NUMERICAL SIMULATIONS OF AIR POLLUTION MODELS IN URBAN STREET CANYONS

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หัวข้อวิทยานิพนธ์	การจำลองแบบเชิงตัวเลขของตัวแบบมลพิษทางอากาศในถนนที่มีอาคารสูงล้อมรอบในเขตเมือง
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บทคัดย่อ

ช่องว่างและความหนาแน่นของอาคารนั้นเป็นตัวแปรที่สำคัญสำหรับการจำลองมลพิษทางอากาศในเมืองอย่างแม่นยำและก่อให้เกิดสิ่งจำเป็นพื้นฐานในการวางผังอาคาร แบบจำลองมลพิษทางอากาศมีจุดมุ่งหมายเพื่อวิเคราะห์การแพร่กระจายของมลพิษทางอากาศในถนนที่มีอาคารสูงล้อมรอบในเขตเมือง โดยระดับสัดส่วนการแพร่กระจายของความเข้มข้นของมลพิษจะถูกจำลองโดยใช้สมการการพาและการแพร่แบบหนึ่งมิติและสองมิติ ในงานวิจัยนี้นำเสนอตัวแบบเชิงตัวเลขและวิธีเชิงตัวเลขสำหรับการจำลองมลพิษทางอากาศโดยเฉลี่ยในถนนที่มีอาคารสูงล้อมรอบในเขตเมือง วิธีเชิงตัวเลขจะถูกคำนวณโดยการใช้วิธีผลต่างอันตะแบบชัดแจ้งในรูปแบบความเร็วการไหลของอากาศ วิธีเชิงตัวเลขนี้สามารถนำไปใช้ได้ในหลายขอบเขตและสามารถสอดแทรกฟังก์ชันความเร็วลมได้

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Abstract

Permeability and building density are important parameters for precisely modeling urban air pollution and influencing regulatory requirements for building planning. An air pollution model is proposed to investigate the dispersion of air pollution in an urban street canyon, the spatial distribution of pollutant concentration level is modeled using one-dimensional and two-dimensional advection-diffusion equation. In this thesis, mathematical models and numerical methods for simulating air pollutant in urban street canyons are proposed model can be used to represent a laterally averaged air pollution model in an urban street canyon. The numerical results are calculated by using an explicit finite difference scheme with the uniform air flow velocity fields. The numerical can be used in several boundary conditions and interpolated wind velocity functions.

Keywords : urban street canyon, dispersion model, air pollution, advection-diffusion equation

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Chapter 1
Introduction

1.1 The air pollution problem

Air contamination has been a critical natural and wellbeing sympathy toward hundreds of years. This presentation is across the board and vital for all populates since it is unavoidable. The Global Burden of Disease 2010 assessed that 3.1, 3.5, and 0.2 million passings happened every year as an after effect of exposures to encompassing particulate matter, family strong energizes and surrounding ozone contamination, separately. With fast urbanization of the total populace, air quality is expected to be on the decay as wellsprings of contamination in total. In profoundly populated urban areas, daily life and human exercises (e.g. control era and vehicle usage) must ascend to stay aware of the requests of developing areas.

Urban air pollution was originally considered as a local problem mainly associated with domestic heating and industrial emissions, which are now controllable to a great extent.

The poisons related to the primary activity are CO, NO_x, hydrocarbons, and particles. CO is a defective fuel for ignition. Burning additionally delivers a blend of NO_2 and NO, of which more than 90% compared to NO. An extensive variety of unburned and artificially changed hydrocarbons (e.g. benzene, toluene, ethane, ethylene, pentane, and so forth) is discharged by engine vehicles through various diverse procedures (e.g. dissipation, fuel tank removal, oil leak, etc.). As last, particles of dense carbonaceous material are transmitted for the most part by diesel and inadequately kept up petrol vehicles.

In urban situations and particularly in the zones where populace and activity thickness are moderately high, human introduction to unsafe substances is relied upon to be altogether expanded. This is regularly the case close to bustling activity pivot in downtown areas, where urban geology and microclimate may add to the production of poor air scattering conditions offering ascension to defilement hotspots. High contamination levels have been seen in road gorge, which is a term habitually used for urban boulevards flanked by structures on both sides. Inside these boulevards, walkers, cyclists, drivers and inhabitants are probably going to be surrounded by pollutant concentrations exceeding current air quality standards.

1.2 Literature review

There are many methods for detecting the level of pollutants in the air, mostly conducted by a field measurement and a mathematical simulation.

In [2] and [4], the air pollution in two dimensional spaces with obstacles domain is also studied.

In [3], the three dimensional fractional step method is applied based on the discretization of the time dependent atmosphere advection-diffusion equation.

In [5], the air pollution problem in three dimensional spaces with multiple sources are presented. The initial conditions in the domain are assumed to be zero everywhere without obstacles.

In [6], they use the finite difference method in the air pollution model of two dimensional spaces with single – point source.

1.3 Objectives

The purpose of this thesis is to investigate the dispersion of air pollution in an urban street canyon, the spatial distribution of pollutant concentration level is modeled using one-dimensional and two-dimensional advection-diffusion equations.

1.4 Scope of the thesis

The scope of the thesis is restricted to the application of the finite difference method to the air pollution problem in an urban street canyon with the boundary conditions and interpolated wind velocity functions. The two-dimensional spatial advection-diffusion equation are used to simulate the air pollutant concentration in the domain.

1.5 Plan of the thesis

The thesis explains the mathematical modeling of air pollution. The process of simulation discusses the pollution levels in the domain.

The first part will study the basic knowledge about the mathematical model for air quality and control by defining the domain of problem in thesis and the domain of study case.

The second part will study the numerical method for solving dispersion model.

The third part is the computation of one-dimensional dispersion model

The fourth part is the computation of two-dimensional dispersion model by using an explicit finite difference scheme with the uniform air flow velocity fields.

1.6 Expected result

The expected result of thesis can be used to a laterally averaged air pollution model in an urban street canyon.

Chapter 2

Mathematical Model of Air Pollution Measurement

In this section, mathematical model is used to simulate pollutant concentration in an urban street canyon. Dispersion model with the governing equation, advection-diffusion equation is used to describe the dispersion of concentration in one dimension and two dimension model.

2.1 The topography of a considered

In this section, we will explain of the domain used in the research include Traditional air pollution measurement model in street canyon and highway and real problem of the Yaowarat road.

2.1.1 Traditional air pollution measurement model in street canyon and highway

Carbon monoxide

The box model and the ATDL model can give the average carbon monoxide concentration over a broad area (say 10 by 10 km). In a street canyon or adjacent to a highway in an urban area, there is additional contribution to the concentration from local sources. In this case the total concentration C is the sum of a spatially averaged C_a and a local ΔC_1 component:

$$C = C_a + \Delta C_1 \tag{2.1}$$

Johnson et al. (1976) outline methods of estimating ΔC_1. Consider the street canyon in Figure 2.1, where the important variables are defined. If the wind is more or less normal to the street, the equations for the concentration ΔC_1 in the street canyon are:

Lee side,

$$\Delta C_1 = \frac{0.1KNS^{-0.75}}{(u+0.5)\left[\left(x^2+z^2\right)^{1/2}+2\right]} \tag{2.2}$$

Windward side,

$$\Delta C_1 = \frac{0.1KNS^{-0.75}}{w(u+0.5)} \tag{2.3}$$

where ΔC = carbon monoxide concentration (ppm)

N = traffic flow (vehicle/hr)

S = average vehicle speed (miles/hr)

u = wind speed at roof level (m/hr)

W = street width (m)

x and y = horizontal distance and height (both m) of the receptor point relative to the traffic lane, respectively

K = dimensionless "best fit" constant

The data suggest that $K \approx 7$. For wind directions nearly parallel to the street,

$$\Delta C_1 = \frac{1}{2}\left[\Delta C_1(\text{windward}) + \Delta C_1(\text{lee})\right] \tag{2.4}$$

Equations 2.2 and 2.3 are subject to some revision since the average emission rate (e.g., grams of carbon monoxide per vehicle mile) changes with the vehicle vintage and type of mixture, which would thus modify the numerator (source term) of both expressions.

The excess concentration ΔC_1 contributed by a major highway in an urban area is important for perhaps 200 or 300 m downwind of the highway. Many highway diffusion models are referenced by Johnson et al. (1976), but most are based on the Gaussian model for an infinite line source:

$$C_1 = \left(\frac{2}{\pi}\right)^{1/2} \frac{Q_l \exp\left(-H^2/2\sigma_z^2\right)}{u\sigma_z \sin\phi} \tag{2.5}$$

where Q_l is line source strength (in mass per unit time per unit length), H is the effective height of emissions (probably 2 or 3 m), and ϕ is the angle between the

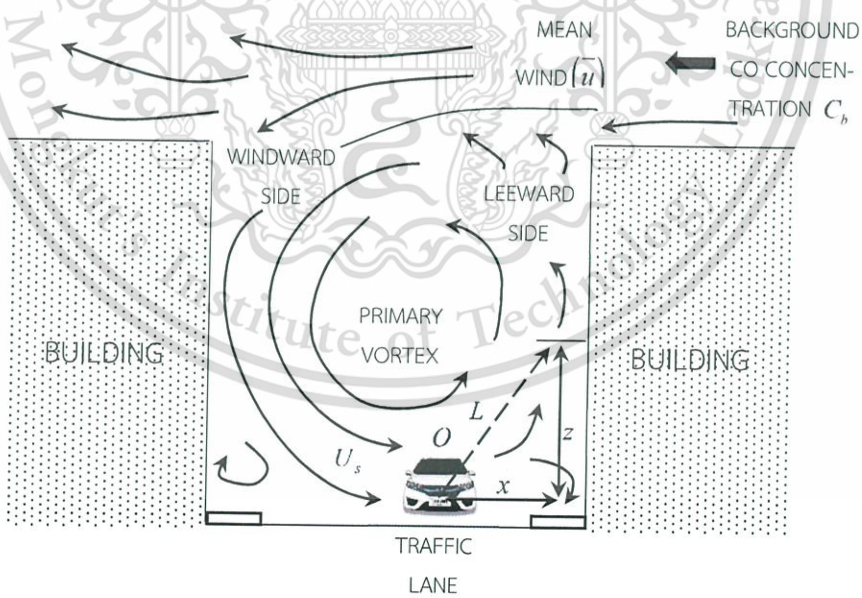


Figure 2.1: Schematic of cross-street air circulation in a street canyon [12]

wind direction and the highway. An initial value of σ_z (e.g., 2 or 3) is assumed to account for the turbulent wake behind moving vehicle. If d is the perpendicular distance from the highway, then the distance x to be used in evaluating σ_z is $d/\sin\phi$. [12]

2.1.2 The Yaowarat road

Yaowarat Road in Samphanthawong district is the main artery of Bangkok's Chinatown. Modern Chinatown now covers a large area around Yaowarat and Charoen Krung Road.

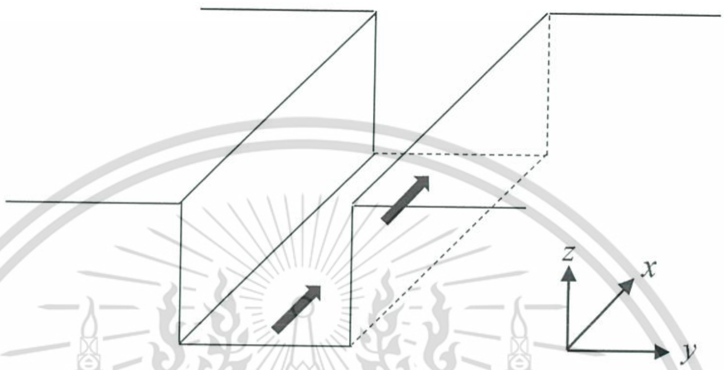
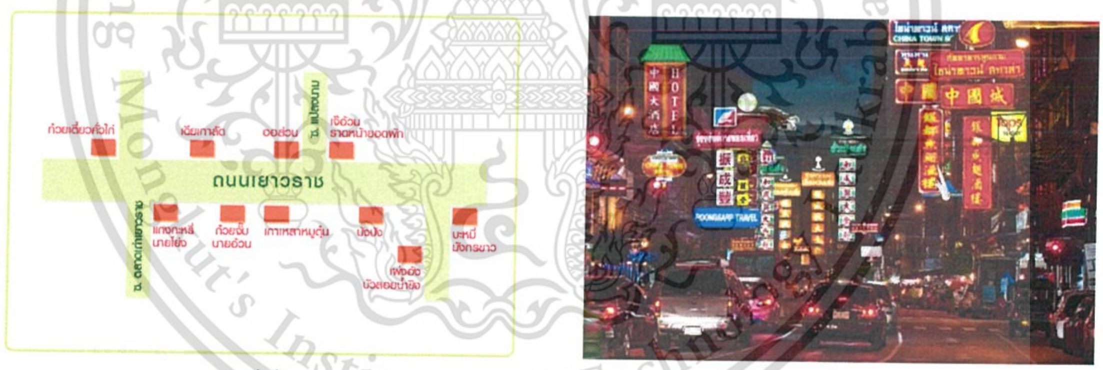


Figure 2.2: Domain of street canyon



(a) (b)

Figure 2.3: Domain of the Yaowarat road

2.2 Air quality

2.2.1 Mobile Sources

While ships, locomotives, and air craft are also included in the mobile sources category, it is motor vehicles which are by far the most important in terms of total emission. There were approximately 100 million gasoline-powered autos and light-duty trucks in use in the United States in 1973, and that number is growing by about 3.4% per year (EPA 1972b). If this rate were to continue for another 20 years, there would be 200 million light-duty vehicles in use. Recall that the three major pollutants emitted by motor vehicles are: carbon monoxide (CO), nitrogen oxides (NO_x), and hydrocarbons (HC). Motor vehicles accounted for 66% of the 1970 emissions of CO; 48% of the HC emissions; and 40% of the NO_x.

In a vehicles which has no emission control equipment, essentially all the CO and NO_x are emitted from the tailpipe while the hydrocarbons are emitted partly from the exhaust, partly from crankcase blowby (gases which slip past the piston rings during the compression and power strokes of the engine cycle), and partly from evaporation, as shown in Figure

The actual quantities of emissions from these various sources are highly dependent on the particular driving conditions encountered. For example, CO and hydrocarbon emission decrease with increasing driving speed, while NO_x emissions remain relatively unaffected. While an engine is just idling, hydrocarbon and CO emission are high but NO_x emissions are low. [7]

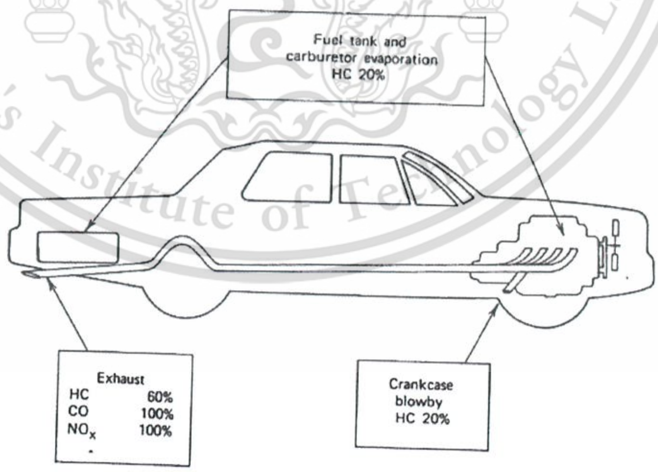


Figure 2.4: Approximate distribution of emissions by source for a vehicle with no emissions [7]

2.2.2 Emission Standards

The history of auto emission controls began in California in 1959 with the adoption of standards to control exhaust hydrocarbon and carbon monoxide, but these standards were not implemented until they became "technologically feasible" in 1966. In 1960, standards to control emissions from crankcase blow-by were added but implementation was not required until 1963. Federal emission standards first became effective on 1968 models. Controls were also required on evaporative emissions from fuel tanks and carburetors, and by 1970 industry had reduced hydrocarbon emissions from new vehicles by almost three-fourths and CO emissions by about two-thirds. Unfortunately, as we shall see in the next section, these improvements in HC and CO emissions were partly made at the expense of increased NO_x emissions. Standards for NO_x were not required until 1971 in California, and not until 1973 for the rest of the country.

The Clean Air Act was written to require that by 1975 emissions of hydrocarbons and carbon monoxide from new vehicles must be 90% less than the emissions allowed in 1970. Similarly, by 1976, the NO_x emissions must be 90% less than the average of vehicles manufactured in 1971. The law allows a 1 year delay in these deadlines if it is adequately proven that the technology is not available to meet them. That delay has been granted by the Environment Protection Agency. [7]

2.3 Governing equations of air quality measurement

Numerous types of water motion transport matter within natural waters. Wind energy and gravity impart motion to the water that leads to mass transport. In the present context within-system motion can be divided into two general categories: advection and diffusion.

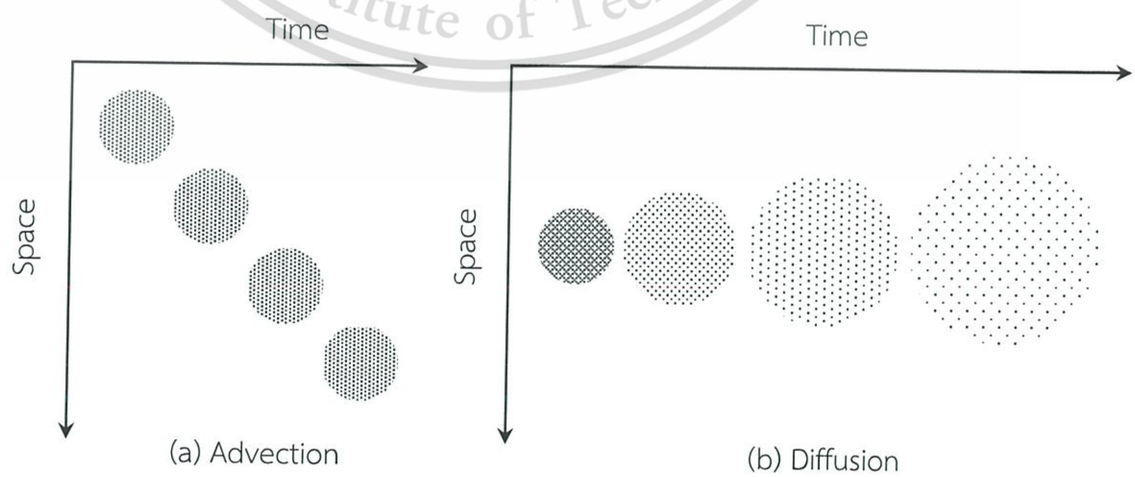


Figure 2.5: The transport of a dye patch in space and time via (a) Advection and (b) Diffusion [11]

Advection results from flow that is unidirectional and does not change the identity of the substance being transported. As in Figure 2.4 (a) advection moves matter from one position in space to another [11].

Diffusion refers to the movement of mass due to mixing. Such transport causes the dye patch depicted in Figure 2.4 (b) to spread out and dilute over time with negligible not movement of its center of mass. Diffusion is the movement molecules from an area of higher concentration to one lower concentration [11].

Dispersion is a related process that also causes pollution to spread. Dispersion is the result of velocity differences in space [11].

Dispersion model (Advection-Diffusion equation)

Advection-Diffusion equation is a combination of the diffusion and advection equation, and describes physical phenomena where particles, energy, or other physical quantities are transferred inside a physical system due to two processes [11].

2.3.1 A one dimensional air pollutant dispersion model with insided air pollutant source

Mathematical model describing the pollutant concentration in the domain of one dimensional an urban street canyon. The concentration of a pollutant can be described by the advection-diffusion equation

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=D\frac{\partial^2 c}{\partial x^2}+Q, \tag{2.6}$$

where $c(x,t)$ is the laterally averaged air pollutant concentration at the point x at time t, u is a wind speed in the x direction, D is the diffusion coefficient of the considered air pollutant matter in x direction, $Q(x,t)$ is the rate of change of concentration at point source or sink which are released or absorbed along the street canyon per unit of time. [1]

2.3.2 A two dimensional form (xz) air pollutant dispersion model with insided air pollutant source

The distributed air pollutant process satisfies a mass transfer equation, which include transportation and diffusion. Averaging the equation over the lateral, we get the advection-diffusion equation

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=D_{xz}\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial z^2}\right)+Q, \tag{2.7}$$

where $c(x,z,t)$ is the laterally averaged air pollutant concentration at the point (x,z) at time t, u is a constant wind speed in the x direction, D_{xz} is the diffusion coefficient of the considered air pollutant matter in x and z direction, $Q(x,z,t)$ is the rate of change of concentration at point source or sink which are released or absorbed along the street canyon per unit of time. [9]

2.3.3 A two dimensional form (xy) air pollutant dispersion model with insided air pollutant source

The distributed air pollutant process satisfies a mass transfer equation, which include transportation and diffusion. Averaging the equation over the lateral, we get the advection-diffusion equation

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D_{xy}\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right)+Q, \qquad (2.8)$$

where $c(x,\mathrm{y},t)$ is the laterally averaged air pollutant concentration at the point (x,y) at time t, u is a constant wind speed in the x direction, v is a constant wind speed in the y-direction, D_{xy} is the diffusion coefficient of the considered air pollutant matter in x and y direction, $Q(x,\mathrm{y},t)$ is the rate of change of concentration at point source or sink which are released or absorbed along the street canyon per unit of time. [9]

Chapter 3

Numerical Methods to a One Dimensional Air Pollution Measurement Models in Street Canyons

In this section, numerical methods are used to solve the governing equation of mathematical models, we use explicit finite different methods for solving advection-diffusion equation. In the final part of this section, the examples used to calculate air pollutant concentration in street canyon.

3.1 Numerical method to a one dimensional air pollutant dispersion model with insided air pollutant source

We use the forward differenced in time and central difference in space in advection diffusion equation. We can approximate c_i^n are the values difference approximation of at point $x = i\Delta x$ and $t = n\Delta t$ where $0 \le i \le L$ and $0 \le n \le N$. Using the forward time center space method to Eq.(2.6), the following finite difference equation can be obtained

$$c \cong c_i^n, \tag{3.1}$$

$$\frac{\partial c}{\partial t} \cong \frac{c_i^{n+1} - c_i^n}{\Delta t}, \tag{3.2}$$

$$\frac{\partial c}{\partial x} \cong \frac{c_{i+1}^n - c_{i-1}^n}{2\Delta x}, \tag{3.3}$$

$$\frac{\partial^2 c}{\partial x^2} \cong \frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{(\Delta x)^2}, \tag{3.4}$$

$$Q \cong Q_i^n \tag{3.5}$$

Substituting Eq. (3.1-3.5) into Eq. (2.6), we have,

$$\left(\frac{c_i^{n+1} - c_i^n}{\Delta t}\right) + u\left(\frac{c_{i+1}^n - c_{i-1}^n}{2\Delta x}\right) = D\left(\frac{c_{i=1}^n - 2c_i^n + c_{i-1}^n}{(\Delta x)^2}\right) + Q_i^n \tag{3.6}$$

we can obtain an explicit form of finite difference equation as follows of Eq. (3.6)

$$c_i^{n+1} = \frac{1}{2}(2\alpha + \beta)c_{i-1}^n + (1 - 2\alpha)c_i^n + \frac{1}{2}(2\alpha - \beta)c_{i+1}^n + Q_i^n \tag{3.7}$$

where

$$\alpha = D\frac{\Delta t}{(\Delta x)^2}, \tag{3.8}$$

$$\beta = u\frac{\Delta t}{\Delta x}. \tag{3.9}$$

If c_i^n are lied on the boundary of the domain, we will approximate by using the boundary conditions and employing the forward different scheme,

$$\frac{\partial c}{\partial x} \approx \frac{c_{i+1}^n - c_i^n}{\Delta x}. \tag{3.10}$$

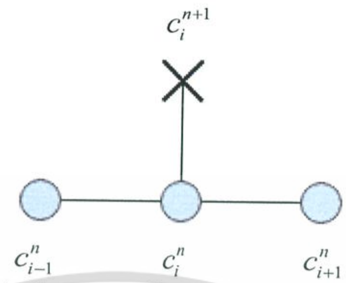


Figure 3.1: The stencil diagram of one - dimensional dispersion model

3.2 Numerical simulation to a one dimensional air pollutant dispersion model with insided air pollutant source

In this section, we show example to find the pollutant concentration is calculated by dispersion model in one dimensional.

Example 3.2.1. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,T)$ in Figure (3.2) with step size $\Delta x = 0.25, \Delta t = 0.01$, diffusion coefficient $D = 0.05$, there is no interior source $Q = 0$, and average air flow velocity $u = 0.25$.

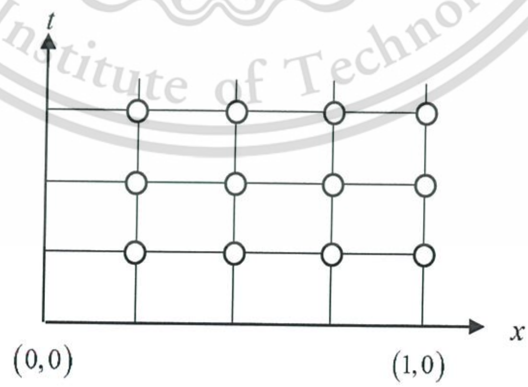


Figure 3.2: Generating grid points of example 3.2.1

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0)=x(1-x)+c_0,$$

the boundary conditions can be assumed by

$$c(0,t)=c_0=1,$$
$$c_x(1,t)=c_1.$$

define $c(x,t)=c_i^n$ with $\Delta x=0.25, \Delta t=0.01, D=0.05, u=0.25,$ and $Q=0.$

approximate differential boundary condition,

$c_x(1,t)=c_1$ or $\frac{\partial c_i^n}{\partial x}=c_1,$

using forward difference,

$$\frac{c_5^n-c_4^n}{\Delta x}=c_1$$

$$c_5^n=c_1\Delta x+c_4^n \qquad i=4 \qquad (3.11)$$

solving problem by finite difference method Eq. (2.6)

$$c_i^{n+1}=\frac{1}{2}(2\alpha+\beta)c_{i-1}^n+(1-2\alpha)c_i^n+\frac{1}{2}(2\alpha-\beta)c_{i+1}^n+Q_i^n$$

$$\alpha=D\frac{\Delta t}{(\Delta x)^2}=0.05\frac{0.01}{(0.25)^2}=0.008,$$

$$\beta=u\frac{\Delta t}{\Delta x}=0.25\frac{0.01}{0.25}=0.01,$$

$$c_i^{n+1}=0.013c_{i-1}^n+0.984c_i^n+0.003c_{i+1}^n \qquad i=1,2,3,4 \qquad (3.12)$$

$n=0,\ i=1 \quad c_1^1=0.013c_0^0+0.984c_1^0+0.003c_2^0=1.18524$

$i=2 \quad c_2^1=0.013c_1^0+0.984c_2^0+0.003c_3^0=1.249$

$i=3 \quad c_3^1=0.013c_2^0+0.984c_3^0+0.003c_4^0=1.187750$

$i=4 \quad c_4^1=0.013c_3^0+0.984c_4^0+0.003c_5^0$

and apply boundary condition $c_5^0=c_1\Delta x+c_4^0$

$$c_4^1=0.013c_3^0+0.984c_4^0+0.003\left(c_1\Delta x+c_4^0\right)$$

$n=1,\ i=1 \quad c_1^2=0.013c_0^1+0.984c_1^1+0.003c_2^1=1.183033$

$i=2 \quad c_2^2=0.013c_1^1+0.984c_2^1+0.003c_3^1=1.247988$

$i=3 \quad c_3^2=0.013c_2^1+0.984c_3^1+0.003c_4^1=1.187990$

$i=4 \quad c_4^2=0.013c_3^1+0.984c_4^1+0.003c_5^1$

and apply boundary condition $c_5^1=c_1\Delta x+c_4^1$

$$c_4^2=0.013c_3^1+0.984c_4^1+0.003\left(c_1\Delta x+c_4^1\right)$$

$n=2,\ i=1 \quad c_1^3=0.013c_0^3+0.984c_1^3+0.003c_2^3=1.180848$

$i=2 \quad c_2^3 = 0.013c_1^2 + 0.984c_2^2 + 0.003c_3^2 = 1.246963$

$i=3 \quad c_3^3 = 0.013c_2^2 + 0.984c_3^2 + 0.003c_4^2 = 1.188221$

$i=4 \quad c_4^3 = 0.013c_3^3 + 0.984c_4^3 + 0.003c_5^3$

and apply boundary condition $c_5^2 = c_1\Delta x + c_4^2$

$$c_4^2 = 0.013c_3^2 + 0.984c_4^2 + 0.003\left(c_1\Delta x + c_4^2\right)$$

Table 3.1 The calculated pollutant concentration of example 3.2.1 with the absorbance boundary ($c_1 = -0.1$), non-slipping boundary ($c_1 = 0$) or releasing boundary ($c_1 = 0.1$) at the exit

Point	Pollutant concentration		
	$c_1 = -0.1$	$c_1 = 0$	$c_1 = 0.1$
c_0^0	1.000000	1.000000	1.000000
c_1^0	1.187500	1.187500	1.187500
c_2^0	1.250000	1.250000	1.250000
c_3^0	1.187500	1.187500	1.187500
c_4^0	1.000000	1.000000	1.000000
c_0^1	1.000000	1.000000	1.000000
c_1^1	1.185250	1.185250	1.185250
c_2^1	1.249000	1.249000	1.249000
c_3^1	1.187750	1.187750	1.187750
c_4^1	1.002363	1.002438	1.002513
c_0^2	1.000000	1.000000	1.000000
c_1^2	1.183033	1.183033	1.183033
c_2^2	1.247988	1.247988	1.247988
c_3^2	1.187990	1.187990	1.187991
c_4^2	1.004698	1.004847	1.004996
c_0^3	1.000000	1.000000	1.000000
c_1^3	1.180848	1.180848	1.180848
c_2^3	1.246963	1.246963	1.246963
c_3^3	1.188220	1.188221	1.188222
c_4^3	1.007005	1.007227	1.007450

Example 3.2.2. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1) \times (0,T)$ in Figure (3.3) with step size $\Delta x = 0.1, \Delta t = 0.01$, diffusion coefficient $D = 0.05$, there is no interior source $Q = 0$, and average air flow velocity $u = 0.25$.

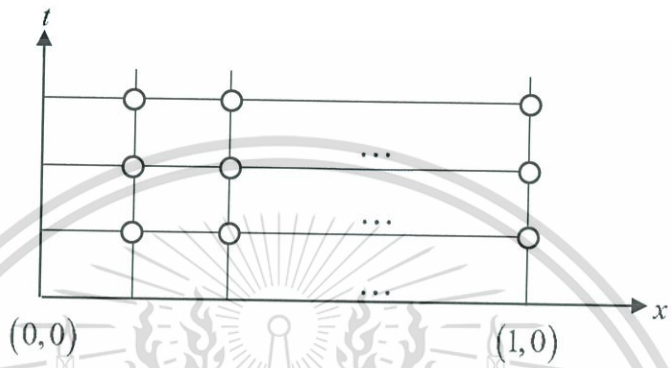


Figure 3.3: Generating grid points of example 3.2.2

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = c_0 = 1,$$

$$c_x(1,t) = c_1.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.1, \Delta t = 0.01, D = 0.05, u = 0.25,$ and $Q = 0.$

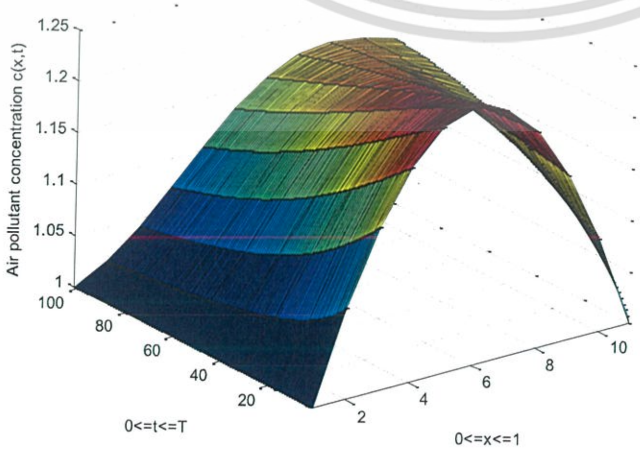


Figure 3.4: The approximated air pollutant concentration $c(x,t)$ for $c_1 = -0.1$ of example 3.2.2

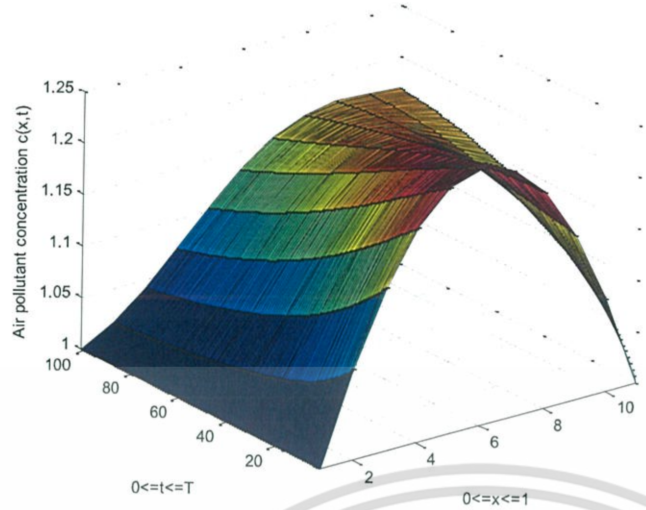


Figure 3.5: The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0$ of example 3.2.2

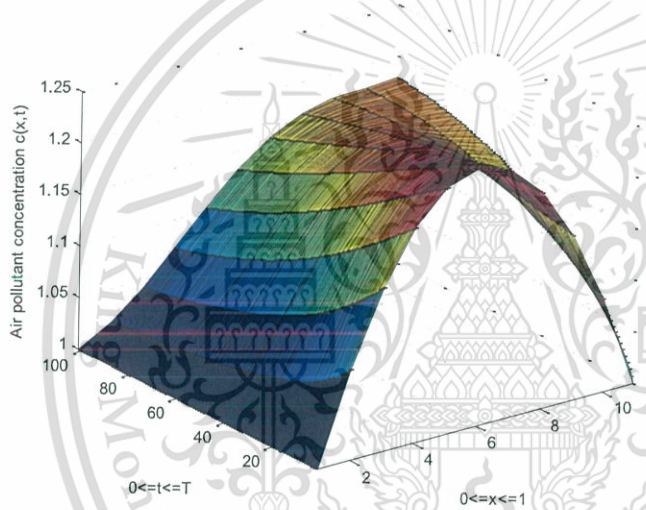
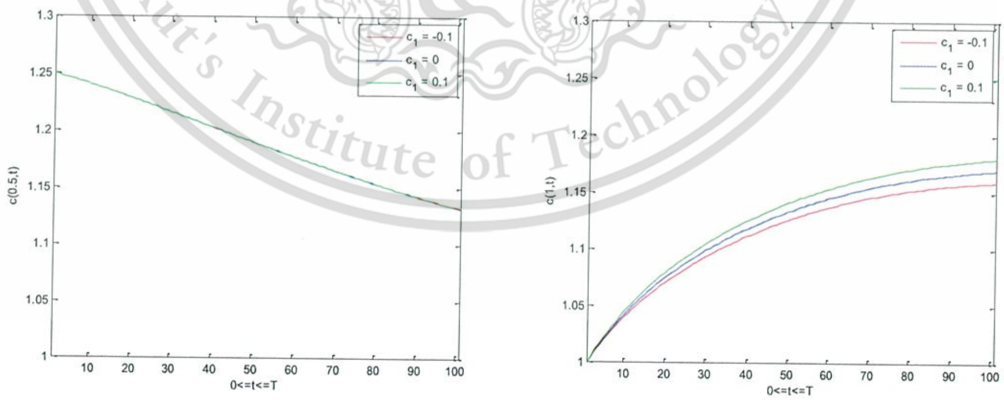


Figure 3.6: The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0.1$ of example 3.2.2



(a) (b)

Figure 3.7: The approximated air pollutant concentration at $t = 100$ sec (a) $c(0.5,t)$ and (b) $c(1,t)$ of example 3.2.2

Example 3.2.3. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,T)$ in Figure (3.8) with step size $\Delta x = 0.25, \Delta t = 0.01$, diffusion coefficient $D = 0.05$, average air pollutant source are added $Q = 0.0001$, and average air flow velocity $u = 0.25$.

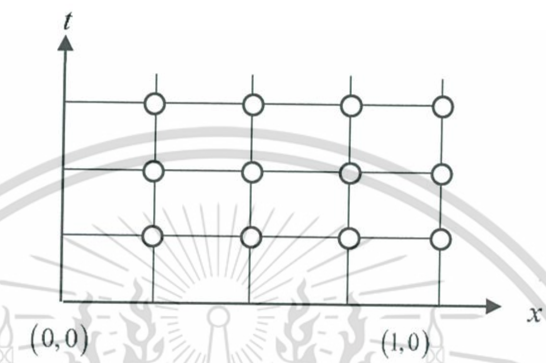


Figure 3.8: Generating grid points of example 3.2.3

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = c_0 = 1,$$

$$c_x(1,t) = c_1.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.25, \Delta t = 0.01, D = 0.05, u = 0.25,$ and $Q = 0.0001.$

approximate differential boundary condition,

$c_x(1,t) = c_1$ or $\frac{\partial c_i^n}{\partial x} = c_1,$

using forward difference,

$$\frac{c_5^n - c_4^n}{\Delta x} = c_1$$

$$c_5^n = c_1 \Delta x + c_4^n \qquad i = 4 \qquad (3.13)$$

solving problem by finite difference method Eq.(2.6)

$$c_i^{n+1} = \frac{1}{2}(2\alpha+\beta)c_{i-1}^n + (1-2\alpha)c_i^n + \frac{1}{2}(2\alpha-\beta)c_{i+1}^n + Q_i^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008,$$

$$\beta = u\frac{\Delta t}{\Delta x} = 0.25\frac{0.01}{0.25} = 0.01,$$

$$c_i^{n+1} = 0.013c_{i-1}^n + 0.984c_i^n + 0.003c_{i+1}^n + 0.0001 \qquad i = 1,2,3,4 \tag{3.14}$$

$n=0,\ i=1$ $\quad c_1^1 = 0.013c_0^0 + 0.984c_1^0 + 0.003c_2^0 + 0.0001 = 1.185350$

$i=2$ $\quad c_2^1 = 0.013c_1^0 + 0.984c_2^0 + 0.003c_3^0 + 0.0001 = 1.249100$

$i=3$ $\quad c_3^1 = 0.013c_2^0 + 0.984c_3^0 + 0.003c_4^0 + 0.0001 = 1.187850$

$i=4$ $\quad c_4^1 = 0.013c_3^0 + 0.984c_4^0 + 0.003c_5^0 + Q_4^0$

and apply boundary condition $c_5^0 = c_1\Delta x + c_4^0$

$$c_4^1 = 0.013c_3^0 + 0.984c_4^0 + 0.003\left(c_1\Delta x + c_4^0\right) + Q_4^0$$

$n=1,\ i=1$ $\quad c_1^2 = 0.013c_0^1 + 0.984c_1^1 + 0.003c_2^1 + 0.0001 = 1.183232$

$i=2$ $\quad c_2^2 = 0.013c_1^1 + 0.984c_2^1 + 0.003c_3^1 + 0.0001 = 1.248188$

$i=3$ $\quad c_3^2 = 0.013c_2^1 + 0.984c_3^1 + 0.003c_4^1 + 0.0001 = 1.188190$

$i=4$ $\quad c_4^2 = 0.013c_3^1 + 0.984c_4^1 + 0.003c_5^1 + Q_4^1$

and apply boundary condition $c_5^1 = c_1\Delta x + c_4^1$

$$c_4^2 = 0.013c_3^1 + 0.984c_4^1 + 0.003\left(c_1\Delta x + c_4^1\right) + Q_4^1$$

$n=2,\ i=1$ $\quad c_1^3 = 0.013c_0^3 + 0.984c_1^3 + 0.003c_2^3 + 0.0001 = 1.096353$

$i=2$ $\quad c_2^3 = 0.013c_1^2 + 0.984c_2^2 + 0.003c_3^2 + 0.0001 = 1.247263$

$i=3$ $\quad c_3^3 = 0.013c_2^2 + 0.984c_3^2 + 0.003c_4^2 + 0.0001 = 1.188521$

$i=4$ $\quad c_4^3 = 0.013c_3^3 + 0.984c_4^3 + 0.003c_5^3 + Q_4^2$

and apply boundary condition $c_5^2 = c_1\Delta x + c_4^2$

$$c_4^2 = 0.013c_3^2 + 0.984c_4^2 + 0.003\left(c_1\Delta x + c_4^2\right) + Q_4^2$$

Table 3.2 The calculated pollutant concentration of example 3.2.3 with the absorbance boundary ($c_1 = -0.1$), non-slipping boundary ($c_1 = 0$) or releasing boundary ($c_1 = 0.1$) at the exit

Point	Pollutant concentration		
	$c_1 = -0.1$	$c_1 = 0$	$c_1 = 0.1$
c_0^0	1.000000	1.000000	1.000000
c_1^0	1.187500	1.187500	1.187500
c_2^0	1.250000	1.250000	1.250000
c_3^0	1.187500	1.187500	1.187500
c_4^0	1.000000	1.000000	1.000000
c_0^1	1.000000	1.000000	1.000000
c_1^1	1.185350	1.185350	1.185350
c_2^1	1.249100	1.249100	1.249100
c_3^1	1.187850	1.187850	1.187850
c_4^1	1.002463	1.002538	1.002613
c_0^2	1.000000	1.000000	1.000000
c_1^2	1.183232	1.183232	1.183232
c_2^2	1.248188	1.248188	1.248188
c_3^2	1.188190	1.188190	1.188191
c_4^2	1.004899	1.005047	1.005196
c_0^3	1.000000	1.000000	1.000000
c_1^3	1.181145	1.181145	1.181145
c_2^3	1.247263	1.247263	1.247263
c_3^3	1.188520	1.188521	1.188522
c_4^3	1.007305	1.007527	1.007550

Example 3.2.4. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,T)$ in Figure (3.9) with step size $\Delta x = 0.1, \Delta t = 0.01$, diffusion coefficient $D = 0.05$, average air pollutant source are added $Q = 0.0001$, and average air flow velocity $u = 0.25$.

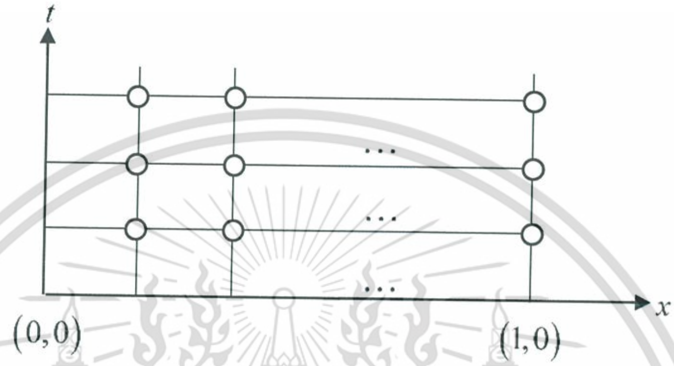


Figure 3.9: Generating grid points of example 3.2.4

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = c_0 = 1,$$

$$c_x(1,t) = c_1.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.05, \Delta t = 0.01, D = 0.05, u = 0.25,$ and $Q = 0.0001.$

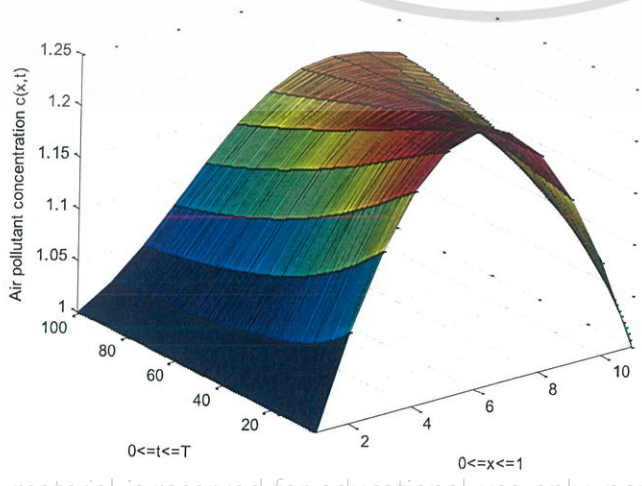


Figure 3.10: The approximated air pollutant concentration $c(x,t)$ for $c_1 = -0.1$ of example 3.2.4

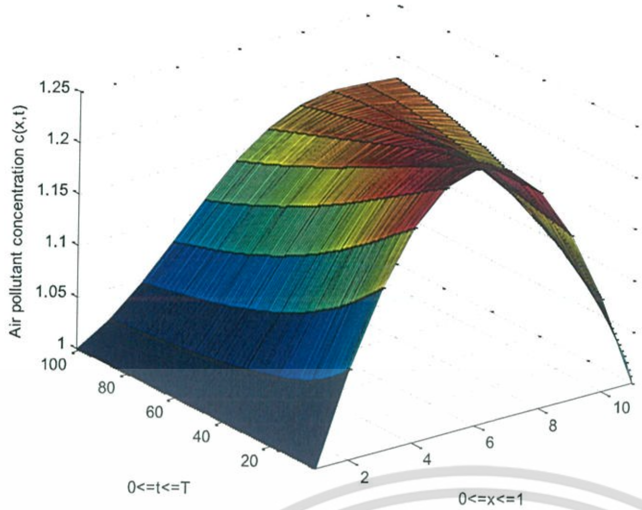


Figure 3.11: The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0$ of example 3.2.4

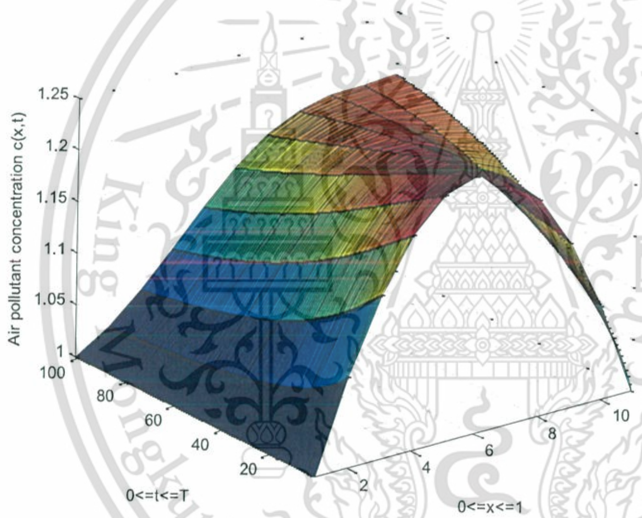


Figure 3.12: The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0.1$ of example 3.2.4

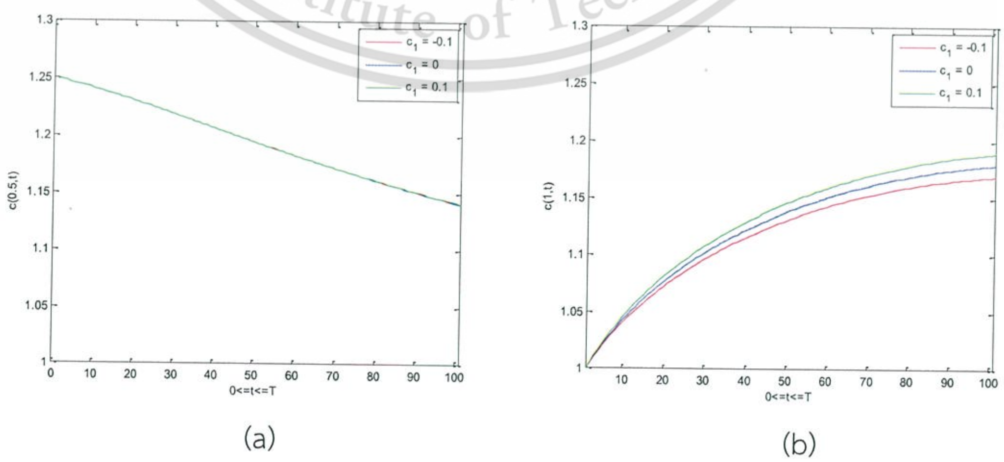


Figure 3.13: The approximated air pollutant concentration at $t = 100$ sec (a) $c(0.5,t)$ and (b) $c(1,t)$ of example 3.2.4

Example 3.2.5. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,T)$ in Figure (3.14) with step size $\Delta x = 0.25, \Delta t = 0.01$, diffusion coefficient $D = 0.05$, interpolated air pollutant source are added $Q = 1 + |\sin x|$, and average air flow velocity $u = 0.25$.

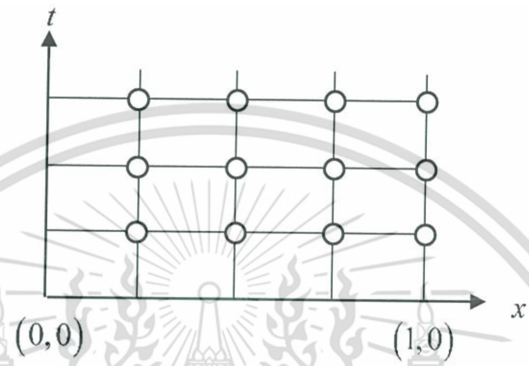


Figure 3.14: Generating grid points of example 3.2.5

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = c_0 = 1,$$
$$c_x(1,t) = c_1.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.25, \Delta t = 0.01, D = 0.05, u = 0.25,$ and $Q = 1 + |\sin x|$.

approximate differential boundary condition,

$c_x(1,t) = c_1$ or $\frac{\partial c_i^n}{\partial x} = c_1,$

using forward difference,

$$\frac{c_5^n - c_4^n}{\Delta x} = c_1$$

$$c_5^n = c_1 \Delta x + c_4^n \qquad i = 4 \qquad (3.15)$$

solving problem by finite difference method Eq.(3.7)

$$i = 1 \quad c_i^{n+1} = \frac{1}{2}(2\alpha + \beta)c_{i-1}^n + (1 - 2\alpha)c_i^n + \frac{1}{2}(2\alpha - \beta)c_{i+1}^n + Q_i^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008,$$

$$\beta = u\frac{\Delta t}{\Delta x} = 0.25\frac{0.01}{0.25} = 0.1,$$

$$Q_1^n = 1 + |\sin x| = 1 + |\sin(0.25)| = 1.247404$$

$$c_i^{n+1} = 0.013c_{i-1}^n + 0.984c_i^n + 0.003c_{i+1}^n + 1.247404 \tag{3.16}$$

$i = 2$ $$c_i^{n+1} = \frac{1}{2}(2\alpha + \beta)c_{i-1}^n + (1 - 2\alpha)c_i^n + \frac{1}{2}(2\alpha - \beta)c_{i+1}^n + Q_i^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008,$$

$$\beta = u\frac{\Delta t}{\Delta x} = 0.25\frac{0.01}{0.25} = 0.1,$$

$$Q_1^n = 1 + |\sin x| = 1 + |\sin(0.5)| = 1.479426$$

$$c_i^{n+1} = 0.013c_{i-1}^n + 0.984c_i^n + 0.003c_{i+1}^n + 1.479426 \tag{3.17}$$

$i = 3$ $$c_i^{n+1} = \frac{1}{2}(2\alpha + \beta)c_{i-1}^n + (1 - 2\alpha)c_i^n + \frac{1}{2}(2\alpha - \beta)c_{i+1}^n + Q_i^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008,$$

$$\beta = u\frac{\Delta t}{\Delta x} = 0.25\frac{0.01}{0.25} = 0.1,$$

$$Q_1^n = 1 + |\sin x| = 1 + |\sin(0.75)| = 1.681639$$

$$c_i^{n+1} = 0.013c_{i-1}^n + 0.984c_i^n + 0.003c_{i+1}^n + 1.681639 \tag{3.18}$$

$i = 4$ $$c_i^{n+1} = \frac{1}{2}(2\alpha + \beta)c_{i-1}^n + (1 - 2\alpha)c_i^n + \frac{1}{2}(2\alpha - \beta)c_{i+1}^n + Q_i^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008,$$

$$\beta = u\frac{\Delta t}{\Delta x} = 0.25\frac{0.01}{0.05} = 0.1,$$

$$Q_1^n = 1 + |\sin x| = 1 + |\sin(0.75)| = 1.841471$$

$$c_i^{n+1} = 0.013c_{i-1}^n + 0.984c_i^n + 0.003c_{i+1}^n + 1.841471 \tag{3.19}$$

$n = 0,\ i = 1$ $\quad c_1^1 = 0.013c_0^0 + 0.984c_1^0 + 0.003c_2^0 + 1.247404$

$i = 2$ $\quad c_2^1 = 0.013c_1^0 + 0.984c_2^0 + 0.003c_3^0 + 1.479426$

$i = 3$ $\quad c_3^1 = 0.013c_2^0 + 0.984c_3^0 + 0.003c_4^0 + 1.681639$

$i = 4$ $\quad c_4^1 = 0.013c_3^0 + 0.984c_4^0 + 0.003c_5^0 + 1.841471$

and apply boundary condition $c_5^0 = c_1\Delta x + c_4^0$

$$c_4^1 = 0.013c_3^0 + 0.984c_4^0 + 0.003(c_1\Delta x + c_4^0) + 1.841471$$

$n = 1,\ i = 1$ $\quad c_1^2 = 0.013c_0^1 + 0.984c_1^1 + 0.003c_2^1 + 1.247404$

$i = 2$ $\quad c_2^2 = 0.013c_1^1 + 0.984c_2^1 + 0.003c_3^1 + 1.479426$

$i = 3$ $\quad c_3^2 = 0.013c_2^1 + 0.984c_3^1 + 0.003c_4^1 + 1.681639$

$i = 4$ $\quad c_4^2 = 0.013c_3^1 + 0.984c_4^1 + 0.003c_5^1 + 1.841471$

and apply boundary condition $c_5^1 = c_1 \Delta x + c_4^1$

$$c_4^2 = 0.013c_3^1 + 0.984c_4^1 + 0.003\left(c_1 \Delta x + c_4^1\right) + 1.841471$$

$n = 2,\ i = 1$ $\quad c_1^3 = 0.013c_0^3 + 0.984c_1^3 + 0.003c_2^3 + 1.247404$

$i = 2$ $\quad c_2^3 = 0.013c_1^2 + 0.984c_2^2 + 0.003c_3^2 + 1.479426$

$i = 3$ $\quad c_3^3 = 0.013c_2^2 + 0.984c_3^2 + 0.003c_4^2 + 1.681639$

$i = 4$ $\quad c_4^3 = 0.013c_3^3 + 0.984c_4^3 + 0.003c_5^3 + 1.841471$

and apply boundary condition $c_5^2 = c_1 \Delta x + c_4^2$

$$c_4^2 = 0.013c_3^2 + 0.984c_4^2 + 0.003\left(c_1 \Delta x + c_4^2\right) + 1.841471$$

Table 3.3 The calculated pollutant concentration of example 3.2.5 with the absorbance boundary ($c_1 = -0.1$), non-slipping boundary ($c_1 = 0$) or releasing boundary ($c_1 = 0.1$) at the exit

Point	Pollutant concentration		
	$c_1 = -0.01$	$c_1 = 0$	$c_1 = 0.01$
c_0^0	1.000000	1.000000	1.000000
c_1^0	1.187500	1.187500	1.187500
c_2^0	1.250000	1.250000	1.250000
c_3^0	1.187500	1.187500	1.187500
c_4^0	1.000000	1.000000	1.000000
c_0^1	1.000000	1.000000	1.000000
c_1^1	2.432654	2.432654	2.432654
c_2^1	2.728426	2.728426	2.728426
c_3^1	2.869389	2.869389	2.869389
c_4^1	2.843834	2.843909	2.843984
c_0^2	1.000000	1.000000	1.000000
c_1^2	3.662321	3.662321	3.662321
c_2^2	4.204430	4.204430	4.204430
c_3^2	4.549119	4.549119	4.549119
c_4^2	4.685562	4.685711	4.685860
c_0^3	1.000000	1.000000	1.000000
c_1^3	4.876741	4.876741	4.876741
c_2^3	5.677843	5.677843	5.677843
c_3^3	6.226686	6.226687	6.226688
c_4^3	6.525184	6.525406	6.525628

Example 3.2.6. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,T)$ in Figure (3.15) with step size $\Delta x = 0.1, \Delta t = 0.01$, diffusion coefficient $D = 0.05$, interpolated air pollutant source are added $Q = 1 + |\sin x|$, and average air flow velocity $u = 0.25$.

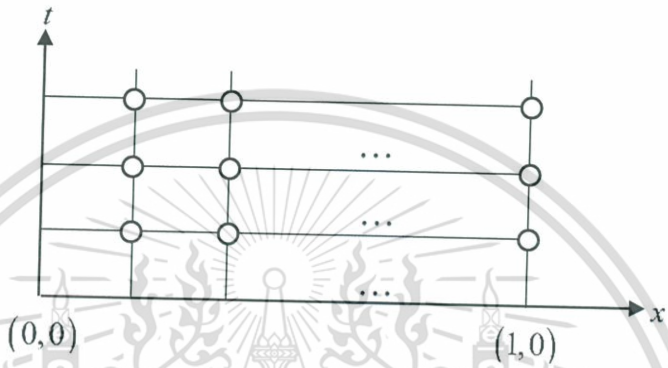


Figure 3.15 Generating grid points of example 3.2.6

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = c_0 = 1,$$

$$c_x(1,t) = c_1.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.1, \Delta t = 0.01, D = 0.05, u = 0.25,$ and $Q = 1 + |\sin x|$.

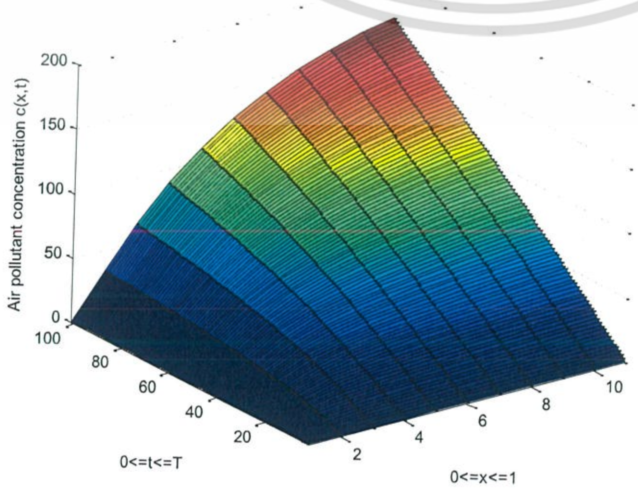


Figure 3.16: The approximated air pollutant concentration $c(x,t)$ for $c_1 = -0.1$ of example 3.2.6

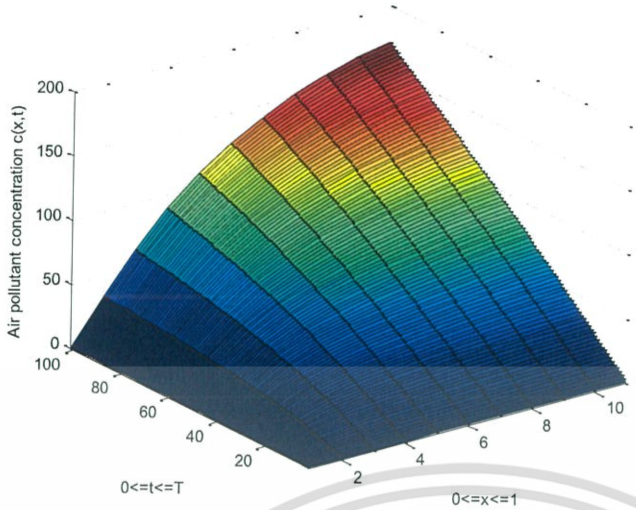


Figure 3.17: The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0$ of example 3.2.6

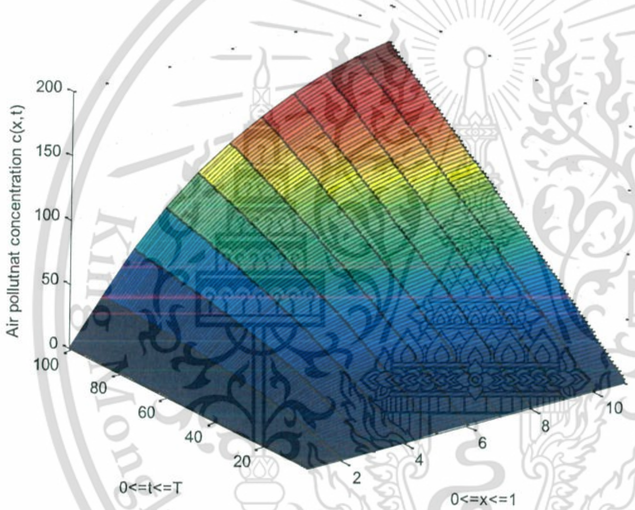
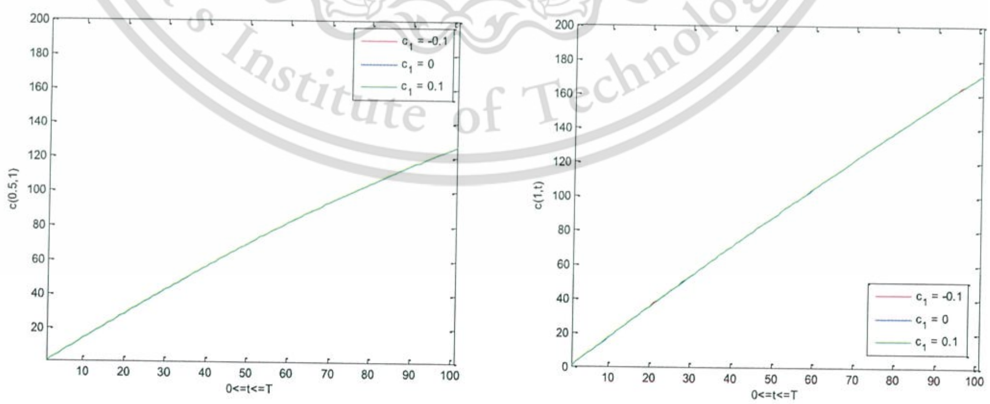


Figure 3.18: The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0.1$ of example 3.2.6



(a) (b)

Figure 3.19: The approximated air pollutant concentration at $t = 100$ sec (a) $c(0.5,t)$ and (b) $c(1,t)$ of example 3.2.6

Example 3.2.7. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the open gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,T)$ in Figure (3.20) with step size $\Delta x = 0.25,\ \Delta t = 0.01$ diffusion coefficient $D = 0.05$, there is no interior source $Q = 0$, and air flow velocity is reduced $u(x) = 0.1 - \frac{x}{100}$.

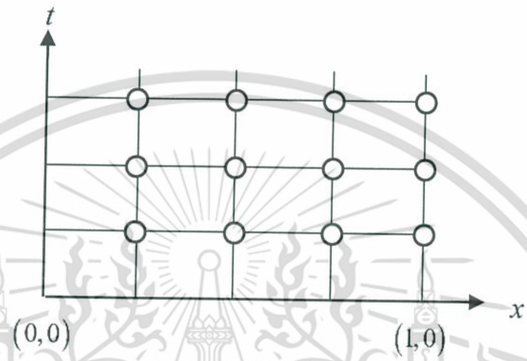


Figure 3.20: Generating grid points of example 3.2.7

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = c_0 = 1,$$
$$c_x(1,t) = c_1.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.25, \Delta t = 0.01, D = 1, u(x) = 0.1 - \frac{x}{100}$, and $Q = 0.$,

approximate differential boundary condition,

$c_x(1,t) = c_1$ or $\frac{\partial c_i^n}{\partial x} = c_1$,

using forward difference,

$$\frac{c_5^n - c_4^n}{\Delta x} = c_1$$

$$c_5^n = c_1 \Delta x + c_4^n \qquad i = 4 \qquad (3.20)$$

solving problem by finite difference method Eq.(3.7)

$i = 1$

$$c_1^{n+1} = \frac{1}{2}(2\alpha + \beta)c_0^n + (1-2\alpha)c_1^n + \frac{1}{2}(2\alpha - \beta)c_2^n + Q_1^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008$$

$$\beta = u(x)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{x}{100}\right)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{(1)(0.25)}{100}\right)\frac{0.01}{0.25} = 0.0039$$

$$c_i^{n+1} = 0.00995c_0^n + 0.984c_1^n + 0.00605c_2^n \quad (3.21)$$

$i = 2$

$$c_2^{n+1} = \frac{1}{2}(2\alpha + \beta)c_1^n + (1 - 2\alpha)c_2^n + \frac{1}{2}(2\alpha - \beta)c_3^n + Q_2^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008$$

$$\beta = u(x)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{x}{100}\right)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{(2)(0.25)}{100}\right)\frac{0.01}{0.25} = 0.0038$$

$$c_2^{n+1} = 0.0099c_1^n + 0.984c_2^n + 0.0061c_3^n \quad (3.22)$$

$i = 3$

$$c_3^{n+1} = \frac{1}{2}(2\alpha + \beta)c_2^n + (1 - 2\alpha)c_3^n + \frac{1}{2}(2\alpha - \beta)c_4^n + Q_3^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008$$

$$\beta = u(x)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{x}{100}\right)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{(3)(0.25)}{100}\right)\frac{0.01}{0.25} = 0.0037$$

$$c_3^{n+1} = 0.00985c_2^n + 0.984c_3^n + 0.00615c_4^n \quad (3.23)$$

$i = 4$

$$c_4^{n+1} = \frac{1}{2}(2\alpha + \beta)c_3^n + (1 - 2\alpha)c_4^n + \frac{1}{2}(2\alpha - \beta)c_5^n + Q_4^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008$$

$$\beta = u(x)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{x}{100}\right)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{(4)(0.25)}{100}\right)\frac{0.01}{0.25} = 0.0036$$

$$c_4^{n+1} = 0.0098c_3^n + 0.984c_4^n + 0.0062c_5^n \quad (3.24)$$

$n = 0,\ i = 1 \quad c_1^1 = 0.00995c_0^0 + 0.984c_1^0 + 0.00605c_2^0$

$i = 2 \quad c_2^1 = 0.0099c_1^0 + 0.984c_2^0 + 0.0061c_3^0$

$i = 3 \quad c_3^1 = 0.00985c_2^0 + 0.984c_3^0 + 0.00615c_4^0$

$i = 4 \quad c_4^1 = 0.0098c_3^0 + 0.984c_4^0 + 0.0062c_5^0$

and apply boundary condition $c_5^0 = c_1\Delta x + c_4^0$

$$c_4^1 = 0.0098c_3^0 + 0.984c_4^0 + 0.0062\left(c_1\Delta x + c_4^0\right)$$

$n = 1,\ i = 1 \quad c_1^2 = 0.00995c_0^1 + 0.984c_1^1 + 0.00605c_2^1$

$i = 2 \quad c_2^2 = 0.0099c_1^1 + 0.984c_2^1 + 0.0061c_3^1$

$i = 3 \quad c_3^2 = 0.00985c_2^1 + 0.984c_3^1 + 0.00615c_4^1$

$i=4 \quad c_4^2 = 0.0098c_3^1 + 0.984c_4^1 + 0.0062c_5^1$

and apply boundary condition $c_5^1 = c_1\Delta x + c_4^1$

$$c_4^2 = 0.0098c_3^1 + 0.984c_4^1 + 0.0062\left(c_1\Delta x + c_4^1\right)$$

$n=2, \; i=1 \quad c_1^3 = 0.00995c_0^2 + 0.984c_1^2 + 0.00605c_2^2$

$i=2 \quad c_2^3 = 0.0099c_1^2 + 0.984c_2^2 + 0.0061c_3^2$

$i=3 \quad c_3^3 = 0.00985c_2^2 + 0.984c_3^2 + 0.00615c_4^2$

$i=4 \quad c_4^3 = 0.0098c_3^2 + 0.984c_4^2 + 0.0062c_4^2$

and apply boundary condition $c_5^2 = c_1\Delta x + c_4^2$

$$c_4^3 = 0.0098c_3^2 + 0.984c_4^2 + 0.0062\left(c_1\Delta x + c_4^2\right)$$

Table 3.4 The calculated pollutant concentration of example 3.2.7 with the absorbance boundary ($c_1 = -0.1$), non-slipping boundary ($c_1 = 0$) or releasing boundary ($c_1 = 0.1$) at the exit

Point	Pollutant concentration		
	$c_1 = -0.1$	$c_1 = 0$	$c_1 = 0.1$
c_0^0	1.000000	1.000000	1.000000
c_1^0	1.187500	1.187500	1.187500
c_2^0	1.250000	1.250000	1.250000
c_3^0	1.187500	1.187500	1.187500
c_4^0	1.000000	1.000000	1.000000
c_0^1	1.000000	1.000000	1.000000
c_1^1	1.186013	1.186013	1.186013
c_2^1	1.249000	1.249000	1.249000
c_3^1	1.186963	1.186963	1.186963
c_4^1	1.001683	1.001838	1.001993
c_0^2	1.000000	1.000000	1.000000
c_1^2	1.184543	1.184543	1.184543
c_2^2	1.247998	1.247998	1.247998
c_3^2	1.186434	1.186435	1.186436
c_4^2	1.003343	1.003652	1.003960
c_0^3	1.000000	1.000000	1.000000
c_1^3	1.183090	1.183090	1.183090
c_2^3	1.246994	1.246994	1.246994
c_3^3	1.185914	1.185917	1.185920
c_4^3	1.005983	1.005443	1.005903

Example 3.2.8. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the open gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1) \times (0,T)$ in Figure (3.21) with step size $\Delta x = 0.1,\ \Delta t = 0.01$ diffusion coefficient $D = 0.05$, there is no interior source $Q = 0$, and air flow velocity is reduced $u(x) = 0.1 - \frac{x}{100}$.

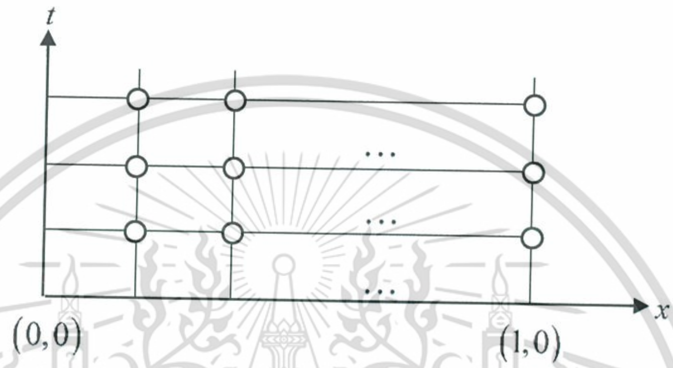


Figure 3.21: Generating grid points of example 3.2.8

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = \overline{c_0} = 1,$$

$$c_x(1,t) = \overline{c_1}.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.1, \Delta t = 0.01, D = 1, u(x) = 0.1 - \frac{x}{100}$ and $Q = 0$.

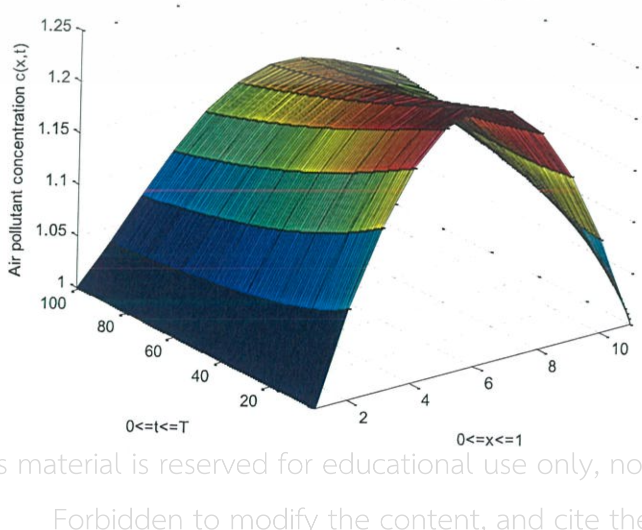


Figure 3.22: The approximated air pollutant concentration $c(x,t)$ for $c_1 = -0.1$ of example 3.2.8

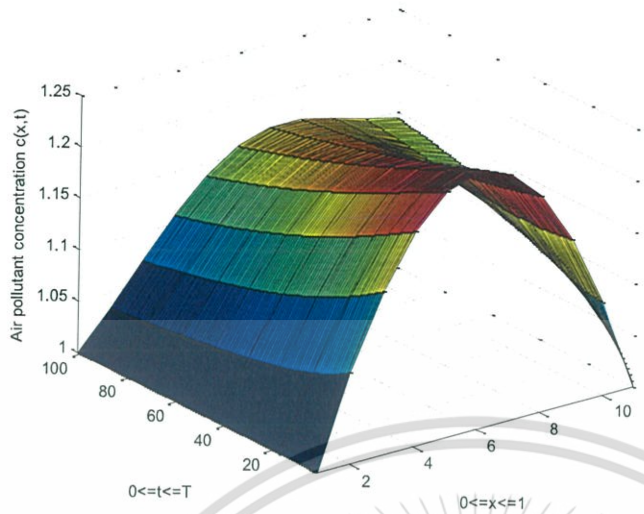


Figure 3.23:
The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0$ of example 3.2.8

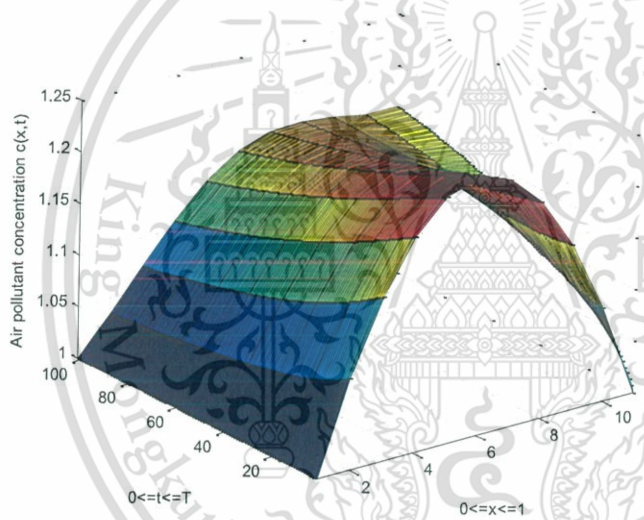
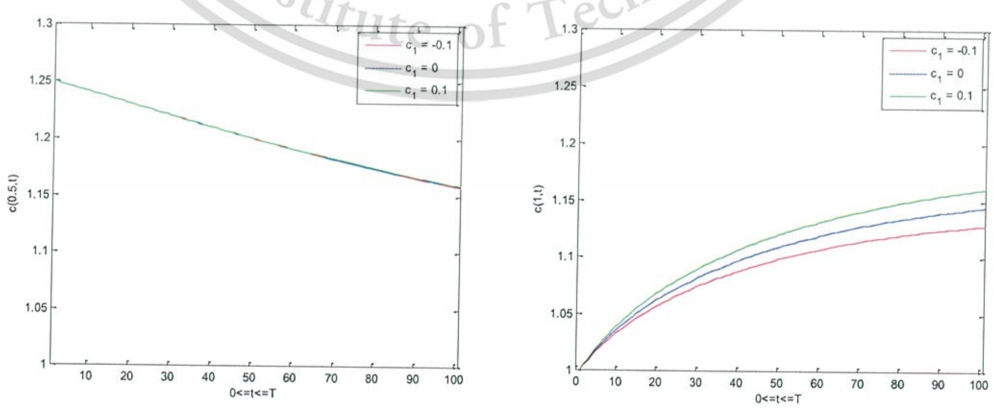


Figure 3.24:
The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0.1$ of example 3.2.8



(a) (b)

Figure 3.25: The approximated air pollutant concentration at $t = 100$ sec (a) $c(0.5,t)$ and (b) $c(1,t)$ of example 3.2.8

Example 3.2.9. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the open gate $\frac{\partial c}{\partial x} = c_1$, domain $\Omega = (0,1)\times(0,T)$ in Figure (3.26) with step size $\Delta x = 0.25,\ \Delta t = 0.01$, diffusion coefficient $D = 0.05$, average air pollutant source are added $Q = 0.0001$, and air flow velocity is reduced $u(x) = 0.1 - \frac{x}{100}$.

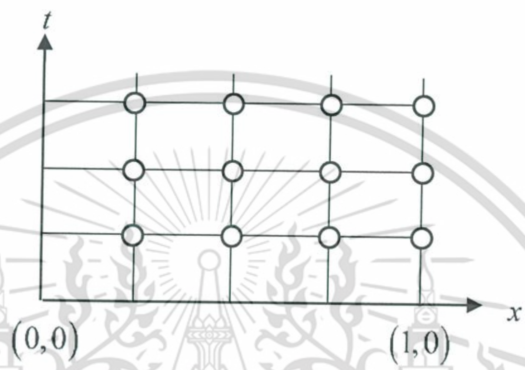


Figure 3.26: Generating grid points of example 3.2.9

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = c_0 = 1,$$
$$c_x(1,t) = c_1.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.25, \Delta t = 0.01, D = 1, u(x) = 0.1 - \frac{x}{100}$ and $Q = 0.0001.$

approximate differential boundary condition,

$c_x(1,t) = c_1$ or $\frac{\partial c_i^n}{\partial x} = c_1,$

using forward difference,

$$\frac{c_5^n - c_4^n}{\Delta x} = c_1$$

$$c_5^n = c_1 \Delta x + c_4^n \qquad i = 4 \qquad (3.25)$$

solving problem by finite difference method Eq.(3.7)

$i = 1$

$$c_1^{n+1} = \frac{1}{2}(2\alpha + \beta)c_0^n + (1 - 2\alpha)c_1^n + \frac{1}{2}(2\alpha - \beta)c_2^n + 0.0001$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008$$

$$\beta = u(x)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{x}{100}\right)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{(1)(0.25)}{100}\right)\frac{0.01}{0.25} = 0.0039$$

$$c_i^{n+1} = 0.00995c_0^n + 0.984c_1^n + 0.00605c_2^n + 0.0001 \quad (3.26)$$

$i = 2$

$$c_2^{n+1} = \frac{1}{2}(2\alpha + \beta)c_1^n + (1 - 2\alpha)c_2^n + \frac{1}{2}(2\alpha - \beta)c_3^n + Q_2^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008$$

$$\beta = u(x)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{x}{100}\right)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{(2)(0.25)}{100}\right)\frac{0.01}{0.25} = 0.0038$$

$$c_2^{n+1} = 0.0099c_1^n + 0.984c_2^n + 0.0061c_3^n + 0.0001 \quad (3.27)$$

$i = 3$

$$c_3^{n+1} = \frac{1}{2}(2\alpha + \beta)c_2^n + (1 - 2\alpha)c_3^n + \frac{1}{2}(2\alpha - \beta)c_4^n + Q_3^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008$$

$$\beta = u(x)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{x}{100}\right)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{(3)(0.25)}{100}\right)\frac{0.01}{0.25} = 0.0037$$

$$c_3^{n+1} = 0.00985c_2^n + 0.984c_3^n + 0.00615c_4^n + 0.0001 \quad (3.28)$$

$i = 4$

$$c_4^{n+1} = \frac{1}{2}(2\alpha + \beta)c_3^n + (1 - 2\alpha)c_4^n + \frac{1}{2}(2\alpha - \beta)c_5^n + Q_4^n$$

$$\alpha = D\frac{\Delta t}{(\Delta x)^2} = 0.05\frac{0.01}{(0.25)^2} = 0.008$$

$$\beta = u(x)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{x}{100}\right)\frac{\Delta t}{\Delta x} = \left(0.1 - \frac{(4)(0.25)}{100}\right)\frac{0.01}{0.25} = 0.0036$$

$$c_4^{n+1} = 0.0098c_3^n + 0.984c_4^n + 0.0062c_5^n + 0.0001 \quad (3.29)$$

$n = 0,\ i = 1 \quad c_1^1 = 0.00995c_0^0 + 0.984c_1^0 + 0.00605c_2^0 + 0.0001 = 1.186113$

$i = 2 \quad c_2^1 = 0.0099c_1^0 + 0.984c_2^0 + 0.0061c_3^0 + 0.0001 = 1.249100$

$i = 3 \quad c_3^1 = 0.00985c_2^0 + 0.984c_3^0 + 0.00615c_4^0 + 0.0001 = 1.187063$

$i = 4 \quad c_4^1 = 0.0098c_3^0 + 0.984c_4^0 + 0.0062c_5^0 + 0.0001 = 1.187063$

and apply boundary condition $c_5^0 = c_1\Delta x + c_4^0$

$c_4^1 = 0.0098c_3^0 + 0.984c_4^0 + 0.0062\left(c_1\Delta x + c_4^0\right) + 0.0001$

$n = 1,\ i = 1 \quad c_1^2 = 0.00995c_0^1 + 0.984c_1^1 + 0.00605c_2^1 + 0.0001 = 1.184742$

$i = 2 \quad c_2^2 = 0.0099c_1^1 + 0.984c_2^1 + 0.0061c_3^1 + 0.0001 = 1.248198$

$i=3 \quad c_3^2 = 0.00985c_2^1 + 0.984c_3^1 + 0.00615c_4^1 + 0.0001 = 1.186635$

$i=4 \quad c_4^2 = 0.0098c_3^1 + 0.984c_4^1 + 0.0062c_5^1 + 0.0001$

and apply boundary condition $c_5^1 = c_1\Delta x + c_4^1$

$$c_4^2 = 0.0098c_3^1 + 0.984c_4^1 + 0.0062\left(c_1\Delta x + c_4^1\right) + 0.0001$$

$n=2,\ i=1 \quad c_1^3 = 0.00995c_0^2 + 0.984c_1^2 + 0.00605c_2^2 + 0.0001 = 1.183387$

$i=2 \quad c_2^3 = 0.0099c_1^2 + 0.984c_2^2 + 0.0061c_3^2 + 0.0001 = 1.247294$

$i=3 \quad c_3^3 = 0.00985c_2^2 + 0.984c_3^2 + 0.00615c_4^2 + 0.0001 = 1.186217$

$i=4 \quad c_4^3 = 0.0098c_3^2 + 0.984c_4^2 + 0.0062c_4^2 + 0.0001$

and apply boundary condition $c_5^2 = c_1\Delta x + c_4^2$

$$c_4^3 = 0.0098c_3^2 + 0.984c_4^2 + 0.0062\left(c_1\Delta x + c_4^2\right) + 0.0001$$

Table 3.5 The calculated pollutant concentration of example 3.2.9 with the absorbance boundary ($c_1 = -0.1$), non-slipping boundary ($c_1 = 0$) or releasing boundary ($c_1 = 0.1$) at the exit

Point	Pollutant concentration		
	$c_1 = -0.1$	$c_1 = 0$	$c_1 = 0.1$
c_0^0	1.000000	1.000000	1.000000
c_1^0	1.187500	1.187500	1.187500
c_2^0	1.250000	1.250000	1.250000
c_3^0	1.187500	1.187500	1.187500
c_4^0	1.000000	1.000000	1.000000
c_0^1	1.000000	1.000000	1.000000
c_1^1	1.186113	1.186113	1.186113
c_2^1	1.249100	1.249100	1.249100
c_3^1	1.187063	1.187063	1.187063
c_4^1	1.001783	1.001938	1.002093
c_0^2	1.000000	1.000000	1.000000
c_1^2	1.184742	1.184742	1.184742
c_2^2	1.248198	1.248198	1.248198
c_3^2	1.186634	1.186635	1.186636
c_4^2	1.003543	1.003851	1.004160
c_0^3	1.000000	1.000000	1.000000
c_1^3	1.183387	1.183387	1.183387
c_2^3	1.247294	1.247294	1.247294
c_3^3	1.186214	1.186217	1.186220
c_4^3	1.005283	1.005743	1.006203

Example 3.2.10. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the open gate $\frac{\partial c}{\partial x} = c_1$,domain $\Omega = (0,1) \times (0,T)$ in Figure (3.27) with step size $\Delta x = 0.1,\ \Delta t = 0.01$, diffusion coefficient $D = 0.05$, average air pollutant source are added $Q = 0.0001$, and air flow velocity is reduced $u(x) = 0.1 - \frac{x}{100}$.

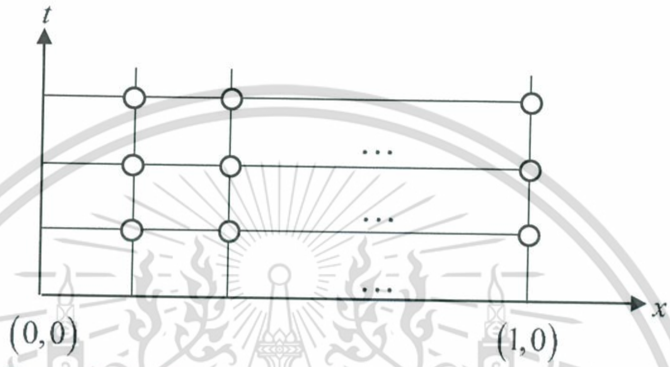


Figure 3.27: Generating grid points of example 3.2.10

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = c_0 = 1,$$

$$c_x(1,t) = c_1.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.1, \Delta t = 0.01, D = 1, u(x) = 0.1 - \frac{x}{100}$ and $Q = 0.0001$

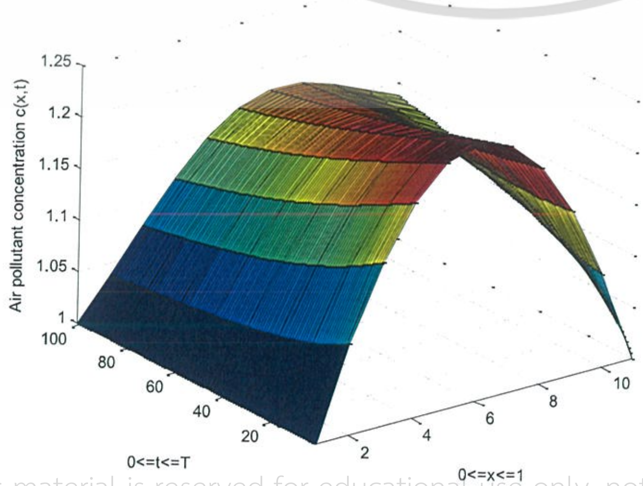


Figure 3.28: The approximated air pollutant concentration $c(x,t)$ for $c_1 = -0.1$ of example 3.2.10

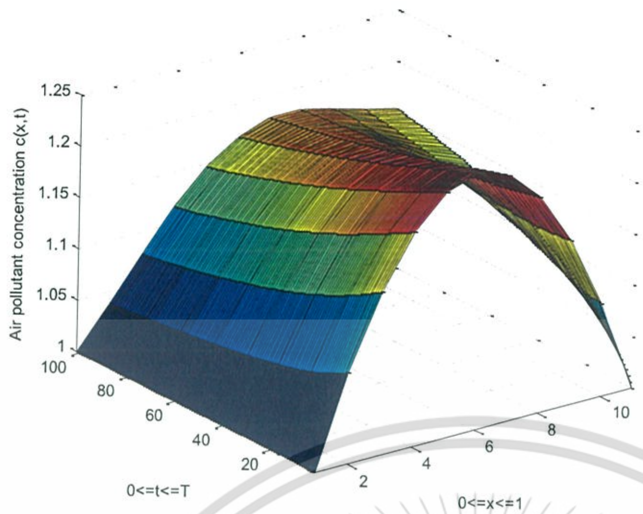


Figure 3.29:
The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0$ of example 3.2.10

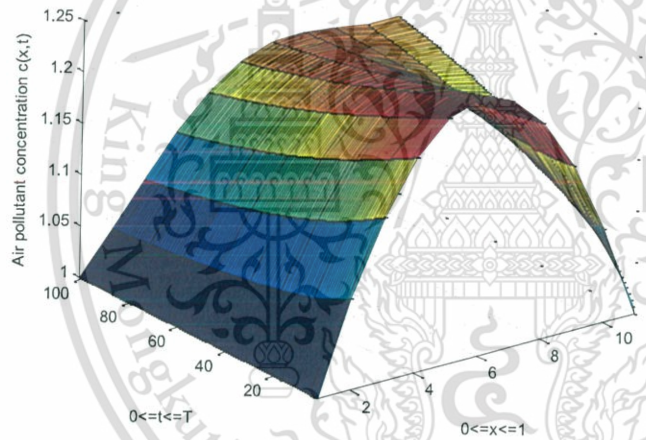
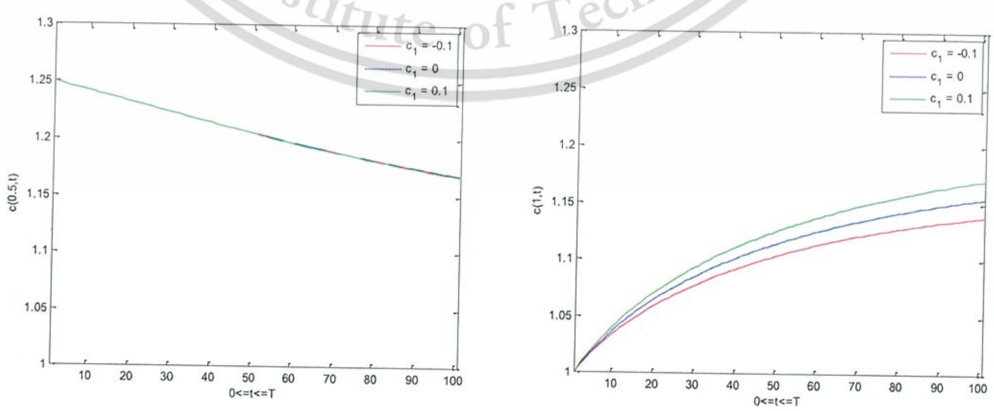


Figure 3.30:
The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0.1$ of example 3.2.10



(a) (b)

Figure 3.31: The approximated air pollutant concentration at $t = 100$ sec (a) $c(0.5,t)$ and (b) $c(1,t)$ of example 3.2.10

Example 3.2.11. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the open gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,T)$ in Figure (3.32) with step size $\Delta x = 0.25,\ \Delta t = 0.01$, diffusion coefficient $D = 0.05$, interpolated air pollutant source are added $Q = 1 + |\sin x|$, and air flow velocity is reduced $u(x) = 0.1 - \frac{x}{100}$.

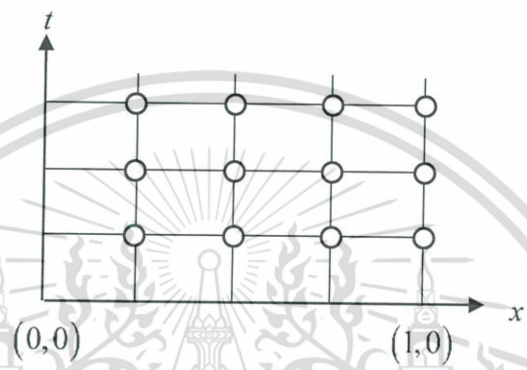


Figure 3.32: Generating grid points of example 3.2.11

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0) = x(1-x) + c_0,$$

the boundary conditions can be assumed by

$$c(0,t) = c_0 = 1,$$

$$c_x(1,t) = c_1.$$

define $c(x,t) = c_i^n$ with $\Delta x = 0.25, \Delta t = 0.01, D = 1, u(x) = 0.1 - \frac{x}{100}$ and $Q = 1 + |\sin x|$.

approximate differential boundary condition,

$c_x(1,t) = c_1$ or $\frac{\partial c_i^n}{\partial x} = c_1$,

using forward difference,

$$\frac{c_5^n - c_4^n}{\Delta x} = c_1$$

$$c_5^n = c_1 \Delta x + c_4^n \qquad i = 4 \qquad (3.30)$$

solving problem by finite difference method Eq.(3.7)

$i=1$

$$c_1^{n+1}=\frac{1}{2}(2\alpha+\beta)c_0^n+(1-2\alpha)c_1^n+\frac{1}{2}(2\alpha-\beta)c_2^n+Q_1^n$$

$$\alpha=D\frac{\Delta t}{(\Delta x)^2}=0.05\frac{0.01}{(0.25)^2}=0.008$$

$$\beta=u(x)\frac{\Delta t}{\Delta x}=\left(0.1-\frac{x}{100}\right)\frac{\Delta t}{\Delta x}=\left(0.1-\frac{(1)(0.25)}{100}\right)\frac{0.01}{0.25}=0.0039$$

$$Q_1^n=1+\left|\sin(0.25)\right|=1.247404$$

$$c_1^{n+1}=0.00995c_0^n+0.984c_1^n+0.00605c_2^n+1.247404 \qquad (3.31)$$

$i=2$

$$c_2^{n+1}=\frac{1}{2}(2\alpha+\beta)c_1^n+(1-2\alpha)c_2^n+\frac{1}{2}(2\alpha-\beta)c_3^n+Q_2^n$$

$$\alpha=D\frac{\Delta t}{(\Delta x)^2}=0.05\frac{0.01}{(0.25)^2}=0.008$$

$$\beta=u(x)\frac{\Delta t}{\Delta x}=\left(0.1-\frac{x}{100}\right)\frac{\Delta t}{\Delta x}=\left(0.1-\frac{(2)(0.25)}{100}\right)\frac{0.01}{0.25}=0.0038$$

$$Q_2^n=1+\left|\sin(0.5)\right|=1.479426$$

$$c_2^{n+1}=0.0099c_1^n+0.984c_2^n+0.0061c_3^n+1.479426 \qquad (3.32)$$

$i=3$

$$c_3^{n+1}=\frac{1}{2}(2\alpha+\beta)c_2^n+(1-2\alpha)c_3^n+\frac{1}{2}(2\alpha-\beta)c_4^n+Q_3^n$$

$$\alpha=D\frac{\Delta t}{(\Delta x)^2}=0.05\frac{0.01}{(0.25)^2}=0.008$$

$$\beta=u(x)\frac{\Delta t}{\Delta x}=\left(0.1-\frac{x}{100}\right)\frac{\Delta t}{\Delta x}=\left(0.1-\frac{(3)(0.25)}{100}\right)\frac{0.01}{0.25}=0.0037$$

$$Q_3^n=1+\left|\sin(0.75)\right|=1.681639$$

$$c_3^{n+1}=0.00985c_2^n+0.984c_3^n+0.00615c_4^n+1.681639 \qquad (3.33)$$

$i=4$

$$c_4^{n+1}=\frac{1}{2}(2\alpha+\beta)c_3^n+(1-2\alpha)c_4^n+\frac{1}{2}(2\alpha-\beta)c_5^n+Q_4^n$$

$$\alpha=D\frac{\Delta t}{(\Delta x)^2}=0.05\frac{0.01}{(0.25)^2}=0.008$$

$$\beta=u(x)\frac{\Delta t}{\Delta x}=\left(0.1-\frac{x}{100}\right)\frac{\Delta t}{\Delta x}=\left(0.1-\frac{(4)(0.25)}{100}\right)\frac{0.01}{0.25}=0.0036$$

$$Q_4^n=1+|\sin(1)|=1.841471$$

$$c_4^{n+1}=0.0098c_3^n+0.984c_4^n+0.0062c_5^n+1.841471 \tag{3.34}$$

$n=0,\ i=1$ $\quad c_1^1=0.00995c_0^0+0.984c_1^0+0.00605c_2^0+1.247404$

$i=2$ $\quad c_2^1=0.0099c_1^0+0.984c_2^0+0.0061c_3^0+1.479426$

$i=3$ $\quad c_3^1=0.00985c_2^0+0.984c_3^0+0.00615c_4^0+1.681639$

$i=4$ $\quad c_4^1=0.0098c_3^0+0.984c_4^0+0.0062c_5^0+1.841471$

and apply boundary condition $c_5^0=c_1\Delta x+c_4^0$

$$c_4^1=0.0098c_3^0+0.984c_4^0+0.0062\left(c_1\Delta x+c_4^0\right)+1.841471$$

$n=1,\ i=1$ $\quad c_1^2=0.00995c_0^1+0.984c_1^1+0.00605c_2^1+1.247404$

$i=2$ $\quad c_2^2=0.0099c_1^1+0.984c_2^1+0.0061c_3^1+1.479426$

$i=3$ $\quad c_3^2=0.00985c_2^1+0.984c_3^1+0.00615c_4^1+1.681639$

$i=4$ $\quad c_4^2=0.0098c_3^1+0.984c_4^1+0.0062c_5^1+1.841471$

and apply boundary condition $c_5^1=c_1\Delta x+c_4^1$

$$c_4^2=0.0098c_3^1+0.984c_4^1+0.0062\left(c_1\Delta x+c_4^1\right)+1.841471$$

$n=2,\ i=1$ $\quad c_1^3=0.00995c_0^2+0.984c_1^2+0.00605c_2^2+1.247404$

$i=2$ $\quad c_2^3=0.0099c_1^2+0.984c_2^2+0.0061c_3^2+1.479426$

$i=3$ $\quad c_3^3=0.00985c_2^2+0.984c_3^2+0.00615c_4^2+1.681639$

$i=4$ $\quad c_4^3=0.0098c_3^2+0.984c_4^2+0.0062c_4^2+1.841471$

and apply boundary condition $c_5^2=c_1\Delta x+c_4^2$

$$c_4^3=0.0098c_3^2+0.984c_4^2+0.0062\left(c_1\Delta x+c_4^2\right)+1.841471$$

Table 3.6 The calculated pollutant concentration of example 3.2.11 with the absorbance boundary ($c_1 = -0.1$), non-slipping boundary ($c_1 = 0$) or releasing boundary ($c_1 = 0.1$) at the exit

Point	Pollutant concentration		
	$c_1 = -0.01$	$c_1 = 0$	$c_1 = 0.01$
c_0^0	1.000000	1.000000	1.000000
c_1^0	1.187500	1.187500	1.187500
c_2^0	1.250000	1.250000	1.250000
c_3^0	1.187500	1.187500	1.187500
c_4^0	1.000000	1.000000	1.000000
c_0^1	1.000000	1.000000	1.000000
c_1^1	2.433417	2.433417	2.433417
c_2^1	2.728426	2.728426	2.728426
c_3^1	2.868602	2.868602	2.868602
c_4^1	2.843154	2.843309	2.843464
c_0^2	1.000000	1.000000	1.000000
c_1^2	3.668343	3.668343	3.668343
c_2^2	4.205787	4.205787	4.205787
c_3^2	4.548703	4.548704	4.548705
c_4^2	4.684719	4.685027	4.685336
c_0^3	1.000000	1.000000	1.000000
c_1^3	4.892448	4.892448	4.892448
c_2^3	5.681984	5.681984	5.681984
c_3^3	6.227801	6.227804	6.227807
c_4^3	6.524702	6.525162	6.525623

Example 3.2.12. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the open gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,T)$ in Figure (3.33) with step size $\Delta x = 0.1,\ \Delta t = 0.01$, diffusion coefficient $D = 0.05$, interpolated air pollutant source are added $Q = 1 + |\sin x|$, and air flow velocity is reduced $u(x) = 0.1 - \frac{x}{100}$.

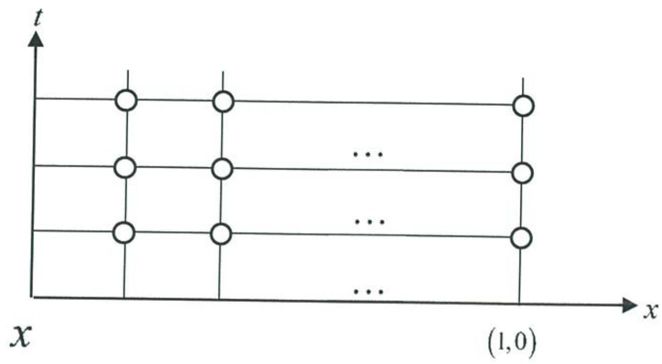


Figure 3.33: Generating grid points of example 3.2.12

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.6) as

$$c(x,0)=x(1-x)+c_0,$$

the boundary conditions can be assumed by

$$c(0,t)=c_0=1,$$

$$c_x(1,t)=c_1.$$

define $c(x,t)=c_i^n$ with $\Delta x=0.05, \Delta t=0.01, D=1, u(x)=0.1-\frac{x}{100}$, and $Q=1+|\sin x|$.

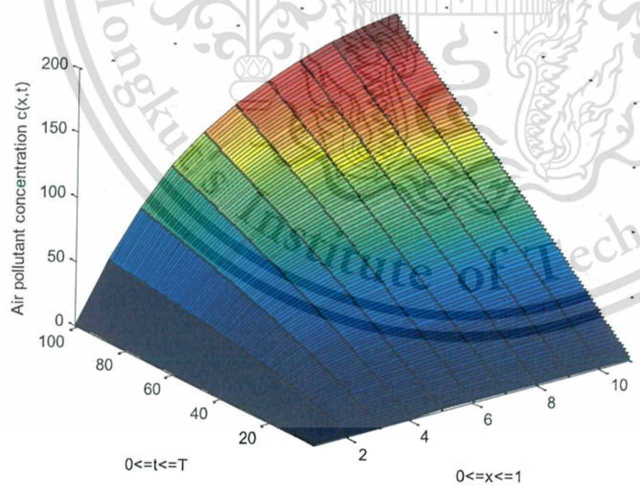


Figure 3.34: The approximated air pollutant concentration $c(x,t)$ for $c_1=-0.1$ of example 3.2.12

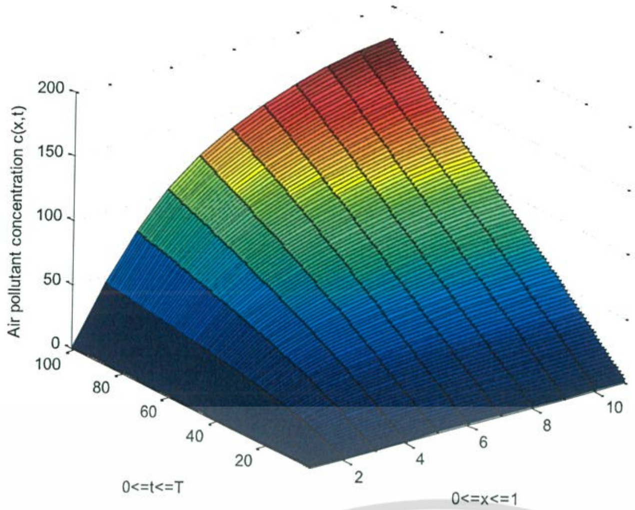


Figure 3.35: The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0$ of example 3.2.12

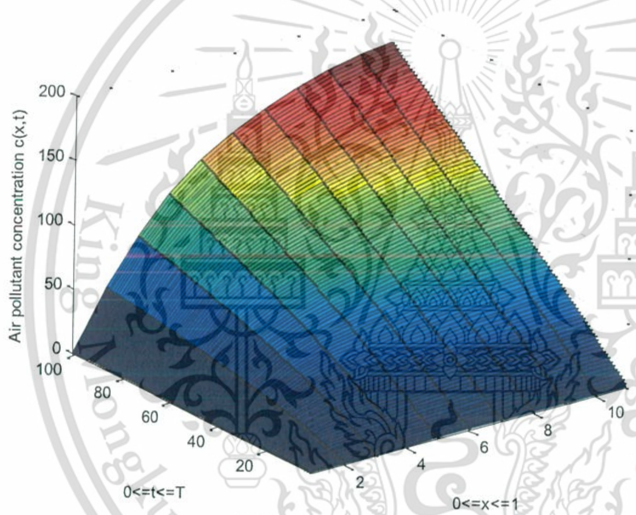
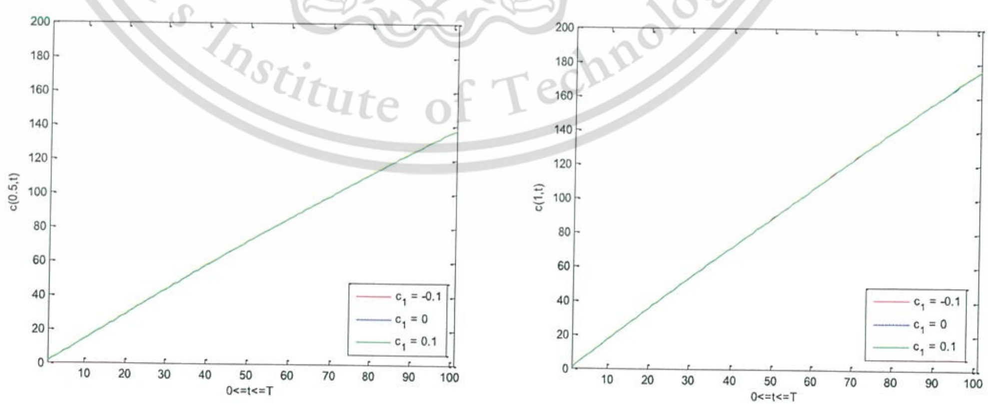


Figure 3.36: The approximated air pollutant concentration $c(x,t)$ for $c_1 = 0.1$ of example 3.2.12



(a)

(b)

Figure 3.37: The approximated air pollutant concentration at $t = 100$ sec (a) $c(0.5,t)$ and (b) $c(1,t)$ of example 3.2.12

Table 3.7 Simulation of example for one dimensional in space (along x-axis)

Simulations	u	Q	RHS		
			- 0.1	0	0.1
3.2.1	K	K	✓	✓	✓
3.2.2	K	K	✓	✓	✓
3.2.3	K	K	✓	✓	✓
3.2.4	K	K	✓	✓	✓
3.2.5	K	f_Q	✓	✓	✓
3.2.6	K	f_Q	✓	✓	✓
3.2.7	f_u	K	✓	✓	✓
3.2.8	f_u	K	✓	✓	✓
3.2.9	f_u	K	✓	✓	✓
3.2.10	f_u	K	✓	✓	✓
3.2.11	f_u	f_Q	✓	✓	✓
3.2.12	f_u	f_Q	✓	✓	✓

where

f_u = function of interpolated air pollutant sources

f_Q = function of interpolated air flow velocity

K = constant

To conclude, we can see that the changing rate of air pollutant concentration will be decreased when $c_1 = -0.1$ and there is not changing rate of air pollutant concentration when $c_1 = 0$ and the changing rate of air pollutant concentration will be increased when $c_1 = 0.1$

Chapter 4

Numerical Methods to a Two Dimensional Air Pollution Measurement Models in Street Canyons

In this section, numerical methods are used to solve the governing equation of mathematical models, we use explicit finite different methods for solving advection-diffusion equation. In the final part of this section, the examples used to calculate air pollutant concentration in street canyon.

4.1 Numerical method to a two-dimensional form (xz) air pollutant dispersion model with insided air pollutant source

We use the forward differenced in time and central difference in space in advection diffusion equation. We can approximate $c_{i,j}^n$ are the values difference approximation of at point $x = i\Delta x, z = j\Delta z$ and $t = n\Delta t$ where $0 \le i \le L, 0 \le j \le H$ and $0 \le n \le N$. Using the forward time center space method to Eq. (2.7), the following finite difference equation can be obtained

$$c \cong c_{i,j}^n \tag{4.1}$$

$$\frac{\partial c}{\partial t} \cong \frac{c_{i,j}^{n+1} - c_{i,j}^n}{\Delta t}, \tag{4.2}$$

$$\frac{\partial c}{\partial x} \cong \frac{c_{i+1,j}^n - c_{i-1,j}^n}{2\Delta x}, \tag{4.3}$$

$$\frac{\partial^2 c}{\partial x^2} \cong \frac{c_{i+1,j}^n - 2c_{i,j}^n + c_{i-1,j}^n}{(\Delta x)^2}, \tag{4.4}$$

$$\frac{\partial^2 c}{\partial z^2} \cong \frac{c_{i,j+1}^n - 2c_{i,j}^n + c_{i,j-1}^n}{(\Delta z)^2}. \tag{4.5}$$

$$Q \cong Q_{i,j}^n \tag{4.6}$$

Substituting Eq. (4.1-4.6) into Eq. (2.7), we have

$$\left(\frac{c_{i,j}^{n+1} - c_{i,j}^n}{\Delta t}\right) + u\left(\frac{c_{i+1,j}^n - c_{i-1,j}^n}{2\Delta x}\right) = D_{xz}\left(\frac{c_{i+1,j}^n - 2c_{i,j}^n + c_{i-1,j}^n}{(\Delta x)^2} + \frac{c_{i,j+1}^n - 2c_{i,j}^n + c_{i,j-1}^n}{(\Delta z)^2}\right) + Q_{i,j}^n \tag{4.7}$$

We can obtain a simply form of Eq. (4.7)

$$c_{i,j}^{n+1} = \frac{1}{2}(2\alpha + \beta)c_{i-1,j}^n + \gamma c_{i,j-1}^n + (1 - 2\alpha - 2\gamma)c_{i,j}^n + \gamma c_{i,j+1}^n + \frac{1}{2}(2\alpha - \beta)c_{i+1,j}^n + Q_{i,j}^n$$

(4.8)

where

$$\alpha = D_{xz} \frac{\Delta t}{(\Delta x)^2}, \tag{4.9}$$

$$\beta = u \frac{\Delta t}{\Delta x}, \tag{4.10}$$

$$\gamma = D_{xz} \frac{\Delta t}{(\Delta z)^2}. \tag{4.11}$$

If $c_{i,j}^n$ are lied on the boundary of the domain, we will approximate by using the boundary conditions and employing the forward and backward different scheme,

$$\frac{\partial c}{\partial x} \approx \frac{c_{i+1,j}^n - c_{i,j}^n}{\Delta x}, \tag{4.12}$$

$$\frac{\partial c}{\partial x} \approx \frac{c_{i,j+1}^n - c_{i,j}^n}{\Delta x}, \tag{4.13}$$

$$\frac{\partial c}{\partial z} \approx \frac{c_{i,j}^n - c_{i-1,j}^n}{\Delta z}, \tag{4.14}$$

$$\frac{\partial c}{\partial z} \approx \frac{c_{i,j}^n - c_{i,j-1}^n}{\Delta z}. \tag{4.15}$$

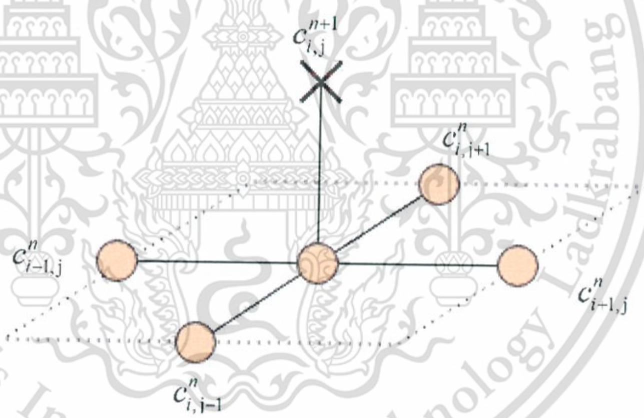


Figure 4.1: The stencil diagram of two-dimensional dispersion model (xz)

we will transform Eq. (2.7) into non-dimensional form below,

$$\frac{1}{st}\frac{\partial C}{\partial T} + U\frac{\partial C}{\partial X} = D\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Z^2}\right) + S \tag{4.16}$$

for all $(X,Z,T) \in [0,1] \times [0,H/L] \times [0,\mathrm{T}]$.

where

$$st = t_{\max} u_{\max} / L, l = \max\{L,H\}, U = u / u_{\max}, D = D_{xz} / Pe, Pe = u_{\max} L / D_{xz},$$
$$X = x / L, Z = z / L, C = c / c_{\max}, S = Q(l / c_{\max} u_{\max}), T = t / t_{\max}$$

we now discretize Eq. (4.16) by dividing the interval $[0,1]$ into M subintervals such that $M\Delta X = 1$ and interval $[0,\mathrm{T}]$ into N subintervals such that $N\Delta T = T$, and similarly defined for $[0, H/L]$. We can then approximate $C(X_i, Z_j, T_n)$ by $C_{i,j}^n$ value of the difference approximation of $C(X,Z,T)$ at point $X = i\Delta X, Z = j\Delta Z$ and $T = n\Delta T,$ where $0 \le i \le M, 0 \le j \le P$ and $0 \le n \le N.$ The grid point (X_i, Z_j, T_n) is defined by $X_i = i\Delta X$ for all $i = 0,1,2,\ldots,M$ and $Z_j = j\Delta Z$ for all $j = 0,1,2,\ldots,P$ and $T_n = n\Delta T$ for all $n = 0,1,2,\ldots,N$ in which M, P and N are positive integers. Using the forward time center space method to Eq. (4.16), the following finite difference equation can be obtained:

$$C \cong C_{i,j}^n, \tag{4.17}$$

$$\frac{\partial C}{\partial T} \cong \frac{C_{i,j}^{n+1} - C_{i,j}^n}{\Delta T}, \tag{4.18}$$

$$\frac{\partial C}{\partial X} \cong \frac{C_{i+1,j}^n - C_{i-1,j}^n}{2\Delta X}, \tag{4.19}$$

$$\frac{\partial^2 C}{\partial X^2} \cong \frac{C_{i+1,j}^n - 2C_{i,j}^n + C_{i-1,j}^n}{(\Delta X)^2}, \tag{4.20}$$

$$\frac{\partial^2 C}{\partial Z^2} \cong \frac{C_{i,j+1}^n - 2C_{i,j}^n + C_{i,j-1}^n}{(\Delta Z)^2}, \tag{4.21}$$

$$S \cong S_{i,j}^n \tag{4.22}$$

Substituting Eq. (4.17-4.22) into Eq. (4.16), we can obtain an explicit form of finite difference equation as follows,

$$C_{i,j}^{n+1} = A_1 C_{i-1,j}^n + A_2 C_{i,j-1}^n + A_3 C_{i,j}^n + A_4 C_{i+1,j}^n + A_5 C_{i,j+1}^n + S_{i,j}^n, \tag{4.23}$$

Where $A_1 = D\dfrac{\Delta T}{(\Delta X)^2} + U\dfrac{\Delta T}{2\Delta X},$ (4.24)

$$A_2 = D\frac{\Delta T}{(\Delta Z)^2}, \tag{4.25}$$

$$A_3 = 1 - 2D\frac{\Delta T}{(\Delta X)^2} - 2D\frac{\Delta T}{(\Delta Z)^2}, \tag{4.26}$$

$$A_4 = D\frac{\Delta T}{(\Delta X)^2} - U\frac{\Delta T}{2\Delta X}, \tag{4.27}$$

$$A_5 = D\frac{\Delta T}{(\Delta Z)^2}. \tag{4.28}$$

If $C_{i,j}^n$ are lied on the boundary of the domain, we will approximate by using the boundary conditions and employing the forward and backward different scheme,

$$\frac{\partial C}{\partial X} \approx \frac{C_{i+1,j}^n - C_{i,j}^n}{\Delta X}, \tag{4.29}$$

$$\frac{\partial C}{\partial X} \approx \frac{C_{i,j+1}^n - C_{i,j}^n}{\Delta X}, \tag{4.30}$$

$$\frac{\partial C}{\partial Z} \approx \frac{C_{i,j}^{n} - C_{i-1,j}^{n}}{\Delta Z}, \tag{4.31}$$

$$\frac{\partial C}{\partial Z} \approx \frac{C_{i,j}^{n} - C_{i,j-1}^{n}}{\Delta Z}. \tag{4.32}$$

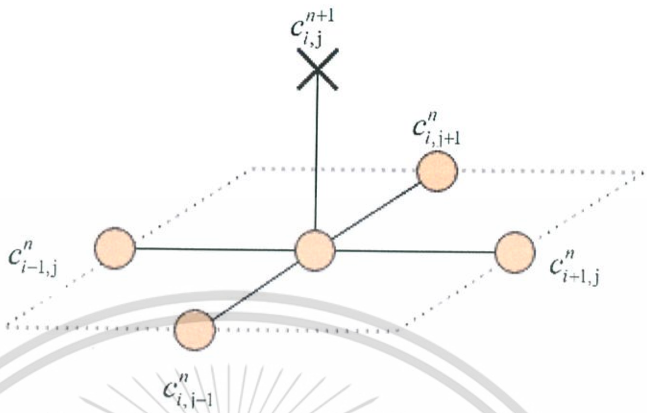


Figure 4.2: The stencil diagram of non-dimensional dispersion model

4.2 Numerical simulation to a two-dimensional form (xz) air pollutant dispersion model with insided air pollutant source

In this section, we show example to find the pollutant concentration at any point is calculated by dispersion model in two dimensional.

Example 4.2.1. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, domain $\Omega = (0,1) \times (0,1)$ in Figure (4.3) with step size $\Delta x = \Delta z = 0.25$, $\Delta t = 0.01$ diffusion coefficient $D_{xz} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, and average air flow velocity $u = 0.1$.

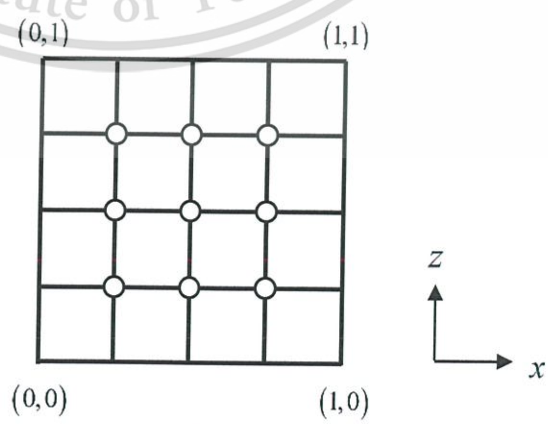


Figure 4.3: Generating grid points of example 4.2.1

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0)=x(1-x)z(1-z)+c_0,$$

the boundary conditions can be assumed by

$$c(0,z,t)=c_0=0,$$

define $c(x,z,t)=c_{i,j}^n$ with $\Delta x=\Delta z=0.25, \Delta t=0.01, D=0.1,$ and $u=0.1.$

solving problem by finite difference method Eq.(4.8)

$$c_{i,j}^{n+1}=\frac{1}{2}(2\alpha+\beta)c_{i-1,j}^n+\gamma c_{i,j-1}^n+(1-2\alpha-2\gamma)c_{i,j}^n+\gamma c_{i,j+1}^n+\frac{1}{2}(2\alpha-\beta)c_{i+1,j}^n+Q_{i,j}^n$$

$$\alpha=D_{xz}\frac{\Delta t}{(\Delta x)^2}=0.1\frac{0.01}{(0.25)^2}=0.016,$$

$$\beta=u\frac{\Delta t}{\Delta x}=0.1\frac{0.01}{0.25}=0.004,$$

$$\gamma=D_{xz}\frac{\Delta t}{(\Delta z)^2}=0.1\frac{0.01}{(0.25)^2}=0.016.$$

$$c_{i,j}^{n+1}=0.018c_{i-1,j}^n+0.016c_{i,j-1}^n+0.936c_{i,j}^n+0.016c_{i,j+1}^n+0.014c_{i+1,j}^n+Q_{i,j}^n$$

$$n=0,1,2,\ldots,\quad i=1,2,3,\quad j=1,2,3 \qquad (4.33)$$

$n=0,$

$j=1,\ i=1\quad c_{1,1}^1=0.018c_{0,1}^0+0.016c_{1,0}^0+0.936c_{1,1}^0+0.016c_{1,2}^0+0.014c_{2,1}^0+Q_{1,1}^0$

$i=2\quad c_{2,1}^1=0.018c_{1,1}^0+0.016c_{2,0}^0+0.936c_{2,1}^0+0.016c_{2,2}^0+0.014c_{3,1}^0+Q_{2,1}^0$

$i=3\quad c_{3,1}^1=0.018c_{2,1}^0+0.016c_{3,0}^0+0.936c_{3,1}^0+0.016c_{3,2}^0+0.014c_{4,1}^0+Q_{3,1}^0$

$j=2,\ i=1\quad c_{1,2}^1=0.018c_{0,2}^0+0.016c_{1,1}^0+0.936c_{1,2}^0+0.016c_{1,3}^0+0.014c_{2,2}^0+Q_{1,2}^0$

$i=2\quad c_{2,2}^1=0.018c_{1,2}^0+0.016c_{2,1}^0+0.936c_{2,2}^0+0.016c_{2,3}^0+0.014c_{3,2}^0+Q_{2,2}^0$

$i=3\quad c_{3,2}^1=0.018c_{2,2}^0+0.016c_{3,1}^0+0.936c_{3,2}^0+0.016c_{3,3}^0+0.014c_{4,2}^0+Q_{3,2}^0$

$j=3,\ i=1\quad c_{1,3}^1=0.018c_{0,3}^0+0.016c_{1,2}^0+0.936c_{1,3}^0+0.016c_{1,4}^0+0.014c_{2,3}^0+Q_{1,3}^0$

$i=2\quad c_{2,3}^1=0.018c_{1,3}^0+0.016c_{2,2}^0+0.936c_{2,3}^0+0.016c_{2,4}^0+0.014c_{3,3}^0+Q_{2,3}^0$

$i=3\quad c_{3,3}^1=0.018c_{2,3}^0+0.016c_{3,2}^0+0.936c_{3,3}^0+0.016c_{3,4}^0+0.014c_{4,3}^0+Q_{3,3}^0$

Table 4.1 The calculated pollutant concentration at time $t = 0.02$ of example 4.2.1

Point	Pollutant concentration	
	$Q = 0$	$Q = 0.0001$
$c_{0,0}^2$	0.000000	0.000000
$c_{0,1}^2$	0.000000	0.000000
$c_{0,2}^2$	0.000000	0.000000
$c_{0,3}^2$	0.000000	0.000000
$c_{0,4}^2$	0.000000	0.000000
$c_{1,0}^2$	0.000000	0.000000
$c_{1,1}^2$	0.033494	0.033691
$c_{1,2}^2$	0.045141	0.045339
$c_{1,3}^2$	0.033858	0.034055
$c_{1,4}^2$	0.000000	0.000000
$c_{2,0}^2$	0.000000	0.000000
$c_{2,1}^2$	0.044898	0.045096
$c_{2,2}^2$	0.060507	0.060707
$c_{2,3}^2$	0.045383	0.045582
$c_{2,4}^2$	0.000000	0.000000
$c_{3,0}^2$	0.000000	0.000000
$c_{3,1}^2$	0.033494	0.033691
$c_{3,2}^2$	0.045141	0.045339
$c_{3,3}^2$	0.033858	0.034055
$c_{3,4}^2$	0.000000	0.000000
$c_{4,0}^2$	0.000000	0.000000
$c_{4,1}^2$	0.000000	0.000000
$c_{4,2}^2$	0.000000	0.000000
$c_{4,3}^2$	0.000000	0.000000
$c_{4,4}^2$	0.000000	0.000000

Example 4.2.2. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, domain $\Omega = (0,1)\times(0,1)$ in Figure (4.4) with step size $\Delta x = \Delta z = 0.1$, $\Delta t = 0.01$ diffusion coefficient $D_{xz} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, and average air flow velocity $u = 0.1$.

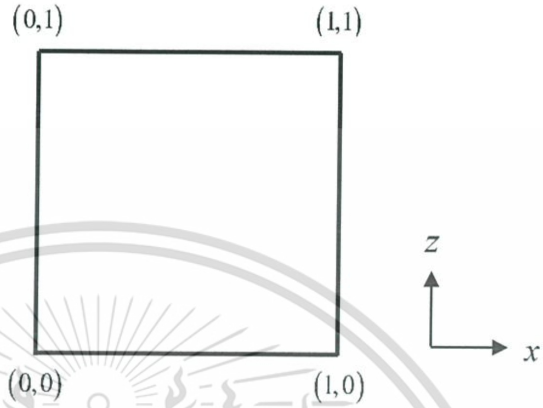


Figure 4.4: Domain of example 4.2.2

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0) = x(1-x)z(1-z) + c_0,$$

the boundary conditions can be assumed by

$$c(0,z,t) = c_0 = 0,$$

define $c(x,z,t) = c_{i,j}^n$ with $\Delta x = \Delta z = 0.1, \Delta t = 0.01, D = 0.1,$ and $u = 0.1$.

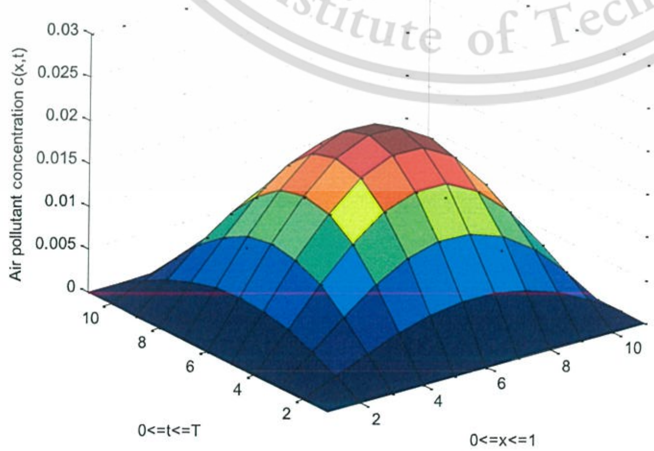


Figure 4.5: The approximated air pollutant concentration for $Q = 0$ and $0 \le t \ge 60$ sec of example 4.2.2

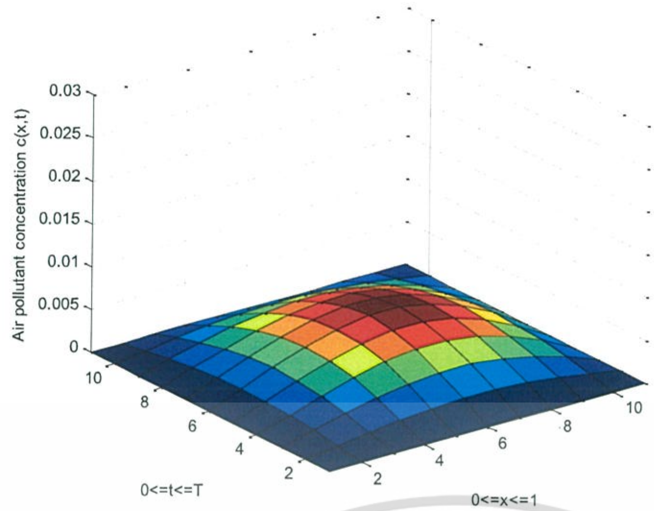


Figure 4.6: The approximated air pollutant concentration for $Q=0$ and $0 \leq t \geq 120$ sec of example 4.2.2

Figure 4.7: The approximated air pollutant concentration for $Q=0.0001$ and $0 \leq t \geq 60$ sec of example 4.2.2

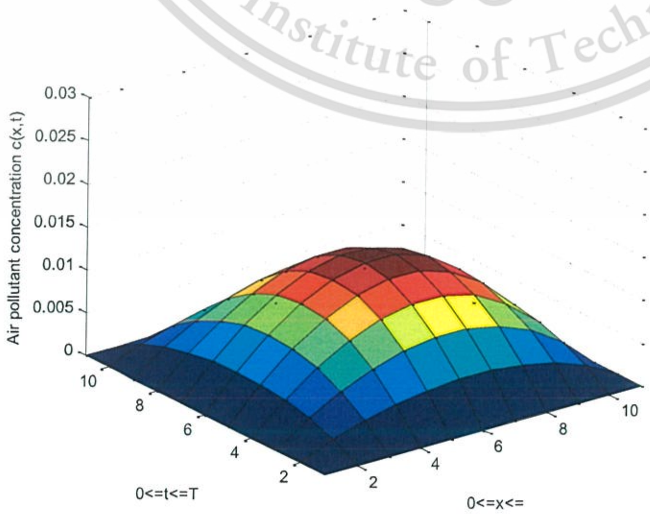


Figure 4.8: The approximated air pollutant concentration for $Q=0.0001$ and $0 \leq t \geq 120$ sec of example 4.2.2

Example 4.2.3. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.9) with step size $\Delta x = \Delta z = 0.25$, $\Delta t = 0.01$ diffusion coefficient $D_{xz} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, and average air flow velocity $u = 0.1$.

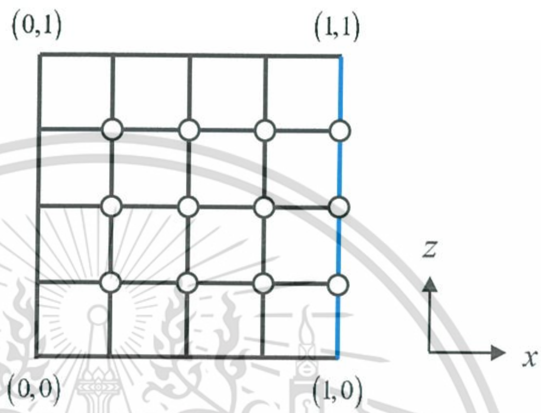


Figure 4.9: Generating grid points of example 4.2.3

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0) = x(1-x)z(1-z) + c_0,$$

the boundary conditions can be assumed by

$$c(0,z,t) = c_0 = 0,$$

$$c_x(1,z,t) = c_1 = 0$$

define $c(x,z,t) = c_{i,j}^n$ with $\Delta x = \Delta z = 0.25, \Delta t = 0.01, D = 0.1,$ and $u = 0.1$.

approximate differential boundary condition,

$c_x(1,z,t) = 0$ or $\frac{\partial c_{i.j}^n}{\partial x} = 0,$

using forward difference,

$$\frac{c_{5,j}^n - c_{4,j}^n}{\Delta x} = 0$$

$$c_{5,j}^n = c_{4,j}^n \qquad j = 1,2,3 \tag{4.34}$$

solving problem by finite difference method Eq.(4.8)

$$c_{i,j}^{n+1} = \frac{1}{2}(2\alpha+\beta)c_{i-1,j}^n + \gamma c_{i,j-1}^n + (1-2\alpha-2\gamma)c_{i,j}^n + \gamma c_{i,j+1}^n + \frac{1}{2}(2\alpha-\beta)c_{i+1,j}^n + Q_{i,j}^n$$

$$\alpha = D_{xz}\frac{\Delta t}{(\Delta x)^2} = 0.1\frac{0.01}{(0.25)^2} = 0.016,$$

$$\beta = u\frac{\Delta t}{\Delta x} = 0.1\frac{0.01}{0.25} = 0.004,$$

$$\gamma = D_{xz}\frac{\Delta t}{(\Delta z)^2} = 0.1\frac{0.01}{(0.25)^2} = 0.016.$$

$$c_{i,j}^{n+1} = 0.018c_{i-1,j}^{n} + 0.016c_{i,j-1}^{n} + 0.936c_{i,j}^{n} + 0.016c_{i,j+1}^{n} + 0.014c_{i+1,j}^{n} + Q_{i,j}^{n}$$

$$i = 1,2,3,4, \quad j = 1,2,3 \qquad (4.35)$$

$n = 0,$

$j = 1, \quad i = 1 \quad c_{1,1}^{1} = 0.018c_{0,1}^{0} + 0.016c_{1,0}^{0} + 0.936c_{1,1}^{0} + 0.016c_{1,2}^{0} + 0.014c_{2,1}^{0} + Q_{1,1}^{0}$

$i = 2 \quad c_{2,1}^{1} = 0.018c_{1,1}^{0} + 0.016c_{2,0}^{0} + 0.936c_{2,1}^{0} + 0.016c_{2,2}^{0} + 0.014c_{3,1}^{0} + Q_{2,1}^{0}$

$i = 3 \quad c_{3,1}^{1} = 0.018c_{2,1}^{0} + 0.016c_{3,0}^{0} + 0.936c_{3,1}^{0} + 0.016c_{3,2}^{0} + 0.014c_{4,1}^{0} + Q_{3,1}^{0}$

$i = 4 \quad c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.936c_{4,1}^{0} + 0.016c_{4,2}^{0} + 0.014c_{5,1}^{0} + Q_{4,1}^{0}$

and apply boundary condition $c_{5,1}^{0} = c_{4,1}^{0}$

$c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.95c_{4,1}^{0} + 0.016c_{4,2}^{0} + Q_{4,1}^{0}$

$j = 2, \quad i = 1 \quad c_{1,2}^{1} = 0.018c_{0,2}^{0} + 0.016c_{1,1}^{0} + 0.936c_{1,2}^{0} + 0.016c_{1,3}^{0} + 0.014c_{2,2}^{0} + Q_{1,2}^{0}$

$i = 2 \quad c_{2,2}^{1} = 0.018c_{1,2}^{0} + 0.016c_{2,1}^{0} + 0.936c_{2,2}^{0} + 0.016c_{2,3}^{0} + 0.014c_{3,2}^{0} + Q_{2,2}^{0}$

$i = 3 \quad c_{3,2}^{1} = 0.018c_{2,2}^{0} + 0.016c_{3,1}^{0} + 0.936c_{3,2}^{0} + 0.016c_{3,3}^{0} + 0.014c_{4,2}^{0} + Q_{3,2}^{0}$

$i = 4 \quad c_{4,2}^{1} = 0.018c_{3,2}^{0} + 0.016c_{4,1}^{0} + 0.936c_{4,2}^{0} + 0.016c_{4,3}^{0} + 0.014c_{5,2}^{0} + Q_{4,2}^{0}$

and apply boundary condition $c_{5,2}^{0} = c_{4,2}^{0}$

$c_{4,2}^{1} = 0.018c_{3,2}^{0} + 0.016c_{4,1}^{0} + 0.95c_{4,2}^{0} + 0.016c_{4,3}^{0} + Q_{4,2}^{0}$

$j = 3, \quad i = 1 \quad c_{1,3}^{1} = 0.018c_{0,3}^{0} + 0.016c_{1,2}^{0} + 0.936c_{1,3}^{0} + 0.016c_{1,4}^{0} + 0.014c_{2,3}^{0} + Q_{1,3}^{0}$

$i = 2 \quad c_{2,3}^{1} = 0.018c_{1,3}^{0} + 0.016c_{2,2}^{0} + 0.936c_{2,3}^{0} + 0.016c_{2,4}^{0} + 0.014c_{3,3}^{0} + Q_{2,3}^{0}$

$i = 3 \quad c_{3,3}^{1} = 0.018c_{2,3}^{0} + 0.016c_{3,2}^{0} + 0.936c_{3,3}^{0} + 0.016c_{3,4}^{0} + 0.014c_{4,3}^{0} + Q_{3,3}^{0}$

$i = 4 \quad c_{4,3}^{1} = 0.018c_{3,3}^{0} + 0.016c_{4,2}^{0} + 0.936c_{4,3}^{0} + 0.016c_{4,4}^{0} + 0.014c_{5,3}^{0} + Q_{4,3}^{0}$

and apply boundary condition $c_{5,3}^{0} = c_{4,3}^{0}$

$c_{4,3}^{1} = 0.018c_{3,3}^{0} + 0.016c_{4,2}^{0} + 0.95c_{4,3}^{0} + 0.016c_{4,4}^{0} + Q_{4,3}^{0}$

Table 4.2 The calculated pollutant concentration at time $t = 0.02$ of example 4.2.3

Point	Pollutant concentration	
	$Q = 0$	$Q = 0.0001$
$c_{0,0}^2$	0.000000	0.000000
$c_{0,1}^2$	0.000000	0.000000
$c_{0,2}^2$	0.000000	0.000000
$c_{0,3}^2$	0.000000	0.000000
$c_{0,4}^2$	0.000000	0.000000
$c_{1,0}^2$	0.000000	0.000000
$c_{1,1}^2$	0.033494	0.033702
$c_{1,2}^2$	0.045141	0.045353
$c_{1,3}^2$	0.033867	0.034076
$c_{1,4}^2$	0.001236	0.001434
$c_{2,0}^2$	0.000000	0.000000
$c_{2,1}^2$	0.044898	0.045096
$c_{2,2}^2$	0.060507	0.060708
$c_{2,3}^2$	0.045395	0.045596
$c_{2,4}^2$	0.001652	0.001852
$c_{3,0}^2$	0.000000	0.000000
$c_{3,1}^2$	0.033494	0.033702
$c_{3,2}^2$	0.045141	0.045353
$c_{3,3}^2$	0.033867	0.034076
$c_{3,4}^2$	0.001236	0.001434
$c_{4,0}^2$	0.000000	0.000000
$c_{4,1}^2$	0.000000	0.000000
$c_{4,2}^2$	0.000000	0.000000
$c_{4,3}^2$	0.000000	0.000000
$c_{4,4}^2$	0.000000	0.000000

Example 4.2.4. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.10) with step size $\Delta x = \Delta z = 0.1$, $\Delta t = 0.01$ diffusion coefficient $D_{xz} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, and average air flow velocity $u = 0.1$.

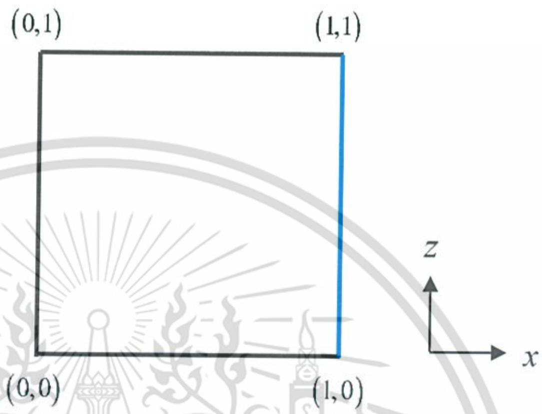


Figure 4.10: Domain of example 4.2.4

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0) = x(1-x)z(1-z) + c_0,$$

the boundary conditions can be assumed by

$$c(0,z,t) = c_0 = 0,$$

$$c_x(1,z,t) = c_1$$

define $c(x,z,t) = c_{i,j}^n$ with $\Delta x = \Delta z = 0.1, \Delta t = 0.01, D = 0.1,$ and $u = 0.1$

Table 4.3 The approximated air pollutant concentration at $0 \le t \ge 60$ sec of example 4.2.4 with the absorbance boundary ($c_1 = -0.1$), non-slipping boundary ($c_1 = 0$) or releasing boundary ($c_1 = 0.1$) at the exit

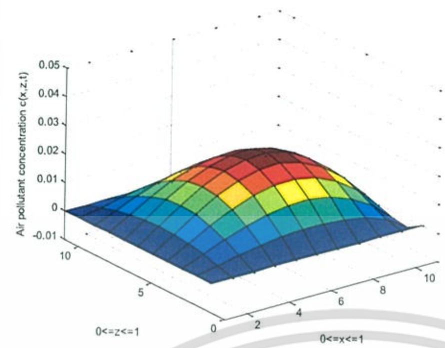
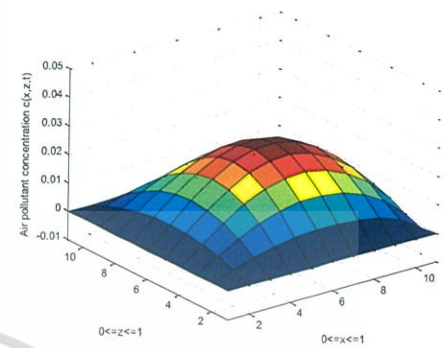
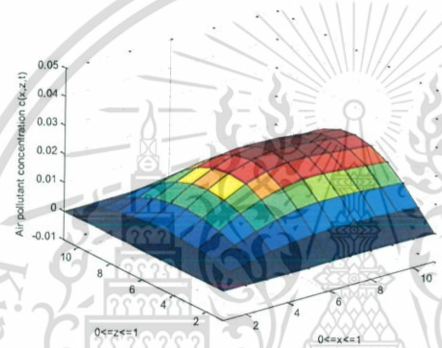
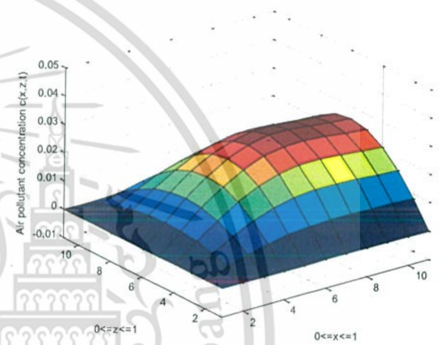
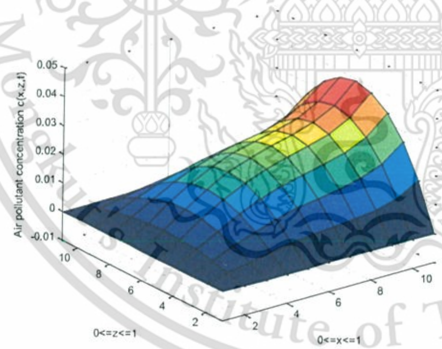
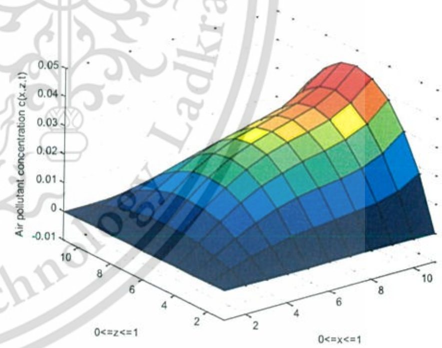
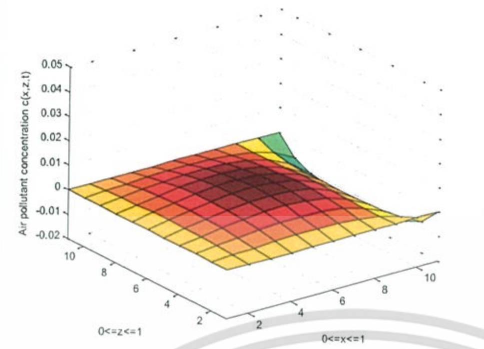
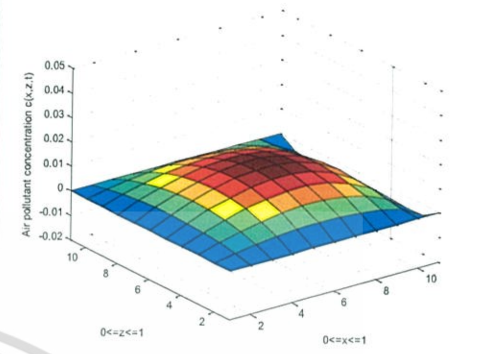
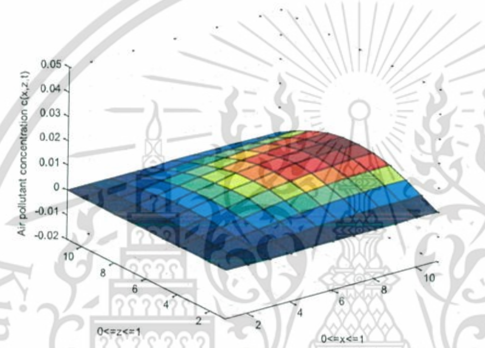
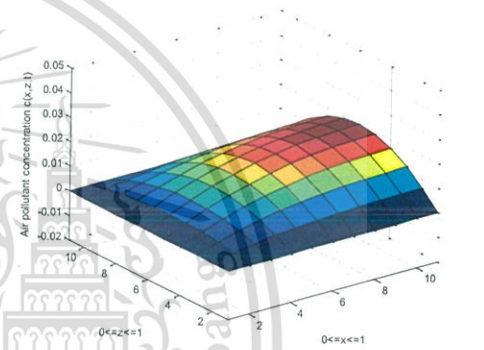
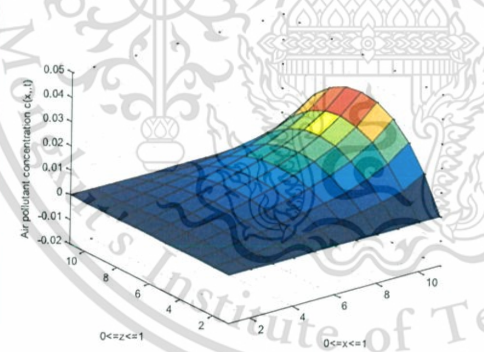
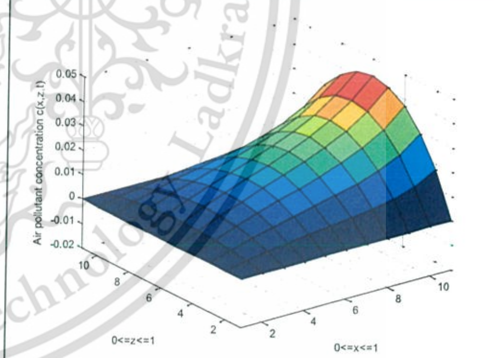
c_1	$Q = 0$	$Q = 0.0001$
-0.1		
0		
0.1		

Table 4.4 The approximated air pollutant concentration at $t=120$ sec of example 4.2.4 with the absorbance boundary ($c_1=-0.1$), non-slipping boundary ($c_1=0$) or releasing boundary ($c_1=0.1$) at the exit

c_1	$Q=0$	$Q=0.0001$
-0.1		
0		
0.1		

Example 4.2.5. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c=c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x}=c_1$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial c}{\partial z}=c_2$ domain $\Omega=(0,1)\times(0,1)$ in Figure (4.11) with step size $\Delta x=\Delta z=0.25$, $\Delta t=0.01$ diffusion coefficient $D_{xz}=0.1$, there is no interior $Q=0$, average air pollutant source are added $Q=0.0001$, and average air flow velocity $u=0.1$.

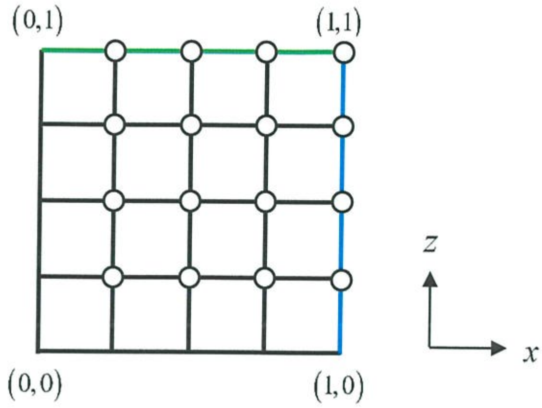


Figure 4.11: Generating grid points of example 4.2.5

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0)=x(1-x)z(1-z)+c_0,$$

the boundary conditions can be assumed by

$$c(0,z,t)=c_0=0,$$
$$c_x(1,z,t)=c_1=0$$
$$c_z(x,1,t)=c_2=0$$

define $c(x,z,t)=c_{i,j}^n$ with $\Delta x=\Delta z=0.25, \Delta t=0.01, D=0.1,$ and $u=0.1$

approximate differential boundary condition,

$$c_x(1,z,t)=0 \text{ or } \frac{\partial c_{5,j}^n}{\partial x}=0,$$

using forward difference,

$$\frac{c_{5,j}^n-c_{4,j}^n}{\Delta x}=0$$

$$c_{5,j}^n=c_{4,j}^n \qquad j=1,2,3,4 \tag{4.36}$$

$$c_z(x,1,t)=0 \text{ or } \frac{\partial c_{i,5}^n}{\partial z}=0,$$

using forward difference,

$$\frac{c_{i,5}^n-c_{i,4}^n}{\Delta z}=0$$

$$c_{i,5}^n=c_{i,4}^n \qquad i=1,2,3,4 \tag{4.37}$$

solving problem by finite difference method Eq.(4.8)

$$c_{i,j}^{n+1}=\frac{1}{2}(2\alpha+\beta)c_{i-1,j}^n+\gamma c_{i,j-1}^n+(1-2\alpha-2\gamma)c_{i,j}^n+\gamma c_{i,j+1}^n+\frac{1}{2}(2\alpha-\beta)c_{i+1,j}^n+Q_{i,j}^n$$

$$\alpha=D_{xz}\frac{\Delta t}{(\Delta x)^2}=0.1\frac{0.01}{(0.25)^2}=0.016,$$

$$\beta = u\frac{\Delta t}{\Delta x} = 0.1\frac{0.01}{0.25} = 0.004,$$

$$\gamma = D_{xz}\frac{\Delta t}{(\Delta z)^2} = 0.1\frac{0.01}{(0.25)^2} = 0.016.$$

$$c_{i,j}^{n+1} = 0.018c_{i-1,j}^{n} + 0.016c_{i,j-1}^{n} + 0.936c_{i,j}^{n} + 0.016c_{i,j+1}^{n} + 0.014c_{i+1,j}^{n} + Q_{i,j}^{n}$$

$$i = 1,2,3,4, \quad j = 1,2,3,4 \tag{4.38}$$

$n = 0,$

$j = 1, \quad i = 1 \quad c_{1,1}^{1} = 0.018c_{0,1}^{0} + 0.016c_{1,0}^{0} + 0.936c_{1,1}^{0} + 0.016c_{1,2}^{0} + 0.014c_{2,1}^{0} + Q_{1,1}^{0}$

$i = 2 \quad c_{2,1}^{1} = 0.018c_{1,1}^{0} + 0.016c_{2,0}^{0} + 0.936c_{2,1}^{0} + 0.016c_{2,2}^{0} + 0.014c_{3,1}^{0} + Q_{2,1}^{0}$

$i = 3 \quad c_{3,1}^{1} = 0.018c_{2,1}^{0} + 0.016c_{3,0}^{0} + 0.936c_{3,1}^{0} + 0.016c_{3,2}^{0} + 0.014c_{4,1}^{0} + Q_{3,1}^{0}$

$i = 4 \quad c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.936c_{4,1}^{0} + 0.016c_{4,2}^{0} + 0.014c_{5,1}^{0} + Q_{4,1}^{0}$

and apply boundary condition $c_{5,1}^{0} = c_{4,1}^{0}$

$c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.95c_{4,1}^{0} + 0.016c_{4,2}^{0} + Q_{4,1}^{0}$

$j = 2, \quad i = 1 \quad c_{1,2}^{1} = 0.018c_{0,2}^{0} + 0.016c_{1,1}^{0} + 0.936c_{1,2}^{0} + 0.016c_{1,3}^{0} + 0.014c_{2,2}^{0} + Q_{1,2}^{0}$

$i = 2 \quad c_{2,2}^{1} = 0.018c_{1,2}^{0} + 0.016c_{2,1}^{0} + 0.936c_{2,2}^{0} + 0.016c_{2,3}^{0} + 0.014c_{3,2}^{0} + Q_{2,2}^{0}$

$i = 3 \quad c_{3,2}^{1} = 0.018c_{2,2}^{0} + 0.016c_{3,1}^{0} + 0.936c_{3,2}^{0} + 0.016c_{3,3}^{0} + 0.014c_{4,2}^{0} + Q_{3,2}^{0}$

$i = 4 \quad c_{4,2}^{1} = 0.018c_{3,2}^{0} + 0.016c_{4,1}^{0} + 0.936c_{4,2}^{0} + 0.016c_{4,3}^{0} + 0.014c_{5,2}^{0} + Q_{4,2}^{0}$

and apply boundary condition $c_{5,2}^{0} = c_{4,2}^{0}$

$c_{4,2}^{1} = 0.018c_{3,2}^{0} + 0.016c_{4,1}^{0} + 0.95c_{4,2}^{0} + 0.016c_{4,3}^{0} + Q_{4,2}^{0}$

$j = 3, \quad i = 1 \quad c_{1,3}^{1} = 0.018c_{0,3}^{0} + 0.016c_{1,2}^{0} + 0.936c_{1,3}^{0} + 0.016c_{1,4}^{0} + 0.014c_{2,3}^{0} + Q_{1,3}^{0}$

$i = 2 \quad c_{2,3}^{1} = 0.018c_{1,3}^{0} + 0.016c_{2,2}^{0} + 0.936c_{2,3}^{0} + 0.016c_{2,4}^{0} + 0.014c_{3,3}^{0} + Q_{2,3}^{0}$

$i = 3 \quad c_{3,3}^{1} = 0.018c_{2,3}^{0} + 0.016c_{3,2}^{0} + 0.936c_{3,3}^{0} + 0.016c_{3,4}^{0} + 0.014c_{4,3}^{0} + Q_{3,3}^{0}$

$i = 4 \quad c_{4,3}^{1} = 0.018c_{3,3}^{0} + 0.016c_{4,2}^{0} + 0.936c_{4,3}^{0} + 0.016c_{4,4}^{0} + 0.014c_{5,3}^{0} + Q_{4,3}^{0}$

and apply boundary condition $c_{5,3}^{0} = c_{4,3}^{0}$

$c_{4,3}^{1} = 0.018c_{3,3}^{0} + 0.016c_{4,2}^{0} + 0.95c_{4,3}^{0} + 0.016c_{4,4}^{0} + Q_{4,3}^{0}$

$j = 4, \quad i = 1 \quad c_{1,4}^{1} = 0.018c_{0,4}^{0} + 0.016c_{1,3}^{0} + 0.936c_{1,4}^{0} + 0.016c_{1,5}^{0} + 0.014c_{2,4}^{0} + Q_{1,4}^{0}$

and apply boundary condition $c_{1,5}^{0} = c_{1,4}^{0}$

$c_{1,4}^{1} = 0.018c_{0,4}^{0} + 0.016c_{1,3}^{0} + 0.952c_{1,4}^{0} + 0.014c_{2,4}^{0} + Q_{1,4}^{0}$

$i = 2 \quad c_{2,4}^{1} = 0.018c_{1,4}^{0} + 0.016c_{2,3}^{0} + 0.936c_{2,4}^{0} + 0.016c_{2,5}^{0} + 0.014c_{3,4}^{0} + Q_{2,4}^{0}$

and apply boundary condition $c_{2,5}^{0} = c_{2,4}^{0}$

$c_{2,4}^{1} = 0.018c_{1,4}^{0} + 0.016c_{2,3}^{0} + 0.952c_{2,4}^{0} + 0.014c_{3,4}^{0} + Q_{2,4}^{0}$

$i = 3 \quad c_{3,4}^{1} = 0.018c_{2,4}^{0} + 0.016c_{3,3}^{0} + 0.936c_{3,4}^{0} + 0.016c_{3,5}^{0} + 0.014c_{4,4}^{0} + Q_{3,4}^{0}$

and apply boundary condition $c_{3,5}^{0}=c_{3,4}^{0}$

$$c_{3,4}^{1}=0.018c_{2,4}^{0}+0.016c_{3,3}^{0}+0.952c_{3,4}^{0}+0.014c_{4,4}^{0}+Q_{3,4}^{0}$$

$i=4$ $\quad c_{4,4}^{1}=\dfrac{c_{3,4}^{1}+c_{4,3}^{1}}{2}$

Table 4.5 The calculated pollutant concentration at time $t=0.02$ of example 4.2.5

Point	Pollutant concentration	
	$Q=0$	$Q=0.0001$
$c_{0,0}^{2}$	0.000000	0.000000
$c_{0,1}^{2}$	0.000000	0.000000
$c_{0,2}^{2}$	0.000000	0.000000
$c_{0,3}^{2}$	0.000000	0.000000
$c_{0,4}^{2}$	0.000000	0.000000
$c_{1,0}^{2}$	0.000000	0.000000
$c_{1,1}^{2}$	0.033495	0.033702
$c_{1,2}^{2}$	0.045141	0.045353
$c_{1,3}^{2}$	0.033867	0.034076
$c_{1,4}^{2}$	0.001236	0.001434
$c_{2,0}^{2}$	0.000000	0.000000
$c_{2,1}^{2}$	0.044898	0.045096
$c_{2,2}^{2}$	0.060508	0.060708
$c_{2,3}^{2}$	0.045396	0.045596
$c_{2,4}^{2}$	0.001652	0.001852
$c_{3,0}^{2}$	0.000000	0.000000
$c_{3,1}^{2}$	0.033504	0.033702
$c_{3,2}^{2}$	0.045153	0.045353
$c_{3,3}^{2}$	0.033876	0.034076
$c_{3,4}^{2}$	0.001236	0.001434
$c_{4,0}^{2}$	0.000000	0.000000
$c_{4,1}^{2}$	0.001095	0.001293
$c_{4,2}^{2}$	0.001468	0.001668
$c_{4,3}^{2}$	0.001101	0.001300
$c_{4,4}^{2}$	0.000598	0.000698

Example 4.2.6. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial c}{\partial z} = c_2$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.12) with step size $\Delta x = \Delta z = 0.1$, $\Delta t = 0.01$ diffusion coefficient $D_{xz} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, and average air flow velocity $u = 0.1$.

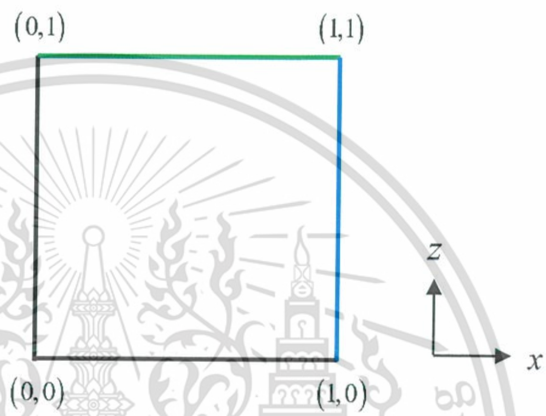


Figure 4.12: Domain of example 4.2.6

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0) = x(1-x)z(1-z) + c_0,$$

the boundary conditions can be assumed by

$$c(0,z,t) = c_0 = 0,$$

$$c_x(1,z,t) = c_1 = 0$$

$$c_z(x,1,t) = c_2 = 0$$

define $c(x,z,t) = c_{i,j}^n$ with $\Delta x = \Delta z = 0.1, \Delta t = 0.01, D = 0.1,$ and $u = 0.1$.

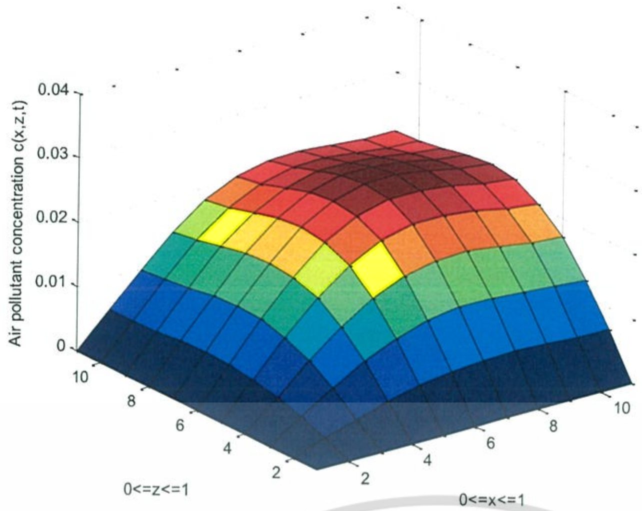


Figure 4.13: The approximated air pollutant concentration for $Q=0$ and $0 \leq t \geq 60$ sec of example 4.2.6

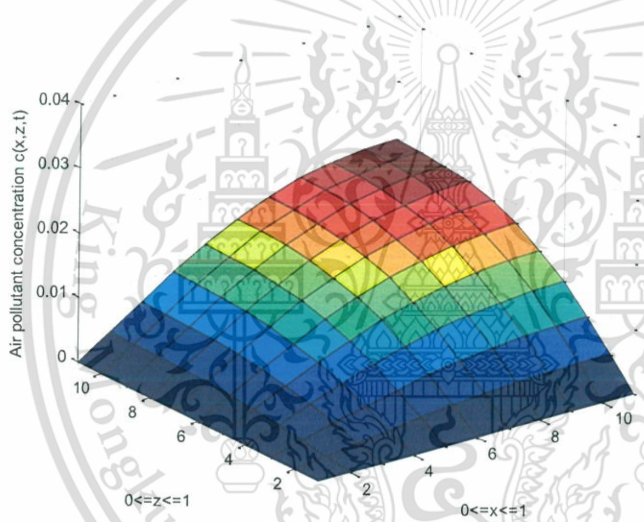


Figure 4.14: The approximated air pollutant concentration for $Q=0$ and $0 \leq t \geq 120$ sec of example 4.2.6

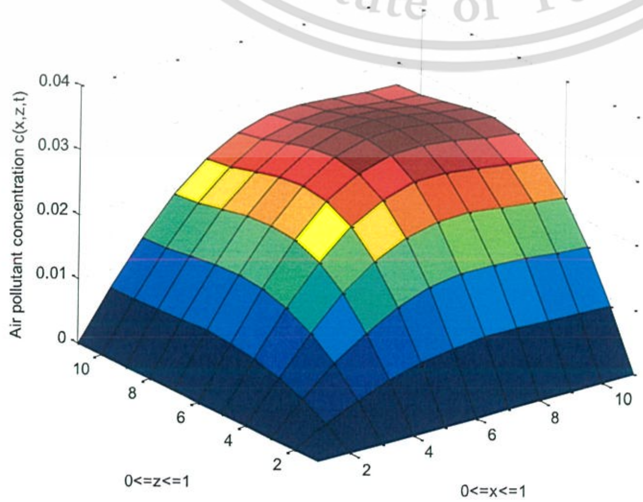


Figure 4.15: The approximated air pollutant concentration for $Q=0.0001$ and $0 \leq t \geq 60$ sec of example 4.2.6

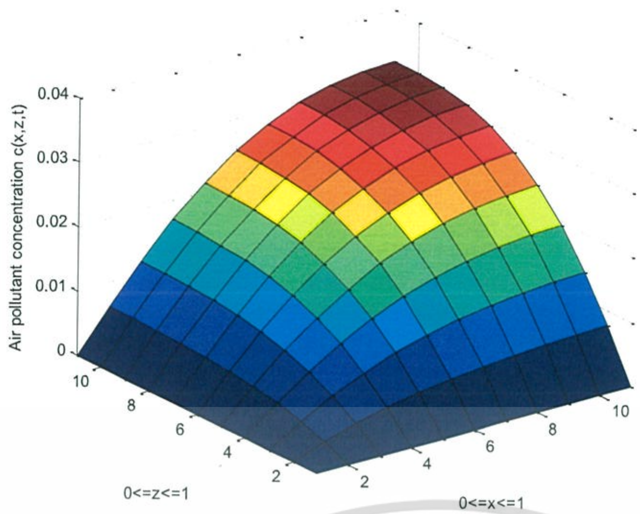


Figure 4.16: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \ge 120$ sec of example 4.2.6

Example 4.2.7. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial c}{\partial z} = c_2$ and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial c}{\partial z} = \overline{c_3}$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.17) with step size $\Delta x = \Delta z = 0.25$, $\Delta t = 0.01$ diffusion coefficient $D_{xz} = 0.1$, source term Q, and average of velocity $u = 0.1$.

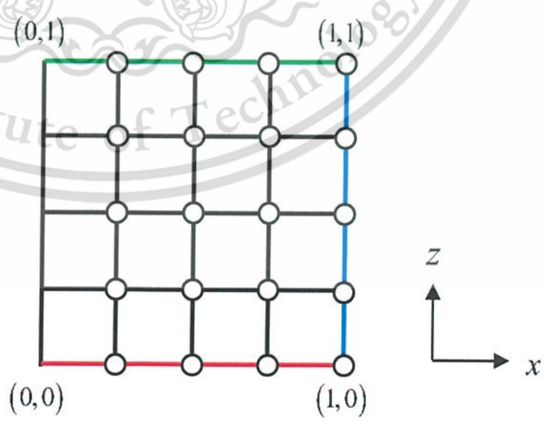


Figure 4.17: Generating grid points of example 4.2.7

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0) = x(1-x)z(1-z) + c_0,$$

the boundary conditions can be assumed by

$$c(0,z,t)=c_0=0,$$
$$c_x(1,z,t)=c_1=0$$
$$c_z(x,1,t)=c_2=0$$
$$c_z(x,0,t)=c_3=0$$

define $c(x,z,t)=c_{i,j}^n$ with $\Delta x=\Delta z=0.25, \Delta t=0.01, D=0.1,$ and $u=0.1$

approximate differential boundary condition,

$c_x(1,z,t)=0$ or $\dfrac{\partial c_{5,j}^n}{\partial x}=0,$

using forward difference,

$$\frac{c_{5,j}^n-c_{4,j}^n}{\Delta x}=0$$
$$c_{5,j}^n=c_{4,j}^n \qquad j=0,1,2,3,4 \tag{4.39}$$

$c_z(x,1,t)=0$ or $\dfrac{\partial c_{i,5}^n}{\partial z}=0,$

using forward difference,

$$\frac{c_{i,5}^n-c_{i,4}^n}{\Delta z}=0$$
$$c_{i,5}^n=c_{i,4}^n \qquad i=1,2,3,4 \tag{4.40}$$

$c_z(x,0,t)=0$ or $\dfrac{\partial c_{i,0}^n}{\partial z}=0,$

using backward difference,

$$\frac{c_{i,0}^n-c_{i,-1}^n}{\Delta z}=0$$
$$c_{i,0}^n=c_{i,-1}^n \qquad i=1,2,3,4 \tag{4.41}$$

solving problem by finite difference method Eq.(4.8)

$$c_{i,j}^{n+1}=\frac{1}{2}(2\alpha+\beta)c_{i-1,j}^n+\gamma c_{i,j-1}^n+(1-2\alpha-2\gamma)c_{i,j}^n+\gamma c_{i,j+1}^n+\frac{1}{2}(2\alpha-\beta)c_{i+1,j}^n+Q_{i,j}^n$$

$$\alpha=D_{xz}\frac{\Delta t}{(\Delta x)^2}=0.1\frac{0.01}{(0.25)^2}=0.016,$$
$$\beta=u\frac{\Delta t}{\Delta x}=0.1\frac{0.01}{0.25}=0.004,$$
$$\gamma=D_{xz}\frac{\Delta t}{(\Delta z)^2}=0.1\frac{0.01}{(0.25)^2}=0.016.$$

$$c_{i,j}^{n+1}=0.018c_{i-1,j}^n+0.016c_{i,j-1}^n+0.936c_{i,j}^n+0.016c_{i,j+1}^n+0.014c_{i+1,j}^n+Q_{i,j}^n$$
$$n=0,1,2,\ldots,\quad i=1,2,3,4,\quad j=0,1,2,3,4 \tag{4.42}$$

$n = 0,$

$j = 0, \quad i = 1 \quad c_{1,0}^{1} = 0.018c_{0,0}^{0} + 0.016c_{1,-1}^{0} + 0.936c_{1,0}^{0} + 0.016c_{1,1}^{0} + 0.014c_{2,0}^{0} + Q_{1,0}^{0}$

and apply boundary condition $c_{1,0}^{0} = c_{1,-1}^{0}$

$c_{1,0}^{1} = 0.018c_{0,0}^{0} + 0.016c_{1,0}^{0} + 0.936c_{1,0}^{0} + 0.016c_{1,1}^{0} + 0.014c_{2,0}^{0} + Q_{1,0}^{0}$

$i = 2 \quad c_{2,0}^{1} = 0.018c_{1,0}^{0} + 0.016c_{2,-1}^{0} + 0.936c_{2,0}^{0} + 0.016c_{2,1}^{0} + 0.014c_{3,0}^{0} + Q_{2,0}^{0}$

and apply boundary condition $c_{2,0}^{0} = c_{2,-1}^{0}$

$c_{2,0}^{1} = 0.018c_{1,0}^{0} + 0.016c_{2,0}^{0} + 0.936c_{2,0}^{0} + 0.016c_{2,1}^{0} + 0.014c_{3,0}^{0} + Q_{2,0}^{0}$

$i = 3 \quad c_{3,0}^{1} = 0.018c_{2,0}^{0} + 0.016c_{3,-1}^{0} + 0.936c_{3,0}^{0} + 0.016c_{3,1}^{0} + 0.014c_{4,0}^{0} + Q_{3,0}^{0}$

and apply boundary condition $c_{3,0}^{0} = c_{3,-1}^{0}$

$c_{3,0}^{1} = 0.018c_{2,0}^{0} + 0.016c_{3,0}^{0} + 0.936c_{3,0}^{0} + 0.016c_{3,1}^{0} + 0.014c_{4,0}^{0} + Q_{3,0}^{0}$

$i = 4 \quad c_{4,0}^{1} = \dfrac{c_{3,0}^{1} + c_{4,1}^{1}}{2}$

$j = 1, \quad i = 1 \quad c_{1,1}^{1} = 0.018c_{0,1}^{0} + 0.016c_{1,0}^{0} + 0.936c_{1,1}^{0} + 0.016c_{1,2}^{0} + 0.014c_{2,1}^{0} + Q_{1,1}^{0}$

$i = 2 \quad c_{2,1}^{1} = 0.018c_{1,1}^{0} + 0.016c_{2,0}^{0} + 0.936c_{2,1}^{0} + 0.016c_{2,2}^{0} + 0.014c_{3,1}^{0} + Q_{2,1}^{0}$

$i = 3 \quad c_{3,1}^{1} = 0.018c_{2,1}^{0} + 0.016c_{3,0}^{0} + 0.936c_{3,1}^{0} + 0.016c_{3,2}^{0} + 0.014c_{4,1}^{0} + Q_{3,1}^{0}$

$i = 4 \quad c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.936c_{4,1}^{0} + 0.016c_{4,2}^{0} + 0.014c_{5,1}^{0} + Q_{4,1}^{0}$

and apply boundary condition $c_{5,1}^{0} = c_{4,1}^{0}$

$c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.936c_{4,1}^{0} + 0.016c_{4,2}^{0} + 0.014c_{4,1}^{0} + Q_{4,1}^{0}$

$j = 2, \quad i = 1 \quad c_{1,2}^{1} = 0.018c_{0,2}^{0} + 0.016c_{1,1}^{0} + 0.936c_{1,2}^{0} + 0.016c_{1,3}^{0} + 0.014c_{2,2}^{0} + Q_{1,2}^{0}$

$i = 2 \quad c_{2,2}^{1} = 0.018c_{1,2}^{0} + 0.016c_{2,1}^{0} + 0.936c_{2,2}^{0} + 0.016c_{2,3}^{0} + 0.014c_{3,2}^{0} + Q_{2,2}^{0}$

$i = 3 \quad c_{3,2}^{1} = 0.018c_{2,2}^{0} + 0.016c_{3,1}^{0} + 0.936c_{3,2}^{0} + 0.016c_{3,3}^{0} + 0.014c_{4,2}^{0} + Q_{3,2}^{0}$

$i = 4 \quad c_{4,2}^{1} = 0.018c_{3,2}^{0} + 0.016c_{4,1}^{0} + 0.936c_{4,2}^{0} + 0.016c_{4,3}^{0} + 0.014c_{5,2}^{0} + Q_{4,2}^{0}$

and apply boundary condition $c_{5,2}^{0} = c_{4,2}^{0}$

$c_{4,2}^{1} = 0.018c_{3,2}^{0} + 0.016c_{4,1}^{0} + 0.936c_{4,2}^{0} + 0.016c_{4,3}^{0} + 0.014c_{4,2}^{0} + Q_{4,2}^{0}$

$j = 3, \quad i = 1 \quad c_{1,3}^{1} = 0.018c_{0,3}^{0} + 0.016c_{1,2}^{0} + 0.936c_{1,3}^{0} + 0.016c_{1,4}^{0} + 0.014c_{2,3}^{0} + Q_{1,3}^{0}$

$i = 2 \quad c_{2,3}^{1} = 0.018c_{1,3}^{0} + 0.016c_{2,2}^{0} + 0.936c_{2,3}^{0} + 0.016c_{2,4}^{0} + 0.014c_{3,3}^{0} + Q_{2,3}^{0}$

$i = 3 \quad c_{3,3}^{1} = 0.018c_{2,3}^{0} + 0.016c_{3,2}^{0} + 0.936c_{3,3}^{0} + 0.016c_{3,4}^{0} + 0.014c_{4,3}^{0} + Q_{3,3}^{0}$

$i = 4 \quad c_{4,3}^{1} = 0.018c_{3,3}^{0} + 0.016c_{4,2}^{0} + 0.936c_{4,3}^{0} + 0.016c_{4,4}^{0} + 0.014c_{5,3}^{0} + Q_{4,3}^{0}$

and apply boundary condition $c_{5,3}^{0} = c_{4,3}^{0}$

$c_{4,3}^{1} = 0.018c_{3,3}^{0} + 0.016c_{4,2}^{0} + 0.936c_{4,3}^{0} + 0.016c_{4,4}^{0} + 0.014c_{4,3}^{0} + Q_{4,3}^{0}$

$j = 4, \quad i = 1 \quad c_{1,4}^{1} = 0.018c_{0,4}^{0} + 0.016c_{1,3}^{0} + 0.936c_{1,4}^{0} + 0.016c_{1,5}^{0} + 0.014c_{2,4}^{0} + Q_{1,4}^{0}$

and apply boundary condition $c_{1,5}^{0} = c_{1,4}^{0}$

$c_{1,4}^{1} = 0.018c_{0,4}^{0} + 0.016c_{1,3}^{0} + 0.936c_{1,4}^{0} + 0.016c_{1,4}^{0} + 0.014c_{2,4}^{0} + Q_{1,4}^{0}$

$i = 2 \quad c_{2,4}^{1} = 0.018c_{1,4}^{0} + 0.016c_{2,3}^{0} + 0.936c_{2,4}^{0} + 0.016c_{2,5}^{0} + 0.014c_{3,4}^{0} + Q_{2,4}^{0}$

and apply boundary condition $c_{2,5}^{0} = c_{2,4}^{0}$

$$c_{2,4}^{1} = 0.018c_{1,4}^{0} + 0.016c_{2,3}^{0} + 0.936c_{2,4}^{0} + 0.016c_{2,4}^{0} + 0.014c_{3,4}^{0} + Q_{2,4}^{0}$$

$i = 3$ $$c_{3,4}^{1} = 0.018c_{2,4}^{0} + 0.016c_{3,3}^{0} + 0.936c_{3,4}^{0} + 0.016c_{3,5}^{0} + 0.014c_{4,4}^{0} + Q_{3,4}^{0}$$

and apply boundary condition $c_{3,5}^{0} = c_{3,4}^{0}$

$$c_{3,4}^{1} = 0.018c_{2,4}^{0} + 0.016c_{3,3}^{0} + 0.936c_{3,4}^{0} + 0.016c_{3,4}^{0} + 0.014c_{4,4}^{0} + Q_{3,4}^{0}$$

$i = 4$ $$c_{4,4}^{1} = \frac{c_{3,4}^{1} + c_{4,3}^{1}}{2}$$

Table 4.6 The calculated pollutant concentration at time $t = 0.02$ of example 4.2.7

Point	Pollutant concentration	
	$Q = 0$	$Q = 0.0001$
$c_{0,0}^{2}$	0.000000	0.000000
$c_{0,1}^{2}$	0.001095	0.001293
$c_{0,2}^{2}$	0.001468	0.001668
$c_{0,3}^{2}$	0.001101	0.001299
$c_{0,4}^{2}$	0.000597	0.000696
$c_{1,0}^{2}$	0.000000	0.000000
$c_{1,1}^{2}$	0.033503	0.033701
$c_{1,2}^{2}$	0.045153	0.045353
$c_{1,3}^{2}$	0.033876	0.034076
$c_{1,4}^{2}$	0.001236	0.001434
$c_{2,0}^{2}$	0.000000	0.000000
$c_{2,1}^{2}$	0.044898	0.045096
$c_{2,2}^{2}$	0.060507	0.060707
$c_{2,3}^{2}$	0.045395	0.045595
$c_{2,4}^{2}$	0.001652	0.001852
$c_{3,0}^{2}$	0.000000	0.000000
$c_{3,1}^{2}$	0.033504	0.033701
$c_{3,2}^{2}$	0.045153	0.045353
$c_{3,3}^{2}$	0.033876	0.034076
$c_{3,4}^{2}$	0.006825	0.001434
$c_{4,0}^{2}$	0.000000	0.000000
$c_{4,1}^{2}$	0.001095	0.001293
$c_{4,2}^{2}$	0.001468	0.001668
$c_{4,3}^{2}$	0.001101	0.001299
$c_{4,4}^{2}$	0.000597	0.000696

Example 4.2.8. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial c}{\partial z} = c_2$ and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial c}{\partial z} = c_3$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.18) with step size $\Delta x = \Delta z = 0.1$, $\Delta t = 0.01$ diffusion coefficient $D_{xz} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, and average air flow velocity $u = 0.1$.

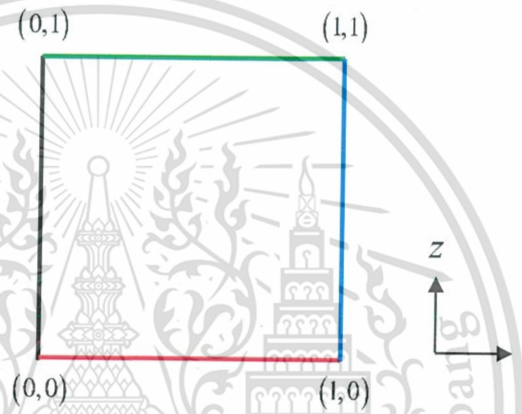


Figure 4.18: Domain of example 4.2.8

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0) = x(1-x)z(1-z) + c_0,$$

the boundary conditions can be assumed by

$$c(0,z,t) = \overline{c_0} = 0,$$
$$c_x(1,z,t) = \overline{c_1} = 0$$
$$c_z(x,1,t) = \overline{c_2} = 0$$
$$c_z(x,0,t) = \overline{c_3}.$$

define $c(x,z,t) = c_{i,j}^n$ with $\Delta x = \Delta z = 0.1, \Delta t = 0.01, D = 0.1,$ and $u = 0.1$

Table 4.7 The approximated air pollutant concentration at $0 \le t \ge 60$ sec of example 4.2.8 with the absorbance boundary ($c_3 = -0.1$), non-slipping boundary ($c_3 = 0$) or releasing boundary ($c_3 = 0.1$) on the ground

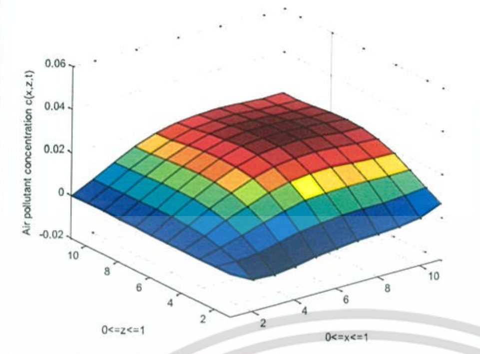
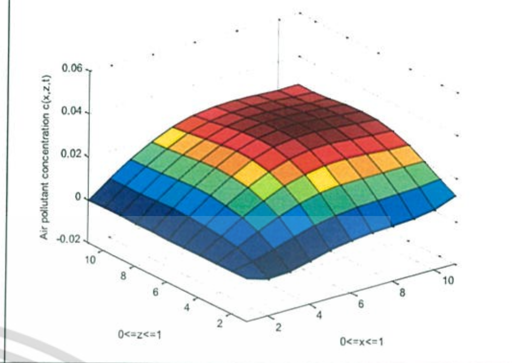
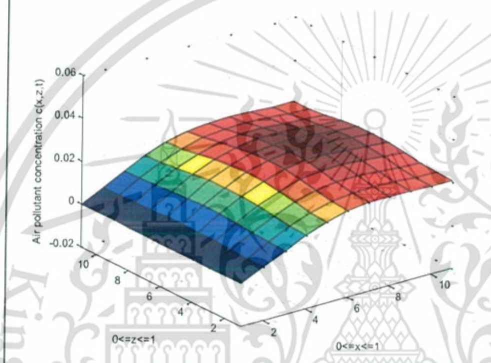
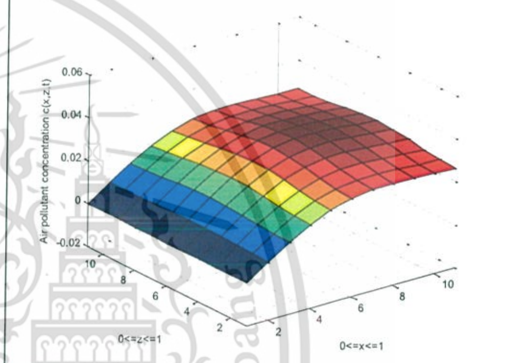
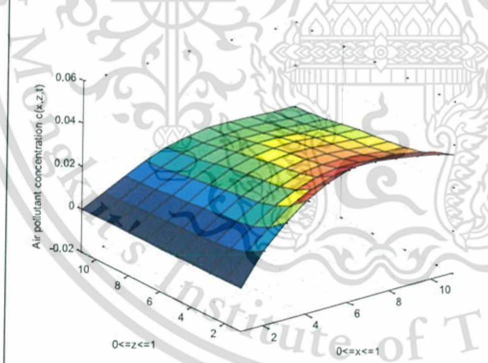
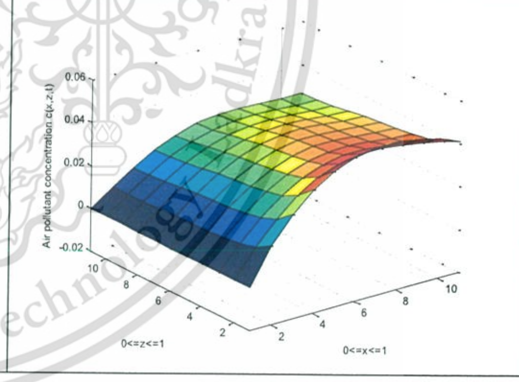
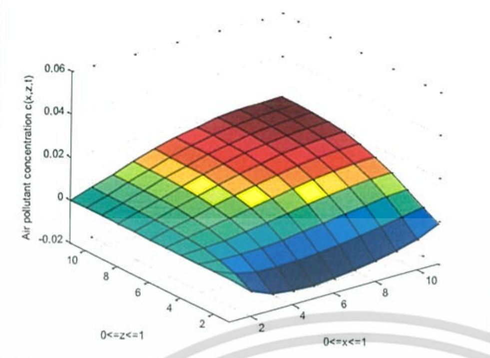
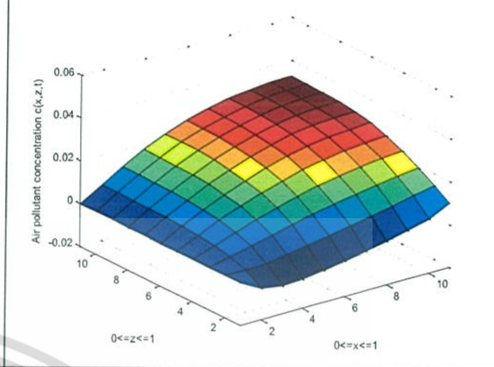
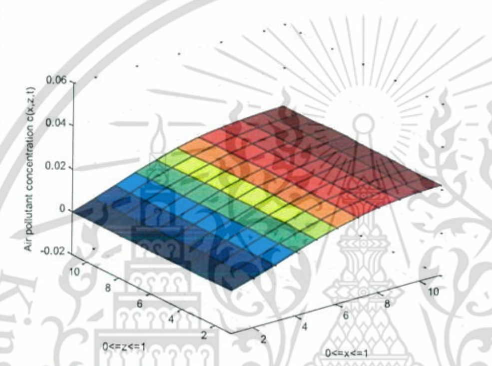
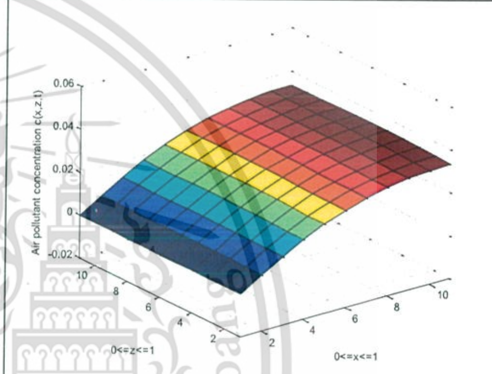
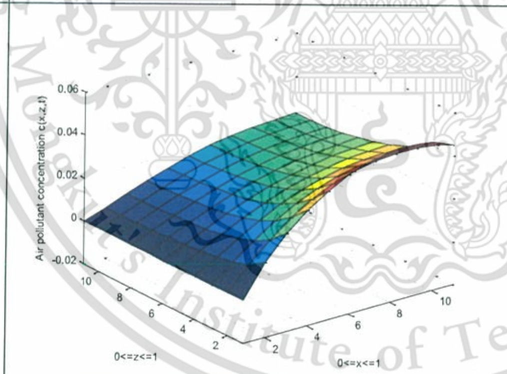
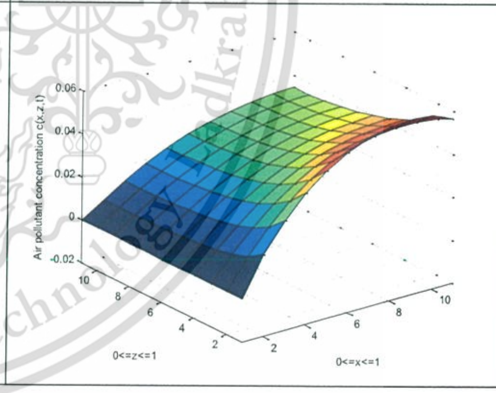
c_3	$Q = 0$	$Q = 0.0001$
-0.1		
0		
0.1		

Table 4.8 The approximated air pollutant concentration at $0 \le t \ge 120$ sec of example 4.2.8 with the absorbance boundary ($c_3 = -0.1$), non-slipping boundary ($c_3 = 0$) or releasing boundary ($c_3 = 0.1$) on the ground

c_3	$Q = 0$	$Q = 0.0001$
-0.1		
0		
0.1		

Example 4.2.9. To find the pollutant concentration in street canyon, at boundary of street canyon that has rate of change of pollutant concentration with respect to x at the entrance gate $\frac{\partial c}{\partial x}=c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x}=c_1$, rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial c}{\partial z}=c_2$, and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial c}{\partial z}=c_3$, domain $\Omega=(0,1)\times(0,1)$ in Figure (4.19) with step size $\Delta x=\Delta z=0.25$, $\Delta t=0.01$ diffusion coefficient $D_{xz}=0.1$, there is no interior $Q=0$, average air pollutant source are added $Q=0.0001$, and average air flow velocity $u=0.1$.

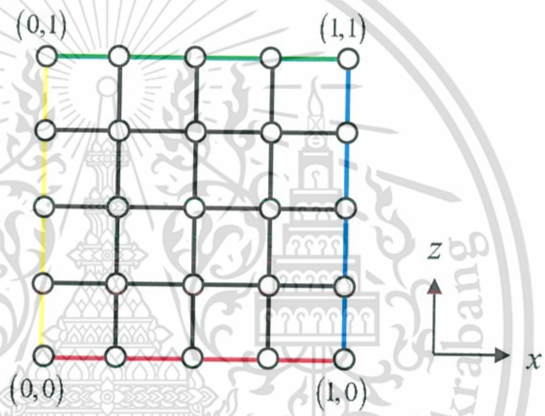


Figure 4.19: Generating grid points of example 4.2.9

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0)=x(1-x)z(1-z)+c_0,$$

the boundary conditions can be assumed by

$$c_x(0,z,t)=c_0=0,$$
$$c_x(1,z,t)=c_1=0,$$
$$c_z(x,1,t)=c_2=0,$$
$$c_z(x,0,t)=c_3=0,$$

define $c(x,z,t)=c_{i,j}^n$ with $\Delta x=\Delta z=0.25, \Delta t=0.01, D=0.1,$ and $u=0.1$

approximate differential boundary condition,

$c_x(0,z,t)=0$ or $\dfrac{\partial c_{0,j}^n}{\partial x}=0,$

using backward difference,

$$\frac{c_{0,j}^n - c_{-1,j}^n}{\Delta x}=0$$

$$c_{0,j}^n = c_{-1,j}^n \qquad j=0,1,2,3,4 \qquad (4.43)$$

$c_x(1,z,t)=0$ or $\dfrac{\partial c_{5,j}^n}{\partial x}=0,$

using forward difference,

$$\frac{c_{5,j}^n - c_{4,j}^n}{\Delta x}=0$$

$$c_{5,j}^n = c_{4,j}^n \qquad j=0,1,2,3,4 \qquad (4.44)$$

$c_z(x,1,t)=0$ or $\dfrac{\partial c_{i,5}^n}{\partial z}=0,$

using forward difference,

$$\frac{c_{i,5}^n - c_{i,4}^n}{\Delta z}=0$$

$$c_{i,5}^n = c_{i,4}^n \qquad i=0,1,2,3,4 \qquad (4.45)$$

$c_z(x,0,t)=0$ or $\dfrac{\partial c_{i,0}^n}{\partial z}=0,$

using backward difference,

$$\frac{c_{i,0}^n - c_{i,-1}^n}{\Delta z}=0$$

$$c_{i,0}^n = c_{i,-1}^n \qquad i=0,1,2,3,4 \qquad (4.46)$$

solving problem by finite difference method Eq.(4.8)

$$c_{i,j}^{n+1} = \frac{1}{2}(2\alpha+\beta)c_{i-1,j}^{n} + \gamma c_{i,j-1}^{n} + (1-2\alpha-2\gamma)c_{i,j}^{n} + \gamma c_{i,j+1}^{n} + \frac{1}{2}(2\alpha-\beta)c_{i+1,j}^{n} + Q_{i,j}^{n}$$

$$\alpha = D_{xz}\frac{\Delta t}{(\Delta x)^2} = 0.1\frac{0.01}{(0.25)^2} = 0.016,$$

$$\beta = u\frac{\Delta t}{\Delta x} = 0.1\frac{0.01}{0.25} = 0.004,$$

$$\gamma = D_{xz}\frac{\Delta t}{(\Delta z)^2} = 0.1\frac{0.01}{(0.25)^2} = 0.016.$$

$$c_{i,j}^{n+1} = 0.018c_{i-1,j}^{n} + 0.016c_{i,j-1}^{n} + 0.936c_{i,j}^{n} + 0.016c_{i,j+1}^{n} + 0.014c_{i+1,j}^{n} + Q_{i,j}^{n}$$

$$i = 0,1,2,3,4, \quad j = 0,1,2,3,4 \tag{4.47}$$

$n = 0,$

$j = 0, \quad i = 0 \quad c_{0,0}^{1} = \frac{c_{1,0}^{1} + c_{0,1}^{1}}{2}$

$i = 1 \quad c_{1,0}^{1} = 0.018c_{0,0}^{0} + 0.016c_{1,-1}^{0} + 0.936c_{1,0}^{0} + 0.016c_{1,1}^{0} + 0.014c_{2,0}^{0} + Q_{1,0}^{0}$

and apply boundary condition $c_{1,0}^{0} = c_{1,-1}^{0}$

$c_{1,0}^{1} = 0.018c_{0,0}^{0} + 0.016c_{1,0}^{0} + 0.936c_{1,0}^{0} + 0.016c_{1,1}^{0} + 0.014c_{2,0}^{0} + Q_{1,0}^{0}$

$i = 2 \quad c_{2,0}^{1} = 0.018c_{1,0}^{0} + 0.016c_{2,-1}^{0} + 0.936c_{2,0}^{0} + 0.016c_{2,1}^{0} + 0.014c_{3,0}^{0} + Q_{2,0}^{0}$

and apply boundary condition $c_{2,-1}^{0} = c_{2,0}^{0}$

$c_{2,0}^{1} = 0.018c_{1,0}^{0} + 0.016c_{2,0}^{0} + 0.936c_{2,0}^{0} + 0.016c_{2,1}^{0} + 0.014c_{3,0}^{0} + Q_{2,0}^{0}$

$i = 3 \quad c_{3,0}^{1} = 0.018c_{2,0}^{0} + 0.016c_{3,-1}^{0} + 0.936c_{3,0}^{0} + 0.016c_{3,1}^{0} + 0.014c_{4,0}^{0} + Q_{3,0}^{0}$

and apply boundary condition $c_{3,-1}^{0} = c_{3,0}^{0}$

$c_{3,0}^{1} = 0.018c_{2,0}^{0} + 0.016c_{3,0}^{0} + 0.936c_{3,0}^{0} + 0.016c_{3,1}^{0} + 0.014c_{4,0}^{0} + Q_{3,0}^{0}$

$i = 4 \quad c_{4,0}^{1} = \frac{c_{3,0}^{1} + c_{4,1}^{1}}{2}$

$j = 1, \quad i = 0 \quad c_{0,1}^{1} = 0.018c_{-1,1}^{0} + 0.016c_{0,0}^{0} + 0.936c_{0,1}^{0} + 0.016c_{0,2}^{0} + 0.014c_{1,1}^{0} + Q_{0,1}^{0}$

and apply boundary condition $c_{0,1}^{0} = c_{-1,1}^{0}$

$c_{0,1}^{1} = 0.018c_{0,1}^{0} + 0.016c_{0,0}^{0} + 0.936c_{0,1}^{0} + 0.016c_{0,2}^{0} + 0.014c_{1,1}^{0} + Q_{0,1}^{0}$

$i = 1 \quad c_{1,1}^{1} = 0.018c_{0,1}^{0} + 0.016c_{1,0}^{0} + 0.936c_{1,1}^{0} + 0.016c_{1,2}^{0} + 0.014c_{2,1}^{0} + Q_{1,1}^{0}$

$i = 2 \quad c_{2,1}^{1} = 0.018c_{1,1}^{0} + 0.016c_{2,0}^{0} + 0.936c_{2,1}^{0} + 0.016c_{2,2}^{0} + 0.014c_{3,1}^{0} + Q_{2,1}^{0}$

$i = 3 \quad c_{3,1}^{1} = 0.018c_{2,1}^{0} + 0.016c_{3,0}^{0} + 0.936c_{3,1}^{0} + 0.016c_{3,2}^{0} + 0.014c_{4,1}^{0} + Q_{3,1}^{0}$

$i = 4 \quad c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.936c_{4,1}^{0} + 0.016c_{4,2}^{0} + 0.014c_{5,1}^{0} + Q_{4,1}^{0}$

and apply boundary condition $c_{5,1}^{0} = c_{4,1}^{0}$

$c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.936c_{4,1}^{0} + 0.016c_{4,2}^{0} + 0.014c_{4,1}^{0} + Q_{4,1}^{0}$

$j=2,\ i=0 \quad c_{0,2}^{1}=0.018c_{-1,2}^{0}+0.016c_{0,1}^{0}+0.936c_{0,2}^{0}+0.016c_{0,3}^{0}+0.014\,c_{1,2}^{0}+Q_{0,2}^{0}$

and apply boundary condition $c_{0,2}^{0}=c_{-1,2}^{0}$

$i=1 \quad c_{1,2}^{1}=0.018c_{0,2}^{0}+0.016c_{1,1}^{0}+0.936c_{1,2}^{0}+0.016c_{1,3}^{0}+0.014\,c_{2,2}^{0}+Q_{1,2}^{0}$

$i=2 \quad c_{2,2}^{1}=0.018c_{1,2}^{0}+0.016c_{2,1}^{0}+0.936c_{2,2}^{0}+0.016c_{2,3}^{0}+0.014\,c_{3,2}^{0}+Q_{2,2}^{0}$

$i=3 \quad c_{3,2}^{1}=0.018c_{2,2}^{0}+0.016c_{3,1}^{0}+0.936c_{3,2}^{0}+0.016c_{3,3}^{0}+0.014\,c_{4,2}^{0}+Q_{3,2}^{0}$

$i=4 \quad c_{4,2}^{1}=0.018c_{3,2}^{0}+0.016c_{4,1}^{0}+0.936c_{4,2}^{0}+0.016c_{4,3}^{0}+0.014\,c_{5,2}^{0}+Q_{4,2}^{0}$

and apply boundary condition $c_{5,2}^{0}=c_{4,2}^{0}$

$c_{4,2}^{1}=0.018c_{3,2}^{0}+0.016c_{4,1}^{0}+0.936c_{4,2}^{0}+0.016c_{4,3}^{0}+0.014c_{4,2}^{0}+Q_{4,2}^{0}$

$j=3,\ i=0 \quad c_{0,3}^{1}=0.018c_{-1,3}^{0}+0.016c_{0,2}^{0}+0.936c_{0,3}^{0}+0.016c_{0,4}^{0}+0.014\,c_{1,3}^{0}+Q_{0,3}^{0}$

and apply boundary condition $c_{0,3}^{0}=c_{-1,3}^{0}$

$c_{0,3}^{1}=0.018c_{0,3}^{0}+0.016c_{0,2}^{0}+0.936c_{0,3}^{0}+0.016c_{0,4}^{0}+0.014\,c_{1,3}^{0}+Q_{0,3}^{0}$

$i=1 \quad c_{1,3}^{1}=0.018c_{0,3}^{0}+0.016c_{1,2}^{0}+0.936c_{1,3}^{0}+0.016c_{1,4}^{0}+0.014\,c_{2,3}^{0}+Q_{1,3}^{0}$

$i=2 \quad c_{2,3}^{1}=0.018c_{1,3}^{0}+0.016c_{2,2}^{0}+0.936c_{2,3}^{0}+0.016c_{2,4}^{0}+0.014\,c_{3,3}^{0}+Q_{2,3}^{0}$

$i=3 \quad c_{3,3}^{1}=0.018c_{2,3}^{0}+0.016c_{3,2}^{0}+0.936c_{3,3}^{0}+0.016c_{3,4}^{0}+0.014\,c_{4,3}^{0}+Q_{3,3}^{0}$

$i=4 \quad c_{4,3}^{1}=0.018c_{3,3}^{0}+0.016c_{4,2}^{0}+0.936c_{4,3}^{0}+0.016c_{4,4}^{0}+0.014\,c_{5,3}^{0}+Q_{4,3}^{0}$

and apply boundary condition $c_{5,3}^{0}=c_{4,3}^{0}$

$c_{4,3}^{1}=0.018c_{3,3}^{0}+0.016c_{4,2}^{0}+0.936c_{4,3}^{0}+0.016c_{4,4}^{0}+0.014c_{4,3}^{0}+Q_{4,3}^{0}$

$j=4,\ i=0 \quad c_{0,4}^{1}=\dfrac{c_{0,3}^{1}+c_{1,4}^{1}}{2}$

$i=1 \quad c_{1,4}^{1}=0.018c_{0,4}^{0}+0.016c_{1,3}^{0}+0.936c_{1,4}^{0}+0.016c_{1,5}^{0}+0.014\,c_{2,4}^{0}+Q_{1,4}^{0}$

and apply boundary condition $c_{1,5}^{0}=c_{1,4}^{0}$

$c_{1,4}^{1}=0.018c_{0,4}^{0}+0.016c_{1,3}^{0}+0.936c_{1,4}^{0}+0.016c_{1,4}^{0}+0.014\,c_{2,4}^{0}+Q_{1,4}^{0}$

$i=2 \quad c_{2,4}^{1}=0.018c_{1,4}^{0}+0.016c_{2,3}^{0}+0.936c_{2,4}^{0}+0.016c_{2,5}^{0}+0.014\,c_{3,4}^{0}+Q_{2,4}^{0}$

and apply boundary condition $c_{2,5}^{0}=c_{2,4}^{0}$

$c_{2,4}^{1}=0.018c_{1,4}^{0}+0.016c_{2,3}^{0}+0.936c_{2,4}^{0}+0.016c_{2,4}^{0}+0.014\,c_{3,4}^{0}+Q_{2,4}^{0}$

$i=3 \quad c_{3,4}^{1}=0.018c_{2,4}^{0}+0.016c_{3,3}^{0}+0.936c_{3,4}^{0}+0.016c_{3,5}^{0}+0.014\,c_{4,4}^{0}+Q_{3,4}^{0}$

and apply boundary condition $c_{3,5}^{0}=c_{3,4}^{0}$

$c_{3,4}^{1}=0.018c_{2,4}^{0}+0.016c_{3,3}^{0}+0.936c_{3,4}^{0}+0.016c_{3,4}^{0}+0.014\,c_{4,4}^{0}+Q_{3,4}^{0}$

$i=4 \quad c_{4,4}^{1}=\dfrac{c_{3,4}^{1}+c_{4,3}^{1}}{2}$

Table 4.9 The calculated pollutant concentration at time $t = 0.02$ of example 4.2.9

Point	Pollutant concentration	
	$Q = 0$	$Q = 0.0001$
$c_{0,0}^2$	0.000527	0.000627
$c_{0,1}^2$	0.001095	0.001294
$c_{0,2}^2$	0.001468	0.001668
$c_{0,3}^2$	0.001101	0.001299
$c_{0,4}^2$	0.000598	0.000696
$c_{1,0}^2$	0.000960	0.001159
$c_{1,1}^2$	0.033512	0.033712
$c_{1,2}^2$	0.045153	0.045353
$c_{1,3}^2$	0.033876	0.034076
$c_{1,4}^2$	0.001236	0.001434
$c_{2,0}^2$	0.001284	0.001484
$c_{2,1}^2$	0.044910	0.045100
$c_{2,2}^2$	0.060507	0.060707
$c_{2,3}^2$	0.045396	0.045595
$c_{2,4}^2$	0.001652	0.001852
$c_{3,0}^2$	0.000960	0.001159
$c_{3,1}^2$	0.033512	0.033712
$c_{3,2}^2$	0.045153	0.045353
$c_{3,3}^2$	0.033876	0.034076
$c_{3,4}^2$	0.001236	0.001434
$c_{4,0}^2$	0.000527	0.000627
$c_{4,1}^2$	0.001095	0.001294
$c_{4,2}^2$	0.001468	0.001668
$c_{4,3}^2$	0.001101	0.001299
$c_{4,4}^2$	0.000598	0.000696

Example 4.2.10. To find the pollutant concentration in street canyon, at boundary of street canyon that has the rate of change of pollutant concentration with respect to x at the entrance gate $\frac{\partial c}{\partial x}=c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x}=c_1$, rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial c}{\partial z}=c_2$, and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial c}{\partial z}=c_3$, domain $\Omega=(0,1)\times(0,1)$ in Figure (4.20) with step size $\Delta x=\Delta z=0.1$, $\Delta t=0.01$ diffusion coefficient $D_{xz}=0.1$, there is no interior $Q=0$, average air pollutant source are added $Q=0.0001$, and average air flow velocity $u=0.1$.

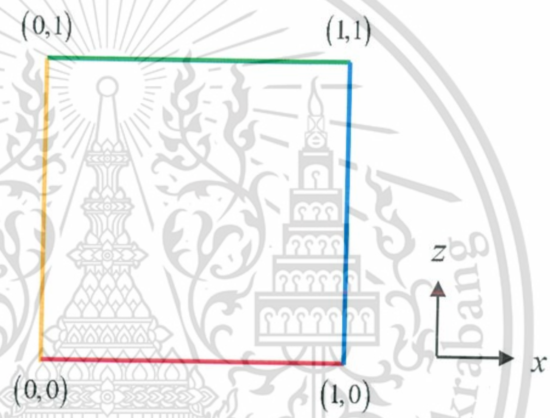


Figure 4.20: Domain of example 4.2.10

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0)=x(1-x)z(1-z)+c_0,$$

the boundary conditions can be assumed by

$$c_x(0,z,t)=c_0=0,$$
$$c_x(1,z,t)=c_1=0,$$
$$c_z(x,1,t)=c_2=0,$$
$$c_z(x,0,t)=c_3=0,$$

define $c(x,z,t)=c_{i,j}^n$ with $\Delta x=\Delta z=0.1, \Delta t=0.01, D=0.1,$ and $u=0.1$

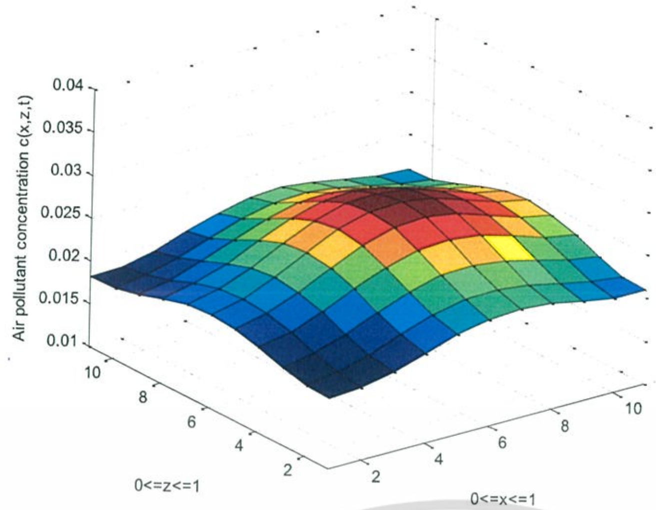


Figure 4.21: The approximated air pollutant concentration for $Q = 0$ and $0 \le t \le 60$ sec of example 4.2.10

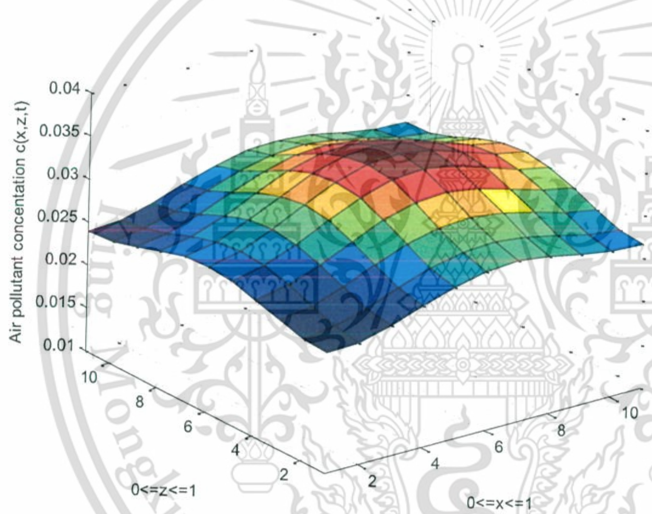


Figure 4.22: The approximated air pollutant concentration for $Q = 0$ and $0 \le t \le 120$ sec of example 4.2.10

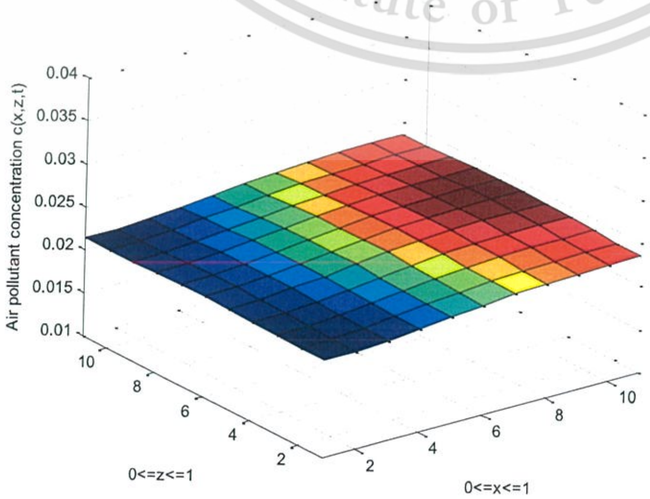


Figure 4.23: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \le 60$ sec of example 4.2.10

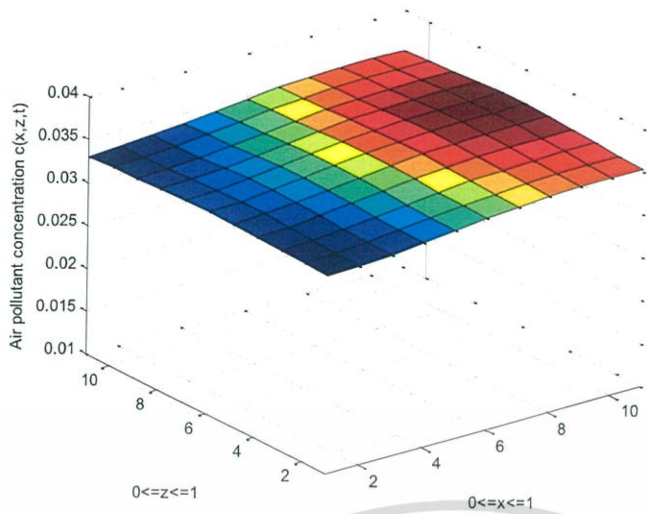


Figure 4.24: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \le 120$ sec of example 4.2.10

Example 4.2.11. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial c}{\partial z} = c_2$ and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial c}{\partial z} = c_3$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.25) with step size $\Delta x = \Delta z = 0.25$, $\Delta t = 0.01$ diffusion coefficient $D_{xz} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, and average air flow velocity $u = 0.1$.

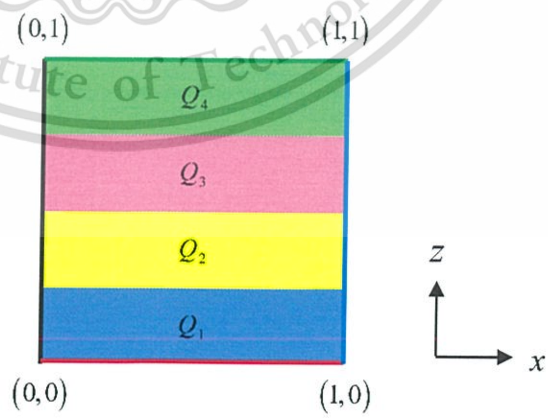


Figure 4.25: The classification of averaged air pollutant sources over the ground of example 4.2.11

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

$$c(x,z,0)=x(1-x)z(1-z)+c_0,$$

the boundary conditions can be assumed by

$$c(0,z,t)=c_0=0,$$
$$c_x(1,z,t)=c_1=0$$
$$c_z(x,1,t)=c_2=0$$
$$c_z(x,0,t)=c_3$$

define $c(x,z,t)=c^n_{i,j}$ with $\Delta x=\Delta z=0.25, \Delta t=0.01, D=0.1, u=0.1, Q_1=0.0001$ and $Q_2=Q_3=Q_4=0$

approximate differential boundary condition,

$c_x(1,z,t)=0$ or $\frac{\partial c^n_{5,j}}{\partial x}=0,$

using forward difference,

$$\frac{c^n_{5,j}-c^n_{4,j}}{\Delta x}=0$$

$$c^n_{5,j}=c^n_{4,j} \qquad j=0,1,2,3,4 \tag{4.48}$$

$c_z(x,1,t)=0$ or $\frac{\partial c^n_{i,5}}{\partial z}=0,$

using forward difference,

$$\frac{c^n_{i,5}-c^n_{i,4}}{\Delta z}=0$$

$$c^n_{i,5}=c^n_{i,4} \qquad i=1,2,3,4 \tag{4.49}$$

$c_z(x,0,t)=c_3$ or $\frac{\partial c^n_{i,0}}{\partial z}=c_3,$

using forward difference,

$$\frac{c^n_{i,0}-c^n_{i,-1}}{\Delta z}=c_3$$

$$c^n_{i,-1}=c^n_{i,0}-c_3\Delta z \qquad i=1,2,3,4 \tag{4.50}$$

Solving problem by finite difference method Eq.(4.8)

$$c_{i,j}^{n+1} = \frac{1}{2}(2\alpha+\beta)c_{i-1,j}^{n} + \gamma c_{i,j-1}^{n} + (1-2\alpha-2\gamma)c_{i,j}^{n} + \gamma c_{i,j+1}^{n} + \frac{1}{2}(2\alpha-\beta)c_{i+1,j}^{n} + Q_{i,j}^{n}$$

$$\alpha = D_{xz}\frac{\Delta t}{(\Delta x)^2} = 0.1\frac{0.01}{(0.25)^2} = 0.016,$$

$$\beta = u\frac{\Delta t}{\Delta x} = 0.1\frac{0.01}{0.25} = 0.004,$$

$$\gamma = D_{xz}\frac{\Delta t}{(\Delta z)^2} = 0.1\frac{0.01}{(0.25)^2} = 0.016.$$

$$c_{i,j}^{n+1} = 0.018c_{i-1,j}^{n} + 0.016c_{i,j-1}^{n} + 0.936c_{i,j}^{n} + 0.016c_{i,j+1}^{n} + 0.014c_{i+1,j}^{n} + Q_{i,j}^{n}$$

$$i = 1,2,3,4, \quad j = 0,1,2,3,4 \tag{4.51}$$

$n = 0,$

$j = 0, \quad i = 1 \quad c_{1,0}^{1} = 0.018c_{0,0}^{0} + 0.016c_{1,-1}^{0} + 0.936c_{1,0}^{0} + 0.016c_{1,1}^{0} + 0.014c_{2,0}^{0} + 0.0001$

and apply boundary condition $c_{1,-1}^{0} = c_{1,0}^{0} - c_3\Delta z$

$$c_{1,0}^{1} = 0.018c_{0,0}^{0} + 0.016(c_{1,0}^{0} - c_3\Delta z) + 0.936c_{1,0}^{0} + 0.016c_{1,1}^{0} + 0.014c_{2,0}^{0} + 0.0001$$

$i = 2 \quad c_{2,0}^{1} = 0.018c_{1,0}^{0} + 0.016c_{2,-1}^{0} + 0.936c_{2,0}^{0} + 0.016c_{2,1}^{0} + 0.014c_{3,0}^{0} + 0.0001$

and apply boundary condition $c_{2,-1}^{0} = c_{2,0}^{0} - c_3\Delta z$

$$c_{2,0}^{1} = 0.018c_{1,0}^{0} + 0.016(c_{2,0}^{0} - c_3\Delta z) + 0.936c_{2,0}^{0} + 0.016c_{2,1}^{0} + 0.014c_{3,0}^{0} + 0.0001$$

$i = 3 \quad c_{3,0}^{1} = 0.018c_{2,0}^{0} + 0.016c_{3,-1}^{0} + 0.936c_{3,0}^{0} + 0.016c_{3,1}^{0} + 0.014c_{4,0}^{0} + 0.0001$

and apply boundary condition $c_{3,-1}^{0} = c_{3,0}^{0} - c_3\Delta z$

$$c_{3,0}^{1} = 0.018c_{2,0}^{0} + 0.016(c_{3,0}^{0} - c_3\Delta z) + 0.936c_{3,0}^{0} + 0.016c_{3,1}^{0} + 0.014c_{4,0}^{0} + 0.0001$$

$i = 4 \quad c_{4,0}^{1} = \dfrac{c_{3,0}^{1} + c_{4,1}^{1}}{2}$

$j = 1, \quad i = 1 \quad c_{1,1}^{1} = 0.018c_{0,1}^{0} + 0.016c_{1,0}^{0} + 0.936c_{1,1}^{0} + 0.016c_{1,2}^{0} + 0.014c_{2,1}^{0} + Q_{1,1}^{0}$

$i = 2 \quad c_{2,1}^{1} = 0.018c_{1,1}^{0} + 0.016c_{2,0}^{0} + 0.936c_{2,1}^{0} + 0.016c_{2,2}^{0} + 0.014c_{3,1}^{0} + Q_{2,1}^{0}$

$i = 3 \quad c_{3,1}^{1} = 0.018c_{2,1}^{0} + 0.016c_{3,0}^{0} + 0.936c_{3,1}^{0} + 0.016c_{3,2}^{0} + 0.014c_{4,1}^{0} + Q_{3,1}^{0}$

$i = 4 \quad c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.936c_{4,1}^{0} + 0.016c_{4,2}^{0} + 0.014c_{5,1}^{0} + Q_{4,1}^{0}$

and apply boundary condition $c_{5,1}^{0} = c_{4,1}^{0}$

$$c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.016c_{4,0}^{0} + 0.936c_{4,1}^{0} + 0.016c_{4,2}^{0} + 0.014c_{4,1}^{0} + Q_{4,1}^{0}$$

$$j=2,\quad i=1\quad c_{1,2}^{1}=0.018c_{0,2}^{0}+0.016c_{1,1}^{0}+0.936c_{1,2}^{0}+0.016c_{1,3}^{0}+0.014\mathrm{c}_{2,2}^{0}+Q_{1,2}^{0}$$

$$i=2\quad c_{2,2}^{1}=0.018c_{1,2}^{0}+0.016c_{2,1}^{0}+0.936c_{2,2}^{0}+0.016c_{2,3}^{0}+0.014\mathrm{c}_{3,2}^{0}+Q_{2,2}^{0}$$

$$i=3\quad c_{3,2}^{1}=0.018c_{2,2}^{0}+0.016c_{3,1}^{0}+0.936c_{3,2}^{0}+0.016c_{3,3}^{0}+0.014\mathrm{c}_{4,2}^{0}+Q_{3,2}^{0}$$

$$i=4\quad c_{4,2}^{1}=0.018c_{3,2}^{0}+0.016c_{4,1}^{0}+0.936c_{4,2}^{0}+0.016c_{4,3}^{0}+0.014\mathrm{c}_{5,2}^{0}+Q_{4,2}^{0}$$

and apply boundary condition $c_{5,2}^{0}=c_{4,2}^{0}$

$$c_{4,2}^{1}=0.018c_{3,2}^{0}+0.016c_{4,1}^{0}+0.936c_{4,2}^{0}+0.016c_{4,3}^{0}+0.014c_{4,2}^{0}+Q_{4,2}^{0}$$

$$j=3,\quad i=1\quad c_{1,3}^{1}=0.018c_{0,3}^{0}+0.016c_{1,2}^{0}+0.936c_{1,3}^{0}+0.016c_{1,4}^{0}+0.014\mathrm{c}_{2,3}^{0}+Q_{1,3}^{0}$$

$$i=2\quad c_{2,3}^{1}=0.018c_{1,3}^{0}+0.016c_{2,2}^{0}+0.936c_{2,3}^{0}+0.016c_{2,4}^{0}+0.014\mathrm{c}_{3,3}^{0}+Q_{2,3}^{0}$$

$$i=3\quad c_{3,3}^{1}=0.018c_{2,3}^{0}+0.016c_{3,2}^{0}+0.936c_{3,3}^{0}+0.016c_{3,4}^{0}+0.014\mathrm{c}_{4,3}^{0}+Q_{3,3}^{0}$$

$$i=4\quad c_{4,3}^{1}=0.018c_{3,3}^{0}+0.016c_{4,2}^{0}+0.936c_{4,3}^{0}+0.016c_{4,4}^{0}+0.014\mathrm{c}_{5,3}^{0}+Q_{4,3}^{0}$$

and apply boundary condition $c_{5,3}^{0}=c_{4,3}^{0}$

$$c_{4,3}^{1}=0.018c_{3,3}^{0}+0.016c_{4,2}^{0}+0.936c_{4,3}^{0}+0.016c_{4,4}^{0}+0.014c_{4,3}^{0}+Q_{4,3}^{0}$$

$$j=4,\quad i=1\quad c_{1,4}^{1}=0.018c_{0,4}^{0}+0.016c_{1,3}^{0}+0.936c_{1,4}^{0}+0.016c_{1,5}^{0}+0.014\mathrm{c}_{2,4}^{0}+Q_{1,4}^{0}$$

and apply boundary condition $c_{1,5}^{0}=c_{1,4}^{0}$

$$c_{1,4}^{1}=0.018c_{0,4}^{0}+0.016c_{1,3}^{0}+0.936c_{1,4}^{0}+0.016c_{1,4}^{0}+0.014\mathrm{c}_{2,4}^{0}+Q_{1,4}^{0}$$

$$i=2\quad c_{2,4}^{1}=0.018c_{1,4}^{0}+0.016c_{2,3}^{0}+0.936c_{2,4}^{0}+0.016c_{2,5}^{0}+0.014\mathrm{c}_{3,4}^{0}+Q_{2,4}^{0}$$

and apply boundary condition $c_{2,5}^{0}=c_{2,4}^{0}$

$$c_{2,4}^{1}=0.018c_{1,4}^{0}+0.016c_{2,3}^{0}+0.936c_{2,4}^{0}+0.016c_{2,4}^{0}+0.014\mathrm{c}_{3,4}^{0}+Q_{2,4}^{0}$$

$$i=3\quad c_{3,4}^{1}=0.018c_{2,4}^{0}+0.016c_{3,3}^{0}+0.936c_{3,4}^{0}+0.016c_{3,5}^{0}+0.014\mathrm{c}_{4,4}^{0}+Q_{3,4}^{0}$$

and apply boundary condition $c_{3,5}^{0}=c_{3,4}^{0}$

$$c_{3,4}^{1}=0.018c_{2,4}^{0}+0.016c_{3,3}^{0}+0.936c_{3,4}^{0}+0.016c_{3,4}^{0}+0.014\mathrm{c}_{4,4}^{0}+Q_{3,4}^{0}$$

$$i=4\quad c_{4,4}^{1}=\frac{c_{3,4}^{1}+c_{4,3}^{1}}{2}$$

Table 4.10 The calculated pollutant concentration at time $t=0.02$ of example 4.2.11

Point	Pollutant concentration		
	$c_3=-0.1$	$c_3=0$	$c_3=0.1$
$c_{0,0}^2$	0.000000	0.000000	0.000000
$c_{0,1}^2$	0.000505	0.001292	0.002078
$c_{0,2}^2$	0.000873	0.001666	0.002460
$c_{0,3}^2$	0.000510	0.001298	0.002086
$c_{0,4}^2$	0.000448	0.000648	0.000848
$c_{1,0}^2$	0.000000	0.000000	0.000000
$c_{1,1}^2$	0.033499	0.033505	0.033512
$c_{1,2}^2$	0.045148	0.045154	0.045161
$c_{1,3}^2$	0.033871	0.033877	0.033884
$c_{1,4}^2$	0.001236	0.001236	0.001236
$c_{2,0}^2$	0.000000	0.000000	0.000000
$c_{2,1}^2$	0.044898	0.044898	0.044898
$c_{2,2}^2$	0.060707	0.060707	0.060707
$c_{2,3}^2$	0.045396	0.045396	0.045396
$c_{2,4}^2$	0.001652	0.001652	0.001652
$c_{3,0}^2$	0.000000	0.000000	0.000000
$c_{3,1}^2$	0.033504	0.033504	0.033504
$c_{3,2}^2$	0.045153	0.045153	0.045153
$c_{3,3}^2$	0.033876	0.033876	0.033876
$c_{3,4}^2$	0.001236	0.001236	0.001236
$c_{4,0}^2$	0.000000	0.000000	0.000000
$c_{4,1}^2$	0.001095	0.001095	0.001095
$c_{4,2}^2$	0.001468	0.001468	0.001468
$c_{4,3}^2$	0.001101	0.001101	0.001101
$c_{4,4}^2$	0.000598	0.000598	0.000598

Example 4.2.12. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c=\overline{c_0}$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x}=\overline{c_1}$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial c}{\partial z}=\overline{c_2}$ and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial c}{\partial z}=\overline{c_3}$ domain $\Omega=(0,1)\times(0,1)$ in Figure (4.26) with step size $\Delta x=\Delta z=0.1$, $\Delta t=0.01$ diffusion coefficient $D_{xz}=0.1$, average air pollutant source are added $Q=0.0001$, and average air flow velocity $u=0.1$.

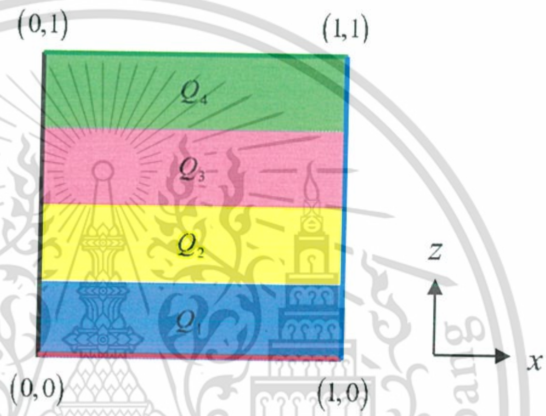


Figure 4.26: The classification of averaged air pollutant sources over the ground example 4.2.12

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.7) as

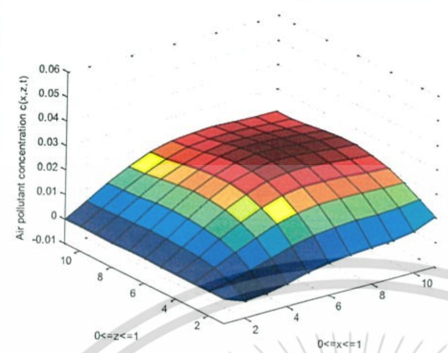
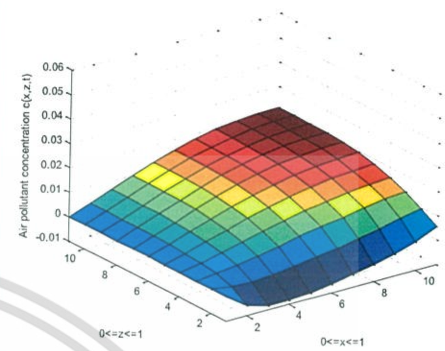
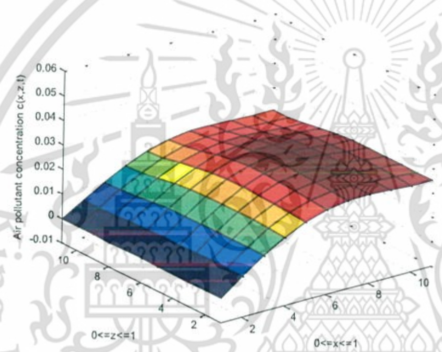
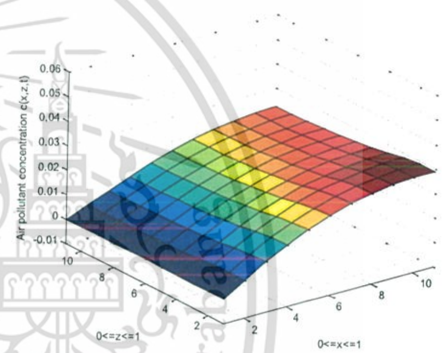
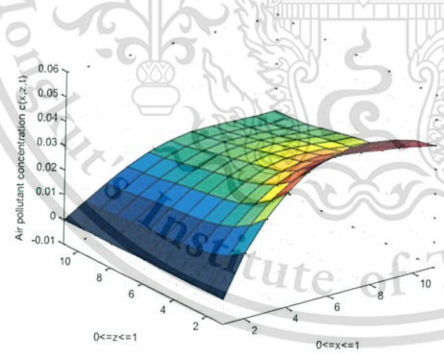
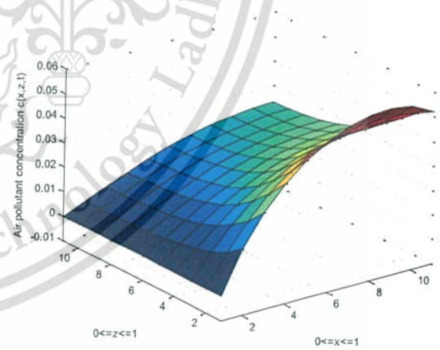
$$c(x,z,0)=x(1-x)z(1-z)+c_0,$$

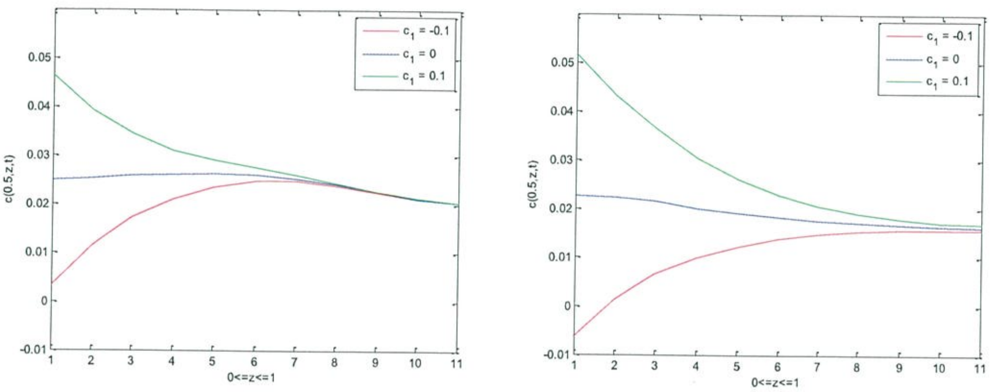
the boundary conditions can be assumed by

$$c(0,z,t)=c_0=0,$$
$$c_x(1,z,t)=c_1=0$$
$$c_z(x,1,t)=c_2=0$$
$$c_z(x,0,t)=c_3.$$

define $c(x,z,t)=c_{i,j}^n$ with $\Delta x=\Delta z=0.1, \Delta t=0.01, D=0.1, u=0.1, Q_1=0.0001$ and $Q_2=Q_3=Q_4=0$

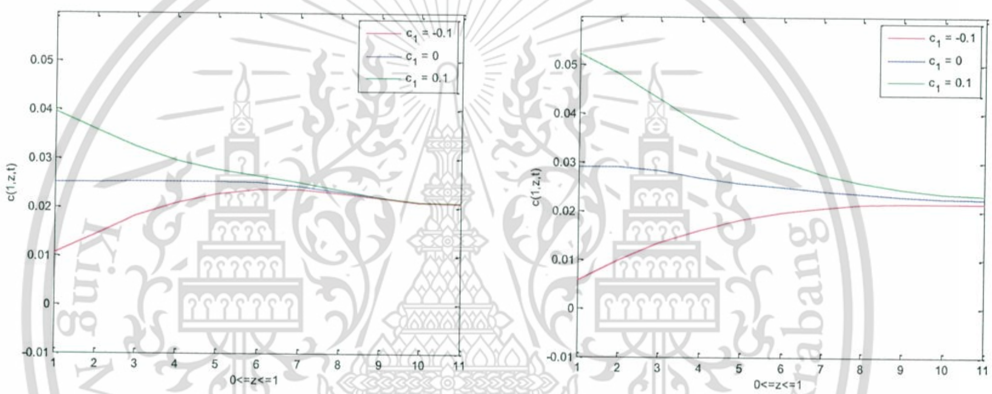
Table 4.11 The approximated air pollutant concentration at $0 \le t \le 60$ sec and $0 \le t \le 120$ sec of example 4.2.12 with the absorbance boundary ($c_3 = -0.1$), non-slipping boundary ($c_3 = 0$) or releasing boundary ($c_3 = 0.1$) on the ground

$\overline{c_3}$	$t = 60$	$t = 120$
-0.1		
0		
0.1		



(a) (b)

Figure 4.27: The approximated air pollutant concentration $c(0.5,z,t)$ at (a) $0 \le t \le 60$ sec and (b) $0 \le t \le 120$ sec of example 4.2.12



(a) (b)

Figure 4.28: The approximated air pollutant concentration $c(1,z,t)$ at (a) $0 \le t \le 60$ sec and (b) $0 \le t \le 120$ sec of example 4.2.12

4.3 Numerical simulation to a two-dimensional using non-dimensional form of the governing equation

In this section, we show example to find the pollutant concentration at any point is calculated by dispersion model in non-dimensional.

Example 4.2.13. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $C = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial C}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial C}{\partial x} = c_2$, and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial C}{\partial z}=c_3$ domain $\Omega=(0,H)\times(0,L)$ in Figure (4.29) with step size $\Delta X=\Delta Z=0.005, \Delta T=0.0001,$ diffusion coefficient $D=0.0005$, there is no interior source $S=0$, and average air flow velocity $U=1$.

Figure 4.29: Domain of example 4.2.13

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (4.16) as

$$C(X,Z,0)=X(1-X)Z(1-Z)+c_0$$

the boundary conditions can be assumed by

$$C(0,Z,T)=c_0=0,$$

$$C_x(1,Z,T)=c_1=0,$$

$$C_z(X,1,T)=c_2=0,$$

$$C_z(X,0,T)=c_3=0,$$

define $C(X,Z,T)=C^n_{i,j}$ with $\Delta X=\Delta Z=0.005, \Delta T=0.0001,$ $D=0.0005, U=1, st=1,$ and $S=0$

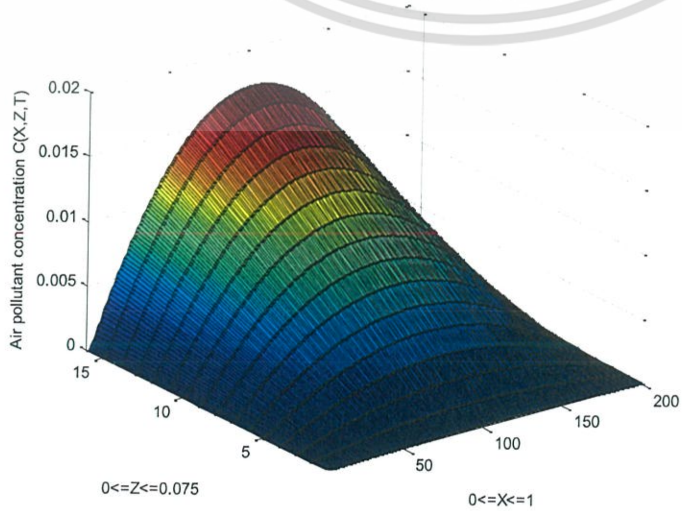


Figure 4.30: The approximated air pollutant concentration $C(X,Z,0.01)$ of example 4.2.13

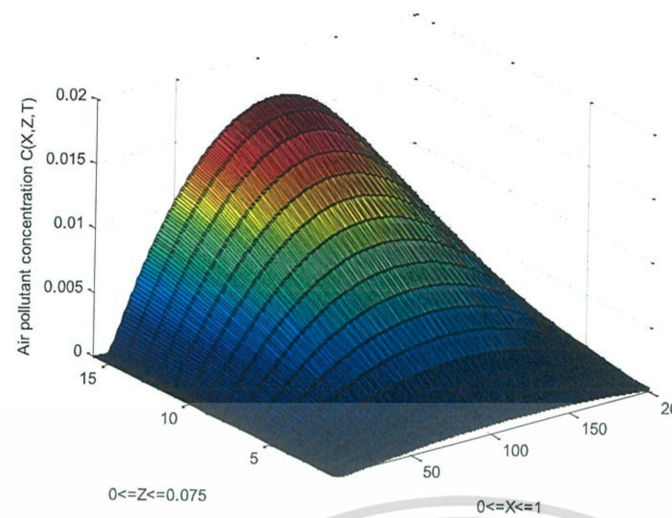


Figure 4.31: The approximated air pollutant concentration $C(X,Z,0.05)$ of example 4.2.13

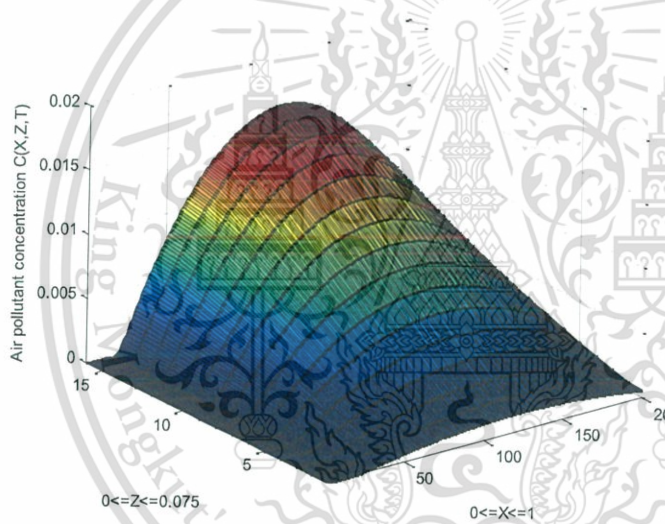


Figure 4.32: The approximated air pollutant concentration $C(X,Z,0.1)$ of example 4.2.13

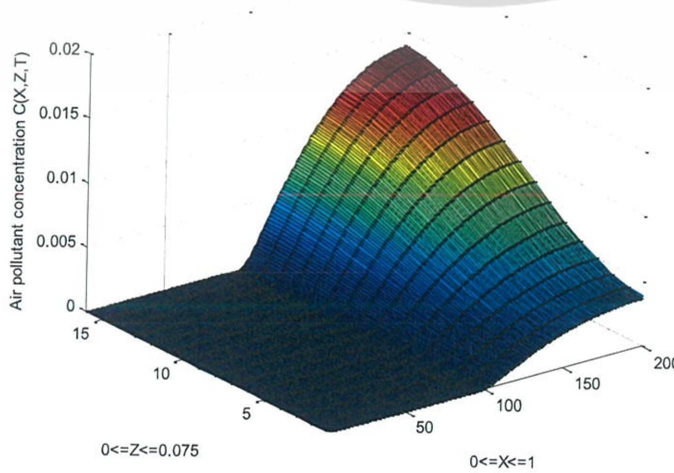


Figure 4.33: The approximated air pollutant concentration $C(X,Z,0.5)$ of example 4.2.13

Example 4.2.14. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $C = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial C}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial C}{\partial x} = c_2$, and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial C}{\partial z} = c_3$ domain $\Omega = (0,H)\times(0,L)$ in Figure (4.34) with step size $\Delta X = \Delta Z = 0.005, \Delta T = 0.0001,$ diffusion coefficient $D = 0.0005$, averages air pollutant source are added S, and average air flow velocity $U = 1$.

Figure 4.34: The classification of averaged air pollutant sources over the ground of example 4.2.14

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (4.16) as

$$C(X,Z,0) = X(1-X)Z(1-Z) + c_0$$

the boundary conditions can be assumed by

$$C(0,Z,T) = c_0 = 0,$$

$$C_x(1,Z,T) = c_1 = 0,$$

$$C_z(X,1,T) = c_2 = 0,$$

$$C_z(X,0,T) = c_3 = 0,$$

define $C(X,Z,T) = C_{i,j}^n$ with $\Delta X = \Delta Z = 0.005, \Delta T = 0.0001, D = 0.0005,$

$U = 1, st = 1,$ and $S_1 = 0.0001, S_2 = S_3 = S_4 = 0$

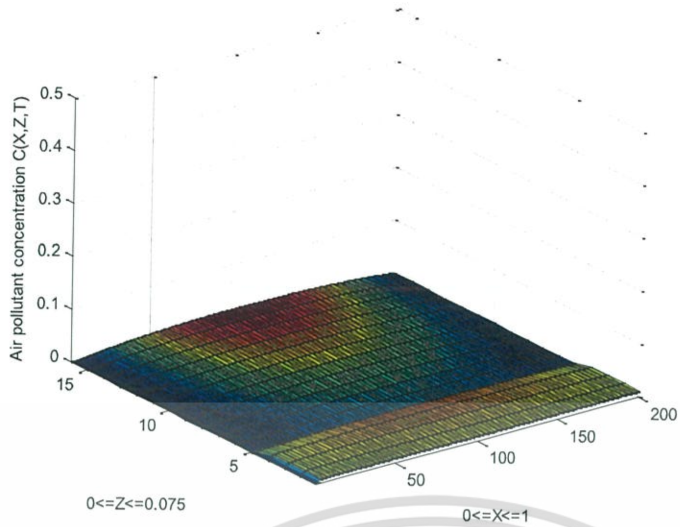


Figure 4.35: The approximated air pollutant concentration $C(X,Z,0.01)$ of example 4.2.14

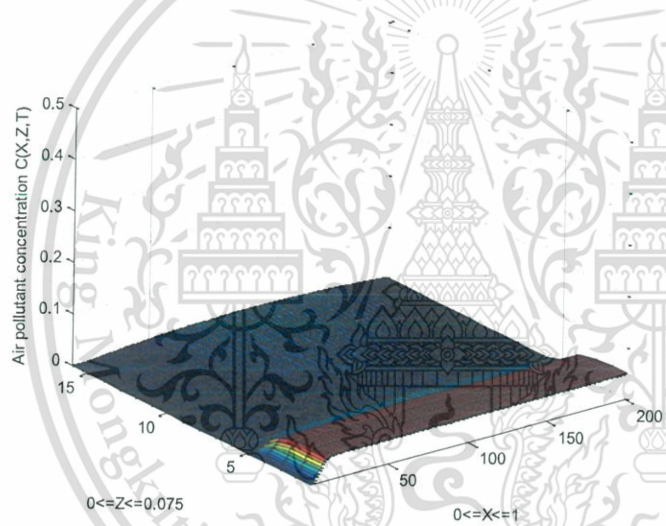


Figure 4.36: The approximated air pollutant concentration $C(X,Z,0.05)$ of example 4.2.14

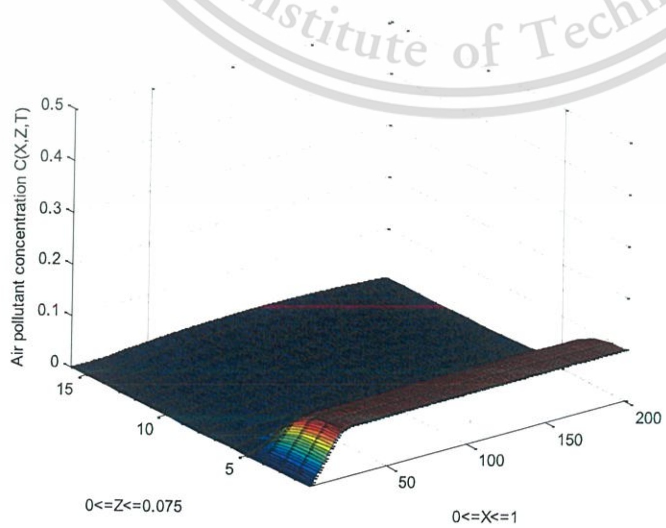


Figure 4.37: The approximated air pollutant concentration $C(X,Z,0.1)$ of example 4.2.14

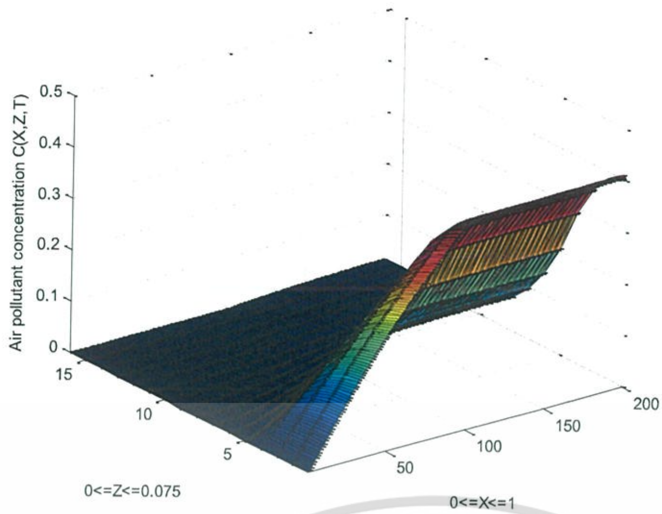


Figure 4.38: The approximated air pollutant concentration $C(X,Z,0.5)$ of example 4.2.14

Example 4.2.15. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $C=c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial C}{\partial x}=c_1$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial C}{\partial x}=c_2$, and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial C}{\partial z}=c_3$ domain $\Omega=(0,H)\times(0,L)$ in Figure (4.39) with step size $\Delta X=\Delta Z=0.005, \Delta T=0.0001,$ diffusion coefficient $D=0.0005$, interpolated air pollutant source are added S, and average air flow velocity $U=1$.

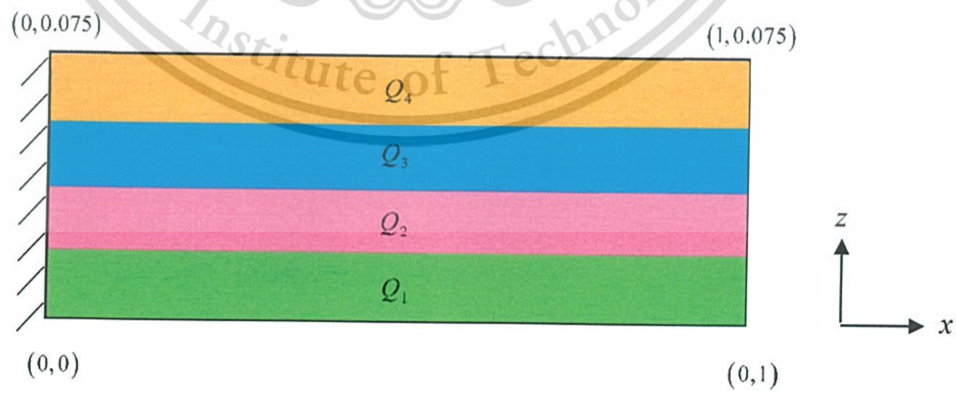


Figure 4.39: The classification of averaged air pollutant sources over the ground of example 4.2.15

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (4.16) as

$$C(X,Z,0)=X(1-X)Z(1-Z)+c_0$$

the boundary conditions can be assumed by

$$C(0,Z,T)=c_0=0,$$
$$C_x(1,Z,T)=c_1=0,$$
$$C_z(X,1,T)=c_2=0,$$
$$C_z(X,0,T)=c_3=0,$$

define $C(X,Z,T)=C_{i,j}^{n}$ with

$\Delta X=\Delta Z=0.005, \Delta T=0.0001, D=0.0005, st=1, U=1,$ and

$S_1=(0.2133\times10^{-4})x^3+(0.22\times10^{4})x^2-(0.3633\times10^{-4})x+(1.2041\times10^{-4}),$

$S_2=S_3=S_4=0$

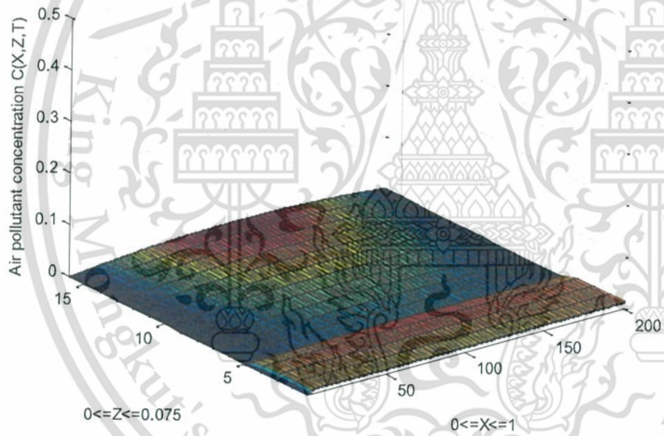


Figure 4.40: The approximated air pollutant concentration $C(X,Z,0.01)$ of example 4.2.15

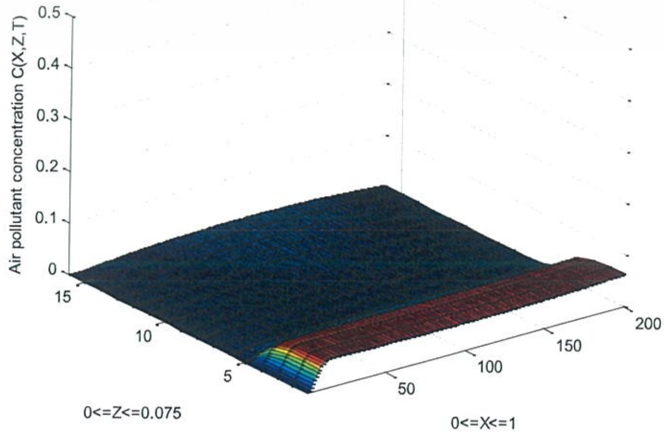


Figure 4.41: The approximated air pollutant concentration $C(X,Z,0.05)$ of example 4.2.15

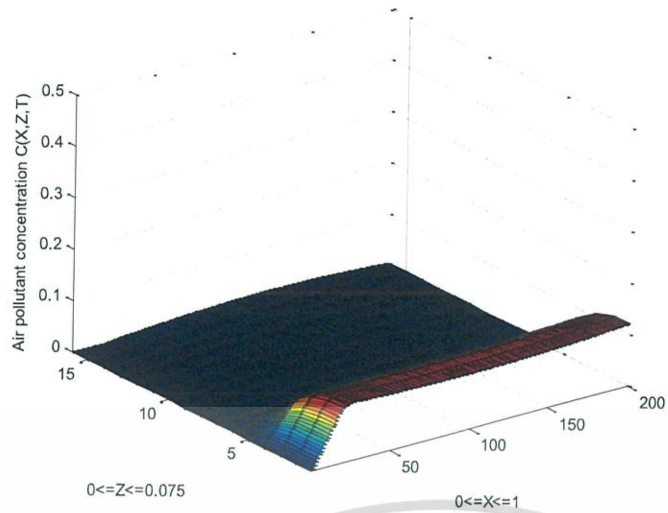


Figure 4.42: The approximated air pollutant concentration $C(X,Z,0.1)$ of example 4.2.15

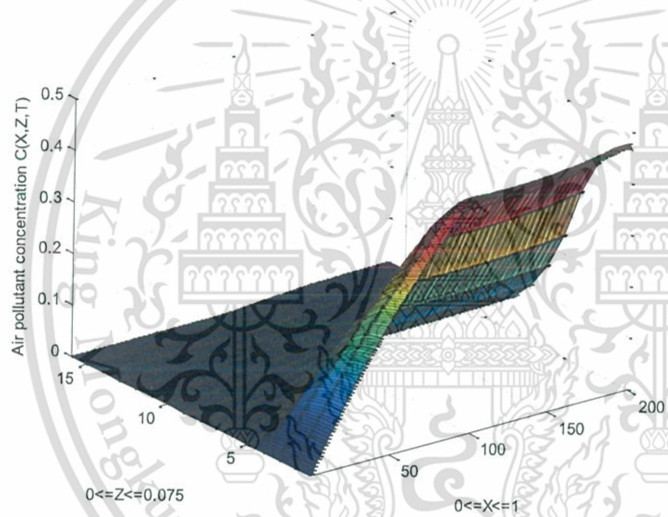


Figure 4.43: The approximated air pollutant concentration $C(X,Z,0.5)$ of example 4.2.15

Example 4.2.16. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $C=c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial C}{\partial x}=c_1$, and rate of change of pollutant concentration with respect to z at the top gate $\frac{\partial C}{\partial x}=c_2$, and rate of change of pollutant concentration with respect to z at the ground gate $\frac{\partial C}{\partial z}=c_3$ domain $\Omega=(0,H)\times(0,L)$ in Figure (4.44) with step size $\Delta X=\Delta Z=0.005, \Delta T=0.0001,$ diffusion coefficient $D=0.0005$, interpolated air pollutant source are added S, and interpolated air flow velocity $U=100z^2+\frac{1}{100}$.

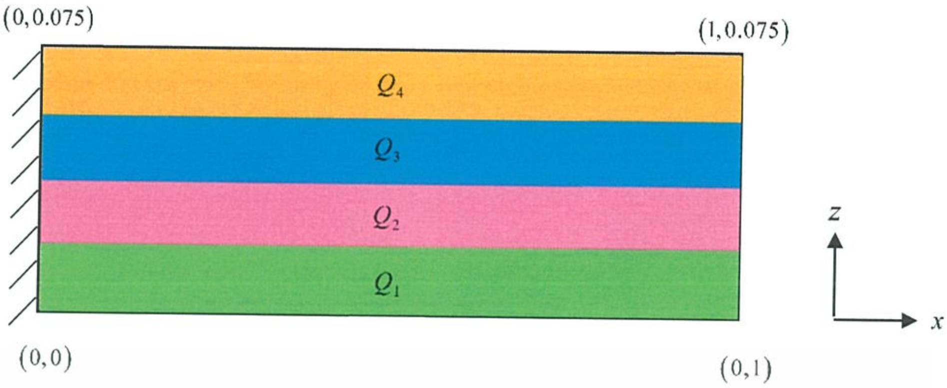


Figure 4.44: The classification of averaged air pollutant sources over the ground of example 4.2.15

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (4.16) as

$$C(X,Z,0) = X(1-X)Z(1-Z)+c_0$$

the boundary conditions can be assumed by

$$C(0,Z,T) = c_0 = 0,$$

$$C_x(1,Z,T) = c_1 = 0,$$

$$C_z(X,1,T) = c_2 = 0,$$

$$C_z(X,0,T) = c_3 = 0,$$

define $C(X,Z,T) = C_{i,j}^n$ with

$\Delta X = \Delta Z = 0.005, \Delta T = 0.0001, D = 0.0005, st = 1 \;\; U = 100z^2 + \frac{1}{100}$, and

$S_1 = (0.2133\times10^{-4})x^3 + (0.22\times10^{4})x^2 - (0.3633\times10^{-4})x + (1.2041\times10^{-4})$,

$S_2 = S_3 = S_4 = 0$

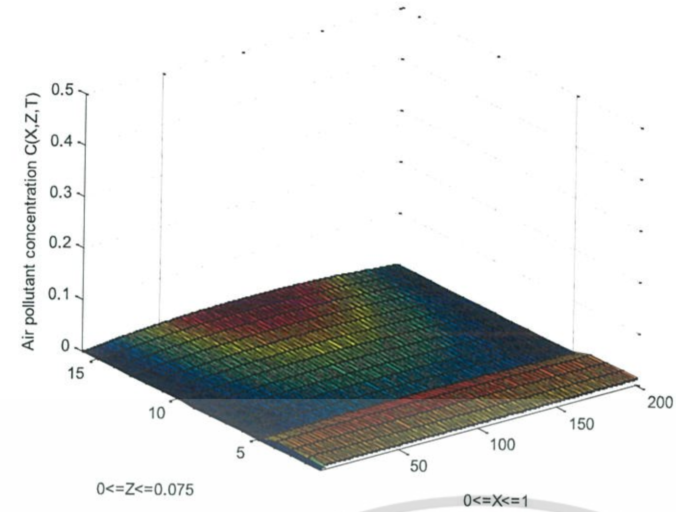


Figure 4.45: The approximated air pollutant concentration $C(X,Z,0.01)$ of example 4.2.16

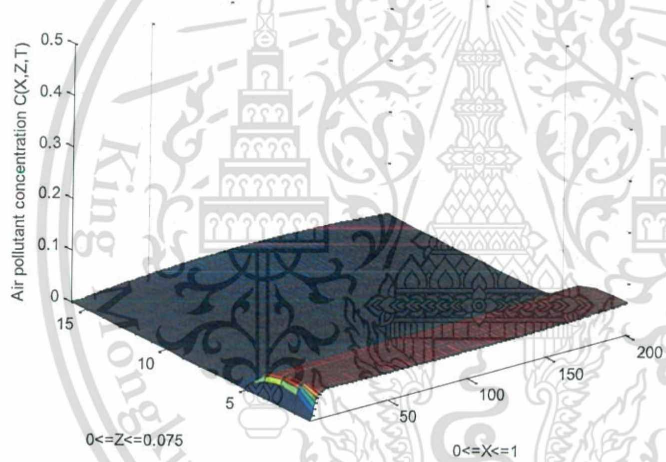


Figure 4.46: The approximated air pollutant concentration $C(X,Z,0.05)$ of example 4.2.16

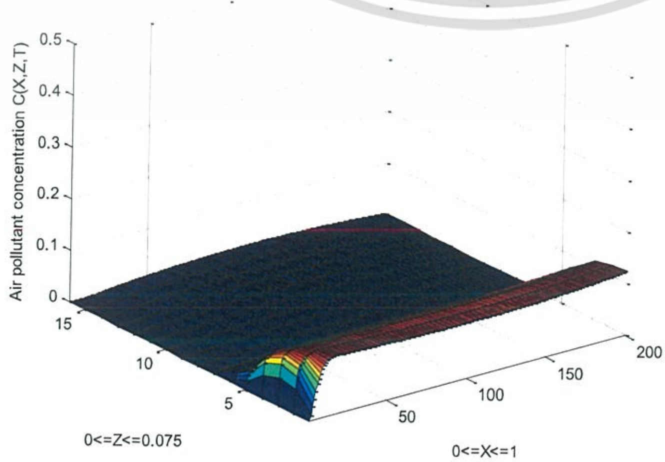


Figure 4.47: The approximated air pollutant concentration $C(X,Z,0.1)$ of example 4.2.16

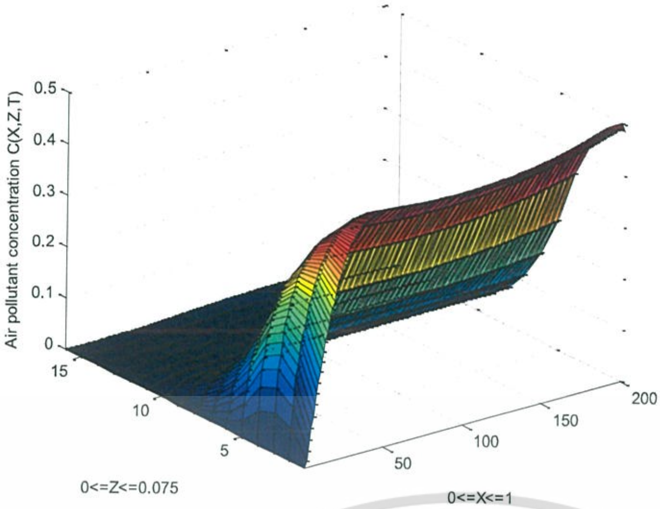


Figure 4.48: The approximated air pollutant concentration $C(X,Z,0.5)$ of example 4.2.16

4.4 Numerical method to a two-dimensional form (xy) air pollutant dispersion model with insided air pollutant source

We use the forward differenced in time and central difference in space in advection diffusion equation. We can approximate $c_{i,j}^{n}$ are the values difference approximation of at point $x = i\Delta x,\ y = j\Delta y$ and $t = n\Delta t$ where $0 \le i \le L, 0 \le j \le H$ and $0 \le n \le N$. Using the forward time center space method to Eq. (2.8), the following finite difference equation can be obtained

$$c \cong c_{i,j}^{n} \tag{4.52}$$

$$\frac{\partial c}{\partial t} \cong \frac{c_{i,j}^{n+1} - c_{i,j}^{n}}{\Delta t}, \tag{4.53}$$

$$\frac{\partial c}{\partial x} \cong \frac{c_{i+1,j}^{n} - c_{i-1,j}^{n}}{2\Delta x}, \tag{4.54}$$

$$\frac{\partial c}{\partial x} \cong \frac{c_{i,j+1}^{n} - c_{i,j-1}^{n}}{2\Delta y}, \tag{4.55}$$

$$\frac{\partial^2 c}{\partial x^2} \cong \frac{c_{i+1,j}^{n} - 2c_{i,j}^{n} + c_{i-1,j}^{n}}{(\Delta x)^2}, \tag{4.56}$$

$$\frac{\partial^2 c}{\partial y^2} \cong \frac{c_{i,j+1}^{n} - 2c_{i,j}^{n} + c_{i,j-1}^{n}}{(\Delta y)^2}. \tag{4.57}$$

$$Q \cong Q_{i,j}^{n}. \tag{4.58}$$

substituting Eq. (4.52-4.58) into Eq. (2.8), we have

$$\frac{c_{i,j}^{n+1}-c_{i,j}^{n}}{\Delta t}+u\left(\frac{c_{i+1,j}^{n}-c_{i-1,j}^{n}}{2\Delta x}\right)+v\left(\frac{c_{i,j+1}^{n}-c_{i,j-1}^{n}}{2\Delta y}\right)$$
$$=D_{xy}\left(\frac{c_{i+1,j}^{n}-2c_{i,j}^{n}+c_{i-1,j}^{n}}{(\Delta x)^2}+\frac{c_{i,j+1}^{n}-2c_{i,j}^{n}+c_{i,j-1}^{n}}{(\Delta y)^2}\right)+Q_{i,j}^{n} \tag{4.59}$$

We can obtain a simply form of Eq. (4.59)

$$c_{i,j}^{n+1}=B_1c_{i-1,j}^{n}+B_2c_{i,j-1}^{n}+B_3c_{i,j}^{n}+B_4c_{i,j+1}^{n}+B_5\,c_{i+1,j}^{n}+Q_{i,j}^{n} \tag{4.60}$$

where

$$B_1=D_{xy}\frac{\Delta t}{(\Delta x)^2}+u\frac{\Delta t}{2\Delta x}, \tag{4.61}$$

$$B_2=D_{xy}\frac{\Delta t}{(\Delta y)^2}+v\frac{\Delta t}{2\Delta y}, \tag{4.62}$$

$$B_3=1-2D_{xy}\frac{\Delta t}{(\Delta x)^2}-2D_{xy}\frac{\Delta t}{(\Delta y)^2}, \tag{4.63}$$

$$B_4=D_{xy}\frac{\Delta t}{(\Delta y)^2}-v\frac{\Delta t}{2\Delta y}, \tag{4.64}$$

$$B_5=D_{xy}\frac{\Delta t}{(\Delta x)^2}-u\frac{\Delta t}{2\Delta x}. \tag{4.65}$$

If $c_{i,j}^{n}$ are lied on the boundary of the domain, we will approximate by using the boundary conditions and employing the forward and backward different scheme,

$$\frac{\partial c}{\partial x}\approx\frac{c_{i+1,j}^{n}-c_{i,j}^{n}}{\Delta x}, \tag{4.66}$$

$$\frac{\partial c}{\partial x}\approx\frac{c_{i,j+1}^{n}-c_{i,j}^{n}}{\Delta x}, \tag{4.67}$$

$$\frac{\partial c}{\partial y}\approx\frac{c_{i,j}^{n}-c_{i-1,j}^{n}}{\Delta y}, \tag{4.68}$$

$$\frac{\partial c}{\partial y}\approx\frac{c_{i,j}^{n}-c_{i,j-1}^{n}}{\Delta y}. \tag{4.69}$$

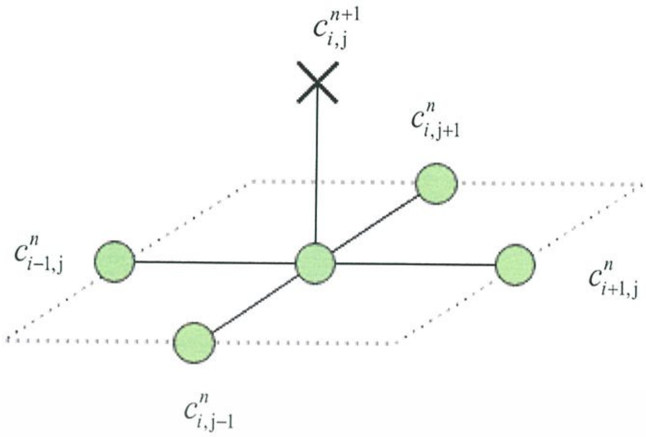


Figure 4.49: The stencil diagram of two - dimensional dispersion model (xy)

4.5 Numerical simulation to a two-dimensional form (xy) air pollutant dispersion model with insided air pollutant source

In this section, we show example to find the pollutant concentration at any point is calculated by dispersion model in two dimensional.

Example 4.5.1. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, domain $\Omega = (0,1)\times(0,1)$ in Figure (4.50) with step size $\Delta x = \Delta y = 0.25$, $\Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction $v = 0.1$.

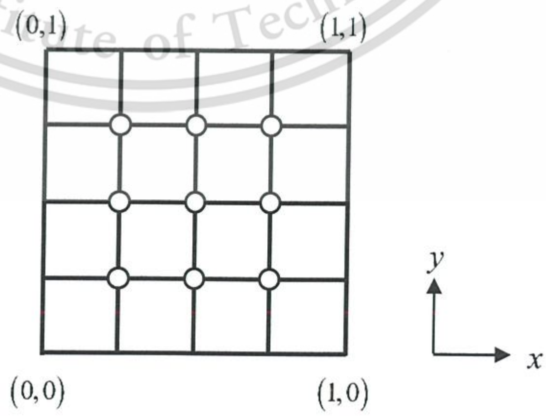


Figure 4.50: Generating grid points of example 4.5.1

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0)=x(1-x)y(1-y)+c_0,$$

the boundary conditions can be assumed by

$$c(0,y,t)=c_0=0,$$

define $c(x,y,t)=c_{i,j}^n$ with $\Delta x=\Delta y=0.25, \Delta t=0.01, D=0.1,$ and $u=v=0.1.$

solving problem by finite difference method Eq.(4.60)

$$c_{i,j}^{n+1}=B_1c_{i-1,j}^n+B_2c_{i,j-1}^n+B_3c_{i,j}^n+B_4c_{i,j+1}^n+B_5\,c_{i+1,j}^n+Q_{i,j}^n$$

$$B_1=D_{xy}\frac{\Delta t}{(\Delta x)^2}+u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}+(0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_2=D_{xy}\frac{\Delta t}{(\Delta y)^2}+v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}+(0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_3=1-2D_{xy}\frac{\Delta t}{(\Delta x)^2}-2D_{xy}\frac{\Delta t}{(\Delta y)^2}=1-2(0.1)\frac{0.01}{(0.25)^2}-2(0.1)\frac{0.01}{(0.25)^2}$$
$$=0.936,$$

$$B_4=D_{xy}\frac{\Delta t}{(\Delta y)^2}-v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}-(0.1)\frac{0.01}{2(0.25)}=0.014,$$

$$B_5=D_{xy}\frac{\Delta t}{(\Delta x)^2}-u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}-(0.1)\frac{0.01}{2(0.25)}=0.014.$$

$$c_{i,j}^{n+1}=0.018c_{i-1,j}^n+0.018c_{i,j-1}^n+0.936c_{i,j}^n+0.014c_{i,j+1}^n+0.014\,c_{i+1,j}^n+Q_{i,j}^n$$
$$n=0,1,2,\ldots,\quad i=1,2,3,\quad j=1,2,3 \qquad (4.70)$$

$n=0,$

$j=1,\ i=1\quad c_{1,1}^1=0.018c_{0,1}^0+0.018c_{1,0}^0+0.936c_{1,1}^0+0.014c_{1,2}^0+0.014\,c_{2,1}^0+Q_{1,1}^0$

$i=2\quad c_{2,1}^1=0.018c_{1,1}^0+0.018c_{2,0}^0+0.936c_{2,1}^0+0.014c_{2,2}^0+0.014\,c_{3,1}^0+Q_{2,1}^0$

$i=3\quad c_{3,1}^1=0.018c_{2,1}^0+0.018c_{3,0}^0+0.936c_{3,1}^0+0.014c_{3,2}^0+0.014\,c_{4,1}^0+Q_{3,1}^0$

$j=2,\ i=1\quad c_{1,2}^1=0.018c_{0,2}^0+0.018c_{1,1}^0+0.936c_{1,2}^0+0.014c_{1,3}^0+0.014\,c_{2,2}^0+Q_{1,2}^0$

$i=2\quad c_{2,2}^1=0.018c_{1,2}^0+0.018c_{2,1}^0+0.936c_{2,2}^0+0.014c_{2,3}^0+0.014\,c_{3,2}^0+Q_{2,2}^0$

$i=3\quad c_{3,2}^1=0.018c_{2,2}^0+0.018c_{3,1}^0+0.936c_{3,2}^0+0.014c_{3,3}^0+0.014\,c_{4,2}^0+Q_{3,2}^0$

$j=3,\ i=1\quad c_{1,3}^1=0.018c_{0,3}^0+0.018c_{1,2}^0+0.936c_{1,3}^0+0.014c_{1,4}^0+0.014\,c_{2,3}^0+Q_{1,3}^0$

$i=2\quad c_{2,3}^1=0.018c_{1,3}^0+0.018c_{2,2}^0+0.936c_{2,3}^0+0.014c_{2,4}^0+0.014\,c_{3,3}^0+Q_{2,3}^0$

$i=3\quad c_{3,3}^1=0.018c_{2,3}^0+0.018c_{3,2}^0+0.936c_{3,3}^0+0.014c_{3,4}^0+0.014\,c_{4,3}^0+Q_{3,3}^0$

Table 4.12 The calculated pollutant concentration at time $t = 0.02$ of example 4.5.1

Point	Pollutant concentration	
	$Q = 0$	$Q = 0.0001$
$c_{0,0}^2$	0.000000	0.000000
$c_{0,1}^2$	0.000000	0.000000
$c_{0,2}^2$	0.000000	0.000000
$c_{0,3}^2$	0.000000	0.000000
$c_{0,4}^2$	0.000000	0.000000
$c_{1,0}^2$	0.000000	0.000000
$c_{1,1}^2$	0.033313	0.033510
$c_{1,2}^2$	0.044898	0.045096
$c_{1,3}^2$	0.033676	0.033873
$c_{1,4}^2$	0.000000	0.000000
$c_{2,0}^2$	0.000000	0.000000
$c_{2,1}^2$	0.044898	0.045096
$c_{2,2}^2$	0.060507	0.060707
$c_{2,3}^2$	0.045384	0.045582
$c_{2,4}^2$	0.000000	0.000000
$c_{3,0}^2$	0.000000	0.000000
$c_{3,1}^2$	0.033676	0.033873
$c_{3,2}^2$	0.045384	0.045582
$c_{3,3}^2$	0.034040	0.034237
$c_{3,4}^2$	0.000000	0.000000
$c_{4,0}^2$	0.000000	0.000000
$c_{4,1}^2$	0.000000	0.000000
$c_{4,2}^2$	0.000000	0.000000
$c_{4,3}^2$	0.000000	0.000000
$c_{4,4}^2$	0.000000	0.000000

Example 4.5.2. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, domain $\Omega = (0,1)\times(0,1)$ in Figure (4.51) with step size $\Delta x = \Delta y = 0.1$, $\Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction $v = 0.1$.

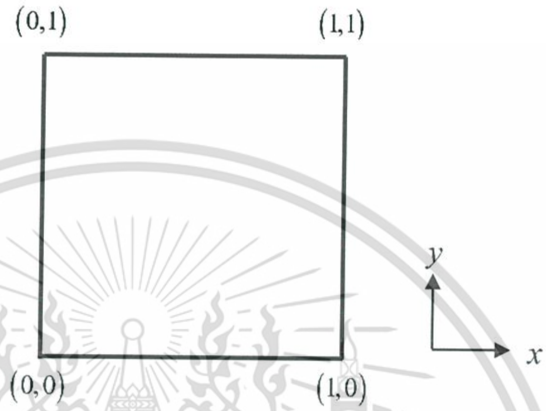


Figure 4.51: Domain of example 4.5.2

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0) = x(1-x)y(1-y) + c_0,$$

the boundary conditions can be assumed by

$$c(0,y,t) = c_0 = 0,$$

define $c(x,y,t) = c_{i,j}^n$ with $\Delta x = \Delta y = 0.1, \Delta t = 0.01, D = 0.1,$ and $u = v = 0.1$.

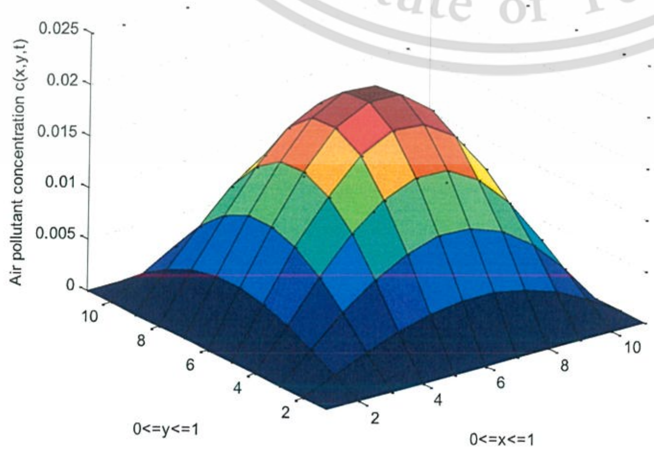


Figure 4.52: The approximated air pollutant concentration for $Q = 0$ and $0 \le t \le 60$ sec of example 4.5.2

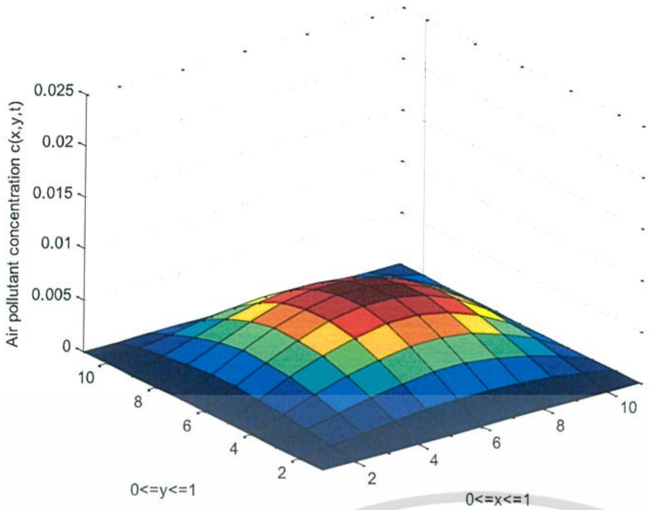


Figure 4.53: The approximated air pollutant concentration for $Q = 0$ and $0 \le t \le 120$ sec of example 4.5.2

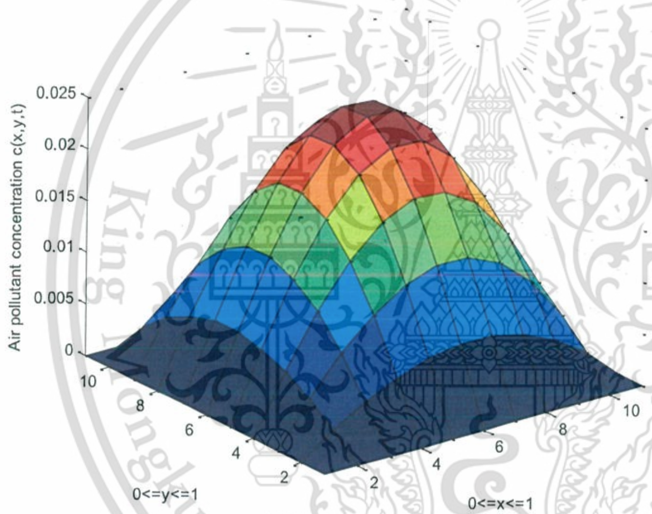


Figure 4.54: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \le 60$ sec of example 4.5.2

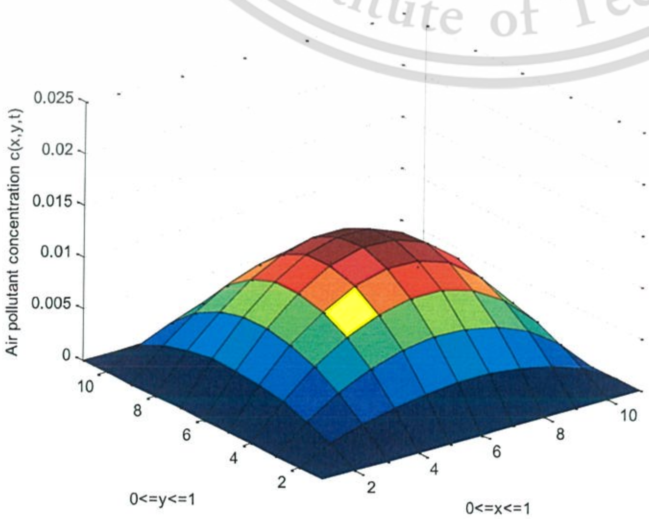


Figure 4.55: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \le 120$ sec of example 4.5.2

Example 4.5.3. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.56) with step size $\Delta x = \Delta y = 0.25$, $\Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction $v = 0.1$.

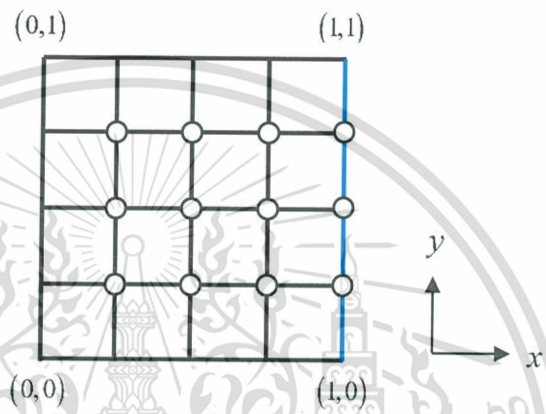


Figure 4.56: Generating grid points of example 4.5.3

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0) = x(1-x)y(1-y) + c_0,$$

the boundary conditions can be assumed by

$$c(0,y,t) = c_0 = 0,$$

$$c_x(1,y,t) = c_1 = 0$$

define $c(x,y,t) = c_{i,j}^n$ with $\Delta x = \Delta y = 0.25$, $\Delta t = 0.01$, $D = 0.1$, and $u = v = 0.1$.

approximate differential boundary condition,

$c_x(1,y,t) = 0$ or $\frac{\partial c_{i.j}^n}{\partial x} = 0$,

using forward difference,

$$\frac{c_{5,j}^n - c_{4,j}^n}{\Delta x} = 0$$

$$c_{5,j}^n = c_{4,j}^n \qquad j = 1,2,3 \qquad (4.71)$$

solving problem by finite difference method Eq.(4.60)

$$c_{i,j}^{n+1} = B_1 c_{i-1,j}^n + B_2 c_{i,j-1}^n + B_3 c_{i,j}^n + B_4 c_{i,j+1}^n + B_5 c_{i+1,j}^n + Q_{i,j}^n$$

$$B_1 = D_{xy}\frac{\Delta t}{(\Delta x)^2} + u\frac{\Delta t}{2\Delta x} = (0.1)\frac{0.01}{(0.25)^2} + (0.1)\frac{0.01}{2(0.25)} = 0.018,$$

$$B_2 = D_{xy}\frac{\Delta t}{(\Delta y)^2} + v\frac{\Delta t}{2\Delta y} = (0.1)\frac{0.01}{(0.25)^2} + (0.1)\frac{0.01}{2(0.25)} = 0.018,$$

$$B_3 = 1 - 2D_{xy}\frac{\Delta t}{(\Delta x)^2} - 2D_{xy}\frac{\Delta t}{(\Delta y)^2} = 1 - 2(0.1)\frac{0.01}{(0.25)^2} - 2(0.1)\frac{0.01}{(0.25)^2}$$
$$= 0.936,$$

$$B_4 = D_{xy}\frac{\Delta t}{(\Delta y)^2} - v\frac{\Delta t}{2\Delta y} = (0.1)\frac{0.01}{(0.25)^2} - (0.1)\frac{0.01}{2(0.25)} = 0.014,$$

$$B_5 = D_{xy}\frac{\Delta t}{(\Delta x)^2} - u\frac{\Delta t}{2\Delta x} = (0.1)\frac{0.01}{(0.25)^2} - (0.1)\frac{0.01}{2(0.25)} = 0.014.$$

$$c_{i,j}^{n+1} = 0.018c_{i-1,j}^n + 0.018c_{i,j-1}^n + 0.936c_{i,j}^n + 0.014c_{i,j+1}^n + 0.014c_{i+1,j}^n + Q_{i,j}^n$$

$$i = 1,2,3,4, \quad j = 1,2,3 \tag{4.72}$$

$n = 0,$

$j = 1, \quad i = 1 \quad c_{1,1}^1 = 0.018c_{0,1}^0 + 0.018c_{1,0}^0 + 0.936c_{1,1}^0 + 0.014c_{1,2}^0 + 0.014c_{2,1}^0 + Q_{1,1}^0$

$i = 2 \quad c_{2,1}^1 = 0.018c_{1,1}^0 + 0.018c_{2,0}^0 + 0.936c_{2,1}^0 + 0.014c_{2,2}^0 + 0.014c_{3,1}^0 + Q_{2,1}^0$

$i = 3 \quad c_{3,1}^1 = 0.018c_{2,1}^0 + 0.018c_{3,0}^0 + 0.936c_{3,1}^0 + 0.014c_{3,2}^0 + 0.014c_{4,1}^0 + Q_{3,1}^0$

$i = 4 \quad c_{4,1}^1 = 0.018c_{3,1}^0 + 0.018c_{4,0}^0 + 0.936c_{4,1}^0 + 0.014c_{4,2}^0 + 0.014c_{5,1}^0 + Q_{4,1}^0$

and apply boundary condition $c_{5,1}^0 = c_{4,1}^0$

$c_{4,1}^1 = 0.018c_{3,1}^0 + 0.018c_{4,0}^0 + 0.936c_{4,1}^0 + 0.014c_{4,2}^0 + 0.014c_{4,1}^0 + Q_{4,1}^0$

$j = 2, \quad i = 1 \quad c_{1,2}^1 = 0.018c_{0,2}^0 + 0.018c_{1,1}^0 + 0.936c_{1,2}^0 + 0.014c_{1,3}^0 + 0.014c_{2,2}^0 + Q_{1,2}^0$

$i = 2 \quad c_{2,2}^1 = 0.018c_{1,2}^0 + 0.018c_{2,1}^0 + 0.936c_{2,2}^0 + 0.014c_{2,3}^0 + 0.014c_{3,2}^0 + Q_{2,2}^0$

$i = 3 \quad c_{3,2}^1 = 0.018c_{2,2}^0 + 0.018c_{3,1}^0 + 0.936c_{3,2}^0 + 0.014c_{3,3}^0 + 0.014c_{4,2}^0 + Q_{3,2}^0$

$i = 4 \quad c_{4,2}^1 = 0.018c_{3,2}^0 + 0.018c_{4,1}^0 + 0.936c_{4,2}^0 + 0.014c_{4,3}^0 + 0.014c_{5,2}^0 + Q_{4,2}^0$

and apply boundary condition $c_{5,2}^0 = c_{4,2}^0$

$c_{4,2}^1 = 0.018c_{3,2}^0 + 0.018c_{4,1}^0 + 0.936c_{4,2}^0 + 0.014c_{4,3}^0 + 0.014c_{4,2}^0 + Q_{4,2}^0$

$j = 3, \quad i = 1 \quad c_{1,3}^1 = 0.018c_{0,3}^0 + 0.018c_{1,2}^0 + 0.936c_{1,3}^0 + 0.014c_{1,4}^0 + 0.014c_{2,3}^0 + Q_{1,3}^0$

$i = 2 \quad c_{2,3}^1 = 0.018c_{1,3}^0 + 0.018c_{2,2}^0 + 0.936c_{2,3}^0 + 0.014c_{2,4}^0 + 0.014c_{3,3}^0 + Q_{2,3}^0$

$i = 3 \quad c_{3,3}^1 = 0.018c_{2,3}^0 + 0.018c_{3,2}^0 + 0.936c_{3,3}^0 + 0.014c_{3,4}^0 + 0.014c_{4,3}^0 + Q_{3,3}^0$

$i = 4 \quad c_{4,3}^1 = 0.018c_{3,3}^0 + 0.018c_{4,2}^0 + 0.936c_{4,3}^0 + 0.014c_{4,4}^0 + 0.014c_{5,3}^0 + Q_{4,3}^0$

and apply boundary condition $c_{5,3}^{0}=c_{4,3}^{0}$

$$c_{4,3}^{1}=0.018c_{3,3}^{0}+0.018c_{4,2}^{0}+0.936c_{4,3}^{0}+0.014c_{4,4}^{0}+0.014c_{4,3}^{0}+Q_{4,3}^{0}$$

Table 4.13 The calculated pollutant concentration at time $t=0.02$ of example 4.5.3

Point	Pollutant concentration	
	$Q=0$	$Q=0.0001$
$c_{0,0}^{2}$	0.000000	0.000000
$c_{0,1}^{2}$	0.000000	0.000000
$c_{0,2}^{2}$	0.000000	0.000000
$c_{0,3}^{2}$	0.000000	0.000000
$c_{0,4}^{2}$	0.000000	0.000000
$c_{1,0}^{2}$	0.000000	0.000000
$c_{1,1}^{2}$	0.033313	0.033510
$c_{1,2}^{2}$	0.044898	0.045096
$c_{1,3}^{2}$	0.033685	0.033883
$c_{1,4}^{2}$	0.001232	0.001430
$c_{2,0}^{2}$	0.000000	0.000000
$c_{2,1}^{2}$	0.044898	0.045096
$c_{2,2}^{2}$	0.060507	0.060707
$c_{2,3}^{2}$	0.045395	0.045595
$c_{2,4}^{2}$	0.001652	0.001852
$c_{3,0}^{2}$	0.000000	0.000000
$c_{3,1}^{2}$	0.033676	0.033873
$c_{3,2}^{2}$	0.045384	0.045582
$c_{3,3}^{2}$	0.034049	0.034248
$c_{3,4}^{2}$	0.001239	0.001438
$c_{4,0}^{2}$	0.000000	0.000000
$c_{4,1}^{2}$	0.000000	0.000000
$c_{4,2}^{2}$	0.000000	0.000000
$c_{4,3}^{2}$	0.000000	0.000000
$c_{4,4}^{2}$	0.000000	0.000000

Example 4.5.4. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x, at the exit gate $\frac{\partial c}{\partial x} = c_1$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.57) with step size $\Delta x = \Delta y = 0.1$, $\Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction $v = 0.1$.

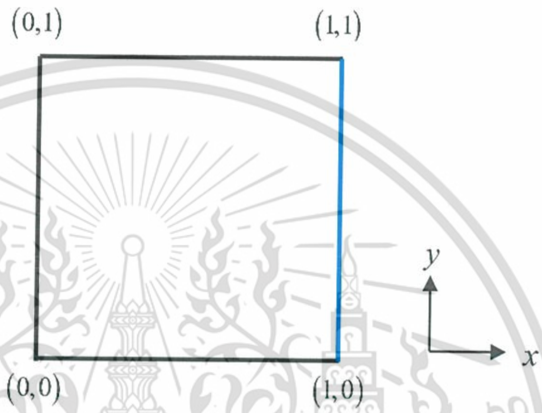


Figure 4.57: Domain of example 4.5.4

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (4.16) as

$$c(x,y,0) = x(1-x)y(1-y) + c_0,$$

the boundary conditions can be assumed by

$$c(0,y,t) = c_0 = 0,$$

$$c_x(1,y,t) = c_1 = 0$$

define $c(x,y,t) = c_{i,j}^n$ with $\Delta x = \Delta y = 0.1, \Delta t = 0.01, D = 0.1,$ and $u = v = 0.1.$

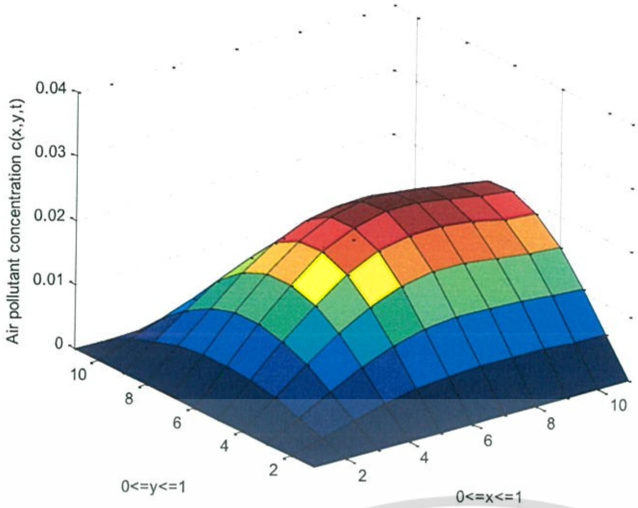


Figure 4.58: The approximated air pollutant concentration for $Q=0$ and $0 \le t \le 60$ sec of example 4.5.4

Figure 4.59: The approximated air pollutant concentration for $Q=0$ and $0 \le t \le 120$ sec of example 4.5.4

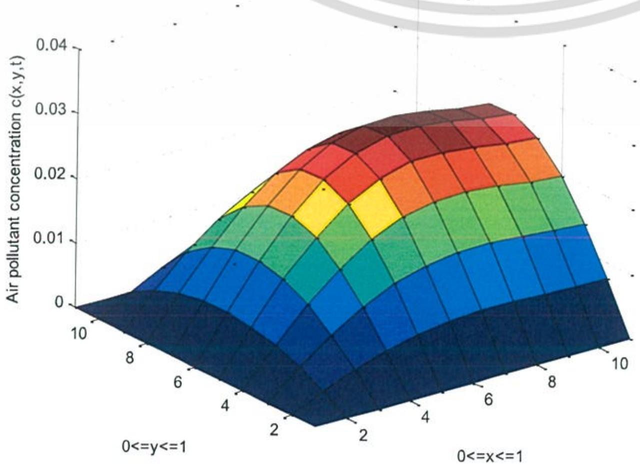


Figure 4.60: The approximated air pollutant concentration for $Q=0.0001$ and $0 \le t \le 60$ sec of example 4.5.4

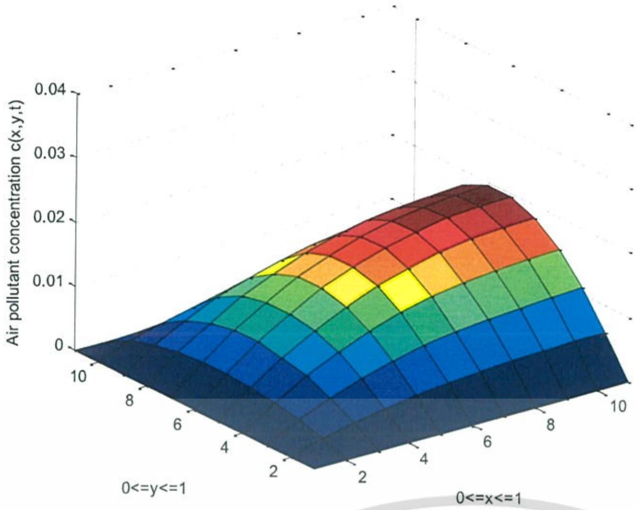


Figure 4.61: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \le 120$ sec of example 4.5.4

Example 4.5.5. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y} = c_2$ domain $\Omega = (0,1) \times (0,1)$ in Figure (4.62) with step size $\Delta x = \Delta y = 0.25$, $\Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction $v = 0.1$.

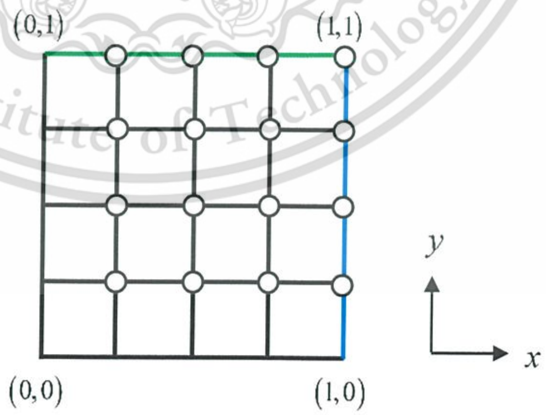


Figure 4.62: Generating grid points of example 4.5.5

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0)=x(1-x)y(1-y)+c_0,$$

the boundary conditions can be assumed by

$$c(0,y,t)=c_0=0,$$

$$c_x(1,y,t)=c_1=0$$

$$c_y(x,1,t)=c_2=0$$

define $c(x,y,t)=c_{i,j}^n$ with $\Delta x=\Delta y=0.25, \Delta t=0.01, D=0.1,$ and $u=v=0.1.$

approximate differential boundary condition,

$c_x(1,y,t)=0$ or $\frac{\partial c_{5,j}^n}{\partial x}=0,$

using forward difference,

$$\frac{c_{5,j}^n-c_{4,j}^n}{\Delta x}=0$$

$$c_{5,j}^n=c_{4,j}^n \qquad j=1,2,3,4 \tag{4.73}$$

$c_y(x,1,t)=0$ or $\frac{\partial c_{i,5}^n}{\partial y}=0,$

using forward difference,

$$\frac{c_{i,5}^n-c_{i,4}^n}{\Delta y}=0$$

$$c_{i,5}^n=c_{i,4}^n \qquad i=1,2,3,4 \tag{4.74}$$

solving problem by finite difference method Eq.(4.60)

$$c_{i,j}^{n+1}=B_1c_{i-1,j}^n+B_2c_{i,j-1}^n+B_3c_{i,j}^n+B_4c_{i,j+1}^n+B_5\,c_{i+1,j}^n+Q_{i,j}^n$$

$$B_1=D_{xy}\frac{\Delta t}{(\Delta x)^2}+u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}+(0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_2=D_{xy}\frac{\Delta t}{(\Delta y)^2}+v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}+(0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_3=1-2D_{xy}\frac{\Delta t}{(\Delta x)^2}-2D_{xy}\frac{\Delta t}{(\Delta y)^2}=1-2(0.1)\frac{0.01}{(0.25)^2}-2(0.1)\frac{0.01}{(0.25)^2}$$
$$=0.936,$$

$$B_4=D_{xy}\frac{\Delta t}{(\Delta y)^2}-v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}-(0.1)\frac{0.01}{2(0.25)}=0.014,$$

$$B_5=D_{xy}\frac{\Delta t}{(\Delta x)^2}-u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}-(0.1)\frac{0.01}{2(0.25)}=0.014.$$

$$c_{i,j}^{n+1}=0.018c_{i-1,j}^n+0.018c_{i,j-1}^n+0.936c_{i,j}^n+0.014c_{i,j+1}^n+0.014\,c_{i+1,j}^n+Q_{i,j}^n$$

$$i=1,2,3,4, \qquad j=1,2,3,4 \tag{4.75}$$

$n=0,$

$j=1,\quad i=1\quad c_{1,1}^{1}=0.018c_{0,1}^{0}+0.018c_{1,0}^{0}+0.936c_{1,1}^{0}+0.014c_{1,2}^{0}+0.014c_{2,1}^{0}+Q_{1,1}^{0}$

$i=2\quad c_{2,1}^{1}=0.018c_{1,1}^{0}+0.018c_{2,0}^{0}+0.936c_{2,1}^{0}+0.014c_{2,2}^{0}+0.014c_{3,1}^{0}+Q_{2,1}^{0}$

$i=3\quad c_{3,1}^{1}=0.018c_{2,1}^{0}+0.018c_{3,0}^{0}+0.936c_{3,1}^{0}+0.014c_{3,2}^{0}+0.014c_{4,1}^{0}+Q_{3,1}^{0}$

$i=4\quad c_{4,1}^{1}=0.018c_{3,1}^{0}+0.018c_{4,0}^{0}+0.936c_{4,1}^{0}+0.014c_{4,2}^{0}+0.014c_{5,1}^{0}+Q_{4,1}^{0}$

and apply boundary condition $c_{5,1}^{0}=c_{4,1}^{0}$

$c_{4,1}^{1}=0.018c_{3,1}^{0}+0.018c_{4,0}^{0}+0.936c_{4,1}^{0}+0.014c_{4,2}^{0}+0.014c_{4,1}^{0}+Q_{4,1}^{0}$

$j=2,\quad i=1\quad c_{1,2}^{1}=0.018c_{0,2}^{0}+0.018c_{1,1}^{0}+0.936c_{1,2}^{0}+0.014c_{1,3}^{0}+0.014c_{2,2}^{0}+Q_{1,2}^{0}$

$i=2\quad c_{2,2}^{1}=0.018c_{1,2}^{0}+0.018c_{2,1}^{0}+0.936c_{2,2}^{0}+0.014c_{2,3}^{0}+0.014c_{3,2}^{0}+Q_{2,2}^{0}$

$i=3\quad c_{3,2}^{1}=0.018c_{2,2}^{0}+0.018c_{3,1}^{0}+0.936c_{3,2}^{0}+0.014c_{3,3}^{0}+0.014c_{4,2}^{0}+Q_{3,2}^{0}$

$i=4\quad c_{4,2}^{1}=0.018c_{3,2}^{0}+0.018c_{4,1}^{0}+0.936c_{4,2}^{0}+0.014c_{4,3}^{0}+0.014c_{5,2}^{0}+Q_{4,2}^{0}$

and apply boundary condition $c_{5,2}^{0}=c_{4,2}^{0}$

$c_{4,2}^{1}=0.018c_{3,2}^{0}+0.018c_{4,1}^{0}+0.936c_{4,2}^{0}+0.014c_{4,3}^{0}+0.014c_{4,2}^{0}+Q_{4,2}^{0}$

$j=3,\quad i=1\quad c_{1,3}^{1}=0.018c_{0,3}^{0}+0.018c_{1,2}^{0}+0.936c_{1,3}^{0}+0.014c_{1,4}^{0}+0.014c_{2,3}^{0}+Q_{1,3}^{0}$

$i=2\quad c_{2,3}^{1}=0.018c_{1,3}^{0}+0.018c_{2,2}^{0}+0.936c_{2,3}^{0}+0.014c_{2,4}^{0}+0.014c_{3,3}^{0}+Q_{2,3}^{0}$

$i=3\quad c_{3,3}^{1}=0.018c_{2,3}^{0}+0.018c_{3,2}^{0}+0.936c_{3,3}^{0}+0.014c_{3,4}^{0}+0.014c_{4,3}^{0}+Q_{3,3}^{0}$

$i=4\quad c_{4,3}^{1}=0.018c_{3,3}^{0}+0.018c_{4,2}^{0}+0.936c_{4,3}^{0}+0.014c_{4,4}^{0}+0.014c_{5,3}^{0}+Q_{4,3}^{0}$

and apply boundary condition $c_{5,3}^{0}=c_{4,3}^{0}$

$c_{4,3}^{1}=0.018c_{3,3}^{0}+0.018c_{4,2}^{0}+0.936c_{4,3}^{0}+0.014c_{4,4}^{0}+0.014c_{4,3}^{0}+Q_{4,3}^{0}$

$j=4,\quad i=1\quad c_{1,4}^{1}=0.018c_{0,4}^{0}+0.018c_{1,3}^{0}+0.936c_{1,4}^{0}+0.014c_{1,5}^{0}+0.014c_{2,4}^{0}+Q_{1,4}^{0}$

and apply boundary condition $c_{1,5}^{0}=c_{1,4}^{0}$

$c_{1,4}^{1}=0.018c_{0,4}^{0}+0.018c_{1,3}^{0}+0.936c_{1,4}^{0}+0.014c_{1,4}^{0}+0.014c_{2,4}^{0}+Q_{1,4}^{0}$

$i=2\quad c_{2,4}^{1}=0.018c_{1,4}^{0}+0.018c_{2,3}^{0}+0.936c_{2,4}^{0}+0.014c_{2,5}^{0}+0.014c_{3,4}^{0}+Q_{2,4}^{0}$

and apply boundary condition $c_{2,5}^{0}=c_{2,4}^{0}$

$c_{2,4}^{1}=0.018c_{1,4}^{0}+0.018c_{2,3}^{0}+0.936c_{2,4}^{0}+0.014c_{2,4}^{0}+0.014c_{3,4}^{0}+Q_{2,4}^{0}$

$i=3\quad c_{3,4}^{1}=0.018c_{2,4}^{0}+0.018c_{3,3}^{0}+0.936c_{3,4}^{0}+0.014c_{3,5}^{0}+0.014c_{4,4}^{0}+Q_{3,4}^{0}$

and apply boundary condition $c_{3,5}^{0}=c_{3,4}^{0}$

$c_{3,4}^{1}=0.018c_{2,4}^{0}+0.018c_{3,3}^{0}+0.936c_{3,4}^{0}+0.014c_{3,4}^{0}+0.014c_{4,4}^{0}+Q_{3,4}^{0}$

$i=4\quad c_{4,4}^{1}=\dfrac{c_{3,4}^{1}+c_{4,3}^{1}}{2}$

Table 4.14 The calculated pollutant concentration at time $t=0.02$ of example 4.5.5

Point	Pollutant concentration	
	$Q=0$	$Q=0.0001$
$c_{0,0}^2$	0.000000	0.000000
$c_{0,1}^2$	0.000000	0.000000
$c_{0,2}^2$	0.000000	0.000000
$c_{0,3}^2$	0.000000	0.000000
$c_{0,4}^2$	0.000000	0.000000
$c_{1,0}^2$	0.000000	0.000000
$c_{1,1}^2$	0.033313	0.033510
$c_{1,2}^2$	0.044898	0.045096
$c_{1,3}^2$	0.033685	0.033883
$c_{1,4}^2$	0.001232	0.001430
$c_{2,0}^2$	0.000000	0.000000
$c_{2,1}^2$	0.044898	0.045096
$c_{2,2}^2$	0.060507	0.060707
$c_{2,3}^2$	0.045395	0.045595
$c_{2,4}^2$	0.001652	0.001852
$c_{3,0}^2$	0.000000	0.000000
$c_{3,1}^2$	0.033684	0.033882
$c_{3,2}^2$	0.045395	0.045595
$c_{3,3}^2$	0.034058	0.034258
$c_{3,4}^2$	0.001239	0.001438
$c_{4,0}^2$	0.000000	0.000000
$c_{4,1}^2$	0.001232	0.001430
$c_{4,2}^2$	0.001652	0.001852
$c_{4,3}^2$	0.001239	0.001438
$c_{4,4}^2$	0.000633	0.000733

Example 4.5.6. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y} = c_2$ domain $\Omega = (0,1) \times (0,1)$ in Figure (4.63) with step size $\Delta x = \Delta y = 0.25$, $\Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction $v = 0.1$.

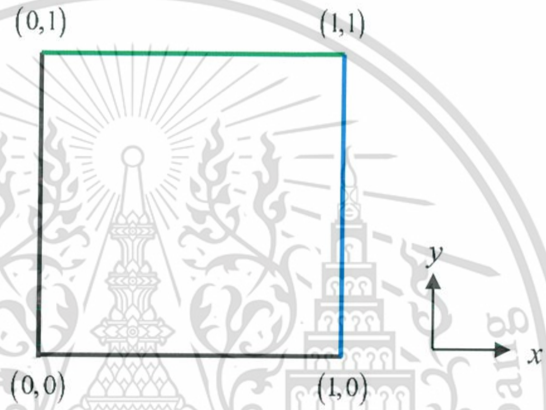


Figure 4.63: Domain of example 4.5.6

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x, y, 0) = x(1-x)y(1-y) + c_0,$$

the boundary conditions can be assumed by

$$c(0, y, t) = c_0 = 0,$$

$$c_x(1, y, t) = c_1 = 0$$

$$c_y(x, 1, t) = c_2 = 0$$

define $c(x, y, t) = c_{i,j}^n$ with $\Delta x = \Delta y = 0.25, \Delta t = 0.01, D = 0.1,$ and $u = v = 0.1$.

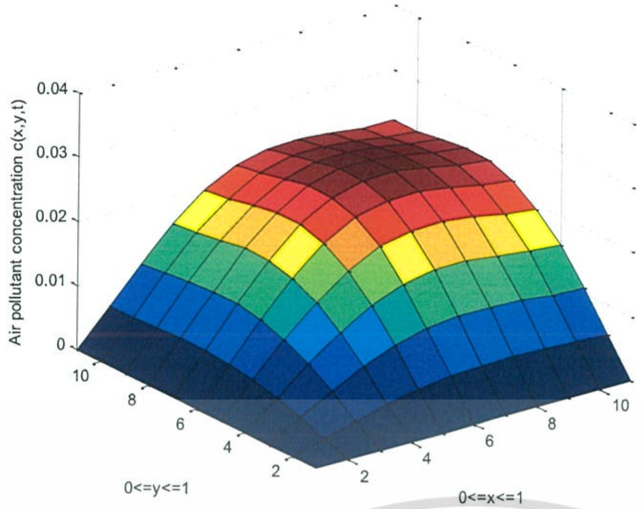


Figure 4.64: The approximated air pollutant concentration for $Q=0$ and $0 \le t \le 60$ sec of example 4.5.6

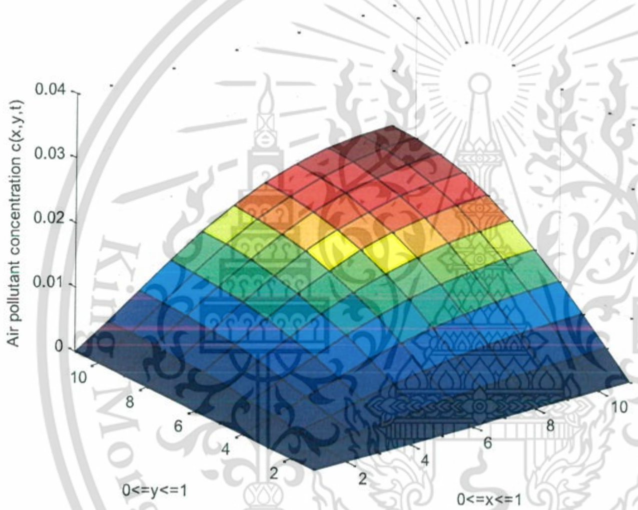


Figure 4.65: The approximated air pollutant concentration for $Q=0$ and $0 \le t \le 120$ sec of example 4.5.6

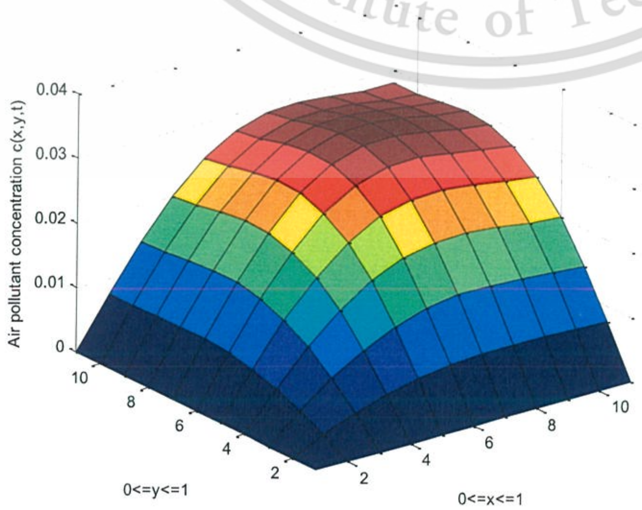


Figure 4.66: The approximated air pollutant concentration for $Q=0.0001$ and $0 \le t \le 60$ sec of example 4.5.6

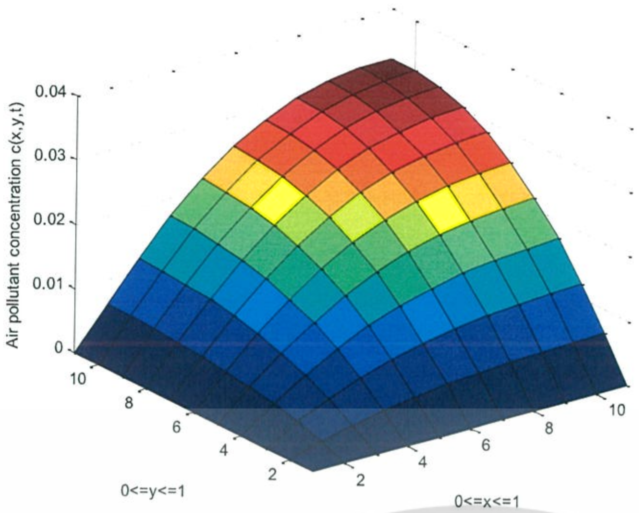


Figure 4.67: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \le 120$ sec of example 4.5.6

Example 4.5.7. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y} = c_2$ and rate of change of pollutant concentration with respect to y at the ground gate $\frac{\partial c}{\partial y} = c_3$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.68) with step size $\Delta x = \Delta y = 0.25$, $\Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction $v = 0.1$.

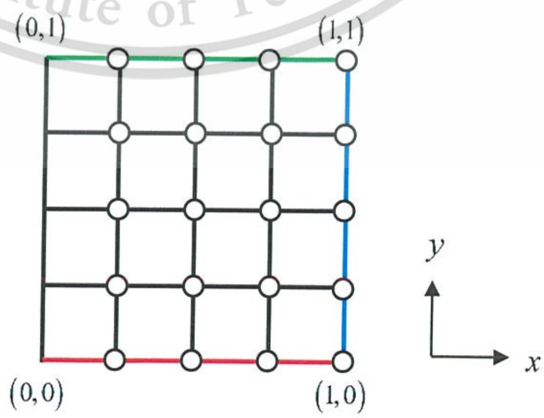


Figure 4.68: Generating grid points of example 4.5.7

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0)=x(1-x)y(1-y)+c_0,$$

the boundary conditions can be assumed by

$$c(0,y,t)=c_0=0,$$

$$c_x(1,y,t)=c_1=0$$

$$c_z(x,1,t)=c_2=0$$

$$c_z(x,0,t)=c_3=0$$

define $c(x,y,t)=c_{i,j}^n$ with $\Delta x=\Delta y=0.25, \Delta t=0.01, D=0.1,$ and $u=v=0.1.$

approximate differential boundary condition,

$c_x(1,y,t)=0$ or $\dfrac{\partial c_{5,j}^n}{\partial x}=0,$

using forward difference,

$$\frac{c_{5,j}^n-c_{4,j}^n}{\Delta x}=0$$

$$c_{5,j}^n=c_{4,j}^n \qquad j=0,1,2,3,4 \tag{4.76}$$

$c_y(x,1,t)=0$ or $\dfrac{\partial c_{i,5}^n}{\partial y}=0,$

using forward difference,

$$\frac{c_{i,5}^n-c_{i,4}^n}{\Delta y}=0$$

$$c_{i,5}^n=c_{i,4}^n \qquad i=1,2,3,4 \tag{4.77}$$

$c_y(x,0,t)=0$ or $\dfrac{\partial c_{i,0}^n}{\partial y}=0,$

using backward difference,

$$\frac{c_{i,0}^n-c_{i,-1}^n}{\Delta y}=0$$

$$c_{i,0}^n=c_{i,-1}^n \qquad i=1,2,3,4 \tag{4.78}$$

solving problem by finite difference method Eq.(4.60)

$$c_{i,j}^{n+1} = B_1 c_{i-1,j}^{n} + B_2 c_{i,j-1}^{n} + B_3 c_{i,j}^{n} + B_4 c_{i,j+1}^{n} + B_5 c_{i+1,j}^{n} + Q_{i,j}^{n}$$

$$B_1 = D_{xy}\frac{\Delta t}{(\Delta x)^2} + u\frac{\Delta t}{2\Delta x} = (0.1)\frac{0.01}{(0.25)^2} + (0.1)\frac{0.01}{2(0.25)} = 0.018,$$

$$B_2 = D_{xy}\frac{\Delta t}{(\Delta y)^2} + v\frac{\Delta t}{2\Delta y} = (0.1)\frac{0.01}{(0.25)^2} + (0.1)\frac{0.01}{2(0.25)} = 0.018,$$

$$B_3 = 1 - 2D_{xy}\frac{\Delta t}{(\Delta x)^2} - 2D_{xy}\frac{\Delta t}{(\Delta y)^2} = 1 - 2(0.1)\frac{0.01}{(0.25)^2} - 2(0.1)\frac{0.01}{(0.25)^2}$$

$$= 0.936,$$

$$B_4 = D_{xy}\frac{\Delta t}{(\Delta y)^2} - v\frac{\Delta t}{2\Delta y} = (0.1)\frac{0.01}{(0.25)^2} - (0.1)\frac{0.01}{2(0.25)} = 0.014,$$

$$B_5 = D_{xy}\frac{\Delta t}{(\Delta x)^2} - u\frac{\Delta t}{2\Delta x} = (0.1)\frac{0.01}{(0.25)^2} - (0.1)\frac{0.01}{2(0.25)} = 0.014.$$

$$c_{i,j}^{n+1} = 0.018c_{i-1,j}^{n} + 0.018c_{i,j-1}^{n} + 0.936c_{i,j}^{n} + 0.014c_{i,j+1}^{n} + 0.014c_{i+1,j}^{n} + Q_{i,j}^{n}$$

$$i = 1,2,3,4, \quad j = 0,1,2,3,4 \tag{4.79}$$

$n = 0,$

$j = 0,$ $i = 1$ $\quad c_{1,0}^{1} = 0.018c_{0,0}^{0} + 0.018c_{1,-1}^{0} + 0.936c_{1,0}^{0} + 0.014c_{1,1}^{0} + 0.014c_{2,0}^{0} + Q_{1,0}^{0}$

and apply boundary condition $c_{1,0}^{0} = c_{1,-1}^{0}$

$$c_{1,0}^{1} = 0.018c_{0,0}^{0} + 0.018c_{1,0}^{0} + 0.936c_{1,0}^{0} + 0.014c_{1,1}^{0} + 0.014c_{2,0}^{0} + Q_{1,0}^{0}$$

$i = 2$ $\quad c_{2,0}^{1} = 0.018c_{1,0}^{0} + 0.018c_{2,-1}^{0} + 0.936c_{2,0}^{0} + 0.014c_{2,1}^{0} + 0.014c_{3,0}^{0} + Q_{2,0}^{0}$

and apply boundary condition $c_{2,0}^{0} = c_{2,-1}^{0}$

$$c_{2,0}^{1} = 0.018c_{1,0}^{0} + 0.018c_{2,0}^{0} + 0.936c_{2,0}^{0} + 0.014c_{2,1}^{0} + 0.014c_{3,0}^{0} + Q_{2,0}^{0}$$

$i = 3$ $\quad c_{3,0}^{1} = 0.018c_{2,0}^{0} + 0.018c_{3,-1}^{0} + 0.936c_{3,0}^{0} + 0.014c_{3,1}^{0} + 0.014c_{4,0}^{0} + Q_{3,0}^{0}$

and apply boundary condition $c_{3,0}^{0} = c_{3,-1}^{0}$

$$c_{3,0}^{1} = 0.018c_{2,0}^{0} + 0.018c_{3,0}^{0} + 0.936c_{3,0}^{0} + 0.014c_{3,1}^{0} + 0.014c_{4,0}^{0} + Q_{3,0}^{0}$$

$i = 4$ $\quad c_{4,0}^{1} = \dfrac{c_{3,0}^{1} + c_{4,1}^{1}}{2}$

$j = 1,$ $i = 1$ $\quad c_{1,1}^{1} = 0.018c_{0,1}^{0} + 0.018c_{1,0}^{0} + 0.936c_{1,1}^{0} + 0.014c_{1,2}^{0} + 0.014c_{2,1}^{0} + Q_{1,1}^{0}$

$i = 2$ $\quad c_{2,1}^{1} = 0.018c_{1,1}^{0} + 0.018c_{2,0}^{0} + 0.936c_{2,1}^{0} + 0.014c_{2,2}^{0} + 0.014c_{3,1}^{0} + Q_{2,1}^{0}$

$i = 3$ $\quad c_{3,1}^{1} = 0.018c_{2,1}^{0} + 0.018c_{3,0}^{0} + 0.936c_{3,1}^{0} + 0.014c_{3,2}^{0} + 0.014c_{4,1}^{0} + Q_{3,1}^{0}$

$i = 4$ $\quad c_{4,1}^{1} = 0.018c_{3,1}^{0} + 0.018c_{4,0}^{0} + 0.936c_{4,1}^{0} + 0.014c_{4,2}^{0} + 0.014c_{5,1}^{0} + Q_{4,1}^{0}$

and apply boundary condition $c_{5,1}^{0}=c_{4,1}^{0}$

$$c_{4,1}^{1}=0.018c_{3,1}^{0}+0.018c_{4,0}^{0}+0.936c_{4,1}^{0}+0.014c_{4,2}^{0}+0.014c_{4,1}^{0}+Q_{4,1}^{0}$$

$j=2,\quad i=1\quad c_{1,2}^{1}=0.018c_{0,2}^{0}+0.018c_{1,1}^{0}+0.936c_{1,2}^{0}+0.014c_{1,3}^{0}+0.014c_{2,2}^{0}+Q_{1,2}^{0}$

$i=2\quad c_{2,2}^{1}=0.018c_{1,2}^{0}+0.018c_{2,1}^{0}+0.936c_{2,2}^{0}+0.014c_{2,3}^{0}+0.014c_{3,2}^{0}+Q_{2,2}^{0}$

$i=3\quad c_{3,2}^{1}=0.018c_{2,2}^{0}+0.018c_{3,1}^{0}+0.936c_{3,2}^{0}+0.014c_{3,3}^{0}+0.014c_{4,2}^{0}+Q_{3,2}^{0}$

$i=4\quad c_{4,2}^{1}=0.018c_{3,2}^{0}+0.018c_{4,1}^{0}+0.936c_{4,2}^{0}+0.014c_{4,3}^{0}+0.014c_{5,2}^{0}+Q_{4,2}^{0}$

and apply boundary condition $c_{5,2}^{0}=c_{4,2}^{0}$

$$c_{4,2}^{1}=0.018c_{3,2}^{0}+0.018c_{4,1}^{0}+0.936c_{4,2}^{0}+0.014c_{4,3}^{0}+0.014c_{4,2}^{0}+Q_{4,2}^{0}$$

$j=3,\quad i=1\quad c_{1,3}^{1}=0.018c_{0,3}^{0}+0.018c_{1,2}^{0}+0.936c_{1,3}^{0}+0.014c_{1,4}^{0}+0.014c_{2,3}^{0}+Q_{1,3}^{0}$

$i=2\quad c_{2,3}^{1}=0.018c_{1,3}^{0}+0.018c_{2,2}^{0}+0.936c_{2,3}^{0}+0.014c_{2,4}^{0}+0.014c_{3,3}^{0}+Q_{2,3}^{0}$

$i=3\quad c_{3,3}^{1}=0.018c_{2,3}^{0}+0.018c_{3,2}^{0}+0.936c_{3,3}^{0}+0.014c_{3,4}^{0}+0.014c_{4,3}^{0}+Q_{3,3}^{0}$

$i=4\quad c_{4,3}^{1}=0.018c_{3,3}^{0}+0.018c_{4,2}^{0}+0.936c_{4,3}^{0}+0.014c_{4,4}^{0}+0.014c_{5,3}^{0}+Q_{4,3}^{0}$

and apply boundary condition $c_{5,3}^{0}=c_{4,3}^{0}$

$$c_{4,3}^{1}=0.018c_{3,3}^{0}+0.018c_{4,2}^{0}+0.936c_{4,3}^{0}+0.014c_{4,4}^{0}+0.014c_{4,3}^{0}+Q_{4,3}^{0}$$

$j=4,\quad i=1\quad c_{1,4}^{1}=0.018c_{0,4}^{0}+0.018c_{1,3}^{0}+0.936c_{1,4}^{0}+0.014c_{1,5}^{0}+0.014c_{2,4}^{0}+Q_{1,4}^{0}$

and apply boundary condition $c_{1,5}^{0}=c_{1,4}^{0}$

$$c_{1,4}^{1}=0.018c_{0,4}^{0}+0.018c_{1,3}^{0}+0.936c_{1,4}^{0}+0.014c_{1,4}^{0}+0.014c_{2,4}^{0}+Q_{1,4}^{0}$$

$i=2\quad c_{2,4}^{1}=0.018c_{1,4}^{0}+0.018c_{2,3}^{0}+0.936c_{2,4}^{0}+0.014c_{2,5}^{0}+0.014c_{3,4}^{0}+Q_{2,4}^{0}$

and apply boundary condition $c_{2,5}^{0}=c_{2,4}^{0}$

$$c_{2,4}^{1}=0.018c_{1,4}^{0}+0.018c_{2,3}^{0}+0.936c_{2,4}^{0}+0.014c_{2,4}^{0}+0.014c_{3,4}^{0}+Q_{2,4}^{0}$$

$i=3\quad c_{3,4}^{1}=0.018c_{2,4}^{0}+0.018c_{3,3}^{0}+0.936c_{3,4}^{0}+0.014c_{3,5}^{0}+0.014c_{4,4}^{0}+Q_{3,4}^{0}$

and apply boundary condition $c_{3,5}^{0}=c_{3,4}^{0}$

$$c_{3,4}^{1}=0.018c_{2,4}^{0}+0.018c_{3,3}^{0}+0.936c_{3,4}^{0}+0.014c_{3,4}^{0}+0.014c_{4,4}^{0}+Q_{3,4}^{0}$$

$i=4\quad c_{4,4}^{1}=\dfrac{c_{3,4}^{1}+c_{4,3}^{1}}{2}$

Table 4.15 The calculated pollutant concentration at time $t=0.02$ of example 4.5.7

Point	Pollutant concentration	
	$Q=0$	$Q=0.0001$
$c_{0,0}^2$	0.000000	0.000000
$c_{0,1}^2$	0.000958	0.001156
$c_{0,2}^2$	0.001284	0.001484
$c_{0,3}^2$	0.000963	0.001162
$c_{0,4}^2$	0.000563	0.000663
$c_{1,0}^2$	0.000000	0.000000
$c_{1,1}^2$	0.033322	0.033520
$c_{1,2}^2$	0.044909	0.045109
$c_{1,3}^2$	0.033693	0.033893
$c_{1,4}^2$	0.001232	0.001430
$c_{2,0}^2$	0.000000	0.000000
$c_{2,1}^2$	0.044898	0.045096
$c_{2,2}^2$	0.060507	0.060707
$c_{2,3}^2$	0.045395	0.045595
$c_{2,4}^2$	0.001652	0.001852
$c_{3,0}^2$	0.000000	0.000000
$c_{3,1}^2$	0.033684	0.033882
$c_{3,2}^2$	0.045395	0.045595
$c_{3,3}^2$	0.034058	0.034258
$c_{3,4}^2$	0.001239	0.001438
$c_{4,0}^2$	0.000000	0.000000
$c_{4,1}^2$	0.001232	0.001430
$c_{4,2}^2$	0.001652	0.001852
$c_{4,3}^2$	0.001239	0.001438
$c_{4,4}^2$	0.000633	0.000733

Example 4.5.8. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y} = c_2$ and rate of change of pollutant concentration with respect to y at the ground gate $\frac{\partial c}{\partial y} = c_3$ domain $\Omega = (0,1)\times(0,1)$ in Figure (4.69) with step size $\Delta x = \Delta y = 0.1,\ \ \Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction $v = 0.1$.

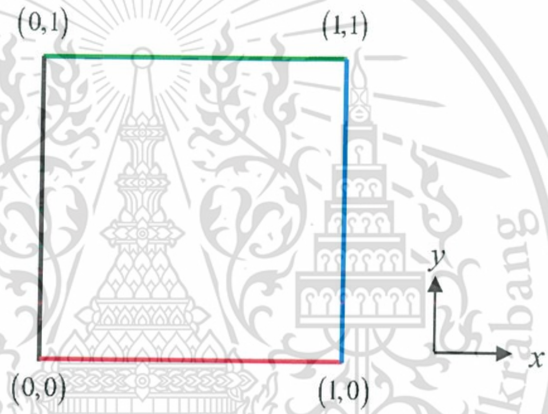


Figure 4.69: Generating grid points of example 4.5.8

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0) = x(1-x)y(1-y) + c_0,$$

the boundary conditions can be assumed by

$$c(0,y,t) = c_0 = 0,$$

$$c_x(1,y,t) = c_1 = 0$$

$$c_z(x,1,t) = c_2 = 0$$

$$c_z(x,0,t) = c_3 = 0$$

define $c(x,y,t) = c_{i,j}^n$ with $\Delta x = \Delta y = 0.1, \Delta t = 0.01, D = 0.1,$ and $u = v = 0.1,$

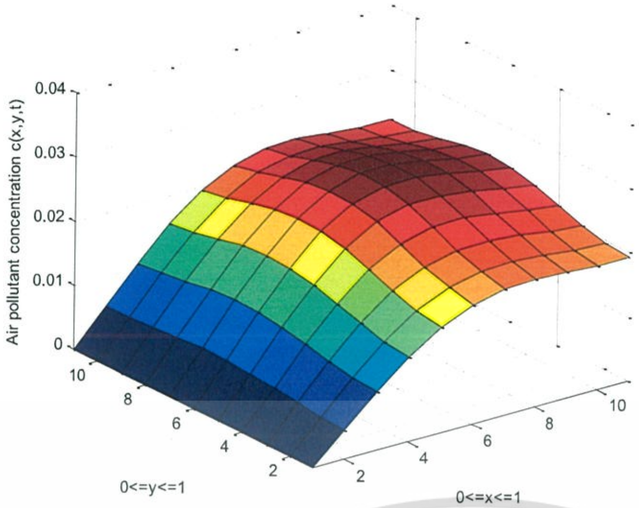


Figure 4.70: The approximated air pollutant concentration for $Q=0$ and $0 \le t \le 60$ sec of example 4.5.8

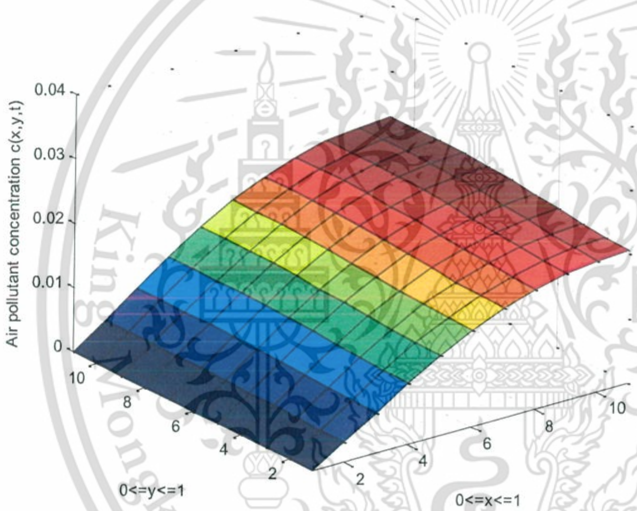


Figure 4.71: The approximated air pollutant concentration for $Q=0$ and $0 \le t \le 120$ sec of example 4.5.8

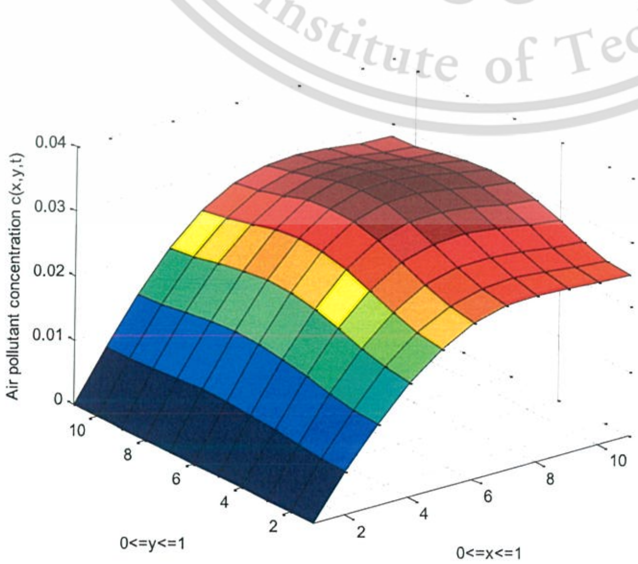


Figure 4.72: The approximated air pollutant concentration for $Q=0.0001$ and $0 \le t \le 60$ sec of example 4.5.8

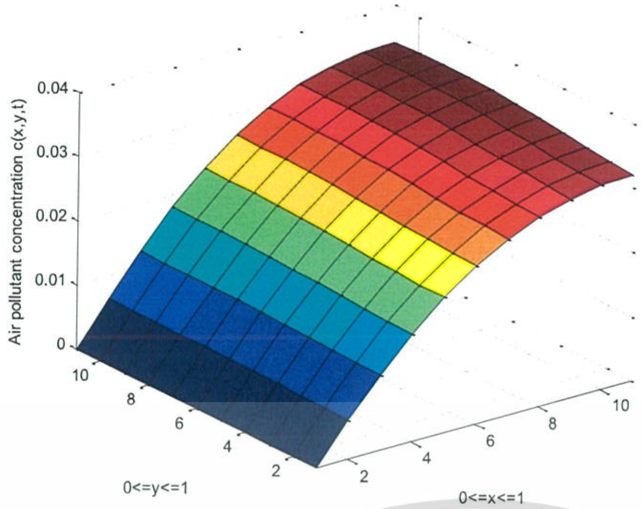


Figure 4.73: The approximated air pollutant concentration for $Q=0.0001$ and $0 \le t \le 120$ sec of example 4.5.8

Example 4.5.9. To find the pollutant concentration in street canyon, at boundary of street canyon that has rate of change of pollutant concentration with respect to x at the entrance gate $\frac{\partial c}{\partial x}=c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x}=c_1$, rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y}=c_2$, and rate of change of pollutant concentration with respect to y at the ground gate $\frac{\partial c}{\partial y}=c_3$, domain $\Omega=(0,1)\times(0,1)$ in Figure (4.74) with step size $\Delta x=\Delta z=0.25$, $\Delta t=0.01$ diffusion coefficient $D_{xy}=0.1$, there is no interior $Q=0$, average air pollutant source are added $Q=0.0001$, average air flow velocity in x-direction $u=0.1$, and average air flow velocity in y-direction $v=0.1$.

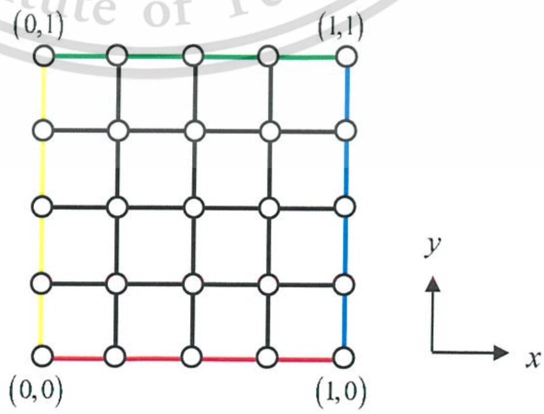


Figure 4.74: Generating grid points of example 4.5.9

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0)=x(1-x)y(1-y)+c_0,$$

the boundary conditions can be assumed by

$$c_x(0,y,t)=c_0=0,$$

$$c_x(1,y,t)=c_1=0,$$

$$c_z(x,1,t)=c_2=0,$$

$$c_z(x,0,t)=c_3=0,$$

define $c(x,y,t)=c_{i,j}^n$ with $\Delta x=\Delta y=0.25, \Delta t=0.01, D=0.1,$ and $u=v=0.1.$

approximate differential boundary condition,

$c_x(0,y,t)=0$ or $\dfrac{\partial c_{0,j}^n}{\partial x}=0,$

using backward difference,

$$\frac{c_{0,j}^n-c_{-1,j}^n}{\Delta x}=0$$

$$c_{0,j}^n=c_{-1,j}^n \qquad j=0,1,2,3,4 \tag{4.80}$$

$c_x(1,z,t)=0$ or $\dfrac{\partial c_{5,j}^n}{\partial x}=0,$

using forward difference,

$$\frac{c_{5,j}^n-c_{4,j}^n}{\Delta x}=0$$

$$c_{5,j}^n=c_{4,j}^n \qquad j=0,1,2,3,4 \tag{4.81}$$

$c_y(x,1,t)=0$ or $\dfrac{\partial c_{i,5}^n}{\partial y}=0,$

using forward difference,

$$\frac{c_{i,5}^n-c_{i,4}^n}{\Delta y}=0$$

$$c_{i,5}^n=c_{i,4}^n \qquad i=0,1,2,3,4 \tag{4.82}$$

$c_y(x,0,t)=0$ or $\frac{\partial c_{i,0}^n}{\partial y}=0,$

using backward difference,

$$\frac{c_{i,0}^n - c_{i,-1}^n}{\Delta y}=0$$

$$c_{i,0}^n = c_{i,-1}^n \qquad i=0,1,2,3,4 \tag{4.83}$$

solving problem by finite difference method Eq.(4.60)

$$c_{i,j}^{n+1}=B_1c_{i-1,j}^n+B_2c_{i,j-1}^n+B_3c_{i,j}^n+B_4c_{i,j+1}^n+B_5\,c_{i+1,j}^n+Q_{i,j}^n$$

$$B_1=D_{xy}\frac{\Delta t}{(\Delta x)^2}+u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}+(0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_2=D_{xy}\frac{\Delta t}{(\Delta y)^2}+v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}+(0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_3=1-2D_{xy}\frac{\Delta t}{(\Delta x)^2}-2D_{xy}\frac{\Delta t}{(\Delta y)^2}=1-2(0.1)\frac{0.01}{(0.25)^2}-2(0.1)\frac{0.01}{(0.25)^2}$$
$$=0.936,$$

$$B_4=D_{xy}\frac{\Delta t}{(\Delta y)^2}-v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}-(0.1)\frac{0.01}{2(0.25)}=0.014,$$

$$B_5=D_{xy}\frac{\Delta t}{(\Delta x)^2}-u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}-(0.1)\frac{0.01}{2(0.25)}=0.014.$$

$$c_{i,j}^{n+1}=0.018c_{i-1,j}^n+0.018c_{i,j-1}^n+0.936c_{i,j}^n+0.014c_{i,j+1}^n+0.014\,c_{i+1,j}^n+Q_{i,j}^n$$
$$i=0,1,2,3,4, \quad j=0,1,2,3,4 \tag{4.84}$$

$n=0,$

$j=0, \quad i=0 \quad c_{0,0}^1=\frac{c_{1,0}^1+c_{0,1}^1}{2}$

$i=1 \quad c_{1,0}^1=0.018c_{0,0}^0+0.018c_{1,-1}^0+0.936c_{1,0}^0+0.014c_{1,1}^0+0.014\,c_{2,0}^0+Q_{1,0}^0$

and apply boundary condition $c_{1,0}^0=c_{1,-1}^0$

$c_{1,0}^1=0.018c_{0,0}^0+0.018c_{1,0}^0+0.936c_{1,0}^0+0.014c_{1,1}^0+0.014\,c_{2,0}^0+Q_{1,0}^0$

$i=2 \quad c_{2,0}^1=0.018c_{1,0}^0+0.018c_{2,-1}^0+0.936c_{2,0}^0+0.014c_{2,1}^0+0.014\,c_{3,0}^0+Q_{2,0}^0$

and apply boundary condition $c_{2,-1}^0=c_{2,0}^0$

$c_{2,0}^1=0.018c_{1,0}^0+0.018c_{2,0}^0+0.936c_{2,0}^0+0.014c_{2,1}^0+0.014\,c_{3,0}^0+Q_{2,0}^0$

$i=3 \quad c_{3,0}^{1}=0.018c_{2,0}^{0}+0.018c_{3,-1}^{0}+0.936c_{3,0}^{0}+0.014c_{3,1}^{0}+0.014c_{4,0}^{0}+Q_{3,0}^{0}$

and apply boundary condition $c_{3,-1}^{0}=c_{3,0}^{0}$

$c_{3,0}^{1}=0.018c_{2,0}^{0}+0.018c_{3,0}^{0}+0.936c_{3,0}^{0}+0.014c_{3,1}^{0}+0.014c_{4,0}^{0}+Q_{3,0}^{0}$

$i=4 \quad c_{4,0}^{1}=\dfrac{c_{3,0}^{1}+c_{4,1}^{1}}{2}$

$j=1, \quad i=0 \quad c_{0,1}^{1}=0.018c_{-1,1}^{0}+0.018c_{0,0}^{0}+0.936c_{0,1}^{0}+0.014c_{0,2}^{0}+0.014c_{1,1}^{0}+Q_{0,1}^{0}$

and apply boundary condition $c_{0,1}^{0}=c_{-1,1}^{0}$

$c_{0,1}^{1}=0.018c_{0,1}^{0}+0.018c_{0,0}^{0}+0.936c_{0,1}^{0}+0.014c_{0,2}^{0}+0.014c_{1,1}^{0}+Q_{0,1}^{0}$

$i=1 \quad c_{1,1}^{1}=0.018c_{0,1}^{0}+0.018c_{1,0}^{0}+0.936c_{1,1}^{0}+0.014c_{1,2}^{0}+0.014c_{2,1}^{0}+Q_{1,1}^{0}$

$i=2 \quad c_{2,1}^{1}=0.018c_{1,1}^{0}+0.018c_{2,0}^{0}+0.936c_{2,1}^{0}+0.014c_{2,2}^{0}+0.014c_{3,1}^{0}+Q_{2,1}^{0}$

$i=3 \quad c_{3,1}^{1}=0.018c_{2,1}^{0}+0.018c_{3,0}^{0}+0.936c_{3,1}^{0}+0.014c_{3,2}^{0}+0.014c_{4,1}^{0}+Q_{3,1}^{0}$

$i=4 \quad c_{4,1}^{1}=0.018c_{3,1}^{0}+0.018c_{4,0}^{0}+0.936c_{4,1}^{0}+0.014c_{4,2}^{0}+0.014c_{5,1}^{0}+Q_{4,1}^{0}$

and apply boundary condition $c_{5,1}^{0}=c_{4,1}^{0}$

$c_{4,1}^{1}=0.018c_{3,1}^{0}+0.018c_{4,0}^{0}+0.936c_{4,1}^{0}+0.014c_{4,2}^{0}+0.014c_{4,1}^{0}+Q_{4,1}^{0}$

$j=2, \quad i=0 \quad c_{0,2}^{1}=0.018c_{-1,2}^{0}+0.018c_{0,1}^{0}+0.936c_{0,2}^{0}+0.014c_{0,3}^{0}+0.014c_{1,2}^{0}+Q_{0,2}^{0}$

and apply boundary condition $c_{0,2}^{0}=c_{-1,2}^{0}$

$i=1 \quad c_{1,2}^{1}=0.018c_{0,2}^{0}+0.018c_{1,1}^{0}+0.936c_{1,2}^{0}+0.014c_{1,3}^{0}+0.014c_{2,2}^{0}+Q_{1,2}^{0}$

$i=2 \quad c_{2,2}^{1}=0.018c_{1,2}^{0}+0.018c_{2,1}^{0}+0.936c_{2,2}^{0}+0.014c_{2,3}^{0}+0.014c_{3,2}^{0}+Q_{2,2}^{0}$

$i=3 \quad c_{3,2}^{1}=0.018c_{2,2}^{0}+0.018c_{3,1}^{0}+0.936c_{3,2}^{0}+0.014c_{3,3}^{0}+0.014c_{4,2}^{0}+Q_{3,2}^{0}$

$i=4 \quad c_{4,2}^{1}=0.018c_{3,2}^{0}+0.018c_{4,1}^{0}+0.936c_{4,2}^{0}+0.014c_{4,3}^{0}+0.014c_{5,2}^{0}+Q_{4,2}^{0}$

and apply boundary condition $c_{5,2}^{0}=c_{4,2}^{0}$

$c_{4,2}^{1}=0.018c_{3,2}^{0}+0.018c_{4,1}^{0}+0.936c_{4,2}^{0}+0.014c_{4,3}^{0}+0.014c_{4,2}^{0}+Q_{4,2}^{0}$

$j=3, \quad i=0 \quad c_{0,3}^{1}=0.018c_{-1,3}^{0}+0.018c_{0,2}^{0}+0.936c_{0,3}^{0}+0.014c_{0,4}^{0}+0.014c_{1,3}^{0}+Q_{0,3}^{0}$

and apply boundary condition $c_{0,3}^{0}=c_{-1,3}^{0}$

$c_{0,3}^{1}=0.018c_{0,3}^{0}+0.018c_{0,2}^{0}+0.936c_{0,3}^{0}+0.014c_{0,4}^{0}+0.014c_{1,3}^{0}+Q_{0,3}^{0}$

$i=1 \quad c_{1,3}^{1}=0.018c_{0,3}^{0}+0.018c_{1,2}^{0}+0.936c_{1,3}^{0}+0.014c_{1,4}^{0}+0.014c_{2,3}^{0}+Q_{1,3}^{0}$

$i=2 \quad c_{2,3}^{1}=0.018c_{1,3}^{0}+0.016c_{2,2}^{0}+0.936c_{2,3}^{0}+0.016c_{2,4}^{0}+0.014c_{3,3}^{0}+Q_{2,3}^{0}$

$i=3 \quad c_{3,3}^{1}=0.018c_{2,3}^{0}+0.018c_{3,2}^{0}+0.936c_{3,3}^{0}+0.014c_{3,4}^{0}+0.014c_{4,3}^{0}+Q_{3,3}^{0}$

$i=4 \quad c_{4,3}^{1}=0.018c_{3,3}^{0}+0.018c_{4,2}^{0}+0.936c_{4,3}^{0}+0.014c_{4,4}^{0}+0.014c_{5,3}^{0}+Q_{4,3}^{0}$

and apply boundary condition $c_{5,3}^{0}=c_{4,3}^{0}$

$c_{4,3}^{1}=0.018c_{3,3}^{0}+0.018c_{4,2}^{0}+0.936c_{4,3}^{0}+0.014c_{4,4}^{0}+0.014c_{4,3}^{0}+Q_{4,3}^{0}$

$j=4, \quad i=0 \quad c_{0,4}^{1}=\dfrac{c_{0,3}^{1}+c_{1,4}^{1}}{2}$

$i=1 \quad c_{1,4}^{1}=0.018c_{0,4}^{0}+0.018c_{1,3}^{0}+0.936c_{1,4}^{0}+0.014c_{1,5}^{0}+0.014c_{2,4}^{0}+Q_{1,4}^{0}$

and apply boundary condition $c_{1,5}^{0}=c_{1,4}^{0}$

$c_{1,4}^{1}=0.018c_{0,4}^{0}+0.018c_{1,3}^{0}+0.936c_{1,4}^{0}+0.014c_{1,4}^{0}+0.014c_{2,4}^{0}+Q_{1,4}^{0}$

$i=2 \quad c_{2,4}^{1}=0.018c_{1,4}^{0}+0.018c_{2,3}^{0}+0.936c_{2,4}^{0}+0.014c_{2,5}^{0}+0.014c_{3,4}^{0}+Q_{2,4}^{0}$

and apply boundary condition $c_{2,5}^{0}=c_{2,4}^{0}$

$c_{2,4}^{1}=0.018c_{1,4}^{0}+0.018c_{2,3}^{0}+0.936c_{2,4}^{0}+0.014c_{2,4}^{0}+0.014c_{3,4}^{0}+Q_{2,4}^{0}$

$i=3 \quad c_{3,4}^{1}=0.018c_{2,4}^{0}+0.018c_{3,3}^{0}+0.936c_{3,4}^{0}+0.014c_{3,5}^{0}+0.014c_{4,4}^{0}+Q_{3,4}^{0}$

and apply boundary condition $c_{3,5}^{0}=c_{3,4}^{0}$

$c_{3,4}^{1}=0.018c_{2,4}^{0}+0.018c_{3,3}^{0}+0.936c_{3,4}^{0}+0.014c_{3,4}^{0}+0.014c_{4,4}^{0}+Q_{3,4}^{0}$

$i=4 \quad c_{4,4}^{1}=\dfrac{c_{3,4}^{1}+c_{4,3}^{1}}{2}$

Table 4.16 The calculated pollutant concentration at time $t=0.02$ of example 4.5.9

Point	Pollutant concentration	
	$Q=0$	$Q=0.0001$
$c_{0,0}^2$	0.000492	0.000592
$c_{0,1}^2$	0.000958	0.001156
$c_{0,2}^2$	0.001284	0.001484
$c_{0,3}^2$	0.000963	0.001162
$c_{0,4}^2$	0.000563	0.000663
$c_{1,0}^2$	0.000958	0.001156
$c_{1,1}^2$	0.033331	0.033531
$c_{1,2}^2$	0.044909	0.045109
$c_{1,3}^2$	0.033693	0.033893
$c_{1,4}^2$	0.001232	0.001430
$c_{2,0}^2$	0.001284	0.001484
$c_{2,1}^2$	0.044909	0.045109
$c_{2,2}^2$	0.060507	0.060707
$c_{2,3}^2$	0.045395	0.045595
$c_{2,4}^2$	0.001652	0.001852
$c_{3,0}^2$	0.000963	0.001162
$c_{3,1}^2$	0.033693	0.033893
$c_{3,2}^2$	0.045395	0.045595
$c_{3,3}^2$	0.034058	0.034258
$c_{3,4}^2$	0.001239	0.001438
$c_{4,0}^2$	0.000563	0.000663
$c_{4,1}^2$	0.001232	0.001430
$c_{4,2}^2$	0.001652	0.001852
$c_{4,3}^2$	0.001239	0.001438
$c_{4,4}^2$	0.000633	0.000733

Example 4.5.10. To find the pollutant concentration in street canyon, at boundary of street canyon that has rate of change of pollutant concentration with respect to x at the entrance gate $\frac{\partial c}{\partial x} = c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y} = c_2$, and rate of change of pollutant concentration with respect to y at the ground gate $\frac{\partial c}{\partial y} = c_3$, domain $\Omega = (0,1)\times(0,1)$ in Figure (4.74) with step size $\Delta x = \Delta y = 0.1, \Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction $v = 0.1$.

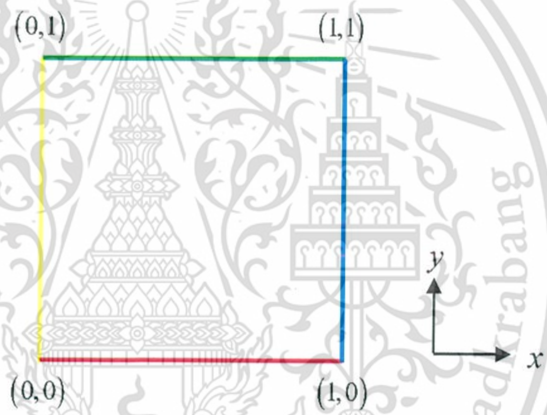


Figure 4.75: Domain of example 4.5.10

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0) = x(1-x)y(1-y) + c_0,$$

the boundary conditions can be assumed by

$$c_x(0,y,t) = c_0 = 0,$$

$$c_x(1,y,t) = c_1 = 0,$$

$$c_z(x,1,t) = c_2 = 0,$$

$$c_z(x,0,t) = c_3 = 0,$$

define $c(x,y,t) = c_{i,j}^n$ with $\Delta x = \Delta y = 0.1, \Delta t = 0.01, D = 0.1,$ and $u = v = 0.1.$

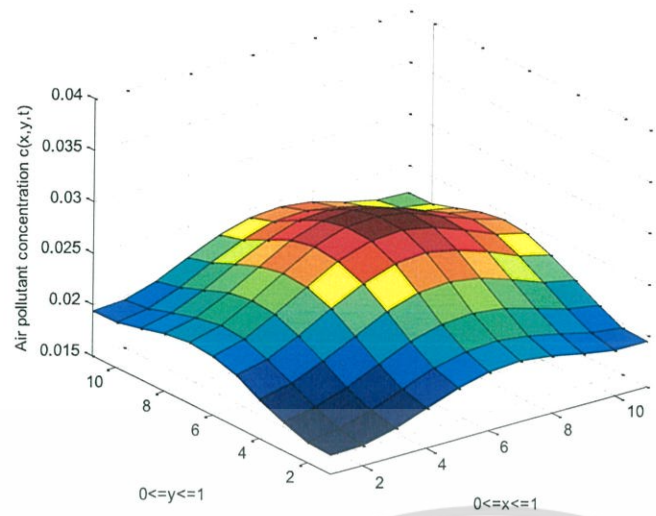


Figure 4.76: The approximated air pollutant concentration for $Q=0$ and $0 \le t \le 60$ sec of example 4.5.10

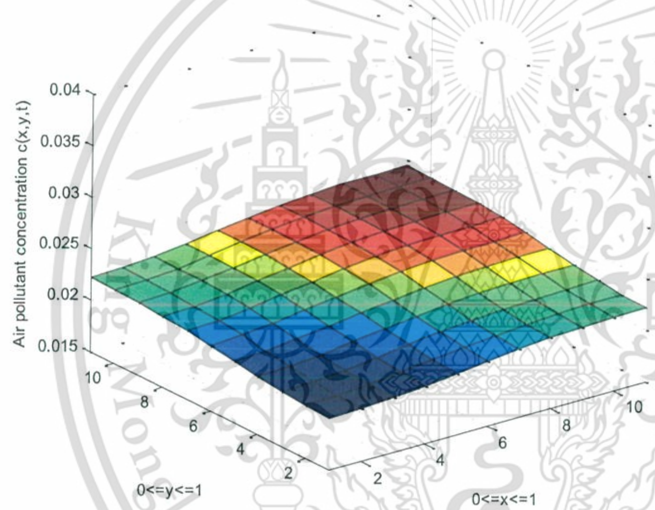


Figure 4.77: The approximated air pollutant concentration for $Q=0$ and $0 \le t \le 120$ sec of example 4.5.10

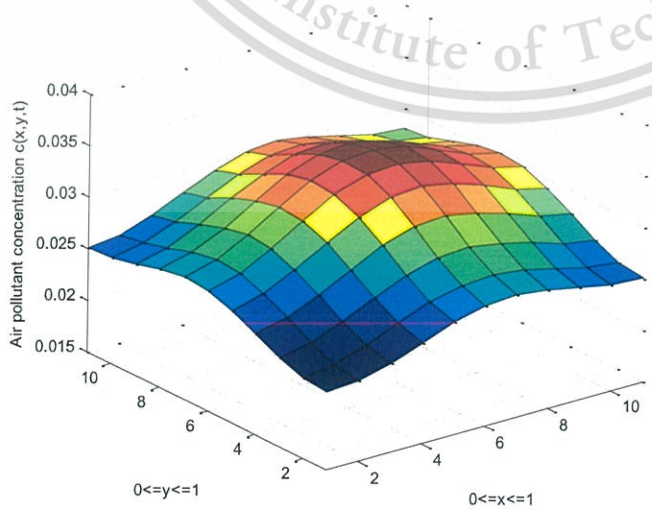


Figure 4.78: The approximated air pollutant concentration for $Q=0.0001$ and $0 \le t \le 60$ sec of example 4.5.10

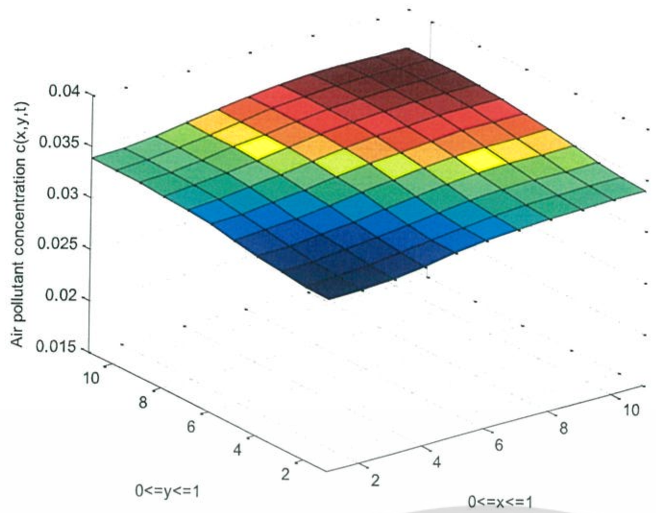


Figure 4.79: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \le 120$ sec of example 4.5.10

Example 4.5.11. To find the pollutant concentration in street canyon, at boundary of street canyon that has rate of change of pollutant concentration with respect to x at the entrance gate $\frac{\partial c}{\partial x} = c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y} = c_2$, and rate of change of pollutant concentration with respect to y at the ground gate $\frac{\partial c}{\partial y} = c_3$, domain $\Omega = (0,1) \times (0,1)$ in Figure (4.74) with step size $\Delta x = \Delta y = 0.1, \Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction v.

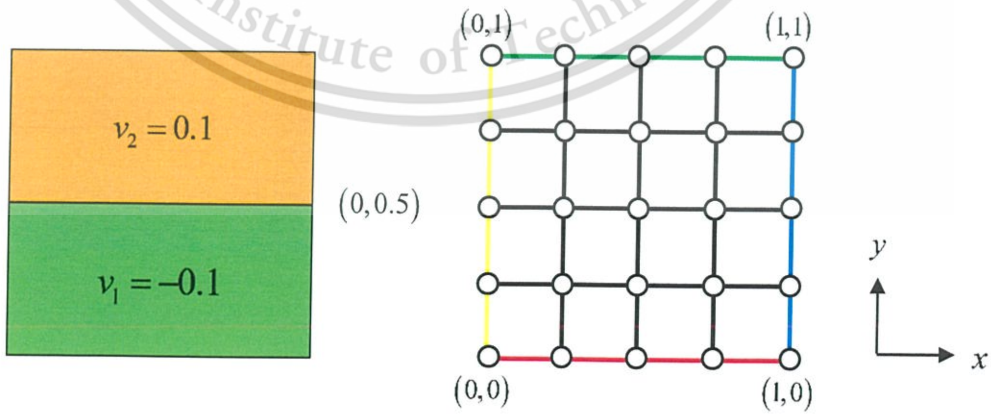


Figure 4.80: The flow velocity y-direction along the left and the right lanes and Generating grid points of example 4.5.11

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0)=x(1-x)y(1-y)+c_0,$$

the boundary conditions can be assumed by

$$c_x(0,y,t)=c_0=0,$$

$$c_x(1,y,t)=c_1=0,$$

$$c_z(x,1,t)=c_2=0,$$

$$c_z(x,0,t)=c_3=0,$$

define $c(x,y,t)=c_{i,j}^n$ with $\Delta x=\Delta y=0.25, \Delta t=0.01, D=0.1, u=0.1, v_1=-0.1,$ and $v_2=0.1.$

approximate differential boundary condition,

$c_x(0,y,t)=0$ or $\dfrac{\partial c_{0,j}^n}{\partial x}=0,$

using backward difference,

$$\frac{c_{0,j}^n-c_{-1,j}^n}{\Delta x}=0$$

$$c_{0,j}^n=c_{-1,j}^n \qquad j=0,1,2,3,4 \qquad (4.85)$$

$c_x(1,z,t)=0$ or $\dfrac{\partial c_{5,j}^n}{\partial x}=0,$

using forward difference,

$$\frac{c_{5,j}^n-c_{4,j}^n}{\Delta x}=0$$

$$c_{5,j}^n=c_{4,j}^n \qquad j=0,1,2,3,4 \qquad (4.86)$$

$c_y(x,1,t)=0$ or $\dfrac{\partial c_{i,5}^n}{\partial y}=0,$

using forward difference,

$$\frac{c_{i,5}^n-c_{i,4}^n}{\Delta y}=0$$

$$c_{i,5}^n=c_{i,4}^n \qquad i=0,1,2,3,4 \qquad (4.87)$$

$$c_y(x,0,t)=0 \text{ or } \frac{\partial c_{i,0}^n}{\partial y}=0,$$

using backward difference,

$$\frac{c_{i,0}^n - c_{i,-1}^n}{\Delta y}=0$$

$$c_{i,0}^n = c_{i,-1}^n \qquad i=0,1,2,3,4 \tag{4.88}$$

solving problem by finite difference method Eq.(4.60)

$$c_{i,j}^{n+1}=B_1c_{i-1,j}^n+B_2c_{i,j-1}^n+B_3c_{i,j}^n+B_4c_{i,j+1}^n+B_5c_{i+1,j}^n+Q_{i,j}^n$$

$v_1=-0.1$

$$B_1=D_{xy}\frac{\Delta t}{(\Delta x)^2}+u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}+(0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_2=D_{xy}\frac{\Delta t}{(\Delta y)^2}+v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}+(-0.1)\frac{0.01}{2(0.25)}=0.014,$$

$$B_3=1-2D_{xy}\frac{\Delta t}{(\Delta x)^2}-2D_{xy}\frac{\Delta t}{(\Delta y)^2}=1-2(0.1)\frac{0.01}{(0.25)^2}-2(0.1)\frac{0.01}{(0.25)^2}$$
$$=0.936,$$

$$B_4=D_{xy}\frac{\Delta t}{(\Delta y)^2}+v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}-(-0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_5=D_{xy}\frac{\Delta t}{(\Delta x)^2}+u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}-(0.1)\frac{0.01}{2(0.25)}=0.014,$$

$$c_{i,j}^{n+1}=0.018c_{i-1,j}^n+0.014c_{i,j-1}^n+0.936c_{i,j}^n+0.018c_{i,j+1}^n+0.014c_{i+1,j}^n+Q_{i,j}^n \tag{4.89}$$

$v_2=0.1$

$$B_1=D_{xy}\frac{\Delta t}{(\Delta x)^2}+u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}+(0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_2=D_{xy}\frac{\Delta t}{(\Delta y)^2}+v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}+(0.1)\frac{0.01}{2(0.25)}=0.018,$$

$$B_3=1-2D_{xy}\frac{\Delta t}{(\Delta x)^2}-2D_{xy}\frac{\Delta t}{(\Delta y)^2}=1-2(0.1)\frac{0.01}{(0.25)^2}-2(0.1)\frac{0.01}{(0.25)^2}$$
$$=0.936,$$

$$B_4=D_{xy}\frac{\Delta t}{(\Delta y)^2}+v\frac{\Delta t}{2\Delta y}=(0.1)\frac{0.01}{(0.25)^2}-(0.1)\frac{0.01}{2(0.25)}=0.014,$$

$$B_5=D_{xy}\frac{\Delta t}{(\Delta x)^2}+u\frac{\Delta t}{2\Delta x}=(0.1)\frac{0.01}{(0.25)^2}-(0.1)\frac{0.01}{2(0.25)}=0.014,$$

$$c_{i,j}^{n+1}=0.018c_{i-1,j}^n+0.018c_{i,j-1}^n+0.936c_{i,j}^n+0.014c_{i,j+1}^n+0.014c_{i+1,j}^n+Q_{i,j}^n \tag{4.90}$$

$$i=0,1,2,3,4, \quad j=0,1,2,3,4$$

Table 4.17 The calculated pollutant concentration at time $t=0.02$ of example 4.5.11

Point	Pollutant concentration	
	$Q=0$	$Q=0.0001$
$c_{0,0}^2$	0.000562	0.000663
$c_{0,1}^2$	0.001232	0.001430
$c_{0,2}^2$	0.001652	0.001852
$c_{0,3}^2$	0.001239	0.001438
$c_{0,4}^2$	0.000633	0.000733
$c_{1,0}^2$	0.000963	0.001162
$c_{1,1}^2$	0.033693	0.033893
$c_{1,2}^2$	0.045395	0.045595
$c_{1,3}^2$	0.034058	0.034258
$c_{1,4}^2$	0.001239	0.001438
$c_{2,0}^2$	0.001284	0.001484
$c_{2,1}^2$	0.044913	0.045113
$c_{2,2}^2$	0.060512	0.060711
$c_{2,3}^2$	0.045399	0.045599
$c_{2,4}^2$	0.001652	0.001852
$c_{3,0}^2$	0.000963	0.001162
$c_{3,1}^2$	0.033693	0.033893
$c_{3,2}^2$	0.045395	0.045595
$c_{3,3}^2$	0.034058	0.034258
$c_{3,4}^2$	0.001652	0.001438
$c_{4,0}^2$	0.000563	0.000663
$c_{4,1}^2$	0.001232	0.001430
$c_{4,2}^2$	0.001652	0.001852
$c_{4,3}^2$	0.001239	0.001438
$c_{4,4}^2$	0.000633	0.000733

Example 4.5.12. To find the pollutant concentration in street canyon, at boundary of street canyon that has rate of change of pollutant concentration with respect to x at the entrance gate $\frac{\partial c}{\partial x} = c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y} = c_2$, and rate of change of pollutant concentration with respect to y at the ground gate $\frac{\partial c}{\partial y} = c_3$, domain $\Omega = (0,1)\times(0,1)$ in Figure (4.74) with step size $\Delta x = \Delta y = 0.1, \Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, there is no interior $Q = 0$, average air pollutant source are added $Q = 0.0001$, average air flow velocity in x-direction $u = 0.1$, and average air flow velocity in y-direction v.

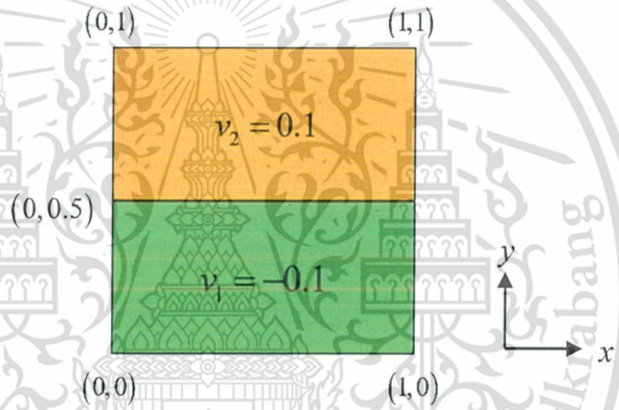


Figure 4.81: The flow velocity y-direction along the left and the right lanes of example 4.5.12

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0) = x(1-x)y(1-y) + c_0,$$

the boundary conditions can be assumed by

$$c_x(0,y,t) = c_0 = 0,$$

$$c_x(1,y,t) = c_1 = 0,$$

$$c_z(x,1,t) = c_2 = 0,$$

$$c_z(x,0,t) = c_3 = 0,$$

define $c(x,y,t) = c_{i,j}^n$ with $\Delta x = \Delta y = 0.1, \Delta t = 0.01, D = 0.1, u = 0.1, v_1 = -0.1,$ and $v_2 = 0.1$.

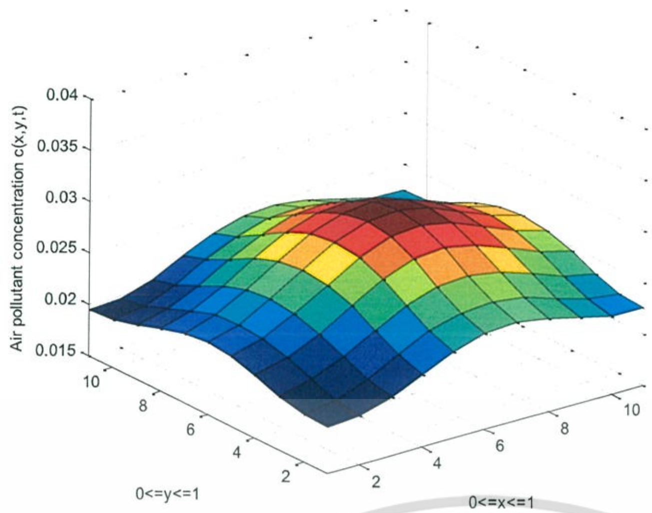


Figure 4.82: The approximated air pollutant concentration for $Q = 0$ and $0 \le t \le 60$ sec of example 4.5.12

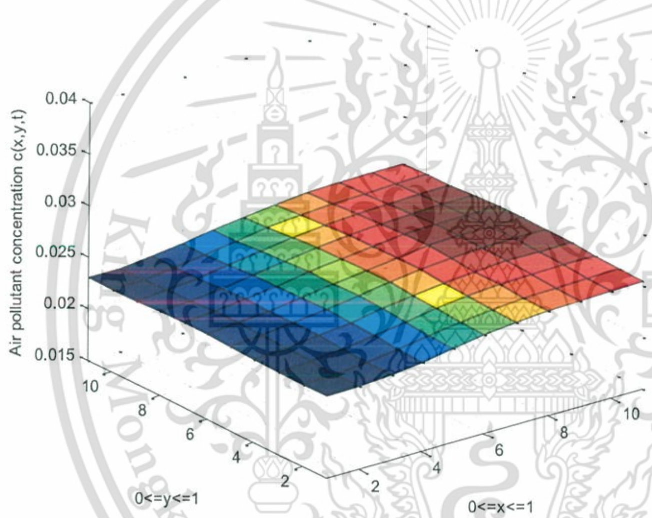


Figure 4.83: The approximated air pollutant concentration for $Q = 0$ and $0 \le t \le 120$ sec of example 4.5.12

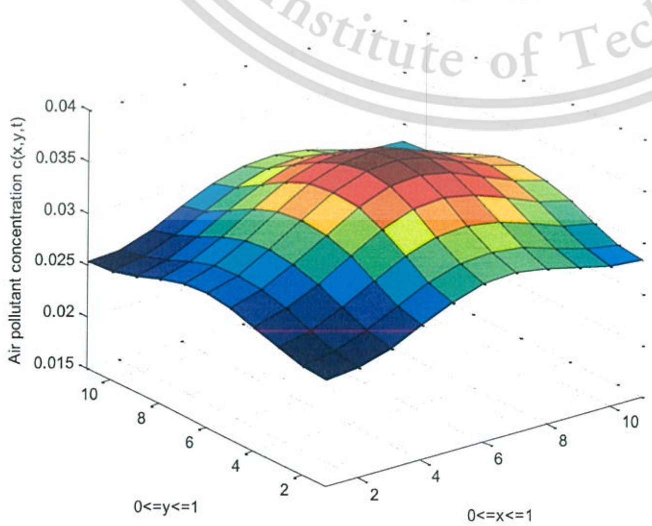


Figure 4.84: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \le 60$ sec of example 4.5.12

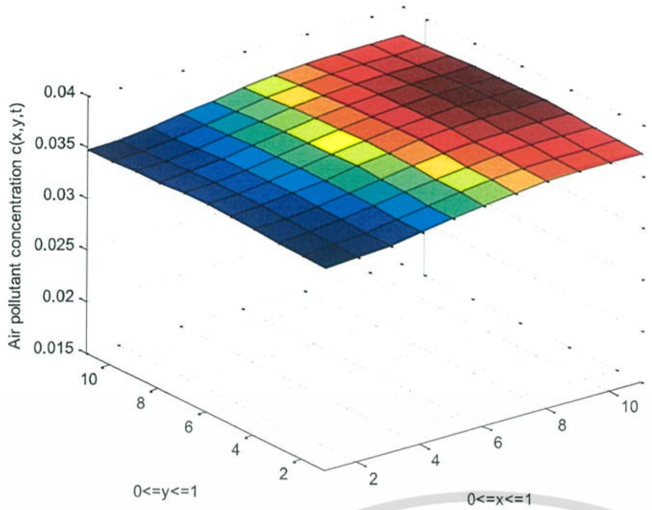


Figure 4.85: The approximated air pollutant concentration for $Q = 0.0001$ and $0 \le t \le 120$ sec of example 4.5.12

Example 4.5.13. To find the pollutant concentration in street canyon, at boundary of street canyon that has the equal of pollutant concentration with inflow $c = c_0$, and rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x} = c_1$, and rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y} = c_2$, and rate of change of pollutant concentration with respect to y at the ground gate $\frac{\partial c}{\partial y} = c_3$, domain $\Omega = (0,1)\times(0,1)$ in Figure (4.69) with step size $\Delta x = \Delta y = 0.1$, $\Delta t = 0.01$ diffusion coefficient $D_{xy} = 0.1$, interpolated air pollutant source are added Q, interpolated air flow velocity in x-direction $u = (2 - x^2) + (2 - y^2)$, and average air flow velocity in y-direction $v = 0.1$.

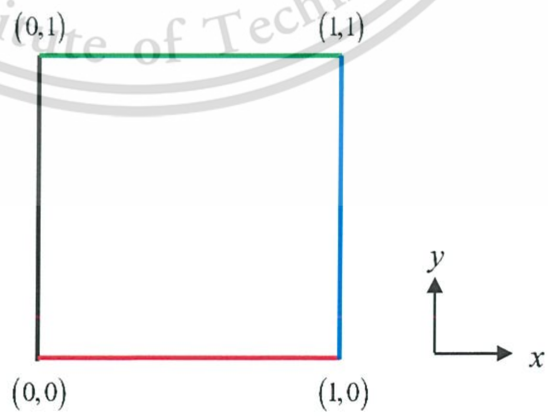


Figure 4.86: Domain of example 4.5.13

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0)=x(1-x)y(1-y)+c_0,$$

the boundary conditions can be assumed by

$$c(0,y,t)=c_0=0,$$

$$c_x(1,y,t)=c_1=0,$$

$$c_z(x,1,t)=c_2=0,$$

$$c_z(x,0,t)=c_3=0,$$

define $c(x,y,t)=c_{i,j}^n$ with $\Delta x=\Delta y=0.1, \Delta t=0.01, D_{xy}=0.1, u=(2-x^2)+(2-y^2)$ $Q=(0.2133\times10^{-4})x^3+(0.22\times10^{-4})x^2-(0.3633\times10^{-4})x+(1.2041\times10^{-4})$ and $v=0.1$.

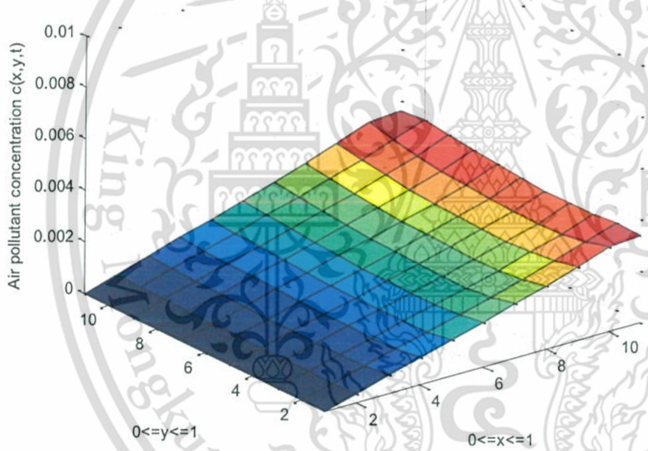


Figure 4.87: The approximated air pollutant concentration for $0\le t\le 60$ sec of example 4.5.13

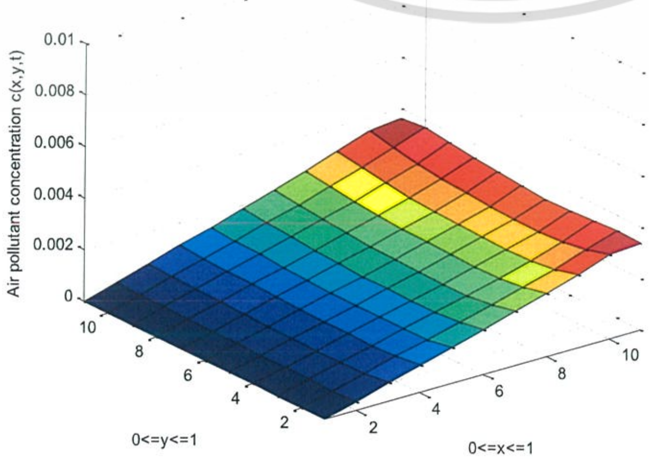


Figure 4.88: The approximated air pollutant concentration for $0\le t\le 120$ sec of example 4.5.13

Example 4.5.14. To find the pollutant concentration in street canyon, at boundary of street canyon that has rate of change of pollutant concentration with respect to x at the entrance gate $\frac{\partial c}{\partial x}=c_0$, rate of change of pollutant concentration with respect to x at the exit gate $\frac{\partial c}{\partial x}=c_1$, rate of change of pollutant concentration with respect to y at the top gate $\frac{\partial c}{\partial y}=c_2$, and rate of change of pollutant concentration with respect to y at the ground gate $\frac{\partial c}{\partial y}=c_3$, domain $\Omega=(0,1)\times(0,1)$ in Figure (4.74) with step size $\Delta x=\Delta y=0.1,\Delta t=0.01$ diffusion coefficient interpolated air pollutant source are added Q, interpolated air flow velocity in x-direction $u=\left(2-x^2\right)+\left(2-y^2\right)$, and average air flow velocity in y-direction $v=0.1$.

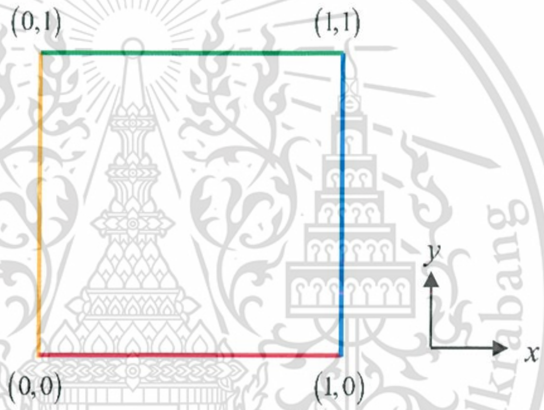


Figure 4.89: Domain of example 4.5.10

The potential air pollutant concentration at initial time can be used to assume the initial condition of Eq. (2.8) as

$$c(x,y,0)=x(1-x)y(1-y)+c_0,$$

the boundary conditions can be assumed by

$$c_x(0,y,t)=c_0=0,$$

$$c_x(1,y,t)=c_1=0,$$

$$c_z(x,1,t)=c_2=0,$$

$$c_z(x,0,t)=c_3=0,$$

define $c(x,y,t)=c_{i,j}^n$ with $\Delta x=\Delta y=0.1,\ \Delta t=0.01,\ D=0.1, u=\left(2-x^2\right)+\left(2-y^2\right)$ $Q=\left(0.2133\times10^{-4}\right)x^3+\left(0.22\times10^{-4}\right)x^2-\left(0.3633\times10^{-4}\right)x+\left(1.2041\times10^{-4}\right)$ and $v=0.1$.

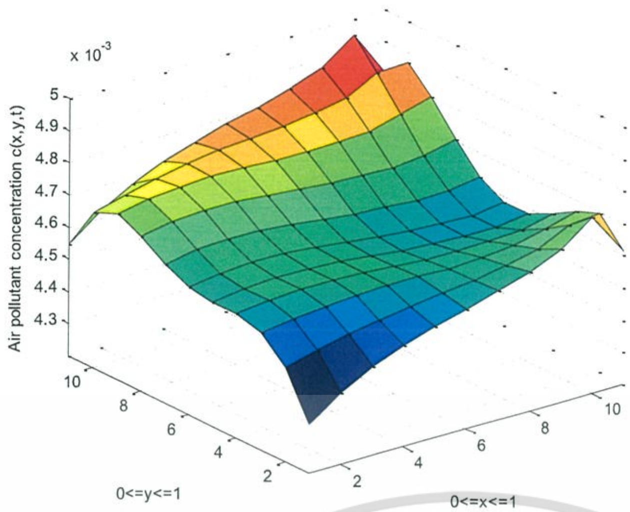


Figure 4.90: The approximated air pollutant concentration for $0 \le t \le 60$ sec of example 4.5.14

Figure 4.91: The approximated air pollutant concentration for $0 \le t \le 120$ sec of example 4.5.14

Table 4.18 Simulation of example for two dimensional in space (Along xz-axis and xy-axis)

Simulation	xz	xy	BCs				u	v	Q
			LHS	RHS	LOWER	TOP			
4.2.1	✓	-	C	C	C	C	K	-	K
4.2.2	✓	-	C	C	C	C	K	-	K
4.2.3	✓	-	C	O	C	C	K	-	K
4.2.4	✓	-	C	O	C	C	K	-	K
4.2.5	✓	-	C	O	C	O	K	-	K
4.2.6	✓	-	C	O	C	O	K	-	K
4.2.7	✓	-	C	O	O	O	K	-	K

4.2.8	✓	-	C	O	O	O	K	-	K
4.2.9	✓	-	O	O	O	O	K	-	K
4.2.10	✓	-	O	O	O	O	K	-	K
4.2.11	✓	-	C	O	O	O	K	-	Z_Q
4.2.12	✓	-	C	O	O	O	K	-	Z_Q
4.2.13	✓	-	C	O	O	O	K	-	K
4.2.14	✓	-	C	O	O	O	K	-	Z_Q
4.2.15	✓	-	C	O	O	O	K	-	Z_Q
4.2.16	✓	-	C	O	O	O	f_u	-	Z_Q
4.4.1	-	✓	C	C	C	C	K	K	K
4.4.2	-	✓	C	C	C	C	K	K	K
4.4.3	-	✓	C	O	C	C	K	K	K
4.4.4	-	✓	C	O	C	C	K	K	K
4.4.5	-	✓	C	O	C	O	K	K	K
4.4.6	-	✓	C	O	C	O	K	K	K
4.4.7	-	✓	C	O	O	O	K	K	K
4.4.8	-	✓	C	O	O	O	K	K	K
4.4.9	-	✓	O	O	O	O	K	K	K
4.4.10	-	✓	O	O	O	O	K	K	K
4.4.11	-	✓	O	O	O	O	K	Z_v	K
4.4.12	-	✓	O	O	O	O	K	Z_v	K
4.4.13	-	✓	C	O	O	O	f_u	K	f_Q
4.4.14	-	✓	O	O	O	O	f_u	K	f_Q

where

C = The street canyon is fully - closed

O = The street canyon is fully - open

K = Constant

f_u = function of interpolated air pollutant sources

f_Q = function of interpolated air flow velocity

Z_Q = The classification of averaged air pollutant sources (Along z-axis)

Z_v = The flow velocity y-direction along the left and the right lanes

Chapter 5
Conclusion

5.1 Conclusions

We have presented how to the concentration of the pollutant in the street canyon can be calculated. However, this calculation of numerical simulation model base on the data of the flow of the pollutant which varies due to shape and location of the street canyon. In this research presents the air pollutant within the street canyon different pollution source configuration. The numerical experiments are presented with the initial mean concentration of Street canyon. Both of two cases, one dimensional and two dimensional. Considered spaces xt, xzt, and xyt are respective

The finite difference scheme is used to solve a laterally averaged air pollution model in a street canyon. The one dimensional advection-diffusion equation is solved by using the forward time center space scheme and employing the for forward space method to their near the right of a laterally average air pollution model in street canyon as the boundary condition.

The two dimensional advection-diffusion equation is solved by using the forward time center space scheme and employing the forward space method and backward space method to their near the left, right, ground and top of a laterally average air pollution model in street canyon as the boundary conditions. The proposed numerical techniques give a reasonable approximation air pollutant concentrations at each point for each times.

5.2 Future works

This research can be applied in three dimensions for virtual reality. The three dimensional Navier-Stokes equation should be introduced to the advanced model. It can be used to obtain the air flow behavior.

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Appendix

Appendix

The research papers

ICMA-MU 2016 Book on the Conference Proceedings

NUMERICAL SIMULATION OF A TWO-DIMENSIONAL AIR POLLUTION MODEL IN AN URBAN STREET CANYON

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Abstract

Permeability and building density are important parameters for precisely modeling urban air pollution and influencing regulatory requirements for building planning. An air pollution model is proposed to investigate the dispersion of air pollution in an urban street canyon. To investigate the level of air pollution in an urban street canyon, the spatial distribution of pollutant concentration level is modeled using a two-dimensional advection-diffusion equation. The proposed model can be use to represent a laterally averaged air pollution model in an urban street canyon. The numerical results are calculated by using an explicit finite difference scheme with the uniform air flow velocity fields. The numerical can be use in several boundary conditions and interpolated wind velocity functions.

1 Introduction

Air contamination has been a critical natural and wellbeing sympathy toward hundreds of years. This presentation is across the board and vital for all populates since it is unavoidable. The Global Burden of Disease 2010 assessed that 3.1, 3.5, and 0.2 million passings happened every year as an aftereffect

Key words: air pollution, urban street canyon, dispersion model, advection-diffusion equation, explicit method.
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of exposures to encompassing particulate matter, family strong energizes and surrounding ozone contamination, separately. With fast urbanization of the total populace, air quality is expected to be on the decay as wellsprings of contamination total. In profoundly populated urban areas, vitality utilization and human exercises (e.g. control era and vehicle utilize) must ascent to stay aware of the requests of developing areas.

Urban air pollution was originally considered as a local problem mainly associated with domestic heating and industrial emissions, which are now controllable to a great extent.

The primary activity related poisons are CO, NOx, hydrocarbons, and particles. CO is a defective fuel ignition item. Burning additionally delivers a blend of NO2 and NO, of which more than 90% is as NO. An extensive variety of unburned and artificially changed hydrocarbons (e.g. benzene, toluene, ethane, ethylene, pentane, and so forth.) is discharged by engine vehicles through various diverse procedures (e.g. dissipation, fuel tank removal, oil leak, and so on.). At last, particles of dense carbonaceous material are transmitted for the most part by diesel and inadequately kept up petrol vehicles.

In [2], [3] and [6], the air pollution problem in three dimensional spaces with multiple sources are presented. The initial conditions in the domains are assumed to be zero everywhere without obstacles. In [4] and [5], the air pollution in two dimensional spaces with obstacles domain are also studied.

In urban situations and particularly in those zones where populace and activity thickness are moderately high, human introduction to unsafe substances is relied upon to be altogether expanded. This is regularly the case close bustling activity pivot in downtown areas, where urban geology and microclimate may add to the production of poor air scattering conditions offering ascend to defilement hotspots. High contamination levels have been seen in road gorge, which is a term habitually utilized for urban boulevards flanked by structures on both sides. Inside these boulevards, walkers, cyclists, drivers and inhabitants are probably going to be exposed to pollutant concentrations exceeding current air quality standards.

2 Governing Equations

2.1 A laterally averaged air pollution model in an urban street canyon

The distributed air pollutant process satisfies a mass transfer equation, which include transportation and diffusion. Averaging the equation over the lateral, we get the advection-diffusion equation

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D_{xz}\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial z^2}\right) + S, \qquad (2.1)$$

for all $(x, z, t) \in [0, L] \times [0, H] \times [0, \tau]$, where L is the length of the considered street canyon, H is the averaged building hight along the street, τ is the stationary of a simulation time, $c(x, z, t)$ (kg/m^3) is the laterally averaged air pollutant concentration at point (x, z) (m,m) and at time t (sec), D_{xz} (m^2/sec) is the diffusion coefficient of the considered air pollutant matter and $S(x, z, t)$ is the rate of change of concentration at point source or sink were released or absorbed along the street canyon per unit of time (kg/m^3 sec^{-1}), respectively. The initial condition is assumed by a cold start technique $c(x, z, 0) = c_0$, where c_0 is a given nonnegative constant. The boundary conditions are assumed by $c(0, z, t) = c_1$, on the left of the street canyon, $\frac{\partial c}{\partial z} = 0$ on the top of the canyon, $\frac{\partial c}{\partial z} = 0$ on the ground of the canyon, and $\frac{\partial c}{\partial x} = c_2$ on the right of the canyon, where c_1 and c_2 are given constants.

2.2 A non-dimensional form of a laterally averaged air pollution model

We will transform Eq.(2.1) into a non-dimensional form as below,

$$\frac{1}{st}\frac{\partial C}{\partial T} + U\frac{\partial C}{\partial X} = D\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Z^2}\right) + Q, \tag{2.2}$$

where $st = t_{\max} u_{\max}/L$, $l = \max\{L, H\}$, $U = u/u_{\max}$,
$D = D_{xz}/Pe$, $Pe = u_{\max} L/D_{xz}$, $X = x/L$, $Z = z/L$,
$C = c/c_{\max}$, $Q = S(t_{\max}/c_{\max})$ and $T = t/t_{\max}$,
for all $(X, Z, T) \in [0, 1] \times [0, H/L] \times [0, \mathcal{T}]$.

3 Numerical Techniques

We now discretize Eq.(2.2) by dividing the interval $[0, 1]$ into M subintervals such that $M\Delta X = 1$ and the interval $[0, \mathcal{T}]$ into N subintervals such that $N\Delta T = \mathcal{T}$, and similarly defined for $[0, H/L]$. We can then approximate $C(X_i, Z_i, T_n)$ by $C^n_{i,j}$, value of the difference approximation of $C(X, Z, T)$ at point $X = i\Delta X, Z = j\Delta Z$ and $T = n\Delta T$, where $0 \le i \le M, 0 \le j \le P$ and $0 \le n \le N$. The grid point (X_i, Z_j, T_n) is defined by $X_i = i\Delta X$ for all $i = 0, 1, 2, \ldots, M$ and $Z_j = j\Delta Z$ for all $j = 0, 1, 2, \ldots, P$ and $T_n = n\Delta T$ for all $n = 0, 1, 2, \ldots, N$ in which M, P and N are positive integers. Using the forward time center space method [1] to Eq.(2.2), the following finite difference equation can be obtained:

$$C \cong C^n_{i,j}, \quad (3.1)$$

$$\frac{\partial C}{\partial T} \cong \frac{C^{n+1}_{i,j} - C^n_{i,j}}{\Delta T}, \quad (3.2)$$

$$\frac{\partial C}{\partial X} \cong \frac{C^n_{i+1,j} - C^n_{i-1,j}}{2\Delta X}, \quad (3.3)$$

$$\frac{\partial^2 C}{\partial X^2} \cong \frac{C^n_{i+1,j} - 2C^n_{i,j} + C^n_{i-1,j}}{(\Delta X)^2}, \quad (3.4)$$

$$\frac{\partial^2 C}{\partial Z^2} \cong \frac{C^n_{i,j+1} - 2C^n_{i,j} + C^n_{i,j-1}}{(\Delta Z)^2}. \quad (3.5)$$

Substituting Eqs(3.1-3.5) into Eq.(2.2), we can obtain an explicit form of finite difference equation as follows,

$$C^{n+1}_{i,j} = \left(\frac{D\Delta T}{(\Delta X)^2} + \frac{U\Delta T}{2\Delta X}\right) C^n_{i-1,j} + \left(1 - \frac{2D\Delta T}{(\Delta X)^2} - \frac{2D\Delta T}{(\Delta Z)^2}\right) C^n_{i,j} \quad (3.6)$$
$$+ \left(\frac{D\Delta T}{(\Delta X)^2} - \frac{U\Delta T}{2\Delta X}\right) C^n_{i+1,j} + \frac{D\Delta T}{(\Delta Z)^2} C^n_{i,j-1} + \frac{D\Delta T}{(\Delta Z)^2} C^n_{i,j+1},$$

for all $1 \leq i \leq M-1$, $2 \leq j \leq P-1$ and $0 \leq n \leq N-1$. For the right boundary, where $i = M$, substituting the approximate unknown value of the right boundary by the forward difference approximation to $\frac{\partial C}{\partial X} = C_2$, we can let $C^n_{M,j} = C_2\Delta X + C^n_{M-1,j}$. For the bottom boundary, where $j = 0$, substituting the approximate unknown value of the bottom boundary by the forward difference approximation to $\frac{\partial C}{\partial Z} = 0$, we can let $C^n_{i,0} = C^n_{i,1}$. For the top boundary, where $j = P$, substituting the approximate unknown value of the top boundary by the forward difference approximation to $\frac{\partial C}{\partial Z} = 0$, we can let $C^n_{i,P} = C^n_{i,P-1}$.

4 Application to an Urban Street Canyon Air Pollution Assessment Problem

Suppose that the measurement of air pollutant concentration c in a uniform air flow street canyon $u = 0.10$ (m/s) is considered. A street canyon is aligned with longitudinal distance, 200 (m) long and 15 (m) high. There is an entrance which discharges waste water into the canyon and the pollutant concentration at the left ended is $c(0, z, t) = c_0 = 1$ (kg/m^3) at $x = 0$ for all $t > 0$, there is no rate of change of air pollutant level at the right ended exit $\frac{\partial C}{\partial x} = 0$ at $x = 200$ m. for all $t > 0$, there is no rate of change of air pollutant level along the top $\frac{\partial C}{\partial z} = 0$ on $z = 15$ m for all $t > 0$, there is no rate of change of air pollutant level along the ground $\frac{\partial C}{\partial z} = 0$ on $z = 0$ m for all $t > 0$, and there is the interpolated initial air pollutant $C(x, z, 0) = x(1-x)z(1-z)$ (kg/m^3) at

$t = 0$. The physical parameters of the considered air pollutant matters are a diffusion coefficient $D = 0.1$ (m^2/s) and an averaged vehicle air pollutant rising rate $S = 0.5 \times 10^{-6}$ (kg/m^3s). In the analysis conducted in this study, meshing the canyon into 200 elements in x-direction with $\Delta x = 1$ m and 15 elements in z-direction with $\Delta z = 1$ m, and the time increment is $\Delta t = 20$ (s). The non-dimensional form of governing equation with the proposed FTCS method can be use to obtain the approximated pollutant concentrations $c(x, z, t)$ as shown in Table 1 and Figure 1.

Table 1: The approximated air pollutant concentration $C(x, z, 0.01)$ (kg/m^3)

z/x (m)	0	25	50	75	100	125	150	175	200
0	1.0000	0.6004	0.6000	0.6000	0.6000	0.6000	0.6000	0.6000	0.5996
5	1.0000	0.6230	0.6000	0.6000	0.6000	0.6000	0.6000	0.6000	0.5999
10	1.0000	0.9289	0.6074	0.6000	0.6000	0.6000	0.6000	0.6000	0.6000
15	1.0000	1.2430	0.7507	0.6000	0.6000	0.6000	0.6000	0.6000	0.6000

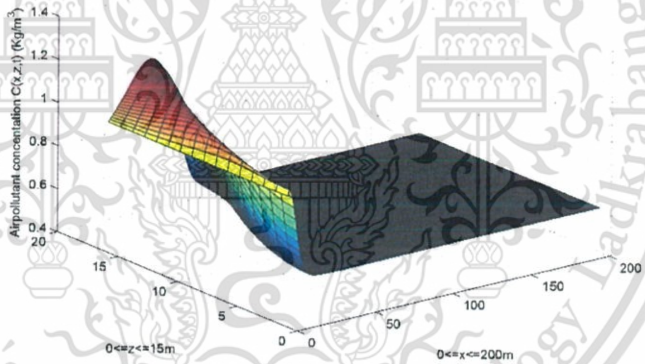

Figure 1: The approximated air pollutant concentration $C(x, z, t)$ (kg/m^3) at $t = 100$ s

5 Discussion and Conclusions

In this paper, the finite difference scheme is used to solved a a laterally averaged air pollution model in a street canyon. The one-dimensional advection-diffusion equation in a uniform air flow in the street canyon is solved by using the forward time center space scheme and employing the forward space method to their near the right, ground and top of a laterally averaged air pollution model in a street canyon as the boundary conditions. The proposed numerical techniques give a good agreement approximated air pollutant concentrations at each point for each times.

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