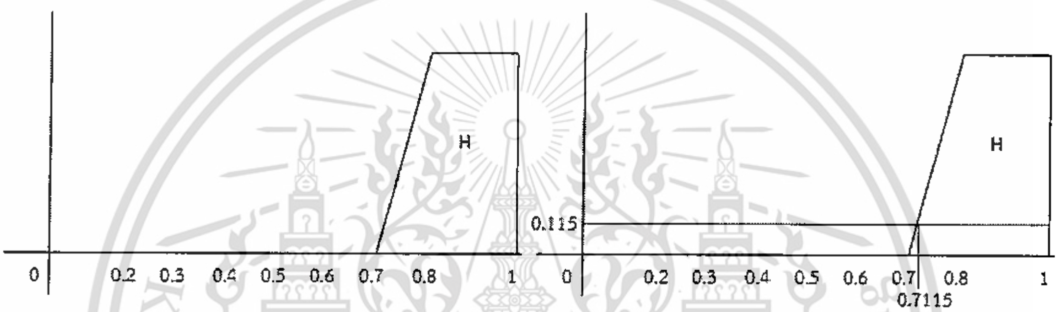

Figure 4.11 The union of truncated fuzzy outputs $\tilde{W}_{out}(w) = \tilde{HW}$,STPI

It is not difficult to find the membership function of $\tilde{W}_{out}(w) = \tilde{HW}$. From Figure 4.11, we have

$$u_{\tilde{W}_{out}}(w) = \begin{cases} \dfrac{w-0.7}{0.1} & ;0.7 \le w \le 0.7115 \\ 0.115 & ;0.7115 \le w \le 1 \end{cases}.$$

Step1-7) Performing defuzzification of the fuzzy output values to crisp values with the centroid method, obtaining the investment weights and normalize investment weight shown in Table 4.7 below.

In this document, we only show defuzzification $\tilde{W}_{out}(w) = \tilde{HW}$ of STPI.

Since $u_{\tilde{W}_{out}}(w) = \begin{cases} \dfrac{w-0.7}{0.1} & ;0.7 \le w \le 0.7115 \\ 0.115 & ;0.7115 \le w \le 1 \end{cases}$, we get

$$\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw = \frac{1}{2}(0.3 + 0.2885)(0.115) = 0.0338$$

and $$\int_{W_{out}} wu_{\tilde{W}_{out}}(w)dw = \int_{0.7}^{0.7115} w\left(\frac{w-0.7}{0.1}\right)dw + \int_{0.7115}^{1} w(0.115)dw.$$

Therefore $$w^{cg} = \frac{0.0289}{0.0338} = 0.855.$$

Note: For convenient, we use the MATLAB to compute the investment weight W.

Calculated investment weight W and normalize investment weight W_1 in industrial group of the property and construction industry G_5, can be conclude in Table 4.7.

Table 4.7 The normalize investment weights from the analysis procedures

Stock	CK	ITD	PREB	SEAFCO	STEC	STPI	SYNTEC	TRC	TTCL	UNIQ
The Investment weights	0.5	0.6285	0.5	0.65	0.65	0.855	0.5	0.674	0.5	0.5
The normalize investment weights W_1	0.084	0.105	0.084	0.1091	0.1091	0.1435	0.084	0.113	0.084	0.084

For the purpose of easy demonstration, the normalize investment weights W_1 of the stocks from the other 5 industrial groups were made up. All of the weights are tabulated in Table 4.8 below.

Table 4.8 The normalize investment weights of all stocks: the ones for G_5 were actually calculated while the rest were made up

G_1		G_2		G_3		G_4		G_5		G_6	
s_{11}	0.0418	s_{12}	0.26	s_{13}	0.1276	s_{14}	0.0518	s_{15} (CK)	0.084	s_{16}	0.0261
s_{21}	0.024	s_{22}	0.169	s_{23}	0.1528	s_{24}	0.1077	s_{25} (ITD)	0.105	s_{26}	0.1258
s_{31}	0.1148	s_{32}	0.1359	s_{33}	0.0282	s_{34}	0.1745	s_{35} (PREB)	0.084	s_{36}	0.0667
s_{41}	0.1704	s_{42}	0.1006	s_{43}	0.0843	s_{44}	0.0528	s_{45} (SEAFCO)	0.1091	s_{46}	0.2034
s_{51}	0.1003	s_{52}	0.004	s_{53}	0.0822	s_{54}	0.1108	s_{55} (STEC)	0.1091	s_{56}	0.0315
s_{61}	0.097	s_{62}	0.1376	s_{63}	0.0841	s_{64}	0.1399	s_{65} (STPI)	0.1435	s_{66}	0.1576
s_{71}	0.0764	s_{72}	0.1825	s_{73}	0.0335	s_{74}	0.0916	s_{75} (SYNTEC)	0.084	s_{76}	0.2068
s_{81}	0.0705	s_{82}	0.0104	s_{83}	0.0421	s_{84}	0.1099	s_{85} (TRC)	0.113	s_{86}	0.0638
s_{91}	0.1484			s_{93}	0.211	s_{94}	0.0825	s_{95} (TTCL)	0.084	s_{96}	0.1215
$s_{10,1}$	0.1565			$s_{10,3}$	0.2517	$s_{10,4}$	0.0986	$s_{10,5}$ (UNIQ)	0.084		

Main step 2): Analysis of industrial groups

Stocks from 6 industrial groups, $G_1, G_2, ..., G_6$, were analyzed. Three decision makers, d_1, d_2, d_3 constructed 4 decision criteria, c_1, c_2, c_3, c_4 calculated in the following steps.

Step2-1) Calculating the weights for decision makers. The preference level of the i^{th} decision maker is compared to that of the j^{th} decision maker with a scale $\left[\frac{1}{9}, 9\right]$, obtaining

$$\tilde{A} = \begin{pmatrix} (1,1,1) & (1,2,3) & (2,3,4) \\ \left(\frac{1}{3}, \frac{1}{2}, 1\right) & (1,1,1) & (1,2,3) \\ \left(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}\right) & \left(\frac{1}{3}, \frac{1}{2}, 1\right) & (1,1,1) \end{pmatrix}.$$

Step2-2) Calculating the fuzzy weight vectors, $\tilde{w}_d = (\tilde{w}_{d,k})_{n\times 1}$, for $\tilde{D} = \left(\tilde{a}_{ij}\right)_{n_1\times n_1}$, and obtaining the following respective vectors for decision makers d_1, d_2, d_3:

Since

$$w_{d,k}^{L} = C_L \cdot \frac{\left(\prod_{j=1}^{3} a_{kj}^{L}\right)^{1/3}}{\sum_{i=1}^{3}\left(\prod_{j=1}^{3} a_{ij}^{M}\right)^{1/3}}, \quad w_{d,k}^{M} = \frac{\left(\prod_{j=1}^{3} a_{kj}^{M}\right)^{1/3}}{\sum_{i=1}^{3}\left(\prod_{j=1}^{3} a_{ij}^{M}\right)^{1/3}}, \quad w_{d,k}^{U} = C_U \cdot \frac{\left(\prod_{j=1}^{3} a_{kj}^{U}\right)^{1/3}}{\sum_{i=1}^{3}\left(\prod_{j=1}^{3} a_{ij}^{M}\right)^{1/3}}$$

That is $\left(\prod_{j=1}^{3} a_{kj}^{L}\right)^{1/3} = \begin{matrix} 1.259921 \\ 0.691042 \\ 0.435429 \end{matrix}$, $\left(\prod_{j=1}^{3} a_{kj}^{M}\right)^{1/3} = \begin{matrix} 1.817121 \\ 1 \\ 0.548481 \end{matrix}$, $\left(\prod_{j=1}^{3} a_{kj}^{U}\right)^{1/3} = \begin{matrix} 2.289428 \\ 1.44125 \\ 0.793701 \end{matrix}$

$$\sum_{i=1}^{3}\left(\prod_{j=1}^{3} a_{ij}^{M}\right)^{1/3} = 3.3650125$$

$$C_L = \min_{i\in I_3}\left\{\frac{\left(\prod_{j=1}^{3} a_{ij}^{M}\right)^{1/3}}{\left(\prod_{j=1}^{3} a_{ij}^{L}\right)^{1/3}}\right\} = 1.25992105, \quad C_U = \max_{i\in I_{n_1}}\left\{\frac{\left(\prod_{j=1}^{n_1} a_{ij}^{M}\right)^{1/n_1}}{\left(\prod_{j=1}^{n_1} a_{ij}^{U}\right)^{1/n_1}}\right\} = 0.79370053$$

Thus $\quad \tilde{w}_{d,1} = \langle 0.47165, 0.53991, 0.53991, 0.53991 \rangle$,

$\tilde{w}_{d,2} = \langle 0.25869, 0.29712, 0.29712, 0.34012 \rangle$,and

$\tilde{w}_{d,3} = \langle 0.16296, 0.16296, 0.16296, 0.18717 \rangle$,

and check a consistency index

$$I_3^9\left(\tilde{A}\right) = C_3^9 \bullet \max_{i,j}\left\{\max\left\{\left|\frac{w_{d,k}^L}{w_{d,k}^U} - a_{ij}^L\right|, \left|\frac{w_{d,k}^M}{w_{d,k}^M} - a_{ij}^M\right|, \left|\frac{w_{d,k}^U}{w_{d,k}^L} - a_{ij}^U\right|\right\}\right\}$$

where $$C_3^9 = \frac{1}{\max\left\{9 - 0^{\frac{2-2(3)}{3}}, 9^{\frac{(3)2-2}{3}} - 9\right\}}; \text{ if } 9 \geq \left(\frac{2}{3}\right)^{\frac{3}{3-2}}$$

$$C_3^9 = 0.103$$

Therefore $I_3^9\left(\tilde{A}\right) = 0.09403$

Step2-3) The 3 decision makers, d_1, d_2, d_3 evaluating 6 industrial groups, $G_1, G_2 ..., G_6$, according to the decision criteria c_1, c_2, c_3, c_4 utilizing linguistic terms $\mathrm{VL, L, ML, M, MH, H, VH}$ represented by trapezoidal fuzzy numbers as in Table 4.9. According to the decision criteria

Table 4.9 Trapezoidal fuzzy numbers representing linguistic terms used for fuzzy evaluation of industrial groups.

Criteria	Industrial group	Decision maker											
		d_1				d_2				d_3			
c_1	G_1	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.8	0.9	1	1
	G_2	0.8	0.9	1	1	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_3	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_4	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
	G_5	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8
	G_6	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8
c_2	G_1	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_2	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8
	G_3	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
	G_4	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_5	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9
	G_6	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
c_3	G_1	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_2	0.8	0.9	1	1	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_3	0.8	0.9	1	1	0.8	0.9	1	1	0.7	0.8	0.8	0.9
	G_4	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8
	G_5	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9
	G_6	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9

c_4	G_1	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8
	G_2	0.6	0.7	0.7	0.8	0.8	0.9	1	1	0.7	0.8	0.8	0.9
	G_3	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_4	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
	G_5	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_6	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9

Step2-4) Decision makers d_1, d_2, d_3 evaluating the decision criteria c_1, c_2, c_3, c_4 utilizing the linguistic terms VL, L, ML, M, MH, H, VH represented by the mentioned trapezoidal fuzzy numbers as in Table 4.10.

Table 4.10 Evaluation of fuzzy decision criteria

Criteria	Decision maker											
	d_1				d_2				d_3			
c_1	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
c_2	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
c_3	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
c_4	0.5	0.6	0.7	0.8	0.8	0.9	1	1	0.7	0.8	0.8	0.9

Step2-5) Calculating fuzzy decision criteria and the evaluation criteria for industrial groups based on the weights of decision makers as in Table 4.11 and Table 4.12, respectively.

Table 4.11 Fuzzy decision criteria

Criteria	Decision maker											
	d_1				d_2				d_3			
c_1	0.3773	0.4859	0.5399	0.5399	0.207	0.2674	0.2971	0.3401	0.1304	0.1467	0.163	0.1872
c_2	0.3302	0.4319	0.4319	0.4859	0.1811	0.2377	0.2377	0.3061	0.1141	0.1304	0.1304	0.1685
c_3	0.3773	0.4859	0.5399	0.5399	0.207	0.2674	0.2971	0.3401	0.1304	0.1467	0.163	0.1872
c_4	0.2358	0.3239	0.3779	0.4319	0.207	0.2674	0.2971	0.3401	0.1141	0.1304	0.1304	0.1685

Table 4.12 Fuzzy evaluation of industrial groups

Criteria	Industrial group	Decision maker											
		d_1				d_2				d_3			
c_1	G_1	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.13	0.147	0.163	0.187
	G_2	0.377	0.486	0.54	0.54	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_3	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_4	0.377	0.486	0.54	0.54	0.207	0.267	0.297	0.34	0.13	0.147	0.163	0.187
	G_5	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.098	0.114	0.114	0.15
	G_6	0.283	0.378	0.378	0.432	0.181	0.238	0.238	0.306	0.098	0.114	0.114	0.15

C_2	G_1	0.283	0.378	0.378	0.432	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_2	0.33	0.432	0.432	0.486	0.155	0.208	0.208	0.272	0.098	0.114	0.114	0.15
	G_3	0.377	0.486	0.54	0.54	0.207	0.267	0.297	0.34	0.13	0.147	0.163	0.187
	G_4	0.283	0.378	0.378	0.432	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_5	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.114	0.13	0.13	0.168
	G_6	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
C_3	G_1	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_2	0.377	0.486	0.54	0.54	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_3	0.377	0.486	0.54	0.54	0.207	0.267	0.297	0.34	0.114	0.13	0.13	0.168
	G_4	0.33	0.432	0.432	0.486	0.155	0.208	0.208	0.272	0.098	0.114	0.114	0.15
	G_5	0.33	0.432	0.432	0.486	0.155	0.208	0.208	0.272	0.114	0.13	0.13	0.168
	G_6	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.114	0.13	0.13	0.168
C_4	G_1	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.098	0.114	0.114	0.15
	G_2	0.283	0.378	0.378	0.432	0.207	0.267	0.297	0.34	0.114	0.13	0.13	0.168
	G_3	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_4	0.377	0.486	0.54	0.54	0.207	0.267	0.297	0.34	0.13	0.147	0.163	0.187
	G_5	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_6	0.33	0.432	0.432	0.486	0.155	0.208	0.208	0.272	0.114	0.13	0.13	0.168

Step2-6) Aggregating the decision criteria and the fuzzy evaluation of industrial groups based on the weights of decision makers. The aggregation results are shown in Table 4.13 and 4.14 below.

Table 4.13 Aggregation of decision criteria

	Criteria															
	C_1				C_2				C_3				C_4			
Weight	0.13	0.3	0.333	0.54	0.114	0.267	0.267	0.486	0.13	0.3	0.333	0.54	0.114	0.241	0.268	0.432

Table 4.14 Aggregation of evaluation of industrial groups

Group	Criteria															
	C_1				C_2				C_3				C_4			
G_1	0.13	0.244	0.25	0.432	0.114	0.249	0.249	0.432	0.114	0.267	0.267	0.486	0.098	0.233	0.233	0.432
G_2	0.114	0.285	0.303	0.54	0.098	0.251	0.251	0.486	0.114	0.285	0.303	0.54	0.114	0.259	0.268	0.432
G_3	0.114	0.267	0.267	0.486	0.13	0.3	0.333	0.54	0.114	0.295	0.322	0.54	0.114	0.267	0.267	0.486
G_4	0.13	0.3	0.333	0.54	0.114	0.249	0.249	0.432	0.098	0.251	0.251	0.486	0.13	0.3	0.333	0.54
G_5	0.098	0.233	0.233	0.432	0.114	0.239	0.239	0.432	0.114	0.257	0.257	0.486	0.114	0.267	0.267	0.486
G_6	0.098	0.243	0.243	0.432	0.114	0.267	0.267	0.486	0.114	0.239	0.239	0.432	0.114	0.257	0.257	0.486

Step2-7) Normalizing the weights of industrial groups for each decision criteria shown in Table 4.13 then multiplying the normalized matrix with the weights of decision criteria from Step2-6), defined by

$\hat{V}=\left[\tilde{v}_{ji}\right]_{6\times4}$, where $\tilde{v}_{ji}=\left\langle v_{ji}^{L}, v_{ji}^{M_1}, v_{ji}^{M_2}, v_{ji}^{U}\right\rangle$ and $\tilde{v}_{ji}=\tilde{r}_{ji}\otimes\tilde{w}_{c,j}$ when $j\in 1,2,...,6$, $i\in 1,2,...,4$;, to obtain a decision matrix shown in Table 4.15 below.

Table 4.15 Decision matrix

Group	Criteria															
	c_1				c_2				c_3				c_4			
G_1	0.031	0.136	0.154	0.432	0.024	0.123	0.123	0.389	0.028	0.148	0.148	0.486	0.021	0.104	0.116	0.346
G_2	0.028	0.158	0.187	0.54	0.021	0.124	0.124	0.437	0.028	0.158	0.168	0.54	0.024	0.115	0.134	0.346
G_3	0.028	0.148	0.165	0.486	0.028	0.148	0.165	0.486	0.028	0.164	0.179	0.54	0.024	0.119	0.133	0.389
G_4	0.031	0.167	0.206	0.54	0.024	0.123	0.123	0.389	0.024	0.14	0.14	0.486	0.028	0.134	0.166	0.432
G_5	0.024	0.13	0.144	0.432	0.024	0.118	0.118	0.389	0.028	0.143	0.143	0.486	0.024	0.119	0.133	0.389
G_6	0.024	0.135	0.15	0.432	0.024	0.132	0.132	0.437	0.028	0.133	0.133	0.432	0.024	0.114	0.128	0.389

Step2-8) Stipulating a positive ideal solution (S^*) and a negative ideal solution (S^-) to be

S^* = [(0.54, 0.54, 0.54, 0.54), (0.486, 0.486, 0.486, 0.486),
(0.54, 0.54, 0.54, 0.54), (0.432, 0.432, 0.432, 0.432)]

S^- = [(0.021, 0.021, 0.021, 0.021), (0.021, 0.021, 0.021, 0.021),
(0.024, 0.024, 0.024, 0.024), (0.021, 0.021, 0.021, 0.021)]

Step2-9) Calculating the distances from the results of industrial groups evaluation in Table 4.15 to the (S^*) and the (S^-) ideal solutions shown in Table 4.16 and Table 4.17, respectively.

$$d\left(\tilde{G}_j, S^{*,-}\right)=\sqrt{\frac{1}{4}\left[\left(a^L-b^L\right)^2+\left(a^{M_1}-b^{M_1}\right)^2+\left(a^{M_2}-b^{M_2}\right)^2+\left(a^U-b^U\right)^2\right]}$$

Table 4.16 Distances between $G_i(1,2,3,4,5,6)$ and S^* for each decision criterion

Distance	Criteria				SUM
	c_1	c_2	c_3	c_4	
$d_1^*=d_v(G_1,S^*)$	0.381572	0.348712	0.378268	0.309819	1.418371
$d_2^*=d_v(G_2,S^*)$	0.364995	0.346628	0.369604	0.301308	1.382533
$d_3^*=d_v(G_3,S^*)$	0.374072	0.326879	0.365443	0.298251	1.364645
$d_4^*=d_v(G_4,S^*)$	0.356869	0.348712	0.384022	0.284309	1.373911
$d_4^*=d_v(G_5,S^*)$	0.388341	0.351266	0.381127	0.298251	1.418985
$d_4^*=d_v(G_6,S^*)$	0.38534	0.341527	0.389187	0.300654	1.416708

Table 4.17 Distances between $G_j(1,2,3,4,5,6)$ and S^- for each decision criterion

Distance	Criteria				SUM
	c_1	c_2	c_3	c_4	
$d_1^- = d_v(G_1, S^-)$	0.223775	0.197715	0.247374	0.174345	0.843208
$d_2^- = d_v(G_2, S^-)$	0.281158	0.220808	0.276399	0.17835	0.956716
$d_3^- = d_v(G_3, S^-)$	0.251745	0.251745	0.278567	0.198531	0.980589
$d_4^- = d_v(G_4, S^-)$	0.285192	0.197715	0.245285	0.225285	0.953478
$d_4^- = d_v(G_5, S^-)$	0.221504	0.196478	0.246015	0.198531	0.862528
$d_4^- = d_v(G_6, S^-)$	0.223065	0.222647	0.218246	0.197315	0.861274

Step2-10) Obtaining the nearness coefficients CC_j to the positive ideal solution and the investment weights W_2 shown in Table 4.18 below.

Table 4.18 Nearness coefficients to the positive ideal solution

Industrial Group	G_1	G_2	G_3	G_4	G_5	G_6
$CC_j = \frac{d_j^-}{d_j^- + d_j^*}$	0.304297	0.38056	0.392965	0.380328	0.318558	0.315015
Weights	0.157599	0.172877	0.176738	0.173169	0.159816	0.159816

Main step 3) Analysis of all stocks from different industrial groups

The two kinds of investment weights obtained from Main Step1) and Main Step 2) were used to calculate the final investment weights for all of the stocks in the market, $W_{OA}(s_{ij})$, where i represents the i^{th} company and j the j^{th} industrial group, and the final weights were ranked as shown in Table 4.19 below.

Table 4.19 The final investment weights of all of the stocks in the market

s_{ij}	s_{12}	s_{102}	s_{93}	s_{76}	s_{46}	s_{72}	s_{34}	s_{22}	s_{23}	s_{41}	s_{66}	s_{101}	s_{64}
$W_{OA}(s_j)$	0.0473	0.0472	0.0396	0.0332	0.0317	0.03114	0.0307	0.03063	0.0287	0.02543	0.02503	0.02478	0.0247
s_{ij}	s_{62}	s_{32}	s_{91}	s_{85}	s_{13}	s_{26}	s_{96}	s_{54}	s_{84}	s_{24}	s_{31}	s_{105}	s_{75}
$W_{OA}(s_j)$	0.0237	0.0227	0.0218	0.0215	0.0201	0.01998	0.0195	0.01894	0.0183	0.01829	0.01792	0.01720	0.02373
s_{ij}	s_{65}	s_{42}	s_{104}	s_{35}	s_{74}	s_{61}	s_{51}	s_{43}	s_{63}	s_{53}	s_{94}	s_{15}	s_{55}
$W_{OA}(s_j)$	0.0167	0.0166	0.0166	0.0166	0.0159	0.01583	0.0157	0.01544	0.015	0.01459	0.01411	0.01279	0.01279
s_{ij}	s_{75}	s_{115}	s_{125}	s_{71}	s_{81}	s_{36}	s_{86}	s_{44}	s_{14}	s_{83}	s_{11}	s_{73}	s_{56}
$W_{OA}(s_j)$	0.0127	0.0127	0.0127	0.0111	0.0102	0.01004	0.0094	0.00960	0.0096	0.0079	0.00608	0.00608	0.00529
s_{ij}	s_{33}	s_{16}	s_{21}	s_{82}	s_{52}								
$W_{OA}(s_j)$	0.0047	0.0039	0.0034	0.0018	0.0007								

From Table 4.19, investors can use the calculated weights to help with their decision-making and strategy-planning. The better stocks to invest in show higher final investment weights.

Chapter 5
Conclusions and Suggestions

5.1 Conclusions

The innovation appeared in this document is to present the tactic of conveying the stock selection to portfolio by using two tactics, fuzzy quantitative analysis and Fuzzy multi-criteria decision making. The two tactics imply the final investment weight. Investors can determine their strategies by using the final investment weights. The final investment weights may be used to select stocks and allocate asset into portfolio. A case study presented in Table 4.19 shows that, if we use the final investment weights as decision criteria to select stocks into portfolio, stock that has the highest weight is the most interesting and is chosen first. In contrast, stock that has the lowest weight is the least interesting and is chosen last. However, decision-making and strategy-planning of each investor may be different and depend on their financial risk tolerance. For example, some investors whose financial risk tolerance is high level maybe invest in only one stock with the highest final investment weights while some investors reduce risk by investing in many stocks with high final investment weights. You should keep in your mind that there is no best tool in the world for financial analysis but you can alter tools that fit for each situation. The purpose of this research is to construct the tool for financial analysis that may be an alternative for investors. At least, we hope that this research will help investors to make an appropriate decision.

5.2 Suggestions

For future work, we will improve our model and compare results with others in each situation. Moreover, the software of this model will also be provided.

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Appendices

Appendix A

Table of Financial Ratios

$\underline{E/P}$

Table A-1 E/P of CNT

CNT stock	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		5.25	10	3.78	4.98
Number of common stocks		1002904144	501452102	501452102	401161682
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		442180000	465780000	165010000	352260000
P/E	22.8100	11.9075	10.7659	11.4871	5.6713
E/P	0.0438	0.0840	0.0929	0.0871	0.1763
E/P (weighted average)	0.0789				
E/P (% weighted average)	7.89				
The first trading day 1/3/1991					

Table A-2 E/P of NWR

NWR stock					
	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		1.87	1.92	0.65	0.73
Number of common stocks		1,974,801,416	1,552,901,243	1,552,901,243	1,552,901,243
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		58360000	538910000	95780000	54270000
P/E	64.0000	63.2776	5.5326	10.5386	20.8885
E/P	0.0156	0.0158	0.1807	0.0949	0.0479
E/P (weighted average)	0.0614				
E/P (% weighted average)	6.14				
The first trading day 27/9/1995					

Table A-3 E/P of PREB

PREB stock					
	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		6.15	9	3.48	3.14
Number of common stocks		308,676,462	231,507,783	215,222,961	201,934,000
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		261500000	188710000	106740000	61750000
P/E	13.5100	7.2595	11.0411	7.0168	10.2684
E/P	0.0740	0.1378	0.0906	0.1425	0.0974
E/P (weighted average)	0.1050				
E/P (% weighted average)	10.5				
The first trading day 2/12/ 2005					

Table A-4 E/P of SEAFCO

SEAFCO stock					
	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		4.52	5.85	3.08	5.05
Number of common stocks		268,730,194	215,000,000	215,000,000	215,000,000
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		138220000	138230000	-58670000	-58880000
P/E	12.3700	8.7879	9.0990	-11.2869	-18.4400
E/P	0.0808	0.1138	0.1099	-0.0886	-0.0542
E/P (weighted average)	0.0638				
E/P (% weighted average)	6.38				
The first trading day 3/9/2004					

Table A-5 E/P of STEC

STEC stock					
	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		13.2	27.25	12.3	13.4
Number of common stocks		1,525,106,540	1,186,208,619	1,186,208,619	1,186,208,619
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		1733200000	1165320000	903500000	443760000
P/E	21.4000	11.6152	27.7385	16.1487	35.8194
E/P	0.0467	0.0861	0.0361	0.0619	0.0279
E/P (weighted average)	0.0559				
E/P (% weighted average)	5.59				
The first trading day 31/8/1992					

Table A-6 E/P of SYNTEC

SYNTEC stock					
	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		1.09	1.04	0.79	1.1
Number of common stocks		1600000000	1600000000	1600000000	1600000000
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		80600000	-132050000	97260000	203480000
P/E	18.3100	21.6377	-12.6013	12.9961	8.6495
E/P	0.0546	0.0462	-0.0794	0.0769	0.1156
E/P (weighted average)	0.0326				
E/P (% weighted average)	3.26				
The first trading day 8/8/1993					

Table A-7 E/P of TRC

TRC stock					
	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		2.92	7.35	3.2	4.16
Number of common stocks		816,586,773	336,585,589	333,558,339	330,668,089
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		126360000	240560000	149040000	182840000
P/E	17.8000	18.8702	10.2839	7.1617	7.5234
E/P	0.0562	0.0530	0.0972	0.1396	0.1329
E/P (weighted average)	0.0798				
E/P (% weighted average)	7.98				
The first trading day 22/12/2005					

Table A-8 E/P of TTCL

TTCL stock					
	14/10/57	27/12/56	28/12/55	30/12/54	30/12/53
Closing price of common stock (baht)		34	34.75	11.8	9.6
Number of common stocks		560,000,000	480,000,000	480,000,000	480,000,000
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		655010000	545660000	399330000	337060000
P/E	27.6900	29.0683	30.5685	14.1838	13.6712
E/P	0.0361	0.0344	0.0327	0.0705	0.0731
E/P (weighted average)	0.0420				
E/P (% weighted average)	4.2				
The first trading day 16/6/2009					

Table A-9 E/P of CK

CK stock					
	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		15.5	13.4	7.55	9.5
Number of common stocks		1652585336	1652585336	1652585336	1652585336
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		7673850000	568400000	927400000	-335060000
P/E	22.0800	3.3380	38.9596	13.4538	-46.8560
E/P	0.0453	0.2996	0.0257	0.0743	-0.0213
E/P (weighted average)	0.1086				
E/P (% weighted average)	10.86				
The first trading day 3/8/1995					

Table A-10 E/P of ITD

ITD stock					
	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		3.88	4.2	3.62	4.64
Number of common stocks		4,860,473,011	4,193,678,180	4,193,678,180	4,193,678,180
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		907370000	126160000	-1698460000	297920000
P/E	38.8600	20.7838	139.6120	-8.9382	65.3151
E/P	0.0257	0.0481	0.0072	-0.1119	0.0153
E/P (weighted average)	0.0089				
E/P (% weighted average)	0.89				
The first trading day 9/8/1996					

Table A-11 E/P of UNIQ

UNIQ stock					
	14/10/14	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)		6.4	3.86	2.06	3.9
Number of common stocks		779,539,289	779,539,289	670,902,000	625,000,000
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		586660000	380300000	-15870000	148240000
P/E	19.0900	8.5042	7.9122	-87.0862	16.4429
E/P	0.0524	0.1176	0.1264	-0.0115	0.0608
E/P (weighted average)	0.0766				
E/P (% weighted average)	7.66				
The first trading day 30/3/2007					

Note

1. The latest price of 2014 (14/10/2014)

2. The closing price of stock 2010-2013, use to the last day of year from Stock Exchange of Thailand, 2010-2014, http://www.settrade.com.

P/BV

Table A-12 P/BV of CNT

CNT stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	5.25	10	3.78	4.98
Number of common stocks	1002904144	501452102	501452102	401161682
Number of preferred stocks	0	0	0	0
Total assets	5,602,628,468	4,550,921,601	4,142,043,466	3,421,241,760
Total liabilities	3,423,329,926	2,723,717,260	2,483,328,626	1,779,096,743
Accounting value per share	2.172987872	3.643826267	3.307823087	4.093474254
P/BV	2.41602821	2.744367944	1.142745516	1.216570495
P/BV of 2014 (2^{nd} quarter)	2.6			
P/BV (weighted average)	2.293285213			
P/BV (highest)	25.18861616			
P/BV (%)	9.10445099			
The first trading day 1/3/1991				

Table A-13 P/BV of NWR

NWR stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	1.87	1.92	0.65	0.73
Number of common stocks	1974801416	1522901243	1552901243	1552901243
Number of preferred stocks	0	0	0	0
Total assets	8987997044	7652648104	6207423312	5550252608
Total liabilities	5960101190	5626139660	4778059463	4038914745
Accounting value per share	1.533265993	1.330689336	0.92044736	0.97323501
P/BV	1.21961878	1.442861191	0.70617835	0.75007577
P/BV of 2014 (2^{nd} quarter)	1.3			
P/BV (weighted average)	1.191299411			
P/BV (highest)	25.18861616			
P/BV (%)	4.729515125			
The first trading day 27/9/1995				

Table A-14 P/BV of PREB

PREB stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	6.15	9	3.48	3.14
Number of common stocks	308,676,462	231,507,783	215,222,961	201,934,000
Number of preferred stocks	0	0	0	0
Total assets	4276212876.40	3118212385.10	2027832427.67	1355169423.95
Total liabilities	3167608844.40	2261196222.24	1356134535.61	753841663.47
Accounting value per share	3.591475763	3.701889205	3.120939741	2.97784306
P/BV	1.712388	2.431191076	1.115048764	1.054454495
P/BV of 2014 (2nd quarter)	2.7			
P/BV (weighted average)	2.06184515			
P/BV (highest)	25.18861616			
P/BV (%)	8.185622969			
The first trading day 2/12/ 2005				

Table A-15 P/BV of SEAFCO

SEAFCO stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	4.52	5.85	3.08	5.05
Number of common stocks	268,730,194	215,000,000	215,000,000	215,000,000
Number of preferred stocks	0	0	0	0
Total assets	1484913281.86	1561939451.56	1540432261.44	1518200024.35
Total liabilities	710269988.03	869468692.41	1030475606.09	933949154.41
Accounting value per share	2.88260609	3.220794229	2.37189142	2.717445907
P/BV	1.56802555	1.816322182	1.298541735	1.858362659
P/BV of 2014 (2nd quarter)	2.2			
P/BV (weighted average)	1.811767658			
P/BV (highest)	25.18861616			
P/BV (%)	7.192803474			
The first trading day 2/9/ 2004				

Table A-16 P/BV of STEP

STEP stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	13.2	27.25	12.3	13.4
Number of common stocks	1,525,106,540	1,186,208,619	1,186,208,619	1,186,208,619
Number of preferred stocks	0	0	0	0
Total assets	25009590132	20295529907	16073528754	10208132456
Total liabilities	17232944917	14184281339	10235877514	5629422762
Accounting value per share	5.099083252	5.151917184	4.921268609	3.859953149
P/BV	2.588700625	5.289293097	2.499355548	3.471544727
P/BV of 2014 (2nd quarter)	5.2			
P/BV (weighted average)	4.046195841			
P/BV (highest)	25.18861616			
P/BV (%)	16.06358926			
The first trading day 31/8/ 1992				

Table A-17 P/BV of SYNTEC

SYNTEC stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	1.09	1.04	0.79	1.1
Number of common stocks	1600000000	1600000000	1600000000	1600000000
Number of preferred stocks	0	0	0	0
Total assets	5448816195	4374783290	4373744971	4339603697
Total liabilities	2884189455	2177701975	2087696823	2014984686
Accounting value per share	1.602891713	1.373175822	1.428780093	1.452886882
P/BV	0.680020984	0.757368418	0.552919238	0.757113313
P/BV of 2014 (2nd quarter)	1.5			
P/BV (weighted average)	0.957009399			
P/BV (highest)	25.18861616			
P/BV (%)	3.799372671			
The first trading day 8/7/ 1993				

Table A-18 P/BV of TRC

TRC stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	2.92	7.35	3.2	4.16
Number of common stocks	816,586,773	336,585,589	333,558,339	330,668,089
Number of preferred stocks	0	0	0	0
Total assets	1976999778	2502835689	2377968299	1082236509
Total liabilities	823141241	1513287606	1580542684	404278040
Accounting value per share	1.413026239	2.939959747	2.390663107	2.050268809
P/BV	2.066486749	2.500034229	1.338540755	2.029002237
P/BV of 2014 (2nd quarter)	2.7			
P/BV (weighted average)	2.264808895			
P/BV (highest)	25.18861616			
P/BV (%)	8.991398658			
The first trading day 22/12/ 2005				

Table A-19 P/BV of TTCL

TTCL stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	34	34.75	11.8	9.6
Number of common stocks	560,000,000	480,000,000	480,000,000	480,000,000
Number of preferred stocks	0	0	0	0
Total assets	14295816690	7658700942	6903631079	3674967924
Total liabilities	8808994838	5337780276	5221441903	2139082927
Accounting value per share	9.797896164	4.835251388	3.504560783	3.19976041
P/BV	3.470132713	7.18680317	3.367041044	3.000224632
P/BV of 2014 (2nd quarter)	3.2			
P/BV (weighted average)	4.078349806			
P/BV (highest)	25.18861616			
P/BV (%)	16.19124203			
The first trading day 16/6/ 2009				

Table A-20 P/BV of CK

CK stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	15.5	13.4	7.55	9.5
Number of common stocks	1,652,585,336	1,652,585,336	1,652,585,336	1,652,585,336
Number of preferred stocks	0	0	0	0
Total assets	72,034,226,062	51,184,836,049	36,639,520,980	30,469,989,695
Total liabilities	55,193,632,977	42,324,463,176	30,044,514,925	24,258,249,267
Accounting value per share	10.19045293	5.361522144	3.990720425	3.758801614
P/BV	1.521031509	2.499290247	1.891888981	2.527401277
P/BV of 2014 (2^{nd} quarter)	2.6			
P/BV (weighted average)	2.192878401			
P/BV (highest)	25.18861616			
P/BV (%)	8.705831186			
The first trading day 3/8/ 1995				

Table A-21 P/BV of ITD

ITD stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	3.88	4.2	3.62	4.64
Number of common stocks	4,860,473,011	4,193,678,180	4,193,678,180	4,193,678,180
Number of preferred stocks	0	0	0	0
Total assets	65,150,761,000	58,982,728,000	53,039,974,000	50,825,850,000
Total liabilities	52,767,334,000	49,562,400,000	44,139,910,000	39,174,432,000
Accounting value per share	2.547782278	2.246316383	2.122257269	2.778329071
P/BV	1.52289308	1.869727716	1.705730994	1.670068549
P/BV of 2014 (2^{nd} quarter)	2.2			
P/BV (weighted average)	1.8521524			
P/BV (highest)	25.18861616			
P/BV (%)	7.353132814			
The first trading day 9/8/ 1994				

Table A-22 P/BV of UNIQ

UNIQ stock				
	27/12/13	28/12/12	30/12/11	30/12/10
Closing price of common stock (baht)	6.4	3.86	2.06	3.9
Number of common stocks	779,539,289	779,539,289	670,902,000	625,000,000
Number of preferred stocks	0	0	0	0
Total assets	12617361372	6162881171	5704666423	5226890700
Total liabilities	10013115304	4098519264	4136728027	3646396560
Accounting value per share	3.340750242	2.648181992	2.33706025	2.528790624
P/BV	1.915737346	1.457603749	0.881449248	1.542239189
P/BV of 2014 (2nd quarter)	3.2			
P/BV (weighted average)	2.089393221			
P/BV (highest)	25.18861616			
P/BV (%)	8.294990117			
The first trading day 30/3/2007				

Note

1. The latest price of 2014 (14/10/2014)

2. The closing price of stock 2010-2013, use to the last day of year from Stock Exchange of Thailand, 2010-2014, http://www.settrade.com.

P/P_n

Table A-23 P/P_n of CNT

CNT stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	5.55	5.25	10	3.78	4.98
Dividend interest rate (%)	3.6	4.95	3	12.7	4.02
Dividend amount (baht)	0.1998	0.2599	0.3	0.4801	0.2002
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	4.9651				
Closing price to target price ratio	1.1178				
The first trading day 1/3/1991					

Table A-24 P/P_n of NWR

NWR stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	1.99	1.87	1.92	0.65	0.73
Dividend interest rate (%)	0	5.05	0	0	0
Dividend amount (baht)	0	0.0944	0	0	0
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	0.8348				
Closing price to target price ratio	2.3838				
The first trading day 27/9/1995					

Table A-25 P/P_n of PREB

PREB stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	9.75	6.15	9	3.48	3.14
Dividend interest rate (%)	4.1	0.49	1.55	2.7	1.58
Dividend amount (baht)	0.3998	0.0301	0.1395	0.094	0.0496
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	3.3164				
Closing price to target price ratio	2.9399				
The first trading day 2/12/2005					

Table A-26 P/P_n of SEAFCO

SEAFCO stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	6.1	4.52	5.85	3.08	5.05
Dividend interest rate (%)	3.03	0.49	0	0	0.99
Dividend amount (baht)	0.1848	0.0221	0	0	0.05
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	6.2659				
Closing price to target price ratio	0.9735				
The first trading day 3/9/2004					

Table A-27 P/P_n of STEC

STEC stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	26.25	13.2	27.25	12.3	13.4
Dividend interest rate (%)	1.9	0.19	2.02	1.79	0.97
Dividend amount (baht)	0.4988	0.0251	0.5505	0.2202	0.13
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	15.7472				
Closing price to target price ratio	1.6670				
The first trading day 31/8/1992					

Table A-28 P/P_n of SYNTEC

SYNTEC stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	2.42	1.09	1.04	0.79	1.1
Dividend interest rate (%)	2.07	0	1.92	3.8	2.73
Dividend amount (baht)	0.0501	0	0.02	0.03	0.03
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	1.2999				
Closing price to target price ratio	1.8617				
The first trading day 8/7/1993					

Table A-29 P/P_n of TRC

TRC stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	4.08	2.92	7.35	3.2	4.16
Dividend interest rate (%)	1.22	1.42	1.62	4.67	1.44
Dividend amount (baht)	0.0498	0.0415	0.1191	0.1494	0.0599
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	4.9166				
Closing price to target price ratio	0.8298				
The first trading day 22/12/2005					

Table A-30 P/P_n of TTCL

TTCL stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	29	34	34.75	11.8	9.6
Dividend interest rate (%)	2.27	1.61	1.24	3.14	3.59
Dividend amount (baht)	0.6583	0.5474	0.4309	0.3705	0.3446
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	10.1073				
Closing price to target price ratio	2.8692				
The first trading day 16/6/ 2009					

Table A-31 P/P_n of CK

CK stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	26.5	15.5	13.4	7.55	9.5
Dividend interest rate (%)	1.48	2.26	2.61	1.32	0.92
Dividend amount (baht)	0.3922	0.3503	0.3497	0.0997	0.0874
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	10.8861				
Closing price to target price ratio	2.4343				
The first trading day 3/8/1995					

Table A-32 P/P_n of ITD

CNT stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	5.54	3.88	4.2	3.62	4.64
Dividend interest rate (%)	0	0	0	1.38	0
Dividend amount (baht)	0	0	0	0.05	0
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	5.8868				
Closing price to target price ratio	0.9411				
The first trading day 9/8/1994					

Table A-33 P/P_n of UNIQ

UNIQ stock					
	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	11.4	6.4	3.86	2.06	3.9
Dividend interest rate (%)	1.06	0.94	0	2.71	2.82
Dividend amount (baht)	0.1208	0.0602	0	0.0558	0.11
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment (1+ r)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	4.7455				
Closing price to target price ratio	2.4023				
The first trading day 30/3/2007					

Note

1. The latest price of 2014 (14/10/2014)

2. The closing price of stock 2010-2013, use to the last day of year from Stock Exchange of Thailand, 2010-2014, http://www.settrade.com.

Appendix B

The inference and Defuzzification

ITD

Input $E/P = 2.14$, $P/BV = 3.66$, $P/P_n = 1.34$, i.e. $x = 2.14$, $y = 3.66$, $z = 1.34$

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

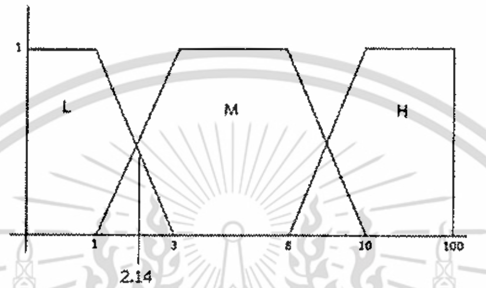

Figure B-1 Intersection between input value and fuzzy set of linguistic terms of E/P , ITD.

For P/BV

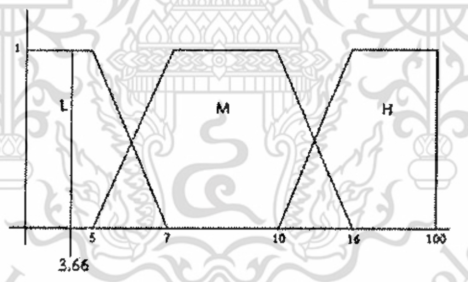

Figure B-2 Intersection between input value and fuzzy set of linguistic terms of P/BV, ITD.

For P/P_n

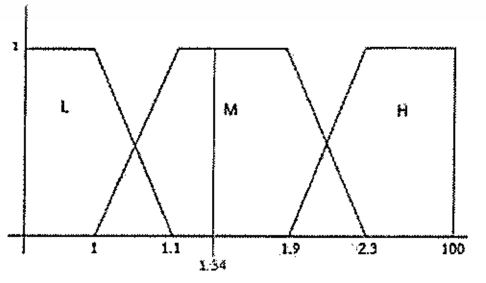

Figure B-3 Intersection between input value and fuzzy set of linguistic terms of P/P_n in ITD.

From Figure B-1-B-3, we have the concerned fuzzy rules as follows:

Rule 2 If x was $\tilde{LX}$ and y was $\tilde{LY}$ and z was $\tilde{MZ}$ then w was $\tilde{MW}$.

Rule 11 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{MZ}$ then w was $\tilde{RHW}$.

Since $u_{\tilde{LX}}(x)=\dfrac{-(x-3)}{2}$ $\quad u_{\tilde{MX}}(x)=\dfrac{(x-1)}{2}$ $\quad u_{\tilde{LY}}(y)=1$ $\quad u_{\tilde{MZ}}(z)=1$

we have

$$h_2=u_{\tilde{LX}}(2.14)\wedge u_{\tilde{LY}}(3.66)\wedge u_{\tilde{MZ}}(1.34)$$
$$=0.43\wedge 1\wedge 1\ =0.43$$
$$h_{11}=u_{\tilde{MX}}(2.14)\wedge u_{\tilde{LY}}(3.66)\wedge u_{\tilde{MZ}}(1.34)$$
$$=0.57\wedge 1\wedge 1\ =0.57$$

Since output of each rule is $\tilde{MW}$, $\tilde{RHW}$, so the union of truncated fuzzy outputs with $h_2=0.43$ is equal $\tilde{MW}$ and equal to $h_{11}=0.57$. $\tilde{RHW}$, that is, $\tilde{W}_{out}(w)=\tilde{MW}\cup\tilde{RHW}$

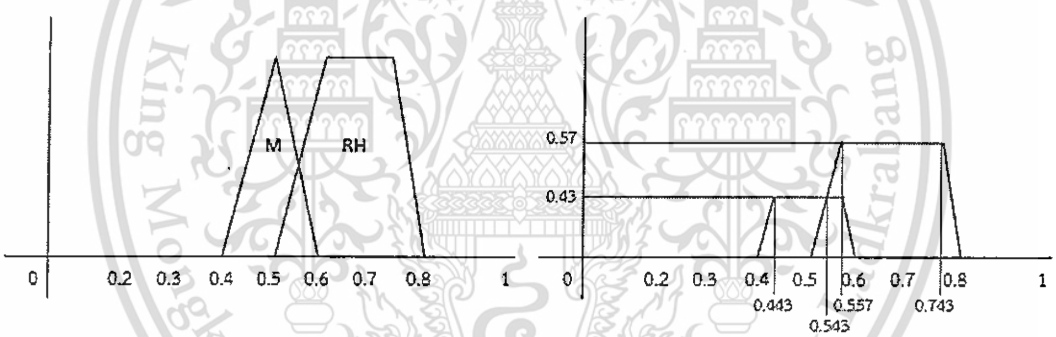

Figure B-4 The union of truncated fuzzy outputs $\tilde{W}_{out}(w)=\tilde{MW}\cup\tilde{RHW}$,ITD.

It is not difficult to find the membership function of $\tilde{W}_{out}(w)=\tilde{MW}\cup\tilde{RHW}$. From Figure B-4, we have

$$u_{\tilde{W}_{out}}(w)=\begin{cases}\dfrac{w-0.4}{0.1} & ;0.4\le w\le 0.443\\ 0.43 & ;0.443\le w\le 0.543\\ \dfrac{w-0.5}{0.1} & ;0.543\le w\le 0.557\\ 0.57 & ;0.557\le w\le 0.743\\ -\dfrac{(w-0.8)}{0.1} & ;0.743\le w\le 0.8\end{cases}$$

Defuzzification

$$w^{cg} = \frac{\int_W w u_{\tilde{W}_{out}}(w)dw}{\int_W u_{\tilde{W}_{out}}(w)dw}$$

Thus

$$\int_W u_{\tilde{W}_{out}}(w)dw = \left[\frac{1}{2}(0.443-0.4)(0.43)\right] + \left[0.43(0.543-0.443)\right] + \left[\frac{1}{2}(0.557-0.543)(0.14)\right] + \left[\frac{1}{2}(0.186+0.243)(0.57)\right]$$

$$= 0.1755$$

$$\int_W w u_{W_{out}}(w)dw = \int_{0.4}^{0.443} \frac{w}{0.1}(w-0.4)dw + \int_{0.443}^{0.543} 0.43w dw + \int_{0.543}^{0.557} \frac{w}{0.1}(w-0.5)dw + \int_{0.557}^{0.743} 0.57w dw - \int_{0.743}^{0.8} \frac{w}{0.1}(w-0.8)dw$$

$$= 0.1103$$

Therefore $w^{cg} = \frac{0.1103}{0.1755} = 0.6285.$

PREB

Input $E/P = 5.76$, $P/BV = 3.41$, $P/P_n = 4.29$,i.e. input $x = 5.76$, $y = 3.41$, $z = 4.29$

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

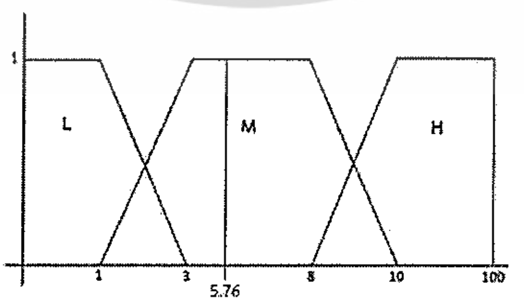

Figure B-5 Intersection between input value and fuzzy set of linguistic terms of E/P, PREB.

For P/BV

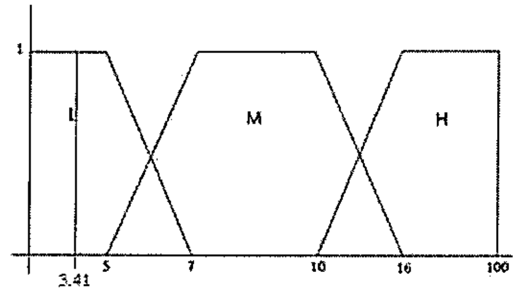

Figure B-6 Intersection between input value and fuzzy set of linguistic terms of P/BV , PREB.

For P/P_n

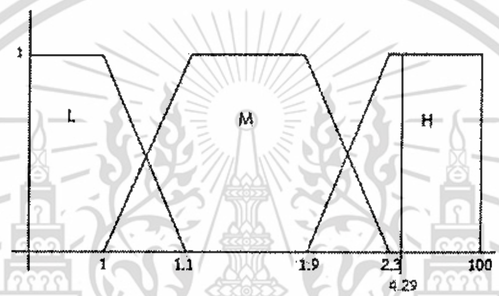

Figure B-7 Intersection between input value and fuzzy set of linguistic terms of P/P_n , PREB.

From Figure B-5-B-7, we have the concerned fuzzy rules as follows:

Rule 12 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{HZ}$ then w was $\tilde{MW}$.

Sine $u_{\tilde{MX}}(x)=1$ $u_{\tilde{LY}}(y)=1$ $u_{\tilde{HZ}}(z)=1$

we have

$$h_{12}=u_{\tilde{MX}}(5.76)\wedge u_{\tilde{LY}}(3.41)\wedge u_{\tilde{HZ}}(4.29)$$
$$=1\wedge 1\wedge 1$$
$$=1$$

Since output of each rule is $\tilde{MW}$, the so truncated fuzzy outputs with $h_{12}=1$ is equal to ~~$\tilde{MW}$~~ ,that is $\tilde{W}_{out}(w)=$ ~~$\tilde{MW}$~~

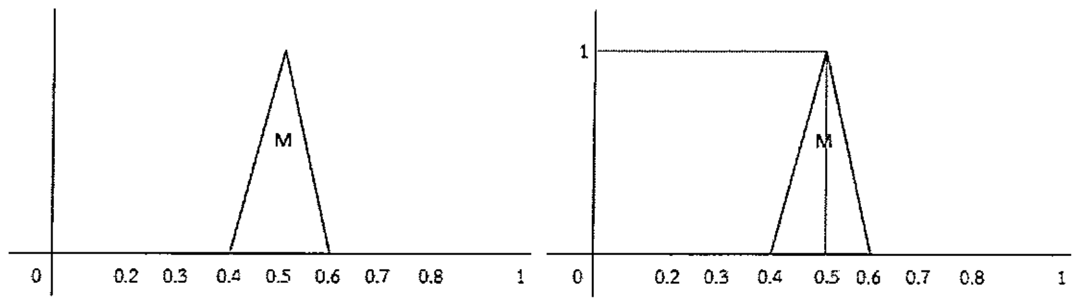

Figure B-8 The union of truncated fuzzy outputs $\tilde{W}_{out}(w) = \tilde{MW}$,PREB.

It is not difficult to find the membership function of $\tilde{W}_{out}(w) = \tilde{MW}$. From Figure B-8, we have

$$u_{\tilde{W}_{out}}(w) = \begin{cases} \dfrac{w-0.4}{0.1} & ;0.4 \le w \le 0.5 \\ -\dfrac{(w-0.6)}{0.1} & ;0.5 \le w \le 0.6 \end{cases}$$

Defuzzification

$$w^{cg} = \frac{\int_{W_{out}} wu_{\tilde{W}_{out}}(w)dw}{\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw}$$

Thus $\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw = \frac{1}{2}(0.6-0.4)(1)$

$$= 0.1$$

$$\int_{W_{out}} wu_{\tilde{W}_{out}}(w)dw = \int_{0.4}^{0.5} w\left(\frac{w-0.4}{0.1}\right)dw - \int_{0.5}^{0.6} w\left(\frac{w-0.6}{0.1}\right)dw$$

$$= 0.05$$

Therefore $w^{cg} = \dfrac{0.05}{0.1} = 0.5.$

<u>SEAFCO</u>

We input $E/P = 5.29$, $P/BV = 3.88$, $P/P_n = 1.79$, i.e. $x = 5.29, y = 3.88$, $z = 1.79$.

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

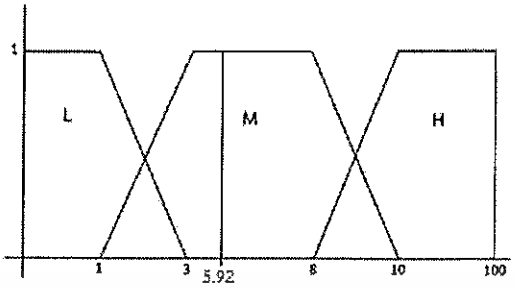

Figure B-9 Intersection between input value and fuzzy set of linguistic terms of E/P , SEAFCO.

For P/BV

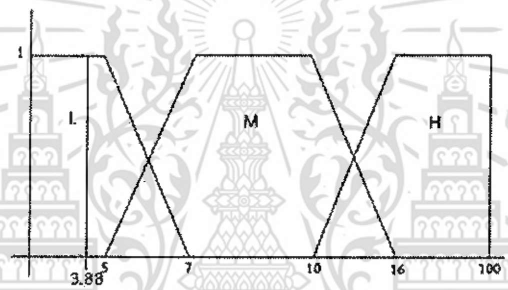

Figure B-10 Intersection between input value and fuzzy set of linguistic terms of P/BV , SEAFCO.

For P/P_n

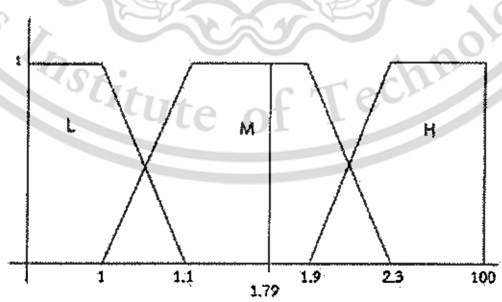

Figure B-11 Intersection between input value and fuzzy set of linguistic terms of P/P_n , SEAFCO.

From Figure B-9-B-11, we have the concerned fuzzy rules as follows:

Rule 11 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{MZ}$ then w was $R\tilde{H}W$.

Since $u_{\tilde{MX}}(x)=1 \quad u_{\tilde{LY}}(y)=1 \quad u_{\tilde{MZ}}(z)=1$

We have

$$h_{11}=u_{\tilde{MX}}(5.92)\wedge u_{\tilde{LY}}(3.88)\wedge u_{\tilde{MZ}}(1.79)$$

$$=1\wedge 1\wedge 1$$

$$=1$$

Since output of each rule is $R\tilde{H}W$, the so truncated fuzzy outputs with $h_{11}=1$ is equal to ~~$R\tilde{H}W$~~ ,that is $\tilde{W}_{out}(w)=$ ~~$R\tilde{H}W$~~

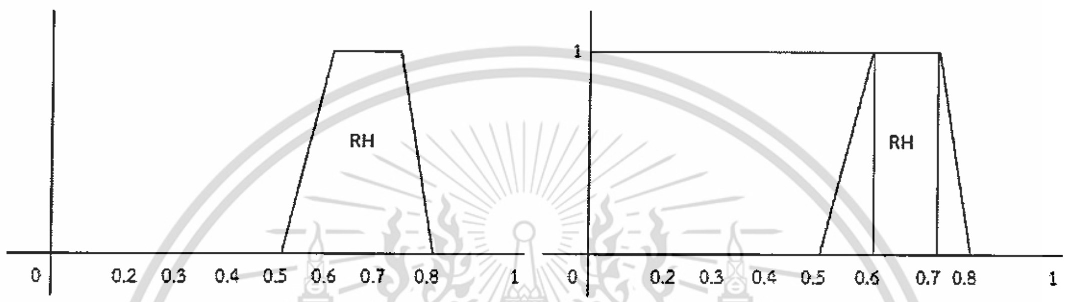

Figure B-12 The union of truncated fuzzy outputs $\tilde{W}_{out}(w)=$ ~~$R\tilde{H}W$~~ , SEAFCO

It is not difficult to find the membership function of $\tilde{W}_{out}(w)=$ ~~$R\tilde{H}W$~~ . From Figure B-12, we have

$$u_{\tilde{W}_{out}}(w)=\begin{cases}\dfrac{w-0.5}{0.1} & ;0.5\le w\le 0.6\\ 1 & ;0.6\le w\le 0.7\\ -\dfrac{(w-0.8)}{0.1} & ;0.7\le w\le 0.8\end{cases}$$

Defuzzification

$$w^{cg}=\frac{\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw}{\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw}$$

Thus $\displaystyle\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw=\frac{1}{2}(0.3+0.1)(1)$

$$=0.2$$

$$\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw=\int_{0.5}^{0.6} w\left(\frac{w-0.5}{0.1}\right)dw+\int_{0.6}^{0.7} w dw-\int_{0.7}^{0.8} w\left(\frac{w-0.8}{0.1}\right)dw$$

$$=0.13$$

Therefore $$w^{cg} = \frac{0.13}{0.2} = 0.65.$$

STEC

We input $E/P = 4.71$, $P/BV = 4.83$, $P/P_n = 1.55$, i.e. input $x = 4.71$, $y = 4.83$, $z = 1.55$

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

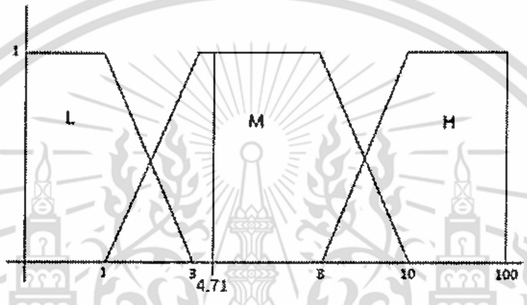

Figure B-13 Intersection between input value and fuzzy set of linguistic terms of E/P, STEC.

For P/BV

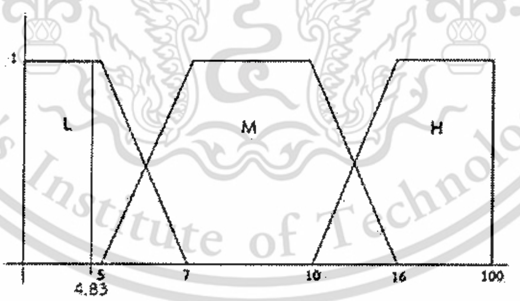

Figure B-14 Intersection between input value and fuzzy set of linguistic terms of P/BV , STEC.

For P/P_n

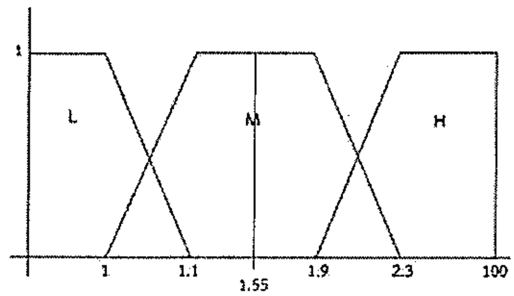

Figure B-15 Intersection between input value and fuzzy set of linguistic terms of P/P_n , STEC

From Figure B-13-B15, we have the concerned fuzzy rules as follows:

Rule 11 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{MZ}$ then w was $R\tilde{H}W$

Since $u_{\tilde{MX}}(x)=1$ $u_{\tilde{LY}}(y)=1$ $u_{\tilde{MZ}}(z)=1$

We have

$$h_{11} = u_{\tilde{MX}}(4.71) \wedge u_{\tilde{LY}}(4.83) \wedge u_{\tilde{MZ}}(1.55)$$
$$= 1 \wedge 1 \wedge 1$$
$$= 1$$

Since output of each rule is $R\tilde{H}W$, the so truncated fuzzy outputs with $h_{11} = 1$ is equal to $R\tilde{H}W$,that is $\tilde{W}_{out}(w) = R\tilde{H}W$

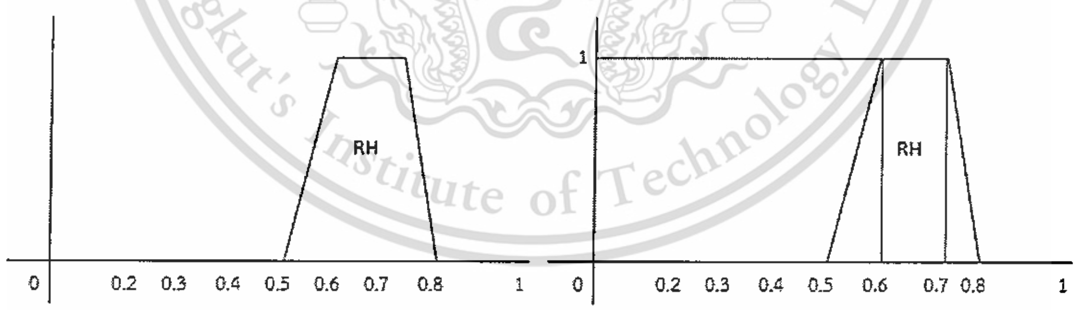

Figure B-16 The union of truncated fuzzy outputs $\tilde{W}_{out}(w) = R\tilde{H}W$,STEC.

It is not difficult to find the membership function of $\tilde{W}_{out}(w) = R\tilde{H}W$. From Figure B-16, we have

$$u_{\tilde{W}_{out}}(w)=\begin{cases}\dfrac{w-0.5}{0.1} & ;0.5\le w\le 0.6\\ 1 & ;0.6\le w\le 0.7\\ -\dfrac{(w-0.8)}{0.1} & ;0.7\le w\le 0.8\end{cases}$$

Defuzzification

$$w^{cg}=\frac{\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw}{\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw}$$

Thus $\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw=\frac{1}{2}(0.3+0.1)(1)$

$=0.2$

$$\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw=\int_{0.5}^{0.6} w\left(\frac{w-0.5}{0.1}\right)dw+\int_{0.6}^{0.7} w dw-\int_{0.7}^{0.8} w\left(\frac{w-0.8}{0.1}\right)dw$$

$=0.13$

Therefore $w^{cg}=\frac{0.13}{0.2}=0.65$

<u>SYNTEC</u>

We input $E/P=4.84$, $P/BV=1.82$, $P/P_n=2.37$. ,i.e. $x=4.84$, $y=1.82$ $z=2.37$.

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

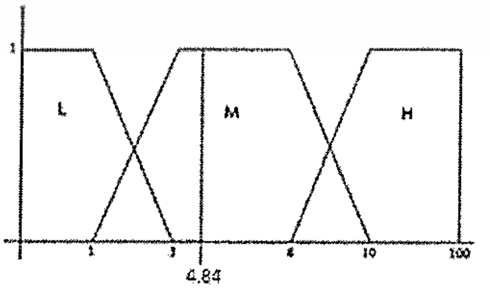

Figure B-17. Intersection between input value and fuzzy set of linguistic terms of E/P, SYNTEC.

For P/BV

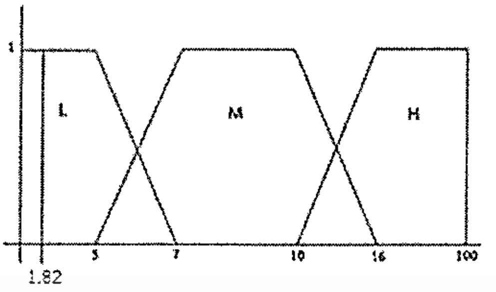

Figure B-18. Intersection between input value and fuzzy set of linguistic terms of P/BV , SYNTEC.

For P/P_n

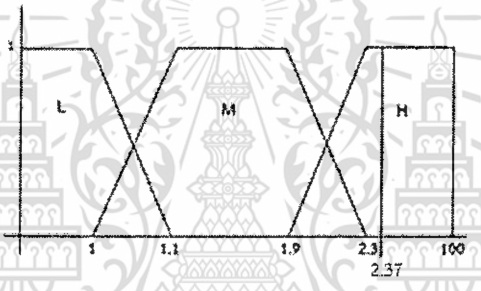

Figure B-19 Intersection between input value and fuzzy set of linguistic terms of P/P_n , SYNTEC.

From Figure B-17-B-19 we have the concerned fuzzy rules as follows:

Rule 12 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{HZ}$ then w was $\tilde{MW}$.

Since $u_{\tilde{LX}}(x)=1$ $\quad u_{\tilde{LY}}(y)=1$ $\quad u_{\tilde{HZ}}(z)=1$

We have $\quad h_{12}=u_{\tilde{MX}}(4.84)\wedge u_{\tilde{LY}}(1.82)\wedge u_{\tilde{HZ}}(2.37)$

$=1\wedge 1\wedge 1$

$=1$

Since output of each rule is $\tilde{MW}$, so the union of truncated fuzzy outputs with $h_{12}=1$-is equal to ~~$\tilde{MW}$~~ , that is, $\tilde{W}_{out}(w)=$ ~~$\tilde{MW}$~~

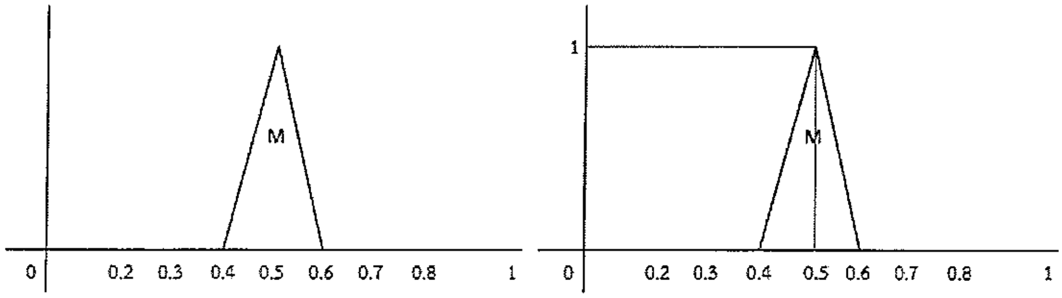

Figure B-20 The union of truncated fuzzy outputs $\tilde{W}_{out}(w) = \tilde{MW}$, SYNTEC.

It is not difficult to find the membership function of $\tilde{W}_{out}(w) = \tilde{MW}$. From Figure B-20, we have

$$u_{\tilde{W}_{out}}(w) = \begin{cases} \dfrac{w-0.4}{0.1} & ;0.4 \le w \le 0.5 \\ -\dfrac{(w-0.6)}{0.1} & ;0.5 \le w \le 0.6 \end{cases}$$

Defuzzification

$$w^{cg} = \frac{\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw}{\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw}$$

Thus $\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw = \frac{1}{2}(0.6-0.4)(1)$

$= 0.1$

$$\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw = \int_{0.4}^{0.5} w\left(\frac{w-0.4}{0.1}\right)dw - \int_{0.5}^{0.6} w\left(\frac{w-0.6}{0.1}\right)dw$$

$= 0.05$

Therefore $w^{cg} = \dfrac{0.05}{0.1} = 0.5$.

TRC

We input $E/P = 5.47$, $P/BV = 3.46$, $P/P_n = 1.09$, i.e. $x = 5.47$, $y = 3.46$, $z = 1.09$

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

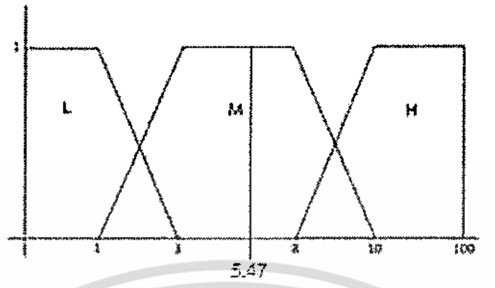

Figure B-21 Intersection between input value and fuzzy set of linguistic terms of E/P , TRC

For P/BV

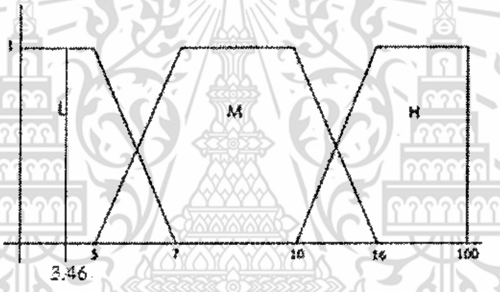

Figure B-22 Intersection between input value and fuzzy set of linguistic terms of P/BV , TRC.

For P/P_n

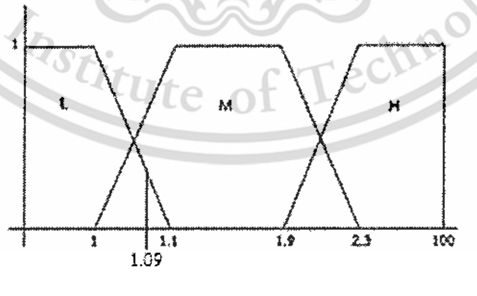

Figure B-23 Intersection between input value and fuzzy set of linguistic terms of P/P_n, TRC.

From Figure 4.8-4.10, we have the concerned fuzzy rules as follows:

Rule 10 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{LZ}$ then w was $\tilde{HW}$.

Rule 11 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{MZ}$ then w was $R\tilde{H}W$.

Sine $\quad u_{\underset{\sim}{LZ}}(z) = -\dfrac{(x-1.1)}{0.1} \quad u_{\underset{\sim}{MZ}}(z) = \dfrac{(x-1)}{0.1} \quad u_{\underset{\sim}{MX}}(x) = 1 \quad u_{\underset{\sim}{LY}}(y) = 1$

We have

$$h_{10} = u_{\underset{\sim}{MX}}(5.47) \wedge u_{\underset{\sim}{LY}}(3.46) \wedge u_{\underset{\sim}{LZ}}(1.09)$$
$$= 1 \wedge 1 \wedge 0.1 = 0.1$$

$$h_{11} = u_{\underset{\sim}{MX}}(5.47) \wedge u_{\underset{\sim}{LY}}(3.46) \wedge u_{\underset{\sim}{MZ}}(1.09)$$
$$= 1 \wedge 1 \wedge 0.9 = 0.9$$

Since output of each rule is $\tilde{HW}$, $R\tilde{H}W$, so the union of truncated fuzzy outputs with $h_{10} = 0.1$ is equal $\underset{\sim}{HW}$ and equal to $h_{11} = 0.9$ $\underset{\sim}{R\tilde{H}W}$, that is, $\tilde{W}_{out}(w) = \underset{\sim}{HW} \cup \underset{\sim}{R\tilde{H}W}$

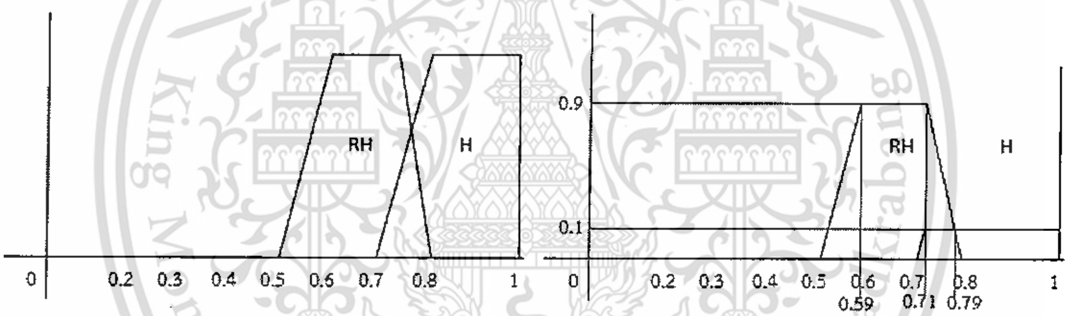

Figure B-24 The union of truncated fuzzy outputs $\tilde{W}_{out}(w) = \underset{\sim}{HW} \cup \underset{\sim}{R\tilde{H}W}$,TRC

It is not difficult to find the membership function of $\tilde{W}_{out}(w) = \underset{\sim}{HW} \cup \underset{\sim}{R\tilde{H}W}$. From Figure B-24, we have

$$u_{\tilde{W}_{out}}(w) = \begin{cases} \dfrac{w-0.5}{0.1} & ;0.5 \le w \le 0.59 \\ 0.9 & ;0.59 \le w \le 0.71 \\ -\left(\dfrac{w-0.8}{0.1}\right) & ;0.71 \le w \le 0.79 \\ 0.1 & ;0.79 \le w \le 1 \end{cases}$$

Defuzzification

$$w^{cg} = \frac{\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw}{\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw}$$

Thus $\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw = \left[\frac{1}{2}(0.12+0.3)(0.9)\right] + \left[\frac{1}{2}(0.2+0.21)(0.1)\right]$

$= 0.2095$

$$\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw = \int_{0.5}^{0.59} \frac{w}{0.1}(w-0.5)dw + \int_{0.59}^{0.71} 0.9wdw - \int_{0.71}^{0.79} \frac{w}{0.1}(w-0.8)dw + \int_{0.79}^{1} 0.1wdw$$

$= 0.1412$

Therefore $w^{cg} = \frac{0.1412}{0.2095} = 0.674$.

TTCL

We input E/P $=3.58$, P/BV $z = 3.04$., P/P_n $= 2.83$,i.e. $x = 3.58$, $y = 3.04$, $z = 3.04$.

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

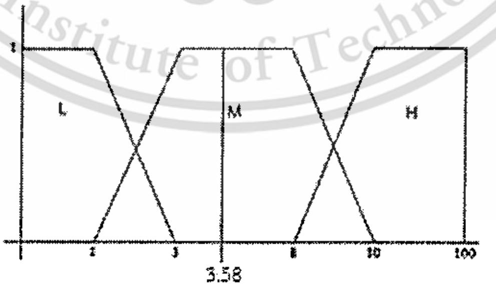

Figure B-25 Intersection between input value and fuzzy set of linguistic terms of E/P, TTCL.

For P/BV

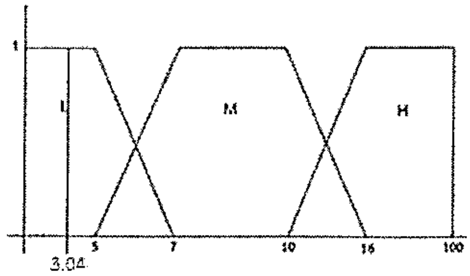

Figure B-26 Intersection between input value and fuzzy set of linguistic terms of P/BV, TTCL.

P/P_n

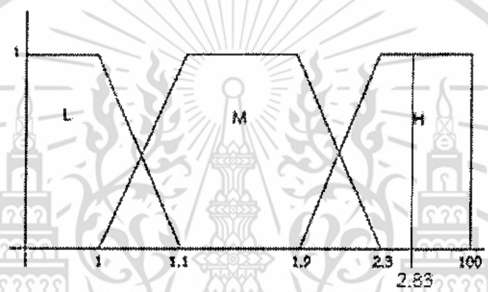

Figure B-27 Intersection between input value and fuzzy set of linguistic terms of P/P_n, TTCL.

Rule 12 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{HZ}$ then w was $\tilde{MW}$

Since $u_{\tilde{MX}}(x)=1$ $u_{\tilde{LY}}(y)=1$ $u_{\tilde{HZ}}(z)=1$

We have

$$h_{12} = u_{\tilde{MX}}(3.58) \wedge u_{\tilde{LY}}(3.04) \wedge u_{\tilde{HZ}}(2.83)$$
$$= 1 \wedge 1 \wedge 1$$
$$= 1$$

Since output of each rule is $\tilde{MW}$, so the union of truncated fuzzy outputs with $h_{12} = 1$ is equal to $\tilde{MW}$, that is, $\tilde{W}_{out}(w) = \tilde{MW}$

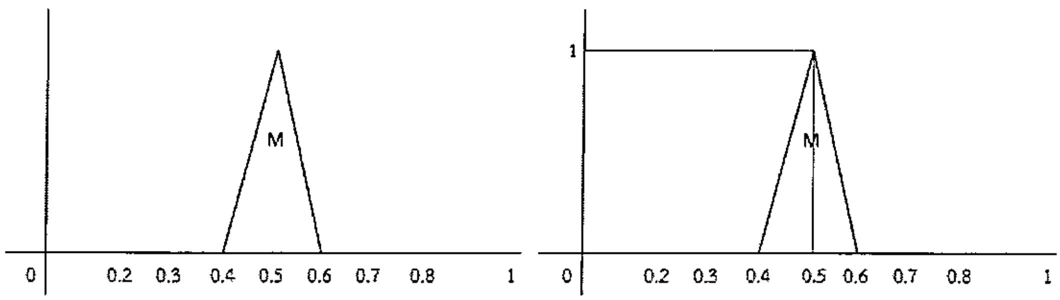

Figure 4.11 The union of truncated fuzzy outputs $\tilde{W}_{out}(w) = \tilde{HW}$,TTCL

It is not difficult to find the membership function of $\tilde{W}_{out}(w) = \tilde{MW}$. From Figure B-28, we have

$$u_{\tilde{MW}}(w) = \begin{cases} \dfrac{w-0.4}{0.1} & ;0.4 \le w \le 0.5 \\ -\dfrac{(w-0.6)}{0.1} & ;0.5 \le w \le 0.6 \end{cases}$$

Defuzzification

$$w^{cg} = \frac{\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw}{\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw}$$

Thus $\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw = \frac{1}{2}(0.6-0.4)(1)$

$$= 0.1$$

$$\int_{W_{out}} w u_{\tilde{W}_{out}}(w)dw = \int_{0.4}^{0.5} w\left(\frac{w-0.4}{0.1}\right)dw - \int_{0.5}^{0.6} w\left(\frac{w-0.6}{0.1}\right)dw$$

$$= 0.05$$

Therefore $w^{cg} = \dfrac{0.05}{0.1} = 0.5$.

<u>UNIQ</u>

We input $E/P = 3.51$, $P/BV = 3.75$, $P/P_n = 3.12$, i.e. $x = 3.51$, $y = 3.75$, $z = 3.12$.

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

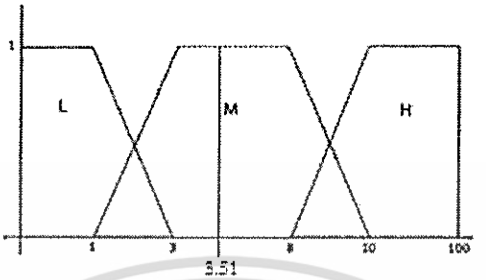

Figure B-29 Intersection between input value and fuzzy set of linguistic terms of E/P, UNIQ.

For P/BV

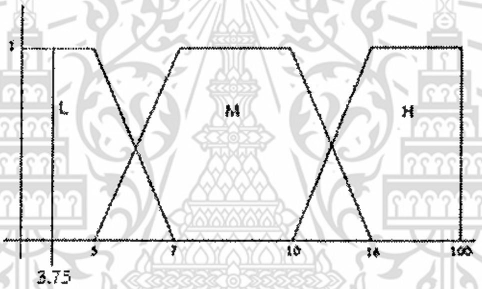

Figure B-30 Intersection between input value and fuzzy set of linguistic terms of P/BV, UNIQ.

For P/P_n

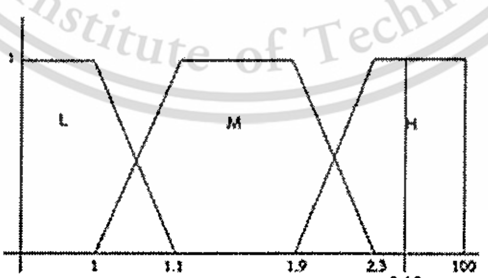

Figure B-31 Intersection between input value and fuzzy set of linguistic terms of P/P_n, UNIQ.

From Figure B-29-B-31, we have the concerned fuzzy rules as follows:

Rule 12 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{HZ}$ then w was $\tilde{MW}$.

Sine $u_{\tilde{MX}}(x)=1 \quad u_{\tilde{LY}}(y)=1 \quad u_{\tilde{HZ}}(z)=1$

We have

$$h_{12}=u_{\tilde{MX}}(3.58)\wedge u_{\tilde{LY}}(3.04)\wedge u_{\tilde{HZ}}(2.83)$$
$$=1\wedge 1\wedge 1$$
$$=1$$

Since output of each rule is $\tilde{MW}$, so the union of truncated fuzzy outputs with $h_{12}=1$-is equal to $\tilde{MW}$, that is, $\tilde{W}_{out}(w)=\tilde{MW}$

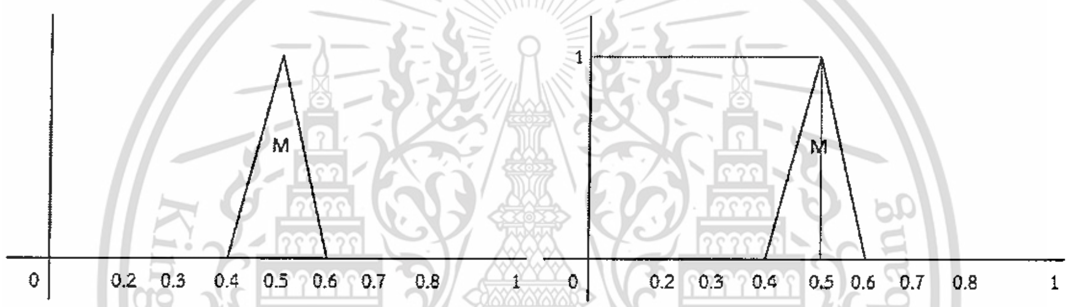

Figure B-32 The union of truncated fuzzy outputs $\tilde{W}_{out}(w)=\tilde{MW}$, UNIQ

It is not difficult to find the membership function of $\tilde{W}_{out}(w)=\tilde{MW}$. From Figure B-32, we have

$$u_{\tilde{W}_{out}}(w)=\begin{cases}\dfrac{w-0.4}{0.1} & ;0.4\le w\le 0.5\\ -\dfrac{(w-0.6)}{0.1} & ;0.5\le w\le 0.6\end{cases}$$

Defuzzification

$$w^{cg}=\frac{\int_{W_{out}} wu_{\tilde{W}_{out}}(w)dw}{\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw}$$

Thus $\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw=\frac{1}{2}(0.6-0.4)(1)$

$$= 0.1$$

$$\int_{W_{out}} wu_{\tilde{W}_{out}}(w)dw = \int_{0.4}^{0.5} w\left(\frac{w-0.4}{0.1}\right)dw - \int_{0.5}^{0.6} w\left(\frac{w-0.6}{0.1}\right)dw$$

$$= 0.05$$

Therefore $\qquad w^{cg} = \dfrac{0.05}{0.1} = 0.5$

CK

We input $E/P = 4.76$, $P/BV = 2.54$, $P/P_n = 2.25$, i.e. $x = 4.76, y = 2.54$, $z = 2.25$

Firstly, we compute weight (w) between input values and conditions of fuzzy rules. For more understanding, let us see these pictures.

For E/P

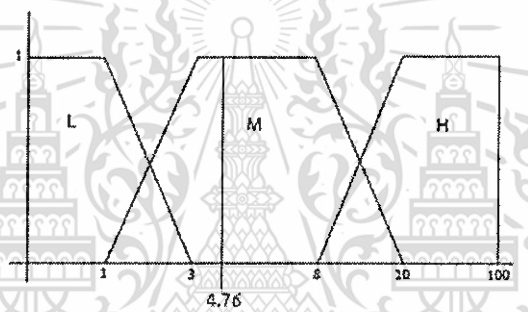

Figure B-33 Intersection between input value and fuzzy set of linguistic terms of E/P , CK.

For P/BV

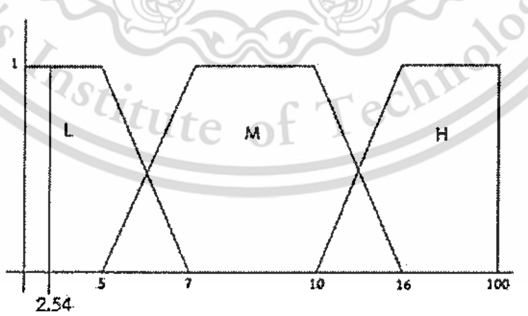

Figure B-34 Intersection between input value and fuzzy set of linguistic terms of P/BV , CK.

For P/P_n

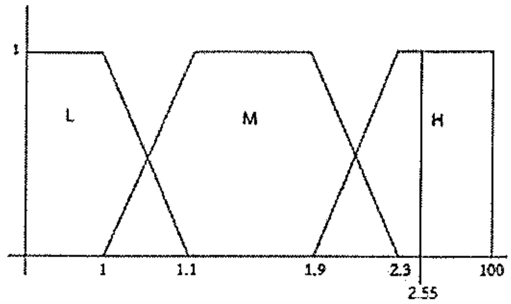

Figure B-35 Intersection between input value and fuzzy set of linguistic terms of P/P_n, CK.

From Figure B-33-B-35, we have the concerned fuzzy rules as follows:

Rule 12 If x was $\tilde{MX}$ and y was $\tilde{LY}$ and z was $\tilde{HZ}$ then w was $\tilde{MW}$.

Consider $u_{\tilde{MX}}(x)=1$ $u_{\tilde{LY}}(y)=1$ $u_{\tilde{HZ}}(z)=1$

Thus
$$h_{12}=u_{\tilde{MX}}(4.76)\wedge u_{\tilde{LY}}(2.54)\wedge u_{\tilde{HZ}}(2.55)$$
$$=1\wedge 1\wedge 1$$
$$=1$$

Since output of each rule is $\tilde{MW}$, so the union of truncated fuzzy outputs with $h_{12}=1$ is equal to $\tilde{MW}$, that is, $\tilde{W}_{out}(w)=\tilde{MW}$

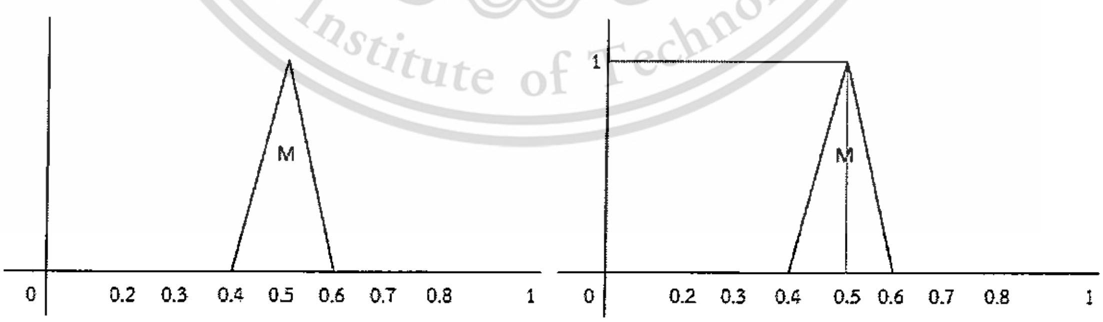

Figure B-36 The union of truncated fuzzy outputs $\tilde{W}_{out}(w)=\tilde{MW}$, CK

It is not difficult to find the membership function of $\tilde{W}_{out}(w)=\tilde{MW}$. From Figure B-36, we have

$$u_{\tilde{W}_{out}}(w)=\begin{cases}\dfrac{w-0.4}{0.1} & ;0.4\le w\le 0.5\\ -\dfrac{(w-0.6)}{0.1} & ;0.5\le w\le 0.6\end{cases}$$

Defuzzification

$$w^{cg}=\frac{\int_{W_{out}} wu_{\tilde{W}_{out}}(w)dw}{\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw}$$

Thus $\int_{W_{out}} u_{\tilde{W}_{out}}(w)dw=\frac{1}{2}(0.6-0.4)(1)=0.1$

$$\int_{W_{out}} wu_{\tilde{W}_{out}}(w)dw=\int_{0.4}^{0.5} w\left(\frac{w-0.4}{0.1}\right)dw-\int_{0.5}^{0.6} w\left(\frac{w-0.6}{0.1}\right)dw=0.05$$

Therefore $w^{cg}=\frac{0.05}{0.1}=0.5$

Appendix C

Stock Selected Technique into Portfolio by Fuzzy Quantitative Analysis and Fuzzy Multi-Criteria Decision Making

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Research Article

Stock Selection into Portfolio by Fuzzy Quantitative Analysis and Fuzzy Multicriteria Decision Making

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This paper presents a stock selection approach assisted by fuzzy procedures. In this approach, stocks are classified into groups according to business types. Within each group, the stocks are screened and then ranked according to their investment weight obtained from fuzzy quantitative analysis. Groups were also ranked according to their group weight obtained from fuzzy analytic hierarchy process (FAHP) and technique for order preference by similarity to ideal solution method (TOPSIS). The overall weight for each stock was then derived from both of these weights and used for selecting a stock into the portfolio. As a demonstration, our analysis procedures were applied to a test set of data.

1. Introduction

Presently, investors are more interested in investing in stocks and bonds than keeping their money in the bank because it yields a higher return. However, this higher return also comes with higher risk; investors may lose some of their investment, get a lower-than-expected return, or get a lower return than that from another type of investment. Therefore, they have to analyze a stock carefully before investing in it.

In addition to several established approaches to stock analysis—such as quantitative fundamental analysis, technical analysis, and stochastic analysis—new analytical tools have been developed and widely used including ones that are based on Brownian movement, fuzzy logic, and the analytic hierarchy process.

The analytic hierarchy process (AHP) is a multicriteria decision-making approach and is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. It was developed by Saaty in the 1970s, to help one make decision when one is faced with the mixture of qualitative, quantitative, and sometimes conflicting factors that are taken into consideration. AHP has been very effective in making complicated, often irreversible decisions. It has been extensively studied and refined since then (e.g., [1–11] and references therein).

Fuzzy sets and fuzzy logic, especially, are of wide interest today. They are effective tools for modeling, in the absence of complete and precise information, complex business, finance, and management systems. The subjective judgement of experts who have used fuzzy logic techniques produces better results than the objective manipulation of inexact data. The concept of a fuzzy set is a reflection of reality reflection which serves as a point of departure for the development of theories which have the capability to model the pervasive imprecision and uncertainty of the real world. As applied to stock analysis (e.g., [12–15] and references therein), fuzzy logic uses integrated experiential knowledge of human experts to make better quantitative estimates, not possible with classical logic, based on robust mathematical principles.

By reason of vagueness of boundaries of stock data in future and the attendant imprecision, uncertainty, and preference of decision makers, therefore, fuzzy logic and AHP seem suitable for this problem. This paper proposes an approach to stock analysis based on calculated weights from fuzzy quantitative analysis and fuzzy multicriteria decision making. The idea of using fuzzy quantitative analysis and fuzzy multicriteria decision making to imply final investment weights for the stock selection into portfolio is different from the previous works. The practicality of the approach was demonstrated by an application to a test set of data.

2. Preliminaries

2.1. Fuzzy Logic: Application and Definitions. Fuzzy logic was introduced by Zadeh [16] and has been widely applied to problems in various fields of study. Many researchers used fuzzy logic in stock market analysis (e.g., [12–15]) and decision making (e.g., [1–4, 6, 7, 9–15, 17]). In this study we use fuzzy logic in both, stock market analysis and decision making.

In this subsection, definitions of the fuzzy logic terms and concepts used in this study are described below.

Definition 1. Given a crisp set A of a universe $\mathcal{U}$, a fuzzy set $\tilde{u}$ on A is defined as

$$\tilde{u} = \{(x, u(x)) \mid x \in A\} \quad \text{where } u(x) \in [0,1] \tag{1}$$

and u is a membership function.

Definition 2. Given a fuzzy set $\tilde{u}$, an α-cut set, denoted by $[\tilde{u}]^\alpha$, for all $\alpha \in [0,1]$, is defined as

$$[\tilde{u}]^\alpha = \begin{cases} \{x \in A \mid u(x) \geq \alpha\}; & 0 < \alpha \leq 1 \\ \overline{\{x \in A \mid u(x) > 0\}}; & \alpha = 0. \end{cases} \tag{2}$$

Definition 3. Let $\tilde{u}$ be a fuzzy set under the membership $u : \mathbb{R} \to [0,1]$, and $\tilde{u}$ is a fuzzy number if it satisfies the following conditions:

(1) $\tilde{u}$ is a normal fuzzy set; that is, $\exists x \in \mathbb{R}$, $u(x) = 1$.

(2) $\tilde{u}$ is a convex fuzzy set; that is, $\forall \lambda \in [0,1], \forall x_1, x_2 \in \mathbb{R}$, $u(\lambda x_1 + (1-\lambda)x_2) \geq \min[u(x_1), u(x_2)]$.

(3) For every $\alpha \in [0,1]$, $[\tilde{u}]^\alpha = [a,b]$ for some closed interval $[a,b]$.

Given an $\mathbb{R}_F$ fuzzy number space, condition (3) of Definition 3 ensures that every $\tilde{u} \in \mathbb{R}_F$ can be represented by a closed interval $[\tilde{u}]^\alpha = [\underline{u}(\alpha), \overline{u}(\alpha)]$, where $\underline{u}, \overline{u} : [0,1] \to \mathbb{R}$ are functions that satisfy the following conditions:

(1) $\underline{u}$ is a bounded, left continuous, and nondecreasing function on $[0,1]$.

(2) $\overline{u}$ is a bounded, right continuous, and no-increasing function on $[0,1]$.

(3) $\underline{u}(\alpha) \leq \overline{u}(\alpha)$ for all $\alpha \in [0,1]$.

Definition 4. $\tilde{u} = [\underline{u}(\alpha), \overline{u}(\alpha)]$ is a positive fuzzy number that can be represented by the expression $\tilde{u} > 0$, if $\underline{u}(0) > 0$.

Definition 5. Given $a^L \leq a^{M_1} \leq a^{M_2} \leq a^U$, a trapezoidal fuzzy number is a fuzzy number $\tilde{z}$ whose membership function $z(x)$ is defined by

$$z(x) = \begin{cases} \dfrac{x - a^L}{a^{M_1} - a^L}; & a^L \leq x \leq a^{M_1} \\ 1; & a^{M_1} \leq x \leq a^{M_2} \\ \dfrac{x - a^U}{a^{M_2} - a^U}; & a^{M_2} \leq x \leq a^U \\ 0; & \text{otherwise} \end{cases} \tag{3}$$

and represented by the expression $\tilde{z} = \langle a^L, a^{M_1}, a^{M_2}, a^U \rangle$.

Definition 6. A trapezoidal fuzzy number $\tilde{z} = \langle a^L, a^M, a^M, a^U \rangle$ is called a triangular fuzzy number and expressed as $\tilde{z} = \langle a^L, a^M, a^U \rangle$.

Note. For any real number a, $a = \langle a, a, a \rangle = \langle a, a, a, a \rangle$.

Definition 7. Given any two positive fuzzy numbers $\tilde{a} = \langle a^L, a^{M_1}, a^{M_2}, a^U \rangle$ and $\tilde{b} = \langle b^L, b^{M_1}, b^{M_2}, b^U \rangle$ and a real positive number $p \in \mathbb{R}^+$, operations $\oplus$, $\ominus$, $\otimes$, and $\oslash$ between $\tilde{a}$ and $\tilde{b}$ and an operation $\odot$ between $\tilde{a}$ and p are defined as follows:

$$\begin{aligned} \tilde{a} \oplus \tilde{b} &= \langle a^L + b^L, a^{M_1} + b^{M_1}, a^{M_2} + b^{M_2}, a^U + b^U \rangle, \\ \tilde{a} \ominus \tilde{b} &= \langle a^L - b^U, a^{M_1} - b^{M_2}, a^{M_2} - b^{M_1}, a^U - b^L \rangle, \\ \tilde{a} \otimes \tilde{b} &= \langle a^L b^L, a^{M_1} b^{M_1}, a^{M_2} b^{M_2}, a^U b^U \rangle, \\ p \odot \tilde{a} &= \langle pa^L, pa^{M_1}, pa^{M_2}, pa^U \rangle, \\ \tilde{a} \oslash \tilde{b} &= \left\langle \frac{a^L}{b^U}, \frac{a^{M_1}}{b^{M_2}}, \frac{a^{M_2}}{b^{M_1}}, \frac{a^U}{b^L} \right\rangle. \end{aligned} \tag{4}$$

Definition 8. Given two trapezoidal fuzzy numbers $\tilde{a} = \langle a^L, a^{M_1}, a^{M_2}, a^U \rangle$ and $\tilde{b} = \langle b^L, b^{M_1}, b^{M_2}, b^U \rangle$, the distance between $\tilde{a}$ and $\tilde{b}$ represented by the symbol $d(\tilde{a}, \tilde{b})$ is defined as

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{4}\left[(a^L - b^L)^2 + (a^{M_1} - b^{M_1})^2 + (a^{M_2} - b^{M_2})^2 + (a^U - b^U)^2\right]}. \tag{5}$$

For convenience, $I_n = \{1, 2, \ldots, n\}$ is defined for further use in this paper.

Definition 9. $\tilde{A} = (\tilde{a}_{ij})_{m \times n}$ is a fuzzy matrix if $\tilde{a}_{ij}$ are fuzzy numbers for all $i \in I_m$ and $j \in I_n$.

Definition 10. $\widetilde{M} = (\widetilde{m}_{ij})_{n \times 1}$ is a fuzzy vector when all $\widetilde{m}_i = \langle m_i^L, m_i^{M_1}, m_i^{M_2}, m_i^U \rangle$, $i \in I_n$ are trapezoidal fuzzy numbers. The aggregation of $\widetilde{M}$, represented by $\widetilde{M}_{agg}$, is defined as

$$\widetilde{M}_{agg} = \left\langle \min_{i=1}^{n} \{m_i^L\}, \frac{1}{n}\sum_{i=1}^{n} m_i^{M_1}, \frac{1}{n}\sum_{i=1}^{n} m_i^{M_2}, \max_{i=1}^{n} \{m_i^U\} \right\rangle. \tag{6}$$

2.2. Consistency Fuzzy Matrix. In this subsection, we introduce the definition of consistency fuzzy matrix and consistency index which was developed by Ramik [3, 4].

Definition 11. Let $A = (a_{ij})_{n\times n}$ be an $n \times n$ matrix where $a_{ij} > 0$ for all $i, j \in I_n$ and A is a reciprocal matrix if $a_{ji} = 1/a_{ij}$ for all $i, j \in I_n$.

Definition 12. Let $A = (a_{ij})_{n\times n}$ be an $n\times n$ matrix where $a_{ij} > 0$ for all $i, j \in I_n$ and A is a consistency matrix if there exist weight vectors $w = (w_i)_{n\times 1}$, $w_i > 0$, for all $i \in I_n$, where $\sum_{i=1}^{n} w_i = 1$ and $a_{ij} = w_i/w_j$ for all $i, j \in I_n$.

Definition 13. Let $\widetilde{A} = (\widetilde{a}_{ij})_{n\times n}$ be an $n\times n$ fuzzy matrix where $\widetilde{a}_{ij} > 0$ are fuzzy numbers for all $i, j \in I_n$ and $\widetilde{A}$ is a reciprocal fuzzy matrix if $\widetilde{a}_{ji} = 1 \oslash \widetilde{a}_{ij}$ for all $i, j \in I_n$.

In particular, if every member of $\widetilde{A} = (\widetilde{a}_{ij})_{n\times n}$ is a triangular fuzzy number $\widetilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^U)$, $\widetilde{A}$ is a reciprocal fuzzy matrix if $\widetilde{a}_{ji} = (1/a_{ij}^U, 1/a_{ij}^M, 1/a_{ij}^L)$ for all $i, j \in I_n$.

Definition 14. Let $\widetilde{A} = (\widetilde{a}_{ij})_{n\times n}$ be an $n\times n$ fuzzy matrix, where $\widetilde{a}_{ij} = [\underline{a}_{ij}(\alpha), \overline{a}_{ij}(\alpha)] > 0$ for all $i, j \in I_n$ and $\widetilde{A}$ is a consistency fuzzy matrix if there exist $a_{ij}^{\alpha} \in [\underline{a}_{ij}(\alpha), \overline{a}_{ij}(\alpha)]$ for all $i, j \in I_n$ and some $\alpha \in [0, 1]$ with which $A = (a_{ij}^{\alpha})_{n\times n}$ is a consistency matrix; that is, there exist $w^{\alpha} = (w_i^{\alpha})_{n\times 1}$, $w_i^{\alpha} > 0$, for all $i \in I_n$, where $\sum_{i=1}^{n} w_i^{\alpha} = 1$ and $a_{ij}^{\alpha} = w_i^{\alpha}/w_j^{\alpha}$ for all $i, j \in I_n$.

According to Definition 14, since $w_i^{\alpha} > 0$ for all $i \in I_n$, there exist fuzzy vectors $\widetilde{w} = (\widetilde{w}_i)_{n\times 1}$, where $w_i^{\alpha} \in [\underline{w}_i(\alpha), \overline{w}_i(\alpha)] > 0$ for all $i \in I_n$. These vectors are called fuzzy weight vectors.

It is clear that if $\widetilde{A}$ is a fuzzy consistency matrix then it is a fuzzy reciprocal fuzzy matrix and $\widetilde{A}$ is not a fuzzy consistency matrix if it is not a fuzzy reciprocal fuzzy matrix. Because of these reasons, construction of a fuzzy consistency matrix usually starts by first constructing a reciprocal fuzzy matrix $\widetilde{A}$. Ramik and Korviny [4] proposed a method for calculating fuzzy weight vector $\widetilde{w} = (\widetilde{w}_i)_{n\times 1}$ for a fuzzy reciprocal matrix $\widetilde{A} = (\widetilde{a}_{ij})_{n\times n}$ where $\widetilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^U)$ for all $i, j \in I_n$ by using the method of geometric mean. $\widetilde{w}_k = (w_k^L, w_k^M, w_k^U)$ are defined for all $k \in I_n$, where

$$w_k^L = C_L \cdot \frac{\left(\prod_{j=1}^{n} a_{kj}^L\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}},$$
$$w_k^M = \frac{\left(\prod_{j=1}^{n} a_{kj}^M\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}}, \qquad (7)$$
$$w_k^U = C_U \cdot \frac{\left(\prod_{j=1}^{n} a_{kj}^U\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}},$$

$$C_L = \min_{i \in I_n}\left\{\frac{\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}}{\left(\prod_{j=1}^{n} a_{ij}^L\right)^{1/n}}\right\},$$
$$C_U = \max_{i \in I_n}\left\{\frac{\left(\prod_{j=1}^{n} a_{ij}^M\right)^{1/n}}{\left(\prod_{j=1}^{n} a_{ij}^U\right)^{1/n}}\right\}. \qquad (8)$$

In addition, Ramik and Korviny [4] defined a consistency index for measuring the nearness of a fuzzy reciprocal matrix to the corresponding fuzzy consistency matrix as follows.

Definition 15. Let $\widetilde{A} = (\widetilde{a}_{ij})_{n\times n}$ be a fuzzy reciprocal matrix, of which $\widetilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^U)$ are triangular fuzzy numbers, evaluated from a scale $S = [1/\sigma, \sigma]$ for some real number $\sigma > 1$; the consistency index of $\widetilde{A}$ represented by the symbol $I_n^{\sigma}(\widetilde{A})$ is defined as

$$I_n^{\sigma}(\widetilde{A}) = C_n^{\sigma} \cdot \max_{i,j}\left\{\max\left\{\left|\frac{w_i^L}{w_j^U} - a_{ij}^L\right|, \left|\frac{w_i^M}{w_j^M} - a_{ij}^M\right|, \left|\frac{w_i^U}{w_j^L} - a_{ij}^U\right|\right\}\right\}, \qquad (9)$$

where $\widetilde{w} = (\widetilde{w}_i)_{n\times 1}$ are fuzzy weight vectors and $\widetilde{w}_i = (w_i^L, w_i^M, w_i^U)$ for all $i \in I_n$ as expressed in (7) and

$$C_n^{\sigma} = \begin{cases} \dfrac{1}{\max\left\{\sigma - \sigma^{(2-2n)/n}, \sigma^2\left((2/n)^{2/(n-2)} - (2/n)^{(n-2)/2}\right)\right\}}; & \sigma < \left(\dfrac{n}{2}\right)^{n/(n-2)} \\ \dfrac{1}{\max\left\{\sigma - \sigma^{(2-2n)/n}, \sigma^{(2-2n)/n} - \sigma\right\}}; & \sigma \geq \left(\dfrac{n}{2}\right)^{n/(n-2)}. \end{cases} \qquad (10)$$

If the consistency index $I_n^{\sigma}(\widetilde{A}) = 0$, the fuzzy reciprocal fuzzy matrix $\widetilde{A}$ is absolutely consistent. The closer the value of $I_n^{\sigma}(\widetilde{A})$ to 0 is, the more consistent the matrix is. Generally, an acceptable value is $I_n^{\sigma}(\widetilde{A}) < 0.1$ or 10%.

Theorem 16 (see [4]). *If $\widetilde{A}$ is an $n \times n$ fuzzy reciprocal matrix with triangular fuzzy elements evaluated with the scale $[1/\sigma, \sigma]$ for some $\sigma > 1$, then $0 \leq I_n^{\sigma}(\widetilde{A}) \leq 1$.*

2.3. Financial Ratios. A sustainable investment and mission requires effective planning and financial management.

The quantitative stock analysis is a useful tool that will improve investment's understanding of financial results and trends over time and provide key indicators of organizational performance. Investor may use the quantitative stock analysis to pinpoint strengths and weaknesses of each company that impact to its stock.

The quantitative stock analysis presented in this study is based on the following financial ratios: price to earnings ratio or P/E Ratio; price to book value ratio or P/BV Ratio; and price to intrinsic ratio or P/P_n Ratio, which are defined as follows.

Definition 17. Let n_1, n_2, and n_3 be the number of common stock, preferred stock, and treasury stock respectively, P_t current price per share, and E_r rth-quarter net profit; *price to earnings ratio or P/E* is defined as

$$\frac{P}{E} = \frac{P_t(n_1 + n_2 - n_3)}{E_r}. \tag{11}$$

P/E denotes the stock price per 1 baht of net profit that the investor is willing to pay for.

Definition 18. Let n be the number of be the number of registered share, A_t and R_t the asset and liability of the company respectively, and P_t current price per share; *price to book value ratio or P/BV* is defined as

$$\frac{P}{BV} = \frac{P_t}{B_t}, \tag{12}$$

where $B_t = (A_t - R_t)/n$.

P/BV denotes how many times the current stock price is compared to its account value.

Definition 19. Let r be the reference interest rate, D_k the kth year-end dividend per share, $k \in I_n$, and P_0 the nth-quarter historical price; the *current target price* P_n is defined as

$$P_n = P_0(1 + r)^n - \sum_{k=1}^{n} D_k (1 + r)^{n-k}. \tag{13}$$

Definition 20. Let P_n be the current target price and P the current stock price; P/P_n is called *price per target price ratio* represented by the symbol P/P_n.

P/P_n denotes how many times the current stock price is compared to the current target price.

3. Stock Selection Procedure

This section presents the proposed stock selection procedure which is done in the following 3 main steps.

Step 1. The first step is analysis of individual stocks within each industrial group from their financial ratios, using fuzzy logic principles to calculate the investment weight for each individual stock.

Step 2. The second step is analysis of industrial groups (e.g., finance, communication, technology, and property) using fuzzy multicriteria decision-making principles to calculate the investment weight for each industrial group.

Step 3. The third step is analysis of individual stocks across all industrial groups using the 2 types of weights from Steps 1 and 2 to calculate the final weight for ranking all individual stocks in the market.

3.1. Step 1: Analysis of Individual Stocks within Each Industrial Group. In this step, we apply the method of Bumlungpong et al. [15] to analyze individual stocks within each industrial group. Price to earnings ratio (P/E ratio), price to book value ratio (P/BV ratio), and price to intrinsic value ratio (P/P_n ratio) are used to calculate the investment weight for each individual stock within an industrial group based on quantitative fuzzy analysis under these assumptions:

(1) A calculated investment weight of an individual stock can be compared only to another one in the same industrial group.

(2) More recent data reflect current trend better than earlier ones.

(3) Fuzzy rules are flexible and depend on expert information.

The specific steps of the fuzzy analysis are as follows.

Step 1.1. This step involves screening in only m individual stocks $(S_1, S_2, \ldots, S_m)$ in the same industrial group of which sufficient financial data are provided for calculating P/E, P/BV, and P/P_n of n earlier years up to the present.

Step 1.2. This step involves calculating $(E/P)(S_k^i)$, $(P/BV)(S_k^i)$, and $(P/P_n)(S_k^i)$ for all $i \in I_n$ and $k \in I_m$, where S_k^i denotes the kth stock in the ith year.

Step 1.3. This step involves calculating the following weighted arithmetic mean: $(E/P)^w(S_k)$, $(P/BV)^w(S_k)$, and $(P/P_n)^w(S_k)$, $k \in I_m$, from the following equations:

$$\left(\frac{E}{P}\right)^w (S_k) = \sum_{i=1}^{n} w_i \left(\frac{E}{P}\right)(S_k^i),$$
$$\left(\frac{P}{BV}\right)^w (S_k) = \sum_{i=1}^{n} w_i \left(\frac{P}{BV}\right)(S_k^i), \tag{14}$$
$$\left(\frac{P}{P_n}\right)^w (S_k) = \sum_{i=1}^{n} w_i \left(\frac{P}{P_n}\right)(S_k^i),$$

$$\text{where } w_i = \frac{2i}{n(n+1)}, \; i \in I_m.$$

Step 1.4. This step involves an expert constructing fuzzy sets in linguistic terms of the ranked financial ratios E/P, P/BV, and P/P_n and a fuzzy set W of the investment weights from $(E/P)^w(S_k)$, $(P/BV)^w(S_k)$, and $(P/P_n)^w(S_k)$, $k \in I_m$.

Step 1.5. This step involves an expert constructing fuzzy rules for estimation based on the fuzzy sets constructed in Step 1.4. These fuzzy rules are in the form of an "if-then" rule as follows:

Rule-1: if x_1 is $\tilde{a}_{11}$ and x_2 is $\tilde{a}_{12}$ and x_3 is $\tilde{a}_{13}$ then y is $\tilde{b}_1$.

Rule-2: if x_1 is $\tilde{a}_{21}$ and x_2 is $\tilde{a}_{22}$ and x_3 is $\tilde{a}_{23}$ then y is $\tilde{b}_2$.

$\vdots$

Rule-q: if x_1 is $\tilde{a}_{q1}$ and x_2 is $\tilde{a}_{q2}$ and x_3 is $\tilde{a}_{q3}$ then y is $\tilde{b}_q$.

x_1, x_2, x_3, and y are fuzzy variables of E/P, P/BV, P/P_a, and W_1, respectively, and $\tilde{a}_{k1}$, $\tilde{a}_{k2}$, and $\tilde{a}_{k3}$, $k \in I_q$, are linguistic terms of E/P, P/BV, P/P_a, and W_1, respectively; that is, $E/P = \{\tilde{a}_{11}, \tilde{a}_{21}, \ldots, \tilde{a}_{q1}\}$, $P/BV = \{\tilde{a}_{12}, \tilde{a}_{22}, \ldots, \tilde{a}_{q2}\}$, $P/P_a = \{\tilde{a}_{13}, \tilde{a}_{23}, \ldots, \tilde{a}_{q3}\}$, and $W = \{\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_q\}$.

Step 1.6. This step involves importing E/P, P/BV, and P/P_a of the latest day and making estimation with Mamdani method using the fuzzy rules constructed in Step 1.5 hence obtaining an output of a fuzzy set $\mathscr{B}$ under the membership $u_{\mathscr{B}}$ on B.

Step 1.7. This step involves performing defuzzification of the fuzzy output to a crisp output by a centroid method. A crisp z^{cg} is the average weight of the weight at each point z on domain B where $w_z = u_{\mathscr{B}}(z) / \int_B u_{\mathscr{B}}(z)\,dz$ for all $z \in B$; that is, the crisp output is $z^{cg} = \int_B z w_z\,dz = \int_B z u_{\mathscr{B}}(z)\,dz / \int_B u_{\mathscr{B}}(z)\,dz$. It is the investment weight of each individual stock in a particular industrial group. These weights are then used to rank stocks in an industrial group.

3.2. Step 2: Analysis of Industrial Groups. Industrial groups are ranked by weights calculated by the method of fuzzy multicriteria decision-making consisting of AHP, fuzzy analytic hierarchy process, and Fuzzy Technique for Order Preference by Similarity to Ideal Solution Method (FTOPSIS).

AHP is a method for calculating decision weights developed by Saaty [11] and Paul Yoon and Hwang [5]. It compares paired data that are metrics of real quantities such as price, weight, and preference. Here, these quantities are preferences. Levels of preferences are represented by numbers in a set $\Omega_n = \{1/n, 1/(n-1), \ldots, 1/3, 1/2, 1, 2, 3, \ldots, n-1, n\}$ expressed as a reciprocal matrix. Generalizing this idea, the set of crisp preference values Ω_n is replaced by a set of fuzzy preference values $\tilde{\Omega}_n^{\delta} = \{1/\tilde{n}_\delta, 1/\widetilde{(n-1)}_\delta, \ldots, 1/\tilde{3}_\delta, 1/\tilde{2}_\delta, 1, \tilde{2}_\delta, \tilde{3}_\delta, \ldots, \widetilde{(n-1)}_\delta, \tilde{n}_\delta\}$, where $\tilde{k}_\delta = (k-\delta, k, k+\delta)$ and $1/\tilde{k}_\delta = 1 \oslash \tilde{k}_\delta = (1/(k+\delta), 1/k, 1/(k-\delta))$ for all $k \in I_n$ and $0 \le \delta \le 1$.

The other technique, FTOPSIS developed by Chan [17] and Balli and Korukoglu [10], is a fuzzy technique for ranking preference levels by comparing the similarity of alternate choice to the ideal choice in order to find the best alternative. It covers diverse alternate choices, decision criteria, and decision makers.

Applying this technique to n_1 decision makers, n_2 decision criteria, and n_3 industrial groups as alternate choices, the analysis steps are as follows.

Step 2.1 (finding weights for decision makers). In this step, a decision maker i, $i = 1, \ldots, n_1$, is compared to another decision maker j in terms of their preference level based on a preference function $\varphi(i, j)$ defined as

$$\varphi(i,j) = \begin{cases} \tilde{c}_{ij}, \ \exists \tilde{c}_{ij} \in \tilde{\Omega}_n; & j > i \\ 1; & j = i \\ 1 \oslash \varphi(j,i); & j < i. \end{cases} \tag{15}$$

The decision maker's preference matrix $\tilde{D} = (\tilde{a}_{ij})_{n_1 \times n_1}$ is a reciprocal matrix where

$$\tilde{a}_{ij} = \begin{cases} \varphi(i,j); & i < j \\ 1; & i = j \\ 1 \oslash \varphi(j,i); & i > j. \end{cases} \tag{16}$$

Step 2.2 (finding a fuzzy weight vector $\tilde{w}_d = (\tilde{w}_{d,k})_{n_1 \times 1}$ for $\tilde{D} = (\tilde{a}_{ij})_{n_1 \times n_1}$). $\tilde{w}_{d,k} = (w_{d,k}^L, w_{d,k}^M, w_{d,k}^U)$ is a fuzzy weight vector for all $k \in I_{n_1}$ where

$$\begin{aligned} w_{d,k}^L &= C_L \cdot \frac{\left(\prod_{j=1}^{n_1} a_{kj}^L\right)^{1/n_1}}{\sum_{i=1}^{n_1}\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}}, \\ w_{d,k}^M &= \frac{\left(\prod_{j=1}^{n_1} a_{kj}^M\right)^{1/n_1}}{\sum_{i=1}^{n_1}\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}}, \\ w_{d,k}^U &= C_U \cdot \frac{\left(\prod_{j=1}^{n_1} a_{kj}^U\right)^{1/n_1}}{\sum_{i=1}^{n_1}\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}} \end{aligned} \tag{17}$$

with

$$\begin{aligned} C_L &= \min_{i \in I_{n_1}} \left\{ \frac{\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}}{\left(\prod_{j=1}^{n_1} a_{ij}^L\right)^{1/n_1}} \right\}, \\ C_U &= \max_{i \in I_{n_1}} \left\{ \frac{\left(\prod_{j=1}^{n_1} a_{ij}^M\right)^{1/n_1}}{\left(\prod_{j=1}^{n_1} a_{ij}^U\right)^{1/n_1}} \right\}. \end{aligned} \tag{18}$$

If its consistency index $I_{n_1}^a(\tilde{D})$ as defined in Definition 15 is less than 0.1, it is accepted as being valid. Otherwise, the decision maker's weight is reevaluated by repeating Step 2.1.

Step 2.3. This step involves decision makers $d_1, d_2, \ldots, d_{n_1}$ constructing decision criteria $c_1, c_2, \ldots, c_{n_2}$ for evaluating industrial groups $G_1, G_2, \ldots, G_{n_3}$, where c_i, $i = 1, \ldots, n_2$, is constructed from investment weight of n_3 individual groups given by decision makers in the term of linguistic terms (see Table 1).

The decision criteria constructed are in the form of a fuzzy matrix $\widetilde{B}$ with members $b_{jik} = (b_{jik}^{L}, b_{jik}^{M_1}, b_{jik}^{M_2}, b_{jik}^{U})$, $j \in I_{n_3}$, $i \in I_{n_2}$, and $k \in I_{n_1}$, which are trapezoidal fuzzy numbers representing the linguistic terms of $c_1, c_2, \ldots, c_{n_2}$ shown in (19).

Decision Criteria for Evaluating Industrial Groups G_1, $G_2, \ldots, G_{n_3}$. Consider

$$
\begin{array}{cc|cccc}
 & & d_1 & d_2 & \cdots & d_{n_1} \\
 & G_1 & \widetilde{b}_{111} & \widetilde{b}_{112} & \cdots & \widetilde{b}_{11n_1} \\
c_1 & G_2 & \widetilde{b}_{211} & \widetilde{b}_{212} & \cdots & \widetilde{b}_{21n_1} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & G_{n_3} & \widetilde{b}_{n_3 11} & \widetilde{b}_{n_3 12} & \cdots & \widetilde{b}_{n_3 1n_1} \\
 & G_1 & \widetilde{b}_{121} & \widetilde{b}_{122} & \cdots & \widetilde{b}_{12n_1} \\
c_2 & G_2 & \widetilde{b}_{221} & \widetilde{b}_{222} & \cdots & \widetilde{b}_{22n_1} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & G_{n_3} & \widetilde{b}_{n_3 21} & \widetilde{b}_{n_3 22} & \cdots & \widetilde{b}_{n_3 2n_1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & G_1 & \widetilde{b}_{1n_2 1} & \widetilde{b}_{1n_2 2} & \cdots & \widetilde{b}_{1n_2 n_1} \\
c_{n_2} & G_2 & \widetilde{b}_{2n_2 1} & \widetilde{b}_{2n_2 2} & \cdots & \widetilde{b}_{2n_2 n_1} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & G_{n_3} & \widetilde{b}_{n_3 n_2 1} & \widetilde{b}_{n_3 n_2 2} & \cdots & \widetilde{b}_{n_3 n_2 n_1}
\end{array}
\boxed{= \widetilde{B}} \qquad (19)
$$

Step 2.4. This step involves decision makers $d_1, d_2, \ldots, d_{n_1}$ evaluating decision criteria $c_1, c_2, \ldots, c_{n_2}$ constructing from the linguistic terms VL, L, ML, M, MH, H, VH as in Step 2.3. A fuzzy matrix $\widetilde{C} = (\widetilde{c}_{ij})_{n_2 \times n_1}$ for evaluation is then obtained where $\widetilde{c}_{ij} \in \{\mathrm{VL, L, ML, M, MH, H, VH}\}$ for all $i \in I_{n_2}$ and $j \in I_{n_1}$ as shown in (20).

Evaluation of Decision Criteria $c_1, c_2, \ldots, c_{n_2}$. Consider

$$
\begin{array}{c|cccc}
 & d_1 & d_2 & \cdots & d_{n_1} \\
c_1 & \widetilde{c}_{11} & \widetilde{c}_{12} & \cdots & \widetilde{c}_{1n_1} \\
c_2 & \widetilde{c}_{21} & \widetilde{c}_{22} & \cdots & \widetilde{c}_{2n_1} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
c_{n_2} & \widetilde{c}_{n_2 1} & \widetilde{c}_{n_2 2} & \cdots & \widetilde{c}_{n_2 n_1}
\end{array}
\boxed{= \widetilde{C}} \qquad (20)
$$

Step 2.5. This step involves calculating decision criteria based on decision makers' weights by multiplying the decision criterion of a decision maker in each column in Step 2.4 (depicted in (20)) with the corresponding decision maker's fuzzy weight vector $\overline{w}_d = (\overline{w}_{d,k})_{n \times 1}$, where $\overline{w}_{d,k} = (w_{d,k}^{L}, w_{d,k}^{M}, w_{d,k}^{U}) = (w_{d,k}^{L}, w_{d,k}^{M}, w_{d,k}^{M}, w_{d,k}^{U})$ calculated from Step 2.2. Equation (21) shows these multiplication results.

Decision Criteria Based on Weights of Decision Makers. Consider

$$
\begin{array}{c|cccc}
 & d_1 & d_2 & \cdots & d_{n_1} \\
c_1 & \widetilde{c}_{11} \otimes \overline{w}_{d,1} & \widetilde{c}_{12} \otimes \overline{w}_{d,2} & \cdots & \widetilde{c}_{1n_1} \otimes \overline{w}_{d,n_1} \\
c_2 & \widetilde{c}_{21} \otimes \overline{w}_{d,1} & \widetilde{c}_{22} \otimes \overline{w}_{d,2} & \cdots & \widetilde{c}_{2n_1} \otimes \overline{w}_{d,n_1} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
c_{n_2} & \widetilde{c}_{n_2 1} \otimes \overline{w}_{d,1} & \widetilde{c}_{n_2 2} \otimes \overline{w}_{d,2} & \cdots & \widetilde{c}_{n_2 n_1} \otimes \overline{w}_{d,n_1}
\end{array}
\boxed{= \widetilde{C}_w} \qquad (21)
$$

Next, we multiply the decision criterion for evaluating industrial groups in the column representing each decision maker constructed in Step 2.3 with the corresponding decision maker's fuzzy weight vector $\overline{w} = (\overline{w}_{d,k})_{n \times 1}$, where $\overline{w}_{d,k} = (w_{d,k}^{L}, w_{d,k}^{M}, w_{d,k}^{U}) = (w_{d,k}^{L}, w_{d,k}^{M}, w_{d,k}^{M}, w_{d,k}^{U})$ calculated from Step 2.2. The multiplication results are in (22).

Decision Criteria for Evaluating Industrial Groups Based on Weights of Decision Makers. Consider

$$
\begin{array}{cc|cccc}
 & & d_1 & d_2 & \cdots & d_{n_1} \\
 & G_1 & \widetilde{b}_{111} \otimes \overline{w}_{d,1} & \widetilde{b}_{112} \otimes \overline{w}_{d,2} & \cdots & \widetilde{b}_{11n_1} \otimes \overline{w}_{d,n_1} \\
c_1 & G_2 & \widetilde{b}_{211} \otimes \overline{w}_{d,1} & \widetilde{b}_{212} \otimes \overline{w}_{d,2} & \cdots & \widetilde{b}_{21n_1} \otimes \overline{w}_{d,n_1} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & G_{n_3} & \widetilde{b}_{n_3 11} \otimes \overline{w}_{d,1} & \widetilde{b}_{n_3 12} \otimes \overline{w}_{d,2} & \cdots & \widetilde{b}_{n_3 1n_1} \otimes \overline{w}_{d,n_1} \\
 & G_1 & \widetilde{b}_{121} \otimes \overline{w}_{d,1} & \widetilde{b}_{122} \otimes \overline{w}_{d,2} & \cdots & \widetilde{b}_{12n_1} \otimes \overline{w}_{d,n_1} \\
c_2 & G_2 & \widetilde{b}_{221} \otimes \overline{w}_{d,1} & \widetilde{b}_{222} \otimes \overline{w}_{d,2} & \cdots & \widetilde{b}_{22n_1} \otimes \overline{w}_{d,n_1} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & G_{n_3} & \widetilde{b}_{n_3 21} \otimes \overline{w}_{d,1} & \widetilde{b}_{n_3 22} \otimes \overline{w}_{d,2} & \cdots & \widetilde{b}_{n_3 2n_1} \otimes \overline{w}_{d,n_1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & G_1 & \widetilde{b}_{1n_2 1} \otimes \overline{w}_{d,1} & \widetilde{b}_{1n_2 2} \otimes \overline{w}_{d,2} & \cdots & \widetilde{b}_{1n_2 n_1} \otimes \overline{w}_{d,n_1} \\
c_{n_2} & G_2 & \widetilde{b}_{2n_2 1} \otimes \overline{w}_{d,1} & \widetilde{b}_{2n_2 2} \otimes \overline{w}_{d,2} & \cdots & \widetilde{b}_{2n_2 n_1} \otimes \overline{w}_{d,n_1} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & G_{n_3} & \widetilde{b}_{n_3 n_2 1} \otimes \overline{w}_{d,1} & \widetilde{b}_{n_3 n_2 2} \otimes \overline{w}_{d,2} & \cdots & \widetilde{b}_{n_3 n_2 n_1} \otimes \overline{w}_{d,n_1}
\end{array}
\boxed{= \widetilde{B}_w} \qquad (22)
$$

Table 1

Linguistic term	Fuzzy number
Very low (VL)	(0, 0, 0.1, 0.2)
Low (L)	(0.1, 0.2, 0.3)
Medium low (ML)	(0.2, 0.3, 0.4)
Medium (M)	(0.3, 0.4, 0.6, 0.7)
Medium high (MH)	(0.6, 0.7, 0.8)
High (H)	(0.7, 0.8, 0.9)
Very high (VH)	(0.8, 0.9, 1, 1)

Step 2.6. This step involves aggregating weights of decision criteria based on the decision makers' weights as follows:

$$\widetilde{w}_{cj} = \left\langle w_{cj}^{L}, w_{cj}^{M_1}, w_{cj}^{M_2}, w_{cj}^{U} \right\rangle, \tag{23}$$

where $w_{cj}^{L} = \min_{k=1}^{n_1}\{c_{w,jk}^{L}\}$, $w_{cj}^{M_1} = (1/n_1)\sum_{k=1}^{n_1} c_{w,jk}^{M_1}$, $w_{cj}^{M_2} = (1/n_1)\sum_{k=1}^{n_1} c_{w,jk}^{M_2}$, $w_{cj}^{U} = \max_{k=1}^{n_1}\{c_{w,jk}^{U}\}$ for all $i \in I_{n_2}$, $\widetilde{C}_w = (\widetilde{c}_{w,jk})_{n_2 \times n_1}$, and n_1 is the number of decision makers. Equation (24) shows these aggregation results.

Weights of Decision Criteria $c_1, c_2, \ldots, c_{n_2}$. Consider

$$\begin{array}{cccccc} & c_1 & c_2 & \cdots & c_{n_2} \\ W_2 & \widetilde{w}_{c,1} & \widetilde{w}_{c,2} & \cdots & \widetilde{w}_{c,n_2} \end{array} \tag{24}$$

Next, we aggregate industrial groups based on the decision makers' weights (see (22)) by the following equations:

$$\widetilde{x}_{ji} = \left\langle x_{ji}^{L}, x_{ji}^{M_1}, x_{ji}^{M_2}, x_{ji}^{U} \right\rangle, \tag{25}$$

where $x_{ji}^{L} = \min_{k=1}^{n_1}\{b_{w,jik}^{L}\}$, $x_{ji}^{M_1} = (1/n_1)\sum_{k=1}^{n_1} b_{w,jik}^{M_1}$, $x_{ji}^{M_2} = (1/n_1)\sum_{k=1}^{n_1} b_{w,jik}^{M_2}$, $x_{ji}^{U} = \max_{k=1}^{n_1}\{b_{w,jik}^{U}\}$ for all $j \in n_3$, $i \in n_2$, $\widetilde{B}_w = (\widetilde{b}_{w,jik})_{n_3 n_2 \times n_1}$, and n_1 is the number of decision makers. These results are shown in (26).

Evaluation Matrix of Industrial Groups $G_1, G_2, \ldots, G_{n_3}$. Consider

$$\begin{array}{cccccc} & c_1 & c_2 & \cdots & c_{n_2} \\ G_1 & \widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1n_2} \\ G_2 & \widetilde{x}_{21} & \widetilde{x}_{22} & \cdots & \widetilde{x}_{2n_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{n_3} & \widetilde{x}_{n_3 1} & \widetilde{x}_{n_3 2} & \cdots & \widetilde{x}_{n_3 n_2} \end{array} \boxed{= \widetilde{X}} \tag{26}$$

Step 2.7. This step involves constructing a decision matrix by normalizing the industrial groups' evaluation matrix $\widetilde{X}$ (see (26)) as follows:

$$\widetilde{R} = \left(\widetilde{r}_{ji}\right)_{n_3 \times n_2},$$
$$\widetilde{r}_{ji} = \left\langle \frac{x_{ji}^{L}}{x_i^{*}}, \frac{x_{ji}^{M_1}}{x_i^{*}}, \frac{x_{ji}^{M_2}}{x_i^{*}}, \frac{x_{ji}^{U}}{x_i^{*}} \right\rangle \text{ where } x_i^{*} = \max_{j}^{n_3}\left\{x_{ji}^{U}\right\}. \tag{27}$$

Then, multiplying the normalized matrix with the decision weights from Step 2.6, $\widetilde{V} = (\widetilde{v}_{ji})_{n_3 \times n_2}$, where $\widetilde{v}_{ji} = (v_{ji}^{L}, v_{ji}^{M_1}, v_{ji}^{M_2}, v_{ji}^{U})$ and $\widetilde{v}_{ji} = \widetilde{r}_{ji} \otimes \widetilde{w}_{c,i}$ when $j \in I_{n_3}, i \in I_{n_2}$.

Industrial Groups' Evaluation Matrix. Consider

$$\begin{array}{cccccc} & c_1 & c_2 & \cdots & c_{n_2} \\ G_1 & \widetilde{v}_{11} & \widetilde{v}_{12} & \cdots & \widetilde{v}_{1n_2} \\ G_2 & \widetilde{v}_{21} & \widetilde{v}_{22} & \cdots & \widetilde{v}_{2n_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{n_3} & \widetilde{v}_{n_3 1} & \widetilde{v}_{n_3 2} & \cdots & \widetilde{v}_{n_3 n_2} \end{array} \boxed{= \widetilde{V}} \tag{28}$$

Step 2.8. This step involves defining positive ideal solution (G^{*}) and negative ideal solution (G^{-}) from (28) as $G^{*} = (\widetilde{v}_1^{*}, \widetilde{v}_2^{*}, \ldots, \widetilde{v}_{n_2}^{*})$ and $G^{-} = (\widetilde{v}_1^{-}, \widetilde{v}_2^{-}, \ldots, \widetilde{v}_{n_2}^{-})$, respectively, where $\widetilde{v}_i^{*} = \max_j^{n_3}\{v_{ji}^{U}\}$ and $\widetilde{v}_i^{-} = \min_j^{n_3}\{v_{ji}^{L}\}$, $j \in I_{n_3}$, $i \in I_{n_2}$, $\widetilde{V} = (\widetilde{v}_{ji})_{n_3 \times n_2}$.

Step 2.9. This step involves calculating the distances between the industrial groups' evaluation results with the positive and negative ideal solutions, as defined by the following:

$$d_j^{*} = \sum_{i=1}^{n_2} d_v\left(\widetilde{v}_{ji}, \widetilde{v}_i^{*}\right), \quad j \in I_{n_3},$$
$$d_j^{-} = \sum_{i=1}^{n_2} d_v\left(\widetilde{v}_{ji}, \widetilde{v}_i^{-}\right), \quad j \in I_{n_3}, \tag{29}$$

where $d_v(\widetilde{v}_{ji}, \widetilde{v}_i^{*,-})$ are calculated in the same way as fuzzy numbers are calculated according to Definition 8 (depicted in (30)).

Distances between the Industrial Groups' Evaluation Results and Positive and Negative Ideal Solutions G^{*} and G^{-}. Consider

$$\begin{array}{cccccc} & c_1 & c_2 & \cdots & c_{n_2} & d_j^{*,-} = \sum_{i=1}^{n_2} d_v\left(\widetilde{v}_{ji}, \widetilde{v}_j^{*,-}\right) \\ G_1 & d_v\left(\widetilde{v}_{11}, \widetilde{v}_1^{*,-}\right) & d_v\left(\widetilde{v}_{12}, \widetilde{v}_2^{*,-}\right) & \cdots & d_v\left(\widetilde{v}_{1n_2}, \widetilde{v}_{n_2}^{*,-}\right) & d_1^{*,-} \\ G_2 & d_v\left(\widetilde{v}_{21}, \widetilde{v}_1^{*,-}\right) & d_v\left(\widetilde{v}_{22}, \widetilde{v}_2^{*,-}\right) & \cdots & d_v\left(\widetilde{v}_{2n_2}, \widetilde{v}_{2n_2}^{*,-}\right) & d_2^{*,-} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{n_3} & d_v\left(\widetilde{v}_{n_3 1}, \widetilde{v}_1^{*,-}\right) & d_v\left(\widetilde{v}_{n_3 2}, \widetilde{v}_2^{*,-}\right) & \cdots & d_v\left(\widetilde{v}_{n_3 n_2}, \widetilde{v}_{n_2}^{*,-}\right) & d_{n_3}^{*,-} \end{array} \tag{30}$$

Table 2: E/P of STPI.

STPI stock	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)		15.7	62.75	28.75	27
Number of common stocks		369360995	368,492,092	367,873,233	367,546,097
Number of preferred stocks		0	0	0	0
Number of treasury stocks		0	0	0	0
Latest 12-month profit		1908520000	1089760000	399510000	2021430000
P/E	14.8500	3.0385	21.2183	26.4733	4.9093
E/P	0.0673	0.3291	0.0471	0.0378	0.2037
E/P (weighted average)	0.1383				
E/P (% weighted average)	13.83				

Step 2.10. This step involves calculating the nearness coefficients to the positive ideal solution, CC_j, and ranking the industrial groups according to them. CC_j are defined as follows:

$$CC_j = \frac{d_j^-}{d_j^- + d_j^*}, \quad j \in I_m. \tag{31}$$

From the calculation, a set of investment weights for industrial groups, $W_3 = (w_1, w_2, \ldots, w_{n_i})$, where $w_1, w_2, \ldots, w_{n_i}$ are weights of individual groups, is obtained. The industrial group of which investment weight value is nearest to one (the closest to the positive ideal solution) is the best industrial group.

3.3. Step 3: Analysis of All Stocks from Different Industrial Groups. In this step, the Correlation-Product Implication is used; the two investment weights from Steps 1 and 2 are used to calculate the integrated final investment weights for all of the stocks in the market, denoted as $W_{OA}(s_{ij})$, where $W_{OA}(s_{ij}) = W_1(s_{ij}) \cdot W_2(G_j)$ and $W_1(s_{ij})$ are the weight of the ith stock from the jth group from Step 1 and $W_2(G_j)$ is the weight of the jth group from Step 2. These weights are then used to rank the stocks for making decisions and planning out strategies.

4. Application of the Analysis Procedures to a Demonstration Case

As a demonstration of the applicability of our analysis procedures, a simulated case of stock selection into a portfolio for a given period of time was conducted. Suppose that the 6 industrial groups of investment interest were the following: agricultural and food industry (G_1), consumer product and service industry (G_2), financial industry (G_3), industrial product and technology industry (G_4), property and construction industry (G_5), and resource industry (G_6). Stocks from each individual industry were analyzed as follows.

Step 1 (analysis of stocks in an industrial group). As an example, the analysis of the property and construction industry, G_5, is shown below.

In this group, G_5, we use the past 5-year financial fact data of the companies from Stock Exchange of Thailand, 2010-2014, http://www.settrade.com/.

Step 1.1. This step involves gathering the past 5-year financial data of the companies in this group and screening in stocks with complete data from 12 companies: CK, CNT, ITD, NWR, PREB, SEAFCO, STEC, STPI, SYNTEC, TRC, TTCL, and UNIQ.

Step 1.2. This step involves calculating the E/P, P/BV, and P/P_n values of each individual stock.

Step 1.3. This step involves calculating the following weighted arithmetic mean of E/P, P/BV, and P/P_n. Tables 2, 3, and 4 show data of some stock (STPI), and Table 5 shows the weighted arithmetic mean of each individual stock in G_5.

Step 1.4. This step involves an expert constructing a fuzzy set based on the latest 5-year financial data of which linguistic terms are represented by trapezoidal and triangular fuzzy numbers.

Values of E/P, P/BV, and P/P_n were grouped into 3 levels: low (L), medium (M), and high (H), and so the fuzzy sets representing these levels were

$$\begin{aligned} L &= \left\langle l^L, l^{M_1}, l^{M_2}, l^U \right\rangle, \\ M &= \left\langle m^L, m^{M_1}, m^{M_2}, m^U \right\rangle, \\ H &= \left\langle h^L, h^{M_1}, h^{M_2}, h^U \right\rangle. \end{aligned} \tag{32}$$

The fuzzy sets of linguistic terms were as follows:

$E/P \Rightarrow LX = \langle 0, 0, 1, 3 \rangle$, $MX = \langle 1, 3, 8, 10 \rangle$, $HX = \langle 8, 10, 100, 100 \rangle$.

$P/BV \Rightarrow LY = \langle 0, 0, 5, 7 \rangle$, $MY = \langle 5, 7, 10, 16 \rangle$, $HY = \langle 10, 16, 100, 100 \rangle$.

$P/P_n \Rightarrow LZ = \langle 0, 0, 1, 1.1 \rangle$, $MZ = \langle 1, 1.1, 1.9, 2.3 \rangle$, $HZ = \langle 1.9, 2.3, 100, 100 \rangle$.

Step 1.5. This step involves an expert constructing fuzzy rules from the fuzzy sets constructed from Step 1.4 as follows:

Table 3: P/BV of STPI.

STPI stock	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	15.7	62.75	28.75	27
Number of common stocks	1477443980	368,492,092	367,873,233	367,546,097
Number of preferred stocks	0	0	0	0
Total assets	10867008638	7347262706	3522893354	4259624240
Total liabilities	4956210154	2922198628	423972604	1021904292
Accounting value per share	4.000692117	12.00857271	8.42388212	8.809017357
P/BV	3.924320978	5.225433658	3.412915754	3.065041072
P/BV of 2014 (2nd quarter)	4.8			
P/BV (weighted average)	4.350963831			
P/BV (highest)	25.18861616			
P/BV	17.27353263			

Table 4: P/P_n of STPI.

STPI stock	14/10/2014	27/12/2013	28/12/2012	30/12/2011	30/12/2010
Closing price of common stock (baht)	20.8	15.7	62.75	28.75	27
Dividend interest rate (%)	1.63	1.59	0.5	12.16	7.86
Dividend amount (baht)	0.339	0.2496	0.3138	3.496	2.1222
Expected interest (r)	0.0703	0.0707	0.0728	0.0750	0.0641
Baht gained from 1 baht investment ($1+r$)	1.0703	1.0707	1.0728	1.0750	1.0641
Target price in 2014	29.3056				
Closing price to target price ratio	0.7098				

Table 5: E/P, P/BV, and P/P_n of stocks in G_1.

Financial ratio	CK	CNT	ITD	NWR	PREB	SEAFCO	STEC	STPI	SYNTEC	TRC	TTCL	UNIQ
E/P (%)	10.86	7.86	0.89	6.14	10.5	6.38	5.59	13.83	3.26	7.98	4.2	7.66
P/BV	8.71	9.1	7.35	4.73	8.19	7.19	16.06	17.27	3.8	8.99	16.19	8.3
P/P_n	2.43	1.12	0.94	2.38	2.94	0.97	1.67	0.71	1.86	0.83	2.87	2.4

Rule 1: if x was LX and y was LY and z was LZ then w was RHW.

Rule 2: if x was LX and y was LY and z was MZ then w was MW.

⋮

Rule 27: if x was HX and y was HY and z was HZ then w was RLW.

Step 1.6. This step involves importing the values of current P/E (inversing to E/P), P/BV, and P/P_n, which, in this study, were the values of the 22nd of January 2015 shown in Table 6.

Note. The E/Ps of CNT and NWR were not applicable, meaning that they suffered a loss, so they were not included in further calculation.

Step 1.7. This step involves performing defuzzification of the fuzzy output values to crisp values with the centroid method, obtaining the investment weights shown in Table 7.

For the purpose of easy demonstration, the investment weights of the stocks from the other 5 industrial groups were made up. All of the weights are tabulated in Table 8.

Step 2 (analysis of industrial groups). Stocks from 6 industrial groups, $G_1, G_2, \ldots, G_6$, were analyzed. Three decision makers, d_1, d_2, d_3 constructed 4 decision criteria, c_1, c_2, c_3, c_4, calculated in the following steps.

TABLE 6: Financial ratios of the 22nd January 2015, http://www.settrade.com/.

Financial ratio	CK	CNT	ITD	NWR	PREB	SEAFCO	STEC	STPI	SYNTEC	TRC	TTCL	UNIQ
E/P (%)	4.76	N.A	2.14	N.A	5.76	5.29	4.71	8.23	4.84	5.47	3.58	3.51
P/BV	2.54	2.36	3.66	1.69	3.41	3.88	4.83	4.09	1.82	3.46	3.04	3.73
P/P_s	2.55	0.91	1.34	2.34	4.29	1.79	1.55	0.66	2.37	1.08	2.83	3.12

TABLE 7: Investment weights from the analysis procedures.

Stock	CK	ITD	PREB	SEAFCO	STEC	STPI	SYNTEC	TRC	TTCL	UNIQ
Investment weights	0.084	0.105	0.084	0.1091	0.1091	0.1435	0.084	0.113	0.084	0.084

TABLE 8: Investment weights of all stocks: the ones for G_5 were actually calculated while the rest were made up.

G_1		G_2		G_3		G_4		G_5		G_6	
s_{11}	0.0418	s_{12}	0.26	s_{13}	0.1276	s_{14}	0.0518	s_{15} (CK)	0.084	s_{16}	0.0261
s_{21}	0.024	s_{22}	0.169	s_{23}	0.1528	s_{24}	0.1077	s_{25} (ITD)	0.105	s_{26}	0.1258
s_{31}	0.1148	s_{32}	0.1359	s_{33}	0.0282	s_{34}	0.1745	s_{35} (PREB)	0.084	s_{36}	0.0667
s_{41}	0.1704	s_{42}	0.1006	s_{43}	0.0843	s_{44}	0.0528	s_{45} (SEAFCO)	0.1091	s_{46}	0.2034
s_{51}	0.1003	s_{52}	0.004	s_{53}	0.0822	s_{54}	0.1108	s_{55} (STEC)	0.1091	s_{56}	0.0315
s_{61}	0.097	s_{62}	0.1376	s_{63}	0.0841	s_{64}	0.1399	s_{65} (STPI)	0.1435	s_{66}	0.1576
s_{71}	0.0764	s_{72}	0.1825	s_{73}	0.0335	s_{74}	0.0916	s_{75} (SYNTEC)	0.084	s_{76}	0.2068
s_{81}	0.0705	s_{82}	0.0104	s_{83}	0.0421	s_{84}	0.1099	s_{85} (TRC)	0.113	s_{86}	0.0638
s_{91}	0.1484			s_{93}	0.211	s_{94}	0.0825	s_{95} (TTCL)	0.084	s_{96}	0.1215
$s_{10,1}$	0.1563			$s_{10,3}$	0.2517	$s_{10,4}$	0.0986	$s_{10,5}$ (UNIQ)	0.084		

Step 2.1. This step involves calculating the weights for decision makers. The preference level of the ith decision maker was compared to that of the jth decision maker with a scale $[1/9, 9]$, obtaining

$$\widetilde{A} = \begin{pmatrix} (1,1,1) & (1,2,3) & (2,3,4) \\ \left(\frac{1}{3},\frac{1}{2},1\right) & (1,1,1) & (1,2,3) \\ \left(\frac{1}{4},\frac{1}{3},\frac{1}{2}\right) & \left(\frac{1}{3},\frac{1}{2},1\right) & (1,1,1) \end{pmatrix}. \quad (33)$$

Step 2.2. This step involves calculating the fuzzy weight vectors, $\widetilde{\omega}_{d} = (\widetilde{\omega}_{d,k})_{3\times1}$, for $\widetilde{D} = (\widetilde{a}_{ij})_{3\times3}$, and obtaining the following respective vectors for decision makers d_1, d_2, d_3: $\widetilde{\omega}_{d,1} = (0.47165, 0.53991, 0.53991, 0.53991)$, $\widetilde{\omega}_{d,2} = (0.25869, 0.29712, 0.29712, 0.34012)$, and $\widetilde{\omega}_{d,3} = (0.16296, 0.16296, 0.16296, 0.18717)$, and a consistency index $I_3^g(\widetilde{A}) = 0.09403$.

Step 2.3. This step involves the 3 decision makers, d_1, d_2, d_3 evaluating 6 industrial groups, $G_1, G_2, \ldots, G_6$, according to the decision criteria c_1, c_2, c_3, c_4 utilizing linguistic terms VL, L, ML, M, MH, H, VH represented by trapezoidal fuzzy numbers as in Table 9.

Step 2.4. This step involves decision makers d_1, d_2, d_3 evaluating the decision criteria c_1, c_2, c_3, c_4 utilizing the linguistic terms VL, L, ML, M, MH, H, VH represented by the mentioned trapezoidal fuzzy numbers as in Table 10.

Step 2.5. This step involves calculating fuzzy decision criteria and the evaluation criteria for industrial groups based on the weights of decision makers as in Tables 11 and 12, respectively.

Step 2.6. This step involves aggregating the decision criteria and the fuzzy evaluation of industrial groups based on the weights of decision makers. The aggregation results are shown in Tables 13 and 14.

Step 2.7. This step involves normalizing the weights of industrial groups for each decision criteria shown in Table 13 and then multiplying the normalized matrix with the weights of decision criteria from Step 2.6, defined by $\widetilde{V} = (\widetilde{v}_{ji})_{6\times4}$, where $\widetilde{v}_{ji} = (v_{ji}^{L}, v_{ji}^{M_1}, v_{ji}^{M_2}, v_{ji}^{U})$ and $\widetilde{v}_{ji} = \widetilde{r}_{ji} \otimes \widetilde{\omega}_{c,i}$, when $j \in 1, 2, \ldots, 6$, $i \in 1, 2, \ldots, 4$, to obtain a decision matrix shown in Table 15.

Step 2.8. This step involves stipulating a positive ideal solution (S^{+}) and a negative ideal solution (S^{-}) to be

$$S^{+} = [(0.54, 0.54, 0.54, 0.54), (0.486, 0.486, 0.486, 0.486), (0.54, 0.54, 0.54, 0.54), (0.432, 0.432, 0.432, 0.432)],$$

$$S^{-} = [(0.021, 0.021, 0.021, 0.021), (0.021, 0.021, 0.021, 0.021),$$

Table 9: Trapezoidal fuzzy numbers representing linguistic terms used for fuzzy evaluation of industrial groups.

Criteria	Industrial group	Decision maker											
		d_1				d_2				d_3			
c_1	G_1	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.8	0.9	1	1
	G_2	0.8	0.9	1	1	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_3	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_4	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
	G_5	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8
	G_6	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8
c_2	G_1	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_2	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8
	G_3	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
	G_4	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_5	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9
	G_6	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
c_3	G_1	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_2	0.8	0.9	1	1	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_3	0.8	0.9	1	1	0.8	0.9	1	1	0.7	0.8	0.8	0.9
	G_4	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8
	G_5	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9
	G_6	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9
c_4	G_1	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.8
	G_2	0.6	0.7	0.7	0.8	0.8	0.9	1	1	0.7	0.8	0.8	0.9
	G_3	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_4	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
	G_5	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
	G_6	0.7	0.8	0.8	0.9	0.6	0.7	0.7	0.8	0.7	0.8	0.8	0.9

Table 10: Evaluation of fuzzy decision criteria.

Criteria	Decision maker											
	d_1				d_2				d_3			
c_1	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
c_2	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9	0.7	0.8	0.8	0.9
c_3	0.8	0.9	1	1	0.8	0.9	1	1	0.8	0.9	1	1
c_4	0.5	0.6	0.7	0.8	0.8	0.9	1	1	0.7	0.8	0.8	0.9

$$(0.024, 0.024, 0.024, 0.024),$$
$$(0.021, 0.021, 0.021, 0.021)]. \quad (34)$$

Step 2.9. This step involves calculating the distances from the results of industrial groups evaluation in Table 14 to the (S^*) and the (S^-) ideal solutions shown in Tables 16 and 17, respectively.

Step 2.10. This step involves obtaining the nearness coefficients CC_j, $j = 1, \ldots, 6$, to the positive ideal solution and the investment weights shown in Table 18.

Step 3 (analysis of all stocks from different industrial groups). The two kinds of investment weights obtained from Steps 1 and 2 were used to calculate the final investment weights for all of the stocks in the market, $W_{OA}(s_{ij})$, where i represents the ith company and j the jth industrial group, and the final weights were ranked as shown in Table 19.

From Table 19, investors can use the calculated weights to help with their decision-making and strategy-planning. The better stocks to invest in show higher final investment weights.

5. Conclusions

The innovation appearing in this paper is to present the tactic of conveying the stock selection to portfolio by using two tactics, fuzzy quantitative analysis and fuzzy hierarchical analysis. The two tactics imply the final investment weight. Investors can determine their strategies by using the final investment weights. The final investment weights may be

Table 11: Fuzzy decision criteria.

Criteria	Decision maker d_1				d_2				d_3			
c_1	0.3773	0.4859	0.5399	0.5399	0.207	0.2674	0.2971	0.3401	0.1304	0.1467	0.163	0.1872
c_2	0.3302	0.4319	0.4319	0.4859	0.1811	0.2377	0.2377	0.3061	0.1141	0.1304	0.1304	0.1685
c_3	0.3773	0.4859	0.5399	0.5399	0.207	0.2674	0.2971	0.3401	0.1304	0.1467	0.163	0.1872
c_4	0.2358	0.3239	0.3779	0.4319	0.207	0.2674	0.2971	0.3401	0.1141	0.1304	0.1304	0.1685

Table 12: Fuzzy evaluation of industrial groups.

Criteria	Industrial group	Decision maker d_1				d_2				d_3			
c_1	G_1	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.13	0.147	0.163	0.187
	G_2	0.377	0.486	0.54	0.54	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_3	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_4	0.377	0.486	0.54	0.54	0.207	0.267	0.297	0.34	0.13	0.147	0.163	0.187
	G_5	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.098	0.114	0.114	0.15
	G_6	0.283	0.378	0.378	0.432	0.181	0.238	0.238	0.306	0.098	0.114	0.114	0.15
c_2	G_1	0.283	0.378	0.378	0.432	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_2	0.33	0.432	0.432	0.486	0.155	0.208	0.208	0.272	0.098	0.114	0.114	0.15
	G_3	0.377	0.486	0.54	0.54	0.207	0.267	0.297	0.34	0.13	0.147	0.163	0.187
	G_4	0.283	0.378	0.378	0.432	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_5	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.114	0.13	0.13	0.168
	G_6	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
c_3	G_1	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_2	0.377	0.486	0.54	0.54	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_3	0.377	0.486	0.54	0.54	0.207	0.267	0.297	0.34	0.114	0.13	0.13	0.168
	G_4	0.33	0.432	0.432	0.486	0.155	0.208	0.208	0.272	0.098	0.114	0.114	0.15
	G_5	0.33	0.432	0.432	0.486	0.155	0.208	0.208	0.272	0.114	0.13	0.13	0.168
	G_6	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.114	0.13	0.13	0.168
c_4	G_1	0.283	0.378	0.378	0.432	0.155	0.208	0.208	0.272	0.098	0.114	0.114	0.15
	G_2	0.283	0.378	0.378	0.432	0.207	0.267	0.297	0.34	0.114	0.13	0.13	0.168
	G_3	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_4	0.377	0.486	0.54	0.54	0.207	0.267	0.297	0.34	0.13	0.147	0.163	0.187
	G_5	0.33	0.432	0.432	0.486	0.181	0.238	0.238	0.306	0.114	0.13	0.13	0.168
	G_6	0.33	0.432	0.432	0.486	0.155	0.208	0.208	0.272	0.114	0.13	0.13	0.168

Table 13: Aggregation of decision criteria.

	Criteria c_1				c_2				c_3				c_4			
Weight	0.13	0.3	0.333	0.54	0.114	0.267	0.267	0.486	0.13	0.3	0.333	0.54	0.114	0.241	0.268	0.432

Table 14: Aggregation of evaluation of industrial groups.

Group	Criteria c_1				c_2				c_3				c_4			
G_1	0.13	0.244	0.25	0.432	0.114	0.249	0.249	0.432	0.114	0.267	0.267	0.486	0.098	0.233	0.233	0.432
G_2	0.114	0.285	0.303	0.54	0.098	0.251	0.251	0.486	0.114	0.285	0.303	0.54	0.114	0.259	0.268	0.432
G_3	0.114	0.267	0.267	0.486	0.13	0.3	0.333	0.54	0.114	0.295	0.322	0.54	0.114	0.267	0.267	0.486
G_4	0.13	0.3	0.333	0.54	0.114	0.249	0.249	0.432	0.098	0.251	0.251	0.486	0.13	0.3	0.333	0.54
G_5	0.098	0.233	0.233	0.432	0.114	0.239	0.239	0.432	0.114	0.257	0.257	0.486	0.114	0.267	0.267	0.486
G_6	0.098	0.243	0.243	0.432	0.114	0.267	0.267	0.486	0.114	0.239	0.239	0.432	0.114	0.257	0.257	0.486

Table 15: Decision matrix.

Group	Criteria															
	c_1				c_2				c_3				c_4			
G_1	0.031	0.136	0.154	0.432	0.024	0.123	0.123	0.389	0.028	0.148	0.148	0.486	0.021	0.104	0.116	0.346
G_2	0.028	0.158	0.187	0.54	0.021	0.124	0.124	0.437	0.028	0.158	0.168	0.54	0.024	0.115	0.134	0.346
G_3	0.028	0.148	0.165	0.486	0.028	0.148	0.165	0.486	0.028	0.164	0.179	0.54	0.024	0.119	0.133	0.389
G_4	0.031	0.167	0.206	0.54	0.024	0.123	0.123	0.389	0.024	0.14	0.14	0.486	0.028	0.134	0.166	0.432
G_5	0.024	0.13	0.144	0.432	0.024	0.118	0.118	0.389	0.028	0.143	0.143	0.486	0.024	0.119	0.133	0.389
G_6	0.024	0.135	0.15	0.432	0.024	0.132	0.132	0.437	0.028	0.133	0.133	0.432	0.024	0.114	0.128	0.389

Table 16: Distances between G_j, $j = 1, \ldots, 6$, and S^* for each decision criterion.

Distance	Criteria				Sum
	c_1	c_2	c_3	c_4	
$d_1^* = d_v(G_1, S^*)$	0.381572	0.348712	0.378268	0.309819	1.418371
$d_2^* = d_v(G_2, S^*)$	0.364995	0.346628	0.369604	0.301308	1.382533
$d_3^* = d_v(G_3, S^*)$	0.374072	0.326879	0.365443	0.298251	1.364645
$d_4^* = d_v(G_4, S^*)$	0.356869	0.348712	0.384022	0.284309	1.373911
$d_5^* = d_v(G_5, S^*)$	0.388341	0.351266	0.381127	0.298251	1.418985
$d_6^* = d_v(G_6, S^*)$	0.38534	0.341527	0.389187	0.300654	1.416708

Table 17: Distances between G_j, $j = 1, \ldots, 6$, and S^- for each decision criterion.

Distance	Criteria				Sum
	c_1	c_2	c_3	c_4	
$d_1^- = d_v(G_1, S^-)$	0.223775	0.197715	0.247374	0.174345	0.843208
$d_2^- = d_v(G_2, S^-)$	0.281156	0.220808	0.276399	0.17835	0.956716
$d_3^- = d_v(G_3, S^-)$	0.251745	0.251745	0.278567	0.198531	0.980589
$d_4^- = d_v(G_4, S^-)$	0.285192	0.197715	0.245285	0.225285	0.953478
$d_5^- = d_v(G_5, S^-)$	0.221504	0.196478	0.246015	0.198531	0.862528
$d_6^- = d_v(G_6, S^-)$	0.223065	0.222647	0.218246	0.197315	0.861274

Table 18: Nearness coefficients to the positive ideal solution.

Industrial group	G_1	G_2	G_3	G_4	G_5	G_6
$CC_j = \frac{d_j^-}{d_j^- + d_j^*}$	0.304297	0.38056	0.392965	0.380328	0.318558	0.315015
Weights	0.157599	0.172877	0.176738	0.173169	0.159816	0.159816

Table 19: The final investment weights of all of the stocks in the market.

s_{ij}	s_{12}	s_{102}	s_{91}	s_{76}	s_{16}	s_{72}	s_{34}	s_{22}	s_{23}	s_{41}	s_{66}	s_{101}	s_{64}
$W_{GA}(s_{ij})$	0.0473	0.0472	0.0396	0.0332	0.0317	0.03114	0.0307	0.03063	0.0287	0.02543	0.02503	0.02478	0.0247
s_{ij}	s_{62}	s_{52}	s_{61}	s_{65}	s_{15}	s_{26}	s_{40}	s_{54}	s_{44}	s_{18}	s_{31}	s_{81}	s_{55}
$W_{GA}(s_{ij})$	0.0237	0.0227	0.0218	0.0215	0.0201	0.01998	0.0195	0.01894	0.0183	0.01829	0.01792	0.01720	0.02373
s_{ij}	s_{45}	s_{42}	s_{104}	s_{28}	s_{74}	s_{61}	s_{51}	s_{43}	s_{63}	s_{53}	s_{94}	s_{13}	s_{35}
$W_{GA}(s_{ij})$	0.0167	0.0166	0.0166	0.0166	0.0159	0.01583	0.0157	0.01544	0.015	0.01459	0.01411	0.01279	0.01279
s_{ij}	s_{75}	s_{93}	s_{103}	s_{71}	s_{31}	s_{36}	s_{56}	s_{14}	s_{84}	s_{83}	s_{11}	s_{25}	s_{56}
$W_{GA}(s_{ij})$	0.0127	0.0127	0.0127	0.0111	0.0102	0.01004	0.0094	0.00960	0.0096	0.0079	0.00608	0.00608	0.00529
s_{ij}	s_{33}	s_{16}	s_{21}	s_{82}	s_{32}								
$W_{GA}(s_{ij})$	0.0047	0.0039	0.0034	0.0018	0.0007								

used to select stocks and allocate asset into portfolio. A case study presented in Table 19 shows that if we use the final investment weights as decision criteria to select stocks into portfolio, stock that has the highest weight is the most interesting and is chosen first. In contrast, stock that has the lowest weight is the least interesting and is chosen last. However, decision-making and strategy-planning of each investor may be different and depend on their financial risk tolerance. For example, some investors whose financial risk tolerance is high level maybe invest in only one stock with the highest final investment weights while some investors reduce risk by investing in many stocks with high final investment weights. You should keep in your mind that there is no best tool in the world for financial analysis but you can alter tools that fit for each situation. The purpose of this research is to construct the tool for financial analysis that may be an alternative for investors. At least, we hope that this research will help investors to make an appropriate decision.

For future work, we will improve our model and compare results with others in each situation. Moreover, the software of this model will also be provided.

Competing Interests

The authors declare that they have no competing interests.

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