EFFICIENCY COMPARISON OF HOMOGENEITY OF VARIANCE TESTS BASED ON MODIFIED CENTRAL TENDENCY FOR THREE POPULATIONS

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Abstract

The objective of this research is to compare the efficiency of Levene's (L) test, O'Brien (OB) test, Jackknife (J) test, Box-Andersen (BA) test, and four modified tests consists of Modified Levene's or Brown-Forsythe (BF) test, Modified O'Brien (MOB) test, Modified Jackknife (MJ) test, and Modified Box-Andersen (MBA) test. For normal, Laplace, uniform, and gamma distributions are considered both equal and unequal sample sizes contain (10, 10, 10), (30, 30, 30), (60, 60, 60), (5, 10, 15), (20, 30, 40), and (45, 60, 75). The data in this research are simulated through the Monte Carlo technique with 1,000 replications for each situation. The efficiency of test statistics of this research is studied in cases of the probability of type I error according to Bradley criterion and power of a test. The significance levels are 0.01 and 0.05. The results are found that MJ test shows the highest power of a test in almost all cases with normal and uniform distributions. For Laplace distribution, L test shows the highest power of a test in almost all cases whereas BA test shows the highest power of a test in case of small sample size. For gamma distribution, MJ and BA tests show the highest power of a test in almost all cases with 0.01 significance level and then J, BA, and MJ tests show the highest power of a test in almost all cases with 0.05 significance level.

Keywords : Homogeneity of variance, Probability of type I error, Power of a test

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Chapter 1
Introduction

1.1 Statement and Significance of the Problem

Statistical inference is the process of drawing conclusions about population from sampling data. Estimation and hypothesis testing are a part of statistical inference which is analyzed the estimator to describe the parameter. Hypothesis testing is used to determine whether that is enough evidence in a sample of data to infer that a certain condition is true for the entire population.

The analysis of variance (ANOVA) is one of the most important and useful techniques for comparing different groups or treatments with respect to their means where there are more than two groups. A set of assumptions include normal distribution, homogeneity of variance and independence of observations. These assumptions are checked before analysis of data. It is now well established that the violation of the assumption of homogeneity of variance can have severe effects on the inference of the population mean, especially in the case of unequal sample sizes and non-normally distributed. These are explained in research papers such as (Mendes, 2004). In fact, the conventional ANOVA F provides generally poor control over both type I and type II error rates under a wide variety of variance heterogeneity conditions (Bishop, 1976). Misleading assumption would impair the utility of the test, leading to wrong and invalid conclusions (Vorapongsathon, et al., 2004).

Therefore, the problem of homogeneity of variances has to be settled before performing an ANOVA. It is well known that the generally statistical packages use Levene's test to check the homogeneity of variance for conducting tests of the equality of means in ANOVA (Oladejo and Adetunde, 2009; Mozahreh et al., 2009; Zeng et al., 2010). Unfortunately, Levene's test for comparing variance is extremely sensitive to the assumption of normality. Specifically, the populations are unequal variance (Welch, 1951; Brown and Forsythe, 1974; Wilcox, 1988; Alexander and Govern, 1994). However, there are many others test statistics for checking homogeneity of variance under different situations (Box and Andersen, 1955; Miller, 1968; O'Brien, 1981) and the modification tests (Layard, 1973; Brown and Forsythe, 1974; Carroll and Schneider, 1985; Tomarken and Serlin, 1986; Gastwirth et al., 2009) that may have been a better choice than Levene's test.

Levene's test is the one-way analysis of variance F-test on $|X_{ij} - \overline{X}_i|$, the absolute deviations of the X_{ij} from their group mean $\overline{X}_i$ Various modifications of Levene's test have been proposed and investigated (Levene, 1960).

Carroll and Schneider (1985) had described that Brown and Forsythe (1974) had considered the median and the 10% trimmed mean, which are more robust estimates of central tendency, as alternatives to the mean in computing the absolute deviations. They found that the alternative formulations of Levene's tests are robust against non-normality.

Layard (1973) had compared the homogeneity of variance tests consists of Bartlett's test, Box test, and Jackknife test in case of small sample sizes. The results found that Bartlett's test showed the best power under normal distribution. Box test and Jackknife test showed the best power as well as Bartlett's test under uniform and double exponential distributions.

Boos and Brownie (1989) had studied the bootstrap versions of Bartlett's test and Layard's (1973) k – sample generalization of Miller's (1968) two-sample jackknife test. Their simulation results show that bootstrap versions of Bartlett's test and jackknife test perform better than original versions.

Lee et al. (2010) had compared Levene's test with modified Levene's test, Z-variance test, Overall-Woodward modified Z-variance test, O'Brien's test, Samiuddin Cobe Root test and F-Max test. The results showed that Levene's test was neither the best nor worst in terms of robustness and power of a test, modified Levene's test showed very good robustness when compared with the other tests but lower power of a test than other tests.

Gorbunova and Lemeshko (2012) had compared F-test with Bartlett's test, Neyman-Pearson test, Hartley test, Z-variance test, Overall-Woodward modified Z-variance test, O'Brien's test, Levene's test and modified Levene's test in term of power for sample were from any distribution. When the data distribution was mesokurtic or platykurtic, the best choice was O'Brien's test or Z-variance test, while the data distribution was leptokurtic or skewed, the modified Levene's test was chosen.

Gogoi, P. and B. (2015) had compared probability of type I error and power of Levene's test, Bartlett's test, Modified Bartlett's test, Box-Andersen test, Jackknife test, and Lepage test in case of small sample sizes. The results found that jackknife test showed the best test as well as Bartlett's test under normal and logistic distributions.

Hatchavanich (2016) had compared probability of type I error and power of parametric tests with nonparametric tests. It was found that Levene's test, modified Bartlett's test and O'Brien's test outperformed the other in term of robustness. Considering the type I error and the power of a test, the finding showed that Gini's test and Bartlett1's test were the best when the data was normally distributed. Moreover, for chi-square and a mixture of Gaussian distribution, modified Bartlett2's test and Levene's test were the best. For uniform and exponential distributions, Jackknife test and Levene's test were the best.

For that reason, the aim of this study is to compare the probability of type I error and power of a test of four original and four modified test statistics. These tests are simulated and analyzed by R program version 3.3.2. The four original homogeneity of variance tests are (1) Levene's test (2) O'Brien's test (3) Jackknife test and (4) Box-Andersen test. These tests are compared with four modified homogeneity of variance tests based on modified central tendency by using the median replace the mean. A straightforward modification of this idea is extracted from the modified Levene test by Brown and Forsythe (1974). "Goodness" of each test is determined by examining the probability of type I error and power of a test for each test. Should general statistic package such as SPSS and other statistical packages consider other tests along with Levene's test?

1.2 Objectives

1.2.1 To compare the probability of type I error (robustness) of four original homogeneity of variance test with four modified homogeneity of variance tests based on modified central tendency by using the median replace the mean for four distribution-types (normal, leptokurtic, platykurtic, skewed).

1.2.2 To compare power of a test of four original homogeneity of variance test with four modified homogeneity of variance tests based on modified central tendency by using the median replace the mean for four distribution-types (normal, leptokurtic, platykurtic, skewed).

1.3 Scope of the Study

1.3.1 Determining the population distributions

Let the population distribution-types are normal distribution and three non-normal distributions consists of Laplace distribution (leptokurtic distribution), uniform distribution (platykurtic distribution), gamma distribution (skewed distribution).

1.3.1.1 Normal distribution

The random variable X is a normal distribution, if its probability density function is given by

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2},-\infty < x < \infty,-\infty < \mu < \infty,\sigma > 0$$

1.3.1.2 Laplace distribution

The random variable X is a Laplace distribution, if its probability density function is given by

$$f(x;\mu,\lambda)=\frac{1}{2\lambda}e^{\left(-\frac{|x-\mu|}{\lambda}\right)},-\infty < x < \infty,-\infty < \mu < \infty,\lambda > 0$$

1.3.1.3 Uniform distribution

The random variable X is a uniform distribution, if its probability density function is given by

$$f(x)=\frac{1}{b-a},\text{ for } x \in [a,b],-\infty < a < b < \infty$$

1.3.1.4 Gamma distribution

The random variable X is a gamma distribution, if its probability density function is given by

$$f(x,\alpha,\beta)=\begin{cases}\dfrac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-\frac{x}{\beta}} & ,0<x<\infty,\alpha>0,\beta>0\\ 0 & \text{, otherwise}\end{cases}$$

1.3.2 Determining the populations and sample sizes

Let three population group cases contain equal sample sizes and unequal sample sizes. The configurations of sample sizes are determined under total sample sizes (30, 90, and 180) for equal and unequal sample sizes. For configurations of equal sample sizes, there are three configurations consists of small sample size (n=10), moderate sample size (n=30), and large sample size (n=60) (Reiczigel, 2003). For configurations of unequal sample size, three configurations are considered under conditions of sample size consists of small sample size (5, 10, 15), medium sample size (20, 30, 40), and large sample size (45, 60, 75). The configurations of sample size are given in **Table 1.1** as follows:

Table 1.1 The population $k = 3$ groups and the ratio of sample size (n_1, n_2, n_3)

Configurations of sample size	The ratios of sample size	
	equal n_i	unequal n_i
Small	10,10,10	5,10,15
Medium	30,30,30	20,30,40
Large	60,60,60	45,60,75

1.3.3 Determining the population variance ratios

Let the different population variance ratios by non-centrality parameter (ϕ), this ϕ is a measure of the different population variance (Game et al., 1972).

The formula is defined by
$$\phi = \frac{\sqrt{\sum_{i=1}^{k}(\sigma_i^2 - \bar{\sigma}^2)^2 / k}}{\sigma_1^2}, \tag{1.1}$$

where σ_1^2 is the population variance with the lowest,

σ_i^2 is the population variance with i^{th} group; $i = 1,2,\dots,k$,

$\bar{\sigma}^2$ is the mean of population variance with k group.

The different population variance ratios are given in **Table 1.2** as follows:

Table 1.2 The different population variance ratios by non-centrality parameter ϕ

The different population variance levels	The population variance ratios	Non-centrality parameter (ϕ)
slightly $(0 < \phi < 1.5)$	$1:2:4$	1.25
moderately $(1.5 \le \phi < 3.0)$	$1:4:8$	2.87
highly $(\phi \ge 3.0)$	$1:8:16$	6.13

1.3.4 Determining significance level

Let the significance levels (α) at 0.05 and 0.01 as the significance level is chosen typically set to 0.05 or much lower. The significance level is chosen on the field of study (Craparo, 2007).

1.3.5 Determining criterion for determined robustness

Let the criterion for determined robustness based on the Bradley limit (Bradley, 1978) as follows:

1.3.5.1 The significance level at 0.05, the probability of type I error (α) has to fall between [0.025, 0.075].

1.3.5.2 The significance level at 0.01, the probability of type I error (α) has to fall between [0.005, 0.015].

1.3.6 Determining the number of replications

Let replications are 1,000 times, since replications are needed to obtain accurate results and the minimum number of simulations assessment is generally considered to be 1,000 times. Therefore, researchers are able to run in excess of 1,000 replications in their studies (Godfrey, 1978; Olejnik et al., 2000; Cahoy, 2010).

1.3.7 Program for data analysis

R program version 3.3.2 is used for simulation and analysis of data as mentioned above.

1.4 Benefits of the study

1.4.1 To guide the selection of the appropriate homogeneity of variance test for different conditions, such as population distribution-types, difference of population variance and equal or unequal sample sizes.

1.4.2 To guide the accurate checking of the homogeneity of variance assumption before performing tests of the equality of mean in ANOVA.

1.5 Definition

1.5.1 Robustness is the probability of type I error (H_0 is rejected when H_0 is true) based on criterion of Bradley at 0.05 and 0.01 significance levels.

1.5.2 Power or power of a test is the probability of H_0 is rejected when H_0 is false at 0.05 and 0.01 significance levels.

1.5.3 Non-centrality Parameter (ϕ) is a measure of the different population variance (Game et al., 1972).

1.6 Process of the Study

1.6.1 R program version 3.3.2 is written to carry out the analyses. The data are generated from normal distribution, Laplace distribution, uniform distribution, and gamma distribution with equal variance. For all tests in this study, there are three groups are arranged as configurations of sample size. The details are given in **Table 1.1**.

1.6.2 Three group experimental data are simulated one thousand times and these experimental data are analyzed for each sample size. All homogeneity of variance tests are computed for each experimental data. The experimental data are counted the number of times the null hypothesis is rejected when it is true at the $\alpha = 0.05$ and 0.01 levels for 1,000 experimental data with R program version 3.3.2.

1.6.3 The probability of type I error is computed with the number of H_0 is rejected when H_0 is true divide by the number of replications 1,000 times for each test statistics.

1.6.4 The probability of type I error is compared under the criterion of Bradley (Bradley, 1978). At significance level ($\alpha = 0.05$), the test statistic is called robustness when the probability of type I error has to fall between [0.025, 0.075] and at significance level ($\alpha = 0.01$), the test statistic is called robustness when the probability of type I error has to fall between [0.005, 0.015] in Bradley limit. If any the probability of type I error is over the limit, it shows that the test statistic isn't called robustness.

1.6.5 The analyses are repeated for the same both sample sizes and significance levels. The data are generated from all distribution-types with unequal variances for only the test statistic is called robustness.

1.6.6 Power of a test (also power) is computed with the number of H_0 is rejected when H_0 is false divide by the number of replications 1,000 times for each test statistics. The maximum power of a test is 1.0 and the minimum is zero. The test statistic is called the best test when it shows the highest power or power of a test.

Chapter 2

Theory and Literature Reviews

In this study, four tests are Levene's test, O'Brien's test, Jackknife test and Box-Andersen test. These tests are compared robustness and power of four modified homogeneity of variance tests based on modified central tendency by using the median replace the mean for four distribution-types contain normal distribution, Laplace distribution (leptokurtic distribution), uniform distribution (platykurtic distribution) and gamma distribution (skewed distribution). These tests are straightforward modification from idea in (Brown and Forsythe, 1974). For the details of this chapter consist of description of the test statistics, Calculated examples of the test statistics, Description of distribution-types, determining different population variance ratios, criterion of efficiency comparison for testing, Measures of skewness and kurtosis, and literature reviews.

2.1 Description of the test statistics

In this section, the homogeneity of variance tests of k populations given sample $\{X_{ij} : i = 1,2,...,k; j = 1,2,...,n_i\}$ from the i^{th} population with mean μ_i, and variance σ_i^2. The null hypothesis is $H_0 : \sigma_1^2 = \sigma_2^2 = ... = \sigma_k^2$ and the alternative hypothesis is $H_1 : \sigma_i^2 \neq \sigma_j^2$ for at least one $i \neq j$,

2.1.1 Levene's test

2.1.1.1 Original Levene's test

Levene's test (Levene, 1960) was used to test if k sample have equal variance. Levene had proposed an alternative method to Bartlett's test (Klotz and Johnson, 1993) for testing the assumption of homogeneity of variance for ANOVA designs. Bartlett's test works well for data that are normally or approximate normally distributed but it doesn't fare well for data that follow a leptokurtic or skewed distribution (Overall and Woodward, 1974). According to Levene, he had proposed that Levene's test is less sensitive to departures from normally. This says that Levene's test has fewer type I errors than Bartlett's test for non-normality distributions (Gastwirth et al., 2009). Brown and Forsythe (1974) had proposed that using the mean provided the best power for symmetric and moderate-tailed distributions.

Levene's test is the one-way analysis of variance F-test based on $Z_{ij} = \left|X_{ij} - \overline{X}_i\right|$, where $\overline{X}_i$ is the mean of the i^{th} group and it is denoted by L.

The test statistic is
$$L = \frac{(N-k)\sum_{i=1}^{k} n_i(\overline{Z}_i - \overline{Z})^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(Z_{ij} - \overline{Z}_i)^2}, \tag{2.1}$$

where $\overline{X}_i = \sum_{j=1}^{n_i} X_{ij} \Big/ n_i$ is the mean of the i^{th} group,

$N = \sum_{i=1}^{k} n_i$ is total sample size, and n_i is sample size of the i^{th} group,

when $\overline{Z}_i = \sum_{j=1}^{n_i} Z_{ij} \Big/ n_i$,

and $\overline{Z} = \sum_{i=1}^{k}\sum_{j=1}^{n_i} Z_{ij} \Big/ N$.

The null hypothesis H_0 is rejected when $L > F_{(\alpha,k-1,N-k)}$ where $F_{(\alpha,k-1,N-k)}$ is the upper critical value of the F-distribution with $k-1$ and $N-k$ degree of freedom at a significance level of α.

2.1.1.2 Modified Levene's test based on median

Modified Levene's test based on median is a statistical test for the equality of group variance based on performing an ANOVA on a transformation of the response variable as Levene's test. Brown and Forsythe (1974) had modified Levene's test and they had proposed that using the median performed skewed distribution. This the modified studies by Brown and Forsythe was referenced in (Carroll and Schneider, 1985). The difference is that the median is used instead of the mean in computing Z_{Mij}. This modification of Levene's test is uaually called Brown-Forsythe's test.

This test is the one-way analysis of variance F-test based on $Z_{Mij} = \left|X_{ij} - M_i\right|$, where M_i is the median of i^{th} group, which is used instead of the mean. The median version of Levene's test is denoted by BF.

The modified test statistic is
$$BF = \frac{(N-k)\sum_{i=1}^{k} n_i(\overline{Z}_{Mi} - \overline{Z}_M)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(Z_{Mij} - \overline{Z}_{Mi})^2}, \tag{2.2}$$

where $\overline{Z}_{Mi} = \sum_{j=1}^{n_i} Z_{Mij} \Big/ n_i$,

and $\overline{Z}_{M} = \sum_{i=1}^{k}\sum_{j=1}^{n_i} Z_{Mij} \Big/ N$.

The null hypothesis H_0 is rejected when $BF > F_{(\alpha,k-1,N-k)}$ where $F_{(\alpha,k-1,N-k)}$ is the upper critical value of the F-distribution with $k-1$ and $N-k$ degree of freedom at a significance level of α.

2.1.2 O'Brien's test

2.1.2.1 Original O'Brien's test

O'Brien's test, O'Brien (1978; 1981) had claimed that this test is a general method and fairly well for behavioral science data. O'Brien (1981) had stated that the test is robust to data that departs from normality. It was also easy to program into statistical packages like SPSS, that was competitive with other tests in terms of power and it can be easily used in different ANOVA designs with equal or unequal sample sizes. O'Brien (1981) stated that not much research had been done on statistic.

The computational operations for this test are straightforward modification. Every raw score, X_{ij} in the study is transformed using formula is defined by

$$V_{ij} = \frac{(n_i - 1.5)n_i(X_{ij} - \overline{X}_i)^2 - 0.5S_i^2(n_i - 1)}{(n_i - 1)(n_i - 2)}, \tag{2.3}$$

where $S_i^2 = \sum_{j=1}^{n_i}(X_{ij} - \overline{X}_{i.})^2 \big/ (n_i - 1)$ is the variance of the i^{th} group. (2.4)

This test is the one-way analysis of variance F-test based on the V_{ij}, where $\overline{X}_i$ is the mean of the i^{th} group and it is denoted by OB.

The test statistic is

$$OB = \frac{(N-k)\sum_{i=1}^{k} n_i(\overline{V}_i - \overline{V})^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(V_{ij} - \overline{V}_i)^2}, \tag{2.5}$$

where $\overline{V}_i = \sum_{j=1}^{n_i} V_{ij} \Big/ n_i$,

and $\overline{V} = \sum_{i=1}^{k}\sum_{j=1}^{n_i} V_{ij} \Big/ N$.

After this transformation the mean of $V-$values will be equal to the variance for original values, that is $\overline{V}_i = \left(\sum_{j=1}^{n_i} V_{ij}\right) \Big/ n_i = S_i^2$.

The null hypothesis H_0 is rejected when $OB > F_{(\alpha,k-1,N-k)}$ where $F_{(\alpha,k-1,N-k)}$ is the upper critical value of the F-distribution with $k-1$ and $N-k$ degree of freedom at a significance level of α.

2.1.2.2 Modified O'Brien's test based on median

Modified O'Brien's test is nearly identical to original O'Brien's test. The median is used instead of the mean. The differences are that the median is used instead of the mean in computing V_{Mij}.

The computational operations for this test are straightforward modification. Every raw score, X_{ij} in the study is transformed using formula is defined by

$$V_{Mij} = \frac{(n_i - 1.5)n_i(X_{ij} - M_i)^2 - 0.5S_i^2(n_i - 1)}{(n_i - 1)(n_i - 2)}, \tag{2.6}$$

where S_i^2 is defined in equation (2.4)

This test is the one-way analysis of variance F-test based on the V_{Mij}, where M_i is the median of the i^{th} group and it is denoted by MOB.

The modified test statistic is

$$MOB = \frac{(N-k)\sum_{i=1}^{k} n_i(\bar{V}_{Mi} - \bar{V}_M)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(V_{Mij} - \bar{V}_{Mi})^2}, \tag{2.7}$$

where $\bar{V}_{Mi} = \sum_{j=1}^{n_i} V_{Mij} \Big/ n_i$,

and $\bar{V}_M = \sum_{i=1}^{k}\sum_{j=1}^{n_i} V_{Mij} \Big/ N$.

The null hypothesis H_0 is rejected when $MOB > F_{(\alpha,k-1,N-k)}$ where $F_{(\alpha,k-1,N-k)}$ is the upper critical value of the F-distribution with $k-1$ and $N-k$ degree of freedom at a significance level of α.

2.1.3 Jackknife test

2.1.3.1 Original Jackknife test

Layard (1973) recommended using the alternative Jackknife pseudovalues that given by $U_{ij} = n_i \log S_i^2 - (n_i - 1)S_{ij}^2$. (2.8)

where $\log$ is natural logarithms or $\log_e$,

S_i^2 is defined in equation (2.4),

and $$S_{ij}^2 = \frac{\left((n_i - 1)S_i^2 - n_i\left(X_{ij} - \bar{X}_i\right)^2 / (n_i - 1)\right)}{(n_i - 2)}.$$

The resulting test statistic, which is the one way analysis of variance F-test based on the U_{ij}, will be called J.

The test statistic is
$$J = \frac{(N-k)\sum_{i=1}^{k} n_i(\overline{U}_i - \overline{U})^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(U_{ij} - \overline{U}_i)^2}, \tag{2.9}$$

where $\overline{U}_i = \sum_{j=1}^{n_i} U_{ij} \Big/ n_i$,

and $\overline{U} = \sum_{i=1}^{k}\sum_{j=1}^{n_i} U_{ij} \Big/ N$.

The null hypothesis H_0 is rejected when $J > F_{(\alpha,k-1,N-k)}$ where $F_{(\alpha,k-1,N-k)}$ is the upper critical value of the F-distribution with $k-1$ and $N-k$ degree of freedom at a significance level of α.

2.1.3.2 Modified Jackknife test based on median

Modified Jackknife test based on median is nearly identical to original Jackknife test. The median is used instead of the mean. The differences are that the median is used instead of the mean in computing U_{Mij}. It is the one-way analysis of variance F-test based on the U_{Mij}. This U_{Mij} is represented the alternative pseudovalues when M_i is the median of the i^{th} group for $j = 1,2,\ldots,n_i$, $i = 1,2,..,k$. The alternative pseudovalues is defined by

$$U_{Mij} = n_i \log S_{Mi}^2 - (n_i - 1)S_{Mij}^2. \tag{2.10}$$

where $\log$ is natural logarithms or $\log_e$,

$$S_{Mi}^2 = \sum_{j=1}^{n_i}(X_{ij} - M_{i.})^2 \big/ (n_i - 1) \tag{2.11}$$

and
$$S_{Mij}^2 = \frac{\left((n_i - 1)S_{Mi}^2 - n_i\left(X_{ij} - \overline{X}_i\right)^2 / (n_i - 1)\right)}{(n_i - 2)}.$$

The resulting test statistic, which is the one way analysis of variance F-test based on the U_{Mij}, will be called MJ.

The modified test statistic is
$$MJ = \frac{(N-k)\sum_{i=1}^{k} n_i(\overline{U}_{Mi} - \overline{U}_M)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(U_{Mij} - \overline{U}_{Mi})^2}, \tag{2.12}$$

where $\overline{U}_{Mi} = \sum_{j=1}^{n_i} U_{Mij} \Big/ n_i$, and $$\overline{U}_M = \sum_{i=1}^{k}\sum_{j=1}^{n_i} U_{Mij} \Big/ N.$$

The null hypothesis H_0 is rejected when $MJ > F_{(\alpha,k-1,N-k)}$ where $F_{(\alpha,k-1,N-k)}$ is the upper critical value of the F-distribution with $k-1$ and $N-k$ degree of freedom at a significance level of α.

2.1.4 Box-Andersen test

2.1.4.1 Original Box-Andersen test

Box-andersen test is another variation of Bartlett's test, which is recommened by Miller (1968). The resulting test statistic will be called BA.

The test statistic is $$BA = \frac{2}{\left(\hat{\beta}_2 - 1\right)}\left((N-k)\ln S_p^2 - \sum_{i=1}^{k}(n_i - 1)\ln S_i^2\right) \tag{2.13}$$

where $\ln$ is natural logarithms or $\log_e$,

$$S_p^2 = \frac{\sum_{i=1}^{k}(n_i - 1)S_i^2}{N-k}, \tag{2.14}$$

$$\hat{\beta}_2 = \frac{N\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(X_{ij} - \overline{X}_i\right)^4}{\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(X_{ij} - \overline{X}_i\right)^2\right)^2} \tag{2.15}$$

and S_i^2 is defined in equation (2.4).

The null hypothesis H_0 is rejected when $BA > \chi^2_{1-\alpha,k-1}$ where $\chi^2_{1-\alpha,k-1}$ is the upper critical value of the χ^2- distribution with $k-1$ degree of freedom at a significance level of α.

2.1.4.2 Modified Box-Andersen test based on median

Modified Box-Andersen test based on median is nearly identical to original Box-Andersen test. The median is used instead of the mean when M_i is the median of the i^{th} group for $j = 1,2,\ldots,n_i$, $i = 1,2,..,k$. The resulting test statistic will be called MBA.

The modified test statistic is $$MBA = \frac{2}{\left(\hat{\beta}_2 - 1\right)}\left((N-k)\ln S_{Mp}^2 - \sum_{i=1}^{k}(n_i - 1)\ln S_{Mi}^2\right) \tag{2.16}$$

where $\ln$ is natural logarithms or $\log_e$,

$$S_{Mp}^2 = \frac{\sum_{i=1}^{k}(n_i - 1)S_{Mi}^2}{N-k}, \tag{2.17}$$

and S_i^2 and $\hat{\beta}_2$ are defined in equation (2.4) and (2.15), respectively.

The null hypothesis H_0 is rejected when $MBA > \chi^2_{1-\alpha,k-1}$ where $\chi^2_{1-\alpha,k-1}$ is the upper critical value of the χ^2- distribution with $k-1$ degree of freedom at a significance level of α.

2.2 Calculated Examples of the test statistics

Let the significance level is 0.05 and random the data set as follows:

X_1	8	3	8	5	8	11	7	8	9	9
X_2	8	6	2	7	6	7	15	11	9	11
X_3	4	12	10	11	10	11	6	12	3	9

2.2.1 Calculated examples of Levene's test

2.2.1.1 Calculated example of original Levene's test (L).

The hypothesis: $H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2$ versus $H_1: \sigma_i^2 \neq \sigma_j^2$ for at least one $i \neq j$.

The test statistic is

$$L = \frac{(N-k)\sum_{i=1}^{k} n_i(\bar{Z}_i - \bar{Z})^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(Z_{ij} - \bar{Z}_i)^2} = \frac{(30-3)\{10(0.53)+10(0.12)+10(0.15)\}}{(3-1)\{1.35+9.24+1.35+\ldots+6.15\}} = 1.43$$

The null hypothesis is rejected when $L > F_{(\alpha,k-1,N-k)}$ where $F_{(0.05,2,27)} = 3.35$ is the upper critical value of the F-distribution with 2 and 27 degree of freedom at a significance level of 0.05. From computing found that $L = 1.43 < 3.35$, that do not reject H_0. So, three population variances are equal.

Its details of the computing are shown as follows:

Computing of

$$\bar{X}_1 = \sum_{j=1}^{n_1} X_{1j} \Big/ n_i = \frac{8+3+8+5+8+\ldots+9}{10} = 7.6,$$

$$\bar{X}_2 = \sum_{j=1}^{n_2} X_{2j} \Big/ n_2 = \frac{8+6+2+7+6+\ldots+11}{10} = 8.2,$$

$$\bar{X}_3 = \sum_{j=1}^{n_3} X_{3j} \Big/ n_3 = \frac{4+12+10+11+10+\ldots+9}{10} = 8.8.$$

Computing of $\bar{X}_i$, $Z_{ij} = |X_{ij} - \bar{X}_i|$, and $\bar{Z}_i$ show in **Table 2.1** as follows:

Table 2.1 Computing of $\overline{X}_i$, Z_{ij}, and $\overline{Z}_i$

X_1	X_2	X_3	$Z_{1j}=\lvert X_{1j}-\overline{X}_1\rvert$	$Z_{2j}=\lvert X_{2j}-\overline{X}_2\rvert$	$Z_{3j}=\lvert X_{3j}-\overline{X}_3\rvert$
8	8	4	$\lvert 8-7.6\rvert=0.4$	$\lvert 8-8.2\rvert=0.2$	$\lvert 4-8.8\rvert=4.8$
3	6	12	$\lvert 3-7.6\rvert=4.6$	$\lvert 6-8.2\rvert=2.2$	$\lvert 12-8.8\rvert=3.2$
8	2	10	$\lvert 8-7.6\rvert=0.4$	$\lvert 2-8.2\rvert=6.2$	$\lvert 10-8.8\rvert=1.2$
5	7	11	$\lvert 5-7.6\rvert=2.6$	$\lvert 7-8.2\rvert=1.2$	$\lvert 11-8.8\rvert=2.2$
8	6	10	$\lvert 8-7.6\rvert=0.4$	$\lvert 6-8.2\rvert=2.2$	$\lvert 10-8.8\rvert=1.2$
11	7	11	$\lvert 11-7.6\rvert=3.4$	$\lvert 7-8.2\rvert=1.2$	$\lvert 11-8.8\rvert=2.2$
7	15	6	$\lvert 7-7.6\rvert=0.6$	$\lvert 15-8.2\rvert=6.8$	$\lvert 6-8.8\rvert=2.8$
8	11	12	$\lvert 8-7.6\rvert=0.4$	$\lvert 11-8.2\rvert=2.8$	$\lvert 12-8.8\rvert=3.2$
9	9	3	$\lvert 9-7.6\rvert=1.4$	$\lvert 9-8.2\rvert=0.8$	$\lvert 3-8.8\rvert=5.8$
9	11	9	$\lvert 9-7.6\rvert=1.4$	$\lvert 11-8.2\rvert=2.8$	$\lvert 9-8.8\rvert=0.2$
$\overline{X}_1=7.6$	$\overline{X}_2=8.2$	$\overline{X}_3=8.8$	$\overline{Z}_1=15.6/10=1.56$	$\overline{Z}_2=26.4/10=2.64$	$\overline{Z}_3=26.8/10=2.68$

Computing of $\overline{Z}=(15.6+26.4+26.8)/30=2.29.$

Computing of $(\overline{Z}_1-\overline{Z})^2=(1.56-2.29)^2=0.53,$

$(\overline{Z}_2-\overline{Z})^2=(2.64-2.29)^2=0.12,$

and $(\overline{Z}_3-\overline{Z})^2=(2.68-2.29)^2=0.15.$

Computing of $(Z_{ij}-\overline{Z}_i)^2$ shows in **Table 2.2** as follows:

Table 2.2 Computing of $(Z_{ij}-\overline{Z}_i)^2$

$(Z_{1j}-\overline{Z}_1)^2$	$(Z_{2j}-\overline{Z}_2)^2$	$(Z_{3j}-\overline{Z}_3)^2$
$(0.4-1.56)^2=1.35$	$(0.2-2.64)^2=5.95$	$(4.8-2.68)^2=4.49$
$(4.6-1.56)^2=9.24$	$(2.2-2.64)^2=0.19$	$(3.2-2.68)^2=0.27$
$(0.4-1.56)^2=1.35$	$(6.2-2.64)^2=12.67$	$(1.2-2.68)^2=2.19$
$(2.6-1.56)^2=1.08$	$(1.2-2.64)^2=2.07$	$(2.2-2.68)^2=0.23$
$(0.4-1.56)^2=1.35$	$(2.2-2.64)^2=0.19$	$(1.2-2.68)^2=2.19$
$(3.4-1.56)^2=3.39$	$(1.2-2.64)^2=2.07$	$(2.2-2.68)^2=0.23$
$(0.6-1.56)^2=0.92$	$(6.8-2.64)^2=17.31$	$(2.8-2.68)^2=0.01$
$(0.4-1.56)^2=1.35$	$(2.8-2.64)^2=0.03$	$(3.2-2.68)^2=0.27$
$(1.4-1.56)^2=0.03$	$(0.8-2.64)^2=3.39$	$(5.8-2.68)^2=9.73$
$(1.4-1.56)^2=0.03$	$(2.8-2.64)^2=0.03$	$(0.2-2.68)^2=6.15$

2.2.1.2 Calculated example of modified Levene's test based on median , and Its called Brown-Forsythe's test (BF).

The hypothesis: $H_0 : \sigma_1^2 = \sigma_2^2 = \sigma_3^2$ versus $H_1 : \sigma_i^2 \neq \sigma_j^2$ for at least one $i \neq j$.

The test statistic is

$$BF = \frac{(N-k)\sum_{i=1}^{k} n_i (\overline{Z}_{Mi} - \overline{Z}_M)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(Z_{Mij} - \overline{Z}_{Mi})^2} = \frac{(30-3)\{10(0.53)+10(0.22)+10(0.07)\}}{(3-1)\{1.96+12.96+1.96+...+1.96\}} = 0.74$$

The null hypothesis is rejected when $BF > F_{(\alpha,k-1,N-k)}$ where $F_{(0.05,2,27)} = 3.35$ is the upper critical value of the F-distribution with 2 and 27 degree of freedom at a significance level of 0.05. From computing found that $BF = 0.74 < 3.35$, that do not reject H_0. So, three population variances are equal. Its details of the computing are shown as follows:

in ascending order X_1: 3,5,7,8,8,8,8,9,9,11 and $M_1 = (8+8)/2 = 8$,

in ascending order X_2: 2,6,6,7,7,8,9,11,11,15 and $M_2 = (7+8)/2 = 7.5$,

in ascending order X_3: 3,4,6,9,10,10,11,11,12,12 and $M_3 = (10+10)/2 = 10$.

Computing of M_i, $Z_{Mij} = |X_{ij} - M_i|$, and $\overline{Z}_{Mi}$ show in **Table 2.3** as follows:

Table 2.3 Computing of M_i, Z_{Mij}, and $\overline{Z}_{Mi}$

X_1	X_2	X_3	$Z_{M1j} = \|X_{1j} - M_1\|$	$Z_{M2j} = \|X_{2j} - M_2\|$	$Z_{M3j} = \|X_{3j} - M_3\|$
8	8	4	$\|8-8\| = 0$	$\|8-7.5\| = 0.5$	$\|4-10\| = 6$
3	6	12	$\|3-8\| = 5$	$\|6-7.5\| = 1.5$	$\|12-10\| = 2$
8	2	10	$\|8-8\| = 0$	$\|2-7.5\| = 5.5$	$\|10-10\| = 0$
5	7	11	$\|5-8\| = 3$	$\|7-7.5\| = 0.5$	$\|11-10\| = 1$
8	6	10	$\|8-8\| = 0$	$\|6-7.5\| = 1.5$	$\|10-10\| = 0$
11	7	11	$\|11-8\| = 3$	$\|7-7.5\| = 0.5$	$\|11-10\| = 1$
7	15	6	$\|7-8\| = 1$	$\|15-7.5\| = 7.5$	$\|6-10\| = 4$
8	11	12	$\|8-8\| = 0$	$\|11-7.5\| = 3.5$	$\|12-10\| = 2$
9	9	3	$\|9-8\| = 1$	$\|9-7.5\| = 1.5$	$\|3-10\| = 7$
9	11	9	$\|9-8\| = 1$	$\|11-7.5\| = 3.5$	$\|9-10\| = 1$
$M_1 = 8$	$M_2 = 7.5$	$M_3 = 10$	$\overline{Z}_{M1} = 14/10 = 1.4$	$\overline{Z}_{M2} = 26/10 = 2.6$	$\overline{Z}_{M3} = 24/10 = 2.4$

Computing of $\overline{Z}_M = (14+26+24)/30 = 2.13$.

Computing of $(\overline{Z}_{M1} - \overline{Z}_M)^2 = (1.4 - 2.13)^2 = 0.53$,

$$(\bar{Z}_{M2}-\bar{Z}_{M})^2=(2.6-2.13)^2=0.22,$$

and $$(\bar{Z}_{M3}-\bar{Z}_{M})^2=(2.4-2.13)^2=0.07.$$

Computing of $(Z_{Mij}-\bar{Z}_{Mi})^2$ shows in **Table 2.4** as follows:

Table 2.4 Computing of $(Z_{Mij}-\bar{Z}_{Mi})^2$

$(Z_{M1j}-\bar{Z}_{M1})^2$	$(Z_{M2j}-\bar{Z}_{M2})^2$	$(Z_{M3j}-\bar{Z}_{M3})^2$
$(0-1.4)^2=1.96$	$(0.5-2.6)^2=4.41$	$(6-2.4)^2=31.36$
$(5-1.4)^2=12.96$	$(1.5-2.6)^2=1.21$	$(2-2.4)^2=0.16$
$(0-1.4)^2=1.96$	$(5.5-2.6)^2=8.41$	$(0-2.4)^2=5.76$
$(3-1.4)^2=2.56$	$(0.5-2.6)^2=4.41$	$(1-2.4)^2=1.96$
$(0-1.4)^2=1.96$	$(1.5-2.6)^2=1.21$	$(0-2.4)^2=5.76$
$(3-1.4)^2=2.56$	$(0.5-2.6)^2=4.41$	$(1-2.4)^2=1.96$
$(1-1.4)^2=0.16$	$(7.5-2.6)^2=24.01$	$(4-2.4)^2=2.56$
$(0-1.4)^2=1.96$	$(3.5-2.6)^2=0.81$	$(2-2.4)^2=0.16$
$(1-1.4)^2=0.16$	$(1.5-2.6)^2=1.21$	$(7-2.4)^2=21.16$
$(1-1.4)^2=0.16$	$(3.5-2.6)^2=0.81$	$(1-2.4)^2=1.96$

2.2.2 Calculated examples of O'Brien's test

2.2.2.1 Calculated example of original O'Brien's test (OB).

The hypothesis: $H_0:\sigma_1^2=\sigma_2^2=\sigma_3^2$ versus $H_1:\sigma_i^2\neq\sigma_j^2$ for at least one $i\neq j$.

The test statistic is

$$OB=\frac{(N-k)\sum_{i=1}^{k}n_i(\bar{V}_i-\bar{V})^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(V_{ij}-\bar{V}_i)^2}=\frac{(30-3)\{10(20.61)+10(9.92)+10(1.88)\}}{(3-1)\{25.50+389.67+25.50+\ldots+131.56\}}=0.79$$

The null hypothesis is rejected when $OB>F_{(\alpha,k-1,N-k)}$ where $F_{(0.05,2,27)}=3.35$ is the upper critical value of the F-distribution with 2 and 27 degree of freedom at a significance level of 0.05. From computing found that $OB=0.79<3.35$, that do not reject H_0. So, three population variances are equal. Its details of the computing are shown as follows:

Computing of
$$\bar{X}_1=\sum_{j=1}^{n_1}X_{1j}\Big/n_i=\frac{8+3+8+5+8+\ldots+9}{10}=7.6,$$

$$\bar{X}_2=\sum_{j=1}^{n_2}X_{2j}\Big/n_2=\frac{8+6+2+7+6+\ldots+11}{10}=8.2,$$

$$\bar{X}_3=\sum_{j=1}^{n_3}X_{3j}\Big/n_3=\frac{4+12+10+11+10+\ldots+9}{10}=8.8.$$

Computing of $$S_1^2=\sum_{j=1}^{n_1}\left(X_{1j}-\bar{X}_1\right)^2\Big/(n_1-1)=\frac{(8-7.6)^2+(3-7.6)^2+...+(9-7.6)^2}{10-1}=4.93,$$

$$S_2^2=\sum_{j=1}^{n_2}\left(X_{2j}-\bar{X}_2\right)^2\Big/(n_2-1)=\frac{(8-8.2)^2+(6-8.2)^2+...+(11-8.2)^2}{10-1}=12.62,$$

$$S_3^2=\sum_{j=1}^{n_3}\left(X_{3j}-\bar{X}_3\right)^2\Big/(n_3-1)=\frac{(4-8.8)^2+(12-8.8)^2+...+(9-8.8)^2}{10-1}=10.84$$

Computing of $\bar{X}_i$, $V_{ij}=\dfrac{(n_i-1.5)n_i(X_{ij}-\bar{X}_i)^2-0.5S_i^2(n_i-1)}{(n_i-1)(n_i-2)}$, and $\bar{V}_i$ show in **Table 2.5** as follows:

Table 2.5 Computing of $\bar{X}_i$, V_{ij}, and $\bar{V}_i$

X_1	X_2	X_3	$V_{ij}=\dfrac{(n_i-1.5)n_i(X_{ij}-\bar{X}_i)^2-0.5S_i^2(n_i-1)}{(n_i-1)(n_i-2)}$		
			V_{1j}	V_{2j}	V_{3j}
8	8	4	−0.12	−0.74	26.52
3	6	12	24.67	4.92	11.41
8	2	10	−0.12	44.59	1.02
5	7	11	7.67	0.91	5.04
8	6	10	−0.12	4.92	1.02
11	7	11	13.34	0.91	5.04
7	15	6	0.12	53.80	8.58
8	11	12	−0.12	8.47	11.41
9	9	3	2.01	−0.03	39.04
9	11	9	2.01	8.47	−0.63
$\bar{X}_1=7.6$	$\bar{X}_2=8.2$	$\bar{X}_3=8.8$	$\bar{V}_1=49.33/10=4.93$	$\bar{V}_2=126.22/10=12.62$	$\bar{V}_3=108.44/10=10.84$

Computing of $\bar{V}=(49.33+126.22+108.44)/30=9.47.$

Computing of $(\bar{V}_1-\bar{V})^2=(4.93-9.47)^2=20.61,$

$(\bar{V}_2-\bar{V})^2=(12.62-9.47)^2=9.92,$

and $(\bar{V}_3-\bar{V})^2=(10.84-9.47)^2=1.88.$

Results after transformation $$\bar{V}_1=\left(\sum_{j=1}^{n_1}V_{1j}\right)\Big/n_1=S_1^2=4.93,$$

$$\bar{V}_2=\left(\sum_{j=1}^{n_2}V_{2j}\right)\Big/n_2=S_2^2=12.62,$$

and $$\bar{V}_3=\left(\sum_{j=1}^{n_3}V_{3j}\right)\Big/n_3=S_3^2=10.84.$$

Computing of $(V_{ij}-\bar{V}_i)^2$ shows in **Table 2.6** as follows:

Table 2.6 Computing of $(V_{ij}-\overline{V}_i)^2$

$(V_{1j}-\overline{V}_1)^2$	$(V_{2j}-\overline{V}_2)^2$	$(V_{3j}-\overline{V}_3)^2$
$(-0.12-4.93)^2=25.50$	$(-0.74-12.62)^2=178.49$	$(26.52-10.84)^2=245.86$
$(24.67-4.93)^2=389.67$	$(4.92-12.62)^2=59.29$	$(11.41-10.84)^2=0.32$
$(-0.12-4.93)^2=25.50$	$(44.59-12.62)^2=1022.08$	$(1.02-10.84)^2=96.43$
$(7.67-4.93)^2=7.51$	$(0.91-12.62)^2=173.12$	$(5.04-10.84)^2=33.64$
$(-0.12-4.93)^2=25.50$	$(4.92-12.62)^2=59.29$	$(1.02-10.84)^2=96.43$
$(13.34-4.93)^2=70.73$	$(0.91-12.62)^2=173.12$	$(5.04-10.84)^2=33.64$
$(0.12-4.93)^2=23.14$	$(53.80-12.62)^2=1635.79$	$(8.58-10.84)^2=5.11$
$(-0.12-4.93)^2=25.50$	$(8.47-12.62)^2=17.22$	$(11.41-10.84)^2=0.32$
$(2.01-4.93)^2=8.53$	$(-0.03-12.62)^2=160.02$	$(39.04-10.84)^2=795.24$
$(2.01-4.93)^2=8.53$	$(8.47-12.62)^2=17.22$	$(-0.63-10.84)^2=131.56$

2.2.2.2 Calculated example of modified O'Brien's test based on the median (MOB).

The hypothesis: $H_0:\sigma_1^2=\sigma_2^2=\sigma_3^2$ versus $H_1:\sigma_i^2\neq\sigma_j^2$ for at least one $i\neq j$.

The test statistic is

$$MOB=\frac{(N-k)\sum_{i=1}^{k}n_i(\overline{V}_{Mi}-\overline{V}_M)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(V_{Mij}-\overline{V}_{Mi})^2}=\frac{(30-3)\{10(26.73)+10(8.47)+10(5.06)\}}{(3-1)\{29.48+580.33+29.48+\ldots+144.96\}}=0.64$$

The null hypothesis is rejected when $MOB>F_{(\alpha,k-1,N-k)}$ where $F_{(0.05,2,27)}=3.35$ is the upper critical value of the F-distribution with 2 and 27 degree of freedom at a significance level of 0.05. From computing found that $MOB=0.64<3.35$, that do not reject H_0. So, three population variances are equal. Its details of the computing are shown as follows:

in ascending order X_1: 3, 5, 7, 8, 8, 8, 8, 9, 9, 11 and $M_1=(8+8)/2=8$,

in ascending order X_2: 2, 6, 6, 7, 7, 8, 9, 11, 11, 15 and $M_2=(7+8)/2=7.5$,

in ascending order X_3: 3, 4, 6, 9, 10, 10, 11, 11, 12, 12 and $M_3=(10+10)/2=10$.

computing of

$$S_1^2=\sum_{j=1}^{n_1}(X_{1j}-\overline{X}_1)^2\Big/(n_1-1)=\frac{(8-7.6)^2+(3-7.6)^2+\ldots+(9-7.6)^2}{10-1}=4.93$$

$$S_2^2=\sum_{j=1}^{n_2}(X_{2j}-\overline{X}_2)^2\Big/(n_2-1)=\frac{(8-8.2)^2+(6-8.2)^2+\ldots+(11-8.2)^2}{10-1}=12.62$$

$$S_3^2=\sum_{j=1}^{n_3}(X_{3j}-\overline{X}_3)^2\Big/(n_3-1)=\frac{(4-8.8)^2+(12-8.8)^2+\ldots+(9-8.8)^2}{10-1}=10.84$$

Computing of M_i, $V_{ij} = \dfrac{(n_i - 1.5)n_i(X_{ij} - M_i)^2 - 0.5S_i^2(n_i - 1)}{(n_i - 1)(n_i - 2)}$, and $\overline{V}_i$ show in **Table 2.7** as follows:

Table 2.7 Computing of M_i, V_{Mij}, and $\overline{V}_{Mi}$

X_1	X_2	X_3	$V_{Mij} = \dfrac{(n_i - 1.5)n_i(X_{ij} - M_i)^2 - 0.5S_i^2(n_i - 1)}{(n_i - 1)(n_i - 2)}$		
			V_{M1j}	V_{M2j}	V_{M3j}
8	8	4	−0.31	−0.49	41.82
3	6	12	29.21	1.87	4.04
8	2	10	−0.31	34.92	−0.68
5	7	11	0.32	−0.49	0.50
8	6	10	−0.31	1.87	−0.68
11	7	11	10.32	−0.49	0.50
7	15	6	0.87	65.62	18.21
8	11	12	−0.31	13.67	4.04
9	9	3	0.87	1.87	57.17
9	11	9	0.87	13.67	0.50
$M_1 = 8$	$M_2 = 7.5$	$M_3 = 10$	$\overline{V}_{M1} = 51.22/10 = 5.12$	$\overline{V}_{M2} = 132.02/10 = 13.20$	$\overline{V}_{M3} = 125.42/10 = 12.54$

Computing of $\overline{V}_M = (51.22 + 132.02 + 125.42)/30 = 10.29.$

Computing of $(\overline{V}_{M1} - \overline{V}_M)^2 = (5.12 - 10.29)^2 = 26.73,$

$(\overline{V}_{M2} - \overline{V}_M)^2 = (13.20 - 10.29)^2 = 8.47,$

and $(\overline{V}_{M3} - \overline{V}_M)^2 = (12.54 - 10.29)^2 = 5.06.$

Computing of $(V_{Mij} - \overline{V}_{Mi})^2$ shows in **Table 2.8** as follows:

Table 2.8 Computing of $(V_{Mij} - \overline{V}_{Mi})^2$

$(V_{M1j} - \overline{V}_{M1})^2$	$(V_{M2j} - \overline{V}_{M2})^2$	$(V_{M3j} - \overline{V}_{M3})^2$
$(-0.31-5.12)^2 = 29.48$	$(-0.49-13.20)^2 = 187.42$	$(41.82-12.54)^2 = 857.32$
$(29.21-5.12)^2 = 580.33$	$(1.87-13.20)^2 = 128.37$	$(4.04-12.54)^2 = 72.25$
$(-0.31-5.12)^2 = 29.48$	$(34.92-13.20)^2 = 471.76$	$(-0.68-12.54)^2 = 174.77$
$(0.32-5.12)^2 = 23.04$	$(-0.49-13.20)^2 = 187.42$	$(0.50-12.54)^2 = 144.96$
$(-0.31-5.12)^2 = 29.48$	$(1.87-13.20)^2 = 128.37$	$(-0.68-12.54)^2 = 174.77$
$(10.32-5.12)^2 = 27.04$	$(-0.49-13.20)^2 = 187.42$	$(0.50-12.54)^2 = 144.96$
$(0.87-5.12)^2 = 18.06$	$(65.62-13.20)^2 = 2747.86$	$(18.21-12.54)^2 = 32.15$
$(-0.31-5.12)^2 = 29.48$	$(13.67-13.20)^2 = 0.22$	$(4.04-12.54)^2 = 72.25$
$(0.87-5.12)^2 = 18.06$	$(1.87-13.20)^2 = 128.37$	$(57.17-12.54)^2 = 1991.84$
$(0.87-5.12)^2 = 18.06$	$(13.67-13.20)^2 = 0.22$	$(0.50-12.54)^2 = 144.96$

2.2.3 Calculated examples of Jackknife test

2.2.3.1 Calculated example of original Jackknife test (J).

The hypothesis: $H_0:\sigma_1^2=\sigma_2^2=\sigma_3^2$ versus $H_1:\sigma_i^2\neq\sigma_j^2$ for at least one $i\neq j$.

The test statistic is

$$J=\frac{(N-k)\sum_{i=1}^{k}n_i(\overline{U}_i-\overline{U})^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(U_{ij}-\overline{U}_i)^2}=\frac{(30-3)\{10(0.27)+10(0.14)+10(0.02)\}}{(3-1)\{1.52+30.56+1.52+...+1.30\}}=0.63$$

The null hypothesis is rejected when $J>F_{(\alpha,k-1,N-k)}$ where $F_{(0.05,2,27)}=3.35$ is the upper critical value of the F-distribution with 2 and 27 degree of freedom at a significance level of 0.05. From computing found that $J=0.63<3.35$, that do not reject H_0. So, three population variances are equal.

Its details of the computing are shown as follows:

Computing of $\overline{X}_1=\sum_{j=1}^{n_1}X_{1j}\Big/n_i=\frac{8+3+8+5+8+...+9}{10}=7.6,$

$$\overline{X}_2=\sum_{j=1}^{n_2}X_{2j}\Big/n_2=\frac{8+6+2+7+6+...+11}{10}=8.2,$$

$$\overline{X}_3=\sum_{j=1}^{n_3}X_{3j}\Big/n_3=\frac{4+12+10+11+10+...+9}{10}=8.8.$$

Computing of $S_1^2=\sum_{j=1}^{n_1}\left(X_{1j}-\overline{X}_1\right)^2\Big/(n_1-1))=\frac{(8-7.6)^2+(3-7.6)^2+...+(9-7.6)^2}{10-1}=4.93,$

$$S_2^2=\sum_{j=1}^{n_{2j}}\left(X_{2j}-\overline{X}_2\right)^2\Big/(n_2-1))=\frac{(8-8.2)^2+(6-8.2)^2+...+(11-8.2)^2}{10-1}=12.62,$$

$$S_3^2=\sum_{j=1}^{n_3}\left(X_{3j}-\overline{X}_3\right)^2\Big/(n_3-1))=\frac{(4-8.8)^2+(12-8.8)^2+...+(9-8.8)^2}{10-1}=10.84$$

Computing of $S_{ij}^2=\frac{\left((n_i-1)S_i^2-n_i\left(X_{ij}-\overline{X}_i\right)^2/(n_i-1)\right)}{(n_i-2)}$, $U_{ij}=n_i\log S_i^2-(n_i-1)S_{ij}^2$. and $\overline{U}_{ij}$ show in Table 2.9 as follows:

Table 2.9 Computing of S_{ij}^2, U_{ij}, and $\bar{U}_{ij}$

$S_{ij}^2 = \frac{\left((n_i-1)S_i^2 - n_i\left(X_{ij}-\bar{X}_i\right)^2/(n_i-1)\right)}{(n_i-2)}$			$U_{ij} = n_i \log S_i^2 - (n_i-1)S_{ij}^2.$		
S_{1j}^2	S_{2j}^2	S_{3j}^2	U_{1j}	U_{2j}	U_{3j}
5.53	14.19	9.00	0.57	1.48	4.06
2.61	13.53	10.78	7.33	1.91	2.44
5.53	8.86	12.00	0.57	5.72	1.47
4.61	14.00	11.53	2.21	1.60	1.83
5.53	13.53	12.00	0.57	1.91	1.47
3.94	14.00	11.53	3.62	1.60	1.83
5.50	7.78	11.11	0.62	6.89	2.17
5.53	13.11	10.78	0.57	2.19	2.44
5.28	14.11	7.53	0.98	1.53	5.67
5.28	13.11	12.19	0.98	2.19	1.33
			$\bar{U}_1 = 18.02/10 = 1.802$	$\bar{U}_2 = 27.02/10 = 2.702$	$\bar{U}_3 = 24.71/10 = 2.471$

Computing of $\bar{U} = (18.02 + 27.02 + 24.71)/30 = 2.325,$

Computing of $(\bar{U}_1 - \bar{U})^2 = (1.802 - 2.325)^2 = 0.27,$

$(\bar{U}_2 - \bar{U})^2 = (2.702 - 2.325)^2 = 0.14,$

and $(\bar{U}_3 - \bar{U})^2 = (2.471 - 2.325)^2 = 0.02.$

Computing of $(U_{ij} - \bar{U}_i)^2$ shows in **Table 2.10** as follows:

Table 2.10 Computing of $(U_{ij} - \bar{U}_i)^2$

$(U_{1j} - \bar{U}_1)^2$	$(U_{2j} - \bar{U}_2)^2$	$(U_{3j} - \bar{U}_3)^2$
$(0.57-1.802)^2 = 1.52$	$(1.84-2.702)^2 = 0.74$	$(4.06-2.471)^2 = 2.52$
$(7.33-1.802)^2 = 30.56$	$(1.91-2.702)^2 = 0.63$	$(2.44-2.471)^2 = 0.001$
$(0.57-1.802)^2 = 1.52$	$(5.72-2.702)^2 = 9.11$	$(1.47-2.471)^2 = 1.00$
$(2.21-1.802)^2 = 0.17$	$(1.60-2.702)^2 = 1.21$	$(1.83-2.471)^2 = 0.41$
$(0.57-1.802)^2 = 1.52$	$(1.91-2.702)^2 = 0.63$	$(1.47-2.471)^2 = 1.00$
$(3.62-1.802)^2 = 3.31$	$(1.60-2.702)^2 = 1.21$	$(1.83-2.471)^2 = 0.41$
$(0.62-1.802)^2 = 1.40$	$(6.89-2.702)^2 = 17.54$	$(2.17-2.471)^2 = 0.09$
$(0.57-1.802)^2 = 1.52$	$(2.19-2.702)^2 = 0.26$	$(2.44-2.471)^2 = 0.001$
$(0.98-1.802)^2 = 0.68$	$(1.53-2.702)^2 = 1.37$	$(5.67-2.471)^2 = 10.23$
$(0.98-1.802)^2 = 0.68$	$(2.19-2.702)^2 = 0.26$	$(1.33-2.471)^2 = 1.30$

2.2.3.2 Calculated example of modified Jackknife test based on the median (MJ).

The hypothesis: $H_0 : \sigma_1^2 = \sigma_2^2 = \sigma_3^2$ versus $H_1 : \sigma_i^2 \neq \sigma_j^2$ for at least one $i \neq j$.

The test statistic is

$$MJ = \frac{(N-k)\sum_{i=1}^{k} n_i(\overline{U}_{Mi} - \overline{U}_M)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(\overline{U}_{Mij} - \overline{U}_{Mi})^2} = \frac{(30-3)\{10(0.28)+10(0.15)+10(0.02)\}}{(3-1)\{1.25+24.44+1.25+\ldots+0.87\}} = 0.83$$

The null hypothesis is rejected when $MJ > F_{(\alpha,k-1,N-k)}$ where $F_{(0.05,2,27)} = 3.35$ is the upper critical value of the F-distribution with 2 and 27 degree of freedom at a significance level of 0.05. From computing found that $MJ = 0.83 < 3.35$, that do not reject H_0. So, three population variances are equal. Its details of the computing are shown as follows:

in ascending order X_1: 3 ,5 ,7 ,8 ,8 ,8 ,8 ,9 ,9 ,11 and $M_1 = (8+8)/2 = 8$,

in ascending order X_2: 2 ,6 ,6 ,7 ,7 ,8 ,9 ,11 ,11 ,15 and $M_2 = (7+8)/2 = 7.5$,

in ascending order X_3: 3 ,4 ,6 ,9 ,10 ,10 ,11 ,11 ,12 ,12 and $M_3 = (10+10)/2 = 10$.

Computing of
$$S_1^2 = \sum_{j=1}^{n_1}(X_{1j} - \overline{X}_1)^2 \Big/ (n_1 - 1) = \frac{(8-7.6)^2 + (3-7.6)^2 + \ldots + (9-7.6)^2}{10-1} = 4.93$$
$$S_2^2 = \sum_{j=1}^{n_2}(X_{2j} - \overline{X}_2)^2 \Big/ (n_2 - 1) = \frac{(8-8.2)^2 + (6-8.2)^2 + \ldots + (11-8.2)^2}{10-1} = 12.62$$
$$S_3^2 = \sum_{j=1}^{n_3}(X_{3j} - \overline{X}_3)^2 \Big/ (n_3 - 1) = \frac{(4-8.8)^2 + (12-8.8)^2 + \ldots + (9-8.8)^2}{10-1} = 10.84$$

Computing of $S_{Mij}^2 = \dfrac{\left((n_i-1)S_{Mi}^2 - n_i\left(X_{ij} - \overline{X}_i\right)^2 / (n_i - 1)\right)}{(n_i - 2)}$, $U_{ij} = n_i \log S_{Mi}^2 - (n_i - 1)S_{Mij}^2$,

and $\overline{U}_{Mij}$ show in **Table 2.11** as follows:

Table 2.11 Computing of S_{Mij}^2, U_{Mij}, and $\overline{U}_{Mij}$

$S_{Mij}^2 = \dfrac{\left((n_i-1)S_{Mi}^2 - n_i\left(X_{ij}-\overline{X}_i\right)^2/(n_i-1)\right)}{(n_i-2)}$			$U_{ij} = n_i \log S_{Mi}^2 - (n_i-1)S_{Mij}^2$		
S_{M1j}^2	S_{M2j}^2	S_{M3j}^2	U_{M1j}	U_{M2j}	U_{M3j}
3.53	9.11	6.72	0.56	1.47	3.65
1.80	8.71	7.77	6.62	1.87	2.34
3.53	5.94	8.50	0.56	5.32	1.53
2.99	8.99	8.22	2.05	1.59	1.84
3.53	8.71	8.50	0.56	1.87	1.53
2.59	8.99	8.22	3.34	1.59	1.84
3.51	5.30	7.97	0.61	6.34	2.11
3.53	8.46	7.77	0.56	2.14	2.34
3.38	9.06	5.84	0.95	1.52	4.91
3.38	8.46	8.61	0.95	2.14	1.42
			$\overline{U}_{M1} = 16.76/10 = 1.676$	$\overline{U}_{M2} = 25.85/10 = 2.585$	$\overline{U}_{M3} = 23.51/10 = 2.351$

Computing of $\overline{U}_M = (16.76 + 25.85 + 23.51)/30 = 2.204$.

Computing of $(\overline{U}_{M1} - \overline{U}_M)^2 = (1.676 - 2.204)^2 = 0.28$,

$(\overline{U}_{M2} - \overline{U}_M)^2 = (2.585 - 2.204)^2 = 0.15$,

and $(\overline{U}_{M3} - \overline{U}_M)^2 = (2.351 - 2.204)^2 = 0.02$.

Computing of $(U_{Mij} - \overline{U}_{Mi})^2$. shows in **Table 2.12** as follows:

Table 2.12 Computing of $(U_{Mij} - \overline{U}_{Mi})^2$

$(U_{M1j} - \overline{U}_{M1})^2$	$(U_{M2j} - \overline{U}_{M2})^2$	$(U_{M3j} - \overline{U}_{M3})^2$
$(0.56-1.676)^2 = 1.25$	$(1.47-2.585)^2 = 1.24$	$(3.65-2.351)^2 = 1.69$
$(6.62-1.676)^2 = 24.44$	$(1.87-2.585)^2 = 0.51$	$(2.34-2.351)^2 = 0.0001$
$(0.56-1.676)^2 = 1.25$	$(5.32-2.585)^2 = 7.48$	$(1.53-2.351)^2 = 0.67$
$(2.05-1.676)^2 = 0.14$	$(1.59-2.585)^2 = 0.99$	$(1.84-2.351)^2 = 0.26$
$(0.56-1.676)^2 = 1.25$	$(1.87-2.585)^2 = 0.51$	$(1.53-2.351)^2 = 0.67$
$(3.34-1.676)^2 = 2.77$	$(1.59-2.585)^2 = 0.99$	$(1.84-2.351)^2 = 0.26$
$(0.61-1.676)^2 = 1.14$	$(6.34-2.585)^2 = 14.10$	$(2.11-2.351)^2 = 0.06$
$(0.56-1.676)^2 = 1.25$	$(2.14-2.585)^2 = 0.20$	$(2.34-2.351)^2 = 0.0001$
$(0.95-1.676)^2 = 0.53$	$(1.52-2.585)^2 = 1.13$	$(4.91-2.351)^2 = 6.55$
$(0.95-1.676)^2 = 0.53$	$(2.14-2.585)^2 = 0.20$	$(1.42-2.351)^2 = 0.87$

2.2.4 Calculated examples of Box-Andersen test

2.2.4.1 Calculated example of original Box-andersen test (BA).

The hypothesis: $H_0 : \sigma_1^2 = \sigma_2^2 = \sigma_3^2$ versus $H_1 : \sigma_i^2 \neq \sigma_j^2$ for at least one $i \neq j$.

The test statistic is

$$BA = \frac{2}{(\hat{\beta}_2 - 1)}\left((N-k)\ln S_p^2 - \sum_{i=1}^{k}(n_i - 1)\ln S_i^2\right)$$

$$= \frac{2}{(2.94-1)}\left(((30-3)\ln(9.47)) - (9*\ln(4.93)) + (9*\ln(12.62)) + (9*\ln(10.84))\right) = 2.13$$

The null hypothesis is rejected when $BA > \chi^2_{(1-\alpha,k-1)}$ where $\chi^2_{(0.95,2)} = 5.991$ is the upper critical value of the χ^2 - distribution with 2 degree of freedom at a significance level of 0.05. From computing found that $BA = 2.13 < 5.991$, that do not reject H_0. So, three population variances are equal.

Its details of the computing are shown as follows:

Computing of $\overline{X}_1 = \sum_{j=1}^{n_1} X_{1j} \Big/ n_i = \frac{8+3+8+5+8+\ldots+9}{10} = 7.6$,

$$\bar{X}_2=\sum_{j=1}^{n_2}X_{2j}\Big/n_2=\frac{8+6+2+7+6+...+11}{10}=8.2,$$

$$\bar{X}_3=\sum_{j=1}^{n_3}X_{3j}\Big/n_3=\frac{4+12+10+11+10+...+9}{10}=8.8.$$

Computing of $$S_1^2=\sum_{j=1}^{n_1}\left(X_{1j}-\bar{X}_1\right)^2\Big/(n_1-1)=\frac{(8-7.6)^2+(3-7.6)^2+...+(9-7.6)^2}{10-1}=4.93,$$

$$S_2^2=\sum_{j=1}^{n_2}\left(X_{2j}-\bar{X}_2\right)^2\Big/(n_2-1)=\frac{(8-8.2)^2+(6-8.2)^2+...+(11-8.2)^2}{10-1}=12.62,$$

$$S_3^2=\sum_{j=1}^{n_3}\left(X_{3j}-\bar{X}_3\right)^2\Big/(n_3-1)=\frac{(4-8.8)^2+(12-8.8)^2+...+(9-8.8)^2}{10-1}=10.84$$

Computing of $$S_p^2=\frac{\sum_{i=1}^{k}(n_i-1)S_i^2}{N-k}=9.47 \text{ and } \hat{\beta}_2=\frac{N\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(X_{ij}-\bar{X}_i\right)^4}{\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(X_{ij}-\bar{X}_i\right)^2\right)^2}=2.94.$$

2.2.4.2 Calculated example of modified Box-andersen test based on median version (MBA).

The hypothesis: $H_0:\sigma_1^2=\sigma_2^2=\sigma_3^2$ versus $H_1:\sigma_i^2\neq\sigma_j^2$ for at least one $i\neq j$.

The modified test statistic is

$$MBA=\frac{2}{(\hat{\beta}_2-1)}\left((N-k)\ln S_{Mp}^2-\sum_{i=1}^{k}(n_i-1)\ln S_{Mi}^2\right)$$

$$=\frac{2}{(2.94-1)}\left(((30-3)\ln(10.24))-(9*\ln(5.11))+(9*\ln(13.17))+(9*\ln(12.44))\right)=2.31$$

The null hypothesis is rejected when $MBA>\chi^2{}_{(1-\alpha,k-1)}$ where $\chi^2{}_{(0.95,2)}=5.991$ is the upper critical value of the χ^2- distribution with 2 degree of freedom at a significance level of 0.05. From computing found that $MBA = 2.31 < 5.991$, that do not reject H_0. So, three population variances are equal.

Its details of the computing are shown as follows:

in ascending order X_1: 3 ,5 ,7 ,8 ,8 ,8 ,8 ,9 ,9 ,11 and $M_1=(8+8)/2=8$,

in ascending order X_2: 2 ,6 ,6 ,7 ,7 ,8 ,9 ,11 ,11 ,15 and $M_2=(7+8)/2=7.5$,

in ascending order X_3: 3 ,4 ,6 ,9 ,10 ,10 ,11 ,11 ,12 ,12 and $M_3=(10+10)/2=10$.

Computing of $$S_{M1}^2=\sum_{j=1}^{n_1}\left(X_{1j}-M_1\right)^2\Big/(n_1-1)=\frac{(8-8)^2+(3-8)^2+...+(9-8)^2}{10-1}=5.11,$$

$$S_{M2}^2=\sum_{j=1}^{n_{2i}}(X_{2j}-M_2)^2\bigg/(n_2-1))=\frac{(8-7.5)^2+(6-7.5)^2+...+(11-7.5)^2}{10-1}=13.17,$$

$$S_{M3}^2=\sum_{j=1}^{n_3}(X_{3j}-M_3)^2\bigg/(n_3-1))=\frac{(4-10)^2+(12-10)^2+...+(9-10)^2}{10-1}=12.44.$$

Computing of $S_{Mp}^2=\frac{\sum_{i=1}^{k}(n_i-1)S_{Mi}^2}{N-k}=10.24$ and $\hat{\beta}_2=\frac{N\sum_{i=1}^{k}\sum_{j=1}^{n_i}(X_{ij}-\bar{X}_i)^4}{\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i}(X_{ij}-\bar{X}_i)^2\right)^2}=2.94.$

2.3 Description of probability distributions

Since the data is used in data analysis may be non-normal distribution, so this study used four population distribution-types consist of normal distribution, Laplace distribution (leptokurtic distribution), uniform distribution (platykurtic distribution) and gamma distribution (skewed distribution).

2.3.1 Normal distribution

The normal distribution was first discovered by De Moivre in 1733. He developed the normal distribution as an approximation to the binomial distribution. Later it was rediscovered by Karl Gauss in 1809 and in 1812 by Laplace. Normal distribution is also called Gaussian distribution. For further discussions on the history of the normal distribution and its development, readers are referred to (Pearson, 1967; Patel and Read, 1982; Johnson et al., 1994; Stigler, 1999) and references therein. Also, see (Wiper et al., 2005) for recent developments.

2.3.1.1 Characteristics of the normal distribution

The random variable X whose distribution has the shape of a normal curve or "bell-shaped curve" is called a normal random variable. This random variable X is said a normal distribution with mean μ and variance σ^2. If its probability density function is given by

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2},-\infty<x<\infty,-\infty<\mu<\infty,\sigma>0 \tag{2.18}$$

Where X is a normal random variable,

μ and σ^2 are location and scale parameters, respectively,

e and π are the mathematical constant approximated by 2.71828 and 3.14159, respectively.

If a random variable X follows the normal distribution, the notation is defined by

$$X \sim N(\mu, \sigma^2).$$

The mean, the variance, skewness, and excess kurtosis are given by

$E(X) = \mu$ (and also its median and mode),

$Var(X) = \sigma^2, \sigma > 0$,

$\gamma_1 = 0,$

and $\gamma_2 = 0,$ respectively.

The normal probability density such as Normal(8,8) shown in **Figure 2.1**.

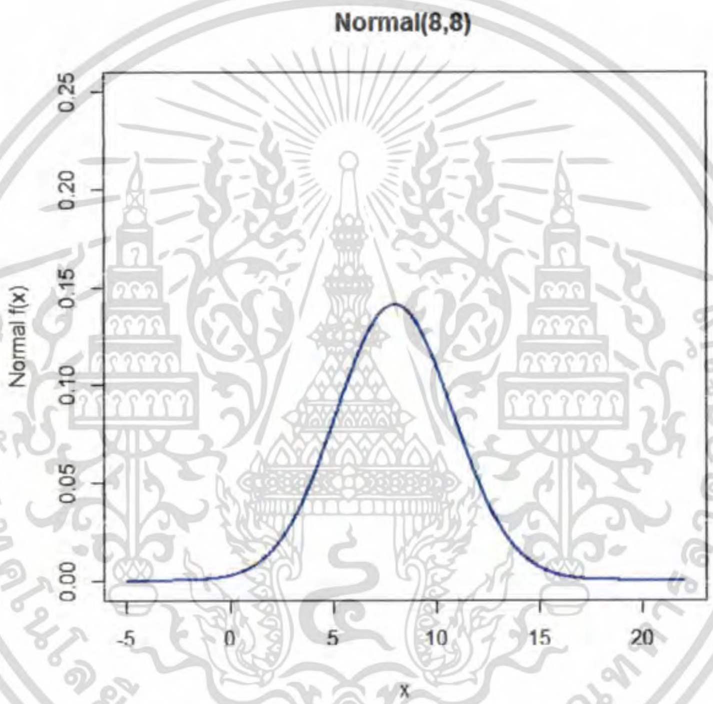


Figure 2.1 Normal probability density

2.3.1.2 Properties of the normal distribution

1. The normal curve is bell-shaped and symmetric about the mean μ, the normal curve approaches, but never foundes the horizontal axis as it extends farther and ferther away from the mean μ. The mean is at the middle and divides the area into halves, and also the mean, mode and median are all equal and its shown in **Figure 2.2**

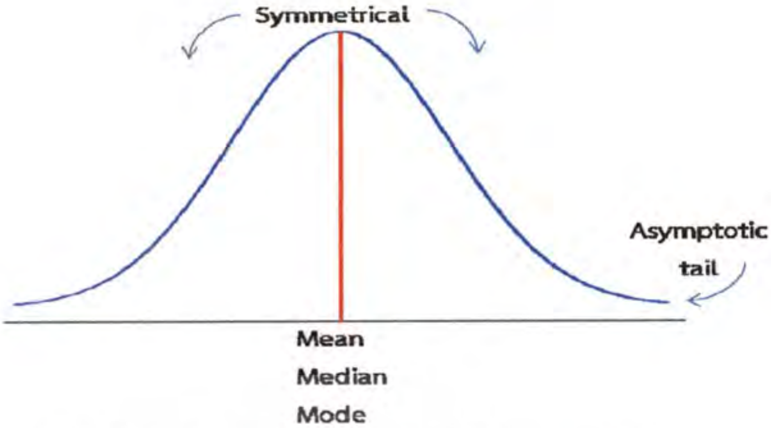


Figure 2.2 The normal curve or Bell-Shaped curve

2. The total area under the curve is equal to 1, It is competely determined by its mean μ and standard deviation σ or variance σ^2 (Patel and Read, 1996) and its shown in **Figure 2.3**.

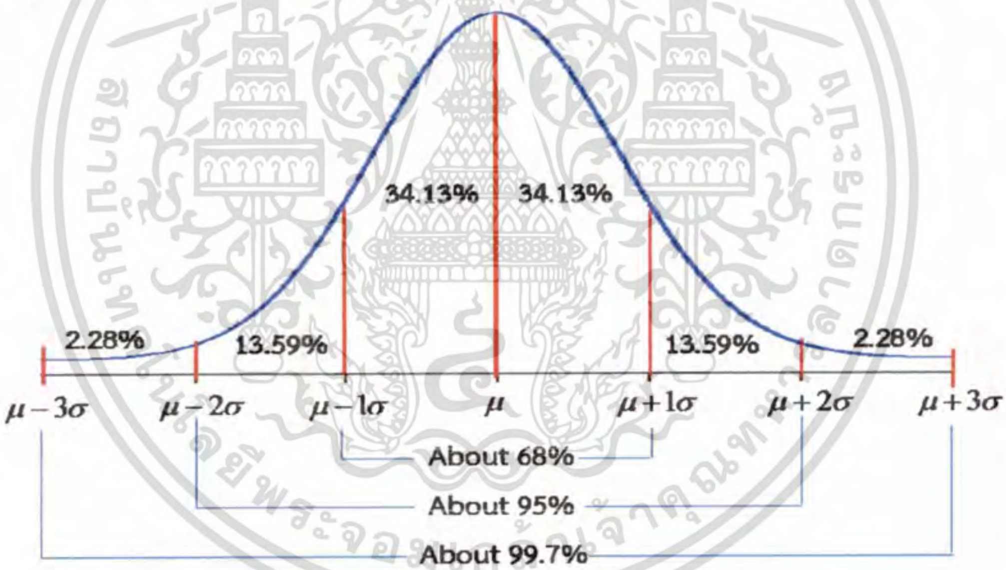


Figure 2.3 The normal curve and the area under the curve between σ units

The empirical rule tells you what percentage of your data falls within a certain number of standard deviations from the mean. The details are as follows:

68% of the data falls within one standard deviations of the mean,

95% of the data falls within two standard deviations of the mean,

99.7% of the data falls within three standard deviations of the mean.

The standard deviation controls the spread of the distribution. A smaller standard deviation means that the data is tightly clustered around the mean; the normal distribution will be taller. A larger standard deviation means that the data is spread out around the mean; the normal distribution will be flatter and wider.

2.3.2 Laplace distribution

The Laplace distribution is a continuous probability distribution. Its named after a French mathematician. That it is known as a double exponential distribution, because it reminds one of an exponential distribution " spliced together back-to-back". The Laplace distribution is used an example of a very leptokurtic distribution.

2.3.2.1 Characteristics of the Laplace distribution

The Laplace distribution is characterized by two main parameters: location $(-\infty < \mu < \infty)$ and scale $\lambda > 0$ (Stockute, et al., 2013). Its probability density function is given by

$$f(x;\mu,\lambda) = \frac{1}{2}\exp\left(-\frac{|x-\mu|}{\lambda}\right), when\ (x \le \mu), -\infty < x < \infty \quad (2.19)$$

and $f(x;\mu,\lambda) = 1 - \frac{1}{2}\exp\left(-\frac{|\mu-x|}{\lambda}\right), when\ (x > \mu), -\infty < x < \infty$ (2.20)

where X is a Laplace random variable,

μ and λ are location and scale parameters, respectively.

If a random variable X follows the logistic distribution, the notation is defined by

$$X \sim L(\mu,\lambda).$$

The mean, the variance, skewness, kurtosis, and excess kurtosis are given by

$E(X) = \mu$ (and also its median and mode),

$Var(X) = 2\lambda^2, \lambda > 0,,$

$\gamma_1 = 0,$

and $\gamma_2 = 3,$ respectively.

The Laplace probability density such as Laplace(8,2) shown in **Figure 2.4**.

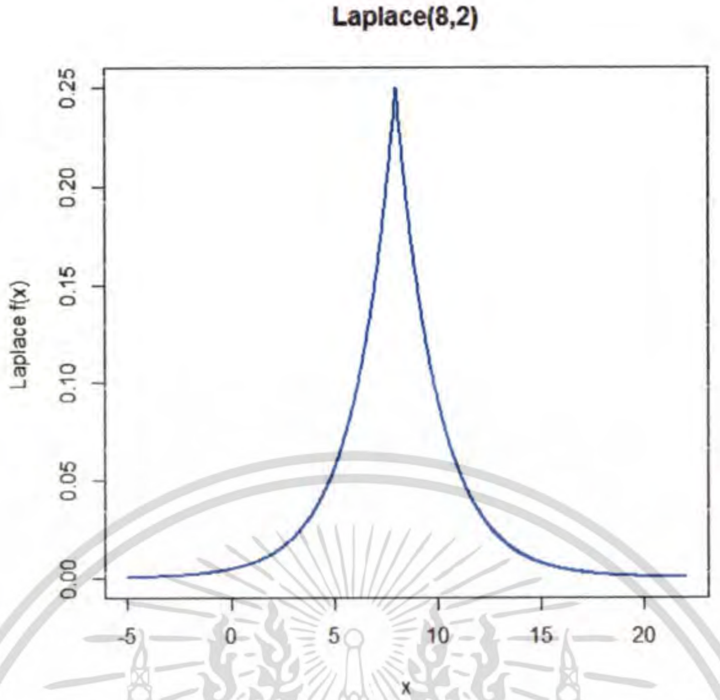


Figure 2.4 Laplace probability density

2.3.3 Uniform distribution (continuous)

The continuous uniform distribution or rectangular distribution is a family of symmetric probability distributions such that for each member of the family, all intervals of the same length on the distribution's support are equally probable. It is used an example of a very platykurtic distribution, which has as mush data in each tail as it does in the peak (Decarlo, 1997).

2.3.3.1 Characteristics of the uniform distribution

The Uniform distribution is characterized by two main parameters: a and b with $b > a$ (Nechval el at., (2002). Its probability density function is given by

$$f(x) = \frac{1}{b-a}, \text{ for } x \in [a,b], -\infty < a < b < \infty \qquad (2.21)$$

where X is a uniform random variable.

If a random variable X follows the uniform distribution, the notation is defined by

$$X \sim U(a,b).$$

The mean, the variance, skewness, kurtosis, and excess kurtosis are given by

$$E(X) = \frac{1}{2}(a+b),$$

$$Var(X) = \frac{1}{12}(b-a)^2,$$

$$\gamma_1 = 0,$$

and $\gamma_2 = -1.2$, respectively.

The uniform probability density such as Uniform(3.1,12.9) shown in **Figure 2.5**.

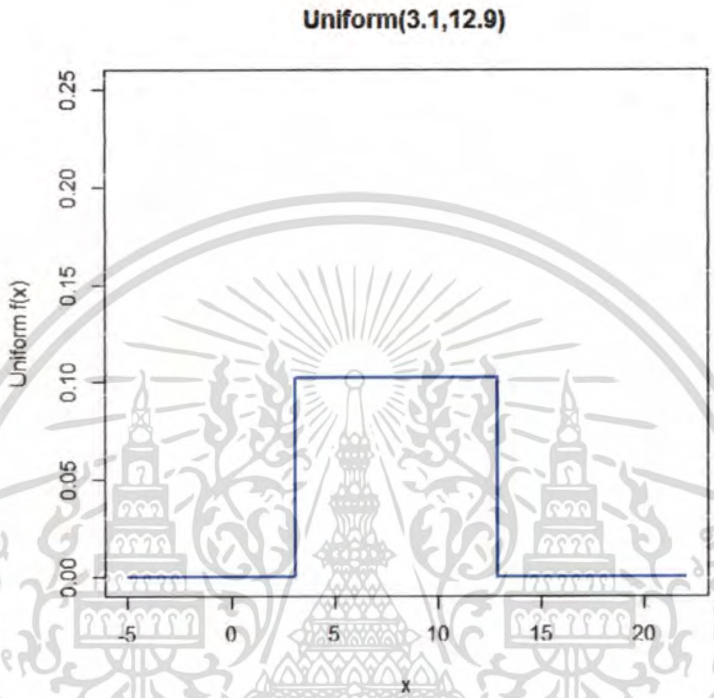


Figure 2.5 Uniform probability density

2.3.4 Gamma distribution

The gamma distribution is a two-parameter family of continuous probability distributions. The common exponential distribution and chi-squared distribution are special cases of the gamma distribution, which is frequently used to model waiting times.

2.3.4.1 Characteristics of the gamma distribution

The gamma distribution is characterized by two main parameters: shape $\alpha > 0$ and scale $\beta > 0$ (A. M. Mathai, 1982). Its probability density function is given by

$$f(x,\alpha,\beta) = \begin{cases} \dfrac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\beta}} & ,0 < x < \infty, \alpha > 0, \beta > 0 \\ 0 & \text{, otherwise} \end{cases} \tag{2.22}$$

where X is a gamma random variable.

If a random variable X follows the gamma distribution, the notation is defined by

$$X \sim G(\alpha, \beta).$$

The mean, the variance, skewness, and excess kurtosis are given by

$$E(X) = \alpha\beta,$$

$$Var(X) = \alpha\beta^2,$$

$$\gamma_1 = \frac{2}{\sqrt{\alpha}},$$

and $\gamma_2 = \dfrac{6}{\alpha}$, respectively.

The gamma probability density such as Gamma(8,1) shown in **Figure 2.6**.

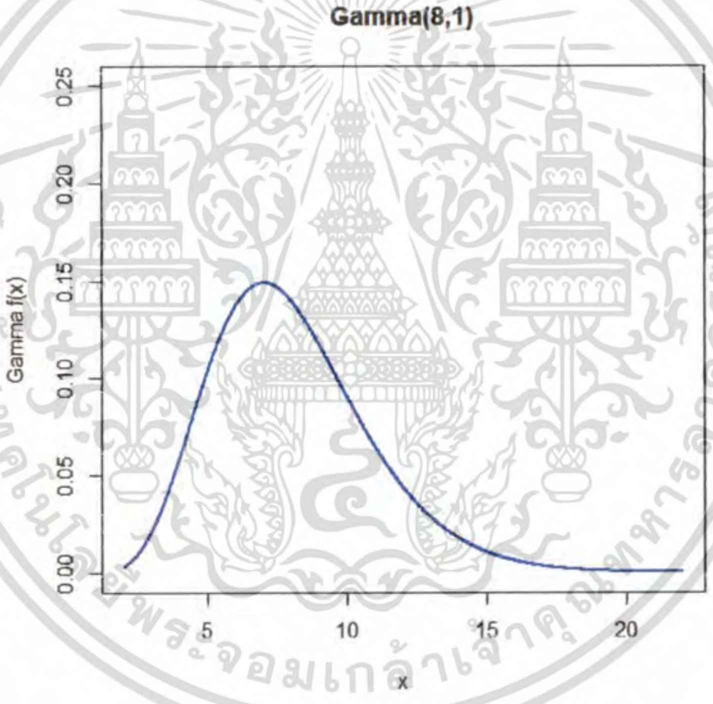


Figure 2.6 Gamma probability density

2.4 Determining the difference of population variance

Let the different population variance ratios by non-centrality parameter (ϕ), this ϕ is a measure of the different population variance ratios (Game et al., 1972). The formula is defined by (Eq. 1.1) in chapter 1.

This study, $k = 3$ groups.

A measure of the difference of population variances, which the details are as follows:

ϕ is between $0<\phi<1.5$ that shown slightly different population variance,

ϕ is between $1.5 \le \phi < 3.0$ that shown moderately different population variance,

ϕ is between $\phi > 3.0$ that shown highly different population variance.

Example of calculation of ϕ in the intervals $0<\phi<1.5$.

Let three population variances σ_1^2, σ_2^2 and σ_3^2 are 1, 2 and 4 , respectively. Computing ϕ, the details are given by

Game et al. (1972) formula $\phi = \dfrac{\sqrt{\sum_{i=1}^{k}\left(\sigma_i^2 - \bar{\sigma}^2\right)^2 / k}}{\sigma_1^2}$

when $\bar{\sigma}^2 = \dfrac{\sigma_1^2+\sigma_2^2+\sigma_3^2}{3} = \dfrac{1+2+4}{3} = \dfrac{7}{3}$

so $\phi = \dfrac{\sqrt{\sum_{i=1}^{3}\left(\sigma_i^2 - \bar{\sigma}^2\right)^2 / 3}}{\sigma_1^2} = \dfrac{\sqrt{\left(1-\frac{7}{3}\right)^2 + \left(2-\frac{7}{3}\right)^2 + \left(4-\frac{7}{3}\right)^2 \Big/ 3}}{1} \approx 1.25$

2.5 Criterion of efficiency comparison for testing

Criterion of efficiency comparison for testing (Piyawan, 2552) is criterion of decision that one of test statistics is the best test among in this study. There are two methods for performance measurement, namely ability to control probability of type I error (also robustness) and power of a test (also power).

2.5.1 The probability of type I error (robustness) is decision probability of rejecting the null hypothesis when it is true.

2.5.2 Power of a test (power) is decision probability of rejecting the null hypothesis when it is false.

The details are shown in **Table 2.13** as follows:

Table 2.13 Table of error types (Sheskin and David, 2004)

The null hypothesis	Judgment of the null hypothesis	
	Fail to reject	Reject
True	Correct decision $1-\alpha$	Type I error α
False	Type II error β	Correct decision $1-\beta$

Criterion of considered ability to control type I error (robustness)

The Bradley limit (Bradley, 1978) is used to consider robustness. The details are as follows:

at 0.05 significance level, the probability of type I error has to fall between [0.025, 0.075],

at 0.01 significance level, the probability of type I error has to fall between [0.005, 0.015].

2.6 Measures of Skewness and Kurtosis

2.6.1 Skewness

In probability theory, and statistics, skewness is a measure of the asymmetry of the probability distribution of a real-valued random variable about its mean. The skewness value can be positive or negative, or even undefined.

The skewness or skew is the dimensionless version of third moment about the mean, the formula is given by

$$m_3 = \frac{\sum (x-\bar{x})^3}{n}, \tag{2.23}$$

Which is rendered dimensionless by dividing by the cube of the standard deviation of x (because this is also measured in units of x^3):

$$s_3 = sd(x)^3 = (\sqrt{s^2})^3. \tag{2.24}$$

The skew is then given by

$$skew = \gamma_1 = \frac{m_3}{s_3}. \tag{2.25}$$

It measures the extent to which a distribution has long, drawn-outtails on one side or the other. A normal distribution is symmetrical and has $\gamma_1 = 0$.
Negative values of γ_1 mean skew to the left (negative skew) and potsitive values mean skew to the right.

There are two kinds of skew distribution consists of negative skew and positive skew. The negative skew is said to be left-skewed, left-tailed, or skewed to the left. The positive skew right tail is said to be right-skewed, right-tailed, or skewed to the right.

The coefficient of skewness is a measure for the degree of symmetry in the variable distribution (Sheskin, 2011). The two skew distributions are shown in **Figure 2.7.**

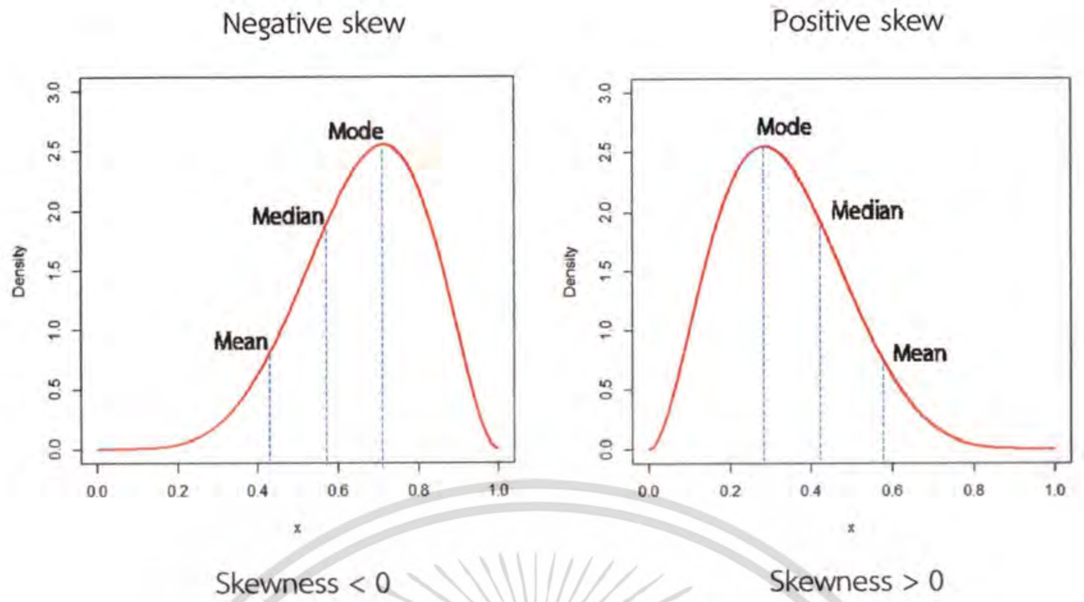


Figure 2.7 General forms of skewness

2.6.2 Kurtosis

In probability theory, and statistics, skewness is a measure of the tailedness of the probability distribution of a real-valued random variable. In a similar way to the concept of skewness, kurtosis is a descriptor of the shape of a probability distribution.

This is a measure of non-normality that has to do with the peakyness, or flat-toppedness, of a distribution is bell-shaped, whereas a kurtotic distribution is other than bell-shaped. In particular, a more flat-topped distribution is said to be platykurtic, and a more pointy distribution is said to be leptokurtic.

The kurtosis is the dimensionless version of fourth moment about the mean,

$$m_4 = \frac{\sum(x-\bar{x})^4}{n}, \tag{2.26}$$

Which is rendered dimensionless by dividing by the square of the variance of x (because this is also measured in units of x^4):

$$s_4 = (\mathrm{var}(x))^2 = (s^2)^2. \tag{2.27}$$

The kurtosis is then given by

$$kurtosis = \gamma_2 = \frac{m_4}{s_4} - 3. \tag{2.28}$$

The minus 3 is included because a normal distribution has $m_4/s_4 = 3$. This formulation therefore has the desirable property of giving zero kurtosis for a normal distribution, while a flat-topped (platykutic) distribution has a negative value of kurtosis, and a pointy (leptokurtic) distribution has a positive value of kurtosis.

A distribution with Zero excess kurtosis are called mesokurtic, or mesokurtotic. The most prominent example of a mesokurtic distribution is the normal distribution family, regardless of the values of its parameters. A few other well-known distributions can be mesokurtic, depending on parameter values: for example, the binomial distribution.

A distribution with positive excess kurtosis is called leptokurtic, or leptokurtotic. "Lepto-" means "slender". In terms of shape, a leptokurtic distribution has fatter tails. Examples of leptokurtic distributions include the Student's t-distribution, Rayleigh distribution, Laplace distribution, exponential distribution, Poisson distribution and the logistic distribution. The most leptokurtic distribution of all is Laplace distribution (also doubleexponential distribution).

A distribution with negative excess kurtosis is called platykurtic. "Platy-" means "broad". In term of shape, a platykurtic distribution has thinner tails. Examples of platykurtic distributions include the continuous or discrete uniform distributions, and the raised cosine distribution. The most platykurtic distribution of all is the Bernoulli distribution with $p = \frac{1}{2}$ (Kahane, 1960).

The coefficient of excess kurtosis is a measure for the degree of tailedness in the variable distribution (Westfall, 2014). The three kutosis are shown in **Figure 2.8.**

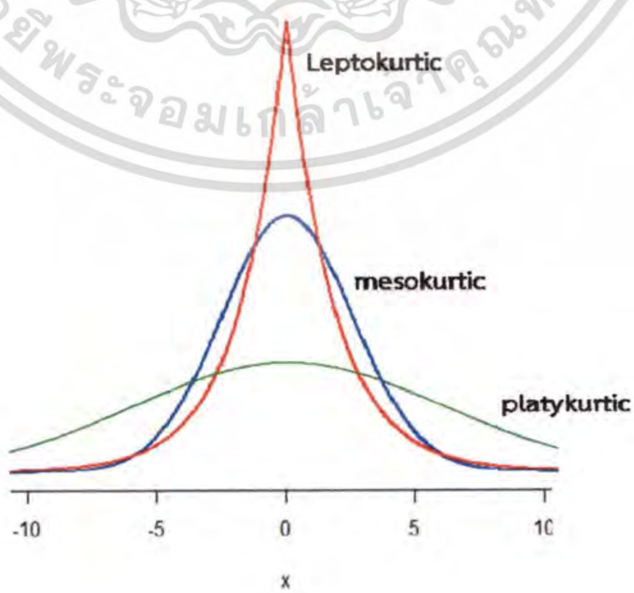


Figure 2.8 General forms of kurtosis

2.7 Literature Reviews

Vorapongsathorn et al. (2004) had compared the probability of type I error and the power of three statistical tests (Bartlett's, Levene's and Cochran's) by varying the sampling distribution, variance and sample size. Monte Carlo methods were used to generate resposes based on sample size and distributions 1000 times. The configurations of sample size were both equal and unequal: 15, 30 and 45. The data distributions were normal, gamma and chi-square. It was found that Bartlett's test was sensitive to the normally assumption whereas Cochran's test and Levene's test were robust when the normal assumption was violated. Moreover, Levene's test was quite good for both equal and small sample size. In the case of power, Bartlett's test had the highest power in all case. When one variance was large, Cochran's test was the best test.

Mu (2006) had discussed the performance of tests in this study under different types of distributions, sample size and true variance ratios of the populations. Monte Carlo methods were used to calculate empirical powers and type I errors under different settings. The results showed that the Bootstrap method showed a consistent performance. For the nominal type I errors of 0.05 in the simulation, the empirical type I errors were at most 0.1 and its power is always among the highest. Levene's test achieved its best performance when the underlying distribution had low kurtosis. All modified editions-Lev/Med, Bhat and F-Shoemaker had lower typr I error than their original versions. Except F-test and Bartlett's test, all other tests could be control type I error pretty well. In the meantime, the F-test and Bartlett's test presented exceptionally high power in simulations. However, they could not be compared with the others because of their poor controlled of type I error. Considering their high probability of type I error, these two tests actually rejected most situations. Bootstrap andLev/Med kept the balance between type I error and power of the test. Bootstrap worked slightly better at smaller sample size(≤ 20), Lev/Med had a better control of type I error (4% vs. 10%). No tests worked well under high kurtosis distribution.

Lee et al. (2010) had described that the unmodified Levene's test was compared to six other tests. In total, seven homogeneity of variance tests used in analysis of Variance (ANOVA) were compared on robustness and power usingMonte Carlo studies. The homogeneity of variance tests were (1) Levene's, (2) modified

Levene's, (3) Z-variance, (4) Overall-Woodward Z-variance, (5) O'Brien's, (6) samiuddin Cube Root and (7) F-Max. Each test was subjected to Monte Carlo analysis through different shaped distributions: (1) normal, (2) platykurtic, (3) leptokurtic, (4) moderate skewed and (5) highly skewed. The Levene's test is the one used in all of the last versions of SPSS. The results from these studies showed that the Levene's test is neither the best nor worst in terms of robustness and power. However, the modified levene's test showed very good robustness when compared to the other tests but lower power than other tests. The Samiuddin test was at its best in terms of robustness and power when the distribution was normal. The results of this study showed the strengths and weakness of the seven tests. No single test outperformed the others in terms of robustness and power. The authors recommend that kurtosis and skewness indices be presented in statistical computer program packages such as SPSS to guide the data analyst in choosing which test would provided the highest robustness and power.

Sittisombat et al. (2010) had studied the test statistics of homogeneity of variance by the Bartlett's test, Scheffe's test, Levene's test, Brown-Forsythe's test, O'Brien's test and Lehmann's test were compared to the power of the six test statistics, when distributions populations were normal and gamma distributions, and the significant level was at 0.05. The data used in research was obtained by the Monte Carlo technique simulation repeated 2,000 times for each situation. As for the result, when the population was normal distribution Lehmann's test had the highest power and followed by Scheffe's test and Brown-Forsythe's test, and the population was gamma distribution, Brown-Forsythe's test had the highest power and followed by Scheffe's test and O'Brien's test respectively.

Othman et al. (2012) had examined the type I error rate (for 0.05) of various modifications of Levene's (1960) and O'Brien's (1981) procedures that could be used to compare variability across groups in independent groups designs, specifically variations not examined by Keselman, et al. (2008). The procedures examined using Levene's or O'Brien's, robust measures of central tendency and/or variability were adopted. The robust values of central tendency and variability (i.e., the trimmed means and winsorized variances) were based on symmetric or asymmetric trimming rule, that is rule that either set a priori the amount of total trimming or determined empirically the amount to be trimmed from the tails based on varied recommendations for total trimming. That is, though the new Levene's modifications all worked very well in controlling type I error rates.

Gorbunova and Lemeshko (2012) had studied that there were a good number of tests that are available for testing a hypothesis that samples come from populations with the same variance. It was well known that classical tests for comparing variances were very sensitive to departures from normally. However, they were more powerful than nonparametric ones. So, the new approach for testing hypothesis of variance homogeneity was proposed. Software for comparing variances using parametric tests (F-test, Cochran's, Bartlett's, Hartley's, Levene's, modified Levene 's, Neyman-Pearson's, Z-variance, Overall-Woodward modified Z-variance and O'Brien tests) when sample were from any distribution (skewed, leptokutic, platykurtic) had been developed. In this case the p-value was defined using a simulated empirical distribution in real-time testing of the hypothesis. Recommendations on choosing the most powerful test for a particular form of data distribution are given.

Sangsawang (2013) had compared power of the test statistics of homogeneity of variance of 3 and 4 populations which were Brown Forsythe's test (BF), Analysis of means test for variances (AV) and Gini's test (GT). The probability of typeI error and power of the test statistics were considered. The normality distributed population with mean equal to zero, small $(n<15)$, medium $(15<n<120)$ and large $(n>120)$ sample size were studied, except for AV, which were studied only for medium sample size. Each compared populations had equal sample size. The three difference levels of variance: low $(0<\phi<1.5)$, medium $(1.5\leq\phi<3.0)$ and high $(\phi>3.0)$ for significant levels at 0.01 and 0.05 were studied. The data were generated by the Monte Carlo technique repeated 1,000 times for each situation. The results showed that, for small sample size, GT controlled the probability of type I error in every cases. For medium sample size, both BF and AV controlled the probability of type error in every cases, but GT controlled the probability of type I error for 4 populations groups at 0.01 significant level. For large sample size, BF controlled the probability of type I error in every cases, GT controlled the probability of type I error for 3 populations groups at 0.01 significant level. However, for 4 populations groups GT controlled the probability of type I error every significant levels. Consequently, only the test statistics that controlled the probability of type I error were compared power of the test. In case of small sample size, GT had low power of the test. When medium sample size, AV had power of the test more than BF for every difference levels of variance, except for 4 population groups at 0.01 significannt level, GT had maximum power of the test for every difference levels of variance. Both BF and GT had high power of the test very close in case of large sample size. Moreover, when the difference of population variance had increased, the power of the three test statistics had increased too. Also, when the significant levels, the number of population groups and the difference levels of variance had increased, power of the test for the three tests had tended to increase.

Hatchavanich (2014) had studied that there were a good number of tests that were available for testing a hypothesis that samples came from populations with the same variance. Many studies reported that there was no test which was uniformly best for all distributions and configurations of sample size. It could be seen that Bartlett's test, Levene's test and O'Brien's test offered different methods for researchers to test data. However, each test had some unigue weak points. To date, there were no studies about these tests when assumptions were violated under different situations. The aim of this paper, These three statistical tests were compared the empirical probability of type I error and the power under the different type of distributions: normal, uniform, student's t, chi-square distribution and nine configurations of group size $(n_1, n_2, n_{3,} n_4)$, the group variances were set as follows the ratio of 1:1:2:2, 1:2:3:4 and 1:1:1:4. It was found that no test outperformed the other in terms of robustness and power. The findings showed that Levene's test was not the best option. Bartlett's test was a good choice to test homogeneity of variance since it was not affected by sample size when the data was normally or uniform distributed. Morever, for low skew distribution, Bartlett's test was a good choice for small equal sample size and equal variance. O'Brien's test was the best for chi-square distributed. When the variance ratio of 1:1:1:4, low skew distribution, Bartlett's test and O'Brien's test would be commended for small unequal sample size, since their still afford high power.

Gogoi P. and Gogoi (2015) had discussed that some tests was used for equality of scale parameters under equal and unequal location parameters. There were Levene's test, Bartlett's test, Box-Anderson test, Jackknife test, test based on bootstrap and Lepage test. These tests were considered in terms of level and power of these tests using simulation technique. Results were displayed in various table and graphs. Discussions and conclusions were made on the basis of results obtained. From the discussion they concluded that Jackknife test J_2 shown more powerful for both normal and logistic distribution and Jackknife test J_1 shown the lowest power for small sample sizes. So it could be concluded that both Jackknife test J_2 and Bartlett test B were the best under normal distribution as well as logistic distribution. Also it is assumed that all the tests were the best for large sample size of the both distributions.

Hatchavanich (2016) had described that there were a good number of tests that were available for testing a hypothesis that samples came from populations with the same variance. Many studies reported that there was no test which was uniformly best for all distributions and configurations of sample size. So, this paper was said about assumptions were violated under different situations. Seven statistical tests were compared to the empirical probability of type I error and the power under 4 and 5 populations with the different types of distributions: normal, a mixture of Gaussian, uniform, student's t, chi-square, exponential distribution and 4 configurations of group equal sample size (6, 16, 30, 60) and the different variances. It was found that for normal, a small sample size had a small effect on the performance of the tests, however, when the sizes increased, the tests performed almost equivalently. Changing the ratio of population variance also seemed to have no effect. Levene's test, modified Bartlett's test and O'Brien outperformed the others in term of robustness. Considering the type I error and the power of a test found that the Gini's and Bartlett's test were the best, when the data was normally distributed. Moreever, for chi-square distribution and a mixture of Gauusian distribution, modified Bartlett's and Levene's were the best. For uniform and exponential distributions Jackknife test and Levene's test were the best.

Chapter 3

Research Methodology

This study is an experimental research. There are four original homogeneity of variance tests consists of Levene's test, O'Brien's test, Jackknife test, and Box-Andersen test then there are four modified homogeneity of variance tests based on modified central tendency by using the median replace the mean of each original tests. These tests are compared efficiency in terms of the probability of type I error and power of a test for three populations in cases of four distribution-types contain normal, Laplace, uniform and gamma. These tests are simulated the data and that are analyzed data by R program version 3.3.2.

3.1 Research Design

3.1.1 Let number of population groups equal to three groups and configurations of sample size consists of small sample size, medium sample size and large sample size. The details are given in **Table 1.1** as above-mentioned.

3.1.2 Let the parameters of population distributions

3.1.2.1 The population distributions are determined parameters of population distribution whereas mean and variances are equal, and skewness (γ_1) and kurtosis (γ_2) are different. The details are given in **Table 3.1** and **Figure 3.1** as follows:

Table 3.1 The parameters of population distributions under equal mean and equal variance

Population distributions	Under the ratio of variances (1: 1: 1)				
	Parameters	Mean	Variance	γ_1	γ_2
Normal	$N(8,8)$	8	8	0	0
Laplace (leptokurtic)	$L(8,2)$	8	8	0	3
Uniform (platykurtic)	$U(3.1,12.9)$	8	8	0	-1.2
Gamma (right skewed)	$G(8,1)$	8	8	0.71	0.75

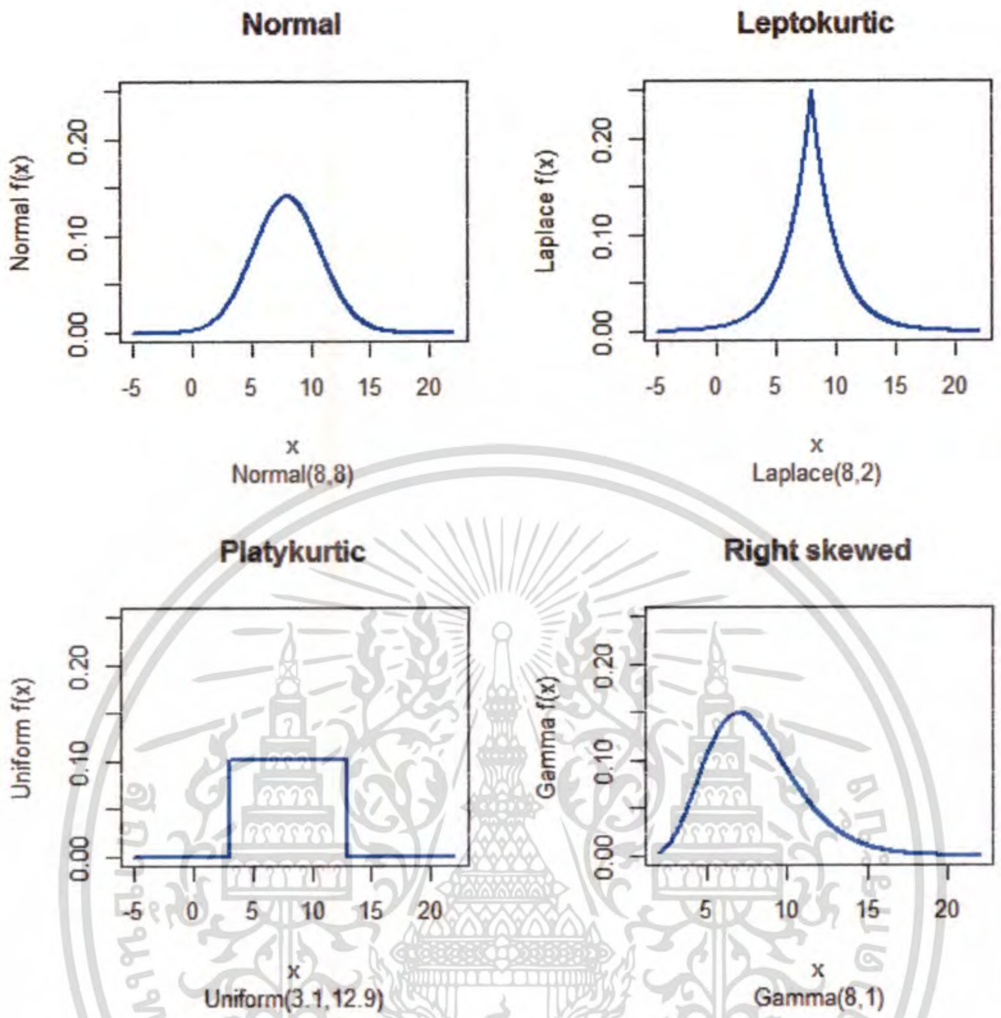


Figure 3.1 Types of population distributions

3.1.2.2 The population distribution when equal mean but unequal variances for normal distribution. The details are given in **Table 3.2** and **Figure 3.2** as follows:

Table 3.2 The normal distributions under equal mean (μ), unequal variances (σ^2) as skewness (γ_1) and excess kurtosis (γ_2) equal to 0

ϕ	Mean (μ)	The ratios of variances $(\sigma_1^2 : \sigma_2^2 : \sigma_3^2)$	Parameters of normal distribution $X_i \sim N(\mu_i, \sigma_i^2); i = 1,2,3$		
			X_1	X_2	X_3
1.25	8	1:2:4	$N(8,1)$	$N(8,2)$	$N(8,4)$
2.87	8	1:4:8	$N(8,1)$	$N(8,4)$	$N(8,8)$
6.13	8	1:8:16	$N(8,1)$	$N(8,8)$	$N(8,16)$

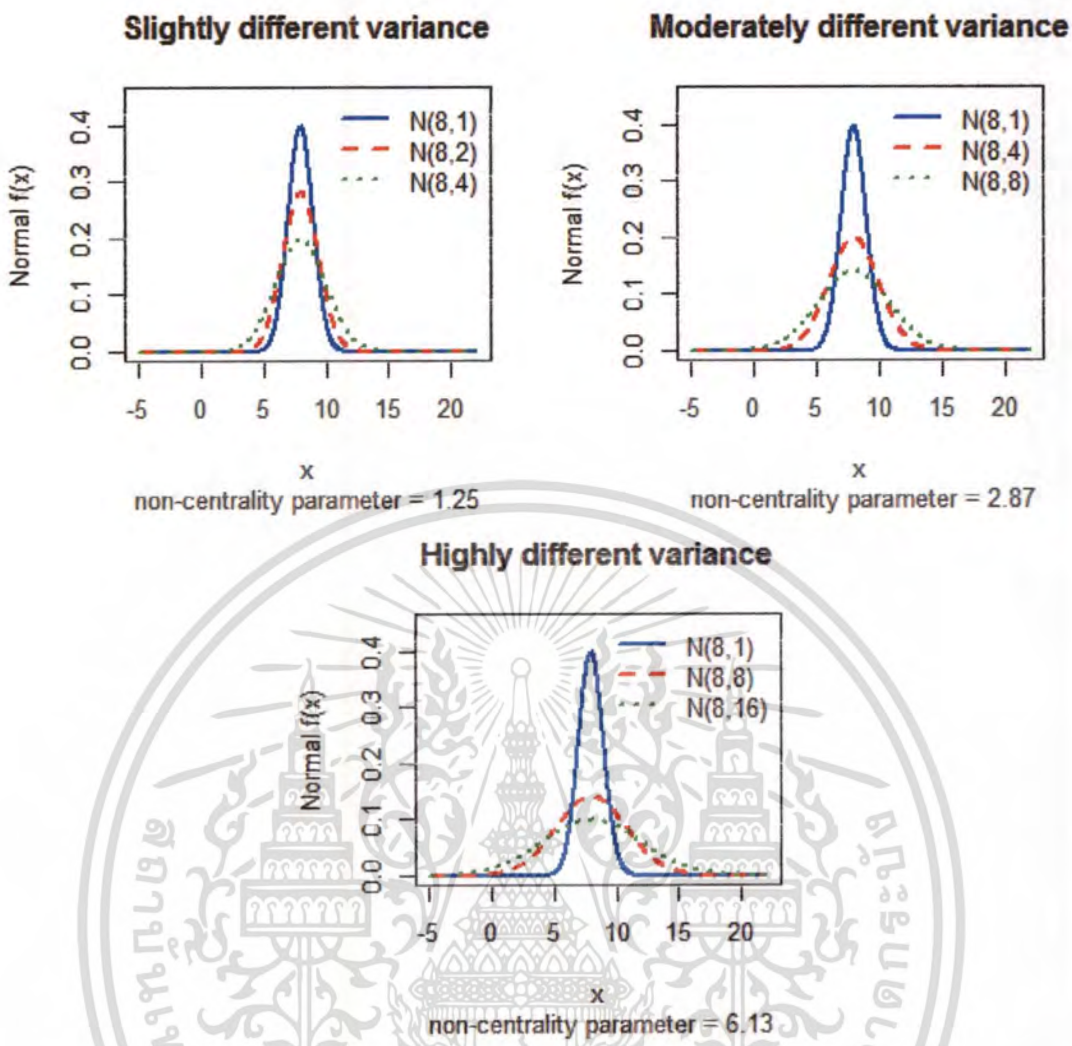


Figure 3.2 Normal probability density with different non-centrality parameters.

3.1.2.3 The population distribution when equal mean but unequal variances for Laplace distribution of k groups. The details are given in **Table 3.3** and **Figure 3.3** as follows:

Table 3.3 The Laplace distribution under equal mean (μ) , unequal variances $\left(2\lambda^2\right)$ as skewness (γ_1) equal to 0 and excess kurtosis (γ_2) equal to 3

ϕ	Mean (μ)	The ratios of variances $(2\lambda_1^2 : 2\lambda_2^2 : 2\lambda_3^2)$	Parameters of Laplace distribution $X_i \sim L(\mu_i, \lambda_i); i = 1,2,3$		
			X_1	X_2	X_3
1.25	8	1:2:4	$L(8,0.7)$	$L(8,1)$	$L(8,1.4)$
2.87	8	1:4:8	$L(8,0.7)$	$L(8,1.4)$	$L(8,2)$
6.13	8	1:8:16	$L(8,0.7)$	$L(8,2)$	$L(8,2.8)$

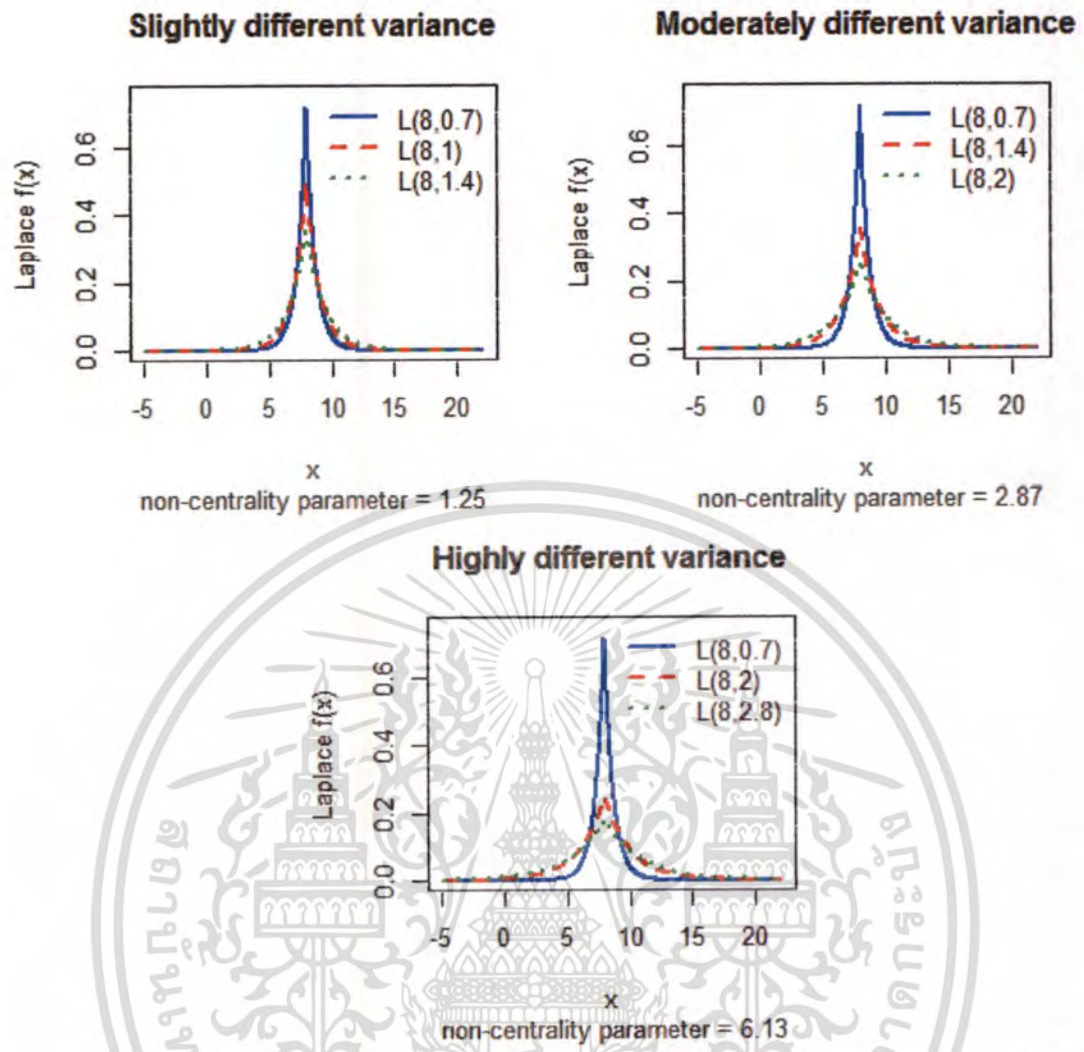


Figure 3.3 Laplace probability density with different non-centrality parameters

3.1.2.4 The population distribution when equal mean but unequal variances for uniform distribution. The details are given in **Table 3.4** and **Figure 3.4** as follows:

Table 3.4 The uniform distribution under equal mean $\left(\frac{a+b}{2}\right)$, unequal variances $\left(\frac{(b-a)^2}{12}\right)$ as skewness (γ_1) equal to 0 and excess kurtosis (γ_2) equal to -1.2

ϕ	Mean $\left(\frac{a+b}{2}\right)$	The ratios of variances $\left(\frac{(b_1-a_1)^2}{12}:\frac{(b_2-a_2)^2}{12}:\frac{(b_3-a_3)^2}{12}\right)$	Parameters of uniform distribution $X_i \sim U(a_i,b_i); i=1,2,3$		
			X_1	X_2	X_3
1.25	8	1:2:4	$U(6.27,9.73)$	$U(5.55,10.45)$	$U(4.54,11.46)$
2.87	8	1:4:8	$U(6.27,9.73)$	$U(4.54,11.46)$	$U(3.10,12.90)$
6.13	8	1:8:16	$U(6.27,9.73)$	$U(3.10,12.90)$	$U(1.07,14.93)$

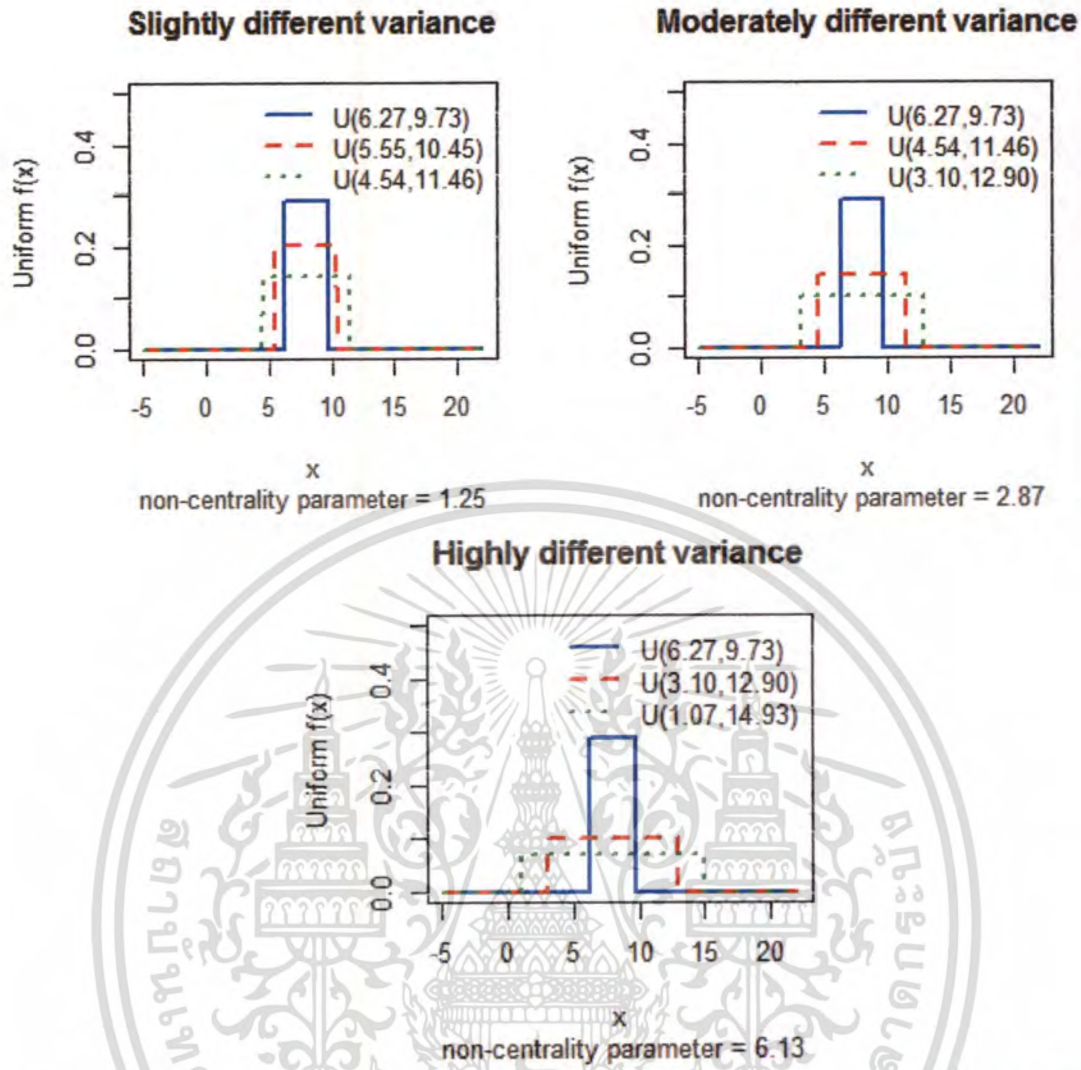


Figure 3.4 Uniform probability density with different non-centrality parameters.

3.1.2.5 The population distribution when equal mean but unequal variances for gamma distribution. The details are given in **Table 3.5 to Table 3.6** and **Figure 3.5** as follows:

Table 3.5 The gamma distribution under equal mean $(\alpha\beta)$, unequal variances $(\alpha\beta^2)$ as different skewness (γ_1) and different excess kurtosis (γ_2)

ϕ	Mean $(\alpha\beta)$	The ratios of variances $(\alpha_1\beta_1^2 : \alpha_2\beta_2^2 : \alpha_3\beta_3^2)$	Parameters of gamma distribution $X_i \sim G(\alpha_i, \beta_i); i = 1,2,3$		
			X_1	X_2	X_3
1.25	8	1:2:4	$G(64,1/8)$	$G(32,1/4)$	$G(16,1/2)$
2.87	8	1:4:8	$G(64,1/8)$	$G(16,1/2)$	$G(8,1)$
6.13	8	1:8:16	$G(64,1/8)$	$G(8,1)$	$G(4,2)$

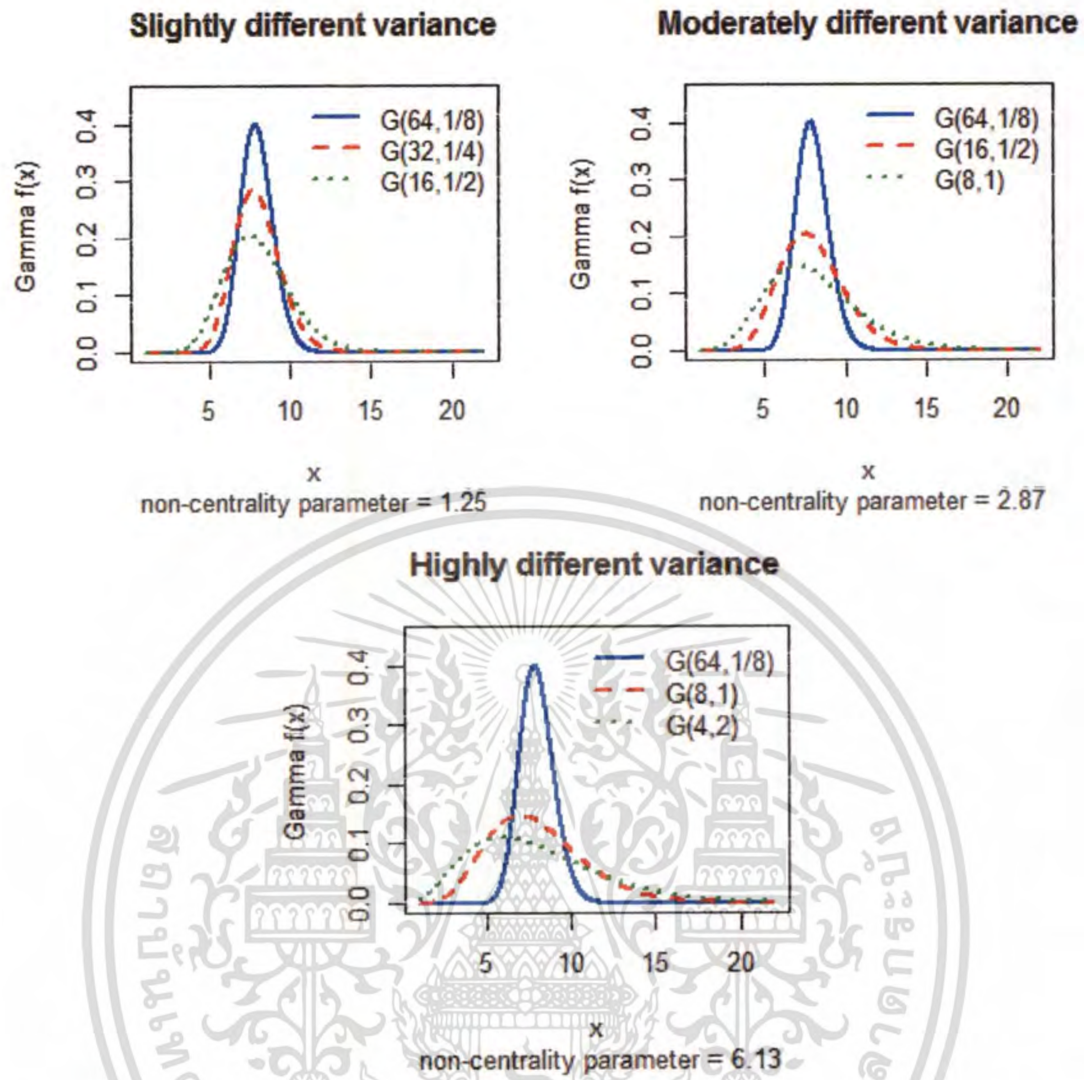


Figure 3.5 Gamma probability density with different non-centrality parameters.

Table 3.6 The different skewness (γ_1) and excess kurtosis (γ_2) of gamma distribution under equal mean $(\alpha\beta)$ and unequal variances $(\alpha\beta^2)$

The ratios of variances $(\alpha_1\beta_1^2 : \alpha_2\beta_2^2 : \alpha_3\beta_3^2)$	Parameters of gamma distribution $X_i \sim G(\alpha_i, \beta_i); i = 1,2,3$					
	X_1		X_2		X_3	
	γ_1	γ_2	γ_1	γ_2	γ_1	γ_2
1:2:4	0.25	0.09	0.35	0.19	0.5	0.38
1:4:8	0.25	0.09	0.5	0.38	0.71	0.75
1:8:16	0.25	0.09	0.71	0.75	1	1.5

3.1.4 Let significance levels (α) at 0.05 and 0.01 (Sproull, 2002; Craparo, 2007).

3.1.5 To test ability to control probability of type I error (also robustness) of homogeneity of variance tests, and then compare to criterion of Bradley in each situations.

3.1.6 Test statistics can evaluated in term of power of the test (also power) for Homogeneity of variance tests when the probability of type I error of these tests have to fall between criterion of Bradley to find the best test statistics in each situations.

3.2 Methodology

In this study, there are research methods as follows:

3.2.1 R program version 3.3.2 is written to carry out the analyses. The data are generated from normal distribution with equal variance. For all tests in this study, there are three groups are arranged as configurations of sample size. The details are given in Table 1.1

3.2.2 Three group experimental data are simulated one thousand times and these experimental data are analyzed for each sample size. All homogeneity of variance tests are computed for each experimental data. The experimental data are counted the number of times the null hypothesis is rejected when it is true at the $\alpha = 0.05$ and 0.01 levels for 1,000 experimental data with R program version 3.3.2.

3.2.3 The probability of Type I error is computed with the number of H_0 is rejected when H_0 is true divide by the number of replications 1,000 times for each test statistics, that is

$$\frac{\text{The number of null hypothesis rejection when it is true}}{\text{The number of replications 1,000 times}}$$

3.2.4 The probability of type I error is compared under the criterion of Bradley (Bradley, 1978). At significance level ($\alpha = 0.05$), the test statistic is called robustness when the probability of type I error has to fall between [0.025, 0.075] and at significance level ($\alpha = 0.01$), the test statistic is called robustness when the probability of type I error has to fall between [0.005, 0.015] in Bradley limit. If any the probability of type I error is over the limit, it shows that the test statistic isn't called robustness.

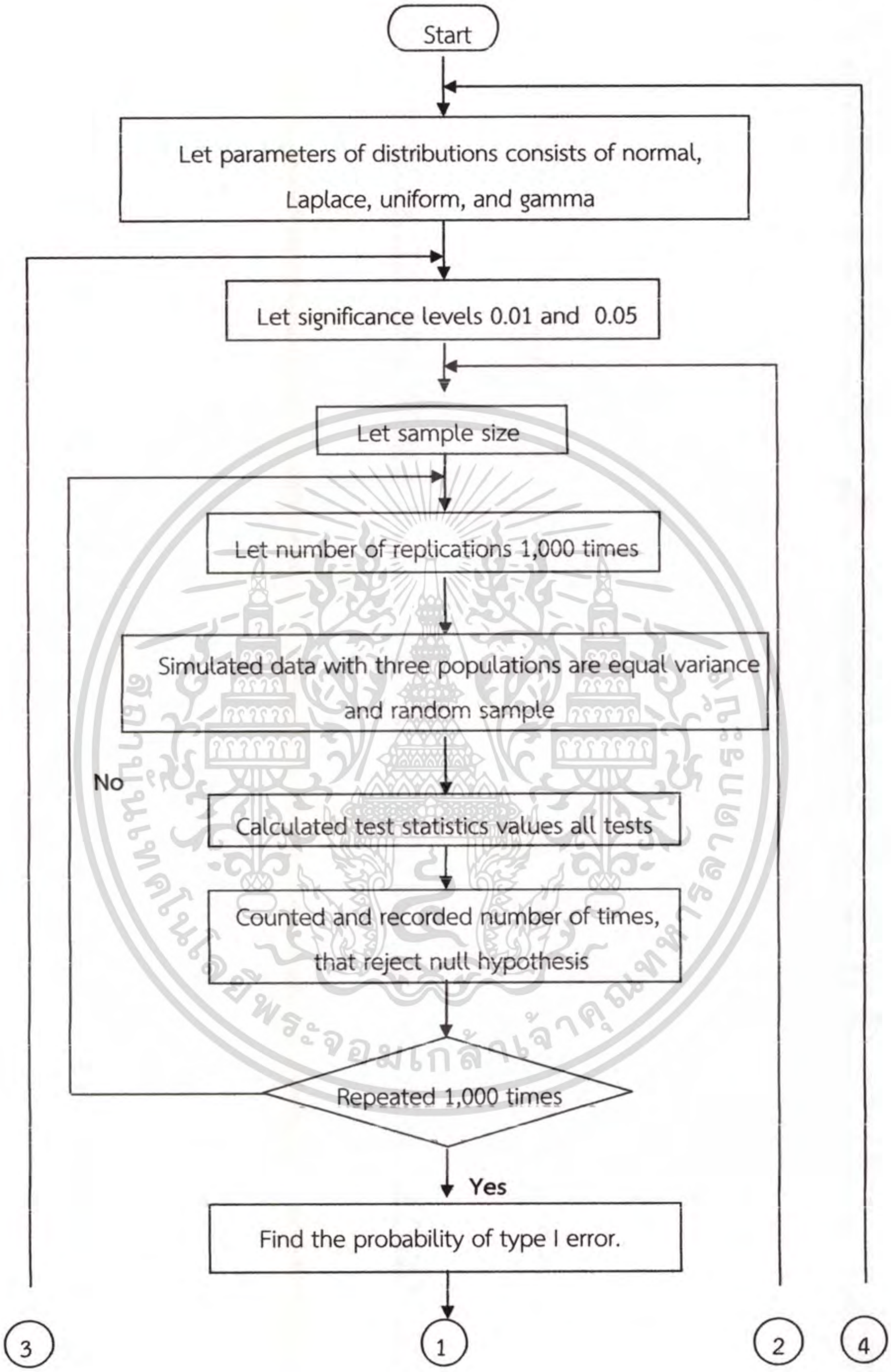
3.2.5 The analyses are repeated for the same sample size with non-normal distributions that still have equal variance. The experimental data are created for Laplace distribution (leptokurtic distribution), uniform distribution (platykurtic distribution) and gamma distribution (skewed distribution).

3.2.6 The data are generated from all distribution-types with unequal variances, the same sample size and significance level for only the test statistic is called robustness.

3.2.7 Power of a test or power is computed with the number of H_0 is rejected when H_0 is false divide by the number of replications 1,000 times for each test statistics, that is

$$\frac{\text{The number of null hypothesis rejection when it is false}}{\text{The number of replications 1,000 times}}$$

3.2.8 The maximum power of a test is 1.0 and the minimum is zero. The test statistic is called the best test when it shows the highest power or power of a test.


Start
Let parameters of distributions consists of normal,
Laplace, uniform, and gamma
Let significance levels 0.01 and 0.05
Let sample size
Let number of replications 1,000 times
Simulated data with three populations are equal variance
and random sample
No
Calculated test statistics values all tests
Counted and recorded number of times,
that reject null hypothesis
Repeated 1,000 times
Yes
Find the probability of type I error.
3
1
2
4

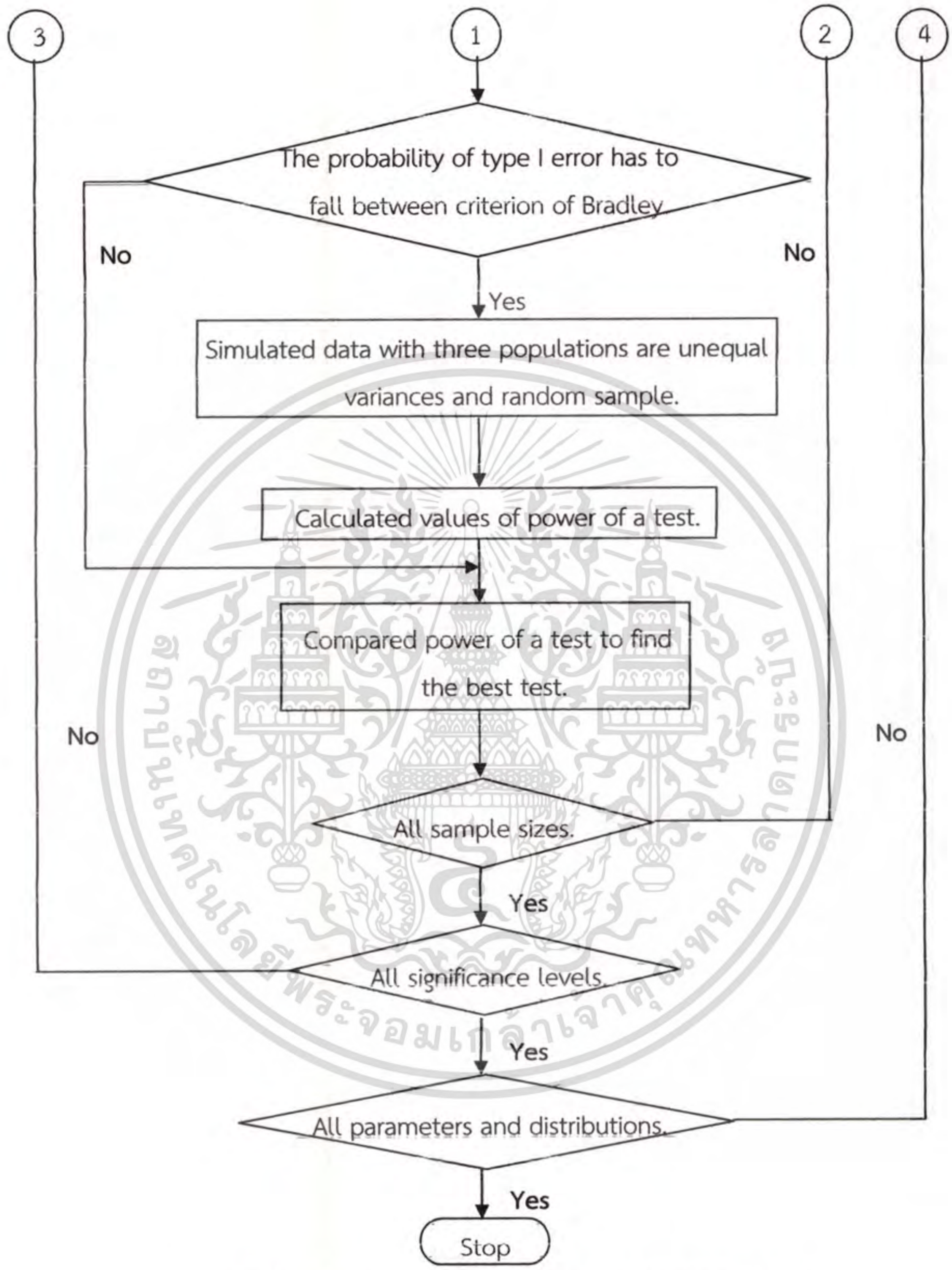


Figure 3.6 Research methodology and data processing

Research methodology and data processing can be described with procedure as diagram shown above.

Chapter 4

Results and Discussion

In this study, to compare the efficiency of four original homogeneity of variance tests consists of Levene's test (L), O'Brien's test (OB), Jackknife test (J) and Box-Andersen test (BA). For these methods, we apply to the modified central tendency as Modified Levene's test or Brown-Forsythe test (BF), Modified O'Brien's test (MOB), Modified Jackknife test (MJ), and Modified Box-Andersen test (MBA). Three populations are simulated four distributions: normal, Laplace, uniform, and gamma distributions. The conditions of sample sizes are determined under total sample sizes (30, 90, and 180) for equal and unequal sample sizes. For equal sample size, there are three conditions contain small sample size (10, 10, 10), medium sample size (30, 30, 30), and large sample size (60, 60, 60). For unequal sample size, there are three conditions contain small sample size (5, 10, 15), medium sample size (20, 30, 40), and large sample size (45, 60, 75). The data in this study is simulated though the Monte Carlo technique with 1,000 replications for each situation. The criterion employed for comparing the efficiency of test statistics are controlling of the probability of type I error according to Bradley criterion and power of a test. The significance levels are 0.01 and 0.05. The Results of this study can be summarized as two parts as follows.

4.1 The probability of Type I Error (also robustness)

Controlling of the probability of type I error of Levene's test (L), O'Brien's test (OB), Jackknife test (J), Box-Andersen test (BA), Modified Levene's test or Brown-Forsythe test (BF), Modified O'Brien's test (MOB), Modified Jackknife test (MJ), and Modified Box-Andersen test (MBA) are considered under 1,000 replications at 0.01 and 0.05 significance levels in cases of normal distribution, Laplace distribution, uniform distribution, and gamma distribution. The details are shown in **Table 4.1**, **Table 4.2** and **Figure 4.1** to **Figure 4.8**.

Table 4.1 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under normal distribution, Laplace distribution, uniform distribution, and gamma distribution at 0.01 significance level

Distribution Types	Sample sizes	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
Normal	(10, 10, 10)	0.013*	0.007*	0.009*	0.010*	0.007*	0.011*	0.014*	0.012*
	(30, 30, 30)	0.011*	0.009*	0.015*	0.010*	0.008*	0.011*	0.018	0.011*
	(60, 60, 60)	0.006*	0.010*	0.010*	0.009*	0.004	0.010*	0.011*	0.006*
	(5, 10, 15)	0.011*	0.007*	0.018	0.014*	0.003	0.021	0.025	0.016
	(20, 30, 40)	0.007*	0.008*	0.012*	0.010*	0.001	0.009*	0.016	0.009*
	(45, 60, 75)	0.010*	0.005*	0.012*	0.011*	0.008*	0.006*	0.012*	0.012*
Laplace	(10, 10, 10)	0.014*	0.003	0.022	0.008*	0.005*	0.010*	0.031	0.011*
	(30, 30, 30)	0.013*	0.007*	0.027	0.009*	0.009*	0.008*	0.030	0.009*
	(60, 60, 60)	0.015*	0.006*	0.017	0.011*	0.012*	0.006*	0.019	0.010*
	(5, 10, 15)	0.020	0.011*	0.020	0.009*	0.003	0.020	0.032	0.013*
	(20, 30, 40)	0.015*	0.007*	0.029	0.008*	0.011*	0.011*	0.034	0.011*
	(45, 60, 75)	0.004	0.002	0.023	0.003	0.003	0.002	0.023	0.003
Uniform	(10, 10, 10)	0.013*	0.004	0.006*	0.009*	0.004	0.018	0.008*	0.017
	(30, 30, 30)	0.014*	0.010*	0.009*	0.013*	0.007*	0.020	0.010*	0.021
	(60, 60, 60)	0.011*	0.009*	0.008*	0.008*	0.007*	0.018	0.009*	0.016
	(5, 10, 15)	0.012*	0.007*	0.017	0.021	0.002	0.040	0.027	0.027
	(20, 30, 40)	0.013*	0.013*	0.006*	0.011*	0.005*	0.019	0.006*	0.017
	(45, 60, 75)	0.005*	0.003	0.001	0.003	0.002	0.007*	0.002	0.006*
Gamma	(10, 10, 10)	0.021	0.011*	0.012*	0.003	0.005*	0.026	0.014*	0.008*
	(30, 30, 30)	0.017	0.006*	0.016	0.009*	0.007*	0.016	0.018	0.012*
	(60, 60, 60)	0.012*	0.005*	0.008*	0.004	0.007*	0.011*	0.011*	0.009*
	(5, 10, 15)	0.024	0.026	0.026	0.014*	0.002	0.048	0.038	0.025
	(20, 30, 40)	0.025	0.015*	0.016	0.012*	0.008*	0.020	0.024	0.017
	(45, 60, 75)	0.018	0.008*	0.011*	0.006*	0.008*	0.012*	0.015*	0.009*

* Refer to the ability to control probability of type I error under Bradley's criterion or the probability of type I error has to fall in between [0.005, 0.015] at 0.01 significance level.

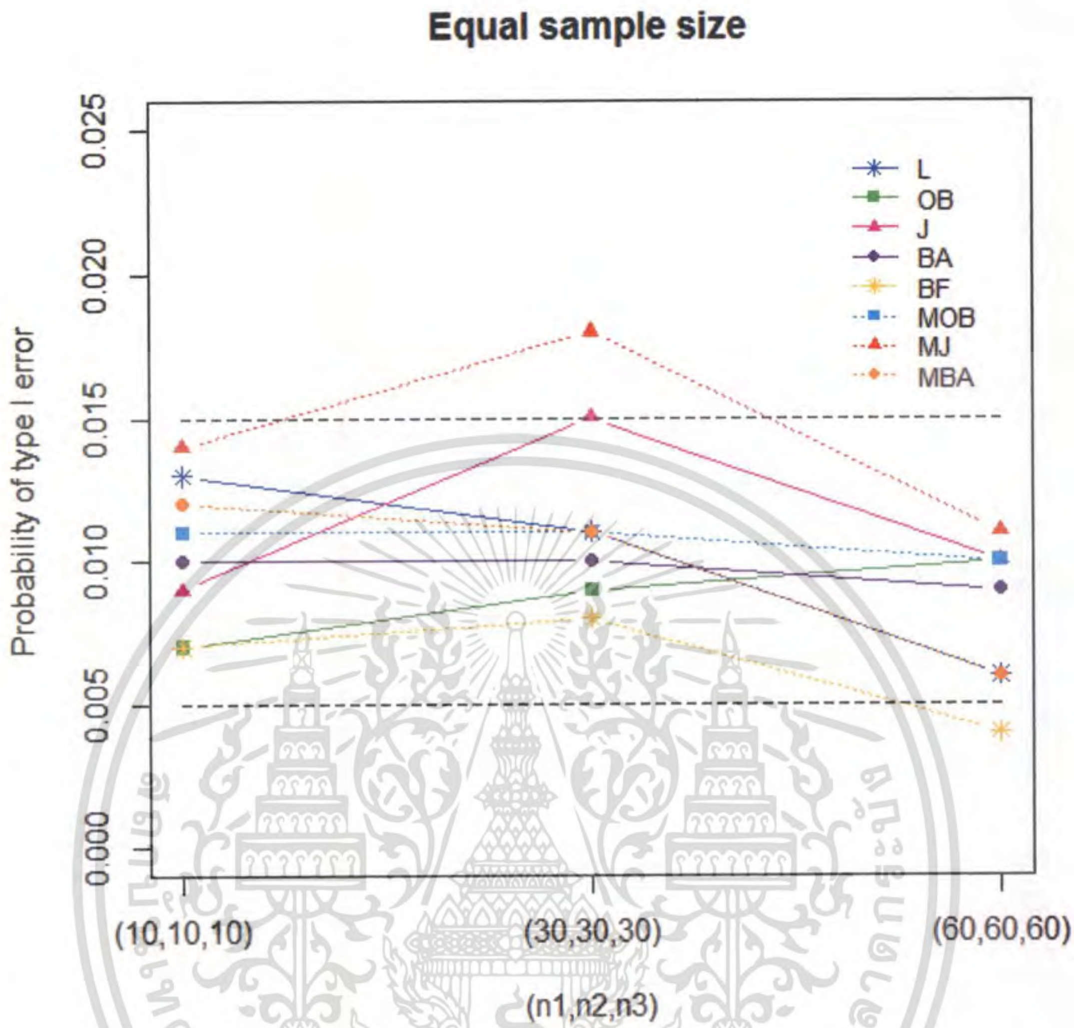


Figure 4.1 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.01) with normal distribution

The probability of type I error of these test statistics is considered under equal sample size at significance level (0.01) with normal distribution. The results in **Table 4.1** and **Figure 4.1** found that, L, OB, J, BA, MOB, and MBA tests can control probability of type I error in all cases. MJ test can not control probability of type I error only in case of medium sample size (30, 30, 30) whereas BF test can not control probability of type I error only in case of large sample size (60, 60, 60). So when sample size increases, it will decrease the probability of type I error.

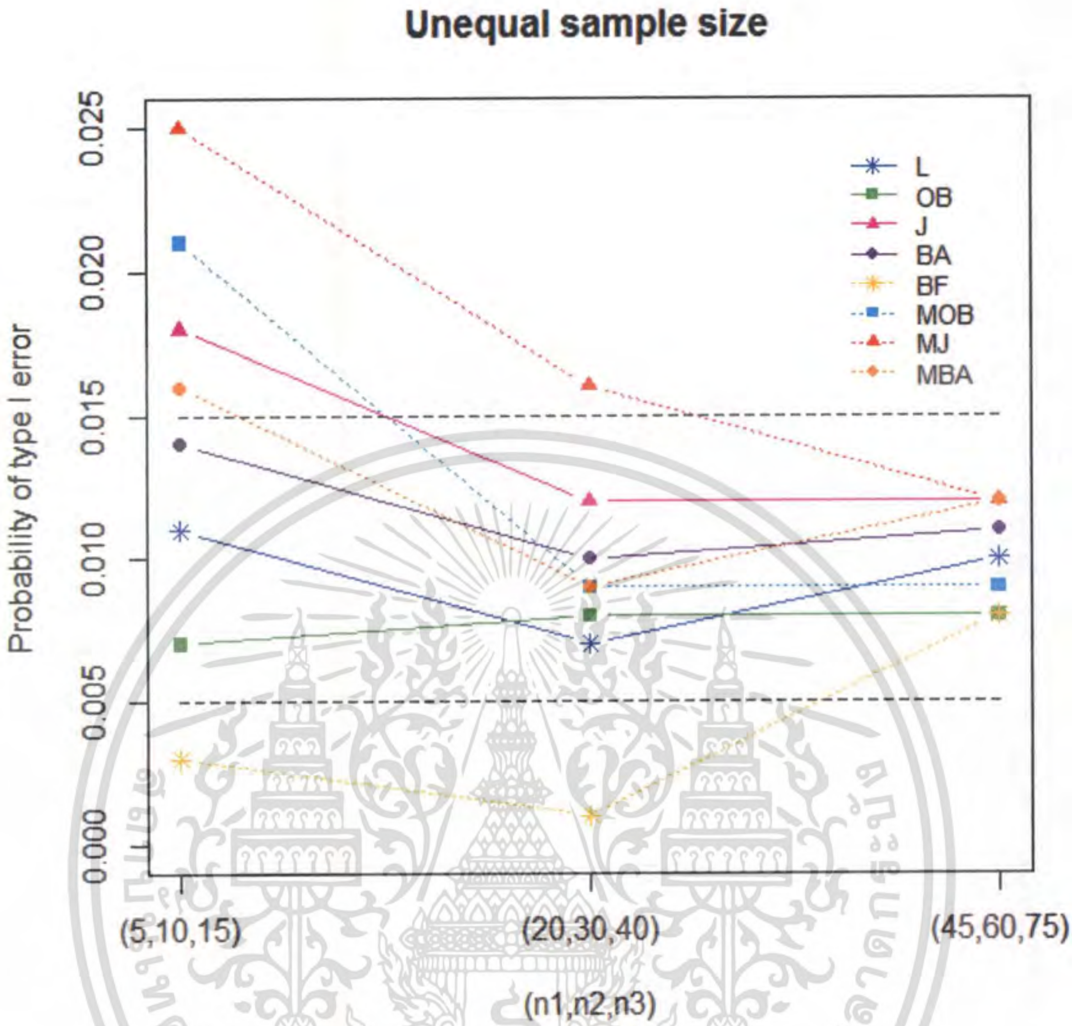


Figure 4.2 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.01) with normal distribution

The probability of type I error of these test statistics is considered under unequal sample size at significance level (0.01) with normal distribution. The results in **Table 4.1** and **Figure 4.2** found that, L, OB, and BA tests can control probability of type I error in all cases. J, MOB, and MBA tests can control probability of type I error in almost all cases expect small sample size (5, 10, 15) whereas BF and MJ tests can control probability of type I error only in case of large sample size (45, 60, 75). So when sample size increases, it will decrease the probability of type I error.

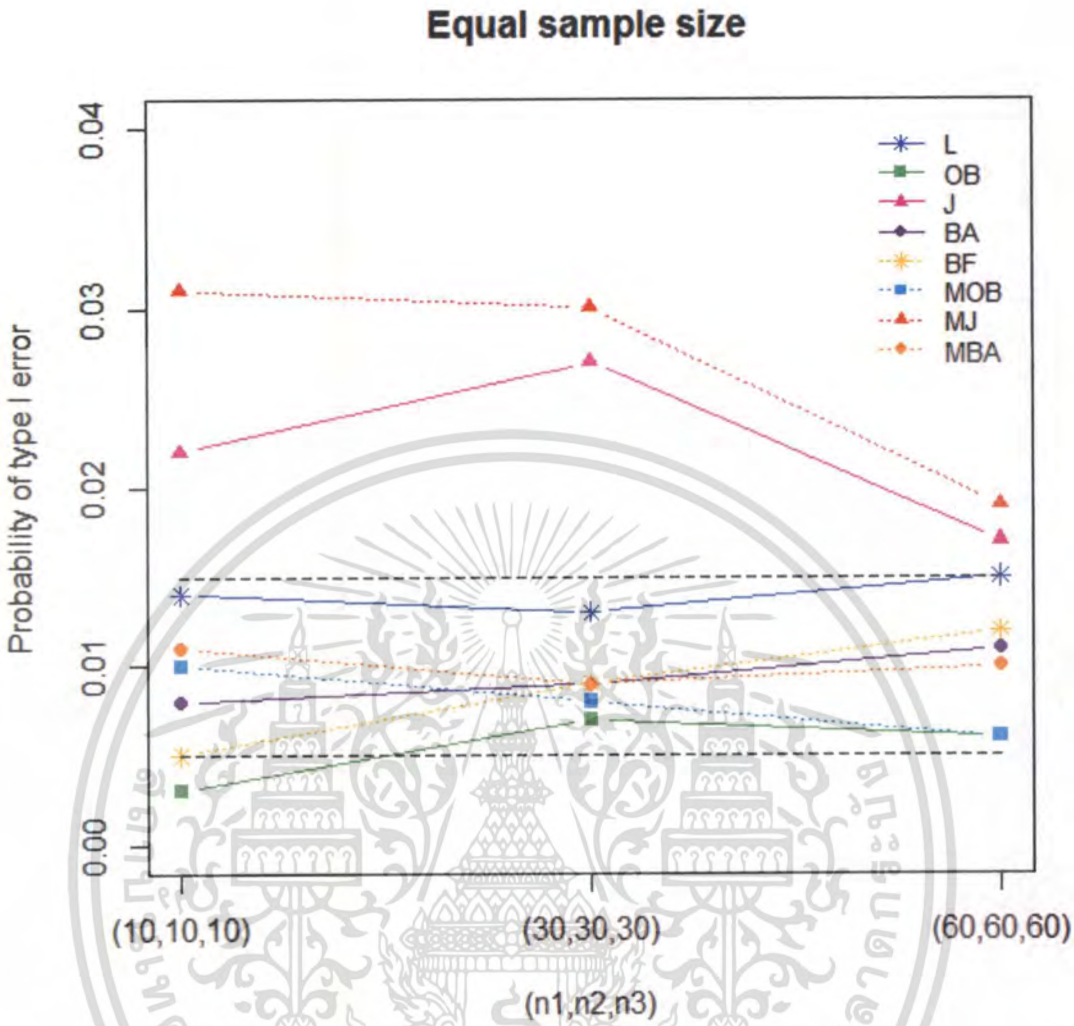


Figure 4.3 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.01) with Laplace distribution

The probability of type I error of these test statistics is considered under equal sample size at significance level (0.01) with Laplace distribution. The results in **Table 4.1** and **Figure 4.3** found that, L, BA, BF, MOB, and MBA tests can control probability of type I error in all cases. OB test can control probability of type I error in almost all cases expect small sample size (10, 10, 10) whereas J and MJ tests can not control probability of type I error in all cases. So when sample size increases, it will decrease the probability of type I error.

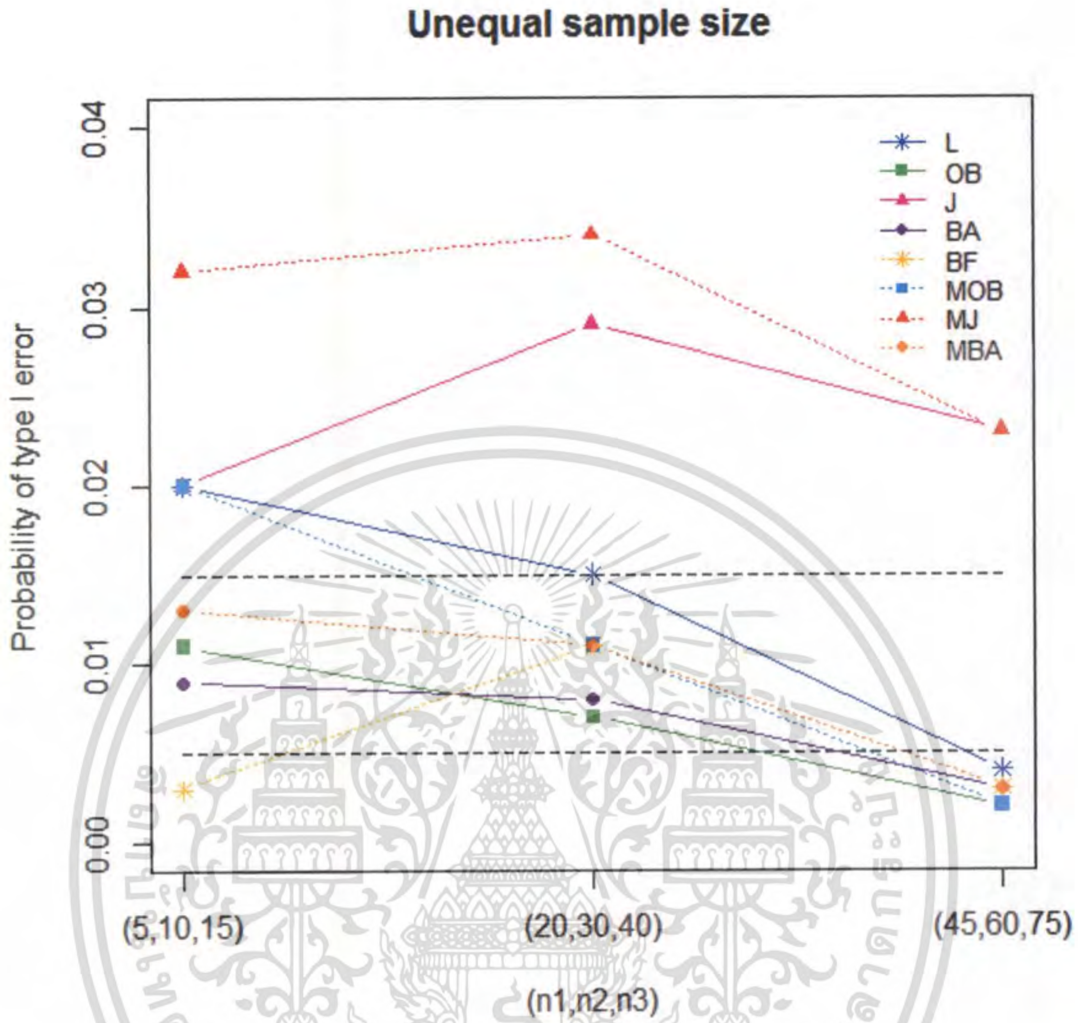


Figure 4.4 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.01) with Laplace distribution

The probability of type I error of these test statistics is considered under unequal sample size at significance level (0.01) with Laplace distribution. The results in **Table 4.1** and **Figure 4.4** found that, OB, BA, and MBA tests can control probability of type I error in almost all cases expect large sample size (45, 60, 75). L, BF, and MOB tests can control probability of type I error only in case of medium sample size (20, 30, 40) whereas J and MJ tests can not control probability of type I error in all cases. So when sample size increases, it will decrease the probability of type I error.

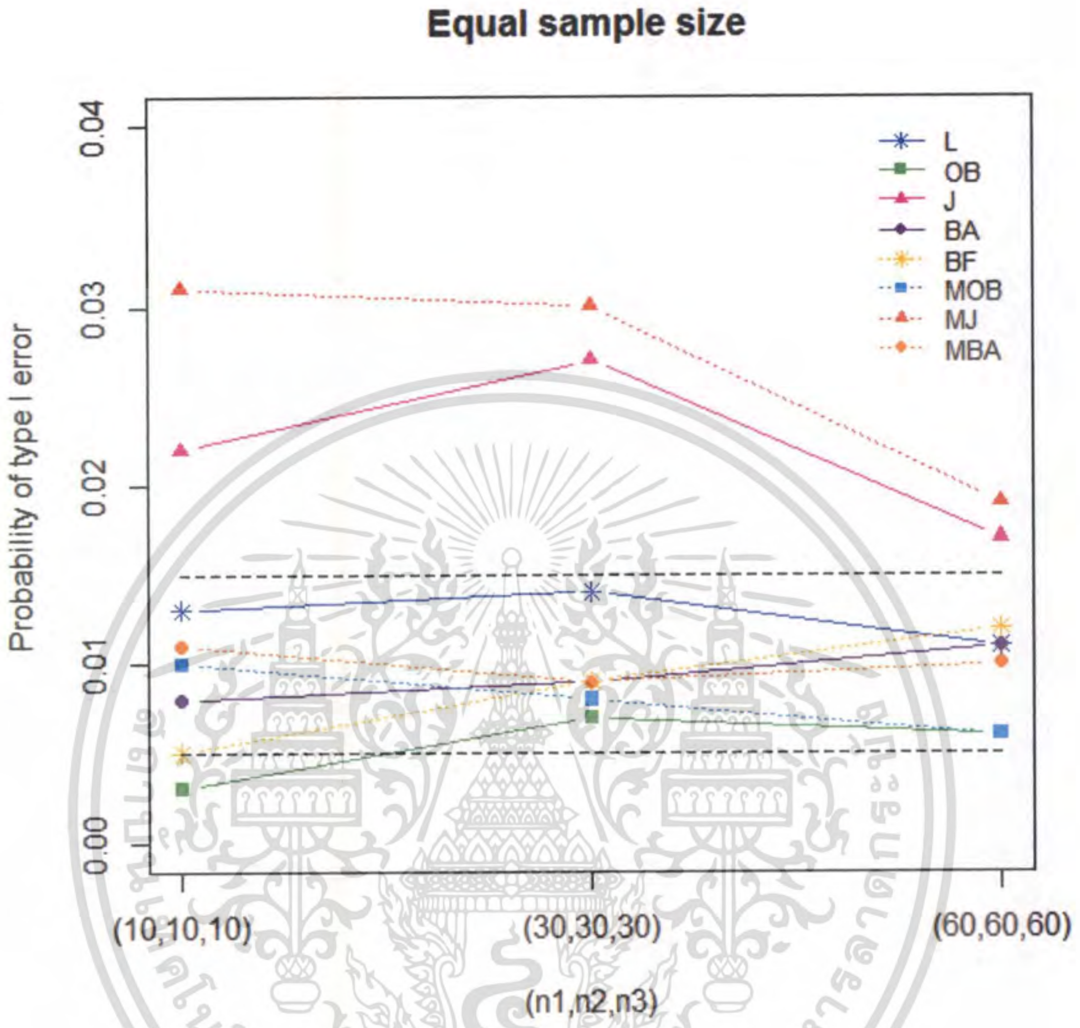


Figure 4.5 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.01) with uniform distribution

The probability of type I error of these test statistics is considered under equal sample size at significance level (0.01) with uniform distribution. The results in **Table 4.1** and **Figure 4.5** found that, L, J, BA, and MJ tests can control probability of type I error in all cases. OB and BF tests can control probability of type I error in almost all cases expect small sample size (10, 10, 10) whereas MOB and MBA tests can not control probability of type I error in all cases. So when sample size increases, it will decrease the probability of type I error.

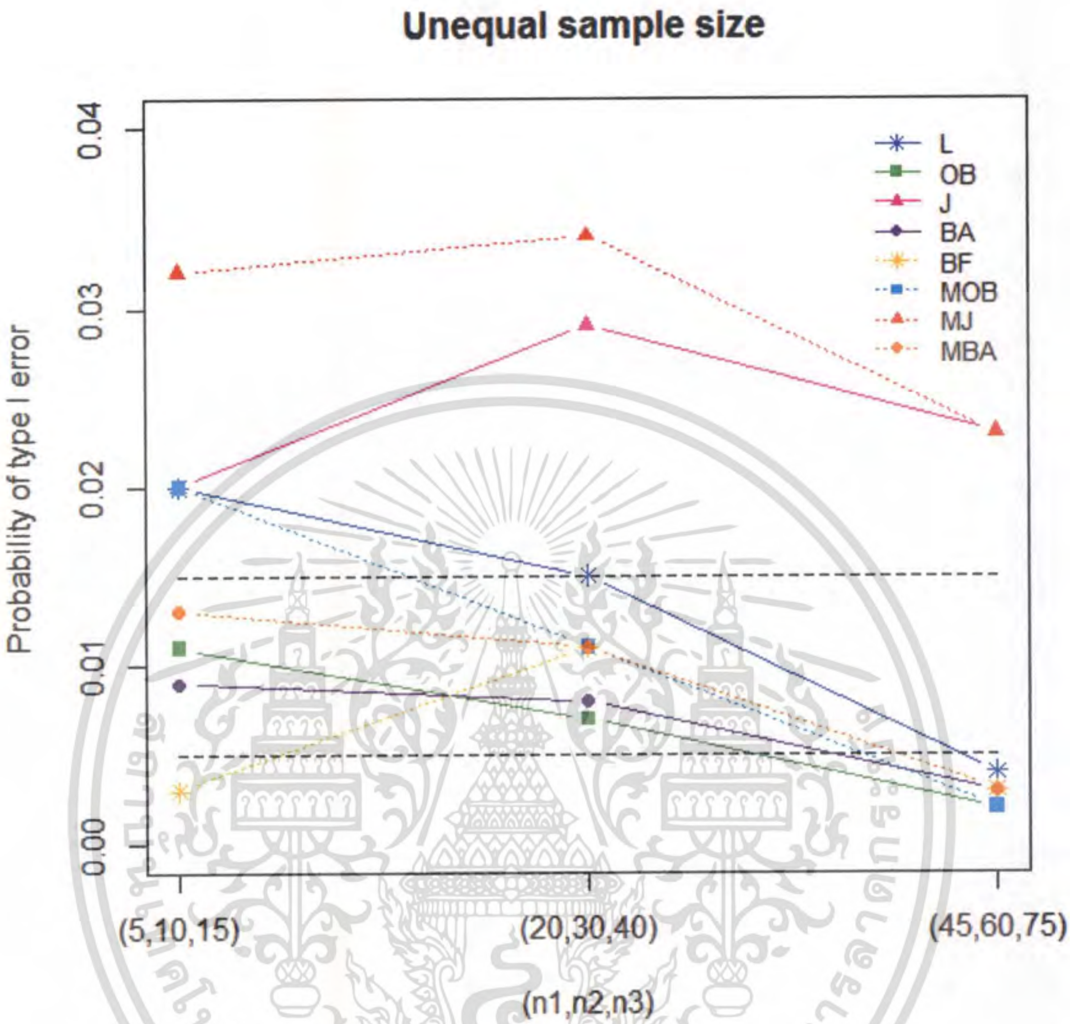


Figure 4.6 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.01) with uniform distribution

The probability of type I error of these test statistics is considered under unequal sample size at significance level (0.01) with uniform distribution. The results in **Table 4.1** and **Figure 4.6** found that, L test can control probability of type I error in all cases. OB test can control probability of type I error in almost all cases expect large sample size (45, 60, 75). J, BA, BF, and MJ tests can control probability of type I error only in cases of medium sample size (20, 30, 40) whereas MOB and MBA tests can control probability of type I error only in case of large sample size (45, 60, 75). So when sample size increases, it will decrease the probability of type I error.

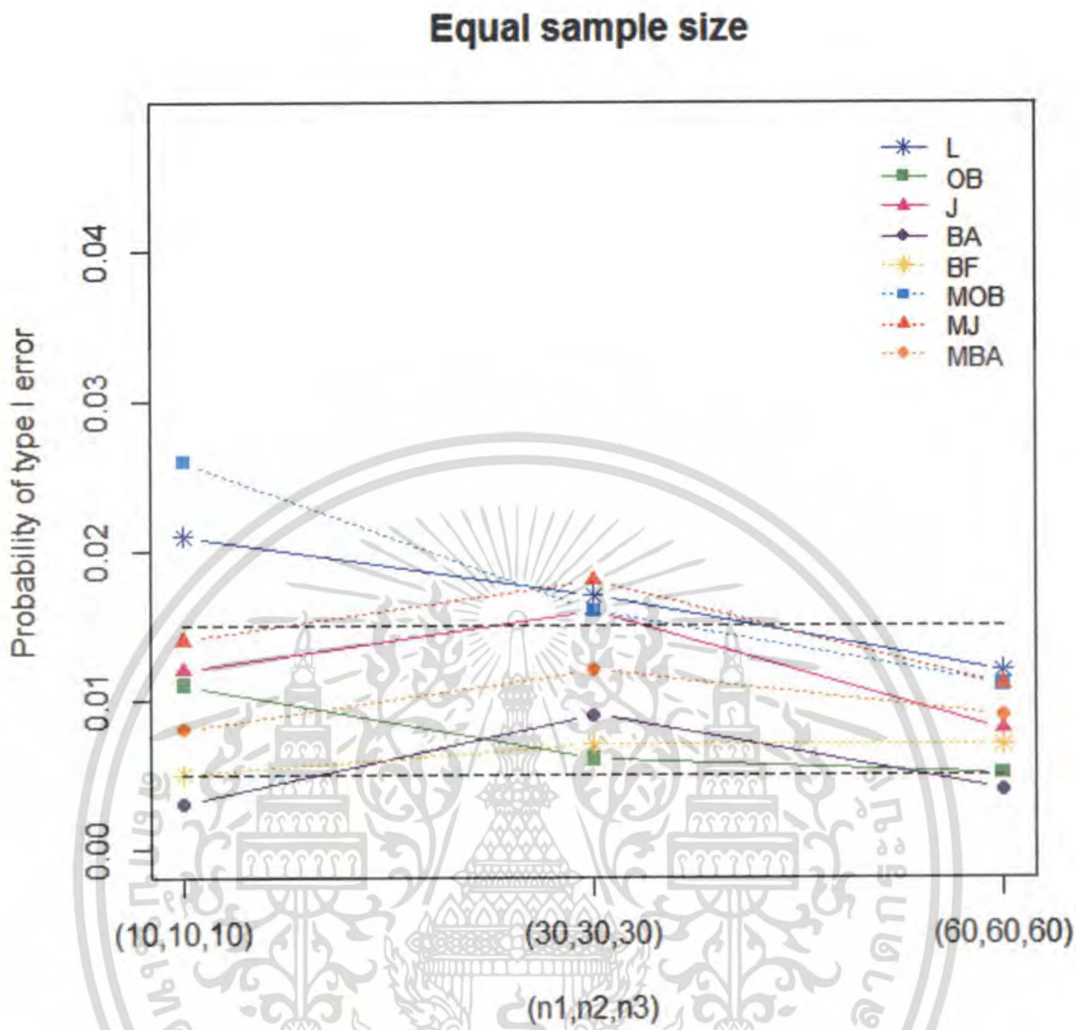


Figure 4.7 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.01) with gamma distribution

The probability of type I error of these test statistics is considered under equal sample size at significance level (0.01) with gamma distribution. The results in **Table 4.1** and **Figure 4.7** found that, OB, BF, and MBA tests can control probability of type I error in all cases. L and MOB tests can control probability of type I error only in case of large sample size (60, 60, 60). J and MJ tests can control probability of type I error in almost all cases expect medium sample size (30, 30, 30) whereas BA test can control probability of type I error only in case of medium sample size (30, 30, 30). So when sample size increases, it will decrease the probability of type I error.

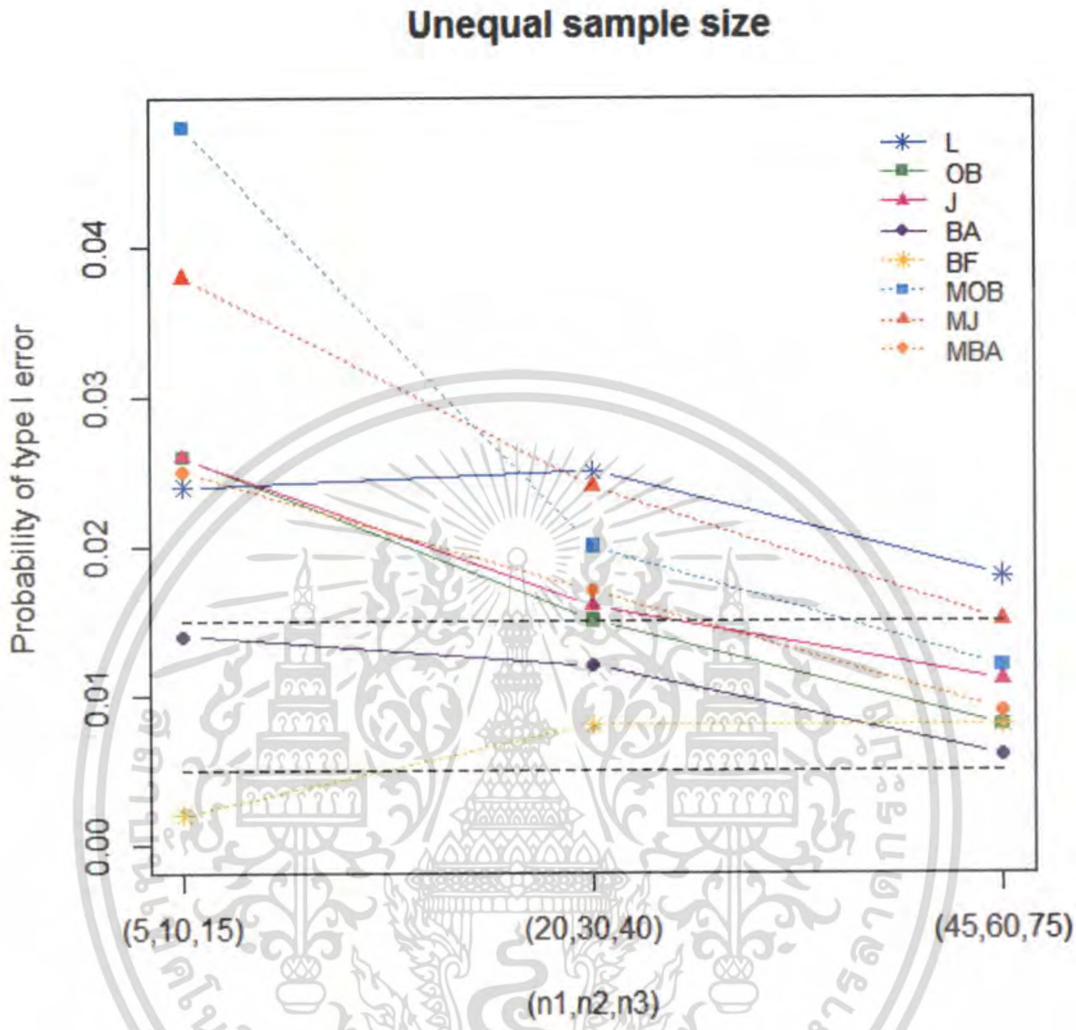


Figure 4.8 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.01) with gamma distribution

The probability of type I error of these test statistics is considered under unequal sample size at significance level (0.01) with gamma distribution. The results in **Table 4.1** and **Figure 4.8** found that, BA test can control probability of type I error in all cases. OB and BF tests can control probability of type I error in almost all cases expect small sample size (5, 10, 15). J, MOB, MJ, and MBA tests can control probability of type I error only in case of large sample size (45, 60, 75) whereas L test can not control probability of type I error in all cases. So when sample size increases, it will decrease the probability of type I error.

Table 4.2 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under normal distribution, Laplace distribution, uniform distribution, and gamma distribution at 0.05 significance level

Distribution Types	Sample sizes	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
Normal	(10, 10, 10)	0.067*	0.039*	0.046*	0.063*	0.040*	0.052*	0.059*	0.067*
	(30, 30, 30)	0.056*	0.050*	0.052*	0.056*	0.041*	0.057*	0.058*	0.059*
	(60, 60, 60)	0.050*	0.047*	0.052*	0.050*	0.047*	0.052*	0.054*	0.052*
	(5, 10, 15)	0.067*	0.036*	0.058*	0.073*	0.028*	0.075*	0.075*	0.078
	(20, 30, 40)	0.055*	0.044*	0.052*	0.053*	0.040*	0.052*	0.055*	0.054*
	(45, 60, 75)	0.050*	0.044*	0.051*	0.050*	0.045*	0.050*	0.053*	0.044*
Laplace	(10, 10, 10)	0.076	0.032*	0.089	0.069*	0.030*	0.043*	0.102	0.071*
	(30, 30, 30)	0.046*	0.046*	0.076	0.059*	0.034*	0.052*	0.083	0.058*
	(60, 60, 60)	0.060*	0.042*	0.065*	0.045*	0.052*	0.046*	0.067*	0.045*
	(5, 10, 15)	0.076	0.045*	0.096	0.065*	0.028*	0.073*	0.119	0.077
	(20, 30, 40)	0.075*	0.052*	0.102	0.064*	0.054*	0.059*	0.112	0.064*
	(45, 60, 75)	0.056*	0.037*	0.065*	0.047*	0.044*	0.042*	0.066*	0.049*
Uniform	(10, 10, 10)	0.054*	0.034*	0.028*	0.047*	0.023	0.060*	0.036*	0.055*
	(30, 30, 30)	0.065*	0.058*	0.035*	0.065*	0.034*	0.074*	0.046*	0.074*
	(60, 60, 60)	0.060*	0.053*	0.044*	0.058*	0.043*	0.070*	0.046*	0.069*
	(5, 10, 15)	0.066*	0.034*	0.039*	0.066*	0.019	0.099	0.059*	0.091
	(20, 30, 40)	0.055*	0.049*	0.028*	0.048*	0.031*	0.069*	0.035*	0.060*
	(45, 60, 75)	0.034*	0.037*	0.031*	0.042*	0.025*	0.054*	0.035*	0.053*
Gamma	(10, 10, 10)	0.077	0.047*	0.059*	0.062*	0.034*	0.078	0.072*	0.077
	(30, 30, 30)	0.079	0.049*	0.067*	0.055*	0.041*	0.068*	0.078	0.069*
	(60, 60, 60)	0.062*	0.039*	0.044*	0.042*	0.038*	0.051*	0.047*	0.049*
	(5, 10, 15)	0.082	0.052*	0.058*	0.078	0.028*	0.091	0.075*	0.090
	(20, 30, 40)	0.080	0.046*	0.052*	0.061*	0.048*	0.068*	0.055*	0.069*
	(45, 60, 75)	0.066*	0.038*	0.051*	0.045*	0.047*	0.052*	0.053*	0.054*

* Refer to the ability to control probability of type I error under Bradley's criterion or the probability of type I error has to fall in between [0.025, 0.075] at 0.05 significance level.

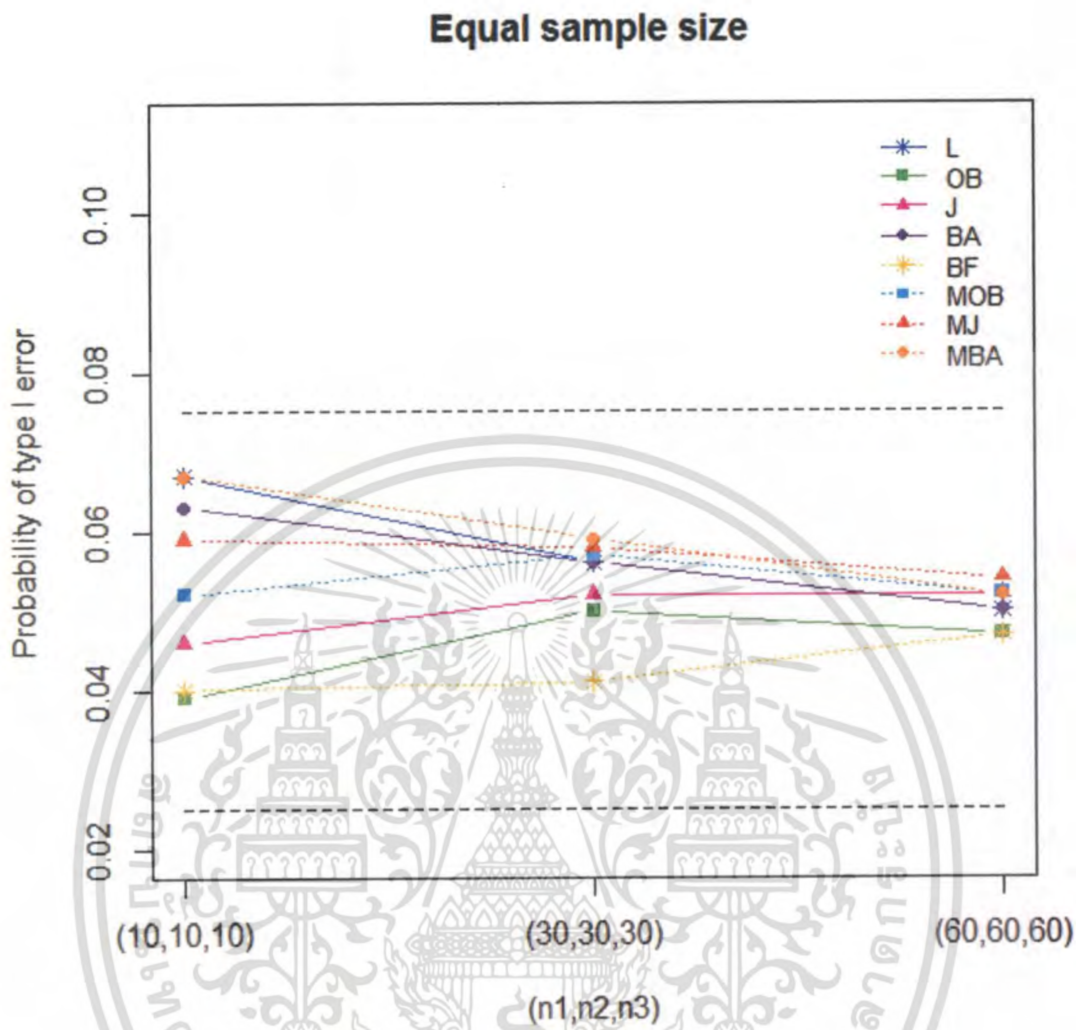


Figure 4.9 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.05) with normal distribution

The probability of type I error of these test statistics is considered under equal sample size at significance level (0.05) with normal distribution. The results in **Table 4.2** and **Figure 4.16** found that, all test can control probability of type I error in all cases. So when sample size increases, it will decrease the probability of type I error.

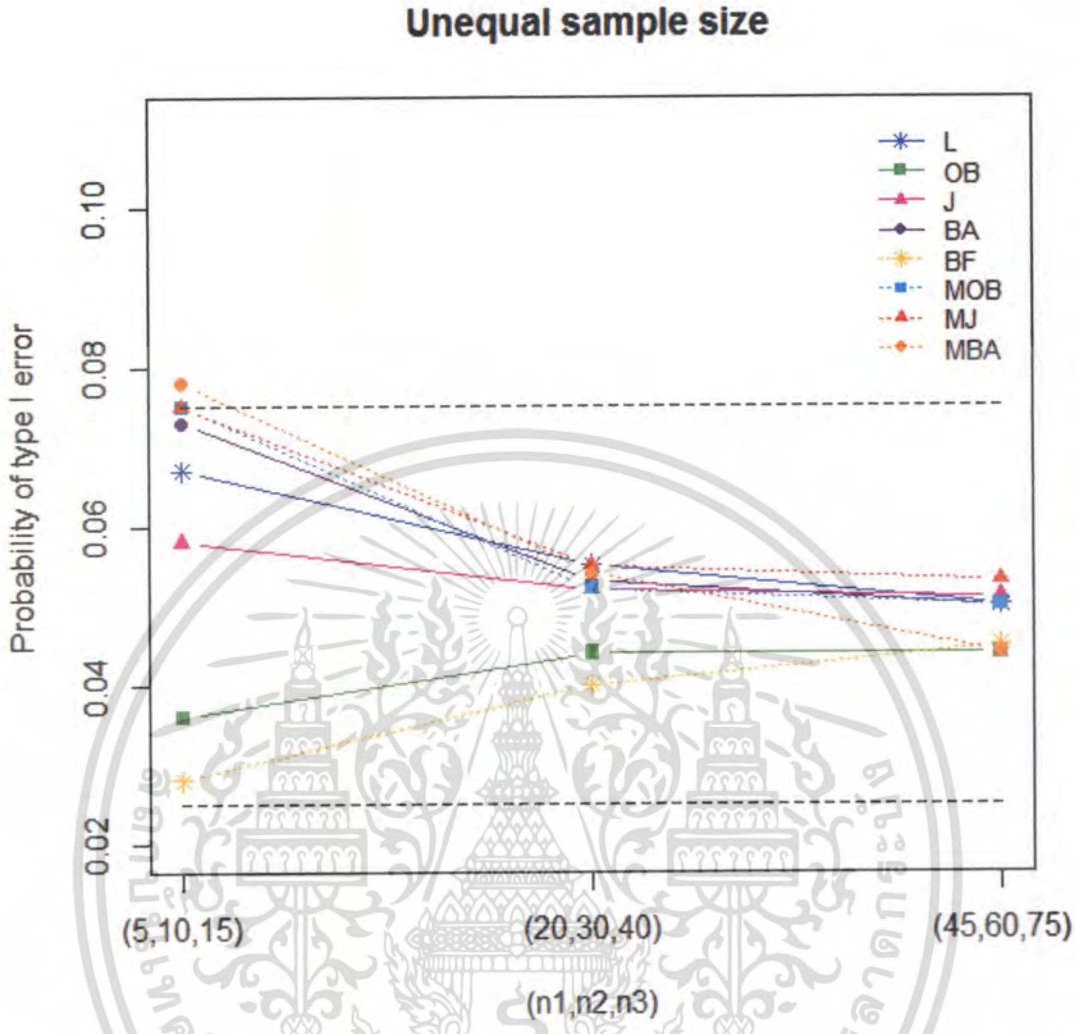


Figure 4.10 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.05) with normal distribution

The probability of type I error of these test statistics is considered under unequal sample size at significance level (0.05) with normal distribution. The results in **Table 4.2** and **Figure 4.10** found that, BL, OB, J, BA, BF, MOB, and MJ tests can control probability of type I error in all cases whereas MBA test can control probability of type I error in almost all cases expect small sample size (5, 10, 15). So when sample size increases, it will decrease the probability of type I error.

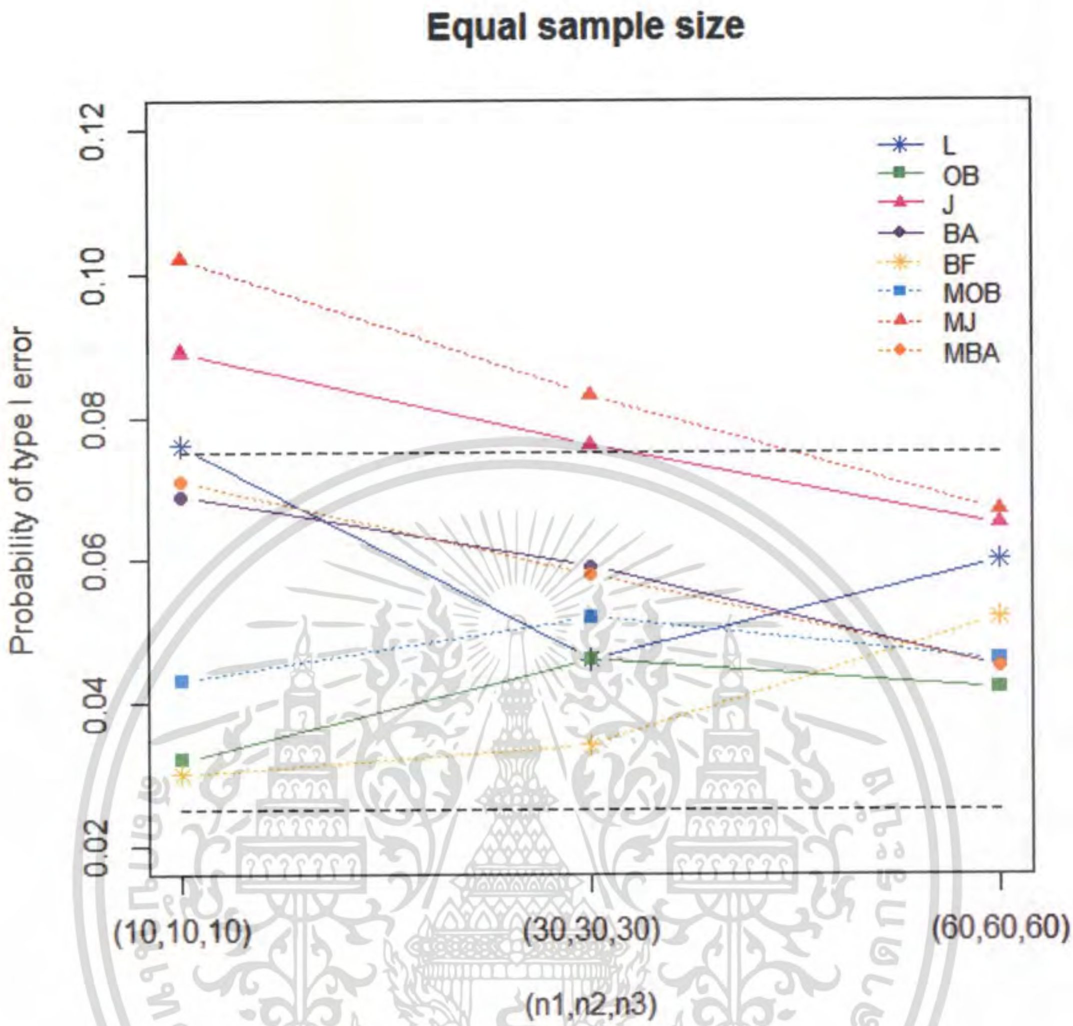


Figure 4.11 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.05) with Laplace distribution

The probability of type I error of these test statistics is considered under equal sample size at significance level (0.05) with Laplace distribution. The results in **Table 4.2** and **Figure 4.11** found that, OB, BA, BF, MOB, and MBA tests can control probability of type I error in all cases. L test can control probability of type I error in almost all cases expect small sample size (10, 10, 10) whereas J and MJ tests can control probability of type I error only in case of large sample size (60, 60, 60). So when sample size increases, it will decrease the probability of type I error.

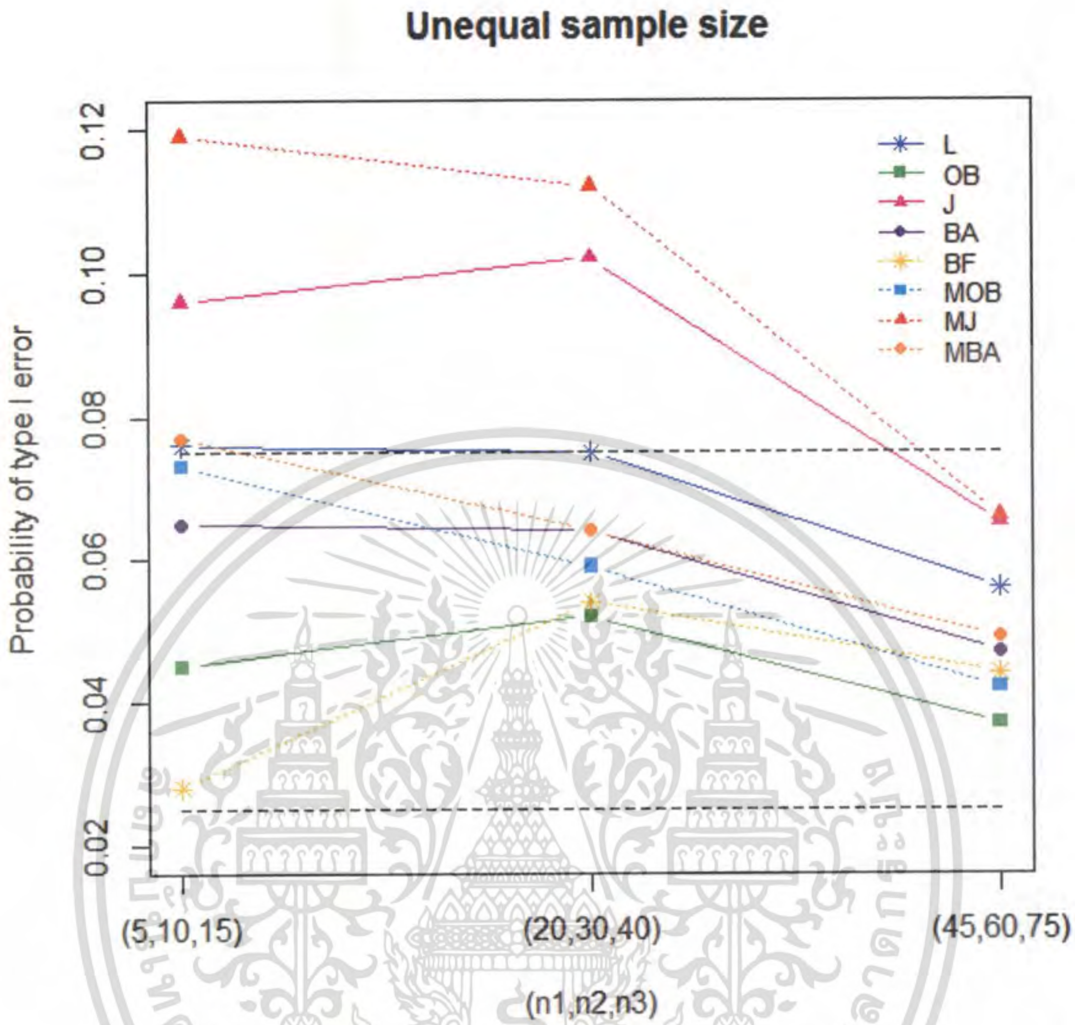


Figure 4.12 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.05) with Laplace distribution

The probability of type I error of these test statistics is considered under unequal sample size at significance level (0.05) with Laplace distribution. The results in **Table 4.2** and **Figure 4.12** found that, OB, BA, BF, MOB, and MBA tests can control probability of type I error in all cases. L test can control probability of type I error in almost all cases expect small sample size (5, 10, 15) whereas J and MJ tests can control probability of type I error only in case of large sample size (45, 60, 75). So when sample size increases, it will decrease the probability of type I error.

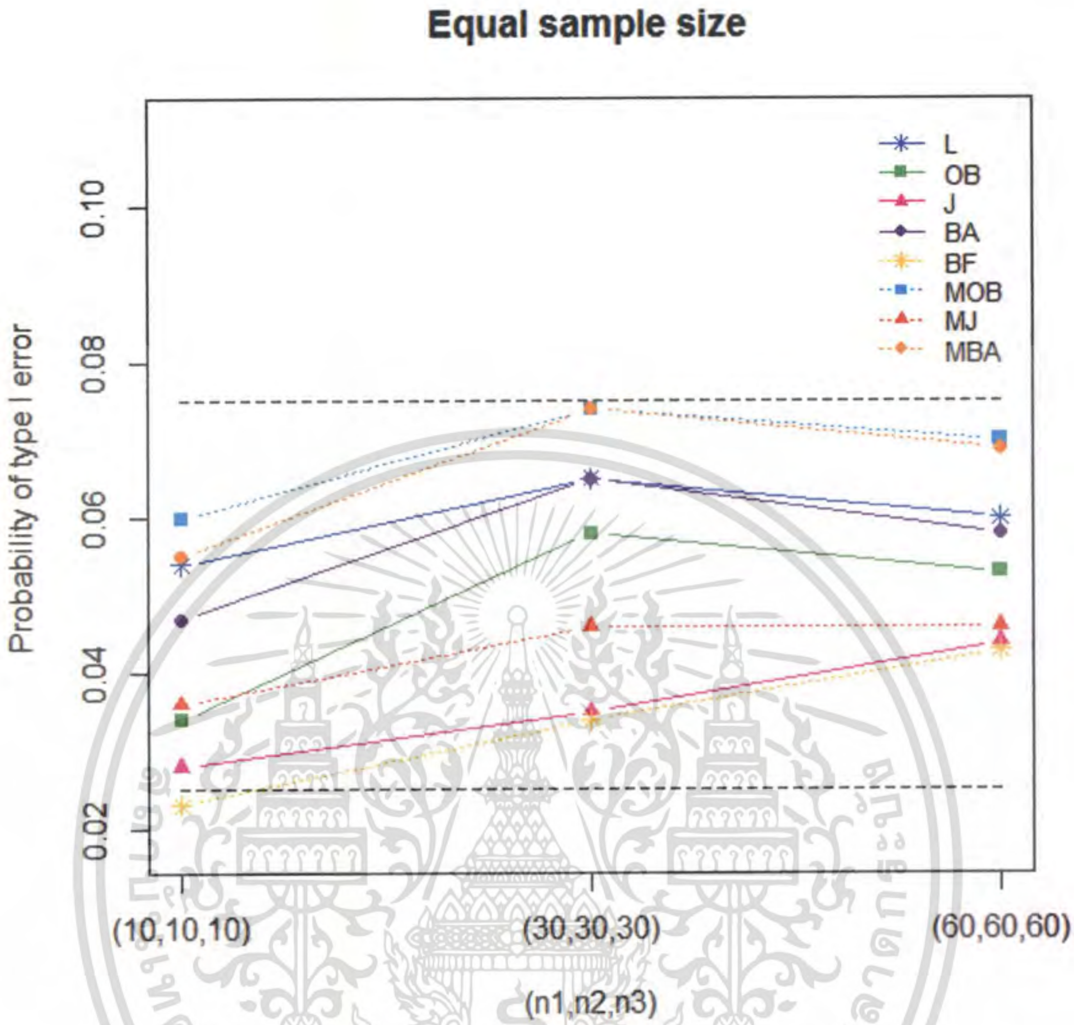


Figure 4.13 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.05) with uniform distribution

The probability of type I error of these test statistics is considered under equal sample size at significance level (0.05) with uniform distribution. The results in **Table 4.2** and **Figure 4.13** found that, L, OB, J, BA, BF, MOB, MJ, and MBA tests can control probability of type I error in all cases whereas BF test can control probability of type I error in almost all cases expect small sample size (10, 10, 10). So when sample size increases, it will decrease the probability of type I error.

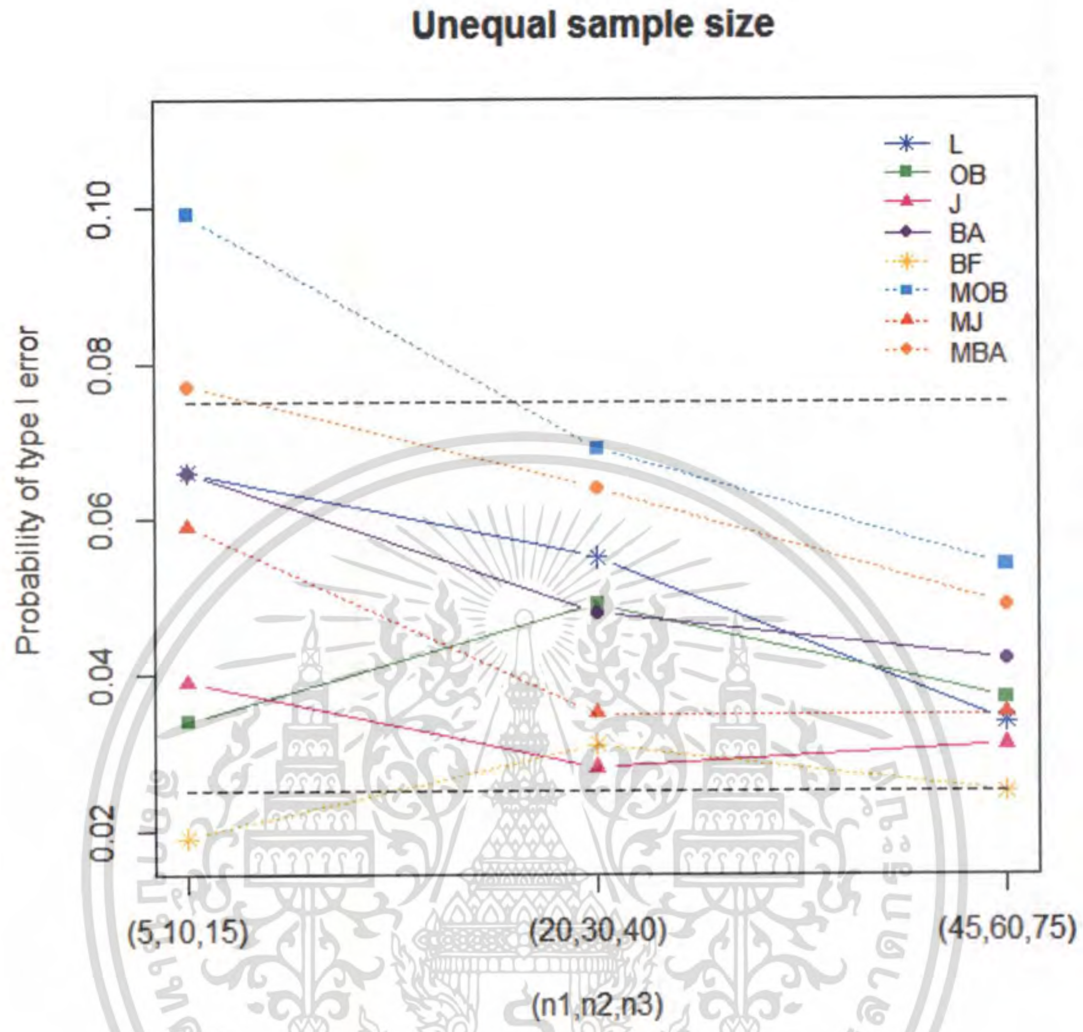


Figure 4.14 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.05) with uniform distribution

The probability of type I error of these test statistics is considered under unequal sample size at significance level (0.05) with uniform distribution. The results in **Table 4.2** and **Figure 4.14** found that, L, OB, J, BA, and MJ tests can control probability of type I error in all cases whereas BF, MOB, and MBA tests can control probability of type I error in almost all cases expect small sample size (5, 10, 15). So when sample size increases, it will decrease type I error.

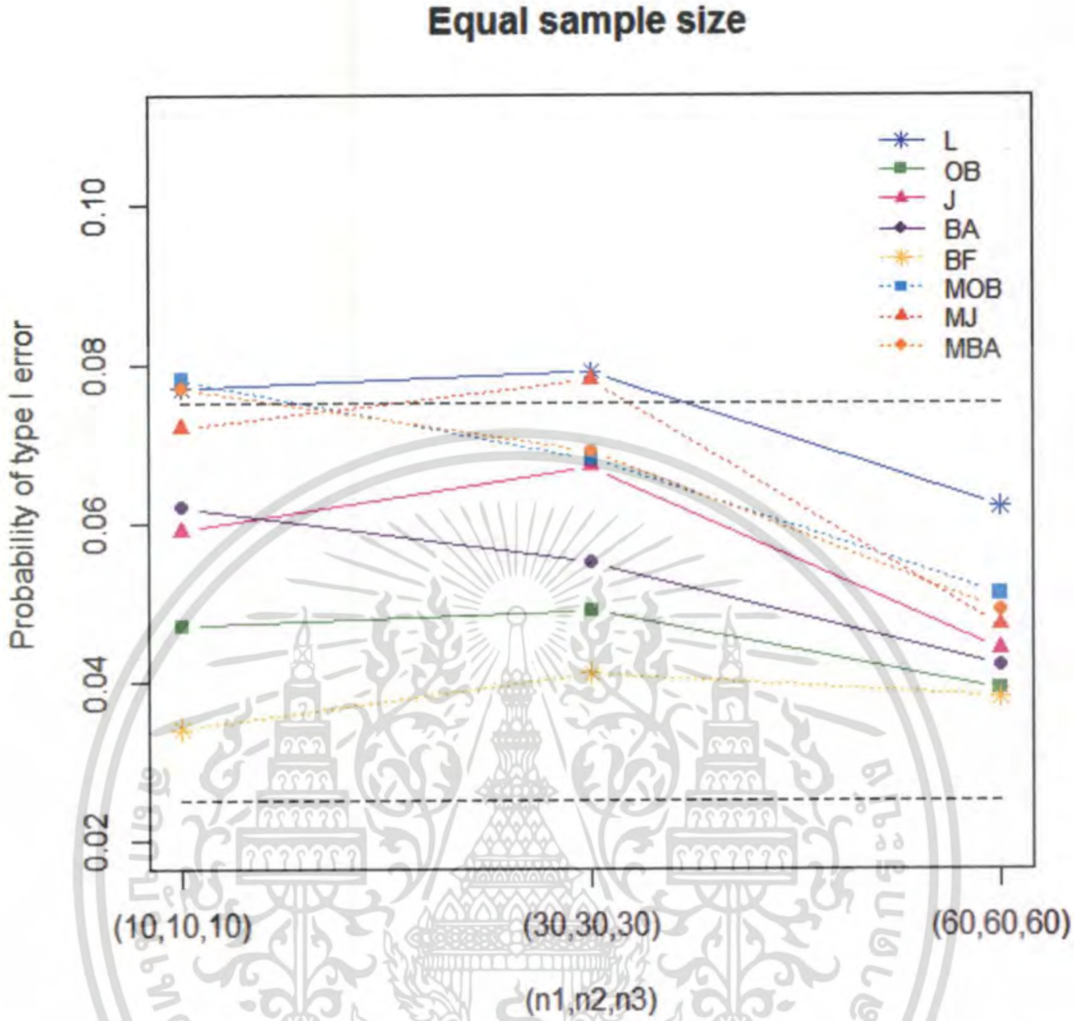


Figure 4.15 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.05) with gamma distribution

The probability of type I error of these test statistics is considered under equal sample size at significance level (0.05) with gamma distribution. The results in **Table 4.2** and **Figure 4.15** found that OB, J, BA, and BF tests can control probability of type I error in all cases. L test can control probability of type I error only in case of large sample size (60, 60, 60). MOB and MBA tests can control probability of type I error in almost all cases expect small sample size (10, 10, 10) whereas MJ test can control probability of type I error in almost all cases expect medium sample size (30, 30, 30). So when sample size increases, it will decrease the probability of type I error.

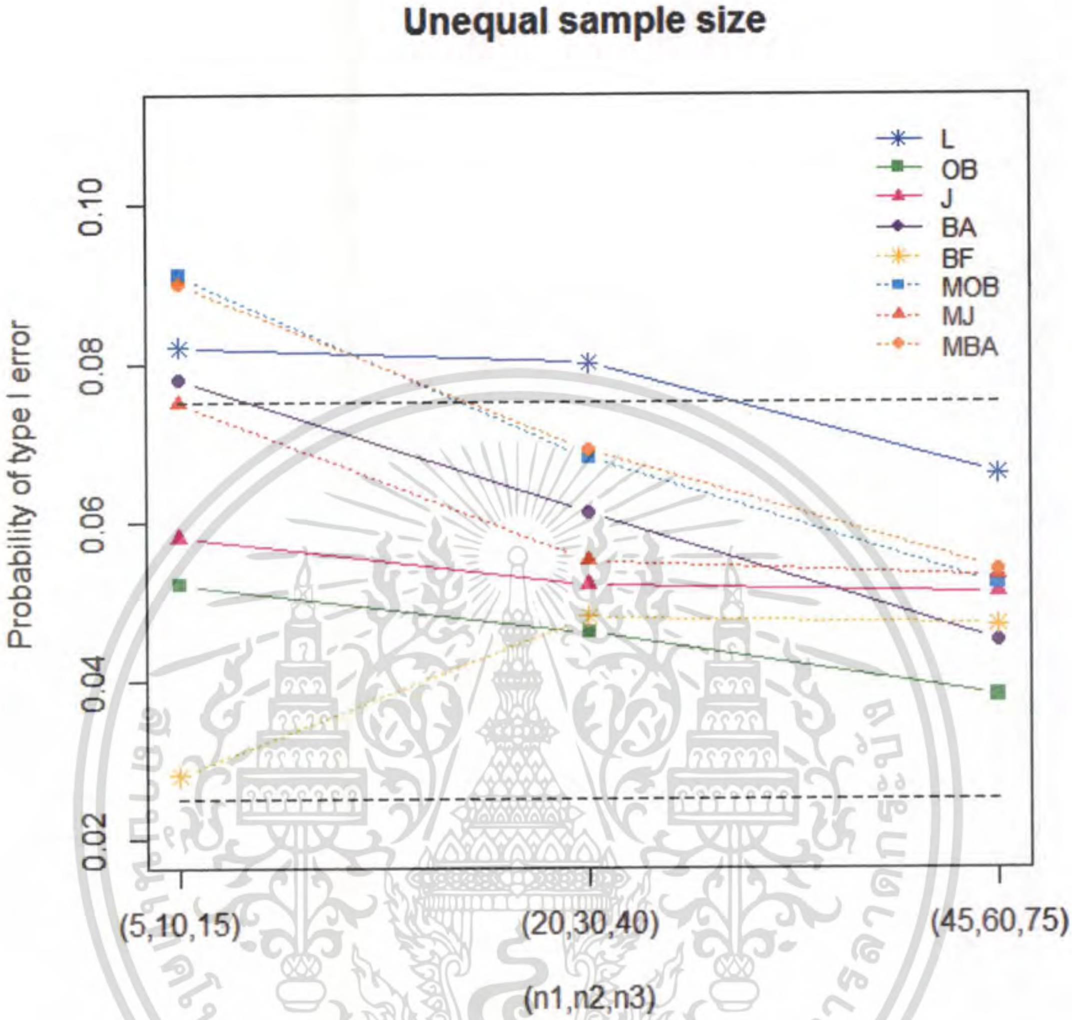


Figure 4.16 The probability of type I error of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.05) with gamma distribution

The probability of type I error of these test statistics is considered under unequal sample size at significance level (0.05) with gamma distribution. The results in **Table 4.2** and **Figure 4.16** found that OB, J, BF, and MJ tests can control probability of type I error in all cases. L test can control probability of type I error only in case of large sample size (45, 60, 75) whereas BA, MOB and MBA tests can control probability of type I error in almost all cases expect small sample size (5, 10, 15). So when sample size increases, it will decrease the probability of type I error.

4.2 Power of a test (also power)

Power of a test of Levene's test (L), O'Brien's test (OB), Jackknife test (J), Box-Andersen test (BA), Modified Levene's test or Brown-Forsythe test (BF), Modified O'Brien's test (MOB), Modified Jackknife test (MJ), and Modified Box-Andersen test (MBA) are considered when under 1,000 replications at 0.01 and 0.05 significance levels in cases of normal distribution, Laplace distribution, uniform distribution, and gamma distribution. The details are shown in **Table 4.3** to **Table 4.10** and **Figure 4.17** to **Figure 4.32**.

Table 4.3 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under normal distribution at significance level (0.01)

Sample sizes	Non-centrality parameter(ϕ)	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
(10, 10, 10)	1.25	0.154	0.055	0.146	0.113	0.071	0.083	0.170*	0.122
	2.87	0.309	0.098	0.402	0.327	0.158	0.133	0.454*	0.326
	6.13	0.528	0.139	0.724	0.605	0.325	0.186	0.762*	0.612
(30, 30, 30)	1.25	0.637	0.558	0.715*	0.645	0.578	0.577	-	0.643
	2.87	0.968	0.859	0.987*	0.970	0.942	0.879	-	0.973
	6.13	0.999	0.956	1.000*	0.999	0.998	0.963	-	0.999
(60, 60, 60)	1.25	0.967	0.970	0.987*	0.975	-	0.971	0.987*	0.975
	2.87	1.000*	0.999	1.000*	1.000*	-	0.999	1.000*	1.000*
	6.13	1.000*	1.000*	1.000*	1.000*	-	1.000*	1.000*	1.000*
(5, 10, 15)	1.25	0.058	0.008	-	0.081*	-	-	-	-
	2.87	0.098	0.011	-	0.181*	-	-	-	-
	6.13	0.164	0.013	-	0.357*	-	-	-	-
(20, 30, 40)	1.25	0.542	0.389	0.649*	0.572	-	0.394	-	0.563
	2.87	0.902	0.607	0.970*	0.948	-	0.629	-	0.944
	6.13	0.993	0.759	0.999*	0.997	-	0.779	-	0.997
(45, 60, 75)	1.25	0.954	0.944	0.982*	0.971	0.951	0.945	0.982	0.965
	2.87	1.000*	0.997	1.000*	1.000*	1.000*	0.997	1.000*	1.000*
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*

- Refer to test statistics are not considered power of a test because it can not control probability of type I error.

* Refer to test statistic shows the highest power of a test and it is called the best test.

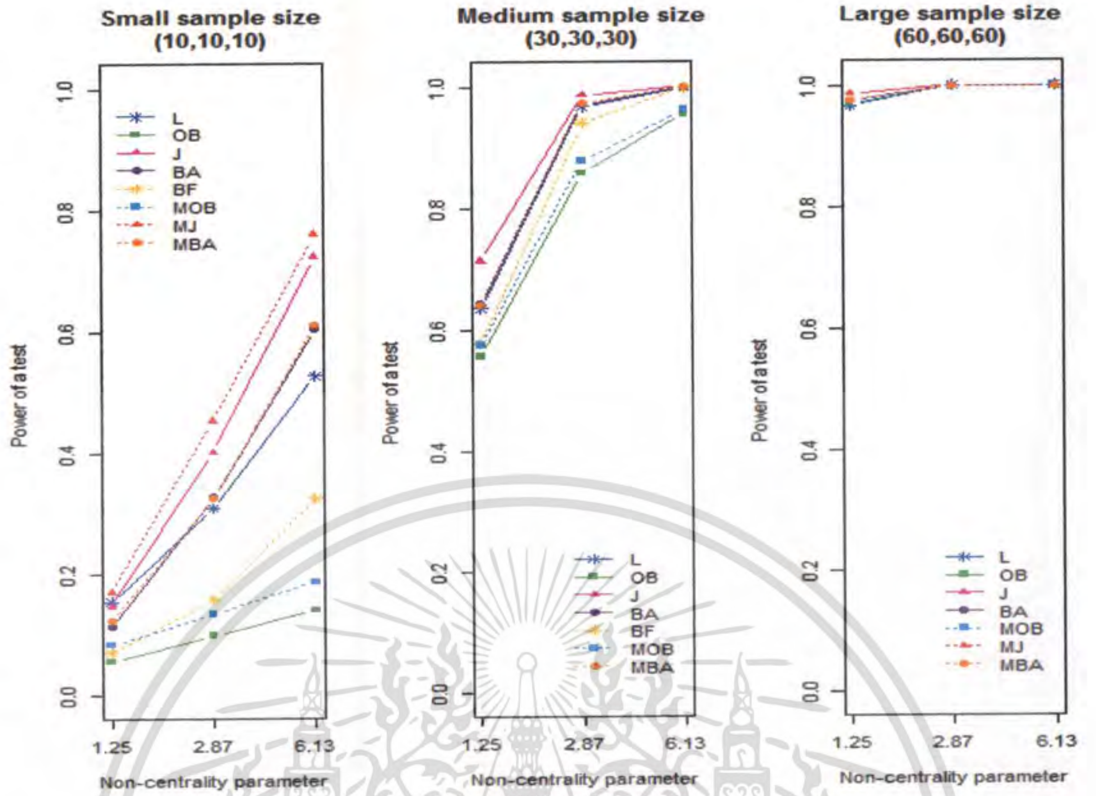


Figure 4.17 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.01) with normal distribution

Power of a test of test statistics is considered under equal sample size at significance level (0.01) with normal distribution. The results in **Table 4.3** and **Figure 4.17** found that under small sample size (10, 10, 10), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (30, 30, 30), J test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under large sample size (60, 60, 60), J and MJ tests are the test statistic that shown the highest power of a test whereas L, BA, and MBA tests are the test statistic that shown the highest power of a test as well as J and MJ tests in cases of non-centrality parameters equal to 2.87 and 6.13 then OB and MOB tests are the test statistic that shown the highest power of a test as well as J and MJ tests in case of non-centrality parameter equal to 6.13. So when sample size increases, it will increase power of a test.

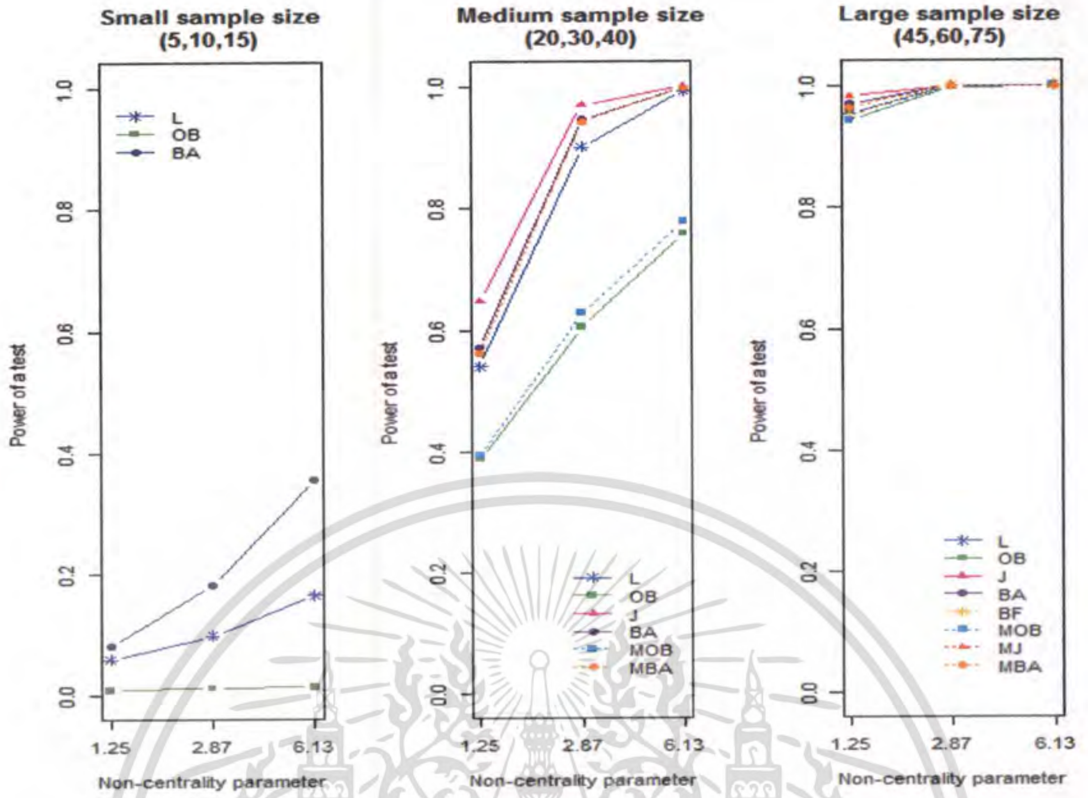


Figure 4.18 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.01) with normal distribution

Power of a test of test statistics is considered under unequal sample size at significance level (0.01) with normal distribution. The results in **Table 4.3** and **Figure 4.18** found that under small sample size (5, 10, 15), BA test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (20, 30, 40), J test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under large sample size (45, 60, 75), J and MJ test are the test statistic that shown the highest power of a test in all case of non-centrality parameters whereas L, BF, BA, and MBA tests are the test statistic that shown the highest power of a test as well as J and MJ tests in cases of non-centrality parameters equal to 2.87 and 6.13. So when sample size increases, it will increase power of a test.

Table 4.4 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under Laplace distribution at significance level (0.01)

Sample sizes	Non-centrality parameter (ϕ)	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
(10, 10, 10)	1.25	0.080*	-	-	0.054	0.034	0.033	-	0.057
	2.87	0.156*	-	-	0.146	0.073	0.046	-	0.153
	6.13	0.262	-	-	0.328	0.135	0.059	-	0.331*
(30, 30, 30)	1.25	0.364*	0.177	-	0.250	0.314	0.187	-	0.253
	2.87	0.795*	0.316	-	0.631	0.701	0.335	-	0.633
	6.13	0.959*	0.442	-	0.904	0.946	0.466	-	0.904
(60, 60, 60)	1.25	0.774*	0.550	-	0.627	0.747	0.565	-	0.624
	2.87	0.995*	0.812	-	0.954	0.994	0.818	-	0.954
	6.13	1.000*	0.898	-	0.997	1.000*	0.905	-	0.997
(5, 10, 15)	1.25	-	0.000	-	0.040	-	-	-	0.041*
	2.87	-	0.000	-	0.074*	-	-	-	0.070
	6.13	-	0.000	-	0.145*	-	-	-	0.137
(20, 30, 40)	1.25	0.274*	0.083	-	0.223	0.239	0.088	-	0.216
	2.87	0.586*	0.147	-	0.538	0.512	0.154	-	0.532
	6.13	0.836*	0.202	-	0.799	0.791	0.223	-	0.796
(45, 60, 75)	1.25	-	-	-	-	-	-	-	-
	2.87	-	-	-	-	-	-	-	-
	6.13	-	-	-	-	-	-	-	-

- Refer to test statistics are not considered power of a test because it can not control probability of type I error.

* Refer to test statistic shows the highest power of a test and it is called the best test.

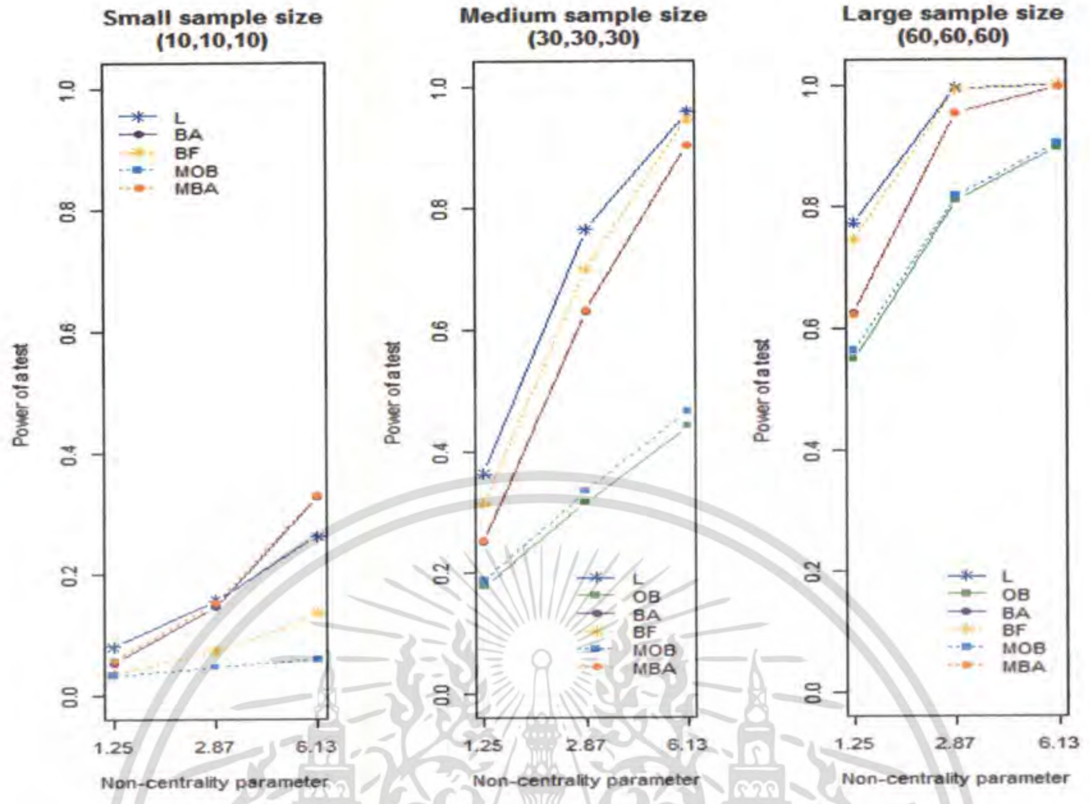


Figure 4.19 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.01) with Laplace distribution

Power of a test of test statistics is considered under equal sample size at significance level (0.01) with Laplace distribution. The results in **Table 4.4** and **Figure 4.19** found that under small sample size (10, 10, 10), L test is the test statistic that shown the highest power of a test in cases of non-centrality parameters equal to 1.25 and 2.87 where as MBA test is the test statistic that shown the highest power of a test in case of non-centrality parameter equal to 6.13. Under medium sample size (30, 30, 30), L test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under large sample size (60, 60, 60), L test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters whereas BF test is the test statistic that shown the highest power of a test as well as L test in case of non-centrality parameters equal to 6.13. So when sample size increases, it will increase power of a test.

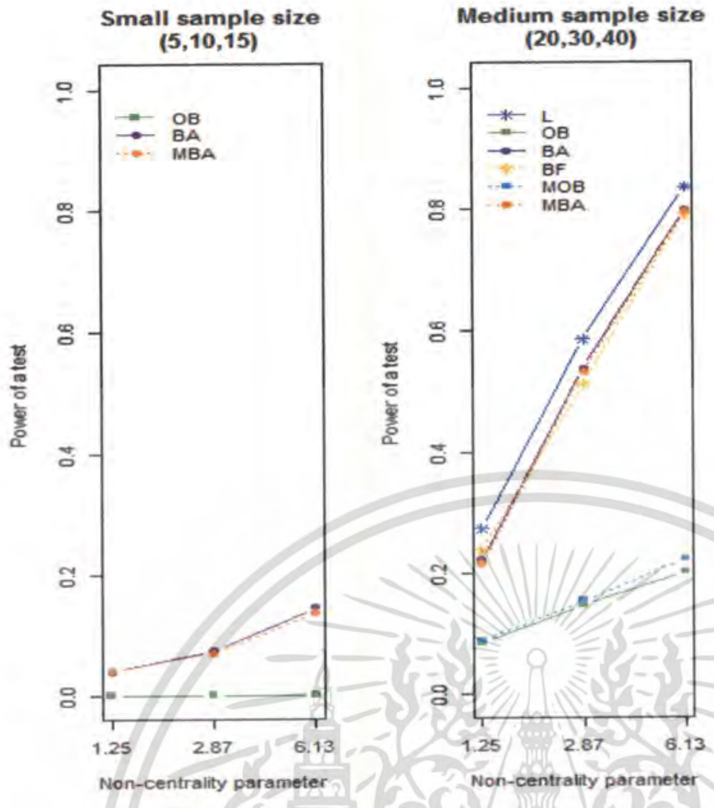


Figure 4.20 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.01) with Laplace distribution

Power of a test of test statistics is considered under unequal sample size at significance level (0.01) with Laplace distribution. The results in **Table 4.4** and **Figure 4.20** found that under small sample size (5, 10, 15), MBA test is the test statistic that shown the highest power of a test in case of non-centrality parameter equal to 1.25 whereas BA test is the test statistic that shown the highest power of a test in cases of non-centrality parameters equal to 2.87 and 6.13. Under medium sample size (20, 30, 40), L test is the test statistic that shown the highest power of a test in all case of non-centrality parameters. Under large sample size (45, 60, 75), no test statistics has power of a test because that can not controlled probability of type I error. So when sample size increases, it will increase power of a test.

Table 4.5 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under uniform distribution at significance level (0.01)

Sample sizes	Non-centrality parameter (ϕ)	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
(10, 10, 10)	1.25	0.238	-	0.207	0.229	-	-	0.260*	-
	2.87	0.494	-	0.640	0.567	-	-	0.720*	-
	6.13	0.747	-	0.924	0.857	-	-	0.944*	-
(30, 30, 30)	1.25	0.884	0.957	0.986	0.969	0.824	-	0.989*	-
	2.87	0.998	1.000*	1.000*	1.000*	0.994	-	1.000*	-
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	-	1.000*	-
(60, 60, 60)	1.25	1.000*	1.000*	1.000*	1.000*	0.998	-	1.000*	-
	2.87	1.000*	1.000*	1.000*	1.000*	1.000*	-	1.000*	-
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	-	1.000*	-
(5, 10, 15)	1.25	0.163*	0.076	-	-	-	-	-	-
	2.87	0.280*	0.116	-	-	-	-	-	-
	6.13	0.410*	0.145	-	-	-	-	-	-
(20, 30, 40)	1.25	0.848	0.914	0.985*	0.964	0.776	-	0.985*	-
	2.87	0.996	0.995	1.000*	1.000*	0.987	-	1.000*	-
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	-	1.000*	-
(45, 60, 75)	1.25	1.000*	-	-	-	-	1.000*	-	
	2.87	1.000*	-	-	-	-	1.000*	-	
	6.13	1.000*	-	-	-	-	1.000*	-	

- Refer to test statistics are not considered power of a test because it can not control probability of type I error.

* Refer to test statistic shows the highest power of a test and it is called the best test.

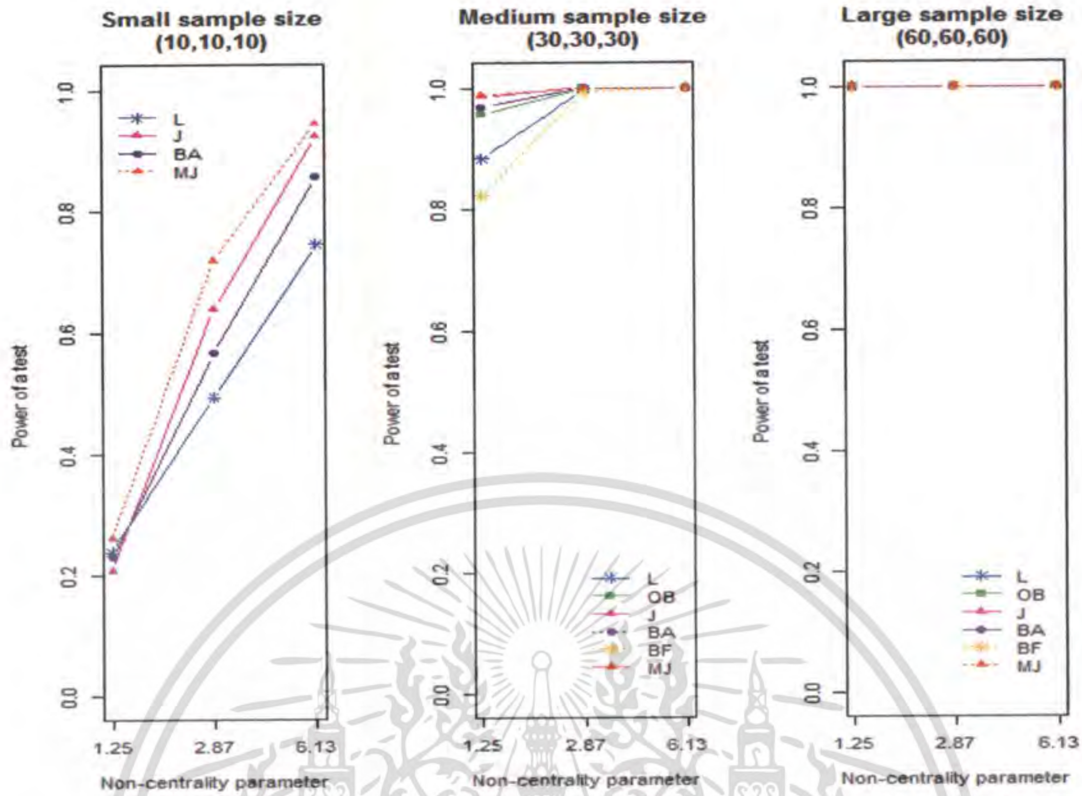


Figure 4.21 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.01) with uniform distribution

Power of a test of test statistics is considered under equal sample size at significance level (0.01) with uniform distribution. The results in **Table 4.5** and **Figure 4.21** found that under small sample size (10, 10, 10), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (30, 30, 30), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters where as OB, J, and BA tests are the test statistics that shown the highest power of a test as well as MJ test in cases of non-centrality parameters equal to 2.87 and 6.13 then L and BF tests are the test statistics that shown the highest power of a test as well as MJ test in cases of non-centrality parameter equal to 6.13. Under large sample size (60, 60, 60), L, OB, J, MJ, and BA tests are the test statistics that shown the highest power of a test in all cases of non-centrality parameters whereas BF test is the test statistic that shown the highest power of a test as well as L, OB, J, MJ, and BA tests in cases of non-centrality parameters equal to 2.87 and 6.13. So when sample size increases, it will increase power of a test.

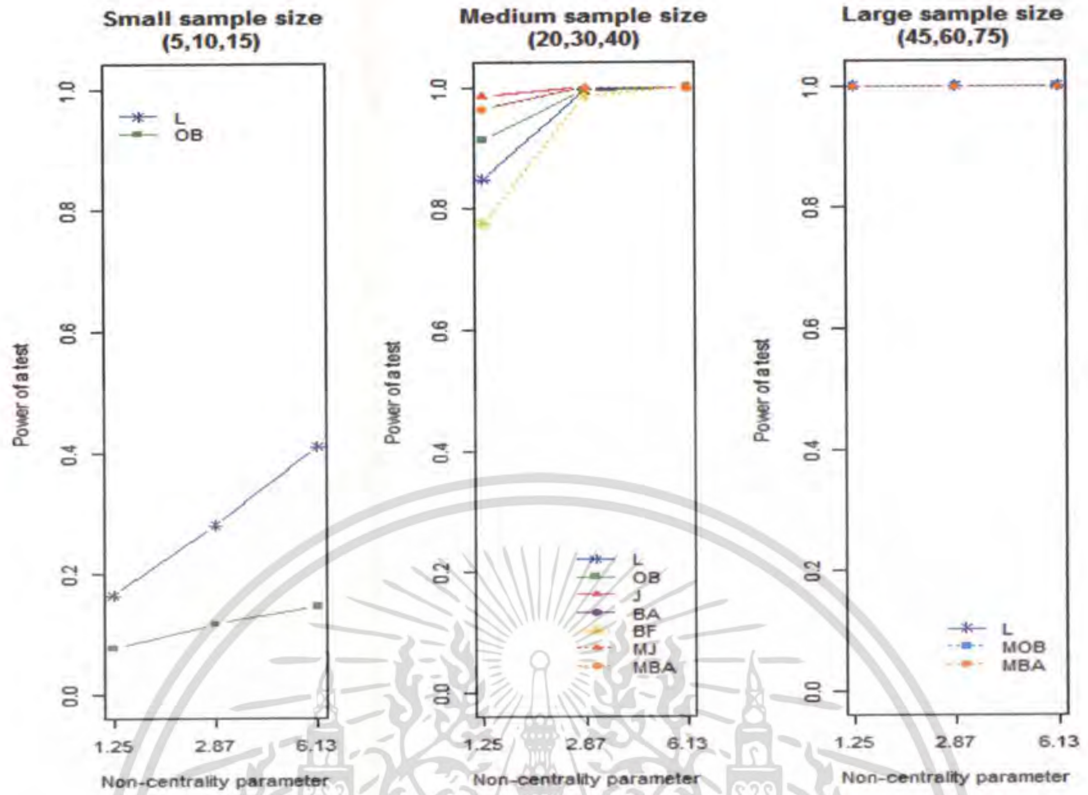


Figure 4.22 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.01) with uniform distribution

Power of a test of test statistics is considered under unequal sample size at significance level (0.01) with uniform distribution. The results in **Table 4.5** and **Figure 4.22** found that under small sample size (5, 10, 15), L test is the test statistic that has the highest power of a test in all cases of non-centrality parameters. Under medium sample size (20, 30, 40), J and MJ tests are the test statistic that have the highest power of a test in all cases of non-centrality parameters whereas BA test is the test statistics that has the highest power of a test as well as J and MJ tests in cases of non-centrality parameters equal to 2.87 and 6.13 then L, OB, and BF tests are the test statistics that have the highest power of a test as well as J and MJ tests in case of non-centrality parameter equal to 6.13. Under large sample size (45, 60, 75), L, MOB, and MBA tests are the test statistics that have the highest power of a test in all cases of non-centrality parameters. So when sample size increases, it will increase power of a test.

Table 4.6 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under gamma distribution at significance level (0.01)

Sample sizes	Non-centrality parameter (ϕ)	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
(10, 10, 10)	1.25	-	0.087	0.146	-	0.062	-	0.182*	0.113
	2.87	-	0.143	0.357	-	0.127	-	0.41*	0.285
	6.13	-	0.162	0.616	-	0.238	-	0.692*	0.542
(30, 30, 30)	1.25	-	0.504	-	0.592	0.563	-	-	0.602*
	2.87	-	0.709	-	0.901	0.914*	-	-	0.902
	6.13	-	0.756	-	0.974	0.996*	-	-	0.976
(60, 60, 60)	1.25	0.970	0.939	0.978	-	0.961	0.950	0.980*	0.96
	2.87	1.000*	0.988	1.000*	-	1.000*	0.990	1.000*	1.000*
	6.13	1.000*	0.977	1.000*	-	1.000*	0.982	1.000*	1.000*
(5, 10, 15)	1.25	-	-	-	0.087*	-	-	-	-
	2.87	-	-	-	0.182*	-	-	-	-
	6.13	-	-	-	0.267*	-	-	-	-
(20, 30, 40)	1.25	-	0.315	-	0.499*	0.458	-	-	-
	2.87	-	0.453	-	0.827*	0.777	-	-	-
	6.13	-	0.463	-	0.942*	0.942*	-	-	-
(45, 60, 75)	1.25	-	0.884	0.968	0.945	0.929	0.888	0.971*	0.942
	2.87	-	0.967	1.000*	0.996	1.000*	0.971	1.000*	0.996
	6.13	-	0.943	1.000*	0.998	1.000*	0.962	1.000*	0.998

- Refer to test statistics are not considered power of a test because it can not control probability of type I error.

* Refer to test statistic shows the highest power of a test and it is called the best test.

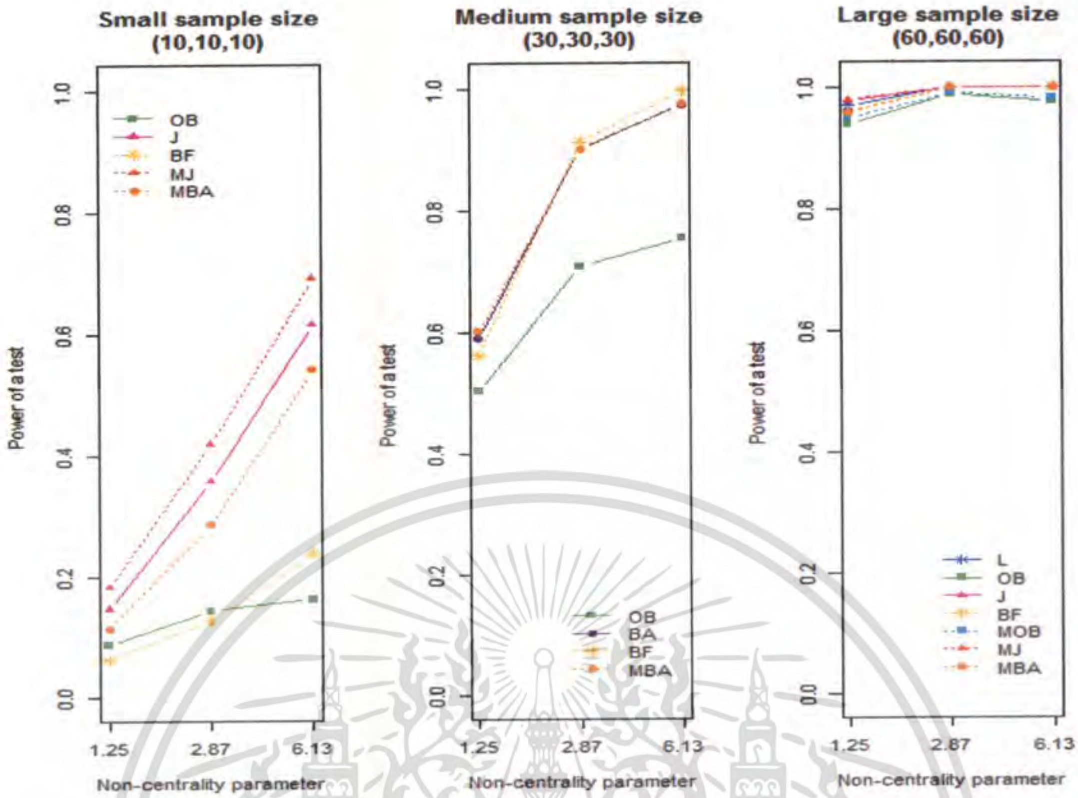


Figure 4.23 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.01) with gamma distribution

Power of a test of test statistics is considered under equal sample size at significance level (0.01) with gamma distribution. The results in **Table 4.6** and **Figure 4.23** found that under small sample size (10, 10, 10), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (30, 30, 30), MBA test is the test statistic that shown the highest power of a test in cases of non-centrality parameter equal to 1.25 whereas BF test is the test statistics that shown the highest power of a test in cases of non-centrality parameter equal to 2.87 and 6.13. Under large sample size (60, 60, 60), MJ test is the test statistics that shown the highest power of a test in all cases of non-centrality parameters whereas L, J, BF, and MBA tests are the test statistics that have the highest power of a test as well as MJ test in cases of non-centrality parameters equal to 2.87 and 6.13. So when sample size increases, it will increase power of a test.

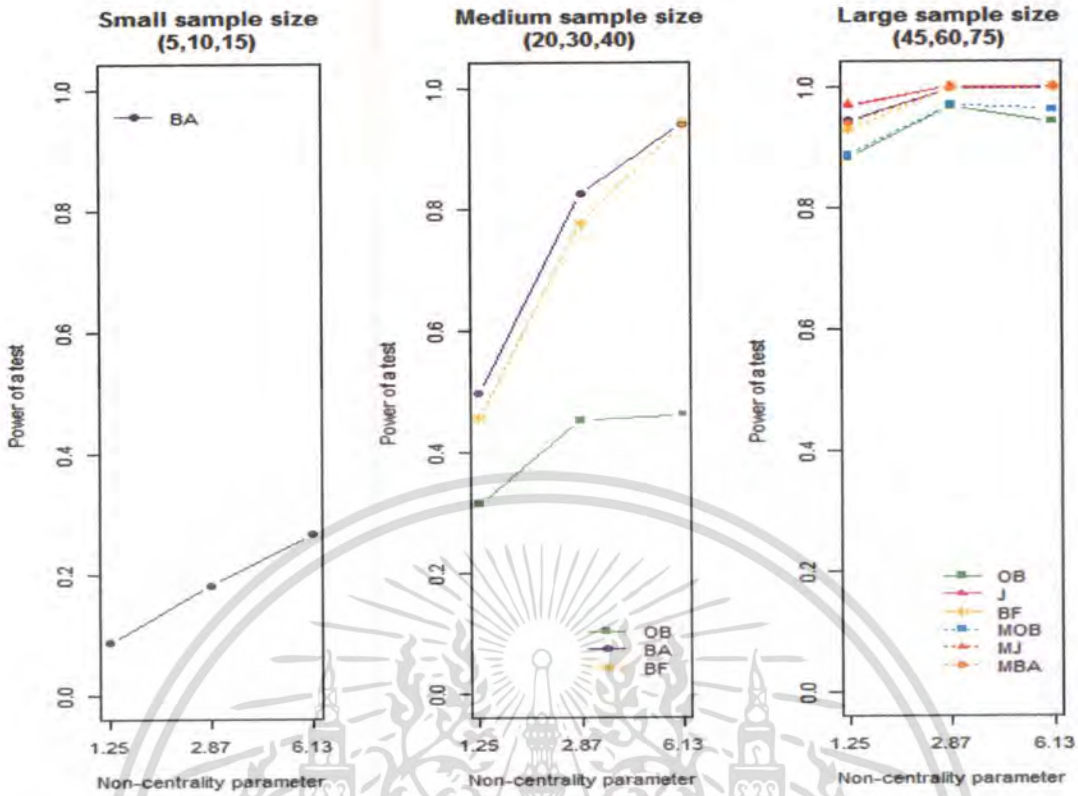


Figure 4.24 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.01) with gamma distribution

Power of a test of test statistics is considered under unequal sample size at significance level (0.01) with gamma distribution. The results in **Table 4.6** and **Figure 4.24** found that under small sample size (5, 10, 15), BA test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (20, 30, 40), BA test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters whereas BF test is the test statistics that shown the highest power of a test as well as BA test in cases of non-centrality parameter equal 6.13. Under large sample size (45, 60, 75), MJ test is the test statistics that shown the highest power of a test in all cases of non-centrality parameters whereas BF and J tests are the test statistics that shown the highest power of a test as well as MJ test in cases of non-centrality parameters equal to 2.87 and 6.13. So when sample size increases, it will increase power of a test.

Table 4.7 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under normal distribution at significance level (0.05)

Sample sizes	Non-centrality parameter (ϕ)	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
(10, 10, 10)	1.25	0.363	0.227	0.323	0.335	0.240	0.272	0.364*	0.337
	2.87	0.619	0.364	0.658	0.654	0.469	0.424	0.710*	0.662
	6.13	0.849	0.474	0.874	0.884	0.697	0.522	0.893*	0.881
(30, 30, 30)	1.25	0.857	0.841	0.891	0.879	0.823	0.853	0.898*	0.881
	2.87	0.995	0.983	0.999*	0.999*	0.991	0.986	0.999*	0.999*
	6.13	1.000*	0.999	1.000*	1.000*	1.000*	0.999	1.000*	1.000*
(60, 60, 60)	1.25	0.989	0.993	0.997*	0.995	0.989	0.992	0.997*	0.995
	2.87	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
(5, 10, 15)	1.25	0.246	0.078	0.263	0.266	0.151	0.109	0.328*	-
	2.87	0.363	0.116	0.454	0.463	0.233	0.146	0.543*	-
	6.13	0.513	0.142	0.675	0.705	0.352	0.179	0.786*	-
(20, 30, 40)	1.25	0.799	0.715	0.847	0.846	0.769	0.718	0.864*	0.837
	2.87	0.984	0.938	0.994	0.995*	0.980	0.945	0.995*	0.995*
	6.13	1.000*	0.983	1.000*	1.000*	0.999	0.984	1.000*	1.000*
(45, 60, 75)	1.25	0.994	0.994	1.000*	0.996	0.991	0.994	1.000*	0.996
	2.87	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*

- Refer to test statistics are not considered power of a test because it can not control probability of type I error.

* Refer to test statistic shows the highest power of a test and it is called the best test.

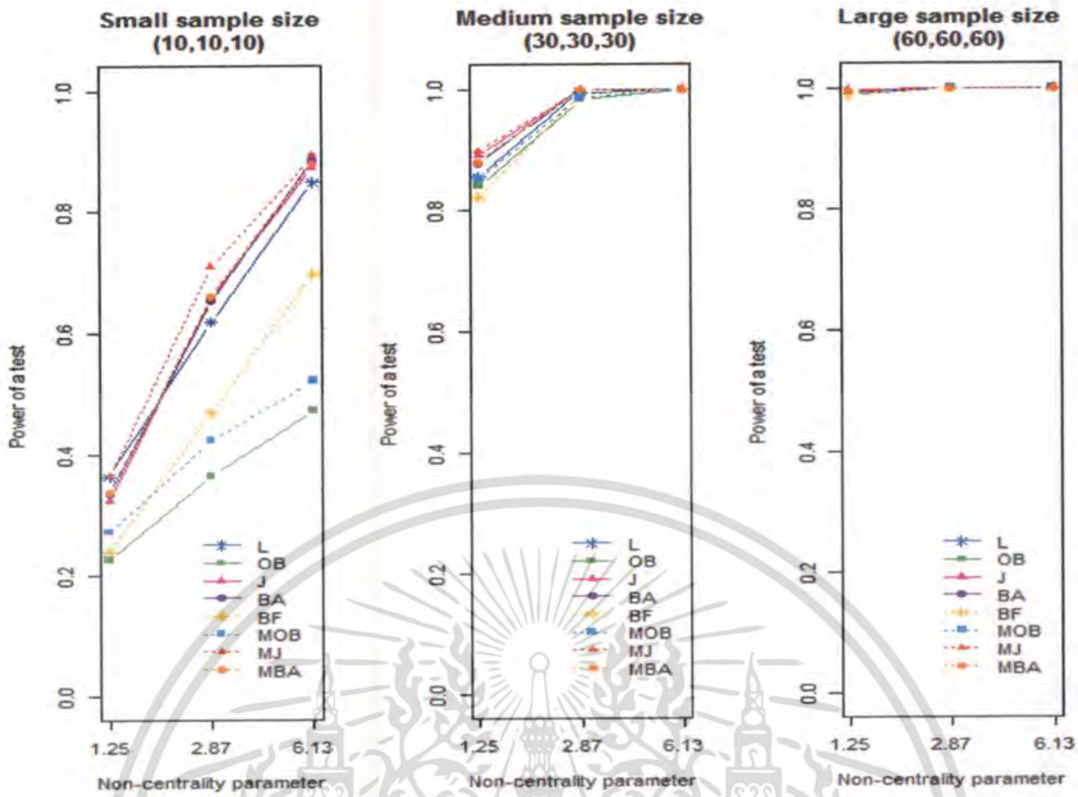


Figure 4.25 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.05) with normal distribution

Power of a test of test statistics is considered under equal sample size at significance level (0.05) with normal distribution. The results in **Table 4.7** and **Figure 4.25** found that under small sample size (10, 10, 10), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (30, 30, 30), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters whereas J, BA, and MBA tests are the test statistics that shown the highest power of a test as well as MJ test in cases of non-centrality parameter equal 2.87 and 6.13 then L and BF tests are the test statistics that shown the highest power of a test as well as MJ test in case of non-centrality parameter equal 6.13. Under large sample size (60, 60, 60), J and MJ tests are the test statistics that shown the highest power of a test in all cases of non-centrality parameters whereas L, OB, BA, BF, MOB, and MBA tests are the test statistics that shown the highest power of a test in cases of non-centrality parameters equal to 2.87 and 6.13. So when sample size increases, it will increase power of a test.

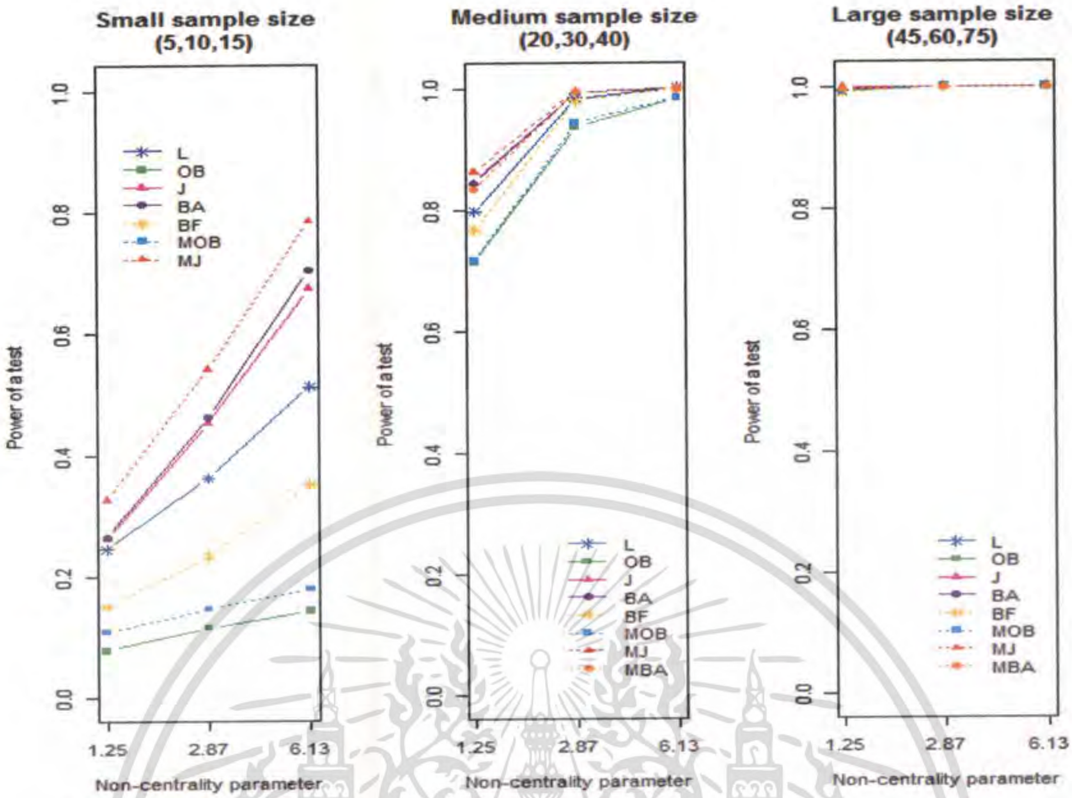


Figure 4.26 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.05) with normal distribution

Power of a test of test statistics is considered under unequal sample size at significance level (0.05) with normal distribution. The results in **Table 4.7** and **Figure 4.26** found that under small sample size (5, 10, 15), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (20, 30, 40), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters whereas BA and MBA tests are the test statistics that shown the highest power of a test as well as MJ test in cases of non-centrality parameter and equal 2.87 and 6.13 then L and J tests are the test statistics that shown the highest power of a test as well as MJ test in case of non-centrality parameter equal 6.13. Under large sample size (45, 60, 75), J and MJ tests are the test statistics that shown the highest power of a test in all cases of non-centrality parameters whereas L, OB, BA, BF, MOB, and MBA tests are the test statistics that shown the highest power of a test in cases of non-centrality parameters equal to 2.87 and 6.13. So when sample size increases, it will increase power of a test.

Table 4.8 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under Laplace distribution at significance level (0.05)

Sample sizes	Non-centrality parameter (ϕ)	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
(10, 10, 10)	1.25	-	0.101	-	0.209*	0.143	0.129	-	0.205
	2.87	-	0.150	-	0.401*	0.275	0.192	-	0.401*
	6.13	-	0.197	-	0.662*	0.419	0.250	-	0.661
(30, 30, 30)	1.25	0.644*	0.434	-	0.543	0.595	0.457	-	0.542
	2.87	0.932*	0.682	-	0.875	0.908	0.708	-	0.874
	6.13	0.998*	0.822	-	0.989	0.994	0.838	-	0.988
(60, 60, 60)	1.25	0.922*	0.796	0.840	0.844	0.916	0.800	0.848	0.844
	2.87	0.999*	0.963	0.989	0.994	0.999*	0.965	0.990	0.994
	6.13	1.000*	0.987	1.000*	1.000*	1.000*	0.987	1.000*	1.000*
(5, 10, 15)	1.25	-	0.021	-	0.154*	0.063	0.030	-	-
	2.87	-	0.029	-	0.275*	0.106	0.040	-	-
	6.13	-	0.036	-	0.431*	0.147	0.047	-	-
(20, 30, 40)	1.25	0.580*	0.311	-	0.516	0.539	0.326	-	0.511
	2.87	0.843*	0.472	-	0.811	0.823	0.488	-	0.807
	6.13	0.979*	0.557	-	0.953	0.971	0.573	-	0.953
(45, 60, 75)	1.25	0.915*	0.743	0.852	0.848	0.906	0.748	0.857	0.844
	2.87	1.000*	0.917	0.986	0.991	1.000*	0.921	0.988	0.991
	6.13	1.000*	0.967	0.998	1.000*	1.000*	0.969	0.999	1.000*

- Refer to test statistics are not considered power of a test because it can not control probability of type I error.

* Refer to test statistic shows the highest power of a test and it is called the best test.

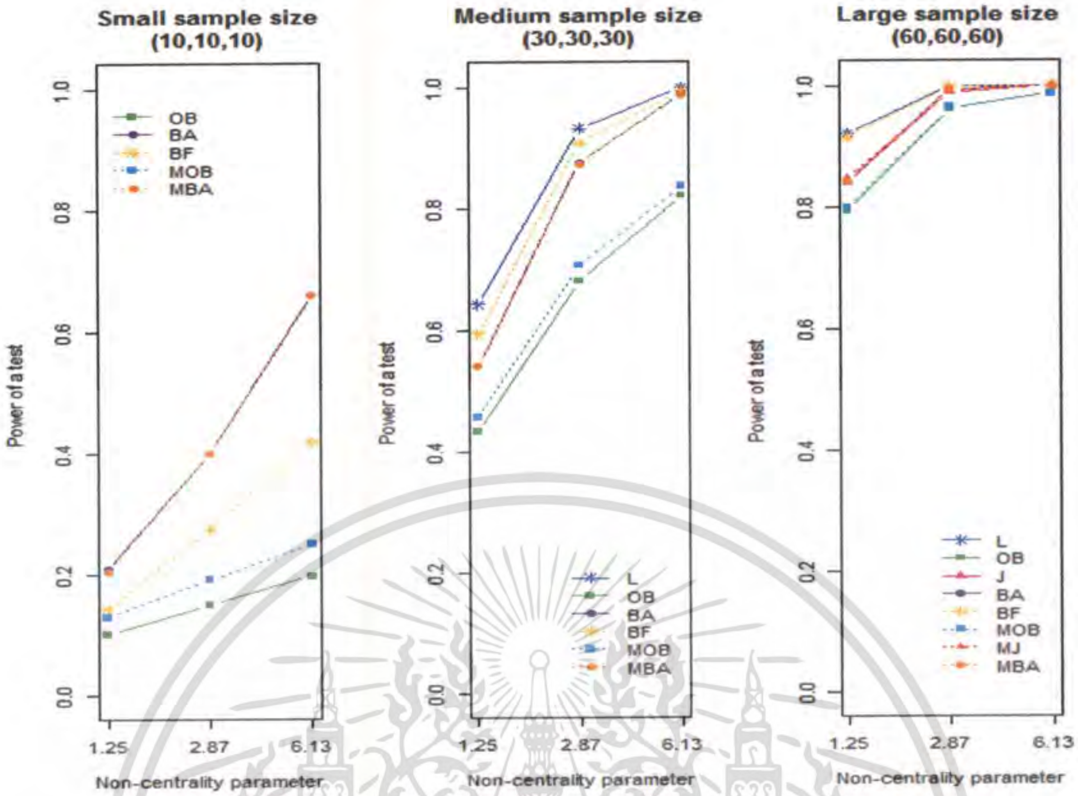


Figure 4.27 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.05) with Laplace distribution

Power of a test of test statistics is considered under equal sample size at significance level (0.05) with Laplace distribution. The results in **Table 4.8** and **Figure 4.27** found that under small sample size (10, 10, 10), BA test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters whereas MBA test is the test statistic that shown the highest power of a test as well as BA test in case of non-centrality parameters equal to 2.87. Under medium sample size (30, 30, 30), L test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under large sample size (60, 60, 60), L test is the test statistics that shown the highest power of a test in all cases of non-centrality parameters whereas BF test is the test statistics that shown the highest power of a test as well as L test in cases of non-centrality parameters equal to 2.87 and 6.13 then J, BA, MJ, and MBA tests are the test statistics that shown the highest power of a test as well as L test in case of non-centrality parameter equal to 6.13. So when sample size increases, it will increase power of a test.

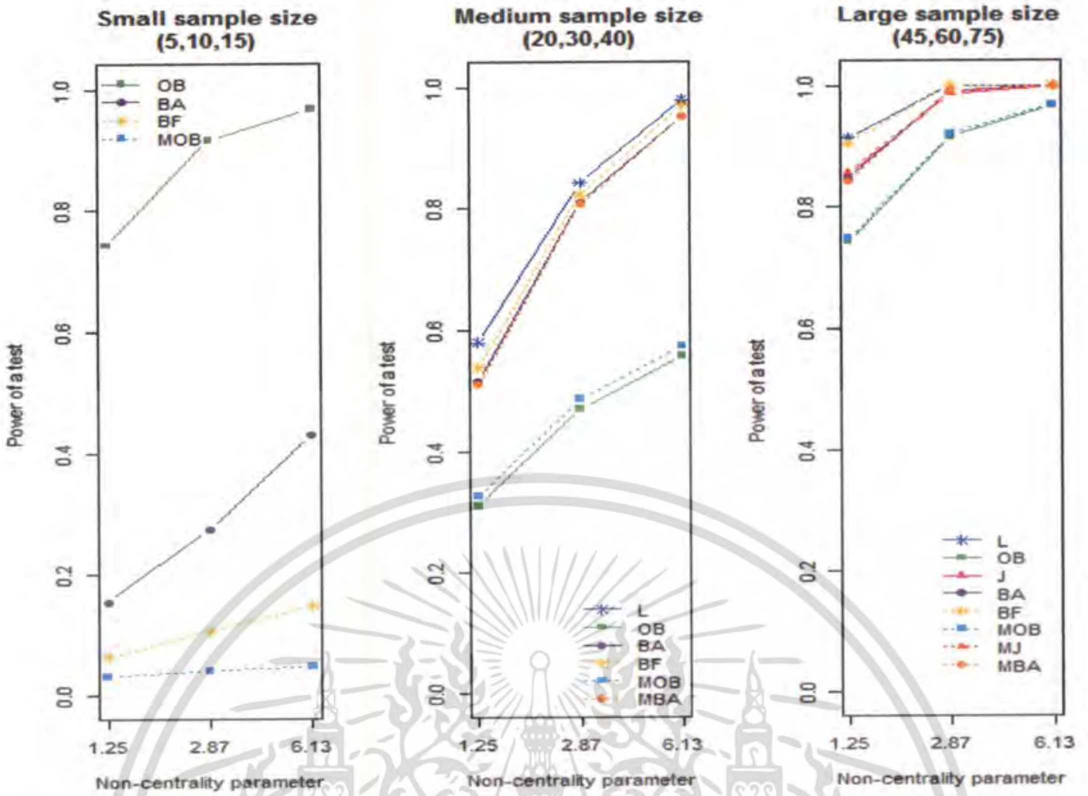


Figure 4.28 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.05) with Laplace distribution

Power of a test of test statistics is considered under unequal sample size at significance level (0.05) with Laplace distribution. The results in **Table 4.8** and **Figure 4.28** found that under small sample size (5, 10, 15), BA test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (20, 30, 40), L test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under large sample size (45, 60, 75), L test is the test statistics that shown the highest power of a test in all cases of non-centrality parameters whereas BF test is the test statistics that shown the highest power of a test as well as L test in cases of non-centrality parameter equal to 2.87 and 6.13 then BA and MBA tests are the test statistics that shown the highest power of a test as well as L test in case of non-centrality parameter equal to 6.13. So when sample size increases, it will increase power of a test.

Table 4.9 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under uniform distribution at significance level (0.05)

Sample sizes	Non-centrality parameter (ϕ)	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
(10, 10, 10)	1.25	0.466	0.434	0.453	0.502	-	0.499	0.514*	0.510
	2.87	0.779	0.643	0.869	0.850	-	0.689	0.903*	0.841
	6.13	0.933	0.746	0.979	0.973	-	0.809	0.983*	0.969
(30, 30, 30)	1.25	0.964	0.992	0.996*	0.994	0.939	0.992	0.996*	0.994
	2.87	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
(60, 60, 60)	1.25	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
	2.87	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
(5, 10, 15)	1.25	0.410	0.278	0.320	0.448*	-	-	0.408	-
	2.87	0.612	0.370	0.612	0.729*	-	-	0.719	-
	6.13	0.788	0.428	0.829	0.912	-	-	0.938*	-
(20, 30, 40)	1.25	0.958	0.990	0.996	0.994	0.934	0.989	0.997*	0.992
	2.87	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
(45, 60, 75)	1.25	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
	2.87	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*

- Refer to test statistics are not considered power of a test because it can not control probability of type I error.

* Refer to test statistic shows the highest power of a test and it is called the best test.

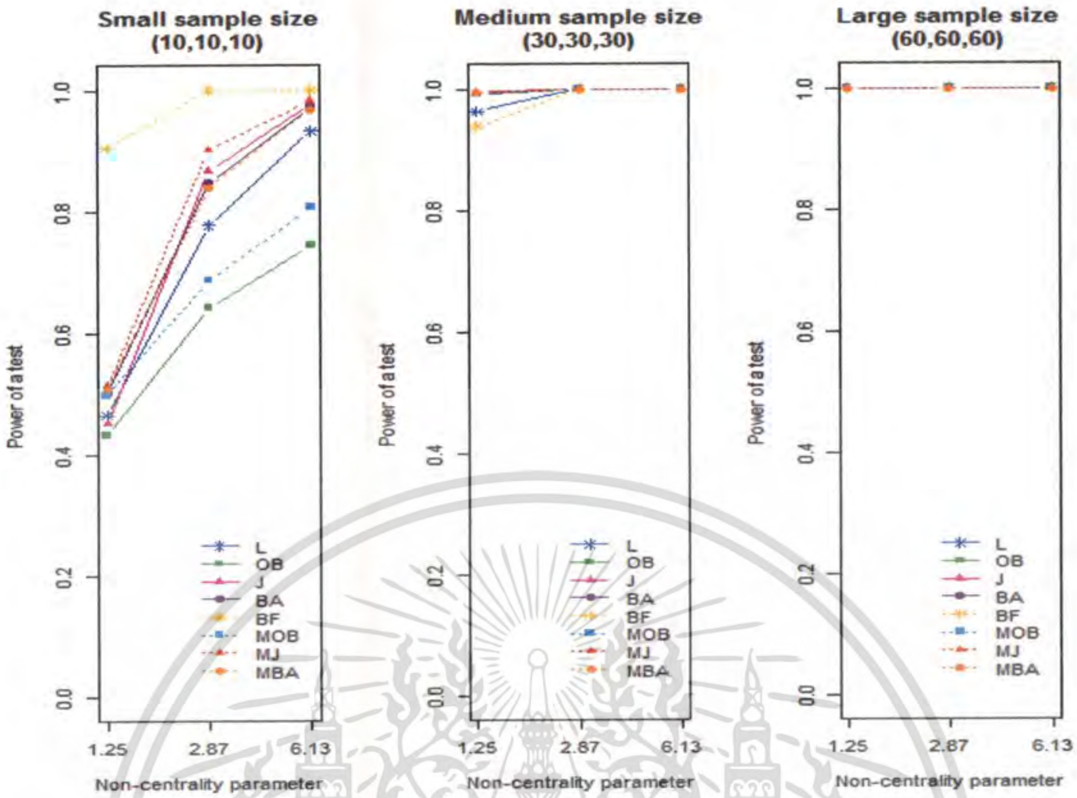


Figure 4.29 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.05) with uniform distribution

Power of a test of test statistics is considered under equal sample size at significance level (0.05) with uniform distribution. The results in **Table 4.9** and **Figure 4.29** found that under small sample size (10, 10, 10), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (30, 30, 30), J and MJ tests are the test statistic that shown the highest power of a test in all cases of non-centrality parameters whereas L, OB, BA, BF, MOB, and MBA tests are the test statistic that shown the highest power of a test as well as J and MJ tests in cases of non-centrality parameter equal to 2.87 and 6.13. Under large sample size (60, 60, 60), All test are the test statistics that shown the highest power of a test in all cases of non-centrality parameters. So when sample size increases, it will increase power of a test.

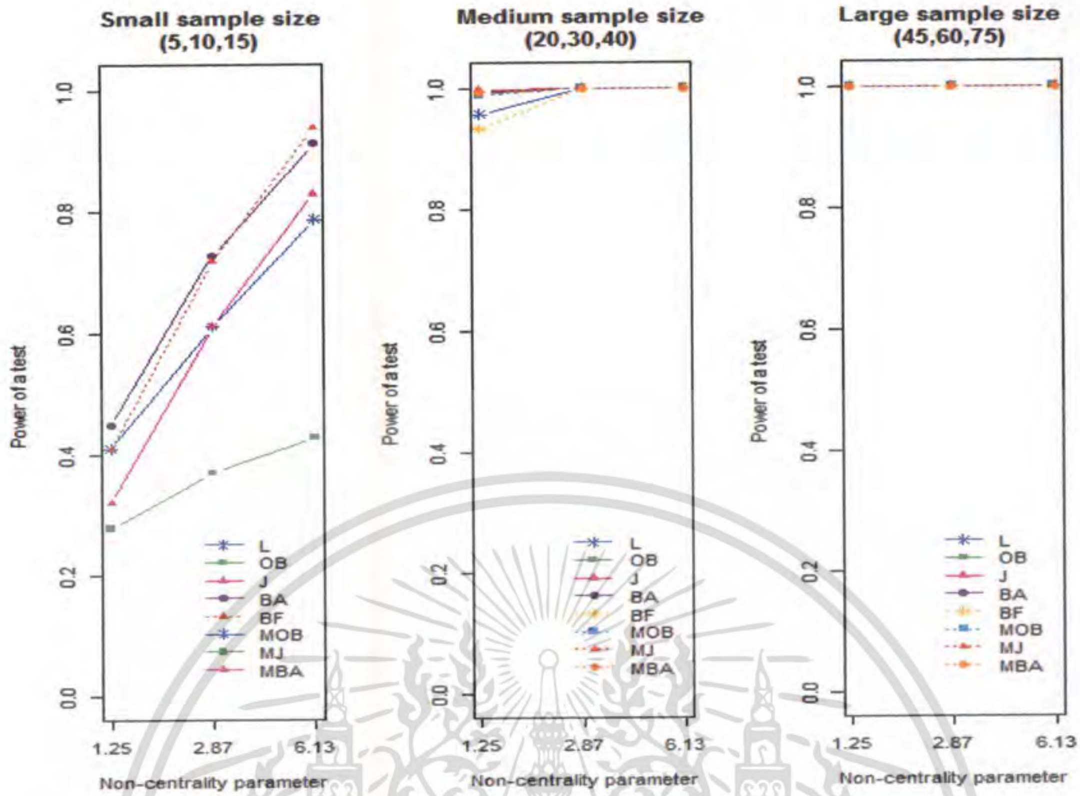


Figure 4.30 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample sizes at significance level (0.05) with uniform distribution

Power of a test of test statistics is considered under unequal sample size at significance level (0.05) with uniform distribution. The results in **Table 4.9** and **Figure 4.30** found that under small sample size (5, 10, 15), BA test is the test statistic that shown the highest power of a test in cases of non-centrality parameters equal to 1.25 and 2.87 whereas MJ test is the test statistic that shown the highest power of a test in case of non-centrality parameters equal to 6.13. Under medium sample size (20, 30, 40), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters whereas L, OB, J, BA, BF, MOB, and MBA tests are the test statistic that shown the highest power of a test as well as MJ test in cases of non-centrality parameters equal to 2.87 and 6.13. Under large sample size (45, 60, 75), All tests are the test statistics that shown the highest power of a test in all cases of non-centrality parameters. So when sample size increases, it will increase power of a test.

Table 4.10 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under gamma distribution at significance level (0.05)

Sample sizes	Non-centrality parameter (ϕ)	Test statistics							
		L	OB	J	BA	BF	MOB	MJ	MBA
(10, 10, 10)	1.25	-	0.217	0.325	0.346	0.231	-	0.361*	-
	2.87	-	0.323	0.607	0.584	0.391	-	0.667*	-
	6.13	-	0.387	0.829	0.810	0.576	-	0.872*	-
(30, 30, 30)	1.25	-	0.788	0.864*	0.839	0.801	0.810	-	0.838
	2.87	-	0.930	0.995*	0.980	0.987	0.945	-	0.981
	6.13	-	0.945	1.000*	0.998	1.000*	0.965	-	0.998
(60, 60, 60)	1.25	0.993	0.990	0.998*	0.997	0.991	0.992	0.998*	0.995
	2.87	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
	6.13	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
(5, 10, 15)	1.25	-	0.089	0.263	-	0.141	-	0.328*	-
	2.87	-	0.116	0.454	-	0.204	-	0.543*	-
	6.13	-	0.117	0.675	-	0.258	-	0.786*	-
(20, 30, 40)	1.25	-	0.658	0.847	0.802	0.739	0.675	0.864*	0.800
	2.87	-	0.800	0.994	0.967	0.955	0.817	0.995*	0.966
	6.13	-	0.834	1.000*	0.989	0.999	0.869	1.000*	0.989
(45, 60, 75)	1.25	0.988	0.979	1.000*	0.993	0.986	0.979	1.000*	0.992
	2.87	1.000*	0.996	1.000*	0.999	1.000*	0.998	1.000*	0.999
	6.13	1.000*	0.995	1.000*	0.999	1.000*	0.996	1.000*	0.999

- Refer to test statistics are not considered power of a test because it can not control probability of type I error.

* Refer to test statistic shows the highest power of a test and it is called the best test.

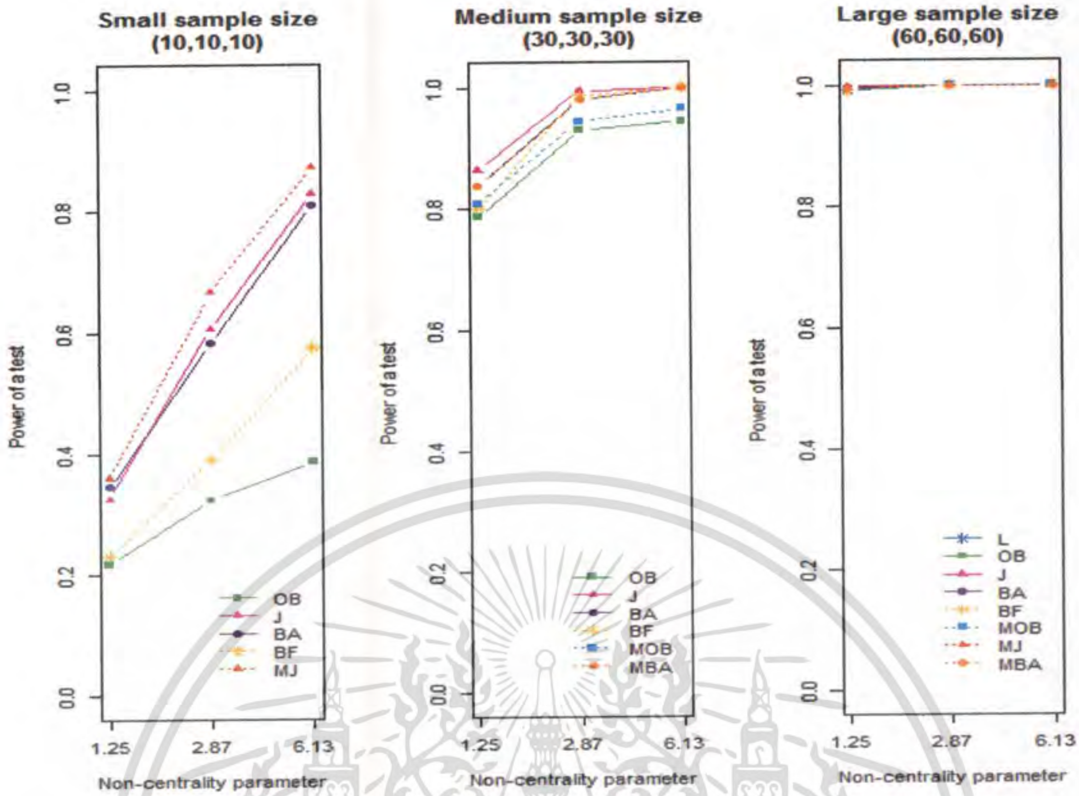


Figure 4.31 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under equal sample size at significance level (0.05) with gamma distribution

Power of a test of test statistics is considered under equal sample size at significance level (0.05) with gamma distribution. The results in **Table 4.10** and **Figure 4.31** found that under small sample size (10, 10, 10), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (30, 30, 30), J test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters whereas BF test is the test statistic that shown the highest power of a test as well as MJ test in case of non-centrality parameter equal to 6.13. Under large sample size (60, 60, 60), J and MJ tests are the test statistics that shown the highest power of a test in all cases of non-centrality parameters whereas L, OB, BA, BF, MOB, and MBA tests are the test statistics that shown the highest power of a test as well as J and MJ tests in cases of non-centrality parameters equal to 2.87 and 6.13. So when sample size increases, it will increase power of a test.

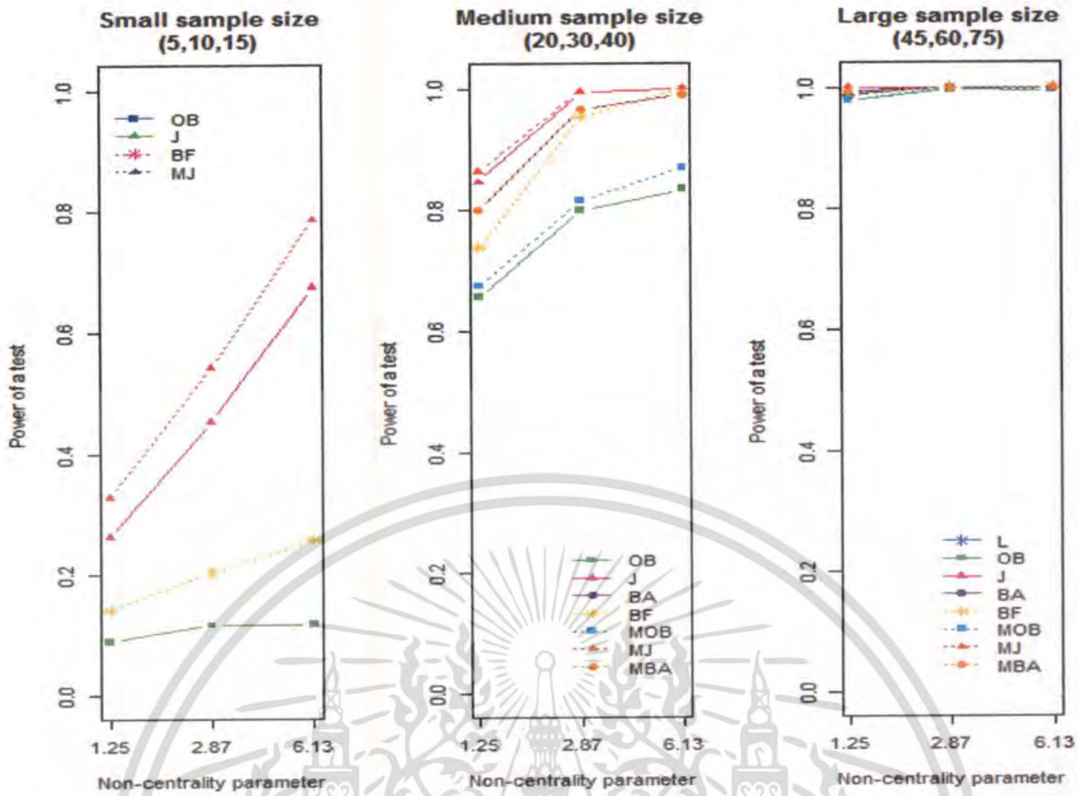


Figure 4.32 Power of a test of L, OB, J, BA, BF, MOB, MJ, and MBA tests under unequal sample size at significance level (0.05) with gamma distribution

Power of a test of test statistics is considered under unequal sample size at significance level (0.05) with gamma distribution. The results in **Table 4.10** and **Figure 4.32** found that under small sample size (5, 10, 15), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters. Under medium sample size (20, 30, 40), MJ test is the test statistic that shown the highest power of a test in all cases of non-centrality parameters whereas J test is the test statistic that shown the highest power of a test as well as MJ test in case of non-centrality parameter equal to 6.13. Under large sample size (45, 60, 75), J and MJ tests are the test statistics that shown the highest power of a test in all cases of non-centrality parameters whereas L and BF tests are the test statistics that shown the highest power of a test as well as J and MJ tests in all cases of non-centrality parameters equal to 2.87 and 6.13. So when sample size increases, it will increase power of a test.

Chapter 5

Conclusions and Suggestions

From the results of data analysis, four original homogeneity of variance tests consists of Levene's test (L), O'Brien's test (OB), Jackknife test (J) and Box-Andersen test (BA) and four modified tests consists of Modified Levene's test or Brown-Forsythe test (BF), Modified O'Brien's test (MOB), Modified Jackknife test (MJ), and Modified Box-Andersen test (MBA). These methods are considered in term of probability of type I error and power of a test for three populations in cases of four distributions: normal, Laplace, uniform, and gamma distributions. The significance levels are 0.01 and 0.05. The conclusion of this study can be summarized as two parts as follows.

5.1 Conclusions

5.1.1 The probability of type I error (also robustness)

Test statistics can control the probability of type I error under 1,000 replications at 0.01 and 0.05 significance levels in cases of normal distribution, Laplace distribution, uniform distribution, and gamma distribution. All test statistics of this study consists of

L	refer to Levene's test
OB	refer to O'Brien's test
J	refer to Jackknife test
BA	refer to Box-Andersen test
BF	refer to Modified Levene's test or Brown-Forsythe test
MOB	refer to Modified O'Brien's test
MJ	refer to Modified Jackknife test
MBA	refer to Modified Box-Andersen test

The details are shown in **Table 5.1**.

Table 5.1 The controlling of test statistics under the probability of type I error in cases of normal distribution, Laplace distribution, uniform distribution, and gamma distribution under 0.01 and 0.05 significance levels

Significance levels	Sample sizes	Distributions			
		Normal	Laplace	Uniform	Gamma
0.01	(10, 10, 10)	L, OB, J, BA, BF, MOB, MJ, MBA	L, BA, BF, MOB, MBA	L, J, BA, MJ	OB, J, BF, MJ, MBA
	(30, 30, 30)	L, OB, J, BA, BF, MOB, MBA	L, OB, BA, BF, MOB, MBA	L, OB, J, BA, BF, MJ	OB, BA, BF, MBA
	(60, 60, 60)	L, OB, J, BA, MOB, MJ, MBA	L, OB, BA, BF, MOB, MBA	L, OB, J, BA, BF, MJ	L, OB, J, BF, MOB, MJ, MBA
	(5, 10, 15)	L, OB, BA	OB, BA, MBA	L, OB	BA
	(20, 30, 40)	L, OB, J, BA, MOB, MBA	L, OB, BA, BF, MOB, MBA	L, OB, J, BA, BF, MJ	OB, BA, BF
	(45, 60, 75)	L, OB, J, BA, BF, MOB, MJ, MBA	-	L, MOB, MBA	OB, J, BA, BF, MOB, MJ, MBA
0.05	(10, 10, 10)	L, OB, J, BA, BF, MOB, MJ, MBA	OB, BA, BF, MOB, MBA	L, OB, J, BA, MOB, MJ, MBA	OB, J, BA, BF, MJ
	(30, 30, 30)	L, OB, J, BA, BF, MOB, MJ, MBA	L, OB, BA, BF, MOB, MBA	L, OB, J, BA, BF, MOB, MJ, MBA	OB, J, BA, BF, MOB, MBA
	(60, 60, 60)	L, OB, J, BA, BF, MOB, MJ, MBA	L, OB, J, BA, BF, MOB, MJ, MBA	L, OB, J, BA, BF, MOB, MJ, MBA	L, OB, J, BA, BF, MOB, MJ, MBA
	(5, 10, 15)	L, OB, J, BA, BF, MOB, MJ	OB, BA, BF, MOB, MBA	L, OB, J, BA, MJ	OB, J, BF, MJ
	(20, 30, 40)	L, OB, J, BA, BF, MOB, MJ, MBA	L, OB, BA, BF, MOB, MBA	L, OB, J, BA, BF, MOB, MJ, MBA	OB, J, BA, BF, MOB, MJ, MBA
	(45, 60, 75)	L, OB, J, BA, BF, MOB, MJ, MBA	L, OB, J, BA, BF, MOB, MJ, MBA	L, OB, J, BA, BF, MOB, MJ, MBA	L, OB, J, BA, BF, MOB, MJ, MBA

- Refer to no test statistics can control probability of type I error.

5.1.2 Power of a test (also power)

Test statistics is the best test statistic that has the highest power of a test under 1,000 replications at 0.01 and 0.05 significance levels in cases of normal distribution, Laplace distribution, uniform distribution, and gamma distribution. All test statistics of this study consists of

L refer to Levene's test

OB refer to O'Brien's test

J refer to Jackknife test

BA refer to Box-Andersen test

BF refer to Modified Levene's test or Brown-Forsythe test

MOB refer to Modified O'Brien's test

MJ refer to Modified Jackknife test

MBA refer to Modified Box-Andersen test

The details are shown in **Table 5.2** and **Table 5.3**.

Table 5.2 The best test statistic under the highest power of a test in cases of normal distribution, Laplace distribution, uniform distribution, and gamma distribution under significance level (0.01)

Sample sizes	Non-centrality parameter(ϕ)	Distributions			
		Normal	Laplace	Uniform	Gamma
(10, 10, 10)	1.25	MJ	L	MJ	MJ
	2.87	MJ	L	MJ	MJ
	6.13	MJ	MBA	MJ	MJ
(30, 30, 30)	1.25	J	L	MJ	MBA
	2.87	J	L	OB, J, BA, MJ	BF
	6.13	J	L	L, OB, J, BA, BF, MJ	BF
(60, 60, 60)	1.25	J, MJ	L	L, OB, J, BA, MJ	MJ
	2.87	L, J, BA, MJ, MBA	L	L, OB, J, BA, BF, MJ	L, J, BF, MJ, MBA
	6.13	L, OB, J, BA, MOB, MJ, MBA	L, BF	L, OB, J, BA, BF, MJ	L, J, BF, MJ, MBA
(5, 10, 15)	1.25	BA	MBA	L	BA
	2.87	BA	BA	L	BA
	6.13	BA	BA	L	BA
(20, 30, 40)	1.25	J	L	J, MJ	BA
	2.87	J	L	J, BA, MJ	BA
	6.13	J	L	L, OB, J, BA, BF, MJ	BF, BA
(45, 60, 75)	1.25	J, MJ	-	L, MOB, MBA	MJ
	2.87	L, J, BA, BF, MJ, MBA	-	L, MOB, MBA	J, BF, MJ
	6.13	L, J, BA, BF, MJ, MBA	-	L, MOB, MBA	J, BF, MJ

- Refer to no the best test statistics.

Table 5.3 The best test statistic under the highest power of a test in cases of normal distribution, Laplace distribution, uniform distribution, and gamma distribution under significance level (0.05)

Sample sizes	Non-centrality parameter(ϕ)	Distributions			
		Normal	Laplace	Uniform	Gamma
(10, 10, 10)	1.25	MJ	BA	MJ	MJ
	2.87	MJ	BA, MBA	MJ	MJ
	6.13	MJ	BA	MJ	MJ
(30, 30, 30)	1.25	MJ	L	J, MJ	J
	2.87	J, BA, MJ, MBA	L	L, OB, J, BA, BF, MOB, MJ, MBA	J
	6.13	L, J, BA, BF, MJ, MBA	L	L, OB, J, BA, BF, MOB, MJ, MBA	J
(60, 60, 60)	1.25	J, MJ	L	L, OB, J, BA, BF, MOB, MJ, MBA	J, MJ
	2.87	L, OB, J, BA, BF, MOB, MJ, MBA	L, BF	L, OB, J, BA, BF, MOB, MJ, MBA	L, J, BA, MJ, MBA
	6.13	L, OB, J, BA, BF, MOB, MJ, MBA	L, J, BA, BF, MJ, MBA	L, OB, J, BA, BF, MOB, MJ, MBA	L, OB, J, BA, MOB, MJ, MBA
(5, 10, 15)	1.25	MJ	BA	BA	BA
	2.87	MJ	BA	BA	BA
	6.13	MJ	BA	MJ	BA
(20, 30, 40)	1.25	MJ	L	MJ	J
	2.87	BA, MJ, MBA	L	L, OB, J, BA, BF, MOB, MJ, MBA	J
	6.13	L, J, BA, MJ, MBA	L	L, OB, J, BA, BF, MOB, MJ, MBA	J
(45, 60, 75)	1.25	J, MJ	L	L, OB, J, BA, BF, MOB, MJ, MBA	J, MJ
	2.87	L, OB, J, BA, BF, MOB, MJ, MBA	L, BF	L, OB, J, BA, BF, MOB, MJ, MBA	L, J, BA, BF, MJ, MBA
	6.13	L, OB, J, BA, BF, MOB, MJ, MBA	L, BA, BF, MBA	L, OB, J, BA, BF, MOB, MJ, MBA	L, J, BA, BF, MJ, MBA

In this study, to compare the efficiency of four original homogeneity of variance tests consists of Levene's test (L), O'Brien's test (OB), Jackknife test (J) and Box-Andersen test (BA) and four modified tests consists of Modified Levene's test or Brown-Forsythe test (BF), Modified O'Brien's test (MOB), Modified Jackknife test (MJ), and Modified Box-Andersen test (MBA) for three populations. From **Table 5.2** and **Table 5.3** found that

- Significance level (0.01)

Under normal distribution, Modified Jackknife test (MJ) is the best test in all cases. Jackknife test (J) is the best test for medium and large sample size both in cases of equal and unequal. Under Laplace distribution, Levene's test (L) is the best test in almost all cases. Under uniform distribution, Modified Jackknife test (MJ) is the best test for sample sizes are equal whereas Levene's test (L) is the best test for sample size are unequal in almost all cases. Modified O'Brien's test (MOB) and Modified Box-Andersen test (MBA) are the best test for large sample size in cases of unequal. Under gamma distribution, Modified Jackknife test (MJ) is the best test for sample sizes are equal in almost all cases whereas Box-Andersen test (BA) is the best test for sample sizes are unequal in most cases.

- Significance level (0.05)

Under normal distribution, Modified Jackknife test (MJ) and Box-Andersen test (BA) are the best test for small sample size in cases of equal and unequal, respectively. Under Laplace distribution, Levene's test (L) is the best test in almost all cases except small sample size whereas Box-Andersen test (BA) is the best test in cases of small sample size. Under uniform distribution, Modified Jackknife test (MJ) is the best test in almost all cases. Under gamma distribution, Modified Jackknife test (MJ) is the best test for sample sizes are equal in almost all cases whereas Box-Andersen test (BA) is the best test for sample sizes are unequal in almost all cases.
The conclusion can be summarized in event of diagram according to **Figure 5.1** to **Figure 5.8** as follows:

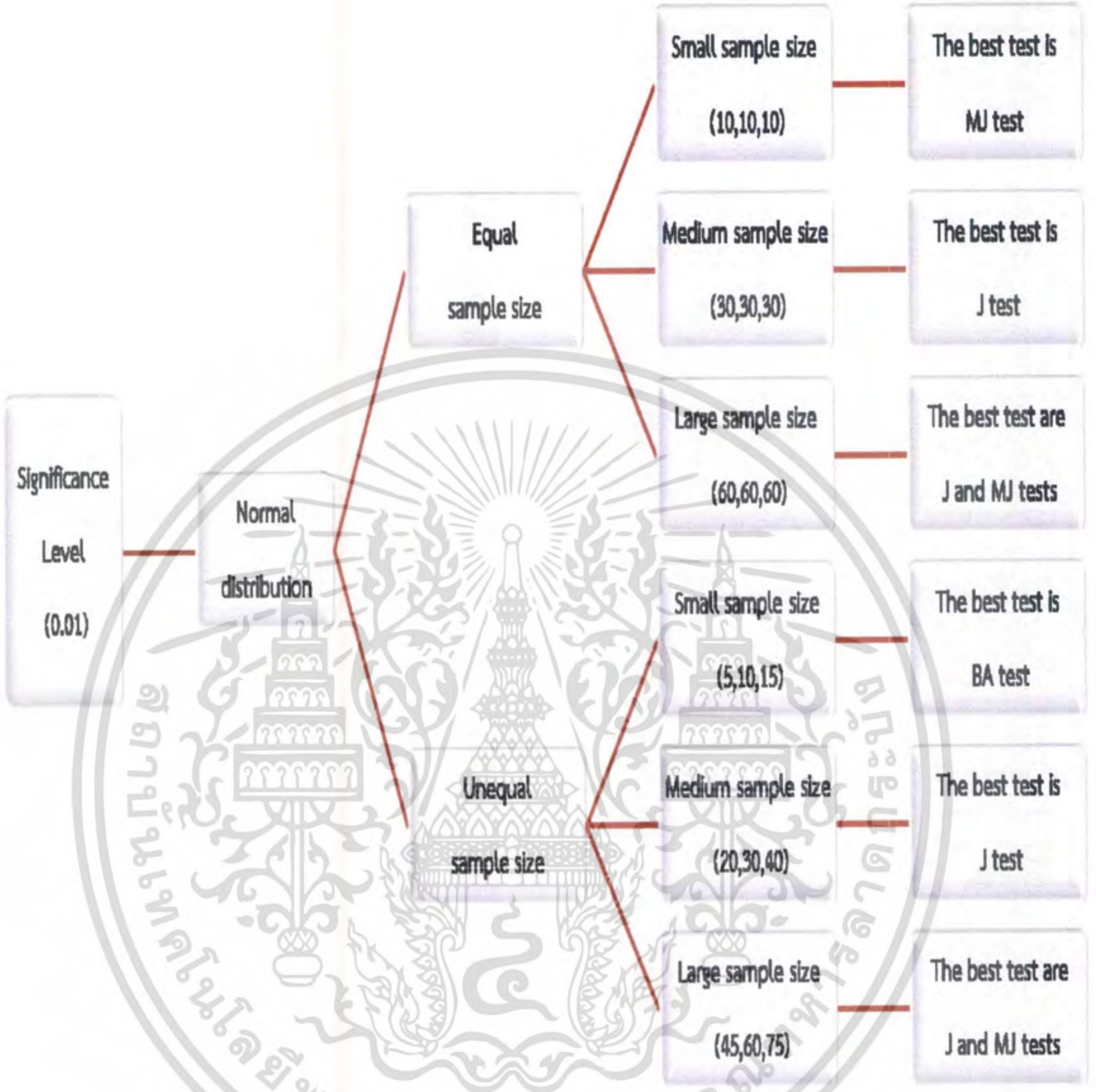


Figure 5.1 Diagram shows the best test statistics under normal distribution at Significance level (0.01)

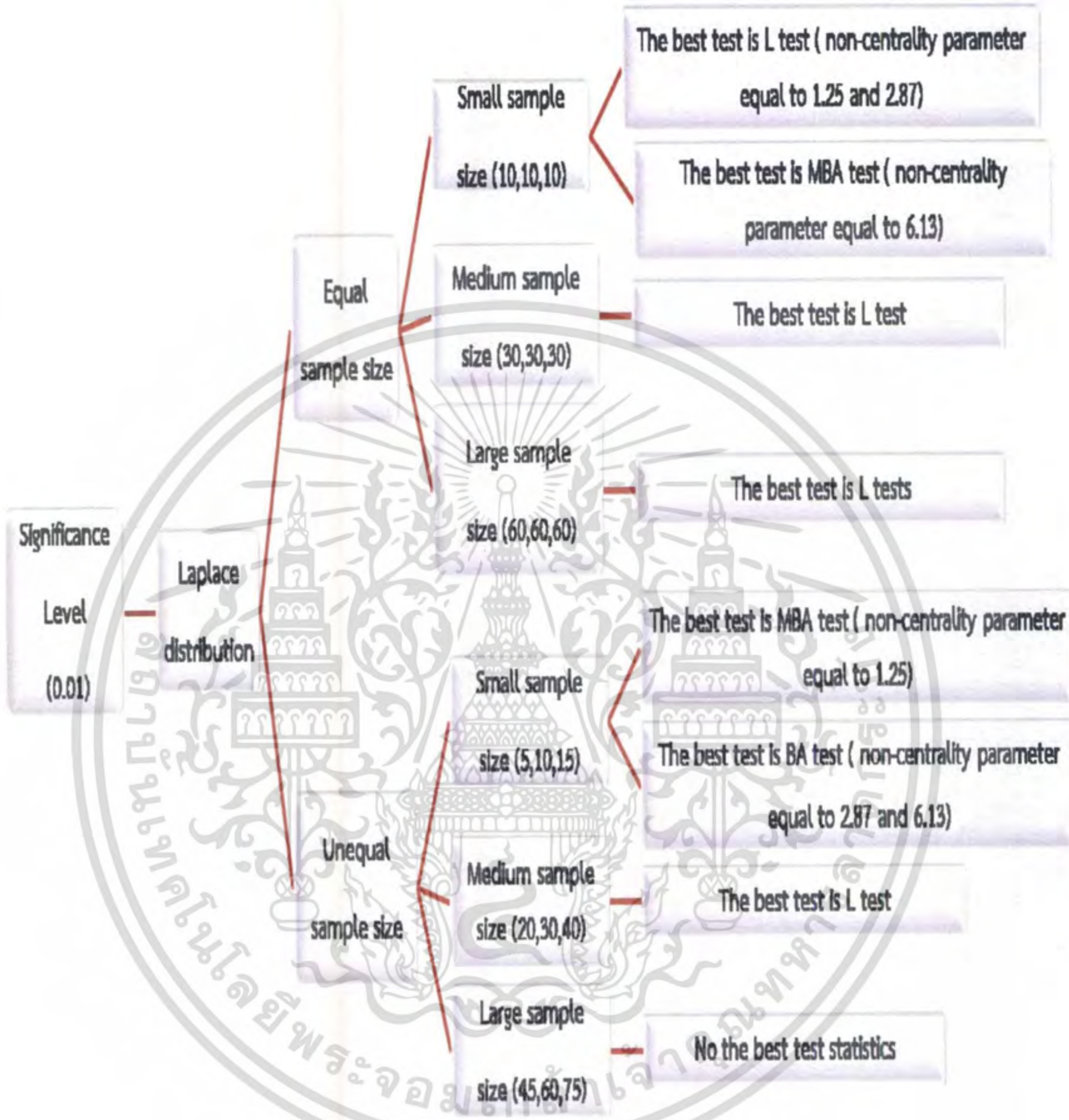


Figure 5.2 Diagram shows the best test statistics under Laplace distribution at significance level (0.01)

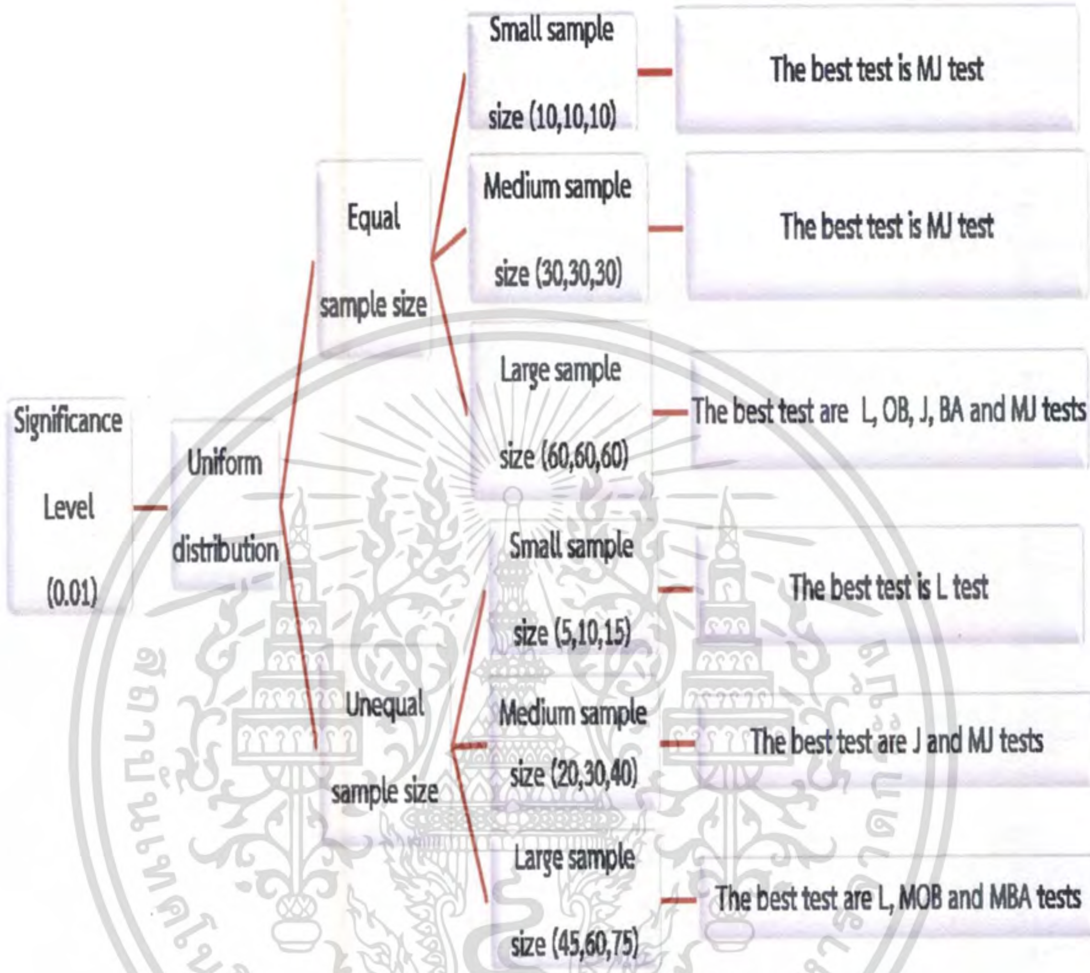


Figure 5.3 Diagram shows the best test statistics under uniform distribution at significance level (0.01)

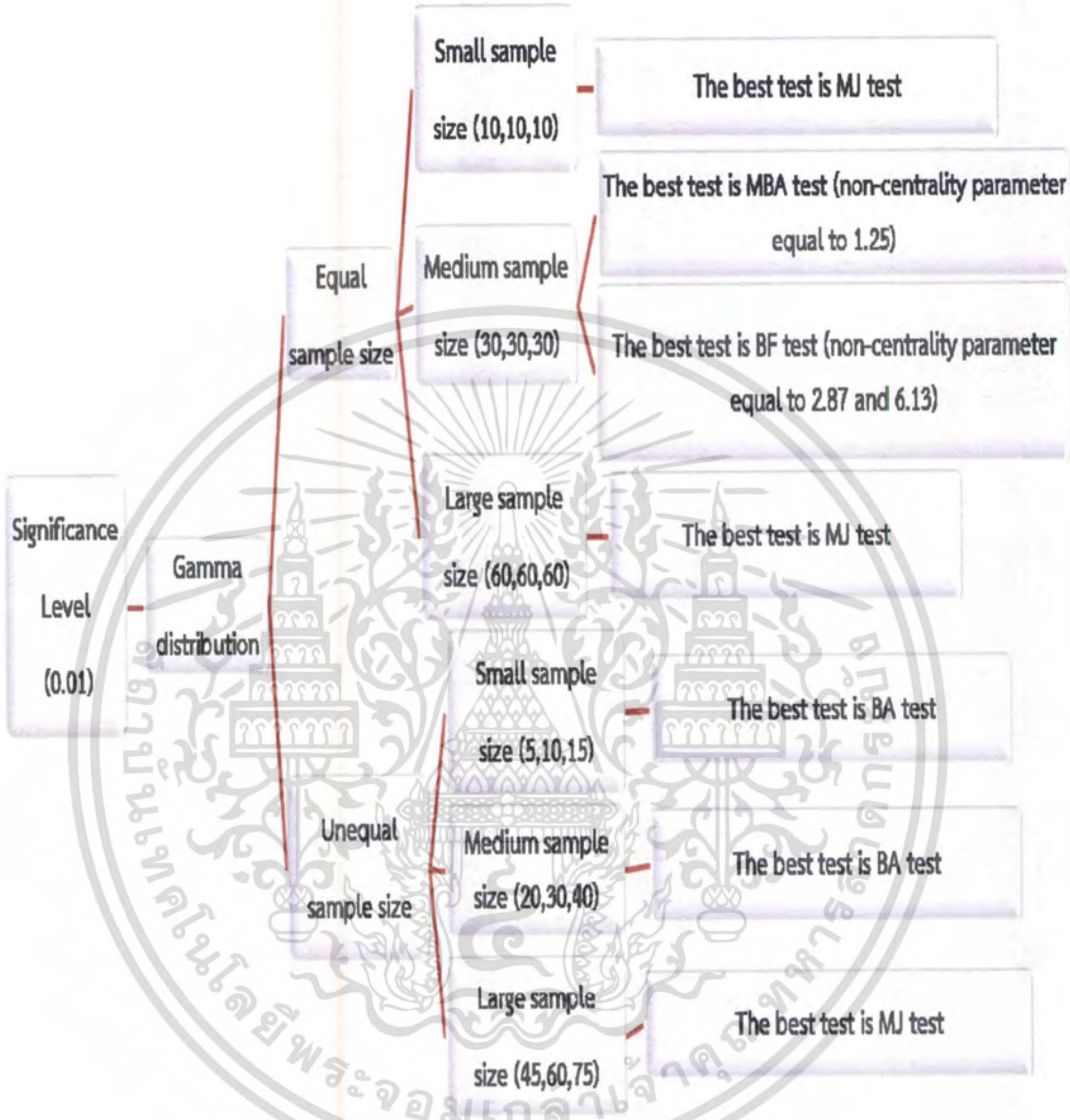


Figure 5.4 Diagram shows the best test statistics under gamma distribution at significance level (0.01)

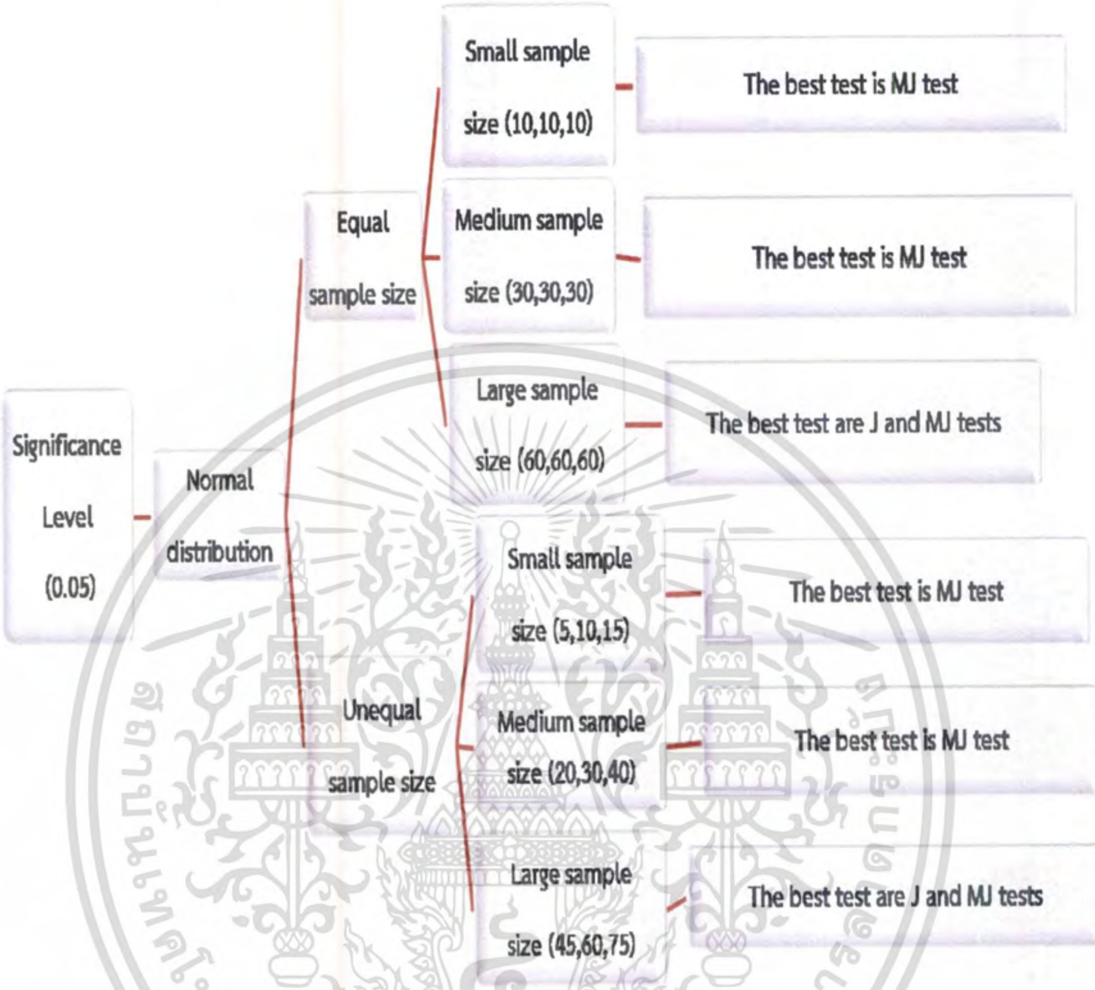


Figure 5.5 Diagram shows the best test statistics under normal distribution at significance level (0.05)

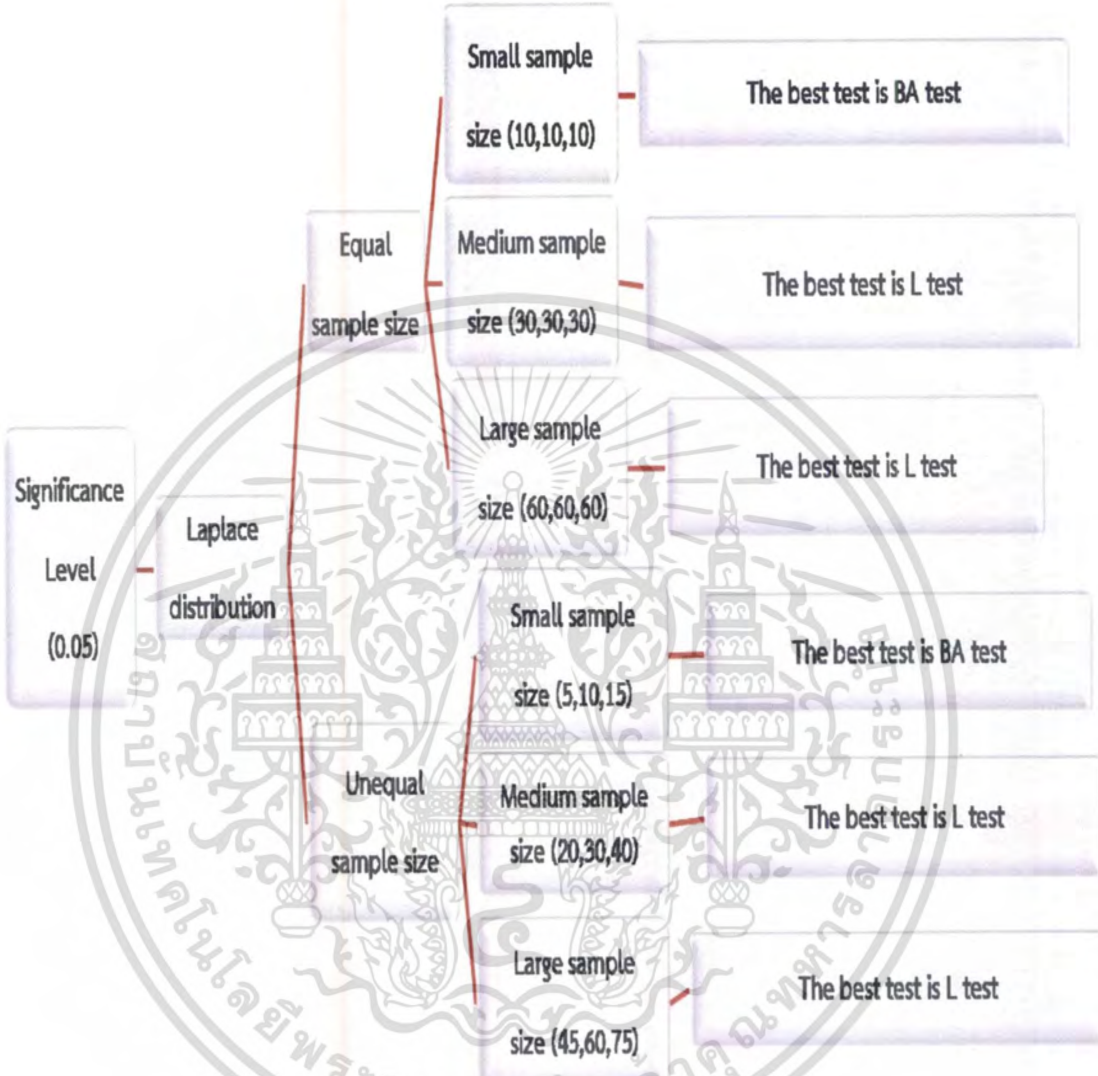


Figure 5.6 Diagram shows the best test statistics under Laplace distribution at significance level (0.05)

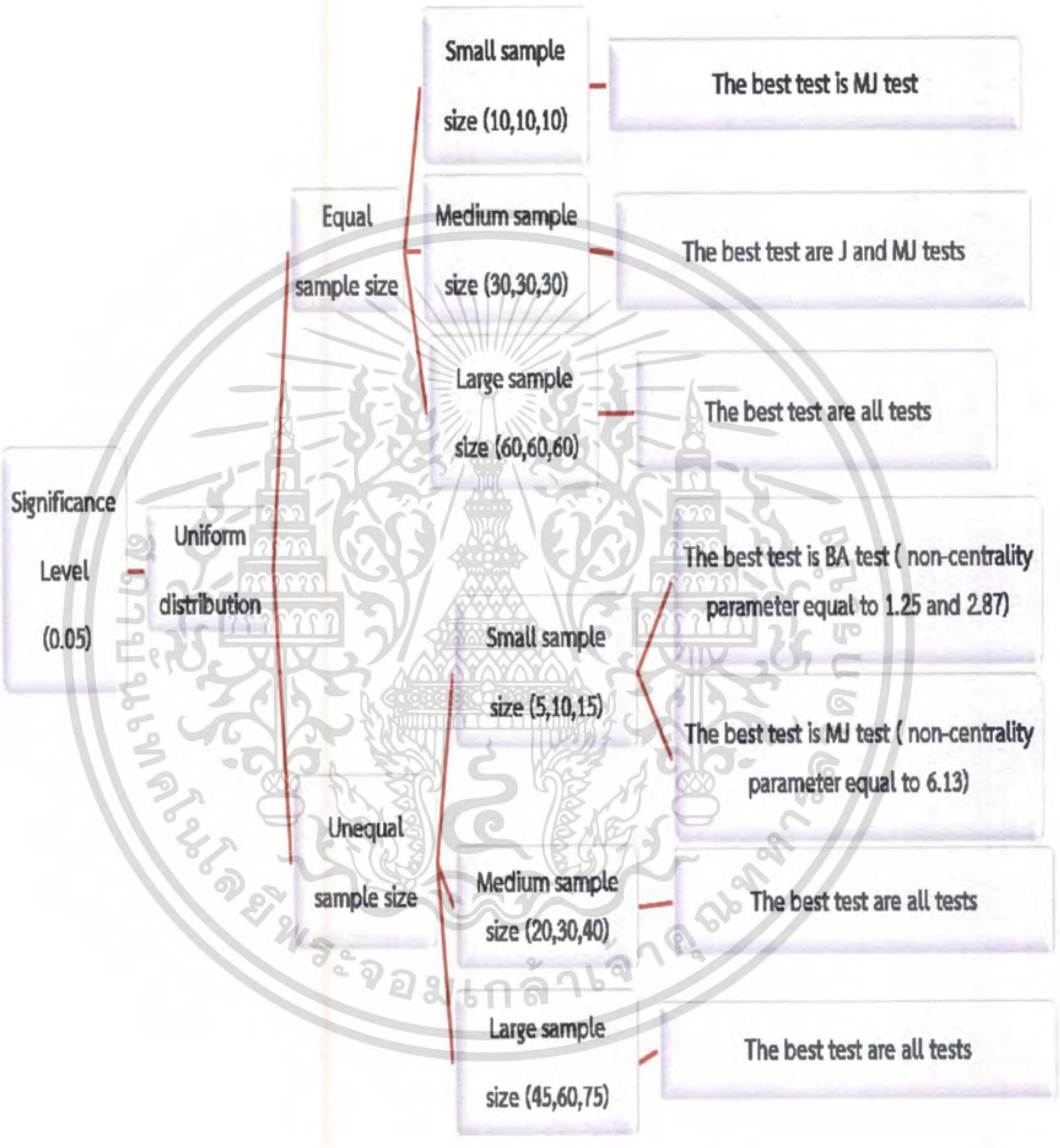


Figure 5.7 Diagram shows the best test statistics under uniform distribution at significance level (0.05)

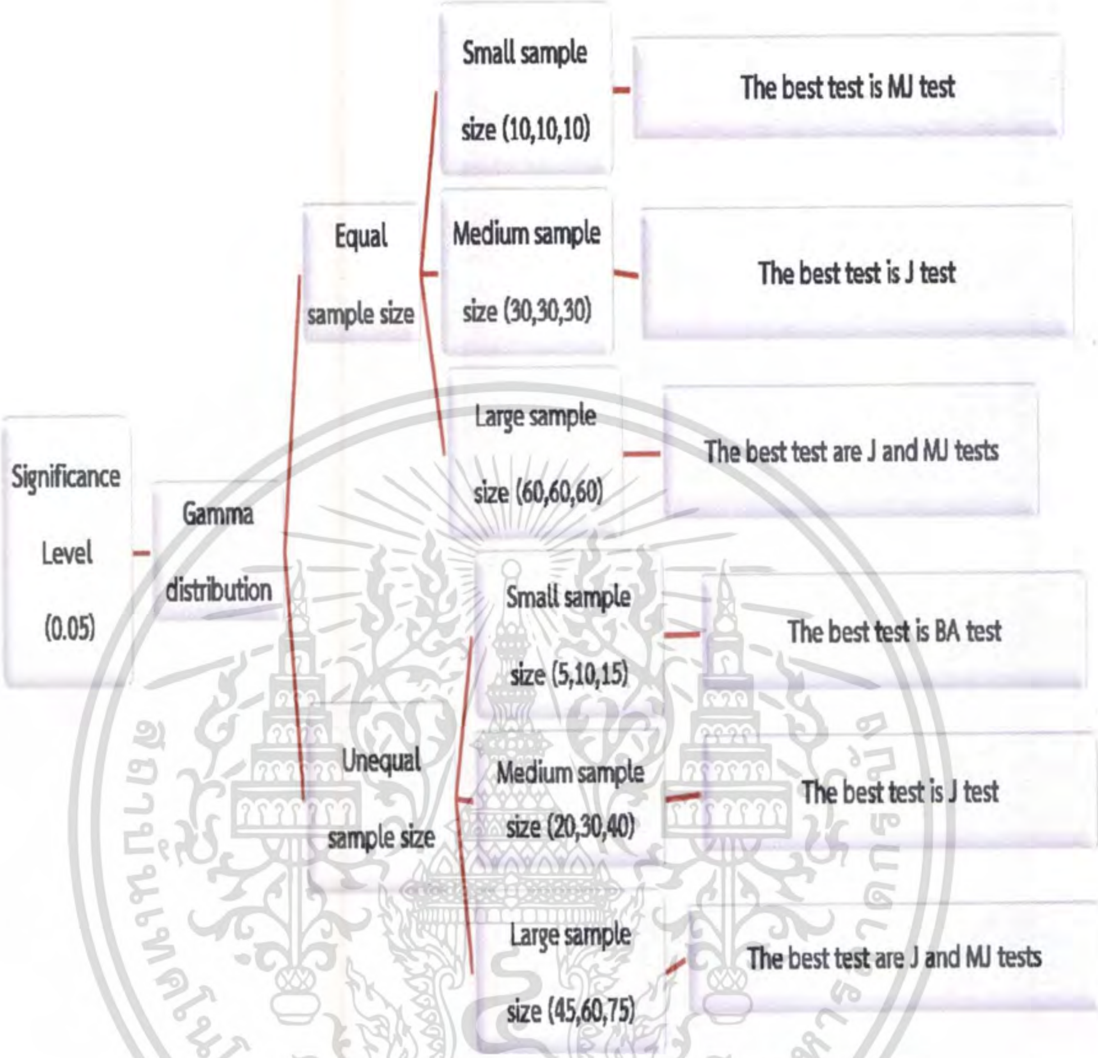


Figure 5.8 Diagram shows the best test statistics under gamma distribution at significance level (0.05)

5.2 Suggestions

5.2.1 Usability

1. This research can be applied to select the appropriate test statistic of homogeneity of variance test with population distributions consists of normal distribution, Laplace distribution, uniform distribution, and gamma distribution.

2. The modified tests in this research can be applied to select the appropriate test statistic of homogeneity of variance test greater than the original tests with

population distributions consists of normal distribution, Laplace distribution, uniform distribution, and gamma distribution.

3. The modified tests in this research can be used replace original test such as Levene's test under normal, uniform and gamma distributions.

5.2.2 Research

1. Study in case the population has other distributions or different distributions in each population.

2. Study in case of population more than 3 population, including 4 and 5 population.

3. Study in case of more sedentary qualities: 7, 8, 9, 10 or more.

4. The modified test can be change to variable formats such as square, log, and square root.

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Appendix

Appendix

The R program command used in this research

The R program command for probability of type I error and power of a test. These R program command can change by variables consists of var1, var2, var3, n1, n2, n3, and alpha.

R-code of Levene's test and Brown-forsythe test under normal distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
var1=8
var2=8
var3=8
n1=10;n2=10;n3=10
alpha=0.05
temp.L=rep(0,m)
temp.BF=rep(0,m)
p.L=c()
p.BF=c()
for(k in 1:m)
{
x1=rnorm(n1,mu1,sqrt(var1))
x2=rnorm(n2,mu2,sqrt(var2))
x3=rnorm(n3,mu3,sqrt(var3))
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
```

```
df=data.frame(x,g)
L=levene.test(df$x,df$g,"mean")
BF=levene.test(df$x,df$g,"median")
p.L[k]=L$p.value
p.BF[k]=BF$p.value
if(p.L[k]<alpha){temp.L[k]=1}
if(p.BF[k]<alpha){temp.BF[k]=1}
cat(c("loop : ",k),fill=T)
}
cat('\t',mean(temp.L),'\t',mean(temp.BF))
```

R-code of Levene's test and Brown-forsythe test under Laplace distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
var1=8
var2=8
var3=8
lamda1=sqrt(var1/2)
lamda2=sqrt(var2/2)
lamda3=sqrt(var3/2)
n1=10;n2=10;n3=10
alpha=0.05
temp.L=rep(0,m)
temp.BF=rep(0,m)
p.L=c()
p.BF=c()
for(k in 1:m)
{
x1=rdoublex(n1,mu1,lamda1)
```

```
x2=rdoublex(n2,mu2,lamda2)
x3=rdoublex(n3,mu3,lamda3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
L=levene.test(df$x,df$g,"mean")
BF=levene.test(df$x,df$g,"median")
p.L[k]=L$p.value
p.BF[k]=BF$p.value
if(p.L[k]<alpha){temp.L[k]=1}
if(p.BF[k]<alpha){temp.BF[k]=1}
cat(c("loop : ",k),fill=T)
}
cat('\t',mean(temp.L),'\t',mean(temp.BF))
```

R-code of Levene's test and Brown-forsythe test under uniform distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
var1=8
var2=8
var3=8
A1=mu1-sqrt(3*var1)
A2=mu2-sqrt(3*var2)
A3=mu3-sqrt(3*var3)
B1=mu1+sqrt(3*var1)
B2=mu2+sqrt(3*var2)
```

```
B3=mu3+sqrt(3*var3)
n1=10;n2=10;n3=10
alpha=0.05
temp.L=rep(0,m)
temp.BF=rep(0,m)
p.L=c()
p.BF=c()
for(k in 1:m)
{
x1=runif(n1,A1,B1)
x2=runif(n2,A2,B2)
x3=runif(n3,A3,B3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
L=levene.test(df$x,df$g,"mean")
BF=levene.test(df$x,df$g,"median")
p.L[k]=L$p.value
p.BF[k]=BF$p.value
if(p.L[k]<alpha){temp.L[k]=1}
if(p.BF[k]<alpha){temp.BF[k]=1}
cat(c("loop : ",k),fill=T)
}
cat('\t',mean(temp.L),'\t',mean(temp.BF))
```

R-code of Levene's test and Brown-forsythe test under gamma distribution as follows:

```
set.seed(55)
m=1000
mu1=8
```

```
mu2=8
mu3=8
var1=8
var2=8
var3=8
B1=var1/mu1
B2=var2/mu2
B3=var3/mu3
A1=(mu1^2)/var1
A2=(mu2^2)/var2
A3=(mu3^2)/var3
n1=10;n2=10;n3=10
alpha=0.05
temp.L=rep(0,m)
temp.BF=rep(0,m)
p.L=c()
p.BF=c()
for(k in 1:m)
{
x1=rgamma(n1,A1,1/B1)
x2=rgamma(n2,A2,1/B2)
x3=rgamma(n3,A3,1/B3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
L=levene.test(df$x,df$g,"mean")
BF=levene.test(df$x,df$g,"median")
p.L[k]=L$p.value
p.BF[k]=BF$p.value
if(p.L[k]<alpha){temp.L[k]=1}
```

```
if(p.BF[k]<alpha){temp.BF[k]=1}
cat(c("loop : ",k),fill=T)
}
cat('\t',mean(temp.L),'\t',mean(temp.BF))
```

R-code of O'Brien test and Modified O'Brien test under normal distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.OB=rep(0,m)
temp.MOB=rep(0,m)
p.OB=c()
p.MOB=c()
for(p in 1:m)
{
x1=rnorm(n1,mu1,sqrt(var1))
x2=rnorm(n2,mu2,sqrt(var2))
x3=rnorm(n3,mu3,sqrt(var3))
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
```

```
df=data.frame(x,g)
w=0.5
a1=(w+n1-2)*n1
a2=(w+n2-2)*n2
a3=(w+n3-2)*n3
aa1=(x1-mean(x1))^2
aa2=(x2-mean(x2))^2
aa3=(x3-mean(x3))^2
aaa1=(n1-1)*w*var(x1)
aaa2=(n2-1)*w*var(x2)
aaa3=(n3-1)*w*var(x3)
aaaa1=(n1-1)*(n1-2)
aaaa2=(n2-1)*(n2-2)
aaaa3=(n3-1)*(n3-2)
b1=(a1*aa1)
b2=(a2*aa2)
b3=(a3*aa3)
v1=(b1-aaa1)/aaaa1
v2=(b2-aaa2)/aaaa2
v3=(b3-aaa3)/aaaa3
v=c(v1,v2,v3)
c1=(mean(v1)-mean(v))^2
c2=(mean(v2)-mean(v))^2
c3=(mean(v3)-mean(v))^2
cc1=(v1-mean(v1))^2
cc2=(v2-mean(v2))^2
cc3=(v3-mean(v3))^2
df1=k-1
df2=N-k
c=sum(n1*c1,n2*c2,n3*c3)
cc=sum(cc1,cc2,cc3)
t1=c/df1
t2=cc/df2
```

```
OB=t1/t2
qf(1-alpha,df1,df2)
1-pf(OB,df1,df2)
a1a=(w+n1-2)*n1
a2a=(w+n2-2)*n2
a3a=(w+n3-2)*n3
aa1a=(x1-median(x1))^2
aa2a=(x2-median(x2))^2
aa3a=(x3-median(x3))^2
aaa1a=(n1-1)*w*var(x1)
aaa2a=(n2-1)*w*var(x2)
aaa3a=(n3-1)*w*var(x3)
aaaa1a=(n1-1)*(n1-2)
aaaa2a=(n2-1)*(n2-2)
aaaa3a=(n3-1)*(n3-2)
b1a=(a1a*aa1a)
b2a=(a2a*aa2a)
b3a=(a3a*aa3a)
v1a=(b1a-aaa1a)/aaaa1a
v2a=(b2a-aaa2a)/aaaa2a
v3a=(b3a-aaa3a)/aaaa3a
va=c(v1a,v2a,v3a)
c1a=(mean(v1a)-mean(va))^2
c2a=(mean(v2a)-mean(va))^2
c3a=(mean(v3a)-mean(va))^2
cc1a=(v1a-mean(v1a))^2
cc2a=(v2a-mean(v2a))^2
cc3a=(v3a-mean(v3a))^2
df1=k-1
df2=N-k
ca=sum(n1*c1a,n2*c2a,n3*c3a)
cca=sum(cc1a,cc2a,cc3a)
t1a=ca/df1
```

```
t2a=cc/df2
MOB=t1a/t2a
qf(1-alpha,df1,df2)
1-pf(MOB,df1,df2)
p.OB[p]=1-pf(OB,df1,df2)
p.MOB[p]=1-pf(MOB,df1,df2)
if(p.OB[p]<alpha){temp.OB[p]=1}
if(p.MOB[p]<alpha){temp.MOB[p]=1}
cat(c("loop : ",p),fill=T)
}
cat('\t',mean(temp.OB),'\t',mean(temp.MOB))
```

R-code of O'Brien test and Modified O'Brien test under Laplace distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
lamda1=sqrt(var1/2)
lamda2=sqrt(var2/2)
lamda3=sqrt(var3/2)
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.OB=rep(0,m)
temp.MOB=rep(0,m)
p.OB=c()
p.MOB=c()
```

```
for(p in 1:m)
{
x1=rdoublex(n1,mu1,lamda1)
x2=rdoublex(n2,mu2,lamda2)
x3=rdoublex(n3,mu3,lamda3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
w=0.5
a1=(w+n1-2)*n1
a2=(w+n2-2)*n2
a3=(w+n3-2)*n3
aa1=(x1-mean(x1))^2
aa2=(x2-mean(x2))^2
aa3=(x3-mean(x3))^2
aaa1=(n1-1)*w*var(x1)
aaa2=(n2-1)*w*var(x2)
aaa3=(n3-1)*w*var(x3)
aaaa1=(n1-1)*(n1-2)
aaaa2=(n2-1)*(n2-2)
aaaa3=(n3-1)*(n3-2)
b1=(a1*aa1)
b2=(a2*aa2)
b3=(a3*aa3)
v1=(b1-aaa1)/aaaa1
v2=(b2-aaa2)/aaaa2
v3=(b3-aaa3)/aaaa3
v=c(v1,v2,v3)
c1=(mean(v1)-mean(v))^2
c2=(mean(v2)-mean(v))^2
```

```
c3=(mean(v3)-mean(v))^2
cc1=(v1-mean(v1))^2
cc2=(v2-mean(v2))^2
cc3=(v3-mean(v3))^2
df1=k-1
df2=N-k
c=sum(n1*c1,n2*c2,n3*c3)
cc=sum(cc1,cc2,cc3)
t1=c/df1
t2=cc/df2
OB=t1/t2
qf(1-alpha,df1,df2)
1-pf(OB,df1,df2)
a1a=(w+n1-2)*n1
a2a=(w+n2-2)*n2
a3a=(w+n3-2)*n3
aa1a=(x1-median(x1))^2
aa2a=(x2-median(x2))^2
aa3a=(x3-median(x3))^2
aaa1a=(n1-1)*w*var(x1)
aaa2a=(n2-1)*w*var(x2)
aaa3a=(n3-1)*w*var(x3)
aaaa1a=(n1-1)*(n1-2)
aaaa2a=(n2-1)*(n2-2)
aaaa3a=(n3-1)*(n3-2)
b1a=(a1a*aa1a)
b2a=(a2a*aa2a)
b3a=(a3a*aa3a)
v1a=(b1a-aaa1a)/aaaa1a
v2a=(b2a-aaa2a)/aaaa2a
v3a=(b3a-aaa3a)/aaaa3a
va=c(v1a,v2a,v3a)
c1a=(mean(v1a)-mean(va))^2
```

```
c2a=(mean(v2a)-mean(va))^2
c3a=(mean(v3a)-mean(va))^2
cc1a=(v1a-mean(v1a))^2
cc2a=(v2a-mean(v2a))^2
cc3a=(v3a-mean(v3a))^2
df1=k-1
df2=N-k
ca=sum(n1*c1a,n2*c2a,n3*c3a)
cca=sum(cc1a,cc2a,cc3a)
t1a=ca/df1
t2a=cc/df2
MOB=t1a/t2a
qf(1-alpha,df1,df2)
1-pf(MOB,df1,df2)
p.OB[p]=1-pf(OB,df1,df2)
p.MOB[p]=1-pf(MOB,df1,df2)
if(p.OB[p]<alpha){temp.OB[p]=1}
if(p.MOB[p]<alpha){temp.MOB[p]=1}
cat(c("loop : ",p),fill=T)
}
cat('\t',mean(temp.OB),'\t',mean(temp.MOB))
```

R-code of O'Brien test and Modified O'Brien test under uniform distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
```

```
A1=mu1-sqrt(3*var1)
A2=mu2-sqrt(3*var2)
A3=mu3-sqrt(3*var3)
B1=mu1+sqrt(3*var1)
B2=mu2+sqrt(3*var2)
B3=mu3+sqrt(3*var3)
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.OB=rep(0,m)
temp.MOB=rep(0,m)
p.OB=c()
p.MOB=c()
for(p in 1:m)
{
x1=runif(n1,A1,B1)
x2=runif(n2,A2,B2)
x3=runif(n3,A3,B3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
w=0.5
a1=(w+n1-2)*n1
a2=(w+n2-2)*n2
a3=(w+n3-2)*n3
aa1=(x1-mean(x1))^2
aa2=(x2-mean(x2))^2
aa3=(x3-mean(x3))^2
aaa1=(n1-1)*w*var(x1)
aaa2=(n2-1)*w*var(x2)
```

```
aaa3=(n3-1)*w*var(x3)
aaaa1=(n1-1)*(n1-2)
aaaa2=(n2-1)*(n2-2)
aaaa3=(n3-1)*(n3-2)
b1=(a1*aa1)
b2=(a2*aa2)
b3=(a3*aa3)
v1=(b1-aaa1)/aaaa1
v2=(b2-aaa2)/aaaa2
v3=(b3-aaa3)/aaaa3
v=c(v1,v2,v3)
c1=(mean(v1)-mean(v))^2
c2=(mean(v2)-mean(v))^2
c3=(mean(v3)-mean(v))^2
cc1=(v1-mean(v1))^2
cc2=(v2-mean(v2))^2
cc3=(v3-mean(v3))^2
df1=k-1
df2=N-k
c=sum(n1*c1,n2*c2,n3*c3)
cc=sum(cc1,cc2,cc3)
t1=c/df1
t2=cc/df2
OB=t1/t2
qf(1-alpha,df1,df2)
1-pf(OB,df1,df2)
a1a=(w+n1-2)*n1
a2a=(w+n2-2)*n2
a3a=(w+n3-2)*n3
aa1a=(x1-median(x1))^2
aa2a=(x2-median(x2))^2
aa3a=(x3-median(x3))^2
aaa1a=(n1-1)*w*var(x1)
```

```
aaa2a=(n2-1)*w*var(x2)
aaa3a=(n3-1)*w*var(x3)
aaaa1a=(n1-1)*(n1-2)
aaaa2a=(n2-1)*(n2-2)
aaaa3a=(n3-1)*(n3-2)
b1a=(a1a*aa1a)
b2a=(a2a*aa2a)
b3a=(a3a*aa3a)
v1a=(b1a-aaa1a)/aaaa1a
v2a=(b2a-aaa2a)/aaaa2a
v3a=(b3a-aaa3a)/aaaa3a
va=c(v1a,v2a,v3a)
c1a=(mean(v1a)-mean(va))^2
c2a=(mean(v2a)-mean(va))^2
c3a=(mean(v3a)-mean(va))^2
cc1a=(v1a-mean(v1a))^2
cc2a=(v2a-mean(v2a))^2
cc3a=(v3a-mean(v3a))^2
df1=k-1
df2=N-k
ca=sum(n1*c1a,n2*c2a,n3*c3a)
cca=sum(cc1a,cc2a,cc3a)
t1a=ca/df1
t2a=cc/df2
MOB=t1a/t2a
qf(1-alpha,df1,df2)
1-pf(MOB,df1,df2)
p.OB[p]=1-pf(OB,df1,df2)
p.MOB[p]=1-pf(MOB,df1,df2)
if(p.OB[p]<alpha){temp.OB[p]=1}
if(p.MOB[p]<alpha){temp.MOB[p]=1}
cat(c("loop : ",p),fill=T)
}
```

```
cat('\t',mean(temp.OB),'\t',mean(temp.MOB))
```

R-code of O'Brien test and Modified O'Brien test under gamma distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
B1=var1/mu1
B2=var2/mu2
B3=var3/mu3
A1=(mu1^2)/var1
A2=(mu2^2)/var2
A3=(mu3^2)/var3
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.OB=rep(0,m)
temp.MOB=rep(0,m)
p.OB=c()
p.MOB=c()
for(p in 1:m)
{
x1=rgamma(n1,A1,1/B1)
x2=rgamma(n2,A2,1/B2)
x3=rgamma(n3,A3,1/B3)
x=c(x1,x2,x3)
g1=rep(1,n1)
```

```
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
w=0.5
a1=(w+n1-2)*n1
a2=(w+n2-2)*n2
a3=(w+n3-2)*n3
aa1=(x1-mean(x1))^2
aa2=(x2-mean(x2))^2
aa3=(x3-mean(x3))^2
aaa1=(n1-1)*w*var(x1)
aaa2=(n2-1)*w*var(x2)
aaa3=(n3-1)*w*var(x3)
aaaa1=(n1-1)*(n1-2)
aaaa2=(n2-1)*(n2-2)
aaaa3=(n3-1)*(n3-2)
b1=(a1*aa1)
b2=(a2*aa2)
b3=(a3*aa3)
v1=(b1-aaa1)/aaaa1
v2=(b2-aaa2)/aaaa2
v3=(b3-aaa3)/aaaa3
v=c(v1,v2,v3)
c1=(mean(v1)-mean(v))^2
c2=(mean(v2)-mean(v))^2
c3=(mean(v3)-mean(v))^2
cc1=(v1-mean(v1))^2
cc2=(v2-mean(v2))^2
cc3=(v3-mean(v3))^2
df1=k-1
df2=N-k
c=sum(n1*c1,n2*c2,n3*c3)
```

```
cc=sum(cc1,cc2,cc3)
t1=c/df1
t2=cc/df2
OB=t1/t2
qf(1-alpha,df1,df2)
1-pf(OB,df1,df2)
a1a=(w+n1-2)*n1
a2a=(w+n2-2)*n2
a3a=(w+n3-2)*n3
aa1a=(x1-median(x1))^2
aa2a=(x2-median(x2))^2
aa3a=(x3-median(x3))^2
aaa1a=(n1-1)*w*var(x1)
aaa2a=(n2-1)*w*var(x2)
aaa3a=(n3-1)*w*var(x3)
aaaa1a=(n1-1)*(n1-2)
aaaa2a=(n2-1)*(n2-2)
aaaa3a=(n3-1)*(n3-2)
b1a=(a1a*aa1a)
b2a=(a2a*aa2a)
b3a=(a3a*aa3a)
v1a=(b1a-aaa1a)/aaaa1a
v2a=(b2a-aaa2a)/aaaa2a
v3a=(b3a-aaa3a)/aaaa3a
va=c(v1a,v2a,v3a)
c1a=(mean(v1a)-mean(va))^2
c2a=(mean(v2a)-mean(va))^2
c3a=(mean(v3a)-mean(va))^2
cc1a=(v1a-mean(v1a))^2
cc2a=(v2a-mean(v2a))^2
cc3a=(v3a-mean(v3a))^2
df1=k-1
df2=N-k
```

```
ca=sum(n1*c1a,n2*c2a,n3*c3a)
cca=sum(cc1a,cc2a,cc3a)
t1a=ca/df1
t2a=cc/df2
MOB=t1a/t2a
qf(1-alpha,df1,df2)
1-pf(MOB,df1,df2)
p.OB[p]=1-pf(OB,df1,df2)
p.MOB[p]=1-pf(MOB,df1,df2)
if(p.OB[p]<alpha){temp.OB[p]=1}
if(p.MOB[p]<alpha){temp.MOB[p]=1}
cat(c("loop : ",p),fill=T)
}
cat('\t',mean(temp.OB),'\t',mean(temp.MOB))
```

R-code of Jackknife test and Modified Jackknife test under normal distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.J=rep(0,m)
p.J=c()
temp.MJ=rep(0,m)
p.MJ=c()
```

```
for(p in 1:m)
{
x1=rnorm(n1,mu1,sqrt(var1))
x2=rnorm(n2,mu2,sqrt(var2))
x3=rnorm(n3,mu3,sqrt(var3))
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
a1=(n1-1)*var(x1)
a2=(n2-1)*var(x2)
a3=(n3-1)*var(x3)
b1=(x1-mean(x1))^2
b2=(x2-mean(x2))^2
b3=(x3-mean(x3))^2
bb1=n1*b1
bb2=n2*b2
bb3=n3*b3
bbb1=bb1/(n1-1)
bbb2=bb2/(n2-1)
bbb3=bb3/(n3-1)
c1=a1-bbb1
c2=a2-bbb2
c3=a3-bbb3
s1=c1/(n1-2)
s2=c2/(n2-2)
s3=c3/(n3-2)
d1=log(var(x1))
d2=log(var(x2))
d3=log(var(x3))
dd1=n1*d1
```

```
dd2=n2*d2
dd3=n3*d3
e1=log(s1)
e2=log(s2)
e3=log(s3)
ee1=(n1-1)*e1
ee2=(n2-1)*e2
ee3=(n3-1)*e3
u1=dd1-ee1
u2=dd2-ee2
u3=dd3-ee3
u=c(u1,u2,u3)
f1=(mean(u1)-mean(u))^2
f2=(mean(u2)-mean(u))^2
f3=(mean(u3)-mean(u))^2
ff1=n1*f1
ff2=n2*f2
ff3=n3*f3
ff=sum(ff1,ff2,ff3)
o1=(u1-mean(u1))^2
o2=(u2-mean(u2))^2
o3=(u3-mean(u3))^2
o=sum(o1,o2,o3)
df1=k-1
df2=N-k
t1=ff/df1
t2=o/df2
J=t1/t2
qf(1-alpha,df1,df2)
1-pf(J,df1,df2)
z1=(x1-median(x1))^2
z2=(x2-median(x2))^2
z3=(x3-median(x3))^2
```

```
zz1=sum(z1)
zz2=sum(z2)
zz3=sum(z3)
zzz1=zz1/(n1-1)
zzz2=zz2/(n2-1)
zzz3=zz3/(n3-1)
ma1=(n1-1)*zzz1
ma2=(n2-1)*zzz2
ma3=(n3-1)*zzz3
mb1=(x1-mean(x1))^2
mb2=(x2-mean(x2))^2
mb3=(x3-mean(x3))^2
mbb1=n1*mb1
mbb2=n2*mb2
mbb3=n3*mb3
mbbb1=mbb1/(n1-1)
mbbb2=mbb2/(n2-1)
mbbb3=mbb3/(n3-1)
mc1=ma1-mbbb1
mc2=ma2-mbbb2
mc3=ma3-mbbb3
ms1=mc1/(n1-2)
ms2=mc2/(n2-2)
ms3=mc3/(n3-2)
md1=log(zzz1)
md2=log(zzz2)
md3=log(zzz3)
mdd1=n1*md1
mdd2=n2*md2
mdd3=n3*md3
me1=log(ms1)
me2=log(ms2)
me3=log(ms3)
```

```
mee1=(n1-1)*me1
mee2=(n2-1)*me2
mee3=(n3-1)*me3
u1m=mdd1-mee1
u2m=mdd2-mee2
u3m=mdd3-mee3
um=c(u1m,u2m,u3m)
mf1=(mean(u1m)-mean(um))^2
mf2=(mean(u2m)-mean(um))^2
mf3=(mean(u3m)-mean(um))^2
mff1=n1*mf1
mff2=n2*mf2
mff3=n3*mf3
mff=sum(mff1,mff2,mff3)
mo1=(u1m-mean(u1m))^2
mo2=(u2m-mean(u2m))^2
mo3=(u3m-mean(u3m))^2
mo=sum(mo1,mo2,mo3)
df1=k-1
df2=N-k
mt1=mff/df1
mt2=mo/df2
MJ=mt1/mt2
qf(1-alpha,df1,df2)
1-pf(MJ,df1,df2)
p.J[p]=1-pf(J,df1,df2)
if(p.J[p]<alpha){temp.J[p]=1}
p.MJ[p]=1-pf(MJ,df1,df2)
if(p.MJ[p]<alpha){temp.MJ[p]=1}
cat(c("loop : ",p),fill=T)
}
cat('\t',mean(temp.J),'\t',mean(temp.MJ))
```

R-code of Jackknife test and Modified Jackknife test under Laplace distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
lamda1=sqrt(var1/2)
lamda2=sqrt(var2/2)
lamda3=sqrt(var3/2)
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.J=rep(0,m)
p.J=c()
temp.MJ=rep(0,m)
p.MJ=c()
for(p in 1:m)
{
x1=rdoublex(n1,mu1,lamda1)
x2=rdoublex(n2,mu2,lamda2)
x3=rdoublex(n3,mu3,lamda3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
a1=(n1-1)*var(x1)
```

```
a2=(n2-1)*var(x2)
a3=(n3-1)*var(x3)
b1=(x1-mean(x1))^2
b2=(x2-mean(x2))^2
b3=(x3-mean(x3))^2
bb1=n1*b1
bb2=n2*b2
bb3=n3*b3
bbb1=bb1/(n1-1)
bbb2=bb2/(n2-1)
bbb3=bb3/(n3-1)
c1=a1-bbb1
c2=a2-bbb2
c3=a3-bbb3
s1=c1/(n1-2)
s2=c2/(n2-2)
s3=c3/(n3-2)
d1=log(var(x1))
d2=log(var(x2))
d3=log(var(x3))
dd1=n1*d1
dd2=n2*d2
dd3=n3*d3
e1=log(s1)
e2=log(s2)
e3=log(s3)
ee1=(n1-1)*e1
ee2=(n2-1)*e2
ee3=(n3-1)*e3
u1=dd1-ee1
u2=dd2-ee2
u3=dd3-ee3
u=c(u1,u2,u3)
```

```
f1=(mean(u1)-mean(u))^2
f2=(mean(u2)-mean(u))^2
f3=(mean(u3)-mean(u))^2
ff1=n1*f1
ff2=n2*f2
ff3=n3*f3
ff=sum(ff1,ff2,ff3)
o1=(u1-mean(u1))^2
o2=(u2-mean(u2))^2
o3=(u3-mean(u3))^2
o=sum(o1,o2,o3)
df1=k-1
df2=N-k
t1=ff/df1
t2=o/df2
J=t1/t2
qf(1-alpha,df1,df2)
1-pf(J,df1,df2)
z1=(x1-median(x1))^2
z2=(x2-median(x2))^2
z3=(x3-median(x3))^2
zz1=sum(z1)
zz2=sum(z2)
zz3=sum(z3)
zzz1=zz1/(n1-1)
zzz2=zz2/(n2-1)
zzz3=zz3/(n3-1)
ma1=(n1-1)*zzz1
ma2=(n2-1)*zzz2
ma3=(n3-1)*zzz3
mb1=(x1-mean(x1))^2
mb2=(x2-mean(x2))^2
mb3=(x3-mean(x3))^2
```

```
mbb1=n1*mb1
mbb2=n2*mb2
mbb3=n3*mb3
mbbb1=mbb1/(n1-1)
mbbb2=mbb2/(n2-1)
mbbb3=mbb3/(n3-1)
mc1=ma1-mbbb1
mc2=ma2-mbbb2
mc3=ma3-mbbb3
ms1=mc1/(n1-2)
ms2=mc2/(n2-2)
ms3=mc3/(n3-2)
md1=log(zzz1)
md2=log(zzz2)
md3=log(zzz3)
mdd1=n1*md1
mdd2=n2*md2
mdd3=n3*md3
me1=log(ms1)
me2=log(ms2)
me3=log(ms3)
mee1=(n1-1)*me1
mee2=(n2-1)*me2
mee3=(n3-1)*me3
u1m=mdd1-mee1
u2m=mdd2-mee2
u3m=mdd3-mee3
um=c(u1m,u2m,u3m)
mf1=(mean(u1m)-mean(um))^2
mf2=(mean(u2m)-mean(um))^2
mf3=(mean(u3m)-mean(um))^2
mff1=n1*mf1
mff2=n2*mf2
```

```
mff3=n3*mf3
mff=sum(mff1,mff2,mff3)
mo1=(u1m-mean(u1m))^2
mo2=(u2m-mean(u2m))^2
mo3=(u3m-mean(u3m))^2
mo=sum(mo1,mo2,mo3)
df1=k-1
df2=N-k
mt1=mff/df1
mt2=mo/df2
MJ=mt1/mt2
qf(1-alpha,df1,df2)
1-pf(MJ,df1,df2)
p.J[p]=1-pf(J,df1,df2)
if(p.J[p]<alpha){temp.J[p]=1}
p.MJ[p]=1-pf(MJ,df1,df2)
if(p.MJ[p]<alpha){temp.MJ[p]=1}
cat(c("loop : ",p),fill=T)
}
cat('\t',mean(temp.J),'\t',mean(temp.MJ))
```

R-code of Jackknife test and Modified Jackknife test under uniform distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
A1=mu1-sqrt(3*var1)
```

```
A2=mu2-sqrt(3*var2)
A3=mu3-sqrt(3*var3)
B1=mu1+sqrt(3*var1)
B2=mu2+sqrt(3*var2)
B3=mu3+sqrt(3*var3)
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.J=rep(0,m)
p.J=c()
temp.MJ=rep(0,m)
p.MJ=c()
for(p in 1:m)
{
x1=runif(n1,A1,B1)
x2=runif(n2,A2,B2)
x3=runif(n3,A3,B3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
a1=(n1-1)*var(x1)
a2=(n2-1)*var(x2)
a3=(n3-1)*var(x3)
b1=(x1-mean(x1))^2
b2=(x2-mean(x2))^2
b3=(x3-mean(x3))^2
bb1=n1*b1
bb2=n2*b2
bb3=n3*b3
bbb1=bb1/(n1-1)
```

```
bbb2=bb2/(n2-1)
bbb3=bb3/(n3-1)
c1=a1-bbb1
c2=a2-bbb2
c3=a3-bbb3
s1=c1/(n1-2)
s2=c2/(n2-2)
s3=c3/(n3-2)
d1=log(var(x1))
d2=log(var(x2))
d3=log(var(x3))
dd1=n1*d1
dd2=n2*d2
dd3=n3*d3
e1=log(s1)
e2=log(s2)
e3=log(s3)
ee1=(n1-1)*e1
ee2=(n2-1)*e2
ee3=(n3-1)*e3
u1=dd1-ee1
u2=dd2-ee2
u3=dd3-ee3
u=c(u1,u2,u3)
f1=(mean(u1)-mean(u))^2
f2=(mean(u2)-mean(u))^2
f3=(mean(u3)-mean(u))^2
ff1=n1*f1
ff2=n2*f2
ff3=n3*f3
ff=sum(ff1,ff2,ff3)
o1=(u1-mean(u1))^2
o2=(u2-mean(u2))^2
```

```
o3=(u3-mean(u3))^2
o=sum(o1,o2,o3)
df1=k-1
df2=N-k
t1=ff/df1
t2=o/df2
J=t1/t2
qf(1-alpha,df1,df2)
1-pf(J,df1,df2)
z1=(x1-median(x1))^2
z2=(x2-median(x2))^2
z3=(x3-median(x3))^2
zz1=sum(z1)
zz2=sum(z2)
zz3=sum(z3)
zzz1=zz1/(n1-1)
zzz2=zz2/(n2-1)
zzz3=zz3/(n3-1)
ma1=(n1-1)*zzz1
ma2=(n2-1)*zzz2
ma3=(n3-1)*zzz3
mb1=(x1-mean(x1))^2
mb2=(x2-mean(x2))^2
mb3=(x3-mean(x3))^2
mbb1=n1*mb1
mbb2=n2*mb2
mbb3=n3*mb3
mbbb1=mbb1/(n1-1)
mbbb2=mbb2/(n2-1)
mbbb3=mbb3/(n3-1)
mc1=ma1-mbbb1
mc2=ma2-mbbb2
mc3=ma3-mbbb3
```

```
ms1=mc1/(n1-2)
ms2=mc2/(n2-2)
ms3=mc3/(n3-2)
md1=log(zzz1)
md2=log(zzz2)
md3=log(zzz3)
mdd1=n1*md1
mdd2=n2*md2
mdd3=n3*md3
me1=log(ms1)
me2=log(ms2)
me3=log(ms3)
mee1=(n1-1)*me1
mee2=(n2-1)*me2
mee3=(n3-1)*me3
u1m=mdd1-mee1
u2m=mdd2-mee2
u3m=mdd3-mee3
um=c(u1m,u2m,u3m)
mf1=(mean(u1m)-mean(um))^2
mf2=(mean(u2m)-mean(um))^2
mf3=(mean(u3m)-mean(um))^2
mff1=n1*mf1
mff2=n2*mf2
mff3=n3*mf3
mff=sum(mff1,mff2,mff3)
mo1=(u1m-mean(u1m))^2
mo2=(u2m-mean(u2m))^2
mo3=(u3m-mean(u3m))^2
mo=sum(mo1,mo2,mo3)
df1=k-1
df2=N-k
mt1=mff/df1
```

```
mt2=mo/df2
MJ=mt1/mt2
qf(1-alpha,df1,df2)
1-pf(MJ,df1,df2)
p.J[p]=1-pf(J,df1,df2)
if(p.J[p]<alpha){temp.J[p]=1}
p.MJ[p]=1-pf(MJ,df1,df2)
if(p.MJ[p]<alpha){temp.MJ[p]=1}
cat(c("loop : ",p),fill=T)
}
cat('\t',mean(temp.J),'\t',mean(temp.MJ))
```

R-code of Jackknife test and Modified Jackknife test under gamma distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
B1=var1/mu1
B2=var2/mu2
B3=var3/mu3
A1=(mu1^2)/var1
A2=(mu2^2)/var2
A3=(mu3^2)/var3
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.J=rep(0,m)
```

```
p.J=c()
temp.MJ=rep(0,m)
p.MJ=c()
for(p in 1:m)
{
x1=rgamma(n1,A1,1/B1)
x2=rgamma(n2,A2,1/B2)
x3=rgamma(n3,A3,1/B3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
a1=(n1-1)*var(x1)
a2=(n2-1)*var(x2)
a3=(n3-1)*var(x3)
b1=(x1-mean(x1))^2
b2=(x2-mean(x2))^2
b3=(x3-mean(x3))^2
bb1=n1*b1
bb2=n2*b2
bb3=n3*b3
bbb1=bb1/(n1-1)
bbb2=bb2/(n2-1)
bbb3=bb3/(n3-1)
c1=a1-bbb1
c2=a2-bbb2
c3=a3-bbb3
s1=c1/(n1-2)
s2=c2/(n2-2)
s3=c3/(n3-2)
d1=log(var(x1))
```

```
d2=log(var(x2))
d3=log(var(x3))
dd1=n1*d1
dd2=n2*d2
dd3=n3*d3
e1=log(s1)
e2=log(s2)
e3=log(s3)
ee1=(n1-1)*e1
ee2=(n2-1)*e2
ee3=(n3-1)*e3
u1=dd1-ee1
u2=dd2-ee2
u3=dd3-ee3
u=c(u1,u2,u3)
f1=(mean(u1)-mean(u))^2
f2=(mean(u2)-mean(u))^2
f3=(mean(u3)-mean(u))^2
ff1=n1*f1
ff2=n2*f2
ff3=n3*f3
ff=sum(ff1,ff2,ff3)
o1=(u1-mean(u1))^2
o2=(u2-mean(u2))^2
o3=(u3-mean(u3))^2
o=sum(o1,o2,o3)
df1=k-1
df2=N-k
t1=ff/df1
t2=o/df2
J=t1/t2
qf(1-alpha,df1,df2)
1-pf(J,df1,df2)
```

```
z1=(x1-median(x1))^2
z2=(x2-median(x2))^2
z3=(x3-median(x3))^2
zz1=sum(z1)
zz2=sum(z2)
zz3=sum(z3)
zzz1=zz1/(n1-1)
zzz2=zz2/(n2-1)
zzz3=zz3/(n3-1)
ma1=(n1-1)*zzz1
ma2=(n2-1)*zzz2
ma3=(n3-1)*zzz3
mb1=(x1-mean(x1))^2
mb2=(x2-mean(x2))^2
mb3=(x3-mean(x3))^2
mbb1=n1*mb1
mbb2=n2*mb2
mbb3=n3*mb3
mbbb1=mbb1/(n1-1)
mbbb2=mbb2/(n2-1)
mbbb3=mbb3/(n3-1)
mc1=ma1-mbbb1
mc2=ma2-mbbb2
mc3=ma3-mbbb3
ms1=mc1/(n1-2)
ms2=mc2/(n2-2)
ms3=mc3/(n3-2)
md1=log(zzz1)
md2=log(zzz2)
md3=log(zzz3)
mdd1=n1*md1
mdd2=n2*md2
mdd3=n3*md3
```

```
me1=log(ms1)
me2=log(ms2)
me3=log(ms3)
mee1=(n1-1)*me1
mee2=(n2-1)*me2
mee3=(n3-1)*me3
u1m=mdd1-mee1
u2m=mdd2-mee2
u3m=mdd3-mee3
um=c(u1m,u2m,u3m)
mf1=(mean(u1m)-mean(um))^2
mf2=(mean(u2m)-mean(um))^2
mf3=(mean(u3m)-mean(um))^2
mff1=n1*mf1
mff2=n2*mf2
mff3=n3*mf3
mff=sum(mff1,mff2,mff3)
mo1=(u1m-mean(u1m))^2
mo2=(u2m-mean(u2m))^2
mo3=(u3m-mean(u3m))^2
mo=sum(mo1,mo2,mo3)
df1=k-1
df2=N-k
mt1=mff/df1
mt2=mo/df2
MJ=mt1/mt2
qf(1-alpha,df1,df2)
1-pf(MJ,df1,df2)
p.J[p]=1-pf(J,df1,df2)
if(p.J[p]<alpha){temp.J[p]=1}
p.MJ[p]=1-pf(MJ,df1,df2)
if(p.MJ[p]<alpha){temp.MJ[p]=1}
cat(c("loop : ",p),fill=T)
```

```
}
cat('\t',mean(temp.J),'\t',mean(temp.MJ))
```

R-code of Box-Andersen test and Modified Box-Andersen test under normal distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.BA=rep(0,m)
p.BA=c()
temp.MBA=rep(0,m)
p.MBA=c()
for(p in 1:m)
{
x1=rnorm(n1,mu1,sqrt(var1))
x2=rnorm(n2,mu2,sqrt(var2))
x3=rnorm(n3,mu3,sqrt(var3))
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
a1=n1-1
```

```
a2=n2-1
a3=n3-1
aa1=a1*var(x1)
aa2=a2*var(x2)
aa3=a3*var(x3)
aa=sum(aa1,aa2,aa3)
aaa=aa/(N-k)
a=(N-k)*log(aaa)
b1=a1*log(var(x1))
b2=a2*log(var(x2))
b3=a3*log(var(x3))
b=sum(b1,b2,b3)
ab=a-b
c1=1/a1
c2=1/a2
c3=1/a3
c=sum(c1,c2,c3)
cc=1/(N-k)
ccc=c-cc
d=3*(k-1)
dd=ccc/d
ddd=1+dd
Bart=ab/ddd
df=k-1
qchisq(1-alpha,df)
1-pchisq(Bart,df)
e1=(x1-mean(x1))^4
e2=(x2-mean(x2))^4
e3=(x3-mean(x3))^4
ee1=(x1-mean(x1))^2
ee2=(x2-mean(x2))^2
ee3=(x3-mean(x3))^2
e=sum(e1,e2,e3)
```

```
ee=sum(ee1,ee2,ee3)
f1=N*e
f2=(ee)^2
f=f1/f2
ff=2/(f-1)
MBart=ff*Bart
df=k-1
qchisq(1-alpha,df)
1-pchisq(MBart,df)
BA=ff*ab
df=(k-1)
qchisq(1-alpha,df)
1-pchisq(BA,df)
s1=(x1-median(x1))^2
s2=(x2-median(x2))^2
s3=(x3-median(x3))^2
ss1=sum(s1)
ss2=sum(s2)
ss3=sum(s3)
sss1=ss1/(n1-1)
sss2=ss2/(n2-1)
sss3=ss3/(n3-1)
a1z=n1-1
a2z=n2-1
a3z=n3-1
aa1z=a1z*sss1
aa2z=a2z*sss2
aa3z=a3z*sss3
aaz=sum(aa1z,aa2z,aa3z)
aaaz=aaz/(N-k)
az=(N-k)*log(aaaz)
b1z=a1z*log(sss1)
b2z=a2z*log(sss2)
```

```
b3z=a3z*log(sss3)
bz=sum(b1z,b2z,b3z)
abz=az-bz
c1z=1/a1z
c2z=1/a2z
c3z=1/a3z
cz=sum(c1z,c2z,c3z)
ccz=1/(N-k)
cccz=cz-ccz
dz=3*(k-1)
ddz=cccz/dz
dddz=1+ddz
BartM=abz/dddz
df=k-1
qchisq(1-alpha,df)
1-pchisq(BartM,df)
e1z=(x1-mean(x1))^4
e2z=(x2-mean(x2))^4
e3z=(x3-mean(x3))^4
ee1z=(x1-mean(x1))^2
ee2z=(x2-mean(x2))^2
ee3z=(x3-mean(x3))^2
ez=sum(e1z,e2z,e3z)
eez=sum(ee1z,ee2z,ee3z)
f1z=N*ez
f2z=(eez)^2
fz=f1z/f2z
ffz=2/(fz-1)
MBartM=ffz*BartM
df=k-1
qchisq(1-alpha,df)
1-pchisq(MBartM,df)
MBA=ffz*abz
```

```
df=(k-1)
qchisq(1-alpha,df)
1-pchisq(MBA,df)
p.BA[p]=1-pchisq(BA,df)
if(p.BA[p]<alpha){temp.BA[p]=1}
p.MBA[p]=1-pchisq(MBA,df)
if(p.MBA[p]<alpha){temp.MBA[p]=1}
cat(c("loop : ",p),fill=T)
}
cat('\t',mean(temp.BA),'\t',mean(temp.MBA))
```

R-code of Box-Andersen test and Modified Box-Andersen test under Laplace distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
lamda1=sqrt(var1/2)
lamda2=sqrt(var2/2)
lamda3=sqrt(var3/2)
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.BA=rep(0,m)
p.BA=c()
temp.MBA=rep(0,m)
p.MBA=c()
for(p in 1:m)
```

```
{
x1=rdoublex(n1,mu1,lamda1)
x2=rdoublex(n2,mu2,lamda2)
x3=rdoublex(n3,mu3,lamda3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
a1=n1-1
a2=n2-1
a3=n3-1
aa1=a1*var(x1)
aa2=a2*var(x2)
aa3=a3*var(x3)
aa=sum(aa1,aa2,aa3)
aaa=aa/(N-k)
a=(N-k)*log(aaa)
b1=a1*log(var(x1))
b2=a2*log(var(x2))
b3=a3*log(var(x3))
b=sum(b1,b2,b3)
ab=a-b
c1=1/a1
c2=1/a2
c3=1/a3
c=sum(c1,c2,c3)
cc=1/(N-k)
ccc=c-cc
d=3*(k-1)
dd=ccc/d
ddd=1+dd
```

```
Bart=ab/ddd
df=k-1
qchisq(1-alpha,df)
1-pchisq(Bart,df)
e1=(x1-mean(x1))^4
e2=(x2-mean(x2))^4
e3=(x3-mean(x3))^4
ee1=(x1-mean(x1))^2
ee2=(x2-mean(x2))^2
ee3=(x3-mean(x3))^2
e=sum(e1,e2,e3)
ee=sum(ee1,ee2,ee3)
f1=N*e
f2=(ee)^2
f=f1/f2
ff=2/(f-1)
MBart=ff*Bart
df=k-1
qchisq(1-alpha,df)
1-pchisq(MBart,df)
BA=ff*ab
df=(k-1)
qchisq(1-alpha,df)
1-pchisq(BA,df)
s1=(x1-median(x1))^2
s2=(x2-median(x2))^2
s3=(x3-median(x3))^2
ss1=sum(s1)
ss2=sum(s2)
ss3=sum(s3)
sss1=ss1/(n1-1)
sss2=ss2/(n2-1)
sss3=ss3/(n3-1)
```

```
a1z=n1-1
a2z=n2-1
a3z=n3-1
aa1z=a1z*sss1
aa2z=a2z*sss2
aa3z=a3z*sss3
aaz=sum(aa1z,aa2z,aa3z)
aaaz=aaz/(N-k)
az=(N-k)*log(aaaz)
b1z=a1z*log(sss1)
b2z=a2z*log(sss2)
b3z=a3z*log(sss3)
bz=sum(b1z,b2z,b3z)
abz=az-bz
c1z=1/a1z
c2z=1/a2z
c3z=1/a3z
cz=sum(c1z,c2z,c3z)
ccz=1/(N-k)
cccz=cz-ccz
dz=3*(k-1)
ddz=cccz/dz
dddz=1+ddz
BartM=abz/dddz
df=k-1
qchisq(1-alpha,df)
1-pchisq(BartM,df)
e1z=(x1-mean(x1))^4
e2z=(x2-mean(x2))^4
e3z=(x3-mean(x3))^4
ee1z=(x1-mean(x1))^2
ee2z=(x2-mean(x2))^2
ee3z=(x3-mean(x3))^2
```

```
ez=sum(e1z,e2z,e3z)
eez=sum(ee1z,ee2z,ee3z)
f1z=N*ez
f2z=(eez)^2
fz=f1z/f2z
ffz=2/(fz-1)
MBartM=ffz*BartM
df=k-1
qchisq(1-alpha,df)
1-pchisq(MBartM,df)
MBA=ffz*abz
df=(k-1)
qchisq(1-alpha,df)
1-pchisq(MBA,df)
p.BA[p]=1-pchisq(BA,df)
if(p.BA[p]<alpha){temp.BA[p]=1}
p.MBA[p]=1-pchisq(MBA,df)
if(p.MBA[p]<alpha){temp.MBA[p]=1}
cat(c("loop : ",p),fill=T)
}
cat('\t',mean(temp.BA),'\t',mean(temp.MBA))
```

R-code of Box-Andersen test and Modified Box-Andersen test under uniform distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
```

```
A1=mu1-sqrt(3*var1)
A2=mu2-sqrt(3*var2)
A3=mu3-sqrt(3*var3)
B1=mu1+sqrt(3*var1)
B2=mu2+sqrt(3*var2)
B3=mu3+sqrt(3*var3)
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.BA=rep(0,m)
p.BA=c()
temp.MBA=rep(0,m)
p.MBA=c()
for(p in 1:m)
{
x1=runif(n1,A1,B1)
x2=runif(n2,A2,B2)
x3=runif(n3,A3,B3)
x=c(x1,x2,x3)
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
a1=n1-1
a2=n2-1
a3=n3-1
aa1=a1*var(x1)
aa2=a2*var(x2)
aa3=a3*var(x3)
aa=sum(aa1,aa2,aa3)
aaa=aa/(N-k)
a=(N-k)*log(aaa)
```

```
b1=a1*log(var(x1))
b2=a2*log(var(x2))
b3=a3*log(var(x3))
b=sum(b1,b2,b3)
ab=a-b
c1=1/a1
c2=1/a2
c3=1/a3
c=sum(c1,c2,c3)
cc=1/(N-k)
ccc=c-cc
d=3*(k-1)
dd=ccc/d
ddd=1+dd
Bart=ab/ddd
df=k-1
qchisq(1-alpha,df)
1-pchisq(Bart,df)
e1=(x1-mean(x1))^4
e2=(x2-mean(x2))^4
e3=(x3-mean(x3))^4
ee1=(x1-mean(x1))^2
ee2=(x2-mean(x2))^2
ee3=(x3-mean(x3))^2
e=sum(e1,e2,e3)
ee=sum(ee1,ee2,ee3)
f1=N*e
f2=(ee)^2
f=f1/f2
ff=2/(f-1)
MBart=ff*Bart
df=k-1
qchisq(1-alpha,df)
```

```
1-pchisq(MBart,df)
BA=ff*ab
df=(k-1)
qchisq(1-alpha,df)
1-pchisq(BA,df)
s1=(x1-median(x1))^2
s2=(x2-median(x2))^2
s3=(x3-median(x3))^2
ss1=sum(s1)
ss2=sum(s2)
ss3=sum(s3)
sss1=ss1/(n1-1)
sss2=ss2/(n2-1)
sss3=ss3/(n3-1)
a1z=n1-1
a2z=n2-1
a3z=n3-1
aa1z=a1z*sss1
aa2z=a2z*sss2
aa3z=a3z*sss3
aaz=sum(aa1z,aa2z,aa3z)
aaaz=aaz/(N-k)
az=(N-k)*log(aaaz)
b1z=a1z*log(sss1)
b2z=a2z*log(sss2)
b3z=a3z*log(sss3)
bz=sum(b1z,b2z,b3z)
abz=az-bz
c1z=1/a1z
c2z=1/a2z
c3z=1/a3z
cz=sum(c1z,c2z,c3z)
ccz=1/(N-k)
```

```
cccz=cz-ccz
dz=3*(k-1)
ddz=cccz/dz
dddz=1+ddz
BartM=abz/dddz
df=k-1
qchisq(1-alpha,df)
1-pchisq(BartM,df)
e1z=(x1-mean(x1))^4
e2z=(x2-mean(x2))^4
e3z=(x3-mean(x3))^4
ee1z=(x1-mean(x1))^2
ee2z=(x2-mean(x2))^2
ee3z=(x3-mean(x3))^2
ez=sum(e1z,e2z,e3z)
eez=sum(ee1z,ee2z,ee3z)
f1z=N*ez
f2z=(eez)^2
fz=f1z/f2z
ffz=2/(fz-1)
MBartM=ffz*BartM
df=k-1
qchisq(1-alpha,df)
1-pchisq(MBartM,df)
MBA=ffz*abz
df=(k-1)
qchisq(1-alpha,df)
1-pchisq(MBA,df)
p.BA[p]=1-pchisq(BA,df)
if(p.BA[p]<alpha){temp.BA[p]=1}
p.MBA[p]=1-pchisq(MBA,df)
if(p.MBA[p]<alpha){temp.MBA[p]=1}
cat(c("loop : ",p),fill=T)
```

```
}
cat('\t',mean(temp.BA),'\t',mean(temp.MBA))
```

R-code of Box-Andersen test and Modified Box-Andersen test under gamma distribution as follows:

```
set.seed(55)
m=1000
mu1=8
mu2=8
mu3=8
k=3
var1=8
var2=8
var3=8
B1=var1/mu1
B2=var2/mu2
B3=var3/mu3
A1=(mu1^2)/var1
A2=(mu2^2)/var2
A3=(mu3^2)/var3
n1=10;n2=10;n3=10
N=n1+n2+n3
alpha=0.05
temp.BA=rep(0,m)
p.BA=c()
temp.MBA=rep(0,m)
p.MBA=c()
for(p in 1:m)
{
x1=rgamma(n1,A1,1/B1)
x2=rgamma(n2,A2,1/B2)
x3=rgamma(n3,A3,1/B3)
x=c(x1,x2,x3)
```

```
g1=rep(1,n1)
g2=rep(2,n2)
g3=rep(3,n3)
g=c(g1,g2,g3)
df=data.frame(x,g)
a1=n1-1
a2=n2-1
a3=n3-1
aa1=a1*var(x1)
aa2=a2*var(x2)
aa3=a3*var(x3)
aa=sum(aa1,aa2,aa3)
aaa=aa/(N-k)
a=(N-k)*log(aaa)
b1=a1*log(var(x1))
b2=a2*log(var(x2))
b3=a3*log(var(x3))
b=sum(b1,b2,b3)
ab=a-b
c1=1/a1
c2=1/a2
c3=1/a3
c=sum(c1,c2,c3)
cc=1/(N-k)
ccc=c-cc
d=3*(k-1)
dd=ccc/d
ddd=1+dd
Bart=ab/ddd
df=k-1
qchisq(1-alpha,df)
1-pchisq(Bart,df)
e1=(x1-mean(x1))^4
```

```
e2=(x2-mean(x2))^4
e3=(x3-mean(x3))^4
ee1=(x1-mean(x1))^2
ee2=(x2-mean(x2))^2
ee3=(x3-mean(x3))^2
e=sum(e1,e2,e3)
ee=sum(ee1,ee2,ee3)
f1=N*e
f2=(ee)^2
f=f1/f2
ff=2/(f-1)
MBart=ff*Bart
df=k-1
qchisq(1-alpha,df)
1-pchisq(MBart,df)
BA=ff*ab
df=(k-1)
qchisq(1-alpha,df)
1-pchisq(BA,df)
s1=(x1-median(x1))^2
s2=(x2-median(x2))^2
s3=(x3-median(x3))^2
ss1=sum(s1)
ss2=sum(s2)
ss3=sum(s3)
sss1=ss1/(n1-1)
sss2=ss2/(n2-1)
sss3=ss3/(n3-1)
a1z=n1-1
a2z=n2-1
a3z=n3-1
aa1z=a1z*sss1
aa2z=a2z*sss2
```

```
aa3z=a3z*sss3
aaz=sum(aa1z,aa2z,aa3z)
aaaz=aaz/(N-k)
az=(N-k)*log(aaaz)
b1z=a1z*log(sss1)
b2z=a2z*log(sss2)
b3z=a3z*log(sss3)
bz=sum(b1z,b2z,b3z)
abz=az-bz
c1z=1/a1z
c2z=1/a2z
c3z=1/a3z
cz=sum(c1z,c2z,c3z)
ccz=1/(N-k)
cccz=cz-ccz
dz=3*(k-1)
ddz=cccz/dz
dddz=1+ddz
BartM=abz/dddz
df=k-1
qchisq(1-alpha,df)
1-pchisq(BartM,df)
e1z=(x1-mean(x1))^4
e2z=(x2-mean(x2))^4
e3z=(x3-mean(x3))^4
ee1z=(x1-mean(x1))^2
ee2z=(x2-mean(x2))^2
ee3z=(x3-mean(x3))^2
ez=sum(e1z,e2z,e3z)
eez=sum(ee1z,ee2z,ee3z)
f1z=N*ez
f2z=(eez)^2
fz=f1z/f2z
```

```
ffz=2/(fz-1)
MBartM=ffz*BartM
df=k-1
qchisq(1-alpha,df)
1-pchisq(MBartM,df)
MBA=ffz*abz
df=(k-1)
qchisq(1-alpha,df)
1-pchisq(MBA,df)
p.BA[p]=1-pchisq(BA,df)
if(p.BA[p]<alpha){temp.BA[p]=1}
p.MBA[p]=1-pchisq(MBA,df)
if(p.MBA[p]<alpha){temp.MBA[p]=1}
cat(c("loop : ",p),fill=T)
}
cat('\t',mean(temp.BA),'\t',mean(temp.MBA))
```

The R program command for plot the probability of type I error graph and power of a test graph under significance levels (0.01 and 0.05).

These R program command can change by variables consists of the probability of type I error according to the results in this study. Every distribution can use similar R-code.

R-code of the probability of type I error graph under equal sample size at significance level (0.01) for normal distribution.

```
set.seed(55)
c=rep(0.015,3)
d=rep(0.005,3)
L=c(0.013,0.011,0.006)
OB=c(0.007,0.009,0.010)
J=c(0.009,0.015,0.010)
BA=c(0.010,0.010,0.009)
BF=c(0.007,0.008,0.004)
MOB=c(0.011,0.011,0.010)
```

```
MJ=c(0.014,0.018,0.011)
MBA=c(0.012,0.011,0.006)
y=c('(10,10,10)','(30,30,30)','(60,60,60)')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0.000,0.025),lwd=1,main="Equal",xlab="(n
1,n2,n3)",ylab="Probability of type I error",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,J,type="b",col="magenta",pch=17,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
lines(x,BF,type="b",col="gold",pch=8,lty=3)
lines(x,MOB,type="b",col="cyan",pch=15,lty=3)
lines(x,MJ,type="b",col="red",pch=17,lty=3)
lines(x,MBA,type="b",col="darkorange",pch=16,lty=3)
lines(x,c,type="l",lty=2)
lines(x,d,type="l",lty=2)
axis(1,at=1:3,labels=y)
labels=c("L","OB","J","BA","BF","MOB","MJ","MBA")
colors=c("blue","limegreen","magenta","darkviolet","gold","cyan","red","darkorange")
pchh=c(8,15,17,16,8,15,17,16)
ltys=c(1,1,1,1,3,3,3,3)
legend("topright",inset=0.05,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",ncol=1
,cex=0.8)
```

R-code of the probability of type I error graph under unequal sample size at significance level (0.01) for normal distribution.

```
set.seed(55)
c=rep(0.005,3)
d=rep(0.015,3)
L=c(0.011,0.007,0.010)
OB=c(0.007,0.008,0.008)
J=c(0.018,0.012,0.012)
BA=c(0.014,0.010,0.011)
BF=c(0.003,0.001,0.008)
```

```
MOB=c(0.021,0.009,0.009)
MJ=c(0.025,0.016,0.012)
MBA=c(0.016,0.009,0.012)
y=c('(5,10,15)','(20,30,40)','(45,60,75)')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0.000,0.025),lwd=1,main="Unequal",xlab="
(n1,n2,n3)",ylab="Probability of type I error",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,J,type="b",col="magenta",pch=17,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
lines(x,BF,type="b",col="gold",pch=8,lty=3)
lines(x,MOB,type="b",col="cyan",pch=15,lty=3)
lines(x,MJ,type="b",col="red",pch=17,lty=3)
lines(x,MBA,type="b",col="darkorange",pch=16,lty=3)
lines(x,c,type="l",lty=2)
lines(x,d,type="l",lty=2)
axis(1,at=1:3,labels=y)
labels=c("L","OB","J","BA","BF","MOB","MJ","MBA")
colors=c("blue","limegreen","magenta","darkviolet","gold","cyan","red","darkorange")
pchh=c(8,15,17,16,8,15,17,16)
ltys=c(1,1,1,1,3,3,3,3)
legend("topright",inset=0.05,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",ncol=1
,cex=0.8)
```

R-code of the probability of type I error graph under equal sample size at significance level (0.05) for normal distribution.

```
set.seed(55)
c=rep(0.075,3)
d=rep(0.025,3)
L=c(0.067,0.056,0.050)
OB=c(0.039,0.050,0.047)
J=c(0.046,0.052,0.052)
BA=c(0.063,0.056,0.050)
```

```
BF=c(0.040,0.041,0.047)
MOB=c(0.052,0.057,0.052)
MJ=c(0.059,0.058,0.054)
MBA=c(0.067,0.059,0.052)
y=c('(10,10,10)','(30,30,30)','(60,60,60)')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0.02,0.11),lwd=1,main="Equal",xlab="(n1,n
2,n3)",ylab="Probability of type I error",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,J,type="b",col="magenta",pch=17,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
lines(x,BF,type="b",col="gold",pch=8,lty=3)
lines(x,MOB,type="b",col="cyan",pch=15,lty=3)
lines(x,MJ,type="b",col="red",pch=17,lty=3)
lines(x,MBA,type="b",col="darkorange",pch=16,lty=3)
lines(x,c,type="l",lty=2)
lines(x,d,type="l",lty=2)
axis(1,at=1:3,labels=y)
labels=c("L","OB","J","BA","BF","MOB","MJ","MBA")
colors=c("blue","limegreen","magenta","darkviolet","gold","cyan","red","darkorange")
pchh=c(8,15,17,16,8,15,17,16)
ltys=c(1,1,1,1,3,3,3,3)
legend("topright",inset=0.02,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",ncol=1
,cex=0.8)
```

R-code of the probability of type I error graph under unequal sample size at significance level (0.05) for normal distribution.

```
set.seed(55)
c=rep(0.075,3)
d=rep(0.025,3)
L=c(0.067,0.055,0.050)
OB=c(0.036,0.044,0.044)
J=c(0.058,0.052,0.051)
```

```
BA=c(0.073,0.053,0.050)
BF=c(0.028,0.040,0.045)
MOB=c(0.075,0.052,0.050)
MJ=c(0.075,0.055,0.053)
MBA=c(0.078,0.054,0.044)
y=c('(5,10,15)','(20,30,40)','(45,60,75)')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0.02,0.11),lwd=1,main="Unequal",xlab="(n
1,n2,n3)",ylab="Probability of type I error",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,J,type="b",col="magenta",pch=17,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
lines(x,BF,type="b",col="gold",pch=8,lty=3)
lines(x,MOB,type="b",col="cyan",pch=15,lty=3)
lines(x,MJ,type="b",col="red",pch=17,lty=3)
lines(x,MBA,type="b",col="darkorange",pch=16,lty=3)
lines(x,c,type="l",lty=2)
lines(x,d,type="l",lty=2)
axis(1,at=1:3,labels=y)
labels=c("L","OB","J","BA","BF","MOB","MJ","MBA")
colors=c("blue","limegreen","magenta","darkviolet","gold","cyan","red","darkorange")
pchh=c(8,15,17,16,8,15,17,16)
ltys=c(1,1,1,1,3,3,3,3)
legend("topright",inset=0.02,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",ncol=1
,cex=0.8)
```

These R program command can change by variables consists of alpha and power of a test according to the results in this study. Every distribution can use similar R-code.
R-code of the graph of power under condition consists of equal sample size, significance level (0.01), and normal distribution.

```
set.seed(55)
par(mfrow=c(1,3))
L=c(0.154,0.309,0.528)
```

```
OB=c(0.055,0.098,0.139)
J=c(0.146,0.402,0.724)
BA=c(0.113,0.327,0.605)
BF=c(0.071,0.158,0.325)
MOB=c(0.083,0.133,0.186)
MJ=c(0.170,0.454,0.762)
MBA=c(0.122,0.326,0.612)
y=c('1.25','2.87','6.13')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0,1),lwd=1,main="Small sample size
(10,10,10)",xlab="Non-centrality parameter",ylab="Power",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,J,type="b",col="magenta",pch=17,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
lines(x,BF,type="b",col="gold",pch=8,lty=3)
lines(x,MOB,type="b",col="cyan",pch=15,lty=3)
lines(x,MJ,type="b",col="red",pch=17,lty=3)
lines(x,MBA,type="b",col="darkorange",pch=16,lty=3)
axis(1,at=1:3,labels=y)
labels=c("L","OB","J","BA","BF","MOB","MJ","MBA")
colors=c("blue","limegreen","magenta","darkviolet","gold","cyan","red","darkorange")
pchh=c(8,15,17,16,8,15,17,16)
ltys=c(1,1,1,1,3,3,3,3)
legend("topleft",inset=0.05,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",ncol=1,
cex=1)
L=c(0.637,0.968,0.999)
OB=c(0.558,0.859,0.956)
J=c(0.715,0.987,1)
BA=c(0.645,0.970,0.999)
BF=c(0.578,0.942,0.998)
MOB=c(0.577,0.879,0.963)
MJ="n"
MBA=c(0.643,0.973,0.999)
```

```
y=c('1.25','2.87','6.13')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0,1),lwd=1,main="Medium sample size
(30,30,30)",xlab="Non-centrality parameter",ylab="Power",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,J,type="b",col="magenta",pch=17,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
lines(x,BF,type="b",col="gold",pch=8,lty=3)
lines(x,MOB,type="b",col="cyan",pch=15,lty=3)
lines(x,MBA,type="b",col="darkorange",pch=16,lty=3)
axis(1,at=1:3,labels=y)
labels=c("L","OB","J","BA","BF","MOB","MBA")
colors=c("blue","limegreen","magenta","darkviolet","gold","cyan","darkorange")
pchh=c(8,15,17,16,8,15,16)
ltys=c(1,1,1,1,3,3,3)
legend("bottomright",inset=0.05,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",nc
ol=1,cex=1)
L=c(0.967,1,1)
OB=c(0.970,0.999,1)
J=c(0.987,1,1)
BA=c(0.975,1,1)
BF="n"
MOB=c(0.971,0.999,1)
MJ=c(0.987,1,1)
MBA=c(0.975,1,1)
y=c('1.25','2.87','6.13')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0,1),lwd=1,main="Large sample size
(60,60,60)",xlab="Non-centrality parameter",ylab="Power",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,J,type="b",col="magenta",pch=17,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
lines(x,MOB,type="b",col="cyan",pch=15,lty=3)
```

```
lines(x,MJ,type="b",col="red",pch=17,lty=3)
lines(x,MBA,type="b",col="darkorange",pch=16,lty=3)
axis(1,at=1:3,labels=y)
labels=c("L","OB","J","BA","MOB","MJ","MBA")
colors=c("blue","limegreen","magenta","darkviolet","cyan","red","darkorange")
pchh=c(8,15,17,16,15,17,16)
ltys=c(1,1,1,1,3,3,3)
legend("bottomright",inset=0.05,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",nc
ol=1,cex=1)
```

R-code of the graph of power under condition consists of unequal sample size, significance level (0.01), and normal distribution.

```
set.seed(55)
par(mfrow=c(1,3))
L=c(0.058,0.098,0.164)
OB=c(0.008,0.011,0.013)
J="n"
BA=c(0.081,0.181,0.357)
BF="n"
MOB="n"
MJ="n"
MBA="n"
y=c('1.25','2.87','6.13')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0,1),lwd=1,main="Small sample size
(5,10,15)",xlab="Non-centrality parameter",ylab="Power",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
axis(1,at=1:3,labels=y)
labels=c("L","OB","BA")
colors=c("blue","limegreen","darkviolet")
pchh=c(8,15,16)
ltys=c(1,1,1)
```

```
legend("topleft",inset=0.05,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",ncol=1,
cex=1)
L=c(0.542,0.902,0.993)
OB=c(0.389,0.607,0.759)
J=c(0.649,0.970,0.999)
BA=c(0.572,0.948,0.997)
BF="n"
MOB=c(0.394,0.629,0.779)
MJ="n"
MBA=c(0.563,0.944,0.997)
y=c('1.25','2.87','6.13')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0,1),lwd=1,main="Medium sample size
(20,30,40)",xlab="Non-centrality parameter",ylab="Power",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,J,type="b",col="magenta",pch=17,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
lines(x,MOB,type="b",col="cyan",pch=15,lty=3)
lines(x,MBA,type="b",col="darkorange",pch=16,lty=3)
axis(1,at=1:3,labels=y)
labels=c("L","OB","J","BA","MOB","MBA")
colors=c("blue","limegreen","magenta","darkviolet","cyan","darkorange")
pchh=c(8,15,17,16,15,16)
ltys=c(1,1,1,1,3,3)
legend("bottomright",inset=0.05,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",nc
ol=1,cex=1)
L=c(0.954,1,1)
OB=c(0.944,0.997,1)
J=c(0.982,1,1)
BA=c(0.971,1,1)
BF=c(0.951,1,1)
MOB=c(0.945,0.997,1)
MJ=c(0.982,1,1)
```

```
MBA=c(0.965,1,1)
y=c('1.25','2.87','6.13')
x=seq(1,3)
plot(x,L,type="b",col="blue",xaxt="n",ylim=c(0,1),lwd=1,main="Large sample size
(45,60,75)",xlab="Non-centrality parameter",ylab="Power",pch=8,lty=1)
lines(x,OB,type="b",col="limegreen",pch=15,lty=1)
lines(x,J,type="b",col="magenta",pch=17,lty=1)
lines(x,BA,type="b",col="darkviolet",pch=16,lty=1)
lines(x,BF,type="b",col="gold",pch=8,lty=3)
lines(x,MOB,type="b",col="cyan",pch=15,lty=3)
lines(x,MJ,type="b",col="red",pch=17,lty=3)
lines(x,MBA,type="b",col="darkorange",pch=16,lty=3)
axis(1,at=1:3,labels=y)
labels=c("L","OB","J","BA","BF","MOB","MJ","MBA")
colors=c("blue","limegreen","magenta","darkviolet","gold","cyan","red","darkorange")
pchh=c(8,15,17,16,8,15,17,16)
ltys=c(1,1,1,1,3,3,3,3)
legend("bottomright",inset=0.05,labels,lwd=1,lty=ltys,col=colors,pch=pchh,bty="n",nc
ol=1,cex=1)
```

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