สำนักหอสมุดกลาง พระจอมเกล้าลาดกระบัง

ON THE DESIGN METHOD OF STRONGLY STABLE SYSTEM FOR ADAPTIVE POLE PLACEMENT CONTROL

PHORNSUK RATIROCH – ANANT

เลขหมู่..........
เลขทะเบียน 35748
วัน, เดือน, ปี 21 ก.ค. 2543

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENT FOR THE DEGREE OF
MASTER OF ENGINEERING IN ELECTRICAL ENGINEERING
SCHOOL OF GRADUATE STUDIES
KING MONGKUT'S INSTITUTE OF TECHNOLOGY LADKRABANG
2000

ISBN 974-622-779-3

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

COPYRIGHT 2000

SCHOOL OF GRADUATE STUDIES

KING MONGKUT'S INSTITUTE OF TECHNOLOGY LADKRABANG

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

หัวข้อวิทยานิพนธ์	วิธีออกแบบระบบควบคุมให้มีเสถียรภาพมั่นคงสำหรับ การควบคุมแบบอะแดพทีฟโพลเพลซเมนท์
นักศึกษา	นางพรสุข รติโรจน์อนันต์
รหัสประจำตัว	38621203
ปริญญา	วิศวกรรมศาสตรมหาบัณฑิต
สาขาวิชา	วิศวกรรมไฟฟ้า
พ.ศ.	2543
อาจารย์ผู้ควบคุมวิทยานิพนธ์	ศ.ดร.วัลลภ สุระกำพลธร
อาจารย์ผู้ควบคุมวิทยานิพนธ์ร่วม	รศ. ฮิโรชิ ฮิราตะ

บทคัดย่อ

ระบบควบคุมแบบอะแดพทิฟสร้างแบบจำลองอ้างอิงที่ออกแบบตัวควบคุมด้วยอะแดพทิฟโพลเพลซเมนท์ มีจุดบกพร่องคือ เมื่อพารามิเตอร์ของระบบเปลี่ยนแปลงในช่วงกว้าง ตัวควบคุมที่ออกแบบไว้จะไม่เสถียรถึงแม้ระบบควบคุมทั้งระบบจะมีเสถียรภาพ เนื่องจากค่าโพลวงปิดที่กำหนดไว้ใช้ออกแบบตัวควบคุมไม่ได้เปลี่ยนแปลงตาม ทำให้ไม่สามารถออกแบบระบบควบคุมที่มีเสถียรภาพมั่นคง

มีบทความจำนวนไม่มากที่พิจารณาเสถียรภาพสัมพัทธ์ของระบบควบคุม อีกประการหนึ่งยังไม่มีรายงานที่กล่าวถึงวิธีการปรับแต่งตัวควบคุมกับโพลวงปิดให้สอดคล้องกับการเปลี่ยนแปลงค่าพารามิเตอร์ในช่วงกว้างของระบบ

วิทยานิพนธ์ฉบับนี้นำเสนอวิธีปรับค่าโพลวงปิดอัตโนมัติเพื่อใช้ออกแบบตัวควบคุมอะแดพทิฟที่มีเสถียรภาพมั่นคง โดยใช้ค่าดัชนีเสถียรภาพตรวจสอบเสถียรภาพสัมพัทธ์ของตัวควบคุม และให้ดัชนีเสถียรภาพสัมพัทธ์นี้สัมพันธ์กับค่าโพลวงปิดและการวินิจฉัยแบบฟัซซีเพื่อหาขอบเขตเสถียรภาพและค่าโพลวงปิดที่เหมาะสมไปใช้ออกแบบตัวควบคุม วิธีนี้สามารถแก้ปัญหาตัวควบคุมไม่เสถียรได้แน่นอน คือสามารถออกแบบระบบควบคุมให้มีเสถียรภาพมั่นคงสำหรับการควบคุมแบบอะแดพทิฟโพลเพลซเมนท์

ได้มีการทดลองเลียนแบบระบบควบคุมอะแดพทิฟโพลเพลซเมนท์ตามวิธีที่นำเสนอหลายระบบเพื่อแสดงสมรรถนะของวิธีการออกแบบที่นำเสนอ

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

Thesis Title	On the Design Method of Strongly Stable System for Adaptive Pole Placement Control
Student	Ms. Phornsuk Ratiroch-anant
Student ID.	38621203
Degree	Master of Engineering
Programme	Electrical Engineering
Year	2000
Thesis Advisor	Prof. Dr. Wanlop Surakampontorn
Thesis Co-advisor	Assoc. Prof. Hiroshi Hirata

ABSTRACT

In conventional adaptive pole placement control system, there is a problem when the plant parameter fluctuates over a wide range, frequently, a series compensator having unstable poles appears, even if the closed-loop system is selected to be a stable system. Under this situation, selecting an appropriate pole to the closed-loop polynomial is difficult and the strongly stable control system can not be achieved.

Few design methods consider the relative stability of the control system, furthermore, adjustment of both the controller and the closed-loop pole according to the fluctuation of the plant parameter have not been reported.

This thesis proposes a method for automatically adjusting the pole of a stable closed-loop system and presents a design method for a strongly stable adaptive system in the case of the plant parameter is widely fluctuate. The stability index is introduced to evaluate the relative stability of the series compensator. This stability index is related to the closed-loop pole and is applied to the fuzzy inference to ensure the resultant stability index of the series compensator within the specified range. By doing so, the designer can easily obtain a unique solution to the problem of unstable controller. A design method for the adaptive system that recursively adjusts the poles of a stable closed-loop system using fuzzy inference is also proposed.

In addition, the numerical simulation is used to prove that the proposed method provides sufficient performance.

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

ACKNOWLEDGMENTS

I wish to express my great thanks to Professor Dr. Wanlop Surakampontorn, Professor of King Mongkut's Institute of technology Ladkrabang and Associate Professor Hiroshi Hirata, Associate professor of Tokai University for their valuable helps, suggestions and encouragement throughout the course of this thesis. Without their strong support, this thesis would not come true.

I am deeply grateful to JICA (Japan International Cooperation Agency) for their support on research tools such as personal computers, workstation computers, research equipments, budgets for attending the international conferences and budgets for doing this research for two months at the Department of Control Engineering, Tokai University, Japan.

I wish to thank Associate Professor Jongkol Ngamwiwit, Associate Professor Vipan Prijapanij, Associate Professor Jirapa Vitayapirak, Associate Professor Suthian Kiatsunthorn, Ajarn Taworn Benjanarasuth and Associate Professor Masatoshi Anabuki for their valuable helps and suggestions.

This thesis have been carried out with the great helps of Ajarn Khanigtha Saetang, special thanks have been owed to her for the kindness helps.

Finally, I would like to thank my sister, my son and my relative for their understandings, patience, loves and encouragement.

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

CONTENTS

	Page
Thai abstract	I
English abstract	II
Acknowledgments	III
Contents	IV
List of Tables	VII
List of Figures	VIII
Chapter 1 Introduction	1
1.1 Motivation	1
1.2 Design Approach	2
Chapter 2 Digital Controller Design	4
2.1 Poles Placement	4
2.1.1 Order of R-S-T controller	6
2.2 Regulation and Tracking	7
2.3 Tracking and Regulation With Independent Objective(MRAC)	10
2.3.1 Regulation	10
2.3.2 Tracking	11
2.4 Conclusion	12
Chapter 3 Design of Digital Controllers in the Presence of Disturbances	13
3.1 Random Disturbance Model	13
3.1.1 Description of Disturbances	13
3.1.2 ARX Model	15
3.1.3 ARMAX Model	16
3.2 Minimum Variance Tracking and Regulation	16
3.3 General Minimum Variance Tracking and Regulation	18
3.3.1 The GMV in ARX Pole Placement	20
3.4 Conclusion	21
Chapter 4 Recursive System Identification	22
4.1 Least Square (LS) Algorithm	22

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

CONTENTS (TO)

Page

4.2 Properties of Least Squares Estimator.............................. 24

4.3 Recursive Least Square (RLS) Algorithm........................... 25

4.4 Time-Varying Parameters.. 27

4.5 Problems in Using RLS Algorithm................................... 28

4.5.1 Estimator Windup....................................... 28

4.5.2 Wide Range Parameter Fluctuation.................. 28

4.5.3 Initial Conditions....................................... 29

4.6 RLS to ARX Model... 30

4.7 System Identification Simulation Examples......................... 31

4.7.1 No Noise and No Constant Disturbance.............. 31

4.7.2 Add Noise and Constant Disturbance................. 34

4.7.3 Plant Parameters Widely Fluctuate................... 36

4.8 Conclusion.. 38

Chapter 5 Adaptive Pole Placement Control.. 39

5.1 Design under Ideal Condition.. 39

5.2 Constant Disturbance Rejection....................................... 41

5.3 Steady State Compensation.. 44

5.4 Stability Index... 45

5.5 Stability Factor and Closed-loop Pole.............................. 46

5.6 Adaptive Pole Placement Simulation Examples..................... 47

5.6.1 No Noise Case... 47

5.6.2 Add Noise Case... 53

5.6.3 Add Constant Disturbance............................ 56

5.7 Conclusion.. 59

Chapter 6 Design of Strongly Stable System.. 60

6.1 Introduction to Fuzzy Control Inference............................ 60

6.2 Fuzzy Inference for Pole Adjustment................................ 61

6.3 Construction of Fuzzy Adaptive Pole Placement................. 63

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

CONTENTS (TO)

Page

6.4 Design Examples and Simulation Results.......................... 64

6.4.1 Application to Second Order Plant................ 64

6.4.1.1 The Order of Diophantine Equation

$nd = 3$.................................. 64

Simulation Results.................... 67

6.4.1.2 The Order of Diophantine Equation

$nd = 2$ 72

Simulation Results.................... 74

6.4.2 Application to Third Order Plant.................. 77

Simulation Results.................................. 81

6.5 Conclusion.. 87

Chapter 7 Conclusion and Suggestion.. 88

References.. 92

Appendix A... 94

Appendix B... 99

Appendix C... 103

Related Publications.. 106

Author Biography.. 107

King Mongkut's Institute of Technology Ladkrabang

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

LIST OF TABLES

Table Page

6.1 Control specification (2^{nd} order plant, $nd = 3$)........ 66

6.2 Control specification (2^{nd} order plant, $nd = 2$)........ 73

6.3 Control specification (3^{rd} order plant, $nd = 3$)........ 80

C.1 103

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

LIST OF FIGURES

Figure		Page
2.1	Pole placement with R-S-T controller	4
2.2	Regulation and tracking performance	8
2.3	Tracking and regulation with independent objectives	11
3.1	Discrete time white noise sequence	14
3.2	Spectral density of the discrete time white noise	14
3.3	The ARX model	15
3.4	The ARMAX model	16
3.5	Effect of a minimum variance control in the presence of random disturbances	17
3.6	Minimum variance tracking and regulation	18
3.7	General minimum variance tracking and regulation	19
3.8	The GMV to ARX system	21
4.1	General discrete plant model (ARX model)	22
4.2	Recursive system identification	25
4.3	The ARX model	30
4.4	Input signal	32
4.5	Plant output	32
4.6	Estimated parameters	33
4.7	Parameter accuracy	33
4.8	Plant output	34
4.9	Estimated parameters	35
4.10	Parameter accuracy	35
4.11	Plant output	36
4.12	Estimated parameters	37
4.13	Parameter accuracy	37
5.1	Block diagram of conventional adaptive pole placement control	40
5.2	Constant disturbance rejection	42
5.3	Block diagram of constant disturbance rejection	44
5.4	Steady state compensation	44
5.5	Simulation system	47

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

LIST OF FIGURES (TO)

Figure		Page
5.6	Input signal	48
5.7	Adaptive manipulated value	48
5.8	Plant output	49
5.9	Estimated model	49
5.10	Error (different) of plant and estimated model	50
5.11	Stability index	50
5.12	Adaptive manipulated value	51
5.13	Plant output	52
5.14	Stability index	52
5.15	Adaptive manipulated value	54
5.16	Plant output	54
5.17	Stability index	55
5.18	Adaptive manipulated value	56
5.19	Plant output	56
5.20	Estimated model	57
5.21	Error (different) of plant and estimated model	57
5.22	Stability index	58
6.1	Fuzzy logic control process	61
6.2	Membership function of antecedent	62
6.3	Membership function of consequent	62
6.4	Block diagram of proposed adaptive control system	63
6.5	Each gain margin g_m of plants Σ_f and Σ_s to stable closed-loop pole α.	65
6.6	Each phase margin p_m of plants Σ_f and Σ_s to stable closed-loop pole α	65
6.7	Each gain-crossover frequency ω_{cg} of plants Σ_f and Σ_s to stable closed-loop pole α	66
6.8	Each index γ of plants Σ_f and Σ_s to stable closed-loop pole α	66
6.9	Plant output controlled by conventional adaptive pole placement	68
6.10	Manipulated variable synthesized by conventional adaptive pole placement	68
6.11	Stability index γ in case of conventional adaptive pole placement	69

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

LIST OF FIGURES (TO)

Figure		Page
6.11-1	Stable closed-loop pole α in case of conventional adaptive pole .... Placement..	70
6.12	Stability index γ in case of proposed adaptive pole placement method (nd=3)	70
6.13	Stable closed-loop pole α in case of proposed method (nd=3).............	71
6.14	Plant output controlled by proposed adaptive pole placement method (nd=3)..	71
6.15	Each gain margin g_m of plants Σ_f and Σ_s to stable closed-loop pole α.	72
6.16	Each phase margin p_m of plants Σ_f and Σ_s to stable closed-loop pole α	72
6.17	Each gain-crossover frequency ω_{cg} of plants Σ_f and Σ_s to stable closed-loop pole α	73
6.18	Each index γ of plants Σ_f and Σ_s to stable closed-loop pole α..........	73
6.19	Manipulated variable synthesized by the proposed adaptive pole placement method (nd=2)..	74
6.20	Plant output controlled by the proposed adaptive pole placement method (nd=2)..	75
6.21	Stability index γ in case of proposed adaptive pole placement method (nd=2).....	75
6.22	Stable closed-loop pole α in case of proposed adaptive pole placement method (nd=2)...	76
6.23	Each index γ of plants Ψ_f and Ψ_s to stable closed-loop pole α.........	78
6.24	Each gain margin g_m of plants Ψ_f and Ψ_s to stable closed-loop pole α	78
6.25	Each phase margin p_m of plants Ψ_f and Ψ_s to stable closed-loop pole . α	79
6.26	Each gain-crossover frequency ω_{cg} of plants and Ψ_s to stable closed-loop pole α	79
6.27	Plant output controlled by conventional adaptive pole placement.........	81
6.28	Manipulated variable synthesized by conventional adaptive pole	

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

LIST OF FIGURES (TO)

Figure		Page
	placement..	82
6.29	Stability index γ in case of conventional adaptive pole placement........	82
6.30	Stable closed-loop pole α in case of conventional adaptive pole placement..	83
6.31	The estimated parameters of both Ψ_{f1} and Ψ_{s1} plants......................	83
6.32	Plant output controlled by proposed adaptive pole placement method....	84
6.33	Manipulated variable synthesized by proposed adaptive pole placement method..	85
6.34	Stability index γ in case of proposed adaptive pole placement method...	85
6.35	Stable closed-loop pole α in case of proposed adaptive pole placement method..	86
6.36	Estimated parameters of both Ψ_{f1} and Ψ_{s1} plants..........................	86
A.1	The General Minimum Variance tracking and regulation.................	98
C.1	Membership function of antecedent..	102
C.2-1	γ=0.4..	103
C.2-2	γ=1.1..	103
C.2-3	γ=1.4..	103
C.2-4	γ=2.2..	104

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Chapter1

Introduction

It is well accepted that the optimal design problem of adaptive system is an important research theme. This is because the realization of the strongly stable system [1] is valuable for the commercialization of adaptive control [2]. Therefore, in this thesis, the design method of strongly stable system for adaptive pole placement control [2],[3],[4] is proposed as the design method that a strongly stable controller can be obtained.

Normally, in order to design the control system to achieve the required specifications, the closed-loop polynomial of the overall adaptive control system is firstly derived. Based on the fluctuation of the plant parameters, the controller parameters are subsequently computed from the closed-loop polynomial. Now an experienced control engineer is able to apply the appropriate design method for a particular application. By this approach, however, the strongly stable control system can not usually be achieved.

In this thesis, the design method that introduce both the stability index [5],[6] and fuzzy inference [7],[8],[9] to adjust both the controller parameter and closed-loop poles of closed-loop system according to the fluctuation of plant parameter is proposed. The detail and discussions on the proposed method will be presented in chapter 5 and chapter 6.

1.1 Motivation

There have been many algorithms for adaptive control system proposed in conferences and journals. Among these algorithms, Model Reference Adaptive Control (MRAC) [2] is an algorithm that is effective in controlling the unknown parameter plant. Because the plant parameters can be identified by measuring from the input and output data [4],[10],[11] and subsequently the appropriate controller can be designed. But the weak point of this method is about the appearance of unstable plant zeros [2],[12].In this thesis, the adaptive pole placement control design method that separately consider the structure of the system identification and control system is proposed. In other words, the identification part and the control part of the

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

proposed adaptive system are independent. Therefore, the designer need not to be worry about the problem of unstable zeros because it does not use unstable pole zeros cancellation.

Next problem is when the plant parameter widely fluctuate, for example in the motor speed control system, the inertia usually fluctuates in a wide range. In such case it is well known that the unstable pole will appear in the series compensator of controller even if the closed-loop is designed as the stable system and the control performance is deteriorated. The problem is how to select the pole of the closed-loop system in order to obtain the strongly stable controller in a full range of parameter fluctuation. Practically, there is no degree of freedom in selecting this pole as a constant value without changing the parameter of the series compensator. In the past, there are few design methods that consider the relative stability of the control system itself and there are several methods [13],[14],[15],[16] that consider the robustness of the adaptive law. However, the design method that consider the adjustment of both the controller parameter and closed-loop poles according to the fluctuation of plant parameter has not yet reported.

1.2 Design Approach

For an unknown and fluctuation parameter plant, the objective for the design of the adaptive control is to realize a compensator that achieves a set of performance requirements, whether they be open-loop characteristics such as gain-phase margin, or closed-loop characteristics such as settling time according to the required control specification. To this end, the design method begins from chapter 2, when the plant parameters are known, the tribranches compensator or controller can be designed by the pole placement method. The order of controller should be selected such that both the plant and controller transfer function are proper. The tracking performance of this control strategy is defined from the disturbance rejection speed of the null reference output response and this value is used as the stable pole in designing the controller. The tracking performance can be achieved with the dynamic model connected to the reference input.

For the case of random disturbances subject to the system output, the disturbance model is needed. ARX or ARMAX model will be selected according to the noise type. The important problem in this case is how to obtain the appropriate

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

control law to satisfy the minimum variance control. Chapter 3 presents both the minimum variance and general minimum variance (GMV) [2],[4],[17],[18],[19] control laws.

When the plant parameters are not known, the parameter estimation should be provided before designing the controller. In some plants, the unknown parameters fluctuate widely and continuously such as the servo motor control and the milling system. In such plants recursive parameter estimation is used to provide the real time estimation. Adaptive control is the real time control system that use the recursive parameter estimation. Chapter 4 presents the Recursive Least Squares method for system identification and problems in using this method [19]. Some simulations on system identification are performed.

Chapter 5 shows the details of controller design in adaptive control system by pole placement method and discusses the relative stability of designed controller with stability index. Disturbance and steady state error compensation also discussed. At the end of this chapter, some simulations on controller design and relative stability evaluation of adaptive pole placement control system are presented.

When the plant parameter fluctuates widely, the designed controller becomes unstable. This problem can overcome with the automatic pole adjustment by evaluating the fine range stability index of designed controller and use it together with the fuzzy inference. Chapter 6 presents the method of evaluating the relative stability and the fine range stability index together with fuzzy inference under the control specifications by using the designed simulations.

Chapter 7 gives the overall discussions and suggestions of the proposed design method.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Chapter 2

Digital Controller Design

The computation of the digital controller parameter is a special case of the pole placement strategy. This strategy enables an R-S-T tribranches canonical form digital controller to be designed for both stable and unstable systems.

2.1 Poles Placement

The R-S-T tribranches canonical form digital controller is shown in figure 2.1, $r(k)$ is the reference input, $u(k)$ is the control input and $y(k)$ is the output of the plant. Since it does not introduce additional zeros, so there is no need to worry about the overshoot of the response. The plant transfer function is

$$G_p(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}, \text{ d = delay time.}$$

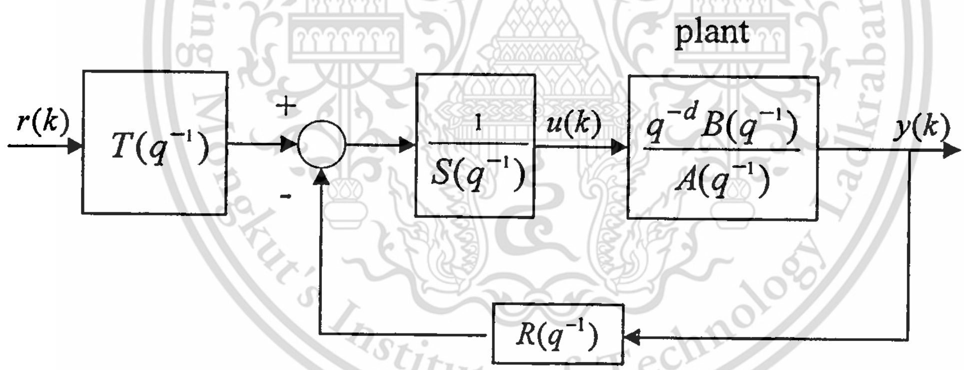

Figure 2.1 Pole placement with R-S-T controller

The series compensator $S(q^{-1})$ is in the forward part, the parallel compensator $R(q^{-1})$ is in the feedback part and $T(q^{-1})$ is the prefilter. The close-loop transfer function is

$$H_{CL}(q^{-1}) = \frac{q^{-d}T(q^{-1})B(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} \tag{2.1}$$

in which

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$$\begin{aligned} A(q^{-1}) &= \textit{plant denominator} \\ &= 1 + a_1 q^{-1} + \ldots + a_n q^{-n} \end{aligned} \tag{2.2}$$

$$\begin{aligned} B(q^{-1}) &= \textit{plant numerator} \\ &= b_0 + b_1 q^{-1} + \ldots + b_m q^{-m} \end{aligned} \tag{2.3}$$

where n and m are the order of $A(q^{-1})$ and $B(q^{-1})$ respectively. The close-loop polynomial is

$$\begin{aligned} D(q^{-1}) &= A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) \\ &= 1 + d_1 q^{-1} + \ldots + d_{nd} q^{-nd}. \end{aligned} \tag{2.4}$$

This equation is known as the Diophantine equation or Bezout identity. The coefficient of $D(q^{-1})$ is linear combination of constant coefficients of $B(q^{-1})$ and $A(q^{-1})$ and unknown coefficient of $S(q^{-1})$ *and* $R(q^{-1})$ which are given as

$$S(q^{-1}) = 1 + s_1 q^{-1} + \ldots + s_{ns} q^{-ns} \tag{2.5}$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \ldots + r_{nr} q^{-nr} \tag{2.6}$$

where *nd, ns* and *nr* are the order of $D(q^{-1}), S(q^{-1})$ and $R(q^{-1})$ respectively. From Eqns. (2.4), (2.5), and (2.6) the coefficients can be matched and the Diophantine equation can be calculated from the following matrix equation.:

$$\underbrace{\left[\begin{array}{cccccc} 1 & 0 & . & . & . & 0 \\ a_1 & 1 & . & & & . \\ a_2 & a_1 & . & . & & . \\ . & . & . & . & . & . \\ . & . & . & . & . & 0 \\ . & . & & . & . & 1 \\ a_n & . & & & . & a_1 \\ 0 & a_n & & & & . \\ . & . & . & & & . \\ . & & . & . & & . \\ . & & & . & . & . \\ 0 & . & . & . & 0 & a_n \end{array}\right.}_{ns+1}\underbrace{\left|\begin{array}{cccccc} b_0 & 0 & . & . & . & 0 \\ b_1 & b_0 & . & & & . \\ b_2 & b_1 & . & . & & . \\ . & . & . & . & . & . \\ . & . & . & . & . & 0 \\ . & . & & . & . & b_0 \\ b_m & . & & & . & b_1 \\ 0 & b_m & & & & . \\ . & . & . & & & . \\ . & & . & . & & . \\ . & & & . & . & . \\ & . & . & . & 0 & b_m \end{array}\right]}_{nr+1}\begin{bmatrix} 1 \\ s_1 \\ . \\ . \\ . \\ s_{ns} \\ r_0 \\ r_1 \\ . \\ . \\ . \\ r_{nr} \end{bmatrix} = \begin{bmatrix} 1 \\ d_1 \\ . \\ . \\ . \\ d_{nd} \end{bmatrix} \tag{2.6a}$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$$\mathbf{MX} = \mathbf{D}. \tag{2.7}$$

Matrix **M** is known as the Sylvester matrix, **D** is the vector of the coefficient of Eqn. (2.4) and **X** is a vector of unknown controller parameters, $D(q^{-1})$ is specified by the designer and **M** is a constant matrix. The solvability of the Diophantine equation can be considered from the Sylvester matrix **M.**

First the plant polynomial must co-prime, it means that $B(q^{-1})$ and $A(q^{-1})$ must have no common factors. If such common factor exits, then the roots also appear as the roots of the Diophantine equation. Thus, the system is forced to accept the roots as the pole and resulting in constrained pole placement. Furthermore, coprimeness of $B(q^{-1})$ and $A(q^{-1})$ ensures that the Sylvester matrix is nonsingular. Therefore, if the Sylvester matrix is nonsingular and square, then the controller parameters can be calculated as follows. :

$$\mathbf{X} = \mathbf{M}^{-1}\mathbf{D}. \tag{2.8}$$

2.1.1 Order of R-S-T Controller

The dimension of the Sylvester matrix depends on the order of the controller. By choosing the proper controller order such that the Sylvester matrix is square, the controller parameter can be uniquely solved if $A(q^{-1})$ and $B(q^{-1})$ do not have any common factors. However, the non-uniqueness solution will be due to the orders of $S(q^{-1})$ and $R(\mathrm{q}^{-1})$ where difference choices of ns and nr will give different $S(q^{-1})$ and $R(\mathrm{q}^{-1})$.

To condition the problem for a unique solution, the number of equation and number of unknown in Eqn.(2.7) are equated. While the number of unknowns is the number of coefficients in the compensator $S(q^{-1})$ and $R(\mathrm{q}^{-1})$, the number of equations is simply the number of coefficients in the $D(q^{-1})$ or it is the order of $A(q^{-1})$ plus the order of $S(q^{-1})$. Therefore,

$$\text{num - unknowns} = ns + nr + 1 \tag{2.9}$$

$$\text{num - equations} = n + ns \tag{2.10a}$$

$$= nd. \tag{2.10b}$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Since both the pant and controller transfer function are assumed to be proper, the order of the $D(q^{-1})$ can be represented by $nd = n + ns$. By equating the number of equations in Eqn.(2.10a) to the number of unknowns, the following relationship is obtained

$$ns + nr + 1 = n + ns \tag{2.11}$$

$$nr = n\text{-}1 \tag{2.12}$$

$$ns \geq nr \qquad \text{for properness.} \tag{2.13}$$

Hence, for a unique solution, the order of $R(q^{-1})$ must be one less than the order of $A(q^{-1})$, ie. $nr = n\text{-}1$. Meanwhile, properness of the controller must be maintained by satisfying $ns \geq nr$.

The fact that ns is cancelled in Eqn.(2.11) implies that $S(q^{-1})$ can be chosen arbitrarily provided that the controller is proper. From Eqn.(2.4) ns is depends on the value of d, such as

$$ns \leq m + d - 1\,. \tag{2.14a}$$

From Eqn.(2.10b), $nd - n = n_s$, then,

$$nd \leq \; n + m + d - 1\,. \tag{2.14b}$$

Given the 4th order plant, for example, if $d = 2$, then, the Sylvester matrix will be squared with the dimension of r x r, ie $r = n + m + d = 9$, for a 3rd order of $R(q^{-1})$ and a 4th order of $S(q^{-1})$.

2.2 Regulation and Tracking[2]

The closed-loop poles which are the roots of the polynomial $D(q^{-1})$ defines the regulation performances of control systems. $D(q^{-1})$ is specified directly from the desired performance which is the disturbance rejection speed of the null reference output response. By doing so, $D(q^{-1})$ can be written in the form of binomial as follow

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$$D(q^{-1})=(1-\alpha q^{-1})^{nd} \tag{2.15}$$

where α is the stable poles of the discrete time close-loop system and ensuring the condition

$$0<\alpha<1. \tag{2.16}$$

Then, the controller $S(q^{-1})$ and $R(q^{-1})$ can be designed under the desired regulation performance.

When the reference input changes, it is desired that the system output follows a desired trajectory $y^*(k)$. This trajectory may be stored or generated each time the reference is changed by using a dynamic model, this is known as "tracking", see figure 2.2.The dynamic model $\dfrac{Bm(q^{-1})}{Am(q^{-1})}$ can be determined from desired performances (rise time, overshoot, and settling time) by passing the step function, for example, through a second order normalized continuous time model and transforming into pulse transfer function. To approach the reference trajectory and the delay d can be compensated, then

$$y^*(k)=\frac{q^{-(d+1)}Bm(q^{-1})}{Am(q^{-1})}r(k) \tag{2.17}$$

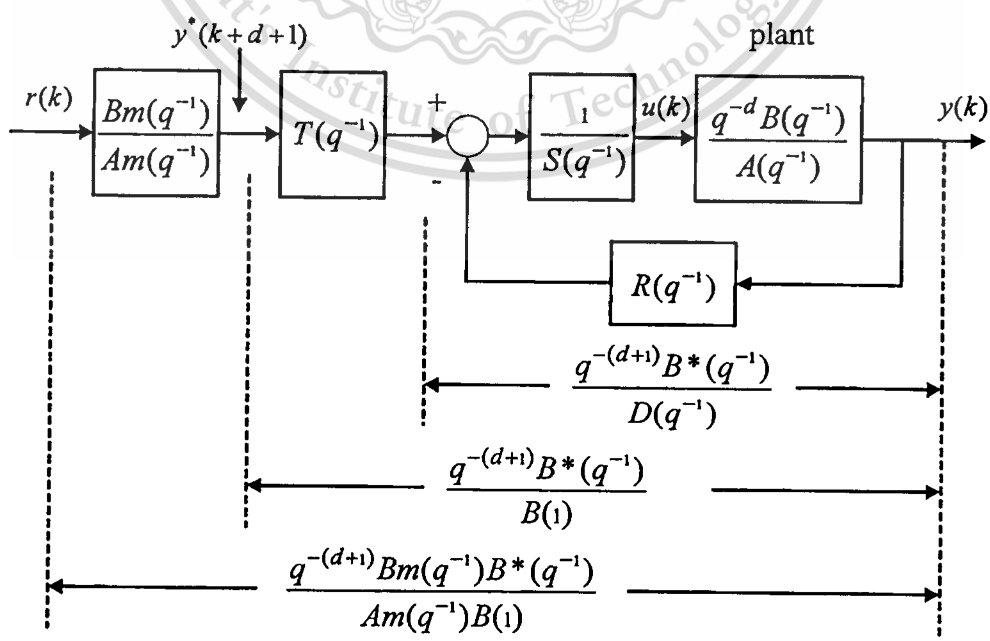

Figure 2.2 Regulation and tracking performance

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

where

$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots$

$A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \ldots$

and $q^{-d}B(q^{-1})$ should be rewritten as $q^{-(d+1)}B^*(q^{-1})$; $B(q^{-1}) = q^{-1}B^*(q^{-1})$.

For this, first $y^*(k+d+1)$ is generated from $r(k)$ as shown below

$$y^*(k+d+1) = \frac{Bm(q^{-1})}{Am(q^{-1})}r(k). \tag{2.18}$$

Ideal tracking is achieved by selecting the prefilter $T(q^{-1})$ properly to ensure a unitary steady state gain of the overall system and the compensation of the regulation dynamics $D(q^{-1})$.

This leads to the choice :

$$T(q^{-1}) = GD(q^{-1}) \tag{2.19}$$

in which

$$G = \frac{1}{B(1)} \quad \text{if } B(1) \neq 0, \qquad = 1 \quad \text{if } B(1) = 0. \tag{2.20}$$

The control law equation becomes

$$S(q^{-1})u(k) + R(q^{-1})y(k) = T(q^{-1})y^*(k+d+1) \tag{2.21}$$

then

$$u(k) = \frac{T(q^{-1})y^*(k+d+1) - R(q^{-1})y(k)}{S(q^{-1})} \tag{2.22}$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

which yields the complete pole placement as shown in figure 2.2. The transfer function between the reference and the output is

$$H_{CL}(q^{-1}) = \frac{q^{-(d+1)}Bm(q^{-1})}{Am(q^{-1})} \times \frac{B^*(q^{-1})}{B(1)}. \tag{2.23}$$

2.3 Tracking and Regulation with Independent Objectives or MRAC

In some cases the designer needs to obtain the desired tracking performance (changing of reference or noise suppression) independent of the desired regulation performance (rejection of disturbance). For example, in a situation for which the desired regulation response time is significantly smaller than the desired response time for a change in reference or vice versa.

This case results in the simplification of the zero $B(q^{-1})$ of the discrete time plant model. By doing so, the strategy can only be applied to discrete time models with stable zeros which implies that the plant must be a minimum phase. The structure of this method is shown in figure 2.3

2.3.1 Regulation

The regulation performance is specified by $D(q^{-1})$. To compensate $B^*(q^{-1})$, $D(q^{-1})$ should has $B^*(q^{-1})$ and in order to compute $R(q^{-1})$ and $S(q^{-1})$, the following equation must be solved

$$A(q^{-1})S(q^{-1}) + q^{-(d+1)}B^*(q^{-1})R(q^{-1}) = B^*(q^{-1})D(q^{-1}). \tag{2.25}$$

In this equation, $S(q^{-1})$ should has $B^*(q^{-1})$

$$\begin{aligned} S(q^{-1}) &= S_0 + S_1 q^{-1} + \ldots + S_{m+d} q^{-(m+d)} \\ &= B^*(q^{-1}) S'(q^{-1}) \end{aligned} \tag{2.26}$$

in which

$$S'(q^{-1}) = 1 + S'_1 q^{-1} + \ldots + S'_d q^{-d}. \tag{2.27}$$

From Eqns. (2.25) and (2.26), one obtains

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$$A(q^{-1})S'(q^{-1})+q^{-(d+1)}R(q^{-1})=D(q^{-1}). \qquad (2.28)$$

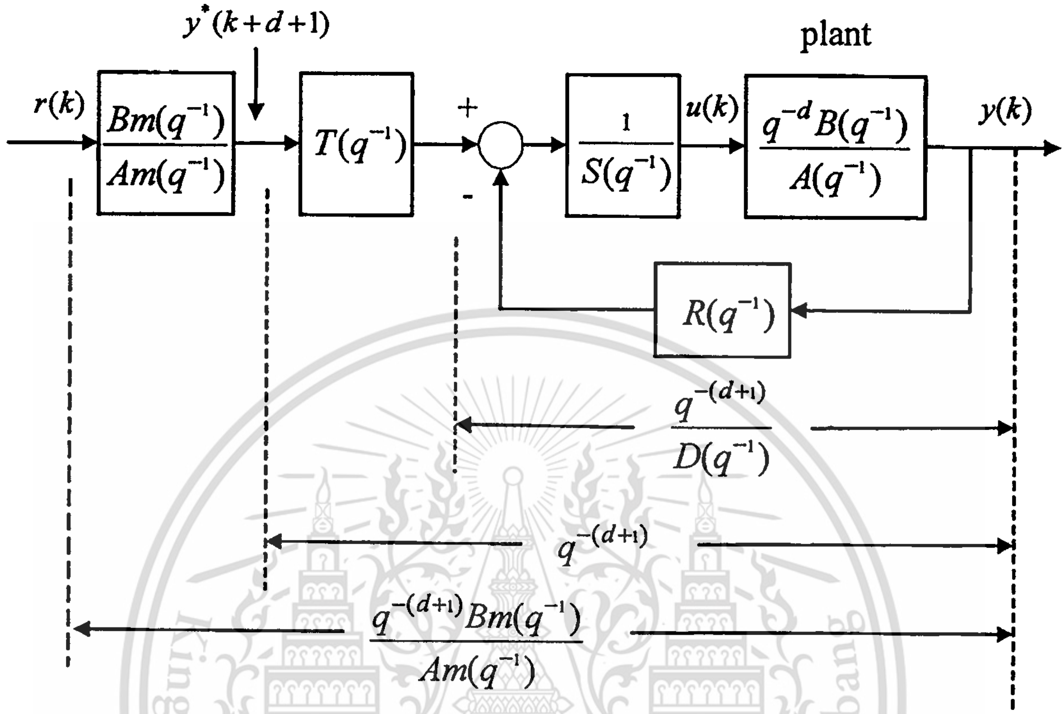

Figure 2.3 Tracking and regulation with independent objectives

This equation has unique solution for

$$\deg D(q^{-1}) \le n+d,\ \deg S'(q^{-1})=d,\ \deg R(q^{-1})=n-1 \qquad (2.29)$$

in which

$$R(q^{-1})=r_0+r_1q^{-1}+\dots+r_{n-1}q^{n-1}.$$

2.3.2 Tracking

From figure 2.3 the tracking performance is defined by the transfer function

$$\frac{y(k)}{r(k)}=\frac{q^{-(d+1)}Bm(q^{-1})}{Am(q^{-1})}. \qquad (2.30)$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

The output of the tracking (dynamic) model specifies the desired trajectory $y*$ with $d+1$ steps in advance to accomplish the asymptotically tracking. To ensure this, the prefilter will be selected as

$$T(q^{-1}) = D(q^{-1}) \tag{2.31}$$

then, from figure 2.3, the controller equation can be obtained :

$$\begin{aligned} &S(q^{-1})u(k) + R(q^{-1})y(k) = D(q^{-1})y*(k+d+1) \\ &\text{where} \quad S(q^{-1}) = B*(q^{-1})S^1(q^{-1}) \end{aligned} \tag{2.32}$$

and $u(k)$ can be written as

$$u(k) = \frac{D(q^{-1})y*(k+d+1) - R(q^{-1})y(k)}{S(q^{-1})}. \tag{2.33}$$

2.5 Conclusion

This chapter presents digital control designs. The controller has a tribranches structure known as R-S-T canonical form, in which the control law equation is

$$u(k) = \frac{T(q^{-1})y*(k+d+1) - R(q^{-1})y(k)}{S(q^{-1})}.$$

From the pole placement strategy, the digital controller is designed in two steps

(1) The computation of the polynomials $S(q^{-1})$ and $R(q^{-1})$ through the Diophantine equation in order to match the desired regulation performances.

(2) The computation of the pre-filter $T(q^{-1})$ in order to approach the desired tracking performances.

The orders of $S(q^{-1})$ and $R(q^{-1})$ polynomials are depend only on the complexity of the plant model.

The tracking and regulation with independent objectives design strategy (MRAC) can exactly match the desire tracking and regulation performance, but **the discrete time plant model must have stable zeros.**

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

Chapter 3

Design of Digital Controllers in the Presence of Disturbances

In the real world, a large number of systems are subjected to disturbances of the random nature. Therefore the models enabling these random disturbances are discussed in this chapter. Two design methods are presented, one is the minimum variance tracking and regulation, the other one is the generalized minimum variance tracking and regulation.

3.1 Random Disturbances Model

In practice, the majority of random disturbances occurring in automatic control systems can be described as passing the discrete time white noise sequence through a filter. This filter is called "random disturbances model". ARX and ARMAX are the practical random disturbances models that used in many methods of system identification and adaptive control system. Both models will be considered later.

3.1.1 Description of Disturbances [4]

In the case of the disturbance is the discrete time white noise sequence, such a sequence is the independent and equally distributed random variables of zero mean value and variance σ^2 with an energy uniformly distributed among all the frequencies between 0 and 0.5 *fs* (the sampling frequencies). The white noise sequence will be noted as $e(k)$ and will be characterized by the parameters $(0,\sigma)$, where such a sequence is presented in figure 3.1.The mean value is given by

$$MV = E\{e(k)\} = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} e(k) = 0 \tag{3.1}$$

and the variance will be given by

$$\text{var} = E\{e^2(k)\} = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} e(k)^2 = \sigma^2 . \tag{3.2}$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

In which E denotes the expectation operator and N denotes the number of data points.

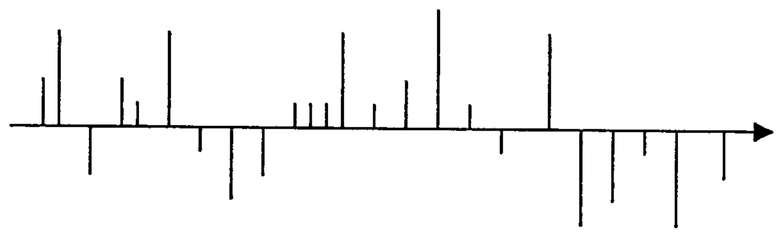

Figure 3.1 Discrete time white noise sequence

The covariance function $R(i)$ is defined as follows

$$R(i) = E\{e(k)e(k-i)\} = \lim_{i\to\infty}\frac{1}{N}\sum_{i=1}^{N} e(k)e(k-i)$$
$$R(i) = 0 \ , \ i \neq 0 \qquad (3.3)$$

and

$$R(0) = \text{var} = \sigma^2. \qquad (3.4)$$

The spectral density function of white noise sequence is given by discrete Fourier transform of $R(i)$ as

$$\phi_e(w) = \frac{1}{2\pi}\sum_{i=-\infty}^{\infty} R(i)e^{-jiw} \qquad (3.5)$$

and is shown in figure 3.2.

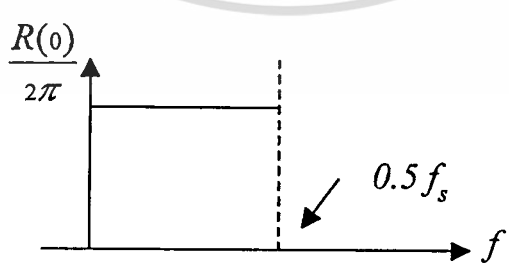

Figure 3.2 Spectral density of the discrete time white noise

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

3.1.2 ARX Model

The ARX model is used to represent the effect of both the control and disturbances on the plant output as shown in figure 3.3. The random disturbance is performed by passing the white noise sequence $e(k)$ through this model. Where AR refers to the auto-regressive part $A(q^{-1})y(k)$ and X to the extra input $B(q^{-1})u(k)$ which is called the exogenous variable in econometrics. The disturbed output $y(k)$ is written by

$$y(k) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k) + \frac{e(k)}{A(q^{-1})} \tag{3.6}$$

where $A(q^{-1}) = 1 + \sum_{i=1}^{n} a_i q^{-1}$ and $B(q^{-1}) = \sum_{i=1}^{m} b_i q^{-1}$.

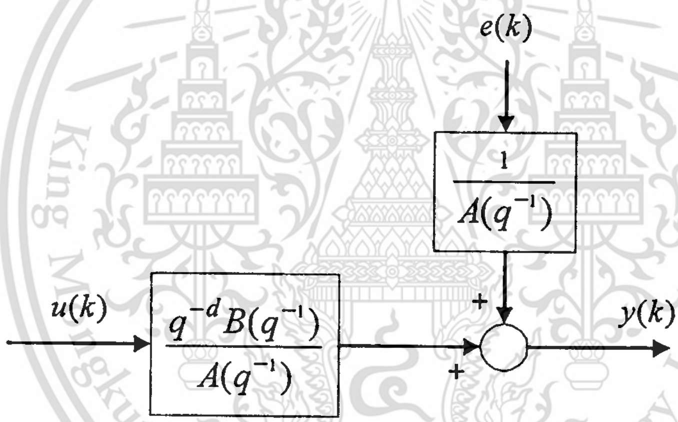

Figure 3.3 The ARX model

The input-output relationship is obtained by describing it as a linear difference equation

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{m} b_i u(k-d-i) + e(k)\,. \tag{3.7}$$

Since the white noise term $e(k)$ enters as an equation disturbance, then the model in Eqn.(3.7) is often called an equation error model.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

3.1.3 ARMAX Model

The basic disadvantage of the ARX model is lack of adequate freedom in describing the properties of the disturbance term. The flexibility to that, by describing the equation error as a moving average of white noise, can be added. This model known as "ARMAX model" as shown in figure 3.4, the added polynomial is $C(q^{-1}) = 1 + \sum_{i=1}^{n} c_i q^{-1}$.

The part $C(q^{-1})e(k)$ is view of the moving average MA. This model becomes a standard tool in control and econometrics for both system description and control design. The disturbed output is

$$y(k) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k) + \frac{C(q^{-1})}{A(q^{-1})} \cdot e(k). \qquad (3.8)$$

The input-output linear difference equation is

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{m} b_i u(k-d-i) + \sum_{i=1}^{n} c_i e(k-i) + e(k). \qquad (3.9)$$

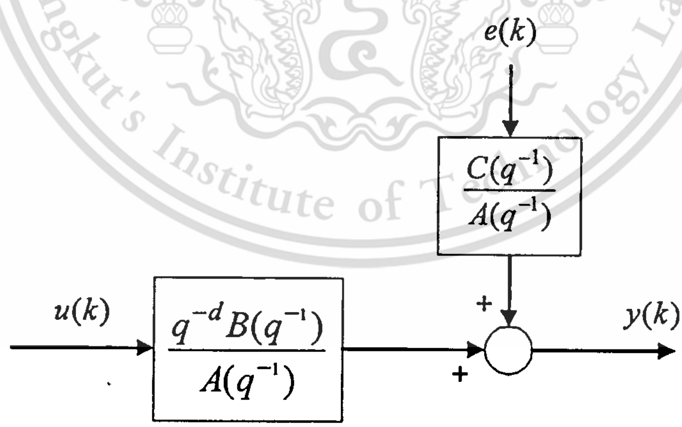

Figure 3.4 The ARMAX model

3.2 Minimum Variance Tracking and Regulation

The idea behind minimum variance control of system is to select the appropriate control law $u(k)$ when the random disturbance is presented to the system.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

สำนักหอสมุดกลาง พระจอมเกล้าลาดกระบัง

So that the output has the smallest possible variance around zero mean value as shown in figure 3.5. This will minimizes the following criterion

$$\min E\{[y(k)-y^*(k)]^2\} = \min \frac{1}{N}\sum_{k=1}^{N}[y(k)-y^*(k)]^2 . \tag{3.10}$$

In which $y(k)-y^*(k)$ represents the difference with respect to the desired value at instant k and $y^*(k)$ is the reference trajectory output.

To solve the minimum variance problem, the ARMAX model are considered, in this case the general process plus the disturbance should be added to the system of tracking and regulation with independent objective as shown in figure 3.6. The output $y(k)$ is described by

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + C(q^{-1})e(k) \tag{3.11}$$

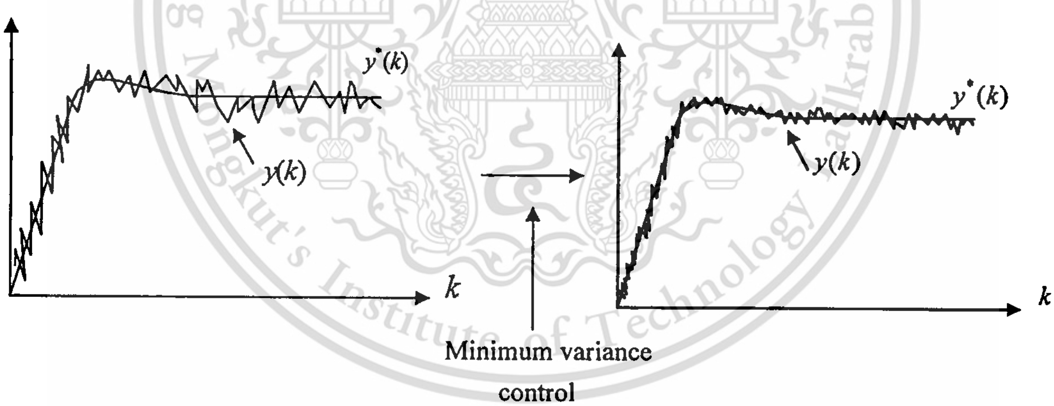

Figure 3.5 Effect of a minimum variance control in the presence of random disturbances

where

$$A(q^{-1}) = 1 + \sum_{i=1}^{n} a_i q^{-1} \tag{3.12}$$

$$B(q^{-1}) = \sum_{i=1}^{m} b_i q^{-1} = q^{-1}B^*(q^{-1}) \tag{3.13}$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

35748

$$C(q^{-1}) = 1 + \sum_{i=1}^{n} c_i q^{-1}. \tag{3.14}$$

Note that $B(q^{-1})$ **must have stable zeros as well as** $C(q^{-1})$ which will specify the close- loop poles, $C(q^{-1})$ is always stable if the disturbance is stationary. The control law is given by

$$u(k) = \frac{C(q^{-1})y*(k+d+1) - R(q^{-1})y(k)}{S(q^{-1})}. \tag{3.15}$$

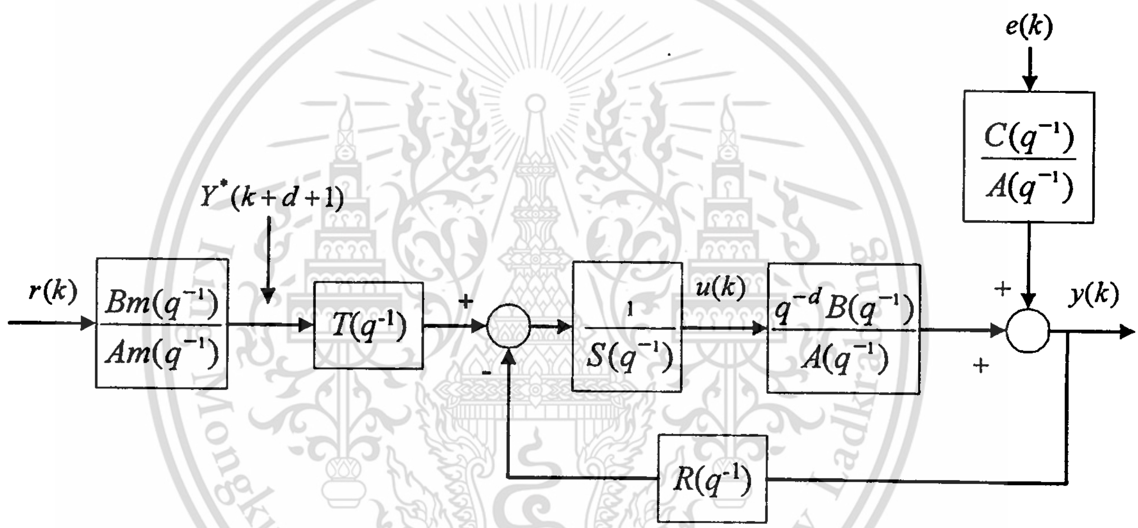

Figure 3.6 Minimum variance tracking and regulation

The structure of the minimum variance control law as shown in Eqn.(3.15) is the same as that for tracking and regulation with independent objective in Eqn. (2.33) if

$$D(q^{-1}) = C(q^{-1}) = T(q^{-1}). \tag{3.16}$$

3.3 General Minimum Variance Tracking and Regulation

The drawback of the minimum variance control is that **all the plant zeros are cancelled.** Therefore, this control strategy is unable to apply to the plants with unstable zeros. In order to improve this control strategy, Clarke and Gawthrop[17], [18] introduced the general minimum variance (GMV) tracking and regulation (see appendix A) as shown in figure 3.7. The ARMAX system is

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + C(q^{-1})e(k). \tag{3.17}$$

The performance index for evaluating the control system is defined as the following equation

$$J \cong E\left[\varepsilon(k+d)^2\right] \tag{3.18}$$

where $\varepsilon(k+d)$ is given by

$$\varepsilon(k+d) \cong P(q^{-1})y(k+d) + Q(q^{-1})u(k) - H(q^{-1})r(k+d). \tag{3.19}$$

The optimal input $u(k)$ that minimizes the variance J is given by

$$u(k) = \frac{C(q^{-1})H(q^{-1})r(k+d) - B(q^{-1})R(q^{-1})y(k)}{B(q^{-1})S(q^{-1}) + C(q^{-1})Q(q^{-1})} \tag{3.20}$$

where the polynomials $S(q^{-1})$ and $R(q^{-1})$ are the solution of the following Diophantine equation

$$P(q^{-1})C(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}). \tag{3.21}$$

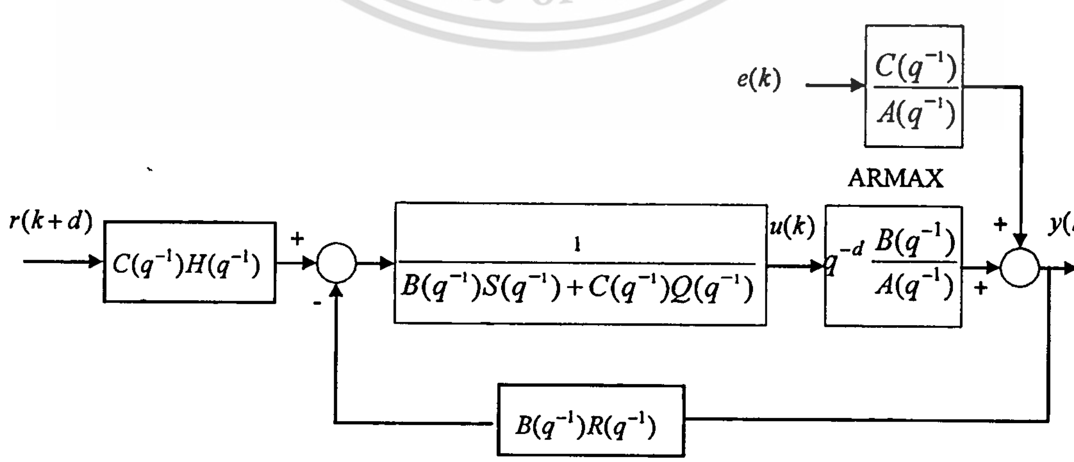

Figure 3.7 General minimum variance tracking and regulation

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

3.3.1 The GMV in ARX Pole Placement Control

In this thesis, the ARX model is proposed. It can be considered as a special case of GMV by the replacement of some variables.
In the ARX system

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + e(k). \tag{3.22}$$

If we assume that

$$P(q^{-1}) = D(q^{-1}), H(q^{-1}) = T(q^{-1})B(q^{-1}), Q(q^{-1}) = 0, C(q^{-1}) = 1. \tag{3.23}$$

The following relations can be obtained:

the performance index

$$J \cong E\left[\varepsilon(k+d)^2\right] \tag{3.24}$$

$$\varepsilon(k+d) = D(q^{-1})y(k+d) - T(q^{-1})B(q^{-1})r(k+d) \tag{3.25}$$

the optimal control law that minimized the index *J*

$$\begin{aligned} u(k) &= \frac{T(q^{-1})B(q^{-1})r(k+d) - B(q^{-1})R(q^{-1})y(k)}{B(q^{-1})S(q^{-1})} \\ &= \frac{1}{S(q^{-1})}\left[T(q^{-1})r(k+d) - R(q^{-1})y(k)\right] \end{aligned} \tag{3.26}$$

Diophantine equation

$$D(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}). \tag{3.27}$$

The GMV in ARX pole placement control is shown in figure 3.8

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

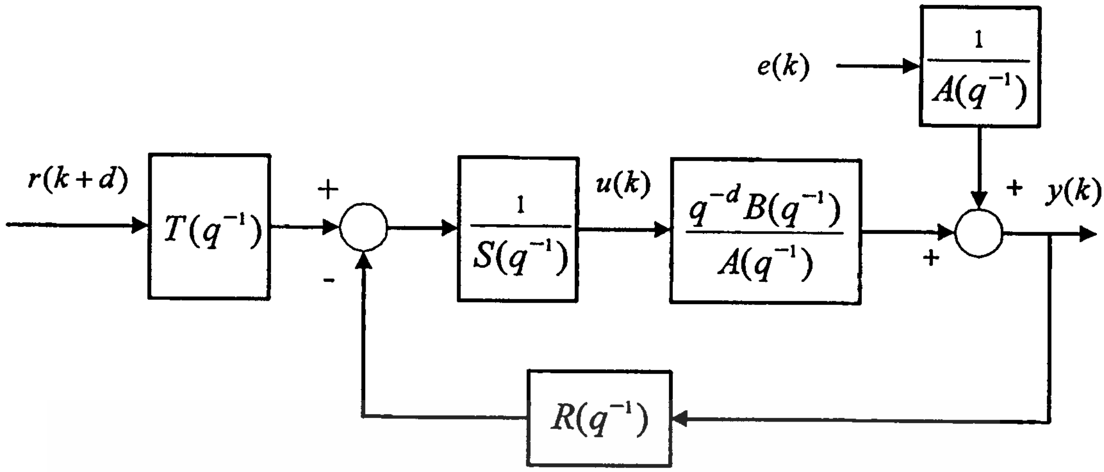

Figure 3.8 The GMV to ARX system

3.4 Conclusion

In this chapter, the digital controllers design method in the presence of random disturbances is considered. The random disturbances encountered in the practice can be modeled as the white noise sequence passed through a filter which known as disturbance model.

The minimum variance tracking and regulation control strategy minimizes the criterion

$$\min E\left\{[y(k+d+1)-y^*(k+d+1)]^2\right\}$$

This control strategy is applicable to the plant models with stable zeros. The resulting closed-loop poles are the zeros of the disturbance polynomial $C(q^{-1})$. The controller is identical to the tracking and regulation independent objectives if the polynomial $D(q^{-1})$ is chosen equal to $C(q^{-1})$.

The generalized minimum variance control or GMV is proposed by Clark and Gawthrop . **This strategy did not use the zeros cancellation.** Then, the designer need not to worry the pole zero cancellation and the unstable zero plant. The ARX model can be considered as a special case of GMV by the replacement of some variables.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Chapter 4

Recursive System Identification

When the plant is an unknown parameter plant or time varying parameter, the stable controller design through pole placement method can not be achieved. Alternatively, system identification can be performed through an experiment approach to estimate the unknown parameters from input-output measured data of the plant. Many identification methods exist, but Recursive Least Squares [2],[10],[11] identification method which is the real-time identification base on the Least Squares algorithm will be discussed in this chapter. This is due to that the real-time identification is the method which can easily be modified to track time varying parameters. Therefore it is appropriate for the time-varying parameter plant.

4.1 Least Squares (LS) Algorithm

The Least Squares algorithm is the concept of the linear regressive which is the simplest type of parametric model. The model structure is written as

$$y(k) = \varphi^T(k)\theta \tag{4.1}$$

where $y(k)$ is a measurable quantity, $\varphi(k)$ is an n-vector of known quantities and θ is an n-vector of unknown parameters or true value vector. The discrete time model for the general plant is shown in figure 4.1.

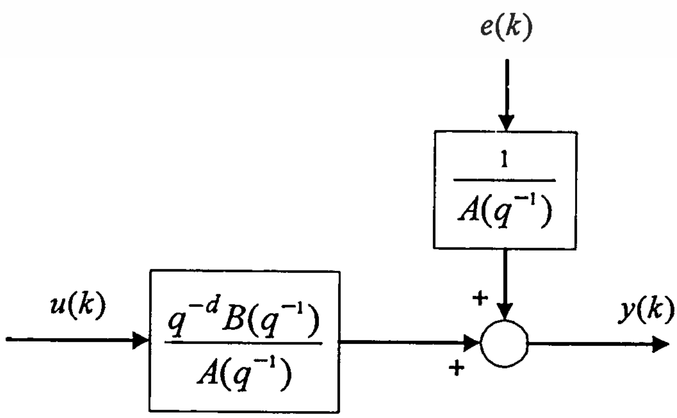

Figure 4.1 General discrete plant model (ARX model)

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Where $e(k)$ is the zero mean value white noise with variance σ^2. The relation between the output $y(k)$ and control input $u(k)$ can be written as

$$\frac{y(k)}{u(k)} = \frac{B(q^{-1})}{A(q^{-1})} = \frac{b_0 + b_1 q^{-1} + ... + b_m q^{-m}}{1 + a_1 q^{-1} + ... + a_n q^{-n}}. \quad (4.2)$$

The plant output can be expressed as the difference equation as

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=0}^{m} b_i u(k-i) \quad (4.3a)$$

or

$$y(k) = -a_1 y(k-1) - ... - a_n y(k-n) + b_0 u(k) + ... + b_m u(k-m) \quad (4.3b)$$

From Eqn.(4.1), the measured vector $\varphi^T(k)$ and parameter vector θ^T are

$$\varphi^T(k) = [-y(k-1) - y(k-2) ... - y(k-n) \; u(k) \; u(k-1) ... u(k-m)] \quad (4.4a)$$

$$\theta^T = [a_1 \; a_2 ... a_n \; b_0 \; b_1 ... b_m]. \quad (4.4b)$$

Then, Eqn.(4.1) can be rewritten as

$$y(k) = \varphi^T(k)\theta + e(k). \quad (4.5)$$

For the estimate model

$$\hat{y}(k) = \varphi^T(k)\hat{\theta} \quad (4.6)$$

where $\hat{y}(k)$ is the predicted output and $\hat{\theta}$ is the estimated parameter vector or estimator

$$\hat{\theta}^T = [\hat{a}_1 \; \hat{a}_2 ... \hat{a}_n \; \hat{b}_0 \; \hat{b}_1 ... \hat{b}_m]. \quad (4.7)$$

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

The prediction error $\varepsilon(k)$ between $y(k)$ and $\hat{y}(k)$ is

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta} \tag{4.8}$$

According to the LS algorithm the parameter vector is determined as the vector that minimizes the sum of squared prediction error as

$$\begin{aligned} V(\theta) &= \sum_{k=1}^{N} \varepsilon^2(k) = \varepsilon^T(k)\varepsilon(k) \\ &= \left[y(k) - \varphi^T(k)\hat{\theta}\right]^T \left[y(k) - \varphi^T(k)\hat{\theta}\right] \end{aligned} \tag{4.9}$$

Eqn.(4.9) is the minimal for parameter θ, then

$$\hat{\theta} = \left[\sum_{k=1}^{N} \varphi(k)\varphi^T(k)\right]^{-1} \sum_{k=1}^{N} \varphi(k)y(k) \tag{4.10a}$$

$$= \left[\mathbf{Z}^T\mathbf{Z}\right]^{-1}\mathbf{Z}^T\mathbf{y} \tag{4.10b}$$

where

$$\mathbf{Z} = \varphi^T(k).$$

4.2 Properties of Least Squares Estimator

The LS estimator $\hat{\theta}$ is a random variable whose properties can be analyzed using Eqn.(4.5). When $e(k)$, $k = 1,2,\ldots$ is the white noise, two important properties [2],[18],[19] hold: bias and covariance.

Bias

An estimate $\hat{\theta}$ is biased if its expected value deviates from the true value as

$$E[\hat{\theta}] \neq \theta. \tag{4.11a}$$

The difference $E[\hat{\theta}] - \theta$ is called the bias, if instead equally, $E[\hat{\theta}] = \theta$, then, $\hat{\theta}$ is said to be unbiased.

Covariance

Covariance is related to the accuracy of estimates which given by

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

$$\mathbf{cov}(\hat{\theta}) = \sigma^2(\mathbf{Z}^T\mathbf{Z})^{-1}. \tag{4.11b}$$

The size of matrix $(\mathbf{Z}^T\mathbf{Z})^{-1}$ gives a measure of the variability and covariability of the parameter estimated which is related to the spread of estimate arising from the random errors.

4.3 Recursive Least Squares (RLS) Algorithm

Since the adaptive control system is an on-line control system, the parameter estimation scheme should be iterative in order to save the computation time. By doing so, the RLS estimation algorithm which known as the recursive system identification will be discussed here as shown in figure 4.2. In this scheme, new input-output data become available at each sample interval.

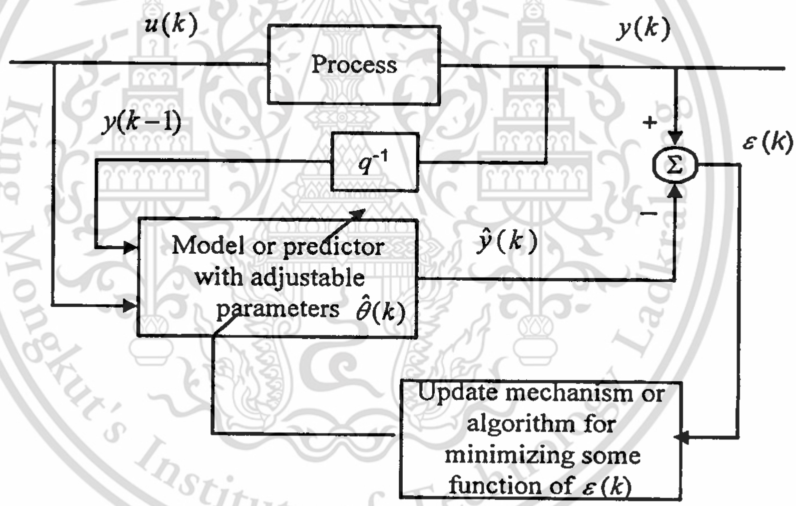

Figure 4.2 Recursive system identification

The model based on past information $\hat{\theta}(k-1)$ is used to obtain an estimate $\hat{y}(k)$. The prediction error, $\varepsilon(k) = y(k) - \hat{y}(k)$ in turn generates an update to the model to correct $\hat{\theta}(k-1)$ to the new value $\hat{\theta}(k)$. This recursive prediction results in saving the computation time. In equation (4.10a), given

$$P(k) = \left[\sum_{s=1}^{k} \varphi(s)\varphi^T(s)\right]^{-1} \tag{4.12}$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

from the notation of $P(k)$, it follows that

$$P^{-1}(k) = P^{-1}(k-1) + \varphi(k)\varphi^{T}(k). \tag{4.13}$$

Let $\hat{\theta}(k-1)$ based on k-1 measurement, then

$$\hat{\theta}^{-1}(k-1) = P(k-1)\sum_{s=1}^{k-1}\varphi(s)y(s). \tag{4.14}$$

From Eqns.(4.12), (4.13) and (4.14) the parameter estimate based on k measurement is

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\varphi(k)\varepsilon(k) \tag{4.15}$$

where $\varepsilon(k) = y(k) - \varphi^{T}(k)\hat{\theta}(k-1)$ which can be interpreted as the prediction error (one-step ahead) of $y(k)$ based on the estimate $\hat{\theta}(k-1)$.To avoid inverting $P(k)$ at each step, the matrix inversion lemma in Eqn. (4.16) is applied to Eqn. (4.13).

$$[\mathbf{A} + \mathbf{BCD}]^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left[\mathbf{DA}^{-1}\mathbf{B} + \mathbf{C}^{-1}\right]^{-1}\mathbf{DA}^{-1} \tag{4.16}$$

Then, the resulting RLS estimation method (see appendix B) is shown in the followings:

parameter adjusting

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1)\varphi(k)}{1 + \varphi^{T}(k)P(k-1)\varphi(k)}\varepsilon(k) \tag{4.17a}$$

the adaptation gain adjusting

$$P(k) = P(k-1) - \frac{P(k-1)\varphi(k)\varphi^{T}(k)P(k-1)}{1 + \varphi^{T}(k)P(k-1)\varphi(k)} \tag{4.17b}$$

the priori error

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1). \qquad (4.17c)$$

4.4 Time-Varying Parameters

In LS problem, parameter θ_i in Eqn. (4.1) are assumed to be constant. But in adaptive problems, the parameters are time-varying. This can be covered by using a forgetting factor λ in LS criterion of Eqn.(4.9) with

$$V(\theta) = \sum_{k=1}^{N} \lambda^{N-k}\left[y(k) - \varphi^T(k)\theta\right]^2 \qquad (4.18)$$

where $0 < \lambda \leq 1$. Eqn.(4.18) implies that a time varying weighting of the data is introduced, the most recent data is given unit weight, but data is N time units old is weighted by λ^N. The method is called exponential forgetting. Repeating the calculation leading to the RLS algorithm for the loss function of Eqn.(4.18), the resulting RLS with exponential forgetting is obtained in the followings:

parameter adjusting

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1)\varphi(k)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}\varepsilon(k) \qquad (4.19a)$$

the adaptation gain adjusting

$$P(k) = \frac{1}{\lambda}\left[P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}\right] \qquad (4.19b)$$

the priori error

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1). \qquad (4.19c)$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

4.5 Problems in Using RLS Algorithm

4.5.1 Estimator Windup

In system identification method, to ensure the unbias estimated parameters, the pseudo random binary sequences (PRBS) is used as the input signal sequences. Because every mode of the system can be excited. But in adaptive pole placement çontrol system, this random signal can not be used, usually the square wave is used instead .Then, the estimator will windup (see appendix.B) in the case when the period of wave is long. To overcome this problem, the value of forgetting factor, $\lambda(k)$ in Eqns.(4.19a) and (4.19b) is modified into the following form

$$\lambda(k) = \lambda \cdot \lambda(k-1) + [1-\lambda] \tag{4.20}$$

where

$0 < \lambda \leq 1$ and $\lambda(0)$ is arbitrary chosen (through the key board)

then

$$\lim_{k\to\infty} \lambda(k) = 1. \tag{4.21}$$

In this thesis, $\lambda = 0.99$ and $\lambda(0) = 0.99$

4.5.2 Wide Range Parameter Fluctuation

There are many control systems that the dynamic characteristics of the controlled process change greatly. For example, the parameter of mechanical system on motor control field changes a wide range. Furthermore, the speed control system on the rolling lines of steel mills is a plant that the load inertia fluctuate very large. Accordingly, system parameters identification usually deteriorates even applying many identification methods. It is well accepted that when the plant parameters are widely fluctuated , it is difficult to keep the unbias estimated parameter and the accuracy of identification will be very low. This is because when the estimated parameter becomes the true value, the adaptation gain converges to the small value. To overcome this problem, in this thesis, every time the plant parameter are widely fluctuated, the initial adaptation gain will be reseted to the initial gain. By doing so, the estimated parameter will no longer biased even the plant parameters change in a wide range.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

4.5.3 Initial Conditions [19]

Normally, to use the RLS algorithm, the initial values for their start-up are required. From Eqn.(4.19b),

$$P(k)=\frac{1}{\lambda}\left[P(k-1)-\frac{P(k-1)\varphi(k)\varphi^{T}(k)P(k-1)}{\lambda+\varphi^{T}(k)P(k-1)\varphi(k)}\right] \qquad (4.19b)$$

the initial condition at time $k = 0$ would be $P(k) = 0$, $\hat{\theta}(k)$ arbitrary. But however, according to the definition of P in Eqns.(4.12) and (4.13), these value can not be used. Because from the RLS algorithm or on-line estimation, $\hat{\theta}(k)$ is derived from the LS or off-time estimation (4.10a) as shown in Eqn.(4.22),

$$\hat{\theta}(k)=\left[P^{-1}(k_0)+\sum_{s=1}^{k}\varphi(s)\varphi^{T}(s)\right]^{-1}\left[P^{-1}(k_0)\hat{\theta}(k_0)+\sum_{s=1}^{k}\varphi(s)y(s)\right] \qquad (4.22)$$

where k_0 is the time instant when $P(k_0)$ has become invertible, then, the initial value of $P(k_0)$ should be very large. So that the on-line estimate will be equal to the off-line one. To achieve the correct initial condition, first choose a time k_0, in this thesis, k_0 = 50 samplings (usually k_0 = 10~50) and put into the LS estimate in Eqn.(4.10a) to obtain $\hat{\theta}(k_0)$ as

$$\hat{\theta}_{off}(k)=\left[\sum_{s=1}^{k_0}\varphi(s)\varphi^{T}(s)\right]^{-1}\left[\sum_{s=1}^{k_0}\varphi(s)y(s)\right], \qquad (4.10a)$$

$$=\hat{\theta}(k_0).$$

Next choose $P(k_0)$ so that

$$P^{-1}(k_0) << \sum_{s=1}^{k_0}\varphi(s)\varphi^{T}(s).$$

In this research, $P(k_0)$ is selected to a value of 10^4I, (I is unit matrix), if $\hat{\theta}(k_0)$ and $P(k_0)$ are put into Eqn.(4.22), then, $\hat{\theta}(k)$ will equal to $\hat{\theta}_{off}$. The value of k_0 also can be the same number of the unknown parameters.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

4.6 RLS to ARX Model

The ARX model of figure 3.3 that rewritten in figure 4.3 is the most appropriate and only one dynamic model that has the linear regression which is the unique feature of the LS criterion, Eqn.(4.10a). The disturbed output of figure 4.3 $y(k)$ as shown in Eqn.(3.11) is rewritten in Eqn.(4.23) below.

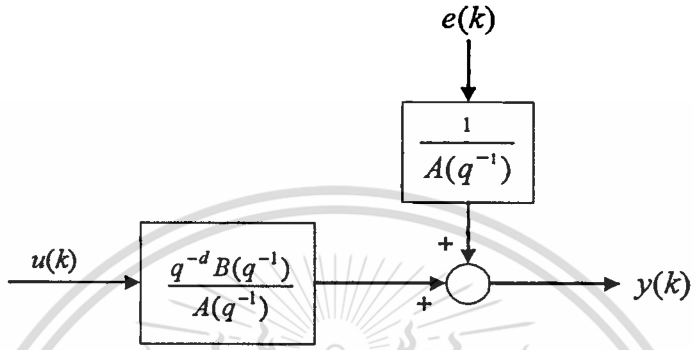

Figure 4.3 The ARX model

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + e(k). \tag{4.23}$$

Since the error $e(k)$ in Eqn.(4.23) is the white noise sequence, then the LS algorithm can be applied to Eqn.(4.23) without problems. To see this approach, neglecting the error in the model (4.23), one can predict the output at time k as

$$\hat{y}(k) = \varphi^T(k)\hat{\theta}$$

Hence, the prediction error can be written as

$$\varepsilon(k) = y(k) - \hat{y}(k). \tag{4.24}$$

Since the error in model (4.23) is

$$e(k) = A(q^{-1})y(k) - B(q^{-1})u(k) \tag{4.25}$$

then,

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$$y(k)-\hat{y}(k)=A(q^{-1})y(k)-B(q^{-1})u(k)$$

$$\hat{y}(k)=y(k)-A(q^{-1})y(k)+B(q^{-1})u(k),$$

$$=\left[1-A(q^{-1})\right]y(k)+B(q^{-1})u(k)=\varphi^{T}(k)\theta \tag{4.26}$$

Eqn (4.26) is called "one step ahead prediction value" which is denoted by

$$\hat{y}(k|\theta)=\left[1-A(q^{-1})\right]\ y(k)+B(q^{-1})u(k)=\varphi^{T}(k)\theta \tag{4.27}$$

this equation shows the linear relationship of the estimated parameters.

4.7 System Identification Simulation Examples

Recursive System Identification of the 2nd order plant using RLS method

$$G(s)=\frac{k\omega_n^{\ 2}}{s^2+2\zeta\omega_n s+\omega_n^{\ 2}}$$

Given $k=1$, $\omega_n=45$, $\zeta=0.15$; sampling time, T = 0.02 sec.

$$G(q^{-1})=\frac{q^{-d}(0.34696q^{-1}+0.31632q^{-2})}{(1-1.2001q^{-1}+0.76338q^{-2})}$$

In this simulation, the square wave of unity amplitude and 100 samplings period is used as the input excitation signal to the plant $\frac{q^{-d}B(k)}{A(k)}$.The initial adaptation gain $P(k_0)$ is 10^4I and the initial parameter vector $\hat{\theta}^T=[0.1\ 0.2\ 0.3\ 0.4]$ Measure the input and output signals of the plant and apply the RLS algorithm to estimate the plant parameters under the following three cases.

4.7.1 No Noise and No Constant Disturbance

Given $d=1$

Simulation results :

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

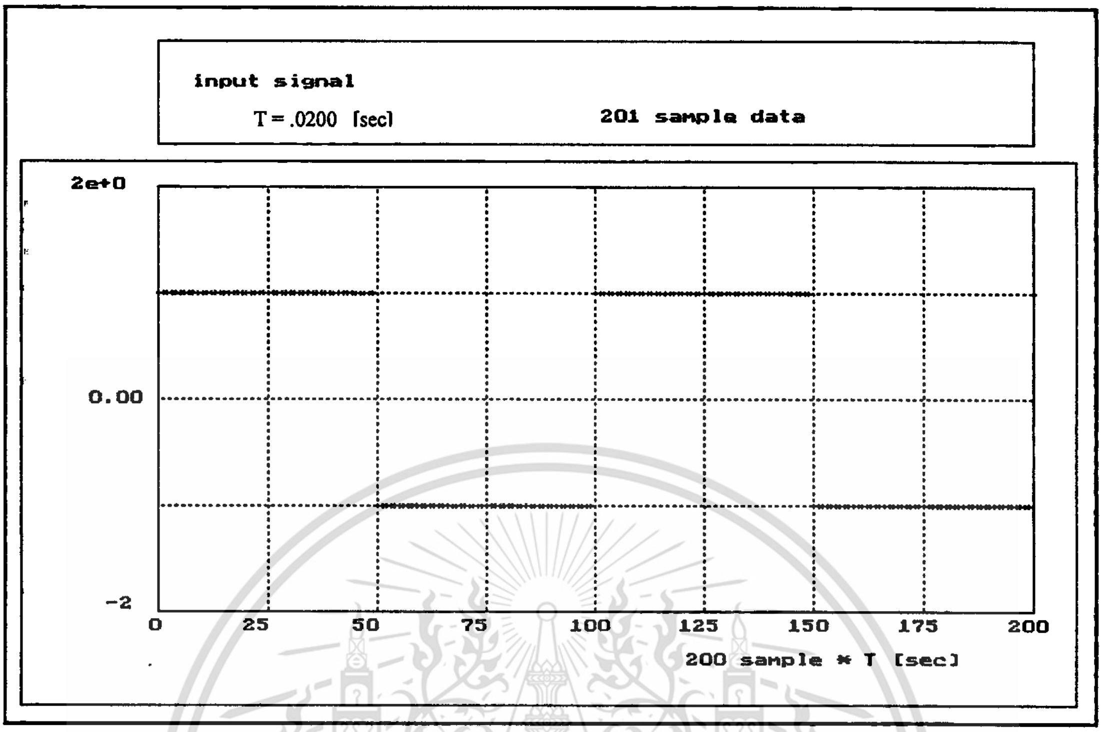

Figure 4.4 Input signal

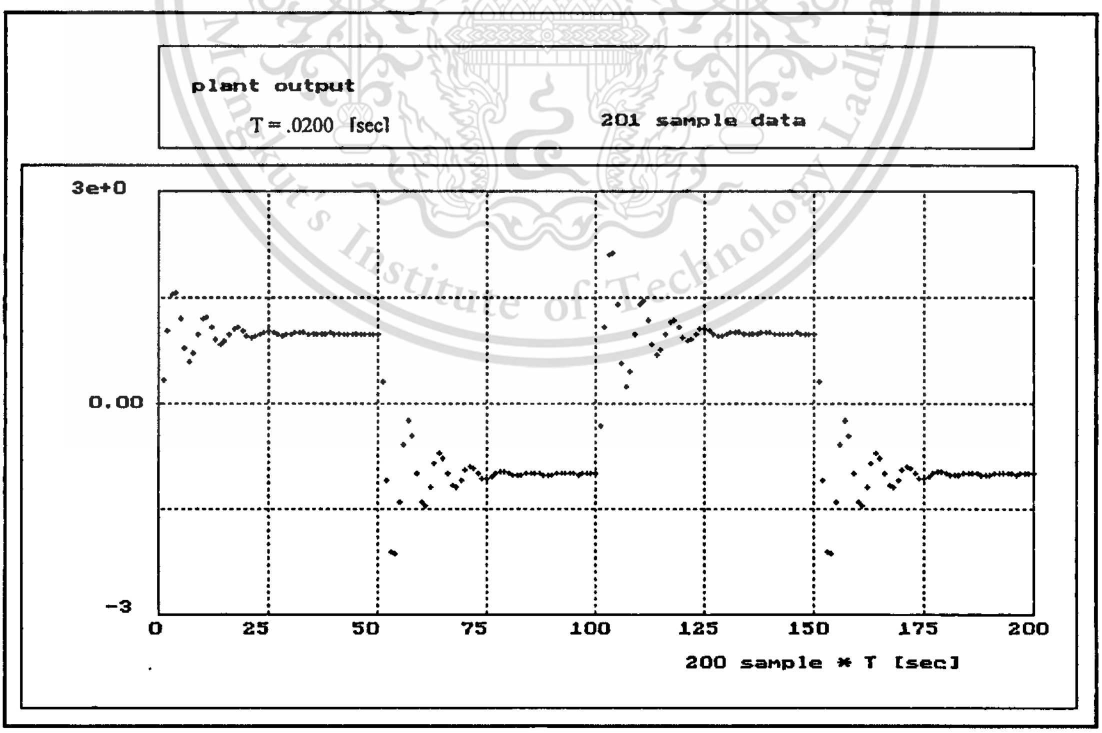

Figure 4.5 Plant output

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

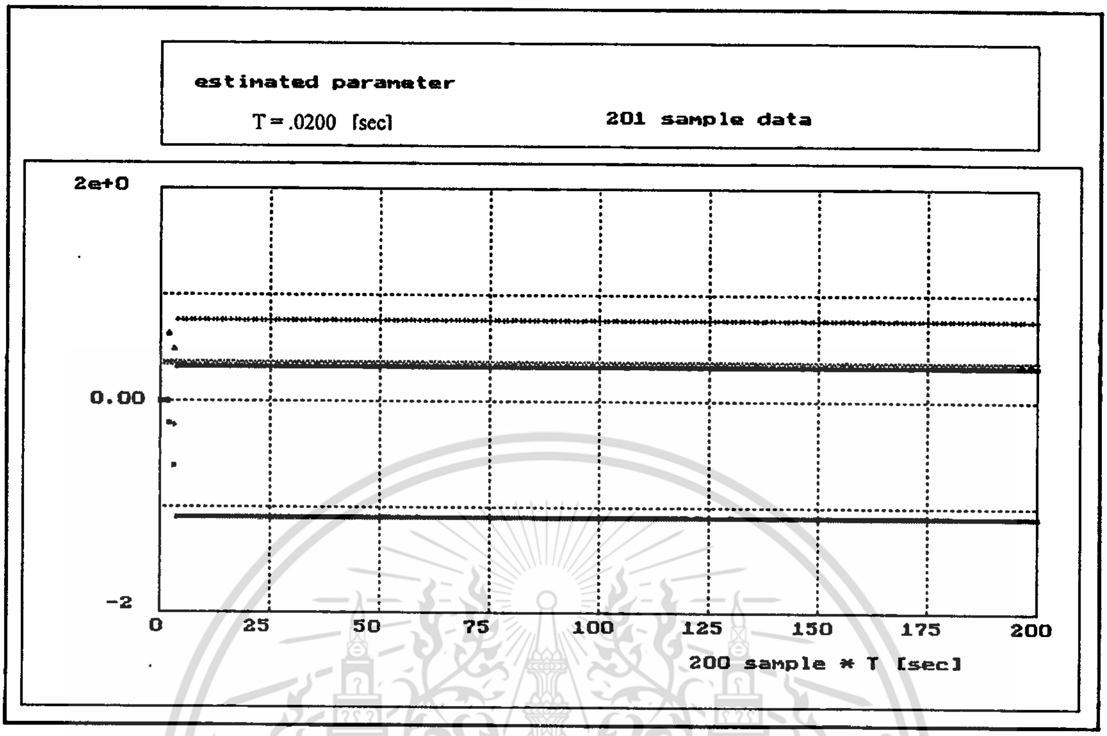

Figure 4.6 Estimated parameters

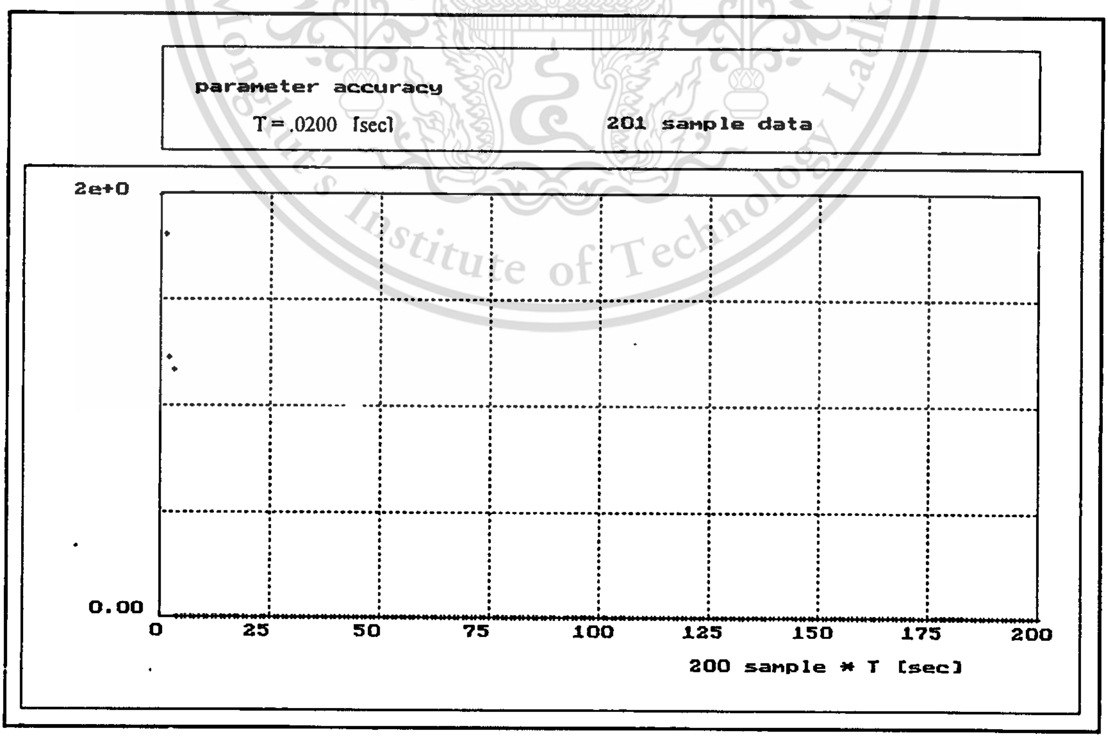

Figure 4.7 Parameter accuracy

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

For 201 sampling steps, plant estimated parameters are as follows:

1-th parameter, $\hat{a}_1$ = -11.00092e-01
2-th parameter, $\hat{a}_2$ = 7.6337934e-01
3-th parameter, $\hat{b}_1$ = 3.4695998e-01
4-th parameter, $\hat{b}_2$ = 3.1632018e-01

4.7.2 Added Noise and Constant Disturbance

Variance = 0.001 , disturbance = 0.5

Simulation results:

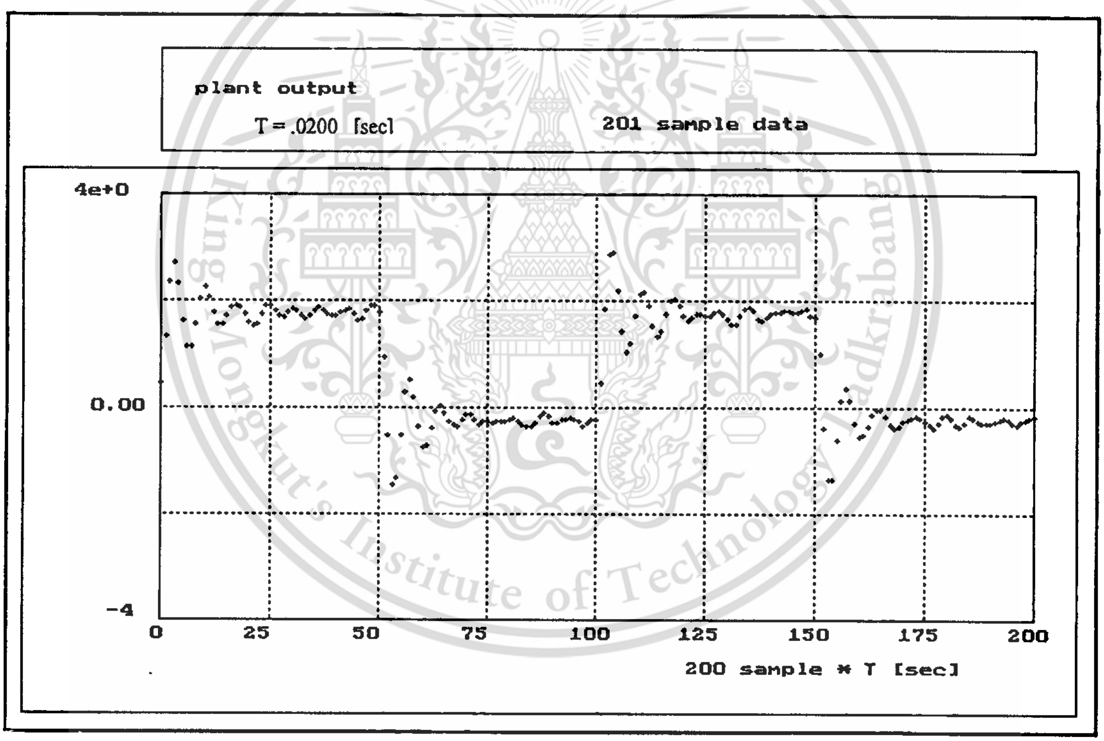

Figure 4.8 Plant output

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

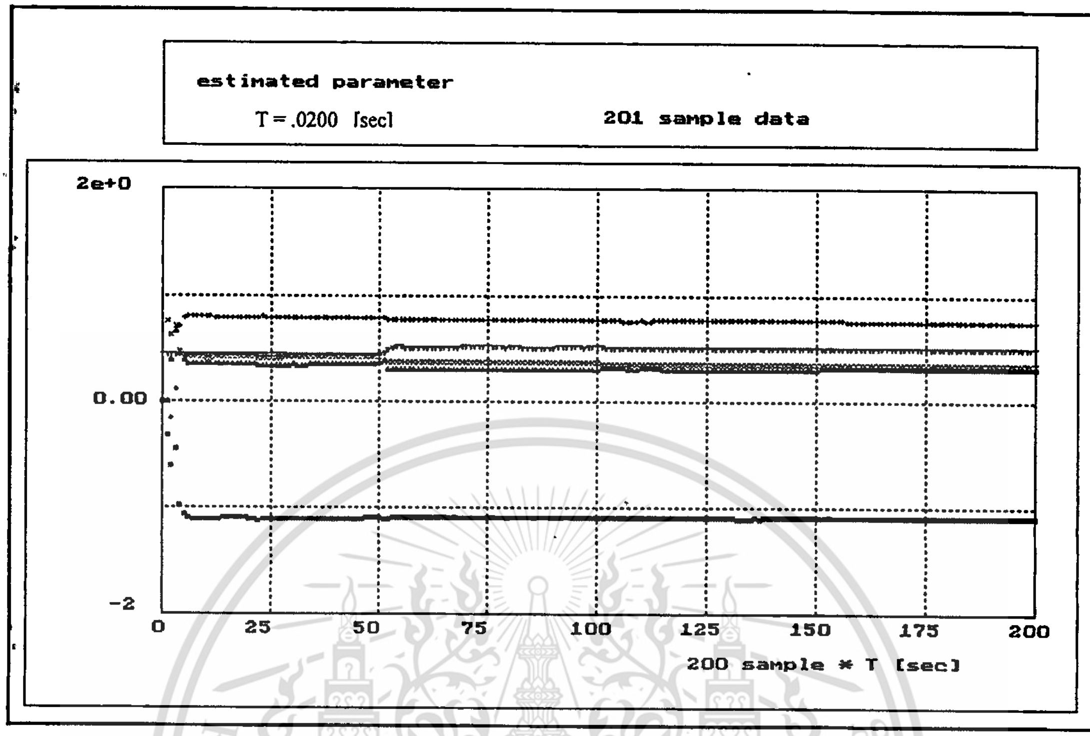

Figure 4.9 Estimated parameters

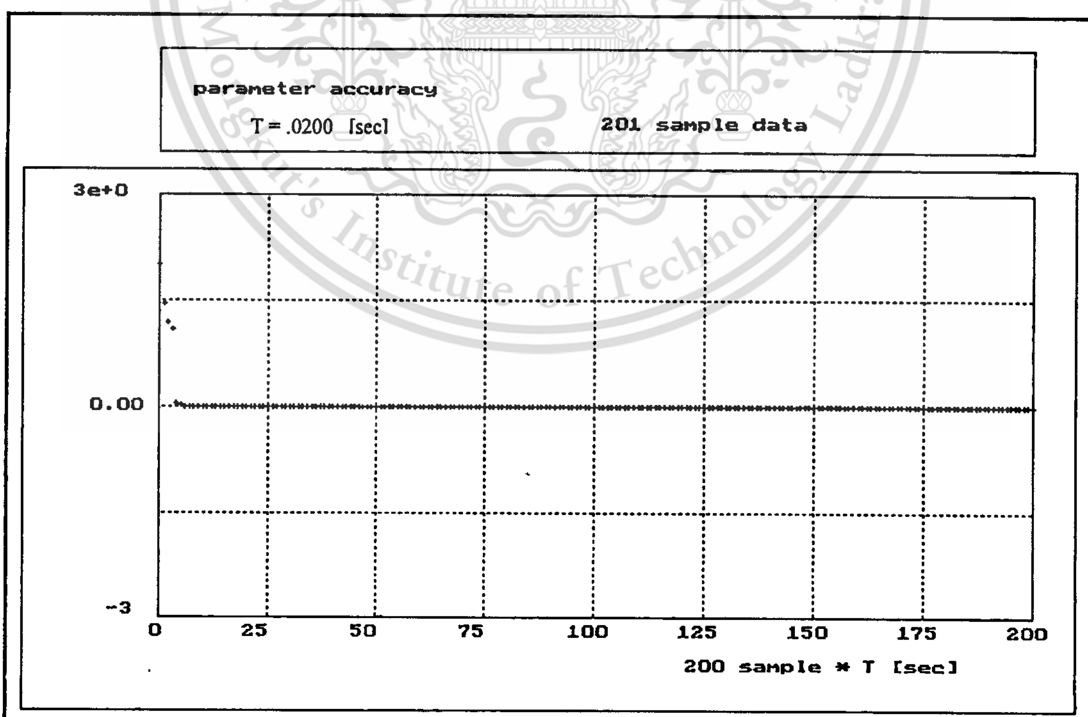

Figure 4.10 Parameter accuracy

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

For 201 sampling steps, plant estimated parameters are as follows:

1-th parameter, $\hat{a}_1$ = -1.1010346e+00
2-th parameter, $\hat{a}_2$ = 7.6233345e-01
3-th parameter, $\hat{b}_1$ = 3.4764892e-01
4-th parameter, $\hat{b}_2$ = 3.1249127e-01
5-th parameter, the constant disturbance = 4.9867558e-01

4.7.3 Plant Parameters Widely Fluctuate

Assume that the plant parameters are changed, then, the fast response plant ($\omega_n = 45, \zeta = 0.15$) becomes the slow response one ($\omega_n = 7$, $\zeta = 0.45$), the same sampling time, T = 0.02 sec is used.

Then,

$$G(q^{-1}) = \frac{q^{-d}(0.0093858q^{-1} + 0.0089996q^{-1})}{1 - 1.8632q^{-1} + 0.88161q^{-2}}$$

Simulation results:

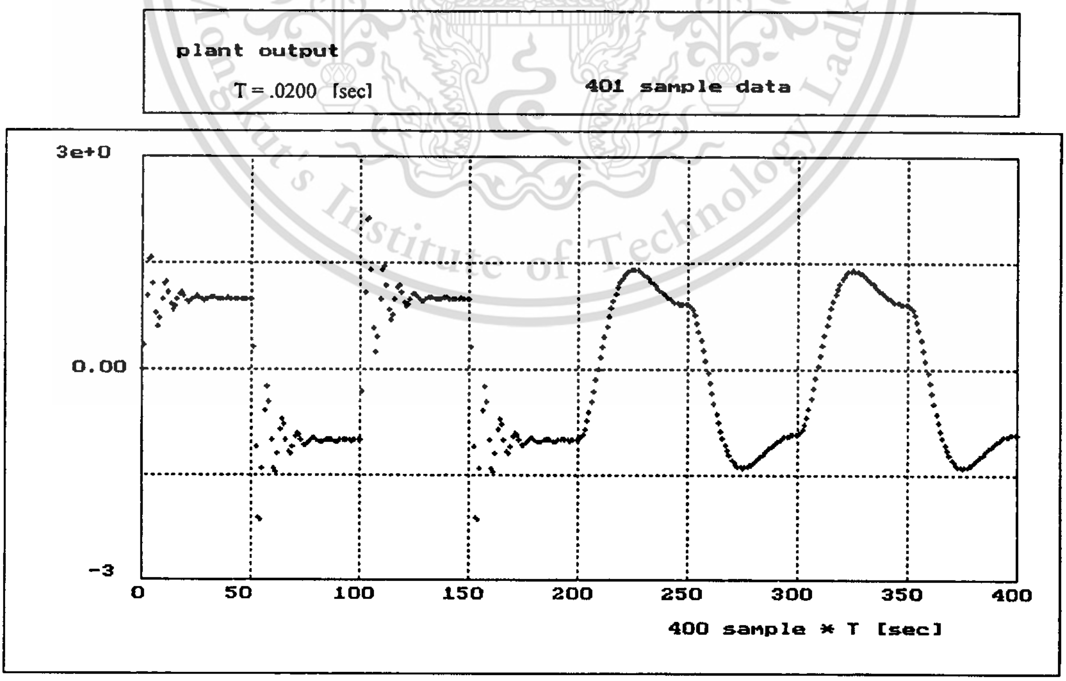

Figure 4.11 Plant output

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

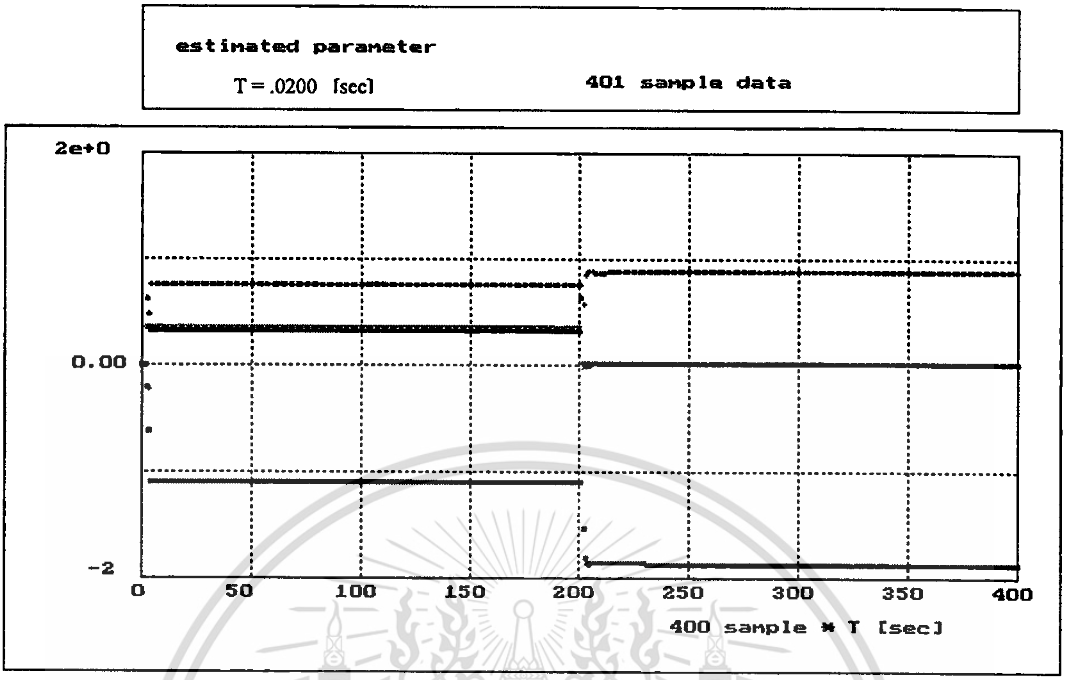

Figure 4.12 Estimated parameters

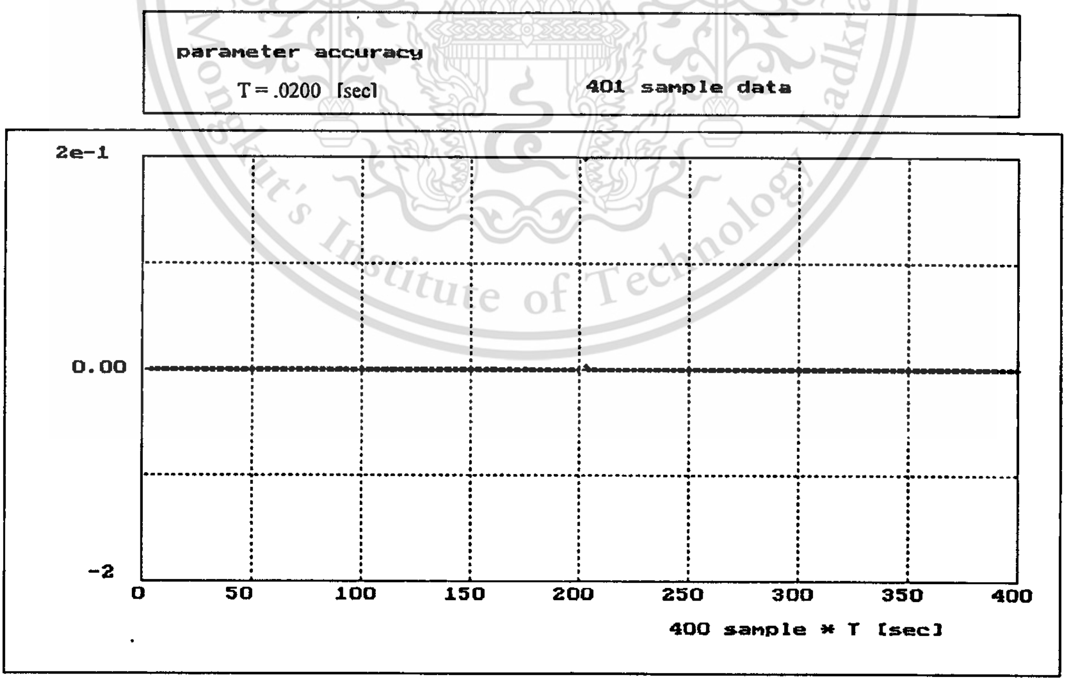

Figure 4.13 Parameter accuracy

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

For 401 sampling steps, plant estimated parameters are as follows:

1-th parameter, $\hat{a}_1$ = -1.8631999e+00
2-th parameter, $\hat{a}_2$ = 8.8160998e-01
3-th parameter, $\hat{b}_1$ = 9.3858000e-03
4-th parameter, $\hat{b}_2$ = 8.9996001e-03

The simulations in section 4.7 are the examples of parameter estimation using RLS algorithm, the simulation shows that the accuracy of parameter estimation is very good in the case of no noise and no disturbance. In the case of added noise and constant disturbance to the simulation system, the estimated parameters are very closed to the true values. The last simulation shows that, even the parameter widely fluctuated, the RLS identification method is also able to deal with.

4.8 Conclusion

In this chapter a recursive system identification algorithms has been derived. It has small requirements in computer time and memory. The recursive algorithm can easily be modified to track time varying parameters by introducing the forgetting factor. The RLS method can be applied only to the ARX model because this model is the linear regression model which is the unique feature of the LS criterion. Section 4.7 shows the application of RLS algorithm to the parameter identification.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Chapter 5

Adaptive Pole Placement Control

In the conventional indirect adaptive control system, the unknown parameters of the plant are adaptively estimated as shown in section 4.3. And the controller is recursively designed with the pole placement method as shown in section 2.1 by assuming that the estimated parameters represent the true values of the plant parameters. This controller design method is known as "Adaptive pole placement control".

5.1 Design under Ideal Condition

When the block diagram of recursive system identification shown in figure 4.2 is applied to the pole placement control, the block diagram of adaptive pole placement control can be written as shown in figure 5.1

In the case of noiseless system, the controlled plant is represented by the following discrete system

$$A(q^{-1})y(y) = q^{-d}B(q^{-1})u(k) \tag{5.1a}$$

where

$$A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_nq^{-n}$$
$$B(q^{-1}) = b_0 + b_1q^{-1} + \ldots + b_mq^{-m}.$$

The reference model is also represent as

$$Am(q^{-1})y_m(k+d) = q^{-d}Bm(q^{-1})x(k) \tag{5.1b}$$

where $y_m(k+d)$ is the reference signal and

$$Am(q^{-1}) = 1 + a_{m1}q^{-1} + \ldots + a_{mn}q^{-n}$$
$$Bm(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots + b_{mn}q^{-n}.$$

This material is reserved for educational use only, not allowed for commercial use.

The prefilter $T(q^{-1})$ can be taken either the constant or the polynomial form as

Forbidden to modify the content, and cite the document when use.

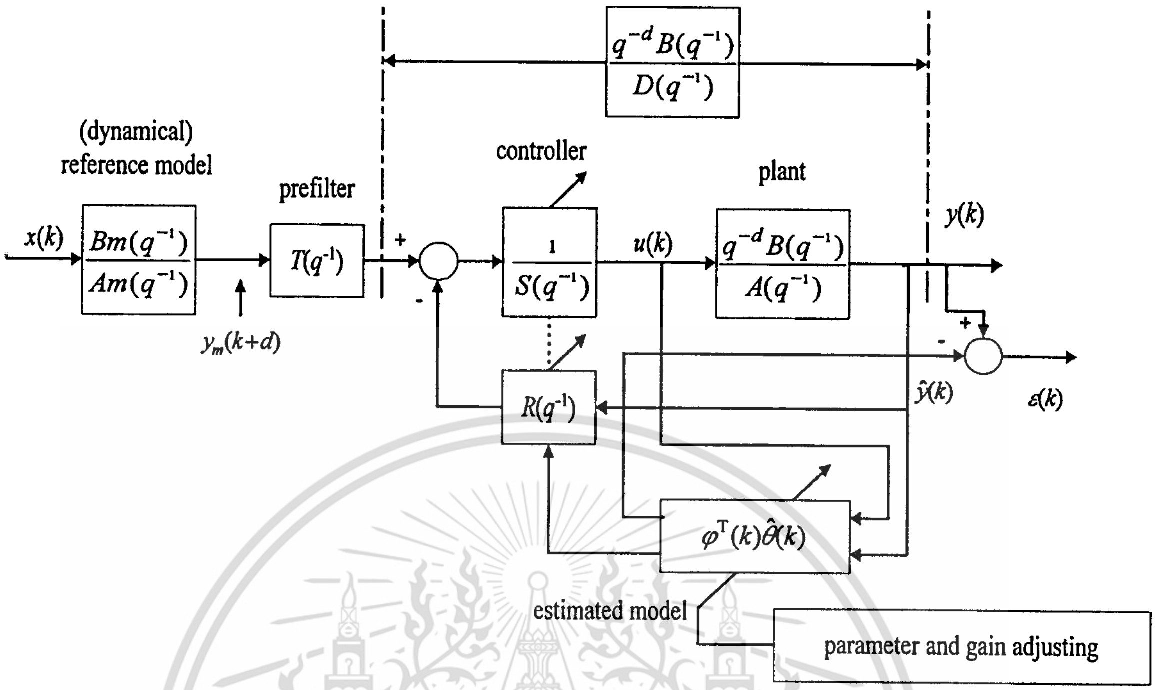

Figure 5.1 Block diagram of conventional adaptive pole placement control

$$T(q^{-1}) = \frac{D(1)}{B(1)} \quad \text{or} \quad T(q^{-1}) = \frac{D(q^{-1})}{B(1)}. \tag{5.2}$$

If the stable closed-loop pole is assigned by the following polynomial

$$D(q^{-1}) = 1 + d_1 q^{-1} + \ldots + d_{nd} q^{-nd} \tag{5.3}$$

then, the plant output is given by

$$D(q^{-1})y(k) = q^{-d}T(q^{-1})B(q^{-1})y_m(k+d). \tag{5.4}$$

The manipulated variable $u(k)$ to the plant is designed as

$$u(k) = \frac{1}{S(q^{-1})}\left[T(q^{-1})y_m(k+d) - R(q^{-1})y(k)\right] \tag{5.5}$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

where

$$S(q^{-1}) = 1 + s_1 q^{-1} + --- + s_{ns} q^{-ns}$$
$$R(q^{-1}) = r_0 + r_1 q^{-1} + --- + r_{nr} q^{-nr}.$$

Put Eqn5.5) into Eqn.(5.1) and compare to Eqn.(5.4), the Diophantine equation can be obtained as

$$D(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}). \tag{5.6}$$

From Eqn.(5.6), the pole placement problem comes down to the problem that solves Eqn.(5.6). When the plant parameters $A(q^{-1})$ and $B(q^{-1})$ are the estimated parameters, the controller polynomial $S(q^{-1})$ and $R(q^{-1})$ can be decided as same as in section 2.1

5.2 Constant Disturbance Rejection

In the case of constant disturbance dn as shown in figure 5.2, the plant output can be expressed in the difference equation as shown in Eqn. (5.7)

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=0}^{m} b_i u(k-d-i) + dn \tag{5.7}$$

which can be rewritten as

$$y(k) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k) + \frac{1}{A(q^{-1})}dn \tag{5.8}$$

let

$dn = q^{-d}B(q^{-1})f$

then, Eqn. (5.8) becomes,

$$y(k) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}\left[u(k) + f\right]. \tag{5.9}$$

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

If $x(k)$ is assumed to be the constant disturbance compensation variable as shown in figure 5.2.

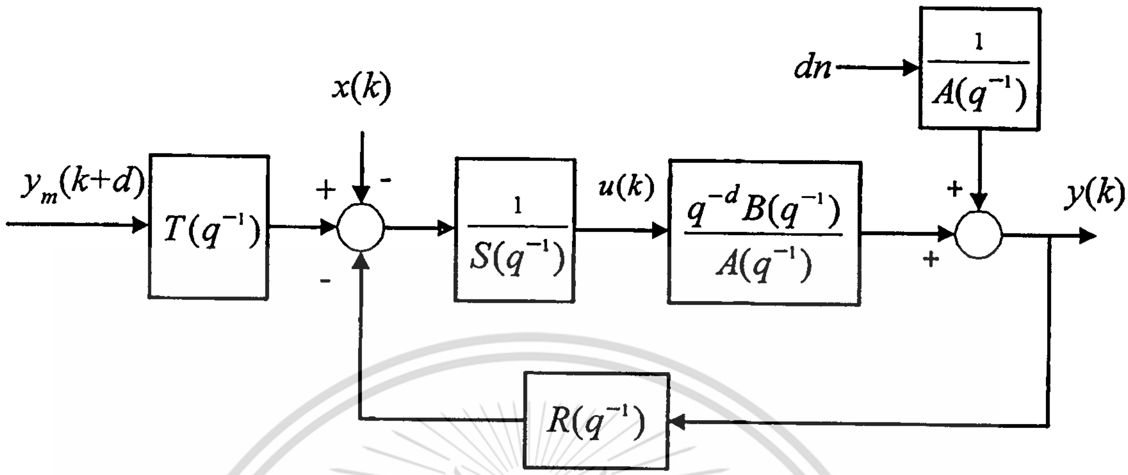

Figure 5.2 Constant disturbance rejection

Then, the manipulate variable $u(k)$ to the plant is defined as

$$u(k) = \frac{1}{S(q^{-1})}\left[T(q^{-1})y_m(k+d) - R(q^{-1})y(k) - x(k)\right]. \tag{5.10}$$

Put Eqn.(5.10) into Eqn.(5.8), $y(k)$ can be rearranged as

$$\left[A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})\right] y(k) = T(q^{-1})q^{-d}B(q^{-1})y_m(k+d) + \left[S(q^{-1})dn - q^{-d}B(q^{-1})x(k)\right] \tag{5.11}$$

In order to reject constant disturbance dn, compared to the case of noiseless in Eqn. (5.4), the constant disturbance compensation variable $x(k)$ will be chosen to make the second term in the right hand side of Eqn.(5.11) equal to zero.

If

$$S(q^{-1})dn - q^{-d}B(q^{-1})x(k) = 0 \tag{5.12}$$

then, $x(k)$ can be calculated as

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$$x(k)=\frac{q^{d}S(q^{-1})}{B(q^{-1})}dn=S(q^{-1})f \tag{5.13}$$

in practice, $x(k)$ is usually chosen as

$$x(k)=\frac{S(1)}{B(1)}dn. \tag{5.14}$$

From Eqn.(5.10) and Eqn.(5.13), an other formula for the manipulated variable can be written as

$$u(k)+f=\frac{1}{S(q^{-1})}\left[T(q^{-1})y_m(k+d)-R(q^{-1})y(k)\right]. \tag{5.15}$$

Putting Eqn.(5.15) into Eqn.(5.9), $y(k)$ can be rewritten as

$$y(k)=\frac{q^{-d}B(q^{-1})}{A(q^{-1})S(q^{-1})}T(q^{-1})y_m(k+d)-\frac{q^{-d}B(q^{-1})R(q^{-1})}{A(q^{-1})S(q^{-1})}y(k)$$

then

$$[A(q^{-1})S(q^{-1})+q^{-d}B(q^{-1})R(q^{-1})]y(k)=T(q^{-1})q^{-d}B(q^{-1})y_m(k+d)$$

in which

$$y(k)=\frac{T(q^{-1})q^{-d}B(q^{-1})}{D(q^{-1})}y_m(k+d) \tag{5.16}$$

Eqn (5.15) can be rearranged as

$$u(k)=T(q^{-1})y_m(k+d)+\left[1-S(q^{-1})\right]u(k)-R(q^{-1})y(k)-S(q^{-1})f \tag{5.17}$$

From this point, the block diagram of constant disturbance rejection can be rewritten as shown in figure 5.3.

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

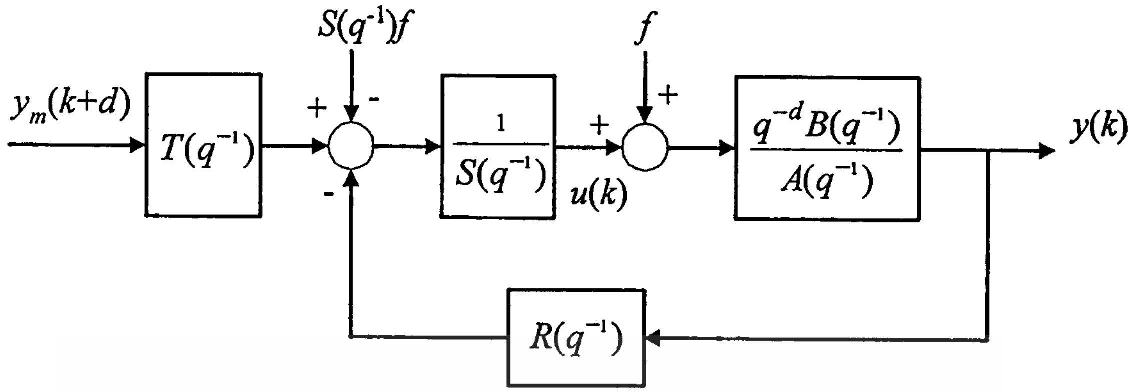

Figure 5.3 Block diagram of constant disturbance rejection.

5.3 Steady State Compensation

In order to have a zero steady state error for a step input or disturbance, the feedforward channel must contains a digital integrator as shown in figure 5.4

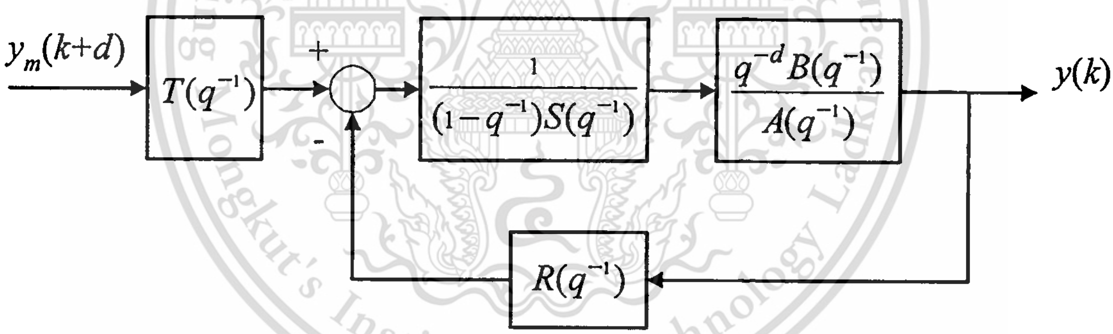

Figure 5.4 Steady state compensation

The manipulate variable in Eqn.(5.5) becomes

$$u(k) = \frac{1}{(1-q^{-1})S(q^{-1})}\left[T(q^{-1})y_m(k+d) - R(q^{-1})y(k)\right] \tag{5.18}$$

Put Eqn.(5.18) into Eqn.(5.8) in the case of noiseless, then $y(k)$ can be obtained as

$$\left[(1-q^{-1})A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})\right]\ y(k) = T(q^{-1})q^{-d}B(q^{-1})y_m(k+d) \tag{5.19}$$

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

From Eqn.(5.19) the Diophantine equation is

$$D(q^{-1}) = (1-q^{-1})A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}). \tag{5.20}$$

Since

$$D(q^{-1}) = 1 + d_1 q^{-1} + d_2 q^{-2} + \ldots + d_n q^{-nd} \ , \ nd \le n+m+d$$

then, the order of $S(q^{-1})$ and $R(q^{-1})$ are as

$$S(q^{-1}) = 1 + s_1 q^{-1} + s_2 q^{-2} + \ldots + s_{ns} q^{-ns}, ns = m+d-1 \tag{5.21}$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + r_2 q^{-2} + \ldots + r_{nr} q^{-nr}, nr = n. \tag{5.22}$$

5.4 Stability Index

In control system design and synthesis, the stability condition is the important factor. However, the exact stability conditions such as the absolute stability of Routh/Hurwitz criteria are too complicated comparing to the relative stability. Gain margin g_m, phase margin p_m, gain crossover frequency ω_g and stability indoces γ_i are often used to represent the relative stability condition of the linear time-invariant system. g_m, p_m and given ω_g can be defined from the Bode diagram of loop transfer function of system, but the stability index is specified by the coefficients of the characteristic polynomial as

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 \ , \ \ n \ge 2. \tag{5.23}$$

The stability indices are generally defined by

$$\gamma_i = \frac{a_i^2}{a_{i+1} \cdot a_{i-1}} \quad \text{for} \ \ i = 1, \ldots, (n-1), \ \gamma_0 \ \text{and} \ \gamma_n = \infty \tag{5.24}$$

where a_i are the coefficients corresponding to the s^i term of $P(s)$, and n is the order of $P(s)$.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

For easy implementation some other different indices can be chosen, that is the resultant stability index γ which is given by the algebraic product of the stability indices as follows :

$$\gamma = \prod_{i=1}^{n-1} \gamma_i \,. \tag{5.25}$$

When the polynomial of the series compensator in Eqn.(5.21) is transformed into the continuous transfer function $S_c(s)$ using the bilinear inverse transformation,

$$q = (2 + Ts)/(2 - Ts) \tag{5.26}$$

where T is the sampling period.
After the bilinear transformation, the stability indices of the denominator polynomial $S_c(s)$ are calculated. Then the relative stability of this compensator or controller can be investigated under the specified value .

5.5 Stability Factor and Closed-loop Pole

The stability index of the series compensator is related to the stable pole of the adaptive pole placement control system. When the poles of stable closed loop system to the unknown plant are arbitrarily selected, the unstable compensator often appears in the pole placement system. It occurs more frequently if the plant parameters widely change and the resultant change of the stability index of series compensator also occurs. From this point of view, if the stability index is evaluated and recursively adjusted into the specified area according to the fluctuation of the plant parameters. Then the stable pole also can be recursively calculated and the strongly stable adaptive pole placement control system can be achieved. The relation between the stability index γ and closed-loop pole α is shown in the numerical examples in section 5.6.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

5.6 Adaptive Pole Placement Simulation Examples

For the 2nd order plant

$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} = \frac{q^{-d}(0.43103q^{-1} + 0.40231q^{-2})}{(1 - 0.98539q^{-1} + 0.81873q^{-2})}$$

given $d = 2$

$$D(q^{-1}) = (1 - \alpha q^{-1})^{nd} \quad , \quad nd = 3$$

$$T(q^{-1}) = \frac{D(1)}{B(1)}$$

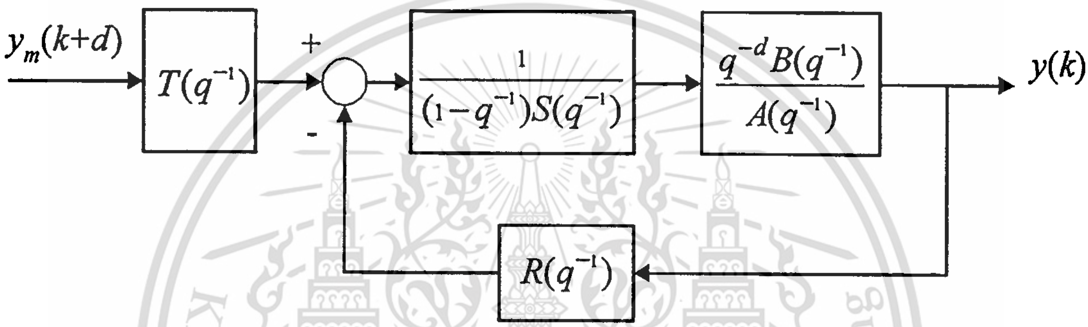

Figure5.5 Simulation system

In this simulation, the square wave of unity amplitude and 100 samplings period is used as the input signal to the system.The initial adaptation gain is 10^4I and the initial parameter vector $\hat{\theta}^T = [0.1\ 0.2\ 0.3\ 0.4]$. Measure the input and output signals of the plant and apply the RLS algorithm to estimate the plant parameters. At the same time, the controller and prefilter are designed according to the conventional adaptive pole placement control system algorithm. The simulations and control results are divided into three cases as shown below.

5.6.1 No Noise Case

Select the stable pole :

1. $\alpha = 0.4$

Simulation results :

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

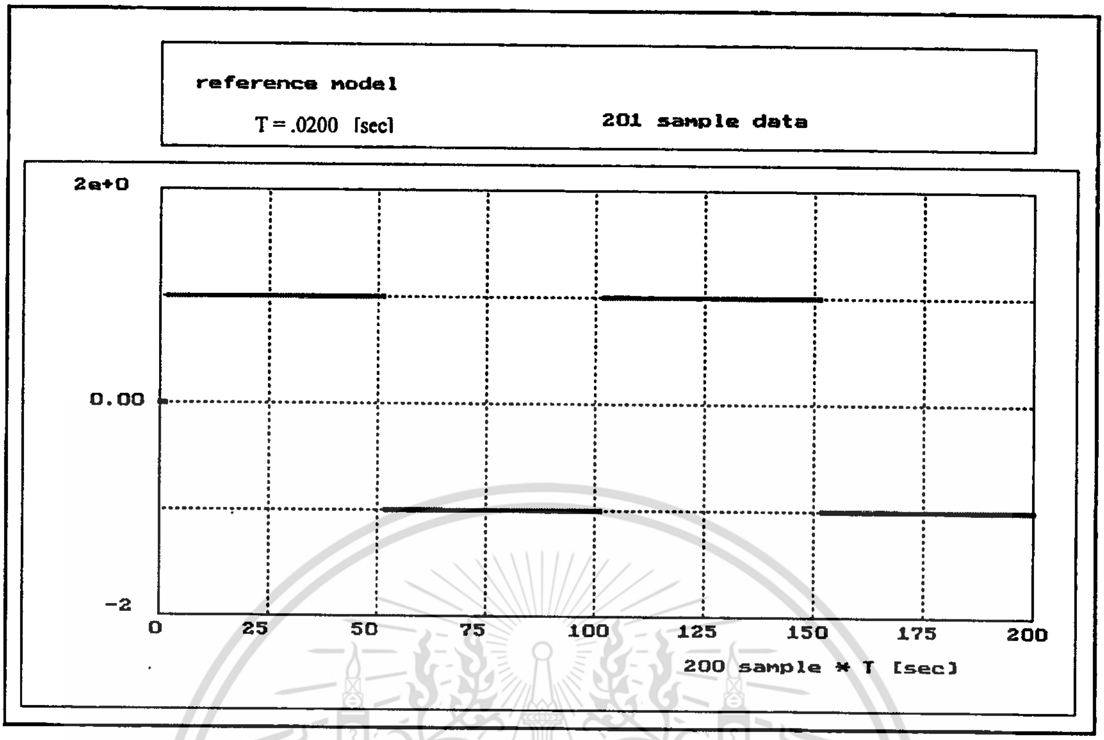

Figure 5.6 Input signal

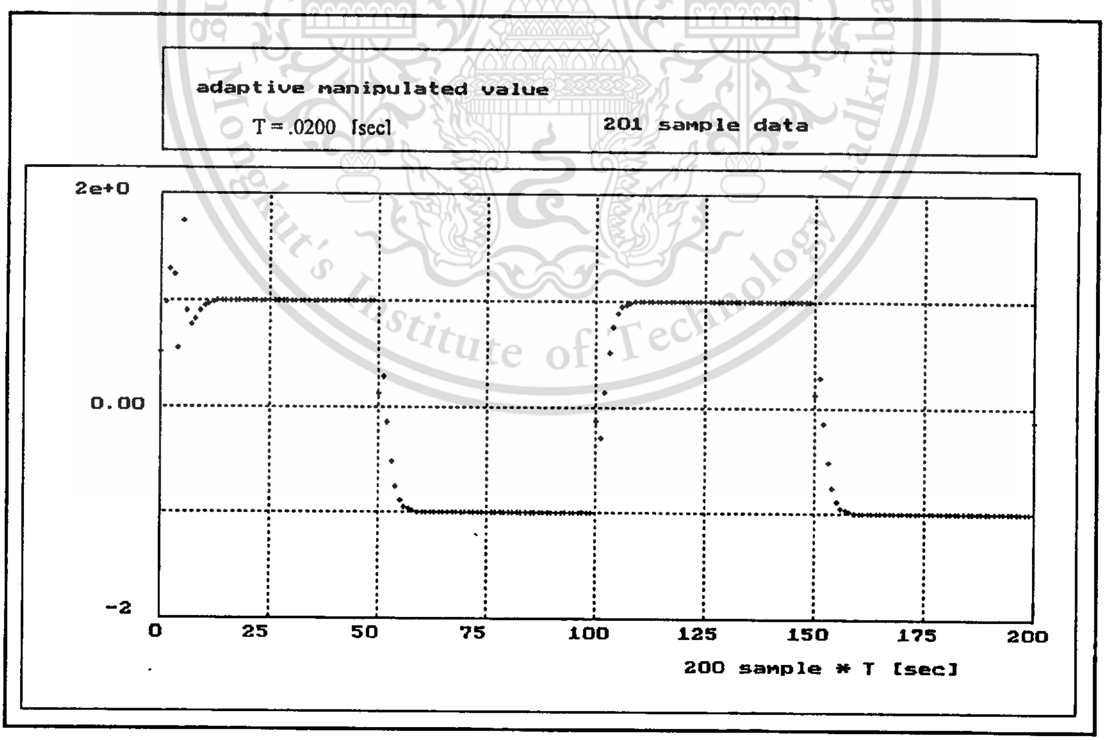

Figure 5.7 Adaptive manipulated value

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

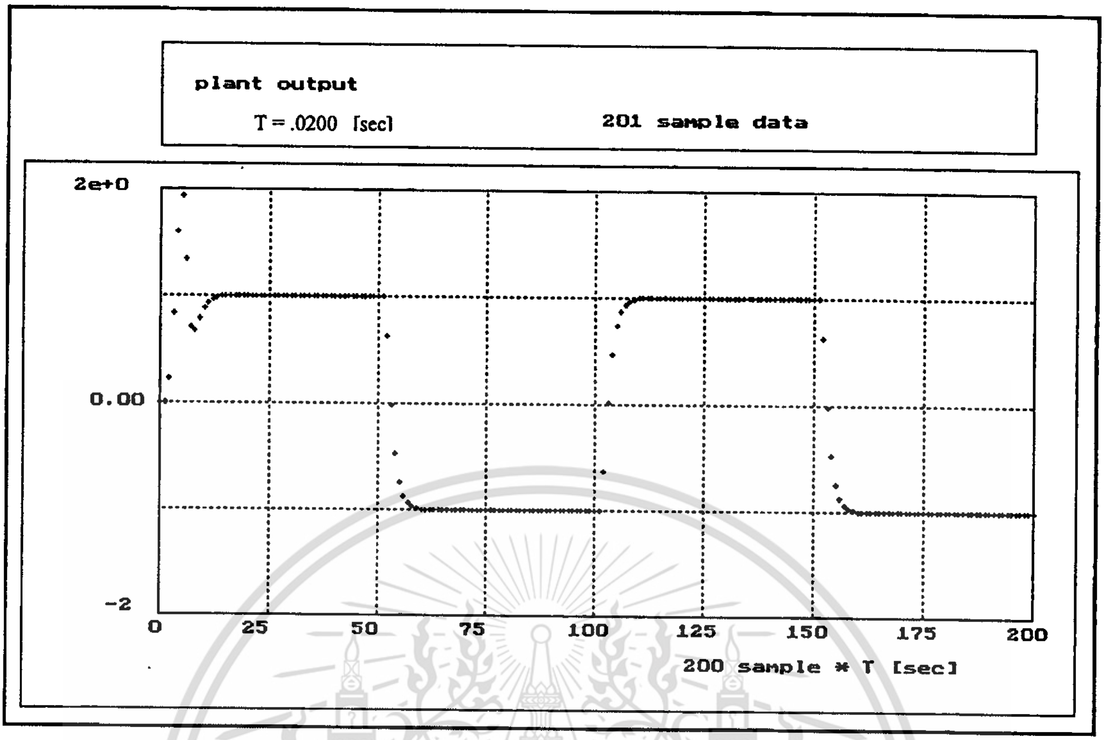

Figure 5.8 Plant output

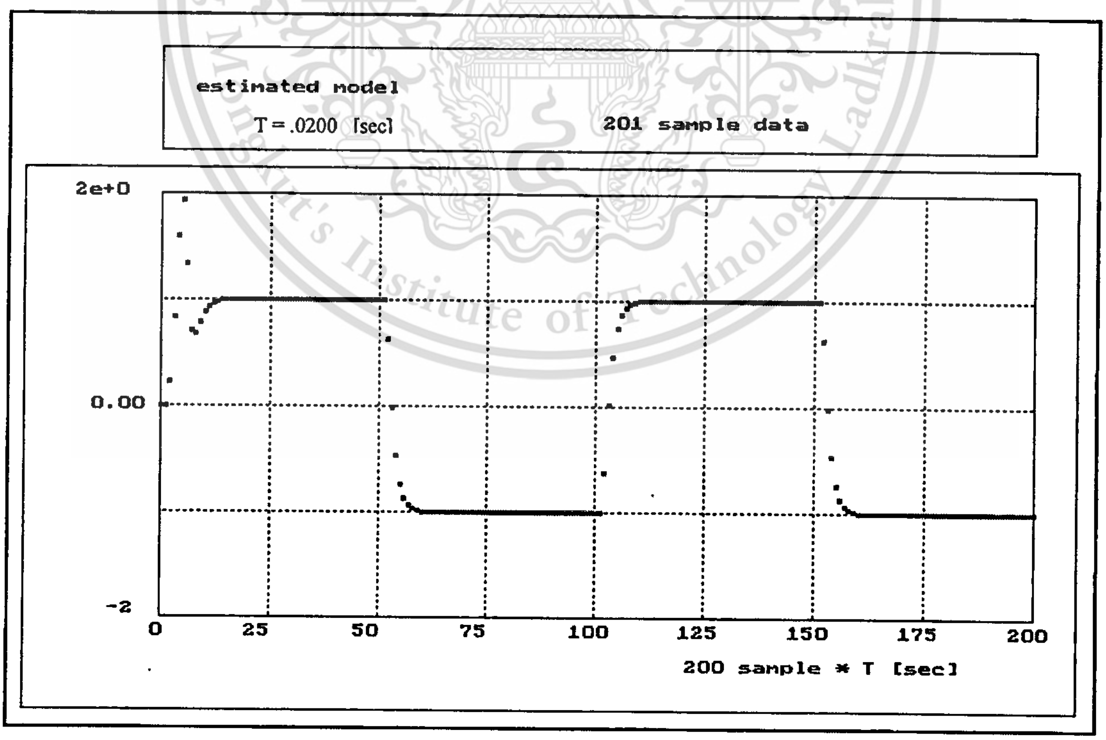

Figure 5.9 Estimated model

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

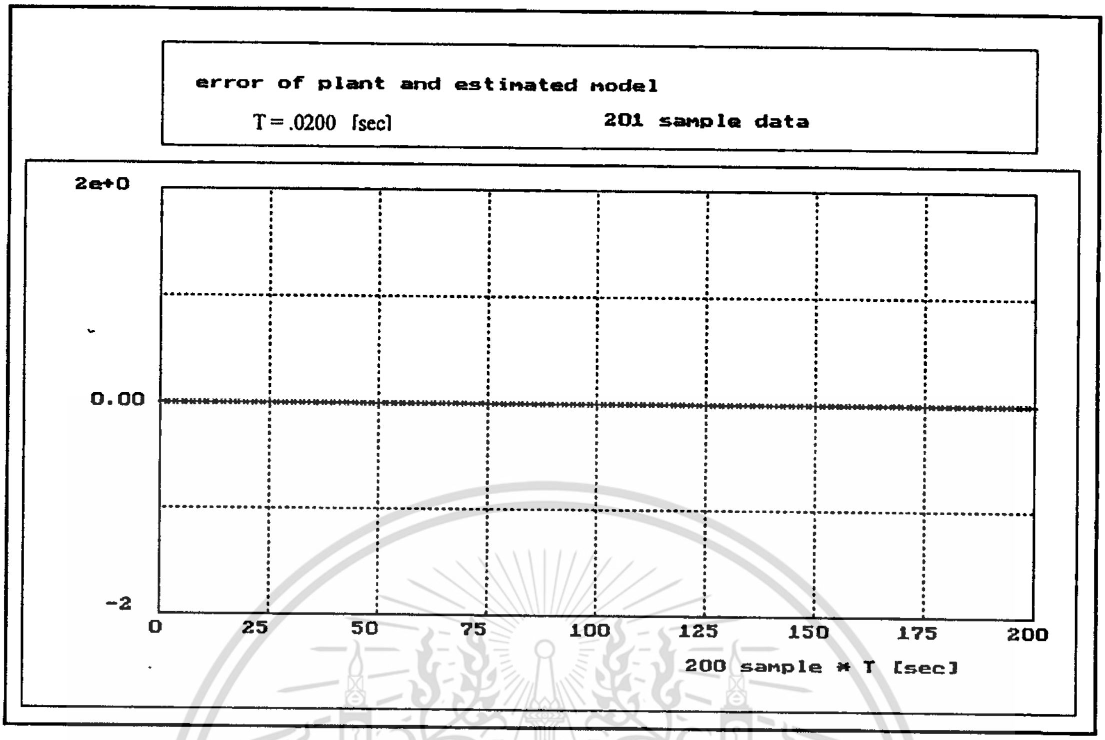

Figure 5.10 Error (different) of plant and estimated model

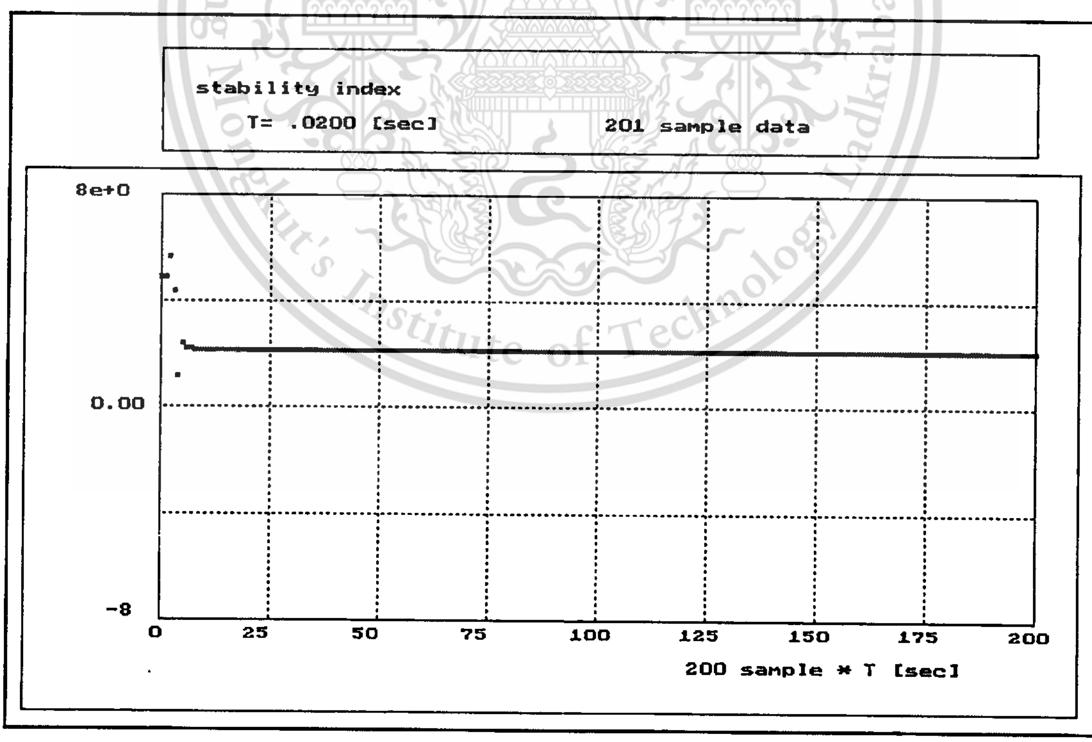

Figure 5.11 Stability index

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

From 201 sampling steps, plant estimated parameters, pre-filter and designed controller parameters are as follows

1-th parameter, $\hat{a}_1$ = -9.8538679e-01

2-th parameter, $\hat{a}_2$ = 8.1872839e-01

3-th parameter, $\hat{b}_1$ = 4.3103108e-01

4-th parameter, $\hat{b}_2$ = 4.0231055e-01

Pre-filter, $T(q^{-1})$ = 2.5919697e-01

Series compensator, s_1 = 7.8538382e-01

s_2 = 2.7119574e-01

Parallel compensator, r_0 = -8.3562076e-02

r_1 = -2.0914184e-01

r_2 = 5.5190086e-01

2. $\alpha = 0.6$

Simulation results :

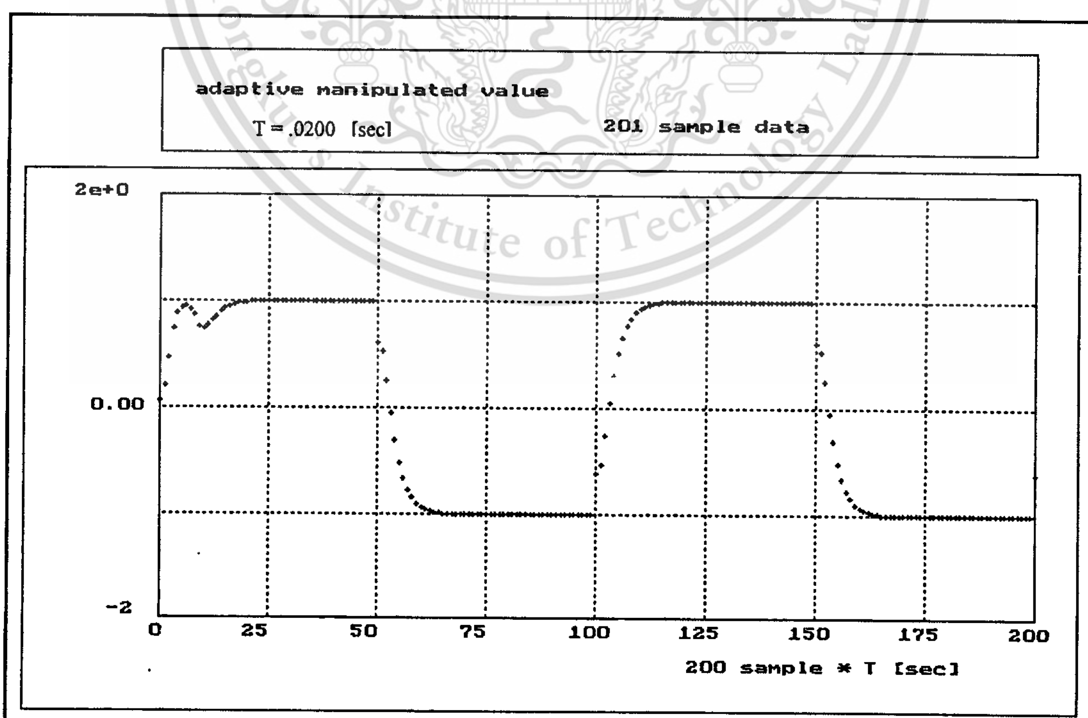

Figure5.12 Adaptive manipulated value

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

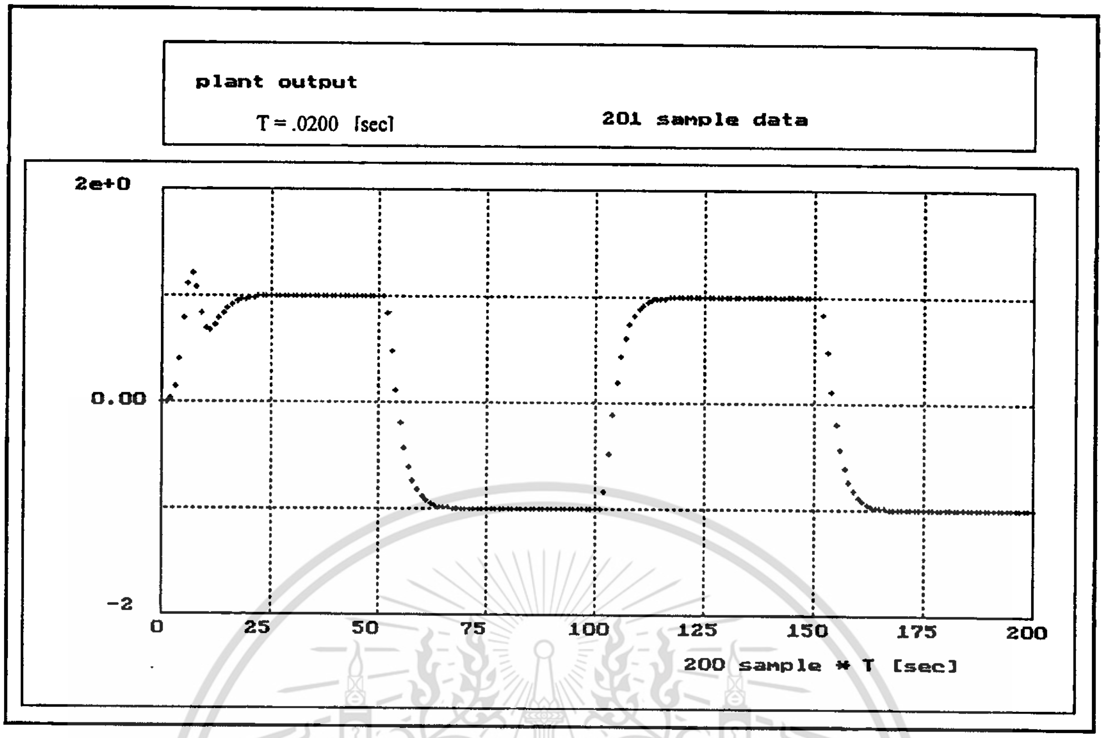

Figure 5.13 Plant output

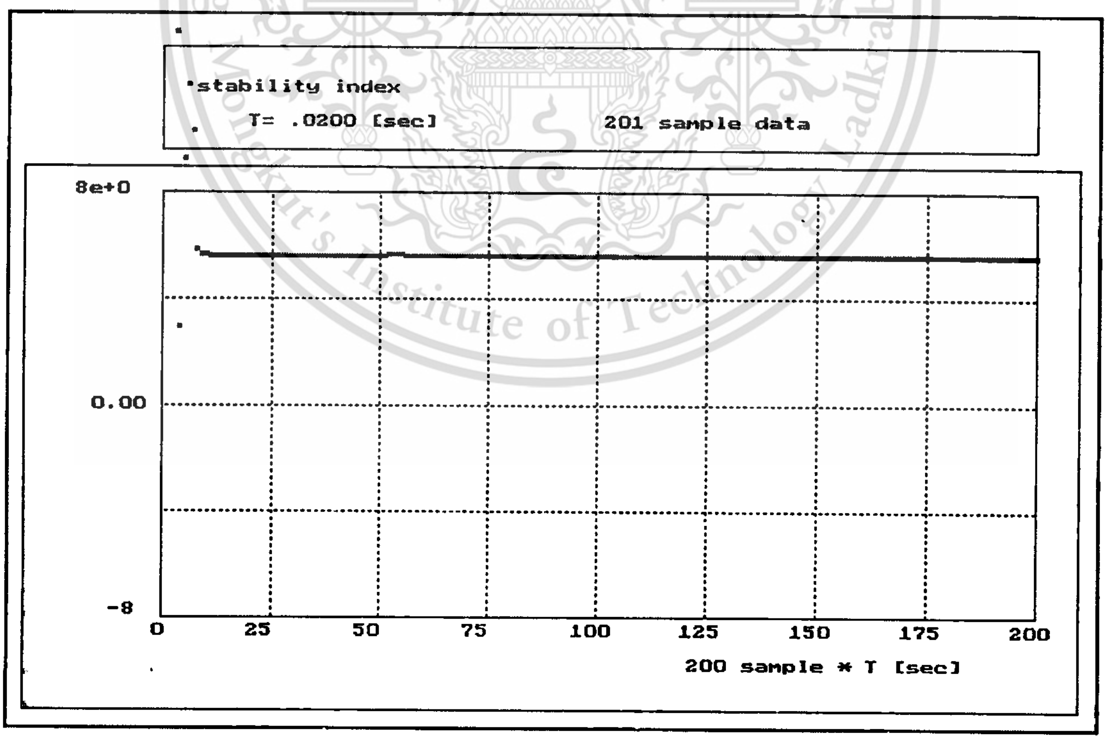

Figure 5.14 Stability index

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

For 201 sampling steps, plant estimated parameters, pre-filter and designed controller parameters are as follows

1-th parameter, $\hat{a}_1$ = -9.8536611e-01

2-th parameter, $\hat{a}_2$ = 8.1872624e-01

3-th parameter, $\hat{b}_1$ = 4.3103865e-01

4-th parameter, $\hat{b}_2$ = 4.0232173e-01

Pre-filter, $T(q^{-1})$ = 1.9199377e-01

Series compensator, s_1 = 7.8536612e-01

s_2 = 2.4079189e-01

Parallel compensator, r_0 = -2.9149345e-01

r_1 = -6.5251975e-03

r_2 = 4.9001241e-01

5.6.2 Added Noise Case

Variance = 0.001, $\alpha = 0.6$

Simulation results :

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

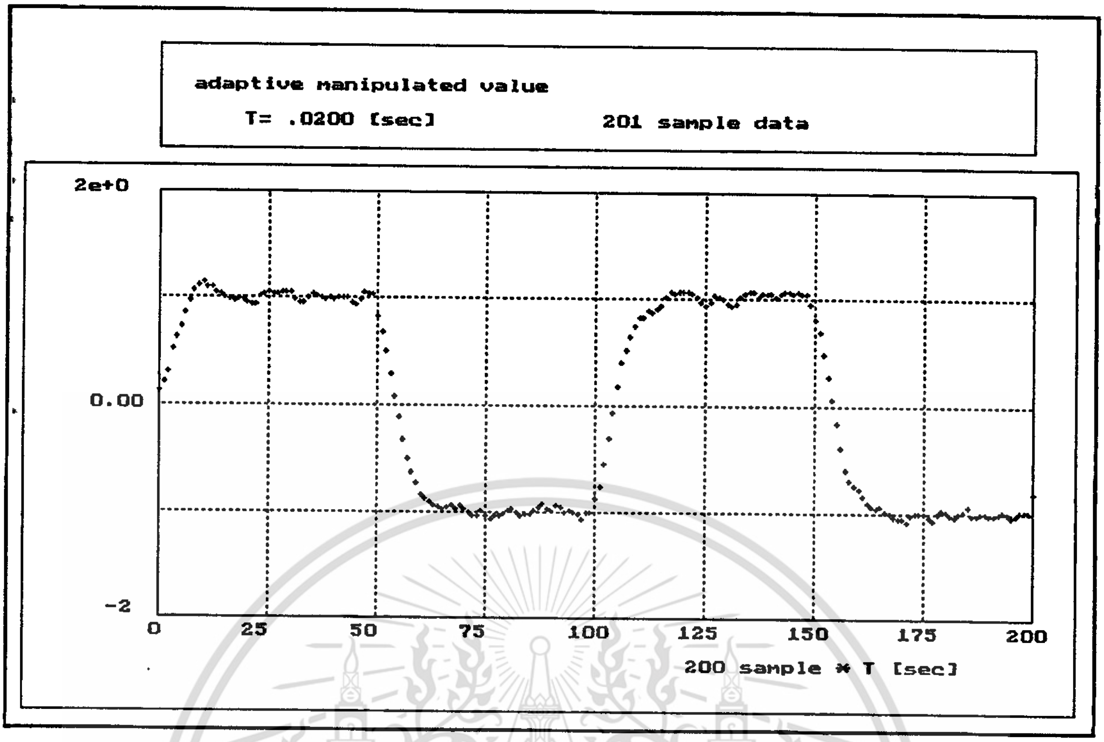

Figure 5.15 Adaptive manipulated value

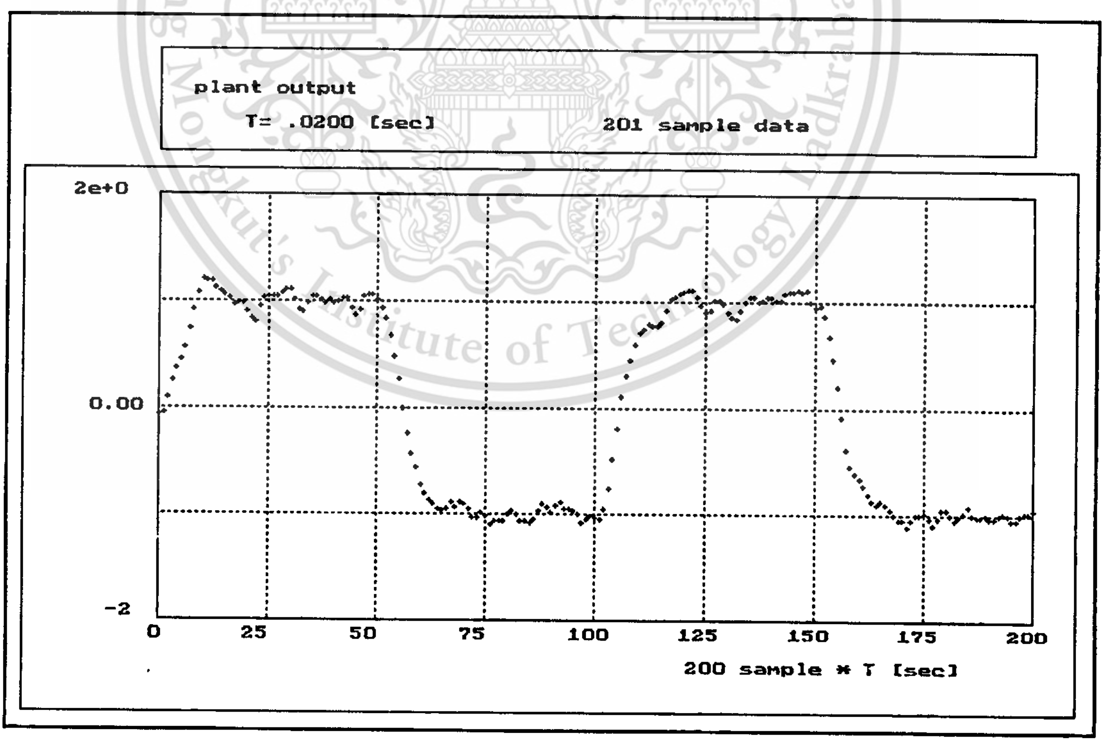

Figure 5.16 Plant output

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

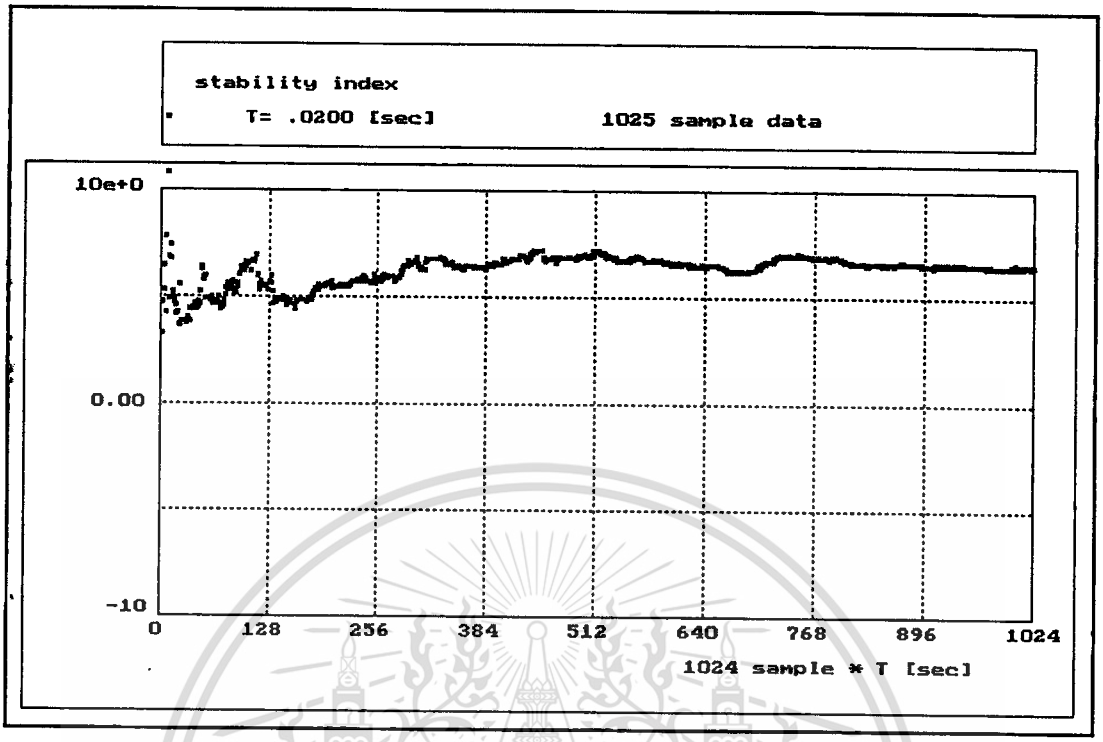

Figure 5.17 Stability index

From 201 sampling steps, plant estimated parameters, pre-filter and designed controller parameters are as follows

1-th parameter, $\hat{a}_1$ = -1.0174792e+00

2-th parameter, $\hat{a}_2$ = 8.7047356e-01

3-th parameter, $\hat{b}_1$ = 3.7457621e-01

4-th parameter, $\hat{b}_2$ = 4.7685355e-01

Pre-filter, $T(q^{-1})$ = 7.5167678e-02

Series compensator, s_1 = 2.1747923e-01

s_2 = -1.0642324e-01

Parallel compensator, r_0 = -7.0151210e-01

r_1 = 9.7095037e-01

r_2 = -1.9427060e-01

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

5.6.3 Add Constant Disturbance

For the constant disturbance of 0.5, $\alpha = 0.4$

Simulation results :

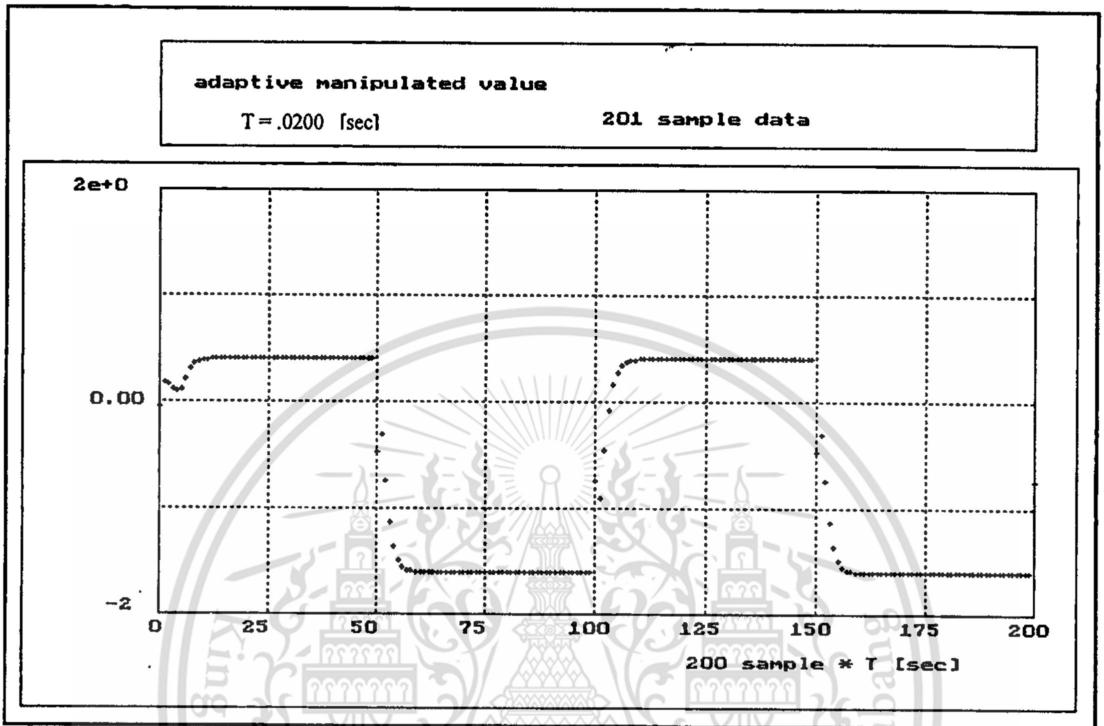

Figure 5.18 Adaptive manipulated value

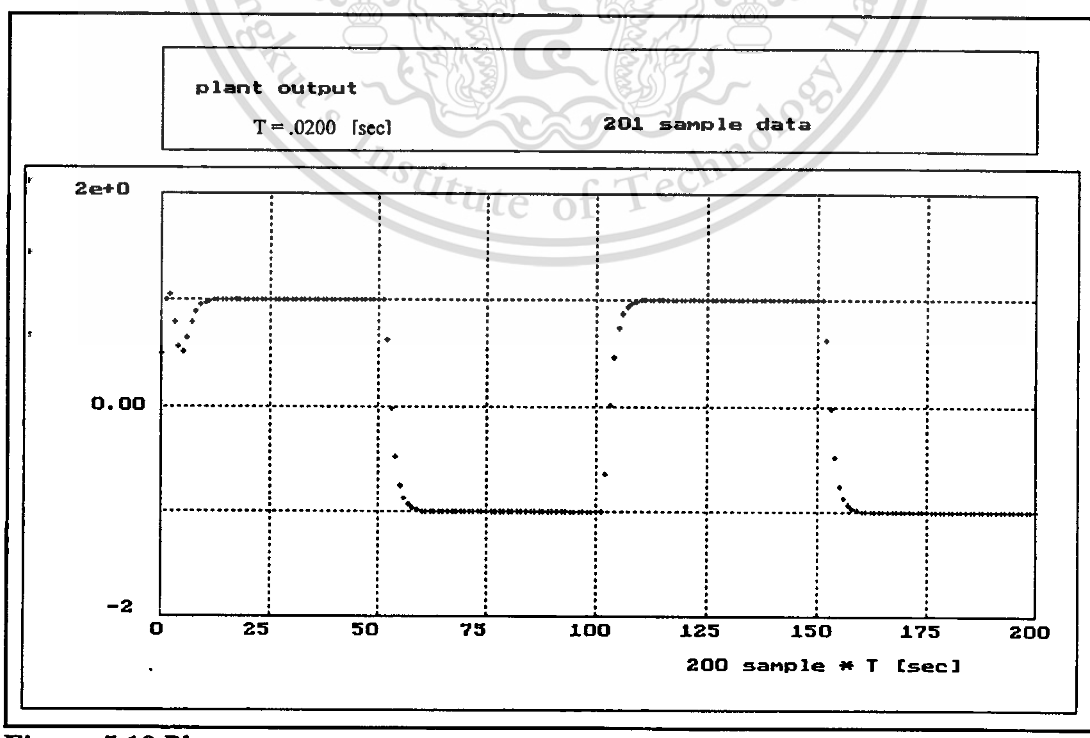

Figure 5.19 Plant output

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

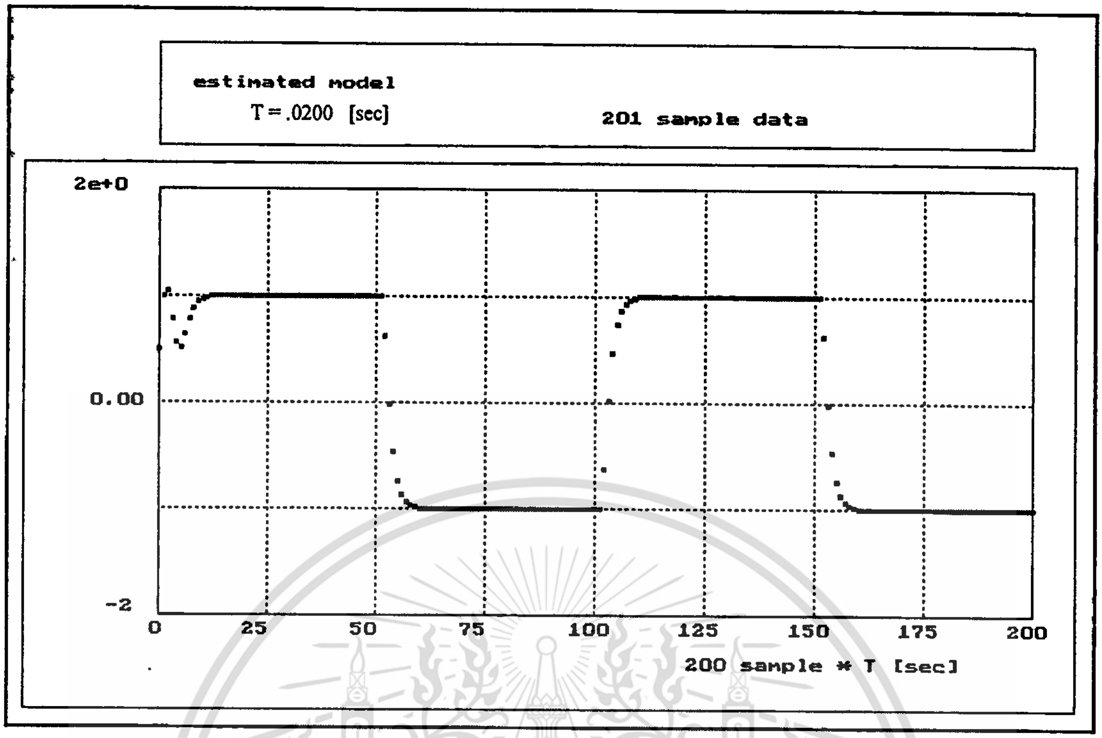

Figure 5.20 Estimated model

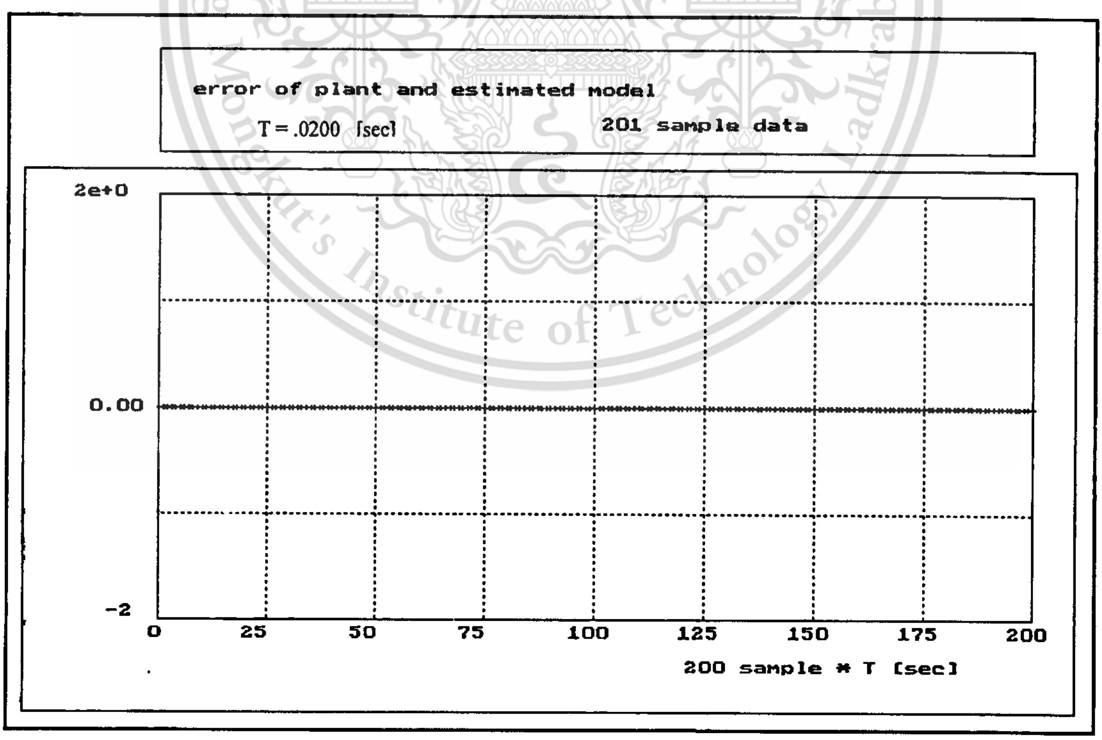

Figure 5.21 Error (different) of plant and estimated model

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

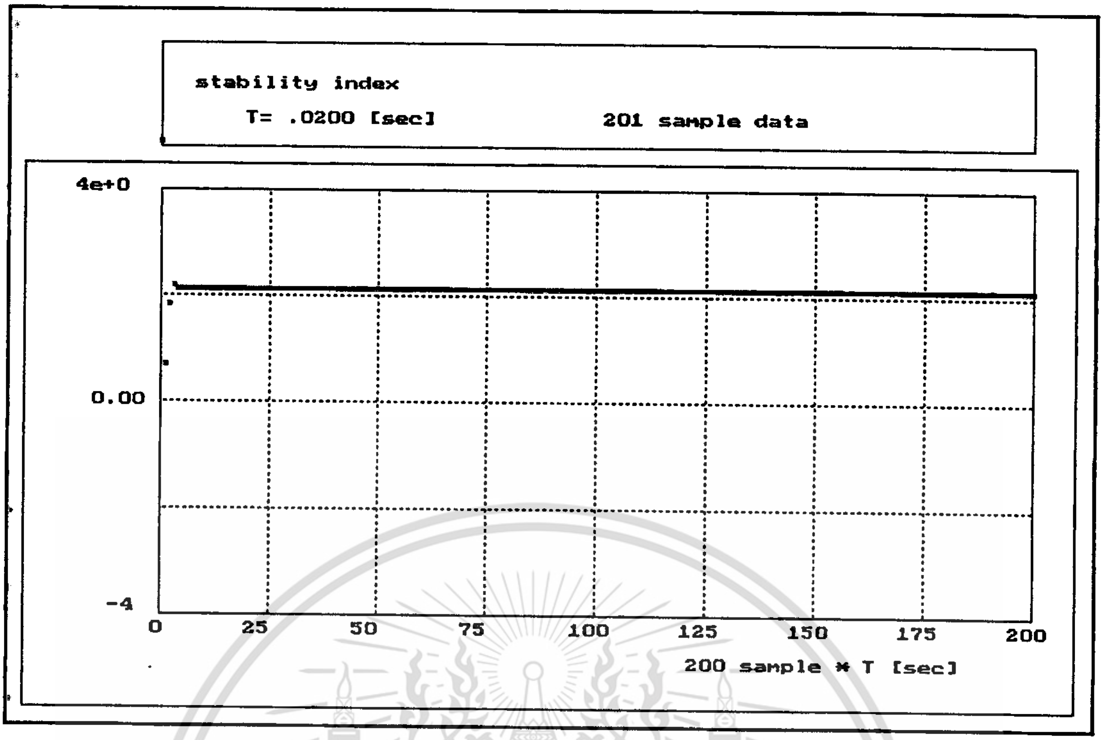

Figure 5.22 Stability index

From 201 sampling steps, plant estimated parameters, pre-filter and designed controller parameters are as follows

1-th parameter, $\hat{a}_1$ = -9.8538452e-01

2-th parameter, $\hat{a}_2$ = 8.1872845e-01

3-th parameter, $\hat{b}_1$ = 4.3103197e-01

4-th parameter, $\hat{b}_2$ = 4.0231207e-01

5-th parameter, = 5.0000250e-01

Pre-filter, $T(q^{-1})$ = 2.5919694e-01

Series compensator, s_1 = 7.8538340e-01

s_2 = 2.7119538e-01

Parallel compensator, r_0 = -8.3562352e-02

r_1 = -2.0914090e-01

r_2 = 5.5190015e-01

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

The simulations in section 5.6 are the examples that show the plant parameter estimation and the design of controller and pre-filter in conventional adaptive pole placement control system. In which, the parameters of the plant are constant. The simulation results show that when the stable pole $\alpha = 0.4$, the stability index of the series compensator is lower than the case when $\alpha = 0.6$, because the low α value means the fast speed of the disturbance rejection which is lead to low stability. The speed of output response of the overall control system also lower in case of small value of α. In cases of no noise and no constant disturbance, the estimated parameters are very closed to the true values. However, when the noise and constant disturbance are added to the simulation, the estimated parameters of the plant have a little error, even the disturbance is compensated.

5.7 Conclusion

This chapter explains the controller design in conventional adaptive pole placement control system under the ideal condition. The compensation for the constant disturbance and steady state error both are presented. At the same time, the stability index are introduced to check the relative stability of the designed series compensator. The last part in this chapter shows some simulation examples to support the theoretical of adaptive pole placement control system.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Chapter 6

Design of Strongly Stable System

This chapter proposes a useful method to design a strongly stable adaptive closed-loop control system where the pole of the system can be automatically adjusted. The closed-loop control system with the system parameters that are widely changed is considered. The recursive adaptive system which safely constructs the strongly stable system by introducing both stability index and fuzzy influence is presented. The simulation example and the usefulness of the proposed method are also discussed.

6.1 Introduction to Fuzzy Control Influence

The aim of fuzzy control is normally to substitute for or replace a skilled human operator with a fuzzy logic rule-based system. Because it is more reliable, more adaptive and provides more accurate information as compared with the crisp rule-based system.

A fuzzy set, the set element can be a member of set with a partial degree of membership between zero and one. There is a gradual transition from membership to non-membership in a fuzzy set and not sharply defined boundaries as in crisp set. The grade of membership in the fuzzy set is expressed by a membership function.

The rules for control applications are used to map input variables to output variables using IF-THEN statements. The conditions or antecedents of the rule follow the word IF and the conclusion or consequence follows the word THEN as the following form,

IF (condition), THEN (action)

Input $\Rightarrow$ output

The i^{th} control rule appears as follows:

IF x_1 is A_i, x_2 is B_i, x_3 is C_i,…, THEN y is Oi

where x_n ;(n = 1,2,3,…)are the inputs, y is the output, A_i, B_i, C_i and O_i are the values of membership functions, and n is the number of antecedents.

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

Membership functions are combined using an inference method based on the compositional rule of inference suggested by Zadeh [1973]. The inference is used to derive a conclusion from implications (facts) and premises. The final fuzzy output level is found using the Max-Min inference techniques. The fuzzy output that results from inference must be defuzzified to obtain a crisp number. In this thesis the centroid of gravity method is used for defuzzification as follows:

$$u_g(k-1) = \left\{\sum x_j \cdot \mu(x_j)\right\} / \sum \mu(x_j) \qquad (6.1)$$

where x_j is non-fuzzy value $\mu(x_j)$ is the value of membership function and $u_g(k\text{-}1)$ is the inference result. The block diagram of the complete process from input to output with two rules being fired is shown in figure 6.1.

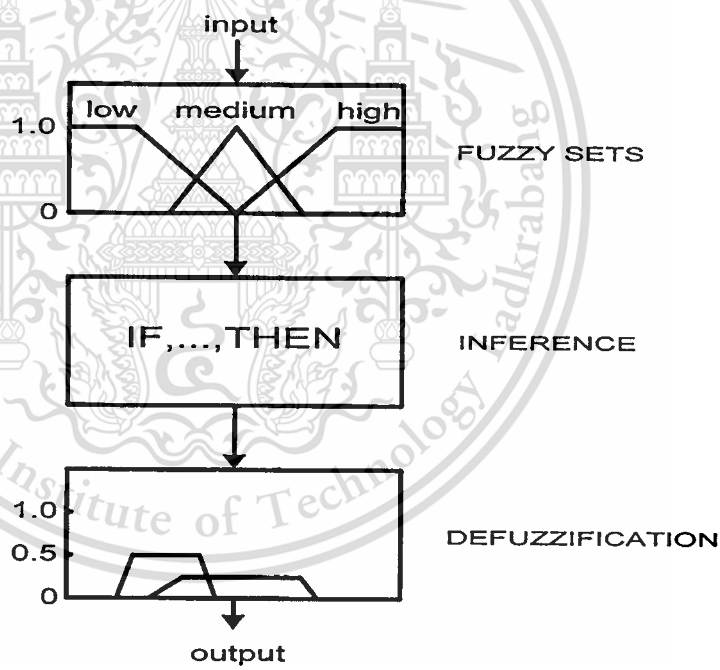

Figure 6.1 Fuzzy logic control process.

6.2 Fuzzy Inference for Pole Adjustment

To achieve the strongly stable adaptive pole placement control as stated in section 5.5, fuzzy inference is introduced to adjust the closed-loop pole of Diophantine equation recursively according to the plant parameter fluctuation and put the stability index into the specified area.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

This index γ is named as a fuzzy variable. The membership function used in the antecedent of the fuzzy rule has the trapezoidal shape to force the stability index into the specified area of the central fuzzy set (JF) as shown in figure 6.2

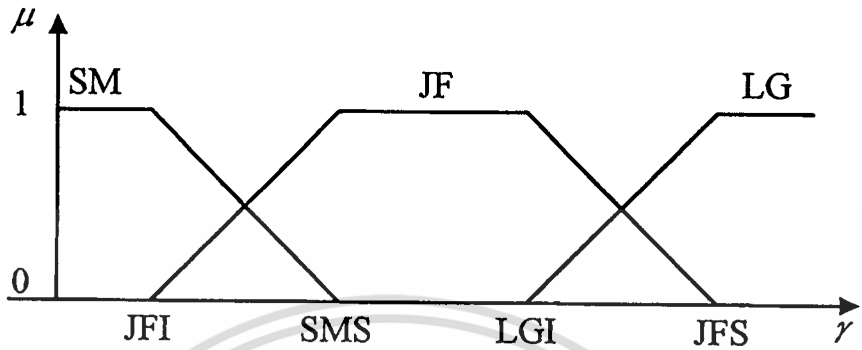

Figure 6.2 Membership function of antecedent.

where SM stands for small

LG stands for large

JF stands for just fit

JFI~JFS is the universe of discourse of fuzzy set JF

SMS is the maximum fuzzy variable of fuzzy set SM

LGI is the minimum fuzzy variable of fuzzy set LG

And the membership function used in the consequent of fuzzy rule has the triangular form as shown in figure 6.3

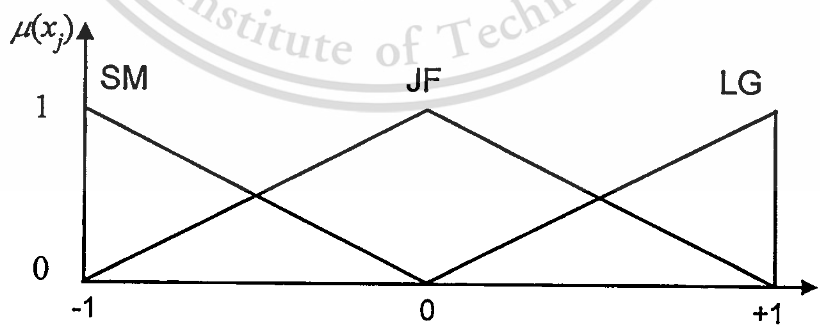

Figure 6.3 Membership function of consequent

The inference rules are

IF γ is μ_{SM}, THEN y is μ_{SM}

IF γ is μ_{JF}, THEN y is μ_{JF}

IF γ is μ_{LG}, THEN y is μ_{LG}

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

where γ is the input, y is the output; μ_{SM}, μ_{JF} and μ_{LG} are the value of membership functions. In order to place the stability index γ of the series compensator $S_c(s)$ into the specific area, the pole α of the closed-loop system is adjusted using the following formula

$$\alpha(k) = \alpha(k-1) + Cu_g(k-1)\text{; C: any constant} \tag{6.2}$$

6.3 Construction of Fuzzy Adaptive Pole Placement

The complete fuzzy adaptive pole placement control system is shown in figure 6.4. In adaptive controller, if the stable closed-loop pole in the Diophantine equation is chosen, the pre-filter $T(q^{-1})$, $S(q^{-1})$ and $R(q^{-1})$ are recursively calculated from Eqns.(5.2), (5.21) and (5.22) respectively according to the recursive least square parameter estimation calculation. At the same time, the stability index of $S_c(s)$ is also recursively calculated from Eqns.(5.24) and (5.25).And each value of stability index

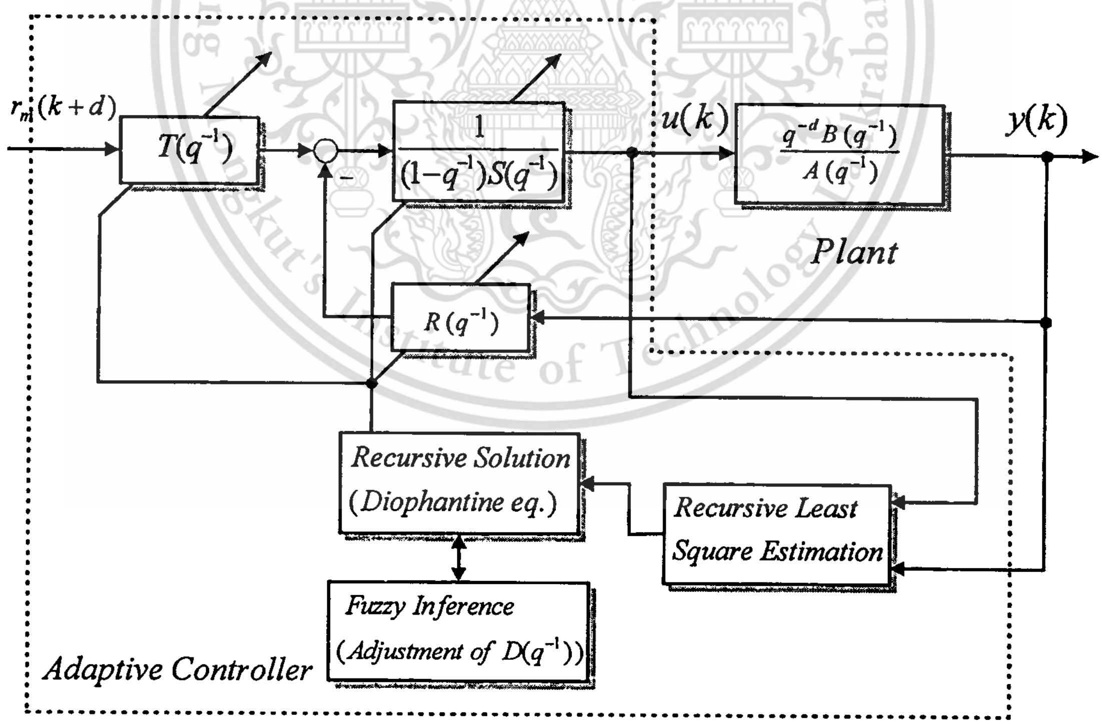

Figure 6.4 Block diagram of proposed adaptive control system.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

is used to adjust the stable pole recursively by the fuzzy inference result as in section 6.2. When the adapted value of stability index is placed into the specified area, which ensures the stable controller condition, the value of stable pole will stop changing. With this appropriate value of stable pole, the **strongly stable adaptive pole placement control system can be achieved.**

6.4 Design Examples and Simulation Results

6.4.1 Application to Second Order Plant

6.4.1.1 The Order of Diophantine Equation $nd = 3$

The second order continuous plant is selected as:

$$G(s) = \frac{K\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}, k = 1 \tag{6.3}$$

Given $\sum_f(\zeta = 0.1, \omega_n = 50)$ is the plant before the plant parameter fluctuation and $\sum_s(\zeta = 0.5, \omega_n = 15)$ is the plant after the plant parameter widely fluctuated.

First, the plant Σ_f is transformed into a discrete system for a sampling time T=0.02 [sec], then

$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} = \frac{q^{-1}(4.3103\times10^{-1} + 4.0231\times10^{-1}q^{-1})}{1 - 9.8539\times10^{-1}q^{-1} + 8.1873\times10^{-1}q^{-2}}$$

Due to the calculation delay, d should be increased by 1, yielding d = 2. When a stable pole in Eqn.(5.3) is designed and Eqn.(5.20) is solved, then the pole placement controller is obtained. Given α = 0.45, the controller $S(q^{-1})$ and $R(q^{-1})$ are

$$S(q^{-1}) = 1 + 6.3538098\times10^{-1}q^{-1} + 1.7898454\times10^{-1}q^{-2}$$
$$R(q^{-1}) = -2.6475137\times10^{-1} + 1.095342\times10^{-1}q^{-1} + 3.6424536\times10^{-1}q^{-2}$$

respectively. The stability index γ in Eqn.(5.24) of the $S(q^{-1})$ is obtained. When the frequency response is calculated from the loop transfer function of this discrete system, the gain- phase margin g_m, p_m and gain crossover frequency ω_{cg} are calculated

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

respectively. The above calculation is repeated while changing the stable pole α from 0.2 to 0.8. The plant Σ_s is calculated in the same procedure as Σ_f, and four important characteristics of both plants can be obtained respectively as shown in figures 6.5, 6.6, and 6.7 and the stability index γ to the closed-loop pole α is shown in figure 6.8. From figures 6.5 to 6.7, the control specification of both plants can be obtained as shown in Table 6.1. Then the range of the closed-loop pole that fills this specification is obtained as

$$0.2 \le \alpha_f \le 0.37, \quad 0.66 \le \alpha_s \le 0.74 \tag{6.4}$$

where the closed-loop poles α_f and α_s are the designed ranges of the plant Σ_f and Σ_s respectively.

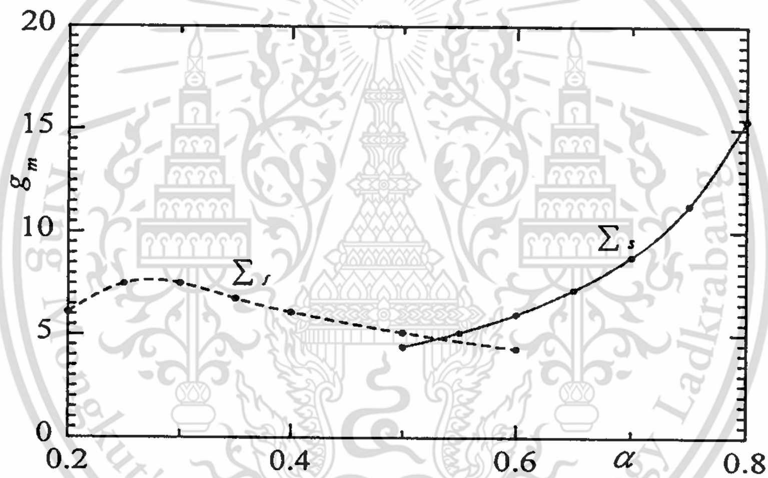

Figure 6.5 Each gain margin g_m of plants Σ_f and Σ_s to stable closed-loop pole α

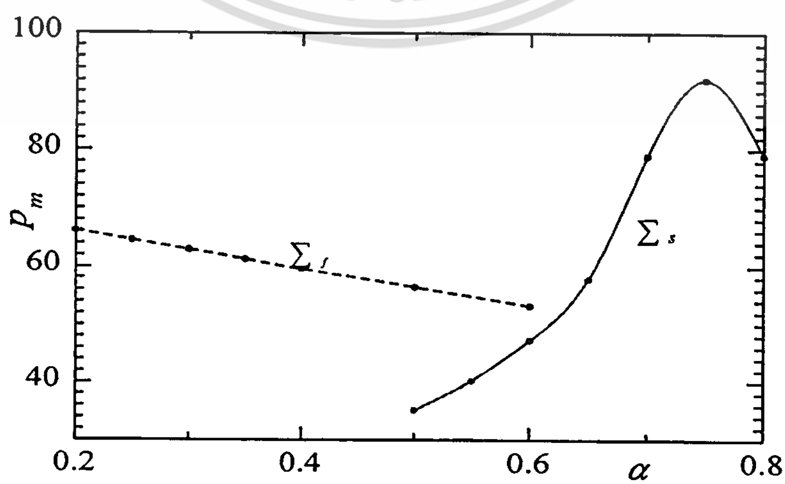

Figure 6.6 Each phase margin p_m of plants Σ_f and Σ_s to stable closed-loop pole α

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

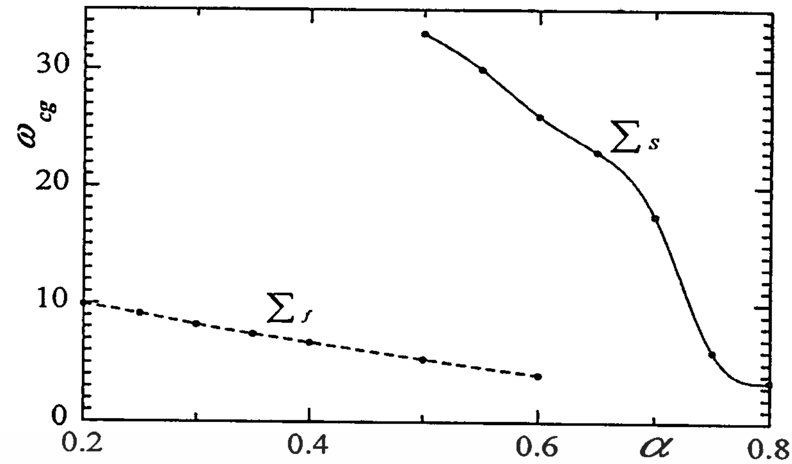

Figure 6.7 Each gain-crossover frequency ω_{cg} of plants Σ_f and Σ_s to stable closed-loop pole α

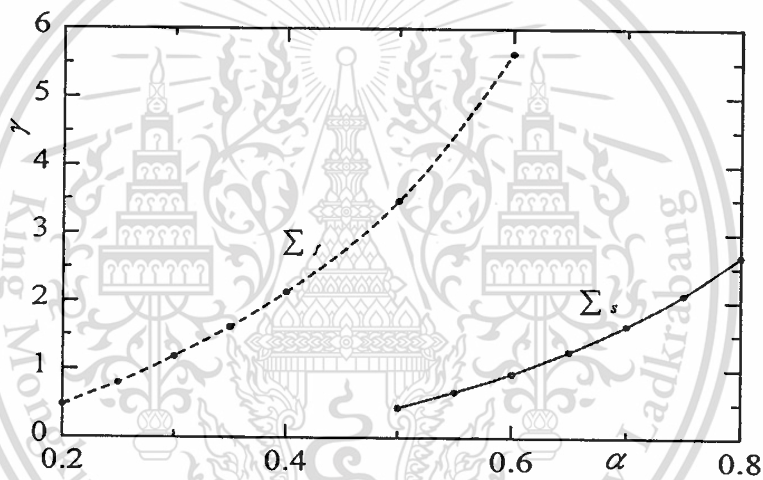

Figure 6.8 Each index γ of plants Σ_f and Σ_s to stable closed-loop pole α

Table 6.1 Control specification (2nd order plant, $nd = 3$)

	g_m [dB]	p_m [dB]	ω_{cg} [rad./sec.]
Σ_f	$g_m \geq 6.5$	$p_m \geq 60$	$\omega_{cg} \geq 7$
Σ_s	$g_m \geq 6.5$	$p_m \geq 60$	$\omega_{cg} \geq 8$

Therefore, if the inequality in(6.4) that fills the control specification is considered, then the range of index γ where has the fine performance of control is decided from figure 6.8 as follows:

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$$
\begin{array}{lll}
l_f \le \gamma_f \le u_f, & l_f = 0.58, & u_f = 1.78 \\
l_s \le \gamma_s \le u_s, & l_s = 1.34, & u_s = 2.0
\end{array}
\tag{6.5}
$$

where the index γ_f and γ_s are the designed ranges of the plant Σ_f and Σ_s respectively. The horizontal scale in membership function of figure 6.2 can be selected as the common part $\gamma_f \cap \gamma_s$ of the inequality in (6.5). In order to reduce the range of SMS and LGI, the next relation can be selected for membership function.

$$
\begin{array}{ll}
\mathrm{JFI} = \min(l_f, l_s) = 0.58, & \mathrm{SMS} = (u_s + l_f)/2 = 1.29 \\
\mathrm{LGI} = (u_f + l_s)/2 = 1.56, & \mathrm{JFS} = \max(u_f, u_s) = 2.0
\end{array}
\tag{6.6}
$$

Then, $LGI - SMS = 0.27$

By using the fuzzy logic rule base in figure 6.2,the complete inference is found in terms of centroid gravity in figure 6.3, which is calculated using Eqn.(6.1) and the adjusting stable pole α is calculated from Eqn.(6.2), the value of C in Eqn.(6.2) is selected to a value of 0.005, see Appendix C.

Simulation Results (2nd order plant with $nd = 3$)

First, the adaptive pole placement control by the conventional method is simulated and the stable closed-loop pole α is chosen with a constant value of 0.45. The square wave of unity amplitude and 100 samplings period is used as the input signal to the system.The initial adaptation gain is 10^4I and the initial parameter vector $\hat{\theta}^T = [0.1\ \ 0.2\ \ 0.3\ \ 0.4]$. And the **other two different parameter plants are selected as the simulation plants,** such as $\Sigma_{f1}(\zeta=0.15, \omega_n=45)$ and $\Sigma_{s1}(\zeta=0.45, \omega_n=15)$. In this simulation, the parameters have been changed into Σ_{s1} from Σ_{f1} at step 400 and the very trifling observation noise of variance $\sigma^2_n = 5\times10^{-6}$ has been added to the simulation. Then the plant output, the manipulated variable and the stability index are shown in figures 6.9, 6.10, and 6.11 respectively.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

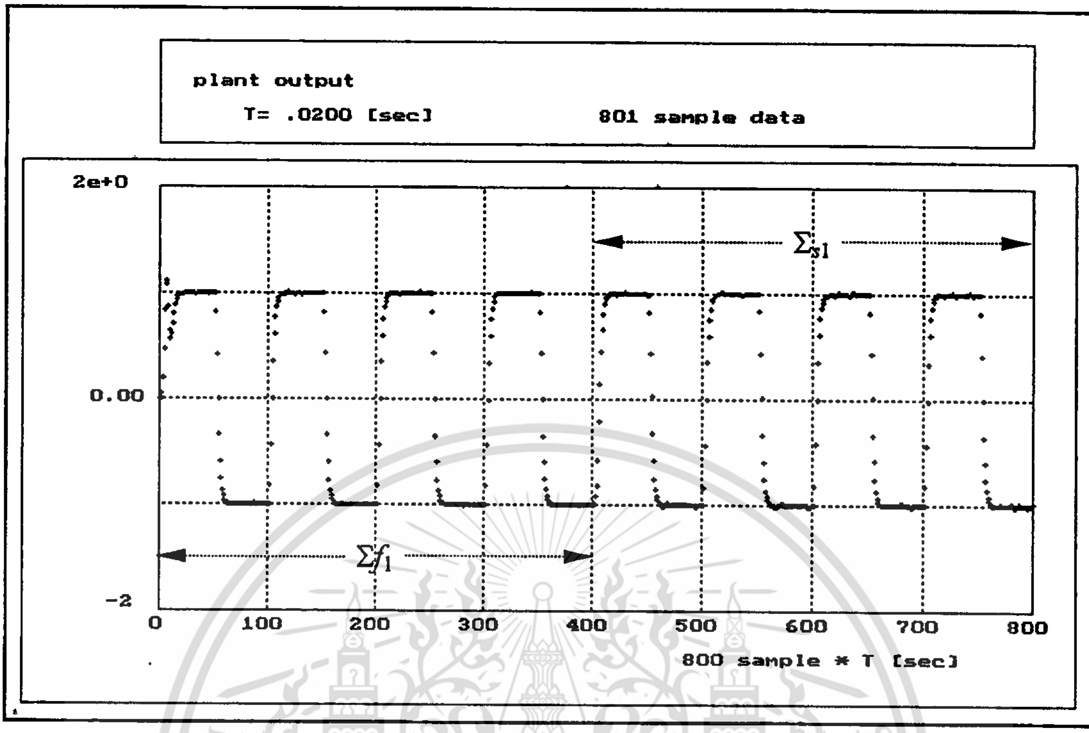

Figure 6.9 Plant output controlled by conventional adaptive pole placement

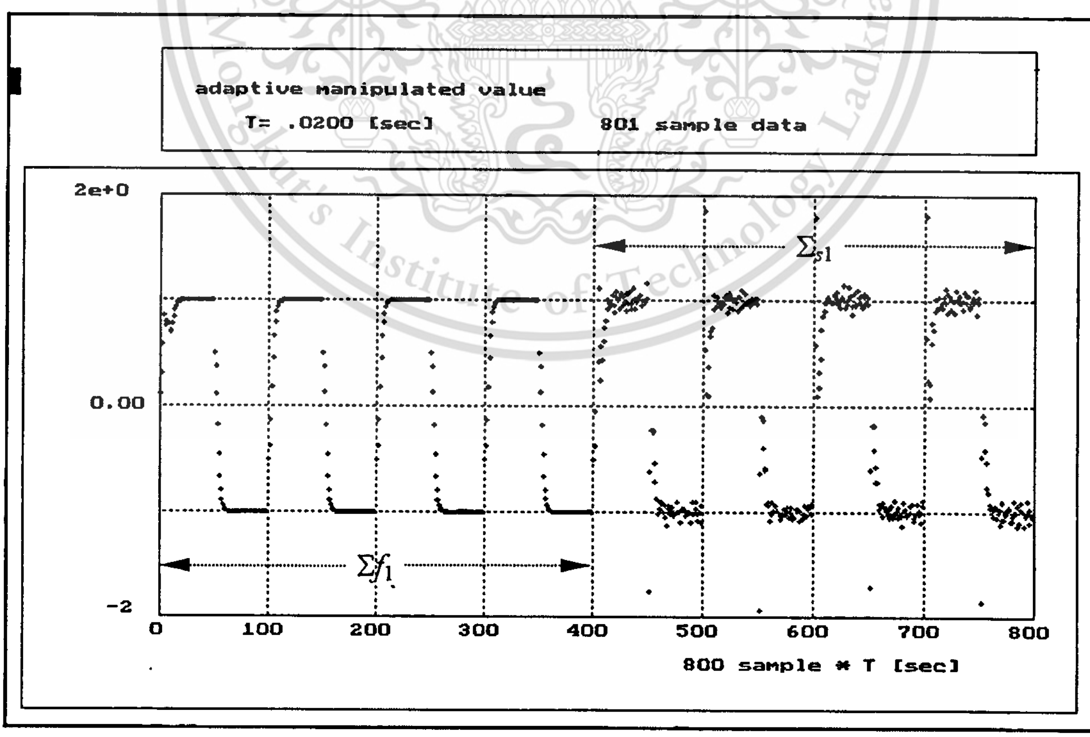

Figure 6.10 Manipulated variable synthesized by conventional adaptive pole placement

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

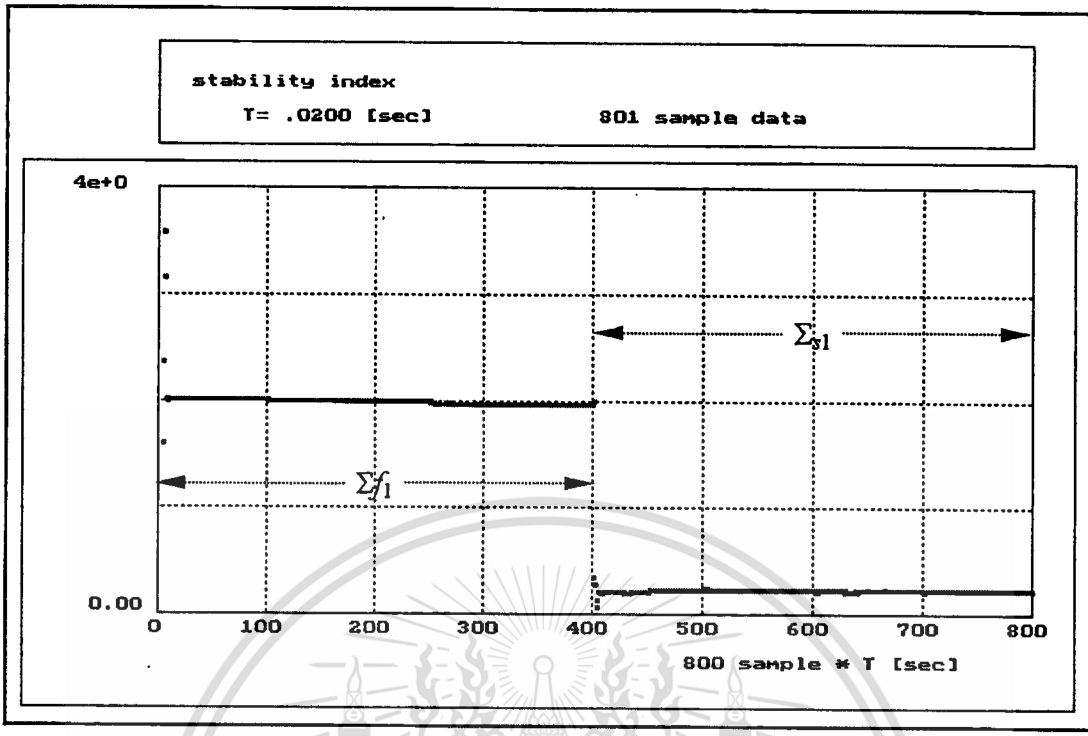

Figure 6.11 Stability index γ in case of conventional adaptive pole placement

From figure 6.9, the control result appears to be good. However, figure 6.11 reveals that the stability index γ is very small in the area of the Σ_{s1} plant because the stable pole is the same value in the area of Σ_{s1} plant as shown in figure 6.11-1 and the series compensator $[S(q^{-1})]^{-1}$ is unstable. When the unstable compensator appears in the pole placement system, the manipulated variable is greatly disturbed for even minimal random disturbance as shown in figure 6.10. Accordingly, a strongly stable system is not realized in the case of conventional method and the appropriate pole selection becomes difficult.

In the proposed method, the simulation results obtained are shown in figure 6.12 to 6.14. Again, the plant parameters are changed to Σ_{s1} from Σ_{f1} at step 400. While the resultant stability index γ is placed in the designed range as shown Fig 6.12. The closed-loop pole is automatically adjusted in figure 6.13 and the plant output is shown in figure6.14.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

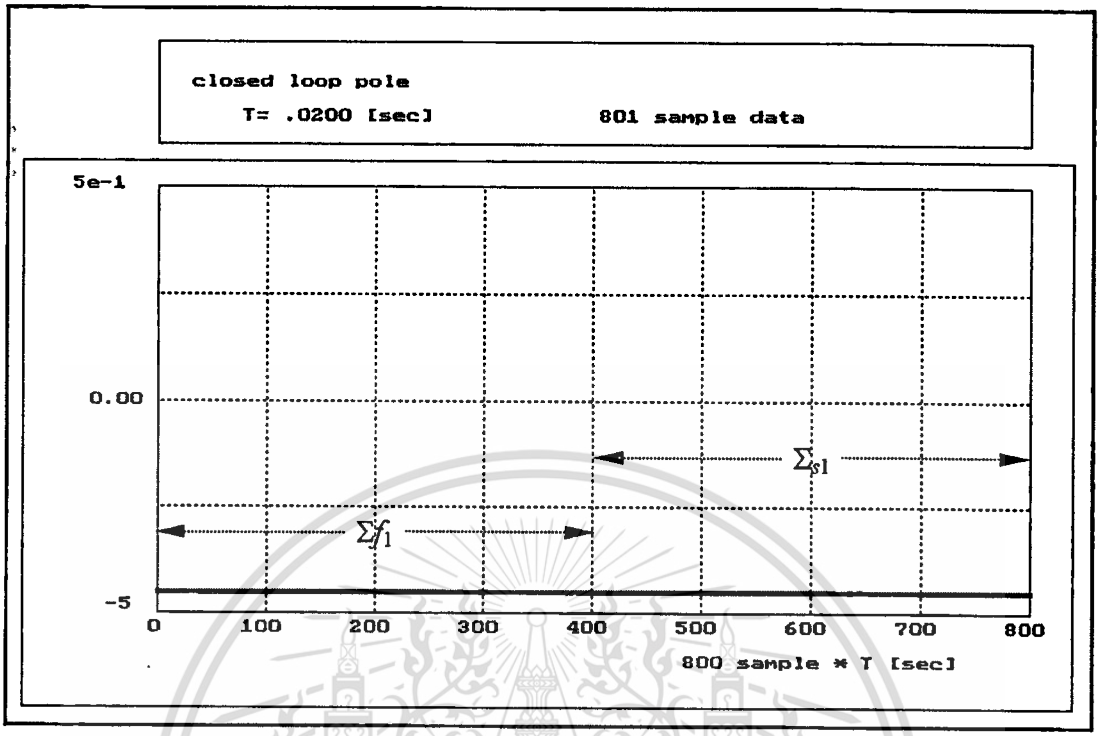

Figure 6.11-1 Stable closed-loop pole α in case of conventional pole placement

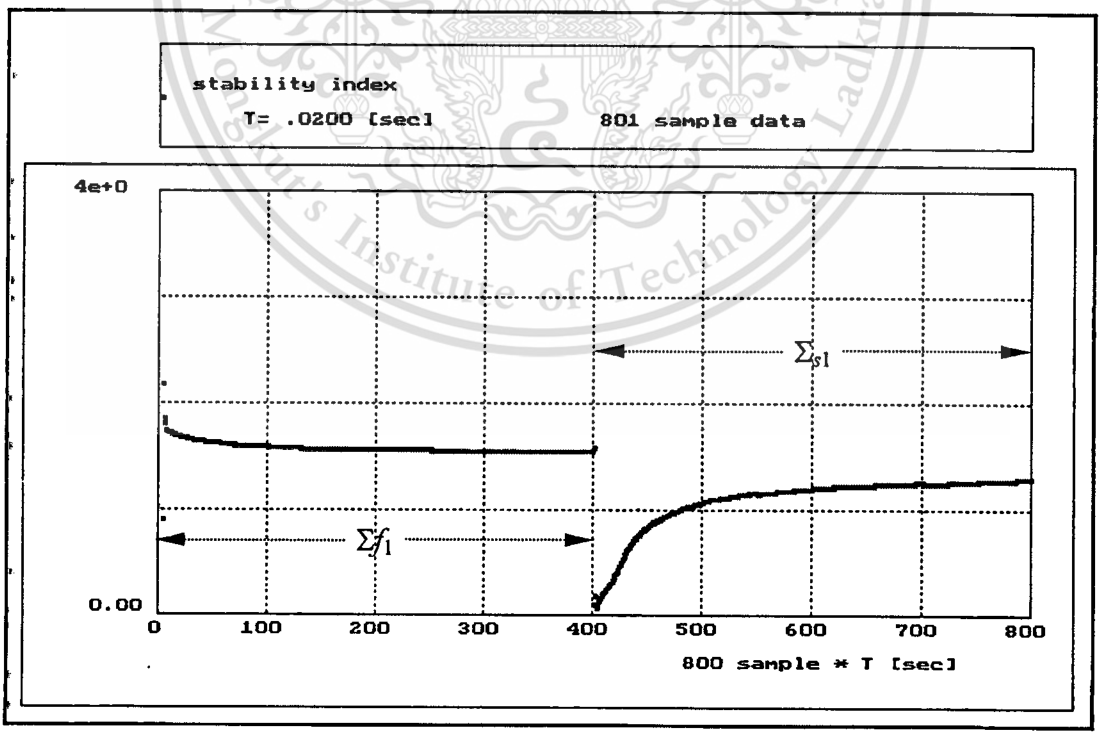

Figure 6.12 Stability index γ in case of proposed adaptive pole placement method (nd=3)

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

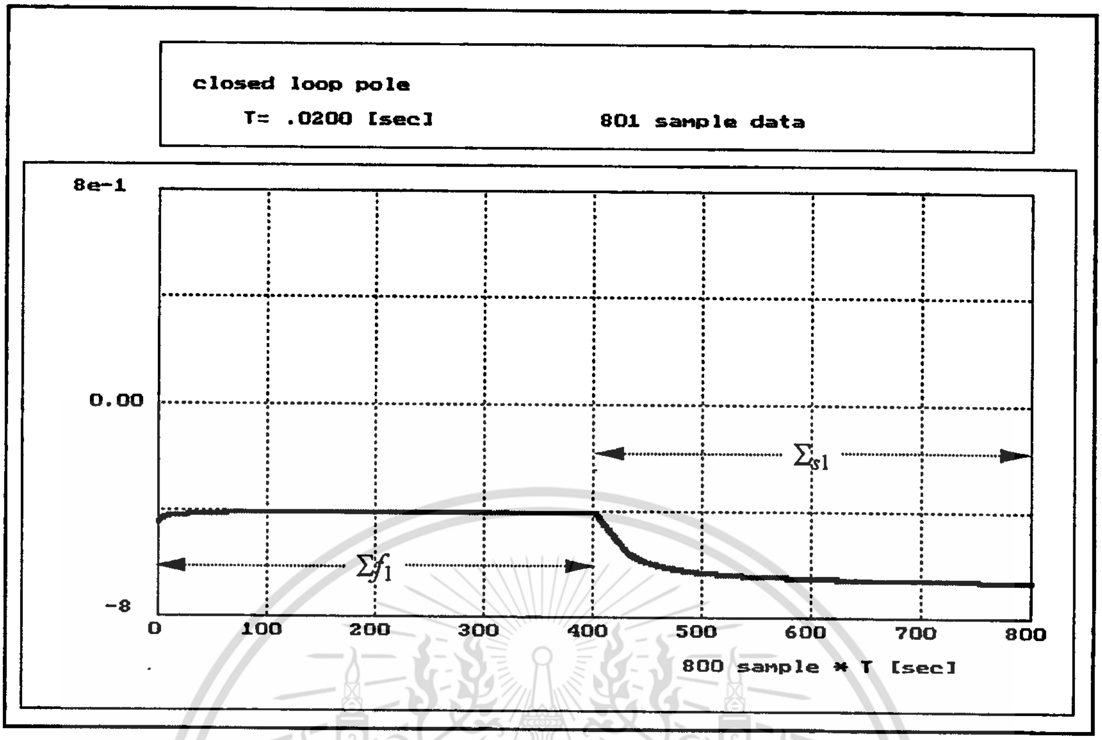

Figure 6.13 Stable closed-loop pole α in case of proposed method (nd=3)

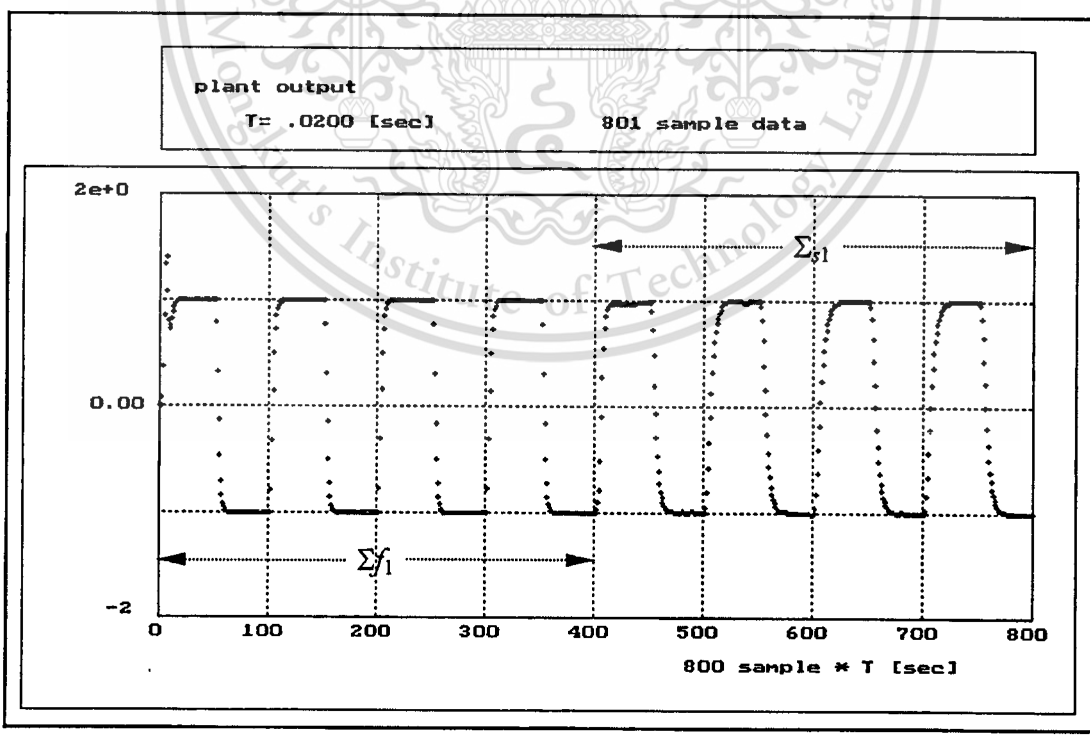

Figure 6.14 Plant output controlled by proposed adaptive pole placement method (nd=3)

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

6.4.1.2 The order of Diophentine Equation $nd = 2$

The same second order plant as in first case is used and the same design procedure is repeated. The four important characteristics of the control system are shown in figure 6.15 to 6.18 respectively. Table 6.2 is the control specification which considering from figure 6.15 to 6.17.

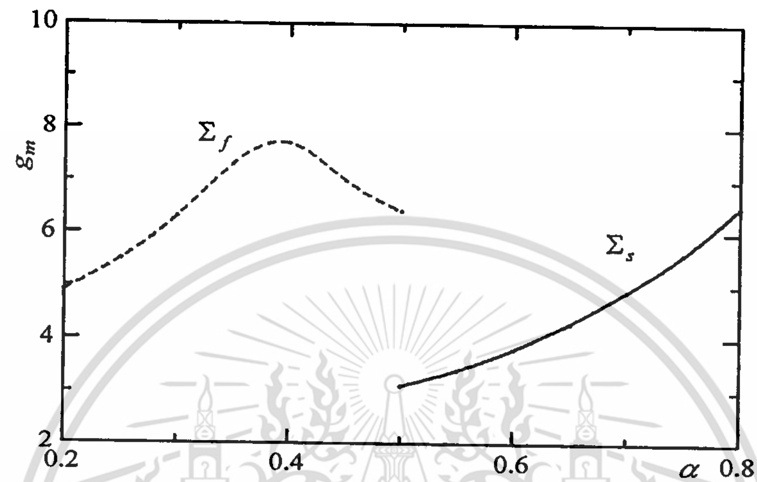

Figure 6.15 Each gain margin g_m of plants Σ_f and Σ_s to stable closed-loop pole α

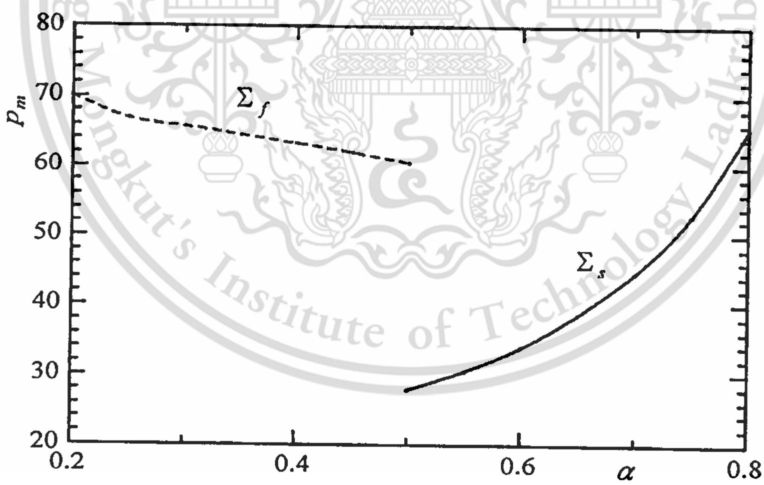

Figure 6.16 Each phase margin p_m of plants Σ_f and Σ_s to stable closed-loop pole α

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

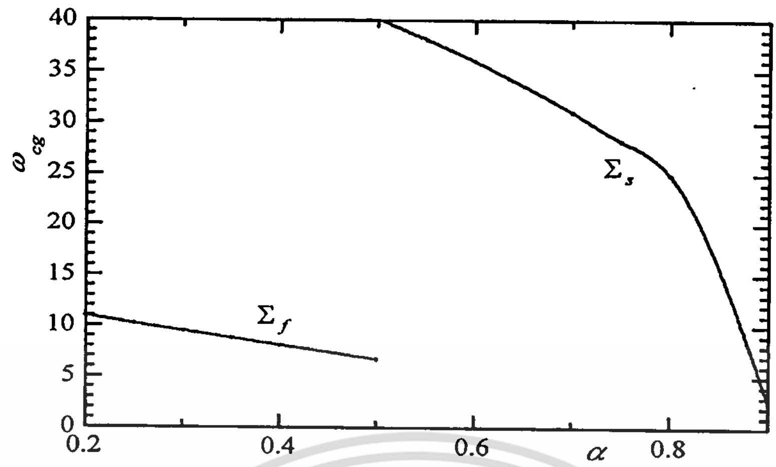

Figure 6.17 Each gain-crossover frequency ω_{cg} of plants Σ_f and Σ_s to stable closed-loop pole α

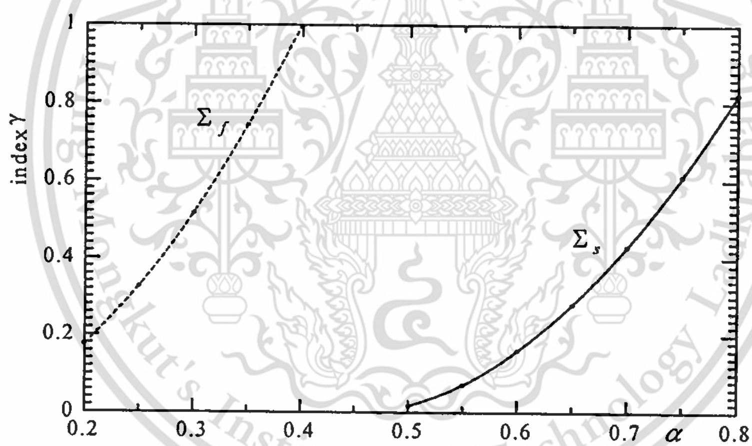

Figure 6.18 Each index γ of plants Σ_f and Σ_s to stable closed-loop pole α

Table 6.2 Control specification (2nd oder plant, $nd = 2$)

	g_m [dB]	p_m [deg.]	ω_{cg} [rad./sec.]
Σ_f	$g_m \geq 5.5$	$p_m \geq 40$	$\omega_{cg} \geq 5$
Σ_s	$g_m \geq 4.3$	$p_m \geq 40$	$\omega_{cg} \geq 5$

The stability index to the closed-loop pole α is shown in figure 6.18. And the range of index γ where has fine performance of control is decided as:

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

$l_f \le \gamma_f \le u_f, \quad l_f = 0.33, \quad u_f = 0.52$

$l_s \le \gamma_s \le u_s, \quad l_s = 0.35, \quad u_s = 0.44$

then,

$\text{JFI} = \min(l_s, l_f) = 0.33$

$\text{SMS} = (u_s + l_f)/2 = 0.385$

$\text{LGI} = (u_f + l_s)/2 = 0.435$

$\text{JFS} = \max(u_f, u_s) = 0.52$

$\text{LGI - SMS} = 0.05$

Simulation Results

The stable closed-loop pole α is chosen as 0.45 and the observation noise of variance σ^2_n=1x10^{-6} is added to the simulation. The square wave of unity amplitude and 100 samplings period is used as the input signal to the system. The initial adaptation gain is 10^{4}I and the initial parameter vector $\hat{\theta}^T$ =[0.1 0.2 0.3 0.4]. The value of C in Eqn.(6.2) is selected to a value of 0.001.And the **other two different parameter plants are selected** The control results are shown in figures 6.19 to 6.22 respectively.

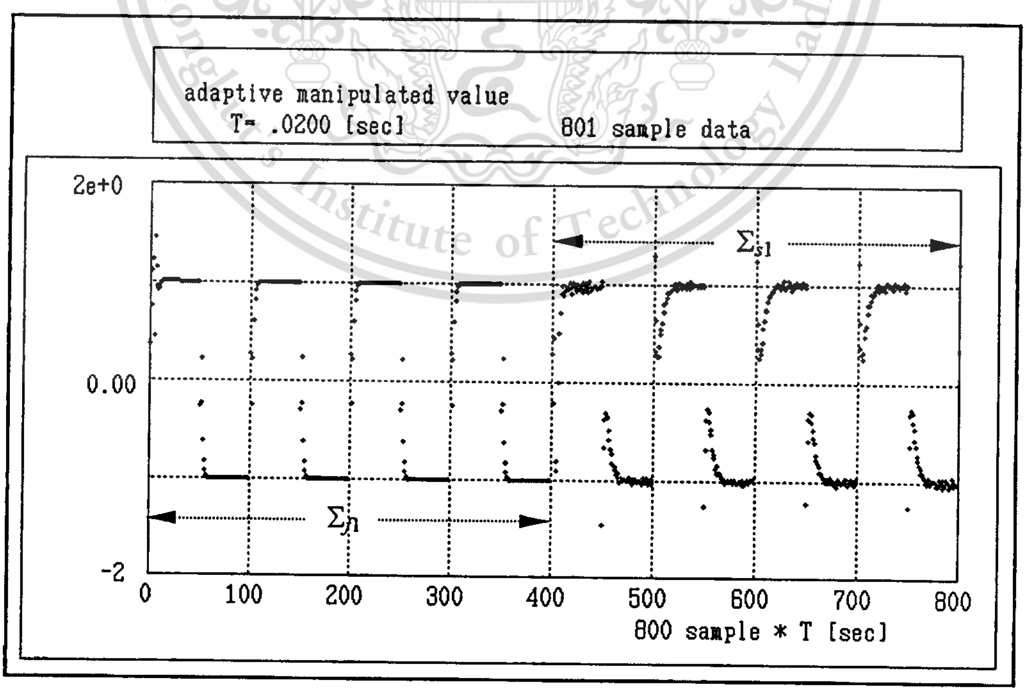

Figure 6.19 Manipulated variable synthesized by the proposed adaptive pole placement method ($nd = 2$)

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

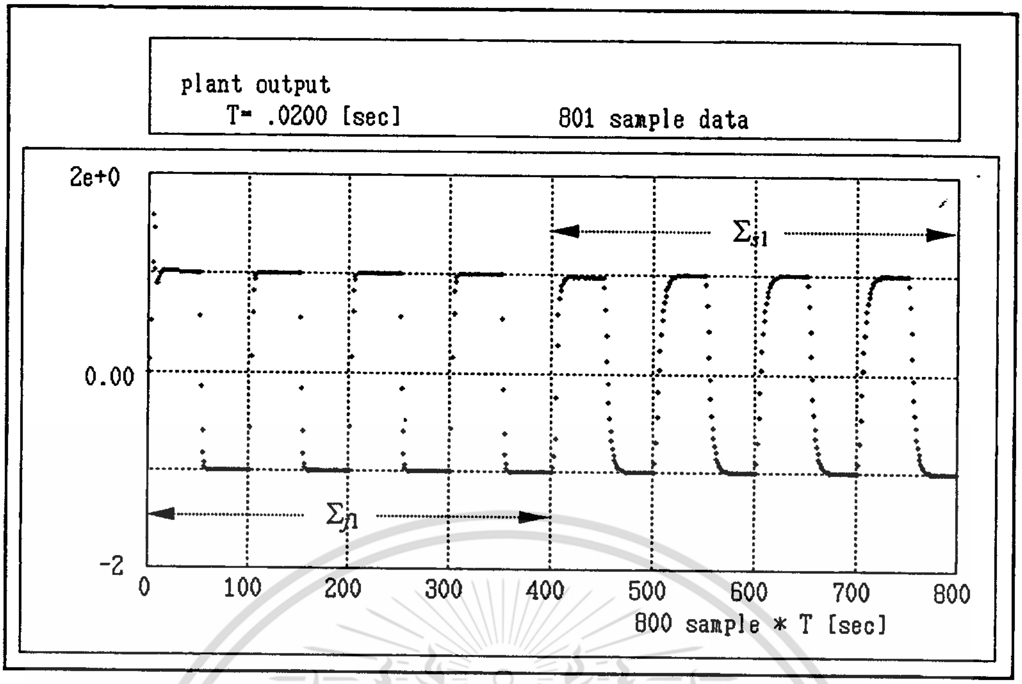

Figure 6.20 Plant output controlled by the proposed adaptive pole placement method ($nd = 2$)

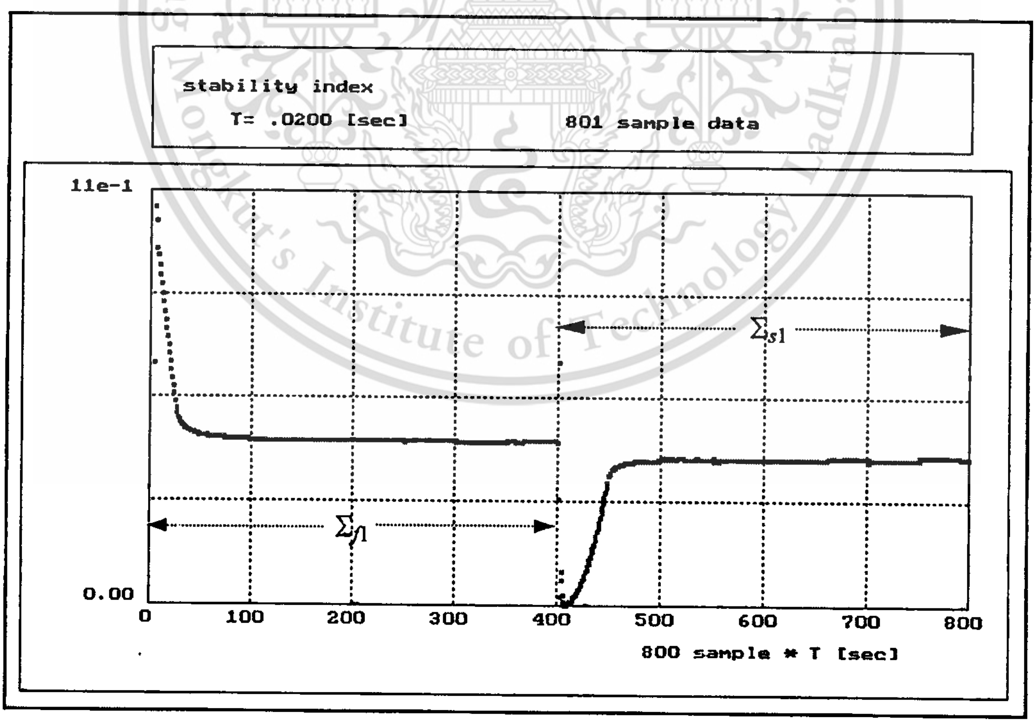

Figure 6.21 Stability index γ in case of proposed adaptive pole placement method ($nd = 2$)

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

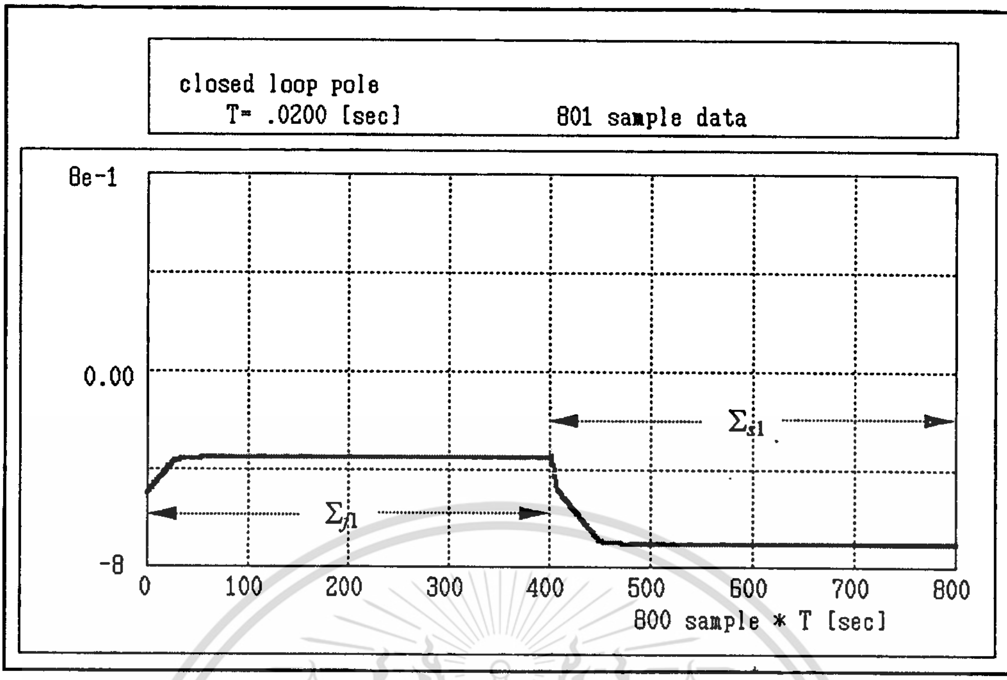

Figure 6.22 Stable closed-loop pole α in case of proposed adaptive pole placement method ($nd = 2$)

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

6.4.2 Application to Third Order Plant

The order of Diophantine equation ***nd* is defined to be 3.**

The third order continuous plant is selected as

$$G(s) = \frac{K\omega_n^2}{(s+l)(s^2 + 2\zeta\omega_n s + \omega_n^2)};$$

K and l are assigned to be constant value of 50.

Given $\Psi_f\,(\zeta = 0.1, \omega_n = 60)$ is the fast response plant and $\Psi_s(\zeta = 0.4, \omega_n = 20)$ is the plant after the plant parameter widely fluctuated and becomes the slow response plant.

First, the plant Ψ_f is transformed into the discrete system for a sampling time $T = 0.02$ [sec], then the discrete model is obtained as

$$\begin{aligned} G_f(q^{-1}) &= \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \\ &= \frac{q^{-1}(0.16573 + 0.46092q^{-1} + 9.0122\times 10^{-2}q^{-2})}{1 - 1.0206q^{-1} + 1.0267q^{-2} - 0.28938q^{-3}} \end{aligned}$$

Due to the calculation delay, d should be increased by 1, yielding $d = 2$. When a stable pole in Eqn.(5.3) is designed and Eqn.(5.20) is solved, then the pole placement controller is obtained. In this simulation $\alpha = 0.45$, then, the controller $S(q^{-1})$ and $R(q^{-1})$ are obtained as

$$\begin{aligned} S(q^{-1}) &= 1 + 0.52058q^{-1} - 0.11896q^{-2} - 0.040074q^{-3} \\ R(q^{-1}) &= -0.76322 + 1.6703q^{-1} - 0.86137q^{-2} + 0.12868q^{-3} \end{aligned}$$

Next, use the same procedure shown in section 6.4.1.1, the four important characteristics of both plants Ψ_f and Ψ_s can be obtained respectively as shown in figures 6.23, 6.24, 6.25 and 6.26 respectively. Then, the control specifications of both plants can be determined from figures 6.24 to 6.26 as shown in Table 6.3.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

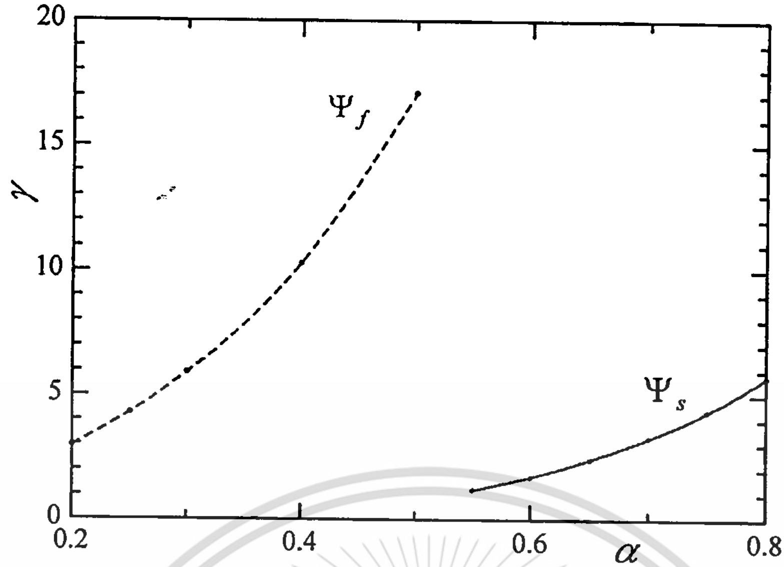

Figure 6.23 Each index γ of plants Ψ_f and Ψ_s to stable closed-loop pole α

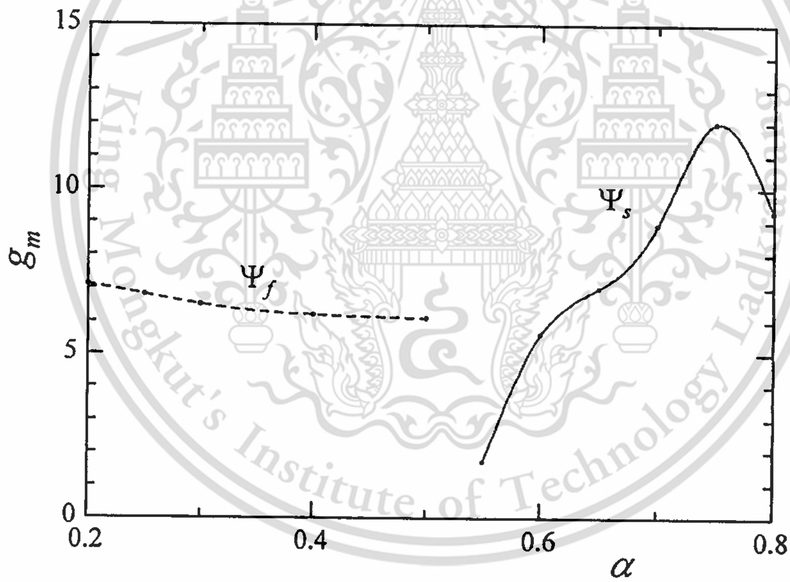

Figure 6.24 Each gain margin g_m of plants Ψ_f and Ψ_s to stable closed-loop pole α

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

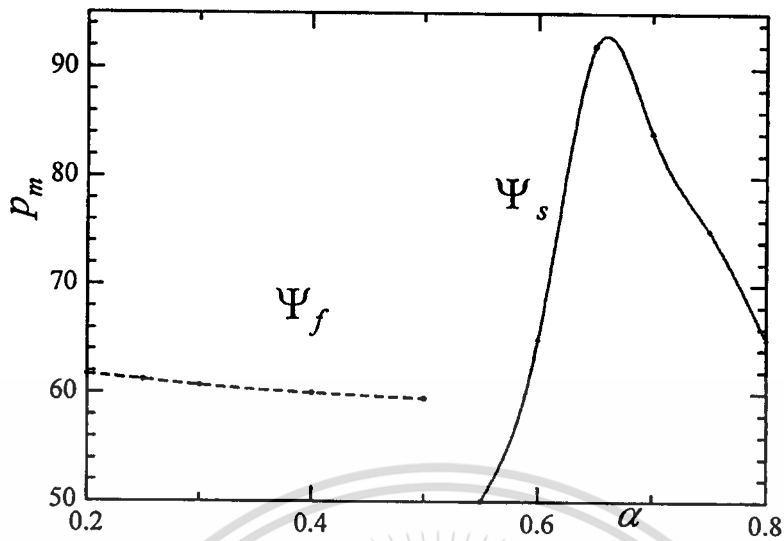

Figure 6.25 Each phase margin p_m of plants Ψ_f and Ψ_s to stable closed-loop pole α

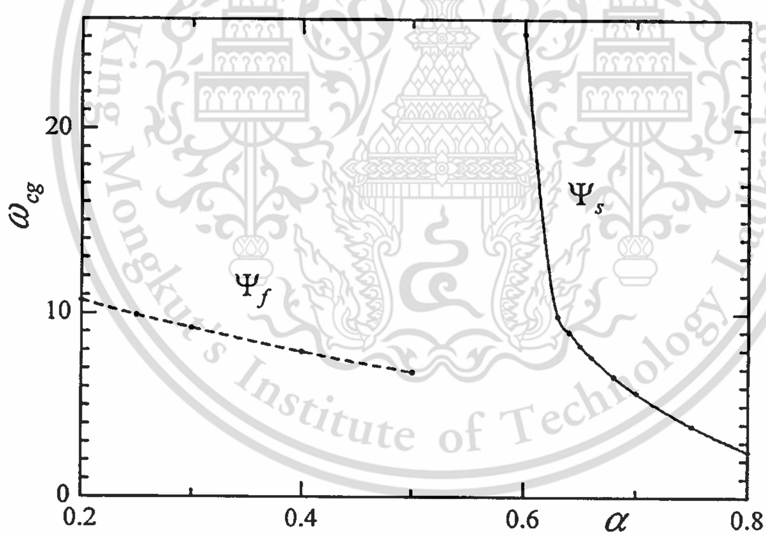

Figure 6.26 Each gain crossover frequency ω_{cg} of plants Ψ_f and Ψ_s to stable closed-loop pole α

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Table 6.3 Control specification (3^{rd} order plant, $nd = 3$)

	g_m [dB]	p_m [dB]	ω_{cg} [rad./sec.]
Σ_f	$g_m \geq 6.5$	$p_m \geq 60$	$\omega_{cg} \geq 9$
Σ_s	$g_m \geq 6.5$	$p_m \geq 60$	$\omega_{cg} \geq 4$

From Table 6.3, the ranges of closed-loop pole of both plants are

$$0.2 \leq \alpha_f \leq 0.32 \text{ and } 0.62 \leq \alpha_s \leq 0.75 \qquad (6.7)$$

From the inequality in (6.7), the ranges of stability index γ where has the fine performance of control are decided from figure 6.23 as shown bellows

$$\begin{aligned} &l_f \leq \gamma_f \leq u_f,\ l_f = 3,\ u_f = 6.6 \\ &l_s \leq \gamma_s \leq u_s,\ l_s = 2,\ u_s = 4.2 \end{aligned} \qquad (6.8)$$

where index γ_f and γ_s represent the designed ranges of the plant Ψ_f and Ψ_s respectively. In this case the horizontal scale in membership function of figure 6.2 can be selected as the common part $\Psi_f \cap \Psi_s$ of the inequality in (6.8). The following relations are also can be selected for membership function to reduce the range of *SMS* and *LGI.*

$$\begin{aligned} &\text{JFI} = \min(l_f, l_s) = 2,\ \ \text{SMS} = (u_s + l_f)/2 = 3.6 \\ &\text{LGI} = (u_f + l_s)/2 = 4.3,\ \ \text{JFS} = \max(u_f, u_s) = 6.6 \end{aligned} \qquad (6.9)$$

Then, $LGI - SMS = 0.7$

Next, find the complete inference in terms of centroid gravity from figure 6.3 and Eqn. (6.1), then, the stable pole is calculated from Eqn.(6.2).In this simulation, the constant value of C in Eqn.(6.2) is selected to a value of 0.005.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Simulation Results (3rd order plant, *nd* = 3)

First, the adaptive pole placement control by the conventional method is simulated and the stable closed-loop pole α is chosen with a constant value of 0.45. The square wave of unity amplitude and 100 samplings period is used as the input signal to the system. The initial adaptation gain is 10^4I and the initial parameter vector $\hat{\theta}^T = [0.1\ 0.2\ 0.3\ 0.4\ 0.5\ 0.6]$. The value of C in Eqn.(6.2) is selected to a value of 0.005. And the **other two different parameter plants are selected as the simulation plants**, such as $\psi_{f1}(\zeta = 0.15,\ \omega_n = 55)$ and $\psi_{s1}(\zeta = 0.35,\ \omega_n = 25)$. In this simulation, the parameters have been changed into ψ_{s1} from ψ_{f1} at step 400 and the observation noise of variance $\sigma_n^2 = 5\times10^{-6}$ has been added to the simulation. Then the plant output, the manipulated variable, stability index, stable closed-loop pole and estimated parameters are shown in figures 6.27, 6.28, 6.29, 6.20 and 6.31 respectively.

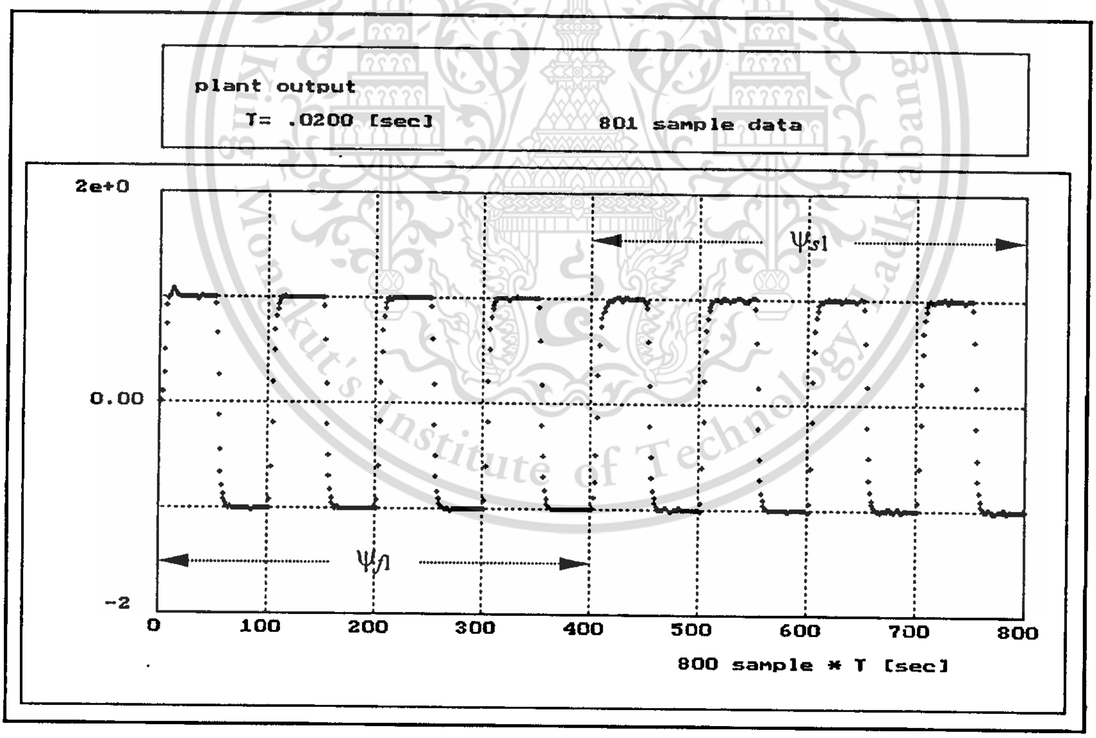

Figure 6.27 Plant output controlled by conventional adaptive pole placement

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

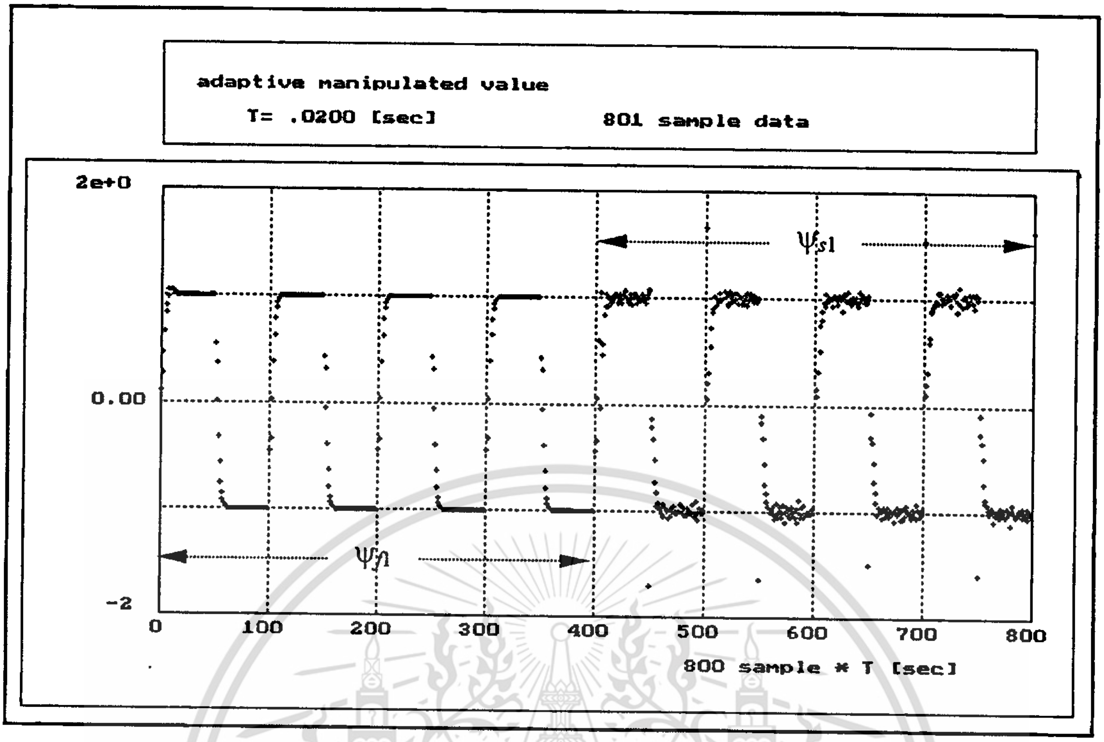

Figure 6.28 Manipulated variable synthesized by conventional adaptive pole placement

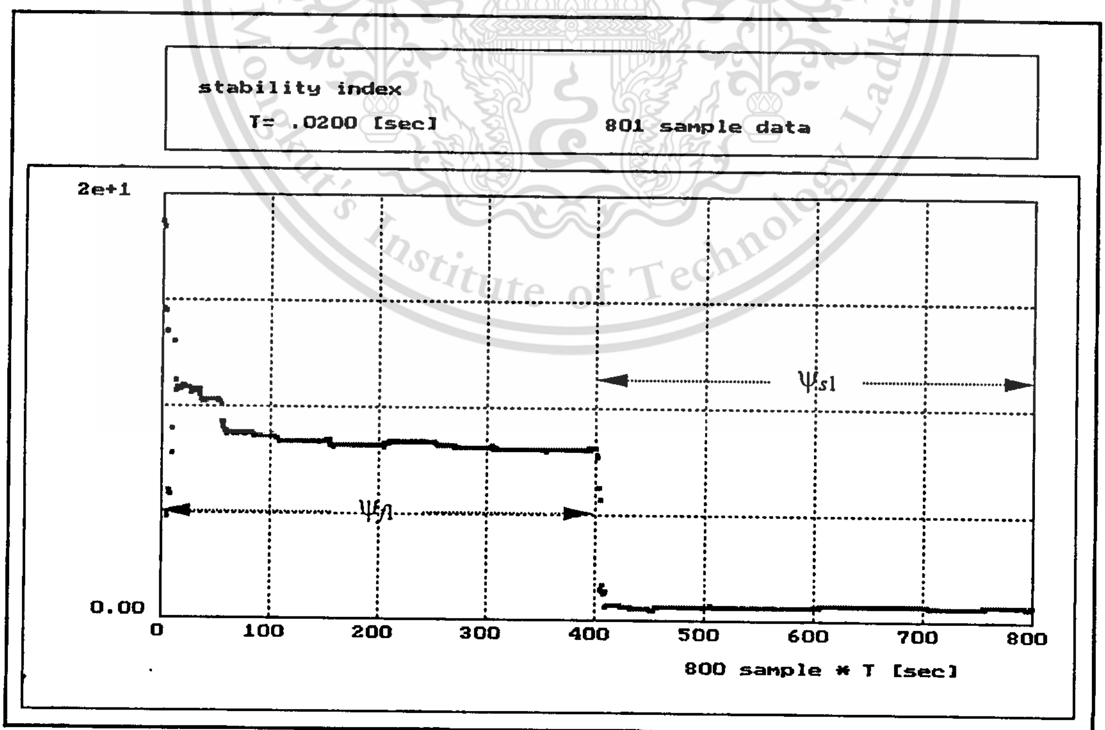

Figure 6.29 Stability index γ in case of conventional adaptive pole placement

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

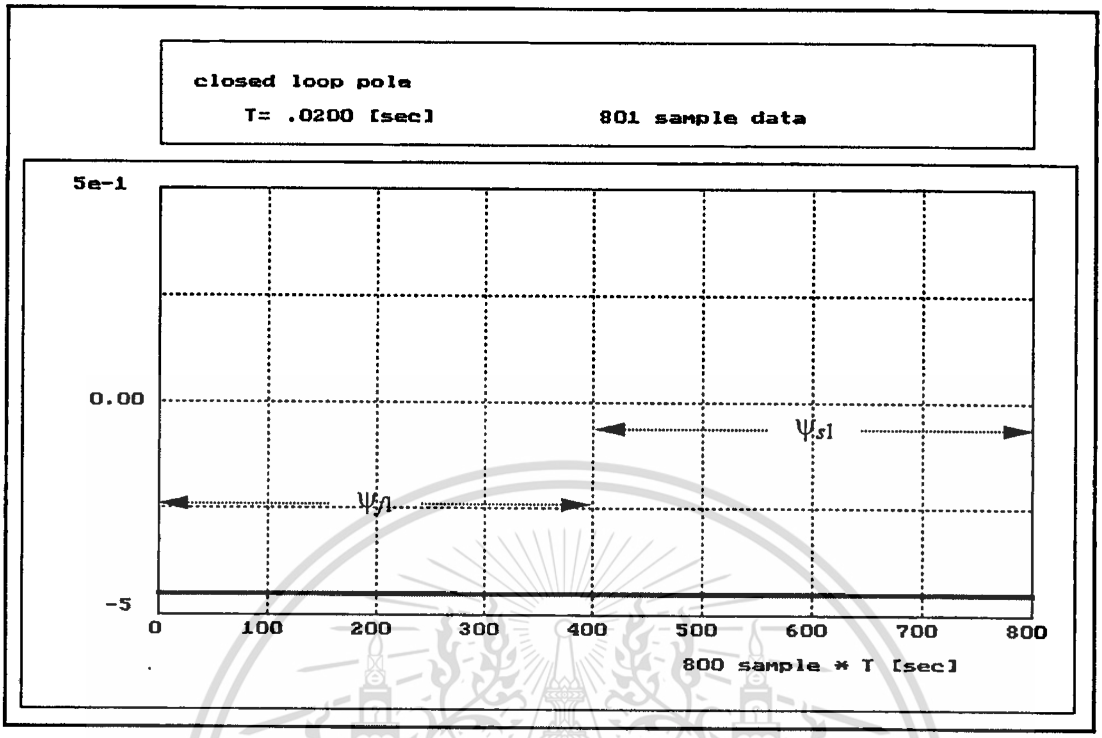

Figure 6.30 Stable closed-loop pole α in case of conventional adaptive pole placement

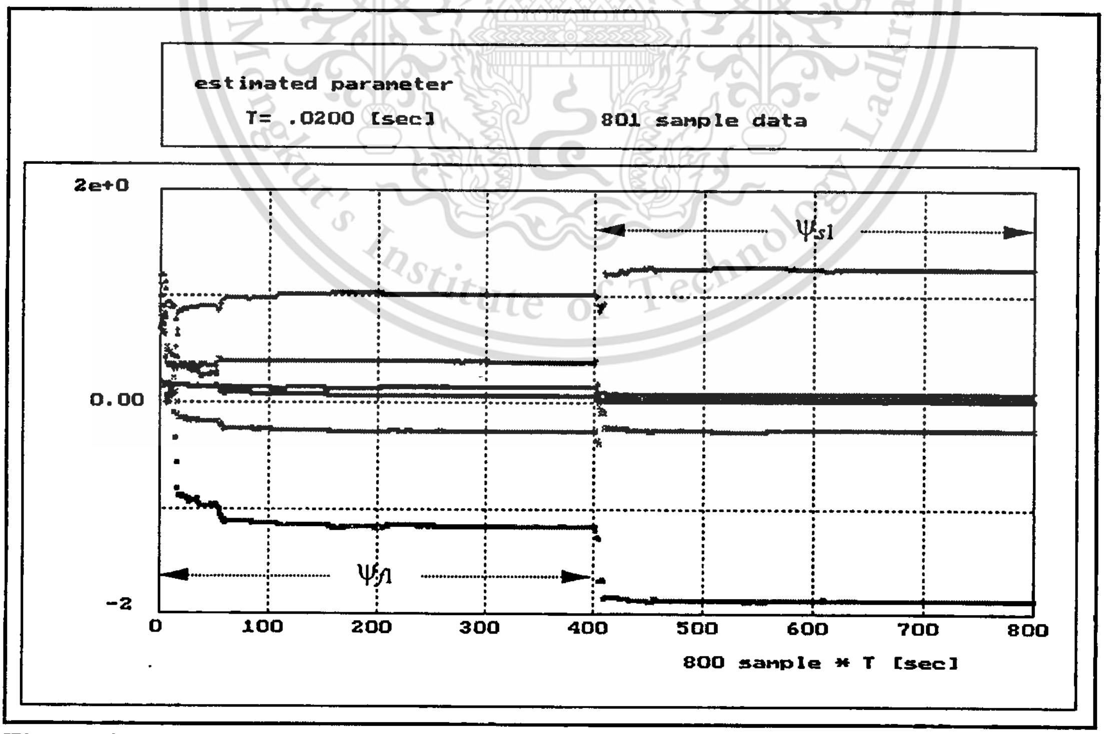

Figure 6.31 The estimated parameters of both ψ_{f1} and ψ_{s1} plants

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

The unstable controller appears in the area of ψ_{s1} plant which is the same situation as in case of second order plant. This is because no automatic stable pole adjustment as shown in figure 6.30 and the stability index is very low. The following figures show the control results of proposed design method. The stability index in figure 6.34 is automatically adjusted into the fine value and then, the stable pole is adjusted accordingly to the appropriate value as shown in figure 6.35. The resultant controller is stable as shown in figure 6.33.

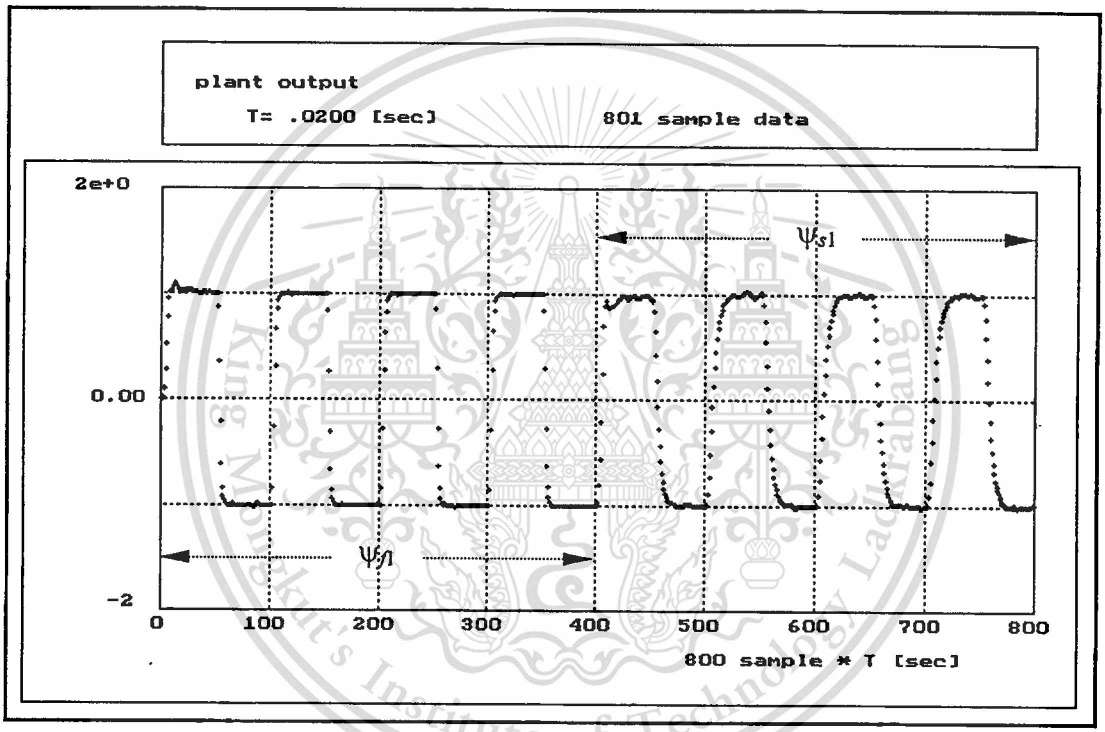

Figure 6.32 Plant output controlled by proposed adaptive pole placement method

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

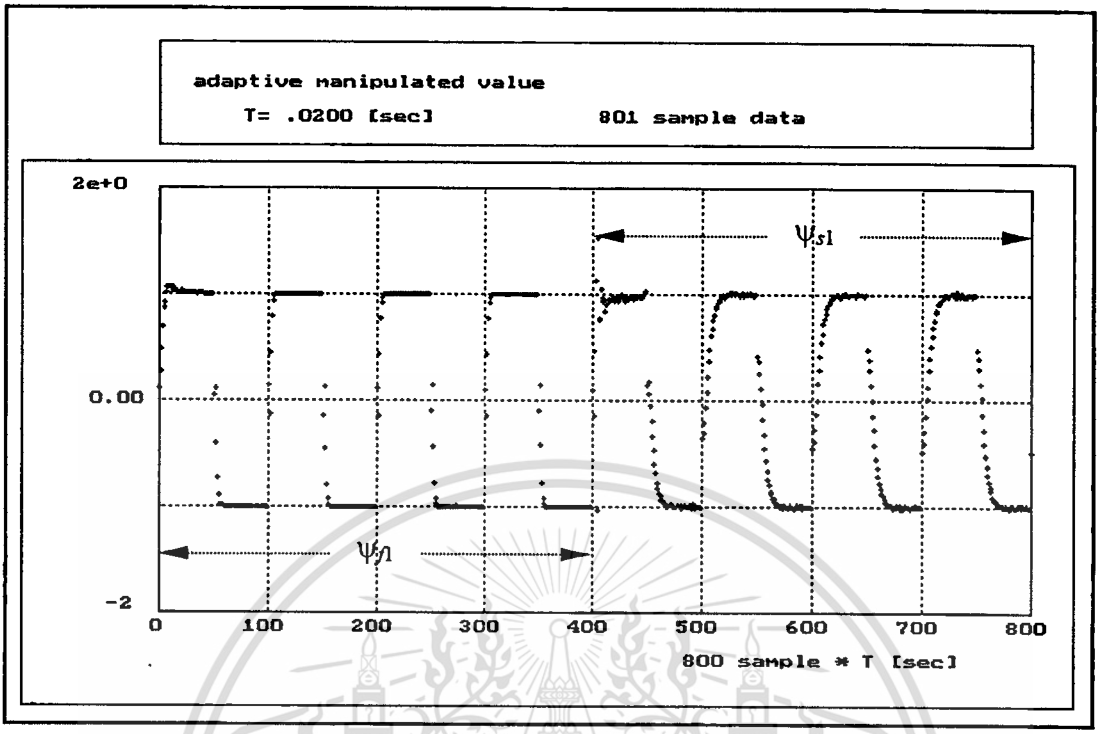

Figure 6.33 Manipulated variable synthesized by proposed adaptive pole placement method

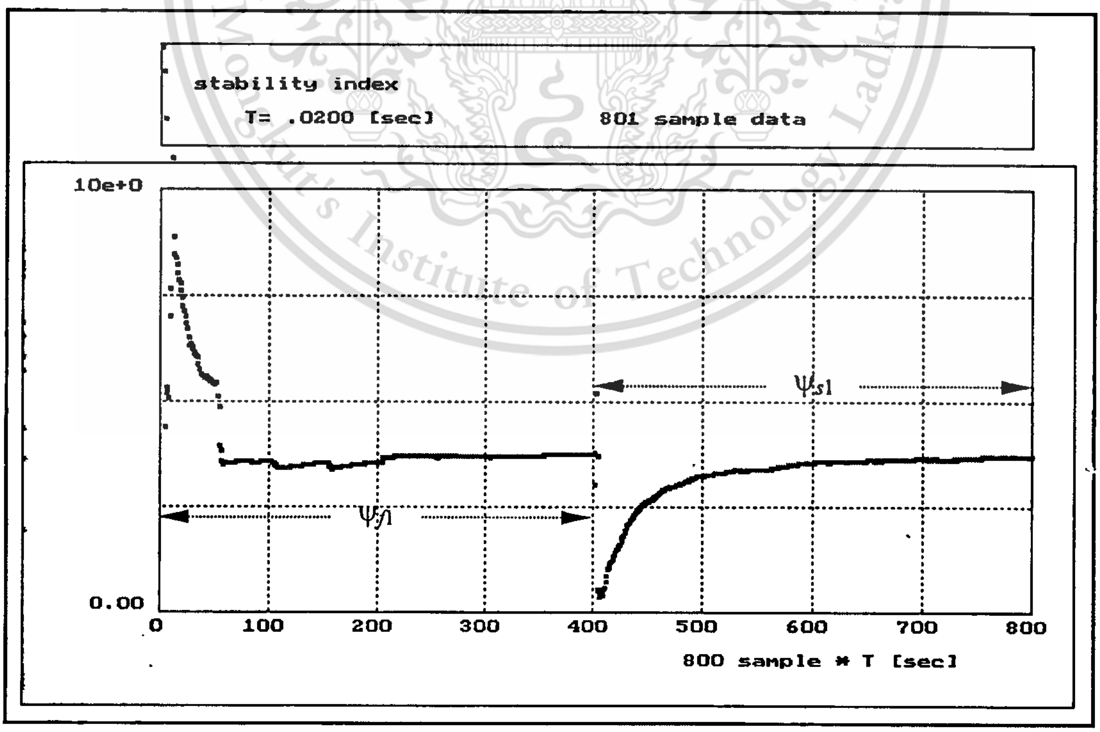

Figure 6.34 Stability index γ in case of proposed adaptive pole placement method

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

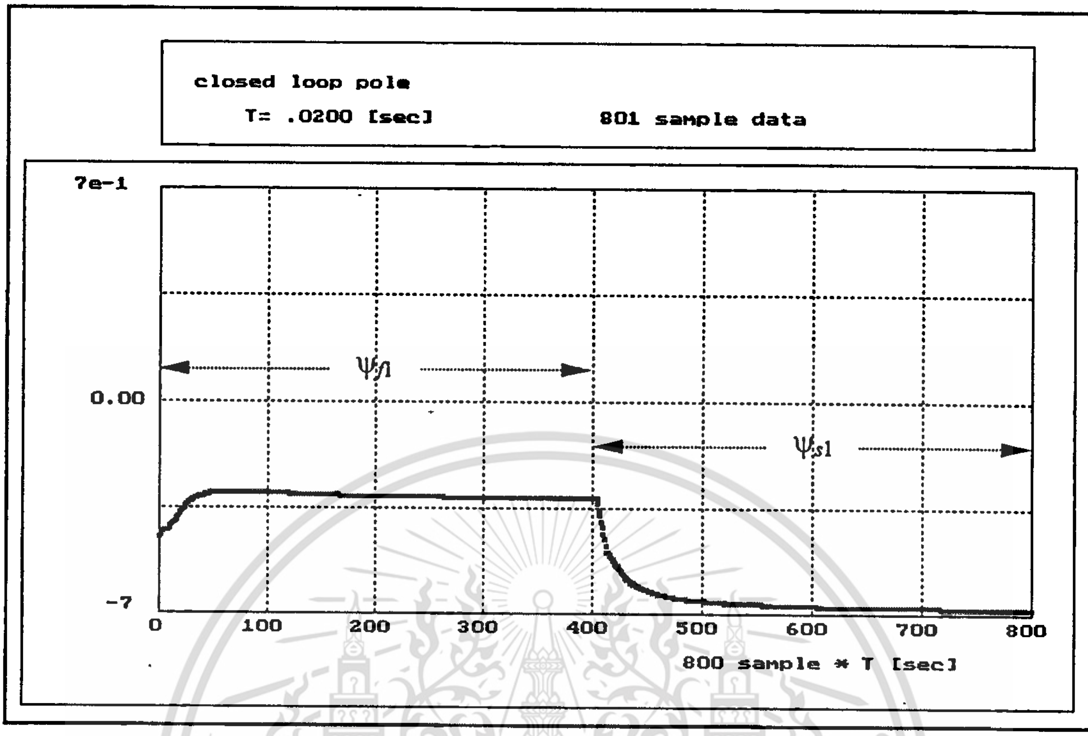

Figure 6.35 Stable closed-loop pole α in case of proposed adaptive pole placement method

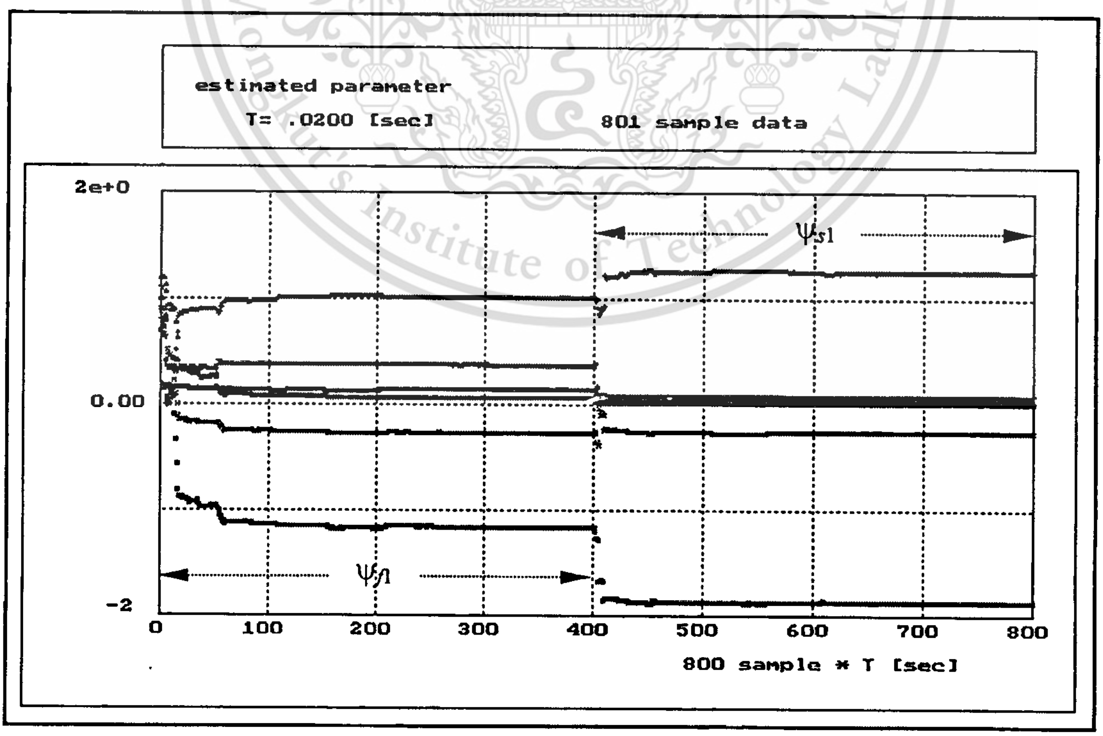

Figure 6.36 Estimated parameters of both ψ_{f1} and ψ_{s1} plants

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

6.5 Conclusion

This chapter presents the design method of strongly stable adaptive pole placement control system by introducing the stability index together with the fuzzy inference. When the gain-phase-margin and gain crossover frequency are calculated from the frequency response of loop transfer function and plotted to the stable pole, the control specifications of the controlled plants can be specified. From these specifications, the fine ranges of both the stable pole and stability index of the series compensator can be defined. By using the stability index as the fuzzy variable in the membership function of antecedent and find the inference result from the membership function of consequent, the stable pole to the Diophantine equation $D(q^{-1})$ can be calculated through Eqn.(6.2) after the crisp number is calculated from the defuzzification in Eqn.(6.1).

$$u_g(k-1) = \left\{\sum x_j \cdot \mu(x_j)\right\} / \sum \mu(x_j) \tag{6.1}$$

$$\alpha(k) = \alpha(k-1) + \mathrm{C}u_g(k-1)\,,\ \mathrm{C} = \text{any constant} \tag{6.2}$$

The appropriate controller and control results can be obtained as shown in the simulation results of the proposed adaptive pole placement control method. The simulation also shows that the proposed method is effective even the order of the Diophantine equation is lower than the over all closed-loop polynomial.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Chapter 7

Conclusion and Suggestion

A strongly stable system design method has been introduced for an adaptive pole placement control system. Usually, in the design procedure, selecting the appropriate pole location to the pole placement technique is often the most difficult task. In this thesis, the stability indices of the series compensator are examined together with the fuzzy inference, subsequently the appropriate pole through the fuzzy inference can be obtained.

Chapter 2 shows the controller design through the pole placement and accomplished both the regulation and tracking performance. In this method, the plant model must have no pole zero cancellation and no unstable zeros which will force the Diophantine equation $D(q^{-1})$ to accept the unstable zeros and resulting in constrained pole placement. An other problem is, when the pole placement is applied to the adaptive pole placement control system, the stable pole α of Eqn.(2.15) is selected to the Diophantine equation It should be selected such that the magnitude of the manipulated variable or the control law equation $u(k)$ should not over the magnitude of the input signal of the D/A converter. Because the magnitude of the manipulated variable depends on the values of the series and parallel compensators $S(q^{-1})$ and $R(q^{-1})$ and the closed loop polynomial $D(q^{-1})$,respectively, from Eqn. (2.33) $u(k)$ can be rewritten as the following

$$u(k) = \frac{D(q^{-1})y^*(k+d+1) - R(q^{-1})y(k)}{S(q^{-1})}$$

When the system is frequently subjected to the random disturbances, the noise model is added to the system output of the pole placement control system. And the minimum variance tracking and regulation are needed. In servo control system, the ARX model is usually used, because the noise is not severe unlike the chemical process control, the ARMAX model is needed. Clark and Gawthrop introduced a General Minimum Variance (GMV) tracking and regulation which obtained the appropriate control law to the pole placement control system. And the GMV has the

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

great advantage that it can deal with the unstable zeros plant, because the strategy dose not use the pole zero cancellation. Furthermore, the ARX model can be considered as the special case of GMV.

In the case of unknown parameter plant or time-varying parameters, the stable controller design under pole placement control can be achieved by working together with the system identification tool. In this thesis, the RLS identification method based on the LS criterion is used. This method is the on-line recursive identification and very useful if the noise to the system is white noise, that is, it can be applied to the ARX model. Anyhow, the user has to be careful in using this algorithm, for example the initial conditions, the input signal and the adaptation gain. If the selection of these matters are not suitable, the bias estimated parameters will be obtained.

Section 4.7 is the examples of parameter estimation using RLS algorithm, the simulation shows that the accuracy of parameter estimation is very good in the case of no noise and no disturbance. In the case of added noise and constant disturbance to the simulation system, the estimated parameters are very closed to the true values. The last simulation shows that, even the parameter widely fluctuated, the RLS identification algorithm is also able to deal with.

The RLS identification algorithm is useful for the system which the signal per noise ratio is in the range of $10 \sim 20$ or $10\log_{10}\dfrac{\max|y(k)|}{\max|n(k)|} \cong 15 \sim 16$ dB, $y(k)$ and $n(k)$ are the magnitudes of plant output and noise respectively.

The disadvantage of RLS identification algorithm is when the noise to the system is not white noise, the estimated parameters could not converge to the true value. In such case, the RLS algorithm should be worked together with other algorithm, for example the instrument variable algorithm. Since the convergence rate of the RLS algorithm is very high at the beginning comparing to the instrument variable algorithm, then, the calculation time can be reduced if these two algorithms are worked together.

Chapter 5 shows the controller design in the conventional adaptive pole placement control system as shown in figure 5.1. The steady state error is compensated by adding the integrator to the controller. By doing so, the high performance controller is obtained. Anyway, the stability of the designed controller can be checked by calculating the stability index in Eqns. 5.24 and 5.25.

This material is reserved for academic use only, and not for commercial use.
Forbidden to modify the content, and cite the document when use.

Section 5.6 is the simulation examples that show the plant parameter estimation and the design of controller and pre-filter in conventional adaptive pole placement control system. The simulation results show that when the stable pole $\alpha = 0.4$, the stability index of the series compensator is lower than the case when $\alpha = 0.6$, because the low α value means the fast speed of the disturbance rejection which is lead to low stability.

The difficulty of conventional adaptive pole placement control is how to place the stable pole α to the Diophantine equation $D(q^{-1})$ to design the appropriate controller according to the plant parameter fluctuation. Chapter 6 presents the design method of strongly stable adaptive pole placement control by introducing both stability index and fuzzy inference. When the fluctuation range of the plant is estimated, the control specifications of the plant can be defined, then, the fine ranges of the stable pole and stability index of the controller are defined. The stability index is used to the fuzzy inference to force to stable pole in the specified area and the resultant strongly stable controller can be obtained.

Since the specified area SMS~LGI for the stable pole is very narrow such as 0.27 in the first simulation and 0.05 in the second simulation, then, the designer should be more careful in defining the value of the constant C in Eqn. (6.2), otherwise, the design method will fail.

Looking the order *nd* of the Diophantine equation $D(q^{-1})$ in the first simulation (section 6.4.1.1), *nd* is equal to 3 the relative stability of the control results is better than in the second simulation (section 6.4.1.2) in which nd is equal to 2. Usually the order of Diophantine equation should be equal to the order of the closed-loop polynomial as shown in section 2.1.1. Anyway, the purpose of the second simulation is to show the **effective ability of the proposed design method**. Because the control results when *nd* equal to 2 is also fine even the stability index is a little low. An other **important effective ability** of this proposed method is shown by the control results of both simulations, the controller that designed for the fluctuation plant (Σ_f and Σ_s)can be used to control the other non-designed fluctuation plant (Σ_{f1} and Σ_{s1}) effectively. This effective ability is also shown in the simulation of the third order plant in section 6.4.2. In such simulation, the value of *nd* is equal to 3 instead of 4. Actually *nd* equal to 4 is better than 3 in control strategy but in

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

parameter estimation *nd* equal to 3 is better than 4, then, better parameter estimation results gives better control performance.

Sampling period is the important variable to the effective ability of the proposed design method because the appropriate sampling period exits for each plant. The control specifications should be defined from the system instead of arbitrary chosen, otherwise, the applicability to this proposed method is occasionally narrow.

This proposed method is effective in motor control application such as the load inertia fluctuates widely, and also no limitation to the order of the controlled plant. The practical range of the stability index γ is normally defined by the inequality $2(n_s-1)\le\gamma\le 3(n_s-1)$ [5],[6] and *nd* is equal to 3. When the designer could not estimate the parameter fluctuation range of the plant, in such case, the proposed method can be applied to one plant design method.

Noting that all the simulation programs in Chapter 4-6 are written on C language. and the Matlab program is used for all design examples in Chapter 6.

King Mongkut's Institute of Technology Ladkrabang

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

References

[1] Doyle J.C., Francis B.A. and Tannenbaum A.R. **Feedback Control Theory**. Macmillan, 1992.

[2] Astrom K.J. and Wittenmark B. **Adaptive Control**. Addison Wesley, 1989.

[3] Fujii S. and Mizuno N. "Theory and Application of Digital Adaptive Control." Systems and Control, Vol.30, No.12, 1986. pp. 727-737 (in Japanese).

[4] Landau I.D. **System Identification and Control Design**. Prentice Hall,1990.

[5] Manabe S. "Coefficient Diagram Method as Applied to the Attitude Control of Controlled-Bias-Momentum Satellite." 13^{th} IFAC symposium on automatic control in aerospace, Palo Alto,CA., 1994. pp. 322-327.

[6] Manabe S. "A Solution of the ACC Benchmark Problem by Coefficient Diagram Method." ISAS 6^{th} Workshop on Astrodynamics and Flight Mechanics, Sagamihara, ISAS, 1996. pp. 108-117.

[7] Terano,T., Asai and Sugeno M. **Fuzzy Systems Theory and its Applications**. Academic Press, 1992.

[8] LI-XIN Wang. **Adaptive Fuzzy Systems and Control**. Design and Stability Analysis, Prentice Hall, 1994.

[9] Jamshidi M., Vadiee N. and Timothy Ross J. **Fuzzy Logic and Control**. Software and Hardware Applications, Prentice Hall, 1993.

[10] Soderstrom T. and Stoica P. **System Identification**. Prentice Hall, 1989.

[11] Ljung L. **System Identification Theory for the User**. Prentice Hall, 1987.

[12] Astrom K.J., Hagander P. and Sternby J. "Zeros of Sampled Systems." Automatica, Vol.20, No.1, Jan., 1984. pp. 31-38.

[13] Narendra K.S. and Annaswamy A.M. "A new Adaptive Law for Robust Adaptive Control Without Persistent Excitation." IEEE Trans. On Automatic Control, AC-32-2, 1987. pp. 150-155.

[14] Middleton R.H. and Goodwin G.C. "Adaptive Control of Time Varying Linear Systems." IEEE Trans. On Automatic Control, AC-33-2, 1988. pp. 150-155.

[15] Narendra K.S. and Annaswamy A.M. **Stable Adaptive Systems**. Prentice Hall, 1989.

[16] Middleton R.H. and Goodwin G.C. **Digital Control and Estimation A Unified Approach**. Prentice Hall, 1990.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

[17] Clarke D.W. "Self-Tuning Control of Nonminimum-Phase Systems." Automatica, Vol.20,No.1, 1984. pp. 31-38.

[18] Clarke D.W. and Gawthrop P.J. "A Self-Tuning Controller." IEE Proc., Vol.122, 1975. pp. 929-934.

[19] Wellstead P.E. and Zarrop M.B. **Self-Tuning Systems**. John Wiley and Sons, 1991.

King Mongkut's Institute of Technology Ladkrabang

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

Appendix A

A.1 Minimum variance tracking and regulation[4]

This strategy concerns optimal controller design, ensuring a minimum variance of the controlled variable around the reference. From figure 3.5, the objective is to compute the control law $u(k)$, which minimizes the following criterion

$$\min E\{[y(k)-y^*(k)]^2\} = \min \frac{1}{N}\sum_{k=1}^{N}[y(k)-y^*(k)]^2 . \tag{A.1}$$

Example

Let the plant and disturbance be presented by the ARMAX model as in figure 3.4,the disturbed output of the plant $y(k)$ is written as

$$\begin{aligned} y(k) &= \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k)+\frac{C(q^{-1})}{A(q^{-1})}e(k) \\ &= -a_1q^{-1}y(k)+(b_1q^{-1}+b_2q^{-2})u(k)+(1+c_1q^{-1})e(k). \end{aligned} \tag{A.2}$$

Then

$$y(k+1) = -a_1y(k)+b_1u(k)+b_2u(k-1)+c_1e(k)+e(k+1) \tag{A.3}$$

and the reference trajectory output is $y^*(k+1)$ which is either stored or generated by a dynamic model from the reference. The variance of the difference $y(k+1)-y^*(k+1)$ is computed, which represents the performance criterion to be minimized as

$$\begin{aligned} E\{[y(k+1)-y^*(k+1)]^2\} &= E\{[-a_1y(k)+b_1u(k)+b_2u(k-1) \\ &+c_1e(k)-y^*(k+1)]^2\}+E\{e^2(k+1)\}+E\{e(k+1)[-a_1y(k) \\ &+b_1u(k)+b_2u(k-1)+c_1e(k)-y^*(k+1)]\}. \end{aligned} \tag{A.4}$$

The third term of the right hand side of Eqn.(A.4) is zero because $u(k+1)$,the white noise at instant k+1, is independent of all signals appearing at instants k,k+1,...Of the two terms which then remain in the criterion given in Eqn.(A.4), $E\{e^2(k+1)\}$ does not depend upon $u(k)$. Therefore, the choice of $u(k)$ will only effect the first term

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

which can be only positive or zero. It follows that the minimization of the criterion (A.4) corresponds to find $u(k)$, such that

$$E\{[-a_1 y(k) + b_1 u(k) + b_2 u(k-1) + c_1 e(k) - y^*(k+1)]^2\} = 0 \qquad \text{(A.5)}$$

which may be obtained by making the bracketed expression zero. The resulting control law is

$$u(k) = \frac{y^*(k+1) - c_1 e(k) - a_1 y(k)}{b_1 + b_2 q^{-1}}. \qquad \text{(A.5)}$$

From Eqn.(A.3), the plant output equation can be expressed as

$$y(k+1) - y^*(k+1) = e(k+1) \qquad \text{(A.6)}$$

and respectively

$$y(k) - y^*(k) = e(k). \qquad \text{(A.7)}$$

This leads to the following comments:

1. The application of the control law given by Eqn.(A.5) leads to a minimum variance for the difference $y(k) - y^*(k)$, which becomes a white noise.
2. $u(k)$ can be replaced in Eqn.(A.5) by the measurable expression given by Eqn.(A.7). This results in the following control law

$$\begin{aligned} u(k) &= \frac{(1 + c_1 q^{-1}) y^*(k+1) - (c_1 - a_1) y(k)}{b_1 + b_2 q^{-1}} \\ &= \frac{T(q^{-1}) y^*(k+1) - R(q^{-1}) y(k)}{S(q^{-1})}. \end{aligned} \qquad \text{(A.8)}$$

The structure of the minimum variance control law is the same as that for tracking and regulation with independent objectives in the deterministic case if $D(q^{-1}) = C(q^{-1})$ which is the desired closed-loop poles.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

A.2 General Minimum Variance (GMV) tracking and regulation

The GMV [17],[18],[19] is defined by Clarke and Gawthrop in such that it can be applied to the plants with both stable zero and unstable zero.

In the following ARMAX system

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + C(q^{-1})e(k) \tag{A.9}$$

the performance for evaluating the control system is defined as follows

$$J \cong E[\varepsilon(k+d)^2] = \sigma^2[\varepsilon(k+d)] \tag{A.10}$$

where $\varepsilon(k+d)$ is given by

$$\varepsilon(k+d) \cong P(q^{-1})y(k+d) + Q(q^{-1})u(k) - H(q^{-1})r(k+d). \tag{A.11}$$

The optimal control law $u(k)$ that minimizes the variance J is given by

$$u(k) = \frac{C(q^{-1})H(q^{-1})r(k+d) - B(q^{-1})R(q^{-1})y(k)}{B(q^{-1})S(q^{-1}) + C(q^{-1})Q(q^{-1})} \tag{A.12}$$

where the polynomials $S(q^{-1})$ and $R(q^{-1})$ are the solution of the following Diophantine equation

$$P(q^{-1})C(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}). \tag{A.13}$$

PROOF

If Eqn.(A.9) is multiplied by $P(q^{-1})$, then, Eqn.(A.14) can be obtained as

$$P(q^{-1})y(k) = \frac{P(q^{-1})B(q^{-1})}{A(q^{-1})}q^{-d}u(k) + \frac{P(q^{-1})C(q^{-1})}{A(q^{-1})}e(k). \tag{A.14}$$

Next, Eqn.(A.13) is transformed as the following equation

$$\frac{P(q^{-1})C(q^{-1})}{A(q^{-1})} = S(q^{-1}) + q^{-d}\frac{B(q^{-1})R(q^{-1})}{A(q^{-1})} \tag{A.15}$$

when Eqn.(A.15) is substituted into Eqn.(A.14), then, Eqn.(A.16) is obtained as

$$P(q^{-1})y(k+d) = \frac{P(q^{-1})B(q^{-1})}{A(q^{-1})}u(k) + S(q^{-1})e(k+d) + \frac{B(q^{-1})R(q^{-1})}{A(q^{-1})}e(k). \tag{A.16}$$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

From Eqn.(A.9), the noise term is

$$e(k)=\frac{A(q^{-1})}{C(q^{-1})}y(k)-\frac{B(q^{-1})}{C(q^{-1})}q^{-d}u(k) \tag{A.17}$$

substitutes Eqn.(A.17) into Eqn.(A.16), one obtains

$$\begin{aligned}P(q^{-1})y(k+d)&=[\frac{P(q^{-1})}{A(q^{-1})}-\frac{B(q^{-1})}{A(q^{-1})}\cdot\frac{R(q^{-1})}{C(q^{-1})}q^{-d}]B(q^{-1})u(k)+\frac{B(q^{-1})R(q^{-1})}{C(q^{-1})}y(k)+S(q^{-1})e(k+d)\\&=\frac{B(q^{-1})S(q^{-1})}{C(q^{-1})}u(k)+\frac{B(q^{-1})R(q^{-1})}{C(q^{-1})}y(k)+S(q^{-1})e(k+d).\end{aligned} \tag{A.18}$$

Furthermore, if substitutes Eqn.(A.18) into Eqn.(A.11), $\varepsilon(k+d)$ is obtained as

$$\begin{aligned}\varepsilon(k+d)&=\frac{B(q^{-1})R(q^{-1})}{C(q^{-1})}y(k)+[\frac{B(q^{-1})S(q^{-1})}{C(q^{-1})}+\frac{C(q^{-1})Q(q^{-1})}{C(q^{-1})}]u(k)\\&\quad-\frac{C(q^{-1})H(q^{-1})}{C(q^{-1})}r(k+d)+S(q^{-1})e(k+d).\end{aligned} \tag{A.19}$$

Therefore, from Eqns.(A.10) and (A.19), the variance is

$$\begin{aligned}E[\varepsilon(k+d)^2]&=E[\{\frac{B(q^{-1})R(q^{-1})}{C(q^{-1})}y(k)+[\frac{B(q^{-1})S(q^{-1})+C(q^{-1})Q(q^{-1})}{C(q^{-1})}]u(k)\\&\quad-\frac{C(q^{-1})H(q^{-1})}{C(q^{-1})}r(k+d)\}^2]+E[\{S(q^{-1})e(k+d)\}^2]\end{aligned} \tag{A.20}$$

Consequently, the optimal control law $u(k)$ that minimizes the variance in Eqn.(A.20) is given by Eqn.(A.12). Furthermore, the minimal value of J becomes as follows

$$J=\sigma_e^2\sum_{i=0}^{ns}S_i^2\ , \quad \text{since } S_0=1. \tag{A.21}$$

---QED---

The closed-loop characteristics of GMV are given as follows.

Substitute Eqn.(A.12) into Eqn.(A.9), one obtains

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

$$A(q^{-1})[B(q^{-1})S(q^{-1})+C(q^{-1})Q(q^{-1})]y(k)=q^{-d}B(q^{-1})C(q^{-1})H(q^{-1})r(k+d)$$
$$-q^{-d}B(q^{-1})B(q^{-1})R(q^{-1})y(k)+C(q^{-1})[B(q^{-1})S(q^{-1})+C(q^{-1})Q(q^{-1})]e(k)$$
(A.22)

rearrange Eqn.(A.22) into Eqn.(A.23) as

$$[A(q^{-1})B(q^{-1})S(q^{-1})+A(q^{-1})C(q^{-1})Q(q^{-1})+q^{-d}B(q^{-1})B(q^{-1})R(q^{-1})]y(k)$$
$$=q^{-d}B(q^{-1})C(q^{-1})H(q^{-1})r(k+d)+C(q^{-1})[B(q^{-1})S(q^{-1})+C(q^{-1})Q(q^{-1})]e(k)$$
(A.23)

substitutes Eqn.(A.13) into Eqn.(23) gives

$$C(q^{-1})[B(q^{-1})P(q^{-1})+A(q^{-1})Q(q^{-1})]y(k)$$
$$=q^{-d}C(q^{-1})B(q^{-1})H(q^{-1})r(k+d)+C(q^{-1})[B(q^{-1})S(q^{-1})+C(q^{-1})Q(q^{-1})]e(k).$$
(A.24)

Therefore, the closed-loop characteristics are given by

$$y(k)=\frac{B(q^{-1})H(q^{-1})q^{-d}}{B(q^{-1})P(q^{-1})+A(q^{-1})Q(q^{-1})}r(k+d)+\frac{B(q^{-1})S(q^{-1})+C(q^{-1})Q(q^{-1})}{B(q^{-1})P(q^{-1})+A(q^{-1})Q(q^{-1})}e(k).$$
(A.25)

The GMV tracking and regulation is shown in figure (3.7) and is shown again in figure A.1

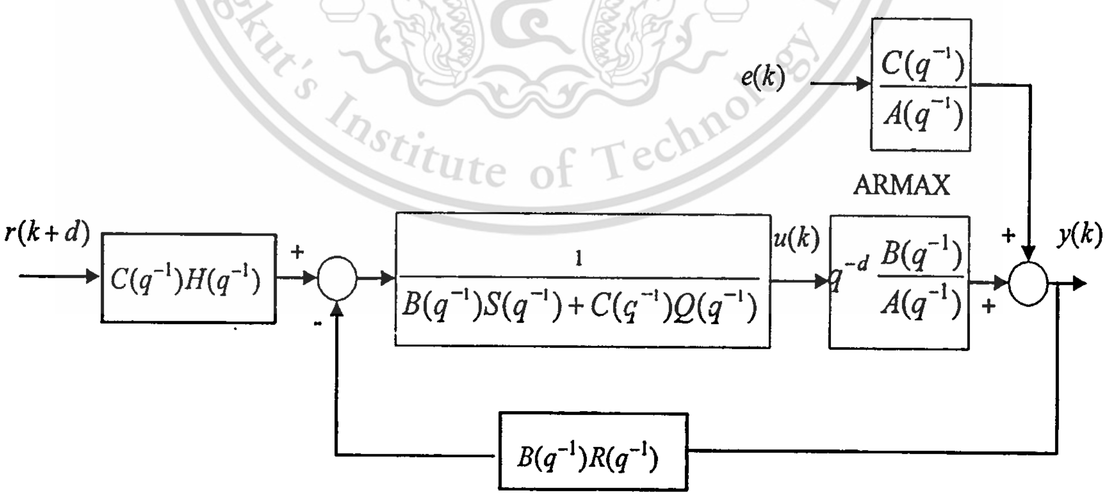

Figure A.1 The General Minimum Variance tracking and regulation

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Appendix B

B.1 Recursive Least Squarer (RLS) Algorithm

The LS estimate in Eqn.(4.10a) is rewritten here, this equation is to stress the dependence of $\hat{\theta}$ on time.

$$\hat{\theta}=\left[\sum_{k=1}^{N}\varphi(k)\varphi^{T}(k)\right]^{-1}\sum_{k=1}^{N}\varphi(k)y(k) \qquad (4.10a)$$

In RLS, it is to express the computation in recursive fashion, then Eqn.(4.10a) can be written as

$$\hat{\theta}(k)=\left[\sum_{s=1}^{k}\varphi(s)\varphi^{T}(s)\right]^{-1}\sum_{s=1}^{k}\varphi(s)y(s). \qquad (4.10a-1)$$

From above equation, given

$$P(k)=\left[\sum_{s=1}^{k}\varphi(s)\varphi^{T}(s)\right]^{-1} \qquad (B.1)$$

from the notation of P(k), it follows that

$$P^{-1}(k)=P^{-1}(k-1)+\varphi(k)\varphi^{T}(k) \qquad (B.2a)$$

or $$P^{-1}(k)=\sum_{s=1}^{k-1}\varphi(s)\varphi^{T}(s)+\varphi(k)\varphi^{T}(k) \qquad (B.2b)$$

then

$$\sum_{s=1}^{k}\varphi(s)y(s)=\sum_{s=1}^{k-1}\varphi(s)y(s)+\varphi(k)y(k). \qquad (B.3)$$

Rearrange Eqn. (B.2a)

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

$$P^{-1}(k-1) = P^{-1}(k) - \varphi(k)\varphi^T(k) \tag{B.4}$$

and Eqn. (4.10a-1) can be rewritten as

$$\hat{\theta}(k) = P(k)\sum_{s=1}^{k}\varphi(s)y(s) \tag{B.5}$$

if $\hat{\theta}(k)$ is based on the measurement k-1, then

$$\hat{\theta}(k-1) = P(k-1)\sum_{s=1}^{k-1}\varphi(s)y(s) \tag{B.6}$$

or

$$P^{-1}(k-1)\hat{\theta}(k-1) = \sum_{s=1}^{k-1}\varphi(s)y(s)\,. \tag{B.7}$$

Put Eqns. (B.3), (B.4) and (B.7) into Eqn. (B.5)

$$\begin{aligned}\hat{\theta}(k) &= P(k)\left[\sum_{s=1}^{k-1}\varphi(s)y(s) + \varphi(k)y(k)\right] \\ &= P(k)\left[P^{-1}(k-1)\hat{\theta}(k-1) + \varphi(k)y(k)\right] \\ &= \hat{\theta}(k-1) + P(k)\varphi(k)\left[y(k) - \varphi^T(k)\hat{\theta}(k-1)\right] \\ &= \hat{\theta}(k-1) + P(k)\varphi(k)\varepsilon(k)\end{aligned} \tag{B.8}$$

where the prediction error is

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1) \tag{B.9}$$

$\varepsilon(k)$ is also a one step ahead of $y(k)$ based on the estimate $\hat{\theta}(k-1)$.

To avoid inverting $P(k)$ at each step, the matrix inversion lemma in the following equation is applied to equation (B.2a)

This material is reserved for educational use only, not allowed for commercial use.

$$[\mathbf{A} + \mathbf{BCD}]^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left[\mathbf{DA}^{-1}\mathbf{B} + \mathbf{C}^{-1}\right]^{-1}\mathbf{DA}^{-1} \tag{B.10}$$

Forbidden to modify the content, and cite the document when use.

where $\mathbf{A} = P(k-1), \mathbf{B} = \mathbf{D}^{\mathbf{T}} = \varphi(k), \mathbf{C} = \mathbf{1}$.

Then an updating equation for $P(k)$ is obtained

$$P(k) = P(k-1) - \frac{p(k-1)\varphi(k)\varphi^T(k)P(k-1)}{1+\varphi^T(k)P(k-1)\varphi(k)} \tag{B.11}$$

and

$$P(k)\varphi(k) = \frac{p(k-1)\varphi(k)}{1+\varphi^T(k)P(k-1)\varphi(k)} \tag{B.12}$$

put equation (B.12) into equation (B.8), the completed RLS method is obtained as.

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1)\varphi(k)}{1+\varphi^T(k)P(k-1)\varphi(k)}\varepsilon(k) \tag{B.13a}$$

$$P(k) = P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{1+\varphi^T(k)P(k-1)\varphi(k)} \tag{B.13b}$$

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1). \tag{B.13c}$$

B.2 Estimator windup [2]

The main property of an adaptive controller is its ability to track variations in process dynamics. This is why the forgetting factor is used in recursive equations, Eqn.(4.19) to discount the old data. Exponential forgetting in Eqn.(4.19) works well only if the process is properly excited all the time, like a PRBS input. In adaptive control system, the square wave is used instead of PRBS. And causes the long periods with no excitation, the estimator will forget the proper values of the parameters, and the uncertainties will grow. This is called estimator windup. From Eqn.(4.19b),

$$P(k) = \frac{1}{\lambda}\left[P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda+\varphi^T(k)P(k-1)\varphi(k)}\right] \tag{4.19b}$$

the negative term in the right hand side presents the reduction in uncertainly due to the last measurement. $P(k)\varphi(k)$ will not change direction if there is no information in the last measurement and Eqn.(4.19b) reduces to

This material is reserved for educational use only, not allowed for commercial use.

Forbidden to modify the content, and cite the document when use.

$$P(k) = \frac{P(k-1)}{\lambda} \tag{B.14}$$

If $\lambda < 1$, $P(k)$ will grows exponentially until φ changes direction and may be very large if there is no excitation for a long period. The estimator windup may then resulting in changing the value of the estimated parameter.

King Mongkut's Institute of Technology Ladkrabang

This material is reserved for educational use only, not allowed for commercial use. Forbidden to modify the content, and cite the document when use.

Appendix C

The inference result is used to calculate the appropriate stable closed-loop pole α. From figure 6.2 which is rewritten here as figure C.1, the value of membership functions μ_{SM}, μ_{JF}, and μ_{LG} of fuzzy set SM, JF and LG to the value of stability index γ are found respectively as shown in Table C.1 by the following example.

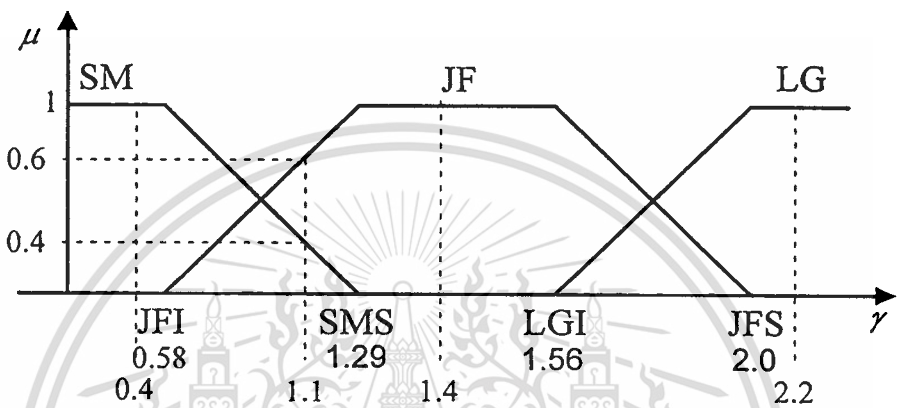

Figure C.1 Membership function of antecedent.

Table C.1

$\gamma = 0.4$	$\gamma = 1.1$	$\gamma = 1.29$	$\gamma = 1.4$	$\gamma = 1.56$	$\gamma = 2.2$
$\mu_{SM} = 1$	$\mu_{SM} = 0.4$	$\mu_{SM} = 0$	$\mu_{SM} = 0$	$\mu_{SM} = 0$	$\mu_{SM} = 0$
$\mu_{JF} = 0$	$\mu_{JF} = 0.6$	$\mu_{JF} = 1$	$\mu_{JF} = 1$	$\mu_{JF} = 1$	$\mu_{JF} = 0$
$\mu_{LG} = 0$	$\mu_{LG} = 0$	$\mu_{LG} = 0$	$\mu_{LG} = 0$	$\mu_{LG} = 0$	$\mu_{LG} = 1$
$\mu_g(k-1) = -0.7$	$\mu_g(k-1) = -0.1$	$\mu_g(k-1) = 0$	$\mu_g(k-1) = 0$	$\mu_g(k-1) = 0$	$\mu_g(k-1) = 0.7$

The values in last row of Table C.1 are the defuzzification results calculated from Eqn.(6.1) and figures C.2-1,C.2-2,C.2-3 and C.2-4 which are rewritten from figure 6.3 as follows

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

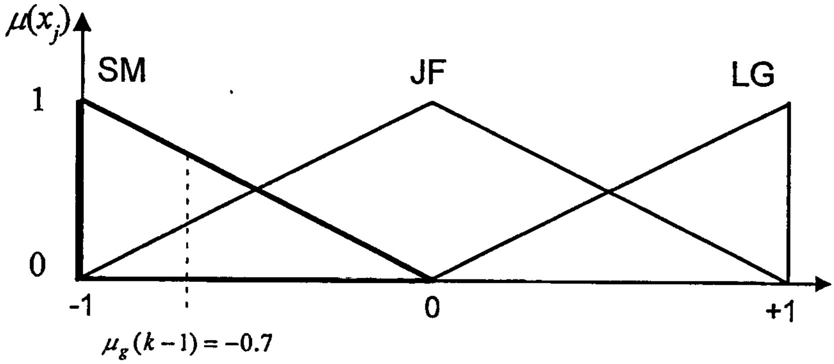

Figure C.2-1 $\gamma = 0.4$

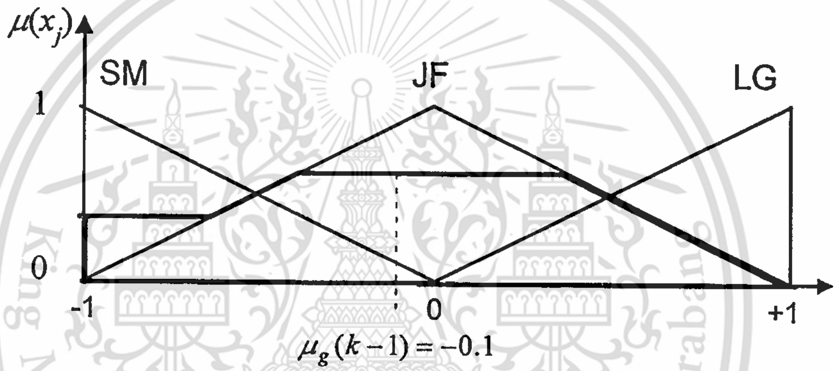

Figure C.2-2 $\gamma = 1.1$

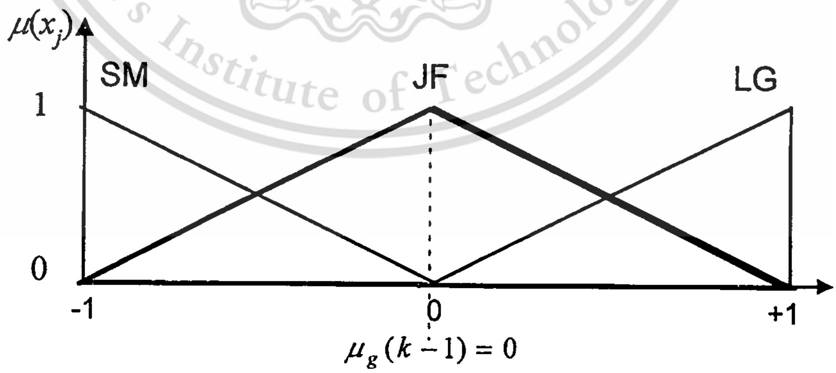

Figure C.2-3 $\gamma = 1.4$

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

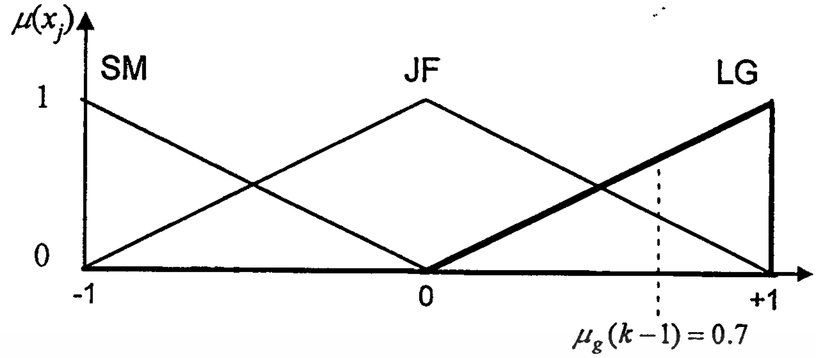

Figure C.2-4 $\gamma = 2.2$

The stable poles for each defuzzification value are calculated from Eqn.(6.2) which is rewritten here in Eqn.(C.1).

$$\alpha(k) = \alpha(k-1) + C\mu_g(k-1). \tag{C.1}$$

For example

$\alpha(1) = 0.45 + 0.005 \times \mu_g(0)$

$\alpha(2) = \alpha(1) + 0.005 \times \mu_g(1)$

$\alpha(3) = \alpha(2) + 0.005 \times \mu_g(2)$

$\alpha(4) = \alpha(3) + 0.005 \times \mu_g(3)$

$\alpha(5) = \alpha(4) + 0.005 \times \mu_g(4)$

.

.

.

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Related Publications

[1] Phornsuk Ratiroch-anant, Wanlop Surakampontorn, Suthian Kiatsunthorn and Hiroshi Hirata. "Strongly Stable Controller Design for Adaptive Pole Placement." 21th Electrical Engineering Conference (EECON-21), November 12-13, 1997. pp.485-488.

[2] Phornsuk Ratiroch-anant, Wanlop Surakampontorn, Hiroshi Hirata, Vipan Prijapanij and Masatoshi Anabuki. "Achieving Strongly Stable Adaptive Pole Placement Control by Considering the Stability Indices." Proceedings of the School of Engineering, Tokai University, vol.23, 1998. pp. 29-44.

King Mongkut's Institute of Technology Ladkrabang

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.

Author Biography

Phornsuk Ratiroch-anant received the Bachelor Degree of Electrical Engineering from King Mongkut's Institute of Technology Ladkrabang and the Graduate Certificate in Electrical Engineering from Graduate school, Chulalongkorn University in 1974 and 1977 respectively.

Currently, she is working at Department of Control Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang as an assistance professor. Her research interests include industrial process control and adaptive control. And her teaching subjects are industrial instrumentation, industrial process control, automatic control system and control system design.

King Mongkut's Institute of Technology Ladkrabang

This material is reserved for educational use only, not allowed for commercial use.
Forbidden to modify the content, and cite the document when use.