SIMPLIFIED DESIGN OF PID-FAMILY CONTROLLER BASED ON CDM

PEERAPON PATTANAVIJIT

A THESIS SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING IN CONTROL ENGINEERING

SCHOOL OF GRADUATE STUDIES

KING MONGKUT'S INSTITUTE OF TECHNOLOGY LADKRABANG

2007

KMITL-2007-EN-080-005

หัวข้อวิทยานิพนธ์	การออกแบบตัวควบคุมในตระกูลพีไอดีอย่างง่าย ด้วยวิธีแผนผังค่าสัมประสิทธิ์
นักศึกษา	นาย พีรพล พัฒนวิจิตร
รหัสนักศึกษา	48060513
ปริญญา	วิศวกรรมศาสตรมหาบัณฑิต
สาขาวิชา	วิศวกรรมระบบควบคุม
พ.ศ.	2550
อาจารย์ที่ปรึกษาวิทยานิพนธ์	รศ.ดร.จงกล งามวิวิทย์

บทคัดย่อ

วิทยานิพนธ์นี้นำเสนอวิธีการออกแบบตัวควบคุมในตระกูลพีไอดีอย่างง่ายสำหรับระบบควบคุมอันดับสูงโดยใช้วิธีการลดอันดับของกระบวนการให้มีอันดับต่ำลง แล้วจึงออกแบบด้วยวิธีแผนผังค่าสัมประสิทธิ์ ซึ่งทำให้ดัชนีเสถียรภาพมีจำนวนน้อยลง ในขั้นแรก ทำการลดอันดับของกระบวนการอันดับสูงโดยใช้วิธีการตัดโพลด้อยทิ้งไป แล้วจึงออกแบบตัวควบคุมให้กับแบบจำลองของกระบวนการที่ถูกลดอันดับให้ต่ำลงด้วยวิธีแผนผังค่าสัมประสิทธิ์ โดยเลือกใช้ค่าดัชนีเสถียรภาพที่เหมาะสมเพื่อให้ได้ผลตอบสนองของระบบควบคุมวงปิดที่ไม่มีค่าพุ่งเกิน จากนั้น จึงนำตัวควบคุมที่ออกแบบไว้แล้วไปควบคุมกระบวนการเดิมซึ่งมีอันดับสูง จากผลการทดลองแสดงให้เห็นว่า ตัวควบคุมที่ถูกออกแบบจากแบบจำลองของกระบวนการที่ลดอันดับแล้วนั้น สามารถนำไปใช้ควบคุมกระบวนการเดิมซึ่งมีอันดับสูงได้โดยมีสมรรถนะใกล้เคียงกันกับการใช้ตัวควบคุมที่ออกแบบจากกระบวนการอันดับสูงเดิม

Thesis Title	Simplified Design of PID-Family Controller Based on CDM
Student	Peerapon Pattanavijit
Student ID.	48060513
Degree	Master of Engineering
Program	Control Engineering
Year	2007
Thesis Advisor	Assoc.Prof. Jongkol Ngamwiwit

ABSTRACT

In this thesis, a simplified designing method of the PID-family controller for a higher-order plant by using the concept of plant model reduction and the Coefficient Diagram Method with the less number of stability index γ_i is presented. By ignoring the non-dominant poles of the higher-order plant model, a lower-order plant model is first obtained. Then, the PID-family controller will be designed for controlling the lower-order plant model by using Coefficient Diagram Method with the appropriate values of stability index in order to achieve the system response without overshoot. Finally, the designed PID-family controller will be employed to control the original higher-order plant. The proposed controller is implemented to control a two-inertia system. The experimental results show that the PID-family controller designed from the lower-order plant model is sufficient for controlling the original higher-order plant with acceptable performance.

ACKNOWLEDGMENTS

First of all, I would like to express my boundless gratitude to my beloved parents for their loves and taking care of me.

I would like to sincerely thank my advisor, Dr. Jongkol Ngamwiwit, for her support and guidance during the course of this research. Without her great suggestions, my research work would not be accomplished.

Thanks to teacher Taworn Benjanarasuth and teacher Songmoung Nundrakwang for the excellent supports and technical advices throughout the course.

Finally, I would like to thank to my friends and all the members of the Control and Mechatronics Lab for any support with friendliness.

CONTENTS

CONTENTS (Cont.)

Page

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURES

$A_c(s)$	Denominator polynomial of controller
$A_p(s)$	Denominator polynomial of plant
$A_p(s)$	Denominator polynomial of the lower-order plant
$A_p^*(s)$	Denominator polynomial of the modified plant
$B_a(s)$	Polynomial of pre-filter
$B_c(s)$	Numerator polynomial of controller
$B_p(s)$	Numerator polynomial of plant
$\tilde{B}_p(s)$	Numerator polynomial of the lower-order plant
$B_p^*(s)$	Numerator polynomial of the modified plant
$C(s)$	Output
$D(s)$	Disturbance
e_a	Input voltage applied to the armature
e_b	Back electromotive force voltage
$G_c(s)$	CDM controller
$G_p(s)$	Plant
$\tilde{G}_p(s)$	The lower-order plant
$G_p^*(s)$	The modified plant
i_a	Armature current
J_L	Moment of load inertia
J_m	Moment of motor inertia
K_d	Derivative gain
K_e	Back electromotive force constant
K_i	Integral gain
K_m	Motor torque constant
K_p	Proportional gain
K_s	Torsional stiffness of drive shaft
L_a	Armature inductance
$P(s)$	Characteristic polynomial
$\tilde{P}(s)$	Characteristic polynomial of the lower-order plant

NOMENCLATURES (Cont.)

$R(s)$	Input
R_a	Armature resistance
T_D	Disturbance torque
T_d	Derivative time
T_i	Integral time
T_L	Load torque
T_m	Motor torque
$U(s)$	Control signal
ω	Angular frequency
ω_L	Load speed
ω_m	Motor speed
θ_L	Angle of load rotation
θ_m	Angle of motor rotation
ζ	Damping ratio
γ_i	Stability index
γ_i^*	Stability limit
τ	Equivalent time constant

Chapter 1

Introduction

1.1 Statement and Significance of the Problems

Since several decades ago, most of the industrial processes have employed the PID-family controllers due to their simplicity and sufficiency in process control applications. It is reported lately that more than 90% of the industrial process controllers used nowadays are the PI controllers [1]. However, the parameters of the controller have to be tuned for acceptable responses. Over the years, there are many formulas derived to tune the parameters of PID-family controllers [2]. Some well-known PID tuning formulas are: 1) Ziegler-Nichols' tuning formula [3-4]; 2) Cohen-Coon tuning method [5]; 3) Gain and phase margin formula (GPM) [6-7]. The most popular tuning method is the Ziegler-Nichols' tuning formula which is designed to have quarterly decayed overshoot characteristic in the step response but the fine tuning is needed for the practical use to reduce the overshoot in the closed-loop response.

In 1998, a new designing method called Coefficient Diagram Method (CDM) [8-9] has been proposed by Professor Shunji Manabe. The CDM is an algebraic design algorithm utilizing polynomial form structure. In CDM, the closed-loop characteristic polynomial is designed based on stability index and equivalent time constant, which are used to determine stability and speed of the closed-loop response. Hence, the unknown parameters of the controllers resulting for over-damped response can be obtained accordingly [10-11]. However, it is quite time consuming in designing the parameters of a controller especially for a higher-order plant.

In this thesis, the simplified design of a PID-family controller for higher-order plant using CDM with less number of stability indexes γ_i is presented. By ignoring the non-dominant poles, the higher-order plant is reduced to be a lower-order plant model. After that, the CDM is used to design the PID-family controller for controlling the lower-order plant model. Finally, the PID-family controller designed for the lower-order plant model is employed for controlling the original higher-order plant. In order to check the control capability of the controller designed by this designing method, the simplified design of PI controller and I-P controller which are the most popular of PID-family controller in speed control of DC motor drive [12-13] will be implemented to control the speed of two-inertia system [14].

1.2 Research Objective and Approach Methodology

The need to design the PID-family controller having a desired system performance and simplicity in design has motivated the following studies:

1) Study of PID-family controller which are the most popular used in industrial process. In practical use, the parameters of the PID-family controller have to be tuned for acceptable performance. There are many formulas derived to tune the parameters of PID-family controllers, but the most popular tuning method is the Ziegler-Nichols' tuning formula. However, the fine tune is needed for overdamp characteristic response.
2) Study of a controller designing method called CDM which is an algebraic design algorithm utilizing polynomial form structure. The plant and the controller are represented in polynomial form. The stability index γ_i and the equivalent time constant τ are defined based on the CDM concept. The coefficients of numerator and denominator polynomials of the closed-loop transfer function obtained from the appropriated value of γ_i and τ are related to the controller parameters.
3) Study of two-inertia system characteristic. Two-inertia system has some problem about the torsional resonance that must be reduced by using a controller. The PI controller and I-P controller are investigated in controlling the speed of two-inertia system.
4) Simplified design method based on CDM for PID-family controller is proposed. The higher-order plant must be reduced first by ignoring the non-dominant poles. A controller for the reduced order plant model is then designed by CDM. Finally, the controller designed for the reduced order plant will be used to control the original plant.
5) Implementation of the simplified design of PI controller and I-P controller for controlling the speed of two-inertia system. The effectiveness of Simplified design PID-family control algorithm used to control the speed of two-inertia system is shown in three aspects: response of speed control system, disturbance rejection capability and the robustness characteristic.

1.3 Summary of Thesis Contents

This thesis is organized in 6 chapters including references and appendices.

Chapter 2 explains the basic of the CDM, which consists of the CDM standard block diagram, basic mathematical relations concerning with the CDM and coefficient diagram.

Chapter 3 describes the fundamentals of two-inertia system, such as the system structure and the mathematical model.

Chapter 4 introduces the structure of PID-family controller and proposes a general form of PID-family control system for designing controller parameters by CDM. In this chapter, the model order reduction will be discussed first. Then the overview of the PID-family controller, the control system structure and the design procedure will be presented respectively. Finally, the controller design for speed control of two-inertia system will be explained. The PI controller and I-P controller which are the most popular of PID-family controller in industrial process will be investigated in controlling the speed of two-inertia system.

Chapter 5 presents the application in controlling the speed of two-inertia system of PI controller and I-P controller. The simulated and experimental results of the PI control system in controlling the speed of two-inertia system will be described first. Then, the controlling result of the I-P control system will be shown.

Chapter 6 concludes the results of this research and the future works.

Chapter 2

Coefficient Diagram Method

The general problem of control system designing is how to choose a proper controller to meet the desired performance specifications. There are three main theories employed for the controller design procedure: Classical Control Theory, Modern Control Theory and Polynomial Approach which is sometimes called the Algebraic Approach. The classical control method, such as frequency response method and root-locus method, uses the transfer function in frequency domain for the system representation. The transfer function seems to be easy to handle, but it will become inaccurate when pole-zero cancellation occurs due to uncontrollable or unobservable situations. The modern control method designed by the pole-placement method and the optimal control uses the state-space representation. This representation is accurate and well-suited in machine computation. The Coefficient Diagram Method (CDM) is an algebraic control design approach. This method uses the polynomial form of transfer function for system representation. The denominator and the numerator of the transfer function are considered independently from each other. Thus, the better results can be achieved against pole-zero cancellation. In this chapter, the CDM standard block diagram and the basic mathematical relations concerning to the CDM method will be described.

2.1 Basic Features of CDM

The CDM is an algebraic control design approach with the following five features.

(1) Polynomials are used for system representation.

(2) Characteristic polynomial and controller are simultaneously designed.

(3) Coefficient diagram is effectively utilized.

(4) The sufficient condition for stability by Lipatov constitutes the theoretical basis of CDM.

(5) Kessler standard form is improved and used as the standard form of CDM.

CDM designing method is based on the stability index γ_i and the equivalent time constant τ as defined later. The equivalent time constant τ specifies the response speed. The stability index γ_i specifies the stability and the wave form of the time response. The variation of stability index γ_i due to the plant parameter variation specifies the robustness of the control system.

2.2 CDM Standard Block Diagram

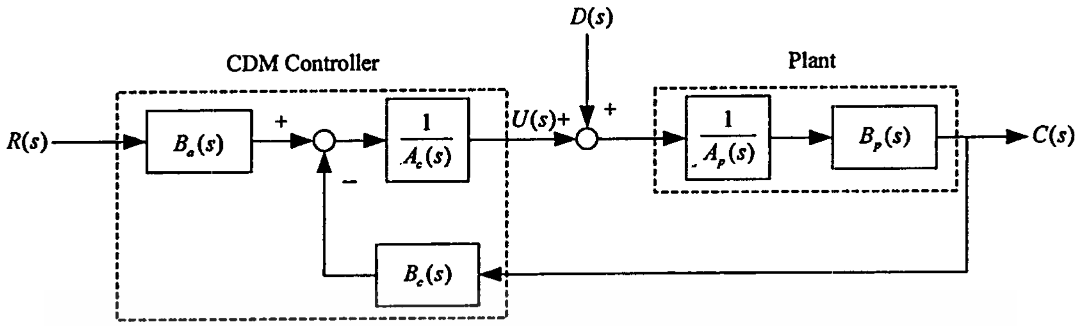


Figure 2.1 CDM standard block diagram of SISO system.

The standard block diagram of the CDM design for a single-input-single-output system is shown in figure 2.1. $A_p(s)$ and $B_p(s)$ are the polynomials of the plant, $A_c(s)$, $B_c(s)$ and $B_a(s)$ are the polynomials of the standard CDM controller. $D(s)$ is the disturbance entering to the controlled system. The transfer function of the plant in polynomial form can be expressed as

$$A_p(s) = p_k s^k + p_{k-1} s^{k-1} + \ldots + p_0 \tag{2.1}$$

$$B_p(s) = q_m s^m + q_{m-1} s^{m-1} + \ldots + q_0 \tag{2.2}$$

and the controller polynomials are given by

$$A_c(s) = l_\lambda s^\lambda + l_{\lambda-1} s^{\lambda-1} + \ldots + l_0 \tag{2.3}$$

$$B_c(s) = k_\lambda s^\lambda + k_{\lambda-1} s^{\lambda-1} + \ldots + k_0 \tag{2.4}$$

$$B_a(s) = p_\lambda s^\lambda + p_{\lambda-1} s^{\lambda-1} + \ldots + p_0 \tag{2.5}$$

where $\lambda < k$ and $m < k$. $B_a(s)$ is called pre-filter. Since the transfer function of the controller has two numerators, it is resembled to a two-degree-of freedom (2DOF) system structure.

The better performance can be expected when using 2DOF structure, because it can compromise on both tracking the desired reference signal and disturbance suppression.

2.3 Characteristic Polynomial

The characteristic polynomial of the closed-loop system without pre-filter is defined as

$$P(s) = A_c(s)A_p(s) + B_c(s)B_p(s)$$

$$= a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 \tag{2.6}$$

$$= \sum_{i=0}^{n} a_i s^i$$

where $a_0, a_1, \ldots, a_n$ are the coefficients of the characteristic polynomial. The stability index γ_i, the equivalent time constant τ and stability limit γ_i^* are defined as follows.

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}} \tag{2.7}$$

$$\tau = \frac{a_1}{a_0} \tag{2.8}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}}; \gamma_0, \gamma_n = \infty \tag{2.9}$$

where $i = 1, \ldots, n-1$. From (2.7) and (2.8), the coefficient a_i can be written by

$$a_i = a_0 \tau^i \frac{1}{\gamma_{i-1} \cdots \gamma_2^{i-2} \gamma_1^{i-1}}$$

$$= a_0 \tau^i \prod_{j=1}^{i-1} \frac{1}{\left(\gamma_{i-j}\right)^j}. \tag{2.10}$$

Substituting each coefficient a_i into equation (2.6), then the characteristic polynomial will be expressed in term of a_0, τ and γ_i as

$$P(s) = a_0\left[\left\{\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\right)(\tau s)^i\right\} + \tau s + 1\right]. \tag{2.11}$$

2.4 Coefficient Diagram

Coefficient diagram is a powerful and useful tool for the controller design. It gives information on stability, time response and robustness of systems in a single diagram. In coefficient diagram, vertical axis logarithmically shows the coefficients of characteristic polynomial a_i, stability index γ_i, stability limits γ_i^* and equivalent time constant τ while the horizontal axis shows the order i values corresponding to each coefficient. The degree of convexity is a measure of stability. The general inclination of the curve is a measure of response speed. The variation of the shape of the curve due to the plant parameter variation is a measure of the robustness.

To give an idea about the coefficient diagram consider the characteristic polynomial

$$P(s) = 0.25s^5 + s^4 + 2s^3 + 2s^2 + s + 0.2 \tag{2.12}$$

for the following CDM parameters

$$\gamma_i = [2\ 2\ 2\ 2.5] \tag{2.13}$$

$$\tau = 5 \tag{2.14}$$

$$\gamma_i^* = [0.5\ 1\ 0.9\ 0.5]. \tag{2.15}$$

These values are shown in a coefficient diagram as in figure 2.2. The control system becomes more stable if the curvature of the a_i curve becomes larger corresponding to larger stability index γ_i as shown in figure 2.3a. If the a_i curve is left-end down, the equivalent time constant τ is small and response is fast as shown in figure 2.3b.

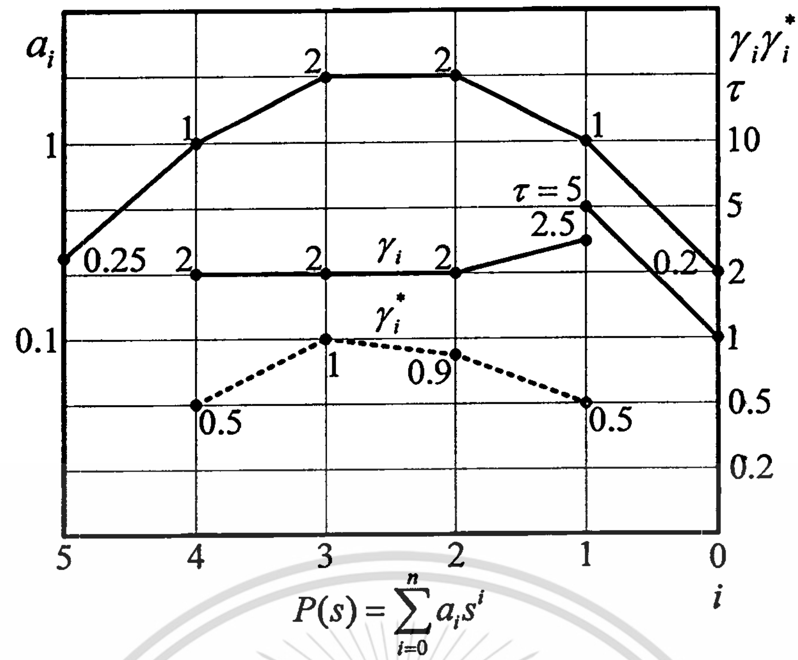
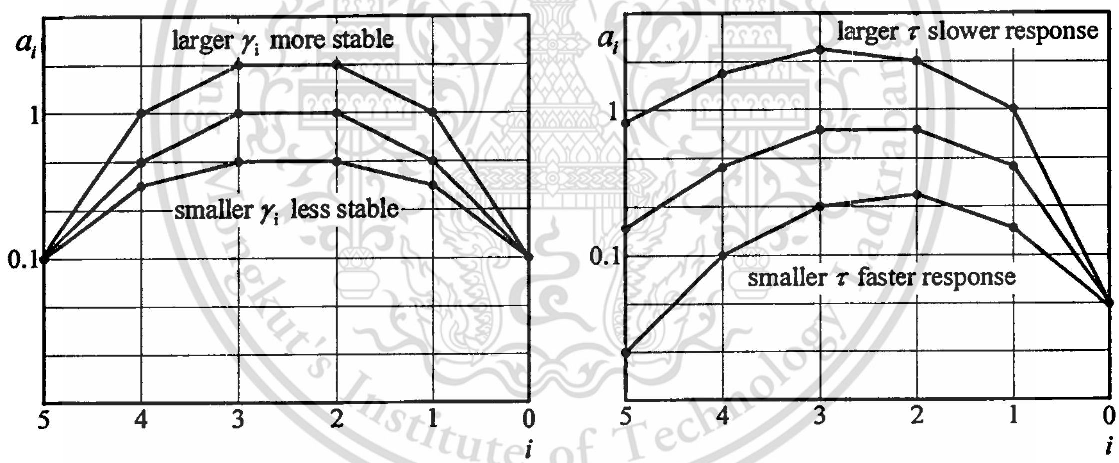


Figure 2.2 Coefficient diagram.



(a) system stability

(b) system response

Figure 2.3 Corresponding of variation of stability index γ_i.

2.5 Stability Condition

Stability analysis of systems with the third or fourth order characteristic polynomial can easily be done using Routh-Hurwitz stability criterion. However, the effect of coefficient variations on stability cannot be seen clearly for systems with higher orders. Necessary conditions for stability or instability of systems with higher orders can be found easily using Lipatov's work.

From the Routh-Hurwitz stability criterion, the stability condition for the third order is given as

$$a_2 a_1 > a_3 a_0 . \tag{2.16}$$

Equivalently, it can be expressed in term of stability index by

$$\gamma_2 \gamma_1 > 1 . \tag{2.17}$$

In the case of fourth order system, the stability condition is given as

$$a_2 > \frac{a_1}{a_3} a_4 + \frac{a_3}{a_1} a_0 \tag{2.18}$$

In term of stability index and stability limit, it can be rewritten by

$$\gamma_2 \gamma_3 > 1. \tag{2.19}$$

Form equations (2.9), (2.17) and (2.19), it is concluded that the stability condition for the third and fourth order is written by

$$\gamma_i > \gamma_i^{*}, \quad \text{for all } i = 2 \square n-2 . \tag{2.20}$$

For the system higher than or including the fifth order, Lipatov gave the sufficient condition for stability and instability in several forms. The conditions most suitable to CDM can be stated as follows;

"The system is stable, if all the partial fourth-order polynomials are stable with the margin of 1.12. The system is unstable if some partial third-order polynomial is unstable."

Thus the sufficient condition for stability is given as

$$a_i > 1.12 \left[\frac{a_{i-1}}{a_{i+1}} a_{i+2} + \frac{a_{i+1}}{a_{i-1}} a_{i-2} \right], \tag{2.21}$$

$$\gamma_i > 1.12\gamma_i^*, \text{ for all } i = 2 \sim n-2. \tag{2.22}$$

The sufficient condition for instability is given as

$$a_{i+1}a_i \le a_{i+2}a_{i-1}, \tag{2.23}$$

$$\gamma_{i+1}\gamma_i \le 1, \text{ for some } i = 1 \sim n-2. \tag{2.24}$$

2.6 Standard Form of CDM

Kessler proposed stability index γ_i to be 2 for all i in order to decrease the oscillation and overshoot in the ITAE (Integral Time Absolute Error) form. But it was later found that better response, no overshoot and shorter settling time can be obtained by increasing γ_1 to 2.5. Thus the standard value of γ_i for the CDM is

$$\gamma_{n-1} = \ldots = \gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5, \tag{2.25}$$

and CDM design has a settling time

$$t_s = 2.5\tau \sim 3\tau. \tag{2.26}$$

As in equation (2.11), the characteristic polynomial is expressed in the powers of τs, and the coefficients are expressed by γ_i. Thus the shapes of response and speed are determined by γ_i and τ respectively.

The features of the CDM standard form can be summarized as follow:

(1) No overshoot for the type 1 system and a proper overshoot about 40 percent for the type 2 system

(2) The shortest settling time for the same equivalent time constant τ

(3) The similar wave form irrespective to the order of system

(4) The low order poles are of the equal decay. The high order poles are in the sector ±50 degrees from the negative real axis. The damping ratio ζ is 0.65.

2.7 Modification of the CDM Standard Form

In the actual design, the choice of the standard stability index is strongly recommended due to stability and response requirements. However, it is not necessary to make $\gamma_4 \sim \gamma_{n-1}$ equal to 2. The condition can be relaxed as

$$\gamma_i > 1.5\gamma_i^{*}. \tag{2.27}$$

Therefore, the designer has the freedom of designing the controller. Sometimes it may be advisable to select the larger value of the stability index, in order to improve the robustness related parameter change. From the sufficient condition for stability by Lipatov, stability is guaranteed when all γ_i's are larger than 1.5. Lipatov proved in his paper [15] that, if all γ_i's are greater than 4, all the roots are negative real. Thus, γ_i's usually chosen between 1.5 and 4. Sometimes the system is built in such manner that some γ_i is already larger than the standard value. In such case it is not necessary to reduce it to the standard value.

Chapter 3

Introduction to Two-Inertia System

Two-inertia system is used in some industrial process such as the rolling mill drive system in the paper and the metal sheet producing process [16] as shown in figure 3.1. Two-inertia system structure and its mathematical model will be described in this chapter respectively.

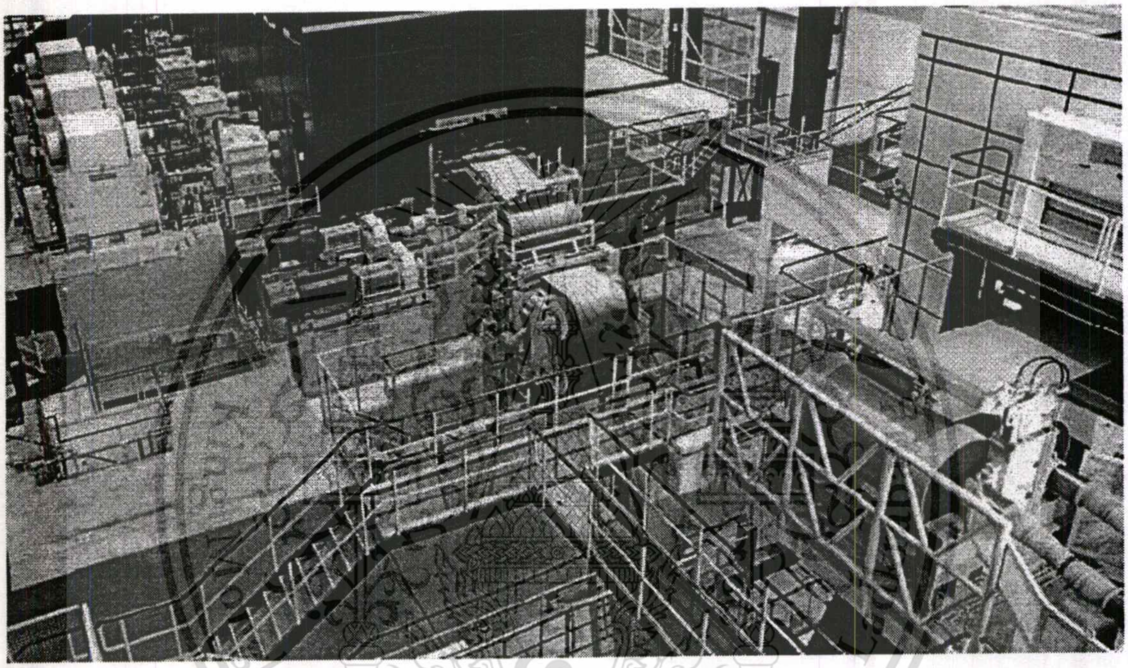


Figure 3.1 Rolling mill drive system.

The main trouble in controlling two-inertia system is the torsional resonance. It is because of the limitation of the stiffness in the shaft connecting between the drive motor and the load [17] that will make the difference of angular position or the torsion along the shaft. This angular position is related with the frequency of the input signal and the parameter of two-inertia system. In some frequency will make the collecting of the kinetic energy which increases the amplitude of the torsion in the shaft. This phenomenon is called the torsional resonance that will make the damage on the system. Thus the controller is needed to control and reduce the unsatisfied torsional resonance in two-inertia system.

3.1 Two-inertia System Structure

The structure of two-inertia system used in this thesis can be shown in the figure 3.2. It consists of two DC motors (Model SS40E8 of SAWAMURA DENKI KOGYO CO., LTD), encoder (Model E6C2-CWZ6C of OMRON), driver and supply. One of the DC motor is the drive motor connecting to the load motor by the shaft. The encoder is the rotary encoder that used for measuring the speed of each motor (More details in the Appendix).

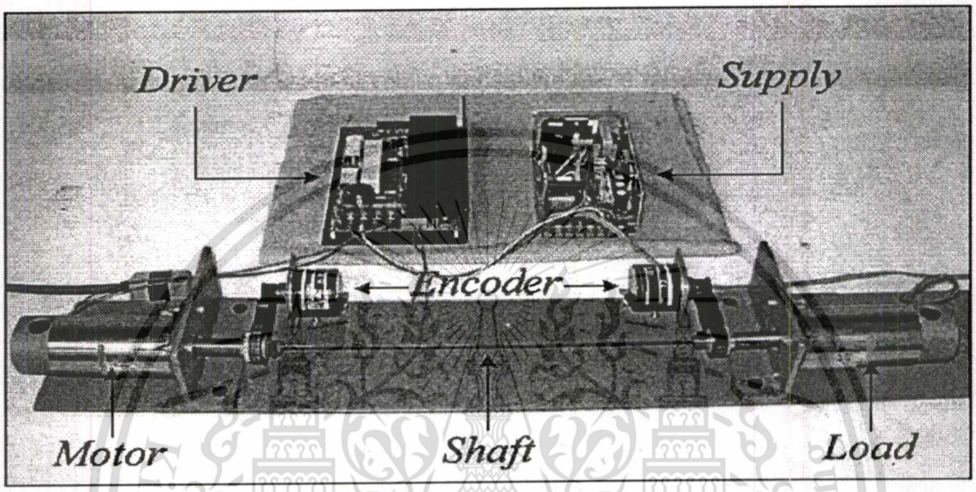


Figure 3.2 Two-inertia system structure.

3.2 Mathematical Model

The model of two-inertia system that shows the motion and the direction of torque of each motor can be depicted in figure 3.3,

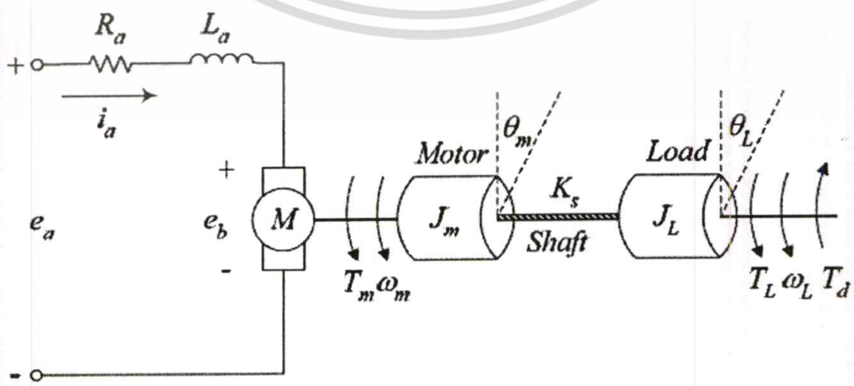


Figure 3.3 Model of two-inertia system.

where the parameters in this model are as follow:

e_a is input voltage applied to the armature

e_b is back electromotive force voltage

R_a is armature resistance

L_a is armature inductance

i_a is armature current

K_e is back electromotive force constant

ω_m is motor speed

K_m is motor torque constant

T_m is motor torque

J_m is moment of motor inertia

θ_m is angle of motor rotation

K_s is torsional stiffness of drive shaft

T_L is load torque

J_L is moment of load inertia

θ_L is angle of load rotation

ω_L is load speed

T_D is disturbance torque

The mathematical model of this two-inertia system is the transfer function of the input voltage applied to the armature e_a and the motor speed ω_m. From the model of two-inertia system when the e_a is applied with DC voltage, the armature current i_a can be calculated by the Kerchoff's law as

$$e_a(t) = R_a i_a(t) + L_a\left(\frac{d}{dt} i_a(t)\right) + e_b(t), \tag{3.1}$$

where e_b is the back electromotive force voltage that occurs in motor where there is relative motion between the armature of the motor and the external magnetic field as the Faraday's law of induction [18]. Its magnitude is depended on the speed of motor with the relation of

$$e_b(t) = K_e \omega_m(t). \tag{3.2}$$

Taking the Laplace transform of equation (3.1) and (3.2), assuming the zero initial condition, the armature current of the motor can be written as

$$I_a(s) = \frac{E_a(s) - K_e\omega_m(s)}{L_a s + R_a}. \tag{3.3}$$

Let the magnetic flux is constant, the torsion of the motor will be depended on the armature current and can be expressed as

$$T_m(s) = K_m I_a(s). \tag{3.4}$$

Replacing the armature current I_a in equation (3.3) to equation (3.4), then

$$T_m(s) = \frac{K_m}{L_a s + R_a}\left(E_a(s) - K_e\omega_m(s)\right). \tag{3.5}$$

The torque of motor T_m in mechanical analysis can be calculated by

$$T_m(s) = T_f(s) + T_L(s) + f_m\omega_m(s) + J_m s\omega_m(s), \tag{3.6}$$

where T_f and f_m are the loss torque from the friction and the viscous friction of the motor which are very small value and can be neglected. Consequently, the motor speed $\omega_m(s)$ can be computed by this equation

$$\omega_m(s) = \frac{T_m(s) - T_L(s)}{J_m s}. \tag{3.7}$$

The motor speed $\omega_m(t)$ is the change of the angle of motor rotation $\theta_m(t)$ per time t. Then the relation of $\omega_m(s)$ and $\theta_m(s)$ can be expressed as

$$\theta_m(s) = \frac{\omega_m(s)}{s}. \tag{3.8}$$

The torque of load motor T_L can also be calculated by the same method of computing the torque of the motor. Hence, the load speed $\omega_L(s)$ and the angle of load rotation $\theta_L(s)$ can be computed by

$$\omega_L(s) = \frac{T_L(s) - T_D(s)}{J_L s} \tag{3.9}$$

and

$$\theta_L(s) = \frac{\omega_L(s)}{s}. \tag{3.10}$$

From studying, torque of load motor T_L will depend on the difference between the angle of motor rotation $\theta_m(s)$ and the angle of load rotation $\theta_L(s)$ and can be written as

$$T_L(s) = K_s \cdot [\theta_m(s) - \theta_L(s)] \tag{3.11}$$

or

$$T_L(s) = \frac{K_s}{s} \cdot [\omega_m(s) - \omega_L(s)]. \tag{3.12}$$

From equation (3.5), (3.7), (3.9) and (3.12), let the disturbance torque $T_D = 0$, the transfer function of two-inertia system can be expressed as

$$G_p(s) = \frac{\omega_m(s)}{E_a(s)} = \frac{a_2 s^2 + a_0}{b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}, \tag{3.12}$$

where $a_2 = \frac{K_m}{J_m L_a}$,

$a_0 = \frac{K_m K_s}{J_m J_L L_a}$,

$b_4 = 1$,

$b_3 = \frac{R_a}{L_a}$,

$b_2 = \frac{K_s L_a (J_m + J_L) + K_m K_e J_L}{J_m J_L L_a}$,

$b_1 = \frac{K_s R_a (J_m + J_L)}{J_m J_L L_a}$,

$b_0 = \frac{K_m K_e K_s}{J_m J_L L_a}$.

Chapter 4

Controller Design

The simplified designing method of the PID-family controller based on CDM is discussed in this chapter. The plant model order reduction, the control system structure, controller design procedure and controller design for speed control of two-inertia system will be described respectively. The order of higher-order plant will be reduced first by ignoring the non-dominant poles. Then the parameters of PID-family controller for a lower-order plant model are designed based on the stability and the speed of the controlled system. Stability is designed from the stability index γ_i, and speed is designed from the equivalent time constant τ. The stability index γ_i and the equivalent time constant τ are defined based on the closed-loop transfer function. These coefficients are related to the controller parameters algebraically in an explicit form. According to the proposed design methodology, the parameters of the PID-family controller can be obtained for the higher-order plant easily and properly.

4.1 Plant Model Order Reduction

The plant model order reduction will be discussed in this section. Consider the k^{th}-order plant with the transfer function given as

$$G_p(s) = \frac{B_p(s)}{A_p(s)} = \frac{K\prod_{j=1}^{m}(s+z_j)}{\prod_{i=1}^{k}(s+p_i)};\ k \geq m\ . \tag{4.1}$$

Without loss of generality, assuming that the plant contains the non-dominant poles $-p_i$ for $i = 1,2,...,l$. By ignoring the non-dominant poles, the k^{th}-order plant can then be reduced to be a lower-order plant model and its transfer function is

$$\tilde{G}_p(s) = \frac{\tilde{B}_p(s)}{\tilde{A}_p(s)} = \frac{\tilde{K}\prod_{j=1}^{m}(s+z_j)}{\prod_{i=l+1}^{k}(s+p_i)};\ \tilde{K} = \frac{K}{\prod_{i=1}^{l}p_i};\ k-l \geq m. \tag{4.2}$$

4.2 PID Control System Structure

The general PID-family control system structure can be shown in figure 4.1. It consists of a higher-order plant $G_p(s)$ and a PID controller where K_p is the proportional gain T_i is the integral time and T_d is the derivative time.

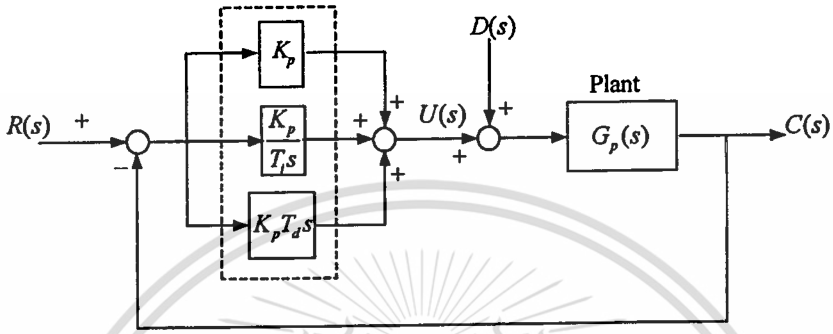


Figure 4.1 PID-family control system structure.

Since $G_p(s)$ is reduced to be the lower-order plant model $\tilde{G}_p(s)$, therefore, the CDM control system structure for the lower-order plant model can be depicted as figure 4.2.

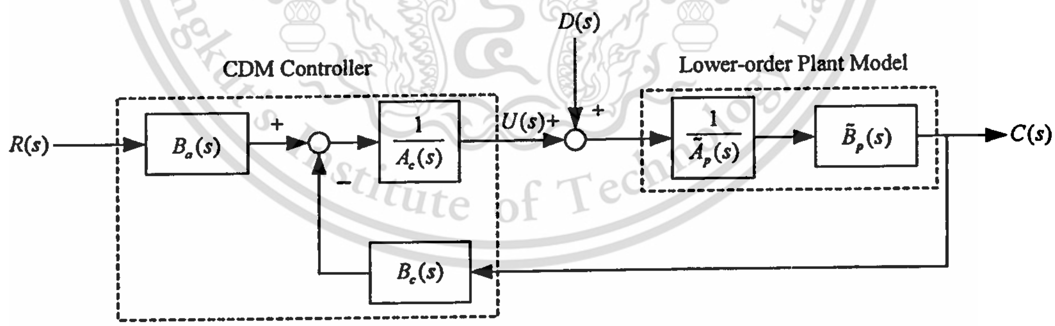


Figure 4.2 CDM control system structure for the lower-order plant model.

In order to be able for PID-family controller designing, the CDM control system structure for the lower-order plant model in figure 4.2 must be rearranged as figure 4.3 such that the PID-family controller can be designed.

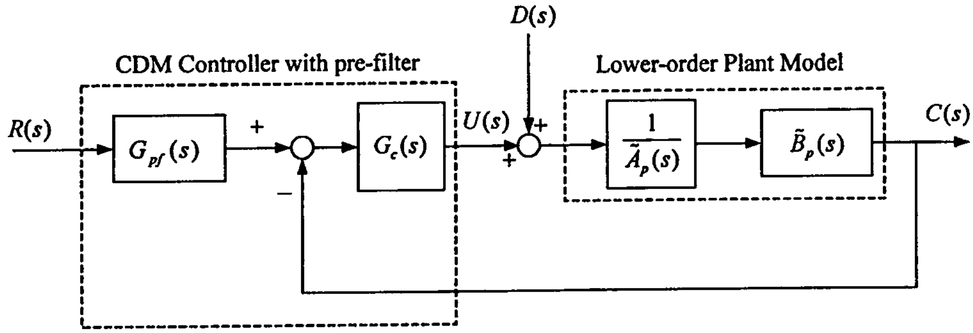


Figure 4.3 Rearranged CDM control system structure for the lower-order plant model.

The rearranged CDM control system structure in figure 4.3 consists of a controller $G_c(s)$ and the pre-filter $G_{pf}(s)$ and the corresponding transfer functions are given as

$$G_c(s) = \frac{B_c(s)}{A_c(s)} \tag{4.3}$$

and

$$G_{pf}(s) = \frac{B_a(s)}{B_c(s)}, \tag{4.4}$$

where $B_a(s)$ is the coefficient of the s^0 of $B_c(s)$. From the block diagram of the rearranged CDM control system structure for the lower-order plant model in the figure 4.3, the closed-loop transfer function from the input $R(s)$ to the output $C(s)$ is then given by

$$\frac{C(s)}{R(s)} = \frac{G_{pf}(s)\left[B_c(s) + B_p(s)\right]}{A_c(s)\tilde{A}_p(s) + B_c(s)\tilde{B}_p(s)} \tag{4.5}$$

and the transfer function from the disturbance $D(s)$ to the output $C(s)$ is

$$\frac{C(s)}{D(s)} = \frac{\tilde{B}_p(s)A_c(s)}{A_c(s)\tilde{A}_p(s) + B_c(s)\tilde{B}_p(s)}. \tag{4.6}$$

From the transfer functions (4.5) and (4.6), it is seen that the pre-filter $G_{pf}(s)$ has an influence on the transfer function from $R(s)$ to $C(s)$ and can be used to increase the speed of the transient response of the controlled system, while the transfer function from $D(s)$ to $C(s)$ is not affected.

In order to derive the closed-loop transfer function of the PID-family control system in term of γ_i and τ, the closed-loop transfer function the PID-family control system given by equation (4.5) might be expressed as

$$\frac{C(s)}{R(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\ldots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+\ldots+a_1 s+a_0}, \tag{4.7}$$

where $n=(k-l)+1$. Then, the characteristic polynomial of the closed-loop system in the term of γ_i, τ and a_0 can be expressed as

$$\begin{aligned}\tilde{P}(s)&=A_c(s)\tilde{A}_p(s)+B_c(s)\tilde{B}_p(s)\\&=a_n s^n+a_{n-1}s^{n-1}+\ldots+a_1 s+a_0\\&=a_0\left[\left\{\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\right)(\tau s)^i\right\}+\tau s+1\right].\end{aligned} \tag{4.8}$$

This characteristic polynomial is a general form for designing the PID-family controller for the lower-order plant model. Then, the PID control system designed for lower-order plant model by the CDM method will be depicted as figure 4.4.

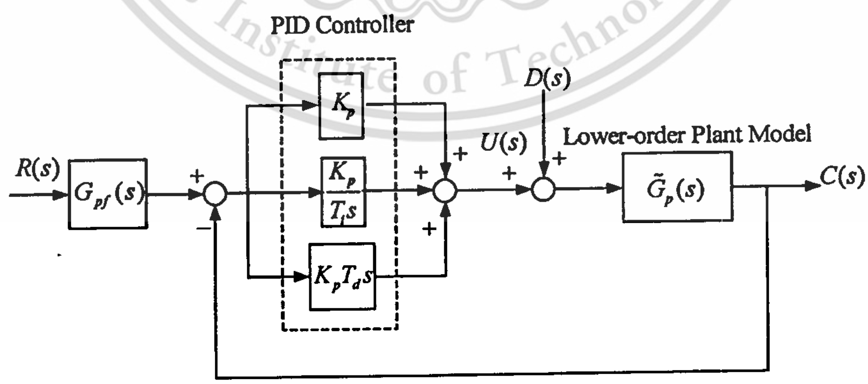


Figure 4.4 PID control system designed by CDM method for the lower order plant model.

The controller $G_c(s)$ in the figure 4.4 is assumed to be the PID controller and its transfer function can be expressed as

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{4.9}$$

or

$$G_c(s) = \frac{B_c(s)}{A_c(s)} = \frac{K_p s + K_i + K_d s^2}{s}, \tag{4.10}$$

where $K_i = \dfrac{K_p}{T_i}$ and $K_d = K_p T_d$.

From the transfer function of lower-order plant model $\tilde{G}_p(s)$ shown in equation (4.2), the characteristic polynomial of the closed-loop system is

$$\begin{aligned}\tilde{P}_{PID}(s) &= A_c(s)\tilde{A}_p(s) + B_c(s)\tilde{B}_p(s) \\ &= \left[\{s\}\left\{\left(\prod_{i=1}^{l}(p_i)\prod_{i=l+1}^{k}(s+p_i)\right)\right\}\right] + \left[\{K_p s + K_i + K_d s^2\}\left\{K\prod_{j=1}^{m}(s+z_j)\right\}\right] \\ &= \sum_{i=0}^{n} a_i s^i,\end{aligned} \tag{4.11}$$

where $n = (k - l) + 1$. This characteristic polynomial is a general form for designing the PID-family controller for the lower-order plant model. With the appropriate values of the stability index γ_i and/or the equivalent time constant τ, the values of the proportional gain K_p, the integral gain K_i and the derivative gain K_d of the PID controller can be obtained by equating the characteristic polynomial in equation (4.11) with the characteristic polynomial in equation (4.7). From the equation (4.4) and (4.10) with $B_a(s) = K_i$, the transfer function of the pre-filter $G_{pf}(s)$ for the PID control system is

$$G_{pf}(s) = \frac{B_a(s)}{B_c(s)} = \frac{K_i}{K_d s^2 + K_p s + K_i}. \tag{4.12}$$

4.3 Controller Design Procedure

The designing steps for the PID-family controller are as follows:

(1) Find the closed-loop characteristic polynomial of the control system with the lower-order plant model and the PID-family controller.

(2) Choosing the appropriate values of the stability index γ_i and/or the equivalent time constant τ to find the closed-loop characteristic polynomial such that the over-damped response of the closed-loop system with lower-order plant model including $G_{pf}(s)$ is obtained, where $B_a(s)$ is the coefficient of s^0 of $B_c(s)$.

(3) Find the parameters of the controller by equating the closed-loop characteristic polynomial obtained in step (2) with the closed-loop characteristic polynomial obtained in step (1).

(4) Applying the controller obtained in step (3) to control the original plant to investigate the control capability of the simplified design controller.

4.4 Controller Design for Speed Control of Two-inertia System

The PI controller and I-P controller which are the most popular controller for the speed control of the DC motor drive are considered. The simplified design of PI controller and I-P controller for controlling the speed of two-inertia system will be demonstrated in this section respectively.

4.4.1 PI Controller Design

The structure of PI control system designed for lower-order plant model by the CDM method will be depicted as figure 4.5.

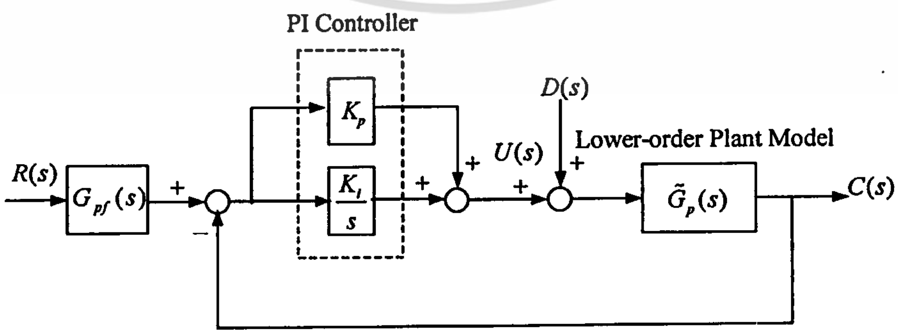


Figure 4.5 Structure of PI control system with the lower-order plant model.

The controller $G_c(s)$ is assumed to be a PI controller. The transfer function of the PI controller can be expressed as

$$G_c(s) = K_p + \frac{K_i}{s} \tag{4.13}$$

or

$$G_c(s) = \frac{B_c(s)}{A_c(s)} = \frac{K_p s + K_i}{s}. \tag{4.14}$$

From the general characteristic polynomial for designing the PID-family controller for controlling the lower-order plant model in equation (4.11), the closed-loop characteristic polynomial of the PI control system can be given by neglecting the derivative term ($K_d s^2$). Thus, the characteristic polynomial for designing the PI controller for controlling the lower-order plant model can be expressed as

$$\begin{aligned}\tilde{P}_{PI}(s) &= \left[\{s\}\left\{\left(\prod_{i=1}^{l}(p_i)\prod_{i=l+1}^{k}(s+p_i)\right)\right\}\right] + \left[\{K_p s + K_i\}\left\{K\prod_{j=1}^{m}(s+z_j)\right\}\right] \\ &= \sum_{i=0}^{n} a_i s^i,\end{aligned} \tag{4.15}$$

where $n = (k-l)+1$. With the appropriate values of the stability index γ_i and/or the equivalent time constant τ, the proportional gain K_p and the integral gain K_i of the PI controller can be obtained by equating the characteristic polynomial in equation (4.15) to the polynomial in equation (4.7) stated previously. In this case, $B_a(s)$ of the PI control system is K_i. Hence, the pre-filter $G_{pf}(s)$ of PI control system can be calculated by

$$G_{pf}(s) = \frac{B_a(s)}{B_c(s)} = \frac{K_i}{K_p s + K_i}. \tag{4.16}$$

4.4.2 I-P Controller Design

It is known that I-P controller is the improvement of PI controller in speed control of the DC motor [13]. The proportional term is moved to the feedback path and it acts like feedback compensation and I-P controller still has the same closed-loop characteristic equation as the PI controller. In this configuration it is possible to avoid large change in control signal due to the suddenly change in the reference input [19]. The structure of the I-P control system designed for lower-order plant model by the CDM method will be depicted as figure 4.6.

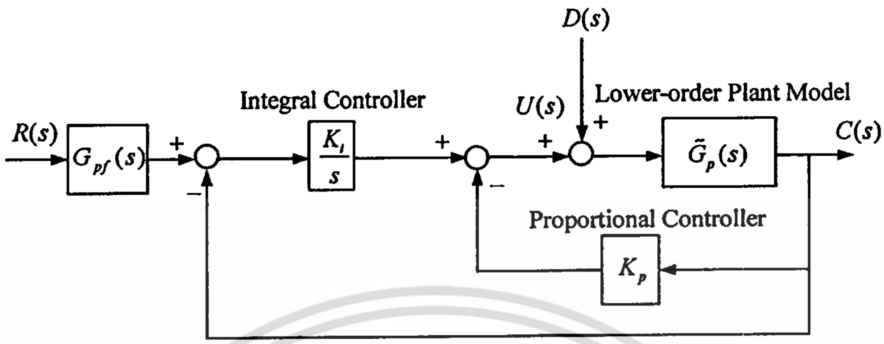


Figure 4.6 Structure of I-P control system with the lower-order plant model.

The polynomials of the modified plant $A_p^*(s)$ and $B_p^*(s)$ shown in figure 4.3 are obtained from the lower-order plant model $\tilde{G}_p(s)$ and the proportional controller K_p.

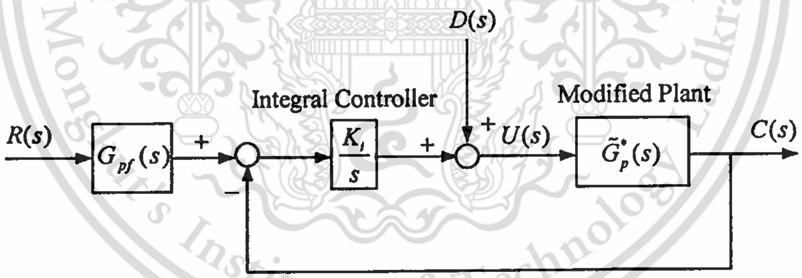


Figure 4.7 Rearrange structure of I-P control system with the lower-order plant model.

In order to assign the parameters of the feedback controller and the integral controller of the I-P control system, the characteristic polynomial of the control system must be first found from the modified plant

$$G_p^*(s) = \frac{\tilde{B}_p^*(s)}{\tilde{A}_p^*(s)} = \frac{\tilde{B}_p(s)}{\tilde{A}_p + K_p\tilde{B}_p(s)}. \tag{4.17}$$

The controller $G_c(s)$ is assumed to be the integral controller and its transfer function is

$$G_c(s)=\frac{B_c(s)}{A_c(s)}=\frac{K_i}{s}. \tag{4.18}$$

The closed-loop transfer function of the simplified design I-P control system can be expressed as

$$\frac{C(s)}{R(s)}=\frac{G_{pf}(s)\left[B_c(s)+\tilde{B}_p^{*}(s)\right]}{A_c(s)\tilde{A}_p^{*}(s)+B_c(s)\tilde{B}_p^{*}(s)}. \tag{4.19}$$

From the general characteristic polynomial for designing the PID-family controller for the lower-order plant model in (4.11), the closed-loop characteristic polynomial of I-P controller for the modified plant can be given by neglecting the proportional term ($K_p s$) and the derivative term ($K_d s^2$). Hence, the characteristic polynomial for designing the I-P controller can be expressed as

$$\begin{aligned}\tilde{P}_{I-P}(s)&=A_c(s)\tilde{A}_p^{*}(s)+B_c(s)\tilde{B}_p^{*}(s)\\&=\left[\{s\}\left\{\left(\prod_{i=1}^{l}(p_i)\prod_{i=l+1}^{k}(s+p_i)\right)+\left(K_p\tilde{K}\prod_{j=1}^{m}(s+z_j)\right)\right\}\right]+\left[\{K_i\}\left\{\tilde{K}\prod_{j=1}^{m}(s+z_j)\right\}\right]\\&=\sum_{i=0}^{n}a_i s^i,\end{aligned} \tag{4.20}$$

where $n=(k-l)+1$. Then, the proportional gain K_p and the integral gain K_i of the I-P controller can then be obtained by equating the characteristic polynomial in equation (4.20) with the polynomial in equation (4.7) assigned by the appropriate values of the stability index γ_i and/or the equivalent time constant τ. From the equation (4.4) and (4.18) with $B_a(s)=K_i$, the transfer function of the pre-filter $G_{pf}(s)$ for the PID control system is

$$G_{pf}(s)=\frac{B_a(s)}{B_c(s)}=1. \tag{4.21}$$

Chapter 5

Simulation and Experimental Results

The effectiveness of the PI controller and I-P controller designed by the simplified design method for the speed control of two-inertia system will be evaluated in this chapter. A series of simulations and experiments are performed. The system responses at a desired speed are investigated first and then the responses when the reference speed is suddenly change will be studied. Finally, the disturbance rejection capability of the simplified designed control system is also investigated.

5.1 Parameters Adjustment of Two-inertia System

The block diagram of two-inertia system can be shown as figure 5.1. The unknown parameters of the system must be found from experimental result.

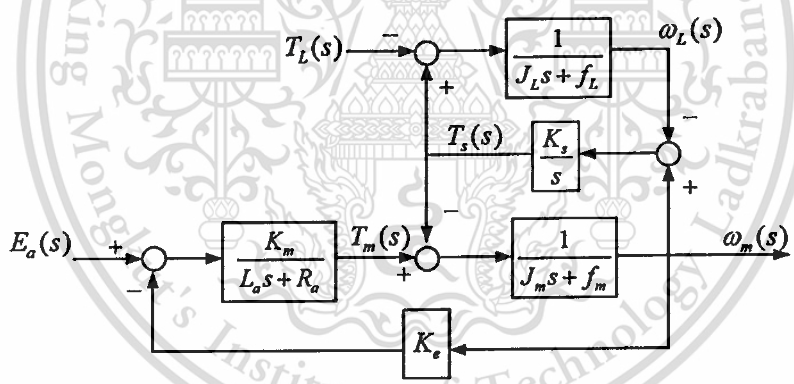


Figure 5.1 Block diagram of two-inertia system.

By using the parameters from the datasheet, the open-loop response of two-inertia system is obtained by simulation and then is compared with the one obtained by experiment. If the simulated response is different from the experimental response, then the parameters of two-inertia system are adjusted until the simulated open-loop response is similar to the experimental one as shown in figure 5.2. The adjusted parameters are shown in Table 5.1. The open-loop responses with torsional resonance shown in figure 5.2 are obtained by applying the voltage at 5 volts to two-inertia system at $E_a(s)$. Note that the solid line is the experimental response while the dashed line is the simulation response of the system used the adjusted parameters.

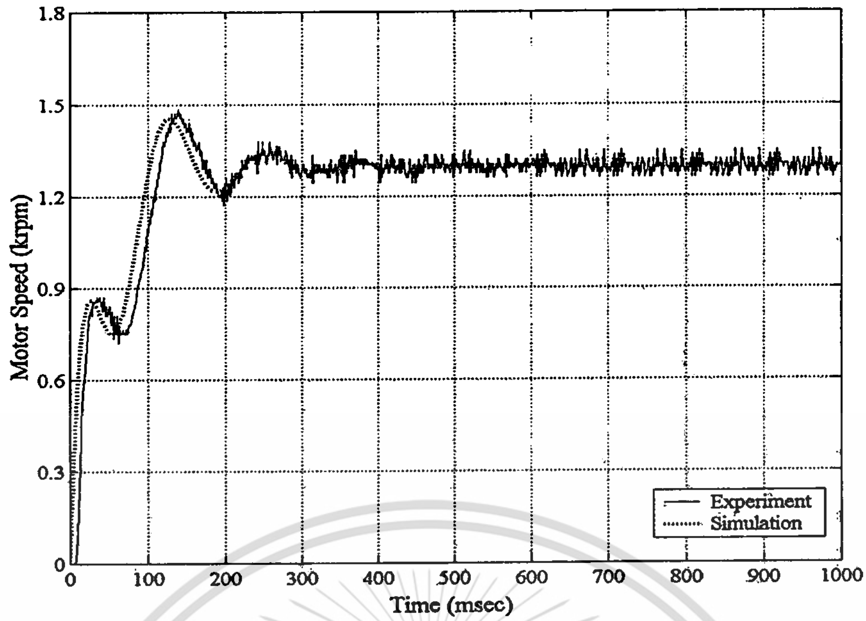


Figure 5.2 Open-loop responses.

Table 5.1 Adjusted parameters of two-inertia system.

R_a	Armature resistance	0.25 Ω
L_a	Armature inductance	0.10 mH
K_e	Back electromotive force constant	3.85 $v / krpm$
K_m	Motor torque constant	3.2 $N - cm / A$
J_m	Moment of motor inertia	0.8 $kg - cm^2$
J_L	Moment of load inertia	0.9 $kg - cm^2$
K_s	Torsional stiffness of drive shaft	15 $N - m / rad$

From the transfer function of two-inertia system indicated in the chapter 3 and the adjusted parameters in Table5.1, the transfer function of this two-inertia system can be expressed as

$$G_p(s) = \frac{4\times10^4 s^2 + 6.67\times10^7}{s^4 + 2.5\times10^3 s^3 + 1.57\times10^5 s^2 + 8.85\times10^6 s + 2.57\times10^8}. \quad (5.1)$$

In order to design the controller for controlling two-inertia system by the proposed simplified designing method, the order of two-inertia system must be reduced first. The open-loop poles of two-inertia system are located at $s = -39.6$, -2436.8 and $-11.8 \pm j50.2$. By ignoring two non-dominant poles at $s = -39.6$ and $s = -2436.8$, the transfer function of lower-order plant model is

$$\tilde{G}_p(s) = \frac{4\times10^4 s^2 + 6.67\times10^7}{9.65\times10^4 s^2 + 2.28\times10^6 s + 2.57\times10^8}. \tag{5.2}$$

Then the lower-order plant model will be used to design the PI controller and I-P controller by CDM. The result of controlling speed of original two-inertia system will be shown next sections.

5.2 PI Controller in Speed Control of Two-inertia System

Based on the PI controller design procedure stated previously, the parameters K_p and K_i of the PI controller including the equivalent time constant τ of the closed-loop system obtained by using the lower-order plant model with $\gamma_1 = 7$ and $\gamma_2 = 0.5$ are shown in the Table 5.2. Note that the gamma values are chosen to give the positive values of K_p and K_i of the PI controller when the simplified design method is employed. For comparison, the parameters of PI controller and the equivalent time constant obtained from the original two-inertia system with $\gamma_1 = 2.5$, $\gamma_2 = 2$, $\gamma_3 = 1.34$ and $\gamma_4 = 32.4$ can also be shown in the Table 5.2. Note that the structure of pre-filter (4.17) of the simplified control system and the direct designed control system are different.

Table 5.2 Parameters of PI controller.

Design Method	K_p	K_i	τ (msec)
Simplified Design	0.41	43.15	87
Direct Design	0.87	54.08	88

5.2.1 Simulation Results

The simulation results in controlling the motor speed of two-inertia system and the corresponding control signal are shown in figure 5.3 and figure 5.4 respectively. It is seen from the figure 5.3 that the proposed PI controller can control the speed at 1300 rpm without torsional resonance, overshoot and steady-state error. It is also seen that the system response speed is quite similar to the response of the system using PI controller obtained from direct design. This can be seen from the value of equivalent time constant τ obtained from the simplified design shown in the Table 5.2 that its value is close to the one obtained from the direct design. The system performances of the simplified designed control system and the direct designed one are also summarized in Table 5.3. It can be noted that the control signals produced from both controllers are similar.

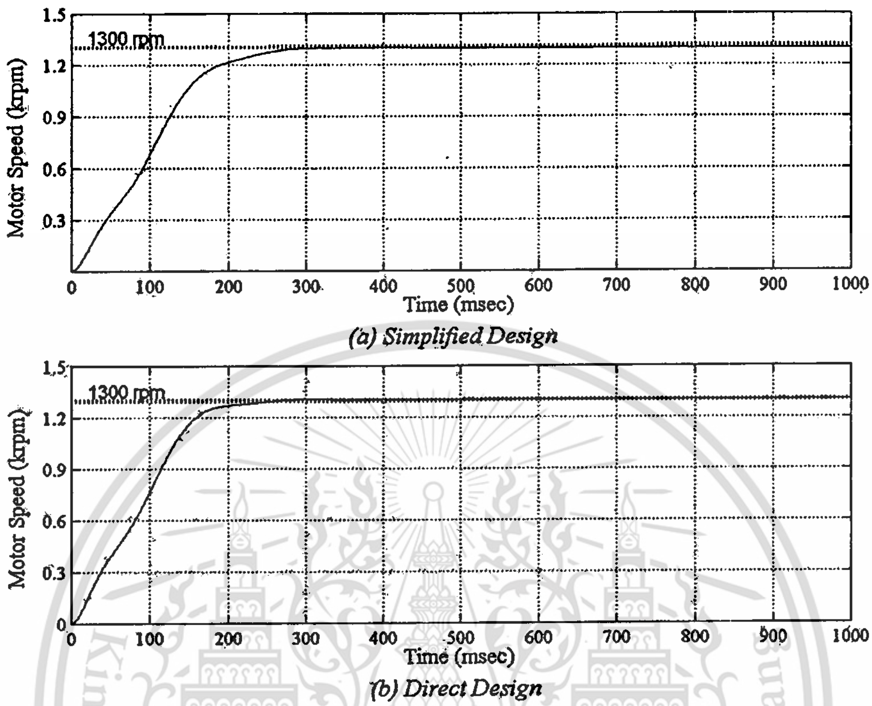


Figure 5.3 Speed responses at 1300 of PI control system (simulation).

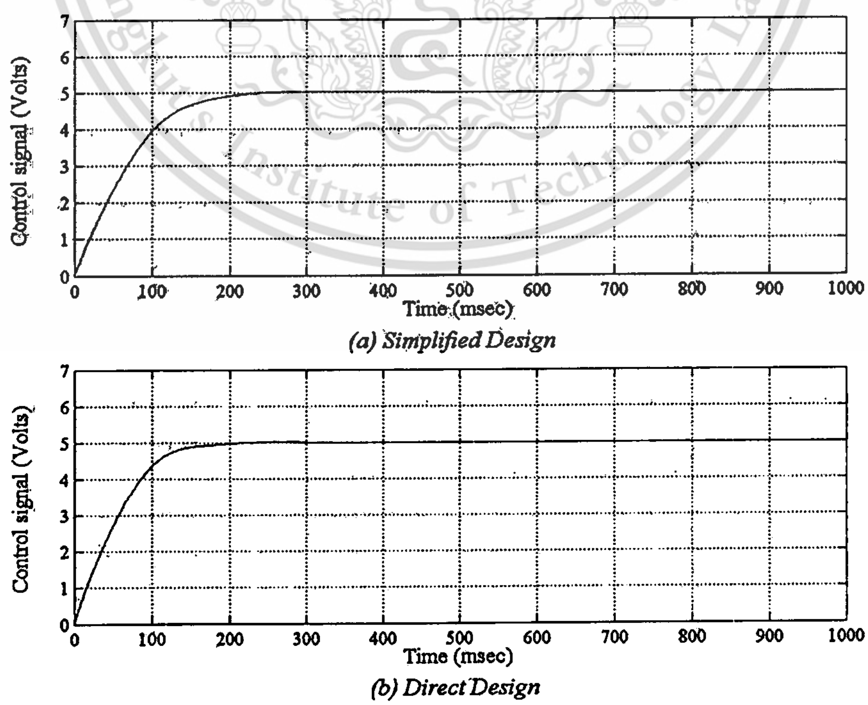


Figure 5.4 Control signal of PI control system in speed control (simulation).

Table 5.3 System performances of PI control system (simulation).

Design Method	t_r *(msec)*	t_s *(msec)*	P_O *(%)*
Simplified Design	152	252	0
Direct Design	132	222	0

The capability of the PI controller designed from the lower-order plant model and the one designed from the original two-inertia system are also investigated in this thesis.

By retarding the motor reference speed of the proposed PI control system at the 500 msec from the speed of 1300 rpm to the speed of 800 rpm, the corresponding responses and control signals are shown in figure 5.5 and figure 5.6 respectively. From the responses of figure 5.5, the motor speed of system using the proposed PI controller can reach to the new reference speed at 800 rpm properly and similarly as the control system using the PI controller obtained from the direct design without error. The control signals of both control system are also decreased from the voltage at 5 volts to around 3 volts in 250 msec.

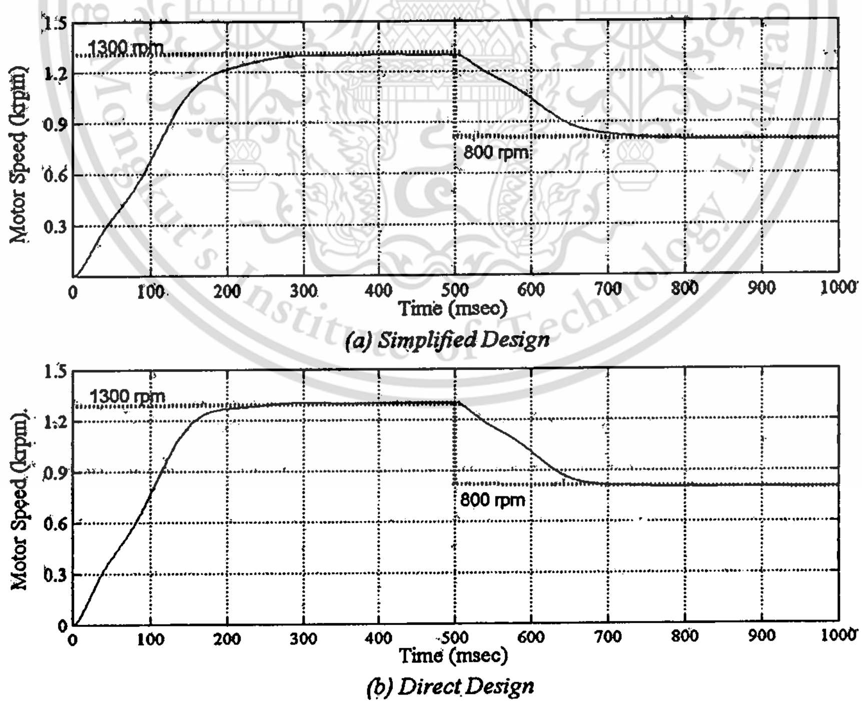
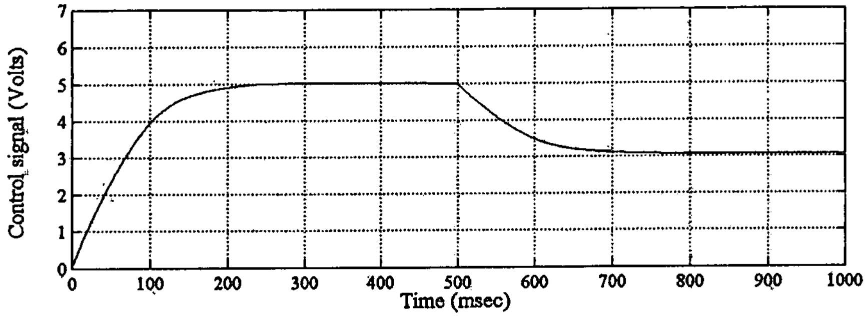
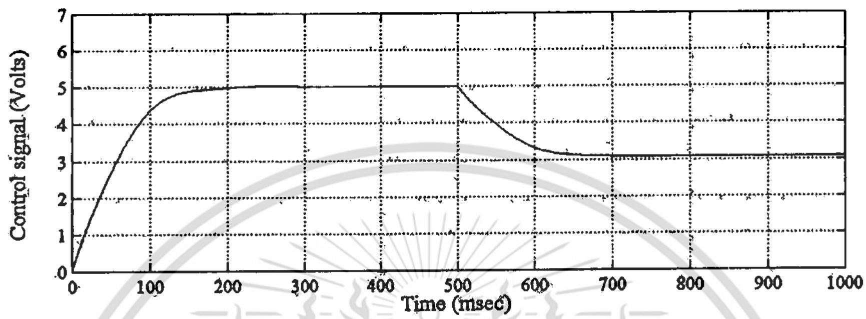


Figure 5.5 Responses of PI control system due to speed retard (simulation).



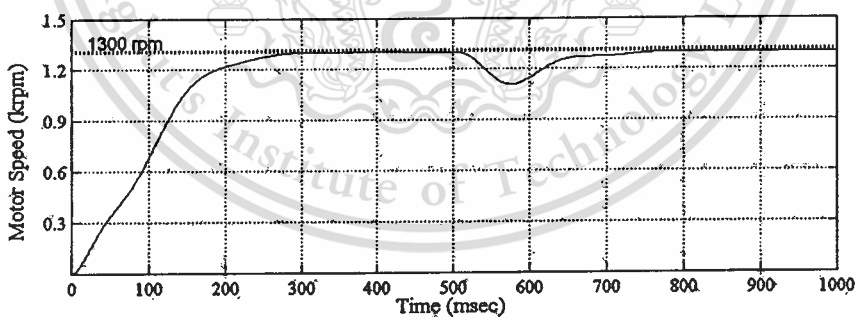
(a) Simplified Design



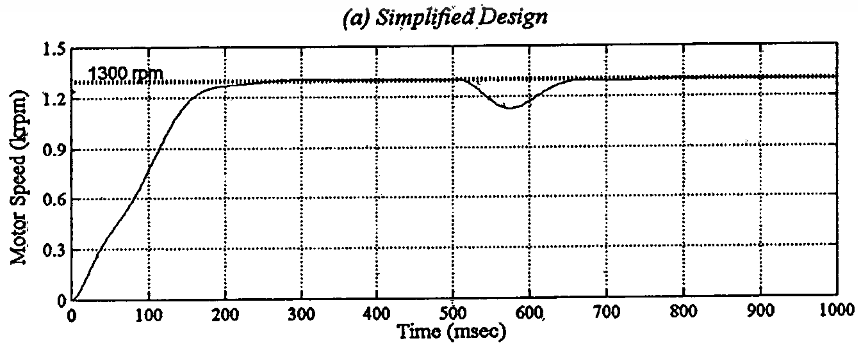
(b) Direct Design

Figure 5.6 Control signal of PI control system due to speed retard (simulation).

In order to compare the disturbance rejection capability of the simplified design PI controller and the direct design PI controller, the motor reference speed at 1300 rpm and the constant load disturbance entering at 500 msec are considered.

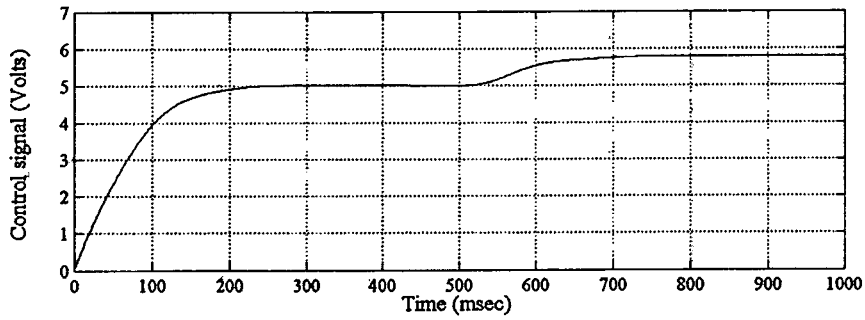


(a) Simplified Design

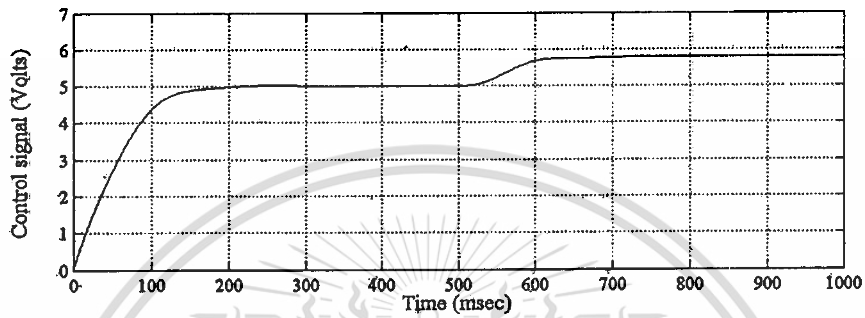


(b) Direct Design

Figure 5.7 Responses of PI control system due to load disturbance (simulation).



(a) Simplified Design



(b) Direct Design

Figure 5.8 Control signal of PI control system due to load disturbance (simulation).

It is seen from the figure 5.7 that the proposed control system can reject the effect of the load disturbance and reach to the speed at 1300 rpm without error similarly, while the control signals are increased to overcome the effects of the load disturbance.

5.2.2 Experimental Results

In this sub-section, the proposed PI controller and the direct designed PI controller are implemented to control the speed of two-inertia system at 1300 rpm. The corresponding experimental results are shown in the figure 5.9 and the figure 5.10. It is seen from the figures 5.9 that the simplified design PI controller can control the speed of two-inertia system at 1300 rpm without torsional resonance, steady-state error and small overshoot. In addition, the response and the control signal obtained from the simplified control system are quite similar to the response and the control signal of the system using PI controller obtained from the direct design. In conclusion, the system performances of both control systems in controlling the speed of two-inertia system can also be shown in Table 5.4.

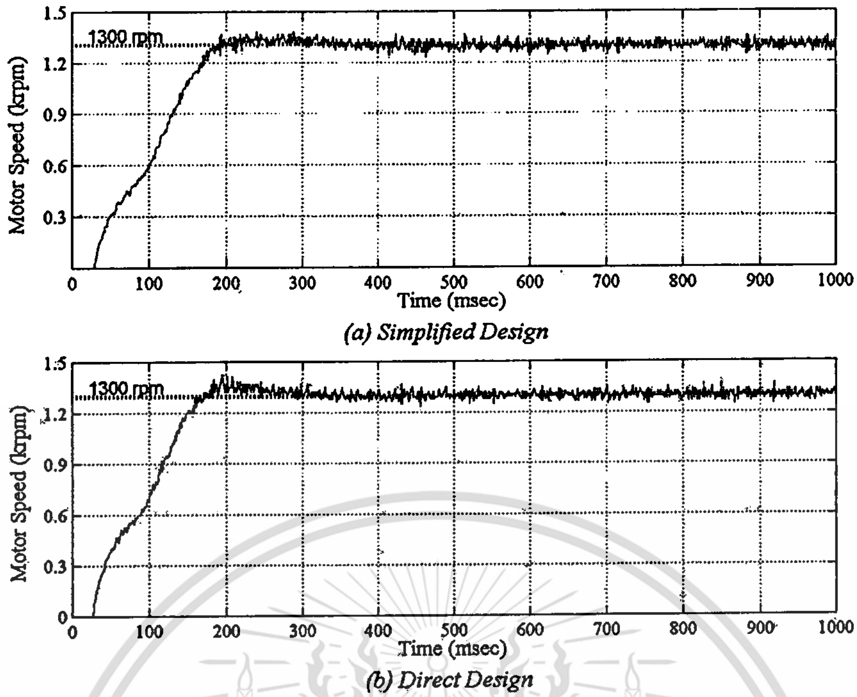


Figure 5.9 Speed responses at 1300 rpm of PI control system (experiment).

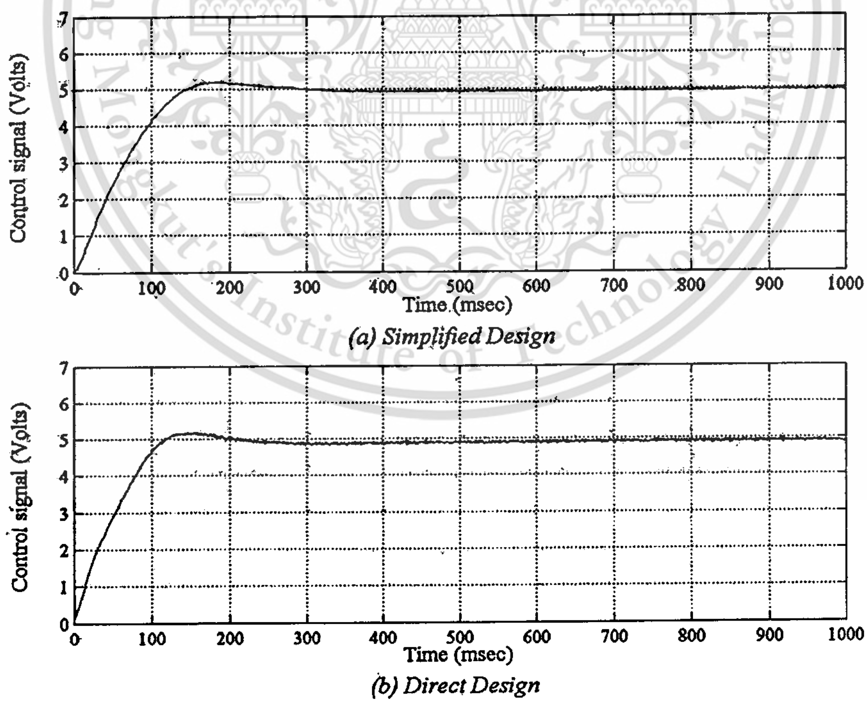


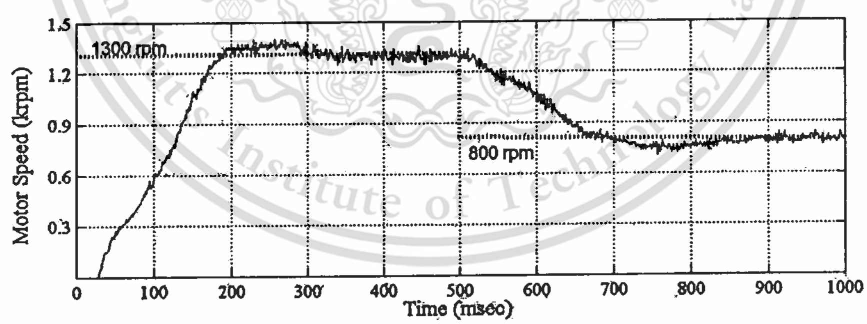
Figure 5.10 Control signal of PI control system in speed control (experiment).

Table 5.4 System performances of PI control system (experiments).

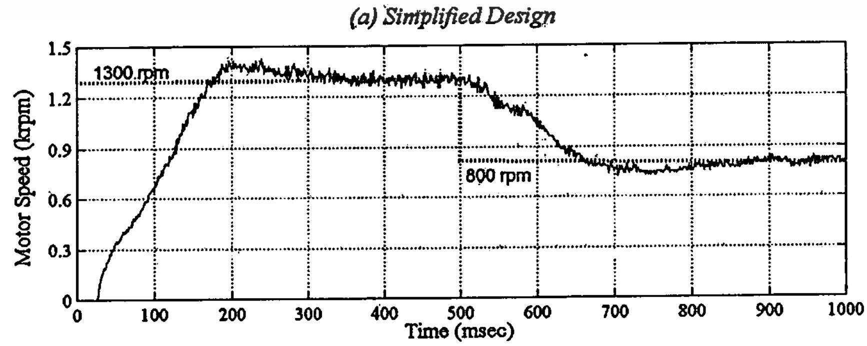
Design Method	t_r *(msec)*	t_s *(msec)*	P_O *(%)*
Simplified Design	176	302	2.3
Direct Design	151	292	3.8

The capability in controlling two-inertia system of the PI controller designed from the lower-order plant model and the one from the direct design is also investigated in two aspects. The first investigation is to retard the reference speed of two-inertia system and observe its effect. The second investigation is to observe the effect when the load disturbance entering to two-inertia system.

By retarding the reference speed of the motor from 1300 to 800 rpm at 500 msec, the motor speed of the proposed PI control system can reach to the new reference speed at 800 rpm properly and similarly as the control system using the PI controller obtained from direct design method as shown in figure 5.11. However, the response of the motor speed controlled by the direct design PI controller reaches to the new reference speed faster than the response obtained from the simplified design PI controller. Note that the control signal produced by the direct design PI controller also reach to 3.5 volts faster than the simplified one.

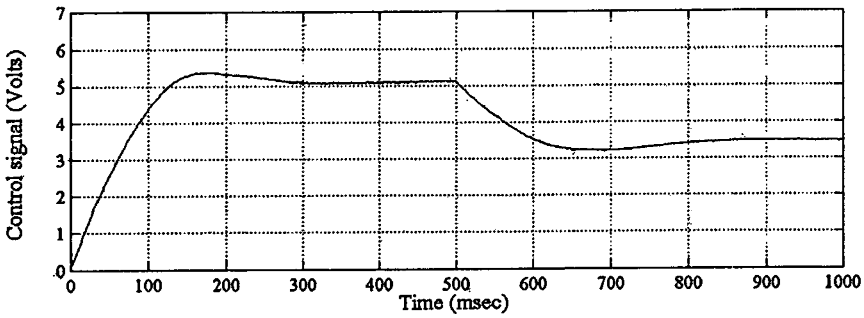


(a) Simplified Design

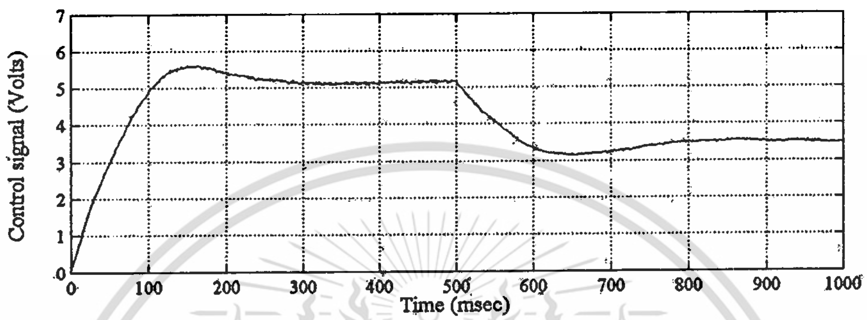


(b) Direct Design

Figure 5.11 Responses of PI control system due to speed retard (experiment).



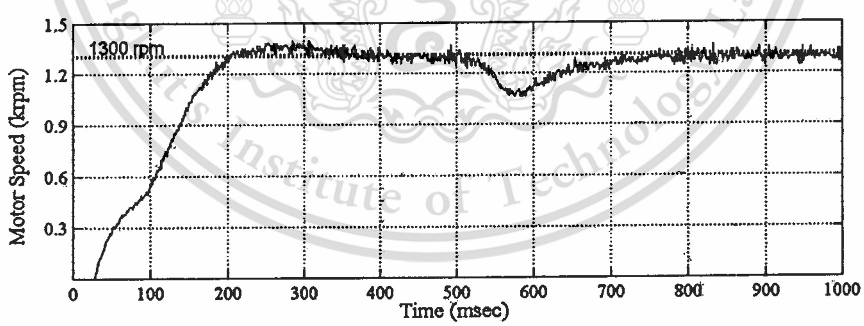
(a) Simplified Design



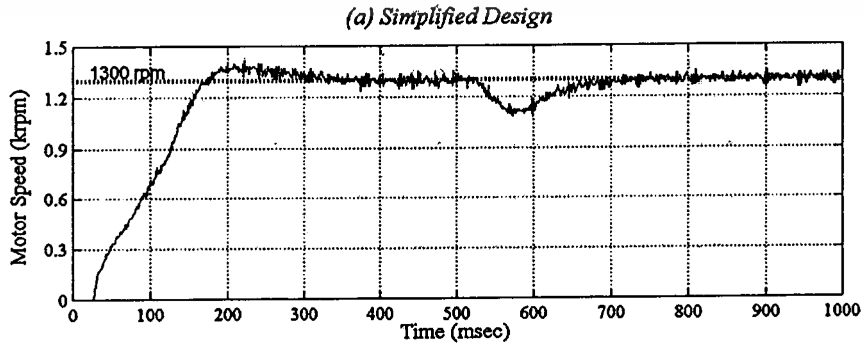
(b) Direct Design

Figure 5.12 Control signal of PI control system due to speed retard (experiment).

In case of the disturbance rejection capability, the proposed PI control system can reject the effect of load disturbance entering to the system at 500 msec and reach back to the reference speed without error similarly to the direct designed PI control system as shown in figure 5.13.



(a) Simplified Design



(b) Direct Design

Figure 5.13 Responses of PI control system due to load disturbance (experiment).

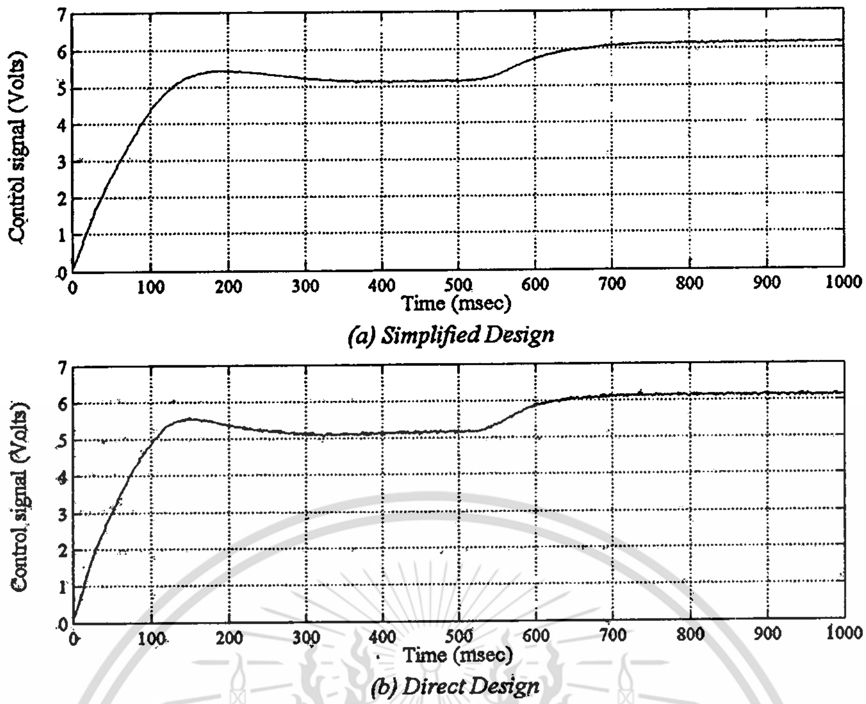


Figure 5.14 Control signal of PI control system due to load disturbance (experiment).

5.3 I-P Controller in Speed Control of Two-inertia System

Based on the controller design procedure stated in sub-section 4.4.2, the parameters K_p and K_i of the I-P controller including the equivalent time constant τ of the closed-loop system obtained by using the simplified design method with $\gamma_1 = 7$ and $\gamma_2 = 0.5$ and the direct design method with $\gamma_1 = 2.5, \gamma_2 = 2, \gamma_3 = 1.34$ and $\gamma_4 = 32.4$ are the same values as PI controller parameters. Since the transfer functions from the input $R(s)$ to the output $C(s)$ including the pre-filter $G_{pf}(s)$ of the either speed control systems are same, therefore, the simulation and experimental results of the two-inertia system controlled by I-P controllers will be omitted here and can refer to the results shown in section 5.2.

Chapter 6

Conclusions and Future Works

6.1 Conclusions

This thesis presents the simplified design method for PID-family controller based on CDM which can be used for designing the PID-family controller with simplicity. The higher order plant must be reduced to be a lower-order plant model first by ignoring the non-dominant poles. Then a controller is designed by CDM for the lower-order plant model. Finally, the PID-family controller designed from the lower-order plant model is employed to control the original higher-order plant. In order to investigate the control capability of the controller designed by this designing method, the simplified design of PI controller and I-P controller which are the most popular of PID-family controller in speed control of DC motor drive will be implemented to control the speed of two-inertia system.

The effectiveness of the PI controller and I-P controller designed by the simplified design method for the speed control of two-inertia system has been evaluated in the previous chapter. The series of simulations and experiments of the simplified design PI controller and I-P controller in controlling the speed of two-inertia system were performed. Not only the simplicity in designing but the speed of the response is also similar to the response of the direct designed control system. Furthermore, the unsatisfied torsional resonance of two-inertia system can be reduced. In addition, the effect of load disturbance is small and can be rejected rapidly. Hence, it can be concluded that the proposed simplified design method based on CDM is successful in designing the PID-family controller.

6.2 Future Works

- The responses of the control system using the controller designed by the CDM method still have a quite slow speed. But, adding the feed-forward controller might be able to give more speed of the responses.
- Applying this simplified design method to design the controller for the others higher-order plant to prove the performance of this designing method.
- Studying of new model order reduction methods that give more performance and simplicity than the method of ignoring the non-dominant poles.

References

[1] C.R. Knospe, "PID Control," *IEEE Control Systems Magazine*, vol. 24, No. 1, pp. 30 – 31, February 2005.

[2] W. K. Ho, et al., "Performance and Gain and Phase Margins of Well-Known PID Tuning Formulas," *IEEE Trans. on Control System Technology*, vol. 4, No.4, July 1996, pp.473-477.

[3] J. G. Ziegler and N. B. Nichols, "Optimum Settings for Automatic Controllers," *ASME Trans.* 64, pp.759-768, 1942.

[4] J.G. Ziegler and N.B. Nichols, "Process Lags in Automatic Control Circuits," *Transactions of the ASME*, vol. 65, pp. 433 – 444, 1943.

[5] G. H. Cohen and G. A. Coon, "Theoretical consideration of retarded control," *Trans. Amer. Soc. Mech. Eng.,* vol. 75, pp. 827-834, 1953.

[6] W. K. Ho, C. C. Hang and L. S. Cao, "Tuning of PID controllers based on gain and phase margins specifications," to appear in *Automatica,* Mar. 1995.

[7] C. C. Hang, W. K. Ho and L. S. Cao, "A comparison of two design methods for PID controllers," *ISA Trans.,* vol. *33,* pp. 147-151, 1994.

[8] S. Manabe, "Coefficient Diagram Method," *Proc. of the 14th IFAC Symposium on Automatic Control in Aerospace*, pp.199-210, 1998.

[9] S. Manabe and Y.C. Kim, "Recent development of Coefficient Diagram Method," *Proc. of the 3rd Asian Control Conference*, Shanghai, 2000.

[10] D. Kumpanya, T. Benjanarasuth, J. Ngamwiwit and N. Komine, "PI Controller Design with Feed forward by CDM for Level Processes," *Proc. of the 2000 IEEE TENCON*, pp. II-65 – II-69, 2000.

[11] S. E. Hamamci, M. Koksal and S. Manabe, "Robust Position Control of a Radar Antenna with the Coefficient Diagram Method," *Proc. of the 4th Asian Control Conference*, pp.1785-1790, 2002.

[12] F. I. Ahmed, et al., "P - I and I - P Controllers in a Closed Loop for DC Motor Drives," *IEEE Trans. of PCC-Nagaoka,* pp., 613-618, 1997.

[13] P. K. NANDAM and P. C. SEN, "Analog and Digital Speed Control of DC Drives Using Proportional-Integral and Integral-Proportional Control Techniques" *IEEE Transaction on Industrial Electronics,* vol. IE-34, No. 2. May 1987.

[14] D. Kumpanya, "Feedforward Controller Design Incorporating PI Controller Designed by Coefficient Diagram Method" Master Thesis, King Mongkut's Institute of Technology Ladkrabang, Bangkok, 2002.

[15] A.Lipatov and N.Sokolov. "Some Sufficient Conditions for Stability and Instability of Continuous Linear Stationary Systems." *Automat. Remote Cont.*, 1979, pp. 1285-1291

[16] G. Zhang, J. Furusho "Speed Control of Two-inertia System by PI/PID Control" *IEEE Conference on Power Electronics and Drive System*, July 1999, pp. 567-572.

[17] G. Zhang, "Comparison of Control Schemes for Two-Inertia System" *IEEE Conference on Power Electronics and Drive System*, July 1999, pp. 573-578.

[18] Wikipedia, Wikipedia contributors (2007), *Electromotive force* [online], Available from: http://en.wikipedia.org/wiki/Electromotive_force [Accessed 19-06-2007].

[19] K. Ogata, **Modern Control Engineering,** second edition, Prentice-Hall, 1990.

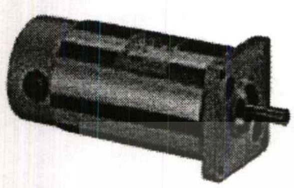
Appendix A

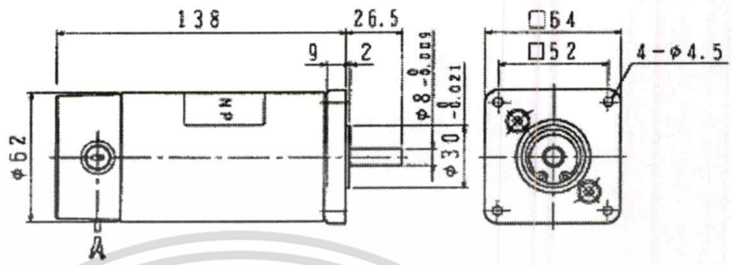
Specification of Two-inertia system

A.1 Specification of motor and load

標準定格電圧 Standard Rated Voltage 12V/24V/100V

DC モータ DC MOTOR 80W MODEL SS40E8

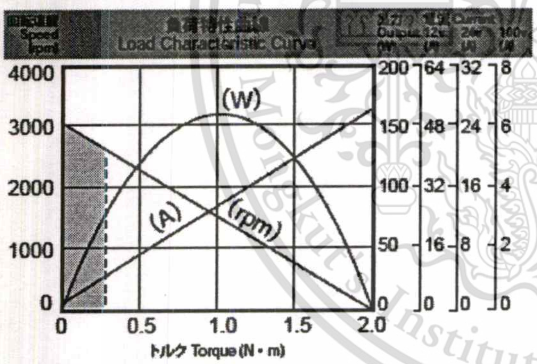




質量 Mass 1.6kg

連続定格 Continuous rating				無負荷 No load		電機子抵抗 Armature R.	電機子慣性モーメント(GD²) R. Inertia	機械的時定数 (tm)	逆起電力定数 (Ke)	トルク定数 (Kt)	ブラシ長さ Brush L
電圧 Voltage DC-V	電流 Current A	回転速度 Speed rpm	トルク Torque N·m (kgf·cm)	電流 Current A	回転速度 Speed rpm	Ω	kg·cm² (kgf·cm²)	ms	V/krpm	N·m/A	mm
12	10	2500	0.31 (3.2)	0.8	3000	0.25	0.7 (2.8)	12	4	0.038	10 限度長さ 4
24	5	2500	0.31 (3.2)	0.4	3000	0.85	0.7 (2.8)	12	8	0.076	
100	1.2	2500	0.31 (3.2)	0.08	3000	15	0.7 (2.8)	12	33.3	0.32	

標準定格電圧以外の電圧も製作いたします。
Ready for offering manufacturing service for a special-voltage products, apart from standard voltage.



連続使用領域 Scope for continual use

許容オーバーハング荷重：98N(10kgf) Permissible Overhung Load

許容スラスト荷重：78N(8kgf) Permissible Thrust Load

オーバーハング荷重の着力点は、軸端部長さの1/2の位置。
Point of application of force for overhung load lies on half as long as shaft edge length.

●適用ドライバ Matching Driver List

電源電圧 Supply Voltage	ドライバ形式名 Driver Model	ページ Page
DC12V	MS-100□10	62
	MS-400□1215	66
DC24V	MS-100□10	62
	MS-400□2408	66
AC100V	MS-300□1001	64

●標準仕様

- 使用温度 ：−10℃〜40℃
- 使用湿度 ：40%〜90%　結露なきこと
- 回転方向 ：可逆
- 耐熱クラス ：B
- 絶縁抵抗 ：DC500Vメガーにて、20MΩ以上
- 耐電圧 ：AC 1000V 1分間
- 外観 ：生地(鉄ケース部めっき)

●取り扱い上の注意

■モータリード線の赤を(+)に、青を(−)に接続するとモータ軸から見て時計方向に回転します。

■DC電源の極性を替えると、正逆相似の特性が得られます。モータが回転中に極性を替えると、モータや相手装置に悪影響を与えます。一旦停止後に逆方向に回転して下さい。

■定格は平滑な直流電源での連続定格です。脈流電源を使用する場合は温度上昇が高くなりますのでモータの温度上昇に注意して下さい。

本カタログ記載の内容は予告なく変更させていただく場合がありますのでご了承下さい。 Content of this catalogue is subject to change without notice.

澤村電気工業株式会社 〒213-0002 神奈川県川崎市高津区二子 6-12-10 TEL (044)811-9331 FAX(044)833-9260
URL http://www.sawamura.co.jp/

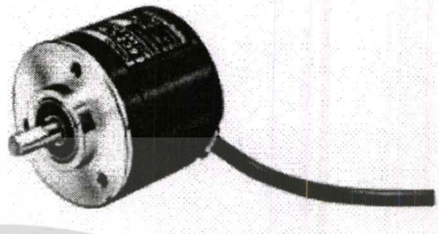
A.2 Specification of encoder

OMRON

Incremental Rotary Encoder — E6C2-C

Industrial Strength Encoder Meets World-Class Standards

- Drip-proof construction
- Shaft withstands heavy loads, 5 kgf radially, 3 kgf thrust (axially)
- Short circuit protection
- Space-saving, A-slant cable protrusion for ease of mounting



Specifications

RATINGS/CHARACTERISTICS

Electrical

Item	E6C2-CWZ6C	E6C2-CWZ3E	E6C2-CWZ1X	E6C2-CWZ5B
Power supply voltage	5 to 24 VDC (allowable range: 4.75 to 27.6 VDC)	5 to 12 VDC (allowable range: 4.75 to 13.2 VDC)	5 VDC±5%	12 VDC –10% to 24 VDC +15%
Current consumption (See Note 1.)	80 mA max.	100 mA max.	160 mA max.	100 mA max.
Resolution	10, 20, 30, 40, 50, 60, 100, 200, 300, 360, 400, 500, 600, 1,000, 1,200, 1,500, 1,800, 2,000 P/R			100, 200, 360, 500, 600, 1,000, 2,000 P/R
Output phases	A, B, and Z (reversible)		A, $\overline{A}$, B, $\overline{B}$, Z, $\overline{Z}$	A, B, and Z (reversible)
Output configuration	NPN open collector output	Voltage output (NPN output)	Line driver (See Note 2.)	PNP open collector output
Output capacity	Applied voltage: 30 VDC max. I_{sink}: 35 mA max. Residual voltage: 0.4 V max. (I_{sink}: 35 mA max.)	Output resistance: 2 kΩ (residual voltage: 0.4 V max. I_{sink}: 20 mA max.)	AM26LS31 Output current: High level (I_0): –20 mA Low level (I_s): 20 mA Output voltage: V_0: 2.5 V min. V_s: 0.5 V max.	I_{sink}: 35 mA max. Residual voltage: 0.4 V max. (I_{sink}: 35 mA max.)
Max. response frequency (See Note 3.)	100 kHz			50 kHz
Phase difference on output	90°±45° between A and B (1/4T±1/8T)			
Rise and fall times of output	1 µs max. (control output voltage: 5 V; load resistance: 1 kΩ; cable length: 2 m)	1 µs max. (cable length: 2 m; I_{sink}: 10 mA max.)	0.1 µs max. (cable length: 2 m; I_0: –20 mA; I_s: 20 mA)	1 µs max. (cable length: 2 m; I_{sink}: 10 mA max.)
Insulation resistance	100 MΩ min. (at 500 VDC) between carry parts and case			
Dielectric strength	500 VAC, 50/60 Hz for 1 min between carry parts and case			

Note: 1. An inrush current of approx. 9 A flows for approx. 0.3 ms right after the E6C2-C is turned on.

2. The line driver output of the E6C2-C is used for data transmission circuitry conforming to RS-422A and ensures long-distance transmission over twisted-pair cable, with quality equivalent to AM26LS31.

3. The maximum electrical response revolution is determined by the resolution and maximum response frequency as follows:
Maximum electrical response frequency (rpm) = maximum response frequency/resolution x 60
This means that the E6C2-C Rotary Encoder will not operate electrically if its revolution exceeds the maximum electrical response revolution.

■ MECHANICAL

Item		E6C2-CWZ6C	E6C2-CWZ3E	E6C2-CWZ1X	E6C2-CWZ5B
Shaft loading	Radial	5 kgf (49.0 N) 11.0 lbf			
	Thrust	3 kgf (29.4 N) 6.6 lbf			
Moment of inertia		10 g • cm² (1 x 10^{-6} kg • m²) max.; 3 g • cm² (3 x 10^{-7} kg • m²) max. at 600 P/R max. 6.6 x 10^{3}lb/in .85 x 10			
Starting torque		100 gf • cm (9.8 mN • m) max. (7.2 m ft • lbf)			
Max. permissible revolution		6,000 rpm			
Vibration resistance		10 to 500 Hz, 150 m/s² (15G) or 2-mm double amplitude for 11 min 3 times each in X, Y, and Z directions			
Shock resistance		1,000 m/s² (100G) 3 times each in X, Y, and Z directions			
Weight		Approx. 400 g max. (cable length: 2 m) 0.88 lbs			

■ ENVIRONMENTAL

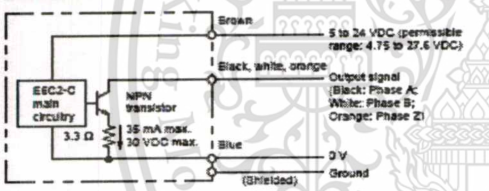
Item		E6C2-CWZ6C	E6C2-CWZ3E	E6C2-CWZ1X	E6C2-CWZ5B
Ambient temperature	Operating	–10°C to 70°C (14°F to 158°F) with no icing			
	Storage	–25°C to 85°C (–13°F to 185°F) with no icing			
Ambient humidity	Operating	35% to 85% (with no condensation)			
Protective circuit		Protection from load short-circuiting and power supply reverse polarity wiring			
Degree of protection		IEC IP64 (JEM IP64f drip-proof) (See Note.)			

Note: The applicable JEM standard is JEM1030 1991.

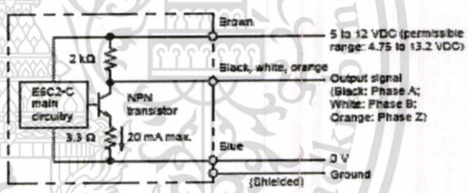
Operation

■ OUTPUT CIRCUIT DIAGRAM

E6C2-CWZ6C



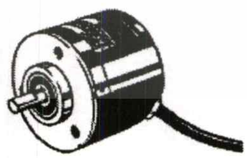
E6C2-CWZ3E

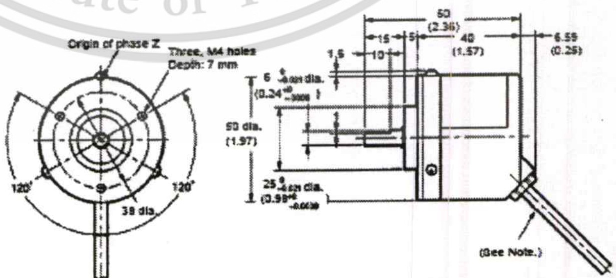


Dimensions

Unit: mm (inch)

■ E6C2-C





Note: 2-m-long, PVC code, 5-dia. (18/0.12 dia.) five conductors and shield (eight conductors for line driver use)

Installation

CONNECTION

E6C2-CWZ6C/-CWZ3E

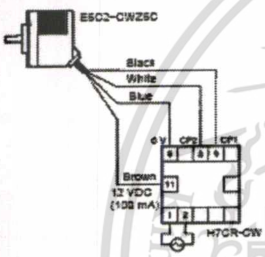
Color	Terminal
Brown	Power supply (+V_{CC})
Black	Output phase A
White	Output phase B
Orange	Output phase Z
Blue	0 V (common)

Note: Receiver: AM26LS32

Connection Examples

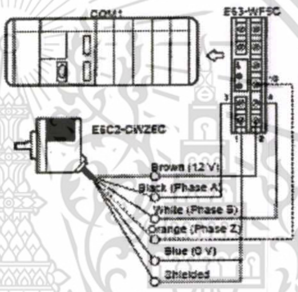
H7CR-CW DIGITAL COUNTER

Applicable Model: E6C2-CWZ6C



CQM1 PROGRAMMABLE CONTROLLER

Applicable Model: E6C2-CWZ6C



NOTE: DIMENSIONS SHOWN ARE IN MILLIMETERS. To convert millimeters to inches divide by 25.4.

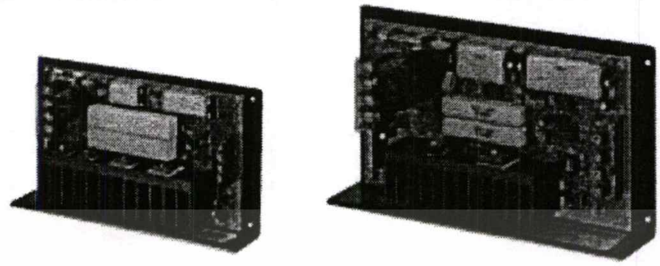
OMRON®
OMRON ELECTRONICS, INC.
One East Commerce Drive
Schaumburg, IL 60173
1-800-55-OMRON

OMRON CANADA, INC.
885 Milner Avenue
Scarborough, Ontario M1B 5V8
416-286-6465

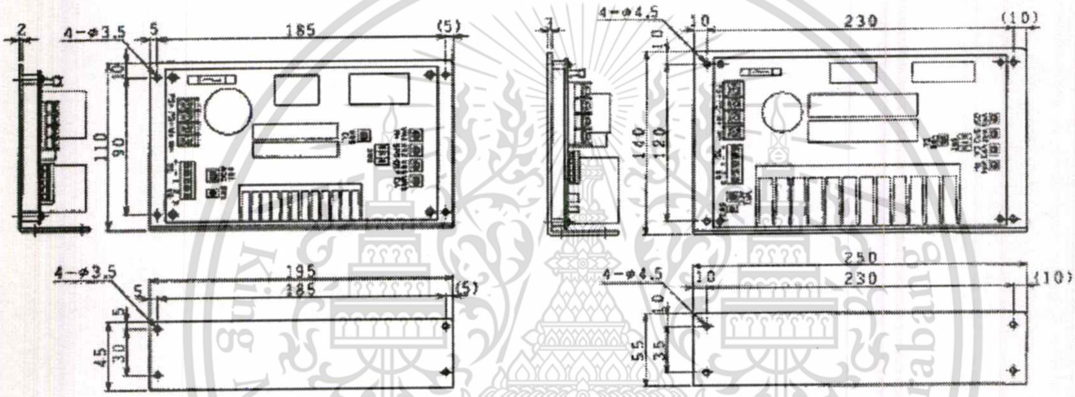
Cat. No. CEDSAX3 10/99 Specifications subject to change without notice. Printed in U.S.A.

A.3 Specification of motor driver

DC モータドライバ DC MOTOR DRIVER MS-400 SERIES



◆寸法図
DIMENSIONS



MS-400□□□04
MS-400□□□08
質量 0.5kg
Mass

MS-400□□□15
MS-400□□□30
質量 1.2kg
Mass

◆ 仕様 SPECIFICATION

形式名	MODEL	MS-400T2430	MS-400T4804	MS-400T4808	MS-400T4815	MS-400C1204	MS-400C1208	MS-400C1215
適用モータ	MATCHING MOTOR	SS60E8-T0	SS40E2-T0 ~SS60E3-T0	SS60E6-T0	SS60E8-T0	SS32G SS40E2	SS40E4 SS40E6	SS40E8 SS60E3
モータ出力	MOTOR OUTPUT	350w	20W~120W	250W	350W	14W,20W	40W,60W	80W,120W
主回路	MAIN CIRCUIT	FET PWM 制御（非可逆） FET PWM control (Non-reversible)						
電源電圧	SUPPLY VOLTAGE	DC 24V±20%	DC 48V±20%			DC 12V±20%		
出力電圧	OUTPUT VOLTAGE	DC 0~23V	DC 0~47V(電源48V Power Supply 48V)			DC 0~11V(電源12V Power Supply 12V)		
定格電流	RATED CURRENT	DC 30A	DC 4A	DC 8A	DC15A	DC 4A	DC 8A	DC 15A
指令電圧	COMMAND VOLTAGE	DC 0~10V						
指令入力抵抗	COMMAND INPUT IMPEDANCE	100kΩ						
速度帰還	SPEED FEEDBACK	タコジェネレータ 3~7V/krpm TACHO-GENERATOR				モータ逆起電圧 Counter Electromotive Force		
調整機能	ADJUSTMENT	スタッティックゲイン、位相補償、ソフトスタート、オフセット、スピード、電流制限 STATIC GAIN, PHASE COMPEN, SOFT START, OFFSET, SPEED, CURRENT LIMIT						
保護機能	PROTECTION	過電流 OVER CURRENT, 電圧低下 LOW VOLTAGE						
使用温度範囲	OPERATING TEMPERATURE	−10~40℃						

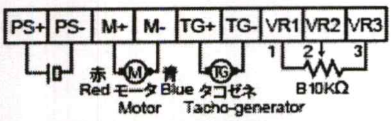
澤村電気工業株式会社 〒213-0002 神奈川県川崎市高津区二子 6-12-10 TEL (044)811-9331 FAX(044)833-9260
URL http://www.sawamura.co.jp/

DC モータドライバ DC MOTOR DRIVER MS-400 SERIES

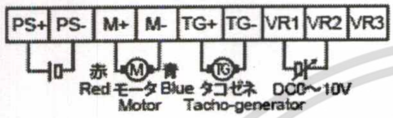
◆ 接続図　CONNECTION DIAGRAM

1.タコジェネレータフィードバック制御
Tacho-genetor Feedback Control

(1) 付属の可変抵抗器による速度調節
Speed Adjustment with Attached Variable Resistor
MS-400端子板　Terminal Board

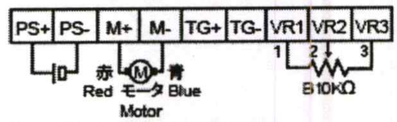


(2) 外部指令電圧による速度調節
Speed Adjustment with External Command Voltage
MS-400端子板　Terminal Board

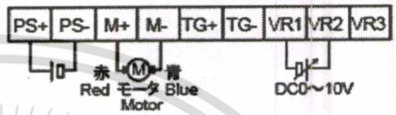


2.逆起電圧フィードバック制御
Counter Electromotive Force Feedback Control

(1) 付属の可変抵抗器による速度調節
Speed Adjustment with Attached Variable Resistor
MS-400端子板　Terminal Board



(2) 外部指令電圧による速度調節
Speed Adjustment with External Command Voltage
MS-400端子板　Terminal Board



沢村電気工業株式会社　〒213-0002 神奈川県川崎市高津区二子 6-12-10　TEL (044)811-9331　FAX(044)833-9260
URL http://www.sawamura.co.jp/

Appendix B

Experimental programming code

File Ex_all.c

```
#include <stdio.h>
#include <conio.h>
#include <math.h>
#include <bios.h>
#include <dos.h>

#define CH      0
#define CH1     1
#define DAADR   0x0300        /* DA ch.1 I/O adress */
#define da_deta 0.00244141    /* da_deta=10.0/4096.0 */
#define ADR1    0x280         /* Counter ch.1 address */
#define ADR2    0x281         /* Counter ch.2 address */
#define cls()   printf("\x1B[2J")
#define locate(x,y)     printf("\x1b[%d;%dH",y+1,x+1)
#define VCT     0x0b    /* Interrupt vector IR3 */
#define IMR     0x21
#define MASK    0x08    /* Interrupt mask IR3 */
#define NN      2700    /* Number of collected data */

        char            s[20];
        int             i,j,n,nn;
        unsigned int    dl,dh,vx1,vx2,dd1,dd2;
        double          cnt_read(unsigned int);
        double          T,x,xd,r,rd,x1[NN],x2[NN],u[NN],y[2],ee[2],volt,volt1;
        double          ref,ref0,ref1,Reref,sp,spm,Kp,Ki,ctrltype,DISTURB,RETARD;
        double          cntnew1,cntold1,cnt1,cnt10,cntnew2,cntold2,cnt2,cnt20,CL;
        FILE            *fp_w;
        int             inter=0;
```

```
void interrupt far insub(void)
{
        inter=1;
        outp(0x20,0x20);
}

void main(void)
{
        void (interrupt far *savevect)(void);

        /* Initialize Timer with Sampling Frequency = 8Mhz/dd1*dd2 */
        dd1=80;  dd2=100;
        outp(DAADR+14,0x36);
        outp(DAADR+8,dd1 & 0x00ff);
        outp(DAADR+8,(dd1 >> 8) & 0x00ff);
        outp(DAADR+14,0x74);
        outp(DAADR+10,dd2 & 0x00ff);
        outp(DAADR+10,(dd2 >> 8) & 0x00ff);
        outp(DAADR+6,0x01);

        /* Enble and Set Interrupt */
        savevect=_dos_getvect(VCT);
        _dos_setvect(VCT,insub);
        outp((IMR),inp(IMR) &~ MASK);

        /* Initial Variables */
        sp=dd1*dd2/8e6;
        spm=sp*1000;
        volt1=0.0;
        n=0.0;
        ctrltype=1.0;
        nn=NN;
```

```
y[1]=0.0;  y[2]=0.0;
ee[1]=0.0;  ee[2]=0.0;
volt=0.0;  volt1-0.0;
RETARD=0.0;  DISTURB=0.0;
ref0=0.0;  ref1=2.5;
r=0.0; rd=0.0; x=0.0; xd=0.0;

/* Initialize DA */
/*outp(DAADR+2,CH*4+0);
outp(DAADR+0,0x00);
outp(DAADR+1,0x00);
outp(DAADR+2,CH*4+2);
outp(DAADR+0,0x00);  */

printf("\n      Reference Speed  (maximum 2.5 (x1000rpm)) = ");  gets(s);  ref1=atof(s);
printf("\n      Time  (max 2700 msec ) = ");  gets(s);  nn=atof(s);
printf("\n      Which controller do you want? ( 1:PI   2:I-P ) = ");  gets(s);
ctrltype=atof(s);
printf("\n      Kp = ");  gets(s);  Kp=atof(s);
printf("\n      Ki  = ");  gets(s);  Ki=atof(s);
printf("\n      Time to Load Disturb (msec) = ");  gets(s);  DISTURB=atof(s);
printf("\n      Time to Speed Retard (max 2700 msec) = ");  gets(s);  RETARD=atof(s);
if(RETARD>100)
{
printf("\n      New Reference speed (max 2.5 (x1000rpm)) = "); gets(s);  Reref=atof(s);
}
printf("That's OK");

/* Initialize Counter */
outp(ADR1+4,0x8c);
outp(ADR2+4,0x8c);
cntold2=cnt_read(ADR1);
```

```
cntold1=cnt_read(ADR2);
/************************ MAIN ROUTINE ***********************/
while(n<nn)
{       if(kbhit() !=0) break;
        if(n<1000) ref=ref1;      /* Step time at 100th sampling */
        if(n>=7000) ref=ref0;
        if(n>=RETARD) ref=Reref;
        if(inter==1)
        {
                r=ref;
                cntnew2=cnt_read(ADR1);
                cnt2=cntnew2-cntold2;
                cntold2=cntnew2;
                cntnew1=cnt_read(ADR2);
                cnt1=cntnew1-cntold1;
                cntold1=cntnew1;

                y[1]=-0.015*cnt1;             /* unit in x1000 rpm */
                y[2]=0.015*cnt2;              /* unit in x1000 rpm */

                x1[n]=y[1];
                x2[n]=y[2];

                T=0.001; /*Sampling Time*/

                if (ctrltype==1)
                {
                x=(Ki*T*r+Kp*xd)/(Kp+Ki*T);  /*Pre-filter = Ki/Kps+Ki*/
                ee[2]=(x-y[1]);
                volt=Kp*(ee[2]-ee[1])+Ki*T*ee[2]+volt1; /* PI Controller */
                }
```

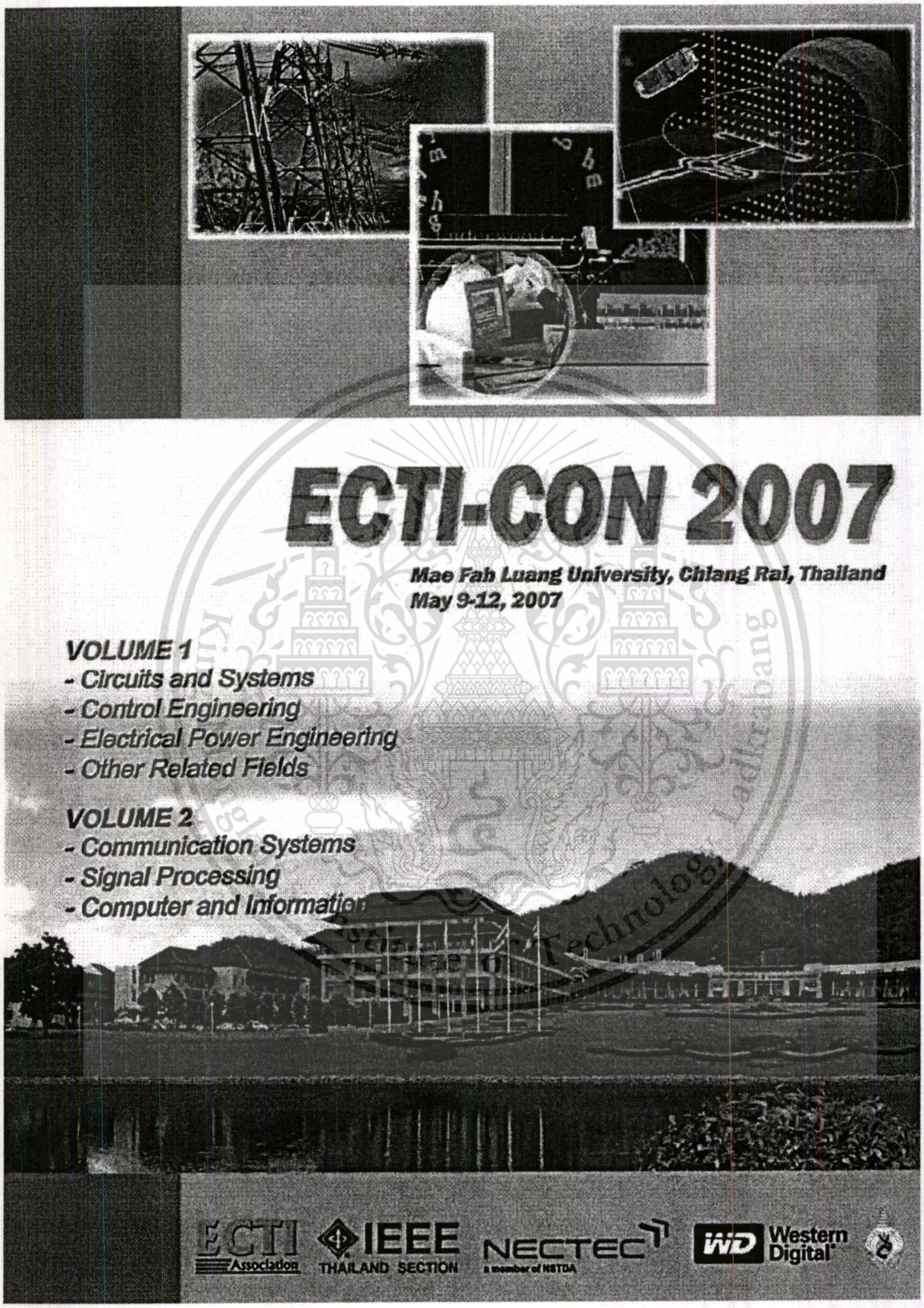
```
if (ctrltype==2)
{
x=r;                              /*Pre-filter = 1 */
ee[2]=(x-y[1]);
volt=Ki*T*ee[2]-Kp*(x1[n]-x1[n-1])+volt1; /* I-P Controller */
}

// for load disturbance
if (n==DISTURB)
 {
  outp(DAADR+2,CH1*4+1);
  outp(DAADR+0,0x1F);
  outp(DAADR+1,0x1F);
  outp(DAADR+2,CH1*4+3);
  outp(DAADR+0,0x1F);
 }

if(volt>10) volt=10;
if(volt<0) volt=0;
u[n]=volt;
vx1=volt1/da_deta;
dl=vx1&0xff;
dh=(vx1>>8)&0x0f;
outp(DAADR+2,CH*4+1);
outp(DAADR+0,dl);
outp(DAADR+1,dh);
outp(DAADR+2,CH*4+3);
outp(DAADR+0,0x00);

ee[1]=ee[2];
volt1=volt;
```

```
                xd=x;
                rd=r;
                inter=0;
                n++;
            }
    }

    /* DISABLE AND RESET INTERRUPT */
    outp((IMR),inp(IMR) | MASK);
    _dos_setvect(VCT,savevect);
    outp(DAADR+6,0x00);

    /*  OUT 0 V. TO DA  CH.1*/
    outp(DAADR+2,CH*4+1);
    outp(DAADR+0,0x00);
    outp(DAADR+1,0x00);
    outp(DAADR+2,CH*4+3);
    outp(DAADR+0,0x00);
    outp(DAADR+2,CH1*4+1);
    outp(DAADR+0,0x00);
    outp(DAADR+1,0x00);
    outp(DAADR+2,CH1*4+3);
    outp(DAADR+0,0x00);

    /* WRITE DATA TO FILE */
    fp_w=fopen("ex_all.dat","w");
    n=0;
    while(n<nn){     fprintf(fp_w,"%f %f\n",x1[n],u[n]);
    n++;     }
    fclose(fp_w);
}
```

```
double cnt_read(unsigned int adr)
{
        double cnt;
        unsigned char lbyt, mbyt, hbyt;
                outp(adr + 2, 0x10);
                while((inp(adr + 2) & 0x08) ==0) ;
                lbyt = inp(adr + 0) ;
                mbyt = inp(adr + 0) ;
                hbyt = inp(adr + 0) ;
                cnt = (double)hbyt*65536.0+(double)mbyt*256.0+(double)lbyt ;
                return(cnt);
}
```

Appendix C

Related Publication


ECTI-CON 2007
Mae Fah Luang University, Chiang Rai, Thailand
May 9-12, 2007
VOLUME 1
- Circuits and Systems
- Control Engineering
- Electrical Power Engineering
- Other Related Fields
VOLUME 2
- Communication Systems
- Signal Processing
- Computer and Information
ECTI Association
IEEE
THAILAND SECTION
NECTEC
a member of NSTDA
WD Western Digital

Simplified PI Controller for Two-Inertia System

Peerapon Pattanavijit[1], Songmoung Nundrakwang[1], Taworn Benjanarasuth[1],
Jongkol Ngamwiwit[1] and Noriyuki Komine[2]
[1] Faculty of Engineering and ReCCIT, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand
(Tel: +662-739-2405, E-mail: knjongko@kmitl.ac.th)
[2] School of Information Science and Technology, Tokai University, Kanagawa, Japan
(Tel: +81-463-58-1211, E-mail: komine@keyaki.cc.u-tokai.ac.jp)

***Abstract*- In this paper, a simplified designing method of a PI controller for a higher-order plant by using the concept of plant model reduction and the Coefficient Diagram Method with the less number of stability index γ_i is presented. By ignoring the non-dominant poles of the higher-order plant model, a lower-order plant model is first obtained. Then, the PI controller will be designed for controlling the lower-order plant model by using Coefficient Diagram Method with the appropriate values of stability index in order to achieve the system response without overshoot. Finally, the designed PI controller will be employed to control the original higher-order plant. The proposed controller is implemented to control a two-inertia system. The experimental results show that the PI controller designed from the lower-order plant model is sufficient for controlling the original higher-order plant with acceptable performance.**

I. Introduction

Since several decades ago, most of industrial processes have employed PID-family controllers due to their simplicity and sufficiency in process control applications. It is reported lately that more than 90% of the industrial controllers used nowadays are PI controllers. However, the parameters of the controller have to be tuned for acceptable responses. The Ziegler-Nichols' tuning formula [1] is the most popular method which is designed to have quarterly decayed overshoot characteristic in the step response but the fine tuning is needed for the practical use to reduce the overshoot in the closed-loop response.

In 1998, a new designing method called Coefficient Diagram Method (CDM) [2] has been proposed. The CDM is an algebraic design algorithm utilizing polynomial form structure. In CDM, the closed-loop characteristic polynomial is designed based on stability index and equivalent time constant, which are used to determine stability and speed of the closed-loop response. Hence, the unknown parameters of the controllers resulting for over-damped response can be obtained accordingly [3-4]. However, it is quite time consuming in designing the parameters of a controller especially for a higher-order plant.

In this paper, the simplified design of a PI controller for higher-order plant using CDM with less number of stability indexes γ_i is presented. By ignoring the non-dominant pole, the higher-order plant model is reduced to be a lower-order plant model. After that, the CDM is used to design the PI controller for controlling the lower-order plant model. Finally, the PI controller designed for the lower-order plant model is employed for controlling the original higher-order plant. In order to check the control capability of this designing method, the simplified PI controller is implemented to control the speed of the two-inertia system.

II. CDM and Proposed Control System

The concept of CDM and the rearranged CDM control system structure are described respectively in this section.

A. CDM Concept

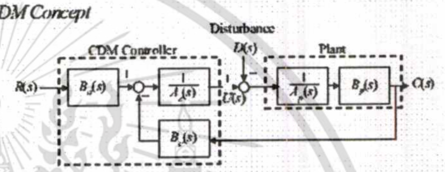


Figure 1. CDM control system structure.

The transfer function of the plant in the polynomial form for each block is

$$A_p(s) = p_k s^k + p_{k-1} s^{k-1} + \ldots + p_0, \tag{1a}$$

$$B_p(s) = q_m s^m + q_{m-1} s^{m-1} + \ldots + q_0, \tag{1b}$$

and the controller polynomials are

$$A_c(s) = l_\lambda s^\lambda + l_{\lambda-1} s^{\lambda-1} + \ldots + l_0, \tag{2a}$$

$$B_c(s) = k_\lambda s^\lambda + k_{\lambda-1} s^{\lambda-1} + \ldots + k_0, \tag{2b}$$

$$B_a(s) = k_0, \tag{2c}$$

where $\lambda < k$ and $m < k$. Hence, the characteristic polynomial of the closed-loop system shown in Fig. 1 can be given as

$$P(s) = A_c(s)A_p(s) + B_c(s)B_p(s) = \sum_{i=0}^{n} a_i s^i. \tag{3}$$

Note that it still has unknown controller's parameters to be solved. Based on CDM concept, the performance specification known as stability index γ_i and the equivalent time constant τ are defined by

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}}, \tag{4}$$

$$\tau = \frac{a_1}{a_0}, \tag{5}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}} \quad ; \quad \gamma_0, \gamma_n = \infty, \tag{6}$$

where $i = 1,\dots, n-1$. The equivalent time constant τ is generally selected according to the specified settling time t_s as $t_s = 2.5\tau$ and the standard stability index is recommended to be

$$\gamma_{n-1} = \dots = \gamma_3 = \gamma_2 = 2, \gamma_1 = 2.5. \quad (7)$$

From (4) to (6), the coefficient a_i can be written by

$$a_i = a_0\tau^i \frac{1}{\gamma_{i-1}\cdots\gamma_2^{i-2}\gamma_1^{i-1}} = a_0\tau^i\prod_{j=1}^{i-1}\frac{1}{(\gamma_{i-j})^j}. \quad (8)$$

Consequently, the characteristic polynomial to be employed to design the parameters of a controller is

$$P(s) = a_0\left[\left\{\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1} 1\Big/\gamma_{i-j}^{j}\right)(\tau s)^i\right\}+\tau s+1\right]. \quad (9)$$

Hence, the parameters of the controller can be obtained by equating the characteristic polynomial in (3) to (9) with the known stability index γ_i and/or the equivalent time constant τ when the mathematical model of the plant is known.

B. Control Systems Structure

For simplicity in design, the original CDM control system structure in Fig. 1 is rearranged to be a rearranged CDM control system structure as shown in Fig. 2.

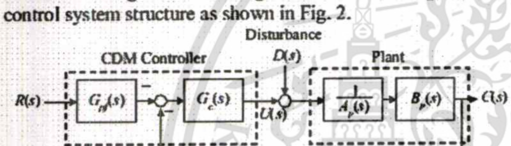


Figure 2. Rearranged CDM control system structure.

The transfer function of controller $G_c(s)$ and the pre-filter $G_{pf}(s)$ of the rearranged control system shown in Fig. 2 are

$$G_c(s) = \frac{B_c(s)}{A_c(s)} \quad (10)$$

and

$$G_{pf}(s) = \frac{B_a(s)}{B_c(s)}. \quad (11)$$

III. Two-Inertia System

The structure of two-inertia system [5] and its block diagram are shown in Fig. 3 and Fig. 4.

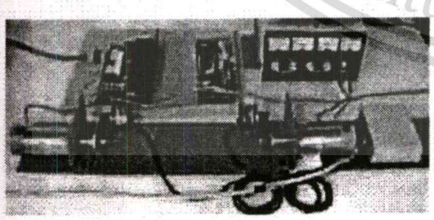


Figure 3. Two-inertia system.

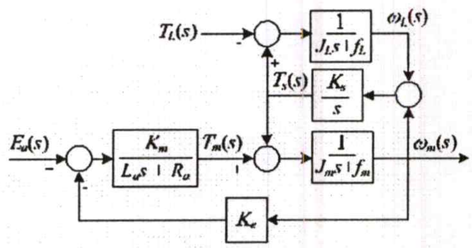


Figure 4. Block diagram of the two-inertia system.

By ignoring the viscous friction of the motor f_m and f_L, the transfer function of the two-inertia system will be

$$G_p(s) = \frac{\omega_m(s)}{E_a(s)} = \frac{a_2 s^2 + a_0}{b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \quad (12)$$

where $b_4 = 1, b_3 = \frac{R_a}{L_a}, b_2 = \frac{K_s L_a(J_m + J_L) + K_m K_e J_L}{J_m J_L L_a}$,

$$b_1 = \frac{K_s R_a(J_m - J_L)}{J_m J_L L_a}, b_0 = \frac{K_m K_e K_s}{J_m J_L L_a}, a_2 = \frac{K_m}{J_m L_a}, a_0 = \frac{K_m K_s}{J_m J_L L_a}$$

and its parameters are shown in the Table I.

TABLE I
PARAMETER VALUES OF TWO-INERTIA SYSTEM

R_a	Armature resistance	0.25 Ω
L_a	Armature inductance	0.10 mH
K_e	Back electromotive force constant	3.85 $v/krpm$
K_m	Motor torque constant	3.2 $N-cm/A$
J_m	Moment of motor inertia	0.8 $kg-cm^2$
J_L	Moment of load inertia	0.9 $kg-cm^2$
K_s	Torsional stiffness of drive shaft	15 $N-m/rad$

IV. Controller Design

In this section, plant model order reduction, PI controller structure and the controller designing procedure will be described respectively.

A. Plant Model Order Reduction

Consider the k-order plant with the transfer function given as

$$G_p(s) = \frac{B_p(s)}{A_p(s)} = \frac{K\prod_{j=1}^{m}(s+z_j)}{\prod_{i=1}^{k}(s+p_i)}; \; k \geq m. \quad (13)$$

Without loss of generality, assuming that the plant contains the non-dominant poles $-p_i$ for $i = 1, 2, \dots, l$. By ignoring the non-dominant poles, the n-order plant model can then be reduced to a lower-order plant model and its transfer function is

$$\tilde{G}_p(s) = \frac{\tilde{K}\prod_{j=1}^{m}(s+z_j)}{\prod_{i=l+1}^{k}(s+p_i)};\ \tilde{K} = \frac{K}{\prod_{i=1}^{l} p_i};\ k-l \ge m. \quad (14)$$

B. PI Controller Structure

The controller (11) in this paper is the conventional PI controller and its transfer function is

$$G_c(s) = K_p\left(1+\frac{1}{T_i s}\right), \quad (15)$$

where K_p and T_i are the proportional gain and the integral time, respectively. The other form of (15) can also be rewritten as

$$G_c(s) = \frac{B_c(s)}{A_c(s)} = \frac{K_p s - K_i}{s}, \quad (16)$$

where the gain K_p and K_i of (16) can be assigned from the controller design procedure stated in the next sub-section.

C. Controller Design Procedure

The designing steps for the PI controller are as follows:

(1) Find the closed-loop characteristic polynomial of the control system with the lower-order plant model (14) and the PI controller (16).

(2) Choosing the appropriate values of the stability index γ_i and/or the equivalent time constant τ to find the characteristic polynomial such that the over-damped response of the closed-loop system with lower-order plant model including $G_{ff}(s)$ is obtained, where $B_a(s)$ is the coefficient of s^0 of $B_c(s)$.

(3) Find the parameters of the controller by equating the characteristic polynomial obtained in step (2) with the characteristic polynomial obtained in step (1).

(4) Applying the PI controller obtained in step (3) to control the original plant.

V. Experimental Results

The experimental results of the speed control of the two-inertia system using the PI controller obtained from simplified design based on CDM approach will be shown in this section. From the transfer function of the two-inertia system in (12) and the values in Table I, the transfer function from input voltage to the motor speed is given by

$$G_p(s) = \frac{3\times10^4 s^2 + 6.67\times10^7}{s^4 + 2.5\times10^3 s^3 + 1.57\times10^5 s^2 + 8.85\times10^6 s + 2.57\times10^8}.$$

It is the fourth-order system and its open-loop poles are located at s = -39.6, -2436.8, -11.8+j50.2, -11.8-j50.2. The open-loop response of the two-inertia system when applying the input voltage at 5 volts is shown in Fig. 5. It is seen that there is an unsatisfied torsional resonance and must be reduced as much as possible by using a controller.

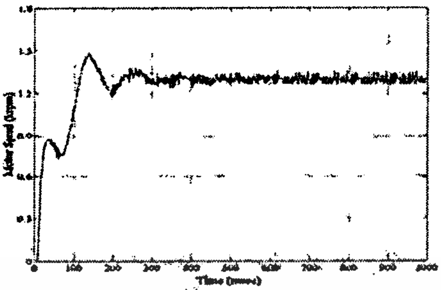


Figure 5. Open-loop step response of two-inertia system.

Since the poles at s = -39.6, -2436.8 are the non-dominant poles, the system is reduced to be a second-order model by ignoring them. Thus, its transfer function can be expressed as

$$\tilde{G}_p(s) = \frac{3\times10^4 s^2 + 6.67\times10^7}{9.65\times10^4 s^2 + 2.28\times10^6 s + 2.57\times10^8}.$$

Based on the controller design procedure stated previously, the parameters K_p and K_i of the PI controller including the equivalent time constant τ of the closed-loop system obtained by using the lower-order plant model with $\gamma_1 = 7$ and $\gamma_2 = 0.5$ are shown in the Table II. Note that the gamma values are chosen to give the positive values of K_p and K_i of the PI controller when the simplified design method is employed. For comparison, the parameters of PI controller and the equivalent time constant τ obtained from the original plant model with $\gamma_1 = 2.5$, $\gamma_2 = 2$, $\gamma_3 = 1.34$ and $\gamma_4 = 32.4$ can also be shown in the Table II.

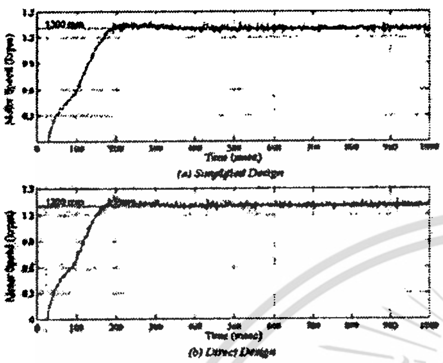
TABLE II
PARAMETERS OF PI CONTROLLERS

Plant Type	K_p	K_i	τ (msec)
Original model	0.87	54.08	87
Reduced model	0.41	43.15	88

The PI controller designed from the lower-order plant model is used to control the speed of the two-inertia system at the speed of 1300 rpm. The proposed control system obtained from simplified design method will be compared to the other one using the PI controller obtained by direct design in order to show the effectiveness of the proposed design method. The results will be shown in two aspects. The first aspect is the system performance and the second aspect is the capability of the simplified design PI controller.

A. System Performances

The experimental results in controlling the motor speed of the two-inertia system are shown in Fig. 6. It is seen that the proposed PI controller can control the speed at 1300 rpm without torsional resonance, overshoot and steady-state error. It is also seen that the system response speed quite similar to the response of the system using PI controller obtained from direct design. This can be seen from the value of equivalent time constant τ obtained from the simplified design shown in the Table II that its value is close to the one obtained from the direct design. The system performances of both control systems are also summarized in Table III.



(a) Simplified Design

(b) Direct Design

Figure 6. Step responses of control systems at 1300 rpm.

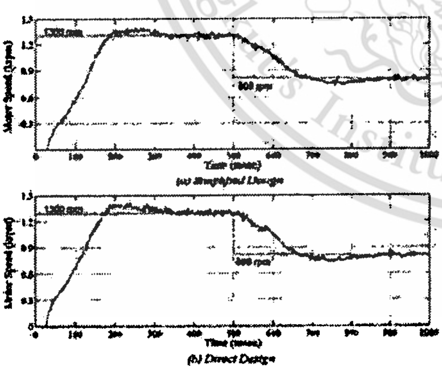
TABLE III

SYSTEM PERFORMANCE AT 1250 RPM.

Design Method	t_r (msec)	t_s (msec)	P_o (%)
Direct	151	292	3.8
Simplified	176	302	2.3

B. Capability of the Simplified Design PI Controller

The capability of the PI controller designed from the lower-order plant model is investigated by retarding the motor reference speed of the control system at 500 msec from 1300 rpm to 800 rpm. Its responses is shown in Fig. 7. From the responses, the motor speed of system using proposed controller can reach to 800 rpm properly and similarly as the control system using the PI controller obtained from direct design.

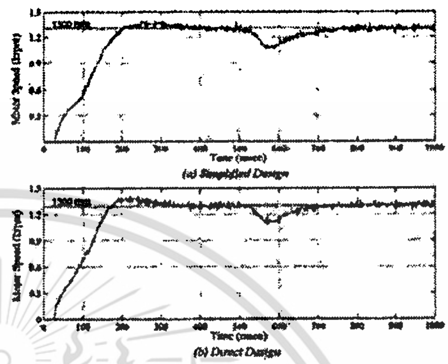


(a) Simplified Design

(b) Direct Design

Figure 7. Responses due to speed retard to 800 rpm.

In order to compare the disturbance rejection capability of the simplified design PI controller and the direct design PI controller, the motor reference speed at 1300 rpm and the constant load disturbance entering at 500 msec are considered. It is seen from the Fig. 8 that the proposed control system can reject the effect of the load disturbance and reach to the speed at 1300 rpm without error similarly.



(a) Simplified Design

(b) Direct Design

Figure 8. Responses with constant load disturbance.

V. CONCLUSIONS

The simplified design of a PI controller for a higher-order plant by using the CDM concept has been proposed in this paper. The proposed controller is implemented to control the speed of the two-inertia system. Not only the simplicity in designing but the speed of the response can still be improved to meet the desired system performance as well. In addition, the unsatisfied torsional resonance can also be reduced. The effectiveness of this designing method has been confirmed by the experimental results.

REFERENCES

[1] J. G Ziegler, and N. B. Nichols, "Optimum Settings for Automatic Controllers," *ASME Trans.* 64, pp.759-68, 1942.

[2] S. Manabe, "Coefficient Diagram Method," *Proc. of the 14th IFAC Symposium on Automatic Control in Aerospace*, pp.199-210, 1998.

[3] D. Kumpanya, T. Benjanarasuth, J. Ngamwiwit and N. Komine, "PI Controller Design with Feed forward by CDM for Level Processes," *Proc. of the 2000 IEEE TENCON*, pp. II-65 – II-69, 2000.

[4] S. E. Hamamci, M. Koksal and S. Manabe, "Robust Position Control of a Radar Antenna with the Coefficient Diagram Method," *Proc. of the 4th Asian Control Conference*, pp.1785-1790, 2002.

[5] D. Kumpanya, "Feedforward Controller Design Incorporating PI Controller Designed by Coefficient Diagram Method" Master Thesis, King Mongkut's Institute of Technology Ladkrabang, Bangkok, 2002.

Author Biography

The author, Peerapon Pattanavijit was born on September 9, 1983 in Bangkok, Thailand. He received his B.Eng degree in Control Engineering from King Mongkut's Institute of Technology Ladkrabang (KMITL) in the year 2004. In consequent year, he enrolled in the Control Engineering Department, KMITL, as a graduate student and become a member of the Control and Mechatronics Laboratory, Research Center for Communications and Information Technology (ReCCIT) to work toward his M.Eng degree. In 2006, he had a chance to participate the "15th International Micro Robot Maze Contest" at Nagoya, Japan and gained the winner awards in "Micro Robot Racing" category.