NUMERICAL METHODS FOR SOLVING SYSTEMS OF LINEAR EQUATIONS

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บทคัดย่อ

เนื้อหาของการทำวิทยานิพนธ์นี้ จะกล่าวถึงการนำเอาวิธีการทำซ้ำของการหาผลเฉลยของระบบสมการไม่เชิงเส้น มาใช้หาผลเฉลยเชิงตัวเลขของระบบสมการเชิงเส้น

$$Ax = b$$

ซึ่ง

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & . & a_{2n} \\ . & & . & & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ . & & & & & & . \\ a_{n1} & . & . & . & . & . & a_{nn} \end{bmatrix}$$

เมื่อ A เป็น nxn เมตริกซ์ และ $x \in R^n$

การหาผลเฉลยของระบบสมการเชิงเส้นที่สะดวกที่สุดได้แก่การใช้เมตริกซ์ผกผันของเมตริกซ์ A นั่นคือ

$$x = A^{-1}b$$

สำหรับในที่นี้จะเป็นการหารูปแบบการทำซ้ำในรูปของ

$$x^{m+1} = x^m - (M(x^m))^{-1} f(x^m)$$

โดยที่

$$f(x)=Ax-b$$

และ M เป็น nxn เมตริกซ์ที่หาอินเวอร์สได้ ดังนั้นสิ่งที่สำคัญในงานวิจัยนี้คือการหาเมตริกซ์ M

จากการวิจัยพบว่า วิธีการเชิงตัวเลขเพื่อหาผลเฉลยเชิงตัวเลขของระบบสมการเชิงเส้นโดยการปรับปรุงวิธีการของ Newton- like สำหรับหาผลเฉลยของระบบสมการไม่เชิงเส้น เมตริกซ์ M ที่ได้จะเป็นเมตริกซ์ $D,U_2,U_3,...,U_n$ และ $L_2,L_3,...,L_n$ จากตัวอย่างที่แสดงผลเฉลย ทำให้ทราบว่าวิธีการ D, $U_2,U_3,...,U_n$ และ $L_2,L_3,...,L_n$ เป็นการหาผลเฉลยของระบบสมการเชิงเส้นที่ใช้ได้ดี

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ABSTRACT

The main purpose of this thesis is to use iterative methods for solving system of nonlinear equations in solving systems of linear equations

$$Ax = b$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & . & a_{2n} \\ . & & . & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & . & . & . \\ a_{n1} & . & . & . & . & . & a_{nn} \end{bmatrix}$$

when A is nxn matrix and $x \in R^n$.

The most convenient method for finding the solution of such system is by using the inverse matrix A that is

$$x = A^{-1}b.$$

In this thesis many iterative methods for solving systems of linear equations are introduced. These methods are the modification of the Newton – like method for solving

systems of nonlinear equations. The matrix M are the matrices $D, U_2, U_3, \ldots, U_n$ and $L_2, L_3, \ldots, L_n$, the examples are illustrated in chapter 4. We find that the D- method, $U_2, U_3, \ldots, U_n$ and $L_2, L_3, \ldots, L_n$ give the reasonable good results. The main task of this thesis is to find matrix M and use it in the iterative algorithm

$$x^{m+1} = x^m - (M(x^m))^{-1} f(x^m)$$

where

$$f(x) = Ax - b$$

and M is nxn invertible matrix.

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Supin marach

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CHAPTER 1

INTRODUCTION

1.1 Statement and Significance of the Problems

Linear systems are among the most important and common problems encountered in engineering, science, economic and business. From the theoretical point of view, the problem is rather easy and explicit solutions. It is well understood when solutions exits, when it does not, and when there are infinitely many solutions. The numerical viewpoint is far more complex. Approximations may be available, but it may be difficult to estimate how accurate they are. This clearly will depend on the data at hand, i.e., primarily on the coefficient matrix. We begin by examining a systematic approach to solving a system of linear equations of the from

$$Ax = b \tag{1.1}$$

or

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n1}x_2 + a_{1n}x_3 + \dots + a_{nn}x_n &= b_n . \end{aligned} \tag{1.2}$$

Where A is nxn invertible real matrix, x is a real unknown vector and b is a given known vector.

Sadd in [1] found when we solve the linear system (1.1), we distinguish three situations.

Case 1 The matrix A is nonsingular. There is unique solution given by $x = A^{-1}b$.

Case 2 The matrix A is singular and $b \in range(A)$. Since be $b \in range(A)$ then there is an x_0 such that $Ax_0 = b$. Then $x + v$ is also solutions for any v in $ker(A)$.

there is an x_0 such that $Ax_0 = b$. Then $x+v$ is also solutions for any v in *ker(A)* . Sine *ker(A)* is at least one – dimensional, there are infinitely many solutions.

Case 3 The matrix A is singular and $b \notin range(A)$. There are no solutions. D.Hoffman' [2] fond there are two fundamentally different approaches for solving systems of linear algebraic equations;

1. Direct methods,
2. Iterative methods.

Direct methods are systematic procedures, based on algebraic elimination, that obtain the solution in a fixed number of operations. Iterative methods, on the other hand; obtain the solution asymptotically by an iterative procedure. A trial solution is assumed, the trail solution is substituted in to the system of equations to determine the mismatch in the trial solution. An improved solution is obtained from the mismatch data. There are many numerical methods for solving systems of linear equations, for example:

Jacobi Iterative Method we use solving system of linear equations is of the form

$$x^{m+1} = \frac{b_i - \sum_{j \neq i} a_{ij} x_j^m}{a_{ii}} \quad ,1 \le i \le n, m \ge 0. \tag{1.3}$$

Gauss- Seidel Iterative Method we use solving system of linear equations is of the form

$$x^{m+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{m+1} - \sum_{j=i+1}^{n} a_{ij} x_j^m}{a_{ii}} \quad ,1 \le i \le n, m \ge 0. \tag{1.4}$$

The numerical methods for solving the systems of nonlinear equations by the help of the Newton-like methods, Rattanametawee [3] proof some theorem to use modified the Newton-like methods which introduced by Podisuk [4] and Podisuk [5]. Then

Kongchouy [6] find more modified the Newton- like methods for solving system of nonlinear equations, that they can use solving the systems of linear equations.

1.2 Goal and Objective

In this thesis, we will find the numerical methods for solving the systems of linear equations by we use iterative methods for solving systems of nonlinear equations. We will treat our system (1.1) as a system of nonlinear equations, that is

$$(A - Ib)x = 0 \tag{1.5}$$

or

$$f(x) = (A - Ib)x = 0\,. \tag{1.6}$$

Then we have the following iterative equation;

$$x^{m+1} = x^m - (M(x^m))^{-1}(Ax^m - b), m = 0,1,2,... \tag{1.7}$$

thus the objective of this thesis is to find a matrix M.

1.3 Hypothesis to be Tested

The matrix M are the matrices D or U_2 or $U_3,\ldots,U_n$ or L_2 or $L_3,\ldots L_n$. We will call them that the D method, U_2- method, U_3-method,…, U_n-method, L_2-method, L_3-method,…, L_n-method respectively, and the U_2- method, U_3-method,…, U_n-method, L_2-method, L_3-method,…, L_n-method can be used for finding solutions of all problems in the form system of linear equations that give a good iterative method.

1.4 Limitation of the Study

We find the good iterative methods that will solve the systems of linear equations in the from of the system (1.1), which we use the iterative equations (1.7) for solving systems of linear equations in many problem, and the matrices M must to be nxn invertible real matrix.

1.5 Definition

The matrix M are the matrices $D, U_2, U_3, \ldots, U_n, L_2, L_3, \ldots L_n$, where

$$D = \begin{bmatrix} a_{11} & 0 & . & . & . & 0 \\ 0 & a_{22} & & & & . \\ . & & . & & & . \\ . & & & . & & . \\ . & & & & . & 0 \\ 0 & . & . & . & 0 & a_{nn} \end{bmatrix}$$

$$U_2 = \begin{bmatrix} a_{11} & a_{12} & 0 & . & . & 0 \\ 0 & a_{22} & a_{23} & & & . \\ . & & . & . & & . \\ . & & & . & . & . \\ . & & & & a_{n-1n-1} & a_{n-1n} \\ 0 & . & . & . & 0 & a_{nn} \end{bmatrix}$$

$$U_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & . & 0 \\ 0 & a_{22} & a_{23} & a_{24} & & . \\ . & & . & . & & . \\ . & & & . & . & a_{n-2n} \\ . & & & & a_{n-1n-1} & a_{n-1n} \\ 0 & . & . & . & 0 & a_{nn} \end{bmatrix}$$

$$\vdots$$

$$U_n = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ 0 & a_{22} & a_{23} & & . & . \\ . & & . & . & & . \\ . & & & . & . & . \\ . & & & & a_{n-1n-1} & a_{n-1n} \\ 0 & . & . & . & 0 & a_{nn} \end{bmatrix}$$

$$L_2 = \begin{bmatrix} a_{11} & 0 & . & . & . & 0 \\ a_{21} & a_{22} & . & & & . \\ 0 & . & . & . & & . \\ . & & . & . & . & . \\ . & & & . & a_{n-1n-1} & 0 \\ 0 & . & . & 0 & a_{nn-1} & a_{nn} \end{bmatrix}$$

$$L_3 = \begin{bmatrix} a_{11} & 0 & . & . & . & 0 \\ a_{21} & a_{22} & . & & & . \\ a_{31} & . & . & . & & . \\ . & & . & . & . & . \\ . & & & . & a_{n-1n-1} & 0 \\ 0 & . & . & a_{nn-2} & a_{nn-1} & a_{nn} \end{bmatrix}$$

$$\vdots$$

$$L_n = \begin{bmatrix} a_{11} & 0 & . & . & . & 0 \\ a_{21} & a_{22} & . & & & . \\ . & & . & . & & . \\ . & & & . & . & . \\ . & & & & a_{n-1n-1} & 0 \\ a_{n1} & . & . & . & a_{nn-1} & a_{nn} \end{bmatrix}$$

CHAPTER 2

LITERATURE REVIEW

In this chapter, we shall give definitions and theorems concern with the modified Newton method for solving systems of linear equation, which will be used in the later chapter.

Definition 2.1 For $n \geq 2$,an nxn matrix A is reducible if there exist an nxn Permutation matrix P such that

$$PAP^T = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

where A_{11} is an $r \; x \; r$ sub matrix and A_{22} is an $(n-r)x\,(n-r)$ sub matrix for $1 \leq r \leq n$.

Definition 2.2 An nxn matrix A is irreducible matrix. If an matrix A is not reducible matrix.

Definition 2.3 An nxn matrix A is irreducible diagonally dominant matrix. If an matrix A is irreducible matrix and diagonally dominant matrix.

Definition 2.4 An nxn matrix A is diagonally dominant matrix. If

$$|a_{ii}| \geq \sum_{\substack{i=1 \\ j=i}}^{n} |a_{ij}| = \Lambda_i$$

for all $1 \leq i \leq n$.

Definition 2.5 An *nxn* matrix A is strictly diagonally dominant matrix. If

$$|a_{ii}| > \sum_{\substack{i=1 \\ j=i}}^{n} |a_{ij}| = \Lambda_i$$

for all $1 \le i \le n$.

Definition 2.6 An *nxn* matrix A is positive definite matrix. If $X^T AX > 0$ for any $x \neq 0$.

Definition 2.7 An *nxn* matrix A is Hermitian matrix. If $A^* = A$ when is complex conjugate transpose of .

Theorem 2.1 If is strictly diagonally dominant matrix or irreducible diagonally dominant matrix then Jacob Iterative method and Gauss Seidel Iterative method converge.

Theorem 2.2 If is Hermitial matrix where is Hermitian and positive definite and is nonsingular then Gauss Seidel Iterative method converge if and only if an matrix is positive definite.

Theorem 2.3 Let is Hermitian matrix. If is Successive over relaxation method converge to if and only if and is positive definite matrix.

We consider of multivariable problems by discusses the system of nonlinear equations in real variables. It is

$$f(x) = 0 \tag{2.1}$$

where X R^n is n-dimensional real vector space, and f is assumed to be continuously differentiable. The system (2.1) is just the standard way of denoting a system of n-nonlinear equations in n-unknowns, with the convention that right-hand side of each equation is zero, and throughout this paper,

$D \subset X$ denotes the domain of $f(x)$,

D_0 denotes closed convex subset of D ,

$N(x,t)$ denotes the open neighborhood of radius t around x ,i.e.

$N(x,t) = \{y \in X : \|x - y\| < t\}$

$L(x ,y)$ denotes a real Banach space,

$f \in Lip_k(D)$ if $\|f'(x) - f'(y)\| \leq K\|x - y\|$ for some constant K ,

$\{x_k\}$ denotes sequence of vectors in R^n ,

$B(x) = f'(x) - A(x)$,

and $M(x) = f'(x) - H(x)$.

In the analysis of Newton's method, it will be necessary to assume that the Jacobian matrix,

$$J(x^k) = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{n1} & f_{n2} & \cdots & f_{nn} \end{bmatrix}. \tag{2.2}$$

Where f_{ij} denotes the partial derivative of $f_i(x^k)$ with respect to the j th variable and evaluated at x, is at least continuous at the solution x^* ; that is

$$\|f'(x^* + h) - f'(x^*)\| \to 0 \quad \text{as} \quad h \to 0 .$$

On occasion, it will also be useful to assume, instead of continuity of f' , that the derivative satisfies the following property.

Definition 2.8 The mapping $f: R^n \to R^n$ is (totally or Frechet) differentiable at x if the Jacobian matrix (2.2) exists at x and

$$\lim_{h \to 0} \frac{\|f(x+h) - f(x) - f'(x)h\|}{\|h\|} = 0. \tag{2.3}$$

Note that if $n = 1$, Definition 2.8 reduces to the usual definition of differentiability. Note also that if f is differentiable at x, then f is continuous at x. This follows from the inequality

$$\|f(x+h) - f(x)\| \le \|f(x+h) - f(x) - f'(x)h\| + \|f'(x)h\|. \tag{2.4}$$

Finally , we note that it is possible to say that if the Jacobian matrix is continuously differentiable at x .

One of the basic tools of nonlinear analysis is the mean value theorem . If f is a differentiable function from R^n to R^1 , this states that

$$f(x) - f(y) = f'(z)(x-y) \tag{2.5}$$

for some point z between x and y. Unfortunately, this result dose not extend verbatim to mappings from R^n to R^n . However we are able to prove some results which are often just as useful. For the first, we define the integral $\int_a^b G(t)dt$ of a mapping G : $[a,b] \subset R^1 \to R^n$ as the vector with components $\int_a^b g_i(t)dt$, $i = 1,2,\ldots,n$ where g_1 ,$g_2,\ldots,g_n$ are the components of G . Thus, for example, if $f: R^n \to R^n$ the relation

$$f(y) - f(x) = \int_0^1 f'(x + t(y-x))(y-x)dt \tag{2.6}$$

is equivalent to

$$f_i(y) - f_i(x) = \int_0^1 \sum_{j=1}^{n} f_{ij}(x + t(y - x))(y_j - x_j)dt \qquad (2.7)$$

for $i = 1,2,\ldots,n$.

For $n = 1$, (2.6) is simply the fundamental theorem of the integral calculus. Hence the next result is a natural extension of that theorem to n-dimensions.

Lemma 2.4 Assume that $f: R^n \to R^n$ is continuously differentiable on a convex set $D \subset R^n$. Then for any $x, y \in D$, (2.6) holds.

Proof : For fixed x, y $\in$ D define the functions $g_i : [0,1] \subset R^1 \to R^1$ by

$$g_i(t) = f_i(x + t(y - x)), \qquad t \in [0,1],\ i = 1,2,\ldots,n . \qquad (2.8)$$

By the convexcity of D , it follows that g_I is continuously differentiable on $[0,1]$ and thus the fundamental theorem of the integral calculus implies that

$$g_i(1) - gi(0) = \int_0^1 g_i'(t)dt. \qquad (2.9)$$

But a simple calculation shown that

$$g_i'(t) = \sum_{j=i}^{n} f_{ij}(x + t(y - x))(y_j - x_j) . \qquad (2.10)$$

So that (2.9) is equivalent to (2.7) .

Lemma 2.5 Assume that $G : [a,b] \subset R^1 \to R^n$ is continuous. Then

$$\left\| \int_a^b G(t)dt \right\| \leq \int_a^b \|G(t)\| dt . \qquad (2.11)$$

proof : Since any norm is continuous function on R^n, both integrals of (2.11) exist and therefore any $\varepsilon > 0$ there is a partition $a < t_0 < \ldots < t_p < b$ *of* $[a,b]$ such that

$$\left\| \int_a^b G(t)dt - \sum_{i=1}^{p} G(t_i)(t_i - t_{i-1}) \right\| < \varepsilon \qquad (2.12)$$

and

$$\left| \int_a^b \| G(t) \| dt - \sum_{i=1}^{p} \| G(t_i) \| (t_i - t_{i-1}) \right| < \varepsilon .$$

Hence
$$\left\| \int_a^b G(t)dt \right\| \le \left\| \sum_{i=1}^{p} G(t_i)(t_i - t_{i-1}) \right\| + \varepsilon$$

$$\le \sum_{i=1}^{p} \| G(t_i) \| (t_i - t_{i-1}) + \varepsilon \le \int_a^b \| G(t) \| dt + 2\varepsilon$$

and, since ε was arbitrary, (2.11) must be valid.

By means of lemma 2.4 and 2.5 we can prove the following useful alternative of the mean value theorem.

Theorem 2.6 Assume that $f : R^n \to R^n$ is continuously differentiable on the convex set $D \subset R^n$. Then for any $x, y \in D$,

$$\| f(x) - f(y) \| \le \sup_{0 \le t \le 1} \| f'(x - t(y - x)) \| \| y - x \|. \qquad (2.13)$$

proof : By lemma 2.4 and 2.5 we have

$$\| f(x) - f(y) \| = \left\| \int_0^1 f'(x + t(y - x))(y - x)dt \right\|$$
$$\le \int_0^1 \| x = t(y - x)) \| \| y - x \| dt$$
$$\le \sup_{0 \le t \le 1} \| f'(x + t(x + t(y - x)) \| \int_0^1 \| y - x \| dt .$$

Lemma 2.7 Let $f' \in Lip_k(D_0)$ and let $x, y \in D_0$; then

$$\|f(y)-f(x)-f'(x)(y-x)\| \le \frac{1}{2}K\|x-y\|^2. \tag{2.14}$$

proof : From $f: R^n \to R^n$ is differentiable and satisfied $f' \in Lip_k(D_0)$, that means

$\|f'(x)-f'(y)\| \le K\|x-y\|$ for all x, y in closed convex subset D_0 of D.

Since $f'(x)(y-x)$ is constant with respect to the integration, we have

$$f(y)-f(x)-f'(x)(y-x) = \int_0^1 [(f'(x+t(y-x))(y-x)-f'(x)(y-x)]dt$$

$$= \int_0^1 [f'(x+t(y-x)-f'(x)](y-x)dt.$$

Hence, the result follows by taking norms of both side,

$$\begin{aligned}\|f(y)-f(x)-f'(x)(y-x)\| &= \left\|\int_0^1 [f'(x+t(y-x)-f'(x)]\right\|(y-x)dt \\ &\le \int_0^1 \|f'(x+t(y-x)-f'(x)\|\|y-x\|dt \\ &\le \int_0^1 K\|x+t(y-x)-x\|\|y-x\|dt \\ &\le \int_0^1 K\|t(y-x)\|\|y-x\|dt \\ &= \int_0^1 K\|t\|\|y-x\|^2 dt \\ &= \mathrm{K}\|y-x\|^2 \int_0^1 t\,dt \\ &= \frac{1}{2}K\|x-y\|^2 .\end{aligned}$$

Definition 2.9 Let t_0 and t' be non-negative real numbers, g be a continuously differentiable real function on $[t_0, t_0+t']$ and G a continuously differentiable operator on $\overline{N}(x_0, t) \subset X$ into X. Then the equation $t = g(t)$ will be said to majorize the equation $x = G(x)$, or g majorizes G, on $N(x_0, t')$ if

$$\|G(x_0)-x_0\| \le g(t_0)-t_0 \tag{2.15}$$

and

$$\|G'(x)\| \le g'(t) \text{ where } \|x-x_0\| \le t-t_0 < t'. \tag{2.16}$$

Definition 2.10 Let t_0 and t' be non- negative real numbers, g be a real function on $[t_0, t_0+t']$, and G an operator sending $N(x_0, t')$ into X. Then the equation $t = g(t)$ will be said to weakly majorize the equation $x = G(x)$, or g weakly majorizes G, if (2.15) holds and in addition

$$\|G(G(x))-G(x)\| \le g(g(t))-g(t) \tag{2.17}$$

when $\|x-x_0\| \le t-t_0 < t'$ and $\|G(x)-x\| \le g(t)-t$.

Lemma 2.8 (The Banach Lemma) Let $M \in L(X,X)$ and $\|I-J\| \le \delta < 1$, then J^{-1} exists in $L(X,X)$ and $J^{-1} \le (1-\delta)^{-1}$.

Lemma 2.9 Let $\{x_k\}$ be a sequence in X and $\{t_k\}$ a sequence of non-negative real numbers such that

$$\|x_{k+1}-x_k\| \le t_{k+1}-t_k \quad , \quad k=0,1,2,\ldots,n \tag{2.18}$$

and $t_k \to t^* < \infty$. By these conditions, there exists a point $s \in X$ such that $x_k \to s$ and

$$\|s-x\| \le t^*-t_k \quad , k=0,1,2,\ldots,n. \tag{2.19}$$

proof : The proof is immediate from

$$\|x_{k+p} - x_k\| \le \sum_{i=1}^{p} \|x_{k+i} - x_{k+i-1}\|$$

$$\le t_{k+p} - t_k$$

$$\le t^* - t_k \ ,$$

which show that $\{x_k\}$ is a Cauchy Sequence.

We shall say that $\{t_k\}$ majorizes $\{x_k\}$ if $\|x_{k+1} - x_k\| \le t_{k+1} - t_k$, $k = 0,1,2,...$ holds.

The following theorem is The Kantorovich Theorem. It is one of the fundamental theorems in mathematics.

Theorem 2.10 (The Kantorovich Theorem) Let x_0 be in D_0 and let $\Gamma_0 = [f'(x_0)]^{-1}$ exist with $\Gamma_0 \in Lip_k(D_0)$,

$$\|\Gamma_0 f(x_0)\| \le \eta \text{ and } h = K\eta \le \frac{1}{2}. \tag{2.20}$$

Define
$$r_0(h) = \frac{1}{h}(1 - \sqrt{1-2h})\eta \tag{2.21}$$

$$r_1(h) = \frac{1}{h}(1 + \sqrt{1-2h})\eta. \tag{2.22}$$

Then if $N(x_0, r_0(h)) \subset D_0$, the sequence of iterates defined by Newton' s method exists, remains in $N(x_0, r_0(h))$ and converges to s in $N(x_0, r_0(h))$ such that $f(s) = 0$. If $h < \frac{1}{2}$, s is the only root in $N(x_0, r_1(h)) \subset D_0$, and if $h = \frac{1}{2}$, s is unique in $N(x_0, r_0(h)) \cap D_0$. Furthermore, the sequence of the iterates satisfy the error bounds

$$\|s - x_m\| \le \frac{1}{h}\frac{1}{2^m}(1 - \sqrt{1-2h})^{2m}\eta. \tag{2.23}$$

This theorem together with Lemma 2.6 and give the convergence of the sequence $\{x_k\}$ in X when the sequence of $\{t_k\}$ converges.

Theorem 2.11 If g majorizes G on $\overline{N}(x_0,t')$ and g has a fixed point in $[t_0,t_0+t']$, then G has a fixed point s in $\overline{N}(x_0,t')$. Furthermore, $x_{k+1} = G(x_k)$ and $t_{k+1} = g(t_k), k = 0,1,2,...,$ converges to s and t^* respectively with the real sequence majorizing the vector sequence.

Lemma 2.12 If $g(t) \in (t,t_0+t')$ when $t \in (t,t_0+t')$ and g weakly majorizing G on $N(x_0,t')$, then there are elements $t^* \in [t_0,t_0+t']$, $s \in \overline{N}(x_0,t')$ such that

$$x_{k+1} = G(x_k) \tag{2.24}$$

and

$$t_{k+1} = g(t_k), \qquad k = 0,1,2,... \tag{2.25}$$

converge to s and t^* respectively with the t sequence majorizing in x sequence.

Theorem 2.13 Let A be a function on $N(x_0,r)$ such that $A(x) \in L(X,Y)$ for each x and $A(x)$ is invertible for each x in $N(x_0,r)$ and that there is a real, nonvanishing, nonincreasing function $a(t)$ on $[0,r)$ such that

$$\|A^{-1}(x)\| \le a(\|x - x_0\|)^{-1}. \tag{2.26}$$

If $f' \in (Lip_k(\overline{N}(x_0,r))$ then if $\sigma > 0$ and $\delta > 0$ are real numbers such that

$$a(t) + \sigma kt \tag{2.27}$$

is isotonic on $(0,r)$, and $\|B(x)\| \le a(\|x - x_0\|) + \sigma k\|x - x_0\| - \delta$ (2.28)

for every $x \in N(x_0,r)$ then

$$g(t) = t + (a(t))^{-1}(0.5\sigma kt^2 - \delta t - a(0))\|(A(x_0))^{-1} - f(x_0)\|) \tag{2.29}$$

weakly majorizes $G(x) = x - (A(x))^{-1}f(x)$ on $N(x_0,r)$.

Theorem 2.14 Let $f' \in (Lip_k(\overline{N}(x_0,r))$ and $[A(x_0)]^{-1}$ exist and be bounded in the norm by $[a(0)]^{-1}$, If $\|B(x_0)\| < a(0)$ and

$$h' = \frac{K}{a(0)}\|[A(x_0)]^{-1} f(x_0)\|a(0) - (\|B(x_0)\|)^2 \le \frac{1}{2} \tag{2.30}$$

and

$$r_0' = \frac{1}{K}(1 - \sqrt{1-2h'})[a(0) - \|B(x_0)\|] \le \frac{1}{2}. \tag{2.31}$$

Then if f has a unique zero $s = \overline{N}(x_0,r_0')$, and

$$x_{m+1}^* = x_m^* - [A(x_0)]^{-1} f(x_m^*) \quad , m = 0,1,2,\ldots$$

converges to s from any $X_0' \in \overline{N}(x_0,r)$ such that

$$\|x_0' - x_0\| < r_1' = \frac{1}{K}(1 - \sqrt{1-2h})[a(0) - \|B(x_0)\|].$$

If, in addition, σ, δ and a satisfy the conditions of Theorem 2.13 and

$$h = \frac{1}{\delta^2}\sigma K \left\| [A(X_0)]^{-1} f(x_0) \right\| a(0) \le \frac{1}{2} \tag{2.32}$$

and

$$r_0 = \frac{1}{\sigma.K}\left(1-\sqrt{1-2h}\right)\delta < r\,, \tag{2.33}$$

then

$$x_{m+1}^{*} = x_m^{*} - [A(x_0)]^{-1} f(x_m^{*}) \quad , m = 0,1,2, \ldots \tag{2.34}$$

converges to s .

In the following theorem, we impose one more condition on $A(x)$ and one condition on $B(x)$ instead of $B(x_0)$.

Theorem 2.15 Let $f' \in Lip_k(\overline{D})$ where $x_0 \in D$ and D is an open convex subset of X . Assume that

$$\left\| [A(x_0)]^{-1} f(x_0) \right\| \le \alpha \tag{2.35}$$

$$\left\| [A(x_0)]^{-1} \right\| \le \beta \tag{2.36}$$

$$\left\| A(x) - A(x_0) \right\| \le \eta_0 + \eta_1 \left\| x - x_0 \right\|, \forall x \in D \tag{2.37}$$

$$\left\| B(x) \right\| \le \delta_0 + \delta_1 \left\| x - x_0 \right\|, \forall x \in D \tag{2.38}$$

then

$$\beta.\delta_0 < 1,\ h_1 = \frac{\beta.K.\alpha}{(1-\beta.\delta_0)^2} \le \frac{1}{2}$$

and $N(x_0, r_1') \subset D$ where $r_1' = \dfrac{1-\sqrt{1-2h_1}}{\beta.K}(1-\beta.\delta_0)$

implied that f has a root $s \in \overline{N}(x_0, r_1')$ which is unique in $D \cap N(x_0, r_1')$. Furthermore

$$x_{m+1}^{*} = x_m^{*} - [A(x_0)]^{-1} f(x_m^{*}) \quad , m = 0,1,2, \ldots$$

Converges to s from any $x_0^* \in \overline{D} \cap N(x_0, r_1')$.

If, in addition, $\beta(\delta_0 + \eta_0) < 1$ and $h_0 = \dfrac{\sigma.\beta.K.\alpha}{(1-\beta.\eta_0 - \beta.\delta_0)^2} \leq \dfrac{1}{2}$ where $\sigma = \max\left(1, \dfrac{\delta_1 + \eta_1}{K}\right)$, and $N(x_0, r_0) \subset D$,

$$r_0 = \frac{1-\sqrt{1-2h_0}}{\sigma.\beta.K}(1-\beta.\eta_0 - \beta.\delta_0)$$

then
$$x_{m+1}^* = x_m^* - \left[A(x_m^*)\right]^{-1} f(x_m^*) \quad , m = 0,1,2,\ldots$$

converges to s for any $x_0^* \in \overline{D} \cap N(x_0, r_0)$.

Theorem 2.16 Let the conditions of the Theorem 2.15 be satisfied and $\delta_0 = 0$, that is

$$\|M(x)\| \leq \delta_1 \|x - x_0\|, \ \forall x \in D. \tag{2.39}$$

Then the order of the convergence of the method is equal to 2.

Proof: The first part of the theorem follows from the result of theorem 2.15. Now for the second part of the theorem, let A be one of the matrices P, U_k, and L_k, k =2,3,... n

$$M(x) = f'(x) - A(x)$$
$$Q = \max\|A(x)\|$$
$$S = Q(\frac{1}{2}K + \delta_1)$$

and

if $e_m = \|s - x^m\|$

$e_{m+1} = \|s - x^{m+1}\|$

$= \|s - x^m + (A(x^m))^{-1} f(x^m)\|$

$$=\left\|s-x^m+(A(x^m)^{-1}f(x^m)(A(x^m)^{-1}f(s)\right\|$$

$$=\left\|(A(x^m)(A(x^m)^{-1}s-x^m+(A(x^m)^{-1}f(x^m)(A(x^m)^{-1}f(s)\right\|$$

$$\leq\left\|(A(x^m))^{-1}\right\|\cdot\left\|f(s)-f(x^m)+A(x^m)(s-x^m)\right\|$$

$$=\left\|(A(x^m))^{-1}\right\|\cdot\left\|f(s)-f(x^m)+f'(x^m)(s-x^m)-f'(x^m)(s-x^m)+A(x^m)(s-x^m)\right\|$$

$$=\left\|(A(x^m))^{-1}\right\|\cdot\left\|f(s)-f(x^m)+f'(x^m)(s-x^m)-(f'(x^m)-A(x^m))(s-x^m)\right\|$$

$$\leq\left\|(A(x^m))^{-1}\right\|(\left\|f(s)-f(x^m)+f'(x^m)(s-x^m)\right\|+\left\|(f'(x^m)-A(x^m))(s-x^m)\right\|).$$

Then by lemma 2.7, we have

$$e_{m+1}\leq\left\|(A(x^m))^{-1}\right\|(\frac{1}{2}Ke_m^2+\left\|M(x^m)e_m\right\|$$

$$\leq\left\|(A(x^m))^{-1}\right\|(\frac{1}{2}Ke_m^2+\delta_1e_m^2)$$

$$\leq Q(\frac{1}{2}K+\delta_1)e_m^2$$

$$=Se_m^2.$$

For the system of equations is of the form (2.1), the function f may be linear function or nonlinear function. In this thesis we consider two numerical iterative methods to solve any nonlinear function f: the Newton method is the iteration equations

$$x^{k+1}=x^k-(f'(x^k))^{-1}f(x^k),k=0,1,2,\ldots \tag{2.40}$$

and $f'(x)$ is the Jacobian matrix of $f(x)$. The Newton –like method for solving (2.1) is the iteration

$$x^{k+1}=x^k-(A(x^k))^{-1}f(x^k),k=0,1,2,\ldots \tag{2.41}$$

where $A(x)$ is an invertible operater.

Pattanametawee [3] found the iterative method for solving a system of nonlinear equations of the form(2.1). She made the modification which introduced M.Podisuk in [4] and [5] one of the Newton – like method for solving (2.40) which is the iterative equation

$$x^{k+1} = x^k - (H(x^k))^{-1} f(x^k), k = 0,1,2,... \tag{2.42}$$

where H is the diagonal matrix

$$H(x) = \begin{bmatrix} f_{11} & & & & \\ & f_{22} & & 0 & \\ & & \cdot & & \\ & & & \cdot & \\ & 0 & & & \cdot \\ & & & & f_{nn} \end{bmatrix}$$

where $f_{ij} = \dfrac{\partial f_i}{\partial x_j}$, $i, j = 1,2,...,n.$

Kongchouy [6] made the modification more than the iteration equation (2.21). The matrix $D(x)$ as the matrix $H(x)$ above and the matrices $U_2(x)$, $U_3(x)$,..., $U_n(x)$ of $f(x)$ as

$$U_2(x) = \begin{bmatrix} f_{11} & f_{12} & & & & \\ & f_{22} & f_{23} & & 0 & \\ & & \cdot & \cdot & & \\ & & & \cdot & \cdot & \\ & & & & \cdot & \cdot \\ & 0 & & & f_{n-1n-1} & f_{n-1n} \\ & & & & & f_{nn} \end{bmatrix}$$

$$U_3(x) = \begin{bmatrix} f_{11} & f_{12} & f_{13} & & & & \\ & f_{22} & f_{23} & f_{24} & & 0 & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & \cdot & \cdot \\ & & & & f_{n-2n-2} & f_{n-2n-1} & f_{n-2n} \\ & 0 & & & & f_{n-1n-1} & f_{n-1n} \\ & & & & & & f_{nn} \end{bmatrix}$$

$$\vdots$$

$$U_n(x) = \begin{bmatrix} f_{11} & f_{12} & f_{13} & \cdot & \cdot & \cdot & \cdot & f_{1n} \\ & f_{22} & f_{23} & f_{24} & \cdot & \cdot & \cdot & f_{2n} \\ & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & & \cdot & f_{n-2n-2} & f_{n-2n-1} & f_{n-2n} \\ & 0 & & & & & f_{n-1n-1} & f_{n-1n} \\ & & & & & & & f_{nn} \end{bmatrix}.$$

the matrices $L_2(x), L_3(x), \ldots, L_n(x)$ of $f(x)$ as

$$L_2(x) = \begin{bmatrix} f_{11} & & & & & \\ f_{21} & f_{22} & & & 0 & \\ & \cdot & \cdot & & & \\ & & \cdot & \cdot & & \\ & & & \cdot & \cdot & \\ & 0 & & f_{n-1n-2} & f_{n-1n-1} & \\ & & & & f_{nn-1} & f_{nn} \end{bmatrix}$$

$$L_3(x) = \begin{bmatrix} f_{11} & & & & & & \\ f_{21} & f_{22} & & & & 0 & \\ f_{31} & f_{32} & f_{33} & & & & \\ & \cdot & \cdot & \cdot & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & 0 & & f_{n-1n-3} & f_{n-1n-2} & f_{n-1n-1} & \\ & & & & f_{nn-2} & f_{nn-1} & f_{nn} \end{bmatrix}$$

$$\vdots$$

$$L_n(x) = \begin{bmatrix} f_{11} & & & & & & & \\ f_{21} & f_{22} & & & & 0 & & \\ f_{31} & f_{32} & f_{33} & & & & & \\ . & . & . & . & & & & \\ . & . & . & . & . & & & \\ . & . & . & . & . & . & & \\ f_{n-11} & . & . & . & . & . & f_{n-1n-1} & \\ f_{n1} & . & . & . & . & . & . & f_{nn} \end{bmatrix}.$$

The matrices $D(x), U_2(x), U_3(x), \ldots, U_n(x), L_2(x), L_3(x), \ldots, L_n(x)$ of the D-method, U_2- method, U_3-method,..., U_n-method, L_2-method, L_3-method,...,L_n-method respectively.

Theorem 2.17 If the matrices $D, U_2, U_3, \ldots, U_n, L_2, L_3, \ldots, L_n$ satisfy all the conditions of the theorem 2.15 – 2.16 then the sequences $\{x^{k+1}\}$ of the following formulas

$$x^{k+1} = x^k - (M(x^k))^{-1} f(x^k) \quad , k = 0,1,2,\ldots \tag{2.43}$$

where M are the matrices $D, U_2, U_3, \ldots, U_n, L_2, L_3, \ldots, L_n$ converge to the solution of the system $f(x) = 0$ and the order of convergence of the method is equal to 2.

CHAPTER 3

RESEARCH METHODOLOGY

In this chapter we shall look for numerical methods for solving system of linear equations of the form

$$Ax = b \qquad (3.1)$$

where A is nxn inevitable real matrix, x is a real unknown vector and b is a given known vector. The simplest way to solve system (3.1) is by using the inverse matrix A^{-1} that is

$$x = A^{-1}b. \qquad (3.2)$$

However, finding inverse of matrix A may cause some difficulties. The purpose of this thesis is to find another nxn matrix B and use its inverse to find the unknown vector x in the iterative fashion. To find matrix B, we will use the help of the Newton-like methods for solving systems of nonlinear equations. We will treat system (3.1) as a system of nonlinear equations,

that is

$$(A - Ib)x = 0 \qquad (3.3)$$

or

$$f(x) = (A - Ib)x = 0. \qquad (3.4)$$

Then we use the following iterative equation;

$$x^{m+1} = x^{m} - (M(x^{m}))^{-1} f(x^{m}), m = 0,1,2,\ldots \qquad (3.5)$$

where the matrices M are $D, U_2, \ldots, U_n$ and $L_2, L_3, \ldots, L_n$. If f satisfies all conditions of Theorem in chapter 2, then we use D-method, U_2- method, U_3- method through U_n- method and L_2- method, L_3- method through L_n.

3.1 First Method

This method uses D- method to solve the problem (3.4). We obtain the iterative equations which we will use, the Newton – like method for solving systems of nonlinear equations of the form

$$x^{m+1} = x^m - (H(x^m))^{-1} f(x^m)\ , m = 0,1,2,3,\dots \qquad (3.6)$$

where H is diagonal matrix

$$H = \begin{bmatrix} f_{11} & 0 & . & . & . & 0 \\ 0 & f_{22} & . & & & . \\ . & . & . & . & & . \\ . & & . & . & . & . \\ . & & & . & . & 0 \\ 0 & . & . & . & 0 & f_{nn} \end{bmatrix}$$

and $f_{ij} = \dfrac{\partial f_i}{\partial f_j}, i.j = 1,2,3,\dots,n.$

We consider the form system of linear equations

$$Ax = b$$

such that

$$H = \begin{bmatrix} a_{11} & 0 & . & . & . & 0 \\ 0 & a_{22} & . & & & . \\ . & . & . & . & & . \\ . & & . & . & . & . \\ . & & & . & . & 0 \\ 0 & . & . & . & 0 & a_{nn} \end{bmatrix}$$

and A is matrix nxn.

$$x^{m+1}=x^m-\begin{bmatrix}\frac{1}{a_{11}} & 0 & . & . & . & 0\\ 0 & \frac{1}{a_{22}} & . & & & .\\ . & . & . & . & & .\\ . & & . & . & . & .\\ . & & & . & . & 0\\ 0 & . & . & . & 0 & \frac{1}{a_{nn}}\end{bmatrix} f(x^m)$$

$$=x^m-\begin{bmatrix}\frac{a_{11}x_1^m+a_{12}x_2^m+\ldots+a_{1n}x_n^m-b_1}{a_{11}}\\ .\\ .\\ .\\ \frac{a_{n1}x_1^m+a_{n2}x_2^m+\ldots+a_{11}x_n^m-b_1}{a_{nn}}\end{bmatrix}$$

that is

$$x^{m+1}=\frac{b_i-\sum_{j\neq i}a_{ij}x_j^m}{a_{ii}}, m=0,1,2,\ldots \qquad (3.7)$$

which is the Jacobi iterative method.

3.2 Second Method

Using the U_2- method for solving system of linear equations, we obtain the iterative equation

$$x^{m+1}=x^m-(U_2(x^m))^{-1}(Ax^m-b) \qquad (3.8)$$

where

$$U_2=\begin{bmatrix}a_{11} & a_{12} & 0 & . & . & 0\\ 0 & a_{22} & a_{23} & & & .\\ . & & . & . & & .\\ . & & & . & . & .\\ . & & & & a_{n-1n-1} & a_{n-1n}\\ 0 & . & . & . & 0 & a_{nn}\end{bmatrix}$$

and

$$(U_2 x^m)^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & \frac{-a_{12}}{a_{11}a_{22}} & \frac{a_{12}a_{23}}{a_{11}a_{22}a_{33}} & . & . & . & \frac{(-1)^{n-1}a_{12}a_{23}\ldots a_{n-1n}}{a_{11}a_{22}\ldots a_{nn}} \\ 0 & \frac{1}{a_{22}} & \frac{-a_{23}}{a_{22}a_{33}} & \frac{a_{23}a_{34}}{a_{22}a_{33}a_{44}} & . & . & \frac{(-1)^{n-1}a_{23}a_{34}\ldots a_{n-1n}}{a_{22}a_{33}\ldots a_{nn}} \\ . & & . & . & . & . & . \\ . & & & . & . & . & . \\ . & & & & . & . & . \\ . & & & . & 0 & \frac{1}{a_{n-1n-1}} & \frac{-a_{n-1n}}{a_{n-1n-1}a_{nn}} \\ 0 & . & . & . & . & 0 & \frac{1}{a_{nn}} \end{bmatrix}.$$

3.3 Third Method

Using the U_k - method, ($k = 3,4,\ldots, n$) to solve system (3.4) and call them and their modification as U_3 - method, U_4 - method, respectively. Where

$$U_k = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1k} & 0 \\ 0 & a_{22} & a_{23} & & . & . \\ . & & . & . & & . \\ . & & & . & . & . \\ . & & & & a_{n-1n-1} & a_{n-1n} \\ 0 & . & . & . & 0 & a_{nn} \end{bmatrix}.$$

3.4 Fourth Method

Using the L_2 - method to solve system (3.4),that is

$$x^{m+1} = x^m - (L_2(x^m))^{-1}(A(x^m) - b)\ ,m=0,1,2,3,\ldots \qquad (3.9)$$

where

$$L_2 = \begin{bmatrix} a_{11} & 0 & . & . & . & 0 \\ a_{21} & a_{22} & . & & & . \\ 0 & . & . & . & & . \\ . & & . & . & . & . \\ . & & & . & a_{n-1n-1} & 0 \\ 0 & . & . & 0 & a_{nn-1} & a_{nn} \end{bmatrix}$$

and

$$(L_2 x^m)^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & . & . & . & . & 0 \\ \frac{-a_{21}}{a_{11}a_{22}} & \frac{1}{a_{22}} & . & & & & . \\ \frac{a_{31}a_{21}}{a_{11}a_{22}a_{33}} & \frac{-a_{32}}{a_{22}a_{33}} & \frac{1}{a_{33}} & . & & & . \\ . & & . & . & . & & . \\ . & & & . & . & . & . \\ . & & & & . & \frac{1}{a_{n-1n-1}} & 0 \\ \frac{(-1)^{n-1} a_{nn-1}a_{n-1n-2}\cdots a_{21}}{a_{11}a_{22}\ldots a_{nn}} & . & . & . & . & \frac{-a_{nn-1}}{a_{n-1}a_{n-1}} & \frac{1}{a_{nn}} \end{bmatrix}.$$

3.5 Fifth Method

Using the L_k - method, ($k = 3,4,\ldots, n$) to solve system (3.4), where

$$U_k = \begin{bmatrix} a_{11} & 0 & . & . & . & 0 \\ a_{21} & a_{22} & . & & & . \\ . & & . & . & & . \\ . & & & . & . & . \\ . & & & & a_{n-1n-1} & 0 \\ 0 & . & a_{nn-k} & . & a_{nn-1} & a_{nn} \end{bmatrix}.$$

3.6 The Examples of the Systems of Linear Equations

We will use the iterative methods above solve some examples of the systems of linear equations.

Example 3.1 The management of Checkers Rent – A-car plans to expand its fleet of rental cars for the next quarter by purchasing compact and full – size car. The average cost of a compact is \$10,000,and the average cost of a full-size is \$ 24,000. If a total of 800 cars is to be purchased with a budget of \$ 12 million, how many cars of each size will be acquired?

Solution Let x and y denote the number of compact and full-size cars to be purchased. Furthermore, let n denote the total number of cars to be acquired and b the amount of money budgeted for the purchased of these cars. Then

$$x + y = 800$$
$$10,000x + 24,000y = 12,000.$$

We use $x = 0, y = 0$ as the initial point and the iteration will stop when $\|Ax^{m+1} - b\| < \varepsilon = 0.0000001$. The results are shown in table 3.1 through table 3.3. Number of iterations from D - method, U_2 - method, L_2 - method are 65,34, 27 respectively, and the solution converges to $x = 514.28571432$, $y = 285.71428570$.

Example 3.2 A company produces three products, each of which must be processed through three different departments. Table summarizes the hours required per unit of each product in each department. In addition, the weekly capacities are stated for each department in terms of work-hours available. What is desired is to determine whether there are any combinations of the three products which would exhaust the weekly capacities of the three departments.

	Product			Hours Available
Department	1	2	3	per Week
A	3	2	5	1,800
B	4	1	3	1,450
C	2	4	1	1,900

Solution If we let x_j = number of units produced per week of product j, the conditions to be satisfied are expressed by the following system of equations.

$$3x_1 + 2x_2 + 5x_3 = 1{,}800 \quad \text{(department } A\text{)}$$
$$4x_1 + x_2 + 3x_3 = 1{,}450 \quad \text{(department } B\text{)}$$
$$2x_1 + 4x_2 + x_3 = 1{,}900 \quad \text{(department } C\text{)}$$

such that

$$4x_1 + x_2 + 3x_3 = 1{,}450$$
$$2x_1 + 4x_2 + x_3 = 1{,}900$$
$$3x_1 + 2x_2 + 5 = 1{,}800.$$

We use $x_1 = 0, x_2 = 0$ and $x_3 = 0$ as the initial point. The iteration will stop when $\left\|Ax^{m+1} - b\right\| < \varepsilon = 0.0000001$. The results are shown in table 3.4 through table 3.18. Number of iterations from D - method, U_2-method, U_3 - method, L_2 - method, L_3 - method are 363, 75, 27, 63, 30 respectively, and the solution converges to A = 200, B = 350, C = 100.

Example 3.3 Find the center and radius of the sphere parsing through point (1,2,3), (2,1,1), (1,-2,1) and (-1,3,2) solution the system we must solve is

$$\begin{aligned} 2x + y + z + w &= 3 \\ -x + 3y + 2z + w &= 7 \\ x + 2y + 3z + w &= 7 \\ x - 2y + z + w &= 3 \end{aligned}$$

$$w = -\left(x^2 + y^2 + z^2 - r^2\right)/2$$

$$r = \sqrt{x^2 + y^2 + z^2 + 2w}\,.$$

We use $x = 0, y = 0, z = 0$ and $w = 0$ as the initial points. The iteration will stop when $\|Ax^{m+1} - b\| < \varepsilon = 0.0000001$. The results are shown in the table 3.9 through the table 3.15. Number of iterations from D - method, U_2- method, U_3- method, U_4- method, L_2 - method, L_3-method, L_4- method are 111, 42, 15, 29, 43, 121, 30 respectively, and the solution converges to x = - 0.66666666, y = 0.22222223, z = 1.55555555, w = 2.55555556, and r = 2.83278862 .

Example 3.4 The five reactors linked by pipes. Its system of linear equation is as follows;

$$\begin{aligned} -c_2 + 9c_3 \qquad\qquad &= 160 \\ -c_2 - 8c_3 + 11c_4 - 2c_5 &= 0 \\ -3c_1 - c_2 + 4c_5 &= 0. \end{aligned}$$

We use $c_1 = 0, c_2 = 0, c_3 = 0, c_4 = 0$ and $c_5 = 0$ as the initial points. The iteration will stop when $\|Ax^{m+1} - b\| < \varepsilon = 0.0000001$. The results are shown in the table 3.16 through the table 3.20. Number of iterations from D - method, U_2- method, U_3-method, L_2 - method, L_3-method are 13,8, 8,7,7 respectively, and the solution converges to c_1 = 11.09090909, c_2 = 11.09090909, c_3 = 16.54545455, c_4 = 15.05785124, c_5=11.09090909.

Example 3.5 A community of five groups consumes and produces five commodities according to the in put - out put table.

	Group 1	Group 2	Group 3	Group 4	Group 5
Group 1	.20	.30	.12	.15	.20
Group 2	.15	.20	.40	.15	.25
Group 3	.20	0	.12	.30	0
Group 4	.18	.14	.09	0	.16
Group 5	0	.12	.13	.15	.22

Suppose the market demands for the commodities are \$ 10,000 \$ 12,000 \$8,000 \$ 24,000 \$15,000 respectively. Find the production schedule that mete the given conditions.

Solution

Let Group1 = x_1

Group2 = x_2

Group3 = x_3

Group4 = x_4

Group5 = x_5.

Such that
$$0.80x_1 - 0.30x_2 - 0.12x_3 - 0.15x_4 - 0.20x_5 = 10{,}000$$
$$-0.15x_1 + 0.80x_2 - 0.40x_3 - 0.15x_4 - 0.25x_5 = 12{,}000$$
$$-0.20x_1 + 0.80x_2 - 0.30x_4 = 8{,}000$$
$$-0.18x_1 - 0.14x_2 - 0.09x_3 - 1.0x_4 - 0.16x_5 = 24{,}000$$
$$-0.12x_2 - 0.13x_3 - 0.15x_4 + 0.78x_5 = 15{,}000.$$

We use $x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0$ and $x_5 = 0$ as the initial points. The iteration will stop when $\|Ax^{m+1} - b\| < \varepsilon = 0.0000001$. The results are shown in the table 3.21 through the table 3.30. Number of iterations from D - method, U_2-method,

U_3 - method, U_4 - method, U_5 - method, L_2 - method, L_3 -method, L_4 - method, L_5 - method are 93, 64, 59, 52, 47, 80, 64, 55, 55 respectively, and the solution converges to Solution of this problem is $x_1 = 73167.035635$, $x_2 = 79116.210439$, $x_3 = 46359.145671$, $x_4 = 60542.136878$, $x_5 = 50771.73683$.

Example 3.6 The potentials at nodes (junctions) 1 through 6 in the electrical network of the values of the resistances are shown in ohms, and the potential applied between A and B is 100 volts.

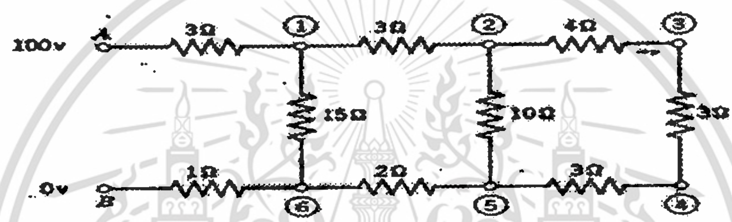


Figure 3.1 The electrical network.

Electrical network

Node	Equation
1	$11v_1 - 5v_2 - v_6 = 500$
2	$-20v_1 + 41v_2 - 15v_3 - 6v_5 = 0$
3	$-3v_2 + 7v_3 - 4v_4 = 0$
4	$-v_3 + 2v_4 + v_5 = 0$
5	$-3v_2 - 10v_4 + 28v_5 - 15v_6 = 0$
6	$-2v_1 - 15v_5 + 47v_6 = 0$

We use $\nu_1 = 0, \nu_2 = 0, \nu_3 = 0, \nu_4 = 0$ and $\nu_5 = 0$ as the initial point. The iteration will stop when $\|Ax^{m+1} - b\| < \varepsilon = 0.0000001$. The results are shown in table 3.31, table 3.32 and table 3.33. Number of iterations from D - method, U_2 - method, L_2 - method, are 159, 80, 88 respectively, and the solution converges to $\nu_1 = 69.99999997$, $\nu_2 = 51.99999995$, $\nu_3 = 39.99999993$, $\nu_4 = 30.99999993$, $\nu_5 = 21.99999995$, $\nu_6 = 9.9999998$.

Exemple 3.7

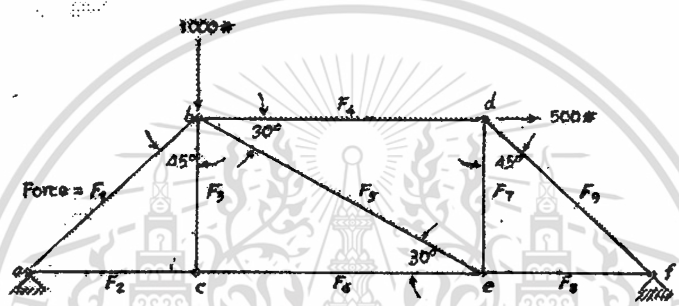
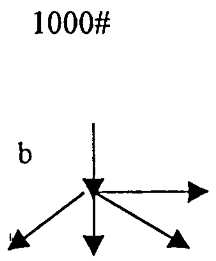


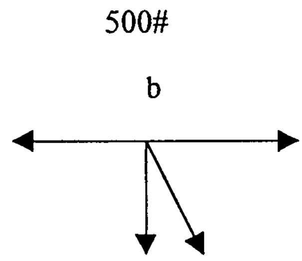
Figure 3.2 The forces on the truss.

In the example truss, there are six pins (joints) and nine members. Each joint Represents two degrees of freedom (signifying that two independent forces act there). We resolve the force acting at each of the pins into x-and y-components.

The forces that act on the truss, in view of the assumptions, are pointed along each of the members. It is conventional to consider the forces to act toward the center of the member, away from the join. Since there are there nine number, we have nine number –forces, F_1, i=1,…9.These forces can be determined by setting the sum of all forces acting on the pins horizontally or vertically to zero, sine the system is at equilibrium(the structure itself does not move).

For instance, at joints b and d, we have





If we start from pin b and progress to each other pin, we can write nine equations: such that

$$\frac{\sqrt{2}}{2} * F_1 - F_4 - \frac{\sqrt{3}}{2} * F_5 = 0$$

$$\frac{\sqrt{2}}{2} * F_1 + F_3 - \frac{1}{2} * F_5 = 1000$$

$$F_2 - F_6 = 0$$

$$-F_3 = 0$$

$$F_4 - \frac{\sqrt{2}}{2} * F_9 = 0$$

$$F_7 + \frac{\sqrt{2}}{2} * F_9 = 500$$

$$\frac{\sqrt{3}}{2} * F_5 + F_6 - F_8 = 0$$

$$\frac{1}{2} * F_5 + F_7 = 500$$

$$F_8 + \frac{\sqrt{2}}{2} * F_9 = 0 .$$

We use $F_1 = F = F_3 = F_4 = F_5 = F_6 = F_7 = F_8 = 0$ and $F_9 = 0$ as the initial point. The iteration will stop when $\|Ax^{m+1} - b\| < \varepsilon = 0.0000001$. The results are shown in the table 3.34, table 3.35 and table 3.36. Number of iterations from D - method, U_2 - method, L_2 - method are 47, 47, 25 respectively, and the solution converges to f_1 = -1035.27618020, f_2 = 303.84757729, f_3 = 0, f_4 = -267.94919243, f_5 = -535.89838487, f_6 = 303.84757729, f_7 = 767.9491919243, f_8 = 767.94919243, f_9 = -1086.04416320.

Example 3.8 Represents the steam distribution system of a chemical plant. The material and energy balances of this system are given below.

$$181.60 - x_3 - 132.57 - x_4 - x_5 = -y_1 - y_2 + y_5 + y_4 = 5.1 \quad (1)$$

$$1.17x_3 - x_6 = 0 \quad (2)$$

$$132.57 - 0.745x_7 = 61.2 \quad (3)$$

$$x_5 + x_7 - x_8 - x_9 - x_{10} + x_{15} = y_7 + y_8 - y_3 = 99.1 \quad (4)$$

$$x_8 + x_9 + x_{10} + x_{11} - x_{12} - x_{13} = -y_7 = -8.4 \quad (5)$$

$$x_6 - x \ = y_6 - y_5 = 24.2 \quad (6)$$

$$-1.15(181.60) + x_3 - x_6 + x_{12} + x_{16} = 1.15y_1 - y_9 + 0.4 = -19.7 \quad (7)$$

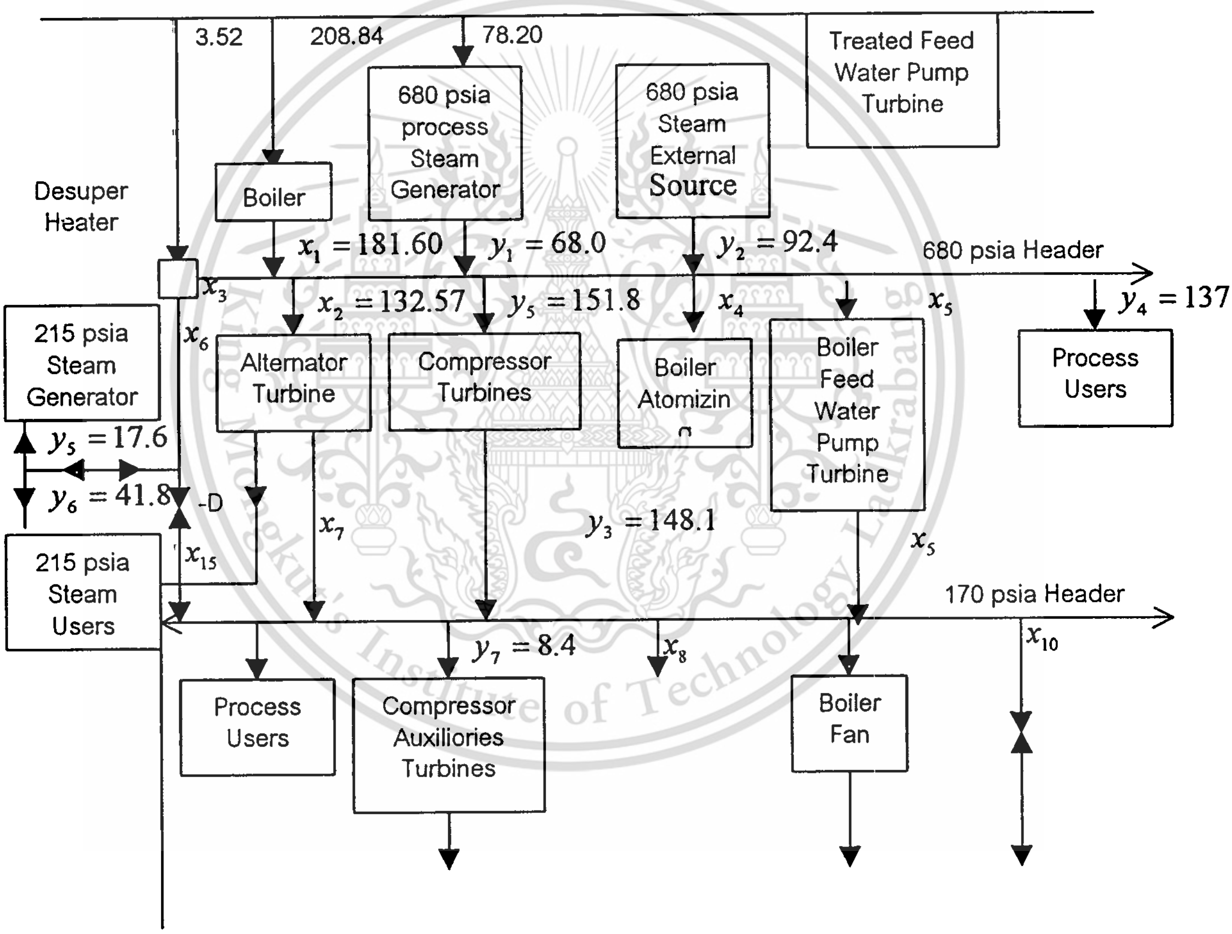


Figure 3.3 The steam distribution system of a chemical plant.

$$181.60 - 4.594x_{12} - 0.11x_{16} = -y_1 + 1.0235y_9 + 2.45 = 35.05 \quad (8)$$

$$-0.0423(181.60) + x_{11} = 0.0423y_1 = 2.88 \quad (9)$$

$$-0.016(181.60) + x_4 = 0 \quad (10)$$

$$x_8 - 0.0147x_{16} = 0 \quad (11)$$

$$x_5 - 0.07x_{14} = 0 \quad (12)$$

$$-0.0805(181.60) + x_9 = 0 \quad (13)$$

$$x_{12} - x_{14} + x_{16} = 0.4 - y_9 = -97.9. \quad (14)$$

There are four levels of steam pressure in this plant: 680, 215, 170, and 37 psia. The fourteen $x_i, i = 3,\ldots,16$ are the unknowns and the y_i are given parameters for the system. Both x_i and y_i have the units of 1000 lb.

We use $x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0, x_6 = 0, x_7 = 0, x_8 = 0, x_9 = 0,$ $x_{10} = 0, x_{11} = 0, x_{12} = 0 x_{13} = 0, x_{14} = 0, x_{15} = 0$ and $x_{16} = 0$, by $x_{12} = a$, $x_{14} = b$, $x_{16} = c$ and $x_2 = x_4$ as the initial point. The iteration will stop when $\|Ax^{m+1} - b\| < \varepsilon = 0.0000001$. The results are shown in the table 3.34 through table 3.41. Number of iterations from D - method, U_2-method, U_3 - method, L_2 - method, L_3 - method are 15, 14, 5, 13, 8 respectively, and the solution converges to x_1 = 7.56738215, x_2 =2.9056, x_3 = 17.56738215, x_4 =2.9056, x_5 =23.45701785, x_6=20.55383712,x_7=95.7986577,x_8=2.94733741,x_9=14.6188,x_{10}= -1.056624744, x_{11}= 10.56168, x_{12}=36.7011114, x_{13}= - 0.173293994, x_{14}= 35.100255, x_{15}= -3.64616288, x_{16}= 200.4991435.

CHAPTER 4

CONCLUSION AND SUGGESTION

In this thesis, we find the method that can solve systems of linear equations

$$(A - Ib)x = 0$$

when A is nxn matrix and $x \in R^n$. There are many iterative methods for solving systems of linear equations. We consider two numerical iterative methods to solve any nonlinear function f: the Newton method is the iteration equations

$$x^{k+1} = x^k - (f'(x^k))^{-1} f(x^k), k = 0,1,2,...$$

and $f'(x)$ is the Jacobian matrix of $f(x)$. The Newton – like method is the iteration equation

$$x^{k+1} = x^k - (H(x^k))^{-1} f(x^k), k = 0,1,2,...$$

where $H(x)$ is an invertible operater. We modified these above two methods to cove up with the new iterative method of the form

$$x^{m+1} = x^m - (M(x^m))^{-1} f(x^m)$$

when $f(x) = Ax - b$ and M is n x n invertible matrix. The matrix M are the matrices $D, U_2, U_3, ..., U_n$ and $L_2, L_3, ..., L_n$.

We have 2n-1 numerical methods, where, the first method is D- method. The second method is U_2- method. The third method is L_2- method. The U_n- method is the 2n-2th method and the L_n- method is the 2n-2th methods. We used the tolerance $\varepsilon = 0.0000001$ to solve the examples in chapter 3. The results from eight examples are somewhat as good as we expected, that is the more information we use, the better results we get. As we can see, the U_2 - method, L_2 - method up to U_n - methods and L_n - methods gave the better results than the Jacobi iterative method, which is the form

$$x^{m+1} = \frac{b_i - \sum_{j \neq i} a_{ij} x_j^m}{a_{ii}} \quad ,1 \leq i \leq n, m \geq 0.$$

.

Through study of this thesis, we find which of the 2n-1 methods. It best to solve the system (3.3). It turns out to be that the D - method is Jacobi Iterative method and the D - method with little modification, as the example in chapter 3, turns out to be the Gauss- Seidel Iterative method, which is the form

$$x^{m+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{m+1} - \sum_{j=i+1}^{n} a_{ij} x_j^m}{a_{ii}} \quad ,1 \leq i \leq n.$$

We may modify the U_2 - method, the U_n - method, the L_2 - method and the L_n - methods in the same manner. Therefore, it is suggested for further research.

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APPENDIX

The table shown the results are number of iterations and convergence of the solution for the examples of the systems of linear equations, which we use the iterative methods found in chapter 3.

Table 3.1 the result of example 3.1 from the D method.

i	x	y	er
1	800.00000000	500.00000000	12000800.0000000
2	300.00000000	166.66666667	8000500.00000000
3	633.33333333	375.00000000	5000333.33330000
4	425.00000000	236.11111111	3333541.66670000
5	563.88888889	322.91666667	2083472.22220000
6	477.08333333	265.04629630	1388975.69450000
7	534.95370370	301.21527778	868113.42593000
8	498.78472222	277.10262346	578739.87269000
9	522.89737654	292.17303241	361713.92747000
10	507.82696759	282.12609311	241141.61364000
11	517.87390689	288.40543017	150714.13645000
12	511.59456983	284.21920546	100475.67236000
13	515.78079454	286.83559590	62797.55686100
14	513.16440410	285.09133561	41864.86349100
15	514.90866439	286.18149829	26165.64869400
16	513.81850171	285.45472317	17443.69311400
17	514.54527683	285.90895762	10902.35360600
18	514.09104238	285.60613465	7268.20547040
19	514.39386535	285.79539901	4542.64733590
20	514.20460099	285.66922277	3028.41895490
21	514.33077723	285.74808292	1892.76971570

Table 3.1 (cont.)

i	x	y	er
22	514.25191708	285.69550949	1261.84123500
23	514.30449051	285.72836788	788.65405964
24	514.27163212	285.70646229	525.76718762
25	514.29353771	285.72015328	328.60585945
26	514.27984672	285.71102595	219.06966024
27	514.28897405	285.71673054	136.91910048
28	514.28326946	285.71292748	91.27903527
29	514.28707252	285.71530439	57.04962520
30	514.28469561	285.71371978	38.03292501
31	514.28628022	285.71471016	23.77067335
32	514.28528984	285.71404991	15.84705972
33	514.28595009	285.71446257	9.90445359
34	514.28553743	285.71418746	6.60295172
35	514.28581254	285.71435940	4.12684676
36	514.28564060	285.71424478	2.75122480
37	514.28575522	285.71431642	1.71950550
38	514.28568358	285.71426866	1.14634239
39	514.28573134	285.71429851	0.71646317
40	514.28570149	285.71427861	0.47764521
41	514.28572139	285.71429104	0.29852759
42	514.28570896	285.71428275	0.19901756
43	514.28571725	285.71428794	0.12439794
44	514.28571207	285.71428448	0.08293670
45	514.28571552	285.71428664	0.05182230
46	514.28571336	285.71428520	0.03454806
47	514.28571480	285.71428610	0.02159263
48	514.28571390	285.71428550	0.01440520
49	514.28571450	285.71428587	0.00898803

Table 3.1 (cont.)

i	x	y	er
50	514.28571413	285.71428562	0.00599708
51	514.28571438	285.71428578	0.00375391
52	514.28571422	285.71428568	0.00251786
53	514.28571432	285.71428574	0.00155650
54	514.28571426	285.71428570	0.00105292
55	514.28571430	285.71428573	0.00064091
56	514.28571427	285.71428571	0.00042727
57	514.28571429	285.71428572	0.00027468
58	514.28571428	285.71428571	0.00018312
59	514.28571429	285.71428572	0.00010682
60	514.28571428	285.71428571	0.00007630
61	514.28571429	285.71428571	0.00004578
62	514.28571429	285.71428571	0.00004578
63	514.28571429	285.71428571	0.00001526
64	514.28571429	285.71428571	0.00001526
65	514.28571429	285.71428571	0.00000000

Table 3.2 the result of example 3.1 from the U_2 method.

i	x	y	er
1	300.00000000	500.00000000	12000800.0000000
2	425.00000000	375.00000000	3000000.00000000
3	477.08333333	322.91666667	1250000.00000000
4	498.78472222	301.21527778	520833.33334000
5	507.82696759	292.17303241	217013.88889000
6	511.59456983	288.40543017	90422.45372000
7	513.16440410	286.83559590	37676.02238500
8	513.81850171	286.18149829	15698.34266700
9	514.09104238	285.90895762	6540.97612000

Table 3.2 the result of example 3.1 from the U_2 method.

i	x	y	er
10	514.20460099	285.79539901	2725.40670780
11	514.25191708	285.74808292	1135.58613590
12	514.27163212	285.72836788	473.16088867
13	514.27984672	285.72015328	197.15037537
14	514.28326946	285.71673054	82.14598083
15	514.28469561	285.71530439	34.22749329
16	514.28528984	285.71471016	14.26145935
17	514.28553743	285.71446257	5.94227600
18	514.28564060	285.71435940	2.47595215
19	514.28568358	285.71431642	1.03164673
20	514.28570149	285.71429851	0.42984009
21	514.28570896	285.71429104	0.17910767
22	14.28571207	285.71428794	0.07463074
23	514.28571336	285.71428664	0.03109741
24	514.28571390	285.71428610	0.01295471
25	514.28571413	285.71428588	0.00540161
26	514.28571422	285.71428578	0.00225830
27	514.28571426	285.71428574	0.00093079
28	514.28571427	285.71428573	0.00039673
29	514.28571428	285.71428572	0.00016785
30	514.28571428	285.71428572	0.00006104
31	514.28571428	285.71428571	0.00003052
32	514.28571429	285.71428571	0.00001526
33	514.28571429	285.71428571	0.00001526
34	514.28571429	285.71428571	0.00000000

Table 3.3 the result of example 3.1 from the L_2 method.

i	x	y	er
1	800.00000000	166.66666667	12000800.000
2	633.33333333	236.11111111	166.66666667
3	563.88888889	265.04629630	69.44444444
4	534.95370370	277.10262346	28.93520044
5	522.89737654	282.12609311	12.05632716
6	517.87390689	284.21920546	5.02346965
7	515.78079454	285.09133561	2.09311235
8	514.90866439	285.45472317	0.87214541
9	514.54527683	285.60613465	0.36340282
10	514.39386535	285.66922277	0.15142674
11	514.33077723	285.69550949	0.06310338
12	514.30449051	285.70646229	0.02628672
13	514.29353771	285.71102595	0.01095280
14	514.28897405	285.71292748	0.00457893
15	514.28707252	285.71371978	0.00191679
16	14.28628022	285.71404991	0.00079230
17	514.28595009	285.71418746	0.00033013
18	514.28581254	285.71424478	0.00015280
19	514.28575522	285.71426866	0.00007257
20	514.28573134	285.71427861	0.00003914
21	514.28572139	285.71428275	0.00002521
22	514.28571725	285.71428448	0.00001941
23	514.28571552	285.71428520	0.00000173
24	514.28571480	285.71428550	0.00001598
25	514.28571450	285.71428562	0.00000030
26	514.28571438	285.71428568	0.00000012
27	514.28571432	285.71428570	0.00000005

Table 3.4 the result of example 3.2 from D method.

i	A	B	C	εr
1	275.00000000	400.00000000	280.00000000	3000.00000000
2	52.50000000	267.50000000	35.00000000	2645.00000000
3	269.37500000	440.00000000	221.50000000	2490.00000000
4	86.37500000	284.93750000	22.37500000	2347.87500000
5	274.48437500	426.21875000	194.20000000	2176.68750000
6	110.29531250	289.20781250	24.82187500	2051.69062500
7	271.58164063	413.64687500	178.13968750	1909.49062500
8	125.48351563	294.67425781	31.59226562	1793.02007810
9	265.13723633	404.36017578	166.84018750	1673.59816410
10	136.27981543	300.72133496	39.17358789	1568.31804490
11	257.93947534	397.06669531	157.94357676	1465.87002540
12	144.77564360	306.54436814	46.40963667	1372.41433610
13	251.05668046	391.00976903	150.51686658	1283.52190060
14	151.85990781	311.84244312	52.96208411	1201.23030660
15	244.81782614	385.82952507	144.14707807	1123.70497090
16	157.93231018	316.55431741	58.77749429	1051.49081330
17	239.27829993	381.33947134	138.61888692	983.73153784
18	163.20096697	320.70612830	63.89723151	920.45098103
19	234.40054429	377.42520864	133.79696849	861.17331554
20	167.79597147	324.35048573	68.38958997	805.75407554
21	230.12018609	374.00461677	129.58222283	753.87654693
22	171.81217869	327.54435125	72.32604164	705.35399766
23	226.36938096	371.01240025	125.89495229	659.94555834
24	175.32568572	330.34157145	75.77341132	617.46580098
25	223.08454864	368.39380431	122.66795999	577.71712644
26	178.40057893	332.79073568	78.79174909	540.52920785
27	220.20850426	366.10177326	119.84335837	505.73389803
28	181.09203791	334.93490828	81.43418814	473.17917651

Table 3.4 (cont.)

i	A	B	C	er
29	217.69063183	364.09543401	117.37081394	442.71960764
30	183.44803104	336.81198060	83.74744730	414.22105003
31	215.48641938	362.33912266	115.20638914	387.55683076
32	185.51042748	338.45519303	85.77249931	362.60913520
33	213.55682726	360.80166143	113.31166630	339.26730764
34	187.31583492	339.89366980	87.54523907	317.42807203
35	211.86765325	359.45577277	111.6530311	296.99464550
36	188.89628346	341.15291559	89.09709894	277.87656881
37	210.38894689	358.27758353	110.20106369	259.98914922
38	190.27980635	342.25526063	90.45559845	243.25317998
39	209.09448601	357.24619721	108.93001194	227.59453238
40	191.49094174	343.22025401	91.64482951	212.94386196
41	207.96131436	356.34332175	107.81733335	99.23628060
42	192.55116955	344.06500948	92.68588268	186.41108165
43	206.96933562	355.55294455	106.84329448	174.41146352
44	193.47929300	344.80450857	93.59722081	163.18428273
45	206.10095725	354.86104830	105.99062077	152.67981568
46	194.29177235	345.45186618	94.39500633	142.85154025
47	205.34077871	354.25536224	105.24419012	133.65592860
48	195.00301685	346.01856312	95.09338788	125.05225511
49	204.67531831	353.72514460	104.59076464	117.00241560
50	195.62564037	346.51464968	95.70475117	109.47075881
51	204.09277420	353.26099202	104.01875591	102.42392836
52	196.17068506	346.94892392	96.23993867	95.83071513
53	203.58281502	352.85467280	103.51801939	89.66191892
54	196.64781726	347.32908764	96.70844187	83.89021928
55	203.13639669	352.49898090	103.07967459	78.49005436
56	197.06549883	347.66188301	97.11856963	73.43750782

Table 3.4 (cont.)

i	A	B	C	ei
57	202.74560203	352.18760818	102.69594750	68.71020281
58	197.43113733	347.95321211	97.47759551	64.28720297
59	202.40350034	351.91503246	102.36003276	60.14891960
60	197.75121732	348.20824164	97.79188682	56.27702503
61	202.10402448	351.67641964	102.06597295	52.65437129
62	198.03141538	348.43149452	98.06701746	49.26491432
63	201.84186328	351.46753795	101.80855296	46.09364281
64	198.27670079	348.62693012	98.30786686	43.12651179
65	201.61236733	351.28468289	101.58320748	40.35038033
66	198.49142367	348.79801447	98.51870645	37.75295348
67	201.41146655	351.12461155	101.38594001	35.32272768
68	198.67939210	348.94778172	98.70327545	33.04893992
69	201.23559798	350.98448509	101.21325205	30.92151996
70	198.84393969	349.07888800	98.86484718	28.93104588
71	201.08164262	350.86181836	101.06208099	27.06870221
72	198.98798467	349.19365844	99.00628708	25.32624097
73	200.94687008	350.75443589	100.92974582	23.69594510
74	199.11408166	349.29412851	99.13010360	22.17059432
75	200.82889018	350.66043327	100.81389960	20.74343312
76	199.22446698	349.38208001	99.23849259	19.40814088
77	200.72561056	350.57814336	100.71248781	18.15880379
78	199.32109830	349.45907277	99.33337632	16.98988880
79	200.63519957	350.50610677	100.62371191	15.89621897
80	199.40568938	349.52647224	99.41643755	14.87295065
81	200.55605378	350.44304592	100.54599748	13.91555195
82	199.47974041	349.58547374	99.48914937	13.01978274
83	200.48676954	350.38784245	100.47796626	12.18167581
84	199.54456469	349.63712367	99.55280129	11.39751935

Table 3.4 (cont.)

i	A	B	C	er
85	200.42611811	350.33951733	100.41841172	10.66384044
86	199.60131188	349.68233801	99.60852220	9.97738978
87	200.37302385	350.29721351	100.36627767	9.33512717
88	199.65098837	349.72191866	99.65730029	8.73420819
89	200.32654512	350.26018074	100.32063951	8.17197143
90	199.69447518	349.75656756	99.70000063	7.64592687
91	200.28585764	350.22776225	100.28068787	7.15374474
92	199.73254354	349.78689922	99.73738052	6.69324528
93	200.25023981	350.19938310	100.24571419	6.26238899
94	199.76586858	349.81345155	99.77010287	5.85926771
95	200.21905996	350.17453999	100.21509823	5.48209606
96	199.79504133	349.83669546	99.79874803	5.12920364
97	200.19176511	350.15279233	100.18829702	4.79902753
98	199.82057915	349.85704319	99.82382400	4.49010547
99	200.16787120	350.13375442	100.16483523	4.20106927
100	199.84293497	349.87485559	99.84577551	3.93063885
101	200.14695447	350.11708864	100.14429678	3.67761653
102	199.86250526	349.89044857	99.86499186	3.44088172
103	200.12864396	350.10249941	100.12631742	3.21938594
104	199.87963708	349.90409867	99.88181386	3.01214826
105	200.11261494	350.08972799	100.11057828	2.81825083
106	199.89463429	349.91604796	99.89653984	2.63683494
107	200.09858313	350.07854790	100.09680024	2.46709710
108	199.90776284	349.92650837	99.90943096	2.30828561
109	200.08629968	350.06876084	100.08473894	2.15969709
110	199.91925558	349.93566542	99.92071586	2.02067350
111	200.07554675	350.06019324	100.07418048	1.89059909
112	199.92931633	349.94368150	99.93059465	1.76889782

Table 3.4 (cont.)

i	A	B	C	er
113	200.06613364	350.05269317	100.06493760	1.65503066
114	199.93812351	349.95069878	99.93924255	1.54849335
115	200.05789339	350.04612761	100.05684638	1.44881404
116	199.94583331	349.95684171	99.94681292	1.35555126
117	200.05067988	350.04038012	100.04976333	1.26829198
118	199.95258247	349.96221923	99.95344002	1.18664974
119	200.04436518	350.03534876	100.04356283	1.11026295
120	199.95849069	349.96692671	99.95924139	1.03879332
121	200.03883728	350.03094431	100.03813490	0.97192431
122	199.96366275	349.97104763	99.96431991	0.90935980
123	200.03399816	350.02708865	100.03338330	0.85082266
124	199.96819036	349.97465510	99.96876564	0.79605367
125	200.02976199	350.02371341	100.02922374	0.74481026
126	199.97215384	349.97781307	99.97265744	0.69686548
127	200.02605365	350.02075872	100.02558247	0.65200698
128	199.97562347	349.98057756	99.97606432	0.61003612
129	200.02280737	350.01817219	100.02239490	0.57076699
130	199.97866078	349.98299759	99.97904670	0.53402569
131	200.01996557	350.01590793	100.01960449	0.49964949
132	199.98131965	349.98511609	99.98165748	0.46748615
133	200.01747787	350.01392581	100.01716178	0.43739322
134	199.98364722	349.98697062	99.98394296	0.40923743
135	200.01530013	350.01219065	100.01502342	0.38289407
136	199.98568477	349.98859408	99.98594366	0.35824649
137	200.01339373	350.01067170	100.01315151	0.33518551
138	199.98746845	349.99001526	99.98769508	0.31360903
139	200.01172487	350.00934201	100.01151283	0.29342144
140	199.98902988	349.99125936	99.98922827	0.27453338

Table 3.4 (cont.)

i	A	B	C	er
141	200.01026396	350.00817799	100.01007833	0.25686116
142	199.99039675	349.99234844	99.99057043	0.24032655
143	200.00898507	350.00715902	100.00882257	0.22485629
144	199.99159332	349.99330182	99.99174535	0.21038189
145	200.00786553	350.00626700	100.00772328	0.19683923
146	199.99264079	349.99413641	99.99277388	0.18416834
147	200.00688549	350.00548614	100.00676096	0.17231308
148	199.99355774	349.99486702	99.99367425	0.16122098
149	200.00602756	350.00480256	100.00591855	0.15084289
150	199.99436045	349.99550659	99.99446244	0.14113287
151	200.00527652	350.00420417	100.00518110	0.13204790
152	199.99506314	349.99606646	99.99515242	0.12354774
153	200.00461907	350.00368033	100.00453553	0.11559474
155	200.00404353	350.00322176	100.00397041	0.10119165
156	199.99621676	349.99698563	99.99628518	0.09467776
157	200.00353971	350.00282033	100.00347569	0.08858317
158	199.99668815	349.99736122	99.99674804	0.08288092
159	200.00309866	350.00246892	100.00304262	0.07754572
160	199.99710080	349.99769001	99.99715324	0.07255397
161	200.00271257	350.00216129	100.00266351	0.06788353
162	199.99746204	349.99797784	99.99750794	0.06351375
163	200.00237458	350.00189199	100.00233164	0.05942526
164	199.99777827	349.99822980	99.99781845	0.05559995
165	200.00207871	350.00165625	100.00204112	0.05202088
166	199.99805510	349.99845037	99.99809027	0.04867220
167	200.00181970	350.00144988	100.00178679	0.04553909
168	199.99829743	349.99864345	99.99832822	0.04260766
169	200.00159297	350.00126923	100.00156416	0.03986492

Table 3.4 (cont.)

i	A	B	C	er
170	199.99850957	349.99881248	99.99853653	0.03729875
171	200.00139448	350.00111108	100.00136927	0.03489775
172	199.99869528	349.99896044	99.99871888	0.03265133
173	200.00122073	350.00097264	100.00119866	0.03054950
174	199.99885785	349.99908997	99.99887850	0.02858298
175	200.00106863	350.00085145	100.00104930	0.02674303
176	199.99900016	349.99920336	99.99901824	0.02502154
177	200.00093548	350.00074536	100.00091856	0.02341085
178	199.99912474	349.99930262	99.99914057	0.02190385
179	200.00081892	350.00065249	100.00080411	0.02049386
180	199.99923380	349.99938951	99.99924765	0.01917464
181	200.00071688	350.00057119	100.00070392	0.01794033
182	199.99932927	349.99946558	99.99934140	0.01678548
183	200.00062756	350.00050002	100.00061621	0.01570496
184	199.99941284	349.99953217	99.99942346	0.01469401
185	200.00054936	350.00043772	100.00053943	0.01374812
186	199.99948600	349.99959046	99.99949530	0.01286314
187	200.00048091	350.00038318	100.00047222	0.01203511
188	199.99955004	349.99964149	99.99955818	0.01126040
189	200.00042099	350.00033543	100.00041338	0.01053554
190	199.99960611	349.99968616	99.99961323	0.00985736
191	200.00036854	350.00029364	100.00036187	0.00922282
192	199.99965519	349.99972526	99.99966142	0.00862913
193	200.00032262	350.00025705	100.00031678	0.00807365
195	200.00028242	350.00022502	100.00027731	0.00706768
196	199.99973576	349.99978946	99.99974054	0.00661272
197	200.00024723	350.00019698	100.00024276	0.00618705
198	199.99976869	349.99981570	99.99977287	0.00578878

Table 3.4 (cont.)

i	A	B	C	er
199	200.00021642	350.00017244	100.00021251	0.00541614
200	199.99979751	349.99983866	99.99980117	0.00506750
201	200.00018946	350.00015095	100.00018603	0.00474129
202	199.99982274	349.99985876	99.99982594	0.00443609
203	200.00016585	350.00013215	100.00016285	0.00415052
204	199.99984482	349.99987636	99.99984763	0.00388336
205	200.00014519	350.00011568	100.00014256	0.00363337
206	199.99986416	349.99989177	99.99986662	0.00339949
207	200.00012710	350.00010127	100.00012480	0.00318065
208	199.99988109	349.99990525	99.99988324	0.00297591
209	200.00011126	350.00008865	100.00010925	0.00278434
210	199.99989590	349.99991706	99.99989778	0.00260511
211	200.00009740	350.00007760	100.00009564	0.00243741
212	199.99990887	349.99992739	99.99991052	0.00228051
213	200.00008526	350.00006793	100.00008372	0.00213371
214	199.99992023	349.99993644	99.99992167	0.00199637
215	200.00007464	350.00005947	100.00007329	0.00186785
216	199.99993017	349.99994436	99.99993143	0.00174762
217	200.00006534	350.00005206	100.00006416	0.00163512
218	199.99993887	349.99995129	99.99993997	0.00152987
219	200.00005720	350.00004557	100.00005616	0.00143138
220	199.99994648	349.99995736	99.99994745	0.00133925
221	200.00005007	350.00003989	100.00004916	0.00125303
222	199.99995315	349.99996267	99.99995400	0.00117238
223	200.00004383	350.00003492	100.00004304	0.00109691
224	199.99995899	349.99996732	99.99995973	0.00102630
225	200.00003837	350.00003057	100.00003768	0.00096023
226	199.99996410	349.99997140	99.99996475	0.00089842

Table 3.4 (cont.)

i	A	B	C	er
227	200.00003359	350.00002676	100.00003298	0.00084059
228	199.99996857	349.99997496	99.99996914	0.00078648
229	200.00002940	350.00002343	100.00002887	0.00073585
230	199.99997249	349.99997808	99.99997299	0.00068848
231	200.00002574	350.00002051	100.00002527	0.00064416
232	199.99997592	349.99998081	99.99997635	0.00060270
233	200.00002253	350.00001795	100.00002213	0.00056390
234	199.99997892	349.99998320	99.99997930	0.00052761
235	200.00001973	350.00001572	100.00001937	0.00049364
236	199.99998154	349.99998530	99.99998188	0.00046186
237	200.00001727	350.00001376	100.00001696	0.00043212
238	199.99998384	349.99998713	99.99998414	0.00040432
239	200.00001512	350.00001204	100.00001484	0.00037829
240	199.99998586	349.99998873	99.99998611	0.00035394
241	200.00001323	350.00001054	100.00001299	0.00033116
242	199.99998762	349.99999014	99.99998784	0.00030984
243	200.00001158	350.00000923	100.00001137	0.00028989
244	199.99998916	349.99999136	99.99998936	0.00027124
245	200.00001014	350.00000808	100.00000996	0.00025377
246	199.99999051	349.99999244	99.99999068	0.00023744
247	200.00000888	350.00000707	100.00000872	0.00022215
248	199.99999169	349.99999338	99.99999184	0.00020786
249	200.00000777	350.00000619	100.00000763	0.00019448
250	199.99999273	349.99999421	99.99999286	0.00018196
251	200.00000680	350.00000542	100.00000668	0.00017025
252	199.99999363	349.99999493	99.99999375	0.00015929
253	200.00000596	350.00000475	100.00000585	0.00014903
254	199.99999443	349.99999556	99.99999453	0.00013945

Table 3.4 (cont.)

i	A	B	C	εr
255	200.00000521	350.00000415	100.00000512	0.00013046
256	199.99999512	349.99999611	99.99999521	0.00012207
257	200.00000456	350.00000364	100.00000448	0.00011420
258	199.99999573	349.99999660	99.99999581	0.00010685
259	200.00000399	350.00000318	100.00000392	0.00009997
260	199.99999626	349.99999702	99.99999633	0.00009354
261	200.00000350	350.00000279	100.00000343	0.00008751
262	199.99999673	349.99999739	99.99999679	0.00008189
263	200.00000306	350.00000244	100.00000301	0.00007661
264	199.99999714	349.99999772	99.99999719	0.00007168
265	200.00000268	350.00000214	100.00000263	0.00006706
266	199.99999749	349.99999800	99.99999754	0.00006275
267	200.00000235	350.00000187	100.00000230	0.00005870
268	199.99999781	349.99999825	99.99999785	0.00005493
269	200.00000205	350.00000164	100.00000202	0.00005139
270	199.99999808	349.99999847	99.99999811	0.00004809
271	200.00000180	350.00000143	100.00000177	0.00004499
272	199.99999832	349.99999866	99.99999835	0.00004210
273	200.00000157	350.00000125	100.00000155	0.00003938
274	199.99999853	349.99999883	99.99999855	0.00003686
275	200.00000138	350.00000110	100.00000135	0.00003448
276	199.99999871	349.99999897	99.99999873	0.00003226
277	200.00000121	350.00000096	100.00000118	0.00003017
278	199.99999887	349.99999910	99.99999889	0.00002824
279	200.00000106	350.00000084	100.00000104	0.00002642
280	199.99999901	349.99999921	99.99999903	0.00002472
281	200.00000092	350.00000074	100.00000091	0.00002313
282	199.99999914	349.99999931	99.99999915	0.00002165

Table 3.4 (cont.)

i	A	B	C	er
283	200.00000081	350.00000064	100.00000079	0.00002025
284	199.99999924	349.99999940	99.99999926	0.00001895
285	200.00000071	350.00000056	100.00000070	0.00001772
286	199.99999934	349.99999947	99.99999935	0.00001658
287	200.00000062	350.00000049	100.00000061	0.00001551
288	199.99999942	349.99999954	99.99999943	0.00001452
289	200.00000054	350.00000043	100.00000053	0.00001357
290	199.99999949	349.99999960	99.99999950	0.00001271
291	200.00000048	350.00000038	100.00000047	0.00001188
292	199.99999956	349.99999965	99.99999956	0.00001112
293	200.00000042	350.00000033	100.00000041	0.00001040
294	199.99999961	349.99999969	99.99999962	0.00000974
295	200.00000036	350.00000029	100.00000036	0.00000911
296	199.99999966	349.99999973	99.99999967	0.00000853
297	200.00000032	350.00000025	100.00000031	0.00000797
298	199.99999970	349.99999976	99.99999971	0.00000747
299	200.00000028	350.00000022	100.00000027	0.00000698
300	199.99999974	349.99999979	99.99999974	0.00000654
301	200.00000024	350.00000019	100.00000024	0.00000611
302	199.99999977	349.99999982	99.99999978	0.00000572
303	200.00000021	350.00000017	100.00000021	0.00000535
304	199.99999980	349.99999984	99.99999980	0.00000500
305	200.00000019	350.00000015	100.00000018	0.00000467
306	199.99999982	349.99999986	99.99999983	0.00000438
307	200.00000016	350.00000013	100.00000016	0.00000410
308	199.99999985	349.99999988	99.99999985	0.00000384
309	200.00000014	350.00000011	100.00000014	0.00000358
310	199.99999987	349.99999989	99.99999987	0.00000336

Table 3.4 (cont.)

i	A	B	C	er
311	200.00000013	350.00000010	100.00000012	0.00000314
312	199.99999988	349.99999991	99.99999988	0.00000294
313	200.00000011	350.00000009	100.00000011	0.00000275
314	199.99999990	349.99999992	99.99999990	0.00000258
315	200.00000010	350.00000008	100.00000009	0.00000241
316	199.99999991	349.99999993	99.99999991	0.00000226
317	200.00000008	350.00000007	100.00000008	0.00000211
318	199.99999992	349.99999994	99.99999992	0.00000198
319	200.00000007	350.00000006	100.00000007	0.00000185
320	199.99999993	349.99999995	99.99999993	0.00000174
321	200.00000007	350.00000005	100.00000006	0.00000161
322	199.99999994	349.99999995	99.99999994	0.00000152
323	200.00000006	350.00000005	100.00000006	0.00000142
324	199.99999995	349.99999996	99.99999995	0.00000133
325	200.00000005	350.00000004	100.00000005	0.00000124
326	199.99999995	349.99999996	99.99999995	0.00000117
327	200.00000004	350.00000003	100.00000004	0.00000108
328	199.99999996	349.99999997	99.99999996	0.00000102
329	200.00000004	350.00000003	100.00000004	0.00000095
330	199.99999996	349.99999997	99.99999996	0.00000090
331	200.00000003	350.00000003	100.00000003	0.00000084
332	199.99999997	349.99999997	99.99999997	0.00000079
333	200.00000003	350.00000002	100.00000003	0.00000073
334	199.99999997	349.99999998	99.99999997	0.00000069
335	200.00000003	350.00000002	100.00000003	0.00000064
336	199.99999998	349.99999998	99.99999998	0.00000061
337	200.00000002	350.00000002	100.00000002	0.00000056*
338	199.99999998	349.99999998	99.99999998	0.00000053

Table 3.4 (cont.)

i	A	B	C	er
339	200.00000002	350.00000002	100.00000002	0.00000049
340	199.99999998	349.99999999	99.99999998	0.00000046
341	200.00000002	350.00000001	100.00000002	0.00000043
342	199.99999998	349.99999999	99.99999998	0.00000041
343	200.00000002	350.00000001	100.00000001	0.00000038
344	199.99999999	349.99999999	99.99999999	0.00000036
345	200.00000001	350.00000001	100.00000001	0.00000033
346	199.99999999	349.99999999	99.99999999	0.00000031
347	200.00000001	350.00000001	100.00000001	0.00000029
348	199.99999999	349.99999999	99.99999999	0.00000028
349	200.00000001	350.00000001	100.00000001	0.00000025
350	199.99999999	349.99999999	99.99999999	0.00000024
351	200.00000001	350.00000001	100.00000001	0.00000022
352	199.99999999	349.99999999	99.99999999	0.00000021
353	200.00000001	350.00000001	100.00000001	0.00000020
354	199.99999999	349.99999999	99.99999999	0.00000019
355	200.00000001	350.00000001	100.00000001	0.00000017
356	199.99999999	349.99999999	99.99999999	0.00000016
357	200.00000001	350.00000000	100.00000001	0.00000015
358	199.99999999	350.00000000	99.99999999	0.00000015
359	200.00000001	350.00000000	100.00000001	0.00000013
360	200.00000000	350.00000000	100.00000000	0.00000012
361	200.00000000	350.00000000	100.00000000	0.00000011
362	200.00000000	350.00000000	100.00000000	0.00000011
363	200.00000000	350.00000000	100.00000000	0.00000009

Table 3.5 the result of example 3.2 from the U_2 method.

i	A	B	C	er
1	198.75000000	355.00000000	280.00000000	3000.00000000
2	64.76562500	350.93750000	98.75000000	1643.75000000
3	189.08105469	397.42578125	180.76562500	1221.79687500
4	137.28472900	358.56420898	87.58105469	960.60058594
5	203.61251755	372.80676575	134.20347900	616.25848389
6	174.09331689	351.01629543	88.70978317	499.99132919
7	206.17542046	359.16896863	115.13749169	327.65803141
8	188.95800630	348.75549970	92.62716027	255.99898983
9	204.59456786	353.74024776	107.12299634	174.44500292
10	194.96627124	348.76592603	95.74716018	131.63981209
11	202.78003045	351.63839767	103.51386684	92.21763502
12	197.56689259	349.19082911	97.67662266	68.11385931
13	201.54986538	350.77067050	101.78353280	48.47255895
14	198.77869686	349.53461417	98.76181257	35.39527714
15	200.83341119	350.38091752	100.91893622	25.39311595
16	199.37419838	349.74639783	99.34758628	18.43754931
17	200.44089270	350.19367035	100.47692184	13.27915325
18	199.67604497	349.86505459	99.65799624	9.61602335
19	200.23153039	350.09988972	100.24835118	6.93824698
20	199.83149828	349.92895333	99.82112588	5.01816219
21	200.12118786	350.05187094	100.12951970	3.62370923
22	199.91216739	349.96277134	99.90653891	2.61946455
23	200.06334118	350.02701855	100.06759103	1.89224388
24	199.95417362	349.98053244	99.95118787	1.36751971
25	200.03308598	350.01409248	100.03528285	0.98801948
26	199.97608057	349.98982916	99.97451142	0.71396681
27	200.01727775	350.00735472	100.01841999	0.51586795
28	199.98751294	349.99468826	99.98669146	0.37276275

Table 3.5 (cont.)

i	A	B	C	er
29	200.00902158	350.00383930	100.00961693	0.26934258
30	199.99348071	349.99722638	99.99305133	0.19462170
31	200.00471040	350.00200439	100.00502103	0.14062700
32	199.99659628	349.99855180	99.99637200	0.10161359
33	200.00245938	350.00104648	100.00262151	0.07342286
34	199.99822290	349.99924387	99.99810578	0.05305338
35	200.00128407	350.00054637	100.00136871	0.03833482
36	199.99907216	349.99960521	99.99901101	0.02769968
37	200.00067043	350.00028527	100.00071462	0.02001498
38	199.99951557	349.99979388	99.99948364	0.01446227
39	200.00035004	350.00014894	100.00037311	0.01045002
40	199.99974707	349.99989238	99.99973040	0.00755089
41	200.00018276	350.00007776	100.00019480	0.00545605
42	199.99986794	349.99994381	99.99985924	0.00394240
43	200.00009542	350.00004060	100.00010171	0.00284866
44	199.99993105	349.99997066	99.99992651	0.00205837
45	200.00004982	350.00002120	100.00005310	0.00148732
46	199.99996400	349.99998468	99.99996163	0.00107470
47	200.00002601	350.00001107	100.00002773	0.00077654
48	199.99998120	349.99999200	99.99997997	0.00056111
49	200.00001358	350.00000578	100.00001448	0.00040544
50	199.99999019	349.99999583	99.99998954	0.00029296
51	200.00000709	350.00000302	100.00000756	0.00021168
52	199.99999488	349.99999782	99.99999454	0.00015296
53	200.00000370	350.00000158	100.00000395	0.00011052
54	199.99999732	349.99999886	99.99999715	0.00007986
55	200.00000193	350.00000082	100.00000206	0.00005770
56	199.99999860	349.99999941	99.99999851	0.00004170

Table 3.5 (cont.)

i	A	B	C	er
57	200.00000101	350.00000043	100.00000108	0.00003012
58	199.99999927	349.99999969	99.99999922	0.00002177
59	200.00000053	350.00000022	100.00000056	0.00001572
60	199.99999962	349.99999984	99.99999959	0.00001137
61	200.00000028	350.00000012	100.00000029	0.00000821
62	199.99999980	349.99999992	99.99999979	0.00000594
63	200.00000014	350.00000006	100.00000015	0.00000428
64	199.99999990	349.99999996	99.99999989	0.00000310
65	200.00000008	350.00000003	100.00000008	0.00000224
66	199.99999995	349.99999998	99.99999994	0.00000162
67	200.00000004	350.00000002	100.00000004	0.00000117
68	199.99999997	349.99999999	99.99999997	0.00000085
69	200.00000002	350.00000001	100.00000002	0.00000061
70	199.99999999	349.99999999	99.99999998	0.00000045
71	200.00000001	350.00000000	100.00000001	0.00000032
72	199.99999999	350.00000000	99.99999999	0.00000024
73	200.00000001	350.00000000	100.00000001	0.00000016
74	200.00000000	350.00000000	100.00000000	0.00000012
75	200.00000000	350.00000000	100.00000000	0.00000008

Table 3.6 the result of example 3.2 from the U_3 method.

i	A	B	C	er
1	-3.75000000	385.00000000	360.00000000	5150.00000000
2	100.10937500	424.81250000	208.25000000	766.25000000
3	166.88222656	392.44296875	130.00937500	598.92187500
4	193.87101318	365.83551758	102.89347656	269.12519531
5	201.06043692	353.72869714	97.34318506	81.72903076
6	201.59537656	350.00171679	97.87225899	17.02421713

Table 3.6 (cont.)

i	A	B	C	er
7	200.85798702	349.44178988	99.04208735	6.91902106
8	200.30766024	349.64388353	99.70849184	4.80680152
9	200.06743535	349.85670727	99.95785044	2.34744659
10	199.99684100	349.96206835	100.01685588	0.77547697
11	199.98787084	349.99731249	100.01706806	0.14224958
12	199.99274151	350.00397646	100.00835250	0.06151810
13	199.99719209	350.00293812	100.00276451	0.03768131
14	199.99929873	350.00127658	100.00050950	0.02017622
15	199.99997413	350.00037310	99.99991013	0.00721014
16	200.00008870	350.00004637	99.99986628	0.00157004
17	200.00006043	349.99997359	99.99992824	0.00053890
18	200.00002522	349.99997621	99.99997431	0.00028691
19	200.00000701	349.99998880	99.99999439	0.00017081
20	200.00000069	349.99999642	100.00000028	0.00006585
21	199.99999939	349.99999940	100.00000102	0.00001636
22	199.99999951	350.00000015	100.00000061	0.00000465
23	199.99999978	350.00000019	100.00000023	0.00000211
24	199.99999993	350.00000010	100.00000006	0.00000143
25	199.99999999	350.00000003	100.00000000	0.00000059
26	200.00000000	350.00000001	99.99999999	0.00000017
27	200.00000000	350.00000000	99.99999999	0.00000004

Table 3.7 the result of example 3.2 from the L_2 method.

i	A	B	C	er
1	362.50000000	293.75000000	242.50000000	5150.00000000
2	107.18750000	360.78125000	-1.81250000	2351.25000000
3	273.66406250	338.62109375	160.23906250	1676.15625000
4	157.66542969	356.10751953	53.35855469	1125.47578120

Table 3.7 (cont.)

i	A	B	C	ei
5	233.45420410	344.93325928	127.42743848	758.03150390
6	180.69610632	352.79508722	78.80944265	512.46759815
7	215.19414621	347.70056623	112.50210971	344.88444870
8	191.19827616	351.27533449	90.37337848	233.17026692
9	206.90113252	348.95608912	105.69859866	156.92776683
10	195.98702873	350.58183597	95.62658610	106.09020443
11	203.13460143	349.52605276	102.59736166	71.40461474
12	198.17046556	350.26542680	98.01306842	48.27003714
13	201.42384198	349.78481190	101.18379590	32.49020652
14	199.16595010	350.12107598	99.09726442	21.96242428
15	200.64678269	349.90229255	100.53951292	14.78353751
16	199.61979217	350.05522568	99.58984011	9.99270836
17	200.29381350	349.95563322	100.24587141	6.72672966
18	199.82668814	350.02518808	99.81363667	4.54659669
19	200.13347548	349.97985309	100.11204588	3.06076016
20	199.92100232	350.01148737	99.91531977	2.06866386
21	200.06063833	349.99085089	100.05105825	1.39268965
22	199.96399359	350.00523864	99.96152154	0.94122552
23	200.02754918	349.99584502	100.02326584	0.63369332
24	199.98358937	350.00238886	99.98251495	0.42825035
25	200.01251657	349.99811298	100.01060119	0.28833918
26	199.99252086	350.00108927	99.99205435	0.19485072
27	200.00568692	349.99914295	100.00483030	0.13119821
28	199.99659154	350.00049666	99.99638918	0.08865568
29	200.00258395	349.99961073	100.00220079	0.05969692
30	199.99844673	350.00022644	99.99835906	0.04033772
31	200.00117410	349.99982319	100.00100269	0.02716287
32	199.99929219	350.00010323	99.99925425	0.01835339

Table 3.7 (cont.)

i	A	B	C	cr
33	200.00053351	349.99991969	100.00045681	0.01235945
34	199.99967747	350.00004706	99.99966107	0.00835068
35	200.00024243	349.99996352	100.00020811	0.00562370
36	199.99985304	350.00002145	99.99984596	0.00379951
37	200.00011017	349.99998343	100.00009481	0.00255885
38	199.99993304	350.00000978	99.99992999	0.00172875
39	200.00005006	349.99999247	100.00004319	0.00116431
40	199.99996949	350.00000446	99.99996818	0.00078657
41	200.00002275	349.99999658	100.00001967	0.00052977
42	199.99998610	350.00000203	99.99998554	0.00035789
43	200.00001034	349.99999845	100.00000896	0.00024105
44	199.99999367	350.00000093	99.99999343	0.00016283
45	200.00000470	349.99999929	100.00000408	0.00010968
46	199.99999712	350.00000042	99.99999701	0.00007409
47	200.00000214	349.99999968	100.00000186	0.00004990
48	199.99999869	350.00000019	99.99999864	0.00003371
49	200.00000097	349.99999985	100.00000085	0.00002270
50	199.99999940	350.00000009	99.99999938	0.00001534
51	200.00000044	349.99999993	100.00000039	0.00001034
52	199.99999973	350.00000004	99.99999972	0.00000698
53	200.00000020	349.99999997	100.00000018	0.00000470
54	199.99999988	350.00000002	99.99999987	0.00000318
55	200.00000009	349.99999999	100.00000008	0.00000214
56	199.99999994	350.00000001	99.99999994	0.00000145
57	200.00000004	349.99999999	100.00000004	0.00000097
58	199.99999997	350.00000000	99.99999997	0.00000066
59	200.00000002	350.00000000	100.00000002	0.00000044
60	199.99999999	350.00000000	99.99999999	0.00000030

Table 3.7 (cont.)

i	A	B	C	er
61	200.00000001	350.00000000	100.00000001	0.00000020
62	199.99999999	350.00000000	99.99999999	0.00000014
63	200.00000000	350.00000000	100.00000000	0.00000009

Table 3.8 the result of example 3.2 from the L_3 method.

i	A	B	C	er
1	362.500000000	293.750000000	25.000000000	5150.000000000
2	270.312500000	333.593750000	64.375000000	393.750000000
3	230.820312500	343.496093750	84.109375000	197.343750000
4	213.543945310	347.200683590	92.993359380	88.839843750
5	205.954809570	348.774255370	96.917412110	39.240527340
6	202.618377080	349.461458440	98.644390380	17.269782720
7	201.151342610	349.763231100	99.403902000	7.595116160
8	200.506265730	349.895891640	99.737883910	3.339819130
9	200.222614160	349.954221940	99.884742730	1.468588190
10	200.097887470	349.979870580	99.949319290	0.645765580
11	200.043042890	349.991148730	99.977714770	0.283954870
12	200.018926740	349.996107940	99.990200780	0.124860100
13	200.008322430	349.998288590	99.995691110	0.054903250
14	200.003659520	349.999247460	99.998105300	0.024141960
15	200.001609160	349.999669100	99.999166870	0.010615650
16	200.000707580	349.999854500	99.999633660	0.004667900
17	200.000311130	349.999936020	99.999838910	0.002052560
18	200.000136810	349.999971870	99.999929170	0.000902550
19	200.000060160	349.999987630	99.999968850	0.000396870
20	200.000026450	349.999994560	99.999986300	0.000174510
21	200.000011630	349.999997610	99.999993980	0.000076740
22	200.000005110	349.999998950	99.999997350	0.000033750
23	200.000002250	349.999999540	99.999998840	0.000014840
24	200.000000990	349.999999800	99.999999490	0.000006530

Table 3.8 (cont.)

i	A	B	C	er
25	200.000000430	349.999999910	99.999999770	0.000002870
26	200.000000190	349.999999960	99.999999900	0.000001260
27	200.000000080	349.999999980	99.999999960	0.000000560
28	200.000000040	349.999999990	99.999999980	0.000000240
29	200.000000020	350.000000000	99.999999990	0.000000110
30	200.000000010	350.000000000	100.00000000	0.000000050

Table 3.9 the result of example 3.3 from the *D* method.

i	x	y	z	w	er
1	-0.88888889	0.44444444	1.33333333	3.000000000	20.00000000
2	-0.86419753	0.09876543	1.18518519	3.44444444	1.33333333
3	-0.74759945	0.02194787	1.59670782	2.87654321	1.25925926
4	-0.57971346	0.1283341	1.83630544	2.19478738	0.83539095
5	-0.53246033	0.29052143	1.77432302	2.00007621	0.7432556
6	-0.61388541	0.35117821	1.53741243	2.33918017	0.75798489
7	-0.72022332	0.28405071	1.37756653	2.7788294	0.56096279
8	-0.7525135	0.18048089	1.41378789	2.91075822	0.47885964
9	-0.70180268	0.13996319	1.56395477	2.69968739	0.48279082
10	-0.63362998	0.18145168	1.66803401	2.41777427	0.36294853
11	-0.24768416	1.6474091	2.32849932	0.30859732	0.61179629
12	-0.64332315	0.27466663	1.55222415	2.45975552	0.30738855
13	-0.6870101	0.24906872	1.48451921	2.64043226	0.23464167
14	-0.70172133	0.2067329	1.49608144	2.70062833	0.19876578
15	-0.68214555	0.18880214	1.55638328	2.61910569	0.19561677
16	-0.65416319	0.20456825	1.60039158	2.50336654	0.15160067
17	-0.64428209	0.23161655	1.59403953	2.4629081	0.1279554
18	-0.65642117	0.2435089	1.55585777	2.51347566	0.12442636
19	-0.67433588	0.23381651	1.52727406	2.58758119	0.09789022
20	-0.68095385	0.21654353	1.53066934	2.61469483	0.08232785

Table 3.9 (cont.)

i	x	y	z	w	er
21	-0.67343684	0.20867036	1.55483174	2.58337158	0.07910515
22	-0.66197304	0.21461683	1.57338343	2.53594583	0.06317208
23	-0.65755203	0.22564209	1.57163871	2.51782327	0.05294308
24	-0.66219996	0.23084563	1.55635678	2.53719751	0.05026679
25	-0.66953228	0.22720536	1.54432475	2.56753445	0.04074402
26	-0.67247867	0.22017131	1.54516776	2.57961826	0.03402899
27	-0.66960931	0.2167376	1.55482751	2.56765352	0.03192562
28	-0.66492173	0.21896076	1.5626257	2.548257	0.02626395
29	-0.66296237	0.22344631	1.56226089	2.54021754	0.02186094
30	-0.66473073	0.22570881	1.55615855	2.54759411	0.0202664
31	-0.66772611	0.22435464	1.55110777	2.5599898	0.01692068
32	-0.66902648	0.2214956	1.55122974	2.56532762	0.0140369
33	-0.66793864	0.2200069	1.55508244	2.56078795	0.0128585
34	-0.66602549	0.22082938	1.55835162	2.55286999	0.01089536
35	-0.66516408	0.22265082	1.55834471	2.54933263	0.00900862
36	-0.66583196	0.2236291	1.5559138	2.55212102	0.00815412
37	-0.66705331	0.22313114	1.55379914	2.55717635	0.00701189
38	-0.66762296	0.22197128	1.5537582	2.55951645	0.00577873
39	-0.66721379	0.22132922	1.55529102	2.55780733	0.00516814
40	-0.66643444	0.22162964	1.55665805	2.5545812	0.00451025
41	-0.66605834	0.22236785	1.55671317	2.55303567	0.00370505
42	-0.66630845	0.22278876	1.55574725	2.55408088	0.00327384
43	-0.66680551	0.22260823	1.55486408	2.5561387	0.00289963
44	-0.66705344	0.22213861	1.55481038	2.5571579	0.00237436
45	-0.66690096	0.22186299	1.55541865	2.55652029	0.00207273
46	-0.66658409	0.22197098	1.55598889	2.5552083	0.00186322
47	-0.66642087	0.22226959	1.55603498	2.55453717	0.00152086
48	-0.66651358	0.22244988	1.55565221	2.55492507	0.00131155

Table 3.9 (cont.)

i	x	y	z	w	er
49	-0.66671549	0.22238561	1.55528423	2.55576114	0.00119665
50	-0.66682279	0.22219584	1.55524726	2.55620248	0.0009737
51	-0.6667666	0.22207802	1.55548797	2.5559672	0.00082944
52	-0.66663801	0.22211605	1.5557253	2.55543467	0.00076816
53	-0.66656755	0.2222366	1.5557537	2.55514481	0.0006231
54	-0.66660149	0.22231351	1.55560244	2.55528704	0.00052424
55	-0.66668335	0.22229117	1.55544947	2.55562607	0.00049286
56	-0.66672956	0.22221463	1.55542827	2.55581622	0.00039855
57	-0.66670913	0.22216446	1.55552325	2.55573055	0.00033114
58	-0.66665705	0.22217749	1.5556218	2.55551481	0.00031607
59	-0.66662678	0.22222605	1.55563728	2.55539023	0.0002548
60	-0.66663902	0.22225874	1.55557769	2.55544161	0.00020905
61	-0.66667214	0.22225123	1.55551424	2.55557882	0.0002026
62	-0.66669195	0.22222043	1.55550311	2.55566036	0.00016282
63	-0.66668465	0.22219914	1.55554047	2.55562969	0.00013189
64	-0.6666636	0.22220343	1.5555813	2.55554247	0.0001298
65	-0.66665065	0.22222295	1.5555892	2.55548915	0.000104
66	-0.66665498	0.2222368	1.5555658	2.55550736	0.00008315
67	-0.66666835	0.22223439	1.55553953	2.55556278	0.00008312
68	-0.66667681	0.22222202	1.55553399	2.55559761	0.00006639
69	-0.66667426	0.22221302	1.55554863	2.55558687	0.00005239
70	-0.66666577	0.22221435	1.55556552	2.55555167	0.0000532
71	-0.66666025	0.22222218	1.55556938	2.55552894	0.00004237
72	-0.66666174	0.22222803	1.55556022	2.55553523	0.00003299
73	-0.66666713	0.22222732	1.55554937	2.55555757	0.00003404
74	-0.66667073	0.22222236	1.5555467	2.55557239	0.00002702
75	-0.66666986	0.22221856	1.55555242	2.55556874	0.00002076
76	-0.66666644	0.22221893	1.55555939	2.55555456	0.00002177

Table 3.9 (cont.)

i	x	y	z	w	er
77	-0.6666641	0.22222207	1.55556122	2.55554491	0.00001723
78	-0.6666646	0.22222453	1.55555765	2.55554701	0.00001305
79	-0.66666676	0.22222435	1.55555318	2.555556	0.00001391
80	-0.66666829	0.22222237	1.55555193	2.55556228	0.00001098
81	-0.66666801	0.22222077	1.55555415	2.55556109	0.0000082
82	-0.66666663	0.22222085	1.55555702	2.5555554	0.00000889
83	-0.66666564	0.2222221	1.55555788	2.55555131	0.00000699
84	-0.6666658	0.22222314	1.55555649	2.55555197	0.00000515
85	-0.66666667	0.22222311	1.55555465	2.55555558	0.00000568
86	-0.66666731	0.22222232	1.55555407	2.55555823	0.00000445
87	-0.66666723	0.22222165	1.55555493	2.55555787	0.00000323
88	-0.66666668	0.22222165	1.55555611	2.55555559	0.00000362
89	-0.66666626	0.22222215	1.5555565	2.55555387	0.00000283
90	-0.6666663	0.22222258	1.55555597	2.55555406	0.00000202
91	-0.66666665	0.22222259	1.55555521	2.5555555	0.00000231
92	-0.66666692	0.22222228	1.55555495	2.55555662	0.0000018
93	-0.6666669	0.222222	1.55555528	2.55555652	0.00000127
94	-0.66666668	0.22222199	1.55555577	2.55555561	0.00000147
95	-0.66666651	0.22222218	1.55555594	2.55555488	0.00000114
96	-0.66666652	0.22222236	1.55555574	2.55555493	0.00000079
97	-0.66666665	0.22222237	1.55555543	2.55555551	0.00000094
98	-0.66666677	0.22222225	1.55555531	2.55555598	0.00000073
99	-0.66666676	0.22222213	1.55555544	2.55555596	0.0000005
100	-0.66666668	0.22222212	1.55555563	2.55555559	0.0000006
101	-0.6666666	0.2222222	1.55555571	2.55555529	0.00000046
102	-0.6666666	0.22222228	1.55555563	2.5555553	0.00000031
103	-0.66666666	0.22222229	1.55555551	2.55555553	0.00000038
104	-0.66666671	0.22222224	1.55555546	2.55555572	0.00000029

Table 3.9 (cont.)

i	x	y	z	w	er
105	-0.66666671	0.22222219	1.5555555	2.55555572	0.0000002
106	-0.66666667	0.22222218	1.55555558	2.55555558	0.00000024
107	0.66666664	0.22222221	1.55555562	2.55555545	0.00000019
108	-0.66666664	0.22222224	1.55555559	2.55555545	0.00000013
109	-0.66666666	0.22222225	1.55555554	2.55555554	0.00000016
110	-0.66666668	0.22222223	1.55555551	2.55555562	0.00000012
111	-0.66666668	0.22222221	1.55555553	2.55555563	0.00000008
r= 2.83278863					

Table 3.10 the result of example 3.3 from the U_2 method.

i	x	y	z	w	er
1	0.77777778	1.7037037	-0.9382716	6.56790123	8.85185185
2	-1.93347051	0.33356196	0.57582686	5.02476757	4.55746075
3	-0.94708208	-0.37763411	2.02878777	1.16302609	5.49816395
4	-0.07318621	0.24585718	1.68178634	1.88311424	1.95394208
5	-0.60102715	0.46120674	1.27898646	3.24445416	1.97246739
6	-0.90590015	0.18673534	1.5106817	2.76868912	0.83354806
7	-0.66461605	0.13028931	1.66317482	2.26201985	0.78238777
8	-0.57015606	0.24660412	1.55891945	2.50444486	0.34165949
9	-0.67785945	0.25645346	1.5110352	2.67973117	0.28840658
10	-0.70409349	0.20867592	1.55877717	2.56266817	0.13615145
11	-0.65803201	0.2099725	1.57296238	2.50501463	0.10341079
12	-0.6525838	0.22900666	1.55231191	2.55828522	0.05759949
13	-0.67167759	0.22639345	1.5489127	2.57555179	0.03513711
14	-0.67176747	0.21904049	1.55758358	2.55226488	0.02699589
15	-0.66409421	0.22089772	1.55799756	2.5478921	0.01114042
16	-0.66490193	0.22364495	1.55447258	2.55771925	0.01205912
17	-0.66789319	0.22259746	1.55469425	2.55839386	0.00379041

Table 3.10 (cont.)

i	x	y	z	w	er
18	-0.66724017	0.2216099	1.55608488	2.55437508	0.0051864
19	-0.6661113	0.22213892	1.55584307	2.55454607	0.00178701
20	-0.66649752	0.22247716	1.55531147	2.55614037	0.00215788
21	-0.66690818	0.22222763	1.55546698	2.55589646	0.0008023
22	-0.66670801	0.2221193	1.55566326	2.55528334	0.00087063
23	-0.66656518	0.22223137	1.55557927	2.55544864	0.00034646
24	-0.66666123	0.22226253	1.55550972	2.55567657	0.00034074
25	-0.666708	0.22221412	1.55555108	2.55558516	0.00014468
26	-0.66666431	0.22220693	1.55557445	2.55550372	0.00012915
27	-0.66665034	0.22222715	1.55555528	2.55554937	0.00005858
28	-0.66666941	0.22222782	1.555548	2.55557704	0.00004722
29	-0.66667292	0.22221963	1.5555565	2.55555568	0.00002301
30	-0.66666488	0.22222026	1.55555848	2.55554692	0.00001653
31	-0.66666435	0.22222348	1.55555486	2.55555646	0.00001067
32	-0.66666764	0.22222287	1.55555446	2.55555892	0.00000546
33	-0.66666749	0.22222165	1.55555596	2.55555482	0.00000488
34	-0.66666619	0.22222203	1.55555595	2.55555428	0.00000167
35	-0.66666639	0.22222248	1.55555535	2.55555599	0.00000214
36	-0.66666689	0.22222227	1.55555542	2.55555602	0.0000007
37	-0.66666675	0.22222212	1.55555565	2.55555533	0.00000091
38	-0.66666657	0.22222221	1.5555556	2.55555539	0.00000032
39	-0.66666664	0.22222227	1.55555551	2.55555566	0.00000037
40	-0.66666671	0.22222222	1.55555554	2.55555561	0.00000014
41	-0.66666667	0.2222222	1.55555557	2.55555551	0.00000015
42	-0.66666665	0.22222222	1.55555556	2.55555554	0.00000006
r = 2.83278861					

Table 3.11 the result of the example 3.3 from the U_3 method.

i	x	y	z	w	er
1	0.61111111	0.85185185	0.19753086	3.89506173	4.12962963
2	-1.11076818	-0.08481939	1.51226947	2.42885993	3.37025606
3	-0.61624456	0.18279593	1.59369525	2.38814116	0.40911856
4	-0.63014582	0.2420458	1.54183736	2.57240006	0.16020745
5	-0.67320095	0.22074703	1.55294975	2.56174526	0.02515904
6	-0.66782391	0.22083337	1.55653875	2.5529519	0.01255936
7	-0.66596916	0.22248783	1.55552914	2.5554157	0.00259732
8	-0.6666758	0.22227222	1.55548009	2.55574016	0.0003827
9	-0.66671137	0.22219514	1.55556651	2.55553515	0.0002474
10	-0.66665904	0.22222251	1.55555883	2.55554522	0.00002672
11	-0.66666461	0.22222402	1.55555425	2.5555584	0.00001559
12	-0.66666748	0.22222193	1.55555551	2.55555584	0.00000308
13	-0.66666669	0.22222214	1.55555565	2.55555531	0.00000069
14	-0.66666661	0.22222225	1.55555554	2.55555557	0.00000029
15	-0.66666667	0.22222222	1.55555555	2.55555557	0.00000003
r = 2.83278862					

Table 3.12 the result of the example 3.3 from the U_4 method.

i	x	y	z	w	er
1	1.5	2.83333333	-0.05555556	7.22222222	24.33333333
2	-3.5	-1.2037037	1.89506173	2.19753086	13.25925926
3	0.05555556	0.35596708	1.34499314	2.31138546	2.22359396
4	-0.50617284	0.49748514	1.39993903	3.10120408	2.67581161
5	-0.99931413	0.03320124	1.61056919	2.45514742	1.77056343
6	-0.54945892	0.25808509	1.52604711	2.539582	0.39384052
7	-0.6618571	0.24882223	1.54154355	2.61795801	0.27235447
8	-0.70416189	0.19826433	1.56322507	2.53746549	0.22699088
9	-0.64947745	0.22886897	1.55142467	2.55579072	0.06073027

Table 3.12 (cont.)

i	x	y	z	w	er
10	-0.66804218	0.22443925	1.55445765	2.56246303	0.02468615
11	-0.67067997	0.2193139	1.55652971	2.55277805	0.02796411
12	-0.66431083	0.2232839	1.55498833	2.5558903	0.00868258
13	-0.66708126	0.22235059	1.55549659	2.55628586	0.00183713
14	-0.66706652	0.22188481	1.55567035	2.5551658	0.0033047
15	-0.66636048	0.22237768	1.55547978	2.55563606	0.00133192
16	-0.66674675	0.22221921	1.55555742	2.55562776	0.00024441
17	-0.6667022	0.22218507	1.5555681	2.55550423	0.0003727
18	-0.6666287	0.22224362	1.55554574	2.5555702	0.0001894
19	-0.66667978	0.22221951	1.55555685	2.55556195	0.00004348
20	-0.66666916	0.2222184	1.55555681	2.55554914	0.00003968
21	-0.66666217	0.22222503	1.55555433	2.5555579	0.00002545
22	-0.66666862	0.22222161	1.55555584	2.55555601	0.00000682
23	-0.66666673	0.22222187	1.55555566	2.55555479	0.0000039
24	-0.66666616	0.22222257	1.55555541	2.5555559	0.00000326
25	-0.66666694	0.22222212	1.5555556	2.55555557	0.00000099
26	-0.66666664	0.22222219	1.55555556	2.55555547	0.00000034
27	-0.66666661	0.22222226	1.55555554	2.5555556	0.0000004
28	-0.6666667	0.22222221	1.55555556	2.55555555	0.00000014
29	-0.66666666	0.22222222	1.55555556	2.55555555	0.00000003
r = 2.83278861					

Table 3.13 the result of the example 3.3 from the L_2 method.

i	x	y	z	w	er
1	-0.60000000	0.30000000	1.30000000	2.10000000	3.10000000
2	-0.35000000	0.65000000	1.40000000	2.80000000	3.10000000
3	-0.92500000	0.15833333	1.41111111	3.23888889	3.45000000
4	-0.90416667	0.01157407	1.55432099	2.68734568	1.04722222

Table 3.13 (cont.)

i	x	y	z	w	er
5	-0.62662037	0.19246399	1.610631	2.31668381	1.66527778
6	-0.5598894	0.30072159	1.56949779	2.44205056	0.56885288
7	-0.65613497	0.25427296	1.53643097	2.62490162	0.57747342
8	-0.70780278	0.19814455	1.54498142	2.61969948	0.31000681
9	-0.68141272	0.20297499	1.56071777	2.54337411	0.1821378
10	-0.65353344	0.22721897	1.56120023	2.52616248	0.16727573
11	-0.65729083	0.23138208	1.55486893	2.55310244	0.05503786
12	-0.66967673	0.22249432	1.55306658	2.56698841	0.07431535
13	-0.67127466	0.21853459	1.55520638	2.55945899	0.02036683
14	-0.66659998	0.22117609	1.55648783	2.551856	0.02804807
15	-0.66475996	0.22346946	1.55593502	2.55301714	0.01225675
16	-0.66621081	0.22296734	1.55526938	2.5564295	0.00870508
17	-0.66733311	0.22189954	1.55532741	2.55681808	0.0067338
18	-0.66702251	0.22183486	1.5556151	2.55551709	0.00287283
19	-0.66648353	0.22225639	1.55566421	2.55502803	0.0032339
20	-0.66647432	0.22238974	1.55555867	2.55543764	0.00076326
21	-0.66669303	0.22225066	1.55551178	2.55574202	0.00131226
22	-0.66675223	0.22216073	1.55554318	2.55565117	0.00047413
23	-0.66667754	0.22219497	1.55557037	2.55550332	0.00044815
24	-0.66663433	0.22224054	1.55556438	2.5555031	0.00025926
25	-0.66665401	0.22223804	1.55555172	2.55556369	0.00014247
26	-0.66667672	0.22221872	1.55555096	2.55557913	0.00013625
27	-0.6666744	0.22221485	1.55555597	2.55555819	0.00004176
28	-0.6666645	0.22222179	1.55555754	2.55554656	0.0000594
29	-0.66666295	0.22222514	1.55555589	2.55555219	0.00001732
30	-0.66666661	0.22222314	1.55555483	2.55555839	0.00002199
31	-0.66666818	0.22222126	1.55555523	2.55555765	0.00001007
32	-0.66666707	0.2222216	1.55555577	2.55555492	0.00000679

Table 3.13 (cont.)

i	x	y	z	w	er
33	-0.66666615	0.22222246	1.55555574	2.55555453	0.00000554
34	-0.66666637	0.22222254	1.55555551	2.55555556	0.00000221
35	-0.6666668	0.2222222	1.55555547	2.55555597	0.00000261
36	-0.66666682	0.22222209	1.55555555	2.55555566	0.00000061
37	-0.66666665	0.2222222	1.55555559	2.55555541	0.00000104
38	-0.6666666	0.22222227	1.55555557	2.55555548	0.00000039
39	-0.66666666	0.22222225	1.55555554	2.55555559	0.00000035
40	-0.66666669	0.22222221	1.55555555	2.5555556	0.00000022
41	-0.66666668	0.22222221	1.55555556	2.55555555	0.00000011
42	-0.66666666	0.22222222	1.55555556	2.55555554	0.00000011
43	-0.66666666	0.22222223	1.55555556	0.55555555	0:00000003

Table 3.14 the result of the example 3.3 from the L_3 method.

i	x	y	z	w	er
1	1.50000000	2.83333333	-0.05555556	5.88888889	20.00000000
2	-2.83333333	-0.53703704	1.67283951	2.12345679	21.66666667
3	-0.12962963	0.46707819	1.35733882	4.40603567	10.44444444
4	-1.61522634	-0.57864655	1.78882792	1.22923335	8.60493827
5	0.28029264	0.82446781	1.28051279	3.58053481	9.72153635
6	-1.34275771	-0.16143937	1.69503388	1.68770193	7.42447798
7	-0.11064822	0.6038607	1.40507496	3.38010407	6.05798574
8	-1.19451987	-0.12825796	1.69031057	1.89594039	5.43943919
9	-0.2289965	0.49814732	1.44558716	3.11882208	4.68065589
10	-1.03127828	-0.01375822	1.64665755	2.06672805	3.89999819
11	-0.34981369	0.43004772	1.47433007	2.97323771	3.35535267
12	-0.93880775	0.04643147	1.6242357	2.20205718	2.88401111
13	-0.43636217	0.37103642	1.49741072	2.85886491	2.45281875
14	-0.86365602	0.09421921	1.6054509	2.2961669	2.09239383

Table 3.14 (cont.)

t	x	y	z	w	er
15	-0.4979185	0.33167093	1.51280325	2.77674292	1.79126736
16	-0.81060855	0.12901401	1.59194587	2.36665758	1.52952263
17	-0.54380873	0.30188065	1.52446328	2.71703993	1.3055182
18	-0.77169193	0.15411386	1.58214143	2.41766181	1.11549911
19	-0.57695855	0.28036559	1.53285518	2.6733162	0.95298312
20	-0.74326849	0.17256831	1.57493855	2.4549539	0.81383782
21	-0.60123039	0.2646462	1.53899469	2.64148831	0.69514672
22	-0.7225646	0.1859859	1.5697015	2.48216099	0.59380975
23	-0.61892419	0.25317061	1.543474	2.61824711	0.50719577
24	-0.70744586	0.19578634	1.56587535	2.50200579	0.43321628
25	-0.63183375	0.24480325	1.54674048	2.60129497	0.37004156
26	-0.69641935	0.2029349	1.56308486	2.51648704	0.31607505
27	-0.6412534	0.23869661	1.54912438	2.58892649	0.26997628
28	-0.68837374	0.20815034	1.56104886	2.52705149	0.23060286
29	-0.64812534	0.23424182	1.55086341	2.57990249	0.1969719
30	-0.68250385	0.21195561	1.55956338	2.5347594	0.16824502
31	-0.65313919	0.23099155	1.55213223	2.57331879	0.14370787
32	-0.67822129	0.21473182	1.55847962	2.54038295	0.1227494
33	-0.65679719	0.22862021	1.55305794	2.56851537	0.10484745
34	-0.67509676	0.21675733	1.55768891	2.54448582	0.08955633
35	-0.65946603	0.22689011	1.55373333	2.56501087	0.07649532
36	-0.67281715	0.21823511	1.55711203	2.54747922	0.06533914
37	-0.66141317	0.22562785	1.55422608	2.56245403	0.05580999
38	-0.67115398	0.21931327	1.55669114	2.54966317	0.04767058
39	-0.66283379	0.22470693	1.55458559	2.56058859	0.04071824
40	-0.66994055	0.22009989	1.55638406	2.55125654	0.03477984
41	-0.66387025	0.22403503	1.55484788	2.55922759	0.0297075
42	-0.66905525	0.2206738	1.55616002	2.55241905	0.02537492

Table 3.14 (cont.)

i	x	y	z	w	er
43	-0.66462644	0.22354482	1.55503925	2.55823463	0.0216742
44	-0.66840935	0.22109251	1.55599657	2.5532672	0.01851321
45	-0.66517814	0.22318717	1.55517886	2.55751017	0.01581322
46	-0.6679381	0.221398	1.55587731	2.553886	0.01350699
47	-0.66558066	0.22292624	1.55528073	2.55698162	0.01153712
48	-0.66759429	0.22162088	1.5557903	2.55433747	0.00985453
49	-0.66587433	0.22273586	1.55535504	2.55659599	0.00841733
50	-0.66734345	0.22178349	1.55572683	2.55466686	0.00718973
51	-0.66608859	0.22259697	1.55540926	2.55631464	0.00614117
52	-0.66716044	0.22190213	1.55568051	2.55490717	0.00524554
53	-0.66624491	0.22249563	1.55544882	2.55610938	0.00448052
54	-0.66702692	0.22198869	1.55564672	2.5550825	0.00382708
55	-0.66635896	0.2224217	1.55547769	2.55595962	0.00326893
56	-0.6669295	0.22205184	1.55562207	2.55521042	0.00279218
57	-0.66644217	0.22236776	1.55549874	2.55585035	0.00238497
58	-0.66685843	0.22209791	1.55560408	2.55530375	0.00203714
59	-0.66650287	0.2223284	1.55551411	2.55577064	0.00174004
60	-0.66680657	0.22213153	1.55559096	2.55537184	0.00148627
61	-0.66654717	0.22229969	1.55552531	2.55571248	0.00126951
62	-0.66676874	0.22215605	1.55558139	2.55542152	0.00108436
63	-0.66657948	0.22227874	1.55553349	2.55567004	0.00092622
64	-0.66674114	0.22217395	1.5555744	2.55545777	0.00079114
65	-0.66660306	0.22226346	1.55553946	2.55563908	0.00067576
66	-0.666721	0.222187	1.55556931	2.55548421	0.0005772
67	-0.66662026	0.22225231	1.55554381	2.5556165	0.00049302
68	-0.66670631	0.22219652	1.55556559	2.5555035	0.00042112
69	-0.66663281	0.22224417	1.55554699	2.55560002	0.0003597
70	-0.66669559	0.22220347	1.55556287	2.55551758	0.00030724

Table 3.14 (cont.)

i	x	y	z	w	er
71	-0.66664196	0.22223824	1.5555493	2.55558799	0.00026244
72	-0.66668777	0.22220854	1.5555609	2.55552785	0.00022416
73	-0.66664864	0.22223391	1.55555099	2.55557922	0.00019147
74	-0.66668206	0.22221224	1.55555945	2.55553534	0.00016355
75	-0.66665352	0.22223075	1.55555223	2.55557282	0.00013969
76	-0.6666779	0.22221494	1.5555584	2.55554081	0.00011932
77	-0.66665707	0.22222844	1.55555313	2.55556815	0.00010192
78	-0.66667486	0.22221691	1.55555763	2.5555448	0.00008705
79	-0.66665967	0.22222676	1.55555378	2.55556475	0.00007436
80	-0.66667265	0.22221835	1.55555707	2.5555477	0.00006351
81	-0.66666156	0.22222553	1.55555426	2.55556226	0.00005425
82	-0.66667103	0.22221939	1.55555666	2.55554983	0.00004634
83	-0.66666294	0.22222464	1.55555461	2.55556045	0.00003958
84	-0.66666985	0.22222016	1.55555636	2.55555138	0.00003381
85	-0.66666395	0.22222398	1.55555487	2.55555913	0.00002888
86	-0.66666899	0.22222072	1.55555614	2.55555251	0.00002467
87	-0.66666468	0.22222351	1.55555505	2.55555816	0.00002107
88	-0.66666836	0.22222112	1.55555598	2.55555333	0.000018
89	-0.66666522	0.22222316	1.55555519	2.55555746	0.00001537
90	-0.6666679	0.22222142	1.55555587	2.55555393	0.00001313
91	-0.66666561	0.22222291	1.55555529	2.55555694	0.00001121
92	-0.66666757	0.22222164	1.55555578	2.55555437	0.00000958
93	-0.6666659	0.22222272	1.55555536	2.55555657	0.00000818
94	-0.66666732	0.2222218	1.55555572	2.55555469	0.00000699
95	-0.6666661	0.22222259	1.55555541	2.55555629	0.00000597
96	-0.66666715	0.22222191	1.55555568	2.55555493	0.0000051
97	-0.66666626	0.22222249	1.55555545	2.55555609	0.00000436
98	-0.66666702	0.222222	1.55555564	2.5555551	0.00000372

Table 3.14 (cont.)

i	x	y	z	w	er
99	-0.66666637	0.22222242	1.55555548	2.55555595	0.00000318
100	-0.66666692	0.22222206	1.55555562	2.55555522	0.00000271
101	-0.66666645	0.22222236	1.5555555	2.55555584	0.00000232
102	-0.66666685	0.2222221	1.5555556	2.55555531	0.00000198
103	-0.66666651	0.22222233	1.55555552	2.55555576	0.00000169
104	-0.6666668	0.22222213	1.55555559	2.55555538	0.00000144
105	-0.66666655	0.2222223	1.55555553	2.55555571	0.00000123
106	-0.66666677	0.22222216	1.55555558	2.55555543	0.00000105
107	-0.66666658	0.22222228	1.55555553	2.55555567	0.0000009
108	-0.66666674	0.22222218	1.55555557	2.55555546	0.00000077
109	-0.6666666	0.22222226	1.55555554	2.55555564	0.00000066
110	-0.66666672	0.22222219	1.55555557	2.55555549	0.00000056
111	-0.66666662	0.22222225	1.55555554	2.55555561	0.00000048
112	-0.66666671	0.2222222	1.55555557	2.5555555	0.00000041
113	-0.66666663	0.22222224	1.55555555	2.5555556	0.00000035
114	-0.66666669	0.2222222	1.55555556	2.55555552	0.0000003
115	-0.66666664	0.22222224	1.55555555	2.55555559	0.00000026
116	-0.66666669	0.22222221	1.55555556	2.55555553	0.00000022
117	-0.66666665	0.22222223	1.55555555	2.55555558	0.00000019
118	-0.66666668	0.22222221	1.55555556	2.55555554	0.00000016
119	-0.66666665	0.22222223	1.55555555	2.55555557	0.00000014
120	-0.66666668	0.22222222	1.55555556	2.55555554	0.00000012
121	-0.66666666	0.22222223	1.55555555	2.55555557	0.0000001
r = 2.83278862					

Table 3.15 the result of the example 3.3 from the L_4 method.

i	x	y	z	w	er
1	1.50000000	2.83333333	-0.05555556	7.22222222	20.00000000
2	-3.50000000	-1.20370370	1.89506173	2.19753086	24.33333333
3	0.05555556	0.35596708	1.34499314	2.31138546	13.25925926
4	-0.50617284	0.49748514	1.39993903	3.10120408	2.22359396
5	-0.99931413	0.03320124	1.61056919	2.45514742	2.67581161
6	-0.54945892	0.25808509	1.52604711	2.53958200	1.77056343
7	-0.66185710	0.24882223	1.54154355	2.61795801	0.39384052
8	-0.70416189	0.19826433	1.56322507	2.53746549	0.27235447
9	-0.64947745	0.22886897	1.55142467	2.55579072	0.22699088
10	-0.66804218	0.22443925	1.55445765	2.56246303	0.06073027
11	-0.67067997	0.21931390	1.55652971	2.55277805	0.02468615
12	-0.66431083	0.22328390	1.55498833	2.55589030	0.02796411
13	-0.66708126	0.22235059	1.55549659	2.55628586	0.00868258
14	-0.66706652	0.22188481	1.55567035	2.55516580	0.00183713
15	-0.66636048	0.22237768	1.55547978	2.55563606	0.00330470
16	-0.66674675	0.22221921	1.55555742	2.55562776	0.00133192
17	-0.66670220	0.22218507	1.55556810	2.55550423	0.00024441
18	-0.66662870	0.22224362	1.55554574	2.55557020	0.00037270
19	-0.66667978	0.22221951	1.55555685	2.55556195	0.00018940
20	-0.66666916	0.22221840	1.55555681	2.55554914	0.00004348
21	-0.66666217	0.22222503	1.55555433	2.55555790	0.00003968
22	-0.66666862	0.22222161	1.55555584	2.55555601	0.00002545
23	-0.66666673	0.22222187	1.55555566	2.55555479	0.00000682
24	-0.66666616	0.22222257	1.55555541	2.55555590	0.00000390
25	-0.66666694	0.22222212	1.55555560	2.55555557	0.00000326
26	-0.66666664	0.22222219	1.55555556	2.55555547	0.00000099
27	-0.66666661	0.22222226	1.55555554	2.55555560	0.00000034
28	-0.66666670	0.22222221	1.55555556	2.55555555	0.00000040

Table 3.15 (cont.)

i	x	y	z	w	er
29	-0.66666666	0.22222222	1.55555556	2.55555555	0.00000014
30	-0.66666666	0.22222223	1.55555555	2.55555556	0.00000003
r = 2.83278862					

Table 3.16 the result of example 3.4 from the *D* method.

i	C_1	C_2	C_3	C_4	C_5	er
1	8.33333333	8.33333333	17.77777778	0.00000000	8.33333333	210.00000000
2	11.29629630	11.29629630	16.85185185	15.20202020	11.29629630	193.33333333
3	11.14197531	11.14197531	16.52263374	15.33670034	11.14197531	5.37037037
4	11.08710562	11.08710562	16.53978052	15.05518144	11.08710562	3.58024691
5	11.08996342	11.08996342	16.54587715	15.05268737	11.08996342	0.09945130
6	11.09097953	11.09097953	16.54555962	15.05790068	11.09097953	0.06630087
7	11.09092660	11.09092660	16.54544672	15.05794687	11.09092660	0.00184169
8	11.09090779	11.909077900	16.54545260	15.05785032	11.09090779	0.00122779
9	11.09090877	11.0900877	16.54545469	15.05784947	11.09090877	0.00003411
10	11.09090912	11.09090912	16.54545458	15.05785126	11.09090912	0.00002274
11	11.09090910	11.09090910	16.54545454	15.05785127	11.09090910	0.00000063
12	11.09090909	11.09090909	16.54545454	15.05785124	11.09090909	0.00000042
13	11.09090909	11.09090909	16.54545455	15.05785124	11.09090909	0.00000001

Table 3.17 the result of example 3.4 from the U_2 method.

i	C_1	C_2	C_3	C_4	C_5	er
1	11.296296300	11.296296300	17.777777780	0.000000000	11.296296300	210.000000000
2	11.087105620	11.087105620	16.522633740	16.010101000	11.087105620	187.407407410
3	11.090979530	11.090979530	16.545877150	15.040217000	11.090979530	10.877914950
4	11.090907790	11.090907790	16.545446720	15.058177800	11.090907790	0.201442870
5	11.090909120	11.090909120	16.545454690	15.057845200	11.090909120	0.003730420

Table 3.17 (cont.)

i	C_1	C_2	C_3	C_4	C_5	er
6	11.090909090	11.090909090	16.545454540	15.057851400	11.090909090	0.000069080
7	11.090909090	11.090909090	16.545454550	15.057851200	11.090909090	0.000001280
8	11.090909090	11.090909090	16.545454550	15.057851200	11.090909090	0.000000020

Table 3.18 the result of example 3.4 from the U_3 method.

i	C_1	C_2	C_3	C_4	C_5	er
1	11.29629630	11.29629630	17.77777778	0.00000000	11.29629630	210.00000000
2	11.08710562	11.08710562	16.52263374	16.01010100	11.08710562	187.40740741
3	11.09097953	11.09097953	16.54587715	15.04021700	11.09097953	10.87791495
4	11.09090779	11.09090779	16.54544672	15.05817780	11.09090779	0.20144287
5	11.09090912	11.09090912	16.54545469	15.05784520	11.09090912	0.00373042
6	11.09090909	11.09090909	16.54545454	15.05785140	11.09090909	0.00006908
7	11.09090909	11.09090909	16.54545455	15.05785120	11.09090909	0.00000128
8	11.09090909	11.09090909	16.54545455	15.05785120	11.09090909	0.00000002

Table 3.19 the result of example 3.4 from the L_2 method.

i	C_1	C_2	C_3	C_4	C_5	er
1	11.29629630	11.29629630	17.77777778	0.00000000	11.29629630	210.00000000
2	11.08710562	11.08710562	16.52263374	16.01010100	11.08710562	187.40740741
3	11.09097953	11.09097953	16.54587715	15.04021700	11.09097953	10.87791495
4	11.09090779	11.09090779	16.54544672	15.05817780	11.09090779	0.20144287
5	11.09090912	11.09090912	16.54545469	15.05784520	11.09090912	0.00373042
6	11.09090909	11.09090909	16.54545454	15.05785140	11.09090909	0.00006908
7	11.09090909	11.09090909	16.54545455	15.05785120	11.09090909	0.00000002

Table 3.20 the result of example 3.4 from the L_3 method.

i	C_1	C_2	C_3	C_4	C_5	er
1	8.333333330	8.333333330	16.851851850	14.528619530	8.33333333 2	10.000000000
2	11.141975310	11.141975310	6.539780520	15.067651830	11.141975310	16.851851850
3	11.089963420	11.089963420	16.545559620	15.057669750	11.089963420	0.312071330
4	11.090926600	11.090926600	16.545452600	15.057854600	11.090926600	0.005779100
5	11.090908770	11.090908770	16.545454580	15.057851180	11.090908770	0.000107020
6	11.090909100	11.090909100	16.545454540	15.057851240	11.090909100	0.000001980
7	11.090909090	11.090909090	16.545454550	15.057851240	11.090909090	0.000000040

Table 3.21 the result of the example 3.5 from the D method.

i	X_1	X_2	X_3	X_4	X_5	er
1	12509.62500000	15011.87500000	9096.59090910	24005.70000000	19235.89743600	79809.68834500
2	28803.98487000	32406.13684100	20117.76704500	32249.83177200	27672.86771600	61390.89989900
3	40635.02675800	45154.24530400	26631.53012000	39959.83430300	33771.18064400	44901.22888600
4	49362.83560000	53980.79546000	31948.81323000	45436.12577300	38300.74807400	32877.50100800
5	55629.58088300	60718.19564600	35799.32369500	49446.13465500	41598.02569400	24161.94243500
6	60309.87859300	65600.74154000	38590.63246900	52391.49519300	44047.45609800	17748.94332000
7	63724.14232100	69191.65385000	40658.43667800	54560.63186000	45830.25062200	13024.91143800
8	66233.31682500	71829.56681700	42173.88411600	56149.27655700	47144.47437100	9565.40335540
9	68076.27812100	73766.32655800	43285.73446800	57316.90185300	48108.39030400	7023.11261760
10	69429.24930300	75187.96044900	44102.64338600	58174.07433100	48816.20505900	5156.50122390
11	70422.57187800	76232.00895500	44702.35472700	58803.41005200	49335.91031300	3786.12339680
12	71151.97353000	76998.52094800	45142.65612600	59265.50176700	49717.51115000	2779.90759720
13	71687.50314300	77561.32691600	45465.96049600	59604.78900400	49997.68343000	2041.09946650
14	72080.71046300	77974.56109700	45703.33787500	59853.90212800	50203.40057000	1498.64914500
15	72369.41788400	78277.97147700	45877.62810300	60036.81093700	50354.44432600	1100.36059430
16	72581.39665200	78500.74580800	46005.59870200	60171.10884700	50465.34598900	807.92327017
17	72737.03888900	78664.31475400	46099.55907300	60269.71505200	50546.77391700	593.20568758
18	72851.31694500	78784.41275000	46168.54806100	60342.11520900	50606.56116400	435.55244333
19	72935.22388300	78872.59292300	46219.20221800	60395.27394700	50650.45905000	319.79789245

Table 3.21 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
20	72996.83130700	78937.33790500	46256.39427400	60434.30495600	50682.69042500	234.80684459
21	73042.06564100	78984.87594400	46283.70198600	60462.96289500	50706.35583000	172.40343004
22	73075.27827700	79019.78004100	46303.75226900	60484.00455900	50723.73180200	126.58465213
23	73099.66416100	79045.40785500	46318.47389000	60499.45408800	50736.48985100	92.94289654
24	73117.56913300	79064.22469500	46329.28302100	60510.79767500	50745.85725800	68.24193829
25	73130.71559300	79078.04068000	46337.21946500	60519.12653400	50752.73513800	50.10562742
26	73140.36818400	79088.18486200	46343.04668100	60525.24187600	50757.78511800	36.78931087
27	73147.45545700	79095.63307600	46347.32522700	60529.73197400	50761.49299100	27.01200408
28	73152.65918100	79101.10181700	46350.46668600	60533.02876200	50764.21544200	19.83316195
29	73156.47993700	79105.11715800	46352.77325500	60535.44937900	50766.21436100	14.56220382
30	73159.28527100	79108.06536200	46354.46682000	60537.22668200	50767.68203700	10.69208157
31	73161.34504600	79110.23003800	46355.71029400	60538.53163900	50768.75965700	7.85050172
32	73162.85740500	79111.81941800	46356.62329700	60539.48978500	50769.55088300	5.76411396
33	73163.96783200	79112.98639700	46357.29365500	60540.19328900	50770.13182900	4.23221493
34	73164.78314600	79113.84323400	46357.78585600	60540.70982700	50770.55838000	3.10744065
35	73165.38177900	79114.47235400	46358.14724700	60541.08908700	50770.87156900	2.28159213
36	73165.82131600	79114.93427600	46358.41259300	60541.36755300	50771.10152300	1.67522496
37	73166.14403900	79115.27343500	46358.60741900	60541.57201300	50771.27036300	1.23000902
38	73166.38099400	79115.52245800	46358.75046800	60541.72213400	50771.39433200	0.90311581
39	73166.55497500	79115.70529900	46358.85549900	60541.83235800	50771.48535400	0.66309929
40	73166.68271800	79115.83954800	46358.93261600	60541.91328900	50771.55218500	0.48687088
41	73166.77651100	79115.93811700	46358.98923900	60541.97271100	50771.60125500	0.35747761
42	73166.84537700	79116.01049100	46359.03081300	60542.01634100	50771.63728400	0.26247275
43	73166.89594100	79116.06363000	46359.06133800	60542.04837500	50771.66373800	0.19271660
44	73166.93306700	79116.10264700	46359.08375100	60542.07189600	50771.68316100	0.14149916
45	73166.96032600	79116.13129400	46359.10020700	60542.08916600	50771.69742300	0.10389382
46	73166.98034100	79116.15232800	46359.11229000	60542.10184600	50771.70789400	0.07628268
47	73166.99503600	79116.16777200	46359.12116100	60542.11115600	50771.71558200	0.05600935
48	73167.00582600	79116.17911100	46359.12767500	60542.11799200	50771.72122700	0.04112381
49	73167.01374800	79116.18743700	46359.13245800	60542.12301100	50771.72537200	0.03019470

Table 3.21 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
50	73167.01956500	79116.19355000	46359.13596900	60542.12669700	50771.72841500	0.02216995
51	73167.02383600	79116.19803800	46359.13854800	60542.12940200	50771.73065000	0.01627815
52	73167.02697200	79116.20133400	46359.14044100	60542.13138900	50771.73229000	0.01195180
53	73167.02927400	79116.20375400	46359.14183100	60542.13284800	50771.73349500	0.00877553
54	73167.03096500	79116.20553100	46359.14285200	60542.13391900	50771.73437900	0.00644332
55	73167.03220600	79116.20683500	46359.14360100	60542.13470500	50771.73502900	0.00473082
56	73167.03311800	79116.20779300	46359.14415100	60542.13528300	50771.73550500	0.00347346
57	73167.03378700	79116.20849600	46359.14455500	60542.13570700	50771.73585500	0.00255042
58	73167.03427800	79116.20901200	46359.14485200	60542.13601800	50771.73611300	0.00187266
59	73167.03463900	79116.20939200	46359.14506900	60542.13624700	50771.73630100	0.00137484
60	73167.03490400	79116.20967000	46359.14522900	60542.13641400	50771.73644000	0.00100952
61	73167.03509800	79116.20987400	46359.14534700	60542.13653700	50771.73654200	0.00074118
62	73167.03524100	79116.21002400	46359.14543300	60542.13662800	50771.73661600	0.00054419
63	73167.03534600	79116.21013400	46359.14549600	60542.13669400	50771.73667100	0.00039953
64	73167.03542300	79116.21021500	46359.14554300	60542.13674300	50771.73671100	0.00029355
65	73167.03547900	79116.21027500	46359.14557700	60542.13677900	50771.73674100	0.00021535
66	73167.03552100	79116.21031800	46359.14560200	60542.13680500	50771.73676300	0.00015795
67	73167.03555100	79116.21035000	46359.14562000	60542.13682500	50771.73677800	0.00011623
68	73167.03557400	79116.21037400	46359.14563400	60542.13683900	50771.73679000	0.00008529
69	73167.03559000	79116.21039100	46359.14564400	60542.13684900	50771.73679900	0.00006282
70	73167.03560200	79116.21040400	46359.14565100	60542.13685700	50771.73680500	0.00004601
71	73167.03561100	79116.21041300	46359.14565600	60542.13686200	50771.73681000	0.00003362
72	73167.03561700	79116.21042000	46359.14566000	60542.13686700	50771.73681300	0.00002491
73	73167.03562200	79116.21042500	46359.14566300	60542.13687000	50771.73681600	0.00001830
74	73167.03562600	79116.21042900	46359.14566500	60542.13687200	50771.73681800	0.00001353
75	73167.03562900	79116.21043200	46359.14566700	60542.13687300	50771.73681900	0.00000978
76	73167.03563000	79116.21043300	46359.14566800	60542.13687500	50771.73682000	0.00000715
77	73167.03563200	79116.21043500	46359.14566900	60542.13687500	50771.73682100	0.00000530
78	73167.03563300	79116.21043600	46359.14566900	60542.13687600	50771.73682100	0.00000387
79	73167.03563300	79116.21043700	46359.14567000	60542.13687700	50771.73682200	0.00000256

Table 3.21 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
80	73167.03563400	79116.21043700	46359.14567000	60542.13687700	50771.73682200	0.00000209
81	73167.03563400	79116.21043800	46359.14567000	60542.13687700	50771.73682200	0.00000149
82	73167.03563500	79116.21043800	46359.14567100	60542.13687700	50771.73682200	0.00000125
83	73167.03563500	79116.21043800	46359.14567100	60542.13687800	50771.73682200	0.00000083
84	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000072
85	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000042
86	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000024
87	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000024
88	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
89	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000024
90	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000018
91	73167.03563600	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000024
92	73167.03563600	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
93	73167.03563600	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000000

Table 3.22 the result of the example 3.5 from the U_2 method.

i	X_1	X_2	X_3	X_4	X_5	er
1	21569.50846800	24169.68924800	18325.62849600	27081.84359000	19235.89743600	110332.56724000
2	41482.55689800	43591.71357700	26916.73423600	37909.41477900	31211.50122200	70729.35347300
3	52727.54164900	56744.37583400	34209.57402500	46026.37920800	37713.58115000	46309.53115400
4	59909.58484400	64586.15293700	38568.59733400	51316.19107900	42513.49564700	29472.56997500
5	64708.78554900	69795.56322700	41310.52570400	54571.15217000	45463.69829200	18955.70310200
6	67721.35961300	73138.04040800	43131.29273900	56712.60166800	47348.08814700	12201.65763200
7	69666.31203000	75269.28872100	44283.28686900	58083.40174000	48577.59378900	7828.50057450
8	70919.70615700	76645.80494200	45024.47110300	58960.90721500	49361.09256500	5032.09883340
9	71722.50600300	77529.04558300	45502.17829700	59526.58556800	49865.14605000	3233.47951760
10	72238.94574400	78096.26051500	45808.39540900	59889.62253100	50189.43138900	2077.19408790
11	72570.77483100	78460.94252800	46005.27733500	60122.84968700	50397.54621100	1334.73500430
12	72783.90747600	78695.18668400	46131.79779200	60272.75697000	50531.31616700	857.57449758
13	72920.87578600	78845.69371700	46213.06566200	60369.05429000	50617.26879600	550.99316073

Table 3.22 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
14	73008.87568700	78942.40255800	46265.28833900	60430.92860500	50672.48708400	354.02402246
15	73065.41525000	79004.53630500	46298.84157200	60470.68482100	50707.96805400	227.46372807
16	73101.74357400	79044.45845500	46320.39934900	60496.22792500	50730.76472300	146.14802468
17	73125.08469800	79070.10901100	46334.25075700	60512.63983900	50745.41169100	93.90197003
18	73140.08164400	79086.58970200	46343.15039600	60523.18469700	50754.82263500	60.33307946
19	73149.71738700	79097.17876800	46348.86851200	60529.95987100	50760.86925700	38.76471984
20	73155.90845400	79103.98236700	46352.54247700	60534.31300800	50764.75428100	24.90679359
21	73159.88629000	79108.35375800	46354.90304100	60537.10995000	50767.25045800	16.00290734
22	73162.44209700	79111.16242900	46356.41973100	60538.90701700	50768.85428000	10.28205752
23	73164.08423300	79112.96703300	46357.39422200	60540.06165400	50769.88475500	6.60634333
24	73165.13932400	79114.12651200	46358.02034400	60540.80352200	50770.54684700	4.24465346
25	73165.81723300	79114.87149100	46358.42263500	60541.28018100	50770.97224900	2.72723991
26	73166.25279700	79115.35014900	46358.68111200	60541.58644000	50771.24557500	1.75228399
27	73166.53265300	79115.65769300	46358.84718600	60541.78321500	50771.42119000	1.12586331
28	73166.71246300	79115.85529300	46358.95389100	60541.90964500	50771.53402500	0.72338045
29	73166.82799300	79115.98225300	46359.02245000	60541.99087800	50771.60652300	0.46478021
30	73166.90222300	79116.06382700	46359.06650000	60542.04307100	50771.65310300	0.29862684
31	73166.94991600	79116.11623900	46359.09480300	60542.07660600	50771.68303200	0.19187135
32	73166.98056000	79116.14991400	46359.11298800	60542.09815300	50771.70226200	0.12327969
33	73167.00024900	79116.17155100	46359.12467100	60542.11199600	50771.71461700	0.07920843
34	73167.01289900	79116.18545300	46359.13217900	60542.12089100	50771.72255500	0.05089247
35	73167.02102700	79116.19438500	46359.13700200	60542.12660600	50771.72765600	0.03269899
36	73167.02624900	79116.20012400	46359.14010100	60542.13027800	50771.73093300	0.02100950
37	73167.02960500	79116.20381200	46359.14209200	60542.13263800	50771.73303800	0.01349890
38	73167.03176100	79116.20618100	46359.14337200	60542.13415300	50771.73439100	0.00867331
39	73167.03314600	79116.20770300	46359.14419400	60542.13512700	50771.73526000	0.00557268
40	73167.03403600	79116.20868100	46359.14472200	60542.13575300	50771.73581900	0.00358027
41	73167.03460800	79116.20930900	46359.14506100	60542.13615500	50771.73617800	0.00230044
42	73167.03497500	79116.20971300	46359.14527900	60542.13641300	50771.73640800	0.00147778
43	73167.03521100	79116.20997200	46359.14541900	60542.13657900	50771.73655600	0.00094980

Table 3.22 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
44	73167.03536300	79116.21013900	46359.14550900	60542.13668600	50771.73665100	0.00061011
45	73167.03546000	79116.21024600	46359.14556700	60542.13675500	50771.73671300	0.00039208
46	73167.03552300	79116.21031500	46359.14560400	60542.13679900	50771.73675200	0.00025195
47	73167.03556300	79116.21035900	46359.14562800	60542.13682700	50771.73677700	0.00016189
48	73167.03558900	79116.21038800	46359.14564300	60542.13684500	50771.73679300	0.00010407
49	73167.03560600	79116.21040600	46359.14565300	60542.13685700	50771.73680400	0.00006700
50	73167.03561600	79116.21041800	46359.14566000	60542.13686400	50771.73681100	0.00004297
51	73167.03562300	79116.21042500	46359.14566400	60542.13686900	50771.73681500	0.00002754
52	73167.03562700	79116.21043000	46359.14566600	60542.13687200	50771.73681800	0.00001764
53	73167.03563000	79116.21043300	46359.14566800	60542.13687400	50771.73681900	0.00001138
54	73167.03563200	79116.21043500	46359.14566900	60542.13687600	50771.73682100	0.00000727
55	73167.03563300	79116.21043600	46359.14567000	60542.13687600	50771.73682100	0.00000459
56	73167.03563400	79116.21043700	46359.14567000	60542.13687700	50771.73682200	0.00000292
57	73167.03563500	79116.21043800	46359.14567100	60542.13687700	50771.73682200	0.00000226
58	73167.03563500	79116.21043800	46359.14567100	60542.13687800	50771.73682200	0.00000125
59	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000083
60	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000060
61	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000036
62	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
63	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
64	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000006

Table 3.23 the result of the example 3.5 from the U_3 method.

i	X_1	X_2	X_3	X_4	X_5	er
1	26220.34172700	29245.65992100	18325.62849600	27081.84359000	19235.89743600	120059.37117000
2	44526.32809800	47621.18051800	28543.99163000	39582.14763000	31992.41978700	72156.69649300
3	55709.25702500	59946.86519800	35396.51286500	47478.88567300	38926.38810100	45191.84119900
4	62322.51528800	67301.28730000	39671.39281400	52563.24757000	43483.33762500	27883.87173400
5	66487.87051900	71810.81276900	42216.29767200	55619.46297900	46305.02907400	17097.69241600
6	69058.43293700	74620.97549600	43804.91619300	57502.50715400	48010.68676500	10558.04553200

Table 3.23 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
7	70632.33162000	76346.38039900	44787.51307300	58671.08305600	49069.91312500	6509.70272830
8	71604.84321900	77408.33036300	45389.96360500	59389.00549400	49723.85231500	4008.77372370
9	72204.52460800	78063.84721600	5761.75301600	59831.24670100	50125.69966200	2471.07620640
10	72573.73585600	78467.67243500	45991.05571600	60104.08036300	50373.55969600	1523.03286280
11	72801.34021400	78716.47689000	46132.26761300	60272.16109400	50526.37178200	938.51352698
12	72941.62662100	78869.84477700	46219.30460200	60375.73335600	50620.50792400	578.39968705
13	73028.07024000	78964.35927100	46272.95131500	60439.57277700	50678.52689100	356.46321422
14	73081.34525800	79022.60397100	46306.00945500	60478.91424200	50714.28551300	219.67794502
15	73114.17841400	79058.50011000	46326.38267500	60503.15900800	50736.32159200	135.38336068
16	73134.41244500	79080.62216100	46338.93858300	60518.10090100	50749.90206700	83.43435788
17	73146.88230300	79094.25543700	46346.67644100	60527.30926300	50758.27155200	51.41883856
18	73154.56726200	79102.65738700	46351.44512700	60532.98416900	50763.42946100	31.68840927
19	73159.30333800	79107.83534300	46354.38398400	60536.48151100	50766.60817700	19.52894819
20	73162.22208700	79111.02641200	46356.19514000	60538.63685300	50768.56716100	12.03530014
21	73164.02085600	79112.99300400	46357.31132000	60539.96514700	50769.77444300	7.41711634
22	73165.12940100	79114.20497700	46357.99920000	60540.78374800	50770.51846700	4.57102215
23	73165.81257500	79114.95189100	46358.42312600	60541.28823600	50770.97699400	2.81703001
24	73166.23360200	79115.41219900	46358.68438400	60541.59914200	50771.25957500	1.73607981
25	73166.49307200	79115.69587800	46358.84539100	60541.79074700	50771.43372500	1.06991160
26	73166.65297900	79115.87070400	46358.94461700	60541.90882900	50771.54104900	0.65936500
27	73166.75152600	79115.97844600	46359.00576800	60541.98160100	50771.60719100	0.40635359
28	73166.81225900	79116.04484500	46359.04345400	60542.02644900	50771.64795300	0.25042760
29	73166.84968700	79116.08576500	46359.06668000	60542.05408800	50771.67307400	0.15433371
30	73166.87275300	79116.11098400	46359.08099300	60542.07112100	50771.68855600	0.09511274
31	73166.88696900	79116.12652500	46359.08981400	60542.08161800	50771.69809700	0.05861634
32	73166.89573000	79116.13610300	46359.09525000	60542.08808800	50771.70397600	0.03612393
33	73166.90112900	79116.14200600	46359.09860000	60542.09207400	50771.70760000	0.02226233
34	73166.90445600	79116.14564400	46359.10066500	60542.09453100	50771.70983300	0.01372010
35	73166.90650600	79116.14788600	46359.10193700	60542.09604600	50771.71121000	0.00845510
36	73166.90777000	79116.14926700	46359.10272200	60542.09697900	50771.71205800	0.00521094
37	73166.90854900	79116.15011900	46359.10320500	60542.09755400	50771.71258000	0.00321144

Table 3.23 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
38	73166.90902900	79116.15064300	46359.10350300	60542.09790800	50771.71290200	0.00197881
39	73166.90932500	79116.15096700	46359.10368600	60542.09812700	50771.71310100	0.00121975
40	73166.90950700	79116.15116600	46359.10379900	60542.09826100	50771.71322300	0.00075161
41	73166.90961900	79116.15128900	46359.10386900	60542.09834400	50771.71329900	0.00046337
42	73166.90968800	79116.15136500	46359.10391200	60542.09839500	50771.71334500	0.00028545
43	73166.90973100	79116.15141100	46359.10393800	60542.09842700	50771.71337400	0.00017595
44	73166.90975700	79116.15144000	46359.10395500	60542.09844600	50771.71339200	0.00010836
45	73166.90977400	79116.15145800	46359.10396500	60542.09845800	50771.71340200	0.00006682
46	73166.90978400	79116.15146900	46359.10397100	60542.09846600	50771.71340900	0.00004113
47	73166.90979000	79116.15147500	46359.10397500	60542.09847000	50771.71341300	0.00002557
48	73166.90979400	79116.15147900	46359.10397700	60542.09847300	50771.71341600	0.00001538
49	73166.90979600	79116.15148200	46359.10397900	60542.09847500	50771.71341700	0.00000966
50	73166.90979700	79116.15148400	46359.10397900	60542.09847600	50771.71341800	0.00000590
51	73166.90979800	79116.15148500	46359.10398000	60542.09847600	50771.71341900	0.00000370
52	73166.90979900	79116.15148500	46359.10398000	60542.09847700	50771.71341900	0.00000250
53	73166.90979900	79116.15148600	46359.10398100	60542.09847700	50771.71342000	0.00000137
54	73166.90979900	79116.15148600	46359.10398100	60542.09847700	50771.71342000	0.00000095
55	73166.90979900	79116.15148600	46359.10398100	60542.09847700	50771.71342000	0.00000036
56	73166.90980000	79116.15148600	46359.10398100	60542.09847700	50771.71342000	0.00000030
57	73166.90980000	79116.15148600	46359.10398100	60542.09847700	50771.71342000	0.00000030
58	73166.90980000	79116.15148600	46359.10398100	60542.09847700	50771.71342000	0.00000024
59	73166.90980000	79116.15148600	46359.10398100	60542.09847700	50771.71342000	0.00000000

Table 3.24 the result of the example 3.5 from the U_4 method.

i	X_1	X_2	X_3	X_4	X_5	er
1	31296.31241800	35253.75287000	18325.62849600	27081.84359000	19235.89743600	131143.43481000
2	48325.88086700	54106.08357900	30346.26880600	41484.84688600	32916.74177900	75986.38710600
3	58841.23310900	64501.05722400	36948.76732600	49499.13024600	40590.34872700	43200.71471600
4	64927.78398700	70646.92206100	40913.89993300	54119.95106300	44831.20020000	25059.22061000
5	68391.04268800	74230.28356300	43221.77318900	56832.01203000	47326.19269000	14561.54691800

Table 3.24 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
6	70400.23032600	76289.20339700	44542.72843100	58397.97493800	48783.67480400	8412.50773500
7	71567.44008900	77479.32697600	45307.16149900	59300.85351200	49621.73762100	4862.70780110
8	72241.76739600	78169.30884100	45750.72342600	59823.82865800	50105.86956300	2814.97818680
9	72631.57739700	78568.46548500	46007.20723900	60126.62963700	50386.51975000	1628.90162500
10	72857.23168800	78799.27994300	46155.49809900	60301.74282700	50548.90672400	942.25977206
11	72987.80562000	78932.83815600	46241.31796600	60403.04490800	50642.80739800	545.15476704
12	73063.33835100	79010.12013800	46290.97738800	60461.66325800	50697.13929600	315.42438251
13	73107.03878300	79054.83044100	46319.70582700	60495.57819300	50728.57816100	182.49297541
14	73132.32361900	79080.69771700	46336.32681300	60515.19946200	50746.76684600	105.58305025
15	73146.95249100	79095.66395400	46345.94345800	60526.55173100	50757.28991200	61.08708912
16	73155.41617000	79104.32295100	46351.50733100	60533.11984300	50763.37831200	35.34306204
17	73160.31299300	79109.33273000	46354.72637600	60536.91992400	50766.90087600	20.44829237
18	73163.14614000	79112.23122200	46356.58881200	60539.11852000	50768.93890400	11.83069831
19	73164.78530300	79113.90819600	46357.66635800	60540.39055600	50770.11803900	6.84485322
20	73165.73366700	79114.87843800	46358.28978900	60541.12651400	50770.80024800	3.96020538
21	73166.28236000	79115.43978700	46358.65048600	60541.55231500	50771.19495200	2.29124314
22	73166.59981500	79115.76456500	46358.85917400	60541.79866900	50771.42331400	1.32563740
23	73166.78348400	79115.95247100	46358.97991300	60541.94120100	50771.55543700	0.76696968
24	73166.88974900	79116.06118700	46359.04976900	60542.02366600	50771.63187900	0.44374323
25	73166.95123000	79116.12408700	46359.09018500	60542.07137700	50771.67610600	0.25673532
26	73166.98680200	79116.16047800	46359.11356900	60542.09898100	50771.70169400	0.14853847
27	73167.00738200	79116.18153300	46359.12709800	60542.11495200	50771.71649800	0.08593941
28	73167.01928900	79116.19371500	46359.13492500	60542.12419200	50771.72506400	0.04972178
29	73167.02617800	79116.20076300	46359.13945400	60542.12953800	50771.73001900	0.02876717
30	73167.03016400	79116.20484100	46359.14207400	60542.13263100	50771.73288600	0.01664388
31	73167.03247000	79116.20720000	46359.14359000	60542.13442100	50771.73454500	0.00962949
32	73167.03380400	79116.20856500	46359.14446700	60542.13545600	50771.73550500	0.00557119
33	73167.03457600	79116.20935500	46359.14497400	60542.13605500	50771.73606000	0.00322360
34	73167.03502200	79116.20981100	46359.14526800	60542.13640200	50771.73638200	0.00186485
35	73167.03528100	79116.21007600	46359.14543800	60542.13660200	50771.73656700	0.00107884
36	73167.03543000	79116.21022900	46359.14553600	60542.13671900	50771.73667500	0.00062436

Table 3.24 (cont.)

k	X_1	X_2	X_3	X_4	X_5	er
37	73167.03551700	79116.21031700	46359.14559300	60542.13678600	50771.73673700	0.00036126
38	73167.03556700	79116.21036900	46359.14562600	60542.13682500	50771.73677300	0.00020903
39	73167.03559600	79116.21039800	46359.14564500	60542.13684700	50771.73679400	0.00012100
40	73167.03561200	79116.21041500	46359.14565600	60542.13686000	50771.73680600	0.00006998
41	73167.03562200	79116.21042500	46359.14566200	60542.13686700	50771.73681300	0.00004041
42	73167.03562800	79116.21043100	46359.14566600	60542.13687200	50771.73681700	0.00002348
43	73167.03563100	79116.21043400	46359.14566800	60542.13687500	50771.73681900	0.00001359
44	73167.03563300	79116.21043600	46359.14566900	60542.13687600	50771.73682100	0.00000769
45	73167.03563400	79116.21043700	46359.14567000	60542.13687700	50771.73682100	0.00000447
46	73167.03563500	79116.21043800	46359.14567000	60542.13687700	50771.73682200	0.00000268
47	73167.03563500	79116.21043800	46359.14567100	60542.13687700	50771.73682200	0.00000149
48	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000089
49	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000054
50	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000036
51	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
52	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000000

Table 3.25 the result of the example 3.5 from the U_5 method.

k	X_1	X_2	X_3	X_4	X_5	er
1	36102.78677700	35253.75287000	18325.62849600	27081.84359000	19235.89743600	135949.90917000
2	52775.31923900	55863.17767100	31733.59208700	42350.01227100	32916.74177900	79638.93387800
3	62057.51590000	66507.35742400	38395.89598500	50777.74872900	41258.26940100	43357.94439200
4	67154.68561100	72264.09843200	42025.79576000	55237.32362800	45626.93804900	23312.05404100
5	69907.59443000	75401.21068700	44011.23592500	57663.16830700	47975.18436500	12649.55223500
6	71399.03857200	77102.08823900	45086.28903900	58981.38489600	49255.23230600	6865.63933830
7	72208.29043900	78023.93623400	45668.81063900	59695.81882800	49949.58448500	3722.40757200
8	72647.14320000	78523.88237400	45984.78576900	60083.17796200	50325.88532700	2018.43400750
9	72885.10761400	78795.00584400	46156.14145200	60293.25279300	50529.95478100	1094.58785130
10	73014.15150800	78942.02802000	46249.06064600	60407.17281900	50640.62437000	593.57487923
11	73084.12981600	79021.75483400	46299.44874100	60468.94863600	50700.63739600	321.88206047

Table 3.25 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
12	73122.07754000	79064.98912800	46326.77327500	60502.44839700	50733.18104100	174.54995805
13	73142.65579200	79088.43418900	46341.59079600	60520.61464200	50750.82882100	94.65485829
14	73153.81495200	79101.14795500	46349.62602600	60530.46581600	50760.39884900	51.32936043
15	73159.86633400	79108.04236700	46353.98336500	60535.80790400	50765.58847500	27.83484471
16	73163.14787200	79111.78106300	46356.34626000	60538.70480700	50768.40270100	15.09425843
17	73164.92738200	79113.80848000	46357.62760900	60540.27573700	50769.92879800	8.18530297
18	73165.89237300	79114.90790700	46358.32245700	60541.12762000	50770.75636800	4.43871963
19	73166.41566800	79115.50410300	46358.69925900	60541.58957800	50771.20514300	2.40702540
20	73166.69944000	79115.82740700	46358.90359100	60541.84008900	50771.44850400	1.30527979
21	73166.85332400	79116.00272900	46359.01439600	60541.97593500	50771.58047300	0.70782632
22	73166.93677200	79116.09780200	46359.07448300	60542.04960200	50771.65203800	0.38383973
23	73166.98202400	79116.14935800	46359.10706700	60542.08955000	50771.69084500	0.20814800
24	73167.00656300	79116.17731600	46359.12473700	60542.11121300	50771.71189000	0.11287433
25	73167.01987000	79116.19247700	46359.13431900	60542.12296000	50771.72330200	0.06120956
26	73167.02708600	79116.20069900	46359.13951500	60542.12933100	50771.72949100	0.03319240
27	73167.03099900	79116.20515700	46359.14233300	60542.13278500	50771.73284700	0.01799971
28	73167.03312100	79116.20757500	46359.14386100	60542.13465800	50771.73466700	0.00976092
29	73167.03427200	79116.20888600	46359.14468900	60542.13567400	50771.73565300	0.00529301
30	73167.03489600	79116.20959600	46359.14513900	60542.13622500	50771.73618900	0.00287044
31	73167.03523500	79116.20998200	46359.14538200	60542.13652400	50771.73647900	0.00155652
32	73167.03541800	79116.21019100	46359.14551500	60542.13668600	50771.73663600	0.00084418
33	73167.03551800	79116.21030500	46359.14558600	60542.13677400	50771.73672200	0.00045770
34	73167.03557100	79116.21036600	46359.14562500	60542.13682100	50771.73676800	0.00024801
35	73167.03560100	79116.21039900	46359.14564600	60542.13684700	50771.73679300	0.00013447
36	73167.03561700	79116.21041700	46359.14565700	60542.13686100	50771.73680600	0.00007319
37	73167.03562500	79116.21042700	46359.14566400	60542.13686900	50771.73681400	0.00003964
38	73167.03563000	79116.21043200	46359.14566700	60542.13687300	50771.73681800	0.00002134
39	73167.03563200	79116.21043500	46359.14566900	60542.13687500	50771.73682000	0.00001174
40	73167.03563400	79116.21043700	46359.14567000	60542.13687600	50771.73682100	0.00000620
41	73167.03563500	79116.21043800	46359.14567000	60542.13687700	50771.73682200	0.00000322
42	73167.03563500	79116.21043800	46359.14567100	60542.13687700	50771.73682200	0.00000209

Table 3.25 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
43	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000095
44	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000054
45	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000036
46	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
47	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000000

Table 3.26 the result of the example 3.5 from the L_2 method.

i	X_1	X_2	X_3	X_4	X_5	er
1	12509.62500000	17355.55468700	9096.59090910	24823.49318200	24007.72304800	87742.98682600
2	31029.15737800	37523.08088700	20396.56017600	34358.43626000	30024.34464100	65538.59251500
3	43578.93231800	49194.14439600	27856.09358400	42149.42321700	36508.63591300	45955.65008700
4	52156.39401800	58019.33624600	33364.33344200	47575.55978800	40590.92751500	32419.32158100
5	58330.05044800	64224.13348900	37163.57584100	51350.12822600	43592.56786000	22953.90485400
6	62684.87744200	68586.02893900	39853.46427000	54052.41041100	45700.02876900	16216.35396700
7	65757.61463700	71672.37082100	41764.43024000	55956.12531600	47185.50336600	11459.23455000
8	67929.95293200	73856.32459300	43111.77332100	57300.24268700	48237.30363200	8099.55278510
9	69466.00913300	75398.71626100	44063.70840100	58250.97930800	48980.68766600	5724.50360300
10	70551.30539500	76488.74747800	44736.92684000	58922.93536300	49505.85685600	4045.67116270
11	71318.33392500	77259.28167900	45212.66100900	59397.73620600	49877.06489400	2859.30578100
12	71860.47154300	77803.82723700	45548.84959900	59733.32638800	50139.43421900	2020.83127360
13	72243.61990600	78188.67542300	45786.46843800	59970.51232500	50324.85457000	1428.22167710
14	72514.40825300	78460.67388100	45954.40645300	60138.13945400	50455.90111000	1009.39848880
15	72705.79009800	78652.90911500	46073.09487200	60256.61054500	50548.51959800	713.39507622
16	72841.04952500	78788.77107300	46156.97861700	60340.34070500	50613.97760600	504.19329965
17	72936.64422900	78884.79198600	46216.26376900	60399.51702100	50660.24013100	356.33960855
18	73004.20603400	78952.65499800	46258.16358200	60441.33998300	50692.93631500	251.84377629
19	73051.95548700	79000.61729000	46287.77636500	60470.89846900	50716.04440500	177.99110484
20	73085.70250300	79034.51474200	46308.70527100	60491.78898700	50732.37609000	125.79557598
21	73109.55327600	79058.47183800	46323.49681500	60506.55340200	50743.91854400	88.90628266
22	73126.40986000	79075.40356400	46333.95076800	60516.98818300	50752.07619700	62.83469796

Table 3.26 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
23	73138.32328600	79087.37009600	46341.33912200	60524.36298600	50757.84163500	44.40855169
24	73146.74312300	79095.82746700	46346.56085600	60529.57514300	50761.91637000	31.38583511
25	73152.69386100	79101.80473200	46350.25132700	60533.25884600	50764.79619800	22.18200338
26	73156.89955700	79106.02917600	46352.85957500	60535.86231100	50766.83152200	15.67717659
27	73159.87194100	79109.01481000	46354.70296000	60537.70231500	50768.26999100	11.07987654
28	73161.97268000	79111.12491400	46356.00577600	60539.00274100	50769.28663200	7.83072579
29	73163.45738100	79112.61623300	46356.92654400	60539.92182000	50770.00514600	5.53438133
30	73164.50669700	79113.67022700	46357.57729800	60540.57138100	50770.51295600	3.91143543
31	73165.24830300	79114.41513900	46358.03722000	60541.03046000	50770.87185300	2.76441497
32	73165.77243400	79114.94160700	46358.36227100	60541.35491500	50771.12550300	1.95375597
33	73166.14286600	79115.31368900	46358.59200200	60541.58422400	50771.30477100	1.38082111
34	73166.40466800	79115.57665900	46358.75436400	60541.74628900	50771.43146900	0.97589815
35	73166.58969800	79115.76251400	46358.86911400	60541.86082800	50771.52101400	0.68971813
36	73166.72046800	79115.89386700	46358.95021400	60541.94177900	50771.58429900	0.48745942
37	73166.81289000	79115.98670100	46359.00753100	60541.99899100	50771.62902600	0.34451324
38	73166.87821000	79116.05231200	46359.04804000	60542.03942600	50771.66063700	0.24348545
39	73166.92437500	79116.09868200	46359.07667000	60542.06800400	50771.68297900	0.17208368
40	73166.95700200	79116.13145400	46359.09690500	60542.08820100	50771.69876800	0.12162054
41	73166.98006100	79116.15461600	46359.11120500	60542.10247500	50771.70992800	0.08595550
42	73166.99635800	79116.17098600	46359.12131200	60542.11256400	50771.71781400	0.06074935
43	73167.00787600	79116.18255600	46359.12845500	60542.11969400	50771.72338900	0.04293460
44	73167.01601700	79116.19073200	46359.13350400	60542.12473300	50771.72732800	0.03034425
45	73167.02177000	79116.19651100	46359.13707200	60542.12829400	50771.73011200	0.02144563
46	73167.02583600	79116.20059500	46359.13959300	60542.13081200	50771.73208000	0.01515681
47	73167.02870900	79116.20348200	46359.14137600	60542.13259000	50771.73347100	0.01071215
48	73167.03074100	79116.20552200	46359.14263500	60542.13384800	50771.73445400	0.00757098
49	73167.03217600	79116.20696400	46359.14352500	60542.13473600	50771.73514800	0.00535077
50	73167.03319000	79116.20798300	46359.14415500	60542.13536400	50771.73563900	0.00378168
51	73167.03390700	79116.20870300	46359.14459900	60542.13580800	50771.73598600	0.00267255
52	73167.03441400	79116.20921200	46359.14491400	60542.13612200	50771.73623200	0.00188899
53	73167.03477200	79116.20957200	46359.14513600	60542.13634400	50771.73640500	0.00133508

Table 3.26 (cont.)

i	X_1	X_2	X_3	X_4	X_5	εr
54	73167.03502500	79116.20982600	46359.14529300	60542.13650000	50771.73652700	0.00094330
55	73167.03520400	79116.21000600	46359.14540400	60542.13661100	50771.73661400	0.00066674
56	73167.03533100	79116.21013300	46359.14548200	60542.13668900	50771.73667500	0.00047135
57	73167.03542000	79116.21022200	46359.14553700	60542.13674500	50771.73671800	0.00033319
58	73167.03548300	79116.21028600	46359.14557700	60542.13678400	50771.73674900	0.00023538
59	73167.03552800	79116.21033100	46359.14560400	60542.13681100	50771.73677000	0.00016630
60	73167.03555900	79116.21036200	46359.14562400	60542.13683100	50771.73678600	0.00011766
61	73167.03558200	79116.21038500	46359.14563800	6542.13684500	50771.73679700	0.00008309
62	73167.03559700	79116.21040100	46359.14564700	60542.13685400	50771.73680400	0.00005871
63	73167.03560900	79116.21041200	46359.14565400	60542.13686100	50771.73681000	0.00004166
64	73167.03561700	79116.21042000	46359.14565900	60542.13686600	50771.73681300	0.00002927
65	73167.03562200	79116.21042500	46359.14566300	60542.13686900	50771.73681600	0.00002074
66	73167.03562600	79116.21042900	46359.14566500	60542.13687200	50771.73681800	0.00001472
67	73167.03562900	79116.21043200	46359.14566700	60542.13687400	50771.73681900	0.00001043
68	73167.03563100	79116.21043400	46359.14566800	60542.13687500	50771.73682000	0.00000745
69	73167.03563200	79116.21043600	46359.14566900	60542.13687600	771.73682100	0.00000507
70	73167.03563300	79116.21043600	46359.14567000	542.13687600	50771.73682100	0.00000364
71	73167.03563400	79116.21043700	46359.14567000	60542.13687700	50771.73682200	0.00000262
72	73167.03563400	79116.21043800	46359.14567000	60542.13687700	50771.73682200	0.00000179
73	73167.03563500	79116.21043800	46359.14567100	60542.13687700	50771.73682200	0.00000119
74	73167.03563500	79116.21043800	46359.14567100	60542.13687700	50771.73682200	0.00000072
75	73167.03563500	79116.21043800	46359.14567100	60542.13687800	50771.73682200	0.00000060
76	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000030
77	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000036
78	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000024
79	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000024
80	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000006

Table 3.27 the result of the example 3.5 from the L_3 method.

k	X_1	X_2	X_3	X_4	X_5	er
1	12509.62500000	17355.55468700	11937.41477300	27507.54498600	26511.78931800	95771.92876400
2	32584.55723800	40520.91071500	25874.06243600	38495.21191000	33616.12367400	75268.93720900
3	47198.33403500	54509.60973100	32941.17088700	45839.85083300	39770.30657200	49168.40608400
4	56419.82795600	63072.49602000	37540.81913800	51067.77234300	43694.39167600	31536.03507500
5	62282.11411100	68678.01067800	40655.40287000	54420.57945300	46175.62668400	20416.42666300
6	66100.32975300	72355.25525000	42666.18157600	56568.57288600	47786.21899900	13264.82466800
7	68586.31012100	74732.82478900	43963.44760200	57963.16015000	48836.34979600	8605.53399440
8	70196.50641300	76273.02059300	44804.82878100	58870.00926300	49516.75398200	5579.02657410
9	71240.42227200	77272.10592300	45351.23549200	59457.75781500	49957.80353500	3618.20600490
10	71917.50551900	77920.29322600	45705.48687300	59838.85944500	50243.83964200	2346.65966830
11	72356.67904700	78340.60679200	45935.22004900	60086.02009100	50429.38052200	1521.92179650
12	72641.48445200	78613.19853900	46084.20786100	60246.32961500	50549.70420400	987.01817036
13	72826.19348500	78789.98457600	46180.83816100	60350.29314900	50627.73943300	640.12413257
14	72945.98476400	78904.63976200	46243.50556500	60417.71820400	50678.34820900	415.14770061
15	73023.67496100	78978.99781700	46284.14824300	60461.44600700	50711.17044100	269.24096417
16	73074.06015400	79027.22229000	46310.50662900	60489.80548100	50732.45695100	174.61403507
17	73106.73711800	79058.49784900	46327.60121400	60508.19774800	50746.26217300	113.24459708
18	73127.92949600	79078.78139400	46338.68775400	60520.12592200	50755.21543400	73.44389868
19	73141.67365500	79091.93612100	46345.87785000	60527.86184200	50761.02200500	47.63147217
20	73150.58731900	79100.46751900	46350.54092800	60532.87891500	50764.78781100	30.89101994
21	73156.36820800	79106.00049000	46353.56513200	60536.13269800	50767.23009500	20.03413111
22	73160.11735800	79109.58885600	46355.52645600	60538.24291400	50768.81401900	12.99298006
23	73162.54884100	79111.91606300	46356.79845700	60539.61147800	50769.84126100	8.42649615
24	73164.12576000	79113.42535500	46357.62340400	60540.49904900	50770.50747100	5.46493852
25	73165.14845800	79114.40419400	46358.15841600	60541.07467700	50770.93953600	3.54424351
26	73165.81172100	79115.03901300	46358.50539500	60541.44799600	50771.21974900	2.29859143
27	73166.24187500	79115.45072000	46358.73042500	60541.69010900	50771.40147800	1.49073374
28	73166.52084900	79115.71772900	46358.87636600	60541.84712900	50771.51933800	0.96680373
29	73166.70177500	79115.89089600	46358.97101600	60541.94896400	50771.59577400	0.62701291
30	73166.81911200	79116.00320100	46359.03240000	60542.01500800	50771.64534700	0.40664434

Table 3.27 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
31	73166.89521100	79116.07603700	46359.07221000	60542.05784000	50771.67749700	0.26372600
32	73166.94456400	79116.12327300	46359.09802800	60542.08561800	50771.69834700	0.17103773
33	73166.97657200	79116.15390800	46359.11477300	60542.10363400	50771.71187000	0.11092514
34	73166.99733000	79116.17377600	46359.12563200	60542.11531800	50771.72063900	0.07193947
35	73167.01079300	79116.18666200	46359.13267500	60542.12289500	50771.72632700	0.04665589
36	73167.01952400	79116.19501800	46359.13724200	60542.12781000	50771.73001600	0.03025818
37	73167.02518700	79116.20043800	46359.14020500	60542.13099700	50771.73240800	0.01962388
38	73167.02885900	79116.20395300	46359.14212600	60542.13306400	50771.73396000	0.01272678
39	73167.03124100	79116.20623200	46359.14337200	60542.13440400	50771.73496600	0.00825381
40	73167.03278500	79116.20771100	46359.14418000	60542.13527400	50771.73561800	0.00535285
41	73167.03378700	79116.20867000	46359.14470400	60542.13583700	50771.73604200	0.00347191
42	73167.03443700	79116.20929100	46359.14504400	60542.13620300	50771.73631600	0.00225145
43	73167.03485800	79116.20969500	46359.14526400	60542.13644000	50771.73649400	0.00146013
44	73167.03513100	79116.20995600	46359.14540700	60542.13659400	50771.73660900	0.00094706
45	73167.03530900	79116.21012600	46359.14550000	60542.13669400	50771.73668400	0.00061423
46	73167.03542300	79116.21023600	46359.14556000	60542.13675900	50771.73673300	0.00039828
47	73167.03549800	79116.21030700	46359.14559900	60542.13680000	50771.73676400	0.00025833
48	73167.03554600	79116.21035300	46359.14562400	60542.13682800	50771.73678500	0.00016755
49	73167.03557800	79116.21038300	46359.14564100	60542.13684500	50771.73679800	0.00010866
50	73167.03559800	79116.21040300	46359.14565100	60542.13685700	50771.73680700	0.00007057
51	73167.03561100	79116.21041600	46359.14565800	60542.13686400	50771.73681200	0.00004560
52	73167.03562000	79116.21042400	46359.14566300	60542.13686900	50771.73681600	0.00002956
53	73167.03562500	79116.21042900	46359.14566600	60542.13687200	50771.73681800	0.00001919
54	73167.03562900	79116.21043200	46359.14566800	60542.13687400	50771.73682000	0.00001240
55	73167.03563100	79116.21043500	46359.14566900	60542.13687500	50771.73682100	0.00000823
56	73167.03563300	79116.21043600	46359.14566900	60542.13687600	50771.73682100	0.00000542
57	73167.03563400	79116.21043700	46359.14567000	60542.13687700	50771.73682200	0.00000328
58	73167.03563400	79116.21043800	46359.14567000	60542.13687700	50771.73682200	0.00000215
59	73167.03563500	79116.21043800	46359.14567100	60542.13687700	50771.73682200	0.00000155
60	73167.03563500	79116.21043800	46359.14567100	60542.13687800	50771.73682200	0.00000095
61	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000054

Table 3.27 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
62	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000030
63	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000018
64	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000000

Table 3.28 the result of the example 3.5 from the L_4 method.

i	X_1	X_2	X_3	X_4	X_5	er
1	12509.62500000	17355.55468700	11937.41477300	29757.47748600	29180.33619300	100690.40814000
2	33673.55630100	41980.88128200	26888.58466600	43027.38992400	37712.69756900	82542.70160200
3	49771.92818400	57629.38246900	35071.13933400	50217.49477000	43041.97715900	52448.81217400
4	59547.96388500	66567.21102700	39744.13773600	54501.73178500	46238.60260600	30867.72512400
5	65203.05015700	71766.27879700	42489.91996200	57006.09727300	48120.34655400	17986.04570400
6	68504.57242100	74815.81884700	44094.02689300	58472.75554300	49220.37690800	10521.85786900
7	70438.77199300	76599.29264400	45033.61484300	59331.16557000	49863.76473600	6159.05917280
8	71570.31169600	77643.26088900	45583.42273900	59833.42303400	50240.38223800	3604.19081080
9	72232.59862300	78254.20988000	45905.16663000	60127.38329000	50460.77255600	2109.33038300
10	72620.18120500	78611.74308200	46093.46730500	60299.41231500	50589.75261100	1234.42553940
11	72847.00171300	78820.98397400	46203.66367900	60400.08821400	50665.23517100	722.41623199
12	72979.74387500	78943.43634700	46268.15368100	60459.00644500	50709.40931900	422.77691519
13	73057.42772000	79015.09865800	46305.89486100	60493.48683000	50735.26119400	247.41959846
14	73102.89030500	79057.03726600	46327.98194300	60513.66563800	50750.39036000	144.79624915
15	73129.49616400	79081.58079700	46340.90786800	60525.47478700	50759.24433100	84.73843384
16	73145.06658500	79095.94429500	46348.47244600	60532.38580000	50764.42590000	49.59107870
17	73154.17879000	79104.35017800	46352.89942900	60536.43030000	50767.45828500	29.02195668
18	73159.51148400	79109.26951300	46355.49021200	60538.79724400	50769.23291400	16.98438448
19	73162.63231100	79112.14843300	46357.00640400	60540.18243900	50770.27147200	9.93969250
20	73164.45869800	79113.83325000	46357.89371800	60540.99309100	50770.87926200	5.81696004
21	73165.52754700	79114.81924800	46358.41299600	60541.46750500	50771.23495600	3.40423226
22	73166.15306400	79115.39627900	46358.71689100	60541.74514400	50771.44311800	1.99224311
23	73166.51913200	79115.73397200	46358.89473800	60541.90762500	50771.56493900	1.16591126
24	73166.73336500	79115.93159800	46358.99881900	60542.00271300	50771.63623200	0.68232083

Table 3.28 (cont.)

i	X_1	X_2	X_3	X_4	X_5	er
25	73166.85873900	79116.04725400	46359.05972900	60542.05836100	50771.67795400	0.39931154
26	73166.93211100	79116.11493900	46359.09537600	60542.09092800	50771.70237100	0.23368704
27	73166.97505000	79116.15455000	46359.11623700	60542.10998700	50771.71666100	0.13675952
28	73167.00018000	79116.17773100	46359.12844500	60542.12114100	50771.72502300	0.08003533
29	73167.01488600	79116.19129800	46359.13559000	60542.12766800	50771.72991700	0.04683858
30	73167.02349200	79116.19923700	46359.13977100	60542.13148800	50771.73278200	0.02741098
31	73167.02852900	79116.20388300	46359.14221800	60542.13372400	50771.73445800	0.01604182
32	73167.03147700	79116.20660200	46359.14365000	60542.13503200	50771.73543900	0.00938785
33	73167.03320200	79116.20819400	46359.14448900	60542.13579800	50771.73601300	0.00549430
34	73167.03421100	79116.20912500	46359.14497900	60542.13624600	50771.73634900	0.00321519
35	73167.03480200	79116.20967000	46359.14526600	60542.13650800	50771.73654500	0.00188172
36	73167.03514800	79116.20998900	46359.14543400	60542.13666100	50771.73666000	0.00110126
37	73167.03535000	79116.21017500	46359.14553200	60542.13675100	50771.73672800	0.00064433
38	73167.03546800	79116.21028500	46359.14559000	60542.13680400	50771.73676700	0.00037706
39	73167.03553800	79116.21034900	46359.14562400	60542.13683500	50771.73679000	0.00022089
40	73167.03557800	79116.21038600	46359.14564300	60542.13685300	50771.73680400	0.00012910
41	73167.03560200	79116.21040800	46359.14565500	60542.13686300	50771.73681100	0.00007564
42	73167.03561600	79116.21042100	46359.14566100	60542.13686900	50771.73681600	0.00004429
43	73167.03562400	79116.21042800	46359.14566500	60542.13687300	50771.73681900	0.00002593
44	73167.03562900	79116.21043300	46359.14566800	60542.13687500	50771.73682000	0.00001514
45	73167.03563200	79116.21043500	46359.14566900	60542.13687600	50771.73682100	0.00000882
46	73167.03563300	79116.21043700	46359.14567000	60542.13687700	50771.73682200	0.00000536
47	73167.03563400	79116.21043800	46359.14567000	60542.13687700	50771.73682200	0.00000304
48	73167.03563500	79116.21043800	46359.14567100	60542.13687700	50771.73682200	0.00000173
49	73167.03563500	79116.21043800	46359.14567100	60542.13687800	50771.73682200	0.00000089
50	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000054
51	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000048
52	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
53	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
54	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
55	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000000

Table 3.29 the result of the example 3.5 from the L_5 method.

i	X_1	X_2	X_3	X_4	X_5	er
1	12509.62500000	17355.55468700	11937.41477300	29757.47748600	29180.33619300	100690.40814000
2	33673.55630100	41980.88128200	26888.58466600	43027.38992400	37712.69756900	82542.70160200
3	49771.92818400	57629.38246900	35071.13933400	50217.49477000	43041.97715900	52448.81217400
4	59547.96388500	66567.21102700	39744.13773600	54501.73178500	46238.60260600	30867.72512400
5	65203.05015700	71766.27879700	42489.91996200	57006.09727300	48120.34655400	17986.04570400
6	68504.57242100	74815.81884700	44094.02689300	58472.75554300	49220.37690800	10521.85786900
7	70438.77199300	76599.29264400	45033.61484300	59331.16557000	49863.76473600	6159.05917280
8	71570.31169600	77643.26088900	45583.42273900	59833.42303400	50240.38223800	3604.19081080
9	72232.59862300	78254.20988000	45905.16663000	60127.38329000	50460.77255600	109.33038300
10	72620.18120500	78611.74308200	46093.46730500	60299.41231500	50589.75261100	1234.42553940
11	72847.00171300	78820.98397400	46203.66367900	60400.08821400	50665.23517100	722.41623199
12	72979.74387500	78943.43634700	46268.15368100	60459.00644500	50709.40931900	422.77691519
13	73057.42772000	79015.09865800	46305.89486100	60493.48683000	50735.26119400	247.41959846
14	73102.89030500	79057.03726600	46327.98194300	60513.66563800	50750.39036000	144.79624915
15	73129.49616400	79081.58079700	46340.90786800	60525.47478700	50759.24433100	84.73843384
16	73145.06658500	79095.94429500	46348.47244600	60532.38580000	50764.42590000	49.59107870
17	73154.17879000	79104.35017800	46352.89942900	60536.43030000	50767.45828500	29.02195668
18	73159.51148400	79109.26951300	46355.49021200	60538.79724400	50769.23291400	16.98438448
19	73162.63231100	79112.14843300	46357.00640400	60540.18243900	50770.27147200	9.93969250
20	73164.45869800	79113.83325000	46357.89371800	60540.99309100	50770.87926200	5.81696004
21	73165.52754700	79114.81924800	46358.41299600	60541.46750500	50771.23495600	3.40423226
22	73166.15306400	79115.39627900	46358.71689100	60541.74514400	50771.44311800	1.99224311
23	73166.51913200	79115.73397200	46358.89473800	60541.90762500	50771.56493900	1.16591126
24	73166.73336500	79115.93159800	46358.99881900	60542.00271300	50771.63623200	0.68232083
25	73166.85873900	79116.04725400	46359.05972900	60542.05836100	50771.67795400	0.39931154
26	73166.93211100	79116.11493900	46359.09537600	60542.09092800	50771.70237100	0.23368704
27	73166.97505000	79116.15455000	46359.11623700	60542.10998700	50771.71666100	0.13675952
28	73167.00018000	79116.17773100	46359.12844500	60542.12114100	50771.72502300	0.08003533
29	73167.01488600	79116.19129800	46359.13559000	60542.12766800	50771.72991700	0.04683858
30	73167.02349200	79116.19923700	46359.13977100	60542.13148800	50771.73278200	0.02741098

Table 3.29 (cont.)

i	x1	x2	x3	x4	x5	er
31	73167.02852900	79116.20388300	46359.14221800	60542.13372400	50771.73445800	0.01604182
32	73167.03147700	79116.20660200	46359.14365000	60542.13503200	50771.73543900	0.00938785
33	73167.03320200	79116.20819400	46359.14448900	60542.13579800	50771.73601300	0.00549430
34	73167.03421100	79116.20912500	46359.14497900	60542.13624600	50771.73634900	0.00321519
35	73167.03480200	79116.20967000	46359.14526600	60542.13650800	50771.73654500	0.00188172
36	73167.03514800	79116.20998900	46359.14543400	60542.13666100	50771.73666000	0.00110126
37	73167.03535000	79116.21017500	46359.14553200	60542.13675100	50771.73672800	0.00064433
38	73167.03546800	79116.21028500	46359.14559000	60542.13680400	50771.73676700	0.00037706
39	73167.03553800	79116.21034900	46359.14562400	60542.13683500	50771.73679000	0.00022089
40	73167.03557800	79116.21038600	46359.14564300	60542.13685300	50771.73680400	0.00012910
41	73167.03560200	79116.21040800	46359.14565500	60542.13686300	50771.73681100	0.00007564
42	73167.03561600	79116.21042100	46359.14566100	60542.13686900	50771.73681600	0.00004429
43	73167.03562400	79116.21042800	46359.14566500	60542.13687300	50771.73681900	0.00002593
44	73167.03562900	79116.21043300	46359.14566800	60542.13687500	50771.73682000	0.00001514
45	73167.03563200	79116.21043500	46359.14566900	60542.13687600	50771.73682100	0.00000882
46	73167.03563300	79116.21043700	46359.14567000	60542.13687700	50771.73682200	0.00000536
47	73167.03563400	79116.21043800	46359.14567000	60542.13687700	50771.73682200	0.00000304
48	73167.03563500	79116.21043800	46359.14567100	60542.13687700	50771.73682200	0.00000173
49	73167.03563500	79116.21043800	46359.14567100	60542.13687800	50771.73682200	0.00000089
50	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000054
51	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682200	0.00000048
52	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
53	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
54	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000012
55	73167.03563500	79116.21043900	46359.14567100	60542.13687800	50771.73682300	0.00000000

Table 3.30 the result of the example 3.6 from the D method.

i	V_1	V_2	V_3	V_4	V_5	V_6	er
1	45.45454545	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	500.00000000
2	45.45454545	22.17294900	0.00000000	0.00000000	0.00000000	1.93423598	1000.00000000
3	55.70899827	22.17294900	9.50269243	0.00000000	3.41187095	1.93423598	274.85021465
4	55.70899827	31.15101409	9.50269243	6.45728169	3.41187095	3.45949066	452.70220179
5	59.92859646	31.15101409	17.04030986	6.45728169	7.49707925	3.45949066	213.56473447
6	59.92859646	36.56485494	17.04030986	12.26869456	7.49707925	4.94283791	303.30762167
7	62.52428297	36.56485494	22.68133472	12.26869456	10.94728854	4.94283791	164.64558556
8	62.52428297	40.39974174	22.68133472	16.81431163	10.94728854	6.15442328	223.26610512
9	64.37755745	40.39974174	26.92235311	16.81431163	13.63066752	6.15442328	125.20775968
10	64.37755745	43.24805977	26.92235311	20.27651031	13.63066752	7.08968357	167.66267057
11	65.75727113	43.24805977	30.12146008	20.27651031	15.67337629	7.08968357	94.76644467
12	65.75727113	45.39042882	30.12146008	22.89741818	15.67337629	7.80032312	126.47900536
13	66.79567884	45.39042882	32.53727989	22.89741818	17.21965411	7.80032312	71.62900245
14	66.79567884	47.00709024	32.53727989	24.87846700	17.21965411	8.33800360	95.51619850
15	67.57940498	47.00709024	34.36216267	24.87846700	18.38842838	8.33800360	54.12084672
16	67.57940498	48.22807585	34.36216267	26.37529553	18.38842838	8.74436672	72.15313373
17	68.17134054	48.22807585	35.74077281	26.37529553	19.27152441	8.74436672	40.88825103
18	68.17134054	49.15042804	35.74077281	27.50614861	19.27152441	9.05139462	54.50845729
19	68.61850316	49.15042804	36.78226837	27.50614861	19.93870320	9.05139462	30.89026371
20	68.61850316	49.84722702	36.78226837	28.36048578	19.93870320	9.28335222	41.17943982
21	68.95631703	49.84722702	37.56908917	28.36048578	20.44274365	9.28335222	23.33683078
22	68.95631703	50.37363756	37.56908917	29.00591641	20.44274365	9.45859125	31.10992786
23	69.21152537	50.37363756	38.16351119	29.00591641	20.82353377	9.45859125	17.63036924
24	69.21152537	50.77132629	38.16351119	29.49352248	20.82353377	9.59097994	23.50271841
25	69.40432831	50.77132629	38.61258126	29.49352248	21.11121081	9.59097994	13.31928003
26	69.40432831	51.07176951	38.61258126	29.86189604	21.11121081	9.69099615	17.75568067
27	69.54998579	51.07176951	38.95184181	29.86189604	21.32854325	9.69099615	10.06236444
28	69.54998579	51.29874640	38.95184181	30.14019253	21.32854325	9.76655575	13.41394710
29	69.66002616	51.29874640	39.20814419	30.14019253	21.49273217	9.76655575	7.60185047
30	69.66002616	51.47022144	39.20814419	30.35043818	21.49273217	9.82363904	10.13388244

Table 3.30 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
31	69.74315875	51.47022144	39.40177386	30.35043818	21.61677256	9.82363904	5.74299713
32	69.74315875	51.59976630	39.40177386	30.50927321	21.61677256	9.86676396	7.65588034
33	69.80596322	51.59976630	39.54805596	30.50927321	21.71048180	9.86676396	4.33868256
34	69.80596322	51.69763402	39.54805596	30.62926888	21.71048180	9.89934369	5.78381526
35	69.85341034	51.69763402	39.65856823	30.62926888	21.78127665	9.89934369	3.27776000
36	69.85341034	51.77157049	39.65856823	30.71992244	21.78127665	9.92395682	4.36951957
37	69.88925539	51.77157049	39.74205732	30.71992244	21.83476029	9.92395682	2.47626106
38	69.88925539	51.82742754	39.74205732	30.78840880	21.83476029	9.94255139	3.30105655
39	69.91633537	51.82742754	39.80513112	30.78840880	21.87516577	9.94255139	1.87074978
40	69.91633537	51.86962607	39.80513112	30.84014844	21.87516577	9.95659909	2.49386098
41	69.93679359	51.86962607	39.85278171	30.84014844	21.90569104	9.95659909	1.41330202
42	69.93679359	51.90150594	39.85278171	30.87923637	21.90569104	9.96721176	1.88404606
43	69.95224922	51.90150594	39.88878047	30.87923637	21.92875207	9.96721176	1.06771232
44	69.95224922	51.92559034	39.88878047	30.90876627	21.92875207	9.97522935	1.42334700
45	69.96392555	51.92559034	39.91597659	30.90876627	21.94617407	9.97522935	0.80662844
46	69.96392555	51.94378547	39.91597659	30.93107533	21.94617407	9.98128643	1.07530105
47	69.97274671	51.94378547	39.93652253	30.93107533	21.95933593	9.98128643	0.60938647
48	69.97274671	51.95753141	39.93652253	30.94792923	21.95933593	9.98586239	0.81236152
49	69.97941086	51.95753141	39.95204445	30.94792923	21.96927937	9.98586239	0.46037537
50	69.97941086	51.96791610	39.95204445	30.96066191	21.96927937	9.98931941	0.61371766
51	69.98444545	51.96791610	39.96377085	30.96066191	21.97679138	9.98931941	0.34780143
52	69.98444545	51.97576146	39.96377085	30.97028111	21.97679138	9.99193110	0.46364747
53	69.98824895	51.97576146	39.97262984	30.97028111	21.98246650	9.99193110	0.26275479
54	69.98824895	51.98168843	39.97262984	30.97754817	21.98246650	9.99390416	0.35027341
55	69.99112239	51.98168843	39.97932256	30.97754817	21.98675390	9.99390416	0.19850430
56	69.99112239	51.98616609	39.97932256	30.98303823	21.98675390	9.99539475	0.26462231
57	69.99329320	51.98616609	39.98437874	30.98303823	21.98999292	9.99539475	0.14996475
58	69.99329320	51.98954885	39.98437874	30.98718583	21.98999292	9.99652086	0.19991516
59	69.99493319	51.98954885	39.98819855	30.98718583	21.99243992	9.99652086	0.11329441
60	69.99493319	51.99210443	39.98819855	30.99031924	21.99243992	9.99737160	0.15103062

Table 3.30 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
61	69.99617216	51.99210443	39.99108432	30.99031924	21.99428856	9.99737160	0.08559093
62	69.99617216	51.99403511	39.99108432	30.99268644	21.99428856	9.99801431	0.11409964
63	69.99710817	51.99403511	39.99326444	30.99268644	21.99568516	9.99801431	0.06466168
64	69.99710817	51.99549368	39.99326444	30.99447480	21.99568516	9.99849987	0.08619926
65	69.99781530	51.99549368	39.99491146	30.99447480	21.99674025	9.99849987	0.04885019
66	69.99781530	51.99659559	39.99491146	30.99582586	21.99674025	9.99886669	0.06512127
67	69.99834951	51.99659559	39.99615574	30.99582586	21.99753735	9.99886669	0.03690503
68	69.99834951	51.99742806	39.99615574	30.99684654	21.99753735	9.99914381	0.04919739
69	69.99875310	51.99742806	39.99709577	30.99684654	21.99813953	9.99914381	0.02788078
70	69.99875310	51.99805697	39.99709577	30.99761765	21.99813953	9.99935317	0.03716733
71	69.99905800	51.99805697	39.99780593	30.99761765	21.99859446	9.99935317	0.02106319
72	69.99905800	51.99853209	39.99780593	30.99820020	21.99859446	9.99951134	0.02807894
73	69.99928835	51.99853209	39.99834244	30.99820020	21.99893815	9.99951134	0.01591268
74	69.99928835	51.99889103	39.99834244	30.99864030	21.99893815	9.99963083	0.02121290
75	69.99946236	51.99889103	39.99874775	30.99864030	21.99919780	9.99963083	0.01202161
76	69.99946236	51.99916221	39.99874775	30.99897278	21.99919780	9.99972110	0.01602578
77	69.99959383	51.99916221	39.99905396	30.99897278	21.99939396	9.99972110	0.00908201
78	69.99959383	51.99936707	39.99905396	30.99922396	21.99939396	9.99978930	0.01210705
79	69.99969315	51.99936707	39.99928529	30.99922396	21.99954215	9.99978930	0.00686122
80	69.99969315	51.99952184	39.99928529	30.99941372	21.99954215	9.99984082	0.00914656
81	69.99976818	51.99952184	39.99946006	30.99941372	21.99965411	9.99984082	0.00518347
82	69.99976818	51.99963876	39.99946006	30.99955708	21.99965411	9.99987974	0.00690998
83	69.99982487	51.99963876	39.99959209	30.99955708	21.99973869	9.99987974	0.00391598
84	69.99982487	51.99972709	39.99959209	30.99966539	21.99973869	9.99990915	0.00522031
85	69.99986769	51.99972709	39.99969183	30.99966539	21.99980259	9.99990915	0.00295842
86	69.99986769	51.99979383	39.99969183	30.99974721	21.99980259	9.99993137	0.00394380
87	69.99990005	51.99979383	39.99976719	30.99974721	21.99985086	9.99993137	0.00223500
88	69.99990005	51.99984424	39.99976719	30.99980902	21.99985086	9.99994815	0.00297944
89	69.99992449	51.99984424	39.99982412	30.99980902	21.99988733	9.99994815	0.00168849
90	69.99992449	51.99988233	39.99982412	30.99985572	21.99988733	9.99996083	0.00225089

Table 3.30 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
91	69.99994295	51.99988233	39.99986712	30.99985572	21.99991488	9.99996083	0.00127561
92	69.99994295	51.99991110	39.99986712	30.99989100	21.99991488	9.99997041	0.00170049
93	69.99995690	51.99991110	39.99989962	30.99989100	21.99993569	9.99997041	0.00096369
94	69.99995690	51.99993284	39.99989962	30.99991766	21.99993569	9.99997764	0.00128467
95	69.99996744	51.99993284	39.99992416	30.99991766	21.99995142	9.99997764	0.00072804
96	69.99996744	51.99994926	39.99992416	30.99993779	21.99995142	9.99998311	0.00097054
97	69.99997540	51.99994926	39.99994271	30.99993779	21.99996330	9.99998311	0.00055002
98	69.99997540	51.99996167	39.99994271	30.99995300	21.99996330	9.99998724	0.00073321
99	69.99998142	51.99996167	39.99995672	30.99995300	21.99997227	9.99998724	0.00041553
100	69.99998142	51.99997104	39.99995672	30.99996449	21.99997227	9.99999036	0.00055392
101	69.99998596	51.99997104	39.99996730	30.99996449	21.99997905	9.99999036	0.00031392
102	69.99998596	51.99997812	39.99996730	30.99997318	21.99997905	9.99999272	0.00041848
103	69.99998939	51.99997812	39.99997530	30.99997318	21.99998417	9.99999272	0.00023716
104	69.99998939	51.99998347	39.99997530	30.99997974	21.99998417	9.99999450	0.00031615
105	69.99999199	51.99998347	39.99998134	30.99997974	21.99998804	9.99999450	0.00017917
106	69.99999199	51.99998751	39.99998134	30.99998469	21.99998804	9.99999584	0.00023884
107	69.99999395	51.99998751	39.99998590	30.99998469	21.99999097	9.99999584	0.00013535
108	69.99999395	51.99999057	39.99998590	30.99998843	21.99999097	9.99999686	0.00018044
109	69.99999543	51.99999057	39.99998935	30.99998843	21.99999318	9.99999686	0.00010226
110	69.99999543	51.99999287	39.99998935	30.99999126	21.99999318	9.99999763	0.00013631
111	69.99999655	51.99999287	39.99999195	30.99999126	21.99999485	9.99999763	0.00007725
112	69.99999655	51.99999462	39.99999195	30.99999340	21.99999485	9.99999821	0.00010299
113	69.99999739	51.99999462	39.99999392	30.99999340	21.99999611	9.99999821	0.00005836
114	69.99999739	51.99999593	39.99999392	30.99999501	21.99999611	9.99999865	0.00007780
115	69.99999803	51.99999593	39.99999541	30.99999501	21.99999706	9.99999865	0.00004409
116	69.99999803	51.99999693	39.99999541	30.99999623	21.99999706	9.99999898	0.00005878
117	69.99999851	51.99999693	39.99999653	30.99999623	21.99999778	9.99999898	0.00003331
118	69.99999851	51.99999768	39.99999653	30.99999715	21.99999778	9.99999923	0.00004440
119	69.99999887	51.99999768	39.99999738	30.99999715	21.99999832	9.99999923	0.00002516
120	69.99999887	51.99999825	39.99999738	30.99999785	21.99999832	9.99999942	0.00003355

Table 3.30 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
121	69.99999915	51.99999825	39.99999802	30.99999785	21.99999873	9.99999942	0.00001901
122	69.99999915	51.99999868	39.99999802	30.99999838	21.99999873	9.99999956	0.00002534
123	69.99999936	51.99999868	39.99999850	30.99999838	21.99999904	9.99999956	0.00001436
124	69.99999936	51.99999900	39.99999850	30.99999877	21.99999904	9.99999967	0.00001915
125	69.99999951	51.99999900	39.99999887	30.99999877	21.99999928	9.99999967	0.00001085
126	69.99999951	51.99999924	39.99999887	30.99999907	21.99999928	9.99999975	0.00001446
127	69.99999963	51.99999924	39.99999915	30.99999907	21.99999945	9.99999975	0.00000820
128	69.99999963	51.99999943	39.99999915	30.99999930	21.99999945	9.99999981	0.00001093
129	69.99999972	51.99999943	39.99999936	30.99999930	21.99999959	9.99999981	0.00000619
130	69.99999972	51.99999957	39.99999936	30.99999947	21.99999959	9.99999986	0.00000826
131	69.99999979	51.99999957	39.99999951	30.99999947	21.99999969	9.99999986	0.00000468
132	69.99999979	51.99999967	39.99999951	30.99999960	21.99999969	9.99999989	0.00000624
133	69.99999984	51.99999967	39.99999963	30.99999960	21.99999976	9.99999989	0.00000353
134	69.99999984	51.99999975	39.99999963	30.99999970	21.99999976	9.99999992	0.00000472
135	69.99999988	51.99999975	39.99999972	30.99999970	21.99999982	9.99999992	0.00000267
136	69.99999988	51.99999981	39.99999972	30.99999977	21.99999982	9.99999994	0.00000356
137	69.99999991	51.99999981	39.99999979	30.99999977	21.99999987	9.99999994	0.00000202
138	69.99999991	51.99999986	39.99999979	30.99999983	21.99999987	9.99999995	0.00000269
139	69.99999993	51.99999986	39.99999984	30.99999983	21.99999990	9.99999995	0.00000152
140	69.99999993	51.99999989	39.99999984	30.99999987	21.99999990	9.99999996	0.00000203
141	69.99999995	51.99999989	39.99999988	30.99999987	21.99999992	9.99999996	0.00000115
142	69.99999995	51.99999992	39.99999988	30.99999990	21.99999992	9.99999997	0.00000154
143	69.99999996	51.99999992	39.99999991	30.99999990	21.99999994	9.99999997	0.00000087
144	69.99999996	51.99999994	39.99999991	30.99999993	21.99999994	9.99999998	0.00000116
145	69.99999997	51.99999994	39.99999993	30.99999993	21.99999996	9.99999998	0.00000066
146	69.99999997	51.99999995	39.99999993	30.99999994	21.99999996	9.99999998	0.00000087
147	69.99999998	51.99999995	39.99999995	30.99999994	21.99999997	9.99999998	0.00000050
148	69.99999998	51.99999997	39.99999995	30.99999996	21.99999997	9.99999999	0.00000066
149	69.99999998	51.99999997	39.99999996	30.99999996	21.99999998	9.99999999	0.00000038
150	69.99999998	51.99999997	39.99999996	30.99999997	21.99999998	9.99999999	0.00000050

Table 3.30 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
151	69.99999999	51.99999997	39.99999997	30.99999997	21.99999998	9.99999999	0.00000028
152	69.99999999	51.99999998	39.99999997	30.99999998	21.99999998	9.99999999	0.00000038
153	69.99999999	51.99999998	39.99999998	30.99999998	21.99999999	9.99999999	0.00000021
154	69.99999999	51.99999999	39.99999998	30.99999998	21.99999999	10.0000000	0.00000029
155	69.99999999	51.99999999	39.99999998	30.99999998	21.99999999	10.0000000	0.00000016
156	69.99999999	51.99999999	39.99999998	30.99999999	21.99999999	10.0000000	0.00000021
157	69.99999999	51.99999999	39.99999999	30.99999999	21.99999999	10.0000000	0.00000012
158	69.99999999	51.99999999	39.99999999	30.99999999	21.99999999	10.0000000	0.00000016
159	70.00000000	51.99999999	39.99999999	30.99999999	21.99999999	0.00000000	0.00000009

Table 3.31 the result of the example 3.6 from the U_2 method.

i	V_1	V_2	V_3	V_4	V_5	V_6	er
1	45.45454545	0	0	0	0	0	45.45454545
2	55.58239198	22.28126237	0.29605653	0.51809892	1.03619784	1.93423598	36.19369817
3	59.81646398	31.20937356	10.78128594	2.15630363	4.01655073	2.69590961	29.02764383
4	62.25931946	36.4313209	18.21703819	8.47278667	6.16428739	3.82725934	24.69612451
5	63.92601662	39.87178471	23.50473729	13.80979958	9.40256096	4.61665851	19.75954571
6	65.21537185	42.55048636	27.30894951	17.88682311	12.26890894	5.72107336	15.82005546
7	66.27149158	44.6530668	30.19032322	20.92020086	14.53145221	6.69073144	12.30565298
8	67.11277336	46.3099551	32.40597428	23.22065489	16.25098656	7.45776098	9.50083907
9	67.76913299	47.60054038	34.12405127	24.9846234	17.56327253	8.04234777	7.32586317
10	68.27658958	48.60002751	35.45592221	26.34745858	18.57086588	8.48909264	5.65598805
11	68.66820925	49.37224182	36.48699519	27.40222095	19.3485197	8.83225888	4.3704894
12	68.9705651	49.96879145	37.28434307	28.21841901	19.94984282	9.09711094	3.37862659
13	69.20418341	50.42978131	37.90071166	28.84965182	20.41496057	9.30188878	2.61210518
14	69.38476903	50.78611412	38.37714981	29.33767618	20.77464069	9.46027182	2.01944409
15	69.52438007	51.0615818	38.74544193	29.71493779	21.05272578	9.58274784	1.56119357
16	69.63231387	51.27454095	39.03014819	30.00657298	21.26770403	9.67743929	1.20690409
17	69.7157556	51.43917446	39.25024343	30.23202029	21.4338924	9.7506423	0.93300918
18	69.78026167	51.56644722	39.4203913	30.40630393	21.56236443	9.80723185	0.72127193

Table 3.31 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
19	69.83012881	51.66483702	39.55192651	30.54103598	21.66168066	9.85097851	0.55758708
20	69.86867912	51.74089836	39.65361149	30.64519234	21.73845817	9.88479718	0.43104915
21	69.89848085	51.79969843	39.73222028	30.72571172	21.79781195	9.91094108	0.33322765
22	69.92151942	51.8451545	39.79298973	30.7879582	21.84369613	9.93115194	0.25760561
23	69.93932966	51.88029485	39.83996826	30.83607858	21.87916744	9.94677619	0.19914507
24	69.95309807	51.90746052	39.87628557	30.87327861	21.90658896	9.9588547	0.15395146
25	69.96374191	51.92846126	39.9043611	30.90203653	21.92778749	9.96819214	0.119014
26	69.97197025	51.94469613	39.92606522	30.92426818	21.94417527	9.97541056	0.09200518
27	69.97833127	51.95724669	39.94284384	30.94145462	21.95684403	9.98099084	0.07112569
28	69.98324873	51.96694905	39.95581475	30.9547408	21.96663777	9.98530474	0.05498456
29	69.98705024	51.97444957	39.96584207	30.96501184	21.97420893	9.98863966	0.04250646
30	69.98998903	51.98024794	39.97359381	30.97295199	21.98006191	9.99121775	0.03286012
31	69.9922609	51.98473044	39.97958638	30.97909022	21.98458662	9.99321078	0.02540291
32	69.9940172	51.98819568	39.98421901	30.98383544	21.98808451	9.99475151	0.01963802
33	69.99537493	51.99087453	39.98780032	30.9875038	21.99078859	9.9959426	0.0151814
34	69.99642453	51.99294545	39.99056889	30.99033966	21.99287901	9.99686338	0.01173616
35	69.99723594	51.9945464	39.99270917	30.99253196	21.99449503	9.9975752	0.00907278
36	69.99786321	51.99578403	39.99436374	30.99422675	21.99574432	9.99812548	0.00701382
37	69.99834813	51.99674079	39.99564282	30.99553692	21.9967101	9.99855088	0.00542212
38	69.998723	51.99748043	39.99663163	30.99654976	21.9974567	9.99887974	0.00419163
39	69.9990128	51.99805222	39.99739604	30.99733275	21.99803387	9.99913397	0.00324039
40	69.99923684	51.99849424	39.99798698	30.99793805	21.99848006	9.9993305	0.00250502
41	69.99941003	51.99883596	39.99844381	30.99840599	21.99882499	9.99948244	0.00193654
42	69.99954391	51.99910012	39.99879697	30.99876773	21.99909165	9.99959989	0.00149706
43	69.99964742	51.99930434	39.99906998	30.99904738	21.99929779	9.99969069	0.00115732
44	69.99972743	51.99946221	39.99928104	30.99926356	21.99945715	9.99976089	0.00089468
45	69.99978929	51.99958426	39.9994442	30.99943069	21.99958034	9.99981515	0.00069164
46	69.99983711	51.9996786	39.99957033	30.99955989	21.99967558	9.9998571	0.00053468
47	69.99987407	51.99975154	39.99966784	30.99965977	21.9997492	9.99988953	0.00041334
48	69.99990265	51.99980793	39.99974322	30.99973698	21.99980612	9.9999146	0.00031954

Table 3.31 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
49	69.99992474	51.99985151	39.99980149	30.99979667	21.99985012	9.99993398	0.00024702
50	69.99994182	51.99988521	39.99984654	30.99984281	21.99988413	9.99994896	0.00019097
51	69.99995502	51.99991126	39.99988137	30.99987848	21.99991043	9.99996054	0.00014763
52	69.99996523	51.9999314	39.99990829	30.99990606	21.99993075	9.9999695	0.00011413
53	69.99997312	51.99994697	39.9999291	30.99992738	21.99994647	9.99997642	0.00008823
54	69.99997922	51.999959	39.99994519	30.99994386	21.99995862	9.99998177	0.0000682
55	69.99998394	51.99996831	39.99995763	30.9999566	21.99996801	9.99998591	0.00005273
56	69.99998758	51.9999755	39.99996725	30.99996645	21.99997527	9.99998911	0.00004076
57	69.9999904	51.99998106	39.99997468	30.99997406	21.99998088	9.99999158	0.00003151
58	69.99999258	51.99998536	39.99998042	30.99997995	21.99998522	9.99999349	0.00002436
59	69.99999426	51.99998868	39.99998487	30.9999845	21.99998857	9.99999497	0.00001883
60	69.99999556	51.99999125	39.9999883	30.99998802	21.99999117	9.99999611	0.00001456
61	69.99999657	51.99999324	39.99999096	30.99999074	21.99999317	9.99999699	0.00001125
62	69.99999735	51.99999477	39.99999301	30.99999284	21.99999472	9.99999767	0.0000087
63	69.99999795	51.99999596	39.9999946	30.99999446	21.99999592	9.9999982	0.00000673
64	69.99999842	51.99999687	39.99999582	30.99999572	21.99999685	9.99999861	0.0000052
65	69.99999878	51.99999758	39.99999677	30.99999669	21.99999756	9.99999893	0.00000402
66	69.99999905	51.99999813	39.9999975	30.99999744	21.99999811	9.99999917	0.00000311
67	69.99999927	51.99999856	39.99999807	30.99999802	21.99999854	9.99999936	0.0000024
68	69.99999943	51.99999888	39.99999851	30.99999847	21.99999887	9.9999995	0.00000186
69	69.99999956	51.99999914	39.99999885	30.99999882	21.99999913	9.99999962	0.00000144
70	69.99999966	51.99999933	39.99999911	30.99999909	21.99999933	9.9999997	0.00000111
71	69.99999974	51.99999948	39.99999931	30.99999929	21.99999948	9.99999977	0.00000086
72	69.9999998	51.9999996	39.99999947	30.99999945	21.9999996	9.99999982	0.00000066
73	69.99999984	51.99999969	39.99999959	30.99999958	21.99999969	9.99999986	0.00000051
74	69.99999988	51.99999976	39.99999968	30.99999967	21.99999976	9.99999989	0.0000004
75	69.99999991	51.99999982	39.99999975	30.99999975	21.99999981	9.99999992	0.00000031
76	69.99999993	51.99999986	39.99999981	30.99999981	21.99999986	9.99999994	0.00000024
77	69.99999994	51.99999989	39.99999985	30.99999985	21.99999989	9.99999995	0.00000018
78	69.99999996	51.99999991	39.99999989	30.99999988	21.99999991	9.99999996	0.00000014

Table 3.31 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
79	69.99999997	51.99999993	39.99999991	30.99999991	21.99999993	9.99999997	0.00000011
80	69.99999997	51.99999995	39.99999993	30.99999993	21.99999995	9.99999998	0.00000008

Table 3.32 the result of the example 3.6 from the L_2 method.

i	V_1	V_2	V_3	V_4	V_5	V_6	er
1	45.45454545	22.17294900	9.50269243	4.75134621	1.69690936	0.54156682	500.00000000
2	55.58239198	30.83828493	15.93146281	8.81418609	5.81372179	3.78967910	450.38188885
3	59.81646398	35.85813543	20.40445009	13.10908594	10.01610360	5.56183698	283.04877922
4	62.25931946	39.30121371	24.33428356	17.17519358	12.95549631	6.68011431	190.67255519
5	63.92601662	41.98213570	27.80674020	20.38111826	15.06844780	7.45841183	146.11045616
6	65.21537185	44.19071285	30.58523022	22.82683901	16.64610624	8.03284312	116.93473930
7	66.27149158	45.95329079	32.73817549	24.69214086	17.85664979	8.47405299	91.94207079
8	67.11277336	47.32848776	34.39343239	26.12504109	18.79360993	8.81802409	71.50141205
9	67.76913299	48.39136110	35.66774967	27.23067980	19.52009366	9.08567982	55.30685237
10	68.27658958	49.21142923	36.65100098	28.08554732	20.08266978	9.29315559	42.72089788
11	68.66820925	49.84451752	37.41082027	28.74674502	20.51782399	9.45362849	33.00134317
12	68.97056510	50.33367195	37.99828514	29.25805456	20.85423306	9.57765775	25.50367160
13	69.20418341	50.71178887	38.45251212	29.65337259	21.11427171	9.67351502	19.71448994
14	69.38476903	51.00411446	38.80369053	29.95898112	21.31528225	9.74760852	15.24083576
15	69.52438007	51.23011349	39.07518071	30.19523148	21.47067093	9.80488515	11.78245612
16	69.63231387	51.40482960	39.28505925	30.37786509	21.59079531	9.84916361	9.10871512
17	69.71575560	51.53989689	39.44730729	30.51905130	21.68365914	9.88339393	7.04163504
18	69.78026167	51.64431214	39.57273452	30.62819683	21.75544886	9.90985626	5.44361927
19	69.83012881	51.72503139	39.66969736	30.71257311	21.81094683	9.93031332	4.20824976
20	69.86867912	51.78743228	39.74465561	30.77780122	21.85385022	9.94612789	3.25323484
21	69.89848085	51.83567201	39.80260299	30.82822660	21.88701719	9.95835353	2.51495094
22	69.92151942	51.87296430	39.84739990	30.86720855	21.91265730	9.96780471	1.94421249
23	69.93932966	51.90179355	39.88203069	30.89734400	21.93247870	9.97511105	1.50299651
24	69.95309807	51.92408034	39.90880243	30.92064056	21.94780186	9.98075930	1.16190927
25	69.96374191	51.94130941	39.92949864	30.93865025	21.95964761	9.98512575	0.89822772

Table 3.32 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
26	69.97197025	51.95462854	39.94549809	30.95257285	21.96880511	9.98850129	0.69438557
27	69.97833127	51.96492506	39.95786665	30.96333588	21.97588442	9.99111078	0.53680297
28	69.98324873	51.97288490	39.96742832	30.97165637	21.98135716	9.99312809	0.41498189
29	69.98705024	51.97903835	39.97482008	30.97808862	21.98558794	9.99468759	0.32080666
30	69.98998903	51.98379535	39.98053436	30.98306115	21.98885858	9.99589317	0.24800338
31	69.99226090	51.98747280	39.98495186	30.98690522	21.99138700	9.99682517	0.19172195
32	69.99401720	51.99031570	39.98836686	30.98987693	21.99334162	9.99754566	0.14821292
33	69.99537493	51.99251344	39.99100686	30.99217424	21.99485266	9.99810264	0.11457775
34	69.99642453	51.99421243	39.99304775	30.99395020	21.99602079	9.99853323	0.08857569
35	69.99723594	51.99552585	39.99462548	30.99532313	21.99692382	9.99886609	0.06847448
36	69.99786321	51.99654120	39.99584516	30.99638449	21.99762192	9.99912342	0.05293501
37	69.99834813	51.99732614	39.99678805	30.99720499	21.99816160	9.99932235	0.04092203
38	69.99872300	51.99793294	39.99751697	30.99783928	21.99857880	9.99947613	0.03163526
39	69.99901280	51.99840203	39.99808046	30.99832963	21.99890133	9.99959502	0.02445601
40	69.99923684	51.99876467	39.99851608	30.99870870	21.99915066	9.99968692	0.01890601
41	69.99941003	51.99904502	39.99885284	30.99900175	21.99934341	9.99975797	0.01461552
42	69.99954391	51.99926174	39.99911317	30.99922829	21.99949241	9.99981290	0.01129870
43	69.99964742	51.99942928	39.99931443	30.99940342	21.99960760	9.99985536	0.00873459
44	69.99972743	51.99955880	39.99947001	30.99953881	21.99969665	9.99988818	0.00675238
45	69.99978929	51.99965892	39.99959028	30.99964347	21.99976549	9.99991356	0.00522001
46	69.99983711	51.99973633	39.99968326	30.99972438	21.99981871	9.99993318	0.00403539
47	69.99987407	51.99979616	39.99975514	30.99978693	21.99985985	9.99994834	0.00311960
48	69.99990265	51.99984242	39.99981071	30.99983528	21.99989166	9.99996006	0.00241164
49	69.99992474	51.99987818	39.99985367	30.99987266	21.99991625	9.99996913	0.00186436
50	69.99994182	51.99990583	39.99988688	30.99990156	21.99993525	9.99997613	0.00144126
51	69.99995502	51.99992720	39.99991255	30.99992390	21.99994995	9.99998155	0.00111418
52	69.99996523	51.99994372	39.99993239	30.99994117	21.99996131	9.99998574	0.00086133
53	69.99997312	51.99995649	39.99994774	30.99995452	21.99997009	9.99998897	0.00066587
54	69.99997922	51.99996637	39.99995960	30.99996484	21.99997688	9.99999148	0.00051475
59	69.99999426	51.99999071	39.99998884	30.99999029	21.99999362	9.99999765	0.00014212

Table 3.32 (cont.)

i	V_1	V_2	V_3	V_4	V_5	V_6	er
60	69.99999556	51.99999282	39.99999138	30.99999250	21.99999506	9.99999818	0.00010987
61	69.99999657	51.99999445	39.99999333	30.99999420	21.99999618	9.99999859	0.00008494
62	69.99999735	51.99999571	39.99999485	30.99999552	21.99999705	9.99999891	0.00006566
63	69.99999795	51.99999668	39.99999602	30.99999653	21.99999772	9.99999916	0.00005076
64	69.99999842	51.99999744	39.99999692	30.99999732	21.99999824	9.99999935	0.00003924
65	69.99999878	51.99999802	39.99999762	30.99999793	21.99999864	9.99999950	0.00003033
66	69.99999905	51.99999847	39.99999816	30.99999840	21.99999895	9.99999961	0.00002345
67	69.99999927	51.99999882	39.99999858	30.99999876	21.99999919	9.99999970	0.00001813
68	69.99999943	51.99999908	39.99999890	30.99999904	21.99999937	9.99999977	0.00001401
69	69.99999956	51.99999929	39.99999915	30.99999926	21.99999951	9.99999982	0.00001083
70	69.99999966	51.99999945	39.99999934	30.99999943	21.99999962	9.99999986	0.00000837
71	69.99999974	51.99999958	39.99999949	30.99999956	21.99999971	9.99999989	0.00000648
72	69.99999980	51.99999967	39.99999961	30.99999966	21.99999978	9.99999992	0.00000500
73	69.99999984	51.99999975	39.99999970	30.99999974	21.99999983	9.99999994	0.00000387
74	69.99999988	51.99999980	39.99999977	30.99999980	21.99999987	9.99999995	0.00000299
75	69.99999991	51.99999985	39.99999982	30.99999984	21.99999990	9.99999996	0.00000231
76	69.99999993	51.99999988	39.99999986	30.99999988	21.99999992	9.99999997	0.00000179
77	69.99999994	51.99999991	39.99999989	30.99999991	21.99999994	9.99999998	0.00000138
78	69.99999996	51.99999993	39.99999992	0.99999993	21.99999995	9.99999998	0.00000107
79	69.99999997	51.99999995	39.99999994	30.99999994	21.99999996	9.99999999	0.00000082
80	69.99999997	51.99999996	39.99999995	30.99999996	21.99999997	9.99999999	0.00000064
81	69.99999998	51.99999997	39.99999996	30.99999997	21.99999998	9.99999999	0.00000049
82	69.99999998	51.99999998	39.99999997	30.99999997	21.99999998	9.99999999	0.00000038
83	69.99999999	51.99999998	39.99999998	30.99999998	21.99999999	10.0000000	0.00000030
84	69.99999999	51.99999999	39.99999998	30.99999998	21.99999999	10.0000000	0.00000023
85	69.99999999	51.99999999	39.99999999	30.99999999	21.99999999	10.0000000	0.00000017
86	69.99999999	51.99999999	39.99999999	30.99999999	21.99999999	10.0000000	0.00000014
87	70.00000000	51.99999999	39.99999999	30.99999999	22.00000000	10.0000000	0.00000010
88	70.00000000	51.99999999	39.99999999	30.99999999	22.00000000	10.0000000	0.00000008

Table 3.33 the result of the example 3.7 from the *D* method.

i	x	y	er
1	-1415.62777590	0.51763809	735.83104653
2	-1414.57958780	-732.78285838	1251.27966800
3	-896.05783408	-732.24027626	269.33285592
4	-896.44149757	-463.83366595	458.00014574
5	-1086.23363180	-464.03226478	98.58266734
6	-1086.09320130	-562.27590270	167.63968828
7	-1016.62445870	-562.20321048	36.08376062
8	-1016.67585980	-526.24354324	61.36038459
9	-1042.10318440	-526.27015044	13.20757305
10	-1042.08437030	-539.43230216	22.45945955
11	-1032.77732350	-539.42256325	4.83430726
12	-1032.78421000	-534.60488135	8.22073275
13	-1036.19082550	-534.60844604	1.76947927
14	-1036.18830490	-536.37184001	3.00899703
15	-1034.94139710	-536.37053524	0.64767436
16	-1034.94231970	-535.72508825	1.10136935
17	-1035.39871960	-535.72556583	0.23706527
18	-1035.39838190	-535.96181582	0.40312916
19	-1035.23132790	-535.96164102	0.08677191
20	-1035.23145160	-535.87516752	0.14755551
21	-1035.29259760	-535.87523150	0.03176072
22	-1035.29255230	-535.90688300	0.05400907
23	-1035.27017130	-535.90685958	0.01162523
24	-1035.27018790	-535.89527433	0.01976869
25	-1035.27837990	-535.89528290	0.00425513
26	-1035.27837380	-535.89952340	0.00723584
27	-1035.27537530	-535.89952026	0.00155749
28	-1035.27537760	-535.89796813	0.00264850

Table 3.33 (cont.)

i	x	y	er
29	-1035.27647510	-535.89796928	0.00057008
30	-1035.27647430	-535.89853740	0.00096942
31	-1035.27607260	-535.89853698	0.00020866
32	-1035.27607280	-535.89832903	0.00035483
33	-1035.27621990	-535.89832918	0.00007638
34	-1035.27621980	-535.89840530	0.00012988
35	-1035.27616600	-535.89840524	0.00002795
36	-1035.27616600	-535.89837738	0.00004753
37	-1035.27618570	-535.89837740	0.00001023
38	-1035.27618570	-535.89838760	0.00001740
39	-1035.27617850	-535.89838759	0.00000374
40	-1035.27617850	-535.89838386	0.00000637
41	-1035.27618110	-535.89838386	0.00000137
42	-1035.27618110	-535.89838523	0.00000233
43	-1035.27618020	-535.89838523	0.00000050
44	-1035.27618020	-535.89838473	0.00000085
45	-1035.27618050	-535.89838473	0.00000018
46	-1035.27618050	-535.89838491	0.00000031
47	-1035.27618040	-535.89838491	0.00000007
f1 = -1035.27618040	f2 = 303.84757728		
f3 = 0.00000000	f4 = -267.94919236		
f5 = -535.89838491	f6 = 303.84757728		
f7 = 767.94919246	f8 = 767.94919246		
f9 = -1086.04416320			

Table 3.34 the result of the example 3.7 from the $\dot{U}_2$ method.

i	x	y	er
1	-1415.62777590	0.51763809	735.83104653
2	-1414.57958780	-732.78285838	1251.27966800
3	-896.05783408	-732.24027626	269.33285592
4	-896.44149757	-463.83366595	458.00014574
5	-1086.23363180	-464.03226478	98.58266734
6	-1086.09320130	-562.27590270	167.63968828
7	-1016.62445870	-562.20321048	36.08376062
8	-1016.67585980	-526.24354324	61.36038459
9	-1042.10318440	-526.27015044	13.20757305
10	-1042.08437030	-539.43230216	22.45945955
11	-1032.77732350	-539.42256325	4.83430726
12	-1032.78421000	-534.60488135	8.22073275
13	-1036.19082550	-534.60844604	1.76947927
14	-1036.18830490	-536.37184001	3.00899703
15	-1034.94139710	-536.37053524	0.64767436
16	-1034.94231970	-535.72508825	1.10136935
17	-1035.39871960	-535.72556583	0.23706527
18	-1035.39838190	-535.96181582	0.40312916
19	-1035.23132790	-535.96164102	0.08677191
20	-1035.23145160	-535.87516752	0.14755551
21	-1035.29259760	-535.87523150	0.03176072
22	-1035.29255230	-535.90688300	0.05400907
23	-1035.27017130	-535.90685958	0.01162523
24	-1035.27018790	-535.89527433	0.01976869
25	-1035.27837990	-535.89528290	0.00425513
26	-1035.27837380	-535.89952340	0.00723584
27	-1035.27537530	-535.89952026	0.00155749
28	-1035.27537760	-535.89796813	0.00264850

Table 3.34 (cont.)

i	x	y	er
29	-1035.27647510	-535.89796928	0.00057008
30	-1035.27647430	-535.89853740	0.00096942
31	-1035.27607260	-535.89853698	0.00020866
32	-1035.27607280	-535.89832903	0.00035483
33	-1035.27621990	-535.89832918	0.00007638
34	-1035.27621980	-535.89840530	0.00012988
35	-1035.27616600	-535.89840524	0.00002795
36	-1035.27616600	-535.89837738	0.00004753
37	-1035.27618570	-535.89837740	0.00001023
38	-1035.27618570	-535.89838760	0.00001740
39	-1035.27617850	-535.89838759	0.00000374
40	-1035.27617850	-535.89838386	0.00000637
41	-1035.27618110	-535.89838386	0.00000137
42	-1035.27618110	-535.89838523	0.00000233
43	-1035.27618020	-535.89838523	0.00000050
44	-1035.27618020	-535.89838473	0.00000085
45	-1035.27618050	-535.89838473	0.00000018
46	-1035.27618050	-535.89838491	0.00000031
47	-1035.27618040	-535.89838491	0.00000007
f1 = -1035.27618040	f2 = 303.84757728		
f3 = 0.00000000	f4 = -267.94919236		
f5 = -535.89838491	f6 = 303.84757728		
f7 = 767.94919246	f8 = 767.94919246		
f9 = -1086.04416320			

Table 3.35 the result of the example 3.7 from the L_2 method.

i	x	y	er
1	-1415.62777590	-732.78285838	1254.35280020
2	-896.05783408	-463.83366595	459.12499019
3	-1086.23363180	-562.27590270	168.05140992
4	-1016.62445870	-526.24354324	61.51108517
5	-1042.10318440	-539.43230216	22.51461979
6	-1032.77732350	-534.60488135	8.24092280
7	-1036.19082550	-536.37184001	3.01638710
8	-1034.94139710	-535.72508825	1.10407430
9	-1035.39871960	-535.96181582	0.40411924
10	-1035.23132790	-535.87516752	0.14791791
11	-1035.29259760	-535.90688300	0.05414171
12	-1035.27017130	-535.89527433	0.01981724
13	-1035.27837990	-535.89952340	0.00725362
14	-1035.27537530	-535.89796813	0.00265501
15	-1035.27647510	-535.89853740	0.00097180
16	-1035.27607260	-535.89832903	0.00035570
17	-1035.27621990	-535.89840530	0.00013020
18	-1035.27616600	-535.89837738	0.00004765
19	-1035.27618570	-535.89838760	0.00001744
20	-1035.27617850	-535.89838386	0.00000638
21	-1035.27618110	-535.89838523	0.00000234
22	-1035.27618020	-535.89838473	0.00000085
23	-1035.27618050	-535.89838491	0.00000031
24	-1035.27618040	-535.89838484	0.00000011
25	-1035.27618040	-535.89838487	0.00000004
f1 = -1035.27618040	f2 = 303.84757729		
f3 = 0.00000000	f4 = -267.94919243		
f5 = -535.89838487	f6 = 303.84757729		

Table 3.35 (cont.)

f7 = 767.94919243	f8 = 767.94919243
f9 = -1086.04416320	

Table 3.36 the result of the example 4.8 from the *D* method.

k	a	b	c	er
1	31.90030475	97.90000000	241.18794800	485.63794800
2	37.67537533	370.98825275	208.12263325	332.68424177
3	36.88365034	343.69800859	199.09781246	39.95224958
4	36.66755755	333.88146280	200.21429136	11.92575498
5	36.69429083	334.78184891	200.54720105	1.35610847
6	36.70226211	335.14149187	200.50975317	0.43371090
7	36.70107210	335.09886758	200.49874957	0.01574274
9	36.70110197	335.09982168	200.49919937	0.00154111
10	36.70111274	335.10030134	200.49915815	0.00057037
11	36.70111175	335.10027089	200.49914167	0.00005147
12	36.70111136	335.10025343	200.49914302	0.00002063
13	36.70111139	335.10025438	200.49914362	0.00000171
14	36.70111141	335.10025502	200.49914358	0.00000074
15	36.70111141	335.10025499	200.49914356	0.00000006
x1 = 17.56738215	x2 = 2.90560000			
x3 = 17.56738215	x4 = 2.90560000			
x5 = 23.45701785	x6 = 20.55383712			
x7 = 95.79865770	x8 = 2.94733741			
x9 = 14.61880000	x10 = -1.05662475			
x11 = 10.56168000	x12 = 36.70111141			
x13 = -0.17329400	x14 = 335.10025499			
x15 = -3.64616288	x16 = 200.49914356			

Table 3.37 the result of the example 3.8 from the U_2 method.

i	a	b	c	er
1	31.90030475	339.087948	241.187948	485.637948
2	37.67537533	335.0528014	205.2524967	94.36643035
3	36.814927	335.1008197	199.5254443	15.45502258
4	36.67779688	335.1002483	200.3853213	2.350301009
5	36.69838601	335.1002551	200.5224582	0.368853495
6	36.70166966	335.100255	200.501869	0.056263401
7	36.70117666	335.100255	200.4985853	0.0088321
8	36.70109804	335.100255	200.4990783	0.001347188
9	36.70110984	335.100255	200.4991569	0.000211479
10	36.70111172	335.100255	200.4991451	0.000032258
11	36.70111144	335.100255	200.4991432	0.000005064
12	36.7011114	335.100255	200.4991435	0.000000772
13	36.7011114	335.100255	200.4991436	0.000000121
14	36.70111141	335.100255	200.4991436	0.000000018
x1 = 17.56738215	x2 = 2.90560000			
x3 = 17.56738215	x4 = 2.90560000			
x5 = 23.45701785	x6 = 20.55383712			
x7 = 95.79865770	x8 = 2.94733741			
x9 = 14.61880000	x10 = -1.05662475			
x11 = 10.56168000	x12 = 36.70111141			
x13 = -0.17329400	x14 = 335.10025499			
x15 = -3.64616288	x16 = 200.49914356			

Table 3.38 the result of the example 3.8 from the U_3 method.

i	a	b	c	er
1	37.65000638	339.028448	199.5035031	0.0088321
2	36.67727152	335.0535095	200.5235397	0.001347188
3	36.70169556	335.1008112	200.4985528	0.000211479

Table 3.38 (cont.)

i	a	b	c	er
4	36.70109726	335.1002484	200.4991578	0.000032258
5	36.70111175	335.100255	200.4991432	0.000005064
x1 = 17.56738215	x2 = 2.90560000			
x3 = 17.56738215	x4 = 2.90560000			
x5 = 23.45701785	x6 = 20.55383712			
x7 = 95.79865770	x8 = 2.94733741			
x9 = 14.61880000	x10 = -1.05662475			
x11 = 10.56168000	x12 = 36.70111141			
x13 = -0.17329400	x14 = 335.10025499			
x15 = -3.64616288	x16 = 200.49914356			

Table 3.39 the result of the example 3.8 from the L_2 method.

i	a	b	c	er
1	31.90030475	129.8003048	239.6433243	485.637948
2	37.63839044	375.1817148	204.8229808	297.9043947
3	36.80464255	339.5276234	199.5091788	44.38866697
4	36.67740742	334.0865862	200.407675	6.73206815
5	36.69892126	335.0065963	200.5239621	1.124565987
6	36.70170567	335.1256677	200.5010313	0.150592393
7	36.7011566	335.1021879	200.4985263	0.0282375
8	36.70109662	335.0996229	200.4991059	0.0033296
9	36.7011105	335.1002164	200.4991588	0.0007034
10	36.70111177	335.1002705	200.4991442	0.0007258
11	36.70111142	335.1002557	200.4991432	0.0001735
12	36.7011114	335.1002546	200.4991435	0.00001595
13	36.7011114	335.1002549	200.4991435	0.00000464
x1 = 17.56738215	x2 = 2.90560000			
x3 = 17.56738215	x4 = 2.90560000			

Table 3.39 (cont.)

x5 = 23.45701785	x6 = 20.55383712
x7 = 95.79865770	x8 = 2.94733741
x9 = 14.61880000	x10 = -1.05662475
x11 = 10.56168000	x12 = 36.70111141
x13 = -0.17329400	x14 = 335.10025499
x15 = -3.64616288	x16 = 200.49914356

Table 3.40 the result of the example 3.8 from the L_3 method.

i	a	b	c	er
1	31.97213757	132.8721376	207.634632	476.31445
2	36.8719655	342.4065975	200.241344	227.14444
3	36.69493858	334.8362825	200.5084576	8.2065497
4	36.70133442	335.109792	200.498807	0.2964962
5	36.70110335	335.0999104	200.4991557	0.0107122
6	36.7011117	335.1002674	200.4991431	0.0003871
7	36.70111139	335.1002545	200.4991435	0.000000432
8	36.7011114	335.100255	200.4991435	0.000000543
x1 = 17.56738215	x2 = 2.90560000			
x3 = 17.56738215	x4 = 2.90560000			
x5 = 23.45701785	x6 = 20.55383712			
x7 = 95.79865770	x8 = 2.94733741			
x9 = 14.61880000	x10 = -1.05662475			
x11 = 10.56168000	x12 = 36.70111141			
x13 = -0.17329400	x14 = 335.10025499			
x15 = -3.64616288	x16 = 200.49914356			

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