รายงานการวิจัยฉบับสมบูรณ์

การรบกวนเอกฐานของสมการเชิงอนุพันธ์ที่มีอิมพัลส์

SINGULAR PERTURBATION OF IMPULSIVE DIFFERENTIAL EQUATIONS

นายวิชัย วิทยาเกียรติเลิศ

ได้รับทุนสนับสนุนงานวิจัยจากทุนส่งเสริมนักวิจัย งบประมาณเงินรายได้
ประจำปีงบประมาณ พ.ศ. 2557

ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์
สถาบันเทคโนโลยีพระจอมเกล้าเจ้าคุณทหารลาดกระบัง

ชื่อโครงการ การรบกวนเอกฐานของสมการเชิงอนุพันธ์ที่มีอิมพัลส์
แหล่งเงิน ทุนส่งเสริมนักวิจัย งบประมาณเงินรายได้
ประจำปีงบประมาณ 2557 จำนวนเงินที่ได้รับการสนับสนุน 50,000 บาท
ระยะเวลาทำการวิจัย 1 ปี ตั้งแต่ 1 ตุลาคม 2556 ถึง 30 กันยายน 2557
หัวหน้าโครงการ นายวิชัย วิทยาเกียรติเลิศ สาขาคณิตศาสตร์ คณะวิทยาศาสตร์
สถาบันเทคโนโลยีพระจอมเกล้าเจ้าคุณทหารลาดกระบัง

บทคัดย่อ

งานวิจัยนี้ศึกษาการรบกวนเอกฐานของสมการเชิงอนุพันธ์ที่มีอิมพัลส์ ในงานวิจัยได้นิยามผลเฉลยของปัญหาการรบกวนเอกฐานของสมการเชิงอนุพันธ์ที่มีอิมพัลส์ นอกจากนี้ยังศึกษาสมบัติปกติ(Regularity)ของผลเฉลยโดยประมาณนี้ด้วย

คำสำคัญ : สมการเชิงอนุพันธ์ที่มีอิมพัลส์, ผลเฉลยละมุน, ผลเฉลยดั้งเดิม, ปัญหาโคชีนามธรรมกึ่งเชิงเส้น

Research Title: Singular perturbation of impusive differential equations
Researcher: Mr. Wichai Witayakiattilerd
Faculty: Science **Department:** Mathematics

ABSTRACT

In this research, we study the singular perturbation of impulsive equations on Banach space. We suggest some approximate solution of the perturbed system. Moreover, we show the regularity of the approximate solution.

Keywords : impulsive differential equation, mild solution, classical solution, semilinear abstract Cauchy problem.

ACKNOWLEDGEMENTS

Beside the efforts I put in writing up this project, many people have involved in this project and contributed to its success. I appreciate their help and I am happy that I had the opportunity to learn from and work with them. I would like to thanks the members of Mathematics Department, faculty of Scienc, King Mongkut's Institute of Technology Ladkrabang. They gave me advices and especially helped me finding fund support for this project. They guided me through the all the necessary paper work. They gave me comments, suggestion and every things. I would like to say from my heart "thank you very much".

Furthermore, we thanks for the money supporting by King Mongkut's Institute of Technology Ladkrabang "The fund Promote Researcher Annual revenue budget for 2013". I hope that I do not forget to mention anyone. If these show to be the case, I do apologize. It does not happen by intention.

Wichai Witayakiattilerd
October 2014

CONTENT

CHAPTER I
INTRODUCTION

The mathematical models of many real world problems can be described by impulsive differential equations. They have been studied quite extensively [2-25 and reference therein] because they have advantage over the traditional initial value problems. They can be used to model other phenomena that cannot be modeled by the traditional initial value, such as the dynamics of the systemic arterial pressure, the dynamics of populations subjected to abrupt changes (harvesting, diseases, etc.). However, some time the systems maybe have perturbed with some small parameters that it seems as a complex problem but views as an interesting problem. In the present, there are many papers done on this problem but in this document we only list some papers as these followings.

In 1989, D. D. Bainov, V. Lakshmikantham, and P. S. Simeonov wrote "Theory of Impulsive Differential Equations". This book is an important book in this field. Many researchers have cited in their papers. In 1992, G.K. Kulev has studied Uniform asymptotic stability in impulsive perturbed systems of differential equations. In 1996, D.D. Bainov, A.B. Dishlievb and I.M. Stamovac have investigated the uniform asymptotic stability in impulsive perturbed systems of differential equations. In 2007, Wei Zhua, Daoyi Xub,Chunde Yanga have investigated the exponential stability of singularly perturbed impulsive delay differential equations. In 2009, W.Witayakiatilerd & A.Chonwerayuth have proved the regularity of piecewise continuous almost periodic solutions for nonlinear impulsive systems. These researches motivate our research.

For this article, we study the abstract Cauchy Problem (ACP) of the singular perturbation of nonlinear-retarded functional impulsive differential equations on a Banach X

$$\begin{cases} \dot{x}(t) = A(\varepsilon)x(t) + f(t, x(t), y(t), \varepsilon), & t \neq t_k \quad (1.1a) \\ \dot{y}(t) = \frac{1}{\varepsilon}\left[B(\varepsilon)y(t) + g(t, x(t), y(t), \varepsilon)\right], & t \neq t_k \quad (1.1b) \\ \Delta x(t_k) = I_k(x(t_k), y(t_k)), & (1.1c) \\ \Delta y(t_k) = J_k(x(t_k), y(t_k)) & \\ x(0) = x^0, \ y(0) = y^0 & (1.1d) \end{cases}$$

where $0 < \varepsilon << 1$, $0 < t_1 < t_2 < \ldots < t_n < T$, $\mathbb{R}_0 = [0,T], A(\varepsilon): X \supseteq D(A(\varepsilon)) \to R(A(\varepsilon)) \subseteq X$ and $B(\varepsilon): X \supseteq D(B(\varepsilon)) \to R(B(\varepsilon)) \subseteq X$ are given continuous operators in ε, $\Delta x(t_k) = x(t_k^+) - x(t_k)$ and $\Delta y(t_k) = y(t_k^+) - y(t_k)$ denote the jump of state x and y at time t_k

with the magnitude of jump I_k and J_k , $k=1,2,\ldots$,respectively. If $f(\cdot,\cdot,\cdot,\varepsilon)$ and $g(\cdot,\cdot,\cdot,\varepsilon)$ are globally Lipschitz and uniformly bounded in ε, then $\dot{y}$ will be of order $\frac{1}{\varepsilon}$ faster than $\dot{x}$. Consequently, we call x the slow variable and y the fast variable of the system.

In this paper we suggest form of the system (1.1) as follows:

$$\begin{cases} \dot{x}(t)=A(\varepsilon)x(t)+f(t,x(t),y(t),\varepsilon), & t\neq t_k \qquad (1.2a) \\ \varepsilon\dot{y}(t)=B(\varepsilon)y(t)+g(t,x(t),y(t),\varepsilon), & t\neq t_k \qquad (1.2b) \\ \Delta x(t_k)=I_k(x(t_k),y(t_k)),\ \Delta y(t_k)=J_k(x(t_k),y(t_k)) & \qquad (1.2c) \\ x(0)=x^0,\ y(0)=y^0. & \qquad (1.2d) \end{cases}$$

We suggest some approximate solution of the perturbed system. Moreover, we show the regularity of the approximate solution.

1.1 The objective of the research project

1.1.1) To study the definitions and theories associated with the nonlinear impulsive systems on Banach.

1.1.2) To study the definitions and theories associated with the singular perturbation of nonlinear impulsive systems on Banach.

1.1.3) To define some estimate solutions of the singular perturbation of nonlinear impulsive systems on Banach.

1.1.4) To prove regularity of some approximate solution of singular perturbation to impulsive differential equations on Banach.

1.2 The scope of the research project

We study the abstract Cauchy Problem of the singular perturbation of nonlinear-retarded functional impulsive differential equations on a Banach X. Then we investigate some approximate solution of the perturbed system and prove the regularity of the approximate solution.

CHAPTER II
PRELIMINARIES

In this chapter, we will state the definitions and theories that use in the research such as some definitions and theories on Banach space, fixed-point theory and fractional calculus theory etc.

2.1 Some definitions and theorems on Banach space

Functional analysis plays a central role in this reseach. In this section, Some of definitions and theories those are required in subsequent chapters, with appropriate references given wherever necessary.

Let X be a Banach space with norm $\|\cdot\|$. Let Y be another Banach space with norm $\|\cdot\|_Y$. A linear transformation from X into Y is bounded on a domain of T, $D(T)$ if there exist a constant c such that $\|Tx\|_Y \le c\|x\|$ for all $x \in D(T)$. The linear space of all bounded linear operators from X into Y, denoted by $\mathcal{L}(X,Y)$ and denote $\mathcal{L}(X,X)$ by $\mathcal{L}(X)$.

Definition 2.1 A sequence x_n in X is said to be a strongly convergent to an element x in X if $\|x_n - x\| \to 0$ as $n \to \infty$. We denote by $x_n \to^s x$.

Theorem 2.2(Uniform Boundedness Principle). Let $\{T_\alpha \mid \alpha \in \Lambda\}$ be a family of operators from $\mathcal{L}(X,Y)$. If for each $x \in X$ there is a constant c_x such that $\sup\{\| T_\alpha \mid \alpha \in \Lambda \|\} \le c_x$, then the operator $\{T_\alpha\}$ are uniformly bounded.

Let X be a Banach and X^* is its dual. Element of X^* can be used to generate a new topology for X called ***weak topology***. Note that the norm topology on X was called ***strong topology***. So the new topology is weaker than the strong (norm) topology. Particularly, the linear functional on X that are continues in the weak topology are precisely the functional in X^*. The concept of open (closed) sets, compactness, convergence, etc., are topological, hence they must be qualified by referring to the topology involved. In the case of norm linear spaces, when one speaks of open (closed) sets, compactness, convergence, etc., one refer to strong (norm) topology, while, with reference to its weak topology, they are called weakly open (weakly closed) sets, weak compactness, weak convergence, etc. Thus a sequence $\{x_n\}$ in X is said to converge weakly to an element x in X if, for every $x^* \in X^*$, $x^*(x_n) \to x^*(x)$, written by $x_n \to^w x$. Every weakly convergent sequence is bound. Every strongly convergent sequence is weakly convergent, but the converse is not true.

2.2 Fixed-point Theorems

Fixed-point theorem on Banach spaces or contraction mapping is an advantage tool that is not difficult applying proving the existence and the uniqueness of equations.

Consider a function $\varphi:\Re\to\Re$ and suppose that we require solving the equation $\varphi(x)=0$. This is equivalent to solving the equation $\psi(x)=x$ where $\psi(x)=\varphi(x)+x$ for all $x\in\Re$.

Thus x is a zero of φ if and only if x is a fixed point of φ, i.e. a point which is left unaltered after the application of ψ. More generally, many problems are equivalent to solving $Af=f$ where $A:D(A)\to R(A)$ is an operator (not necessarily linear), acting in some normed vector spaces , $D(A)$ and $R(A)$ are domain and range of A in X respectively i.e. we seek a fixed point $f\in D(A)$ of the operator A (for simplicity, we write Af rather than $A(f)$).

There are many fixed-point theorems, which guarantee existence and/or uniqueness of fixed points. We state here what is used in this thesis.

Definition 2.3 Let X be a normed vector space and let $A:D(A)\to R(A)$ be an operator (not necessarily linear). Then

1) A is a contraction if there exists a constant c with $0\le c\le 1$ such that

$$\| Af_1 - Af_2 \| \le \| f_1 - f_2 \| \quad \text{for all } f_1, f_2 \in D(A) \tag{1}$$

2) A is strictly contraction if there exists a constant c with $0\le c<1$ such that (1) holds.

Theorem 2.4 (The contraction mapping theorem; Banach fixed point theorem) Let X be a Banach space and let $A:X\to X$ be a strictly contraction. Then the equation $Af=f$ has a unique solution in X .i.e. A has a unique fixed point f.

The result of this theorem can easily generalize as follows:

Theorem 2.5 Let X_0 be a closed subset of the Banach space X and assume that the operator A map X_0 into itself and a strictly contraction on X_0. Then the equation $Af=f$ has a unique solution $f\in X_0$.

2.3 Semigroup of linear operators

Consider a dynamic system, the state of which is evolving with time according to some law. For example, we may be interested in the temperature distribution along a rod which is being heated at one end. Suppose the initial state of the system is x_0; in this case $x_0(z)$ would measure the initial temperature at the point z of the rod. At a subsequent time $t>0$, the state of the system will be given by $x(z,t)$; this state would measure the temperature at the point x at time t. Since, for each $t>0$, the state $x(z,t)$ is an element of a

Banach space X .We shall use the symbol $u(t)$ to indicate such a state, i.e. $x(t)(z) = x(z,t)$. The state $x(t)$ will be related to the original state x_0 by some transition operator $T(t)$ so that

$$x(t) = T(t)x_0, \quad \text{for all } t \geq 0$$

We shall thus obtain a family $\{T(t)\}_{t\geq 0}$ of such operators. It is natural to ask what properties this family should have.

Firstly, each operator $T(t)$ acts in a set of "state x_0", where the states can typically represented by functions. Hence the domain of $T(t)$ will be a subspaces of function.

Next, it is clear that $T(0)$ must be I, the identity operator on X since at $t = 0$ there is no transition. Further, for any $s,t \geq 0$ we should require that $T(s+t)x_0 = T(s)T(t)x_0$. Indeed, the left hand side describes the evolution over a time interval of length $s+t$. The right hand side effectively say that the system evolves from x_0 to $T(t)x_0$ in t units of time and then continues to evolve from $T(t)x_0$ to $T(s)[T(t)x_0]$ in a subsequent time interval of length s, from t to $s+t$. The net effect should be the same as going nonstop from 0 to $s+t$, without taking a snapshot at time t.

Thus, we led to the two conditions

$$T(0) = I, \ T(s)T(t) = T(s+t) \quad \text{for all } s,t \geq 0$$

Finally, it is natural to expect that if s is close to t, and then $T(s)x_0$ should be close to $T(t)x_0$ in some sense. This is concept to define a family of transition operator say "semigroup of operators". We are now ready to make the following formal definition. Throughout this section X be a Banach space.

Definition 2.6 A one-parameter family $\{T(t)\}_{t\geq 0}$ of bounded linear operators from X into X is a semigroup of bounded linear operators on X if

1) $T(0) = I$, (I is the identity operator on X);

2) $T(t+s) = T(t)T(s)$ for every $t,s \geq 0$ (the semigroup property).

A semigroup of bounded linear operators $\{T(t)\}_{t_0}$ is uniformly continuous if

$$\lim_{t\to 0^+} \| T(t) - I \|_{L(X)} = 0.$$

The linear operator A defined by

$$D(A)=\{x\in X \mid \lim_{t\to 0^+}\frac{T(t)x-x}{t} \text{ exists}\}$$

and

$$Ax=\lim_{t\to 0^+}\frac{T(t)x-x}{t}=\frac{d^+T(t)x}{dt}\Big|_{t=0} \qquad \text{for all } x\in D(A)$$

is called the infinitesimal generator of semigroup $\{T(t)\}_{t\geq 0}$, $D(A)$ is the domain of A.

Definition 2.7 A one-parameter family $\{T(t)\}_{t\in\mathbb{R}}$ of bounded linear operators from X into X is a group of bounded linear operators on X if

1) $T(0)=I$, (I is the identity operator on X) ;

2) $T(t+s)=T(t)T(s)$ for every $t,s\in\mathbb{R}$ (the semigroup property).

3) $\lim_{t\to 0}T(t)x=x$

Definition 2.8 The infinitesimal generator A of a group $\{T(t)\}_{t\in\mathbb{R}}$ is defined by

$$Ax=\lim_{t\to 0}\frac{T(t)x-x}{t}=\frac{dT(t)x}{dt}\Big|_{t=0} \qquad \text{for all } x\in D(A)$$

where $D(A)=\{x\in X \mid \lim_{t\to 0}\frac{T(t)x-x}{t} \text{ exists}\}$.

From definition 2.6, we have a semigroup $\{T(t)\}_{t\geq 0}$ with a unique infinitesimal generator. If $T(t)$ is uniformly continuous, its infinitesimal generator is a bounded operator. On the other hand, every bounded linear operator A is the infinitesimal generator of a uniformly continuous semigroup $\{T(t)\}_{t\geq 0}$ and this semigroup is unique.

Definition 2.9 A semigroup $\{T(t)\}_{t\geq 0}$ of a bounded linear operator on X is a strongly continuous semigroup of bounded linear operators if $\lim_{t\to 0^+}T(t)x=x$ for all $x\in X$.

A strongly continuous semigroup of bounded linear operators on X will be called a semigroup of a C_0 – semigroup.

Theorem 2.9 Let A be an infinitesimal generator of the $C_0-semigroup$ $\{T(t)\}_{t\geq 0}$. Then

a) for all $x \in X$, $\lim_{h\to 0^+} \frac{1}{h}\int_t^{t+h} T(s)xds = T(t)x$;

b) for all $x \in X, \int_0^t T(s)xds \in D(A)$ and $A\int_0^t T(s)xds = x - T(t)x$;

c) for all $x \in D(A)$,$T(t)x \in D(A)$ and $\frac{d}{dt}T(t)x = -AT(t)x = -T(t)Ax$;

d) for all $x \in D(A)$, $T(s)x - T(t)x = \int_s^t T(\tau)Axd\tau = \int_s^t AT(\tau)xd\tau$.

Theorem 2.9 has some simple consequences that we now state.

Theorem 2.10 If A is the infinitesimal generator of a C_0 – semigroup $\{T(t)\}_{t\geq 0}$ then $D(A)$ is dense in X and A is a closed linear operator.

CHAPTER III

REGULARITY OF SOME APPROXIMATE SOLUTION OF SINGULAR PERTURBATION TO IMPULSIVE DIFFERENTIAL EQUATIONS ON BANACH SPACE.

In section, we study the abstract Cauchy Problem (ACP) of the singular perturbation of nonlinear-retarded functional impulsive differential equations on a Banach X

$$\begin{cases} \dot{x}(t) = A(\varepsilon)x(t) + f(t, x(t), y(t), \varepsilon), & t \neq t_k \quad (3.1a) \\ \dot{y}(t) = \dfrac{1}{\varepsilon}\left[B(\varepsilon)y(t) + g(t, x(t), y(t), \varepsilon)\right], & t \neq t_k \quad (3.1b) \\ \Delta x(t_k) = I_k(x(t_k), y(t_k)), & \quad (3.1c) \\ \Delta y(t_k) = J_k(x(t_k), y(t_k)) & \\ x(0) = x^0,\ y(0) = y^0 & \quad (3.1d) \end{cases}$$

where $0 < \varepsilon << 1$, $0 < t_1 < t_2 < ... < t_n < T$, $\mathbb{R}_0 = [0,T], A(\varepsilon): X \supseteq D(A(\varepsilon)) \to R(A(\varepsilon)) \subseteq X$ and $B(\varepsilon): X \supseteq D(B(\varepsilon)) \to R(B(\varepsilon)) \subseteq X$ are given continuous operators in ε, $\Delta x(t_k) = x(t_k^+) - x(t_k)$ and $\Delta y(t_k) = y(t_k^+) - y(t_k)$ denote the jump of state x and y at time t_k with the magnitude of jump I_k and J_k, $k = 1, 2, \ldots$, respectively. If $f(\cdot,\cdot,\cdot,\varepsilon)$ and $g(\cdot,\cdot,\cdot,\varepsilon)$ are globally Lipschitz and uniformly bounded in ε, then $\dot{y}$ will be of order $\frac{1}{\varepsilon}$ faster than $\dot{x}$. Consequently, we call x the slow variable and y the fast variable of the system.

A suggestive form of the system (3.1) is as follows:

$$\begin{cases} \dot{x}(t) = A(\varepsilon)x(t) + f(t, x(t), y(t), \varepsilon), & t \neq t_k \quad (3.2a) \\ \varepsilon\dot{y}(t) = B(\varepsilon)y(t) + g(t, x(t), y(t), \varepsilon), & t \neq t_k \quad (3.2b) \\ \Delta x(t_k) = I_k(x(t_k), y(t_k)),\ \Delta y(t_k) = J_k(x(t_k), y(t_k)) & \quad (3.2c) \\ x(0) = x^0,\ y(0) = y^0 & \quad (3.2d) \end{cases}$$

3.1 Approximate solution

Suppose that (x_0, y_0) is arbitrary initial value. Then we can choose an initial (x_0, y_0) such that the fast equation becomes stationary, i.e. $\dot{y}(t; y_0) = 0$. This implies that

$$B(\varepsilon)y + g(t, x, y, \varepsilon) = 0. \tag{3.3}$$

Accordingly, for small enough ε, it seem reasonable to substitute (3.2) with the differential algebraic equation

$$\begin{cases} \dot{x} = Ax(t) + F(t, x(t), y(t)) , & t \neq t_k & (3.4a) \\ 0 = By(t) + G(t, x(t), y(t)) , & t \neq t_k & (3.4b) \\ \Delta x(t_k) = I_k(x(t_k), y(t_k)) , \; \Delta y(t_k) = J_k(x(t_k), y(t_k)) & & (3.4c) \\ x(0) = x^0 , \; y(0) = y^0 & & (3.4d) \end{cases}$$

where $A \equiv A(0)$, $B \equiv B(0)$, $F(t, x(t), y(t)) \equiv f(t, x(t), y(t), 0)$ and $G(t, x(t), y(t)) \equiv g(t, x(t), y(t), 0)$. Suppose that $y(t) = h(t, x(t)) \in D(B) \subseteq X$ solve the system

$$\begin{cases} 0 = By(t) + G(t, x(t), y(t)) , & t \neq t_k & (3.5a) \\ \Delta y(t_k) = J_k(x(t_k), y(t_k)) & & (3.5b) \\ y(0) = y^0 & & (3.5c) \end{cases}$$

Therefore, by substituting $y(t) = h(t, x(t))$ into (3.4), we have

$$\begin{cases} \dot{x}(t) = Ax(t) + F(t, x(t), h(t, x(t))) , & t \neq t_k & (3.6a) \\ \Delta x(t_k) = I_k(x(t_k), h(t_k, x(t_k))) & & (3.6c) \\ x(0) = x^0 & & (3.6d) \end{cases}$$

In other words, we can approximate the slow equation by the inhomogeneous ACP (3.6).

First we need to be solve semilinear ACP,

$$\begin{cases} \dot{x}(t) = Ax(t) + F(t, x(t), h(t, x(t))) , & t_0 \leq t \leq \theta & (3.7a) \\ x(t_0) = x^0 & & (3.7b) \end{cases}$$

Definition 3.1 A mild solution on $[t_0, \theta]$ of the semilinear ACP (3.7) is a continuous function $x : [t_0, \tau] \to X$ such that $x(t)$ satisfies the integral equation

$$x(t) = S(t - t_0)x^0 + \int_{t_0}^{t} S(t - s)F(s, x(s), h(s, x(s))ds \qquad (3.8)$$

where $\{S(t)\}_{t \geq 0}$ is the C_0- semigroup generated by the operator A.

We start with the following classical result which assure the existence and uniqueness of mild solutions of (3.7) for Lipschitz continuous function h and F.

Theorem 3.2 Let $h:[t_0,\theta]\times X\rightarrow X$, $F:[t_0,\theta]\times X\times X\rightarrow X$ be continuous in t on $[t_0,\theta]$ and uniformly Lipschitz continuous (with constant L_h and L_F) on X. If A is infinitesimal generator of a C_0-semigroup $\{S(t)\}_{t\geq 0}$ on X, then for every $x^0\in X$ the semilinear ACP (3.7) has a unique mild solution $x\in C([t_0,\theta],X)$. Moreover, the mapping $x^0\rightarrow x$ is Lipchitz continuous from X into $C([t_0,\theta],X)$.

Proof. For a given $x^0\in X$. Define a mapping $Q:C([t_0,\theta],X)\rightarrow C([t_0,\theta],X)$ by

$$(Qx)(t)=S(t-t_0)x^0+\int_{t_0}^{t}S(t-s)F(s,x(s),h(s,x(s))ds\text{, for all } t\in[t_0,\theta]. \quad (3.9)$$

Denoting by $\|x\|_\infty$ the norm of x as an element of $C([t_0,\theta],X)$ it follows from the definition of Q and the uniformly Lipschitz continuous of h and F that

$$\begin{aligned}\|(Qx_1)(t)-(Qx_2)(t)\| &\leq ML_F\int_{t_0}^{t}\|x_1(s)-x_2(s)\|+\|h(s,x_1(s)-h(s,x_2(s)\|ds\\ &\leq ML_F(1+L_h)(t-t_0)\|x_1-x_2\|_\infty \end{aligned}\quad (3.10)$$

where M is a bound of $\|S(t)\|$ on $[t_0,\theta]$. Using (1.9) and (1.10) and induction on n, we have

$$\|(Q^n x_1)(t)-(Q^n x_2)(t)\|\leq\frac{\left(ML_F(1+L_h)(t-t_0)\right)^n}{n!}\|x_1-x_2\|_\infty. \quad (3.11)$$

This implies that

$$\|Q^n x_1-Q^n x_2\|_\infty\leq\frac{\left(ML_F(1+L_h)\theta\right)^n}{n!}\|x_1-x_2\|_\infty. \quad (3.12)$$

For n sufficiently large, $\frac{\left(ML_F(1+L_h)\theta\right)^n}{n!}<1$ and by a well-known extension the contraction mapping principle, there is a unique $x\in C([t_0,\theta],X)$ such that

$$x(t)=(Qx)(t)=S(t-t_0)x^0+\int_{t_0}^{t}S(t-s)F(s,x(s),h(s,x(s))ds \quad (3.13)$$

Therefore, the semilinear ACP (3.7) has a unique mild solution $x\in C([t_0,\theta],X)$.

The Lipchitz continuity of the mapping $x^0\rightarrow x$ is consequences of the following argument. Let x_1 and x_2 be the mild solutions of (3.7) with the initial value x_1^0 and x_2^0, respectively. Then it follows from the definition of mild solution and the uniformly Lipschitz continuous of h and F that

$$\|x_1(t)-x_2(t)\|\leq M\|x_1^0-x_2^0\|+ML_F(1+L_h)\int_{t_0}^{t}\|x_1(s)-x_2(s)\|ds. \quad (3.14)$$

By using Granwall's Lemma, it implies that

$$\|x_1(t)-x_2(t)\|\leq\left(M\exp\left(ML_F(1+L_h)\theta\right)\right)\|x_1^0-x_2^0\|. \quad (3.15)$$

Thus,

$$\|x_1 - x_2\|_\infty \le \left(M \exp\left(ML_F(1+L_h)\theta\right)\right)\|x_1^0 - x_2^0\| \tag{3.16}$$

which yields the Lipchitz continuity of the mapping $x^0 \to x$. □

Corollary 3.3 Under the conditions of Theorem 3.2, for every $\varphi \in C([t_0,\theta],X)$ the integral equation

$$u(t) = \varphi(t) + \int_{t_0}^{t} S(t-s)F(s,u(s),h(s,u(s))ds \tag{3.17}$$

has a unique solution $u \in C([t_0,\theta],X)$.

Proof. The proof is similar to the proof of Theorem 1.2 by defining

$$(Qx)(t) = \varphi(t) + \int_{t_0}^{t} S(t-s)F(s,x(s),h(s,x(s))ds\text{, for all } t \in [0,\theta]. \tag{3.18}$$

□

3.2 Regularity of the system without impulses

Definition 1.4 A classical solution of the inhomogeneous ACP (3.7) is a function $x \in C([t_0,\theta],X)$ such that

(i) x is continuously differentiable on (t_0,θ)

(ii) $x(t) \in D(A)$ for all $t \in [t_0,\theta]$

(iii) x satisfies the semilinear ACP (1.7).

Theorem 1.5 (Regularity)Let A be the infinitesimal generator of a C_0-group $\{S(t)\}$ on a Banach space X If $h:[t_0,\theta]\times X \to X$, $F:[t_0,\theta]\times X\times X \to X$ are continuously differentiable on $[t_0,\theta]\times X$ and $[t_0,\theta]\times X\times X$, respectively, then the mild solution $x \in C([t_0,\theta],X)$ of (3.7) with $x^0 \in D(A)$ is a classical solution of the semilinear ACP (3.7).

Proof. Let x be the solution $x \in C([t_0,\theta],X)$ of (3.7) with $x^0 \in D(A)$. Set $H(t,x(t)) \equiv F(t,x(t),h(t,x(t))$. The continuity differentiable of h and F implies the continuity differentiable of H. Define a function φ by

$$\varphi(t) = S(t-t_0)H(t_0,x(t_0)) + AS(t-t_0)x^0 + \int_{t_0}^{t} S(t-s)\frac{\partial}{\partial s}H(s,x(s))ds. \tag{3.18}$$

It follows from the assumptions that $\varphi \in C([t_0,\theta],X)$ and that the function $\alpha(t,x) \equiv \left(\frac{\partial}{\partial x}H(t,x)\right)x$ is continuous in t on $[t_0,\theta]$ and uniformly Lipschitz in x.

Corollary 3.3 assures that the integral equation

$$u(t) = \varphi(t) + \int_{t_0}^{t} S(t-s)\alpha(s,u(s))ds. \tag{3.19}$$

has a unique solution $u \in C([t_0,\theta],X)$.

Moreover, from our assumption, we obtain

$$H\big(t,x(t+\delta)\big)-H\big(t,x(t)\big)=\frac{\partial}{\partial x}H\big(t,x(t)\big)\big(x(t+\delta)-x(t)\big)+o(t,\delta) \tag{3.20}$$

and

$$H\big(t+\delta,x(t+\delta)\big)-H\big(t,x(t+\delta)\big)=\frac{\partial}{\partial t}H\big(t,x(t+\delta)\big)\cdot\delta+o(t,\delta) \tag{3.21}$$

This implies that

$$H\big(t+\delta,x(t+\delta)\big)-H\big(t,x(t)\big)=\frac{\partial}{\partial x}H\big(t,x(t)\big)\big(x(t+\delta)-x(t)\big)+\frac{\partial}{\partial t}H\big(t,x(t+\delta)\big)\cdot\delta+o(t,\delta) \tag{3.22}$$

where $o(t,\cdot)$ denote a little-o notation and $\delta^{-1}\|o(t,\delta)\|\to 0$ as $\delta\to 0$ uniformly on $[0,\theta]$. We will show that $x(t)$ is differentiable on $[0,\theta]$ by regarding the convergence of $v_\delta(t)\equiv\delta^{-1}\cdot\big(x(t+\delta)-x(t)\big)-u(t)$ as $\delta\to 0$ in norm $\|\cdot\|$. It follows from the definition of x ,(3.18), (3.19), 3.20 and (3.21), we get

$$\begin{aligned}
\|v_\delta(t)\| &= \left\|\delta^{-1}\cdot\big(x(t+\delta)-x(t)\big)-u(t)\right\| \\
&= \Big\|\delta^{-1}\cdot\Big(S(t+\delta)x^0+\int_{t_0}^{t+\delta}S(t+\delta-s)H(s,x(s))-S(t)x^0-\int_{t_0}^{t}S(t-s)H(s,x(s))\Big) \\
&\quad -S(t-t_0)H(t_0,x(t_0))-AS(t-t_0)x^0-\int_{t_0}^{t}S(t-s)\frac{\partial}{\partial s}H(s,x(s))ds \\
&\quad -\int_{t_0}^{t}S(t-s)\left(\frac{\partial}{\partial x}H\big(t,x(s)\big)\right)u(s)ds\Big\| \\
&= \Big\|\Big[\delta^{-1}\cdot\big(S(t-t_0+\delta)x^0-S(t-t_0)x^0\big)-AS(t-t_0)x^0\Big] \\
&\quad +\delta^{-1}\cdot\int_{t_0-\delta}^{t_0}S(t-s)H\big(s+\delta,x(s+\delta)\big)-S(t-t_0)H\big(t_0,x(t_0)\big)ds \\
&\quad +\delta^{-1}\cdot\int_{t_0}^{t}S(t-s)\Big(H\big(s+\delta,x(s+\delta)\big)-H\big(s,x(s)\big)\Big)ds \\
&\quad -\int_{t_0}^{t}S(t-s)\frac{\partial}{\partial s}H(s,x(s))ds-\int_{t_0}^{t}S(t-s)\left(\frac{\partial}{\partial x}H\big(t,x(s)\big)\right)u(s)ds\Big\| \\
&= \Big\|\Big[\delta^{-1}\cdot\big(S(t-t_0+\delta)x^0-S(t-t_0)x^0\big)-AS(t-t_0)x^0\Big] \\
&\quad +\delta^{-1}\cdot\int_{t_0-\delta}^{t_0}S(t-s)H\big(s+\delta,x(s+\delta)\big)-S(t)H\big(t_0,x(t_0)\big)ds
\end{aligned}$$

$$+\delta^{-1}\cdot\int_{t_0}^{t} S(t-s)\left(\frac{\partial}{\partial x}H\left(s,x(s)\right)\left(x(s+\delta)-x(s)\right)+\frac{\partial}{\partial s}H\left(s,x(s+\delta)\right)\cdot\delta+o(s,\delta)\right)ds$$

$$-\int_{t_0}^{t} S(t-s)\frac{\partial}{\partial s}H(s,x(s))ds-\int_{t_0}^{t} S(t-s)\left(\frac{\partial}{\partial x}H\left(t,x(s)\right)\right)u(s)ds\Bigg\|$$

$$=\left[\delta^{-1}\cdot\left(S(t-t_0+\delta)x^0-S(t-t_0)x^0\right)-AS(t-t_0)x^0\right]$$

$$+\delta^{-1}\cdot\int_{t_0-\delta}^{t_0} S(t-s)H\left(s+\delta,x(s+\delta)\right)-S(t-t_0)H\left(t_0,x(t_0)\right)ds$$

$$+\delta^{-1}\cdot\int_{t_0}^{t} S(t-s)\left(\frac{\partial}{\partial x}H\left(s,x(s)\right)\left(x(s+\delta)-x(s)\right)\right)ds+\delta^{-1}\cdot\int_{t_0}^{t} S(t-s)o(s,\delta)ds$$

$$-\int_{t_0}^{t} S(t-s)\left(\frac{\partial}{\partial x}H\left(t,x(s)\right)\right)u(s)ds$$

$$=\Bigg\|\left[\delta^{-1}\cdot\left(S(t-t_0+\delta)x^0-S(t-t_0)x^0\right)-AS(t-t_0)x^0\right]$$

$$+\delta^{-1}\cdot\int_{t_0-\delta}^{t_0} S(t-s)H\left(s+\delta,x(s+\delta)\right)-S(t-t_0)H\left(t_0,x(t_0)\right)ds$$

$$+\int_{0}^{t} S(t-s)\left(\frac{\partial}{\partial x}H\left(s,x(s)\right)\left(\delta^{-1}\cdot\left(x(s+\delta)-x(s)\right)-u(s)\right)\right)ds+\delta^{-1}\cdot\int_{0}^{t} S(t-s)o(s,\delta)ds\Bigg\|$$

$$\leq\left\|\delta^{-1}\cdot\left(S(t-t_0+\delta)x^0-S(t-t_0)x^0\right)-AS(t-t_0)x^0\right\|$$

$$+\left\|\delta^{-1}\cdot\int_{t_0-\delta}^{t_0} S(t-s)H\left(s+\delta,x(s+\delta)\right)-S(t-t_0)H\left(t_0,x(t_0)\right)ds\right\|$$

$$+\left\|\delta^{-1}\cdot\int_{t_0}^{t} S(t-s)o(s,\delta)ds\right\|+\left\|\int_{t_0}^{t} S(t-s)\frac{\partial}{\partial x}H\left(s,x(s)\right)v_\delta(s)ds\right\| \tag{3.23}$$

Since $\left\|\delta^{-1}\cdot\left(S(t-t_0+\delta)x^0-S(t-t_0)x^0\right)-AS(t-t_0)x^0\right\|\to 0$, $\left\|\delta^{-1}\cdot\int_{t_0}^{t} S(t-s)o(s,\delta)ds\right\|\to 0$ and $\left\|\delta^{-1}\cdot\int_{t_0-\delta}^{t_0} S(t-s)H\left(s+\delta,x(s+\delta)\right)-S(t)H\left(t_0,x(t_0)\right)ds\right\|\to 0$ as $\delta\to 0$, so

$$\left\|\delta^{-1}\cdot\left(S(t-t_0+\delta)x^0-S(t-t_0)x^0\right)-AS(t-t_0)x^0\right\|+\left\|\delta^{-1}\cdot\int_{t_0}^{t} S(t-s)o(s,\delta)ds\right\|$$

$$+\left\|\delta^{-1}\cdot\int_{-\delta}^{t_0} S(t-s)H\left(s+\delta,x(s+\delta)\right)-S(t-t_0)H\left(t_0,x(t_0)\right)ds\right\|\leq\beta(\delta) \tag{3.24}$$

for some nonnegative function $\beta(\delta)\to 0$ as $\delta\to 0$.

Therefore we have

$$\|v_\delta(t)\| \le \beta(\delta) + M\int_{t_0}^{t}\|v_\delta(s)\|ds \tag{3.25}$$

where $M = \sup_{t_0\le s\le\theta}\left\{\|S(t-s)\|\left\|\frac{\partial}{\partial x}H\left(s,x(s)\right)\right\|\right\}$.

By applying Gronwall Lemma to (3.25) , we have

$$\|v_\delta(t)\| \le \beta(\delta)\exp\left(\theta M\right). \tag{3.26}$$

Thus $\|v_\delta(t)\| = \left\|\delta^{-1}\cdot\left(x(t+\delta)-x(t)\right)-u(t)\right\| \to 0$ as $\delta\to 0$. This implies that $x(t)$ is differentiable on $[t_0,\theta]$ and its derivative is $u(t)$ which is an element in $C([t_0,\theta],X)$. Consequently, $x(t)$ is continuously differentiable on $[t_0,\theta]$.

Next, we will show that $x(t)$ satisfies the semilinear ACP (3.7). Since $x(t)$ is differentiable on $[t_0,\theta]$ and $\{S(t)\}$ is C_0 − group,

$$\begin{aligned}
x'(t) &= \frac{d}{dt}S(t)x^0 + \frac{d}{dt}\int_0^t S(t-s)F(s,x(s),h(s,x(s)))ds \\
&= AS(t)x^0 + \frac{d}{dt}S(t)\int_0^t S(-s)F(s,x(s),h(s,x(s)))ds \\
&= AS(t)x^0 + S(t)S(-t)F(t,x(t),h(t,x(t))) + AS(t)\int_0^t S(-s)F(s,x(s),h(s,x(s)))ds \\
&= A\left(S(t)x^0 + S(t)\int_0^t S(-s)F(s,x(s),h(s,x(s)))ds\right) + F(t,x(t),h(t,x(t))) \\
&= Ax(t) + F(t,x(t),h(t,x(t)))
\end{aligned} \tag{3.27}$$

Hence $x(t)$ satisfies the semilinear ACP (3.7). Finally, we show that $x(t)\in D(A)$ for all $t\in[t_0,\theta]$. It follows from (3.27) and $x^0\in D(A)$ that $Ax(t) = x'(t) - F(t,x(t),h(t,x(t)))$. Using the assumption of F and h, and the continuity of x', we obtain that $Ax(t)$ is continuous . This implies that $x(t)\in D(A)$ for all $t\in[t_0,\theta]$. □

3.3 System with impulses

Next, we will prove the existence of solution of the impulsive system (3.6). Throughout this section, we denote $PC([0,T],X)$ the all type I-piecewise continuous function from $[0,T]$ to X such that discontinuous at $t_i, i = 1,\ldots,n$.

Definition 3.6 A mild solution on $[0,T]$ of the semilinear impulsive ACP (3.6) is a function $x \in PC([0,T],X)$ such that $x(t)$ satisfies the integral equation

$$x(t) = S(t)x^0 + \int_0^t S(t-s)F(s,x(s),h(s,x(s))ds + \sum_{0<t_k\le t} S(t-t_k)I_k(x(t_k),h(t_k,x(t_k))) \quad (3.28)$$

where $\{S(t)\}_{t\ge 0}$ is the C_0- semigroup generated by the operator A.

Definition 3.7 A classical solution of the inhomogeneous ACP (3.6) is a function $x \in PC([0,T],X)$ such that

(i) x is continuously differentiable on $(0,t_1)$, (t_{k-1},t_k) and (t_n,T) for all $k=1,2,...,n$

(ii) $x(t) \in D(A)$ for all $t \in [0,T]$

(iii) x satisfies the semilinear ACP (1.6).

Theorem 3.8 Let A be the infinitesimal generator of a C_0-group $\{S(t)\}$ on a Banach space X If $h:[t_0,\theta]\times X \to X$ is continuously differentiable on $(0,t_1)\times X$, $(t_{k-1},t_k)\times X$ and $(t_n,T)\times X$ for all $k=1,2,...,n$ and $F:[t_0,\theta]\times X\times X \to X$ is continuously differentiable on $(0,t_1)\times X\times X$, $(t_{k-1},t_k)\times X\times X$ and $(t_n,T)\times X\times X$ for all $k=1,2,...,n$, then the mild solution $x \in PC([0,T],X)$ of (3.7) with $x^0, I_k(x(t_k),h(t_k,x(t_k))) \in D(A)$ for all $k=1,2,...,n$, is a classical solution of the semilinear ACP (3.7).

Proof. Let us consider the system,

$$\begin{cases} \dot{x}(t) = Ax(t) + F(t,x(t),h(t,x(t))), \quad 0 \le t \le t_1 & (3.29a) \\ x(0) = x^0 & (3.29b) \end{cases}$$

Theorem 3.2 and Theorem 3.5 imply that System (3.29) has a unique mild solution $x \in C([0,t_1],X)$ such that

$$x(t) = S(t)x^0 + \int_0^t S(t-s)F(s,x(s),h(s,x(s))ds.$$

and the mild solution $x(t)$ is a classical solution on $[0,t_1]$.

So $$x(t_1) = S(t_1)x^0 + \int_0^{t_1} S(t_1-s)F(s,x(s),h(s,x(s))ds$$

Next, we consider the system,

$$\begin{cases} \dot{x}(t) = Ax(t) + F(t, x(t), h(t, x(t))) , & t_1 \le t \le t_2 \quad (3.30a) \\ x(t_1) = S(t_1)x^0 + \int_0^{t_1} S(t_1 - s)F(s, x(s), h(s, x(s))ds + I_1(x(t_1), h(t_1, x(t_1))) & (3.30b) \end{cases}$$

By applying Theorem 3.2 and Theorem 3.5, we have

$$\begin{aligned} x(t) &= S(t-t_1)\left[S(t_1)x^0 + \int_0^{t_1} S(t_1 - s)F(s, x(s), h(s, x(s))ds + I_1(x(t_1), h(t_1, x(t_1))) \right] \\ &\quad + \int_{t_1}^{t} S(t-s)F(s, x(s), h(s, x(s))ds \\ &= S(t-t_1)S(t_1)x^0 + S(t-t_1)\int_0^{t_1} S(t_1 - s)F(s, x(s), h(s, x(s))ds + S(t-t_1)I_1(x(t_1), h(t_1, x(t_1))) \\ &\quad + \int_{t_1}^{t} S(t-s)F(s, x(s), h(s, x(s))ds \\ &= S(t)x^0 + \int_0^{t_1} S(t-s)F(s, x(s), h(s, x(s))ds + S(t-t_1)I_1(x(t_1), h(t_1, x(t_1))) \\ &\quad + \int_{t_1}^{t} S(t-s)F(s, x(s), h(s, x(s))ds \\ &= S(t)x^0 + \int_0^{t} S(t-s)F(s, x(s), h(s, x(s))ds + S(t-t_1)I_1(x(t_1), h(t_1, x(t_1))) \end{aligned}$$

That is,

$$x(t) = S(t)x^0 + \int_0^{t} S(t-s)F(s, x(s), h(s, x(s))ds + S(t-t_1)I_1(x(t_1), h(t_1, x(t_1)))$$

is the classical solution on $[t_1, t_2]$.

We continues these processes, we have

$$x(t) = S(t)x^0 + \int_0^{t} S(t-s)F(s, x(s), h(s, x(s))ds + \sum_{0 < t_k \le t} S(t-t_k)I_k(x(t_k), h(t_k, x(t_k)))$$

is the classical solution on $[0, T]$. □

CHAPTER IV

CONCLUSION

In this research, we considered the impulsive differential with singular perturbation, when A is the infinitesimal generator of a C_0- semigroup $\{S(t)\}_{t\geq 0}$. We defined an approximate solution of this problem. Moreover, we investigated the regularity of the approximate solution.

Finally yet importantly, we should be interested in developing this method and use weakly assumptions to prove the existence and uniqueness of PC-mild solution a little further. Moreover, we should be interested in studying the others the solution behaviors for example; the stable property. Even though it seems likely that efforts in this direction can be successful, there no guarantee for that. Therefore, we can only hope for the best, but have to expect the worst.

BIBLIOGRAPHY

[1] A.Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations,Springer-Verlag,New York Inc.,1983.

[2] C. Lizama. Regularized solutions for abstract Volterra equations. J. Math. Anal. Appl. 243 (2000), 278-292.

[3] Danhua He and Liguang Xu, Integrodifferential Inequality for Stability of Singularly Perturbed Impulsive Delay Integrodifferential Equations,Journal of Inequalities and Applications,Volume 2009, Article ID 369185, 11 pages,doi:10.1155/2009/369185.

[4] D.D. Bainov, a, A.B. Dishlievb, I.M. Stamovac,Lipschitz quasistability of impulsive differential-difference equations with variable impulsive perturbations,Journal of Computational and Applied Mathematics,Volume 70, Issue 2, 28 June 1996, Pages 267–277, doi:10.1016/0377-0427(95)00207-3

[5] D. D. Bainov, V. Lakshmikantham, and P. S. Simeonov, Theory of Impulsive Differential Equations, vol.6 of Series in Modern Applied Mathematics, World Scientific, Singapore, (1989).

[6] D. Smith, Singular perturbation theory, Cambridge University Press, Cambridge, 1985.

[7] D. S. Naidu and A. J. Calise, Singular perturbations and time scales in guidance and control of aerospace, Journal of Guidance, Control, and Dynamics 24 (2001), no. 6, 1057–1078.

[8] F.A. Bornemann. Homogenization in Time of Singularly Perturbed Mechanical Systems. Springer, 1998.

[9] G.K. Kulev,Uniform asymptotic stability in impulsive perturbed systems of differential equations,Journal of Computational and Applied Mathematics,Volume 41, Issues 1–2, 20 August 1992, Pages 49–55,doi:10.1016/0377-0427(92)90237-R

[10] G. W. Desch and R. Grimmer, Propagation of singularities for integrodifferential equations, Journal of Differential Equations, vol. 65, no. 3, pp. 411–426, 1986.

[11] James H. Liu, Singular perturbations of integrodifferential equations in banach space, Proceedings of the Americanm Athematical Society, Vol 122, No 3, November 1994.

[12] J. Goldstein, Semigroups of linear operators and applications, Oxford Univ. Press, New York,1985

[13] Jin Liang, James Liu, and Ti-Jun Xiao, Convergence for Hyperbolic Singular Perturbation Of Integrodifferential Equations,Journal of Inequalities and Applications,Vol. 2007, Article ID 80935, 11 pages doi:10.1155/2007/80935.

[14] J. PrÄuss.Positivity and regularity of hyperbolic Volterra equations in Banach spaces. Math. Ann. 279 (1987), 317-344.

[15] K.J. Engel. On singular perturbations on second order Cauchy problems. Pacific J. Math. 152 (1) (1992), 79-91.

[16] N. Fenichel. Geometric singular perturbation theory for ordinary differential equations. J. Differential Equations31:53-98, 1979.

[17] P.S.Theocaris, G. Tsamasphyros, A numerical solution of singular integro-differential equations with vaeiable coefficients. Appl.Math.Comput. 159 (1984) 47-59.

[18] Sh. Sadigh Behzadia, A.Yildirim, A Method to Estimate the Solution of a Weakly Singular Non-linear Integro-differential Equations by Applying the Homotopy Methods, Int. J. Industrial Mathematics Vol. 4, No. 1 (2012) 41-51.

[19] T. KATO. Perturbation Theory for Linear Operators, Springer-Verlag, New York/ Berlin, 1980.

[20] Wei Zhua, b, Daoyi Xub,Chunde Yanga,Exponential stability of singularly perturbed impulsive delay differential equations ,Journal of Mathematical Analysis and Applications,Volume 328, Issue 2, 15 April 2007, Pages 1161-1172, doi:10.1016/j.jmaa.2006.05.082

[21] W.Witayakiatilerd & A.Chonwerayuth , Fractional integro-differential equations of mixed type with solution operator and optimal controls , Journal of Mathematics Research(JMR), August 2011.

[22] W.Witayakiatilerd & A.Chonwerayuth ,Regularity of piecewise continuous almost periodic solutions for nonlinear impulsive systems, Pushpa Publishing House ,November 2009.

[23] W.Witayakiattilerd, Optimal Regulation of Impulsive Fractional Differential Equation with Delay and Application to Nonlinear Fractional Heat Equation , Journal of Mathematics Research(JMR), Vol 5, No 2 (2013)

[24] X.B Lin, Heteroclinic Bifurcation and Singularly perturbed Boundary Value Problems, Journal of differential equations 84,319-382 (1990).

[25] X. B. LIN, Shadowing lemma and singularly perturbed boundary value problems, SIAM J. Appl. Math. 49 (1989), 2654.

VITAE

Wichai Witayakiattilerd, Ph.D.

PERSONAL INFORMATION

Date of birth: Febuary 5, 1981

Place of birth: Bangkok, Thailand

Professional address: Department of Mathematics

Faculty of Science

King Mongkut's Institute of Technology Ladkrabang

Chalongkrung Rd.Ladkrabang

Bangkok Thailand, Postal code : 10520

Phone number +66(0) 2329 8000 - 2329 8099

E-mail address: kwwichai@kmitl.ac.th, waichi.mum@gmail.com

PRESENT POSITIONS Lecturer, Department of Mathematics

King Mongkut's Institute of Technology Ladkrabang

EDUCATION

Undergraduate B.Sc. (Mathematics)

Silapakorn University, Nakhorn Pathom, Thailand, 2000-2003

Graduate: M.Sc. (Mathematics)

Chulalongkorn University, Bangkok, Thailand, 2004-2007

Doctorate: Ph.D.(Mathematics)

Chulalongkorn University, Bangkok, Thailand, 2008-2011

FIELD OF RESEARCH

- Mathematical control theory
- Fractional calculus

- Applied mathematics
- Financial mathematics

ABSTRACTS/PRESENTATIONS

1. Sirikul Siriteerakul& W.Witayakiatilerd "A Study on Heat Diffusion of Tile" The 5th Thailand-Japan International AcademicConference (5th TJIA 2012), on Oct 20, 2012, The University of Tokyo, Tokyo, Japan. (Oral)
2. W.Witayakiatilerd., "Optimal Regulation of Impulsive FractionalDifferential Equations with Delays" International conference on global optimization and its applications 2012, on Dec 13-15, 2012, Faculrty of Mathematics and Natural Sciences, University of Padjadjaran, Bundung, Indonesia. (Oral)

ORIGINAL ARTICLES

Local Publications

1. W.Witayakiatilerd "Fractional Calculus : Fractional Derivative and Application"Scientific journal Lad Krabang, Vol. 20 (2) pp.81-90,2012.

International Publications

1. W.Witayakiatilerd & A.Chonwerayuth ,Regularity of piecewise continuous almost periodic solutions for nonlinear impulsive systems, Pushpa Publishing House ,November 2009
2. W.Witayakiatilerd & A.Chonwerayuth , Fractional integro-differential equations of mixed type with solution operator and optimal controls , Journal of Mathematics Research(JMR), August 2011
3. W.Witayakiatilerd, Optimal Regulation of Impulsive Fractional Differential Equation with Delay and Application to Nonlinear Fractional Heat Equation , Journal of Mathematics Research(JMR), Vol 5, No 2 (2013)