MULTIVARIATE SECANT METHOD FOR SOLVING SYSTEMS OF NONLINEAR EQUATIONS

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หัวข้อวิทยานิพนธ์	วิธีซีแคนท์เพื่อแก้ปัญหาระบบสมการไม่เชิงเส้น
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บทคัดย่อ

วิทยานิพนธ์ฉบับนี้ เป็นการศึกษาและปรับปรุงวิธีซีแคนท์ของหลายตัวแปร เพื่อแก้ปัญหาระบบสมการไม่เชิงเส้นที่อยู่ในรูป

$$f(x) = 0$$

เมื่อ $x = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix}$ และ $f(x) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ . \\ . \\ . \\ f_n(x_1, x_2, \ldots, x_n) \end{bmatrix}$

โดยศึกษาและจำกัดเงื่อนไขของเวกเตอร์เริ่มต้น $n+1$ เวกเตอร์ ซึ่งวิทยานิพนธ์ฉบับนี้ได้เสนอแนวทางใหม่สองแนวทาง เพื่อปรับปรุงวิธีซีแคนท์ของหลายตัวแปรให้มีการลู่เข้าที่เร็วขึ้น พร้อมทั้งแสดงการหาผลเฉลยของปัญหาตัวอย่าง ด้วยวิธีซีแคนท์ของหลายตัวแปร และวิธีการใหม่สองวิธีที่นำเสนอ แล้วทำการเปรียบเทียบจำนวนการทำซ้ำของทั้งสามวิธี

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ABSTRACT

The contents of this thesis will be study and improving of the multivariate secant method for solving the system of nonlinear equations of the form

$$f\,(x) = 0$$

where $x = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$ and $f(x) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ \cdot \\ \cdot \\ \cdot \\ f_n(x_1, x_2, \ldots, x_n) \end{bmatrix}$

We study and impose the condition of the $n+1$ initial vectors and we will state the new alternatives, which include 2 methods for improving the multivariate secant method will has a better convergence. We also find some examples and solve their solution from the multivariate secant method and the new methods, then their the number of iterations are compared.

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Prathana Wettasak

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CHAPTER 1

INTRODUCTION

A system of nonlinear equations is a set of nonlinear equations, which may be algebraic functional, ordinary differential, partial differential etc, in otherwise systems of nonlinear equations that involve two or more independent variables. Nonlinear systems are used to described a variety of phenomena i.e. astronomy, in the social and life sciences as well as the physical, earth sciences and engineering. Many problems of branches in science and technology are of the form of system of nonlinear equations

$$f(x) = 0 \tag{1.1}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and $f(x) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ \vdots \\ f_n(x_1, x_2, \ldots, x_n) \end{bmatrix}$.

We need to know how to find approximated value of x, that is a root of the system (1.1), when f is real function. We also say that x is a zero of the system (1.1).

There are two groups of numerical methods for finding the root of the system, first, the method by using memory, this method has two kinds i.e. a one point and multipoint iteration function with memory. For a one point iteration function with memory, the best-known is the secant method. So, we can find new data from old data are reused. Finally, the method, which does not use memory.

We have many iterative methods for solving the system (1.1) base on Newton method i.e. the Newton-Raphson Method, the Newton-like method and the Modified Newton method. With these methods use only one initial vector and have to find the Jacobian matrix or others kind of matrices of the system (1.1), that may be complicated for some system which is difficult in finding such matrices.

So, the purpose of this thesis is to study and improve the multivariate secant method will has a better convergence. Although the multivariate secant method is not very popular, but it is a good and appropriate method to solve the systems of nonlinear equations, which are difficult to find the Jacobian matrices .

In this thesis, by learning the multivariate secant method described by *Janina Jankowska* in [10]. We will study and impose more conditions on the $n+1$ initial vectors, $x^{(0)}, x^{(1)}, \ldots, x^{(n)}$. Then we obtain the new alternatives, that are two methods, the first method is called the modified multivariate secant method, and the second method is called the mid modified multivariate secant method. So the new methods will have a better convergence, thus we consider from comparing their the number of iterations.

Many problems that are above-mentioned, where it is necessary to be solved by a nonlinear system of equations. We examine some examples, first, let us consider from astronomy [3], which is the example 4.6 in chapter 4. A binary or double star system is a pair of stars that are orbiting around each other. If one star is much more massive than the other, we can say that the smaller star is in orbit around the larger one, as shown in Figure 1.1. the mean distance between the stars will be denoted by a, and this distance is measured in AU (astronomical units), where 1 AU is the mean distance from the earth to our sun. Hence 1 AU is approximately 92,900,000 miles.

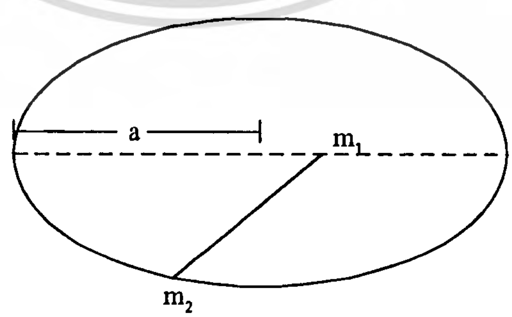

Figure 1.1 A double star system.

As seen from the earth, the stars appear very close in the sky, with some angular separation α that will be measured in seconds of arc. Typical values for some well-know systems range from a few seconds of arc to a few thousandths of a second of arc. Let us assume that the stars have masses μ_1 and μ_2 measured in multiples of the sun's mass. For example, a star with a mass of 2 solar masses has twice the mass of our sun. Each binary system has an orbital period T that is the length of time for each member to complete revolution about the other member. The period T is measured in earth years. If the star is observed from the earth, its apparent direction changes as the earth revolves around the sun. The angular difference between positions measured at six month intervals is called *the parallax* of the star and is denoted by p. Using the diameter of the earth's orbit and simple trigonometry, it is possible to compute the distance of a star from its parallax. For nearby stars, the parallax can be measured directly, even though the values are very small factions of second of arc. For the more distant stars, however , unable values are not obtainable by direct measurement, so indirect methods are needed.

In the case of binary systems, a method know as the method of *dynamical parallaxes* is used. The method rests on the fact that certain theoretical relations exists between visual magnitudes (apparent brightness as seen from earth) m_1 and m_2 of the component stars in the system, the parallax p, and the individual masses μ_1 and μ_2. These relations provide us with a nonlinear system to solve for the three unknowns μ_1 μ_2 and p after observable values of m_1, m_2, α and T are given. Detail of these relationship are supplied in most introductory astronomy texts. They are as follows:

$$p^3 = \frac{\alpha^3}{T^2(\mu_1 + \mu_2)} \tag{1.2}$$

$$M_1 = m_1 + 5 + 5\log p \quad \text{and} \quad M_2 = m_2 + 5 + 5\log p \tag{1.3}$$

$$0.4(M_0 - M_1) = 3.5\log \mu_1 \quad \text{and} \quad 0.4(M_0 - M_2) = 3.5\log \mu_2 . \tag{1.4}$$

The usual textbook procedure for determining p, μ_1 and μ_2 is to begin by assuming that the sum of the masses is 2. We substitute that in Equation (1.2) and solve for p (we know α and T). We substitute this value of p in Equation (1.3) to determine M_1 and M_2, the absolute magnitudes of the stars. In Equation (1.4), the number M_0 is the absolute magnitude of the sun, which is about 4.8. We use this value of M_0 and the values of M_1 and M_2 found from Equation (1.3) to compute μ_1 and μ_2 from Equation (1.4). Now we substitute these values in Equation (1.2) to recompute p. We continuous this iterative procedure until the new value of p agrees well with the preceding one. In this case, we have five equations and five unknowns. We can improve on this procedure by substituting the values of the absolute magnitude M_1 and M_2 from Equation (1.4). We obtained the equations

$$M_0 - m_1 - 5 = 5 \log p + 8.75 \log \mu_1 \tag{1.5}$$

$$M_0 - m_2 - 5 = 5 \log p + 8.75 \log \mu_2 . \tag{1.6}$$

Substituting $M_0 = 4.8$ and combining these Equation (1.2), we obtained the system of three equations

$$p^3 = \frac{\alpha^3}{T^2(\mu_1 + \mu_2)}$$

$$-m_1 - 0.2 = 5 \log p + 8.75 \log \mu_1 \tag{1.7}$$

$$-m_2 - 0.2 = 5 \log p + 8.75 \log \mu_2$$

to solve for the three unknowns p, μ_1 and μ_2.

The second example [1], which is example 4.5 in chapter 4, that is the relationship between the surface area M and the greatest volume V of bucket are given by the formulas

$$M = \pi(sx + sy + x^2) \tag{1.8}$$

$$V = \frac{\pi}{3}(x^2 + xy + y^2)h \tag{1.9}$$

where x, y, h and s are shown in Figure 1.2.

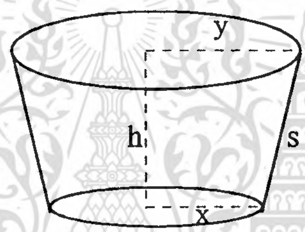

Figure 1.2 Model of a bucket.

We want to find the bucket which has the greatest volume V^* for the given surface area $M = \pi$, where the radii of this bucket are denoted by x^* and y^*, so we obtained the radii from the following system

$$F(x, y) = \begin{pmatrix} (1 - 4x^2)(x + 2y - 2y^3) + 4xy^2(x^2 - 1) + 3xy^4 \\ 2x(1 - x^2)^2 + y(1 - 2x^2 + 4x^4) + 6xy^2(x^2 - y^2) - 3y^5 \end{pmatrix} \tag{1.10}$$

then we can compute the optimal radii by using iterative methods, those radii which give the greatest volume under the condition $x = y$ (cylindrical bucket; what is its volume?). Determine x^*, y^* and the corresponding volume V^* and height h^*. What is the percentage gain compared with the cylindrical bucket? This is useful, which we obtained from nonlinear systems.

Let us consider the Largrange mutiplier method [5] to the problem of maximizing a function $f(x, y, z)$ subject to one constraint of the $g(x, y, z) = 0$, i.e. the third example, which is the problem 4.8 in chapter 4, we want to find the maximum value of rectangular box inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{1.11}$$

with its face parallel to the coordinate planes. From (1.11), we see that it is a nonlinear equation subject to the constraint

$$g(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0. \tag{1.12}$$

Let (x, y, z) be the vertex of the box that lies in the first octant (where x, y and z are all positive). We want to maximize the volume $V(x, y, z) = 8xyz$ subject to (1.12), then from Theorem for the method of Lagrange multipliers [5], we obtained the system of four nonlinear equations, for four unknowns λ, x, y and z

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0 \tag{1.13}$$

$$8yz - \frac{2\lambda x}{a^2} = 0 \tag{1.14}$$

$$8xz - \frac{2\lambda y}{b^2} = 0 \tag{1.15}$$

$$8xy - \frac{2\lambda z}{c^2} = 0 \tag{1.16}$$

then we can compute four unknowns by iterative methods.

In order to find the solutions of three problems and other problems, we need to solve the systems of nonlinear equations and their numerical solution that will be show in chapter 4.

CHAPTER 2

LITERATURE REVIEWS

We shall review in this chapter about definitions, lemma, theorem and corollary, that are support this thesis. We shall consider the problem of solving a system of nonlinear equations

$$f(x) = 0 \tag{2.1}$$

for $f : D_f \subset C^n \to C^n$, where C^n denotes n - dimensional complex vector space and D_f is an open and convex subset in C^n. The system (2.1) could be solved by iteration, of course in this thesis solved by multivariate secant method. So that we construct a sequence $\{z_i\}_{i=0}^{\infty}$ of complex vectors which converges under certain assumption to a zero $\alpha \in D_f$ of the function f .

Definition 2.1 Let $z_d, z_{d-1}, \ldots, z_{d-m}$ be sufficiently close approximations to simple zero α, the standard information β defined by

$$\begin{aligned}\beta(z_d; f) &= \beta(z_d, z_{d-1}, \ldots, z_{d-m}; f)\\ &= \left\{\, f^k(z_j) : j = d, d-1, \ldots, d-m \;\; ; \; k = 0,1,\ldots,s \,\right\},\end{aligned}$$

where f^k denotes the k^{th} Frechet derivative of f .

The evaluation cost of $f^k(x_j)$ in general increases quickly with k, (for $n \geq 2$). Therefore the most common iterations use the standard information with small s, usually $s = 0$ or $s = 1$. For instance the Newton method uses $f(z_d), f'(z_d)$ in every iterative step, which show that $s = 1$ and $m = 0$. The multivariate secant method uses $f(z_d)$, $f(z_{d-1})$,…, $f(z_{d-n})$ per step, that are $s = 0$ and $m = n$.

In this thesis we restrict ourselves to $s = 0$ which means, that β contains only the values of function, i.e.

$$\beta(z_d; f) = \{ f(z_j) : j = d, d-1, \ldots, d-m \}.$$

One can prove that the convergence of stationary iterative methods using this information is at most linear and does not depend on the number m of forming point (see [15]). To get better convergence we have to assume suitable positions of the forming points z_d, $d = 0, 1, \ldots, m$. We shall show that for the multivariate secant method $z_d - z_{d-1}, \ldots, z_{d-n+1} - z_{d-n}$ must be sufficiently linearly independent. It leads us to the definition of a set of admissible approximations κ for the multivariate secant method (see [16]).

Definition 2.2 A set of admissible approximations κ is defined as

$$\kappa(c, \xi) = \left\{ (z_0, z_1, \ldots, z_n) : \left| \det\left[\frac{z_1 - z_0}{\|z_1 - z_0\|}, \ldots, \frac{z_n - z_{n-1}}{\|z_n - z_{n-1}\|} \right] \right| \geq c\|z_n - z_0\|^{\xi} \right\},$$

where $\|\cdot\|$ denotes an Euclidean norm of vectors.

In this thesis we assume that f in the problem (2.1) satisfies the following conditions

(i) there exists a simple zero $\alpha = \alpha(f) \in D_f$

$$\text{i.e. } f(\alpha) = 0 \ , \quad M = M(f) \overset{df}{=} \left\| [f'(\alpha)]^{-1} \right\|_2 \ \langle +\infty, \tag{2.2}$$

(ii) The Frechet derivative f' of f is Lipschitz function on D_f with a constant $L = L(f)$, i.e.

$$\| f'(x) - f'(y) \|_2 \ \leq \ L\| x - y \| \quad \text{for } x, y \in D_f \ ,$$

where $\|\cdot\|$ denotes the Euclidean norm of vectors and $\|\cdot\|_2$ the spectral norm of matrices. We shall briefly say that f has property (2.2) and we shall use D_f, M, L in the above meaning.

We solve the system (2.1) by one step of the multivariate secant method, is defined as follows. Let

(i) $\beta(x_d; f) = \{ f(x_j) : j = d, d-1, \ldots, d-n \}$,

(ii) $\kappa \ = \ \left\{ (x_0, x_1, \ldots, x_n) : \det\left[\dfrac{x_1 - x_0}{\|x_1 - x_0\|}, \ldots, \dfrac{x_n - x_{n-1}}{\|x_n - x_{n-1}\|} \right] \neq 0 \right\}$,

(iii) $(x_d, x_{d-1}, \ldots, x_{d-n}) \in \kappa$ be approximations of α,

(iv) W be interpolatory polynomial of f of at most first degree, such that $W(x_{d-k}) \ = \ f(x_{d-k})$ for $k = 0, 1, 2, \ldots, n$, and let $\varphi_{\beta,\kappa}$ denote the multivariate secant method.

The next approximation of α in $\varphi_{\beta,\kappa}$ is the unique zero of the polynomial W, which is defined as

$$W(x) \ = \ f(x_d) + F_d X_d^{-1} (x - x_d) \, ,$$

where

$$X_d = [x_{d-n+1} - x_{d-n}, \ldots, x_d - x_{d-1}],$$

$$F_d = [f_{d-n+1} - f_{d-n}, \ldots, f_d - f_{d-1}],$$

and $$f_k = f(x_k) \quad \forall\ k.$$

Hence

$$\varphi_{\beta,\kappa}(x_d; f) = x_d - X_d F_d^{-1} f(x_d) \tag{2.3}$$

provided F_d is a nonsingular matrix.

Let us consider two lemmas, which are necessary for prove the first theorem of the multivariate secant method. In lemma 2.1 we estimate the Schur norm of the matrix A^{-1} in terms of the norm of the column of A. The Schur norm of matrices is denoted by $\|\cdot\|$.

Lemma 2.1 Let A be a nonsingular complex matrix $n \times n$ and let $A^{-1} = [b_1, b_2, \ldots, b_n]^T$. If

(i) $$|\det A| = d\prod_{j=1}^{n} \|a_j\| \quad , d > 0 \quad ,$$

(ii) $$0 < a_{\min} \le \|a_j\| \le a_{\max} \quad , j = 1,2,\ldots,n \ ,$$

then

(i) $$\frac{1}{a_{\max}} \le \|b_j\| \le \frac{\sqrt{n}}{da_{\min}} \qquad , j = 1,2,\ldots,n \ ,$$

(ii) $$\frac{\sqrt{n}}{a_{\max}} \le \|A^{-1}\| \le \frac{n}{da_{\min}} \qquad .$$

Let $\quad X = [x_1 - x_0, x_2 - x_1, \ldots, x_n - x_{n-1}],$

$$F = [f_1 - f_0, f_2 - f_1, \ldots, f_n - f_{n-1}].$$

If $x_0, x_1, \ldots, x_n$ are sufficiently good approximation of α then the matrix $J = XF^{-1}$ is the approximation of $[f'(\alpha)]^{-1}$. We shall next denote

$$d(x_0, x_1, \ldots, x_n) = \det\left[\frac{x_1 - x_0}{\|x_1 - x_0\|}, \frac{x_2 - x_1}{\|x_2 - x_1\|}, \ldots, \frac{x_n - x_{n-1}}{\|x_n - x_{n-1}\|}\right].$$

Lemma 2.2

Let

(i) $\quad f$ has property (2.2)

(ii) $\quad x_0, x_1, \ldots, x_n \in D_f : \| x_0 - \alpha \| \ge \ldots \ge \| x_n - \alpha \|,$

(iii) $\quad X = [x_1 - x_0, \ldots, x_n - x_{n-1}],$

$$F = [f_1 - f_0, f_2 - f_1, \ldots, f_n - f_{n-1}],$$

where $f_k = f(x_k)$, $k = 0, 1, \ldots, n,$

(iv) $d = d(x_0, x_1, \ldots, x_n) > 0$, $(d \le 1)$,

(v) $M \cdot L \cdot n \cdot \sqrt{n} \| x_0 - \alpha \| / d < 1$.

Then

(i) F is nonsingular,

(ii) $F = f'(\alpha) \cdot (I + B) \cdot X$, where $\| B \| \le \dfrac{M \cdot L \cdot n \cdot \sqrt{n}}{d} \| x_0 - \alpha \| < 1$,

(iii) $F^{-1} = X^{-1}(I - C) \cdot [f'(\alpha)]^{-1}$, where $\| C \| \le \dfrac{\|B\|}{1 - \|B\|}$,

(iv) $\left\| J - [f'(\alpha)]^{-1} \right\| \le \| C \| \cdot \left\| [f'(\alpha)]^{-1} \right\|$.

Theorem 2.3

Let

(i) f has property (2.2) ,

(ii) $x_0, x_1, \ldots, x_n \in D_f$, where $\|x_0 - \alpha\| \ge \|x_1 - \alpha\| \ge \ldots \ge \|x_n - \alpha\|$,

(iii) $d = d(x_0, x_1, \ldots, x_n) > 0$, $(d \le 1)$,

(iv) $M \cdot L \cdot n \cdot \sqrt{n} \| x_0 - \alpha \| / d < 1$.

Then

$$\left\|\varphi_{\beta,\kappa}(x_n; f) - \alpha\right\| \leq \frac{\frac{3}{2} M \cdot L \cdot n \cdot \sqrt{n}\|x_0 - \alpha\|}{d(1 - \frac{M \cdot L \cdot n \cdot \sqrt{n}}{d}\|x_n - \alpha\|)}\|x_n - \alpha\|.$$

The assumptions of theorem 2.3 mean that $x_0, x_1, \ldots, x_n$ are close enough to α and moreover the vectors $x_1 - x_0, \ldots, x_n - x_{n-1}$ are sufficiently linearly independent (see Ortega [12], Barnes [2]), so that $d > M \cdot L \cdot n \cdot \sqrt{n}\|x_0 - \alpha\|$.

Corollary 2.4

Let

(i) f has property (2.2)

(ii) $y_0, y_1, \ldots, y_{n-1}$ are fixed points of D_f such that

$$\|y_0 - \alpha\| \geq \|y_1 - \alpha\| \geq \ldots \geq \|y_{n-1} - \alpha\|,$$

$$M \cdot L \cdot n\sqrt{n}\|y_0 - \alpha\| \overset{df}{=} \eta < 1,$$

$$d(y_0, y_1, \ldots, y_{n-1}, \alpha) > \eta/s \quad \text{for some } s \in (\eta, 1),$$

(iii) $S = \{x \in D_f : d(y_0, y_1, \ldots, y_{n-1}, x) \geq \eta/s\}$

(iv) $\varphi_{\beta,s}(x, f) = \varphi(x; y_{n-1}, \ldots, y_0; \beta(x; y_{n-1}, \ldots, y_0, f))$

where an operator φ is defined as in (2.3)

and $\beta(x; y_{n-1}, \ldots, y_0; f) \equiv \beta(x; f) = \{f(x)\}$,

then

(i) $\lim_{x \to \alpha} \varphi_{\beta,s}(x; f) = \alpha \;,\;\; x \in S \,,$

(ii) $\left\| \varphi_{\beta,s}(x; f) - \alpha \right\| \le \dfrac{3/2\, s}{1-s} \left\| x - \alpha \right\|$ for $x \in S$.

Corollary 2.5

Let

(i) f has property (2.2)

(ii) $$T(x) = \{(y_0, y_1, \ldots, y_{n-1}) : y_i \in D_f \,, i = 0,1,\ldots,n-1 \,,$$
$$\left\| y_0 - \alpha \right\| \ge \left\| y_1 - \alpha \right\| \ge \ldots \ge \left\| y_{n-1} - \alpha \right\| ,$$
$$\frac{M \cdot L \cdot n \cdot \sqrt{n} \left\| y_0 - \alpha \right\|}{d(y_0, y_1, \ldots, y_{n-1}, x)} \le \tau < 1 \} ,$$

(iii) $$\kappa_T = \bigcup_{x \in D_f} (\{x\} \times T(x)) ,$$

$$\varphi_{\beta,\kappa_T}(x; f) = \varphi\,(x, y_{n-1}(x), \ldots, y_0(x) ; f)$$

where $y_i(x), i = 0,1,\ldots,n-1$ are some functions of x such that $(y_0(x), \ldots, y_{n-1}(x)) \in T(x) \;\forall\, x \in D_f$.

If $\left\| x - \alpha \right\| \le \left\| \, y_0(x) - \alpha \, \right\| \;\forall\, x \in D_f$ then

(i) $\lim_{x \to \alpha} \varphi_{\beta,\kappa_T}(x; f) = \alpha \;,$

(ii) $$\left\|\varphi_{\beta,\kappa_T}(x;f)-\alpha\right\| \leq \frac{3/2\,\tau(x)}{1-\tau(x)}\left\|x-\alpha\right\| \text{ , } \forall x \in D_f$$

$$\text{where } \tau(x) = \frac{M \cdot L \cdot n \cdot \sqrt{n}\left\|y_0(x)-\alpha\right\|}{d(y_0(x),\ldots,y_{n-1}(x),x)} \text{ .}$$

Theorem 2.6

Let

(i) f has property (2.2) ,

(ii) $J = \{x : \| x - \alpha \| \leq \Gamma \} \subset D_f$,

(iii) $\{x_i\}_{i=0}^{\infty} \subset J$ is of q^{th} order in $\kappa(c,1-\varepsilon)$, $c \neq 0$,

(iv) $\mu_i = \frac{5}{2} M \cdot L \cdot n \cdot \frac{\sqrt{n}}{d_i}$, where $d_i = d(x_i,\ldots,x_{i-n})$.

Then for at will small $\varepsilon > 0$

(i) $\limsup_{i\to\infty} \mu_i \| x_{i-n} - \alpha \| = 0$,

(ii) $\lim_{i\to\infty} \varphi_{\beta,\kappa(c,1-\varepsilon)}(x_i;f) = \alpha$; $\varphi_{\beta,\kappa(c,1-\varepsilon)}(x_i;f) \in J$ *for* $i \geq i_0$,

(iii) $\| \varphi_{\beta,\kappa(c,1-\varepsilon)}(x_i;f) - \alpha \| \leq \beta_i \| x_{i-n} - \alpha \| \cdot \| x_i - \alpha \|$ *for* $i \geq i_0$.

CHAPTER 3

MULTIVARIATE SECANT METHOD FOR SOLVING SYSTEMS OF NONLINEAR EQUATIONS

In this part, we shall introduce multivariate secant method for solving a system of nonlinear equations.

Let us consider the system of nonlinear equations is of the form

$$f(x) = 0 \tag{3.1}$$

for $f : D_f \subset R^n \to R^n$ is a vector function from the open subset and convex of the real Euclidean space R^n to R^n. We solve the system (3.1) by iteration, that is, we construct a sequence $\{z_i\}_{i=0}^{\infty}$ of real vectors which converges under certain assumptions to a zero $\alpha \in D_f$ on the function f.

Given a set of $n+1$ initial vectors, $x^{(0)}, x^{(1)}, x^{(2)}, \ldots, x^{(n)}$, the multivariate secant method for solving the system (3.1) is defined by

$$x^{(k+n+1)} = x^{(k+n)} - X^{(k+n)}\left(F^{(k+n)}\right)^{-1} f^{(k+n)} \quad , \; k = 0,1,2,\ldots \tag{3.2}$$

where

$$f^{(m)} = f\left(x^m\right),$$

$$df^{(j)} = f^{(j+1)} - f^{(j)},$$

$$dx^{(j)} = x^{(j+1)} - x^{(j)},$$

$$X^{(m)} = \left[dx^{(m-n)}, \ldots, dx^{(m-1)}\right] \quad , \; m \geq n,$$

$$F^{(m)} = \left[df^{(m-n)}, \ldots, df^{(m-1)}\right] \quad , \; m \geq n.$$

$X^{(m)}$ and $F^{(m)}$ are $n \times n$ nonsingular matrices for all $m \geq n$, having as column vectors $dx^{(j)}$ and $df^{(j)}$ respectively. We can write $X^{(k+n)}$ and $F^{(k+n)}$ are in form

$$X^{(k+n)} = \begin{bmatrix} x_1^{(k+1)} - x_1^{(k)} & x_1^{(k+2)} - x_1^{(k+1)} & \cdots & x_1^{(k+n)} - x_1^{(k+n-1)} \\ x_2^{(k+1)} - x_2^{(k)} & x_2^{(k+2)} - x_2^{(k+1)} & \cdots & x_2^{(k+n)} - x_2^{(k+n-1)} \\ \vdots & \vdots & & \vdots \\ x_n^{(k+1)} - x_n^{(k)} & x_n^{(k+2)} - x_n^{(k+1)} & \cdots & x_n^{(k+n)} - x_n^{(k+n-1)} \end{bmatrix}, \qquad (3.3)$$

$$F^{(k+n)} = \begin{bmatrix} f_1^{(k+1)} - f_1^{(k)} & f_1^{(k+2)} - f_1^{(k+1)} & \cdots & f_1^{(k+n)} - f_1^{(k+n-1)} \\ f_2^{(k+1)} - f_2^{(k)} & f_2^{(k+2)} - f_2^{(k+1)} & \cdots & f_2^{(k+n)} - f_2^{(k+n-1)} \\ \vdots & \vdots & & \vdots \\ f_n^{(k+1)} - f_n^{(k)} & f_n^{(k+2)} - f_n^{(k+1)} & \cdots & f_n^{(k+n)} - f_n^{(k+n-1)} \end{bmatrix}, \qquad (3.4)$$

where $k = 0, 1, 2, \ldots$

Hence, the equation (3.2) could be written in form

$$\begin{bmatrix} x_1^{(k+n+1)} \\ x_2^{(k+n+1)} \\ \vdots \\ x_n^{(k+n+1)} \end{bmatrix} = \begin{bmatrix} x_1^{(k+n)} \\ x_2^{(k+n)} \\ \vdots \\ x_n^{(k+n)} \end{bmatrix} - \begin{bmatrix} x_1^{(k+1)} - x_1^{(k)} & x_1^{(k+2)} - x_1^{(k+1)} & \cdots & x_1^{(k+n)} - x_1^{(k+n-1)} \\ x_2^{(k+1)} - x_2^{(k)} & x_2^{(k+2)} - x_2^{(k+1)} & \cdots & x_2^{(k+n)} - x_2^{(k+n-1)} \\ \vdots & \vdots & \cdots & \vdots \\ x_n^{(k+1)} - x_n^{(k)} & x_n^{(k+2)} - x_n^{(k+1)} & \cdots & x_n^{(k+n)} - x_n^{(k+n-1)} \end{bmatrix} \bullet$$

$$\begin{bmatrix} f_1^{(k+1)} - f_1^{(k)} & f_1^{(k+2)} - f_1^{(k+1)} & \cdots & f_1^{(k+n)} - f_1^{(k+n-1)} \\ f_2^{(k+1)} - f_2^{(k)} & f_2^{(k+2)} - f_2^{(k+1)} & \cdots & f_2^{(k+n)} - f_2^{(k+n-1)} \\ \vdots & \vdots & \cdots & \vdots \\ f_n^{(k+1)} - f_n^{(k)} & f_n^{(k+2)} - f_n^{(k+1)} & \cdots & f_n^{(k+n)} - f_n^{(k+n-1)} \end{bmatrix}^{-1} \bullet \begin{bmatrix} f_1^{(k+n)} \\ f_2^{(k+n)} \\ \vdots \\ f_n^{(k+n)} \end{bmatrix}$$

where $k = 0,1,2,\ldots$

In this thesis, we will modify the multivariate secant method by study the strict condition for the next iteration.

3.1 Definitions

Definition 3.1 The value $[f(x)]$ of a vector function

$$f(x) = (f_1(x), f_2(x), \ldots, f_n(x))$$

is defined by the formula

$$[f(x)] = f_1(x) + f_2(x) + \ldots + f_n(x).$$

Definition 3.2 The set of $n+1$ vectors, $\{x^{(i)}, x^{(i+1)}, \ldots, x^{(i+n)}\}$ is said to be a mixture if $[f(x^{(i)})], [f(x^{(i+1)})], \ldots, [f(x^{(i+n)})]$ have both nonnegative and negative values.

Definition 3.3 The ordered set of $n+1$ vectors,

$$\{x^{(i)}, x^{(i+1)}, x^{(i+2)}, \ldots, x^{(i+n)}\},$$

is said to be well-ordered provide that

$$\| f(x^{(i)}) \| \geq \| f(x^{(i+1)}) \| \geq \ldots \geq \| f(x^{(i+n)}) \| .$$

Definitions 3.4 The ordered set of $n+1$ vector is said to be primary if it is both mixture and well - ordered.

3.2 Algorithms

In this part we shall show algorithms for solving the systems of nonlinear equations, which include the following three algorithms and we use absolute error is $\varepsilon = 10^{-7}$.

3.2.1 Algorithm of Multivariate Secant Method

Step 1 Using formula (3.2) to find x^{n+1}.

Step 2 If $\left\| f^{n+1} \right\| < \varepsilon$, go to step 4, else go to step 3.

Step 3 Delete x^0, then set $x^i = x^{i+1}$ and go to step 1.

Step 4 Stop, the approximated solution of the system (3.1) is x^{n+1}.

3.2.2 Algorithm of Modified Multivariate Secant Method

For any given $\varepsilon > 0$ and the primary set, $A = \{x^0, x^1, x^2, \ldots, x^n\}$, of the initial $n+1$ vectors. We will modify the multivariate secant method described by Janina Jankowska in [10] by doing the iterations by the following steps.

Step 1 Compute $\| f(x) \|$, then arrange initial vectors along the norm of functions.

Step 2 Using formula (3.2) to find x^{n+1}. If $\left\| f^{n+1} \right\| < \varepsilon$, go to step 5, else go to step3.

Step 3 Set $\overline{A} = \{x^0, x^1, x^2, \ldots, x^n, x^{n+1}\}$. Choose the first i for which the set $A_i = \{x^0, x^1, \ldots, x^{i-1}, \hat{x}^i, x^{i+1}, \ldots, x^{n+1}\}$, $i = 0,1,2,\ldots,n$, $\hat{x}^i$ means that x^i is deleted, is a mixture. There must be at least one value of $i, i \in \{0,1,2,\ldots,n\}$, for which A_i is a mixture since the initial set A is primary. Then go to step 4.

Step 4 Rearrange A_i to obtain the set A_i^*, which is primary. Set $A = A_i^*$ and go to step 1.

Step 5 Stop, the approximated solution of the system (3.1) is x^{n+1}.

3.2.3 Algorithm of Mid Modified Multivariate Secant Method

For any given $\varepsilon > 0$ and the primary set, $A = \{x^0, x^1, x^2, \ldots, x^n\}$, of the initial $n+1$ vectors.

Step1 Compute $\| f(x) \|$, then arrange initial vectors along the norm of functions.

Step 2 Using formula (3.2) to find x^{n+1}. If $\left\| f^{n+1} \right\| < \varepsilon$, go to step 4, else go to step 3.

Step 3 Delete x^0, then set $x^i = x^{i+1}$ and go to step 2.

Step 4 Stop, the approximated solution of the system (3.1) is x^{n+1}.

CHAPTER 4

SOME EXAMPLES AND APPLICATION PROBLEMS OF SYSTEMS OF NONLINEAR EQUATIONS

In this chapter, we shall solve the systems of nonlinear equations by using the multivariate secant method, the modified multivariate secant method and the mid modified multivariate secant method. The examples and the results of method are compared.

Example 4.1 Find a solution of system [4]

$$
\begin{aligned}
&0.5\,sin(xy) - \frac{0.25y}{\pi} - 0.5x = 0 \\
&(1 - \frac{0.25}{\pi})(e^{2y} - e) + \frac{ey}{\pi} - 2ex = 0
\end{aligned}
\qquad (4.1)
$$

We choose following initial vectors

$$
\text{A}=\left\{\begin{bmatrix}0.25\\3\end{bmatrix},\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right\},\ \text{B}=\left\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}0.35\\3\end{bmatrix}\right\} \text{ and C}=\left\{\begin{bmatrix}1\\-1\end{bmatrix},\begin{bmatrix}0.25\\3\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right\}.
$$

Repeat the method until $\|f(x^{m+1})\| < 10^{-7}$. We obtain the results from the multivariate secant method, the modified multivariate secant method and the mid modified multivariate secant method in the table 4.1.

Table 4.1 Give the results with initial vectors of example 4.1.

Initial vectors (x^0, y^0)	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate secant method	Mid modified Multivariate secant Method	
A	21	16	8	$x = 0.2994487$ $y = 2.8369278$
B	30	16	30	$x = 0.2994487$ $y = 2.8369278$
C	17	12	9	$x = 0.2994487$ $y = 2.8369278$

Estimate values are $x = 0.2994487$ and $y = 2.8369278$. Let us consider the most initial vectors, we see that the mid modified multivariate secant method has the number of iteration less than the modified multivariate secant method and the multivariate secant method, respectively. Showing the iterations of table 4.1 in appendix.

Example 4.2 Find the solution of the system [9]

$$\begin{aligned} 0.5\cos x + 0.35\cos(x+y) - 0.2 &= 0 \\ 0.5\sin x + 0.35\sin(x+y) - 0.4 &= 0 \end{aligned} \tag{4.2}$$

We choose following initial vectors

$$A=\left\{\begin{bmatrix}0.2\\1.5\end{bmatrix},\begin{bmatrix}-1\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right\},\ B=\left\{\begin{bmatrix}1.5\\0.25\end{bmatrix},\begin{bmatrix}-1\\0\end{bmatrix},\begin{bmatrix}2\\1\end{bmatrix}\right\} \text{ and } C=\left\{\begin{bmatrix}1.5\\0.25\end{bmatrix},\begin{bmatrix}2\\1\end{bmatrix},\begin{bmatrix}-1\\0\end{bmatrix}\right\}.$$

Repeat the method until $\|f(x^{m+1})\| < 10^{-7}$. We obtain the results from the multivariate secant method, the modified multivariate secant method and the mid modified multivariate secant method in the table 4.2.

Table 4.2 Give the results with initial points of example 4.2.

Initial vectors (x^0, y^0)	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate secant method	Mid modified Multivariate secant method	
A	12	8	8	x = 0.3580637 y = 2.0861667
B	12	10	9	x = 1.8562337 y = -2.0861667
C	diverge	10	9	x = 1.8562337 y = -2.0861667

We obtain two results are different, that are x = 0.3580637, y = 2.0861667 and x= 1.8562337, y = - 2.0861667 . Let us consider the most initial vectors, we see that the mid modified multivariate secant method has the number of iteration less than or equal to the modified multivariate secant method and less than the multivariate secant method, respectively. Showing the iterations of table 4.2 in appendix.

Example 4.3 Find the solution of the system [4]

$$
\begin{aligned}
\frac{x-50}{\sqrt{|z-50|}} + \frac{x-y}{\sqrt{|x-y|}} &= 0 \\
\frac{y-x}{\sqrt{|y-x|}} + \frac{y-z}{\sqrt{|y-z|}} &= 0 \qquad (4.3) \\
\frac{0.9\sqrt{2}(z-50)}{\sqrt{|z-50|}} + \frac{0.9\sqrt{2}z}{\sqrt{|z|}} + \frac{3.2(z-y)}{\sqrt{|z-y|}} &= 0
\end{aligned}
$$

We choose following initial vectors

$$
A = \left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 4 \end{bmatrix} \right\}, B = \left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \\ 3 \end{bmatrix} \right\},
$$

$$
C = \left\{ \begin{bmatrix} 1 \\ 8 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix} \right\} \text{ and } D = \left\{ \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix} \right\}.
$$

Repeat the method until $\|f(x^{m+1})\| < 10^{-7}$. We obtain the results from the multivariate secant method, the modified multivariate secant method and the mid modified multivariate secant method in the table 4.3.

Table 4.3 Give the results with initial points of example 4.3.

Initial vectors (x^0, y^0, z^0)	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate secant method	Mid modified Multivariate secant method	
A	87	16	27	$x =$ 45.8978764 $y =$ 43.8468146 $z =$ 41.7957528
B	27	16	27	$x =$ 45.8978764 $y =$ 43.8468146 $z =$ 41.7957528
C	23	15	22	$x =$ 45.8978764 $y =$ 43.8468146 $z =$ 41.7957528
D	35	12	15	$x =$ 45.8978764 $y =$ 43.8468146 $z =$ 41.7957528

Estimate values are $x =$ 45.8978764, $y =$ 43.8468146 and $z =$ 41.7957528. Let us consider all initial vectors, we see that the modified multivariate secant method has the number of iteration less than the mid modified multivariate secant method and the multivariate secant method. Showing the iterations of table 4.3 in appendix.

Example 4.4 Find the solution of the system

$$
\begin{aligned}
2\log x + e^{y} - 2z + 1 &= 0 \\
xy - \sin y + z - 2 &= 0 \qquad (4.4) \\
\log x \cos y - z + 1 &= 0
\end{aligned}
$$

We choose following initial vectors

$$A = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}, B = \left\{ \begin{bmatrix} 1 \\ -1.1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1.5 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2.5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix} \right\}$$

$$\text{and } C = \left\{ \begin{bmatrix} 1 \\ 1.5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Repeat the method until $\|f(x^{m+1})\| < 10^{-7}$. We obtain the results from the multivariate secant method, the modified multivariate secant method and the mid modified multivariate secant method in the table 4.4.

Table 4.4 Give the results with initial points of example 4.4.

Initial vectors (x^0, y^0, z^0)	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate secant method	Mid modified Multivariate secant method	
A	13	12	22	$x = 2.7182818$ $y = 0.0000000$ $z = 2.0000000$
B	20	13	13	$x = 2.7182819$ $y = 0.0000000$ $z = 2.0000000$
C	21	16	16	$x = 2.7182818$ $y = 0.0000000$ $z = 2.0000000$

Estimate values are $x =$ 2.718282818 , $y =$ 0.0000000 and z = 2.0000000. Let us consider the most initial vectors, we see that the modified multivariate secant method and the mid modified multivariate secant method have the same number of iteration and they have the number of iteration less than the multivariate secant method. Showing the iterations of table 4.4 in appendix.

Example 4.5 The greatest volume of bucket. [1]

In this example is relationship between the surface area M and the volume V of bucket are given by the formulas

$$M = \pi(sx + sy + x^2) \tag{4.5}$$

$$V = \frac{\pi}{3}(x^2 + xy + y^2)h \tag{4.6}$$

where x, y, h and s are shown in Figure 4.1.

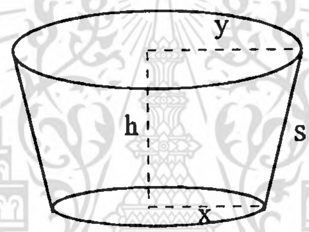

Figure 4.1 Model of a bucket.

We want to find the bucket which has the greatest volume V^* for the given surface area $M = \pi$, where the radii of this bucket are denoted by x^* and y^*, so we obtained the radii from the following system

$$F(x,y) = \begin{pmatrix} (1-4x^2)(x+2y-2y^3)+4xy^2(x^2-1)+3xy^4 \\ 2x(1-x^2)^2+y(1-2x^2+4x^4)+6xy^2(x^2-y^2)-3y^5 \end{pmatrix} \tag{4.7}$$

then we can compute the optimal radii by using iterative methods, those radii which give the greatest volume under the condition $x = y$ (cylindrical bucket; what is its volume?). Determine x^*, y^* and the corresponding volume V^* and height h^*. So we choose following initial vectors

$$A=\left\{\begin{bmatrix}0.4\\0.6\end{bmatrix},\begin{bmatrix}0.5\\1.2\end{bmatrix},\begin{bmatrix}0.05\\1.50\end{bmatrix}\right\}, B=\left\{\begin{bmatrix}1.0\\3.0\end{bmatrix},\begin{bmatrix}0.0\\2.0\end{bmatrix},\begin{bmatrix}0.5\\1.0\end{bmatrix}\right\} \text{ and } C=\left\{\begin{bmatrix}0.5\\1.0\end{bmatrix},\begin{bmatrix}1.0\\0.6\end{bmatrix},\begin{bmatrix}2.0\\3.0\end{bmatrix}\right\}.$$

Repeat the method until $\|f(x^{m+1})\| < 10^{-7}$. We obtain the results in the table 4.5.

Table 4.5 Give the results with initial vectors of example 4.5 .

Initial vectors (x^0, y^0)	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate secant method	Mid modified Multivariate secant Method	
A	10	8	8	$x = 0.4033783$ $y = 0.7353084$
B	13	13	13	$x = 0.4033783$ $y = 0.7353084$
C	16	14	14	$x = 0.4033783$ $y = 0.7353084$

Estimate values are $x = 0.4033783$ and $y = 0.7353084$. Let us consider all initial vectors, we see that the modified multivariate secant method and the mid modified multivariate secant method have the same number of iteration and they have the number of iteration less than the multivariate secant method. Showing the iterations of table 4.5 in appendix.

Hence the bucket which has the greatest volume V^*, where the radii of this bucket are x^*=0.4033783 and $y^* = 0.7353084$ for the given surface area $M = \pi$. So from equation (4.5) we obtain equation $sx + sy + x^2 = 1$, substituting x^*=0.4033783 and $y^* = 0.7353084$, then compute s from this equation, we obtain $s = 0.7353084$.

From Pythagorean proposition, h^* is computed and $h^* = 0.6561256$. Thus the bucket which has the greatest volume $V^* = \frac{\pi}{3}(x^2 + xy + y^2)h = 0.6871$ unit cubic.

Let us consider the greatest volume of the bucket under the additional condition $x = y$ (cylindrical bucket). Since surface area equal to π, so we have $\pi r^2 + 2\pi r h = \pi$, then $h = \frac{1 - r^2}{2r}$. Thus the formula for finding the volume is $V = \frac{\pi r(1 - r^2)}{2}$ and from Ordinary Differential Equation, we have $r = \frac{1}{\sqrt{3}}$ for the greatest volume, hence the greatest volume of the bucket under addition $x = y$ is $V = \pi r^2 h = 0.6046$ unit cubic.

Example 4.6 The problem of a double star system. [3]

A binary or double star system is a pair of stars that are orbiting around each other. If one star is much more massive than the other, we can say that the smaller star is in orbit around the larger one, as shown in Figure 4.2. The mean distance between the stars will be denoted by *a*, and this distance is measured in AU (astronomical units), where 1 AU is the mean distance from the earth to our sun. Hence 1 AU is approximately 92,900,000 miles.

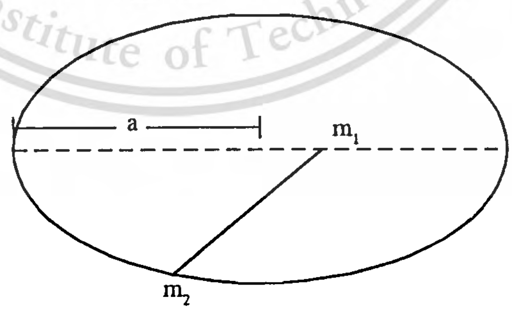

Figure 4.2 A double star system.

As seen from the earth, the stars appear very close in the sky, with some angular separation α that will be measured in seconds of arc. Typical values for some well-know systems range from a few seconds of arc to a few thousandths of a second of arc. Let us assume that the stars have masses μ_1 and μ_2 measured in multiples of the sun's mass. For example, a star with a mass of 2 solar masses has twice the mass of our sun. Each binary system has an orbital period T that is the length of time for each member to complete revolution about the other member. The period T is measured in earth years. If the star is observed from the earth, its apparent direction changes as the earth revolves around the sun. The angular difference between positions measured at six month intervals is called *the parallax* of the star and is denoted by p. Using the diameter of the earth's orbit and simple trigonometry, it is possible to compute the distance of a star from its parallax. For nearby stars, the parallax can be measured directly, even though the values are very small factions of second of arc. For the more distant stars, however , unable values are not obtainable by direct measurement, so indirect methods are needed.

In the case of binary systems, a method know as the method of *dynamical parallaxes* is used. The method rests on the fact that certain theoretical relations exists between visual magnitudes (apparent brightness as seen from earth) m_1 and m_2 of the component stars in the system, the parallax p, and the individual masses μ_1 and μ_2. These relations provide us with a nonlinear system to solve for the three unknowns μ_1 μ_2 and p after observable values of m_1, m_2, α and T are given. Detail of these relationship are supplied in most introductory astronomy texts. They are as follows:

$$p^3 = \frac{\alpha^3}{T^2(\mu_1 + \mu_2)} \tag{4.8}$$

$$M_1 = m_1 + 5 + 5\log p \quad \text{and} \quad M_2 = m_2 + 5 + 5\log p \tag{4.9}$$

$$0.4(M_0 - M_1) = 3.5\log \mu_1 \quad \text{and} \quad 0.4(M_0 - M_2) = 3.5\log \mu_2 . \tag{4.10}$$

The usual textbook procedure for determining p, μ_1 and μ_2 is to begin by assuming that the sum of the masses is 2. We substitute that in Equation (4.8) and solve for p (we know α and T). We substitute this value of p in Equation (4.9) to determine M_1 and M_2, the absolute magnitudes of the stars. In Equation (4.10), the number M_0 is the absolute magnitude of the sun, which is about 4.8. We use this value of M_0 and the values of M_1 and M_2 found from Equation (4.9) to compute μ_1 and μ_2 from Equation (4.10). Now we substitute these values in Equation (4.8) to recompute p. We continuous this iterative procedure until the new value of p agrees well with the preceding one. In this case, we have five equations and five unknowns. We can improve on this procedure by substituting the values of the absolute magnitude M_1 and M_2 from Equation (4.10). We obtained the equations

$$M_0 - m_1 - 5 = 5\log p + 8.75\log \mu_1 \tag{4.11}$$

$$M_0 - m_2 - 5 = 5\log p + 8.75\log \mu_2 . \tag{4.12}$$

Substituting $M_0 = 4.8$ and combining these Equation (4.8), we obtained the system of three equations

$$p^3 = \frac{\alpha^3}{T^2(\mu_1 + \mu_2)}$$

$$-m_1 - 0.2 = 5\log p + 8.75\log \mu_1 \tag{4.13}$$

$$-m_2 - 0.2 = 5\log p + 8.75\log \mu_2$$

where

α is the angle separation

p is the parallax of the star

m_1 is visual magnitude of the first star

m_2 is visual magnitude of the second star

μ_1 is the mass of the first star

μ_2 is the mass of the second star

T is time in year on the earth

to solve for the three unknowns p, μ_1 and μ_2. Given values $T = 88, \alpha = 4.6$, m_1=4.2, m_2= 6.0 for a certain double star system, then find dynamical parallax. We choose the following initial vectors

$$\mathrm{A} = \left\{ \begin{bmatrix} 0.03 \\ 4.00 \\ 4.00 \end{bmatrix}, \begin{bmatrix} 0.05 \\ 10.0 \\ 5.00 \end{bmatrix}, \begin{bmatrix} 0.05 \\ 5.00 \\ 5.00 \end{bmatrix}, \begin{bmatrix} 1.00 \\ 10.00 \\ 20.00 \end{bmatrix} \right\}, \mathrm{B} = \left\{ \begin{bmatrix} 0.1 \\ 2.0 \\ 5.0 \end{bmatrix}, \begin{bmatrix} 0.2 \\ 4.0 \\ 6.0 \end{bmatrix}, \begin{bmatrix} 0.01 \\ 3.00 \\ 3.00 \end{bmatrix}, \begin{bmatrix} 1.0 \\ 8.0 \\ 10.0 \end{bmatrix} \right\}$$

$$\text{and } \mathrm{C} = \left\{ \begin{bmatrix} 0.04 \\ 4.75 \\ 3.42 \end{bmatrix}, \begin{bmatrix} 0.02 \\ 2.50 \\ 2.40 \end{bmatrix}, \begin{bmatrix} 0.20 \\ 2.20 \\ 1.45 \end{bmatrix}, \begin{bmatrix} 0.03 \\ 3.65 \\ 1.45 \end{bmatrix} \right\}.$$

Repeat the method until $\left\| f(x^{m+1}) \right\| < 10^{-7}$. We obtain the results from the multivariate secant method, the modified multivariate secant method and the mid modified multivariate secant method in the table 4.6.

Table 4.6 Give the results with initial vectors of example 4.6.

Initial vectors (μ_1^0, μ_2^0, p^0)	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate secant method	Mid modified multivariate secant method	
A	15	10	10	$\mu_1 = 0.0251253$ $\mu_2 = 4.9640298$ $p = 4.041061$
B	18	15	15	$\mu_1 = 0.0251253$ $\mu_2 = 4.9640298$ $p = 4.041061$
C	11	10	10	$\mu_1 = 0.0251253$ $\mu_2 = 4.9640298$ $p = 4.041061$

Hence we estimate values are μ_1=0.0251253, μ_2=4.9640298 and p = 4.041061. We also see that the modified multivariate secant method and the mid modified multivariate secant method have the same number of iterations and they have the number of iterations less than the multivariate secant method. Showing the iterations of table 4.6 in appendix.

Example 4.7 The hydrogen spectrum in atomic physics.[3]

This application is taken from the Bohr theory of the hydrogen spectrum in atomic physics. A constant known as the Rydberg constant, usually denoted by R_∞, plays an important part in that theory, but it cannot be measured directly. The values R_H and R_{He} can be determined from measurements of the spectrum of hydrogen and helium, respectively. If we knew the exact mass M_H of the nucleus of a hydrogen atom and the exact mass m of an electron, then theoretically, we could determine R_∞ from the equation

$$R_\infty = \left(1 + \frac{m}{M_H}\right) R_H$$

Suppose, for the moment, that m and M_H are unknown (because even our best values are inferred from other measured variables). The atomic weight A_H of hydrogen can be measured. Also, $A_H = M_H + m$. Similar equations hold for the helium atom. So we have a system of four equations:

$$R_\infty = \left(1 + \frac{m}{M_H}\right) R_H \quad (4.14)$$

$$R_\infty = \left(1 + \frac{m}{M_{He}}\right) R_{He} \quad (4.15)$$

$$A_H = M_H + m \quad (4.16)$$

$$A_{He} = M_{He} + m \quad (4.17)$$

In this system, the numbers A_H, A_{He}, R_H and R_{He} can be taken as known because their values can be determined from laboratory measurements. The number R_H and masses m, M_H and M_{He} will be regarded as unknown. Their values can be determined by solving the system of four equations. From the system of equations (4.14), (4.15), (4.16) and (4.17), we make some algebraic changes in the system. Let $w = R_\infty$, $x = m$, $y = M_H$ and $z = M_{He}$. Then system becomes

$$w-\left(1+\frac{x}{y}\right)R_H = 0$$

$$w-\left(1+\frac{x}{z}\right)R_{He} = 0$$

(4.18)

$$y+x-A_H = 0$$

$$2x+z-A_{He} = 0$$

Since $w = R_\infty$ and R_∞ is approximated by the measured number R_H, we have an initial estimate of w. Similarly, since x is the mass of the electron and we know that the value is somewhere around $\frac{1}{2000}$ of a unit of atomic mass, an initial estimate of 0.0005 for x seems reasonable. We need values of A_H, A_{He}, R_H and R_{He} to perform the calculations. Suppose laboratory measurements produced the following values: A_H = 1.00812, A_{He} = 4.00388, R_H = 109677.68, and R_{He}=109722.34. What are the corresponding values for Rydberg constant R_∞ and masses m, M_H, and M_{He}?

Then we find the solution of the system by multivariate secant method, the modified multivariate secant method and mid modified multivariate secant method, then we choose the following initial vectors

$$A = \left\{ \begin{bmatrix} 10900 \\ 0.0005 \\ 1.0075 \\ 4.0000 \end{bmatrix}, \begin{bmatrix} 10871 \\ 0.0003 \\ 1.0020 \\ 3.0000 \end{bmatrix}, \begin{bmatrix} 10800 \\ 0.0004 \\ 1.0300 \\ 3.5000 \end{bmatrix}, \begin{bmatrix} 10753 \\ 0.0003 \\ 1.0050 \\ 4.0000 \end{bmatrix}, \begin{bmatrix} 10725 \\ 0.0002 \\ 1.0000 \\ 3.2000 \end{bmatrix} \right\},$$

$$B = \left\{ \begin{bmatrix} 1073 \\ 0.002 \\ 1.000 \\ 3.200 \end{bmatrix}, \begin{bmatrix} 1080 \\ 0.004 \\ 1.025 \\ 3.500 \end{bmatrix}, \begin{bmatrix} 1087 \\ 0.003 \\ 1.015 \\ 3.000 \end{bmatrix}, \begin{bmatrix} 1075 \\ 0.003 \\ 1.040 \\ 4.000 \end{bmatrix}, \begin{bmatrix} 1090 \\ 0.005 \\ 1.007 \\ 4.000 \end{bmatrix} \right\},$$

$$C = \left\{ \begin{bmatrix} 102200 \\ 0.0035 \\ 1.2000 \\ 3.005 \end{bmatrix}, \begin{bmatrix} 106200 \\ 0.005 \\ 1.200 \\ 3.025 \end{bmatrix}, \begin{bmatrix} 103500 \\ 0.0015 \\ 1.2000 \\ 3.0050 \end{bmatrix}, \begin{bmatrix} 104200 \\ 0.002 \\ 1.500 \\ 3.500 \end{bmatrix}, \begin{bmatrix} 103000 \\ 0.0025 \\ 1.6000 \\ 3.0500 \end{bmatrix} \right\}.$$

Repeat the method until $\left\| f(x^{m+1}) \right\| < 10^{-7}$. We obtain the results in the table 4.7.

Table 4.7 Give the results with initial vectors of example 4.7.

Initial vectors (w^0, x^0, y^0, z^0)	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate secant method	Mid modified Multivariate secant Method	
A	10	5	5	$w = 109737.37154$ $x = 0.0005484$ $y = 1.0075716$ $z = 4.0027833$
B	16	11	11	$w = 109737.37154$ $x = 0.0005484$ $y = 1.0075716$ $z = 4.0027833$
C	11	7	7	$w = 109737.37154$ $x = 0.0005484$ $y = 1.0075716$ $z = 4.0027833$

Showing the iterations of table 4.7 in appendix.

Hence from the table 4.7 we obtain

R_∞ = 109737.37

electron mass : $m = 0.0005484$

mass of helium nucleus : $M_{He} = 4.0027833$

mass of hydrogen nucleus : $M_H = 1.0075716$

hydrogen mass / electron mass : $M_H / m = 1837.41$.

The value of one atomic mass unit is 1.6599×10^{-24} g. So the computed mass of the electron in this case is equivalent to $0.0005484 \times 1.6599 \times 10^{-24} = 9.103 \times 10^{-28}$ g. The accepted value is 9.11×10^{-28} g.

Moreover, we also see that the modified multivariate secant method and the mid modified multivariate secant method have the same number of iterations and they have the number of iteration less than the multivariate secant method.

Example 4.8 The problem of maximizing a function. [5]

We want to find the maximum value of rectangular box inscribed in the ellipsoid

$$f(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad (4.19)$$

with its face parallel to the coordinate planes. From (4.19), we see that it is a nonlinear equation subject to the constraint

$$g(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0. \qquad (4.20)$$

Let (x, y, z) be the vertex of the box that lies in the first octant (where x, y and z are all positive). We want to maximize the volume $V(x,y,z) = 8xyz$ subject to (4.20), then from Theorem for the method of Lagrange multipliers [5], we obtained the system of four nonlinear equations, for four unknowns λ, x, y and z

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1 = 0 \tag{4.21}$$

$$8yz-\frac{2\lambda x}{a^2} = 0 \tag{4.22}$$

$$8xz-\frac{2\lambda y}{b^2} = 0 \tag{4.23}$$

$$8xy-\frac{2\lambda z}{c^2} = 0 \tag{4.24}$$

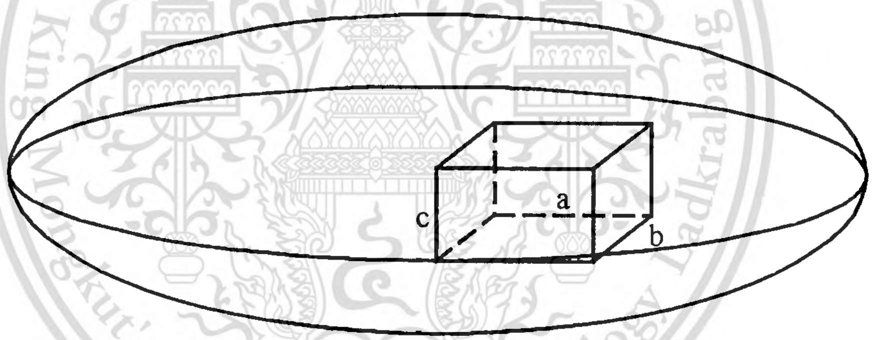

Figure 4.3 The rectangular box inscribed in the ellipsoid.

In this problem, the numbers a, b and c can be given, then suppose that $a=6$, $b=3$ and $c=1$. Then the system becomes

$$x^2+4y^2+36z^2-36 = 0$$

$$144yz-\lambda x = 0$$

$$36xz - \lambda y = 0 \tag{4.25}$$

$$4xy - \lambda z = 0$$

We can compute four unknowns by iterative methods. We choose the following initial vectors

$$A = \left\{ \begin{bmatrix} 35.02 \\ 3.00 \\ 0.25 \\ 0.20 \end{bmatrix}, \begin{bmatrix} 38.50 \\ 3.40 \\ 0.05 \\ 0.31 \end{bmatrix}, \begin{bmatrix} 30.0 \\ 4.3 \\ 0.6 \\ 0.4 \end{bmatrix}, \begin{bmatrix} 33.1 \\ 3.60 \\ 0.85 \\ 0.45 \end{bmatrix}, \begin{bmatrix} 38.01 \\ 4.30 \\ 0.75 \\ 0.49 \end{bmatrix} \right\},$$

$$B = \left\{ \begin{bmatrix} 29.0 \\ 2.5 \\ 2.0 \\ 0.4 \end{bmatrix}, \begin{bmatrix} 33.0 \\ 2.0 \\ 1.5 \\ 1.0 \end{bmatrix}, \begin{bmatrix} 32.0 \\ 1.5 \\ 2.0 \\ 1.0 \end{bmatrix}, \begin{bmatrix} 28.0 \\ 3.0 \\ 0.5 \\ 0.4 \end{bmatrix}, \begin{bmatrix} 30.0 \\ 1.0 \\ 1.0 \\ 0.0 \end{bmatrix} \right\},$$

$$C = \left\{ \begin{bmatrix} 27.0 \\ 1.5 \\ 1.0 \\ 0.5 \end{bmatrix}, \begin{bmatrix} 35.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{bmatrix}, \begin{bmatrix} 22.0 \\ 2.0 \\ 1.5 \\ 0.3 \end{bmatrix}, \begin{bmatrix} 25.0 \\ 2.5 \\ 1.5 \\ 0.45 \end{bmatrix}, \begin{bmatrix} 30.0 \\ 1.0 \\ 0.5 \\ 1.0 \end{bmatrix} \right\}.$$

Repeat the method until $\left\| f(x^{m+1}) \right\| < 10^{-7}$. We obtain the results from the multivariate secant method, the modified multivariate secant method and the mid modified multivariate secant method in the table 4.8.

Table 4.8 Give the results with initial vectors of example 4.8.

Initial vectors $(\lambda^0, x^0, y^0, z^0)$	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate secant method	Mid modified Multivariate secant Method	
A	18	13	5	λ =41.5692194 $x = 3.4641016$ $y = 1.7320508$ $z = 0.5773503$
B	24	16	16	λ =41.5692194 $x = 3.4641016$ $y = 1.7320508$ $z = 0.5773503$
C	13	12	11	λ =41.5692194 $x = 3.4641016$ $y = 1.7320508$ $z = 0.5773503$

Showing the iterations of table 4.8 in appendix. Therefore, the box of maximum volume has volume

$$V = V_{\max} = 8xyz = 27.7128 \text{ unit cubic.}$$

In this example, let us consider the number of iterations for each initial vector, we see that, the modified multivariate secant method and the mid modified multivariate secant method are the most appropriate method more than the multivariate secant method.

Example4.9 The problem of hanging chain.[8]

This problem has been used to test the minimizer. Given a chain of 8 sticks of unit weight and unit length, we are required to find the shape it will assume when suspended between 2 points spaced 6 units horizontally apart. In other words, we must find the angles α , β , γ and δ . Solving this by the energy method involves finding an expression for the chain's energy, depending on α , β and γ let the nonlinear minimizer do the rest. A different approach comes from statics. We introduce a 5^{th} unknown, the horizontal component x of the force going though the chain, which is in all joints. Then there is a system of equations for α , β , γ , δ and x:

$$2x\sin\alpha - 7\cos\alpha = 0 \tag{4.26}$$

$$2x\sin\beta - 5\cos\beta = 0 \tag{4.27}$$

$$2x\sin\gamma - 3\cos\gamma = 0 \tag{4.28}$$

$$2x\sin\delta - \cos\alpha = 0 \tag{4.29}$$

$$\cos\alpha + \cos\beta + \cos\gamma + \cos\delta - 3 = 0 \tag{4.30}$$

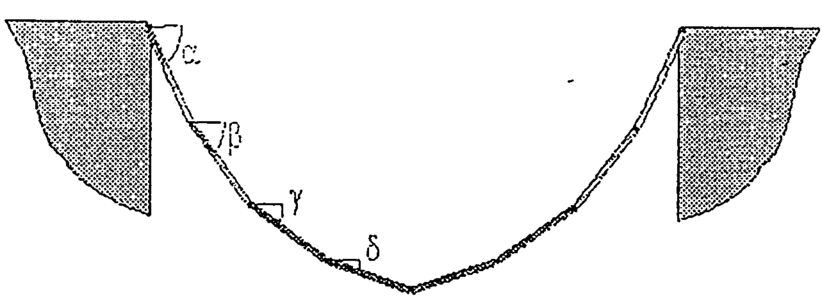

Figure 4.4 Model of a hanging chain.

Equations (4.26) to (4.30) arise from the requirement that, in the state of equilibrium, the torque on each stick equals 0. Equation (4.30) is purely geometric, stating that the horizontal length of the chain is 6 units.

Next we choose the following initial vectors for solving the system by multivariate secant method, modified multivariate secant method and mid modified multivariate secant method

$$A = \left\{ \begin{bmatrix} 0.50 \\ 0.50 \\ 0.50 \\ 0.50 \\ 1.00 \end{bmatrix}, \begin{bmatrix} 0.40 \\ 0.80 \\ 0.63 \\ 0.15 \\ 1.50 \end{bmatrix}, \begin{bmatrix} 0.80 \\ 1.10 \\ 0.45 \\ 0.20 \\ 1.70 \end{bmatrix}, \begin{bmatrix} 0.40 \\ 1.20 \\ 1.00 \\ 0.10 \\ 1.30 \end{bmatrix}, \begin{bmatrix} 0.35 \\ 0.50 \\ 1.00 \\ 0.50 \\ 1.00 \end{bmatrix}, \begin{bmatrix} 1.50 \\ 2.00 \\ 1.00 \\ 1.00 \\ 2.00 \end{bmatrix} \right\},$$

$$B = \left\{ \begin{bmatrix} 0.50 \\ 0.50 \\ 0.50 \\ 0.50 \\ 2.00 \end{bmatrix}, \begin{bmatrix} 0.60 \\ 0.98 \\ 0.63 \\ 0.15 \\ 2.00 \end{bmatrix}, \begin{bmatrix} 0.80 \\ 1.00 \\ 0.45 \\ 0.20 \\ 2.20 \end{bmatrix}, \begin{bmatrix} 0.40 \\ 1.00 \\ 1.00 \\ 0.10 \\ 2.00 \end{bmatrix}, \begin{bmatrix} 0.35 \\ 0.50 \\ 1.00 \\ 0.50 \\ 1.50 \end{bmatrix}, \begin{bmatrix} 1.50 \\ 2.00 \\ 1.00 \\ 1.00 \\ 2.00 \end{bmatrix} \right\},$$

$$C = \left\{ \begin{bmatrix} 1.00 \\ 1.00 \\ 1.00 \\ 1.00 \\ 1.50 \end{bmatrix}, \begin{bmatrix} 2.00 \\ 1.00 \\ 1.00 \\ 0.50 \\ 2.00 \end{bmatrix}, \begin{bmatrix} 0.50 \\ 1.00 \\ 0.50 \\ 0.10 \\ 1.00 \end{bmatrix}, \begin{bmatrix} 1.50 \\ 0.60 \\ 0.40 \\ 0.20 \\ 1.50 \end{bmatrix}, \begin{bmatrix} 0.75 \\ 0.50 \\ 0.30 \\ 0.50 \\ 1.00 \end{bmatrix}, \begin{bmatrix} 0.65 \\ 0.60 \\ 0.40 \\ 0.15 \\ 1.00 \end{bmatrix} \right\}.$$

Repeat the method until $\|f(x^{m+1})\| < 10^{-7}$. We obtain the results in the table 4.9.

Table 4.9 Give the results with initial vectors of example 4.9.

Initial vectors $(\alpha^0, \beta^0, \gamma^0, \delta^0, x^0)$	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate Secant method	Mid modified Multivariate secant method	
A	37	11	12	$\alpha = 1.0071538$ $\beta = 0.8464150$ $\gamma = 0.5958635$ $\delta = 0.2222919$ $x = 2.2121240$
B	55	9	9	$\alpha = 1.0071538$ $\beta = 0.8464150$ $\gamma = 0.5958635$ $\delta = 0.2222919$ $x = 2.2121240$
C	diverge	14	17	$\alpha = 1.0071538$ $\beta = 0.8464150$ $\gamma = 0.5958635$ $\delta = 0.2222919$ $x = 2.2121240$

Estimate the values are $\alpha = 1.0071538$, $\beta = 0.8464150$, $\gamma = 0.5958635$, $\delta = 0.2222919$ and $x = 2.2121240$. In this example, we see that the multivariate secant method is so slow or divergence. Otherwise, the number of iterations of the modified multivariate secant method and the mid modified multivariate secant method are not much different or mostly the same. Showing the iterations of table 4.9 in appendix.

Example 4.10 An alligator -filled moat problem.[5]

This problem shows an alligator –filled moat of width $w = 10$ ft bounded on each side by a wall of height $h = 6$ ft. Soldiers plan to bridge this moat by scaling a ladder that is placed across the wall as indicated and anchored at the ground by handy boulder, with the upper end directly above the wall on the opposite side of the moat. What is the minimal length L of a ladder that will suffice for this purpose?

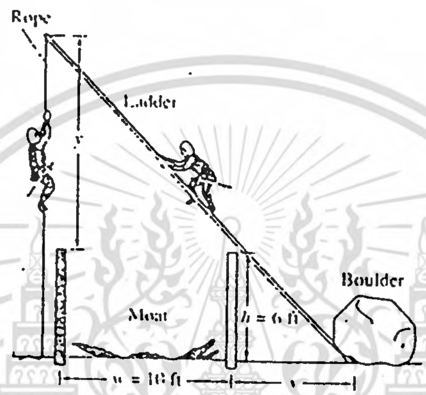

Figure 4.5 An alligator -filled moat problem.

Given $z = L$ for the length of the ladder, observe directly from Figure 4.4 that we need to minimize the function

$$f(x, y, z) = z$$

subject to the two constrains

$$g(x, y, z) = xy - 60 = 0$$

$$h(x, y, z) = (x+10)^2 + (y+6)^2 - z^2 = 0 \ .$$

This leads to a system of five equations in five unknown x, y, z and two Lagrange multipliers, then the system is

$$xy - 60 = 0$$

$$(x+10)^2 + (y+6)^2 - z^2 = 0$$

$$my + 2n(x+10) = 0 \tag{4.31}$$

$$mx + 2n(y+6) = 0$$

$$2nz + 1 = 0$$

Next we choose the following initial vectors for solving the system by multivariate secant method, modified multivariate secant method and the mid modified multivariate secant method

$$A = \left\{ \begin{bmatrix} 0.06 \\ -0.01 \\ 6.90 \\ 8.22 \\ 18.22 \end{bmatrix}, \begin{bmatrix} 0.70 \\ -0.01 \\ 9.10 \\ 7.23 \\ 17.10 \end{bmatrix}, \begin{bmatrix} 0.05 \\ -0.02 \\ 8.45 \\ 6.21 \\ 20.51 \end{bmatrix}, \begin{bmatrix} 0.08 \\ -0.03 \\ 9.20 \\ 6.60 \\ 21.25 \end{bmatrix}, \begin{bmatrix} 0.22 \\ -0.04 \\ 6.85 \\ 6.50 \\ 21.20 \end{bmatrix}, \begin{bmatrix} 0.12 \\ -0.02 \\ 9.50 \\ 6.25 \\ 22.00 \end{bmatrix} \right\},$$

$$B = \left\{ \begin{bmatrix} 0.00 \\ 0.50 \\ 7.85 \\ 9.50 \\ 15.20 \end{bmatrix}, \begin{bmatrix} 0.03 \\ 0.35 \\ 5.20 \\ 9.60 \\ 13.00 \end{bmatrix}, \begin{bmatrix} 0.01 \\ 0.30 \\ 5.10 \\ 9.23 \\ 15.10 \end{bmatrix}, \begin{bmatrix} 0.00 \\ 0.25 \\ 6.50 \\ 10.25 \\ 18.25 \end{bmatrix}, \begin{bmatrix} 0.02 \\ 0.40 \\ 4.45 \\ 10.21 \\ 16.51 \end{bmatrix}, \begin{bmatrix} 0.02 \\ 0.25 \\ 3.90 \\ 9.22 \\ 20.22 \end{bmatrix} \right\},$$

$$C = \left\{ \begin{bmatrix} 0.45 \\ 0.02 \\ 6.90 \\ 9.22 \\ 20.22 \end{bmatrix}, \begin{bmatrix} 0.50 \\ 0.01 \\ 5.10 \\ 9.23 \\ 15.10 \end{bmatrix}, \begin{bmatrix} 0.55 \\ 0.02 \\ 4.45 \\ 8.21 \\ 16.51 \end{bmatrix}, \begin{bmatrix} 0.65 \\ 0.03 \\ 5.20 \\ 9.60 \\ 13.00 \end{bmatrix}, \begin{bmatrix} 0.50 \\ 0.10 \\ 7.85 \\ 9.50 \\ 15.20 \end{bmatrix}, \begin{bmatrix} 0.25 \\ 0.35 \\ 6.50 \\ 7.25 \\ 23.25 \end{bmatrix} \right\}.$$

Repeat the method until $\|f(x^{m+1})\| < 10^{-7}$. We obtain the results in the table 4.10.

Table 4.10 Give the results with initial points of example 4.10.

Initial vectors $(m^0, n^0, x^0, y^0, z^0)$	Number of iterations (k)			Solutions
	Multivariate secant method	Modified multivariate secant method	Mid modified Multivariate secant method	
A	15	10	11	$m = 0.0906309$ $n = -0.0223332$ $x = 7.1137866$ $y = 8.4343266$ $z = 22.3881995$
B	106	20	106	$m = 0.0906309$ $n = -0.0223332$ $x = 7.1137866$ $y = 8.4343266$ $z = 22.3881995$
C	166	13	19	$m = 0.0906309$ $n = -0.0223332$ $x = 7.1137866$ $y = 8.4343266$ $z = 22.3881995$

Estimate values are $m = 0.0906309$, $n = -0.0223332$, $x = 7.1137866$, $y = 8.4343266$ and $z = 22.3881995$. We see that the multivariate secant method is so slow for convergence. Otherwise, the modified multivariate secant method and the mid modified multivariate secant method are not much different for the initial vector at A, but the mid modified multivariate secant method is so slow for convergence at initial vector at B. Thus the modified multivariate secant method is appropriate method for this example. Showing the iterations of table 4.10 in appendix.

CHAPTER 5

CONCLUSION AND SUGGESTIONS

In this part, we will conclude and suggest of the results from some examples and application problems in chapter 4.

5.1 Conclusion

In this thesis, we found the methods that can solve systems of nonlinear equations $f(x)=0$ in the iterative fashion. We used three methods for solve the systems in chapter 4, the first method is multivariate secant method. The second method is modified multivariate secant method and the last method is mid modified multivariate secant method.

The method, that we used for solving the systems of nonlinear equations in this thesis is one-point stationary iterative method with memory and the order of convergence of the multivariate secant method is not less than $(1+\sqrt{5})/2$ and does not depend on n.

Whether or not the iteration procedure converges quickly, or in deed at all depends on the choice of the function f, as well as the initial vectors. In this thesis we will consider the number of iterations of methods from assuming that the absolute error is $\varepsilon = 10^{-7}$ for all examples, and iterated until $\| f(x^{m+1}) \| \langle \varepsilon$.

From the results of most examples in this thesis, as well as we expected, and they are consistent of theorem 2.3. We can see that the convergence of the modified multivariate secant method and the mid modified multivariate secant method are faster than the multivariate secant method. Moreover we can see that arranging in order of norm of function have effect for convergence i.e. in example 4.2, number of iterations of the modified multivariate secant and the mid modified multivariate secant method at the initial vectors of B and C are 10 and 9 respectively, otherwise number of iterations of the multivariate secant at the initial vectors of B and C are 12 and divergence respectively. The other notice, we see that the number of iterations of the modified multivariate secant method and the mid modified multivariate secant method are not much different or mostly the same. Hence we can say that the modified multivariate secant method has in one step more operation than the mid modified multivariate secant method. In many problems, in order to get the solutions of the system of equations the arranging of initial vectors is very important. The other important thing is that technique to choose the initial vectors, we have to choose the initial vectors, which is close to the root or say that by guessing. But guessing initial vectors must be efficacious, that is, we need to understand the mathematical modeling of real problem, we roughly know the solution, hence we can easily guess initial vectors.

The nonlinear systems have more than one solution, which all obtained solutions do not satisfy the real world problem. In this case, engineers or scientists who model the problem need to resolve or improve upon a better model and the mathematics create new method and guess initial vectors to solve the systems of nonlinear equations.

5.2 Suggestions

The arrangement of initial vectors in order to norm of functions can do only in first step is enough, so the mid modified multivariate secant is appropriate method.

Furthermore, we have technique to choose the initial vectors, that is, we can use the Modified Newton method to find n initial vectors, so we have $n+1$ initial vectors, then using the modified multivariate secant method or the mid modified multivariate secant method to solve the systems of nonlinear equations.

Also, we think that may be have an new alternative, that is interested method for solving the system of nonlinear equations, that is the multivariate secant method, which is multipoint stationary iterative method with memory.

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APPENDIX

The iterations of table 4.1 at the initial vectors of A

The result from multivariate secant method.

k	x^k	y^k	ER
0	0.25	3.00	0.2532182
0	0.00	0.00	1.5815453
0	1.00	1.00	0.3151753
1	0.1016793	2.7458345	0.4679571
2	-1.2143499	3.2051506	6.9887932
3	2.4030082	-2.2653504	94.9918787
4	-0.1304954	2.890305	1.4596357
5	-0.5697475	3.1698229	3.660937
6	1.5925808	1.5514936	12.4449239
7	4.3798353	-0.2421945	5838.42162
8	-6.9790651	7.9678114	42.4596563
9	0.2480273	2.5571023	0.1300933
10	0.1795285	2.6083714	0.1205558
11	0.2378555	2.5647227	0.1016459
12	0.127621	2.6483518	0.3039026
13	0.1187649	2.6329709	0.319222
14	0.2817257	2.8615431	0.0598142
15	0.7398164	1.7354328	0.9809641
16	0.3124714	2.8361292	0.0278151
17	0.3034119	2.838338	0.007206
18	0.3002926	2.8368519	0.0018601
19	0.2994118	2.8368822	0.0000413
20	0.2994501	2.8369294	0.0000015
21	0.2994487	2.8369278	0.0000000

Estimates values are $x = 0.2994487$ and $y = 2.8369278$ after 21 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	ER
0	0.00	0.00	1.5815453
0	1.00	1.00	0.3151753
0	0.25	3.00	0.2532182
1	0.1016793	2.7458345	0.4679571
2	0.5369764	2.5939086	0.4831767
3	0.0754188	3.3571425	1.0781419
4	0.2367777	3.3360509	0.5759456
5	0.6107322	2.4365338	0.5918149
6	0.421642	2.9310124	0.1224063
7	0.3900113	2.9531149	0.0649254
8	0.3209017	2.7735767	0.0982799
9	0.2467904	2.7683751	0.065785
10	0.357959	2.9139067	0.0483677
11	0.3101679	2.8505451	0.011249
12	0.3018085	2.8397796	0.0026637
13	0.301113	2.8391793	0.001697
14	0.2993731	2.8368323	0.0000829
15	0.2994492	2.8369284	0.0000006
16	0.2994487	2.8369278	0.0000000

Estimates values are $x = 0.2994487$ and $y = 2.8369278$ after 16 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	ER
0	0.00	0.00	1.5815453
0	1.00	1.00	0.3151753
0	0.25	3.00	0.2532182
1	0.1016793	2.7458345	0.4679571
2	0.5369764	2.5939086	0.4831767
3	0.3551335	2.8946049	0.0590367
4	0.3263886	2.8668034	0.0300290

k	x^k	y^k	ER
5	0.2935799	2.8282340	0.0055785
6	0.2981519	2.8351787	0.0013404
7	0.2994877	2.8369780	0.0000418
8	0.2994486	2.8369277	0.0000001

Estimates values are $x = 0.2994486$ and $y = 2.8369277$ after 8 iterations.

The iterations of table 4.1 at the initial vectors of B

The result from multivariate secant method.

k	x^k	y^k	ER
0	0.00	0.00	1.5815453
0	1.00	1.00	0.3151753
0	0.35	3.00	0.0487849
1	0.4201999	2.760628	0.2664144
2	0.4254766	2.767108	0.266889
3	-0.1314738	2.3167192	0.9631599
4	1.8419657	4.5856527	28.1005179
5	0.3902191	2.7426712	0.2428707
6	0.3626558	2.7239138	0.2165338
7	0.3034879	2.6882595	0.1370156
8	-0.1421122	2.3288675	1.0167946
9	0.9323755	2.8981409	1.0021391
10	0.0914918	2.562493	0.3495812
11	-0.0780372	2.5314766	0.9370356
12	-0.0875352	2.4801204	0.9299264
13	0.4906188	3.9216672	0.6851686
14	0.3120207	1.6209288	1.0786897
15	0.6745881	3.7578399	0.7209942
16	0.3710258	3.0584417	0.0650847
17	0.3818027	2.9812054	0.0345239
18	0.4050527	3.014392	0.0381262

k	x^k	y^k	ER
19	-0.0226702	2.3574632	0.5775533
20	-19.2558988	-24.7637001	81.5418528
21	0.369533	2.9589313	0.0334132
22	0.3411921	2.9146835	0.0216569
23	0.2809484	2.8203678	0.0269104
24	0.2059412	2.7039597	0.120302
25	0.2745333	2.80942	0.0326284
26	0.2909954	2.829486	0.0120931
27	0.3008824	2.8380195	0.0021385
28	0.2994538	2.8369354	0.0000048
29	0.2994495	2.8369288	0.000001
30	0.2994487	2.8369278	0.000000

Estimates values are $x = 0.2994487$ and $y = 2.8369278$ after 30 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	ER
0 0 0	0.00 1.00 0.35	0.00 1.00 3.00	1.5815453 0.3151753 0.0487849
1	0.4201999	2.7606280	0.2664144
2	0.4254766	2.7671080	0.2668890
3	-2.9565980	0.0321271	13.6768385
4	0.5311861	2.1886650	0.8332690
5	0.5907961	1.5872257	1.3404685
6	0.8674402	7.8461046	4.8559776
7	0.4117837	2.0021097	0.9110329
8	0.3189618	2.3198489	0.4868677
9	0.1703355	2.6455263	0.1735431
10	0.2148694	2.7305350	0.1172826
11	0.2884527	2.8359266	0.0230238
12	0.3048804	2.8354312	0.0127355

k	x^k	y^k	ER
13	0.2994226	2.8369347	0.0000614
14	0.2994491	2.8369282	0.0000004
15	0.2994489	2.8369280	0.0000002
16	0.2994487	2.8369278	0.0000000

Estimates values are $x = 0.2994487$ and $y = 2.8369278$ after 16 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	ER
0	0.00	0.00	1.5815453
0	1.00	1.00	0.3151753
0	0.35	3.00	0.0487849
1	0.4201999	2.760628	0.2664144
2	0.4254766	2.767108	0.266889
3	-0.1314738	2.3167192	0.9631599
4	1.8419657	4.5856527	28.1005179
5	0.3902191	2.7426712	0.2428707
6	0.3626558	2.7239138	0.2165338
7	0.3034879	2.6882595	0.1370156
8	-0.1421122	2.3288675	1.0167946
9	0.9323755	2.8981409	1.0021391
10	0.0914918	2.562493	0.3495812
11	-0.0780372	2.5314766	0.9370356
12	-0.0875352	2.4801204	0.9299264
13	0.4906188	3.9216672	0.6851686
14	0.3120207	1.6209288	1.0786897
15	0.6745881	3.7578399	0.7209942
16	0.3710258	3.0584417	0.0650847
17	0.3818027	2.9812054	0.0345239
18	0.4050527	3.014392	0.0381262
19	-0.0226702	2.3574632	0.5775533
20	-19.2558988	-24.7637001	81.5418528
21	0.369533	2.9589313	0.0334132

k	x^k	y^k	ER
22	0.3411921	2.9146835	0.0216569
23	0.2809484	2.8203678	0.0269104
24	0.2059412	2.7039597	0.120302
25	0.2745333	2.80942	0.0326284
26	0.2909954	2.829486	0.0120931
27	0.3008824	2.8380195	0.0021385
28	0.2994538	2.8369354	0.0000048
29	0.2994495	2.8369288	0.000001
30	0.2994487	2.8369278	0.000000

Estimates values are $x = 0.2994487$ and $y = 2.8369278$ after 30 iterations.

The iterations of table 4.1 at the initial vectors of C

The result from multivariate secant method.

k	x^k	y^k	ER
0	1.00	-1.00	2.1722084
0	0.25	3.00	0.2532182
0	1.00	1.00	0.3151753
1	1.5453753	0.3009905	9.6173669
2	0.1751395	3.1771962	0.6062914
3	-0.2327841	4.1806631	3.0249962
4	0.1617282	3.1903408	0.6571028
5	-4.0081748	-4.2236655	15.7459691
6	0.2920839	3.0746717	0.2212536
7	0.3557754	3.119249	0.1395177
8	0.3878532	3.1837095	0.145563
9	0.0026302	2.3426106	0.4734605
10	-0.1316389	2.149548	0.8184549
11	0.3583733	3.0510454	0.0776689
12	0.3348137	2.9690637	0.0474375
13	0.2859016	2.8089553	0.0081886
14	0.2922192	2.8249588	0.0060015

k	x^k	y^k	ER
15	0.2981371	2.8351219	0.0013277
16	0.2994392	2.8369157	0.0000104
17	0.2994487	2.8369278	0.0000000

Estimates values are $x = 0.2994487$ and $y = 2.8369278$ after 17 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	ER
0	1.00	-1.00	2.1722084
0	1.00	1.00	0.3151753
0	0.25	3.00	0.2532182
1	1.5453753	0.3009905	9.6173669
2	0.1751395	3.1771962	0.6062914
3	-0.2327841	4.1806631	3.0249962
4	0.1617282	3.1903408	0.6571028
5	0.15627	3.1806362	0.6651534
6	0.1834434	2.6342229	0.1285051
7	0.229323	2.715493	0.0690916
8	0.2895593	2.8343239	0.0192392
9	0.2928374	2.8339115	0.0117205
10	0.2990018	2.8364378	0.0005487
11	0.2994567	2.8369376	0.000009
12	0.2994487	2.8369277	0.0000000

Estimates values are $x = 0.2994487$ and $y = 2.8369277$ after 12 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	ER
0	1.00	-1.00	2.1722084
0	1.00	1.00	0.3151753
0	0.25	3.00	0.2532182
1	1.5453753	0.3009905	9.6173669

k	x^k	y^k	ER
2	0.1751395	3.1771962	0.6062914
3	0.2453946	3.0043730	0.2683160
4	0.2457939	2.9949419	0.2592159
5	0.2834530	2.8103074	0.0136772
6	0.2987952	2.8376126	0.0019761
7	0.2993607	2.8368032	0.0000862
8	0.2994447	2.8369229	0.0000045
9	0.2994487	2.8369278	0.0000000

Estimates values are $x = 0.2994487$ and $y = 2.8369278$ after 9 iterations.

The iterations of table 4.2 at the initial vectors of A

The result from multivariate secant method.

k	x^k	y^k	ER
0	0.20	1.50	0.2492971
0	-1.00	0.00	1.1449880
0	1.00	1.00	0.3472956
1	0.5377711	0.7598777	0.3770624
2	0.5356033	0.7678641	0.3756543
3	-0.3982829	3.4188427	0.558412
4	-0.0107691	2.6508265	0.2372152
5	0.2773056	2.272625	0.0685841
6	0.3742376	2.112641	0.0129006
7	0.4703666	2.2355153	0.0760626
8	0.3539172	2.0882262	0.0018266
9	0.3546589	2.0878704	0.0015007
10	0.358059	2.0861785	0.0000042
11	0.3580633	2.0861652	0.0000006
12	0.3580637	2.0861667	0.0000000

Estimates values are $x = 0.3580637$ and $y = 2.0861667$ after 12 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	ER
0	-1.00	0.00	1.144988
0	1.00	1.00	0.3472956
0	0.20	1.50	0.2492971
1	0.5377711	0.7598777	0.3770624
2	0.1513567	2.1834537	0.0888083
3	0.3133917	2.0319906	0.0311468
4	0.3206948	2.1236318	0.0187005
5	0.360442	2.0905277	0.00205
6	0.358304	2.0860221	0.0001077
7	0.3580612	2.0861626	0.000002
8	0.3580637	2.0861667	0.0000000

Estimates values are $x = 0.3580637$ and $y = 2.0861667$ after 8 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	ER
0	-1.00	0.00	1.1449880
0	1.00	1.00	0.3472956
0	0.20	1.50	0.2492971
1	0.5377711	0.7598777	0.3770624
2	0.1513567	2.1834537	0.0888083
3	0.4741279	1.8817415	0.0758924
4	0.3217077	2.0975399	0.0157678
5	0.3608109	2.0936619	0.0031388
6	0.3580526	2.0856535	0.0001808
7	0.3580656	2.0861626	0.0000015
8	0.3580637	2.0861667	0.0000000

Estimates values are $x = 0.3580637$ and $y = 2.0861667$ after 8 iterations.

The iterations of table 4.2 at the initial vectors of B

The result from multivariate secant method.

k	x^k	y^k	ER
0 0 0	1.50 -1.00 2.00	0.25 0.00 1.00	0.4979079 1.144988 0.7617096
1	0.6428289	0.0662055	0.4830121
2	0.9815862	0.4011722	0.3870189
3	1.1708182	0.3787353	0.4104613
4	1.5073326	-0.9968177	0.3028179
5	3.4930257	-5.8834635	1.2303675
6	2.0402218	-2.5654497	0.1789117
7	1.8082482	-1.986841	0.0358536
8	1.8552737	-2.0961446	0.0036137
9	1.8891904	-2.1247041	0.0175535
10	1.8562124	-2.0857573	0.0001414
11	1.8562325	-2.0861653	0.0000006
12	1.8562337	-2.0861667	0.0000000

Estimates values are $x = 1.8562337$ and $y = -2.0861667$ after 12 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	ER
0 0 0	-1.00 2.00 1.50	0.00 1.00 0.25	1.144988 0.7617096 0.4979079
1	0.6428289	0.0662055	0.4830121
2	1.1296217	0.5475512	0.4008458
3	2.032723	-2.2174019	0.0805333
4	1.8396192	-1.9135094	0.0585051
5	1.9650267	-2.2678076	0.0720078

k	x^k	y^k	ER
6	1.8461851	-2.0872794	0.0045976
7	1.857177	-2.0847876	0.0007111
8	1.8562368	-2.08612	0.0000167
9	1.8562339	-2.086167	0.0000001
10	1.8562337	-2.081667	0.0000000

Estimates values are $x = 1.8562337$ and $y =$ -2.0861667 after 10 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	ER
0 0 0	-1.00 2.00 1.50	0.00 1.00 0.25	1.1449880 0.7617096 0.4979079
1	0.6428289	0.0662055	0.4830121
2	1.1296217	0.5475512	0.4008458
3	2.0327230	-2.2174019	0.0805333
4	1.8396192	-1.9135094	0.0585051
5	1.9650267	-2.2678076	0.0720078
6	1.8461851	-2.0872794	0.0045976
7	1.8553836	-2.0851942	0.0004477
8	1.8562423	-2.0861316	0.0000137
9	1.8562336	-2.0861665	0.0000001

Estimates values are $x = 1.8562336$ and $y =$ -2.0861665 after 9 iterations.

The iterations of table 4.2 at the initial vectors of C

The result from multivariate secant method.

k	x^k	y^k	ER
0	1.50	0.25	0.4979079
0	2.00	1.00	0.7617096
0	-1.00	0.00	1.1449880
1	0.6428289	0.0662055	0.4830121
2	0.9815862	0.4011722	0.3870189
3	0.5138228	0.4566343	0.4535766
4	-3.429314	10.5051959	0.4338618
5	-1.8681368	6.2647283	1.2934906
6	-1.3692762	5.2417071	1.1799359
7	-0.5895089	4.3379425	0.8805134
8	29.6944052	-85.8107744	0.7489152
9	759.2634472	-2271.4944900	0.5120221
10	1602.821928	-4797.5876719	0.2345384
11	1997.600863	-5978.0214110	0.6186411
12	-4870.329844	14578.62219	0.6546886
13	-622.4056035	1862.3241248	0.2955031
14	507.8452711	-1522.353172	0.9735728
15	1524.744471	-4564.962759	0.6134507
16	2751.582275	-8235.257383	0.3729183
17	3254.672737	-9740.949562	0.7368241
18	134.6972414	-3397.357894	0.9706507
19	782.6424007	-2343.346918	1.2360413
20	7936.093938	-23747.75694	0.2052558
21	9710.049308	-29055.6245	0.2804349
22	9881.983458	-29570.16493	1.0788728
23	7458.670978	-22319.23313	0.618984
24	8929.521602	-26720.18035	0.3246479
25	8219.073416	-24594.42183	0.5072667

k	x^k	y^k	ER
26	7814.196717	-23383.03124	0.6059403
27	9049.454045	-27079.00595	0.6031523
28	9049.069781	-27077.82863	0.3432287
29	10681.44063	-31961.92408	0.6001456
30	9984.528485	-29876.76354	0.5588278
31	8780.049347	-26272.93531	0.3672808
32	8398.187011	-25130.41331	0.7964056
33	9321.785311	-27893.81525	0.5852127
34	7731.684565	-23136.21805	0.7985926
35	16338.51038	-48887.88703	0.8115961
36	-112402.7744	336306.7122	1.051167
37	572184.2341	-1711981.215	0.8781614
38	503016.9958	-1505032.47	0.7397903
39	3575469.154	-10697825.44	1.1391648
40	-4093493.441	12247750.86	0.6483696
41	-3663656.013	10961675	0.7343142
42	-5173200.463	15478239.91	0.7607315
43	-3486758.616	10432397.07	0.3612094
44	-2340124.576	7001662.017	1.2370932
45	622926103.1	-1863798168	0.4621473
46	327988098	-981342111.1	0.3344479
47	407632800.6	-1219639479	0.3761792
48	461835221.9	-1381813407	0.7263241
49	357827721.7	-1070622422	0.6524661
50	367282298.3	-1098910564	0.0252943
51	370143917.8	-1107472545	0.3869211
52	367285511.3	-1098920178	0.8119914
53	367464941.6	-1099457034	0.5463369
54	368921699.7	-1103815662	0.2948749
55	368346082.3	-1102093412	1.1402019
56	368770837.5	-1103364281	0.6699927
57	368792254.6	-1103428362	0.4992933

k	x^k	y^k	ER
58	367827343.8	-1100541343	0.6467228
59	368438555.8	-1102370093	0.2163296
60	368353290.9	-1102114980	0.4629667
61	368438896.2	-1102371111	0.5760246
62	368412571.4	-1102292347	0.8845347
63	368386381.7	-1102213988	0.5774044
64	366371441.7	-1096185277	0.8281705
65	374782276.1	-1121350537	1.1077614
66	248224364.9	-742688602.3	1.0540669
67	437361273.6	-1308587227	0.6643938
68	784049220.6	-2345879385	0.5333258
69	599330559.2	-1793200180	0.2792294
70	719267214.6	-2152051283	0.5888265
71	2028933862	-6070580769	0.6302135
72	978773144.2	-2928494387	0.339027
73	1612708685	-4825232854	0.937769
74	-227524802.6	680755405.8	0.8760804
75	139713974.7	-418024944.6	0.9670409
76	-4887603937	14623736653	0.9526175
77	-8557550512	25604236080	0.7663904
78	-19309660619	57774605996	0.2292894
79	-23443000192	70141579710	1.1559906
80	-18428548392	55138313586	0.7800742
81	-18299469371	54752108476	0.9301371
82	-1687139704	5047930858	0.397756
83	10364940004	-31011954933	0.5086356
84	37268412.88	-111507285.7	0.2114507
85	-62204605284	1.86117E+11	0.3030962
86	-43914114146	1.31391E+11	0.2959653
87	-7775712624	23264973014	1.0702594
88	-1.34204E+11	4.01539E+11	0.8532664
89	1798728937	-5381806426	0.6193203

k	x^k	y^k	ER
90	5.82424E+11	-1.74262E+12	0.3995872
91	-5.38821E+12	1.61215E+13	0.4472136

We see that the result from secant method is divergence but the result from modified multivariate secant method and the mid modified multivariate secant method are convergence, they are shown in below table.

The result from modified multivariate secant method.

k	x^k	y^k	ER
0	-1.00	0.00	1.144988
0	2.00	1.00	0.7617096
0	1.50	0.25	0.4979079
1	0.6428289	0.0662055	0.4830121
2	1.1296217	0.5475512	0.4008458
3	2.032723	-2.2174019	0.0805333
4	1.8396192	-1.9135094	0.0585051
5	1.9650267	-2.2678076	0.0720078
6	1.8461851	-2.0872794	0.0045976
7	1.857177	-2.0847876	0.0007111
8	1.8562368	-2.08612	0.0000167
9	1.8562339	-2.086167	0.0000001
10	1.8562337	-2.081667	0.0000000

Estimates values are $x = 1.8562337$ and $y = -2.0861667$ after 10 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	ER
0	-1.00	0.00	1.1449880
0	2.00	1.00	0.7617096
0	1.50	0.25	0.4979079
1	0.6428289	0.0662055	0.4830121
2	1.1296217	0.5475512	0.4008458
3	2.0327230	-2.2174019	0.0805333
4	1.8396192	-1.9135094	0.0585051
5	1.9650267	-2.2678076	0.0720078
6	1.8461851	-2.0872794	0.0045976
7	1.8553836	-2.0851942	0.0004477
8	1.8562423	-2.0861316	0.0000137
9	1.8562336	-2.0861665	0.0000001

Estimates values are $x = 1.8562336$ and $y = -2.0861665$ after 9 iterations.

The iterations of table 4.3 at the initial vectors of A

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	3.00	2.00	1.00	12.251052
0	3.00	2.00	4.00	6.5900442
0	7.00	1.00	3.00	5.7900338
0	5.00	2.00	4.00	6.0311568
1	23.3881432	106.4535718	94.2836727	35.380986
2	34.1598663	53.3019436	46.702158	21.8759652
3	40.6655404	57.6930974	50.4504094	26.1355138
4	-5.0915898	23.051463	22.8634257	23.5513112
5	-2471.647568	-1252.052327	-935.7097356	154.3560734
6	392.8684356	199.6913299	152.2378055	61.3486775

k	x^k	y^k	z^k	ER
7	-513.933057	-256.7905455	-189.3222702	71.2292082
8	14839.08532	7578.388127	5691.619577	376.5047773
9	4791.113407	2453.994342	1845.933493	213.6193702
10	8105.17988	4143.576348	3113.800718	278.1062563
11	-28373.32423	-14468.18022	-10855.25984	521.0946244
12	4978.130272	2546.333134	1914.617807	217.8440887
13	-3680.588409	-1869.536204	-1399.290294	188.0953785
14	-16011.39462	-8161.104156	-6121.540055	391.5480704
15	66877.28833	34128.41161	25618.02795	799.6453077
16	14954.26181	7637.306351	5735.489647	377.9443806
17	61488.13159	31379.28943	23555.11011	766.7447257
18	44505.57324	22715.4502	17053.21533	652.2939761
19	-54595.73789	-27845.93456	-20894.48524	722.7040162
20	-10907.16553	-5554.552246	-4162.774902	323.1835872
21	137980.4556	70408.76245	52851.09033	1148.702888
22	43436.8092	22172.97955	16648.92175	644.4512603
23	-20802.68201	-10602.4303	-7950.532623	446.1745388
24	-3054.171875	-1547.15625	-1154.285156	171.2549467
25	8511.804688	4354.064453	3275.079102	285.1234277
26	3126.488281	1606.534668	1213.02832	172.604721
27	27699.36328	14142.81348	10620.96777	514.57241
28	4957.503418	2540.292725	1913.358154	217.4718895
29	-18497.30273	-9425.683533	-7066.671921	420.7241307
30	2236.235107	1151.987671	871.5078125	145.8479436
31	-7657.389404	-3895.594757	-2916.723663	270.831201
32	-744.5270386	-368.7886047	-269.9103088	85.1016511
33	-28170.62488	-14360.19159	-10769.02277	519.1306322
34	-15599.77368	-7946.871826	-5955.999634	386.3651375
35	22546.44934	11513.39545	8646.986847	464.2105868
36	-4344.895386	-2205.440094	-1647.9814	204.1161673
37	-8350.464844	-4248.809204	-3181.225983	282.7782712
38	7248.898621	3709.232178	2790.519592	263.0682549
39	-71853.90137	-36643.53253	-27488.28314	828.9542995
40	35942.74023	18346.2041	13772.625	586.1350594

k	x^k	y^k	z^k	ER
41	-49491.3125	-25236.04688	-18928.71875	687.9923639
42	-8358.564209	-4253.003906	-3184.433716	282.917008
43	-17508.35059	-8920.528076	-6686.641602	409.3030459
44	-1770.60791	-892.3032227	-662.7265625	130.5760362
45	13105.18512	6696.164505	5031.102707	353.8183509
46	5812.117188	2975.796875	2239.585938	235.4870442
47	-3570.03125	-1810.260742	-1351.555176	185.0840848
48	-12824.95313	-6531.421875	-4893.96875	350.3517493
49	-2217.886818	-1120.500622	-834.0068665	146.041072
50	1200.247803	623.1748657	474.329895	106.5819605
51	-12772.67529	-6504.764648	-4873.933472	349.6356412
52	1868.038239	963.8209839	729.943573	133.2076786
53	908.0369873	474.1076202	362.4768829	92.5579883
54	-2227.480421	-1125.387952	-837.6579407	146.3531811
55	916.4493103	478.4037018	365.7108154	92.9926987
56	-279.2244186	-131.5110283	-91.8655529	52.8624712
57	-289.5695835	-136.7862527	-95.8205802	53.7883729
58	-137.9596603	-59.461331	-37.8393735	38.0033454
59	104.1133611	64.0646158	54.9062766	36.6272274
60	383.2114445	206.2243893	161.3222194	59.6399306
61	27.817049	25.0285667	25.4851081	7.4416154
62	46.850554	34.7641122	32.8084276	7.2570965
63	-196.7681325	-89.2896398	-59.9968575	44.7392568
64	52.1259345	37.528614	34.9813076	9.1057668
65	-56.6416934	-16.5189258	-3.6371102	23.9042518
66	37.6306463	32.0196867	33.3891199	10.1170679
67	102.1625214	64.3014166	56.7676932	33.7302843
68	16.9279731	20.5207194	23.5012099	13.6319333
69	0.6611639	12.9421073	18.7839626	19.5621031
70	63.8733884	40.3168979	32.7650908	17.0929346
71	28.1461712	25.3429816	25.9107001	7.8490612
72	48.989499	34.7216325	31.2528131	9.8141253
73	39.5576191	27.9239991	23.4896991	9.810852
74	61.7806793	76.370128	109.6068092	45.8395703

k	x^k	y^k	z^k	ER
75	42.5548231	37.7872113	42.0604574	15.9926393
76	41.5221645	36.817806	40.7495137	15.3691712
77	41.3076142	36.6563484	40.6055002	15.3917902
78	35.9080842	35.6795907	36.3438249	10.2059885
79	44.6974498	41.6420597	36.0526454	5.6208322
80	46.0638602	43.4773237	39.6565403	3.0640382
81	45.7718852	43.7472487	41.522422	0.3753044
82	45.785372	43.7586113	41.552515	0.3357748
83	45.8988764	43.847172	41.7941064	0.003958
84	45.8976195	43.8466123	41.7958196	0.0005115
85	45.8978823	43.8468165	41.7957814	0.0000508
86	45.8978763	43.8468146	41.7957524	0.0000007
87	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 87 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	3.00	2.00	1.00	12.251052
0	3.00	2.00	4.00	6.5900442
0	5.00	2.00	4.00	6.0311568
0	7.00	1.00	3.00	5.7900338
1	23.3881432	106.4535718	94.2836727	35.380986
2	34.1598663	53.3019436	46.702158	21.875965
3	88.1268234	89.7281778	77.7952805	17.568645
4	39.5327036	33.6323912	29.9118874	5.5096856
5	45.0588445	51.9222262	47.1882001	10.715592
6	59.167104	43.3978378	40.1265267	10.775416
7	37.4028117	35.4156745	33.6878809	3.7639646
8	52.2912423	48.3397752	47.1878038	7.4429269
9	44.832489	42.8845362	41.1908135	0.6661559
10	47.7990244	45.3531167	43.0377754	0.8944408

k	x^k	y^k	z^k	ER
11	46.1152683	44.0507728	41.9688462	0.0934258
12	45.8695491	43.8217642	41.7725799	0.0148193
13	45.8966744	43.8457155	41.7946721	0.0007166
14	45.8978572	43.8467989	41.7957334	0.0000191
15	45.8978768	43.8468149	41.7957531	0.0000002
16	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 16 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	3.00	2.00	1.00	12.2510523
0	3.00	2.00	4.00	6.5900442
0	5.00	2.00	4.00	6.0311568
0	7.00	1.00	3.00	5.7900338
1	23.3881432	106.4535718	94.2836727	35.3809860
2	34.1598663	53.3019436	46.7021580	21.8759652
3	40.6655404	57.6930974	50.4504094	26.1355138
4	-1.0739368	26.0931324	25.2856695	24.3994158
5	1122.5085588	578.1893136	434.0810460	102.4301008
6	194.2291805	100.4760852	77.7167667	44.6454778
7	-987.6850766	-498.7457017	-367.2585448	97.2543480
8	122.0432389	68.9981204	55.4561049	42.4046637
9	130.8287571	73.6267139	58.9857284	39.6106939
10	-89.5312569	-22.7147039	-8.7733624	32.3943869
11	-29.8036461	-3.1516948	3.1981947	21.1028536
12	34.5746272	35.9224703	33.7709723	10.0424484
13	-67.0714993	-13.5476992	-2.6330774	28.1925616
14	-8.0643290	-5.8657627	0.3068148	11.0383525
15	55.9543446	32.0963412	28.5092257	14.3245613
16	63.1340952	32.1906687	29.0687142	16.8441550

k	x^k	y^k	z^k	ER
17	-2.8164563	17.6432537	31.1798763	30.9009371
18	57.7349196	58.6572203	55.0050643	11.5402806
19	42.0963427	37.9972293	36.1188116	2.2293369
20	43.8172728	41.2910114	39.3082806	1.1705176
21	44.2756318	41.6595408	39.6835210	0.9449635
22	45.6545878	43.5152355	41.4572023	0.1677763
23	45.9598924	43.9355942	41.8763001	0.0342247
24	45.8877115	43.8338839	41.7839896	0.0053945
25	45.9010929	43.8506756	41.7992633	0.0016731
26	45.8978721	43.8468094	41.7957480	0.0000023
27	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 27 iterations.

The iterations of table 4.3 at the initial vectors of B

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	3.00	2.00	1.00	12.251052
0	3.00	2.00	4.00	6.5900442
0	5.00	2.00	4.00	6.0311568
0	7.00	1.00	3.00	5.7900338
1	23.3881432	106.4535718	94.2836727	35.380986
2	34.1598663	53.3019436	46.702158	21.875965
3	40.6655404	57.6930974	50.4504094	26.135514
4	-1.0739368	26.0931324	25.2856695	24.399416
5	1122.508559	578.1893136	434.081046	102.4301
6	194.2291805	100.4760852	77.7167667	44.645478
7	-987.6850766	-498.7457017	-367.2585448	97.254348
8	122.0432389	68.9981204	55.4561049	42.404664

k	x^k	y^k	z^k	ER
9	130.8287571	73.6267139	58.9857284	39.610694
10	-89.5312569	-22.7147039	-8.7733624	32.394387
11	-29.8036461	-3.1516948	3.1981947	21.102854
12	34.5746272	35.9224703	33.7709723	10.042448
13	-67.0714993	-13.5476992	-2.6330774	28.192562
14	-8.064329	-5.8657627	0.3068148	11.038353
15	55.9543446	32.0963412	28.5092257	14.324561
16	63.1340952	32.1906687	29.0687142	16.844155
17	-2.8164563	17.6432537	31.1798763	30.900937
18	57.7349196	58.6572203	55.0050643	11.54028
19	42.0963427	37.9972293	36.1188116	2.229337
20	43.8172728	41.2910114	39.3082806	1.170518
21	4.2756318	41.6595408	39.683521	0.944964
22	45.6545878	43.5152355	41.4572023	0.167776
23	45.9598924	43.9355942	41.8763001	0.034225
24	45.8877115	43.8338839	41.7839896	0.005395
25	45.9010929	43.8506756	41.7992633	0.001673
26	45.8978721	43.8468094	41.795748	0.0000023
27	45.8978764	43.8468146	41.7957528	0.000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 27 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	3.00	2.00	1.00	12.2510523
0	3.00	2.00	4.00	6.5900442
0	5.00	2.00	4.00	6.0311568
0	7.00	1.00	3.00	5.7900338
1	23.3881432	106.4535718	94.2836727	35.380986
2	34.1598663	53.3019436	46.702158	21.8759652

k	x^k	y^k	z^k	ER
3	88.1268234	89.7281778	77.7952805	17.5686445
4	39.5327036	33.6323912	29.9118874	5.5096856
5	45.0588445	51.9222262	47.1882001	10.7155924
6	59.167104	43.3978378	40.1265267	10.7754158
7	37.4028117	35.4156745	33.6878809	3.7639646
8	52.2912423	48.3397752	47.1878038	7.4429269
9	44.832489	42.8845362	41.1908135	0.6661559
10	47.7990244	45.3531167	43.0377754	0.8944408
11	46.1152683	44.0507728	41.9688462	0.0934258
12	45.8695491	43.8217642	41.7725799	0.0148193
13	45.8966744	43.8457155	41.7946721	0.0007166
14	45.8978572	43.8467989	41.7957334	0.0000191
15	45.8978768	43.8468149	41.7957531	0.0000002
16	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 16 iterations.

The result from mid multivariate secant method.

k	x^k	y^k	z^k	ER
0	3.00	2.00	1.00	12.2510523
0	3.00	2.00	4.00	6.5900442
0	5.00	2.00	4.00	6.0311568
0	7.00	1.00	3.00	5.7900338
1	23.3881432	106.4535718	94.2836727	35.3809860
2	34.1598663	53.3019436	46.7021580	21.8759652
3	40.6655404	57.6930974	50.4504094	26.1355138
4	-1.0739368	26.0931324	25.2856695	24.3994158
5	1122.5085588	578.1893136	434.0810460	102.4301008
6	194.2291805	100.4760852	77.7167667	44.6454778
7	-987.6850766	-498.7457017	-367.2585448	97.2543480

k	x^k	y^k	z^k	ER
8	122.0432389	68.9981204	55.4561049	42.4046637
9	130.8287571	73.6267139	58.9857284	39.6106939
10	-89.5312569	-22.7147039	-8.7733624	32.3943869
11	-29.8036461	-3.1516948	3.1981947	21.1028536
12	34.5746272	35.9224703	33.7709723	10.0424484
13	-67.0714993	-13.5476992	-2.6330774	28.1925616
14	-8.0643290	-5.8657627	0.3068148	11.0383525
15	55.9543446	32.0963412	28.5092257	14.3245613
16	63.1340952	32.1906687	29.0687142	16.8441550
17	-2.8164563	17.6432537	31.1798763	30.9009371
18	57.7349196	58.6572203	55.0050643	11.5402806
19	42.0963427	37.9972293	36.1188116	2.2293369
20	43.8172728	41.2910114	39.3082806	1.1705176
21	44.2756318	41.6595408	39.6835210	0.9449635
22	45.6545878	43.5152355	41.4572023	0.1677763
23	45.9598924	43.9355942	41.8763001	0.0342247
24	45.8877115	43.8338839	41.7839896	0.0053945
25	45.9010929	43.8506756	41.7992633	0.0016731
26	45.8978721	43.8468094	41.7957480	0.0000023
27	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 27 iterations.

The iterations of table 4.3 at the initial vectors of C

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	8.00	5.00	15.63298
0	2.00	4.00	6.00	8.68736
0	1.00	2.00	6.00	8.5145605
0	5.00	4.00	1.00	14.272339

k	x^k	y^k	z^k	ER
1	63.8305424	51.4671835	6.231362	35.58201
2	56.2517145	47.7323754	15.6403735	27.286561
3	55.4419984	47.2755746	16.6839818	26.320596
4	42.9457993	40.3022677	49.3507668	29.304554
5	10.3553598	7.5341036	65.0861783	57.281441
6	29.5003576	28.1781928	41.4660025	26.80515
7	73.0547923	58.9332141	54.3319042	21.614315
8	99.8536519	90.7443517	53.7256117	39.596507
9	44.0287237	53.4955958	27.1131786	28.436222
10	68.24012	13.7949721	69.8346713	66.598915
11	40.4359763	49.7363949	49.2493746	23.432448
12	43.8358957	25.7038954	31.1952313	18.526476
13	46.5381784	38.5358242	33.5852019	7.4787955
14	50.0199465	41.215723	43.951645	18.197433
15	48.6896912	46.4593918	42.5072677	3.0565862
16	45.6347712	41.2468935	40.2633057	2.7298299
17	45.7013722	44.063945	42.4904407	1.1089339
18	45.9208025	43.8374237	41.8488817	0.1352828
19	45.8968204	43.8609834	41.8221708	0.0312279
20	45.8975654	43.8452106	41.7943768	0.0011801
21	45.8978264	43.8468121	41.795825	0.0001563
22	45.8978762	43.8468147	41.7957527	0.0000005
23	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 23 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	8.00	5.00	15.63298
0	5.00	4.00	1.00	14.272339
0	2.00	4.00	6.00	8.68736
0	1.00	2.00	6.00	8.5145605
1	63.8305424	51.4671835	6.231362	35.58201
2	56.2517145	47.7323754	15.6403735	27.286561
3	50.7203234	44.611845	22.7695453	20.3354
4	47.961687	43.2435045	28.8181038	14.534254
5	46.4914433	42.6536948	33.6295826	9.5184214
6	44.707786	42.6418171	44.153214	12.731903
7	47.5619767	45.6323682	40.2561557	4.8548175
8	44.050528	42.3237533	40.535089	0.7350716
9	47.7230082	45.6333505	43.5596932	1.1161447
10	45.8089676	43.7528629	41.6071567	0.2084528
11	45.8737073	43.8213953	41.792133	0.0390103
12	45.8921947	43.8416779	41.7909656	0.0029682
13	45.898098	43.8470269	41.7959609	0.0001264
14	45.8978762	43.8468145	41.7957522	0.0000009
15	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 15 iterations.

The result from mid multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	8.00	5.00	15.6329801
0	5.00	4.00	1.00	14.2723387
0	2.00	4.00	6.00	8.6873600
0	1.00	2.00	6.00	8.5145605
1	63.8305424	51.4671835	6.2313620	35.5820097

k	x^k	y^k	z^k	ER
2	56.2517145	47.7323754	15.6403735	27.2865613
3	34.1755781	35.2781385	44.0933550	24.3394845
4	76.6686591	67.6621585	41.8481869	26.0799971
5	51.9135111	47.2008181	34.9743082	12.5860898
6	61.2734825	52.6394937	14.8412524	30.3666235
7	36.1832072	35.9500161	53.9995522	36.6508538
8	44.1544993	41.8574702	43.8842967	13.6271650
9	47.8939474	44.2317158	39.6072074	4.4716617
10	46.3202631	43.0106017	41.4478206	1.6018061
11	45.8113813	42.5416623	42.2427577	4.5438295
12	50.4986234	50.7823045	32.8997063	16.6683393
13	46.0881779	43.6084850	41.0388935	1.0822156
14	46.1739287	44.0149540	40.7774649	2.0347899
15	45.6401757	44.1590247	41.8648266	0.8510312
16	46.1318974	43.7684733	41.2052251	1.0243101
17	45.8930856	43.8088473	41.8067519	0.0961993
18	45.9109323	43.9320347	41.7614655	0.2270497
19	45.8978546	43.8467485	41.7952386	0.0008591
20	45.8978830	43.8484452	41.8057309	0.0163218
21	45.8978780	43.8468121	41.7957304	0.0000388
22	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 22 iterations.

The iterations of table 4.3 at the initial vectors of D

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	2.00	4.00	6.00	8.68736
0	1.00	8.00	5.00	15.63298
0	1.00	2.00	6.00	8.5145605
0	5.00	4.00	1.00	14.272339
1	63.8305424	51.4671835	6.231362	35.58201
2	56.243757	47.728454	15.6502526	27.277578
3	55.4421555	47.2757192	16.6840709	26.320622
4	42.860438	40.2245423	49.3040629	29.08917
5	7.7702168	4.6136787	67.4769269	59.178004
6	29.5103737	28.192335	41.9846976	27.47894
7	79.0599046	65.510687	55.8930306	18.912891
8	93.693177	86.0574118	50.5860941	72.060173
9	104.1629519	78.1651693	67.8348628	25.381901
10	122.3749735	88.1855452	77.5007666	29.646372
11	15.3525883	24.7393976	21.3693548	21.239705
12	7.8906543	22.1669427	19.8179574	22.985096
13	192.4393844	108.7630472	100.4199088	48.015711
14	117.9579173	46.3760681	50.2674811	166.28815
15	106.6068586	68.8272432	60.7507088	31.71506
16	70.6834117	55.372656	47.2042809	19.86703
17	15.2353449	28.0109292	18.1043985	28.296178
18	85.4095723	62.7573247	52.6117658	29.340582
19	80.6230172	53.0480426	46.891859	26.858996
20	67.8069162	103.2867027	85.8331092	19.158172
21	79.4789904	71.1147972	71.0418514	27.642643
22	64.5143051	56.7830328	56.1073818	20.661134
23	13.8781161	16.826478	17.6869747	9.9471577
24	24.9891399	22.8947623	22.3871151	6.993155

k	x^k	y^k	z^k	ER
25	25.3467262	23.3073168	22.7461259	6.944782
26	41.4636467	39.0240094	37.5199467	1.814566
27	48.0462775	45.6967024	43.6708575	1.520173
28	47.7183226	45.4037933	43.3899098	1.306625
29	43.9132067	42.1156194	40.1536695	1.069801
30	45.427455	43.4156919	41.4204974	0.205192
31	45.8097808	43.7701523	41.7297491	0.038387
32	45.9402891	43.8863717	41.8300722	0.018824
33	45.8970479	43.8460861	41.7950876	0.000421
34	45.8981053	43.847018	41.7959369	0.000112
35	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 35 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	y^k	ER
0 0 0 0	1.00 5.00 2.00 1.00	8.00 4.00 4.00 2.00	5.00 1.00 6.00 6.00	15.63298 14.272339 8.68736 8.5145605
1	-8.1421161	-14.2057064	-10.3197795	17.1234
2	36.7827115	33.0576332	43.6448522	24.10893
3	67.3288497	65.8000461	59.7288373	13.941624
4	33.2402963	31.9077248	29.2980856	7.0613913
5	40.9174229	38.9817264	36.8842663	2.6875501
6	47.5977086	45.4085536	43.1524644	0.8079275
7	44.8598842	42.8412865	40.8273513	0.5414185
8	45.9178992	43.8646789	41.8114823	0.0090443
9	45.8969972	43.8459481	41.7949717	0.0004237
10	45.8977584	43.8467104	41.7956593	0.0000568

k	x^k	y^k	z^k	ER
11	45.8978775	43.8468155	41.7957536	0.0000005
12	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 12 iterations.

The result from mid multivariate secant method.

k	x^k	y^k	y^k	ER
0	1.00	8.00	5.00	15.6329801
0	5.00	4.00	1.00	14.2723387
0	2.00	4.00	6.00	8.6873600
0	1.00	2.00	6.00	8.5145605
1	-8.1421161	-14.2057064	-10.3197795	17.1234001
2	36.7827115	33.0576332	43.6448522	24.1089303
3	67.3288497	65.8000461	59.7288373	13.9416244
4	30.7692534	29.4509065	27.0921928	7.6389138
5	40.3889623	38.5717702	36.2796981	3.3054485
6	47.7849563	45.5851498	43.3335586	0.9332221
7	47.6806337	45.4870026	43.2413079	0.8717116
8	48.4253058	46.1799759	43.8562287	1.2851753
9	45.2796603	43.2793395	41.2958913	0.2742182
10	45.8352856	43.7913668	41.7464612	0.0292507
11	45.8750221	43.8264612	41.7775175	0.0108659
12	45.8886967	43.8386806	41.7885264	0.0042891
13	45.8979010	43.8468369	41.7957726	0.0000113
14	45.8978779	43.8468159	41.7957540	0.0000007
15	45.8978764	43.8468146	41.7957528	0.0000000

Estimates values are $x = 45.8978764$, $y = 43.8468146$ and $z = 41.7957528$ after 15 iteration.

The iterations of table 4.4 at the initial vectors of A

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	1.00	1.00	1.9132605
0	1.00	-1.00	2.00	2.8201401
0	2.00	1.00	2.00	1.7185799
0	1.00	0.00	1.00	1.0000000
1	1.7254820	0.0177785	1.2716996	0.9521400
2	2.2883038	-0.2709204	1.9335399	0.6287005
3	2.9957632	-0.2057793	2.2857351	0.6146735
4	3.3926851	0.0074428	2.2591061	0.2874808
5	3.0762749	-0.0001527	2.0911186	0.1163576
6	3.6291633	0.0431719	2.4376687	0.6248093
7	2.7478561	-0.0147725	2.0109366	0.0210584
8	2.6897320	-0.0016886	1.9911620	0.0128821
9	2.7199524	0.0000096	1.9996687	0.0021464
10	2.7184043	-0.0000108	1.9998672	0.0004166
11	2.7182813	0.0000000	2.0000004	0.0000014
12	2.7182818	0.0000000	2.0000001	0.0000002
13	2.7182818	0.0000000	2.0000000	0.0000000

Estimates values are $x = 2.7182818$, $y = 0.0000000$ and $z = 2.0000000$ after 13 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	-1.00	2.00	2.8201401
0	1.00	1.00	1.00	1.9132605
0	2.00	1.00	2.00	1.7185799
0	1.00	0.00	1.00	1.0000000
1	1.7254820	0.0177785	1.2716996	0.9521400

k	x^k	y^k	z^k	ER
2	2.0270876	-0.1369298	1.6263673	0.5209520
3	2.7572465	-0.1635607	2.1553078	0.4786127
4	3.1523607	-0.0296189	2.2244699	0.2545385
5	2.9029965	0.0007472	2.0376188	0.0745909
6	2.8686701	-0.0246098	2.0884990	0.1085946
7	2.6060540	0.0143159	1.9720593	0.0206461
8	2.7187757	-0.0003059	2.0005187	0.0010362
9	2.7182849	0.0000032	1.9999846	0.0000410
10	2.7182900	-0.0000004	2.0000022	0.0000022
11	2.7182825	0.0000000	2.0000002	0.0000002
12	2.7182818	0.0000000	2.0000000	0.0000000

Estimates values are $x = 2.7182818$, $y = 0.0000000$ and $z = 2.0000000$ after 12 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	-1.00	2.00	2.8201401
0	1.00	1.00	1.00	1.9132605
0	2.00	1.00	2.00	1.7185799
0	1.00	0.00	1.00	1.0000000
1	1.7254820	0.0177785	1.2716996	0.9521400
2	2.0270876	-0.1369298	1.6263673	0.5209520
3	2.7572465	-0.1635607	2.1553078	0.4786127
4	3.1523607	-0.0296189	2.2244699	0.2545385
5	2.9029965	0.0007472	2.0376188	0.0745909
6	2.8686701	-0.0246098	2.0884990	0.1085946
7	2.6060540	0.0143159	1.9720593	0.0206461
8	2.7187757	-0.0003059	2.0005187	0.0010362
9	2.7182849	0.0000032	1.9999846	0.0000410
10	2.7182900	-0.0000004	2.0000022	0.0000022

k	x^k	y^k	z^k	ER
11	2.7182825	0.0000000	2.0000002	0.0000002
12	2.7182818	0.0000000	2.0000000	0.0000000

Estimates values are $x = 2.7182818$, $y = 0.0000000$ and $z = 2.0000000$ after 12 iterations.

The iterations of table 4.4 at the initial vectors of B

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	-1.10	3.00	5.1388813
0	1.50	-1.00	0.00	3.6470713
0	2.50	1.00	0.00	5.7588134
0	3.00	2.00	3.00	8.0105169
1	1.7436480	0.1037902	1.4504749	0.5797248
2	1.8592062	0.5356728	1.7386570	0.5610865
3	1.8788756	1.6356810	1.9401146	4.1681100
4	2.0220193	-0.3269548	1.5344987	0.8184677
5	1.5725676	0.6086190	1.5600741	0.6533754
6	2.5345133	-0.2510106	1.9196423	0.5099404
7	12.5680019	-7.0343940	5.8499709	84.1184900
8	2.2412988	-0.0982778	1.7738446	0.3505926
9	2.2381094	-0.0508427	1.7897791	0.2741756
10	2.2229003	0.0217108	1.8079355	0.1658156
11	2.2236385	0.0339930	1.8121385	0.1471284
12	2.5044337	0.0451057	1.9397620	0.0240431
13	2.7886238	-0.0382760	2.0289316	0.0595371
14	2.3676255	0.0094777	1.8881412	0.1109933
15	2.7742535	-0.0040895	2.0180581	0.0110621
16	2.7190652	0.0004922	1.9989188	0.0035170
17	2.7208883	0.0000615	2.0001753	0.0018280

k	x^k	y^k	z^k	ER
18	2.7180092	0.0000038	1.9999554	0.0001271
19	2.7182835	0.0000000	2.0000003	0.0000007
20	2.7182818	0.0000000	2.0000000	0.0000000

Estimates values are $x = 2.7182818$, $y = 0.0000000$ and $z = 2.0000000$ after 20 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	3.00	2.00	3.00	8.0105169
0	2.50	1.00	0.00	5.7588134
0	1.00	-1.10	3.00	5.1388813
0	1.50	-1.00	0.00	3.6470713
1	1.7436480	0.1037902	1.4504749	0.5797248
2	1.9098405	0.7249114	1.8649302	0.9401762
3	1.8766911	0.5134086	1.7678363	0.5112341
4	2.1764628	0.3650038	1.8384834	0.4363790
5	2.1260077	0.2143845	1.7939578	0.1735263
6	2.4747827	-0.0599960	1.9236030	0.1903533
7	2.5480370	0.0005343	1.9418819	0.0590219
8	2.6918988	-0.0059891	1.9950894	0.0222498
9	2.6932274	0.0035609	1.9929171	0.0025484
10	2.7145152	-0.0027817	1.9972381	0.0076551
11	2.7180010	-0.0000183	2.0000553	0.0003719
12	2.7182857	0.0000005	2.0000024	0.0000037
13	2.7182819	0.0000000	2.0000000	0.0000000

Estimates values are $x = 2.7182819$, $y = 0.0000000$ and $z = 2.0000000$ after 13 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	3.00	2.00	3.00	8.0105169
0	2.50	1.00	0.00	5.7588134
0	1.00	-1.10	3.00	5.1388813
0	1.50	-1.00	0.00	3.6470713
1	1.7436480	0.1037902	1.4504749	0.5797248
2	1.9098405	0.7249114	1.8649302	0.9401762
3	1.8766911	0.5134086	1.7678363	0.5112341
4	2.1764628	0.3650038	1.8384834	0.4363790
5	2.1260077	0.2143845	1.7939578	0.1735263
6	2.4747827	-0.0599960	1.9236030	0.1903533
7	2.5480370	0.0005343	1.9418819	0.0590219
8	2.6918988	-0.0059891	1.9950894	0.0222498
9	2.6932274	0.0035609	1.9929171	0.0025484
10	2.7145152	-0.0027817	1.9972381	0.0076551
11	2.7180010	-0.0000183	2.0000553	0.0003719
12	2.7182857	0.0000005	2.0000024	0.0000037
13	2.7182819	0.0000000	2.0000000	0.0000000

Estimates values are $x = 2.7182819$, $y = 0.0000000$ and $z = 2.0000000$ after 13 iterations.

The iterations of table 4.4 at the initial vectors of C

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	1.50	1.00	3.5170527
0	3.00	2.00	1.00	9.5219263
0	1.00	4.00	3.00	49.9711643
0	2.00	1.00	1.00	3.1310991
1	1.3511566	1.4865573	0.7808885	4.4734729

k	x^k	y^k	z^k	ER
2	0.6861392	2.5101483	-0.1947358	13.0726365
3	1.8159828	0.9126070	1.0836798	2.5328665
4	1.9401570	0.4972941	1.4079503	1.1716782
5	2.0217227	0.3514350	1.5498775	0.7424085
6	2.1453214	0.1892256	1.7096085	0.3263701
7	2.0917610	0.1353320	1.7749786	0.1133832
8	1.5773400	0.3392125	1.5679104	0.3226488
9	2.3521995	0.0274239	1.8846066	0.0891620
10	2.3572204	0.0260349	1.8865418	0.0892839
11	3.0535898	-0.0413886	2.1220421	0.0641827
12	2.2081789	0.0446676	1.8311066	0.1257767
13	2.4163985	0.0082392	1.9296845	0.1147978
14	5.3227362	-0.0754792	2.6243599	0.3019224
15	3.0096772	-0.0056169	2.0584832	0.1033536
16	2.7530047	-0.0001648	2.0040514	0.0195414
17	2.7558989	-0.0002736	2.0052810	0.0192857
18	2.7328453	-0.0001560	2.0025725	0.0064794
19	2.7182467	-0.0000002	2.0000025	0.0000348
20	2.7182867	0.0000000	2.0000002	0.0000035
21	2.7182818	0.0000000	2.0000000	0.0000000

Estimates values are $x = 2.7182818$, $y = 0.0000000$ and $z = 2.0000000$ after 21 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	4.00	3.00	49.9711643
0	3.00	2.00	1.00	9.5219263
0	1.00	1.50	1.00	3.5170527
0	2.00	1.00	1.00	3.1310991
1	1.3511566	1.4865573	0.7808885	4.4734729

k	x^k	y^k	z^k	ER
2	1.9347119	0.5883524	1.6370024	0.8795290
3	1.9501958	0.5487492	1.6503681	0.7956923
4	2.1109981	0.1665709	1.7735404	0.1396728
5	2.0893998	0.1810141	1.7710453	0.1414453
6	2.1067553	0.1736402	1.7761281	0.1379458
7	2.4067397	0.0400970	1.8975739	0.0502241
8	2.5269422	0.0135679	1.9412071	0.0432526
9	2.7991825	-0.0102973	2.0295229	0.0153000
10	2.3212573	0.0538800	1.8508939	0.0871496
11	2.7428100	-0.0009097	2.0063034	0.0070170
12	2.7298452	-0.0004024	2.0028888	0.0034615
13	2.7287898	-0.0003712	2.0026427	0.0031185
14	2.7182192	0.0000005	1.9999913	0.0000327
15	2.7181607	0.0000008	1.9999835	0.0000638
16	2.7182818	0.0000000	2.0000000	0.0000000

Estimates values are $x = 2.7182818$, $y = 0.0000000$ and $z = 2.0000000$ after 16 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	4.00	3.00	49.9711643
0	3.00	2.00	1.00	9.5219263
0	1.00	1.50	1.00	3.5170527
0	2.00	1.00	1.00	3.1310991
1	1.3511566	1.4865573	0.7808885	4.4734729
2	1.9347119	0.5883524	1.6370024	0.8795290
3	1.9501958	0.5487492	1.6503681	0.7956923
4	2.1109981	0.1665709	1.7735404	0.1396728
5	2.0893998	0.1810141	1.7710453	0.1414453
6	2.1067553	0.1736402	1.7761281	0.1379458
7	2.4067397	0.0400970	1.8975739	0.0502241

k	x^k	y^k	z^k	ER
8	2.5269422	0.0135679	1.9412071	0.0432526
9	2.7991825	-0.0102973	2.0295229	0.0153000
10	2.3212573	0.0538800	1.8508939	0.0871496
11	2.7428100	-0.0009097	2.0063034	0.0070170
12	2.7298452	-0.0004024	2.0028888	0.0034615
13	2.7287898	-0.0003712	2.0026427	0.0031185
14	2.7182192	0.0000005	1.9999913	0.0000327
15	2.7181607	0.0000008	1.9999835	0.0000638
16	2.7182818	0.0000000	2.0000000	0.0000000

Estimates values are $x = 2.7182818$, $y = 0.0000000$ and $z = 2.0000000$ after 16 iterations.

The iterations of table 4.5 at the initial vectors of A

The result from multivariate secant method.

k	x^k	y^k	ER
0	0.40	0.60	0.6345680
0	0.50	1.20	11.1837160
0	0.05	1.50	22.9524324
1	0.3906385	0.6242453	0.5630385
2	0.3794180	0.6435164	0.5012710
3	0.3904237	0.6321898	0.5354834
4	0.3926777	0.7859716	0.3944590
5	0.4076187	0.7140357	0.1371020
6	0.4036816	0.7325008	0.0190743
7	0.4035828	0.7333228	0.0135239
8	0.4033772	0.7353062	0.0000164
9	0.4033784	0.7353078	0.0000041
10	0.4033783	0.7353084	0.0000000

Estimates values are $x = 0.4033783$ and $y = 0.7353084$ after 10 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	ER
0	0.05	1.50	22.9524324
0	0.50	1.20	11.1837160
0	0.40	0.60	0.6345680
1	0.3906385	0.6242453	0.5630385
2	0.5087156	0.4214498	0.9175099
3	0.3887071	0.7170133	0.1357868
4	0.4001892	0.7287171	0.0473900
5	0.4031768	0.7385051	0.0221328
6	0.4033631	0.7353656	0.0003895
7	0.4033780	0.7353096	0.0000078
8	0.4033783	0.7353084	0.0000000

Estimates values are $x = 0.403378$ and $y = 0.7353084$ after 8 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	ER
0	0.05	1.50	22.9524324
0	0.50	1.20	11.1837160
0	0.40	0.60	0.6345680
1	0.3906385	0.6242453	0.5630385
2	0.5087156	0.4214498	0.9175099
3	0.3887071	0.7170133	0.1357868
4	0.4006792	0.7292166	0.0435591
5	0.4031849	0.7384482	0.0217378
6	0.4033661	0.7353533	0.0003057
7	0.4033780	0.7353094	0.0000062
8	0.4033783	0.7353084	0.0000000

Estimates values are $x = 0.4033783$ and $y = 0.7353084$ after 8 iterations.

The iterations of table 4.5 at the initial vectors of B

The result from multivariate secant method.

k	x^k	y^k	ER
0	1.00	3.00	1214.3146215
0	0.00	2.00	94.7628619
0	0.50	1.00	3.9375000
1	0.5151440	0.9667809	3.1869532
2	0.5500050	0.8924682	1.8201143
3	0.5266873	0.9243437	2.3480014
4	0.3628844	0.8596266	1.1461961
5	0.3253343	0.7952338	0.4578779
6	0.7082320	0.5941564	1.9643676
7	0.3575580	0.7483158	0.1530072
8	0.3829036	0.7372535	0.0659757
9	0.3826192	0.7372965	0.0668614
10	0.4039165	0.7351670	0.0019273
11	0.4032867	0.7353281	0.0003179
12	0.4033790	0.7353083	0.0000026
13	0.4033783	0.7353084	0.0000000

Estimates values are $x = 0.4033783$ and $y = 0.7353084$ after 13 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	ER
0	1.00	3.00	1214.3146215
0	0.00	2.00	94.7628619
0	0.50	1.00	3.9375000
1	0.5151440	0.9667809	3.1869532
2	0.5500050	0.8924682	1.8201143
3	0.5266873	0.9243437	2.3480014
4	0.3628844	0.8596266	1.1461961

k	x^k	y^k	ER
5	0.3286666	0.8009483	0.5037446
6	0.6735273	0.6171477	1.6513478
7	0.3605352	0.7495596	0.1502623
8	0.3856825	0.7377060	0.0576031
9	0.3896849	0.7367883	0.0446871
10	0.4033821	0.7352456	0.0004312
11	0.4033595	0.7353124	0.0000649
12	0.4033787	0.7353083	0.0000015
13	0.4033783	0.7353084	0.0000000

Estimates values are $x = 0.4033783$ and $y = 0.7353084$ after 13 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	ER
0	1.00	3.00	1214.3146215
0	0.00	2.00	94.7628619
0	0.50	1.00	3.9375000
1	0.5151440	0.9667809	3.1869532
2	0.5500050	0.8924682	1.8201143
3	0.5266873	0.9243437	2.3480014
4	0.3628844	0.8596266	1.1461961
5	0.3253343	0.7952338	0.4578779
6	0.7082320	0.5941564	1.9643676
7	0.3575580	0.7372535	0.0659757
9	0.3826192	0.7372965	0.0668614
10	0.4039165	0.7351670	0.0019273
11	0.4032867	0.7353281	0.0003179
12	0.4033790	0.7353083	0.0000026
13	0.4033783	0.7353084	0.0000000

Estimates values are $x = 0.4033783$ and $y = 0.7353084$ after 13 iterations.

The iterations of table 4.5 at the initial vectors of C

The result from multivariate secant method.

k	x^k	y^k	ER
0	0.50	1.00	3.9375000
0	1.00	0.60	5.7320590
0	2.00	3.00	1750.8592176
1	1.1315075	0.5089156	8.4582204
2	0.6355153	0.8492571	1.2935444
3	2.1387396	-0.1378536	55.3421459
4	0.5009753	0.9452978	2.7232072
5	0.9090452	0.6464691	4.1299593
6	-0.6691821	1.8668610	26.7707757
7	0.6814758	0.8079869	1.2120292
8	0.6775695	0.8063076	1.1962187
9	0.3729527	0.6779859	0.3619271
10	0.3756831	0.6761859	0.3668996
11	0.3885602	0.7438212	0.0701613
12	0.4041488	0.7323561	0.0199633
13	0.4032038	0.7356329	0.0022255
14	0.4033818	0.7353065	0.0000165
15	0.4033783	0.7353084	0.0000005
16	0.4033783	0.7353084	0.0000000

Estimates values are $x = 0.4033783$ and $y = 0.7353084$ after 16 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	ER
0	2.00	3.00	1750.859218
0	1.00	0.60	5.732059
0	0.50	1.00	3.9375000

k	x^k	y^k	ER
1	1.1315075	0.5089156	8.4582204
2	0.7295159	0.7847555	1.5152061
3	0.7374634	0.7824665	1.5776545
4	0.9361782	0.9091996	1.6768347
5	1.1897176	1.2239017	11.1643291
6	0.9393323	0.9732387	0.8388388
7	0.9981002	0.9393992	1.7859562
8	0.9787589	0.9783803	0.4759756
9	0.9884929	0.9878182	0.2793486
10	1.0011639	1.0011532	0.0278077
11	0.9998139	0.999834	0.0040092
12	0.9999886	0.9999894	0.0002554
13	1.0000000	1.0000000	0.0000005
14	1.0000000	1.0000000	0.0000000

Estimates values are $x = 1.000000$and $y = 1.0000000$ after 14 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	ER
0 0 0	1.00 1.50 1.50	2.00 1.00 0.50	181.1215062 29.6875000 23.5835036
1	1.4415957	0.8182208	23.5120915
2	0.1132927	2.9754433	750.0381281
3	1.3250912	1.2732314	19.3509309
4	1.3752011	1.0118822	18.0009298
5	1.3458195	1.0564464	15.4803168
6	1.0176564	1.1531101	6.7675652
7	1.0349972	1.0103582	0.7100829
9	1.0120890	1.0015352	0.2634546
10	1.0006947	0.9995999	0.0279850

k	x^k	y^k	ER
11	0.9997755	0.9999141	0.0039104
12	0.9999901	0.9999985	0.0002051
13	1.0000000	1.0000000	0.0000001
14	1.0000000	1.0000000	0.0000000

Estimates values are $x = 1.000000$ and $y = 1.0000000$ after 14 iterations.

The iterations of table 4.6 at the initial vectors of A

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	0.03	4.00	4.00	1.2810544
0	0.05	10.00	5.00	10.9405912
0	0.05	5.00	5.00	6.3568098
0	1.00	10.00	20.00	50.5287781
1	0.0264596	5.3069910	3.8230634	0.8731733
2	0.0274244	4.5093907	3.8712677	0.4074767
3	0.0272411	4.7570053	3.8621488	0.0325239
4	0.0272140	4.7468531	3.8608169	0.0078496
5	0.0265059	2.1318489	3.8470591	7.1301477
6	0.0273981	3.3847309	3.8458167	2.9177494
7	0.0290808	2.5125767	3.7074129	5.2269911
8	0.0250558	4.6097570	4.0257359	0.6633843
9	0.0252679	4.6734511	4.0209511	0.4997373
10	0.0250683	5.1959128	4.0560021	0.3886884
11	0.0251147	4.9830096	4.0424824	0.0313065
12	0.0251260	4.9634836	4.0409816	0.0008239
13	0.0251252	4.9640711	4.0410483	0.0000690
14	0.0251253	4.9640321	4.0410463	0.0000036
15	0.0251253	4.9640298	4.0410461	0.0000000

Estimates values are $x = 0.0251253$, $y = 4.9640298$ and $z = 4.0410461$ after 15 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	10.00	20.00	50.5287781
0	0.05	10.00	5.00	10.9405912
0	0.05	5.00	5.00	6.3568098
0	0.03	4.00	4.00	1.2810544
1	0.0297722	4.8568968	3.9887954	0.9859636
2	0.0275265	4.7712947	3.87646	0.1436636
3	0.0270115	4.740412	3.850715	0.0730556
4	0.026899	4.7422807	3.8446974	0.1115219
5	0.0268344	4.7346812	3.8496044	0.1278467
6	0.0248572	4.990602	4.0622708	0.0104138
7	0.0251076	4.9677892	4.0440851	0.0043552
8	0.0251238	4.9644761	4.0414068	0.0006834
9	0.0251253	4.964028	4.0410446	0.0000038
10	0.0251253	4.9640297	4.041046	0.0000001

Estimates values are $x = 0.0251253$, $y = 4.9640298$ and $z = 4.0410461$ after 10 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	10.00	20.00	50.5287781
0	0.05	10.00	5.00	10.9405912
0	0.05	5.00	5.00	6.3568098
0	0.03	4.00	4.00	1.2810544
1	0.0297722	4.8568968	3.9887954	0.9859636
2	0.0275265	4.7712947	3.87646	0.1436636
3	0.0270115	4.740412	3.850715	0.0730556
4	0.026899	4.7422807	3.8446974	0.1115219
5	0.0268344	4.7346812	3.8496044	0.1278467
6	0.0248572	4.990602	4.0622708	0.0104138

k	x^k	y^k	z^k	ER
7	0.0251076	4.9677892	4.0440851	0.0043552
8	0.0251238	4.9644761	4.0414068	0.0006834
9	0.0251253	4.964028	4.0410446	0.0000038
10	0.0251253	4.9640297	4.041046	0.0000001

Estimates values are $x = 0.0251253$, $y = 4.9640297$ and $z = 4.041046$ after 10 iterations.

The iterations of table 4.6 at the initial vectors of B

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	0.10	2.00	5.00	8.8320432
0	0.20	4.00	6.00	16.2251088
0	0.01	3.00	3.00	11.5438869
0	1.00	8.00	10.00	39.0989906
1	0.1037927	4.3326544	4.1795402	9.4557684
2	0.0347417	4.0399858	3.0528339	0.8530829
3	0.0447180	3.9833903	3.2536137	1.3739516
4	0.0307323	3.8159481	3.0657639	1.9136918
5	0.0379676	3.8944348	3.1661715	0.0920028
6	0.0361316	3.8660353	3.1445110	0.5299096
7	0.0362236	3.8652279	3.1443658	0.5134853
8	0.0311000	4.2405696	3.4493372	0.4457116
9	0.0258879	4.7530683	3.8687238	0.3268879
10	0.0410072	1.9629478	1.5920047	8.0398150
11	0.0251091	4.8815449	3.9735345	0.2124784
12	0.0250560	4.9030618	3.9911009	0.1729224
13	0.0250645	4.9061558	3.9936380	0.1626913
14	0.0251223	4.9581282	4.0362121	0.0155995
15	0.0251251	4.9640609	4.0410715	0.0000392

k	x^k	y^k	z^k	ER
16	0.0251253	4.9640275	4.0410442	0.0000035
17	0.0251253	4.9640334	4.0410490	0.0000056
18	0.0251253	4.9640298	4.0410461	0.0000000

Estimates values are $x = 0.0251253$, $y = 4.9640298$ and $z = 4.0410461$after 18 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	8.00	10.00	39.0989906
0	0.20	4.00	6.00	16.2251088
0	0.01	3.00	3.00	11.5438869
0	0.10	2.00	5.00	8.8320432
1	0.1074984	3.1561328	4.6962672	9.1973135
2	0.0497764	4.1280708	3.2920696	2.4283379
3	0.0405298	3.9711384	3.1848117	0.5351844
4	0.0378293	3.9086030	3.1599305	0.1154022
5	0.0408512	3.9753647	3.1920997	0.6096497
6	0.0361964	3.9105575	3.1857092	0.3658483
7	0.0242624	4.8192072	3.9151178	0.6263072
8	0.0220995	5.2559169	4.2755913	0.2048260
9	0.0239845	5.2057916	4.2376032	0.2595166
10	0.0243867	5.1273922	4.1737462	0.1892783
11	0.0250966	4.9626293	4.0398419	0.0116604
12	0.0251315	4.9629556	4.0401752	0.0009118
13	0.0251252	4.9640708	4.0410794	0.0000854
14	0.0251253	4.9640123	4.0410318	0.0000374
15	0.0251253	4.9640298	4.0410461	0.0000000

Estimates values are $x = 0.0251253$, $y = 4.9640298$ and $z = 4.0410461$ after 15 iterations.

The result from mid modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	1.00	8.00	10.00	39.0989906
0	0.20	4.00	6.00	16.2251088
0	0.01	3.00	3.00	11.5438869
0	0.10	2.00	5.00	8.8320432
1	0.1074984	3.1561328	4.6962672	9.1973135
2	0.0497764	4.1280708	3.2920696	2.4283379
3	0.0405298	3.9711384	3.1848117	0.5351844
4	0.0378293	3.9086030	3.1599305	0.1154022
5	0.0408512	3.9753647	3.1920997	0.6096497
6	0.0361964	3.9105575	3.1857092	0.3658483
7	0.0242624	4.8192072	3.9151178	0.6263072
8	0.0220995	5.2559169	4.2755913	0.2048260
9	0.0239845	5.2057916	4.2376032	0.2595166
10	0.0243867	5.1273922	4.1737462	0.1892783
11	0.0250966	4.9626293	4.0398419	0.0116604
12	0.0251315	4.9629556	4.0401752	0.0009118
13	0.0251252	4.9640708	4.0410794	0.0000854
14	0.0251253	4.9640123	4.0410318	0.0000374
15	0.0251253	4.9640298	4.0410461	0.0000000

Estimates values are $x = 0.0251253$, $y = 4.9640298$ and $z = 4.0410461$ after 15 iterations.

The iterations of table 4.6 at the initial vectors of C

The result from multivariate secant method.

k	x^k	y^k	z^k	ER
0	0.04	4.75	3.42	2.1235353
0	0.02	2.50	2.40	9.1380319
0	0.20	2.20	1.45	3.5420785
0	0.03	3.65	1.45	8.2804919

k	x^k	y^k	z^k	ER
1	0.0342720	4.3136726	3.3561112	0.3316119
2	0.0327800	4.2578366	3.4069905	0.1641958
3	0.0320450	4.2783829	3.4604394	0.1641605
4	0.0253262	4.8477501	4.0237836	0.1675961
5	0.0246983	5.0178577	4.1004044	0.0427782
6	0.0247353	5.0145541	4.0963210	0.0419622
7	0.0253621	4.9115540	3.9961105	0.0686742
8	0.0251263	4.9639699	4.0410167	0.0001801
9	0.0251252	4.9640425	4.0410584	0.0000115
10	0.0251253	4.9640298	4.0410460	0.0000001
11	0.0251253	4.9640298	4.0410461	0.0000000

Estimates values are $x = 0.0251253$, $y = 4.9640298$ and $z = 4.0410461$ after 11 iterations.

The result from modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	0.02	2.50	2.40	9.1380319
0	0.03	3.65	1.45	8.2804919
0	0.20	2.20	1.45	3.5420785
0	0.04	4.75	3.42	2.1235353
1	0.0341365	4.2784589	3.0683275	0.9071959
2	0.0322192	4.2381289	3.2876043	0.5792211
3	0.0315877	4.3277488	3.5039986	0.1173578
4	0.0287190	4.5193979	3.7453527	0.1527087
5	0.0253977	4.8848578	4.0040475	0.0907357
6	0.0249401	4.9908539	4.0632539	0.0149457
7	0.0250931	4.9706595	4.0468437	0.0080918
8	0.0251360	4.9613267	4.0389986	0.0034979
9	0.0251253	4.9640222	4.0410427	0.0000118
10	0.0251253	4.9640298	4.0410461	0.0000001

Estimates values are $x = 0.0251253$, $y = 4.9640298$ and $z = 4.0410461$ after 10 iterations

The result from mid modified multivariate secant method.

k	x^k	y^k	z^k	ER
0	0.02	2.50	2.40	9.1380319
0	0.03	3.65	1.45	8.2804919
0	0.20	2.20	1.45	3.5420785
0	0.04	4.75	3.42	2.1235353
1	0.0341365	4.2784589	3.0683275	0.9071959
2	0.0322192	4.2381289	3.2876043	0.5792211
3	0.0315877	4.3277488	3.5039986	0.1173578
4	0.0287190	4.5193979	3.7453527	0.1527087
5	0.0253977	4.8848578	4.0040475	0.0907357
6	0.0249401	4.9908539	4.0632539	0.0149457
7	0.0250931	4.9706595	4.0468437	0.0080918
8	0.0251360	4.9613267	4.0389986	0.0034979
9	0.0251253	4.9640222	4.0410427	0.0000118
10	0.0251253	4.9640298	4.0410461	0.0000001

Estimates values are $x = 0.0251253$, $y = 4.9640298$ and $z = 4.0410461$ after 10 iterations.

The iterations of table 4.7 at the initial vectors of A

The result from multivariate secant method.

k	w^k	x^k	y^k	z^k	ER
0	10900.0000	0.0005	1.0075	4.0000	139772.5006
0	10871.0000	0.0003	1.002	3.0000	139796.3055
0	10800.0000	0.0004	1.03	3.5000	139904.7207
0	10753.0000	0.0003	1.005	4.0000	139961.1735
0	10725.0000	0.0002	1.000	3.2000	139992.1629

k	w^k	x^k	y^k	z^k	ER
1	111492.28398	0.0152309	0.9928891	3.9734182	1355.811934
2	109488.93560	-0.00179	1.00991	4.0074599	184.4828545
3	109736.74910	0.0005087	1.0076113	4.0028627	3.7317981
4	109737.49338	0.0005561	1.0075639	4.0027677	0.7305723
5	109737.37882	0.0005488	1.0075712	4.0027823	0.0436693
6	109737.37490	0.0005486	1.0075714	4.0027828	0.0201849
7	109737.37154	0.0005484	1.0075716	4.0027833	0.0000212
8	109737.37154	0.0005484	1.0075716	4.0027833	0.0000083
9	109737.37154	0.0005484	1.0075716	4.0027833	0.0000003
10	109737.37154	0.0005484	1.0075716	4.0027833	0.0000000

Estimates values are w =109737.37154, $x = 0.0005484$, $y = 1.0075716$ and $z = 4.0027833$ after 10 iterations.

The result from modified multivariate secant method.

k	w^k	x^k	y^k	z^k	ER
0	10725.0000	0.0002	1.0000	3.2000	139992.1629
0	10753.0000	0.0003	1.0050	4.0000	139961.1735
0	10800.0000	0.0004	1.0300	3.5000	139904.7207
0	10871.0000	0.0003	1.0020	3.0000	139796.3055
0	10900.0000	0.0005	1.0075	4.0000	139772.5006
1	111492.28398	0.0152309	0.9928891	3.9734182	1355.811931
2	109735.43966	0.0003044	1.0078156	4.0032712	25.0879079
3	109737.30655	0.0005442	1.0075758	4.0027916	0.3892297
4	109737.37148	0.0005484	1.0075716	4.0027833	0.0003690
5	109737.37154	0.0005484	1.0075716	4.0027833	0.0000000

Estimates values are w =109737.37154, $x = 0.0005484$, $y = 1.0075716$ and $z = 4.0027833$ after 5 iterations.

The result from mid modified multivariate secant method.

k	w^k	x^k	y^k	z^k	ER
0	10725.0000	0.0002	1.0000	3.2000	139992.1629000
0	10753.0000	0.0003	1.0050	4.0000	139961.1735300
0	10800.0000	0.0004	1.0300	3.5000	139904.7207400
0	10871.0000	0.0003	1.0020	3.0000	139796.3054800
0	10900.0000	0.0005	1.0075	4.0000	139772.5005600
1	111492.2839800	0.0152309	0.9928891	3.9734182	1355.8119312
2	109735.4396600	0.0003044	1.0078156	4.0032712	25.0879079
3	109737.3065500	0.0005442	1.0075758	4.0027916	0.3892297
4	109737.3714800	0.0005484	1.0075716	4.0027833	0.0003690
5	109737.3715400	0.0005484	1.0075716	4.0027833	0.0000000

Estimates values are w =109737.37154, $x = 0.0005484$, $y = 1.0075716$ and $z = 4.0027833$ after 5 iterations.

The iterations of table 4.7 at the initial vectors of B

The result from multivariate secant method.

k	w^k	x^k	y^k	z^k	ER
0	1073.000	0.002	1.000	3.200	153825.4076
0	1080.000	0.004	1.025	3.500	154003.3174
0	1087.000	0.003	1.015	3.000	153908.8472
0	1075.000	0.003	1.040	4.000	153900.9227
0	1090.000	0.005	1.007	4.000	154080.0181
1	119205.65361	-0.1519294	1.1600494	4.3077388	27370.50798
2	109878.87047	-0.0145068	1.0226268	4.0328936	1841.49125
3	109777.17948	0.0032855	1.0048345	3.9973089	261.5156972
4	109738.57885	0.0006344	1.0074856	4.0026112	8.2445529
5	109737.99366	0.0005927	1.0075273	4.0026946	4.2467409
6	109737.33786	0.0005459	1.0075741	4.0027881	0.2319582
7	109737.36985	0.0005483	1.0075717	4.0027835	0.0101383

k	w^k	x^k	y^k	z^k	ER
8	109737.37152	0.0005484	1.0075716	4.0027833	0.0000948
9	109737.37155	0.0005484	1.0075716	4.0027833	0.0000586
10	109737.37152	0.0005484	1.0075716	4.0027833	0.0000156
11	109737.37152	0.0005484	1.0075716	4.0027833	0.0000154
12	109737.37153	0.0005484	1.0075716	4.0027833	0.0000031
13	109737.37154	0.0005484	1.0075716	4.0027833	0.000001
14	109737.37154	0.0005484	1.0075716	4.0027833	0.0000001
15	109737.37154	0.0005484	1.0075716	4.0027833	0.0000001
16	109737.37154	0.0005484	1.0075716	4.0027833	0.0000000

Estimates values are $w = 109737.37154$, $x = 0.0005484$, $y = 1.0075716$ and $z = 4.0027833$ after 16 iterations.

The result from modified multivariate secant method.

k	w^k	x^k	y^k	z^k	ER
0	1090	0.005	1.007	4.0	154080.0181
0	1080	0.004	1.025	3.5	154003.3174
0	1087	0.003	1.015	3.0	153908.8472
0	1075	0.003	1.040	4.0	153900.9227
0	1073	0.002	1.000	3.2	153825.4076
1	119205.65361	-0.1519294	1.1600494	4.3077388	27370.50797
2	109896.51567	-0.0147668	1.0228868	4.0334136	1891.958865
3	109777.93217	0.0033265	1.0047935	3.9972271	265.2612901
4	109738.69492	0.0006425	1.0074775	4.002595	9.0160025
5	109737.32474	0.000545	1.007575	4.00279	0.3221451
6	109737.37141	0.0005484	1.0075716	4.0027833	0.0007625
7	109737.37157	0.0005484	1.0075716	4.0027833	0.0001763
8	109737.37157	0.0005484	1.0075716	4.0027833	0.0001761
9	109737.37153	0.0005484	1.0075716	4.0027833	0.0000333
10	109737.37154	0.0005484	1.0075716	4.0027833	0.0000001
11	109737.37154	0.0005484	1.0075716	4.0027833	0.0000000

Estimates values are w =109737.37154, $x = 0.0005484$, $y = 1.0075716$ and $z = 4.0027833$ after 11 iterations.

The result from mid modified multivariate secant method.

k	w^k	x^k	y^k	z^k	ER
0	1090.0000	0.0050	1.0070	4.0000	154080.0180800
0	1080.0000	0.0040	1.0250	3.5000	154003.3174400
0	1087.0000	0.0030	1.0150	3.0000	153908.8471800
0	1075.0000	0.0030	1.0400	4.0000	153900.9226800
0	1073.0000	0.0020	1.0000	3.2000	153825.4076300
1	119205.6536100	-0.1519294	1.1600494	4.3077388	27370.5079730
2	109896.5156700	-0.0147668	1.0228868	4.0334136	1891.9588652
3	109777.9321700	0.0033265	1.0047935	3.9972271	265.2612901
4	109738.6949200	0.0006425	1.0074775	4.0025950	9.0160025
5	109737.3247400	0.0005450	1.0075750	4.0027900	0.3221451
6	109737.3714100	0.0005484	1.0075716	4.0027833	0.0007625
7	109737.3715700	0.0005484	1.0075716	4.0027833	0.0001763
8	109737.3715700	0.0005484	1.0075716	4.0027833	0.0001761
9	109737.3715300	0.0005484	1.0075716	4.0027833	0.0000333
10	109737.3715400	0.0005484	1.0075716	4.0027833	0.0000001
11	109737.3715400	0.0005484	1.0075716	4.0027833	0.0000000

Estimates values are w =109737.37154, $x = 0.0005484$, $y = 1.0075716$ and $z = 4.0027833$ after 11 iterations.

The iterations of table 4.7 at the initial vectors of C

The result from multivariate secant method.

k	w^k	x^k	y^k	z^k	ER
0	102200.0000	0.0035	1.2000	3.0500	10922.35738
0	106200.0000	0.0050	1.2000	3.0250	5403.611724
0	103500.0000	0.0015	1.2000	3.0050	8903.848571

k	w^k	x^k	y^k	z^k	ER
0	104200.0000	0.0020	1.5000	3.5000	7925.976072
0	103000.0000	0.0025	1.6000	3.0500	9660.052535
1	109692.35447	-0.0002987	1.0084187	4.0044774	51.9555576
2	109747.75991	0.0007344	1.0073856	4.0024112	11.2035949
3	109737.36510	0.0005481	1.0075719	4.0027838	0.0207519
4	109737.37159	0.0005484	1.0075716	4.0027833	0.0003194
5	109737.37155	0.0005484	1.0075716	4.0027833	0.0000854
6	109737.37159	0.0005484	1.0075716	4.0027833	0.0003160
7	109737.37154	0.0005484	1.0075716	4.0027833	0.0000001
8	109737.37154	0.0005484	1.0075716	4.0027833	0.0000001
9	109737.37154	0.0005484	1.0075716	4.0027833	0.0000003
10	109737.37154	0.0005484	1.0075716	4.0027833	0.0000002
11	109737.37154	0.0005484	1.0075716	4.0027833	0.0000000

Estimates values are w =109737.37154, $x = 0.0005484$, $y = 1.0075716$ and $z = 4.0027833$ after 11 iterations.

The result from modified multivariate secant method.

k	w^k	x^k	y^k	z^k	ER
0	102200	0.0035	1.2000	3.0500	10922.35738
0	103000	0.0025	1.6000	3.0500	9660.052535
0	103500	0.0015	1.2000	3.0050	8903.848571
0	104200	0.002	1.5000	3.5000	7925.976072
0	106200	0.005	1.2000	3.0250	5403.611724
1	109692.35457	-0.0002987	1.0084187	4.0044774	51.9555578
2	109747.75991	0.0007344	1.0073856	4.0024112	11.203595
3	109737.35837	0.000548	1.007572	4.002784	0.0274219
4	109737.37169	0.0005484	1.0075716	4.0027833	0.0003168
5	109737.37154	0.0005484	1.0075716	4.0027833	0.0000055
6	109737.37154	0.0005484	1.0075716	4.0027833	0.0000059
7	109737.37154	0.0005484	1.0075716	4.0027833	0.0000000

Estimates values are w =109737.37154, $x = 0.0005484$, $y = 1.0075716$ and $z = 4.0027833$ after 7 iterations.

The result from mid modified multivariate secant method.

k	w^k	x^k	y^k	z^k	ER
0	102200.0000	0.0035	1.2000	3.0500	10922.3573840
0	103000.0000	0.0025	1.6000	3.0500	9660.0525352
0	103500.0000	0.0015	1.2000	3.0050	8903.8485713
0	104200.0000	0.0020	1.5000	3.5000	7925.9760716
0	106200.0000	0.0050	1.2000	3.0250	5403.6117244
1	109692.3544700	-0.0002987	1.0084187	4.0044774	51.9555578
2	109747.7599100	0.0007344	1.0073856	4.0024112	11.2035950
3	109737.3582700	0.0005480	1.0075720	4.0027840	0.0274219
4	109737.3715900	0.0005484	1.0075716	4.0027833	0.0003168
5	109737.3715400	0.0005484	1.0075716	4.0027833	0.0000055
6	109737.3715400	0.0005484	1.0075716	4.0027833	0.0000059
7	109737.3715400	0.0005484	1.0075716	4.0027833	0.0000000

Estimates values are w =109737.37154, $x = 0.0005484$, $y = 1.0075716$ and $z = 4.0027833$ after 7 iterations.

The iterations of table 4.8 at the initial vectors of A

The result from multivariate secant method.

k	λ^k	x^k	y^k	z^k	ER
0	35.02	3.00	0.25	0.20	101.9715732
0	38.50	3.40	0.05	0.31	135.717557
0	30.00	4.30	0.60	0.40	104.6756825
0	33.10	3.60	0.85	0.45	72.0403307
0	38.01	4.30	0.75	0.49	120.5544957
1	52.0293769	2.8611366	2.4811962	0.7791258	140.1964202

k	λ^k	x^k	y^k	z^k	ER
2	11.6389745	5.164294	-0.0408106	0.0473247	61.8040371
3	17.5689429	4.8342706	0.622196	0.2282533	71.6663803
4	27.6698808	4.2587612	1.1199392	0.3758875	64.1818976
5	29.1872968	4.1670117	1.1833968	0.3969875	60.4912464
6	30.0636803	4.1155161	1.2212853	0.4094787	57.9540019
7	47.8837756	3.0839237	1.9962834	0.6653606	49.537077
8	25.2518466	4.4498037	1.0492954	0.3499456	67.5596414
9	38.3617704	3.6596552	1.5983708	0.5327916	20.1919045
10	38.7170115	3.6378747	1.6131486	0.5377201	18.1200187
11	39.8130749	3.5711102	1.6588462	0.5529507	11.4725814
12	41.6380277	3.4598997	1.7349166	0.5783056	0.4701914
13	41.5657261	3.4643152	1.7319054	0.5773018	0.0238315
14	41.5691083	3.4641084	1.7320462	0.5773487	0.0007577
15	41.5691409	3.4641064	1.7320475	0.5773492	0.0005356
16	41.5692313	3.4641009	1.7320513	0.5773504	0.0000815
17	41.5692194	3.4641016	1.7320508	0.5773503	0.0000005
18	41.5692194	3.4641016	1.7320508	0.5773503	0.0000000

Estimates values are $\lambda = 41.5692194$, $x = 3.4641016$, $y = 1.7320508$ and $z = 0.5773503$ after 18 iterations.

The result from modified multivariate secant method.

k	λ^k	x^k	y^k	z^k	ER
0	38.50	3.40	0.05	0.31	135.717557
0	38.01	4.30	0.75	0.49	120.5544957
0	30.00	4.30	0.60	0.40	104.6756825
0	35.02	3.0	0.25	0.20	101.9715732
0	33.10	3.6	0.85	0.45	72.0403307
1	52.0293769	2.8611366	2.4811962	0.7791258	140.1964202
2	25.2041966	4.3907726	0.806212	0.293102	81.9600600
3	49.2085074	3.0046239	2.4081667	0.7719359	126.4127897

k	λ^k	x^k	y^k	z^k	ER
4	35.2651772	3.8033839	1.3992245	0.4672329	43.2451392
5	51.2135823	2.8931515	2.0883806	0.6934997	70.9344474
6	42.6068096	3.4088188	1.8130752	0.6039532	13.0107360
7	40.5901735	3.5205687	1.6886875	0.5628948	6.7507841
8	41.5305279	3.4669622	1.7306284	0.5768971	0.2555987
9	41.5195963	3.4670002	1.7298704	0.5766249	0.3488758
10	41.5708126	3.4640078	1.7321189	0.577373	0.0109673
11	41.5689627	3.4641169	1.7320400	0.5773467	0.0017574
12	41.5692382	3.4641005	1.7320516	0.5773505	0.0001287
13	41.5692194	3.4641016	1.7320508	0.5773503	0.0000000

Estimates values are $\lambda = 41.5692194$, $x = 3.4641016$, $y = 1.7320508$ and $z = 0.5773503$ after 13iterations.

The result from mid modified multivariate secant method.

k	λ^k	x^k	y^k	z^k	ER
0	38.50	3.40	0.05	0.31	135.717557
0	38.01	4.30	0.75	0.49	120.5544957
0	30.00	4.30	0.60	0.40	104.6756825
0	35.02	3.00	0.25	0.20	101.9715732
0	33.10	3.60	0.85	0.45	72.0403307
1	52.0293769	2.8611366	2.4811962	0.7791258	140.1964202
2	25.2041966	4.3907726	0.8062120	0.2931020	81.9600600
3	34.8285767	3.8350048	1.4485058	0.4850875	37.0106634
4	36.0654145	3.7631349	1.4837192	0.4963150	33.2601369
5	34.4910631	3.8512240	1.4088222	0.4717268	41.4513722
6	40.2731087	3.5312473	1.6698575	0.5571346	9.1403314
7	40.5270574	3.5271665	1.6883947	0.5628261	6.9371620
8	41.6677725	3.4584445	1.7362817	0.5787556	0.6753658
9	41.6157543	3.4613541	1.7340275	0.5780070	0.3188744
10	41.5684900	3.4641462	1.7320204	0.5773401	0.0049759

k	λ^k	x^k	y^k	z^k	ER
11	41.5690597	3.4641113	1.7320441	0.5773480	0.0010900
12	41.5692196	3.4641016	1.7320508	0.5773503	0.0000018
13	41.5692194	3.4641016	1.7320508	0.5773503	0.0000000

Estimates values are $\lambda = 41.5692194$, $x = 3.4641016$, $y = 1.7320508$ and $z = 0.5773503$ after 13 iterations.

The iterations of table 4.8 at the initial vectors of B

The result from multivariate secant method.

k	λ^k	x^k	y^k	z^k	ER
0	29.0	2.5	2.0	0.4	49.4134607
0	33.0	2.0	1.5	1.0	153.6757951
0	32.0	1.5	2.0	1.0	241.7293166
0	28.0	3.0	0.5	0.4	65.8511777
0	28.0	3.0	0.5	0.4	65.8511777
1	44.1489847	-0.2530566	-0.2530566	0.5651414	233.2261354
2	93.5757964	-19.3935160	3.8905865	-0.0459648	1886.7610051
3	22.3797444	-1.5659412	0.0562900	-0.8288846	57.2566680
4	23.3648093	-1.9073520	-0.0313287	-0.9401180	84.4836862
5	39.3448230	1.4285175	1.0182523	0.0622199	66.8385575
6	41.8266324	0.1613569	0.4749110	-0.3347850	50.2238319
7	42.5636141	2.1941030	1.5587268	0.3092644	51.5446394
8	38.6744726	2.2095244	1.6227288	0.3031503	44.8589210
9	47.4177140	0.7826781	1.0309729	-0.0667988	76.0894221
10	62.6359896	2.0271858	0.7339886	0.1578527	119.1564942
11	25.5113787	4.7478805	2.2683527	0.8294372	176.0068730
12	642.5102237	-24.0836320	-12.5256023	-4.2650740	26335.8766010
13	39.5110508	3.9231818	1.8466934	0.6675705	32.4119252
14	42.5384886	3.2519330	1.7110883	0.5371712	12.0473212
15	42.7893330	3.4348397	1.7217097	0.5725061	5.8858255

k	λ^k	x^k	y^k	z^k	ER
16	43.0121711	3.4431078	1.7177449	0.5741288	6.7591495
17	43.1995073	3.4223555	1.7157170	0.5705485	7.9979227
18	40.4271311	3.5575198	1.7515151	0.5930542	7.9403848
19	41.7922080	3.4603345	1.7293472	0.5767222	1.1012400
20	41.6163226	3.4633910	1.7314615	0.5772323	0.2323894
21	41.5693776	3.4640997	1.7320480	0.5773500	0.0008292
22	41.5692229	3.4641016	1.7320508	0.5773503	0.0000161
23	41.5692194	3.4641016	1.7320508	0.5773503	0.0000002
24	41.5692194	3.4641016	1.7320508	0.5773503	0.0000000

Estimates values are $\lambda = 41.5692194$, $x = 3.4641016$, $y = 1.7320508$ and $z = 0.5773503$ after 24 iterations.

The result from modified multivariate secant method.

k	λ^k	x^k	y^k	z^k	ER
0	32.0	1.5	2.0	1.0	241.7293166
0	33.0	2.0	1.5	1.0	153.6757951
0	28.0	3.0	0.5	0.4	65.8511777
0	28.0	3.0	0.5	0.4	65.8511777
0	29.0	2.5	2.0	0.4	49.4134607
1	44.1489847	-0.2530566	-0.2530566	0.5651414	233.2261354
2	29.6457886	5.3632845	1.9225133	0.7444569	104.2434020
3	19.6071566	6.8864179	2.4445644	1.1730331	381.2974556
4	29.8454422	4.4749486	1.9254412	0.7651624	105.0683158
5	24.8874919	4.8819192	1.9409429	0.8420617	154.9045839
6	29.6503251	4.4276330	1.8714371	0.7536582	98.8905513
7	25.0519572	4.8605334	1.9083712	0.8342215	148.9745109
8	38.8828484	3.5855277	1.7740495	0.5942025	14.9331434
9	40.4644194	3.5855186	1.7414363	0.5991013	8.8212735
10	41.7562329	3.3605855	1.7410863	0.5587968	5.2758172
11	41.6814689	3.4656013	1.7277090	0.5773767	0.8157249

k	λ^k	x^k	y^k	z^k	ER
12	41.5917062	3.4647943	1.7314036	0.5774792	0.1316252
13	41.5699234	3.4640249	1.7320393	0.5773377	0.0052985
14	41.5690024	3.4641329	1.7320549	0.5773553	0.0018900
15	41.5692195	3.4641018	1.7320508	0.5773503	0.0000079
16	41.5692194	3.4641016	1.7320508	0.5773503	0.0000000

Estimates values are $\lambda = 41.5692194$, $x = 3.4641016$, $y = 1.7320508$ and $z = 0.5773503$ after 16 iterations.

The result from mid multivariate secant method.

k	λ^k	x^k	y^k	z^k	ER
0	32.00	1.50	2.00	1.00	241.7293166
0	33.00	2.00	1.50	1.00	153.6757951
0	28.00	3.00	0.50	0.40	65.8511777
0	30.00	1.00	1.00	0.00	52.6972485
0	29.00	2.50	2.00	0.40	49.4134607
1	44.1489847	-0.2530566	2.3689914	0.5651414	233.2261354
2	29.6457886	5.3632845	1.9225133	0.7444569	104.2434020
3	-6.7720733	10.8888643	3.8163955	2.2992334	1670.5363677
4	34.7711939	3.4524429	1.6719733	0.5222704	10.5905585
5	36.4230276	3.6858185	1.7381699	0.5877507	20.0790498
6	37.7786521	3.6381448	1.7783600	0.5985678	19.8752455
7	39.4298915	3.5994942	1.7787130	0.6018750	14.8983734
8	40.5029118	3.5230606	1.7516512	0.5868936	6.5365859
9	41.5004483	3.4650219	1.7324469	0.5773274	0.2620500
10	41.5532802	3.4643611	1.7321968	0.5773912	0.0751227
11	41.5668595	3.4641575	1.7320810	0.5773592	0.0119590
12	41.5688043	3.4641126	1.7320562	0.5773519	0.0020967
13	41.5692186	3.4641016	1.7320508	0.5773503	0.0000040
14	41.5692194	3.4641016	1.7320508	0.5773503	0.0000000

Estimates values are $\lambda = 41.5692194$, $x = 3.4641016$, $y = 1.7320508$ and $z = 0.5773503$ after 16 iterations.

The iterations of table 4.8 at the initial vectors of C

The result from multivariate secant method.

k	λ^k	x^k	y^k	z^k	ER
0	27.00	1.50	1.00	0.50	38.4585816
0	35.00	1.00	1.00	1.00	113.4372073
0	22.00	2.00	1.50	0.30	31.3403510
0	25.00	2.50	1.50	0.45	37.5276445
0	30.00	1.00	0.50	1.00	54.7083175
1	18.1911633	1.9359805	1.3819714	0.5415571	74.9748787
2	45.0399335	3.9697885	1.6894900	0.5745040	39.6195178
3	0.3744935	-1.1408969	2.5152707	0.3593577	132.1236944
4	34.3289473	2.8784532	1.9229352	0.4920317	40.8912705
5	37.2367935	3.1114923	1.7657797	0.5486186	24.2617336
6	42.0960390	3.5022906	1.7362663	0.5763894	3.3610575
7	39.3225492	3.2805251	1.7704693	0.5566609	13.6446292
8	42.0175092	3.5012180	1.7199898	0.5826634	3.0759802
9	41.5864634	3.4655265	1.7316957	0.5775006	0.1169866
10	41.5718030	3.4643174	1.7319391	0.5773987	0.0189665
11	41.5693241	3.4641103	1.7320476	0.5773516	0.0007291
12	41.5692230	3.4641019	1.7320507	0.5773503	0.0000253
13	41.5692194	3.4641016	1.7320508	0.5773503	0.0000000

Estimates values are $\lambda = 41.5692194$, $x = 3.4641016$, $y = 1.7320508$ and $z = 0.5773503$ after 13 iterations.

The result from modified multivariate secant method.

k	λ^k	x^k	y^k	z^k	ER
0	35.00	1.00	1.00	1.00	113.4372073
0	30.00	1.00	0.50	1.00	54.7083175
0	27.00	1.50	1.00	0.50	38.4585816
0	25.00	2.50	1.50	0.45	37.5276445

k	λ^k	x^k	y^k	z^k	ER
0	22.00	2.00	1.50	0.30	31.3403510
1	18.1911633	1.9359805	1.3819714	0.5415571	74.9748787
2	30.5764449	2.8741719	1.5238291	0.5567554	36.7474320
3	37.6644612	3.1489796	1.8246056	0.4882125	17.6379511
4	40.1908743	3.2980676	1.8430270	0.5011136	15.3715443
5	41.6158805	3.4571973	1.7005472	0.5892190	2.7844432
6	41.6478790	3.4719739	1.7188705	0.5843211	1.5293393
7	41.5497936	3.4623056	1.7323073	0.5771077	0.1142583
8	41.5569470	3.4629951	1.7321762	0.5771980	0.0677787
9	41.5860310	3.4656039	1.7318576	0.5775631	0.0935072
10	41.5692193	3.4641016	1.7320510	0.5773502	0.0000151
11	41.5692193	3.4641016	1.7320508	0.5773503	0.0000002
12	41.5692194	3.4641016	1.7320508	0.5773503	0.0000000

Estimates values are $\lambda = 41.5692194$, $x = 3.4641016$, $y = 1.7320508$ and $z = 0.5773503$ after 12 iterations.

The result from mid modified multivariate secant method.

k	λ^k	x^k	y^k	z^k	ER
0	35.00	1.00	1.00	1.00	113.4372073
0	30.00	1.00	0.50	1.00	54.7083175
0	27.00	1.50	1.00	0.50	38.4585816
0	25.00	2.50	1.50	0.45	37.5276445
0	22.00	2.00	1.50	0.30	31.3403510
1	18.1911633	1.9359805	1.3819714	0.5415571	74.9748787
2	30.5764449	2.8741719	1.5238291	0.5567554	36.7474320
3	39.0235661	3.2016731	1.8822786	0.4750697	20.1769769
4	42.9023951	3.5222276	1.7439614	0.5650567	9.7454197
5	41.4261965	3.4542602	1.7182553	0.5875132	3.0116267
6	41.7424672	3.4774727	1.7347059	0.5760927	1.2898931
7	41.5513333	3.4626675	1.7320985	0.5772174	0.0953676

k	λ^k	x^k	y^k	z^k	ER
8	41.5690137	3.4640842	1.7320468	0.5773501	0.0010841
9	41.5697552	3.4641468	1.7320661	0.577349	0.0029408
10	41.5692223	3.4641019	1.7320509	0.5773503	0.0000160
11	41.5692194	3.4641016	1.7320508	0.5773503	0.0000000

Estimates values are $\lambda = 41.5692194$, $x = 3.4641016$, $y = 1.7320508$ and $z = 0.5773503$ after 11 iterations.

The iterations of table 4.9 at the initial vectors of A

The result from multivariate secant method.

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	0.50	0.50	0.50	0.50	1.00	6.4578358
0	0.40	0.80	0.63	0.15	1.50	5.5260837
0	0.80	1.10	0.45	0.20	1.70	2.8482445
0	0.40	1.20	1.00	0.10	1.30	5.5504628
0	0.35	0.50	1.00	0.50	1.00	6.8201150
0	1.50	2.00	1.00	1.00	2.00	7.8145131
1	0.9883986	0.7363378	0.8145528	0.2832113	2.3183622	1.4809611
2	1.0565268	0.8502888	0.6049266	0.2507006	2.1123621	0.2904906
3	1.0082294	0.8696901	0.5771588	0.1889538	2.2016337	0.2365989
4	0.9921614	0.8444250	0.5731279	0.2150507	2.2594938	0.1058162
5	1.0012003	0.8361182	0.6060135	0.2314706	2.2277802	0.1009999
6	1.0133783	0.8448529	0.6050460	0.2287992	2.1953705	0.0573655
7	1.0103061	0.8503858	0.5943234	0.2190132	2.2031205	0.0314806
8	1.0052744	0.8477141	0.5913584	0.2192455	2.2175943	0.0281688
9	1.0053919	0.8449622	0.5958331	0.2230205	2.2170540	0.0104644
10	1.0076529	0.8455834	0.5977188	0.2237633	2.2107238	0.0130072
11	1.0079769	0.8468738	0.5962374	0.2222388	2.2098016	0.0034661
12	1.0070981	0.8468482	0.5951418	0.2216676	2.2122929	0.0055728

k	α^k	β^k	γ^k	δ^k	x^k	ER
13	1.0067935	0.8462999	0.5955758	0.2221975	2.2131394	0.0016311
14	1.0071118	0.8462102	0.5961151	0.2225396	2.2122400	0.0022369
15	1.0072982	0.8464259	0.5960318	0.2223770	2.2117161	0.0009176
16	1.0071982	0.8465035	0.5957879	0.2222029	2.2120003	0.0008343
17	1.0071009	0.8464266	0.5957785	0.2222395	2.2122737	0.0004957
18	1.0071254	0.8463796	0.5958798	0.2223199	2.2122038	0.0002874
19	1.0071710	0.8464036	0.5959024	0.2223191	2.2120753	0.0002469
20	1.0071690	0.8464278	0.5958637	0.2222850	2.2120813	0.0000919
21	1.0071494	0.8464221	0.5958471	0.2222791	2.2121367	0.0001139
22	1.0071466	0.8464108	0.5958604	0.2222924	2.2121444	0.0000308
23	1.0071544	0.8464112	0.5958699	0.2222974	2.2121223	0.0000491
24	1.0071570	0.8464160	0.5958660	0.2222926	2.2121150	0.0000142
25	1.0071542	0.8464167	0.5958613	0.2222897	2.2121230	0.0000197
26	1.0071525	0.8464148	0.5958620	0.2222911	2.2121276	0.0000080
27	1.0071534	0.8464142	0.5958642	0.2222926	2.2121250	0.0000074
28	1.0071543	0.8464149	0.5958642	0.2222923	2.2121226	0.0000043
29	1.0071541	0.8464153	0.5958633	0.2222916	2.2121233	0.0000026
30	1.0071537	0.8464151	0.5958632	0.2222916	2.2121244	0.0000022
31	1.0071537	0.8464148	0.5958635	0.2222919	2.2121243	0.0000008
32	1.0071539	0.8464149	0.5958636	0.2222920	2.2121239	0.0000010
33	1.0071539	0.8464150	0.5958635	0.2222918	2.2121238	0.0000003
34	1.0071538	0.8464150	0.5958634	0.2222918	2.2121240	0.0000004
35	1.0071538	0.8464149	0.5958635	0.2222918	2.2121240	0.0000001
36	1.0071538	0.8464149	0.5958635	0.2222919	2.2121240	0.0000002
37	1.0071538	0.8464150	0.5958635	0.2222919	2.2121239	0.0000001

Estimates values are $\alpha = 1.0071538$, $\beta = 0.8464150$, $\gamma = 0.5958635$, $\delta = 0.2222919$ and $x = 2.2121239$ after 37 iterations.

The result from modified multivariate secant method.

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	1.50	2.00	1.00	1.00	2.00	7.8145131
0	0.35	0.50	1.00	0.50	1.00	6.8201150
0	0.50	0.50	0.50	0.50	1.00	6.4578358
0	0.40	1.20	1.00	0.10	1.30	5.5504628
0	0.40	0.80	0.63	0.15	1.50	5.5260837
0	0.80	1.10	0.45	0.20	1.70	2.8482445
1	0.9883986	0.7363378	0.8145528	0.2832113	2.3183622	1.4809611
2	1.1939181	1.0800888	0.1821823	0.1851378	1.6969305	2.5223519
3	1.0450493	0.8972149	0.4891865	0.1878266	2.1423462	0.7255058
4	1.0193842	0.8605525	0.5638415	0.2127495	2.1937510	0.2198338
5	1.0058360	0.8458620	0.5992346	0.2229138	2.2124335	0.0215593
6	1.0072165	0.8464856	0.5961417	0.2224575	2.2115092	0.0011946
7	1.0072179	0.8464764	0.5961172	0.2224428	2.2115545	0.0010771
8	1.0071748	0.8464334	0.5958888	0.2222958	2.2120508	0.0000890
9	1.0071580	0.8464188	0.5958705	0.2222952	2.2121025	0.0000193
10	1.0071542	0.8464153	0.5958642	0.2222922	2.2121218	0.0000019
11	1.0071538	0.8464150	0.5958635	0.2222919	2.2121240	0.0000000

Estimates values are $\alpha = 1.0071538$, $\beta = 0.8464150$, $\gamma = 0.5958635$, $\delta = 0.2222919$ and $x = 2.2121240$ after 11 iterations.

The result from mid modified multivariate secant method.

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	1.50	2.00	1.00	1.00	2.00	7.8145131
0	0.35	0.50	1.00	0.50	1.00	6.8201150
0	0.50	0.50	0.50	0.50	1.00	6.4578358
0	0.40	1.20	1.00	0.10	1.30	5.5504628
0	0.40	0.80	0.63	0.15	1.50	5.5260837
0	0.80	1.10	0.45	0.20	1.70	2.8482445
1	0.9883986	0.7363378	0.8145528	0.2832113	2.3183622	1.4809611

k	α^k	β^k	γ^k	δ^k	x^k	ER
2	1.1939181	1.0800888	0.1821823	0.1851378	1.6969305	2.5223519
3	1.0229576	0.8700769	0.5347450	0.1882256	2.2084446	0.4149202
4	1.0242922	0.8743566	0.5239638	0.1773168	2.2134322	0.4942767
5	1.0034265	0.8435547	0.6066211	0.2258843	2.2130170	0.0698820
6	1.0073952	0.8463578	0.5957494	0.2224473	2.2117017	0.0020345
7	1.0072106	0.8464939	0.5958107	0.2222076	2.2120108	0.0007503
8	1.0071915	0.8464642	0.5958388	0.2222494	2.2120263	0.0004132
9	1.0071630	0.8464195	0.5958612	0.2222921	2.2121018	0.0000554
10	1.0071500	0.8464133	0.5958643	0.2222915	2.2121325	0.0000229
11	1.0071538	0.8464149	0.5958635	0.2222919	2.2121240	0.0000003
12	1.0071538	0.8464150	0.5958635	0.2222918	2.2121240	0.0000000

Estimates values are $\alpha = 1.0071538$, $\beta = 0.8464150$, $\gamma = 0.5958635$, $\delta = 0.2222918$ and $x = 2.2121240$ after 9 iterations.

The iterations of table 4.9 at the initial vectors of B

The result from multivariate secant method.

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	0.50	0.50	0.50	0.50	2.00	5.0802871
0	0.60	0.98	0.63	0.15	2.00	3.5860277
0	0.80	1.00	0.45	0.20	2.20	2.1465076
0	0.40	1.00	1.00	0.10	2.00	5.2679085
0	0.35	0.50	1.00	0.50	1.50	6.3760863
0	1.50	2.00	1.00	1.00	2.00	7.8145131
1	0.8940353	0.6834760	0.6909786	0.2785635	2.3569161	1.3863723
2	1.3823037	1.4011573	0.4169787	0.1533756	1.5191779	3.2457894
3	0.9092118	0.6923691	0.6134555	0.2139780	2.4685876	0.9156974
4	1.0988670	1.0255087	0.5065261	0.2046659	2.1422113	1.3728326
5	0.9498762	0.7615652	0.6202219	0.2280016	2.3097371	0.5985270
6	1.0578931	0.9310182	0.5620815	0.2137543	2.1413411	0.6076158

k	α^k	β^k	γ^k	δ^k	x^k	ER
7	0.9749918	0.7958137	0.6114100	0.2255865	2.2684719	0.3499747
8	1.0331055	0.8895609	0.5792776	0.2183317	2.1740986	0.3081293
9	0.9895906	0.8184742	0.6051087	0.2243708	2.2413744	0.1955182
10	1.0206393	0.8685300	0.5876861	0.2203559	2.1915449	0.1570077
11	0.9977185	0.8312773	0.6010340	0.2234725	2.2275109	0.1063094
12	1.0142149	0.8579269	0.5916924	0.2213132	2.2011423	0.0814893
13	1.0021233	0.8383091	0.5986791	0.2229401	2.2202255	0.0570512
14	1.0108679	0.8524506	0.5937010	0.2217869	2.2062908	0.0426576
15	1.0044821	0.8421000	0.5973753	0.2226413	2.2163980	0.0304041
16	1.0091123	0.8495923	0.5947319	0.2220283	2.2090323	0.0224379
17	1.0057377	0.8441251	0.5966694	0.2224785	2.2143812	0.0161448
18	1.0081880	0.8480913	0.5952684	0.2221535	2.2104870	0.0118325
19	1.0064040	0.8452017	0.5962915	0.2223911	2.2133169	0.0085567
20	1.0077003	0.8473004	0.5955497	0.2222189	2.2112577	0.0062483
21	1.0067570	0.8457727	0.5960904	0.2223445	2.2127546	0.0045305
22	1.0074427	0.8468829	0.5956978	0.2222534	2.2116657	0.0033019
23	1.0069439	0.8460751	0.5959836	0.2223197	2.2124574	0.0023975
24	1.0073066	0.8466623	0.5957759	0.2222715	2.2118816	0.0017455
25	1.0070428	0.8462352	0.5959271	0.2223066	2.2123003	0.0012684
26	1.0072346	0.8465458	0.5958172	0.2222811	2.2119958	0.0009230
27	1.0070951	0.8463199	0.5958971	0.2222997	2.2122172	0.0006709
28	1.0071965	0.8464841	0.5958390	0.2222862	2.2120562	0.0004881
29	1.0071228	0.8463647	0.5958813	0.2222960	2.2121733	0.0003549
30	1.0071764	0.8464515	0.5958506	0.2222888	2.2120881	0.0002581
31	1.0071374	0.8463884	0.5958729	0.2222940	2.2121501	0.0001877
32	1.0071658	0.8464343	0.5958567	0.2222903	2.2121050	0.0001365
33	1.0071451	0.8464009	0.5958685	0.2222930	2.2121378	0.0000993
34	1.0071601	0.8464252	0.5958599	0.2222910	2.2121139	0.0000722
35	1.0071492	0.8464075	0.5958661	0.2222925	2.2121313	0.0000525
36	1.0071572	0.8464204	0.5958616	0.2222914	2.2121187	0.0000382
37	1.0071514	0.8464110	0.5958649	0.2222922	2.2121278	0.0000278
38	1.0071556	0.8464178	0.5958625	0.2222916	2.2121212	0.0000202

k	α^k	β^k	γ^k	δ^k	x^k	ER
39	1.0071525	0.8464129	0.5958642	0.2222920	2.2121260	0.0000147
40	1.0071548	0.8464165	0.5958630	0.2222917	2.2121225	0.0000107
41	1.0071531	0.8464139	0.5958639	0.2222919	2.2121250	0.0000078
42	1.0071543	0.8464158	0.5958632	0.2222918	2.2121232	0.0000056
43	1.0071535	0.8464144	0.5958637	0.2222919	2.2121245	0.0000041
44	1.0071541	0.8464154	0.5958633	0.2222918	2.2121236	0.0000030
45	1.0071536	0.8464146	0.5958636	0.2222919	2.2121243	0.0000022
46	1.0071540	0.8464152	0.5958634	0.2222918	2.2121237	0.0000016
47	1.0071537	0.8464148	0.5958636	0.2222919	2.2121241	0.0000011
48	1.0071539	0.8464151	0.5958635	0.2222918	2.2121239	0.0000008
49	1.0071538	0.8464149	0.5958635	0.2222919	2.2121241	0.0000006
50	1.0071539	0.8464150	0.5958635	0.2222918	2.2121239	0.0000004
51	1.0071538	0.8464149	0.5958635	0.2222919	2.2121240	0.0000003
52	1.0071538	0.8464150	0.5958635	0.2222918	2.2121239	0.0000002
53	1.0071538	0.8464149	0.5958635	0.2222919	2.2121240	0.0000002
54	1.0071538	0.8464150	0.5958635	0.2222918	2.2121240	0.0000001
55	1.0071538	0.8464149	0.5958635	0.2222919	2.2121240	0.0000001

Estimates values are $\alpha = 1.0071538$, $\beta = 0.8464149$, $\gamma = 0.5958635$, $\delta = 0.2222919$ and $x = 2.2121240$ after 55 iterations.

The result from modified multivariate secant method.

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	1.50	2.00	1.00	1.00	2.00	7.8145131
0	0.35	0.50	1.00	0.50	1.50	6.3760863
0	0.40	1.00	1.00	0.10	2.00	5.2679085
0	0.50	0.50	0.50	0.50	2.00	5.0802871
0	0.60	0.98	0.63	0.15	2.00	3.5860277
0	0.80	1.00	0.45	0.20	2.20	2.1465076
1	0.8940353	0.6834760	0.6909786	0.2785635	2.3569161	1.3863723
2	1.0013862	0.8412658	0.6307369	0.2510396	2.1727306	0.2321070

k	α^k	β^k	γ^k	δ^k	x^k	ER
3	0.9919189	0.8322157	0.6415333	0.2613807	2.1818166	0.3467264
4	1.0112117	0.8492041	0.5952338	0.2213819	2.2083756	0.0320421
5	1.0070010	0.8462678	0.5954483	0.2220602	2.2133812	0.0016510
6	1.0070462	0.8463295	0.5957418	0.2222337	2.2125941	0.0003183
7	1.0071391	0.8464040	0.5958532	0.2222879	2.2121715	0.0000501
8	1.0071538	0.8464149	0.5958634	0.2222918	2.2121242	0.0000002
9	1.0071538	0.8464150	0.5958635	0.2222919	2.2121240	0.0000000

Estimates values are $\alpha = 1.0071538$, $\beta = 0.8464150$, $\gamma = 0.5958635$, $\delta = 0.2222919$ and $x = 2.2121240$ after 9 iterations.

The result from mid modified multivariate secant method.

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	1.50	2.00	1.00	1.00	2.00	7.8145131
0	0.35	0.50	1.00	0.50	1.50	6.3760863
0	0.40	1.00	1.00	0.10	2.00	5.2679085
0	0.50	0.50	0.50	0.50	2.00	5.0802871
0	0.60	0.98	0.63	0.15	2.00	3.5860277
0	0.80	1.00	0.45	0.20	2.20	2.1465076
1	0.8940353	0.6834760	0.6909786	0.2785635	2.3569161	1.3863723
2	1.0013862	0.8412658	0.6307369	0.2510396	2.1727306	0.2321070
3	1.0199854	0.8590451	0.6095266	0.2307240	2.1548806	0.0344304
4	1.0136642	0.8514922	0.5916566	0.2180491	2.2075673	0.0641119
5	1.0059773	0.8454953	0.5959597	0.2226423	2.2141384	0.0081370
6	1.0069871	0.8462860	0.5958508	0.2223218	2.2124832	0.0009788
7	1.0071478	0.8464102	0.5958639	0.2222938	2.2121362	0.0000394
8	1.0071534	0.8464146	0.5958635	0.2222920	2.2121248	0.0000030
9	1.0071538	0.8464150	0.5958635	0.2222919	2.2121240	0.0000000

Estimates values are $\alpha = 1.0071538$, $\beta = 0.8464150$, $\gamma = 0.5958635$, $\delta = 0.2222919$ and $x = 2.2121240$ after 9 iterations.

The iterations of table 4.9 at the initial vectors of C

The result from multivariate secant method.

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	1.00	1.00	1.00	1.00	1.50	2.6589038
0	2.00	1.00	1.00	0.50	2.00	7.0426686
0	0.50	1.00	0.50	0.10	1.00	5.6064746
0	1.50	0.60	0.40	0.20	1.50	3.8584102
0	0.75	0.50	0.30	0.50	1.00	5.5913096
0	0.65	0.60	0.40	0.15	1.00	5.7191859
1	1.6704649	0.6224276	-0.3323411	0.5328270	0.4360257	5.0219145
2	0.9226391	0.9040003	1.1190995	0.0805760	2.2097323	2.8774917
3	0.8272192	0.7667725	0.3288387	0.9743701	2.7834332	4.2340586
4	0.9860218	0.8915709	0.7191547	-0.3363672	2.2565269	2.5685167
5	0.6721122	-0.1718414	-0.5819194	2.6991145	4.8042259	11.4421994
6	1.4312325	1.0879114	0.5751908	0.3973353	-0.0364863	3.8253836
7	0.9148943	0.8179601	0.5714457	0.0591121	2.2192001	1.0825166
8	0.9971901	0.7808677	0.5243395	0.3791701	2.4157186	0.9287584
9	0.9959271	0.8807789	0.6373055	0.0604698	2.1914217	0.7950415
10	1.0008579	0.7789104	0.5191844	0.4233092	2.3405761	1.0967161
11	0.9980707	0.8901329	0.6500686	0.0294819	2.1737491	0.9449824
12	0.9985048	0.7599157	0.4978835	0.4789228	2.3765376	1.4131310
13	0.9943480	0.8981733	0.6613565	-0.0167540	2.1740088	1.1640039
14	0.9920843	0.7268376	0.4609980	0.5641150	2.4486488	1.9262414
15	0.9870592	0.9079765	0.6766782	-0.0865695	2.1848729	1.4931377
16	0.9768186	0.6616118	0.3882953	0.7229686	2.5983095	2.9324903
17	0.9703234	0.9210756	0.7020886	-0.2138494	2.2270422	2.0971722
18	0.9198017	0.4558948	0.1588807	1.1942707	3.0953865	6.0476372
19	0.8861983	0.9372332	0.7790416	-0.6391356	2.5362606	4.2762934
20	1.6448479	2.6833015	2.6647866	-3.5682170	-2.6288279	7.8701368
21	1.0454498	0.9404974	0.5895037	0.5789042	1.8295335	1.3209692
22	1.0437334	0.9121606	0.7136891	-0.0253762	2.0221798	1.1813980
23	0.9867409	0.7059354	0.4320066	0.7019462	2.4823445	2.6080516

k	α^k	β^k	γ^k	δ^k	x^k	ER
24	0.9788148	0.9196594	0.6973946	-0.1796216	2.1930632	1.9183223
25	0.9403686	0.5204331	0.2322011	1.0511540	2.9325580	5.0962731
26	0.9194404	0.9355503	0.7529488	-0.4927420	2.4031449	3.4904321
27	-4.5305161	-16.7463576	-19.1184300	38.4122612	46.7438286	141.4679087
28	0.8394029	0.3998776	0.5203513	0.4440532	2.2441874	3.3570177
29	0.8876021	0.9573782	0.6466005	-0.0453745	2.5433539	1.9562363
30	1.0008266	0.6984169	0.4481880	0.5262381	2.4720111	1.8738340
31	0.9814517	0.9074577	0.6722238	-0.0719992	2.2055263	1.4484911
32	0.9810864	0.6787872	0.4076018	0.6753231	2.5585016	2.6395077
33	0.9753175	0.9176777	0.6949053	-0.1771974	2.2138189	1.9232823
34	0.9418208	0.5327905	0.2445310	1.0209837	2.9074767	4.9099567
35	0.9254863	0.9344537	0.7481835	-0.4657570	2.3822852	3.3500802
36	-0.5239716	-4.1178235	-4.9664068	11.1233082	14.6771289	52.8327013
37	0.7116750	3.4778528	3.7730926	-5.7208888	-1.3235932	10.5632646
38	1.0335282	0.7614261	0.3242045	1.2876487	2.0388944	4.0682795
39	0.8549939	0.8810099	0.7966710	-0.7580577	2.6281505	4.7780118
40	1.4095112	1.8252701	1.7397914	-1.7757855	-0.6092916	4.4551307
41	1.0631929	0.9163541	0.5485832	0.7504319	1.7899956	1.9035072
42	1.0254799	0.9302369	0.7514739	-0.1623053	2.0507947	1.7956561
43	0.9197957	0.4675464	0.1612973	1.2939244	3.0498233	6.2119432
44	0.8687709	0.9319266	0.7862782	-0.7012967	2.5961050	4.5982844
45	1.4939075	2.1976093	2.1222594	-2.5108437	-1.4201931	6.4865053
46	1.0645675	0.9212904	0.5536198	0.7636748	1.7843371	1.9301508
47	1.0242313	0.9311686	0.7552358	-0.1705906	2.0523277	1.8350991
48	0.9113661	0.4416427	0.1311106	1.3599784	3.1144868	6.5722621
49	0.8467804	0.9295685	0.7992391	-0.7795412	2.6864359	5.0449692
50	1.4065631	1.9181989	1.8120717	-1.9018063	-0.7205913	4.9005817
51	1.0649608	0.9179487	0.5497408	0.7607117	1.7837907	1.9280038
52	1.0244677	0.9313012	0.7547483	-0.1700790	2.0524168	1.8325188
53	0.9122106	0.4435187	0.1335480	1.3543245	3.1088661	6.5430276
54	0.8486396	0.9298735	0.7981479	-0.7729483	2.6785511	5.0070996
55	1.4119371	1.9356542	1.8313226	-1.9398212	-0.7638792	5.0221746

k	α^k	β^k	γ^k	δ^k	x^k	ER
56	1.0651929	0.9179015	0.5494882	0.7642955	1.7833245	1.9389684
57	1.0239347	0.9315072	0.7554928	-0.1729319	2.0535908	1.8456334
58	0.9095290	0.4350375	0.1237943	1.3748194	3.1299267	6.6555682
59	0.8410138	0.9290035	0.8025462	-0.7992825	2.7101432	5.1593762
60	1.3917819	1.8710692	1.7598890	-1.7982271	-0.6031066	4.5655460
61	1.0641415	0.9182637	0.5507680	0.7484918	1.7854011	1.8901948
62	1.0262519	0.9305529	0.7522185	-0.1603681	2.0485587	1.7879998
63	0.9207932	0.4709085	0.1650057	1.2880697	3.0410729	6.1705727
64	0.8707648	0.9320073	0.7849528	-0.6937676	2.5877498	4.5555302
65	1.5061006	2.2363633	2.1654142	-2.5950381	-1.5180525	6.6259237
66	1.0638521	0.9223612	0.5553808	0.7558217	1.7857936	1.9041827
67	1.0254255	0.9306099	0.7535327	-0.1638199	2.0495462	1.8040611
68	0.9174192	0.4610427	0.1533513	1.3132491	3.0666304	6.3115113
69	0.8629962	0.9312250	0.7896882	-0.7222697	2.6196868	4.7166388
70	1.4644026	2.1030698	2.0171920	-2.3054313	-1.1831909	6.0586763
71	1.0656473	0.9192262	0.5504713	0.7750668	1.7822617	1.9688105
72	1.0223935	0.9319723	0.7576999	-0.1807914	2.0568001	1.8819550
73	0.9015605	0.4103767	0.0952764	1.4348617	3.1917467	6.9763662
74	0.8161835	0.9257473	0.8164460	-0.8823201	2.8139314	5.6446941
75	1.3466762	1.7261865	1.6005317	-1.4780176	-0.2456094	3.5744081
76	1.0597757	0.9208509	0.5568414	0.6879450	1.7938302	1.6987689
77	1.0341923	0.9259848	0.7397648	-0.1135638	2.0333651	1.5756391
78	0.9528622	0.5774450	0.2867962	1.0280423	2.7810287	4.6152616
79	0.9323048	0.9327827	0.7427750	-0.4401686	2.3464703	3.1909691
80	0.0594728	-2.2985191	-2.9181027	7.1697034	10.0555342	20.1290897
81	1.3717276	1.2205000	1.3353682	0.1691272	-0.6156110	4.6394335

We see that the iteration is diverge for initial vector at C.

The result from modified multivariate secant method.

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	2.00	1.00	1.00	0.50	2.00	7.0426686
0	0.65	0.60	0.40	0.15	1.00	5.7191859
0	0.50	1.00	0.50	0.10	1.00	5.6064746
0	0.75	0.50	0.30	0.50	1.00	5.5913096
0	1.50	0.60	0.40	0.20	1.50	3.8584102
0	1.00	1.00	1.00	1.00	1.50	2.6589038
1	1.6704649	0.6224276	-0.3323411	0.5328270	0.4360257	5.0219145
2	0.9122949	0.9078951	1.1391763	0.0743203	2.2342668	3.0316025
3	0.9173677	0.8966244	1.0788414	0.1267815	2.2405180	2.7050763
4	1.0783673	0.9663390	0.4683522	0.0398512	1.8819474	1.3218358
5	1.0362598	0.8647265	0.5946674	0.2014654	2.1111255	0.1977812
6	1.2342559	0.8688713	0.4410534	0.1633192	1.6900506	1.7358636
7	1.0088131	0.8497798	0.6052716	0.2283031	2.1935573	0.0407473
8	1.0097420	0.8511028	0.6067964	0.2294661	2.1872923	0.0447490
9	1.0073682	0.8465711	0.5956322	0.2218935	2.2117310	0.0028598
10	1.0071556	0.8464306	0.5958950	0.2223308	2.2120254	0.0002182
11	1.0071543	0.8464168	0.5958701	0.2223006	2.2121077	0.0000461
12	1.0071536	0.8464148	0.5958628	0.2222910	2.2121256	0.0000040
13	1.0071542	0.8464152	0.5958645	0.2222932	2.2121216	0.0000061
14	1.0071538	0.8464150	0.5958635	0.2222919	2.2121240	0.0000000

Estimates values are $\alpha = 1.0071538$, $\beta = 0.8464150$, $\gamma = 0.5958635$, $\delta = 0.2222919$ and $x = 2.2121240$ after 14 iterations.

The result from mid modified multivariate secant method.

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	2.00	1.00	1.00	0.50	2.00	7.0426686
0	0.65	0.60	0.40	0.15	1.00	5.7191859
0	0.50	1.00	0.50	0.10	1.00	5.6064746
0	0.75	0.50	0.30	0.50	1.00	5.5913096
0	1.50	0.60	0.40	0.20	1.50	3.8584102

k	α^k	β^k	γ^k	δ^k	x^k	ER
0	1.00	1.00	1.00	1.00	1.50	2.6589038
1	1.6704649	0.6224276	-0.3323411	0.5328270	0.4360257	5.0219145
2	0.9122949	0.9078951	1.1391763	0.0743203	2.2342668	3.0316025
3	0.9040588	0.9261943	1.2371363	-0.0108559	2.2241173	3.5599984
4	1.0067321	0.9390296	0.7506312	0.1032462	2.0274969	0.9400938
5	1.0472926	0.8529637	0.5988375	0.2672457	2.0902301	0.2667514
6	1.0408828	0.8329929	0.5960424	0.2833807	2.1388740	0.3549962
7	0.7848621	1.0714323	0.6737703	-0.3790526	2.4706574	3.7509800
8	0.8718435	0.9692709	0.6334950	-0.1270974	2.3937492	2.1638871
9	1.0102135	0.8497774	0.5996948	0.2287599	2.1970630	0.0243858
10	1.0095083	0.8484395	0.5978895	0.2242618	2.2016370	0.0072801
11	1.0073726	0.8472129	0.5966620	0.2223034	2.2098644	0.0036338
12	1.0070288	0.8463217	0.5956221	0.2217610	2.2128570	0.0022402
13	1.0072039	0.8463724	0.5958675	0.2224779	2.2119961	0.0009506
14	1.0071561	0.8464147	0.5958655	0.2222993	2.2121169	0.0000341
15	1.0071537	0.8464151	0.5958635	0.2222917	2.2121239	0.0000018
16	1.0071539	0.8464149	0.5958635	0.2222919	2.2121240	0.0000003
17	1.0071538	0.8464150	0.5958635	0.2222919	2.2121240	0.0000000

Estimates values are $\alpha = 1.0071538$, $\beta = 0.8464150$, $\gamma = 0.5958635$, $\delta = 0.2222919$ and $x = 2.2121240$ after 17 iterations.

The iterations of table 4.10 at the initial vectors of A

The result from multivariate secant method.

k	m^k	n^k	x^k	y^k	z^k	ER
0	0.06	-0.01	6.90	8.22	18.22	155.8859804
0	0.70	-0.01	9.10	7.23	17.10	247.6210830
0	0.05	-0.02	8.45	6.21	20.51	69.2382814
0	0.08	-0.03	9.20	6.60	21.25	75.8439860
0	0.22	-0.04	6.85	6.50	21.20	18.0585262
0	0.12	-0.02	9.50	6.25	22.00	46.3214430

k	m^k	n^k	x^k	y^k	z^k	ER
1	0.1169264	-0.0209820	6.3104959	9.4049129	22.3015085	6.0368923
2	0.0916086	-0.0221725	7.5004908	8.3511616	22.8045838	8.2587870
3	0.0906396	-0.0223783	7.1602725	8.2851002	22.2729674	2.5455622
4	0.0905961	-0.0223540	7.0860734	8.4413483	22.3634839	0.4052280
5	0.0906016	-0.0223286	7.1111089	8.4449446	22.3966823	0.1731965
6	0.0906356	-0.0223320	7.1153954	8.4339565	22.3896552	0.0234992
7	0.0906328	-0.0223335	7.1139800	8.4336822	22.3877264	0.0096645
8	0.0906307	-0.0223333	7.1136936	8.4343358	22.3881015	0.0016344
9	0.0906308	-0.0223332	7.1137726	8.4343661	22.3882260	0.0005487
10	0.0906310	-0.0223332	7.1137920	8.4343270	22.3882061	0.0001130
11	0.0906310	-0.0223332	7.1137876	8.4343242	22.3881981	0.0000311
12	0.0906309	-0.0223332	7.1137863	8.4343266	22.3881991	0.0000076
13	0.0906309	-0.0223332	7.1137865	8.4343268	22.3881996	0.0000017
14	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881996	0.0000005
15	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000001

Estimates values are $m = 0.0906309$, $n = -0.0223332$, $x = 7.1137866$, $y = 8.4343266$ and $z = 22.3881995$ after 15 iterations.

The result from modified multivariate secant method.

k	m^k	n^k	x^k	y^k	z^k	ER
0	0.70	-0.01	9.10	7.23	17.10	247.6210830
0	0.06	-0.01	6.90	8.22	18.22	155.8859804
0	0.08	-0.03	9.20	6.60	21.25	75.8439860
0	0.05	-0.02	8.45	6.21	20.51	69.2382814
0	0.12	-0.02	9.50	6.25	22.00	46.3214430
0	0.22	-0.04	6.85	6.50	21.20	18.0585262
1	0.1169264	-0.0209820	6.3104959	9.4049129	22.3015085	6.0368923
2	0.0867722	-0.0223999	7.7278150	8.1498640	22.9006861	10.3847765
3	0.0924286	-0.0222594	7.0504548	8.3548301	22.1880612	4.6012534
4	0.0908627	-0.0223470	7.1033164	8.4199428	22.3588326	0.5733544

k	m^k	n^k	x^k	y^k	z^k	ER
5	0.0905235	-0.0223219	7.1148880	8.4420493	22.3971105	0.1525599
6	0.0906333	-0.0223334	7.1135220	8.4345915	22.3881666	0.0003580
7	0.0906309	-0.0223332	7.1138032	8.4343245	22.3882134	0.0001713
8	0.0906309	-0.0223332	7.1137964	8.4343163	22.3882003	0.0000090
9	0.0906310	-0.0223332	7.1137847	8.4343280	22.3881988	0.0000086
10	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000000

Estimates values are m = 0.0906309, n = -0.0223332, x = 7.1137866, y = 8.4343267 and z = 22.3881995 after 10 iterations.

The result from mid modified multivariate secant method.

k	m^k	n^k	x^k	y^k	z^k	ER
0	0.70	-0.01	9.10	7.23	17.10	247.6210830
0	0.06	-0.01	6.90	8.22	18.22	155.8859804
0	0.08	-0.03	9.20	6.60	21.25	75.8439860
0	0.05	-0.02	8.45	6.21	20.51	69.2382814
0	0.12	-0.02	9.50	6.25	22.00	46.3214430
0	0.22	-0.04	6.85	6.50	21.20	18.0585262
1	0.1169264	-0.0209820	6.3104959	9.4049129	22.3015085	6.0368923
2	0.0867722	-0.0223999	7.7278150	8.1498640	22.9006861	10.3847765
3	0.0906562	-0.0223035	7.2626967	8.2906066	22.4113529	0.2190053
4	0.0907475	-0.0223592	7.1006456	8.4256962	22.3664912	0.3226120
5	0.0905920	-0.0223328	7.1110366	8.4373822	22.3881722	0.0049460
6	0.0906392	-0.0223330	7.1140883	8.4343761	22.3885779	0.0059403
7	0.0906316	-0.0223332	7.1139394	8.4340876	22.3881331	0.0013656
8	0.0906314	-0.0223332	7.1138985	8.4341524	22.3881517	0.0009862
9	0.0906310	-0.0223332	7.1137874	8.4343255	22.3881993	0.0000059
10	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000001
11	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000000

Estimates values are m = 0.0906309, n = -0.0223332, x = 7.1137866, y = 8.4343267 and z = 22.3881995 after 11 iterations.

The iterations of table 4.10 at the initial vectors of B

The result from multivariate secant method.

k	m^k	n^k	x^k	y^k	z^k	ER
0	0.00	0.50	7.85	9.50	15.20	329.4053600
0	0.03	0.35	5.20	9.60	13.00	306.1288542
0	0.01	0.30	5.10	9.23	15.10	232.8919515
0	0.00	0.25	6.50	10.25	18.25	203.9388312
0	0.02	0.40	4.45	10.21	16.51	200.7947457
0	0.02	0.25	3.90	9.22	20.22	32.6768262
1	0.0987484	-0.0264314	5.2609144	10.6560791	20.9610277	71.0664824
2	0.1362995	-0.0407654	5.0155519	11.8417313	23.5970215	13.1006006
3	0.1557007	-0.0345320	4.2725370	14.1240612	26.3168412	83.9095763
4	0.4464784	-0.0302597	-12.1072950	64.4582149	72.8636715	907.2215829
5	0.0604749	0.0409728	3.9057158	3.3398564	12.8746465	124.1019377
6	0.1528280	-0.0871761	6.7824977	5.8355328	19.8535044	34.4728553
7	0.1223584	-0.0473427	7.1857211	8.1785146	22.1708946	5.1640938
8	0.1090879	-0.0289504	7.0581210	8.4834212	22.4091841	1.4614627
9	0.1035931	-0.0255568	7.0669698	8.4899621	22.4734954	3.8204500
10	0.0992174	-0.0241774	7.0986872	8.4518048	22.4596497	3.2172801
11	0.0960923	-0.0234692	7.1244806	8.4194365	22.4326704	2.0574790
12	0.0940512	-0.0230705	7.1395499	8.4007759	22.4114157	1.1257963
13	0.0927786	-0.0228317	7.1461481	8.3928454	22.3979399	0.5243896
14	0.0920004	-0.0226828	7.1474677	8.3914650	22.3902600	0.1761158
15	0.0915259	-0.0225869	7.1459325	8.3935672	22.3862390	0.0280243
16	0.0912342	-0.0225234	7.1430645	8.3972699	22.3843607	0.1084509
17	0.0910518	-0.0224799	7.1397316	8.4015140	22.3836784	0.1460380
18	0.0909350	-0.0224490	7.1363950	8.4057407	22.3836335	0.1551017
19	0.0908581	-0.0224264	7.1332797	8.4096787	22.3839060	0.1494285
20	0.0908056	-0.0224093	7.1304797	8.4132152	22.3843172	0.1368943
21	0.0907686	-0.0223959	7.1280192	8.4163225	22.3847701	0.1218251
22	0.0907416	-0.0223853	7.1258868	8.4190159	22.3852141	0.1065059
23	0.0907212	-0.0223767	7.1240551	8.4213303	22.3856250	0.0920698

k	m^k	n^k	x^k	y^k	z^k	ER
24	0.0907053	-0.0223697	7.1224904	8.4233080	22.3859931	0.0790118
25	0.0906927	-0.0223639	7.1211588	8.4249917	22.3863163	0.0674824
26	0.0906826	-0.0223591	7.1200281	8.4264219	22.3865966	0.0574540
27	0.0906743	-0.0223550	7.1190695	8.4276349	22.3868376	0.0488144
28	0.0906674	-0.0223516	7.1182575	8.4286626	22.3870438	0.0414174
29	0.0906616	-0.0223488	7.1175701	8.4295329	22.3872196	0.0351099
30	0.0906569	-0.0223464	7.1169883	8.4302696	22.3873692	0.0297458
31	0.0906528	-0.0223443	7.1164960	8.4308931	22.3874961	0.0251919
32	0.0906494	-0.0223426	7.1160795	8.4314207	22.3876038	0.0213304
33	0.0906466	-0.0223412	7.1157271	8.4318672	22.3876951	0.0180582
34	0.0906442	-0.0223399	7.1154289	8.4322450	22.3877724	0.0152868
35	0.0906421	-0.0223389	7.1151766	8.4325647	22.3878379	0.0129402
36	0.0906404	-0.0223380	7.1149631	8.4328353	22.3878934	0.0109536
37	0.0906390	-0.0223373	7.1147824	8.4330643	22.3879403	0.0092720
38	0.0906377	-0.0223367	7.1146295	8.4332581	22.3879801	0.0078486
39	0.0906367	-0.0223361	7.1145001	8.4334222	22.3880138	0.0066438
40	0.0906358	-0.0223357	7.1143906	8.4335610	22.3880423	0.0056240
41	0.0906351	-0.0223353	7.1142979	8.4336785	22.3880664	0.0047608
42	0.0906344	-0.0223350	7.1142194	8.4337780	22.3880868	0.0040301
43	0.0906339	-0.0223347	7.1141530	8.4338622	22.3881041	0.0034116
44	0.0906334	-0.0223345	7.1140967	8.4339334	22.3881188	0.0028881
45	0.0906331	-0.0223343	7.1140492	8.4339938	22.3881311	0.0024449
46	0.0906327	-0.0223341	7.1140089	8.4340448	22.3881416	0.0020698
47	0.0906325	-0.0223340	7.1139748	8.4340881	22.3881505	0.0017522
48	0.0906322	-0.0223338	7.1139459	8.4341247	22.3881580	0.0014834
49	0.0906320	-0.0223337	7.1139215	8.4341557	22.3881644	0.0012558
50	0.0906319	-0.0223337	7.1139008	8.4341819	22.3881698	0.0010631
51	0.0906317	-0.0223336	7.1138833	8.4342041	22.3881743	0.0009000
52	0.0906316	-0.0223335	7.1138684	8.4342229	22.3881782	0.0007620
53	0.0906315	-0.0223335	7.1138559	8.4342388	22.3881815	0.0006451
54	0.0906314	-0.0223334	7.1138453	8.4342523	22.3881842	0.0005461
55	0.0906313	-0.0223334	7.1138363	8.4342637	22.3881866	0.0004623

k	m^k	n^k	x^k	y^k	z^k	ER
56	0.0906313	-0.0223334	7.1138286	8.4342734	22.3881886	0.0003914
57	0.0906312	-0.0223333	7.1138222	8.4342815	22.3881903	0.0003314
58	0.0906312	-0.0223333	7.1138167	8.4342885	22.3881917	0.0002805
59	0.0906312	-0.0223333	7.1138121	8.4342943	22.3881929	0.0002375
60	0.0906311	-0.0223333	7.1138082	8.4342993	22.3881939	0.0002011
61	0.0906311	-0.0223333	7.1138049	8.4343035	22.3881948	0.0001702
62	0.0906311	-0.0223333	7.1138021	8.4343070	22.3881955	0.0001441
63	0.0906311	-0.0223332	7.1137997	8.4343100	22.3881961	0.0001220
64	0.0906310	-0.0223332	7.1137977	8.4343126	22.3881966	0.0001033
65	0.0906310	-0.0223332	7.1137960	8.4343147	22.3881971	0.0000874
66	0.0906310	-0.0223332	7.1137946	8.4343166	22.3881975	0.0000740
67	0.0906310	-0.0223332	7.1137933	8.4343181	22.3881978	0.0000627
68	0.0906310	-0.0223332	7.1137923	8.4343194	22.3881980	0.0000531
69	0.0906310	-0.0223332	7.1137914	8.4343205	22.3881983	0.0000449
70	0.0906310	-0.0223332	7.1137907	8.4343215	22.3881985	0.0000380
71	0.0906310	-0.0223332	7.1137901	8.4343223	22.3881986	0.0000322
72	0.0906310	-0.0223332	7.1137895	8.4343229	22.3881988	0.0000273
73	0.0906310	-0.0223332	7.1137891	8.4343235	22.3881989	0.0000231
74	0.0906310	-0.0223332	7.1137887	8.4343240	22.3881990	0.0000195
75	0.0906310	-0.0223332	7.1137884	8.4343244	22.3881991	0.0000165
76	0.0906310	-0.0223332	7.1137881	8.4343247	22.3881991	0.0000140
77	0.0906310	-0.0223332	7.1137879	8.4343250	22.3881992	0.0000119
78	0.0906310	-0.0223332	7.1137877	8.4343253	22.3881992	0.0000100
79	0.0906310	-0.0223332	7.1137875	8.4343255	22.3881993	0.0000085
80	0.0906310	-0.0223332	7.1137874	8.4343257	22.3881993	0.0000072
81	0.0906310	-0.0223332	7.1137873	8.4343258	22.3881994	0.0000061
82	0.0906310	-0.0223332	7.1137872	8.4343260	22.3881994	0.0000052
83	0.0906310	-0.0223332	7.1137871	8.4343261	22.3881994	0.0000044
84	0.0906310	-0.0223332	7.1137870	8.4343261	22.3881994	0.0000037
85	0.0906310	-0.0223332	7.1137869	8.4343262	22.3881994	0.0000031
86	0.0906310	-0.0223332	7.1137869	8.4343263	22.3881995	0.0000026
87	0.0906310	-0.0223332	7.1137868	8.4343263	22.3881995	0.0000022

k	m^k	n^k	x^k	y^k	z^k	ER
88	0.0906310	-0.0223332	7.1137868	8.4343264	22.3881995	0.0000019
89	0.0906310	-0.0223332	7.1137868	8.4343264	22.3881995	0.0000016
90	0.0906310	-0.0223332	7.1137868	8.4343265	22.3881995	0.0000014
91	0.0906309	-0.0223332	7.1137867	8.4343265	22.3881995	0.0000012
92	0.0906309	-0.0223332	7.1137867	8.4343265	22.3881995	0.0000010
93	0.0906309	-0.0223332	7.1137867	8.4343265	22.3881995	0.0000008
94	0.0906309	-0.0223332	7.1137867	8.4343266	22.3881995	0.0000007
95	0.0906309	-0.0223332	7.1137867	8.4343266	22.3881995	0.0000006
96	0.0906309	-0.0223332	7.1137867	8.4343266	22.3881995	0.0000005
97	0.0906309	-0.0223332	7.1137867	8.4343266	22.3881995	0.0000004
98	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000004
99	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000003
100	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000003
101	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000002
102	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000002
103	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000002
104	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000001
105	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000001
106	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000001

Estimates values are $m = 0.0906309$, $n = -0.0223332$, $x = 7.1137866$, $y = 8.4343266$ and $z = 22.3881995$ after 106 iterations.

The result from modified multivariate secant method.

k	m^k	n^k	x^k	y^k	z^k	ER
0	0.00	0.50	7.85	9.50	15.20	329.4053600
0	0.03	0.35	5.20	9.60	13.00	306.1288542
0	0.01	0.30	5.10	9.23	15.10	232.8919515
0	0.00	0.25	6.50	10.25	18.25	203.9388312
0	0.02	0.40	4.45	10.21	16.51	200.7947457
0	0.02	0.25	3.90	9.22	20.22	32.6768262

k	m^k	n^k	x^k	y^k	z^k	ER
1	0.0987484	-0.0264314	5.2609144	10.6560791	20.9610277	71.0664824
2	0.1362995	-0.0407654	5.0155519	11.8417313	23.5970215	13.1006006
3	0.1388934	-0.0399320	4.9162118	12.1468758	23.9606579	22.3497933
4	0.1401603	-0.0415726	4.6088379	12.9069986	24.3467851	21.9308239
5	0.1699471	-0.0555760	2.8401685	17.4088893	27.3337103	36.0267235
6	0.1065709	-0.0226535	6.5917757	7.4580050	20.6813123	30.6676165
7	0.1039099	-0.0203964	6.9777570	6.4344142	19.9940256	45.6681728
8	0.1017054	-0.0246843	6.0716678	8.7806623	21.5395988	14.4530458
9	0.0992703	-0.0243258	6.0402662	8.8786090	21.6003669	13.6644984
10	0.1128658	-0.0273067	6.3725127	8.5916592	21.7482085	9.5636122
11	0.1863629	-0.0437388	4.5170044	8.1031488	19.3213064	43.2200998
12	0.0858848	-0.0218955	6.9161023	8.7373447	22.4620954	1.2771185
13	0.0911823	-0.0223383	7.0963359	8.4403975	22.3736567	0.2518703
14	0.0912649	-0.0224044	7.1252178	8.4112387	22.3787214	0.1645575
15	0.0900968	-0.0222694	7.1010766	8.4585480	22.3972432	0.1544102
16	0.0906365	-0.0223333	7.1137710	8.4343540	22.3881971	0.0003753
17	0.0906315	-0.0223333	7.1138201	8.4342789	22.3881912	0.0001532
18	0.0906310	-0.0223332	7.1137879	8.4343251	22.3881995	0.0000009
19	0.0906310	-0.0223332	7.1137871	8.4343261	22.3881995	0.0000006
20	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000000

Estimates values are $m = 0.0906309$, $n = -0.0223332$, $x = 7.1137866$, $y = 8.4343266$ and $z = 22.3881995$ after 20 iterations.

The result from mid modified multivariate secant method.

k	m^k	n^k	x^k	y^k	z^k	ER
0	0.00	0.50	7.85	9.50	15.20	329.4053600
0	0.03	0.35	5.20	9.60	13.00	306.1288542
0	0.01	0.30	5.10	9.23	15.10	232.8919515
0	0.00	0.25	6.50	10.25	18.25	203.9388312
0	0.02	0.40	4.45	10.21	16.51	200.7947457
0	0.02	0.25	3.90	9.22	20.22	32.6768262

k	m^k	n^k	x^k	y^k	z^k	ER
1	0.0987484	-0.0264314	5.2609144	10.6560791	20.9610277	71.0664824
2	0.1362995	-0.0407654	5.0155519	11.8417313	23.5970215	13.1006006
3	0.1557007	-0.0345320	4.2725370	14.1240612	26.3168412	83.9095763
4	0.4464784	-0.0302597	-12.1072950	64.4582149	72.8636715	907.2215829
5	0.0604749	0.0409728	3.9057158	3.3398564	12.8746465	124.1019377
6	0.1528280	-0.0871761	6.7824977	5.8355328	19.8535044	34.4728553
7	0.1223584	-0.0473427	7.1857211	8.1785146	22.1708946	5.1640938
8	0.1090879	-0.0289504	7.0581210	8.4834212	22.4091841	1.4614627
9	0.1035931	-0.0255568	7.0669698	8.4899621	22.4734954	3.8204500
10	0.0992174	-0.0241774	7.0986872	8.4518048	22.4596497	3.2172801
11	0.0960923	-0.0234692	7.1244806	8.4194365	22.4326704	2.0574790
12	0.0940512	-0.0230705	7.1395499	8.4007759	22.4114157	1.1257963
13	0.0927786	-0.0228317	7.1461481	8.3928454	22.3979399	0.5243896
14	0.0920004	-0.0226828	7.1474677	8.3914650	22.3902600	0.1761158
15	0.0915259	-0.0225869	7.1459325	8.3935672	22.3862390	0.0280243
16	0.0912342	-0.0225234	7.1430645	8.3972699	22.3843607	0.1084509
17	0.0910518	-0.0224799	7.1397316	8.4015140	22.3836784	0.1460380
18	0.0909350	-0.0224490	7.1363950	8.4057407	22.3836335	0.1551017
19	0.0908581	-0.0224264	7.1332797	8.4096787	22.3839060	0.1494285
20	0.0908056	-0.0224093	7.1304797	8.4132152	22.3843172	0.1368943
21	0.0907686	-0.0223959	7.1280192	8.4163225	22.3847701	0.1218251
22	0.0907416	-0.0223853	7.1258868	8.4190159	22.3852141	0.1065059
23	0.0907212	-0.0223767	7.1240551	8.4213303	22.3856250	0.0920698
24	0.0907053	-0.0223697	7.1224904	8.4233080	22.3859931	0.0790118
25	0.0906927	-0.0223639	7.1211588	8.4249917	22.3863163	0.0674824
26	0.0906826	-0.0223591	7.1200281	8.4264219	22.3865966	0.0574540
27	0.0906743	-0.0223550	7.1190695	8.4276349	22.3868376	0.0488144
28	0.0906674	-0.0223516	7.1182575	8.4286626	22.3870438	0.0414174
29	0.0906616	-0.0223488	7.1175701	8.4295329	22.3872196	0.0351099
30	0.0906569	-0.0223464	7.1169883	8.4302696	22.3873692	0.0297458
31	0.0906528	-0.0223443	7.1164960	8.4308931	22.3874961	0.0251919
32	0.0906494	-0.0223426	7.1160795	8.4314207	22.3876038	0.0213304

k	m^k	n^k	x^k	y^k	z^k	ER
33	0.0906466	-0.0223412	7.1157271	8.4318672	22.3876951	0.0180582
34	0.0906442	-0.0223399	7.1154289	8.4322450	22.3877724	0.0152868
35	0.0906421	-0.0223389	7.1151766	8.4325647	22.3878379	0.0129402
36	0.0906404	-0.0223380	7.1149631	8.4328353	22.3878934	0.0109536
37	0.0906390	-0.0223373	7.1147824	8.4330643	22.3879403	0.0092720
38	0.0906377	-0.0223367	7.1146295	8.4332581	22.3879801	0.0078486
39	0.0906367	-0.0223361	7.1145001	8.4334222	22.3880138	0.0066438
40	0.0906358	-0.0223357	7.1143906	8.4335610	22.3880423	0.0056240
41	0.0906351	-0.0223353	7.1142979	8.4336785	22.3880664	0.0047608
42	0.0906344	-0.0223350	7.1142194	8.4337780	22.3880868	0.0040301
43	0.0906339	-0.0223347	7.1141530	8.4338622	22.3881041	0.0034116
44	0.0906334	-0.0223345	7.1140967	8.4339334	22.3881188	0.0028881
45	0.0906331	-0.0223343	7.1140492	8.4339938	22.3881311	0.0024449
46	0.0906327	-0.0223341	7.1140089	8.4340448	22.3881416	0.0020698
47	0.0906325	-0.0223340	7.1139748	8.4340881	22.3881505	0.0017522
48	0.0906322	-0.0223338	7.1139459	8.4341247	22.3881580	0.0014834
49	0.0906320	-0.0223337	7.1139215	8.4341557	22.3881644	0.0012558
50	0.0906319	-0.0223337	7.1139008	8.4341819	22.3881698	0.0010631
51	0.0906317	-0.0223336	7.1138833	8.4342041	22.3881743	0.0009000
52	0.0906316	-0.0223335	7.1138684	8.4342229	22.3881782	0.0007620
53	0.0906315	-0.0223335	7.1138559	8.4342388	22.3881815	0.0006451
54	0.0906314	-0.0223334	7.1138453	8.4342523	22.3881842	0.0005461
55	0.0906313	-0.0223334	7.1138363	8.4342637	22.3881866	0.0004623
56	0.0906313	-0.0223334	7.1138286	8.4342734	22.3881886	0.0003914
57	0.0906312	-0.0223333	7.1138222	8.4342815	22.3881903	0.0003314
58	0.0906312	-0.0223333	7.1138167	8.4342885	22.3881917	0.0002805
59	0.0906312	-0.0223333	7.1138121	8.4342943	22.3881929	0.0002375
60	0.0906311	-0.0223333	7.1138082	8.4342993	22.3881939	0.0002011
61	0.0906311	-0.0223333	7.1138049	8.4343035	22.3881948	0.0001702
62	0.0906311	-0.0223333	7.1138021	8.4343070	22.3881955	0.0001441
63	0.0906311	-0.0223332	7.1137997	8.4343100	22.3881961	0.0001220
64	0.0906310	-0.0223332	7.1137977	8.4343126	22.3881966	0.0001033

k	m^k	n^k	x^k	y^k	z^k	ER
65	0.0906310	-0.0223332	7.1137960	8.4343147	22.3881971	0.0000874
66	0.0906310	-0.0223332	7.1137946	8.4343166	22.3881975	0.0000740
67	0.0906310	-0.0223332	7.1137933	8.4343181	22.3881978	0.0000627
68	0.0906310	-0.0223332	7.1137923	8.4343194	22.3881980	0.0000531
69	0.0906310	-0.0223332	7.1137914	8.4343205	22.3881983	0.0000449
70	0.0906310	-0.0223332	7.1137907	8.4343215	22.3881985	0.0000380
71	0.0906310	-0.0223332	7.1137901	8.4343223	22.3881986	0.0000322
72	0.0906310	-0.0223332	7.1137895	8.4343229	22.3881988	0.0000273
73	0.0906310	-0.0223332	7.1137891	8.4343235	22.3881989	0.0000231
74	0.0906310	-0.0223332	7.1137887	8.4343240	22.3881990	0.0000195
75	0.0906310	-0.0223332	7.1137884	8.4343244	22.3881991	0.0000165
76	0.0906310	-0.0223332	7.1137881	8.4343247	22.3881991	0.0000140
77	0.0906310	-0.0223332	7.1137879	8.4343250	22.3881992	0.0000119
78	0.0906310	-0.0223332	7.1137877	8.4343253	22.3881992	0.0000100
79	0.0906310	-0.0223332	7.1137875	8.4343255	22.3881993	0.0000085
80	0.0906310	-0.0223332	7.1137874	8.4343257	22.3881993	0.0000072
81	0.0906310	-0.0223332	7.1137873	8.4343258	22.3881994	0.0000061
82	0.0906310	-0.0223332	7.1137872	8.4343260	22.3881994	0.0000052
83	0.0906310	-0.0223332	7.1137871	8.4343261	22.3881994	0.0000044
84	0.0906310	-0.0223332	7.1137870	8.4343261	22.3881994	0.0000037
85	0.0906310	-0.0223332	7.1137869	8.4343262	22.3881994	0.0000031
86	0.0906310	-0.0223332	7.1137869	8.4343263	22.3881995	0.0000026
87	0.0906310	-0.0223332	7.1137868	8.4343263	22.3881995	0.0000022
88	0.0906310	-0.0223332	7.1137868	8.4343264	22.3881995	0.0000019
89	0.0906310	-0.0223332	7.1137868	8.4343264	22.3881995	0.0000016
90	0.0906310	-0.0223332	7.1137868	8.4343265	22.3881995	0.0000014
91	0.0906309	-0.0223332	7.1137867	8.4343265	22.3881995	0.0000012
92	0.0906309	-0.0223332	7.1137867	8.4343265	22.3881995	0.0000010
93	0.0906309	-0.0223332	7.1137867	8.4343265	22.3881995	0.0000008
94	0.0906309	-0.0223332	7.1137867	8.4343266	22.3881995	0.0000007
95	0.0906309	-0.0223332	7.1137867	8.4343266	22.3881995	0.0000006
96	0.0906309	-0.0223332	7.1137867	8.4343266	22.3881995	0.0000005

k	m^k	n^k	x^k	y^k	z^k	ER
97	0.0906309	-0.0223332	7.1137867	8.4343266	22.3881995	0.0000004
98	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000004
99	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000003
100	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000003
101	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000002
102	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000002
103	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000002
104	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000001
105	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000001
106	0.0906309	-0.0223332	7.1137866	8.4343266	22.3881995	0.0000001

Estimates values are $m = 0.0906309$, $n = -0.0223332$, $x = 7.1137866$, $y = 8.4343266$ and $z = 22.3881995$ after 106 iterations.

The iterations of table 4.10 at the initial vectors of C

The result from multivariate secant method.

k	m^k	n^k	x^k	y^k	z^k	ER
0	0.45	0.02	6.90	9.22	20.22	108.6561674
0	0.50	0.01	5.10	9.23	15.10	232.3860495
0	0.55	0.02	4.45	8.21	16.51	140.2600495
0	0.65	0.03	5.20	9.60	13.00	305.6856453
0	0.50	0.10	7.85	9.50	15.20	328.3618024
0	0.25	0.35	6.50	7.25	23.25	96.7683841
1	0.1688342	-0.0475569	6.5671699	9.2795791	23.5026935	44.4684002
2	0.0955782	-0.0271154	6.5766859	9.1316322	22.7854914	15.4289304
3	0.0856298	-0.0225742	6.6188860	9.0274230	22.5889351	8.2537298
4	0.0830446	-0.0215162	6.6729028	8.9490770	22.5242163	5.8873766
5	0.0828790	-0.0212518	6.7235468	8.8818026	22.4930525	4.8015370
6	0.0833394	-0.0212497	6.7691510	8.8247532	22.4737802	4.1024030
7	0.0839819	-0.0213198	6.8092997	8.7759393	22.4599996	3.5796183

k	m^k	n^k	x^k	y^k	z^k	ER
8	0.0846335	-0.0214093	6.8444716	8.7340956	22.4493097	3.1500994
9	0.0852457	-0.0214989	6.8752506	8.6980551	22.4406774	2.7848379
10	0.0858037	-0.0215827	6.9022238	8.6668832	22.4335466	2.4684818
11	0.0863066	-0.0216593	6.9259093	8.6398101	22.4275747	2.1921396
12	0.0867578	-0.0217285	6.9467533	8.6162098	22.4225246	1.9494673
13	0.0871618	-0.0217908	6.9651350	8.5955686	22.4182225	1.7356233
14	0.0875234	-0.0218467	6.9813760	8.5774627	22.4145352	1.5466914
15	0.0878469	-0.0218969	6.9957500	8.5615401	22.4113589	1.3794267
16	0.0881365	-0.0219419	7.0084910	8.5475060	22.4086104	1.2310937
17	0.0883956	-0.0219823	7.0197994	8.5351118	22.4062229	1.0993624
18	0.0886277	-0.0220185	7.0298482	8.5241471	22.4041420	0.9822325
19	0.0888354	-0.0220509	7.0387870	8.5144319	22.4023226	0.8779767
20	0.0890215	-0.0220801	7.0467456	8.5058124	22.4007276	0.7850958
21	0.0891882	-0.0221062	7.0538374	8.4981557	22.3993260	0.7022836
22	0.0893376	-0.0221296	7.0601611	8.4913472	22.3980917	0.6283978
23	0.0894714	-0.0221506	7.0658037	8.4852873	22.3970025	0.5624363
24	0.0895913	-0.0221694	7.0708414	8.4798890	22.3960399	0.5035182
25	0.0896988	-0.0221863	7.0753411	8.4750766	22.3951878	0.4508667
26	0.0897951	-0.0222014	7.0793623	8.4707838	22.3944324	0.4037960
27	0.0898814	-0.0222150	7.0829571	8.4669521	22.3937620	0.3616993
28	0.0899588	-0.0222272	7.0861720	8.4635303	22.3931663	0.3240388
29	0.0900282	-0.0222381	7.0890480	8.4604731	22.3926365	0.2903374
30	0.0900904	-0.0222479	7.0916216	8.4577405	22.3921650	0.2601714
31	0.0901462	-0.0222567	7.0939251	8.4552971	22.3917448	0.2331637
32	0.0901962	-0.0222646	7.0959873	8.4531116	22.3913703	0.2089789
33	0.0902411	-0.0222716	7.0978340	8.4511562	22.3910362	0.1873181
34	0.0902813	-0.0222780	7.0994878	8.4494063	22.3907380	0.1679148
35	0.0903174	-0.0222837	7.1009692	8.4478398	22.3904717	0.1505312
36	0.0903497	-0.0222888	7.1022964	8.4464372	22.3902338	0.1349552
37	0.0903787	-0.0222934	7.1034855	8.4451812	22.3900211	0.1209973
38	0.0904047	-0.0222975	7.1045510	8.4440562	22.3898310	0.1084880
39	0.0904280	-0.0223011	7.1055060	8.4430485	22.3896609	0.0972761

k	m^k	n^k	x^k	y^k	z^k	ER
40	0.0904489	-0.0223044	7.1063619	8.4421456	22.3895088	0.0872262
41	0.0904677	-0.0223074	7.1071290	8.4413366	22.3893726	0.0782172
42	0.0904845	-0.0223101	7.1078167	8.4406116	22.3892507	0.0701407
43	0.0904996	-0.0223124	7.1084332	8.4399618	22.3891416	0.0628999
44	0.0905132	-0.0223146	7.1089859	8.4393795	22.3890439	0.0564080
45	0.0905253	-0.0223165	7.1094814	8.4388575	22.3889564	0.0505872
46	0.0905362	-0.0223182	7.1099257	8.4383895	22.3888780	0.0453679
47	0.0905460	-0.0223198	7.1103241	8.4379700	22.3888077	0.0406878
48	0.0905547	-0.0223211	7.1106813	8.4375939	22.3887448	0.0364911
49	0.0905626	-0.0223224	7.1110016	8.4372567	22.3886884	0.0327276
50	0.0905696	-0.0223235	7.1112888	8.4369543	22.3886378	0.0293527
51	0.0905759	-0.0223245	7.1115464	8.4366832	22.3885925	0.0263261
52	0.0905816	-0.0223254	7.1117774	8.4364401	22.3885519	0.0236119
53	0.0905867	-0.0223262	7.1119846	8.4362221	22.3885155	0.0211776
54	0.0905912	-0.0223269	7.1121704	8.4360267	22.3884829	0.0189945
55	0.0905953	-0.0223276	7.1123370	8.4358513	22.3884536	0.0170365
56	0.0905990	-0.0223281	7.1124864	8.4356941	22.3884274	0.0152805
57	0.0906023	-0.0223287	7.1126204	8.4355531	22.3884039	0.0137055
58	0.0906053	-0.0223291	7.1127406	8.4354267	22.3883828	0.0122930
59	0.0906079	-0.0223296	7.1128484	8.4353133	22.3883639	0.0110261
60	0.0906103	-0.0223299	7.1129451	8.4352116	22.3883470	0.0098897
61	0.0906124	-0.0223303	7.1130318	8.4351204	22.3883317	0.0088706
62	0.0906143	-0.0223306	7.1131096	8.4350386	22.3883181	0.0079565
63	0.0906160	-0.0223308	7.1131794	8.4349652	22.3883059	0.0071366
64	0.0906176	-0.0223311	7.1132420	8.4348994	22.3882949	0.0064012
65	0.0906189	-0.0223313	7.1132981	8.4348404	22.3882851	0.0057416
66	0.0906202	-0.0223315	7.1133484	8.4347874	22.3882763	0.0051500
67	0.0906213	-0.0223317	7.1133936	8.4347399	22.3882684	0.0046193
68	0.0906223	-0.0223318	7.1134341	8.4346974	22.3882613	0.0041433
69	0.0906232	-0.0223320	7.1134704	8.4346592	22.3882549	0.0037164
70	0.0906240	-0.0223321	7.1135030	8.4346249	22.3882492	0.0033335
71	0.0906247	-0.0223322	7.1135322	8.4345942	22.3882441	0.0029900

k	m^k	n^k	x^k	y^k	z^k	ER
72	0.0906253	-0.0223323	7.1135584	8.4345666	22.3882395	0.0026820
73	0.0906259	-0.0223324	7.1135819	8.4345419	22.3882354	0.0024056
74	0.0906264	-0.0223325	7.1136030	8.4345197	22.3882317	0.0021578
75	0.0906269	-0.0223326	7.1136219	8.4344998	22.3882284	0.0019355
76	0.0906273	-0.0223326	7.1136389	8.4344820	22.3882254	0.0017361
77	0.0906277	-0.0223327	7.1136541	8.4344660	22.3882227	0.0015572
78	0.0906280	-0.0223327	7.1136678	8.4344516	22.3882203	0.0013968
79	0.0906283	-0.0223328	7.1136800	8.4344387	22.3882182	0.0012528
80	0.0906286	-0.0223328	7.1136910	8.4344272	22.3882163	0.0011238
81	0.0906288	-0.0223329	7.1137008	8.4344168	22.3882145	0.0010080
82	0.0906291	-0.0223329	7.1137097	8.4344075	22.3882130	0.0009041
83	0.0906293	-0.0223329	7.1137176	8.4343992	22.3882116	0.0008110
84	0.0906294	-0.0223330	7.1137247	8.4343917	22.3882104	0.0007274
85	0.0906296	-0.0223330	7.1137311	8.4343850	22.3882092	0.0006525
86	0.0906297	-0.0223330	7.1137368	8.4343790	22.3882082	0.0005853
87	0.0906299	-0.0223330	7.1137419	8.4343736	22.3882073	0.0005250
88	0.0906300	-0.0223330	7.1137465	8.4343688	22.3882065	0.0004709
89	0.0906301	-0.0223331	7.1137507	8.4343644	22.3882058	0.0004224
90	0.0906302	-0.0223331	7.1137544	8.4343605	22.3882052	0.0003789
91	0.0906302	-0.0223331	7.1137577	8.4343571	22.3882046	0.0003398
92	0.0906303	-0.0223331	7.1137607	8.4343539	22.3882041	0.0003048
93	0.0906304	-0.0223331	7.1137633	8.4343511	22.3882036	0.0002734
94	0.0906304	-0.0223331	7.1137657	8.4343486	22.3882032	0.0002452
95	0.0906305	-0.0223331	7.1137679	8.4343463	22.3882028	0.0002200
96	0.0906305	-0.0223331	7.1137698	8.4343443	22.3882025	0.0001973
97	0.0906306	-0.0223331	7.1137716	8.4343425	22.3882022	0.0001770
98	0.0906306	-0.0223331	7.1137731	8.4343409	22.3882019	0.0001588
99	0.0906307	-0.0223331	7.1137745	8.4343394	22.3882016	0.0001424
100	0.0906307	-0.0223332	7.1137757	8.4343381	22.3882014	0.0001277
101	0.0906307	-0.0223332	7.1137769	8.4343369	22.3882012	0.0001146
102	0.0906307	-0.0223332	7.1137779	8.4343358	22.3882011	0.0001028
103	0.0906308	-0.0223332	7.1137788	8.4343349	22.3882009	0.0000922

k	m^k	n^k	x^k	y^k	z^k	ER
104	0.0906308	-0.0223332	7.1137796	8.4343340	22.3882008	0.0000827
105	0.0906308	-0.0223332	7.1137803	8.4343333	22.3882006	0.0000742
106	0.0906308	-0.0223332	7.1137809	8.4343326	22.3882005	0.0000665
107	0.0906308	-0.0223332	7.1137815	8.4343320	22.3882004	0.0000597
108	0.0906308	-0.0223332	7.1137821	8.4343314	22.3882003	0.0000535
109	0.0906308	-0.0223332	7.1137825	8.4343309	22.3882002	0.0000480
110	0.0906309	-0.0223332	7.1137829	8.4343305	22.3882002	0.0000431
111	0.0906309	-0.0223332	7.1137833	8.4343301	22.3882001	0.0000386
112	0.0906309	-0.0223332	7.1137837	8.4343298	22.3882000	0.0000346
113	0.0906309	-0.0223332	7.1137840	8.4343294	22.3882000	0.0000311
114	0.0906309	-0.0223332	7.1137842	8.4343291	22.3881999	0.0000279
115	0.0906309	-0.0223332	7.1137845	8.4343289	22.3881999	0.0000250
116	0.0906309	-0.0223332	7.1137847	8.4343287	22.3881999	0.0000224
117	0.0906309	-0.0223332	7.1137849	8.4343285	22.3881998	0.0000201
118	0.0906309	-0.0223332	7.1137851	8.4343283	22.3881998	0.0000180
119	0.0906309	-0.0223332	7.1137852	8.4343281	22.3881998	0.0000162
120	0.0906309	-0.0223332	7.1137854	8.4343280	22.3881997	0.0000145
121	0.0906309	-0.0223332	7.1137855	8.4343278	22.3881997	0.0000130
122	0.0906309	-0.0223332	7.1137856	8.4343277	22.3881997	0.0000117
123	0.0906309	-0.0223332	7.1137857	8.4343276	22.3881997	0.0000105
124	0.0906309	-0.0223332	7.1137858	8.4343275	22.3881997	0.0000094
125	0.0906309	-0.0223332	7.1137859	8.4343274	22.3881997	0.0000084
126	0.0906309	-0.0223332	7.1137860	8.4343273	22.3881996	0.0000076
127	0.0906309	-0.0223332	7.1137860	8.4343273	22.3881996	0.0000068
128	0.0906309	-0.0223332	7.1137861	8.4343272	22.3881996	0.0000061
129	0.0906309	-0.0223332	7.1137861	8.4343271	22.3881996	0.0000055
130	0.0906309	-0.0223332	7.1137862	8.4343271	22.3881996	0.0000049
131	0.0906309	-0.0223332	7.1137862	8.4343270	22.3881996	0.0000044
132	0.0906309	-0.0223332	7.1137863	8.4343270	22.3881996	0.0000039
133	0.0906309	-0.0223332	7.1137863	8.4343270	22.3881996	0.0000035
134	0.0906309	-0.0223332	7.1137863	8.4343269	22.3881996	0.0000032
135	0.0906309	-0.0223332	7.1137864	8.4343269	22.3881996	0.0000028

k	m^k	n^k	x^k	y^k	z^k	ER
136	0.0906309	-0.0223332	7.1137864	8.4343269	22.3881996	0.0000025
137	0.0906309	-0.0223332	7.1137864	8.4343269	22.3881996	0.0000023
138	0.0906309	-0.0223332	7.1137864	8.4343268	22.3881996	0.0000021
139	0.0906309	-0.0223332	7.1137865	8.4343268	22.3881996	0.0000018
140	0.0906309	-0.0223332	7.1137865	8.4343268	22.3881996	0.0000017
141	0.0906309	-0.0223332	7.1137865	8.4343268	22.3881995	0.0000015
142	0.0906309	-0.0223332	7.1137865	8.4343268	22.3881995	0.0000013
143	0.0906309	-0.0223332	7.1137865	8.4343268	22.3881995	0.0000012
144	0.0906309	-0.0223332	7.1137865	8.4343267	22.3881995	0.0000011
145	0.0906309	-0.0223332	7.1137865	8.4343267	22.3881995	0.0000010
146	0.0906309	-0.0223332	7.1137865	8.4343267	22.3881995	0.0000009
147	0.0906309	-0.0223332	7.1137865	8.4343267	22.3881995	0.0000008
148	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000007
149	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000006
150	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000006
151	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000005
152	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000004
153	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000004
154	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000004
155	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000003
156	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000003
157	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000003
158	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000002
159	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000002
160	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000002
161	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000002
162	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000002
163	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000001
164	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000001
165	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000001
166	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000001

Estimates values are $m = 0.0906309$, $n = -0.0223332$, $x = 7.1137866$, $y = 8.4343267$ and $z = 22.3881995$ after 166 iterations.

The result from modified multivariate secant method.

k	m^k	n^k	x^k	y^k	z^k	ER
0	0.50	0.10	7.85	9.50	15.20	328.3618024
0	0.65	0.03	5.20	9.60	13.00	305.6856453
0	0.50	0.01	5.10	9.23	15.10	232.3860495
0	0.55	0.02	4.45	8.21	16.51	140.2600495
0	0.45	0.02	6.90	9.22	20.22	108.6561674
0	0.25	0.35	6.50	7.25	23.25	96.7683841
1	0.1688342	-0.0475569	6.5671699	9.2795791	23.5026935	44.4684002
2	0.1148978	-0.0325064	6.5741762	9.1706499	22.9746376	22.9903330
3	0.0972368	-0.0251356	6.6296998	9.0235585	22.6594567	11.1996321
4	0.0849542	-0.0221554	6.9054433	8.6696318	22.4308116	2.1542477
5	0.0910850	-0.0221259	7.2189659	8.2823795	22.3481986	1.0585664
6	0.0894758	-0.0223684	7.0569130	8.5123608	22.4111446	0.7162613
7	0.0904971	-0.0223251	7.1127974	8.4365248	22.3898023	0.0428029
8	0.0906269	-0.0223313	7.1145071	8.4336291	22.3881707	0.0059210
9	0.0906306	-0.0223331	7.1138382	8.4343196	22.3882442	0.0005821
10	0.0906309	-0.0223332	7.1137895	8.4343385	22.3882142	0.0002428
11	0.0906310	-0.0223332	7.1137870	8.4343236	22.3881970	0.0000421
12	0.0906309	-0.0223332	7.1137865	8.4343271	22.3881999	0.0000056
13	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000000

Estimates values are $m = 0.0906309$, $n = -0.0223332$, $x = 7.1137866$, $y = 8.4343267$ and $z = 22.3881995$ after 13 iterations.

The result from mid modified multivariate secant method.

k	m^k	n^k	x^k	y^k	z^k	ER
0	0.50	0.10	7.85	9.50	15.20	328.3618024
0	0.65	0.03	5.20	9.60	13.00	305.6856453
0	0.50	0.01	5.10	9.23	15.10	232.3860495
0	0.55	0.02	4.45	8.21	16.51	140.2600495
0	0.45	0.02	6.90	9.22	20.22	108.6561674
0	0.25	0.35	6.50	7.25	23.25	96.7683841
1	0.1688342	-0.0475569	6.5671699	9.2795791	23.5026935	44.4684002
2	0.1148978	-0.0325064	6.5741762	9.1706499	22.9746376	22.9903330
3	0.0490863	-0.0050402	6.7810774	8.6225335	21.8001586	20.2515364
4	0.0574298	-0.0124763	7.1042656	8.3247353	22.1033789	9.2457732
5	0.0837114	-0.0201001	7.4342248	8.0287699	22.2835861	4.2137528
6	0.0771146	-0.0183094	7.2926505	8.1805301	22.2549486	4.8562623
7	0.0834955	-0.0200050	7.2482420	8.2431909	22.3063438	2.8109155
8	0.1036221	-0.0271227	6.7595749	8.8984258	22.5252385	4.5491011
9	0.0891963	-0.0220000	7.1507882	8.3959623	22.3847187	0.3203046
10	0.0907503	-0.0223039	7.1231369	8.4240351	22.3890912	0.0180036
11	0.0907676	-0.0224216	7.1073645	8.4431745	22.3910929	0.0943631
12	0.0906996	-0.0223509	7.1132989	8.4351047	22.3888390	0.0229258
13	0.0906502	-0.0223401	7.1134501	8.4348115	22.3884397	0.0083068
14	0.0905545	-0.0223058	7.1151191	8.4324060	22.3872438	0.0330753
15	0.0906309	-0.0223332	7.1137873	8.4343257	22.3881993	0.0000092
16	0.0906310	-0.0223332	7.1137866	8.4343267	22.3881996	0.0000016
17	0.0906310	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000008
18	0.0906310	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000008
19	0.0906309	-0.0223332	7.1137866	8.4343267	22.3881995	0.0000000

Estimates values are $m = 0.0906309$, $n = -0.0223332$, $x = 7.1137866$, $y = 8.4343267$ and $z = 22.3881995$ after 19 iterations.

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