การจดจำรูปแบบตัวพิมพ์อักษรภาษาไทย

Printed Thai Characters Recognition

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บทคัดย่อ

วิทยานิพนธ์นี้เสนอวิธีการพัฒนาระบบการจดจำอัตโนมัติ สำหรับเครื่องอ่านอักษรตัวพิมพ์ภาษาไทย ในการเตรียมข้อมูลให้กับคอมพิวเตอร์โดยอัตโนมัติ ในสามขั้นตอนหลักของระบบการจดจำ ในขั้นตอนของการเปลี่ยนสัญญาณแสงไปเป็นสัญญาณไฟฟ้า จะไม่สนใจสำหรับวิทยานิพนธ์นี้

ในขั้นตอนการจัดการล่วงหน้า เพื่อปรับปรุงข้อมูลให้เหมาะสมสำหรับการทำงานในขั้นตอนของการจดจำต่อไป จะบรรยายเกี่ยวกับเทคนิคในการแยกตัวอักษรภาษาไทยออกจากประโยค และเทคนิคสำหรับการลดทอนข้อมูลของแต่ละตัวอักษรให้น้อยลง เพื่อให้เกิดการประหยัดหน่วยความจำในคอมพิวเตอร์สำหรับเก็บข้อมูลให้น้อยที่สุดเท่าที่จะเป็นไปได้

วิทยานิพนธ์นี้จะเน้นหนักในขั้นตอนของการจดจำ เพื่อให้ได้วิธีที่ดีที่สุดสำหรับการจดจำรูปแบบอักษรตัวพิมพ์ภาษาไทย จะเสนอทั้งวิธีการจดจำแบบการซ้อนทับ (matching method) และวิธีการแบบวิเคราะห์โครงสร้าง (structural analysis method)

สำหรับแบบการซ้อนทับ เนื่องจากลักษณะของอักษรตัวพิมพ์ภาษาไทยมีส่วนคล้ายกันมาก และเพื่อให้การจดจำมีประสิทธิภาพสูง ระบบการจดจำจึงถูกแบ่งออกเป็นสองขั้นตอนด้วยกัน

ในขั้นตอนของการแบ่งกลุ่มตัวอักษร (Rough classification stage) รูปร่างปลีกย่อยที่ไม่จำเป็นและสัญญาณระกวนจะถูกกำจัดออกไปโดยการทำตัวอักษรให้เบลอ ตัวอักษรที่เบลอแล้วจะถูกแบ่งออกเป็นกลุ่ม ๆ เงื่อนไขสำหรับการแบ่งกลุ่มใช้วิธีการวัดค่าสัมประสิทธิของความเหมือนของแต่ละรูปแบบ แล้วใช้การกระจายแบบคาร์ฮูเนน-โลบ (Karhunen Loève expansion) หารูปแบบมาตรฐานสำหรับเก็บไว้เป็นตัวแทนของแต่ละกลุ่ม

ในขั้นตอนของการแยกตัวอักษรออกจากกลุ่ม (Fine classification stage) มีวิธีการอยู่สองวิธีสำหรับแยกตัวอักษรออกจากกัน วิธีแรกใช้การซ้อนทับเป็นส่วน ๆ (subpattern matching) ในการแยกตัวอักษรออกจากกัน วิธีที่สองใช้วิธีการสร้างฟังค์ชั่นการตัดสินใจแบบเชิงเส้น (linear decision functions) บนระนาบของไอเกนเวคเตอร์ที่ได้จากการกระจายแบบคาร์ฮูเนน-โลบ สำหรับแยกตัวอักษรในแต่ละกลุ่มออกจากกัน

สำหรับการจดจำแบบวิเคราะห์โครงสร้าง ระบบการจดจำก็จะแบ่งออกเป็นสองขั้นตอนด้วยเช่นกัน ในขั้นตอนของการแบ่งกลุ่มตัวอักษร เทคนิคการทำตัวอักษรให้บางจะถูกนำมาใช้เพื่อกำจัดสัญญาณรบกวนและกำจัดลักษณะปลีกย่อยที่ไม่จำเป็นออกไป

ตัวอักษระที่ถูกทำให้บางแล้วจะถูกแบ่งออกเป็นกลุ่ม ๆ โดยใช้ฟังค์ชั่นการตัดสินใจแบบเชิงเส้นที่ยึดหลักความสูง และความกว้างของตัวอักษรเป็นเกณฑ์ในการแบ่งแยกตัวอักษรออกเป็นกลุ่ม ๆ ในขั้นตอนของการแยกตัวอักษรออกจากกลุ่ม ค่า "ความสัมพันธ์ระ

หว่างจุด" ของแต่ละจุดในรูปร่างตัวอักษรจะถูกคำนวณออกมา

คุณสมบัติทางเรขาคณิตศาสตร์ของตัวอักษรที่อยู่ในฟอร์มของค่าความสัมพันธ์ระหว่างจุดที่แทนจุดปลาย จุดแยก จุดต่อเนื่อง และอื่นๆ จะให้ความแตกต่างระหว่างตัวอักษรต่างๆ ซึ่งคุณสมบัติเหล่านี้จะนำมาใช้ในการแยกตัวอักษรออกจากกันในแต่ละกลุ่ม

การจดจำรูปแบบอักษรตัวพิมพ์ภาษาไทยที่มีประสิทธิภาพสูงจะได้รับจากการจดจำแบบการซ้อนทับ เพราะว่าไม่มีการกำกวมในกระบวนการของการจดจำที่ใช้การกระจายแบบคาร์ฮูเนน-โลบ เป็นเครื่องมือ

ถึงแม้ว่าการจดจำรูปแบบอักษรตัวพิมพ์ภาษาไทยที่ใช้คุณสมบัติทางเรขาคณิตศาสตร์ของตัวอักษร จะมีความยุ่งยากและต้องทำงานหนักในกระบวนการของการจดจำก็ตาม แต่ประสิทธิภาพของการจดจำยังสูงพอ ที่จะสามารถยอมรับได้ในสายงานของการจดจำรูปแบบตัวอักษร

ผลของการจดจำรูปแบบอักษรตัวพิมพ์ภาษาไทยทั้งสองวิธี และทั้งขั้นตอนการแบ่งกลุ่มตัวอักษร และขั้นตอนการแยกตัวอักษรออกจากกลุ่ม จะแสดงให้เห็นอย่างชัดเจนในวิทยานิพนธ์นี้

ABSTRACT

In this thesis, the development of methods of automatic recognition system to directly read machine printed Thai characters for preparing the data of computer is presented. In the three major subsystems of the recognition system, on the electrooptical converter subsystem, it does not take account in this thesis.

On the preprocessor subsystem, to reform the data suitability for the operation in the recognition subsystem, techniques of segmentation of Thai characters from sentences are described, and then the techniques to reduce the data of each character as small as possible for saving the storage area of computer are also described.

Thesis is emphasized on the recognition subsystem, and both matching method and structure analysis method are presented to get the best method for printed Thai characters recognition. For the matching method, since there are many similar characters in printed Thai characters, and in order to get a high recognition rate, the recognition system is separated into two stages.

In the rough classification stage, fine features and noises are ignored by adding blur to characters. The blurred characters are separated into some cluster domains. The clustering criterion used is based on selection of the patterns by measuring the similarity coefficient. The Karhunen-Loève expansion is applied to get a standard pattern of each category.

In the fine classification stage, there are two methods to discriminate between the characters, the first one, subpattern matching is used to discriminate the characters. The second one,

the linear decision functions on the eigenvector space of Karhunen-Loève expansion are constructed to discriminate the characters in a category.

For the structure analysis method, the recognition system is also divided into two stages. In the rough classification stage, a thinning technique is applied to eliminate noises and fine features.

The thinned characters are separated into cluster domains by using a linear decision function based on height and width of each character. In the fine classification stage , the "connected numbers" of points in the character pattern are computed.

Topological properties of characters in the form of "connected number" which represent end strokes, branch strokes, connecting strokes and so on show difference among the characters. These properties are used to discriminate the characters in a category.

A high recognition rate of printed Thai characters recognition is obtained from a matching method, because no ambiguity occurs in the procedure of recognition by using K-L expansion.

Even though there was much labour with printed Thai characters recognition by using topological properties of characters, but the recognition rate is high enough for character recognition, it is still valid approach for printed Thai characters recognition.

The results of recognition of both methods and of both rough and fine classification are shown in this thesis.

Content

1. INTRODUCTION

1.1 Character pattern recognition background

Recently, due to the widespread propagation of computers and their high performance, amount of data to be input to computers is increasing year by year. In spite of development of new data entry equipment such as Key-to disc and Key-to-MT, data entry is still predominately dependent upon Key-punchers. However, because of the shortage and high cost of key-punchers, data entry has become the serious bottleneck in data processing.

One of the best probable solution to solve this problem is the development of an OCR (optical character reader) which directly reads machine printed characters and handprinted characters without key-punchers. In the U.S.A and Europe, the OCR's for machine printed characters have made great progress because of the wide use of typewriters. The situation in Japan is different in that almost all source data are mainly handwritten chinese characters. Therefore, the OCR's for handwritten characters have been mainly developed in Japan.

In the case of Thailand , typewriter has also been widely used, therefore, this thesis has proposed the procedure to develop the OCR for printed Thai characters recognition.

1.2 Character pattern recognition method

An OCR system (1) such as shown in Fig. 1, is made up of three major subsystems: an electrooptical converter, a preprocessor, and recognition method. The electrooptical converter scans the characters and converts them into electrical signals for further processing, and at the same time, to filter the noise out from them.

The preprocessor examines the output of the scanner and performs quantization. In addition, the preprocessor will reform the signal suitable for the operation of recognition method and for easier to store them.

The recognition method examines the output of the preprocessor, locates and separates the data derived from each character, and then processes these data to determine the identity of each character.

Fig.1 Major components of an optical character recognition system are shown between "input" and "output".

In this thesis, on the preprocessor, some techniques are used to reduce the data as small as possible for saving the storage area of computer , and some techniques to reform the data suitability for recognition method. This thesis emphasizes on recognition method, we proposed three procedures to develop the OCR for printed Thai characters which will be described in next section.

Character pattern recognition methods (2) are generally divided into matching method and structure analysis method. The matching method is an effective tool for the solution of problems in which the patterns include noise but exclude rotation, whereas structure analysis is employed in the opposite case. As there exists noise, arising from an imperfect paper surface and interruption in the line of printed characters, a matching method is usually used. For the case of handwritten characters, their shapes are rotated according to the writer, and structure analysis is usually used.

Since there are many similar characters in printed Thai characters, and each character is composed of many convex and concave arcs , it looks like the handwritten character, therefore in this thesis, both of the recognition methods have been tested.

The structure analysis method for printed Thai characterers recognition has been proposed in the paper (3), (4). In the paper (3), a recognition method of printed Thai character by the structural analysis of their contours has been described. Digital contours of characters are encoded according to directional differences of contour tracing, then, arithmetic operations are applied for extracting concavities and convexities of the contours. Several geometric features of concave and convex arcs are used to calculate similarities of the arcs, then, a pair of the most similar arcs between a model and an input character is determined from the similarities. However the concavity and convexity extraction did not work well for low resolution characters. Further, the features used in it were not sufficient for classifying similar characters.

As a modification of the method (3), an effective extraction method of concavities, convexities and features for the recognition of low resolution Thai characters has been reported in the paper (4). Freeman chain code and directional differences of contour tracing of characters are utilized for extracting concavities and convexities. Each arc is then segmented by points at which the arc bends. Several local features of arcs are extracted, and are used to calculate similarities between arcs. Then, a pair of the most similar arcs between a model and an input characters is detected. The recognition is made by detecting similar arc pairs from the characters. However, the detecting similar arc pairs from the characters of this methods is very complex, then, it is very difficult to obtain the optimum result in recognition, and it takes much of the computation time of matching.

The recognition method by tree grammars has been done with Korean characters (5). This method is very difficult to do with Thai characters, because there are no fundamental strokes in Thai characters and no rules to compose the fundamental strokes be the character. Thai characters almost compose of only on stroke.

Further more, the structure analysis by feature concentration method has been treated with handprinted numeral rcognition at the electrical communication laboratories of Nippon telegraph and telephone public corporation Japan (6), that method examined on both their strokes and background of strokes of character.

The structure analysis method by using topological properties of characters for printed Thai characters recognition in this thesis examined only the strokes of characters, hence the complexity in procedure is decreased.

To reduce the recognition time and to get high recognition rate in this method, it is divided into two stages which is the original idea in this method.

In the rough classification stage, all printed Thai characters are separated into cluster domains by using a linear decision function based on height and width of each character.

In the fine classification stage, topological properties of characters in the form of " connected number" which represent end strokes, branch strokes, connecting strokes and so on show difference among the characters. These properties are used to discriminate the characters in a group.

The matching method has never been used with printed Thai characters recognition. Therefore, in this thesis, printed Thai characters recognition method emphasizes matching method.

In the view of character recognition, Thai characters are said to have roundness and are relatively simpler than Chinese/ Japanese, and then, there are smaller number of effective features to discriminate "similar characters" than Chinese/ Japanese characters. Most of characters have small holes (called head of character), and few characters are separated into two subpatterns. Therefore, it is necessary to develop efficient feature selection method. The K-L expansion is one of the best way to have features from the pattern set in which patterns are distributed in the form of normal distribution (7) , Hence, in matching method of this thesis, the experimental approach has used the K-L expansion being a tool for discriminating printed Thai characters.

Since there are many similar characters in printed Thai characters, and in order to get a high recognition rate, the recognition system in matching method for printed Thai characters is separated into two stages (which is the original idea in this method). In the rough classification stage, fine features and noises are ignored by adding blur (original idea) to characters. The blurred characters are separated into some cluster domains. The clustering criterion used is based on selection of the patterns by measuring the similarity coefficient. The Karhunen-Loeve expansion is applied to get a standard pattern of each category.

In the fine classification stage, there are two methods to discriminate the characters in a category, the first one, subpattern matching is used to discriminate the characters. The second one, the linear decision functions on the eigenvector space of K-L expansion are constructed to discriminate the characters in a category.

1.3 Summary of printed Thai characters recognition method

Printed Thai characters recognition method can be summarized by comparing with each other as shown in table 1 and table 2.

Table 1 : Summary of printed Thai characters recognition using structure analysis method

Characteristic	Method	
	Using contour trace (3),(4)	Using topological properties(in thesis)
1. Feature selection	convex and concave of strokes	end,branch,connecting strokes, etc.
2. Using code	Freeman chain code	connected number code
3. Using process	only one process	rough and fine classification stages
4. Noise eliminating method	one arithmetic operation	thinning process
5. Complexity in the process	much	less
6. Implementation programming	not simple	simple
7. Classification of most similar characters	successful	not successful for some characters, it is necessary to use other criterion classify again.
8. Application for on-line recognition	easy	not easy
9. Dimension of experimental data	50X50 pixels	128X64 pixels
10. Both methods can be applied for recognizing other characters.		

Table 2: Summary of printed Thai characters recognition using matching method

Characteristic	Method	
	Subpattern matching	Classification using K-L expansion
1. Feature selection	explicit pieces showing most different among characters	linear decision function on eigen-vector space of K-L expansion
2. Using process	rough and fine classification stages	rough and fine classification stages
3. Kind of method	hueristic method	mathematic method
4. Noise eliminating method	reducing gray level, blurring method	reducing gray level, blurring method
5. Standard pattern	get by using K-L expansion	get by using K-L expansion
6. Improving rough classification	by using blurring method	by using blurring method
7. Saving recognition time	by matching with standard pattern and subpattern	by matching with standard pattern and by using decision function
8. Experimental approach	yes	yes
9. Ambiguity in the process for getting the best result	yes	no

Table 2: Summary of printed Thai characters recognition
(continue) using matching method

Characteristic	Method	
	subpattern matching	classification using K-L expansion
10. Classification of most similar characters	successful	successful but it is necessary to use subpattern including in K-L expansion
11. Application for handwritten character recognition	not easy	easy
12. Application for other characters recognition	easy	easy

1.4 Thai characters and experimental data

Thai characters, as shown in Fig.2, consists of 44 consonants, 19 vowels, 4 controled voice tones, 10 Thai numerals and 3 special symbols. The Thai language sentences, such as that in Fig.3, are composed of consonants, vowels and controled voice tones, on different levels. The vertical level can be divided into four parts as shown in Fig.4. However, as can be seen by inspection, sometime, depending on the printing machine or hand

Fig.2 Thai character set

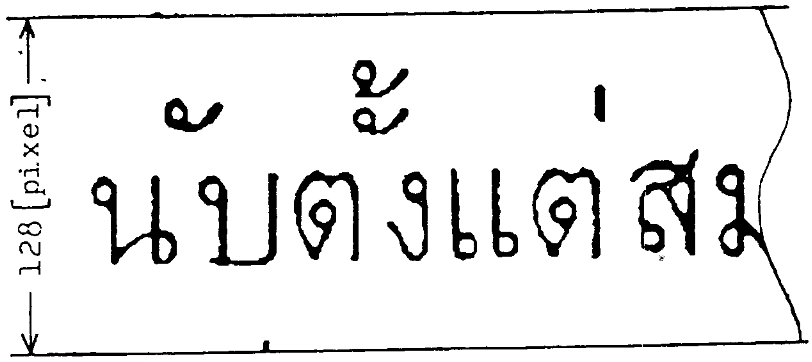


Fig.3 An example of Thai language sentence

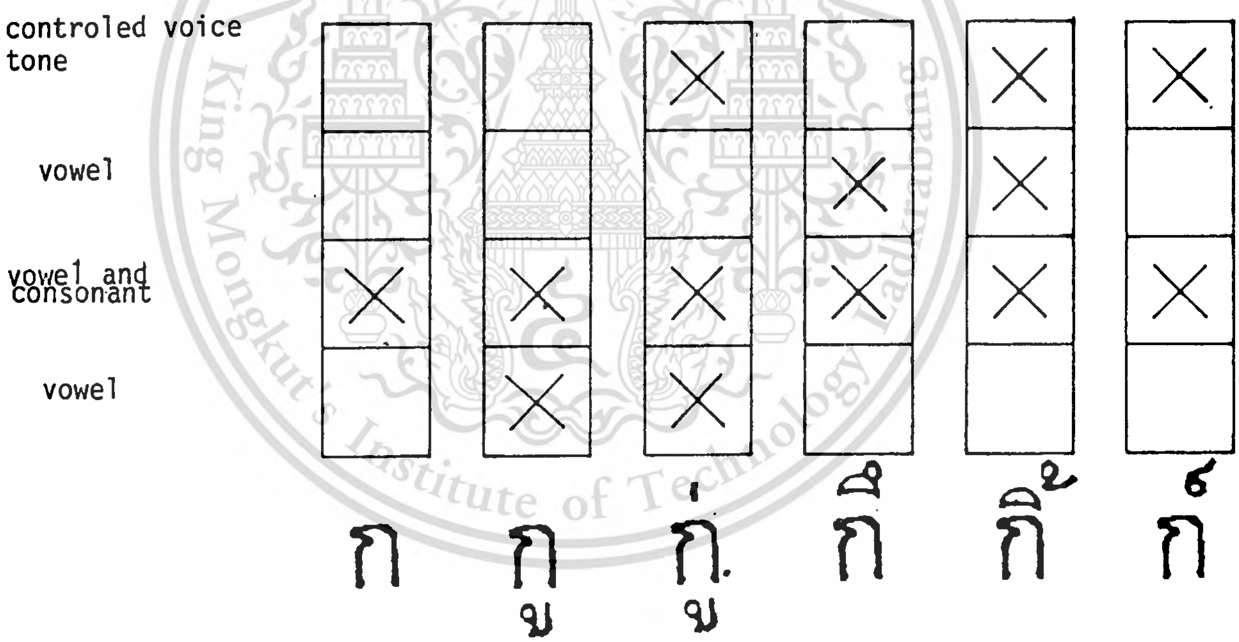


Fig. 4 Composition of Thai character word or sentence

writting (see Fig.5), a vowel can be situated above two consonants. As another special case, if there are no vowels above consonant, controled voice tones can replace those vowels.

The consideration of Thai word is different from an English word. The blank space can not be used as the separation between two words, because the blank in Thai language indicates the termination of a sentence. There are many rules for separation the word. The history of Thai language was influenced by Pali and Sanskrit language. So, the composition and pronunciation of the word are very complex. The complexity of the word is one of the cause of difficulties of Thai characters recognition. There are, nevertheless, difficulties, particularly where there are many similar characters.

All Thai characters and some sentences of Thai languages are printed on ordinary paper and then fed to be input of a kind of optical character reader. This machine automatically changes the grey level of each character to be electrical analog signals. These signals are free from the noise by spectal band-pass filter. The new class RC active filter as shown in appendix 3 can be used as this band-pass filter.

The filtered signals are converted into digital signals by Analog to Digital converters circuit which each pixel is represented by one bit of hexadecimal code. Each character is divided into a matrix of 128X64 pixels as shown in Fig.6. Each point of these matrices is represented by four bits of binary or one hexadecimal number corresponding to its grey level. These binary data are recorded on magnetic tape as experimental data. The recording format is shown in Fig.7.

To give further experimental data, the hexadecimal value for

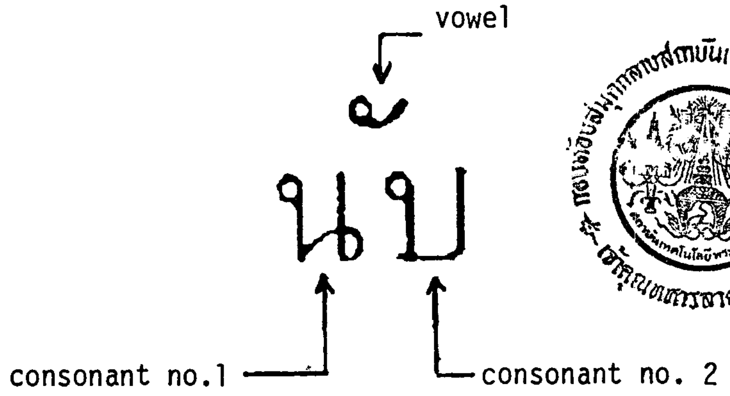


Fig. 5 Example of Thai word

Vowel is situated
above both consonant
no.1 and no.2

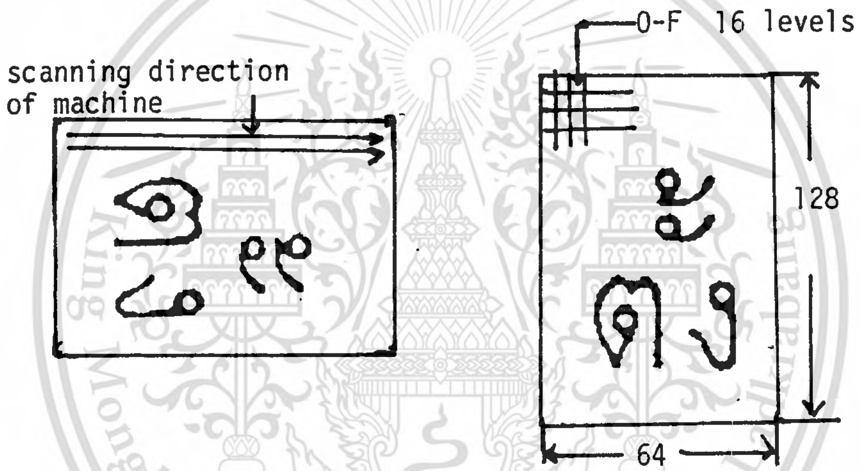


Fig. 6 Bit pattern of a character

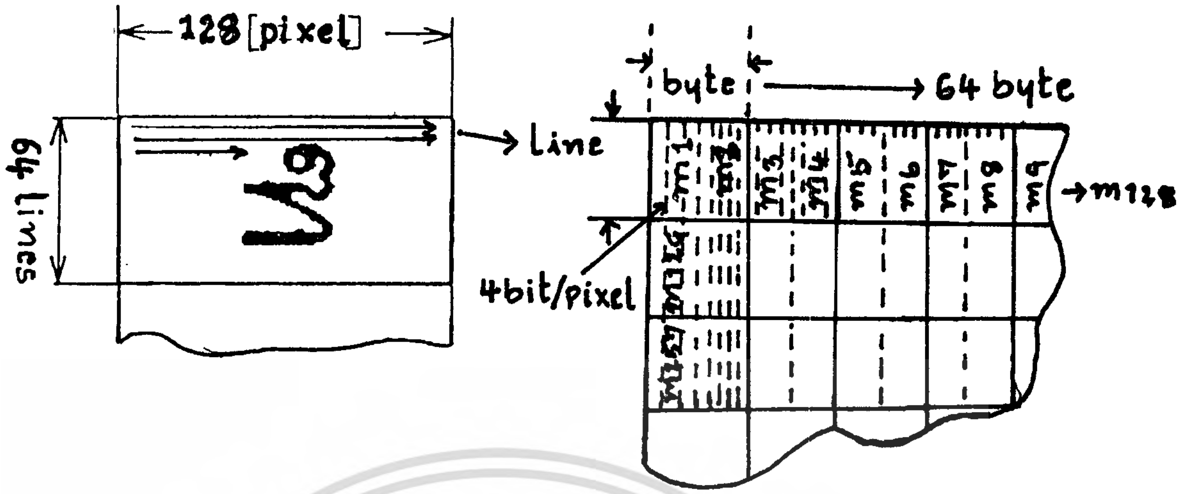


Fig. 7 Recording format of magnetic tape

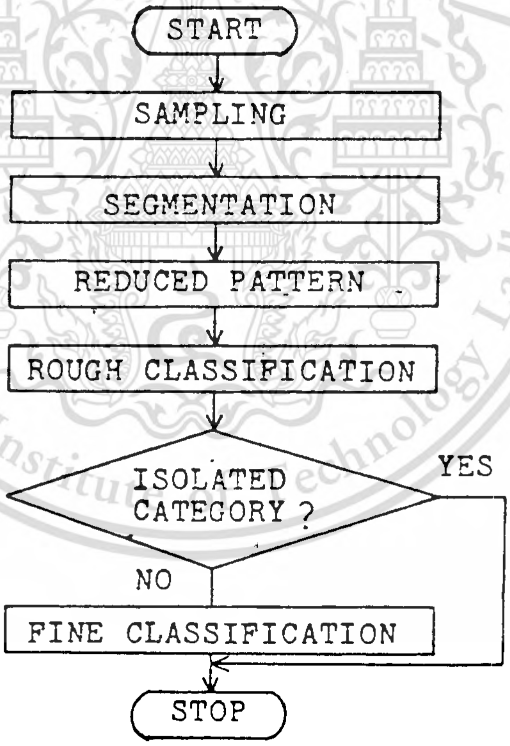


Fig. 8 Sequence of recognition method

each point in the matrix is converted to one binary bit, i.e. 0 in hexademal becomes 0 in binary but other hexadecimal values become 1 in binary. Therefore, 0 represents a white level, and 1 represents black level. However, in some cases when noise elimination is necessary, 1, 2, 3, 4 and 5 in hexadecimal may become 0 in binary, and other hexadecimal values become 1 in binary.

2. SEQUENCE OF RECOGNITION METHOD

Sequence of recognition method generally can be divided into two stages, dictionary design stage and the decision unknown stage. In dictionary design stage, the sequence of recognition of printed Thai characters is shown by a flowchart in Fig.8.

The sample Thai characters in a form of sentence are transfered from magnetic tape into main memory of computer. A pattern of each character is pulled out from the sentence by a technique in which the features are extracted by one-directional scanning, performed in the vertical and horizontal directions, respectively.

After we have the pattern of all Thai characters, the reduced pattern procedure can be started for eliminating the noises and for saving the storage area of computer. This procedure will be described in the following section. Each pattern of all Thai characters has been stored individual (not in the form of sentence) in magnetic tape. In other ways, eliminating the noise can be done by, an example, converted hexadecimal 1, 2, 3, 4, 5 into 0 in binary, and other hexadecimal values into 1 in binary.

In the rough classification stage, the reduced patterns are grouped by the clustering techniques. If a category has only one member, the pattern of that member is kept to be a standard pattern in dictionary. If a group has two members or more, a standard pattern will be determined by a special technique.

In the fine classification stage, we must set new criterion again to find a standard pattern of each member, and that pattern is kept to be a standard pattern in dictionary again.

In the decision unknown stage, the sequence of procedure is as the same in the design dictionary stage, but the unknown pattern input must be in the form of sentence. Each pattern in that sentence will be compared with a standard pattern of dictionary in both rough classification and fine classification stage.

The method of comparision depends on the algorithm of decision method which will be described in the following section. The sampling, segmentation and reduced pattern processes of this flowchart are treated in the preprocessor subsystem, and rough classification, fine classification stages are treated in the recognition method subsystem of Fig. 1.

3. PREPROCESSING

Preprocessing performs (i) eliminating the noise (ii) saving the storage area of computer (iii) improving the data or features for increasing the reliability of the decision of unknown pattern. The procedures of preprocessing are described in the following section.

3.1 Segmentation of Thai characters from the sentence

The segmentation of Thai characters from the sentence must be done before the decision unknown pattern process. Because the construction of a Thai word, as shown in Fig.4, is different from an English word, we cannot separate the characters by scanning only in the vertical direction. In order to separate characters from the sentence, it is necessary to scan in both vertical and horizontal directions. The procedure of segmentation as shown in Fig.9 is as follows:

(i) scan along the pattern of the sentence in a vertical direction to find the space, and pull out the data between two spaces

(ii) scan again along the data taken by item(i) in a horizontal direction, the pattern of each character is to be found between two spaces

(iii) in particular cases, if the width of pattern of the character is larger than a threshold value , it is necessary to scan to find the space one more in a vertical direction along that pattern.

The procedure (ii) is difficult to use with a pattern such as that in Fig.10(a), because the scanning only in a horizontal direction does not succeed. In such a case, as scanning traverses the pattern in a horizontal direction and encounters 1's data, the scanning must then turn to the vertical direction above or below that datum to find 0's data before returning again to horizontal scanning. So a combination of vertical and horizontal scanning is necessary in this case to separate one character from the other. If this fails, then it is treated as another standard pattern for furthur matching with unknown patterns, such as the pattern as shown in Fig.10(b).

Before the operation in the procedure(iii) will be started, the maximum width of Thai characters must be known. We can find the maximum width of characters from a graph which shows the relation the height and width of characters. This graph is shown in the following section.

From the data which are the results of this segmentation, we can obtain other experimental data by shifting or rotating those data. We will also show the operation of shifting or rotating the data in the following section.

3.2 Reduced pattern

The best recognition method is necessary to use the storage area of computer as small as possible. The reduced pattern is the results of ones of the process in the preprocessing, and then the storage area for storing these patterns is decreased. To obtain reduced pattern, it can be done in two ways;

1) The procedure of reduction keeping only one of four

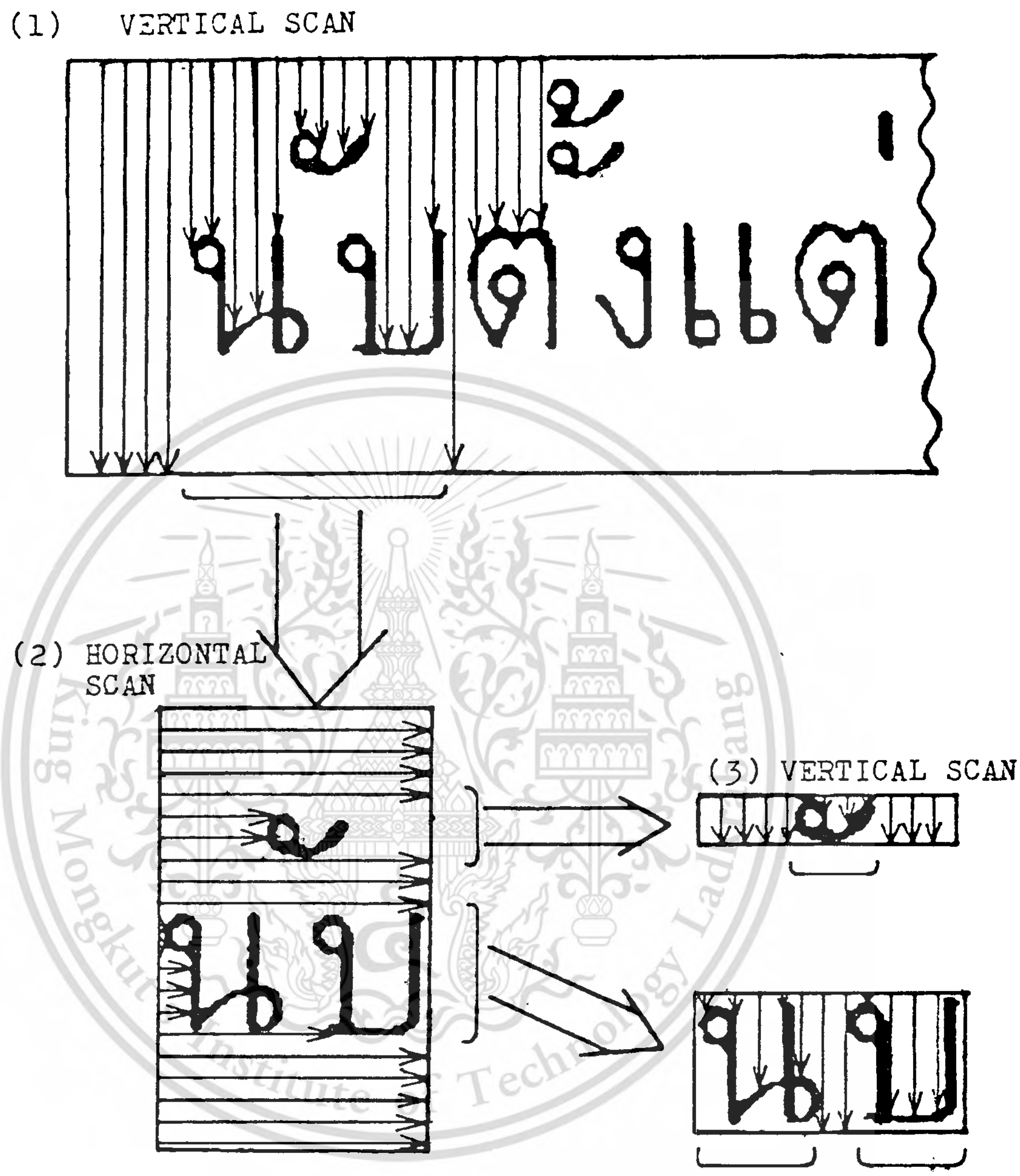
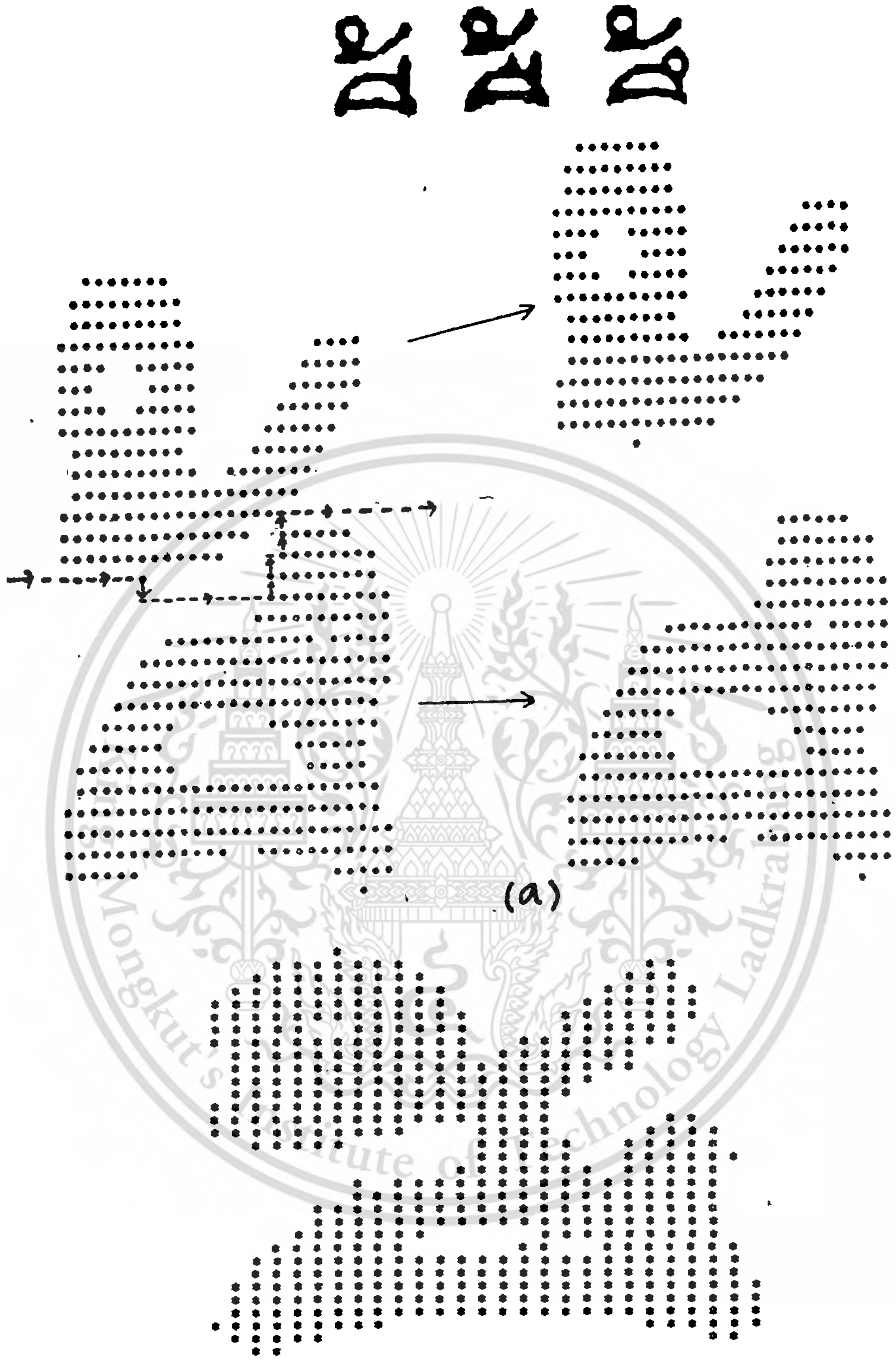


Fig. 9 A method of segmentation



(a)

(b)

Fig. 10 Special case of pattern in which procedure (ii) is very difficult to use.

Scaning trace : - - → - - - - - →

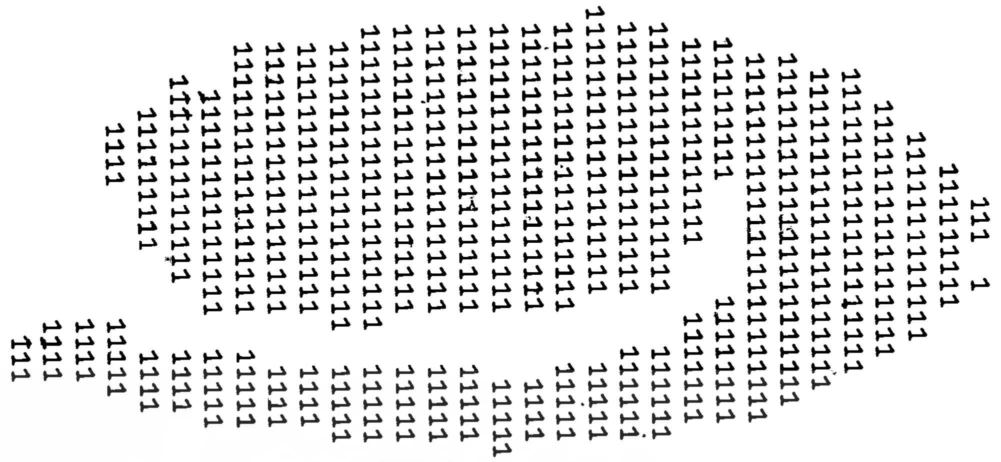
neighbouring 1's is used, i.e. if the binary values $q(i,j)$, $q(i,j+1)$, $q(i+1,j)$ and $q(i+1,j+1)$ are neighbours with each other, and RP is identified by

$$RP = (q(i,j) + q(i,j+1) + q(i+1,j) + q(i+1,j+1))/4 + 0.5$$

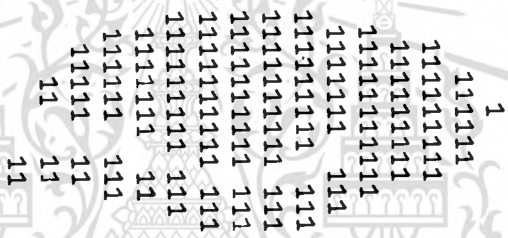
The result of reduction is the integer part of RP. If the method to store only 1's data is used, then the storage area of computer is decreased.

2) The pattern of a character is reduced by reducing the gray level, for example, the gray levels represented by 0001, 0010 are reduced to 0000, others are reduced to 0001 etc. Then the method to store only 1's data is used, the storage area of computer is decreased and also noise is eliminated.

Fig. 11(a) shows original pattern data. Fig. 11(b) shows reduced pattern data which are the result of RP. Fig. 11(c) shows reduced pattern data which are the result of reducing the gray level of data.



(a)



(b)

Fig. 11

(a): Original pattern data of character No.1 of Fig.2, 1 represents black gray level and blank represents white gray level of each pixel.

(b): Reduced pattern data of character No.1, reducing by equation RP as described in the text.

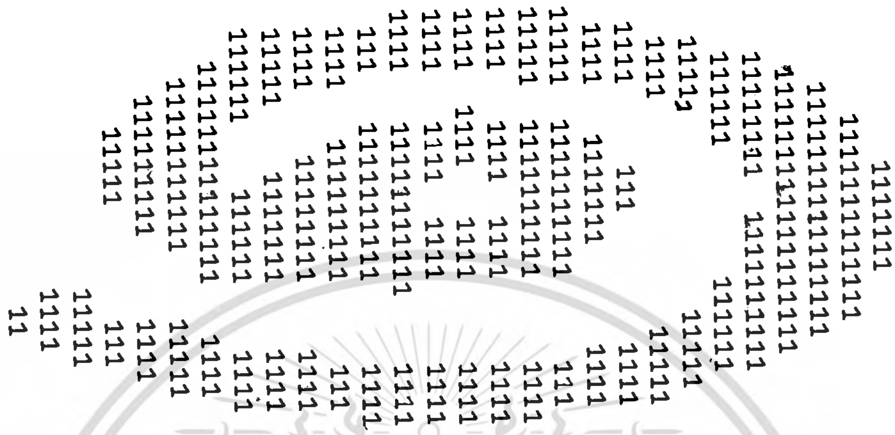


Fig. 11(c)
Reduced pattern data of character No.1
by reducing gray level lower than 8 to 0 and other levels to 1 .

4. MATCHING METHOD FOR PRINTED THAI CHARACTERS RECOGNITION

4.1 Rough classification in the matching method

Since there are many similar characters in printed Thai characters and in order to get a high recognition rate, to save the reconition time, it is necessary that the recognition system is separated into rough and fine classification stages.

The rough classification of printed Thai characters recognition by matching method is the operation of clustering all printed Thai characters by dividing into the groups. The operation of clustering is first to measure the similarity between all characters. If we let f_{ns} be a given pattern, f_x be the unknown pattern and $S(f_{ns}, f_x)$ be the similarity between two characters. The definition of $S(f_{ns}, f_x)$ is

$$S(f_{ns}, f_x) = \frac{(f_{ns} \cdot f_x)}{\|f_{ns}\| \|f_x\|} \quad (1)$$

where the scalar product $(f_{ns} \cdot f_x)$ is represented by

$$(f_{ns} \cdot f_x) = \sum_i \sum_j (f_{ns(i,j)} \cdot f_{x(i,j)}) \quad (2)$$

and the norm $\|f\|$ is

$$\|f\| = \sqrt{(f.f)} \quad (3)$$

If we let one character to be a given pattern and other characters be to unknown patterns, then the similarity value is computed by equation (1).

The second, after the similarity of all characters are known, the clustering can be done by setting the criterion based on the following equation:

$$S(f_{ns}, f_x) \geq T \qquad (4)$$

Where T_n is the criterion decision value (threshold) of n th group , and if equation (4) is satisfied, f_x is a member of n th group. A group is called "a category", and the data f_x which represent each character or each member are called "a class". The relation between category and class can be shown clearly by the Fig. 12.

4.2 Blurring method

In order to get more easily the criterion decision value of the group, and increase the reliability of the decision of unknown pattern, it is necessary to eliminate the different fine features between Thai characters. The blurring method is one of processes to serve this purpose, and also eliminate the noise.

A blurring method is illustrated in Fig.13(a). The bit pattern is scanned in a horizontal direction from the first to the last row, as 1's data are encountered, the neighbouring data are also converted to 1. The blurring method ascribes the name 'blurred level 1' as scanning is completed once on the pattern, in the same way 'blurred level 2' is the name of a pattern which

Numbers refer to character numbers given in Fig. 2

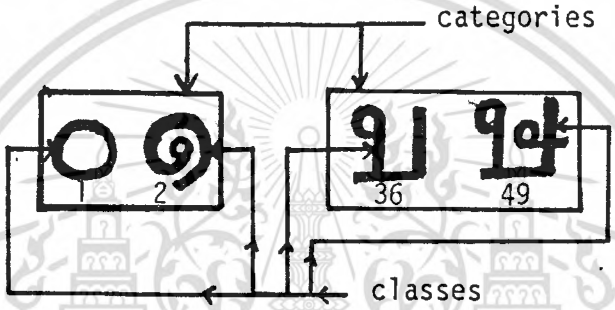
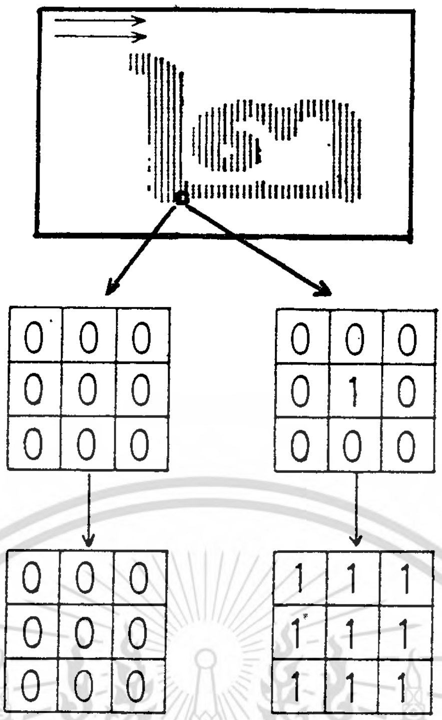
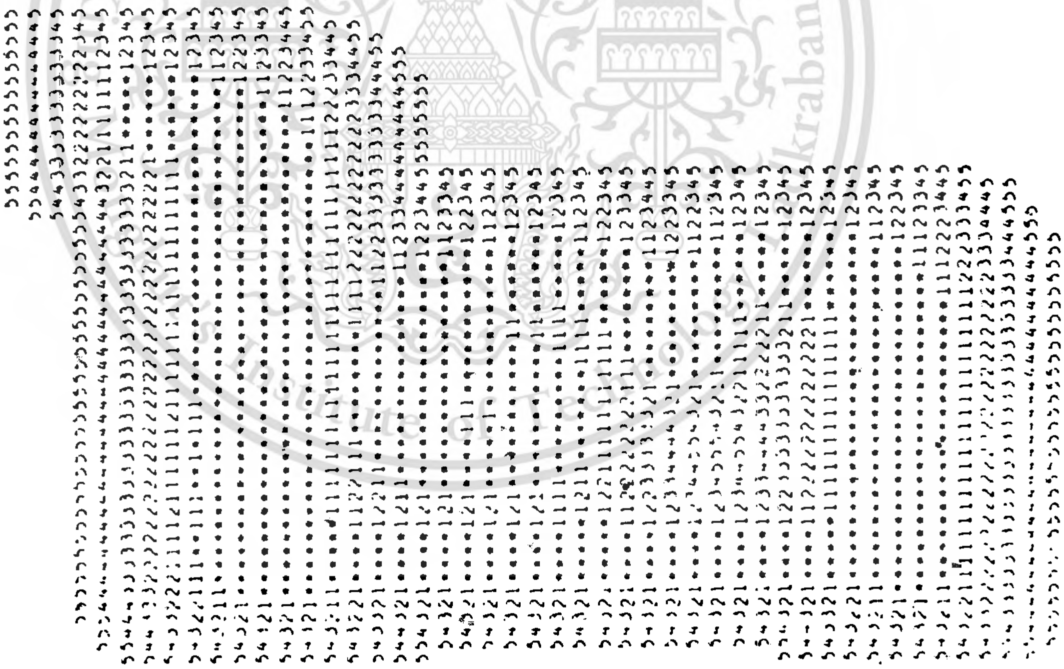


Fig. 12 The relation between category and class



(a)



(b)

Fig.13: (a) Blurring method
(b) The result of blurring method

is blurred twice and so on. As the blurred level is increased, the fine features of a character are decreased, and noise is eliminated.

The example of blurred pattern character as shown in Fig.13(b). "X" means the original bit pattern, "1" means the bit pattern from the result of "blurred level 1", in the same way "2" means the bit pattern from the result of "blurred level 2" and so on.

In order to improve the results of blurring method, the reduced pattern by reducing the gray level is used in the procedure of blurring. This is called " improved blurring method". And then the similarity between blurred characters is improved. Table 3 shows examples of similarity between some characters. Table 4 shows similarity values between blurred characters at any level which derived from original pattern (original pattern means the pattern without reducing gray level), and shows similarity values between blurred characters at any level which are derived from reduced pattern (reduced pattern means the pattern with reducing gray level). It is clear from Table 4 that the similarity values between reduced patterns at blurred level 2 become the same as the similarity values between original patterns at blurred level 5, indicating that the recognition time is reduced if the reduced pattern is used.

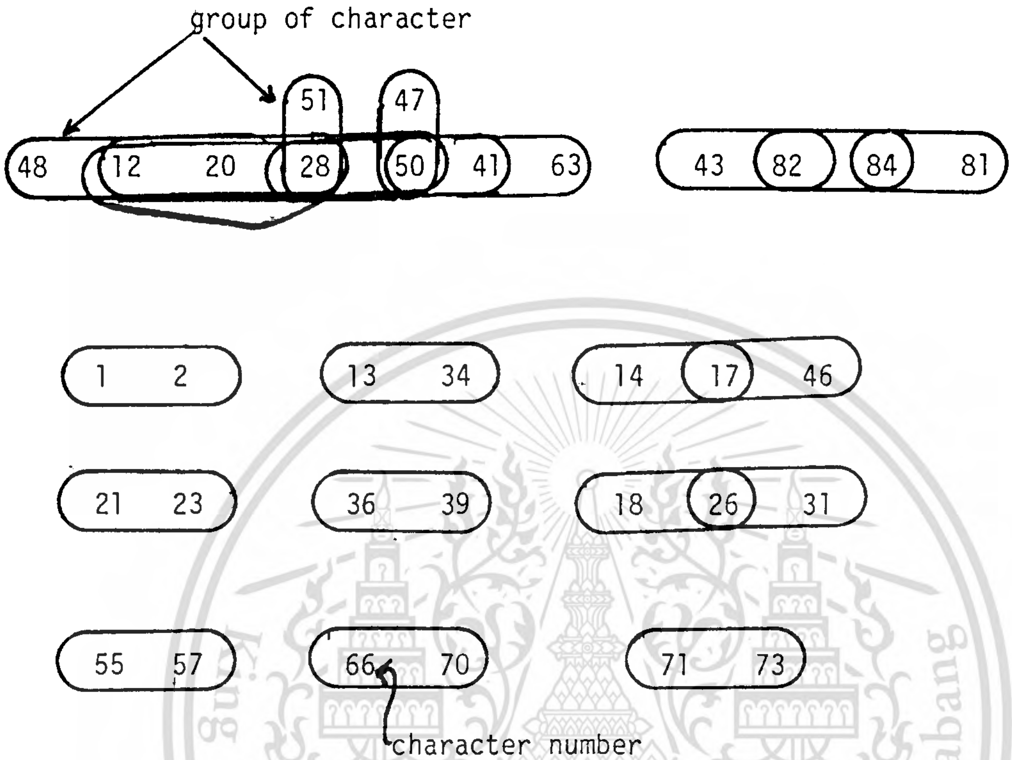
The results of clustering by using equation (4) are shown in Fig. 14. These results are not good enough for the aim of rough classification because some characters belong to the member of many categories. It is necessary to adjust the members in each category again by using their height and width of characters. By investigation, the characters which have nearly

Table 3 : Examples of similarity with blurring level 5

ก 11	ค 12	ฆ 13	ข 14	ง 15	จ 16	ช 17	ฌ 18	ฉ 19	ฅ 20	
1.00	0.93	0.85	0.86	0.75	0.83	0.85	0.78	0.83	0.90	ก 11
	1.00	0.88	0.88	0.82	0.86	0.86	0.83	0.88	0.96	ค 12
		1.00	0.87	0.75	0.80	0.88	0.91	0.88	0.96	ฆ 13
			1.00	0.80	0.88	0.95	0.81	0.87	0.88	ข 14
				1.00	0.85	0.77	0.72	0.79	0.82	ง 15
					1.00	0.85	0.76	0.84	0.86	จ 16
						1.00	0.82	0.84	0.88	ช 17
							1.00	0.90	0.86	ฌ 18
								1.00	0.89	ฉ 19
									1.00	ฅ 20

Table 4 : example of similarity (characters 0 and 1 of Fig.2)

Blurred level	0	1	2	3	4	5
Original data	0.72	0.82	0.87	0.91	0.92	0.94
Reduced data	0.75	0.89	0.94	0.95	0.95	0.95



threshold value = 0.94 blurring level = 5

Fig. 14 Example of grouping characters

Numbers refer to character numbers given in Fig. 2

same or same height and width must belong to only one category.

And then the results of clustering of characters are shown in Fig. 15(a). In categories which have only one member, rough classification stage is the final recognition as shown in Fig. 15(b)

4.3 Standard pattern by using Karhunen-Loève expansion

To save the recognition time in the process of fine classification stage by matching method, it is necessary to find the standard pattern which represents a category for matching method. One of processes to find the standard pattern is the K-L expansion.

The adjusted members in each category of Fig. 15(a) are transformed to a unique appropriate pattern by the Karhunen-Loève expansion.

The Karhunen-Loeve expansion is a method that presents the expansion of a random vector in the eigenvectors of the covariance matrix. In the context of pattern recognition, the coeficients $\mathbb{F}_1$, $\mathbb{F}_2$, ... , $\mathbb{F}_n$ are viewed as a feature representing the observed vector. The covariance matrix is defined by

$$\mathbb{V} = (\sum_i \sum_j \mathbb{F}_i \ \mathbb{F}_j^T)/n \qquad (5)$$

An eigenvalue (λ) and eigenvector (ρ) of these matrics are

Fig. 15(a) category numbers of rough classification

No.25	ฒ	No.36	ๆ
No.26	ฅ	No.37	า
No.27	ฎ	No.38	0
No.28	ธ	No.39	1
No.29	ฑ	No.40	2
No.30	ภ	No.41	3
No.31	ย	No.42	4
No.32	ท	No.43	5
No.33	บ	No.44	7
No.34	เ	No.45	9
No.35	๙	No.46	,

Fig. 15(b)

Final recognition for one-member categories.

determined from the equation

$$\mathbb{V}\mathbb{P} = \lambda\mathbb{P} \qquad (6)$$

The eigenvector of the maximum eigenvalue is a first principal component (equivalent to standard pattern) of each category which is used to be a dictionary of rough classification.

The characters in each category of rough classification are transformed by the Karhunen-Loève expansion to function as a standard pattern of each category. There are two ways to calculate the eigenvectors and eigenvalues of category.

Let n be the number of pixells in a pattern $\mathbb{F}_i$, m be the number of patterns in a category i . A pattern $\mathbb{F}_i$ is presented as a vector $\mathbb{F}_i$ or $\mathbb{G}_i$ as follows; (see Fig. 16)

$$\mathbb{F}_i = (f_{i11} , f_{i12}, \ldots, f_{i1n} , f_{i21} , \ldots, f_{i2n} , \ldots, f_{imn})^t \qquad (7)$$

$$\mathbb{G}_i = (g_{i1} , g_{i2} , \ldots\ldots\ldots\ldots\ldots\ldots, g_{iN})^t \qquad (8)$$

where N = mXn , a matrix $\mathbb{G}$ which represents the pattern of a category is defined as follows;

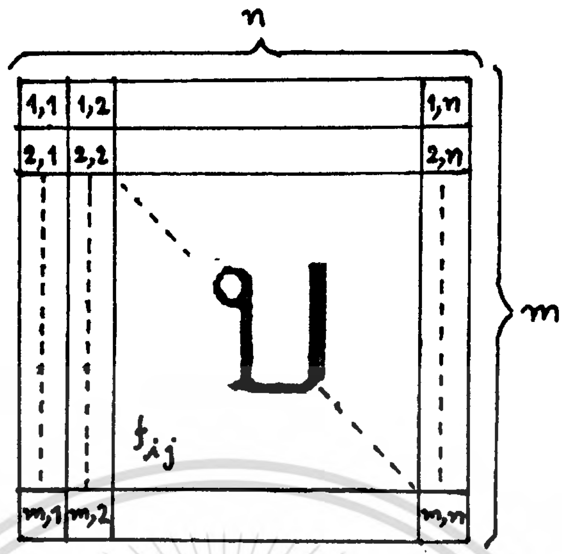


Fig. 16 Schematic presentation of $\mathbb{F}_i$ or $\mathbb{G}_i$

f_{ij} =1 or 0

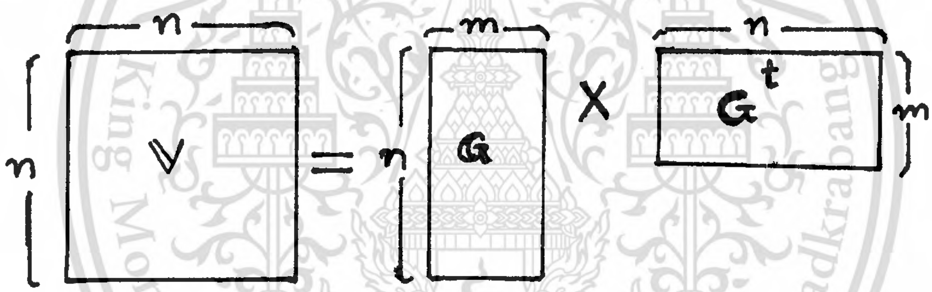


Fig. 17 Dimension of matrix $\mathbb{V} = \mathbb{G} \cdot \mathbb{G}^T$

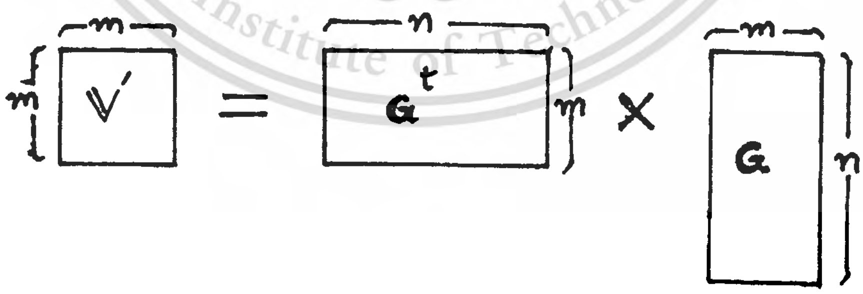


Fig. 18 Dimension of matrix $\mathbb{V}' = \mathbb{G}^T \cdot \mathbb{G}$

$$\mathbb{G} = \begin{bmatrix} g_{11} & g_{21} & g_{31} \cdots\cdots\cdots\cdots & g_{m1} \\ g_{12} & g_{22} & g_{32} \cdots\cdots\cdots\cdots & g_{m2} \\ \cdot & \cdot & \cdot \cdots\cdots\cdots\cdots & \\ \cdot & \cdot & \cdot \cdots\cdots\cdots\cdots & \\ g_{1j} & g_{2j} & g_{3j} \cdots\cdots\cdots\cdots & g_{mj} \\ \cdot & \cdot & \cdot \cdots\cdots\cdots\cdots & \\ \cdot & \cdot & \cdot \cdots\cdots\cdots\cdots & \\ g_{1n} & g_{2n} & g_{3n} \cdots\cdots\cdots\cdots & g_{mn} \end{bmatrix} \qquad (9)$$

The elements in each column represent the pattern of each member in a category.

A matrix which is used for calculating eigenvalues and eigenvectors can be defined by two ways. The first one is defined as follows; (see Fig.17)

$$\mathbb{V} = \mathbb{G}\,\mathbb{G}^{T} \qquad (10)$$

$$v_{ij} = \sum_{k=1}^{m} g_{ki} \cdot g_{kj} \qquad (11)$$

Eigenvalues of higher order than m+1 in the matrix become 0, the one can write

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m , \lambda_{m+1} = \lambda_{m+2} \; \ldots\ldots = \lambda_n = 0 \qquad (12)$$

$$\rho_i = (\rho_{i1} , \rho_{i2} , \ldots\ldots\ldots\ldots , \rho_{in})^t \qquad (13)$$

In second way, a matrix is defined as follows; (see Fig.18)

$$\mathbb{V}' = \mathbb{G}^T \cdot \mathbb{G} \qquad (14)$$

$$v_{ij} = \sum_{k=1}^{n} g_{ik} \; g_{jk} \qquad (15)$$

In the second case, it is easy to calculate the eigenvalues and eigenvectors, because the size of matrix $\mathbb{V}'$ is much smaller than the case of $\mathbb{V}$. To obtain the eigenvectors in the N = mXn pixells form, it is necessary to develop ρ' to ρ with a next equation.

$$\mathbb{V}'\rho' = \lambda' \rho' \qquad (16)$$

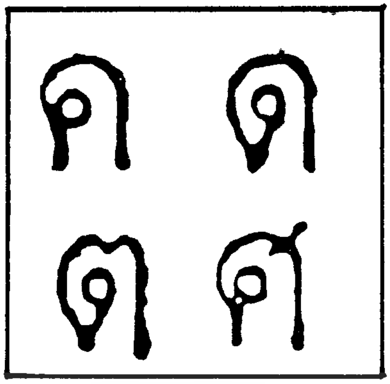
$$\rho'_i = (\rho'_{i1} , \rho'_{i2} \; , \ldots\ldots , \rho'_{im})^t \qquad (17)$$

$$\lambda'_1 \geq \lambda'_2 \geq \lambda'_3 \geq \ldots\ldots\ldots\ldots \geq \lambda'_m \tag{18}$$

$$\varphi_i = \frac{1}{\sqrt{\lambda'}} G \cdot \varphi'_i \quad , \begin{bmatrix} \varphi_{i1} \\ \vdots \\ \varphi_{in} \end{bmatrix} = \frac{1}{\sqrt{\lambda'_i}} \begin{bmatrix} g_{11}, g_{21} \cdots g_{m1} \\ \vdots \\ g_{1n}, g_{2n}, \ldots, g_{mn} \end{bmatrix} \begin{bmatrix} \varphi'_{i1} \\ \vdots \\ \varphi'_{im} \end{bmatrix} \tag{19}$$

Fig. 19(a) shows patterns in a category No.5, and Fig.19(b) shows an eigenvalue of category No.5 which derived from the Karhunen-Loève expansion. Fig 19(c) shows a standard pattern of category No.5.

The learning pattern of characters (all of characters in Fig. 2) are matched with standard patterns of each category to test the recognition rate. In other words, a rotation of -5 of the learning pattern of characters are also matched with standard patterns. The results of testing are shown in Table 5, the percentage of rejection and error are high. To improve rejection and error, the rotations by -5 of the learning pattern of characters are included to the K-L expansion to have new eigenvectors which will be used to new standard pattern. The results of testing with new standard patterns are shown in Table 6.



(a)

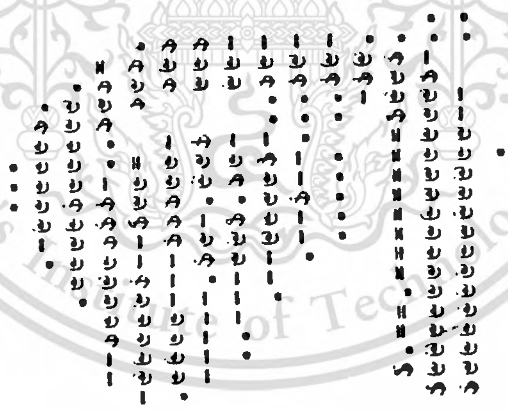
$\lambda_1 = 115.9$

$\lambda_2 = 12.2$

$\lambda_3 = 8.3$

$\lambda_4 = 4.7$

(b)



(c)

Fig. 19 Category no.5

(a) Patterns in a category no.5 (refers to Fig.16(a))

(b) Eigenvalue of category no.5

(c) Eigenvector of maximum eigenvalue of category no.5 which equivalent to a standard pattern of category no.5

Table 5: Result of testing for rough classification

Pattern	Correct %	Reject %	Error %
Learning	100	-	-
Rotation (-5)	94.0	4.8	1.2
Rotation (+5)	94.0	6.0	-

Table 6: Result of testing for classification

Rotation deg.	Correct %	Reject %	Error %
0	100	-	-
-5	98.8	-	1.2
+5	98.8	-	1.2

Karhunen-Loève expansion including rotation pattern

4.4 Conclusions of rough classification

1.) The advantage of rough classification in the matching method is that the recognition time is decreased by matching with only standard pattern, does not match with all pattern in a category.

2.) It can be said that the K-L expansion is suitable to perform a standard pattern of a category in printed Thai character recognition.

3.) From Table 4, it can be summarized that

(i) The similarity value between reduced data is greater than the similarity value between original data.

(ii) If the blurring method is treated with reduced data, and then similarity value between this result is computed. The similarity value increases more.

(iii) It indicates that the determination criterion decision value of clustering characters is easy, and a high recognition rate can be obtained.

(iv) Using blurring method and reduced pattern as a tool for clustering characters is another advantage of recognition method in this thesis.

4.) The result of testing for rough classification is 100% successful of only learning pattern without rotation.

5.) Even though the result of testing of learning pattern with rotation for rough classification is not good enough, if rotated patterns are included in the process of deriving K-L expansion, rough classification is improved. It can be shown that this rough classification method can be used in the practical case.

6.) Training patterns for testing rough classification compose of original pattern (the pattern is derived in section 1.3), +5 degrees rotated pattern (original pattern which is rotated +5 degrees) and -5 degrees rotated pattern (original pattern which is rotated -5 degrees) of each character in Fig. 2.

7.) The result of testing for rough classification is high enough for character recognition as shown in table 6.

8.) FORTRAN programming is used in the implementation of rough classification with FACOM M-360R FUJITSU computer, 4MB.

5. FINE CLASSIFICATION IN MATCHING METHOD

5.1 Fine classification by using subpattern matching

After we have the category of printed Thai characters as shown in Fig. 15(a), the next step in the sequence of character recognition confronted the method of fine classification to discriminate the character pattern in each category.

5.1.1 Partial matching

One method can be done by template matching (partial matching). How can we design the template or features which can be used in the matching process ? The heuristic approach is based on human intuition and experience, making use of the detection of features or explicit pieces such as the number and sequence of particular strokes which are most different among the characters in a category.

The results of design features as shown in Fig.20 are used as standard patterns for matching with the pieces of unknown pattern which are in the same position. Which piece is determined depends on the particular category of which the unknown pattern is a member. The position is measured from the centre of gravity of the pattern, and the dimension of explicit pieces are selected by visual inspection.

The number of explicit pieces in each character are varied with respect to the categories, for example, Fig.20(a) uses only one piece, Fig.20(d) uses two pieces etc. The summary of a design explicit pieces can be devided into mode 1 and mode 2 as

Fig. 20 Pattern for subpattern matching

shown in Fig.21.

5.1.2 Centre of gravity of character pattern

The centre of gravity of pattern of each character is computed by the following equation

$$I_x = \frac{\sum_j \sum_i iF(i,j)}{\sum_j \sum_i F(i,j)} \qquad (20)$$

$$I_y = \frac{\sum_i \sum_j jF(i,j)}{\sum_i \sum_j F(i,j)} \qquad (21)$$

where F(i,j) is the pattern of the character and I_x and I_y are the coordinates of the centre of gravity.

5.1.3 The procedure of partial matching

The procedure of subpattern matching (partial matching) is done by the calculation of similarity between the explicit pieces of Fig. 20 and the explicit pieces of unknown pattern (by using the equation(1)), and then set the clustering criterion based on the equation (4).

5.1.4 Conclusion of partial matching

To get the best fine classification, in some categories we tried several times to change the position, dimension and the

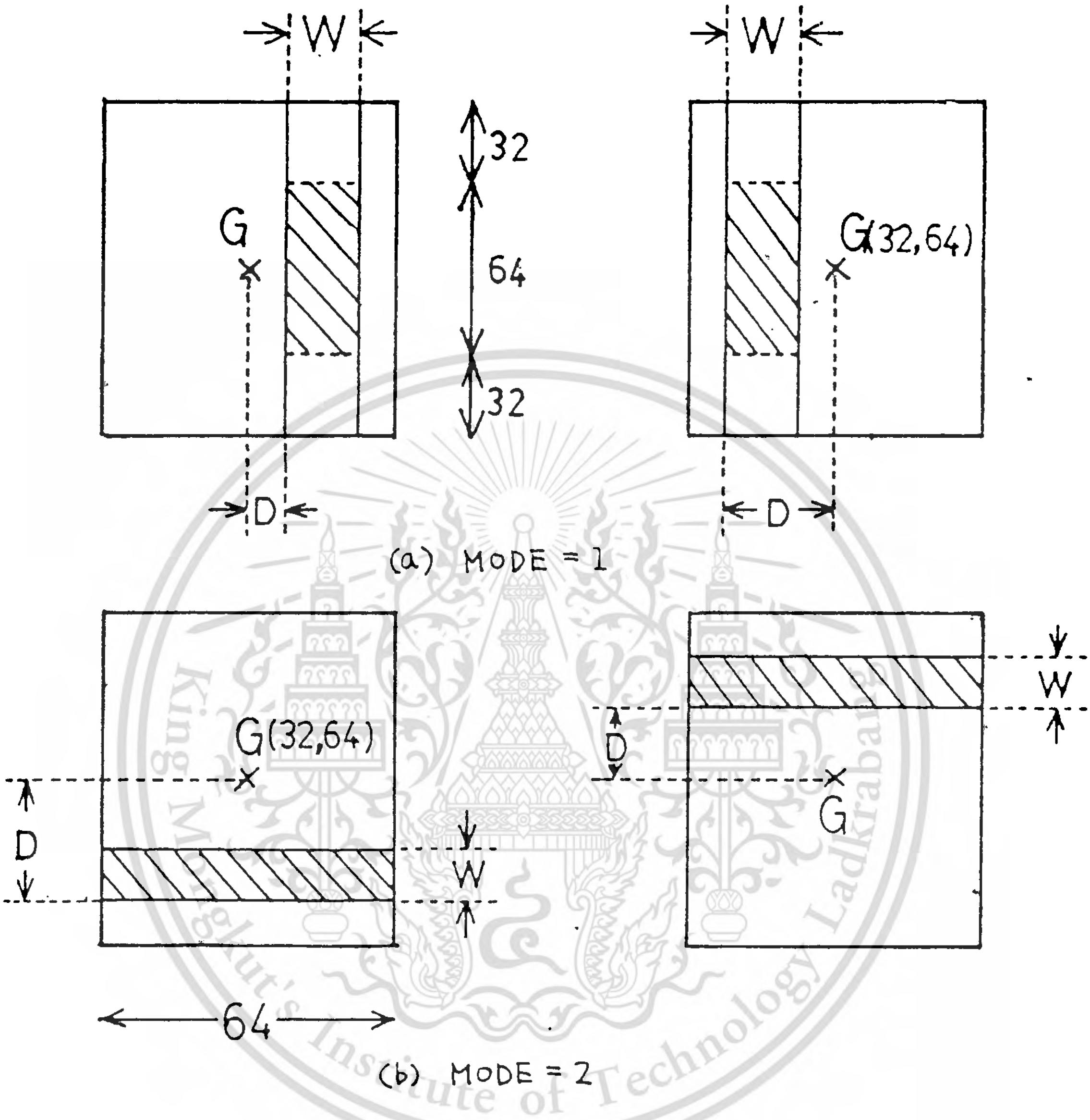


Fig. 21 Explicit pieces of subpattern matching

number of explicit pieces. The results of fine classification are shown in Table 7, the percentage of recognition correctness of learning pattern and of the rotation by +5 is 100%, only that of the rotation by -5 is less than 100%. The training patterns for testing fine classification are the same as used in rough classification. Even though fine classification by partial matching gives the results 100% but an ambiguity in the process of selecting explicit peices can occur.

If we can reduce the dimension of explicit pieces, and can reduce number of explicit pieces in the category, more saving of storage area in recognition dictionary and more reducing the recognition time are obtained.

5.2 Fine classification by using Karhunen-Loève expansion

In the section 5.1, the fine classification stage, there is an ambiguity in the process of selecting the optimum explicit pieces of subpattern matching. The higher order eigenvectors apart from the maximum one which have been computed in the rough classification stage are not used.

In addition, it has been known that the K-L expansion is the method of optimum feature selection from a character set in which patterns are distributed as a normal distribution (7) . Therefore in this section, the eigenvectors in higher order of the K-L expansion are used in the fine classification stage. Even though the distributions of Thai characters are not known a priori on the original pattern space, the real distributions of the pattern are known by plotting each pattern on the eigenvector spaces derived from the K-L expansion having lower dimension than the

Table 7 Result of fine classification by subpattern matching

Pattern	Correct %	Reject %	Error %
Learning	100	-	-
Rotation (-5°)	98.2	1.2	-
Rotation (+5°)	100	-	-

original pattern space.

In our experiment, the scores (the inner product between a character pattern and an eigenvector of a category) are plotted on eigenvector space of the K-L expansion. By investigation of the score plot of pattern, the linear decision function for discrimination the pattern classes can be constructed. By using this process, the fine classification would be done successfully.

5.2.1 Karhunen-Loève expansion with score plot on eigenvector space

The application of discrete K-L expansion to feature selection is based on the optimal results where the pattern classes under consideration are normally distributed. However, if this assumption is not valid, only suboptimum results may be expected from the K-L expansion (7). We will make use of the K-L expansion in carrying out fine classification.

The covariance matrix in the K-L expansion as shown in equation (5) , it is in the case of the expected value $E(\mathbb{F}) \neq 0$. So, we can define the covariance matrix in the case of the expected value $E(\mathbb{F}) = 0$ by the following equation

$$\mathbb{V} = \left(\sum_{i=1}^{m} (\mathbb{F}_i - \bar{\mathbb{F}}_i)(\mathbb{F}_i - \bar{\mathbb{F}}_i)^T\right) / m \qquad (22)$$

where m is the number of patterns. The eigenvalue (λ) and eigenvector ($\wp$) of this matrix is also determined from the equation (6).

Score plot of the character patterns on the eigenvector space of the K-L expansion will be used to know the distribution of character patterns in a category. The score of character patterns can be defined by the equation

$$x_{ij} = \mathbb{F}_i \, \varphi_j \tag{23}$$

In our experiment, to choose good results of fine classification process, "the score plot" has been done in four cases. In addition, the artificial patterns are added into the experimental character set of a category for all four cases. The artificial patterns are obtained by 1-bit shifting (shift right, shift left, shift up, and shift down, respectively), by rotating -2 degrees, and by rotating +2 degrees followed by shifting 1-bit of the original patterns, respectively. The four cases of K-L expansion for fine classification can be described as follows.

The first case, covariance matrix is computed directly by using equation (5). Eigenvectors are computed by using equation (6) , and the scores are computed by using equation (23), respectively.

The second case, covariance matrix is computed by using the equation

$$\mathbb{V} = \sum_{k=1}^{m} (\tilde{\mathbb{F}}_k \, \tilde{\mathbb{F}}_k^{T}) \;/m \tag{24}$$

$\tilde{\mathbb{F}}_k$ is the mean of $\mathbb{F}_i$ of the artificial patterns together with the original patterns. The number of patterns for the calculation of the matrix in the category of this case is different from the first case, that is, the number of patterns in this case is equal to the number of characters in a category, but the number of patterns in first case equal to the number of characters in a category including artificial patterns.

The third case, covariance matrix is computed directly by using equation (22).

The fourth case, covariance matrix is computed by using equation

$$\mathbb{V} = \Big(\sum_{i=1}^{m} (\tilde{\mathbb{F}}_i - \tilde{\tilde{\mathbb{F}}}_i)(\tilde{\mathbb{F}}_i - \tilde{\tilde{\mathbb{F}}}_i)^T \Big) / m \qquad (25)$$

where $\tilde{\tilde{\mathbb{F}}}_i$ is the mean of $\tilde{\mathbb{F}}_i$ in a category. The eigenvectors are computed by using equation (6) , and the scores are computed by using equation (23).

In equation (24), the expected value $E(\tilde{\mathbb{F}}_k) \neq 0$, but in equation (25) the expected value $E(\tilde{\mathbb{F}}_k) = 0$.

The expansion of all four cases can be illustrated by the Fig. 22.

The results of "the score plot" in the first case are shown in Fig. 23(a) through (d), in the second case is shown in Fig.24(a) through (d). The typical result of score plot in the third case is shown in Fig.25, and the fourth case is shown in Fig.26.

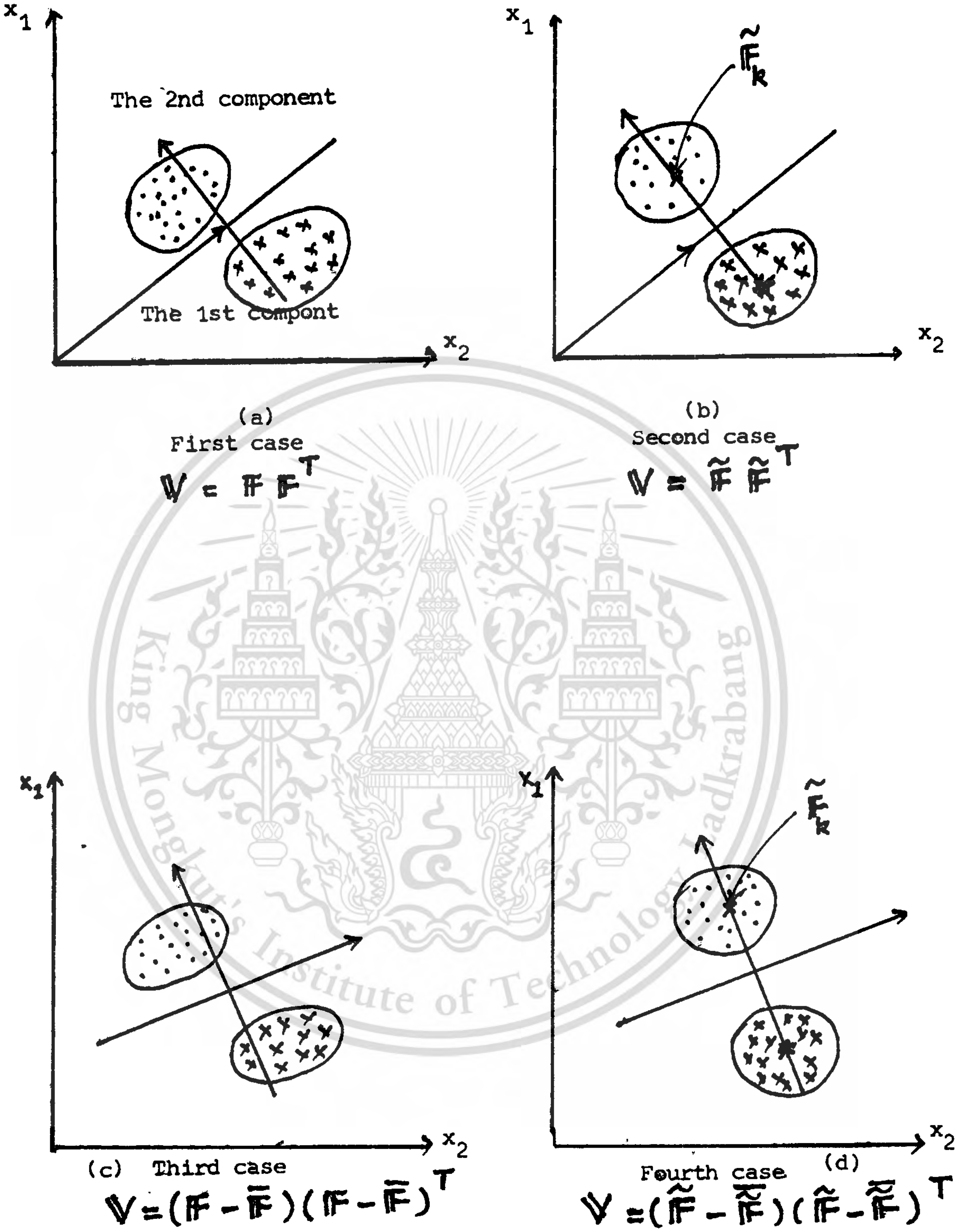


Fig. 22 Expansion of scores of all four case

Symbols in the figure 23-30

●	:	score of original pattern
▸	:	score of original pattern shifted right by 1 bit
◂	:	score of original pattern shifted left by 1 bit
▴	:	score of original pattern shifted up by 1 bit
▾	:	score of original pattern shifted down by 1 bit
□	:	score of orignal pattern rotated by + 2^{o}
■	:	score of orignal pattern rotated by - 2^{o}
▷	:	score of original pattern shifted right by 1 bit and then rotated by + 2^{o}.
◁	:	score of original pattern shifted left by 1 bit and then rotated by + 2^{o}.
△	:	score of original pattern shifted Up by 1 bit and then rotated by + 2^{o}.
▽	:	score of original pattern shifted down by 1 bit and then rotated by + 2^{o}.
▶	:	score of original pattern shifted right by 1 bit and then rotated by - 2^{o}.
◀	:	score of original pattern shifted left by 1 bit and then rotated by - 2^{o}.
▲	:	score of original pattern shifted up by 1 bit and then rotated by - 2^{o}.
▼	:	score of original pattern shifted down by 1 bit and then rotated by - 2^{o}.

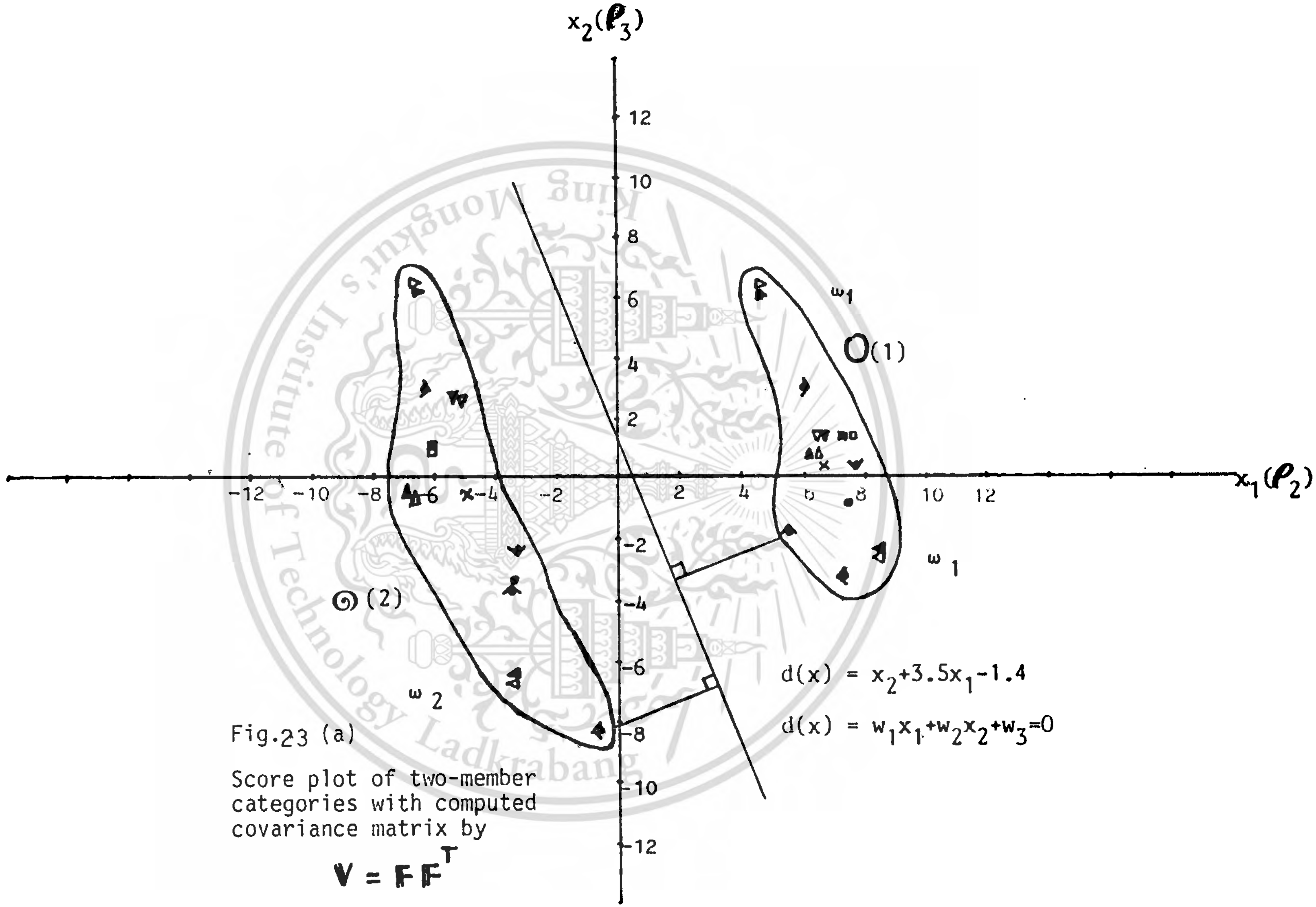


Fig.23 (a)

Score plot of two-member categories with computed covariance matrix by

$$\mathbf{V} = \mathbf{F}\mathbf{F}^T$$

First case

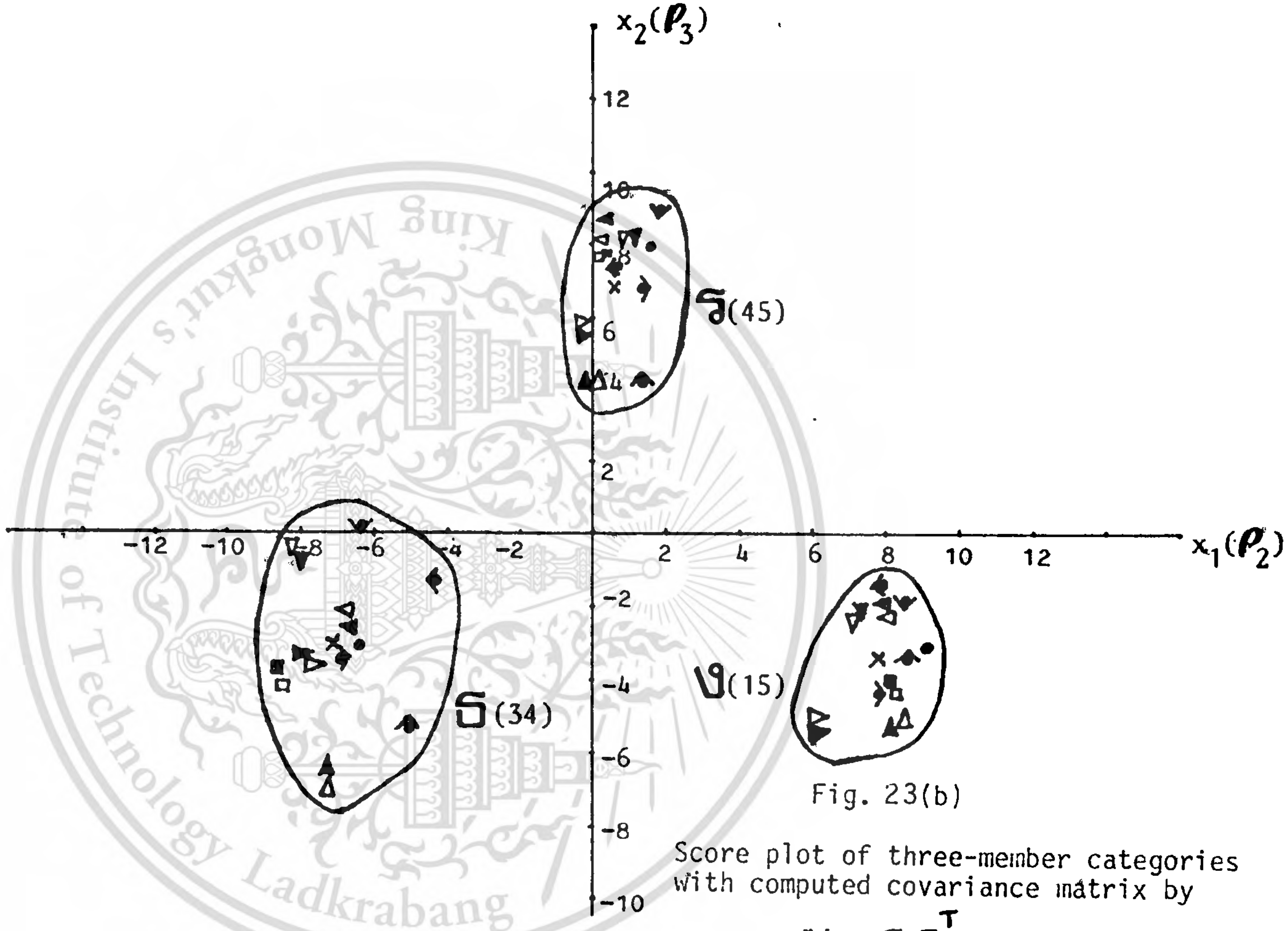


Fig. 23(b)

Score plot of three-member categories with computed covariance matrix by

$$\mathbb{V} = \mathbb{F}\mathbb{F}^T$$

First case

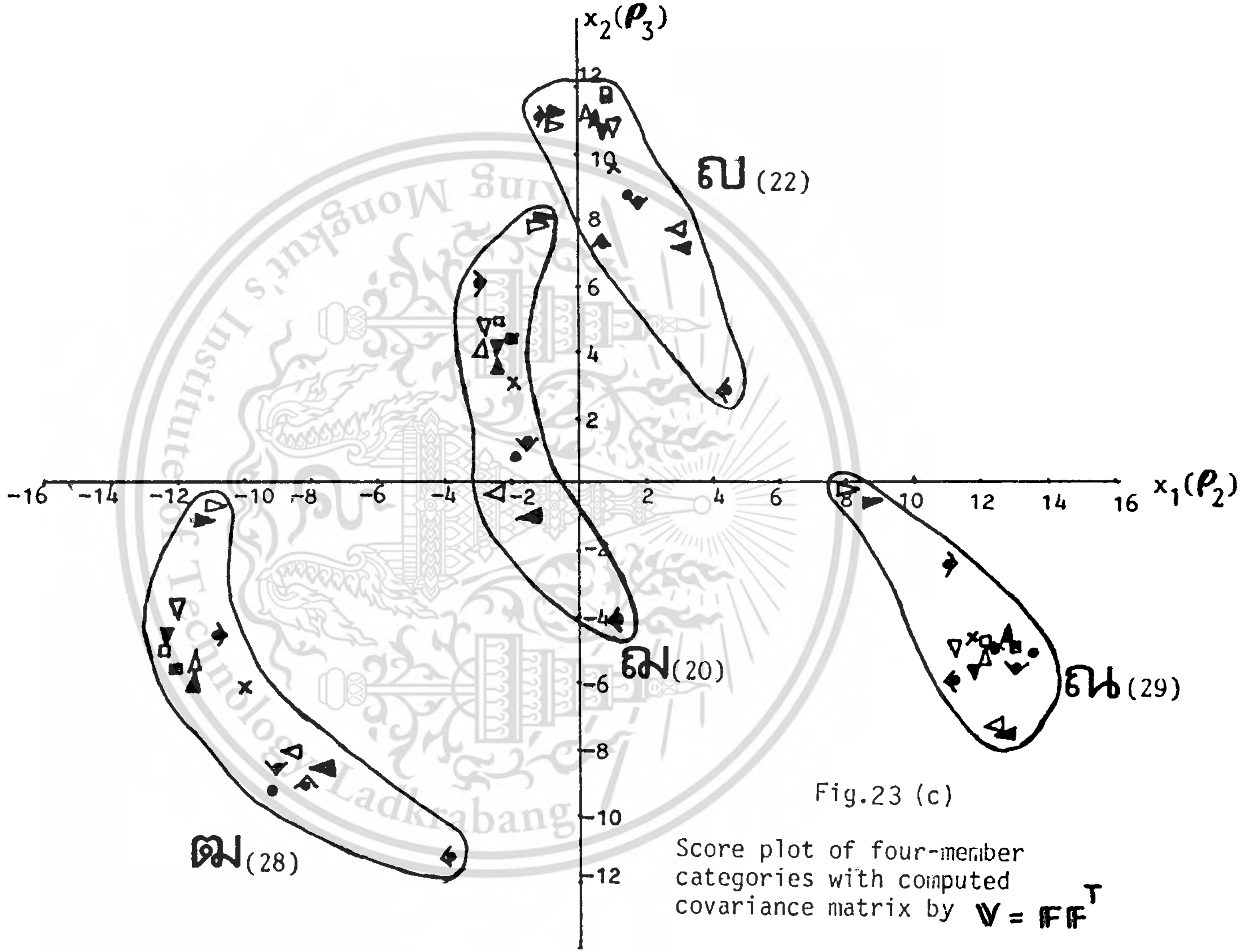


Fig.23 (c)

Score plot of four-member categories with computed covariance matrix by $\mathbb{V} = \mathbb{F}\mathbb{F}^T$

First case

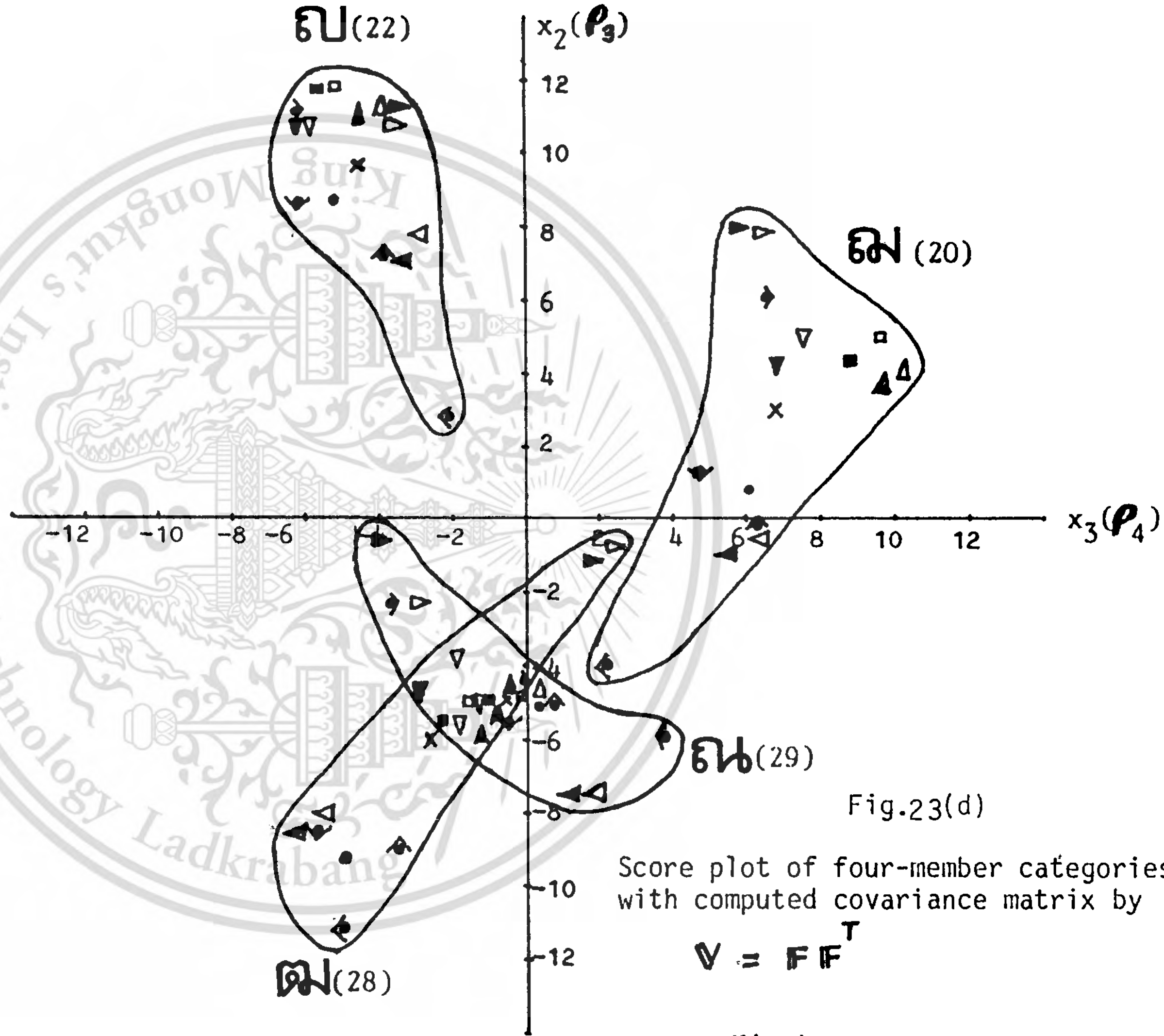


Fig.23(d)

Score plot of four-member categories with computed covariance matrix by

$$\mathbb{V} = \mathbb{F}\mathbb{F}^T$$

First case

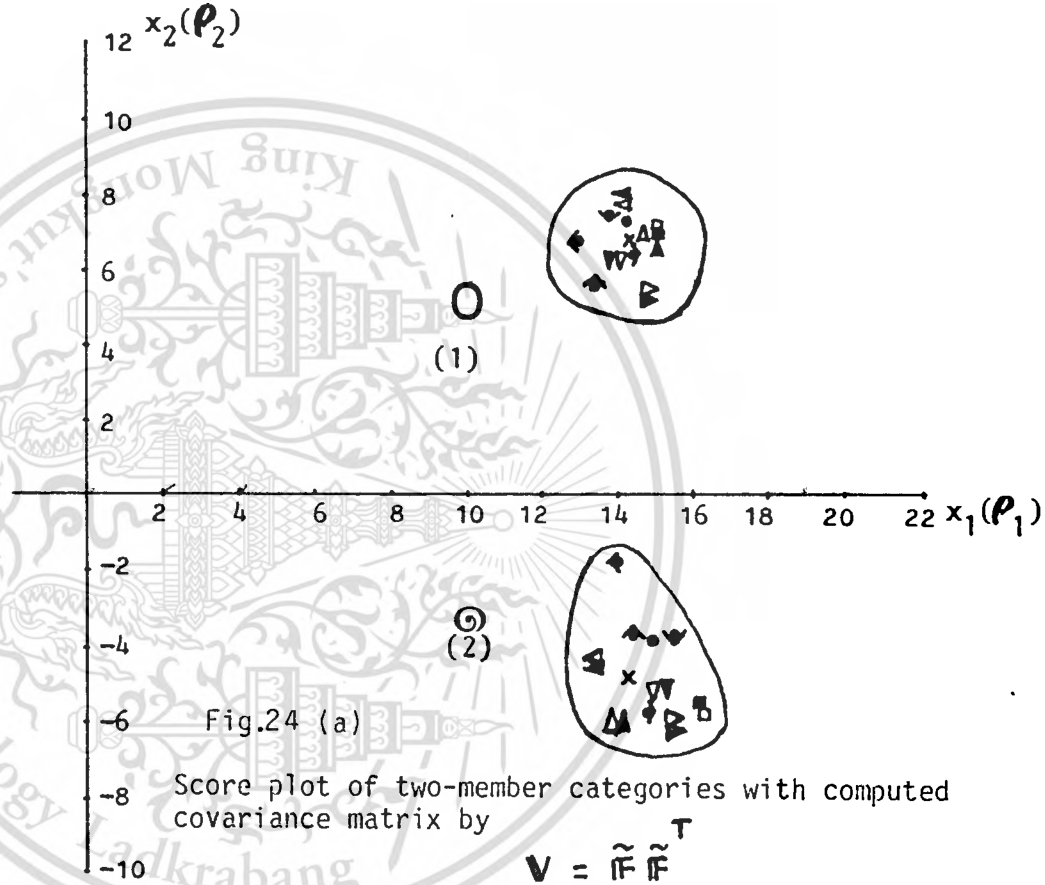


Fig.24 (a)

Score plot of two-member categories with computed covariance matrix by

$$\mathbb{V} = \tilde{\mathbb{F}}\tilde{\mathbb{F}}^T$$

Second case

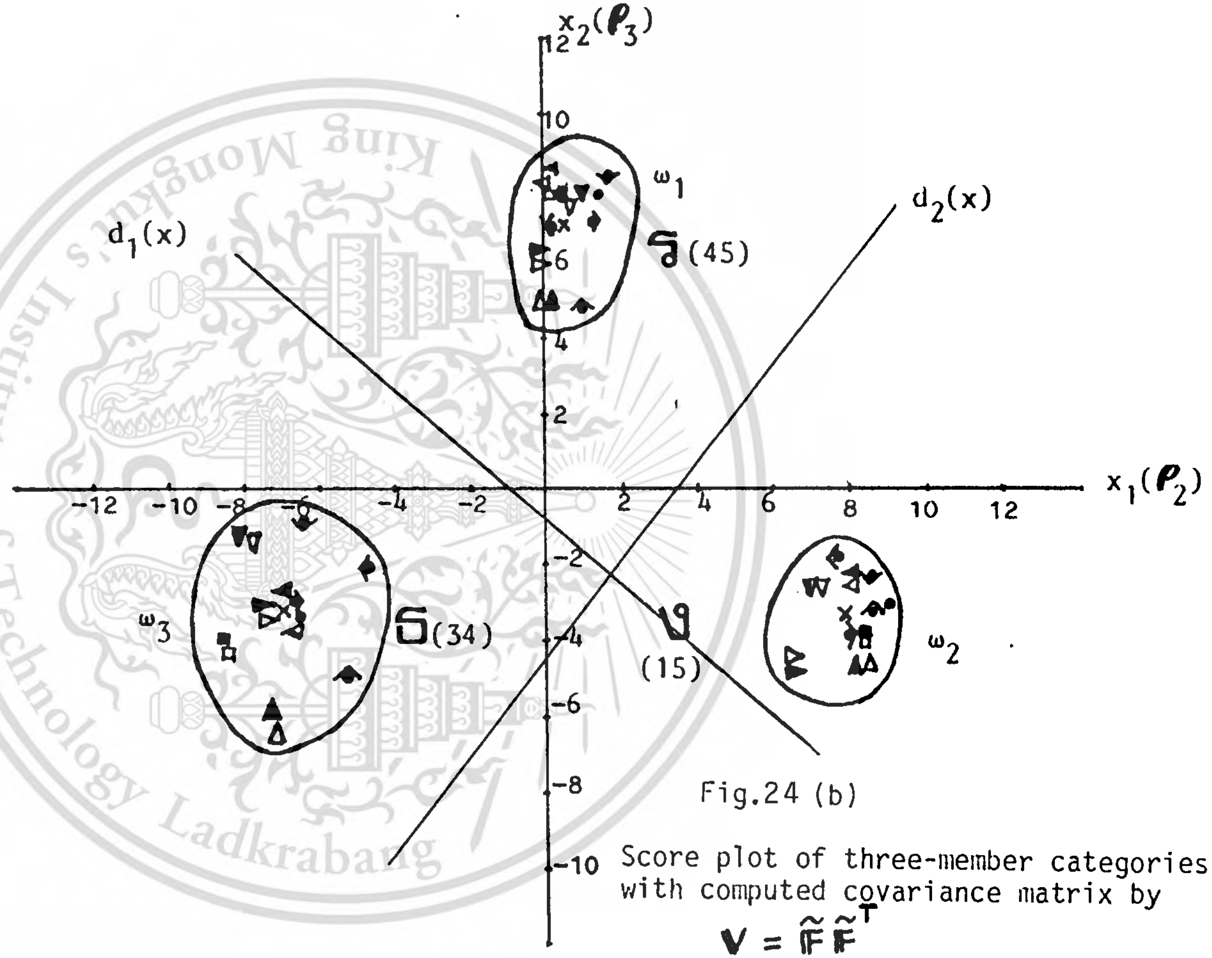


Fig.24 (b)

Score plot of three-member categories with computed covariance matrix by

$$\mathbf{V} = \tilde{\mathbb{F}}\tilde{\mathbb{F}}^T$$

Second case

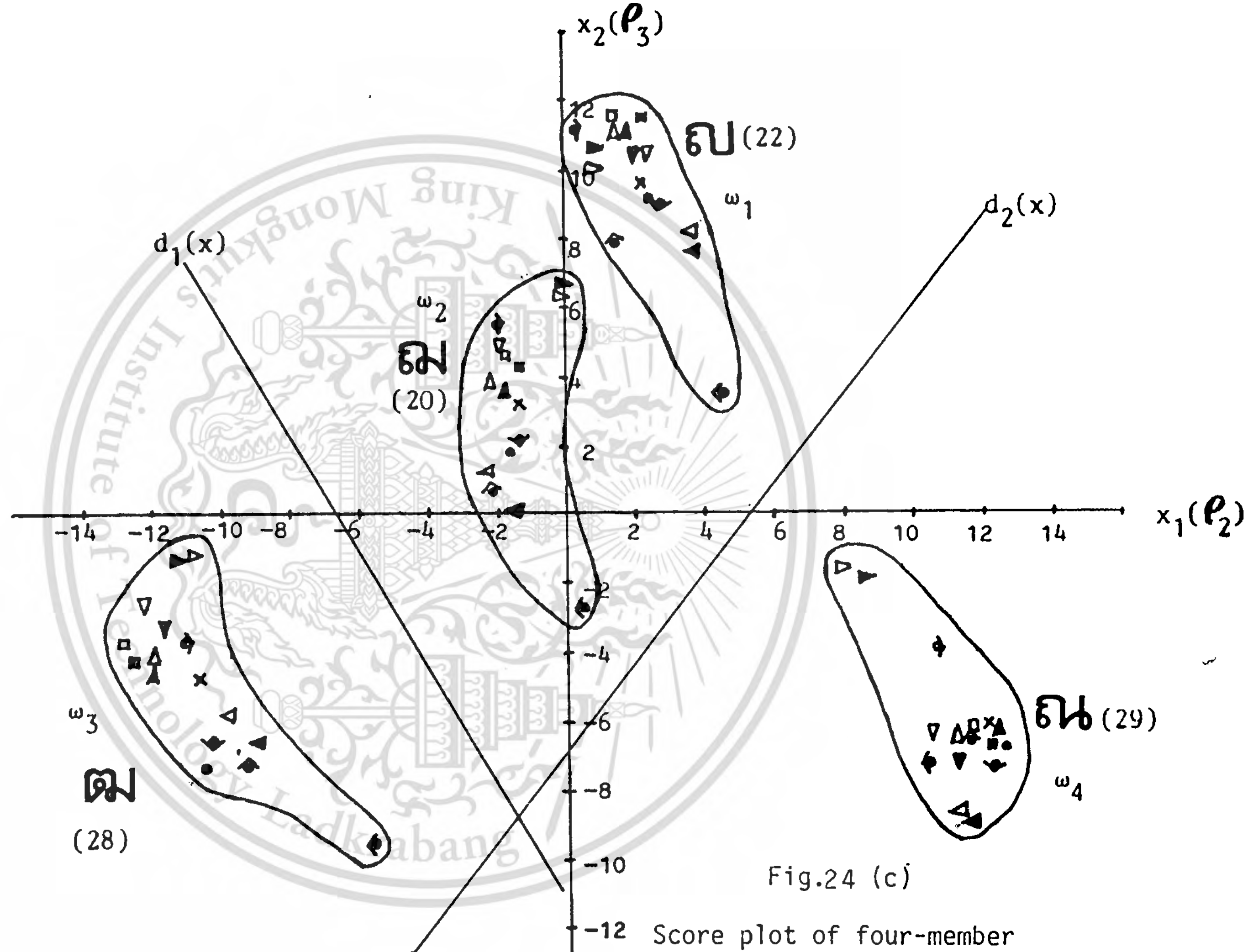


Fig.24 (c)

Score plot of four-member categories with computed covariance matrix by $\mathbb{V} = \tilde{\mathbb{F}}\tilde{\mathbb{F}}^T$
Second case

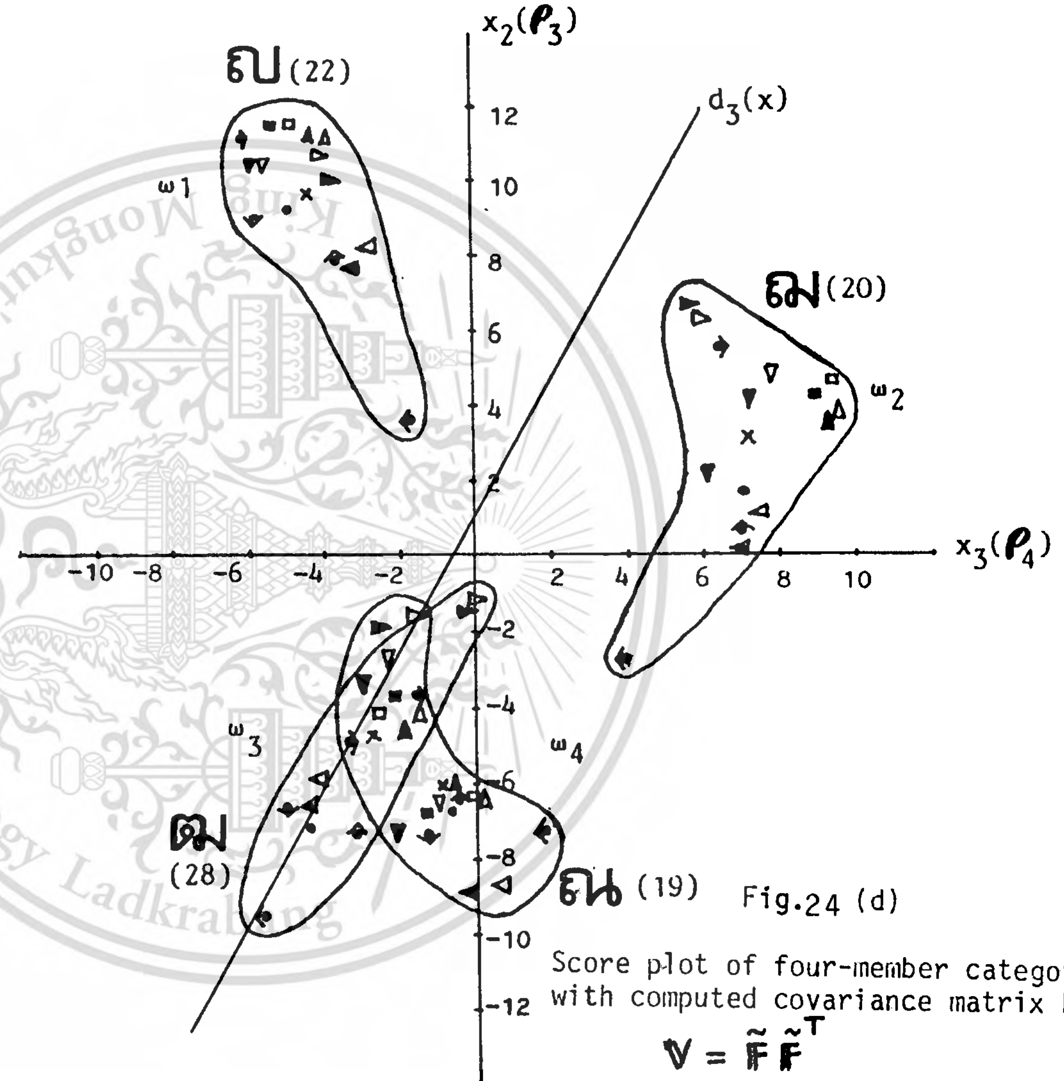


Fig.24 (d)

Score plot of four-member categories with computed covariance matrix by

$$\mathbb{V} = \tilde{\mathbb{F}}\tilde{\mathbb{F}}^T$$

Second case

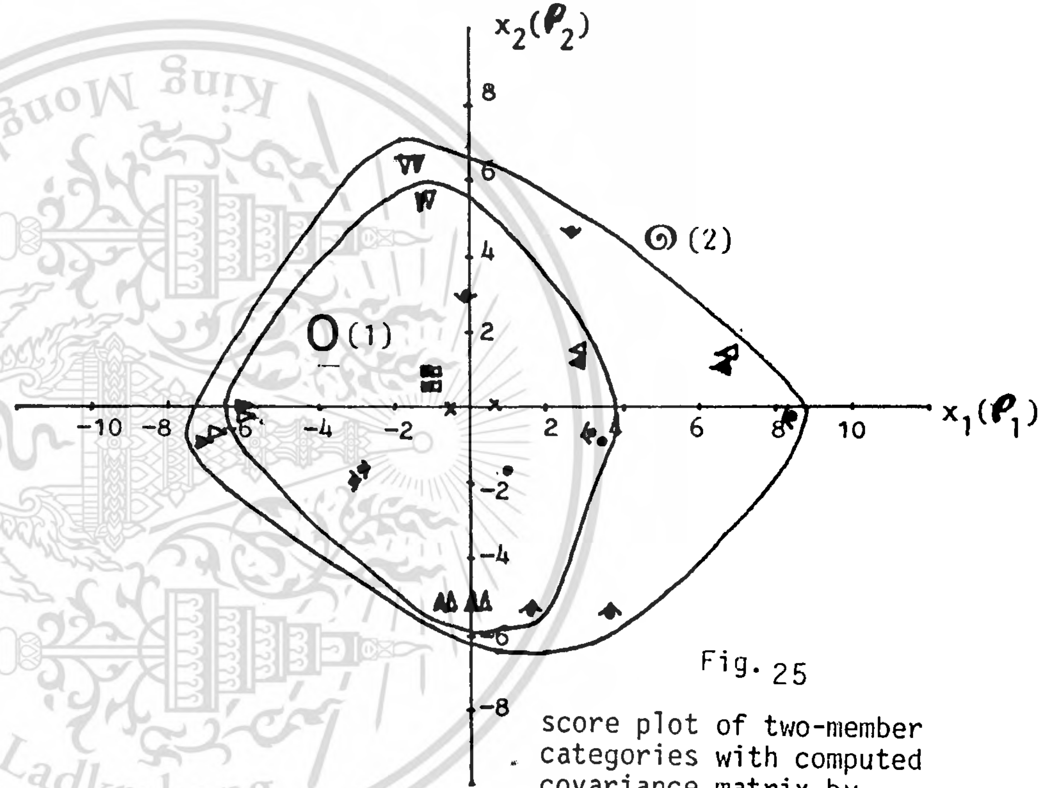


Fig. 25

score plot of two-member categories with computed covariance matrix by

$$\mathbb{V} = (\mathbb{F} - \bar{\mathbb{F}})(\mathbb{F} - \bar{\mathbb{F}})^T$$

Third case

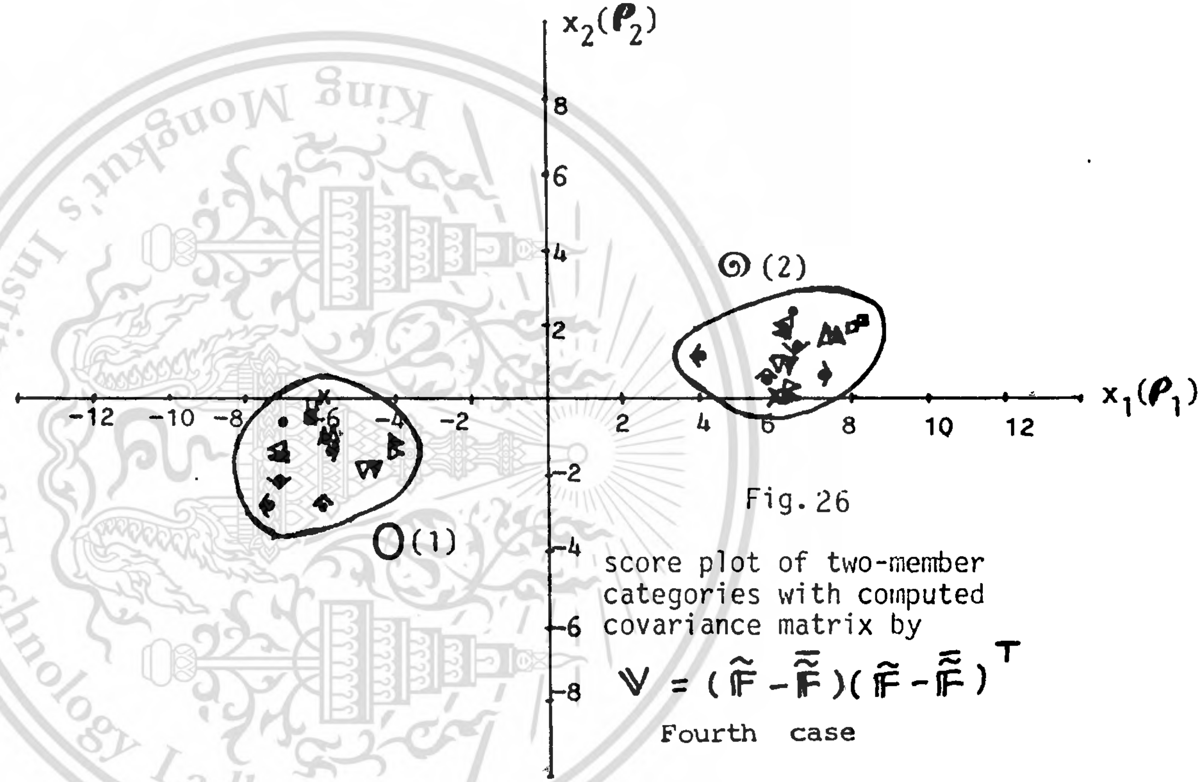


Fig. 26

score plot of two-member categories with computed covariance matrix by

$$\mathbb{V} = (\tilde{\mathbb{F}} - \tilde{\bar{\mathbb{F}}})(\tilde{\mathbb{F}} - \tilde{\bar{\mathbb{F}}})^T$$

Fourth case

5.2.2 Linear decision function on eigenvector space

On the fine classification stage, in order to discriminate the character in a category, it is necessary to establish some rules. One important approach to this problem is the use of linear decision function. The case of a category which has two classes, is illustrated in Fig.23(a) where two patterns classes are shown. It is seen in the figure that the population of pattern can be conveniently separated by a line.

Let $d(x) = w_1 x_1 + w_2 x_2 + w_3 = 0$ is the equation of a separating line on the eigenvector space in which x_1 (represents ρ_2), x_2 (represents ρ_3) are the coordinate variables. It is clear from the figure that any pattern belonging to ω_1 class will yield a positive quantity when the pattern x is substituted into d(x). The d(x) can be used as a decision (or discrimination) function, we may say that x belongs to ω_1 if $d(x) > 0$, or to ω_2 if $d(x) < 0$. The procedure to perform the decision function d(x) is done by drawing the straight line between classes.

The perpendicular distance from this straight line to the nearest two points which sit on the different side of the line must be equal. The coefficients of linear decision function is performed directly from the score plot by measuring under the definition of straight line. An example of this result is shown in Fig.23(a).

In the case of a category which has three members denoted by $\omega_1, \omega_2, \omega_3$, each pattern class is separable by some decision lines. To determine W's and $d_i(x)$, we can use the formula of straight line as in a category which has two members. A typical result is shown in Fig.24(b).

In the case of a category which has four members, denoted by $\omega_1, \omega_2, \omega_3$ and ω_4, we can consider in multiclass of 3-dimension. Some characters can be separated on the plane of ρ_2, ρ_3, but other character can be separated on the plane of ρ , ρ , as shown in Fig.24(c) and Fig.24(d).

5.2.3 Results of experiment and analysis of fine classification by using K-L expansion

If the results of "the score plot" on eigenvector space are clearly distributed, the decision function can be constructed by drawing the straight line, so that the decision condition can be given definitely. The discrimination results of character from a category will be successful.

The problem of discrimination in a category will be focused on the distribution of score on eigenvector space. The results of "the score plot" as described in section 5.2.1 are analysed as follows;

(1) : The distribution of scores of the shifted patterns in the first case of experiment is not good enough, and the extent of distribution depends on the shape of acharacter. If the shape of characters in a category is very close to one another, the distribution of score of shifted patterns is closed together. If the large number of learning patterns are used, the distribution of scores would be more improved but computation of eigenvectors will be difficult to execute practically because of the memory limitation in the computer.

To eliminate the shifting effect, the centre of gravity of pattern has been set into the center of the frame. However, in the digitalization process, 1 bit shifted pattern may occur. In this case, some categories are not well separated as shown in Fig. 23(c) and (d). The scores distribution of pattern class of character ฌ is closed to ญ in Fig. 23(c) (on the plane ρ_2 , ρ_3), but it is closed to ณ and ฆ in Fig. 23(d) (on the plane ρ_4 ρ_3).

(2) : The distribution of scores of the second case in experiments were more condensed than the distribution of scores of the first case. In the second case, noises from the shifted pattern are decreased, because eigenvector are derived from the mean patterns of each class which will be able to absorb the noise caused by deformation. Then, spreading of distribution in score plot is suppressed as shown in Fig. 24(a) through (d). The distribution of scores in this case is suitable for discrimination the characters which have nearly same shape.

(3) : The distribution of scores of the third case are very close together. It is very difficult to determine the discrimination function as shown in Fig.25.

(4) : The distribution of scores of the fourth case is nearly the same as of the second case. Also, in this case the distribution of patterns on the eigenvector space is clustered as well as the second case for each category (see Fig.26).

In all four cases, when the distribution of scores on the lower order eigenvector space is not clearly separable, the distribution of scores on the higher order eigenvector space is also not separable. In the experiment, the distribution of scores on eigenvector space not more than the fourth order is used to determine the discrimination function.

On analysis of the results of score distribution as described above, it is found that the linear decision functions are conveniently constructed on eigenvector space derived from the first case for categories which have two members. The examples are shown in Table 8.

The linear decision function are conveniently constructed on eigenvector space derived from the second case for categories which have three and four members. The examples are shown in Table 9 and Table 10, respectively.

When the characters in a category show nearly the same shape as shown in Fig. 27, it is not easy to determine the linear decision functions for all four cases. Then, we tried to use subpattern for K-L expansion. It is seen from Fig. 27 that only the upper part of these characters is different. The score plot of all four cases of this category are shown in Fig. 28(a), (b), (c) and (d), respectively. From the score distribution of this category as shown in Fig. 28, it is clear that the linear decision function can not separate the character No.59 (ไ) from character No.60 (ใ).

Fig. 29 shows the score plot of these two characters on eigenvector space which newly derived from the 1'st case by using characters of classes ไ and ใ. The linear decision function does not separate these two characters also. If only the upper part of these two characters are used for expanding by K-L expansion and for plotting the scores as shown in Fig. 30, the linear decision function easily separates these two characters.

Table 8 The result of linear decision function

Category NO.	Decision function	Condition	member
1	$d(x) = x_2 + 2.86x_1 - 0.4$	$d(x) > 0$ $d(x) < 0$	0 ๑
3	$d(x) = x_2 - 9.0x_1 + 12.0$	$d(x) > 0$ $d(x) < 0$	ถ ก
6	$d(x) = x_2 - 3.9x_1 - 0.3$	$d(x) > 0$ $d(x) < 0$	ฆ ท

Table 9 The result of linear decision function

Category NO.	Decision function	condition	member
4	$d_1(x) = x_2 + 0.3x_1 + 1.6$ $d_2(x) = x_2 + 5.0x_1 - 13.1$	$d_1(x) < 0$	ร
		$d_1(x) > 0$ and $d_2(x) > 0$	ธ
		$d_1(x) > 0$ and $d_2(x) < 0$	ธ

Table 10 The result of linear decision function

Category No.	Decision function	condition	member
2	$d_1(x) = x_2 + 2.0x_1 - 0.8$	$d(x) > 0$ and $d_2(x) > 0$	[illegible]
	$d_2(x) = x_2 - 0.6x_1 + 1.5$	$d_1(x) > 0$ and $d_2(x) < 0$	[illegible]
	$d_3(x) = x_2 - 1.55x_3 + 4.5$	$d_2(x) < 0$ and $d_3(x) < 0$	[illegible]
		$d_2(x) < 0$ and $d_3(x) > 0$	[illegible]

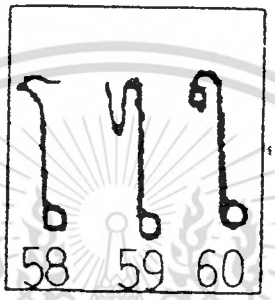


Fig. 27

The shape of characters in category No. 19.

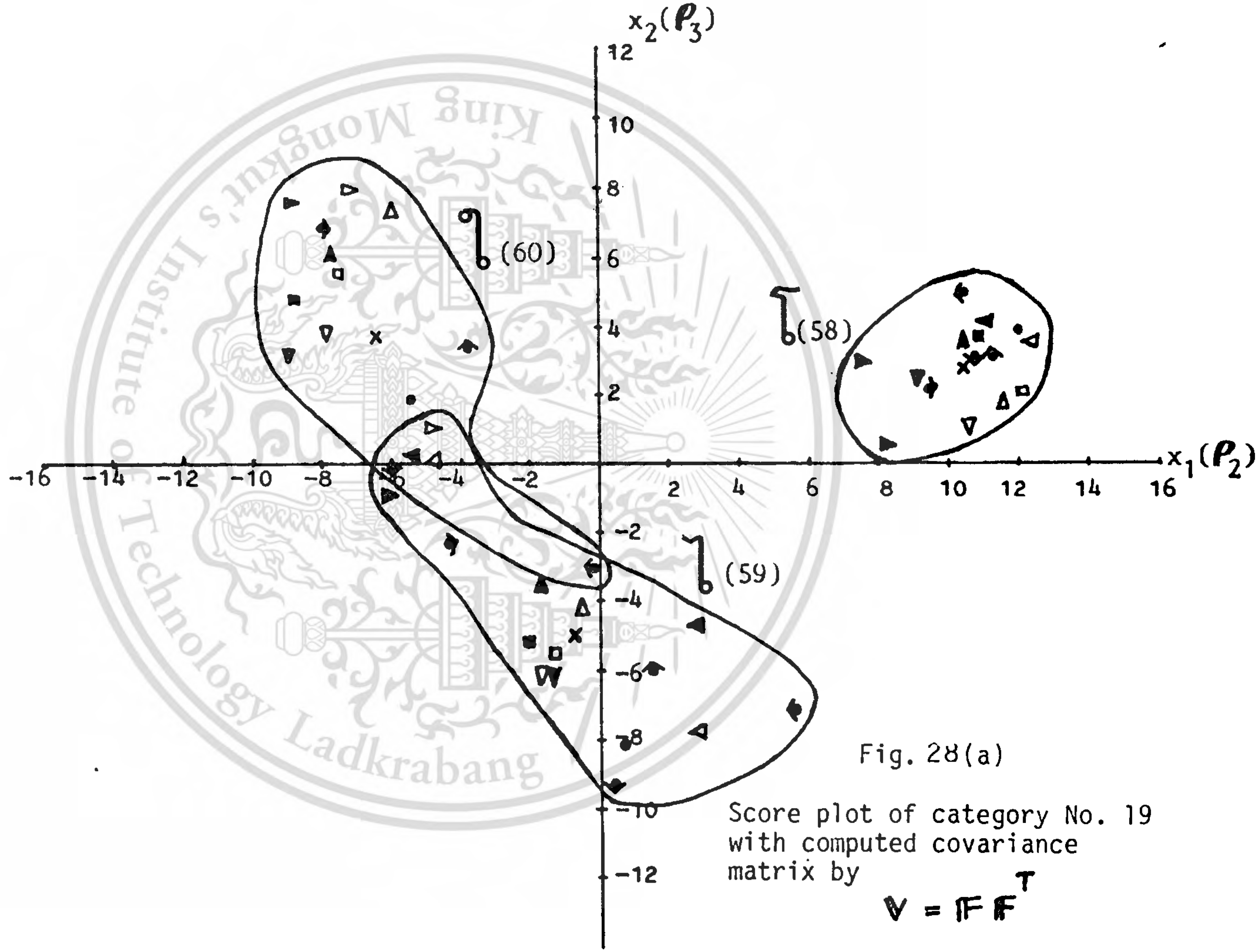


Fig. 28(a)

Score plot of category No. 19 with computed covariance matrix by

$$\mathbb{V} = \mathbb{F}\mathbb{F}^{T}$$

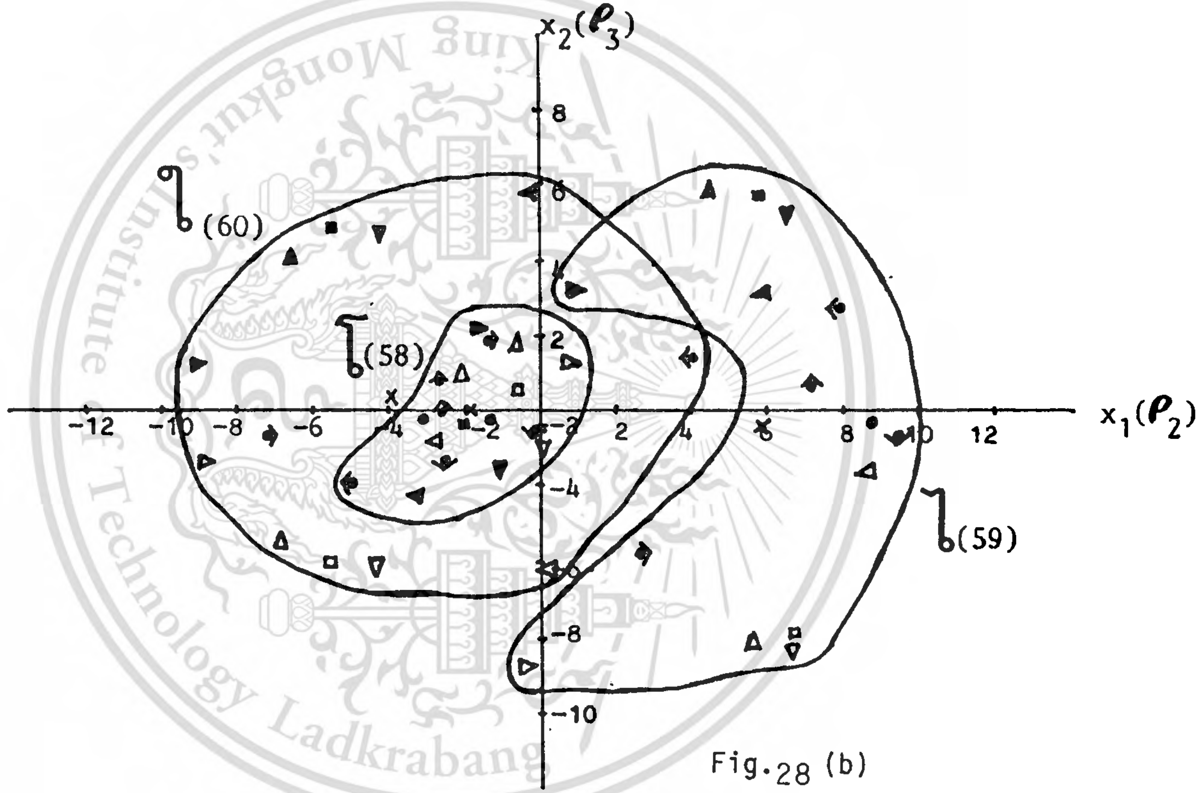


Fig. 28 (b)

Score plot of category No. 19 with computed covariance matrix by

$$\mathbb{V} = (\mathbb{F} - \bar{\mathbb{F}})(\mathbb{F} - \bar{\mathbb{F}})^T$$

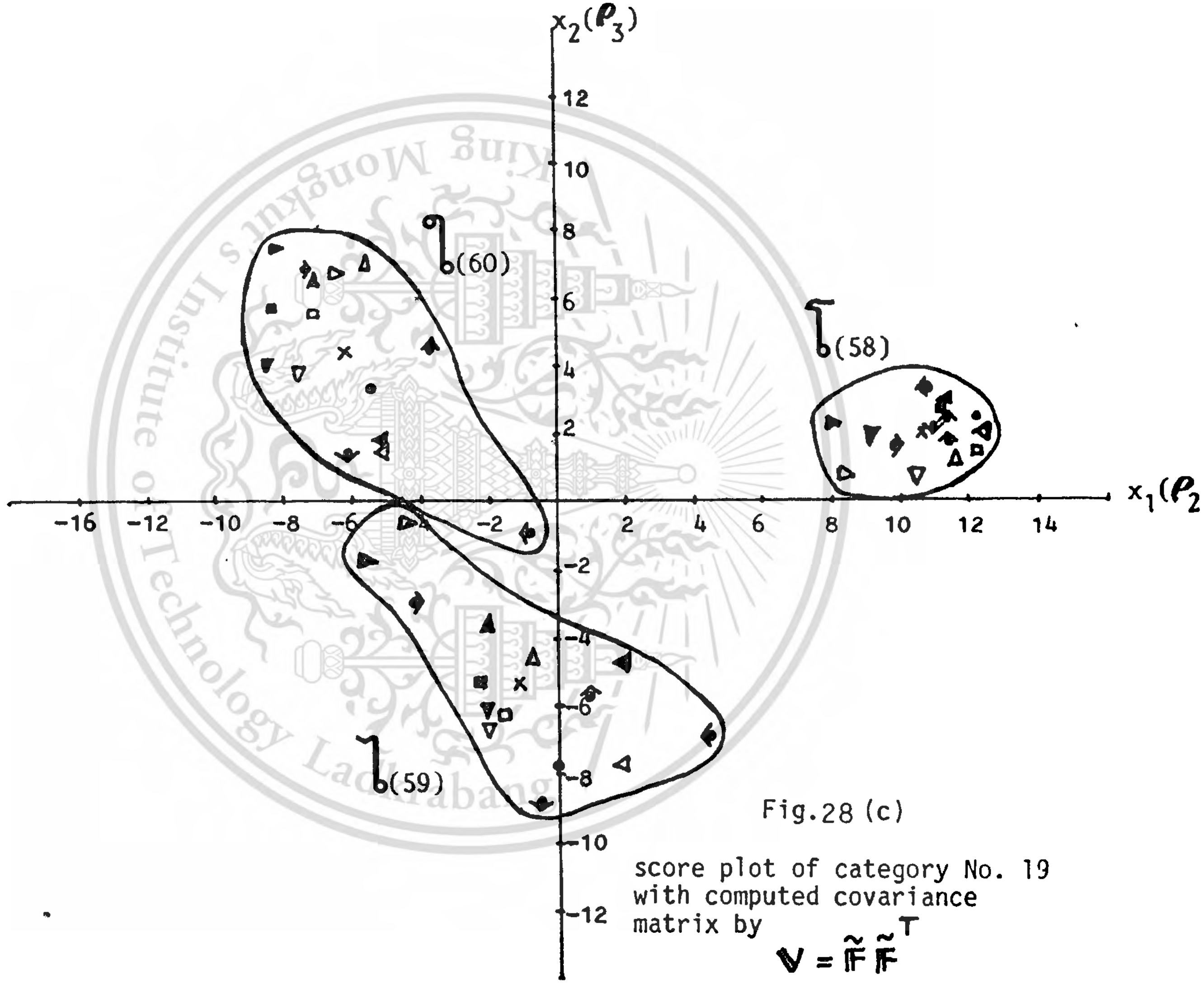


Fig.28 (c)

score plot of category No. 19
with computed covariance
matrix by

$$\mathbb{V} = \tilde{\mathbb{F}}\tilde{\mathbb{F}}^T$$

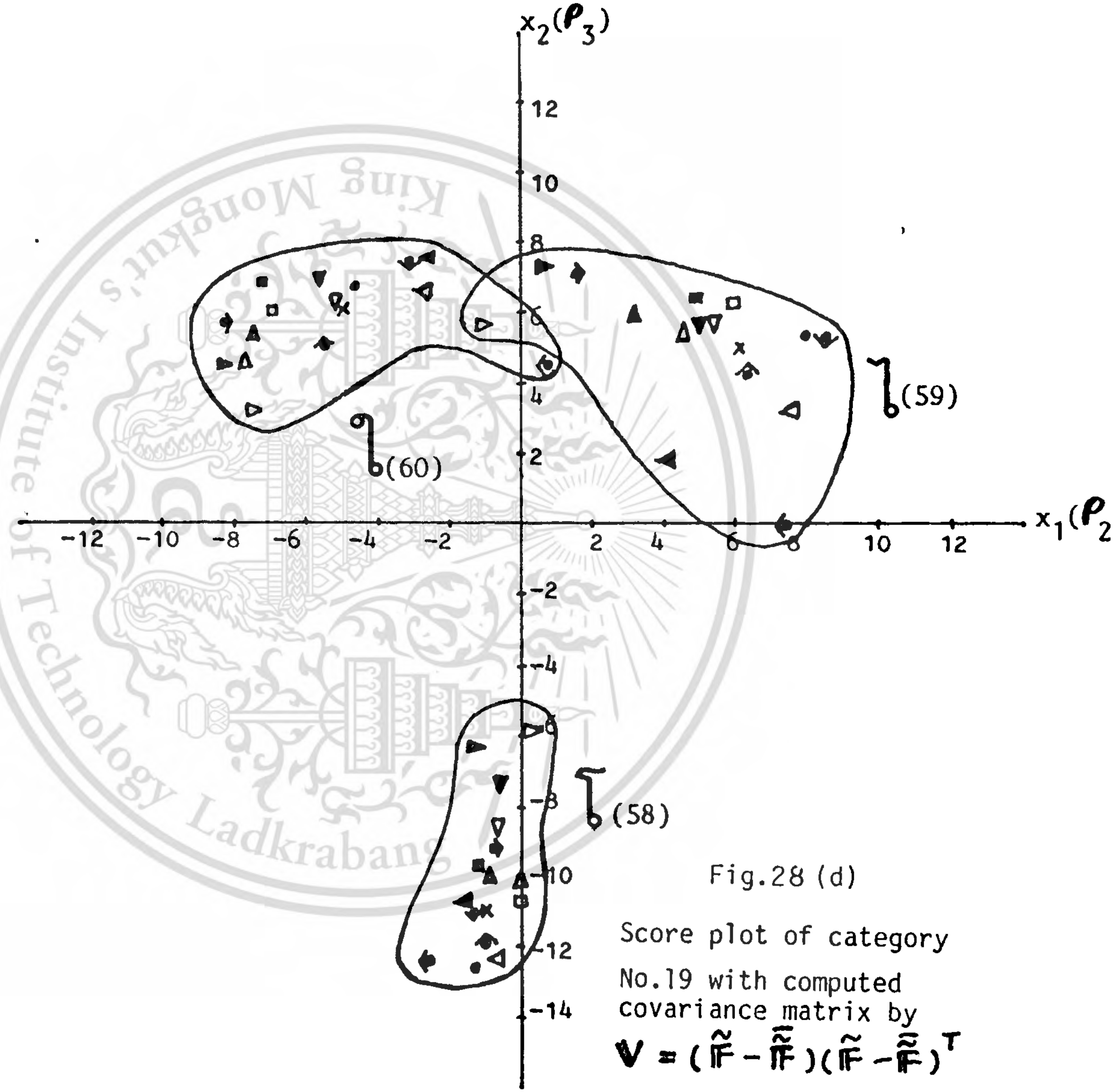


Fig.28 (d)

Score plot of category No.19 with computed covariance matrix by

$$\mathbb{V} = (\tilde{\mathbb{F}} - \bar{\tilde{\mathbb{F}}})(\tilde{\mathbb{F}} - \bar{\tilde{\mathbb{F}}})^T$$

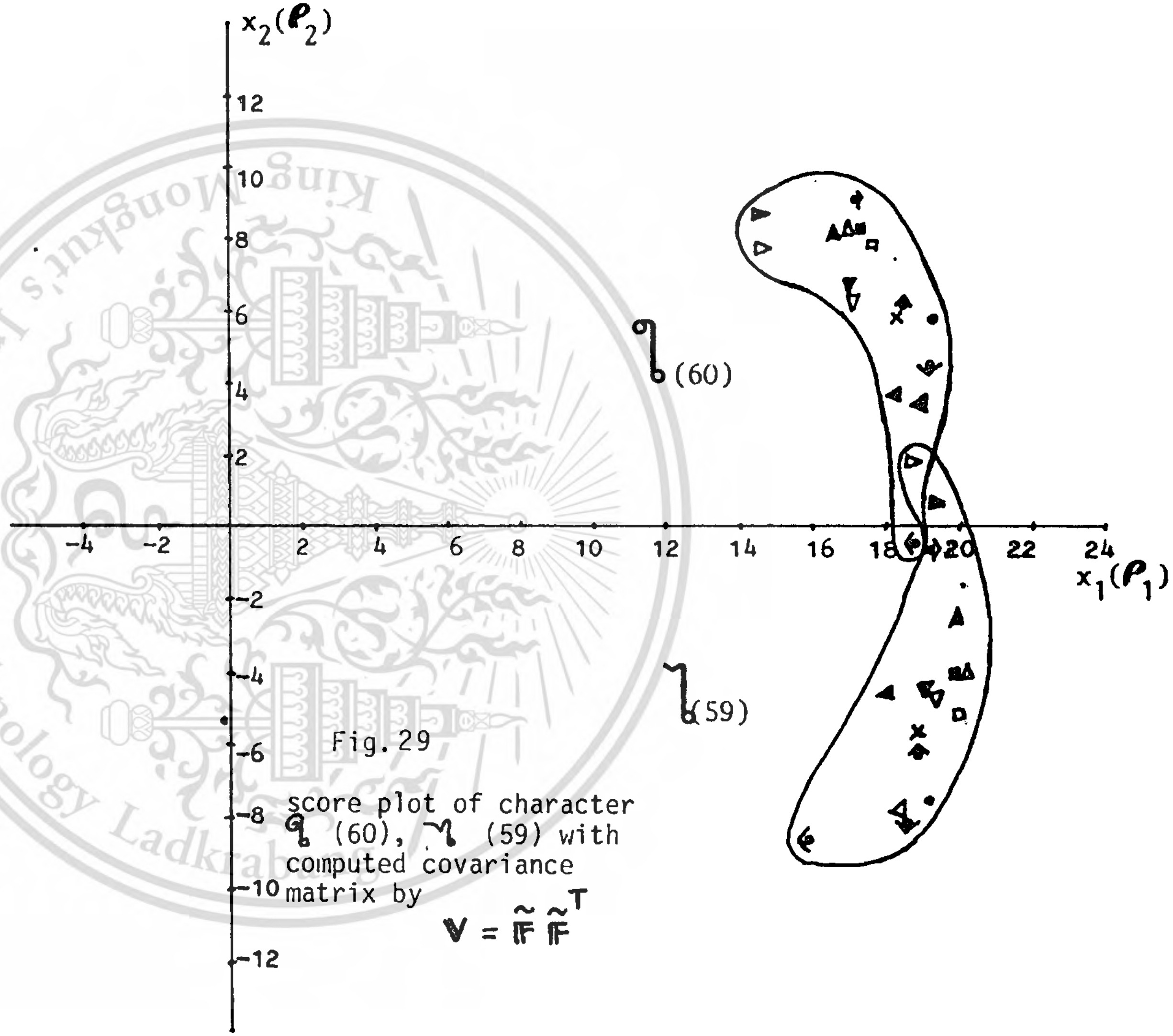


Fig. 29

score plot of character ใ (60), ไ (59) with computed covariance matrix by

$$\mathbb{V} = \tilde{\mathbb{F}}\,\tilde{\mathbb{F}}^T$$

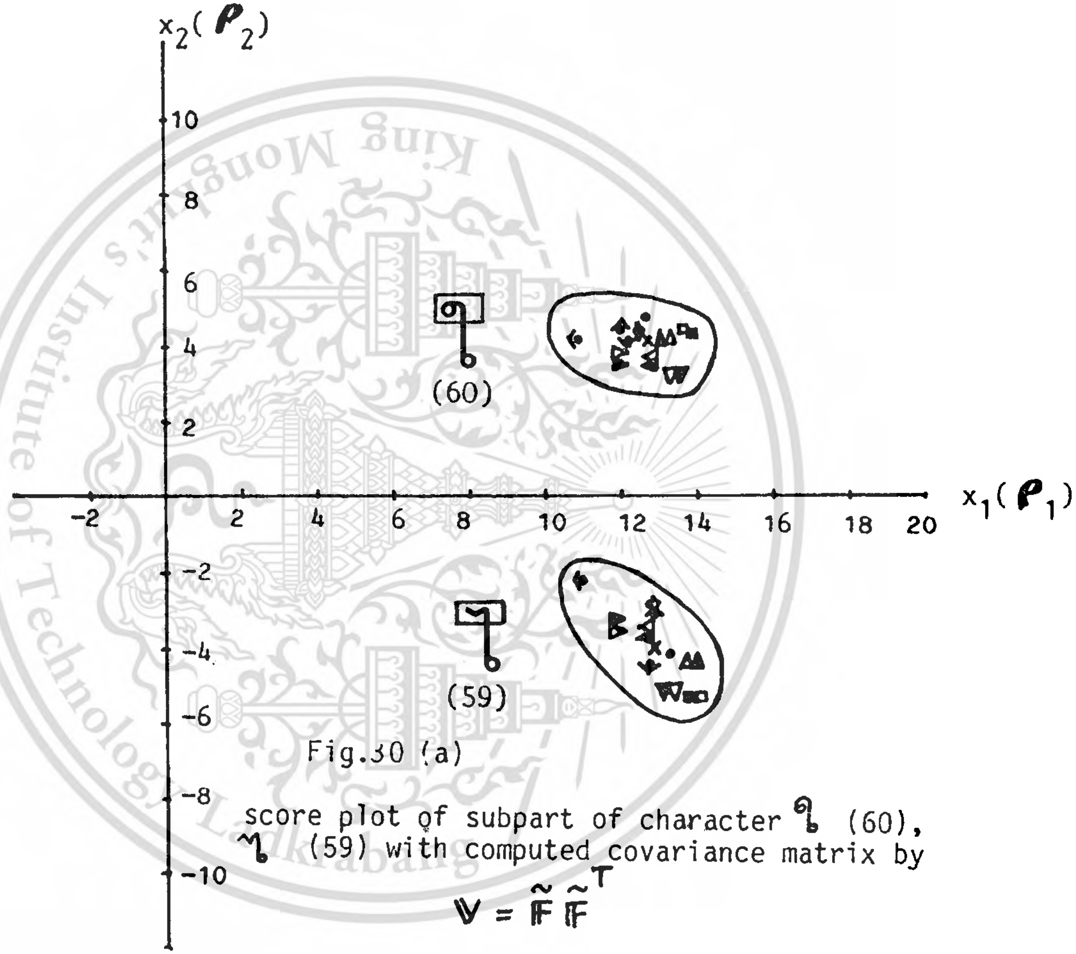


Fig.30 (a)

score plot of subpart of character (60), (59) with computed covariance matrix by

$$\mathbb{V} = \tilde{\mathbb{F}}\tilde{\mathbb{F}}^T$$

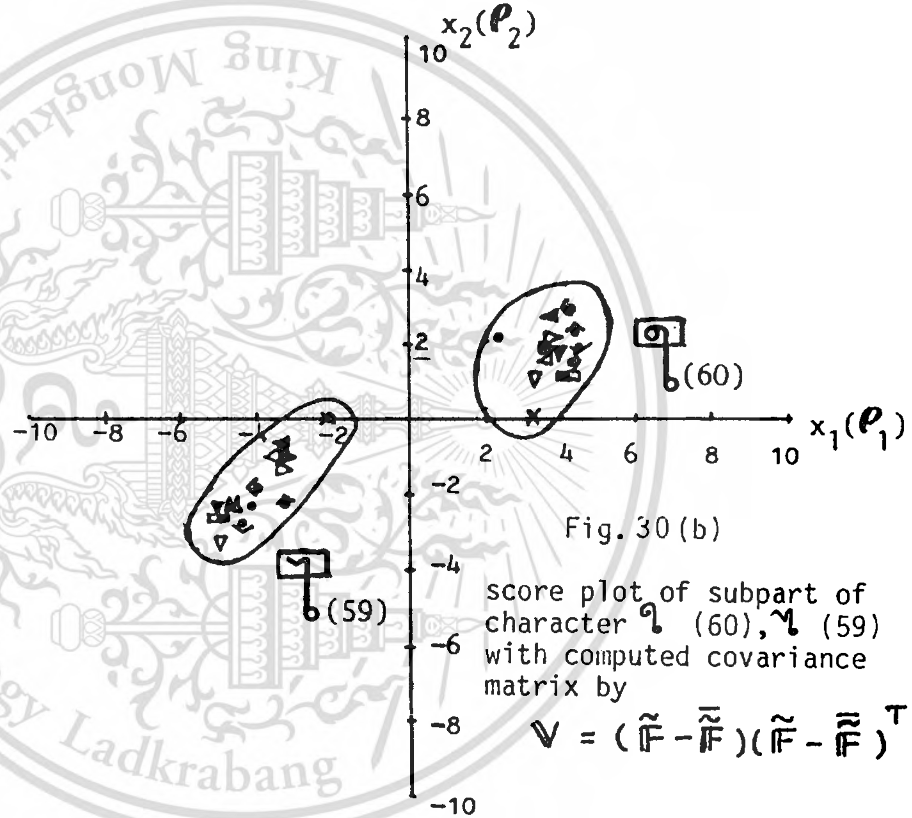


Fig. 30 (b)

score plot of subpart of character (60), (59) with computed covariance matrix by

$$\mathbb{V} = (\tilde{\mathbb{F}} - \bar{\tilde{\mathbb{F}}})(\tilde{\mathbb{F}} - \bar{\tilde{\mathbb{F}}})^T$$

From these results, even though the K-L expansion is used, it is necessary and effective to use subpatterns for achieving the precise fine classification for very similar characters. The K-L expansion is useful tool for classification of the characters. The score plot is one successful criterion for studying the distribution of characters.

5.3 Conclusions of fine classification

5.3.1 Conclusions of fine classification by using subpattern matching

1.) The percentage of recognition correctness of printed Thai characters as described in the preceeding text obtained 100% success in fine classification stage of learning-pattern input data.

2.) In the case of rotation input data, the percentage of correctness is high enough for character recognition.

3.) Even though fine classification by subpattern matching give the results 100%, but an ambiguity in the process of selecting explicit pieces are still occurs.

4.) If we can reduce the dimension of explicit pieces, and number of explicit pieces in the category, more saving of storage area in recognition dictionary and more reducing the recognition time are obtained.

5.) Training set (learning pattern) is each original pattern of printed Thai characters which are derived in section 1.3 and each ±5 degrees rotated original patterns.

6.) FORTRAN programming is used in the implementation of fine classification with FACOM M-360R FUJITSU computer,4MB.

5.3.2 Conclusions of fine classification by using K-L expansion

1.) The fine classification of printed Thai characters using the K-L expansion has been investigated and it is shown that this method is 100% successful to separate the characters which has been used in the process of the K-L expansion.

2.) In the case of shift and rotated input pattern, the recognition is successful 100% for 1-bit shifted patterns and 2-degree rotated patterns.

3.) In practical cases, we can protect input data so that shifts are not more than 1 bit, and rotation are not more than + 2 degree by controlling the machine in the scanning process.

4.) Among four cases for K-L expansion as described in section 5.2, for two member categories, the 1 st case is the most useful method. For 3 or 4 member categories, the second case gives good results because the average patterns of each class which are used in the expansion absorb the deformation.

5.) It is necessary and effective to use subpatterns for achieving the precise fine classification for very similar characters.

6.) There is no an ambiguity in the procedure of this method, it can give the optimum results.

7.) Training set (learning pattern) is each original pattern of printed Thai characters which is derived in section 1.3 and each artificial pattern which is derived in section 5.2.

8.) FORTRAN programming is used in the implementation of fine classification with FACOM M-360R FUJITSU computer, 4MB.

6. STRUCTURAL ANALYSIS METHOD FOR PRINTED THAI CHARACTERS RECOGNITION

6.1 Rough classification by using linear decision function based on their height and width of characters

The main problem of rough classification as described in matching method is the calculation of eigenvalues and eigenvectors. The calculation will take long time and will use a large size of memory in computer if that category has many members.

In other words, the height and width of printed Thai characters are not the same with each other, the rough classification by using linear decision function based on their height and width of characters will be useful. And then, fine classification by using topological properties of characters can be followed.

6.2 Thinning process (8)

Thinning process can be used to eliminate noises and improve the features, to make easier in measuring the height and width of characters. The thinning process used in this thesis is a J.D. Hilditch's algorithm, as shown in Appendix 1. The connected number of a character pattern which describes a relationship among pixels of the character pattern is computed, and then, to leave only skeletal pixels. The algorithm of connected number will be described in next section.

The amount of 1's data of result of thinning process is decreased, and noises are eliminated. An example of a bit pattern of a character is shown in Fig. 31(a), and the result of application of the thinning process is shown in Fig. 31(b).

6.3 Connected number (9,10)

The connected number will be referred by thinning algorithm and topological properties of characters, therefore in this section, its algorithm will be described. The connected number shows the relationship between the pixels in the character pattern. It may be computed from the following algorithm.

Let x_0 be a point of a digital picture that is to be analysed. Neighbouring points are x_1, x_2, x_3, x_4, x_5, x_6, x_7 and x_8 as shown in Fig.32. The connected numbers NC(4) and NC(8) can be computed from the following equations

$$NC(4) = \sum_{k \in S_i} (x_k - x_k x_{k+1} x_{k+2}) \qquad (26)$$

$$NC(8) = \sum_{k \in S_i} (\bar{x}_k - \bar{x}_k \bar{x}_{k+1} \bar{x}_{k+2}) \qquad (27)$$

where $s_i = 1,3,5,7$ for $k \leq 8$; and for $k \geq 9 \rightarrow k = k - 8$.

For example, in Fig.33(a) we have

$$\begin{aligned} NC(4) &= x_1 + x_3 + x_5 + x_7 - x_1 x_2 x_3 - x_3 x_4 x_5 - x_5 x_6 x_7 - x_7 x_8 x_1 \\ &= 1 + 1 + 1 + 1 - 1.0.1 - 1.0.1 - 1.0.1 - 1.0.1 \\ &= 4 \end{aligned}$$

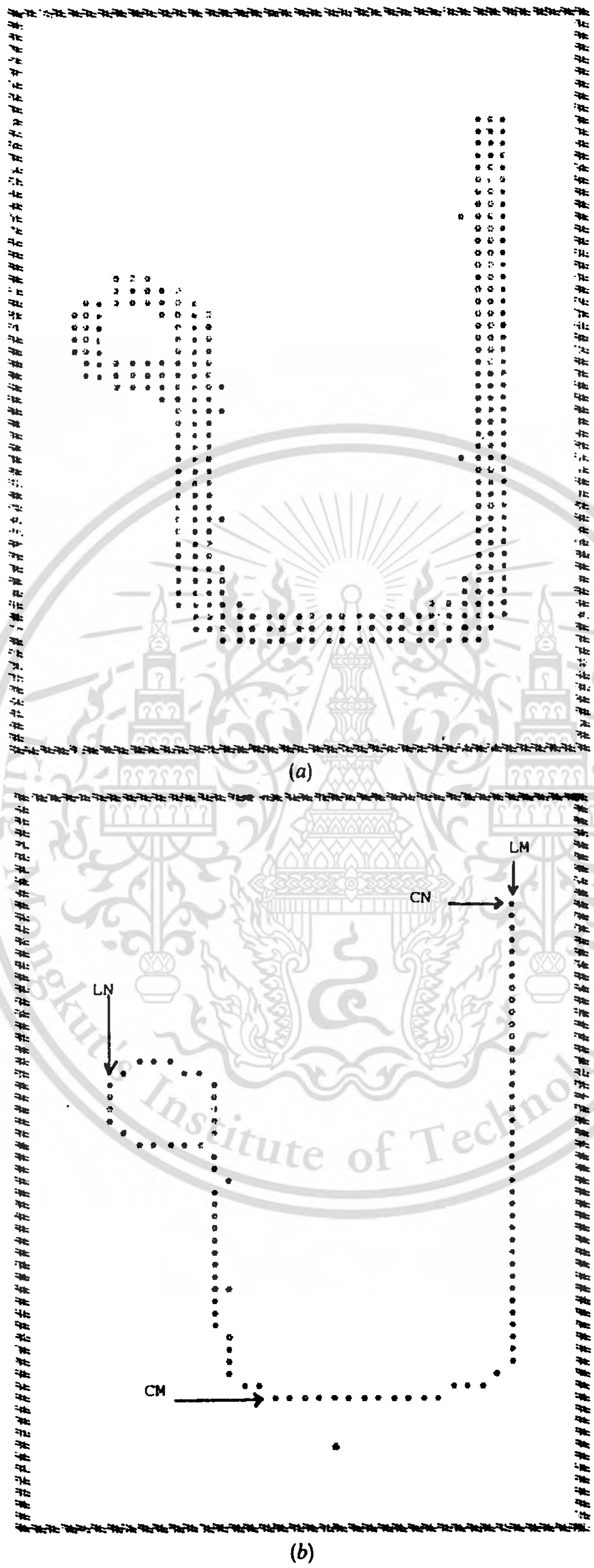


Fig. 31 (a) Bit pattern of character (b) Result of thinning algorithm

x_4	x_3	x_2
x_5	x_0	x_1
x_6	x_7	x_8

Fig. 32 Connectivity structure

	1	
1	1	1
	1	

(a) NC(4) = 4
NC(8) = 0

1		1
	1	
1		1

(b) NC(4) = 0
NC(8) = 4

Fig. 33 Intersection strokes

$$NC(8) = \bar{x}_1 + \bar{x}_3 + \bar{x}_5 + \bar{x}_7 - \bar{x}_1\bar{x}_2\bar{x}_3 - \bar{x}_3\bar{x}_4\bar{x}_5 - \bar{x}_5\bar{x}_6\bar{x}_7 - \bar{x}_7\bar{x}_8\bar{x}_1$$
$$= 0 + 0 + 0 + 0 - 0.1.0 - 0.1.0 - 0.1.0 - 0.1.0$$
$$= 0$$

Similarly, from Fig. 33(b) we obtain

$$NC(4) = 0 + 0 + 0 + 0 - 0.1.0 - 0.1.0 - 0.1.0 - 0.1.0$$
$$= 0$$
$$NC(8) = 1 + 1 + 1 + 1 - 1.0.1 - 1.0.1 - 1.0.1 - 1.0.1$$
$$= 4$$

6.4 The clustering of all printed Thai characters

Because the widths and heights of printed Thai characters are different from each other, the thinned characters can be separated into cluster domains by using linear decision functions based on height and width. A technique in which the bit pattern is scanned in both the horizontal and vertical directions is used.

The width of a character in pixels is the distance between the left-most and the right-most columns containing 1's; and the height of a character in pixels is the distance between the upper-most and the lower-most rows containing 1's (see Fig. 31(b)).

Let LN be the left-most column, LM the right-most column, CN the upper-most row, CM the lower-most row; and let x_1 be the width and x_2 the height. Then the width and the height of the thinned character are expressed by the following equation

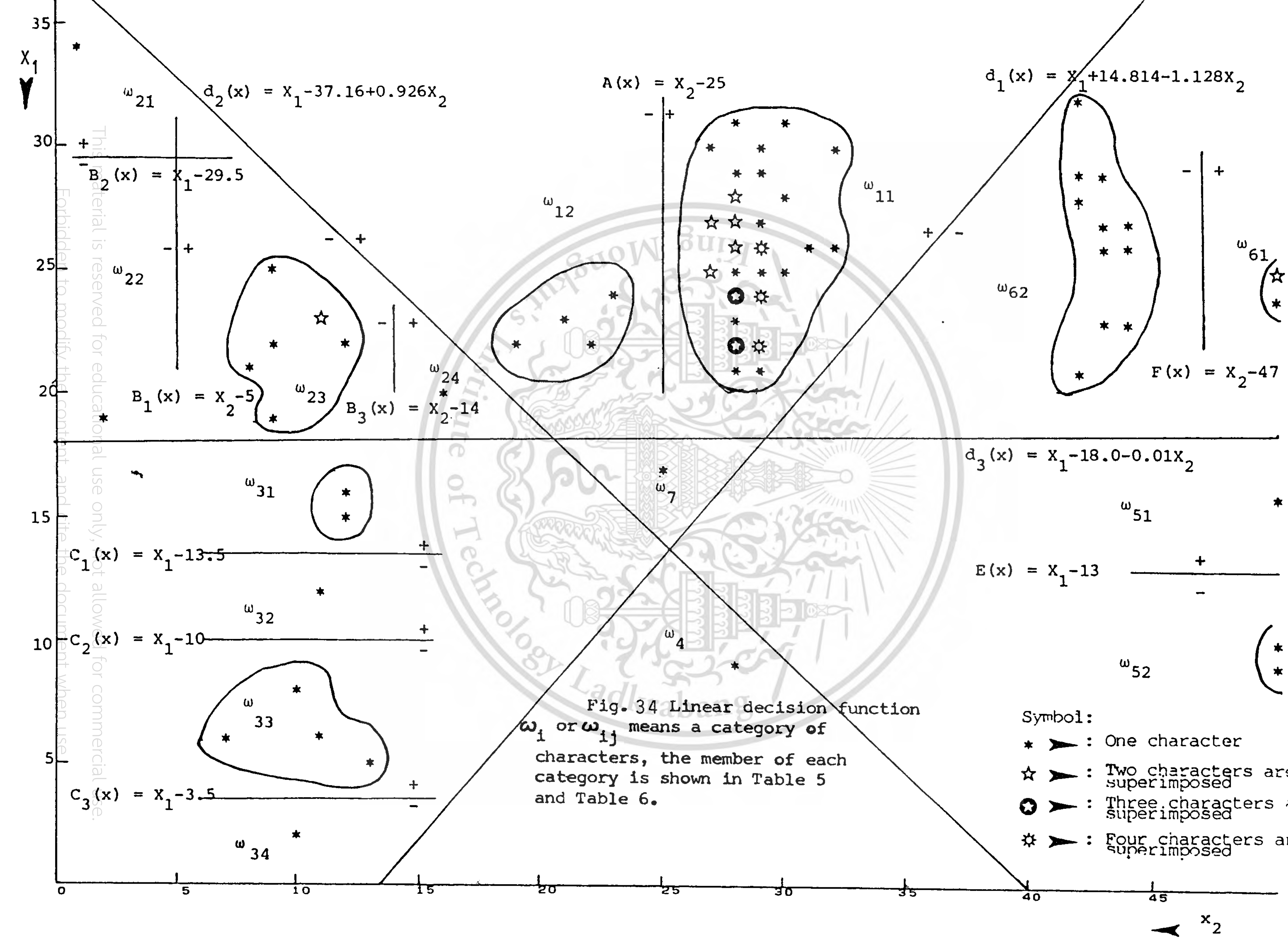


Fig. 34 Linear decision function
ω_i or ω_{ij} means a category of characters, the member of each category is shown in Table 5 and Table 6.

$$x_1 = LM - LN + 1 \qquad (28)$$

$$x_2 = CM - CN + 1 \qquad (29)$$

If x_1 and x_2 are the coordinates of the plane, we can plot a graph of all printed Thai characters as shown in Fig. 34. ω_i in Fig. 34 means the group of characters. The characters which are the members of each group are shown in Table 11 and Table 12.

The principal function of a pattern recognition system is to yield decisions concerning the separation of membership of the pattern characters on this graph with which it is confronted. In order to accomplish this task, it is necessary to establish some rules upon which to base these decisions. One important approach to this problem is the use of linear decision functions (7).

As a way of introduction to this relatively simple concept, consider Fig. 35, where two hypothetical pattern classes are shown. It is seen in this figure that the two pattern populations can be conveniently separated by a line.

Let $d(x) = w_1x_1 + w_2x_2 + w_3 = 0$ be the equation of a separating line where the w's are parameters and x_1, x_2 are the general coordinate variables. It is clear from the figure that any pattern x belonging to category ω_1 will yield a positive quantity when substituted into d(x). Similarly, d(x) becomes negative upon substitution of any pattern from ω_2.

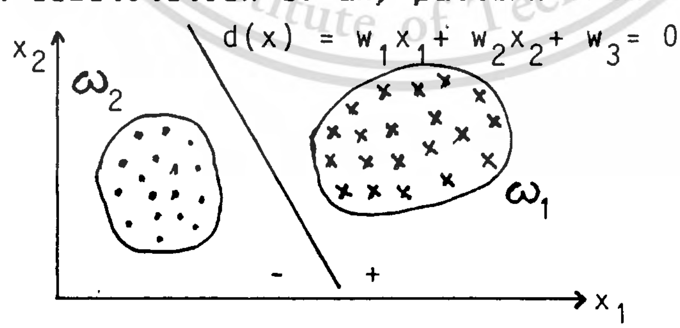


Fig. 35 A simple decision function for two pattern classes

Table 11 Main categories

Main categories	Condition	member
ω_1	$d_1(x) > 0$ $d_2(x) > 0$ $d_3(x) > 0$	ก ข ค ฆ ง จ ฉ ช ซ ฌ ฑ ฒ ณ ด ต ถ ท ธ น บ ผ พ ภ ม ย ร ล ว ส ศ ษ ห ฟ อ ฮ ะ า ฯ แ ๖ ๑ ๒ ๓ ๔ ๕ ๖ ๗ ๘ ๙ ๐
ω_2	$d_1(x) > 0$ $d_2(x) < 0$ $d_3(x) > 0$	- — ๊ ั ิ ี ึ ื ่ ็
ω_3	$d_1(x) > 0$ $d_2(x) < 0$ $d_3(x) < 0$	ุ ู ่ + ' ● , "
ω_4	$d_1(x) < 0$ $d_2(x) < 0$ $d_3(x) < 0$	เ
ω_5	$d_1(x) < 0$ $d_2(x) > 0$ $d_3(x) < 0$	( ) /
ω_6	$d_1(x) < 0$ $d_2(x) > 0$ $d_3(x) > 0$	ไ ใ โ ๆ ำ % ป ฝ ญ ฎ ฏ ฐ ฬ ฤ ฦ
ω_7	$d_1(x) > 0$ $d_2(x) > 0$ $d_3(x) < 0$	?

Table 12 sub-category

sub-category	condition	member
ω_{11}	A(x) 0	ก ข ค ฆ ง จ ฉ ช ซ ฌ ฑ ฒ ณ ด ต ถ ท ธ น บ ผ พ ภ ม ย ร ล ว ศ ษ ส ห ฟ อ ฮ ะ า ฯ แ ๒ ๔ ๕ ๖ ๗ ๘ ๙
ω_{12}	A(x)·0	 ๑ ๓ ๐
ω_{21}	$B_1(x)<0$ $B_2(x)>0$ $B_3(x)<0$	—
ω_{22}	$B_1(x)<0$ $B_2(x)<0$ $B_3(x)<0$	–
ω_{23}	$B_1(x)>0$ $B_2(x)<0$ $B_3(x)<0$	
ω_{24}	$B_1(x)>0$ $B_2(x)<0$ $B_3(x)>0$	
ω_{31}	$C_1(x)>0$ $C_2(x)>0$ $C_3(x)>0$	
ω_{32}	$C_1(x)<0$ $C_2(x)>0$ $C_3(x)>0$	·
ω_{33}	$C_1(x)<0$ $C_2(x)<0$ $C_3(x)>0$	ฯ + • ,
ω_{34}	$C_1(x)<0$ $C_2(x)<0$ $C_3(x)<0$	
ω_{51}	E(x)>0	/
ω_{52}	E(x)<0	( )
ω_{61}	F(x)>0	ไ ใ โ
ω_{62}	F(x)<0	ๆ ป % ำ ญ ฎ ฏ ฐ ฬ ฤ ฦ

Therefore, d(x) can be used as a decision (or discriminating) function since, given a pattern x of unknown classification, we may say that x belongs to ω_1 if $d(x) > 0$, or to ω_2 if $d(x) < 0$. If the pattern lies on the separating boundary, we obtain the indeterminate condition $d(x) = 0$. As will be seen in the following section, these concepts need not be restricted to two categories. In addition, they can be easily extended to encompass the more general case of nonlinear boundaries in any finite dimensional Euclidean space.

The success of pattern grouping scheme depends on two factors: (1) the form of d(x), and (2) one's ability to determine its coefficients. The first problem is directly related to the geometrical properties of the pattern classes under consideration. It is not difficult to visualize situations in which boundaries that are considerably more complicated than the linearly separable example discussed above might be necesary to separate the given pattern populations. If the dimensionality of the patterns is higher than three, our powers of visualization are no longer of assistance in determining these boundaries. Under these conditions, the only reasonable recourse is a strictly analytical approach. Unfortunately, unless some a priori information is available, the only way to establish the effectiveness of a chosen decision function is by direct trial.

Once a certain function (or functions if more than two categories are involved) has been selected, the problem becomes the determination of the coeffcients. If the pattern classes under consideration are separable by the specified decision functions, it is possible to utilize sample patterns in order to determine the coefficients which characterize these functions.

The simple two-dimensional linear decision function can be easily generalized to the n-dimensional case. Thus, a general linear decision function is of the form

$$d(x) = w_1 x_1 + w_2 x_2 + \dots + w_n x_n + w_{n+1}$$
$$= W_0^T X + w_{n+1} \qquad (30)$$

where $W_0 = (w_1, w_2, \dots, w_n)^t$. This vector is referred to as the weight or parameter vector.

It is widely accepted convention to append a 1 after the last component of all pattern vectors and express equation (30) in the form

$$d(x) = W^T X \qquad (31)$$

where $X = (x_1, x_2, \dots, x_n, 1)^t$ and $W = (w_1, w_2, \dots, w_n, w_{n+1})^t$ are called the augmented pattern and weight vectors, respectively. Since the same quantity is equally appended to all patterns, the basic geometrical properties of the pattern classes are not disturbed. Whether or not weight vector has to be augmented can usually be determined from the context. We will normally refer to X and W in equation (31) simply as pattern and weight vectors, respectively.

In the two-category case a decision function d(x) is assumed to have the property

$$d(x) = W^T X \begin{cases} > 0 \text{ if } X \in \omega_1 \\ < 0 \text{ if } X \in \omega_2 \end{cases} \qquad (32)$$

When we have more than two categories, denoted by $\omega_1, \omega_2, \ldots, \omega_M$, we consider the following multiclass cases.

Case 1 : Each pattern class is separable from the other classes by a single decision surface. In this case there are M decision functions with the property

$$d_i(x) = W_i^T X = \begin{cases} > 0 \text{ if } X \in \omega_i \\ < 0 \text{ otherwise} \end{cases}, \quad i = 1,2,\ldots,M \qquad (33)$$

where $W_i = (w_{i1}, w_{i2}, \ldots, w_{in}, w_{i,n+1})^t$ is the weight vector associated with the decision function.

Case 2 : Each pattern class is separable from every other individual class by a distinct decision surface, that is, the classes are pairwise separable. In this case there are M(M-1)/2 (the combination of M classes taken two at a time) decision surfaces. The decision functions here are of the form $d_{ij}(x) = W_{ij}^t X$ and have the property that, if x belongs to category ω_i, then

$$d_{ij}(X) > 0 \quad \text{for all} \quad j \neq i \qquad (34)$$

These functions also have the property that $d_{ij}(X) = -d_{ji}(X)$. It is not uncommon to find problems involving a combination of Cases 1 and 2. These situations require fewer than the M(M-1)/2 decision surfaces which would be needed if all the classes were only pairwise separable.

Case 3 : There exist M decision functions $d_k(X) = W_k^T X$, $k = 1, 2, \ldots, M$, with the property that, if X belongs to category ω_i,

$$d_i(X) > d_j(X) \quad \text{for all} \quad j = i \qquad (35)$$

This is a special instance of Case 2 since we may define

$$\begin{aligned} d_{ij}(X) &= d_i(X) - d_j(x) \\ &= (W_i - W_j)^t X \\ &= W_{ij}^t X \end{aligned} \qquad (36)$$

where $W_{ij} = W_i = W_j$. It is easily verified that, if $d_i(X) > d_j(X)$ for all $j \neq i$, then $d_{ij}(X) > 0$ for all $j \neq i$, that is, if the classes are separable under Case 3 conditions, they are automatically separable under Case 2. The converse, however, is in general not true. If the pattern classes in a given situation are classifiable by any of the linear decision function, the classes are said to be linearly separable.

According to the explaination said above, the linear decision function $d_i(x)$ for grouping of membership on Fig. 34 can be obtained by follow the determinating $d_i(x)$ of Case 1. Hence, form of $d_i(x)$ is very simple as

$$d_i(x) = w_1 x_1 + w_2 x_2 + w_3 = 0$$

The coefficients w_i of this function are performed by using linear regression formula as followed.

$$a = \bar{y} - b\bar{x} \qquad (37)$$

where $\bar{y} = \Sigma x_1 / N$ and $\bar{x} = \Sigma x_2 / N$ and

$$b = \frac{N \Sigma x_2 x_1 - \Sigma x_2 \Sigma x_1}{N \Sigma x_2^2 - (\Sigma x_2)^2} \qquad (38)$$

where N is the number of x_1 or x_2 (number of x_1 must equal number of x_2), and then the form of $d_i x$ can be changed in

$$d_i(x) = x_1 - a - bx_2 \qquad (39)$$

The procedure to compute a and b can be explained as follows.

Because $d_i(x)$ is a straight line and completly divides all memberships into two groups, by inspection, first we can try to draw $d_i(x)$ dividing memberships into two groups at any where. Second, adjust this $d_i(x)$ by select coordinate (the value of x_1 and x_2) of some memberships from both groups, and then the coefficient a and b can be obtained by substitute x_1 and x_2 into equation (37) and (38).

The computation a and b of $d_i(x)$ can be shown as an example by selecting :

$$\left.\begin{array}{ll} x_1 : & 17,\ 26 \\ x_2 : & 25,\ 32 \end{array}\right\} \text{ from first group}$$

$$\left.\begin{array}{ll} x_1 : & 9,\ 32 \\ x_2 : & 28,\ 42 \end{array}\right\} \text{ from second group}$$

and then

$$\Sigma x = 84$$

$$\Sigma x = 127$$

$$\bar{x} = \frac{\Sigma x_2}{N} = \frac{127}{4} = 31.75$$

$$\bar{y} = \frac{\Sigma x_1}{N} = \frac{84}{4} = 21$$

$$\Sigma x_2^2 = 25^2 + 32^2 + 28^2 + 42^2$$

$$= 625 + 1024 + 784 + 1764$$

$$= 4197$$

$$\Sigma x_2 x_1 = (25 \times 17) + (32 \times 26) + (28 \times 9) + (42 \times 32)$$

$$= 425 + 832 + 252 + 1344$$

$$= 2853$$

$$b = \frac{N \, \Sigma x_2 x_1 - \Sigma x_2 \, \Sigma x_1}{N \, \Sigma x_2^2 - (\Sigma x_2)^2}$$

$$= \frac{4(2853) - 127(84)}{4(4197) - (127)}$$

$$= \frac{11412 - 10668}{16788 - 16129} = \frac{744}{659}$$

$$b = 1.128$$

$$a = \bar{y} - b\bar{x}$$

$$= 21 - 1.128(31.75)$$

$$= 21 - 35.814$$

$$= - 14.814$$

Substitute a and b into equation (37), the result is

$$d_1(x) = x_1 + 14.814 - 1.128x_2 \qquad (40)$$

by the same way, we can obtain

$$d_2(x) = x_1 - 37.161 + 0.926x_2 \qquad (41)$$

$$d_3(x) = x_1 - 18.0 - 0.01x_2 \qquad (42)$$

$d_1(x)$, $d_2(x)$ and $d_3(x)$ are shown in Fig. 34.

By using equation (33) we can obtain seven main categories by the separating of $d_1(x)$, $d_2(x)$ and $d_3(x)$. The boundary condition and members of each category are as shown in Table 11. We see that, in this case, there are no classes in a category because one point on the graph represents one character. But we can divide the main category into sub-category (can say be a class) by performing new decision function by the same way as described above. The results of $d_i(x)$ are as following

$$A(x) = x_2 - 25 \quad \text{for category } \omega_1 \tag{43}$$

$$\left.\begin{aligned} B_1(x) &= x_2 - 5 \\ B_2(x) &= x_1 - 29.5 \\ B_3(x) &= x_2 - 14 \end{aligned}\right\} \text{for category } \omega_2 \tag{44}$$

$$\left.\begin{aligned} C_1(x) &= x_1 - 13.5 \\ C_2(x) &= x_1 - 10 \\ C_3(x) &= x_1 - 3.5 \end{aligned}\right\} \text{for category } \omega_3 \tag{45}$$

$$E(x) = x_1 - 13 \quad \text{for category } \omega_4 \tag{46}$$

$$F(x) = x_1 - 47 \quad \text{for category } \omega_5 \tag{47}$$

The boundary condition and members of each sub-category are as shown in Table 12.

6.5 Conclusion of rough classification in structural analysis method

1.) The advantage of rough classification by using decision funtion based on their height and width of characters is saving the storage area of dictionary of recognition, because it keeps only decision functions and their boundary condition.

2.) Thinning process is used to eliminate noises and improve the features, to make easier measuring the height and width of characters. But the decision function for clustering characters is determined by inspecting, it is very difficult to obtain the best decision function.

3.) In the case of high recognition rate, rough classification by using K-L expansion (in matching method) gives better results than rough classification by using decision function based on their height and width of characters.

4.) In the case of saving area of computer, rough classification by using linear decision functions is better than rough classification by using K-L expansion.

5.) 100% successful in rough classification of learning pattern input data.

6.) Learning patterns are each original pattern character which are derived in section 1.3.

7.) FORTRAN programming is used in the implementation of rough classification with NEC system 300 main frame computer, 2MB.

6.6 Fine classification by using topological properties of characters

6.6.1 Topological properties of characters

We have obtained the connected number of the point in the character pattern as described in section 6.3, those connected number can be converted into the topological properties of characters, and then, they can be used to classify the characters.

In other words, the main problem of fine classification by using K-L expansion as described in the section 5.2 is the calculation of eigenvalues and eigenvectors. The calculation will take a long times and will use the large size of memory in computer if that category has many members. Therefore, the fine classification by topological properties of characters will be described in this section.

The topological properties of characters which are represented in the form of numerical digits allow the separation of various aspects such as end strokes, branch strokes, intersection strokes, connecting strokes, independent or inner strokes. All of these properties can be used to discriminate between characters. By using equation (26) and (27) o compute NC(4) and NC(8) of these properties, the results can be shown in Fig. 36. The results of NC(4) and NC(8) in this figure can be written in the form of table as shown in Table 13, and then also adds the value of NC(4) and NC(8) of intersection stroke into this table. This table shows NC(4) and NC(8) of all features of stroke of characters which can appear.

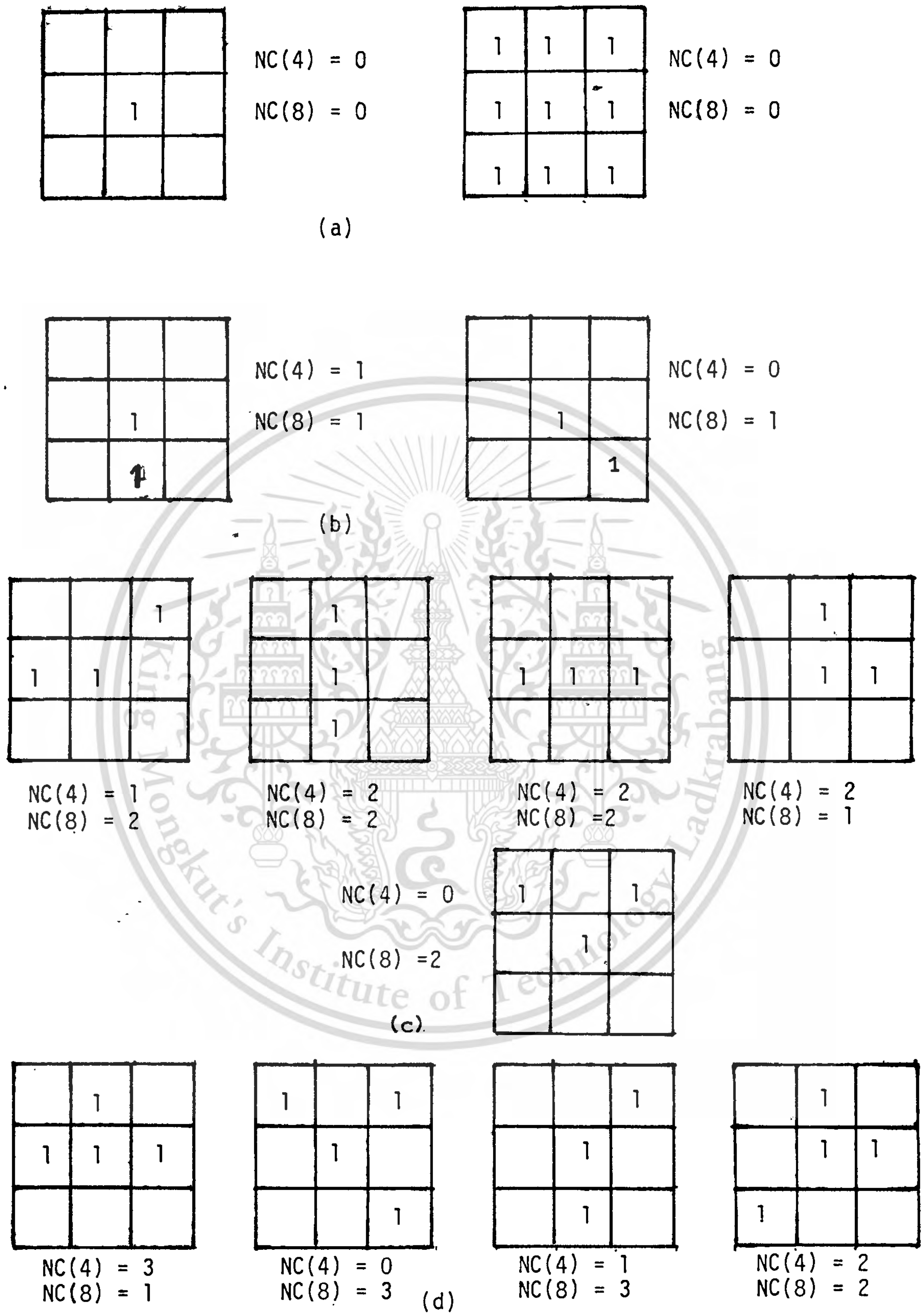


Fig. 36 : (a) NC(4) and NC(8) of independent or inner strokes.
(b) NC(4) and NC(8) of end strokes. (c) NC(4) and NC(8) of connecting strokes. (d) NC(4) and NC(8) of branch strokes.

Table 13 Summary of topological properties.

Connectivity	Inner and independent stro.	End strokes		Connecting strokes					Branch strokes				Intersection strokes	
NC(4)	0	1	0	1	2	2	2	0	3	0	1	2	4	0
NC(8)	0	1	1	2	2	2	1	2	1	3	3	2	0	4

Note: the value of inner strokes = 0, end strokes = 1, connecting strokes = 2, branch strokes = 3, intersection strokes = 4.

6.6.2 The procedure conversion of patterns into topological properties

Let 0 represents inner or independent strokes, 1 represents end strokes, 2 represents connecting strokes, 3 represents branch strokes, 4 represents intersection strokes, then by using a computer program to compute NC(4) and NC(8) of each point in the patterns of thinned Thai characters, we can convert the patterns into the form of topological properties, as shown in Fig. 37, by let P be the topological properties of a character; then the conversion rules are as follows

If NC(4) = 0 and NC(8) = 0, then

$$P = 0 \qquad (48)$$

If NC(4) = 1 and NC(8) = 1, or NC(4) = 0 and NC(8) = 1, then

$$P = 1 \qquad (49)$$

If NC(4) = 1 and NC(8) = 2, or NC(4) = 2 and NC(8) = 2, or NC(4) = 2 and NC(8) = 1, or NC(4) = 0 and NC(8) ≐ 2, then

$$P = 2 \qquad (50)$$

If NC(4) = 2 and NC(8) = 1, or NC(4) = 0 and NC(8) = 3, or NC(4) = 1 and NC(8) = 3, then

$$P = 3 \qquad (51)$$

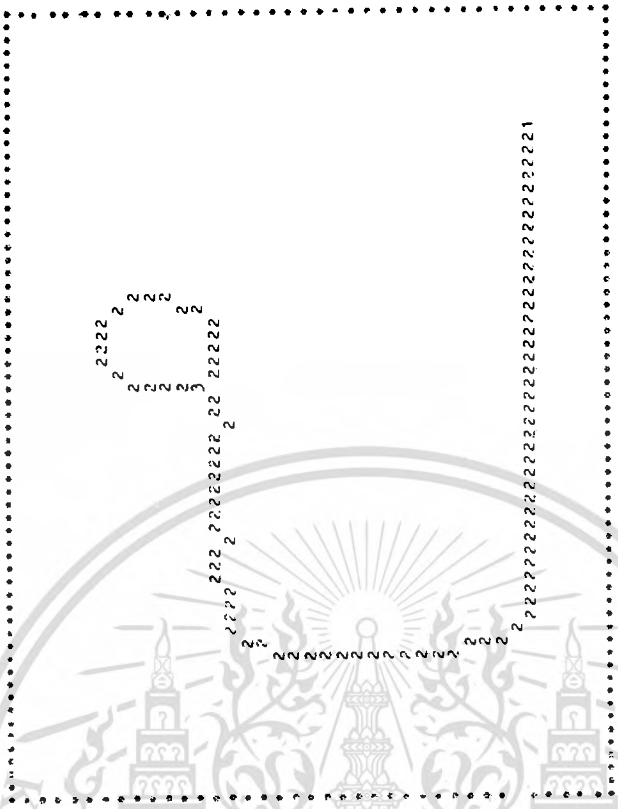
If NC(4) = 4 and NC(8) = 0, or NC(4) = 0 and NC(8) = 4, then

$$P = 4 \qquad (52)$$

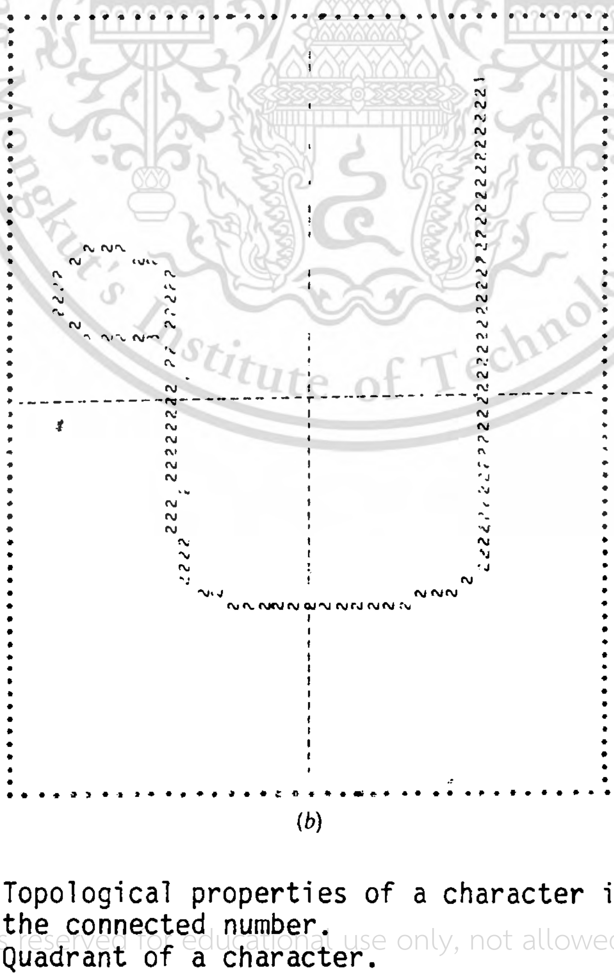
Because intersection strokes rarely occur in printed Thai characters, they may be expressed in branch strokes by converting the points which have NC(4) = 4 and NC(8) = 0 or NC(4) = 0 and NC(8) = 4 into 3.

6.6.3 Decision table of fine classification

The results of properties of each character in each category which are obtained from the operation at the fine classification stage can be shown in the form of a decision table, as shown in Table 14. To design the decision table, we divide the pattern of each character into four parts (each part is called quadrant) by drawing two the straight lines to perpendicular with each other at the centre of gravity as shown in Fig. 37(b). However, before making this, the centre of gravity of a character must also be the centre of the frame of a character pattern. Then, we can list the numbers which represent the topological properties of each part into the decision table. For example, (let see the Fig. 38(a)) after the pattern is divided into four parts, namely lower-right hand-part is Q1, upper-right hand-part is Q2, upper-left hand-part is Q3 and lower-left hand-part is Q4, respectively. In table, P1,P2 and P3 mean the topological properties, and numerals 1, 2 and 3 in the pattern mean end



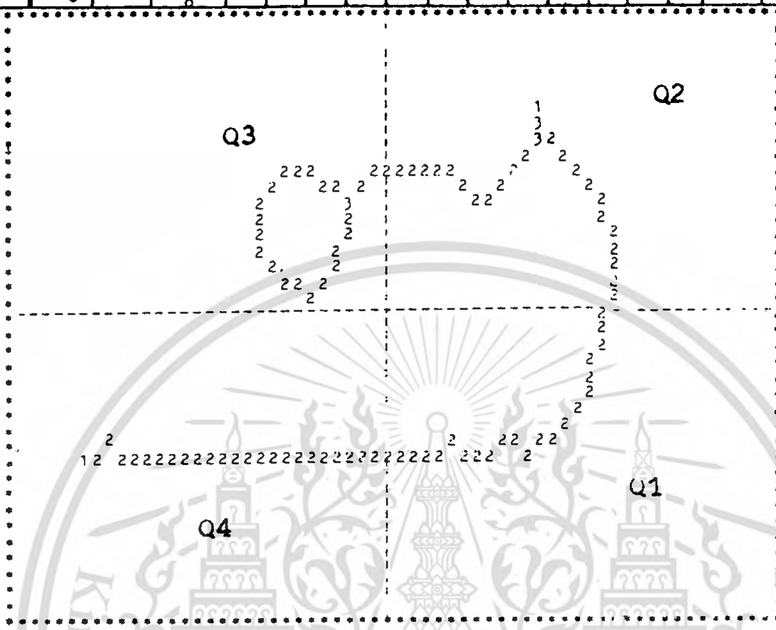
(a)



(b)

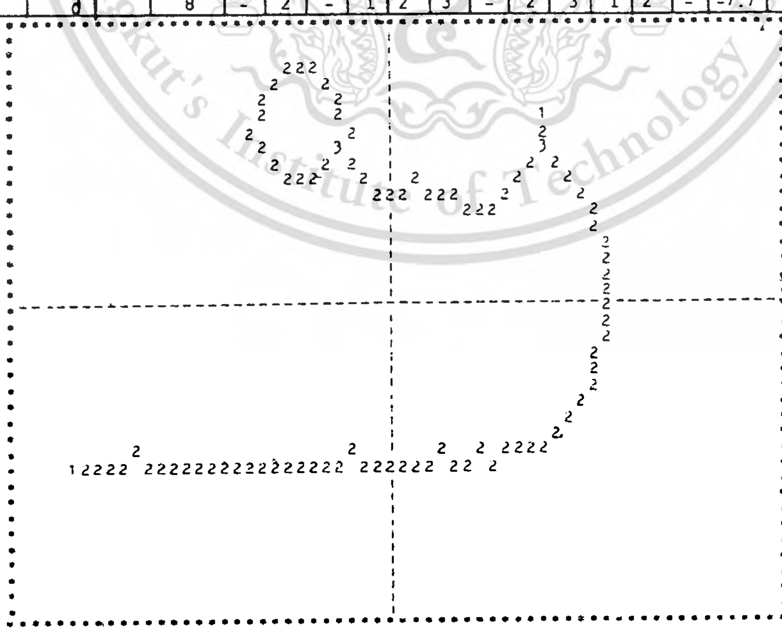
Fig.37 : (a) Topological properties of a character in the form of the connected number.
(b) Quadrant of a character.

sub-category	Characters	Weight	Quadrant Q_1 Properties P1	P2	P3	Q_2 P1	P2	P3	Q_3 P1	P2	P3	Q_4 P1	P2	P3	$d_1(x)$	$d_2(x)$	$d_3(x)$
	ฏ	8	-	2	-	1	2	3	-	2	3	1	2	-	-10.7	26	3.8
	ฎ	8	-	2	-	1	2	3	-	2	3	1	2	-	-7.7	29	6.8



(a)

sub-category	Characters	Weight	Quadrant Q_1 Properties P1	P2	P3	Q_2 P1	P2	P3	Q_3 P1	P2	P3	Q_4 P1	P2	P3	$d_1(x)$	$d_2(x)$	$d_3(x)$
	ฏ	8	-	2	-	1	2	3	-	2	3	1	2	-	-10.7	26	3.8
	ฎ	8	-	2	-	1	2	3	-	2	3	1	2	-	-7.7	29	6.8



(b)

Figure 38 (a) and (b) Distinguishing the characters by $d_i(x)$.

sub-category	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{11}	า	6	-	2	-	1	2	-	-	2	-	1	2	-	5.2	10	3.5
	ฯ	6	-	2	-	-	2	3	-	2	-	1	2	-	7.2	12	6.5
	ก	6	-	2	-	-	2	-	1	2	-	1	2	-	4.0	11	3.5
	ข	6	1	2	-	-	2	3	-	2	-	-	2	-	5.9	15	6.5
	ง	6	-	2	3	1	2	-	-	2	-	-	2	-	4.0	11	3.5
	๒	6	-	2	-	1	2	3	-	2	-	-	2	-	10.2	15	8.6
	ป	6	1	2	-	-	2	3	-	2	-	-	2	-	8.0	15	7.5
	ว	6	-	2	-	1	2	-	-	2	-	-	2	3	6.0	13	5.5
	ภ	6	-	2	-	-	2	-	-	2	3	1	2	-	7.0	14	6.5
	อ	6	-	2	-	1	2	-	-	2	3	-	2	-	6.0	13	5.5
	ย	6	1	2	-	-	2	3	-	2	-	-	2	-	5.2	10	3.6
	ะ	6	1	2	-	-	2	-	-	2	-	-	2	3	4.2	9	2.6
	๔	7	1	2	-	-	2	-	-	2	-	1	2	3	10.2	15	8.6
	๖	7	-	2	-	1	2	3	-	2	-	-	2	3	9.2	14	7.6
	จ	7	-	2	-	1	2	-	-	2	3	-	2	3	7.2	12	5.6

Table 14 Decision table.

sub-category	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
	ฆ	7	1	2			2	3		2	3		2		11.2	16	9.6
	ช	7	1	2	3		2	3		2			2		8.9	18	9.5
	ษ	7	1	2	3		2	3		2			2		12.0	19	11.5
	ธ	7	1	2		1	2	3		2			2		4.0	11	3.5
	ฆ	7	1	2			2	3	-	2	3		2		8.0	15	7.5
^{w}11	ร	7	1	2		1	2	3		2			2	3	3.0	10	2.5
	ฮ	7	1	2	3		2			2	3		2		6.0	13	5.5
	ซ	7	1	2			2	3		2		-	2		8.6	22	11.4
	ค	7		2		-	2		1	2	3	1	2		6.0	13	5.5
	ด	7		2			2		1	2	3	1	2		7.2	12	5.6
	ต	7		2			2	-	1	2	3	1	2		8.2	13	6.6
	๕	8	1	2	3		2			2	3	1	2	3	8.0	15	7.5
	๘	8	1	2		-	2		1	2	3	-	2	3	11.3	12	6.6
	๙	8	1	2	-	-	2	3	-	2	3	1	2	-	9.3	12	6.6
	ฉ	8	-	2	-	1	2	-	1	2	3	-	2	3	8.0	15	7.5

Table 14 Continued

sub-category	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{11}	ฑ	8	-	2	-	-	2	3	1	2	3	1	2	-	14.3	17	11.6
	ท	8	-	2	-	-	2	3	1	2	3	1	2	-	9.2	14	7.6
	ล	8		2	-	1	2		-	2	3	1	2	3	6.2	11	4.6
	ถ	8		2		1	2	3	-	2	3	1	2	-	4.0	11	3.5
	แ	8	1	2	-	1	2	-	-	2	3	-	2	3	7.2	12	5.6
	ห	9		2	3		2	3	1	2	3	1	2	-	11.3	14	8.6
	ศ	9	1	2	3		2	-	1	2	3	1	2	-	4.6	18	7.4
	น	9	1	2	-		2	3	1	2	3	1	2	3	11.2	16	9.6
	ฒ	9	1	2	-	-	2	-	1	2	3	1	2	3	14.2	19	12.6
	ณ	10	1	2		1	2	3		2	3	1	2	3	11.7	18	10.5
	ผ	10	1	2	-		2	3	1	2	3	1	2	3	5.2	10	3.6
	ส	10	1	2	3	1	2	-	-	2	3	1	2	3	5.8	17	7.4
	๗	10	1	2	-	1	2	3	-	2	3	1	2	3	11.3	14	8.6
	ฆ	11	1	2	-	1	2	3	1	2	3	1	2	3	12.2	17	10.6
	ฬ	11	1	2	3		2	3	1	2	3	1	2	3	11.9	21	12.5

Table 14 Continued

sub-category	Characters	Weight	Quadrant Q_1 P1	Q_1 P2	Q_1 P3	Q_2 P1	Q_2 P2	Q_2 P3	Q_3 P1	Q_3 P2	Q_3 P3	Q_4 P1	Q_4 P2	Q_4 P3	$d_1(x)$	$d_2(x)$	$d_3(x)$
			Properties														
ω_{11}	ฬ	11	1	2	3	-	2	3	1	2	3	1	2	3	9.0	16	8.5
ω_{12}	O		-	2	-	-	2	-	-	2	-	-	2	-	14.1	4	4.9
	๑		-	2	-	-	2	-	-	2	3	-	2	-	11.9	4	3.9
	[illegible]		1	2	-	-	2	-	-	2	3	1	2	--	12.8	7	5.8
	[illegible]		1	2	3	-	2	-	-	2	3	1	2	-	15.3	1	4.0
ω_{21}	—		-	-	-	-	-	-	-	-	-	-	-	-	47.68	-5	16.9
ω_{22}	—		-	2	-	1	2	-	-	-	-	-	-	-	31.55	-19	1.9
ω_{23}	[illegible]	7	1	2	-	-	2	-	1	2	3	-	2	-	23.65	-12	1.5
	[illegible]	6	-	2	-	-	2	-	1	2	3	-	2	-	26.87	-11	3.6
	[illegible]	7	1	2	-	-	2	-	1	2	3	-	2	-	23.65	-12	1.5
	[illegible]	7	1	2	3	-	2	-	-	2	3	-	2	-	25.39	-6	5.4
	[illegible]	8	1	2	-	-	2	-	-	2	3	1	2	3	26.65	-9	4.4
	[illegible]	8	-	2	3	-	2	-	1	2	3	-	2	3	25.39	-6	5.4
	[illegible]	10	1	2	3	-	2	-	1	2	3	1	2	3	23.26	-6	4.4
ω_{24}	[illegible]		1	2	-	-	2	-	1	2	3	-	2	-	16.75	-6	5.4

Table 14 Continued

sub-category	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{31}			1	2	-	-	2	-	-	2	3	-	2	-	17.26	-12	-1.6
			1	2	-	-	2	-	-	2	-	-	2	-	16.26	-13	-2.6
ω_{32}			1	2	-	-	-	-	-	-	-	1	2	-	5.52	-28	-15
ω_{33}			-	-	-	-	-	-	-	-	-	1	2	-	8.39	-23	-11.5
			1	2	-	1	-	-	1	-	-	1	-	-	11.52	-22	-9.5
			1	2	-	1	-	-	1	-	-	1	-	-	12.91	-27	-11.3
			1	2	-	-	-	-	1	2	-	-	2	-	5.13	-22	-12.6
ω_{34}			1	2	-	1	2	-	1	2	-	1	2	-	14.39	-17	-5.5
ω_{51}			1	2	-	-	-	-	1	2	-	-	-	-	-29.0	29	-3.6
ω_{52}			1	2	-	-	2	-	-	2	-	1	2	-	-39.4	25	-10.8
			-	2	-	1	2	-	1	2	-	-	2	-	-40.6	28	-9.9
ω_{61}			-	2	-	1	2	3	-	-	-	-	2	3	-16.6	33	3.5
			-	2	-	-	2	3	-	-	-	-	2	3	-19.8	36	4.4
			1	2	-	1	2	3	-	-	-	-	2	3	-17.7	36	5.4

Table 1 4 Continued

sub-category	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{62}	ๆ	5	-	2		-	2	3	-	-	-	1	2	-	-11.5	23	1.9
	ป	6	1	2	-	-	2	3	-	2	-	-	2	-	-4.5	30	8.9
	%	6	1	2	-	-	2	-	1	2	-	-	2	-	-0.5	34	12.9
	ำ	6	-	2	-	-	2	-	1	2	-	1	2	-	-8.8	30	6.8
	ฎ	7	-	2	-	-	2	3	-	2	3	1	2	-	-6.7	30	7.8
	ญ	8	1	2	-	-	2	-	-	2	3	1	2	3	-4.7	32	9.8
	ฏ	8	-	2	-	-	2	3	-	2	3	1	2	3	-7.8	31	7.8
	ฤ	8	-	2	-	1	2	3	-	2	3	1	2	-	-10.7	26	3.8
	ฦ	8	-	2	-	1	2	3		2	3	1	2	-	-7.7	29	6.8
	ฬ	11	1	2	3	-	2	3	1	2	3	1	2	3	-3.5	31	9.9
	ฐ	12	1	2	3	1	2	3	1	2	3	1	2	3	-11.8	27	3.8

Table 14 Continued

stroke, connecting stroke and branch stroke, respectively. It appears only numeral 2 in Q1, so 2 is written into the table under P2. It appears numeral 1, 2 and 3 in Q2, so 1, 2 and 3 are written into the table under P1, P2 and P3,respectively. It appears numeral 2 and 3 in Q3, so 2 and 3 are written into the table under P2 and P3, respectively. It appears numeral 1 and 2 in Q4, so 1 and 2 are written into the table under P1 and P2, respectively. By the same way, we can write the topological properties of other thinned Thai characters, the results are shown in Table 14.

If we let one property giving value (called weight) 1 in each quadrant, and then we can compute the weight of the character by adding together the weights in every quadrants. The characters weight can be used for clustering the characters in the sub-category into subclass. Table 14 also shows the value of d_i (x)(compute by substitute their width and height of characters pattern into the equation of $d_i(x)$). When the properties of characters are all the same in every quadrant, these $d_i(x)$ can be used to discriminate characters; for example sub-category ω_{62} as shown in Fig. 38.

In the case shown in Fig. 39, since neither the properties nor $d_i(x)$ can be used to distinguish between these two characters, a special method must be used. The bit pattern of the thined character is scanned in a vertical direction through its centre of gravity from the top of the pattern until 1's data are encountered. It is then back-stepped by one bit position, and the bit pattern is then scanned in a horizontal direction. If 1's data are encountered, this indicates character No. 2; otherwise it is character No. 1.

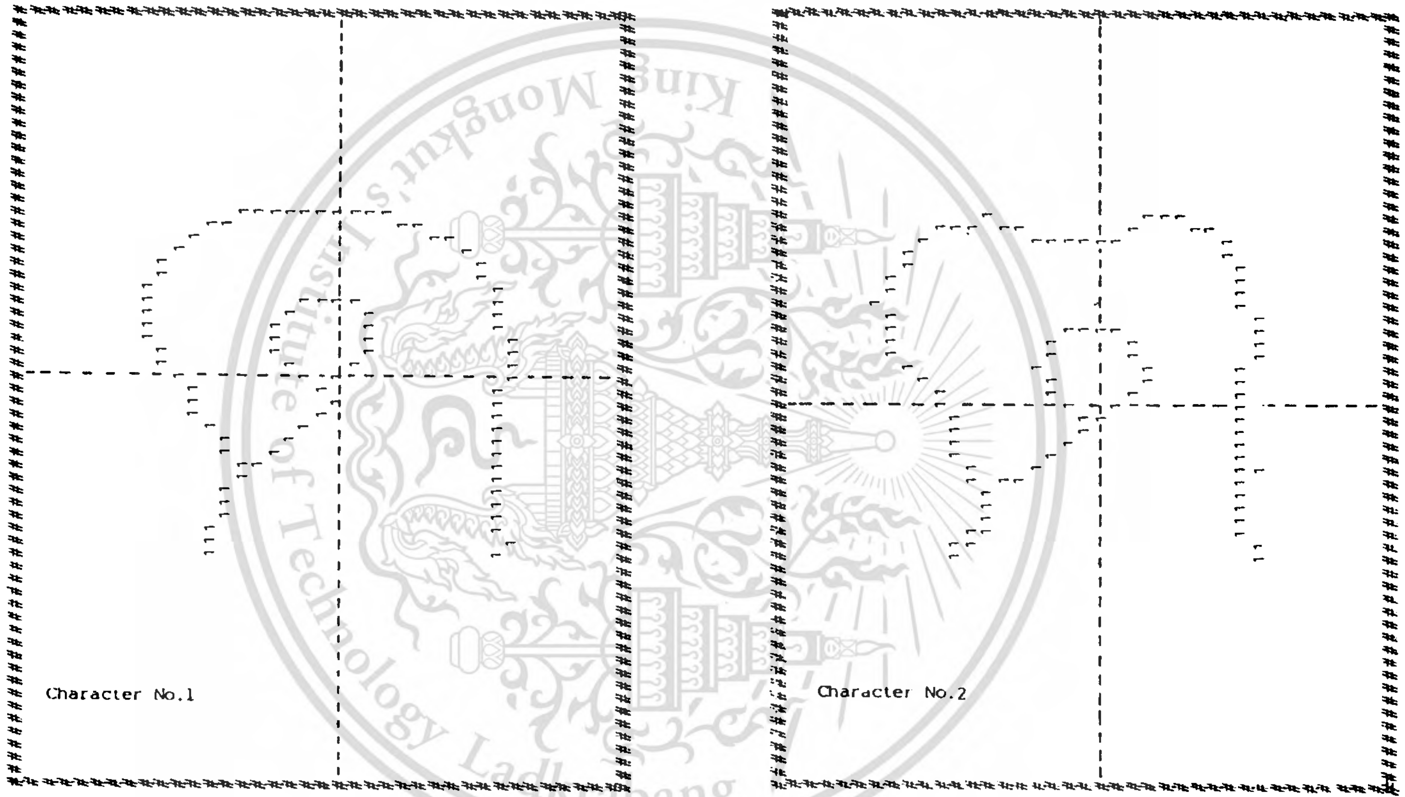


Figure 39 Distinguishing the characters by a special method.

6.6.4 Fine classification procedure of using decision table

We can perform the classification procedure by inspection of the decision table. After the unknown input data passed the rough classification stage, the sub-category of this input data is identified, and the fine classification procedure is started at that sub-category by inspecting the weight of the character. The properties of the character are then inspected starting from quadrants 1, 2, 3 and 4 respectively as, for example, shown by the flow chart of sub-category ω_{62} in Fig. 40.

6.7 Conclusions of fine classification by using topological properties of characters

1.) Successful recognition of printed Thai characters by using topological properties has been achieved for fine classification of learning-pattern input data.

2.) With corrupted line pattern input data, as shown in Fig. 41, the percentage of correctness is still high enough for character recognition.

3.) The weak point of this fine classification is that if a corrupted line occurs in the pattern input data, the skeletal shape of the character will be slightly distorted after the thinning process, and hence the properties will be also distorted in some quadrants (see Fig. 41).

4.) Consequently there will be an error in the weight of the character, and the cluster of this character will also be in error. To avoid this, we should not use weight to cluster the

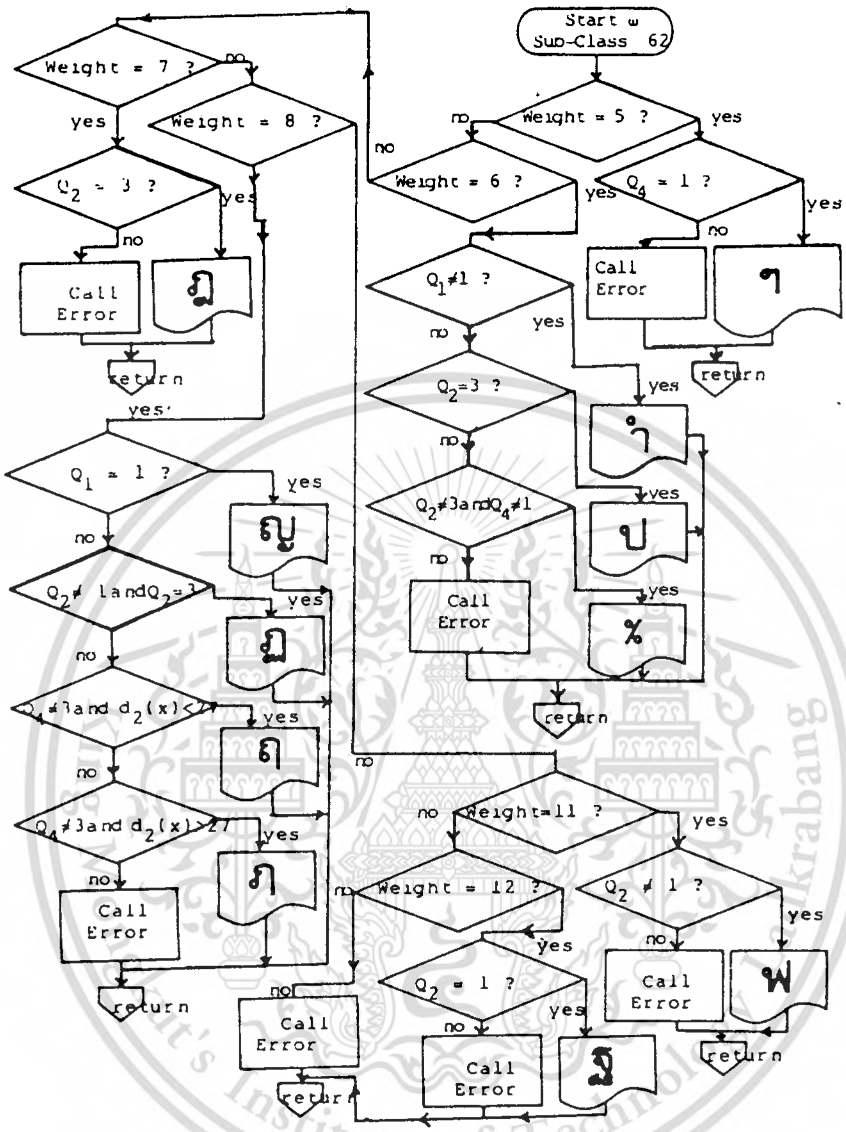


Figure 40 Procedure of the fine classification stage.

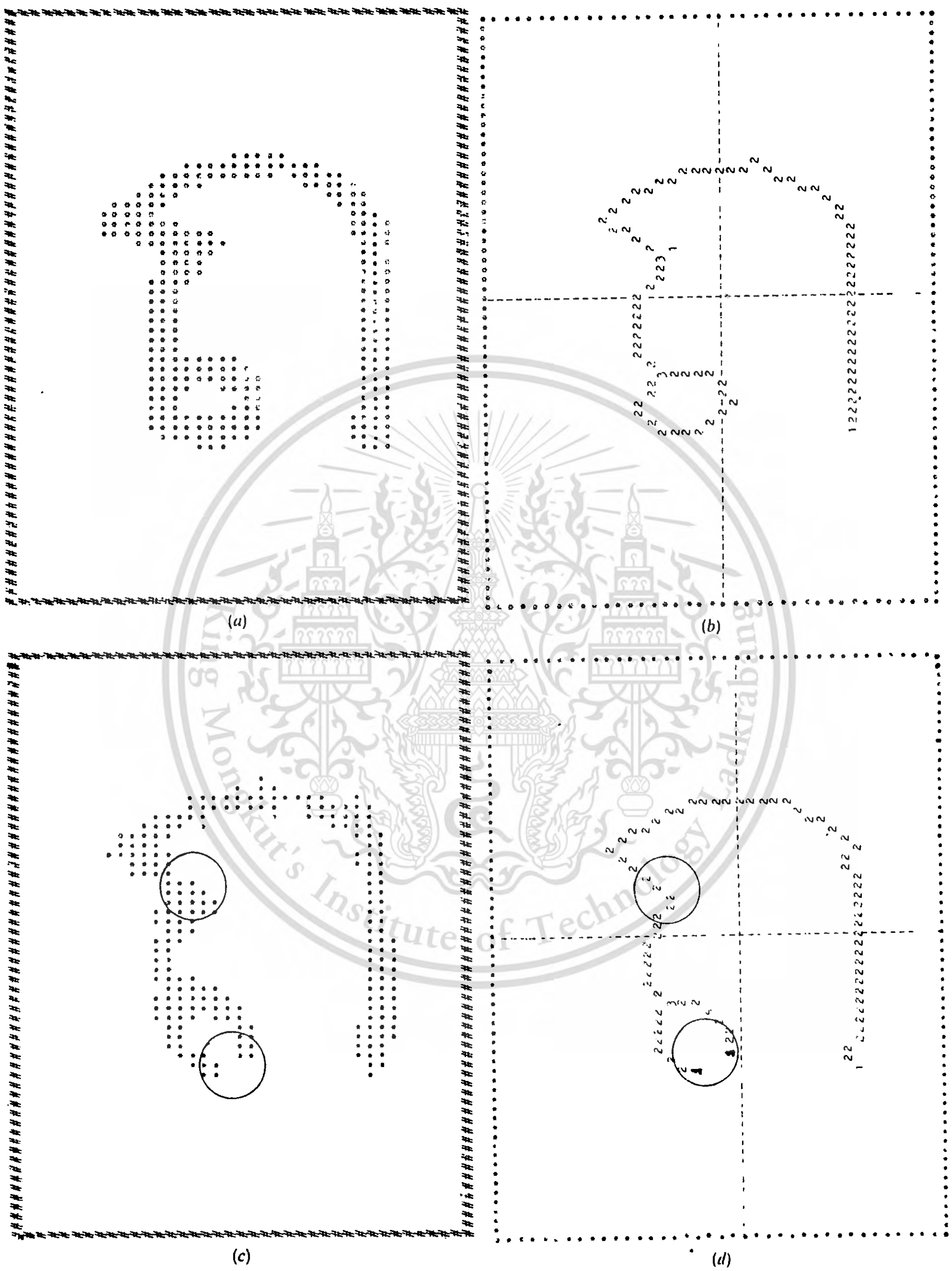


Fig. 4[1] : (a) Normal pattern, (b) Normal properties,
(c) Abnormal pattern, (d) Abnormal properties.

character, and discard those properties in that quadrant.

5.) Other cases which we must pay careful attention to are the small circle called the head of the character. If the head of character is broken, the properties indicating the head at that quadrant will disappear, and the cluster of that character will definitely give an error.

6.) The labour involved in the process of fine classification by using topological properties.

7.) The problems as said above have been solved; and hence saving of storage area of computer and high enough recognition rate for character recognition are obtained.

8.) Training patterns (learning pattern) are each original pattern character which is derived in section 1.3.

9.) FORTRAN programming is used in the implementation of fine classification with NEC system 300 main frame computer, 2MB.

7. CONCLUSION OF PRINTED THAI CHARACTERS RECOGNITION

1.) In this thesis, the attempt has been made to use topological properties of characters, subpattern matching and K-L expansion for recognizing printed Thai characters. Successful recognition has been achieved for learning-pattern input data and for single-font characters.

2.) The advantage of recognition method which has been used in this thesis is dividing the recognition method into rough and fine classification stage. This method can improve the recognition rate, and the recognition time. This is the original idea of this thesis.

3.) The best results are obtained from the recognition by using K-L expansion, it has been proved that K-L expansion is a useful tool for printed Thai characters recognition.

4.) Eventhough successful recognition is obtained from subpattern matching method, there is still an ambiguity in the process of selecting the optimum explicit pieces of fine classification stage.

5.) There are many problems as described in the preceding text in the recognition by using topological properties of characters, and also much labour involved in this method. But such problems have been solved, and the labour was overcome. Hence high enough recognition rate is obtained. It is still valid approach for printed Thai characters recognition.

The method to overcome the problem in the recognition by using topological properties of characters is the original idea of this thesis.

6.) Reduced pattern method is used to reduce 1's data of character pattern, if a technique store only 1's data is used, saving of storage area of computer is obtained.

7.) Blurring method is a useful tool to eliminate noise, and to eliminate fine features difference between printed Thai characters. Increasing the similarity values between blurred characters, the method can get the best criterion decision value of clustering characters. Then, the recognition rate is improved. This is the original idea of this thesis.

8.) The best results are obtained in the process for determining a standard pattern of category by using K-L expansion. Then, recognition rate and recognition time are improveed, and storage area of computer is decreased. This is the original idea of this thesis.

9.) There is only simple mathematics in the recognition by using topological properties of characters. Program of computer for the implementation of this method is simple, so it can be said that printed Thai characters recognition by using topological properties of characters is a simple method.

10.) This thesis is emphasized on experimental approach in recognizing single-font printed Thai characters.

11.) K-L expansion has never been used with character recognition. In this thesis, the distribution of printed Thai character patterns is investigated on eigenvector space of K-L expansion, it is an original idea, useful also for other character character recognition.

12.) K-L expansion can be applied for handwritten character recognition, if the problem in calculating eigenvector of large dimemsion data is overcome.

13.) Match only with subpattern but does not match with all patterns in a category is an original idea to save the recognition time and to save the storage area of dictionary in recognition by using subpattern matching.

14.) In the future, three methods of recognizing printed Thai characters in this thesis will be tried in a realistic environment with noise coming from equiptment and from the paper material. The different kinds of font characters will be used as input data for both rough classification and fine classification. Machine level language in the implementation of these method will also be used.

15.) In future, these methods in this thesis will be tried to recognize handwritten Thai characters.

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APPENDIX 1

COMPUTER PROGRAMMING FOR THINNING ALGORITHM

```
      SUBROUTINE HILDITCH (ML,F,IE,JE,JSTEP)
C     *** THINNING ALGORITHM BY J.D.HILDITH
C     --- F(IE,JE) = BINARY FIGURE (INPUT)
C     --- ML(IE,JE) = MEDIAL LINE (OUTPUT)
C     --- JSTEP = NUMBER OF ITERATIONS
      IMPLICIT INTEGER (A-Z)
      INTEGER F(IE,JE),ML(IE,JE),A(9),B(4),C(9)
C******  PREPROCESSING FOR FRAME-BOUNDARY  ****
      DO 1 I = 1,IE
      F(I,1) = -1
      F(I,JE) = -1
    1 CONTINUE
      DO 2 J = 1,JE
      F(1,J) = -1
      F(IE,J) = -1
    2 CONTINUE
C---------------------------------------------
      M = 1
 1000 R = 0
      DO 200 J = 2,JE-1
      DO 100 I = 2,IE-1
C*****   CONDITION 1  ************************
      IF(F(I,J).LE.0) GO TO 100
      A(1) = F(I+1,J)
      A(2) = F(I+1,J-1)
      A(3) = F(I,J-1)
      A(4) = F(I-1,J-1)
      A(5) = F(I-1,J)
      A(6) = F(I-1,J+1)
      A(7) = F(I,J+1)
      A(8) = F(I+1,J+1)
C*****   CONDITION 2  ************************
      DO 20 II = 1,8
      IF(A(II).NE.-M) GO TO 21
      A(II) = 1
      C(II) = 0
      GO TO 20
   21 IF(A(II).LT.0) A(LI) = 0
      C(II) = A(II)
   20 CONTINUE
      A(9) = A(1)
      C(9) = C(1)
      U = 4-(A(1)+A(3)+A(5)+A(7))
      IF(U.EQ.0) GO TO 100
C*******   CONDITION 3&4   ***********************
      V = 0
      W = 0
      DO 34 II = 1,8
      IF(A(II).EQ.1) V = V+1
      IF(C(II).EQ.1) W = W+1
   34 CONTINUE
```

```
      IF(V.LE.1) GO TO 100
      IF(W.EQ.0) GO TO 100
C********   CONDITION 5    ***********************
      CALL CONNEC (A,X,8)
      IF(X.NE.1) GO TO 100
C********   CONDITION 6    ***********************
      IF(F(I,J-1).NE.-M) GO TO 61
      A(3) = 0
      CALL CONNEC (A,X1,8)
      IF(X1.NE.1) GO TO 100
      A(3) = 1
   61 IF(F(I-1,J).NE.-M) GO TO 62
      A(5) = 0
      CALL CONNEC (A,XI,8)
      IF(XI.NE.1) GO TO 100
   62 F(I,J) = -M
      R = R+1
  100 CONTINUE
  200 CONTINUE
      IF(M.EQ.JSTEP) GO TO 999
      IF(R.EQ.0) GO TO 99
      M = M+1
      GO TO 1000
C**********  POSTPROCESSING FOR BINARIZATION  ***********
   99 JSTEP = M-1
  999 CONTINUE
      DO 400 J = 1,JE
      DO 300 I = 1,IE
      IF(F(I,J).LT.0) GO TO 310
      ML(I,J) = F(I,J)
      GO TO 300
  310 F(I,J) = 1
      ML(I,J) = 0
  300 CONTINUE
  400 CONTINUE
      RETURN
      END

      SUBROUTINE CONNEC (A,X,CN)
C     *** CALCULATION OF CONNECTING NUMBER ***
C     --- A = NEIBOURHOOD POINTS
C     --- X = CONNECTING NUMBER
C     --- CN = CONNECTEDNESS (4 OR 3)
      IMPLICIT INTEGER (A-Z)
      INTEGER A(9),B(4)
C     ***   CONNECTIVITY CHECK  **************
      IF(CN.EQ.4) GO TO 4
      IF(CN.NE.8) GO TO 9999
      DO 10 II = 1,4
      B(II) = (1-A(II*2-1))*MAX0(A(II*2),A(II*2+1))
   10 CONTINUE
      GO TO 30
    4 DO 20 II = 1,4
      B(II) = A(II*2-1)*(1-A(II*2)*A(II*2+1))
   20 CONTINUE
```

```
   30 X = B(1)+B(2)+B(3)+B(4)
      RETURN
 9999 PRINT," CONNEC--1NVALID CONNECTIVITY"
      RETURN
      END
```

APPENDIX 2

COMPUTER PROGRAMMING FOR COMPUTING EIGENVALUE AND EIGENVECTOR OF K-L EXPANSION

```
      DIMENSION IP(64,32,4),A(2048,4),EV(4,4),C(4,4),E(4),IM(4),
     *EVS(64,32,4)
      N = 2
      DO 5 M = 1,N
      READ(5,100)((IP(J,I,M),I=1,8),J=1,10)
  100 FORMAT(8I1)
    5 CONTINUE
      DO 60 M=1,N
      L=0
      DO 60 J=1,10
      DO 60 I=1,8
      L=L+1
      A(L,M)=FLOAT(IP(J,I,M))
   60 CONTINUE
      WRITE(6,8)
    8 FORMAT(1H1,23H**** INPUT PATTERN ****/)
      WRITE(6,9)((A(L,M),L=1,80),M=1,2)
    9 FORMAT(1H ,10(8F2.0/1X))
      TRC=0.0
      DO 80 I=1,N
      DO 80 J=1,I
      C(I,J)=0.0
      DO 81 K=1,80
      C(I,J)=C(I,J)+A(K,I)*A(K,J)
   81 CONTINUE
      IF(I.EQ.J) TRC=TRC+C(I,J)
      IF(I.NE.J) C(J,I)=C(I,J)
   80 CONTINUE
      WRITE(6,600) TRC
  600 FORMAT(1H ,6HTRACE=,E15.7)
      WRITE(6,999) C
  999 FORMAT(1HO,18HCORRELATION MATRIX/1H ,4(4E15.7/1H ))
      OUT=0.0
      MAX=30
      CALL JAC(C,E,EV,4,4,OUT,MAX)
      WRITE(6,601) OUT
  601 FORMAT(1HO,4HOUT=,E15.7)
      AMXVL=TRC*2.0
      DO 90 J=1,N
      AMX=-1.E10
      DO 91 I=1,N
      IF(E(I).LE.AMX.OR.E(I).GE.AMXVL) GO TO 91
```

```
      IF(OUT.GE.0.5) WRITE(6,3) A,V
    3 FORMAT(1H ,5E15.7)
      P=1
      Q=2
      DO 5 I=1,NM1
      IP1=I+1
      DO 5 J=IP1,N
      IF(ABS(A(I,J)).LE.ABS(A(P,Q))) GO TO 5
      P=I
      Q=J
    5 CONTINUE
      IF(OUT.GE.0.5) WRITE(6,6) P,Q
    6 FORMAT(1H0,2I5)
      IF(ABS(A(P,Q)).LT.EPS) GO TO 73
      IF(ABS(A(P,P)-A(Q,Q)).LT.EPS) GO TO 7
      R=2.0*A(P,Q)/(A(P,P)-A(Q,Q))
      T=0.5*ATAN(R)
      GO TO 9
    7 T=0.7853982
    9 CONTINUE
      IF(OUT.GE.0.5) WRITE(6,10) T
   10 FORMAT(1H ,E15.7/)
      S=SIN(T)
      C=COS(T)
      DO 11 J=1,N
      APJ=A(P,J)
      AQJ=A(Q,J)
      A(P,J)=APJ*C+AQJ*S
      A(Q,J)=-APJ*S+AQJ*C
      VPJ=V(P,J)
      VQJ=V(Q,J)
      V(P,J)=VPJ*C+VQJ*S
      V(Q,J)=-VPJ*S+VQJ*C
   11 CONTINUE
      DO 12 I=1,N
      AIP=A(I,P)
      AIQ=A(I,Q)
      A(I,P)=AIP*C+AIQ*S
      A(I,Q)=-AIP*S+AIQ*C
   12 CONTINUE
  100 CONTINUE
   73 DO 13 I=1,N
   13 E(I)=A(I,I)
      RETURN
      END
```

```
**** INPUT PATTERN ****
0.0.0.0.0.0.0.0.
0.0.0.0.0.0.0.0.
0.0.1.1.1.1.1.0.
0.0.1.0.0.0.1.0.
0.0.1.0.0.0.1.0.
0.0.1.1.1.0.1.0.
```

```
      AMX=E(I)
      IM(J)=I
   91 CONTINUE
      AMXVL=AMX
   90 CONTINUE
      CRAMD=0.0
      DO 20 I=1,N
      J=IM(I)
      RAMUDA=E(J)/TRC*100.0
      CRAMD=CRAMD+RAMUDA
      WRITE(6,602) E(J),RAMUDA,CRAMD
  602 FORMAT(1H ,12HEIGEN VALUE=,E15.7,3X,13HCONTRIBUTION=,F7.3,
     *3X,17HCUM.CONTRIBUTION=,F7.3)
      WRITE(6,660) (EV(J,M),M=1,N)
  660 FORMAT(1H ,14HEIGEN VECTOR :=,2(E15.7,2X))
   20 CONTINUE
      WRITE(6,612)
  612 FORMAT(1H1)
      DO 95 M=1,2
      K=IM(M)
      L=0
      DO 92 J=1,10
      DO 92 I=1,8
      L=L+1
      EVS(J,I,M)=0.0
      DO 92 IJ=1,N
      EVS(J,I,M)=EVS(J,I,M)+EV(K,IJ)*A(L,IJ)
   92 CONTINUE
      ANORM=SQRT(ANORM)
      DO 94 I=1,8
      DO 94 J=1,10
      EVS(J,I,M)=EVS(J,I,M)/ANORM
   94 CONTINUE
      WRITE(6,611)M
  611 FORMAT(1H ,25HPRINCIPAL COMPORNENT NO.=,I2)
      WRITE(6,610) ((EVS(J,I,M),I=1,8),J=1,10)
  610 FORMAT(1H0,8F7.4)
   95 CONTINUE
      STOP
      END

      SUBROUTINE JAC(A,E,V,N,M,OUT,MAX)
      DIMENSION A(M,M),E(M),V(M,M)
      INTEGER P,Q
      NM1=N-1
      W=0.0
      DO 14 I=1,N
      DO 14 J=I,N
   14 W=AMAX1(W,ABS(A(I,J)))
      EPS=0.0000001*W
      DO 1 I=1,N
      DO 2 J=1,N
    2 V(I,J)=0.0
    1 V(I,I)=1.0
      DO 100 KAISUU=1,MAX
```

```
0.0.1.0.1.0.1.0.
0.0.1.0.0.0.1.0.
0.0.1.0.0.0.1.0.
0.0.0.0.0.0.0.0.


0.0.0.0.0.0.0.0.
0.0.0.0.0.0.0.0.
0.0.1.1.1.1.1.0.
0.0.1.0.0.0.1.0.
0.0.1.0.0.0.1.0.
0.0.1.1.1.0.1.0.
0.0.1.0.1.0.1.0.
0.0.1.0.1.0.1.0.
0.0.1.1.1.0.1.0.
0.0.0.0.0.0.0.0.

TRACE=  0.4300000E+02

CORRELATION MATRIX
  0.2000000E+02  0.2000000E+02  0.0           0.0
  0.2000000E+02  0.2300000E+02  0.0           0.0
  0.0            0.0            0.0           0.0
  0.0            0.0            0.0           0.0


OUT=  0.0

EIGENVALUE=0.4155615E+02CONTRIBUTION=96.642CUM.CONTRIBUTION=96.64
EIGENVECTOR := 0.6801507E+00     0.7330723E+00
EIGENVALUE=0.1443829E+01CONTRIBUTION= 3.358CUM.CONTRIBUTION=100.0
EIGENVECTOR := 0.7330723E+00    -0.6801507E+00


PRINCIPAL COMPORNENT NO.=1
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0     0.0     0.2192  0.2192  0.2192  0.2192  0.2192  0.0
0.0     0.0     0.2192  0.0     0.0     0.0     0.2192  0.0
0.0     0.0     0.2192  0.0     0.0     0.0     0.2192  0.0
0.0     0.0     0.2192  0.2192  0.2192  0.0     0.2192  0.0
0.0     0.0     0.2192  0.0     0.2192  0.0     0.2192  0.0
0.0     0.0     0.2192  0.0     0.1137  0.0     0.2192  0.0
0.0     0.0     0.2192  0.1137  0.1137  0.0     0.2192  0.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0

PRINCIPAL COMPORNENT NO.=2
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0     0.0     0.0440  0.0440  0.0440  0.0440  0.0440  0.0
0.0     0.0     0.0440  0.0     0.0     0.0     0.0440  0.0
0.0     0.0     0.0440  0.0     0.0     0.0     0.0440  0.0
0.0     0.0     0.0440  0.0440  0.0440  0.0     0.0440  0.0
0.0     0.0     0.0440  0.0     0.0440  0.0     0.0440  0.0
0.0     0.0     0.0440  0.0    -0.5660  0.0     0.0440  0.0
```

```
0.0     0.0     0.0440-0.5660-0.5660 0.0     0.0440 0.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
```

Multiple critical pole maximally flat *RC* active filter with sharp cutoff

Prof. P. Thajchayapong, Prof. C. Kimpan and Y. Rungsunseri

Indexing terms: Filters and filtering, Design, Circuit theory and design, Active networks

Abstract: A maximally flat *RC* active filter with sharp cutoff can be achieved by including single or multiple pairs of coincident transmission zeros but at the expense of high *Q*-factor of the critical pole pair. A novel design technique is proposed here to reduce the *Q*-factor of such filters by increasing the multiplicity of the critical pole pairs. Hence a new class of sharp cutoff maximally flat *RC* active filters with low sensitivity is obtained.

1 Introduction

It has been demonstrated by Premoli [1] that the sensitivity performances of an *RC* active filter can be enhanced by using multiple critical pole pairs. Owing to the all-pole nature, this class of filters, however, exhibits a slow cutoff rate. On the other hand, another class of maximally flat filters with sharp cutoff has also been described by Budak [2] and Dutta Roy [3]. This is achieved by including a single or multiple pair of imaginary-conjugate zeros. The latter, however, does not take the sensitivity performance consideration into account. The purpose of this paper is to propose a novel design technique to obtain a maximally flat *RC* active filter with a multiple imaginary-conjugate zero pair and also a multiple critical pole pair. A class of filters with both sharp cutoff and low sensitivity is therefore obtained. In comparison to the previous results [4] which aim at reducing the number of operational amplifiers, this paper is seen as an attempt to reduce the filter sensitivity.

2 Design procedure

Let the squared magnitude of the required filter be defined as

$$|H(j\omega)|^2 = \frac{(1 - \omega^2/\omega_0^2)^{2r}}{\{(\alpha\omega)^4 - h_c(\alpha\omega)^2 + 1\}^{\mu_c}\left[1 + \sum_{i=1}^{m-2} a_i(\alpha\omega)^{2i}\right]} \tag{1}$$

where r is the number of coincident pairs of the imaginary zeros at $\pm j\omega_0$, μ_c is the multiplicity of the critical pole pair, and $m - 2$ is the remaining distinct poles, and α is the normalisation factor which gives $|H(j)|^2 = \frac{1}{2}$.

If n is the function order and δ is the degree increment due to μ_c, we then have $n = m + \delta$ and $\delta = 2(\mu_c - 1)$.

The realisation condition [5] also requires that the degree of the numerator polynomial cannot exceed that of the denominator, i.e. $r \leqslant n/2$.

Now for $|H(j\omega)|^2$ to satisfy the maximally flat criteria around $\omega = 0$, it is necessary to expand the numerator and denominator of eqn. 1 into a power series of ω^2. By equating the corresponding coefficients of the two power series, a matrix equation is formulated as

$$Ma = b - d \tag{2}$$

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where

$$M = \overbrace{\begin{bmatrix} \alpha^2 c_0 & 0 & 0 & \cdots & \\ \alpha^4 c_1 & \alpha^4 c_0 & 0 & \cdots & \\ \alpha^6 c_2 & \alpha^6 c_1 & \alpha^6 c_0 & \cdots & \\ \vdots & \vdots & \vdots & & \\ \alpha^{2(m-2)} c_{m-3} & \alpha^{2(m-2)} c_{m-4} & \alpha^{2(m-2)} c_{m-5} & \cdots & \alpha^{2(m-2)} c_0 \end{bmatrix}}^{m-2} \; m-2$$

$$a^T = [a_1, a_2, \ldots, a_{m-2}]^T$$

$$b^T = [b_1, b_2 \ldots b_{m-2}]^T$$

and

$$d^T = [\alpha^2 c_1, \alpha^4 c_2 \ldots \alpha^{2(m-2)} c_{m-2}]^T$$

The matrix element c_k in M is derived from expanding $(\omega^4 - h_c\omega^2 + 1)^{\mu_c}$ into an explicit polynomial e.g.

$$(\omega^4 - h_c\omega^2 + 1)^{\mu_c} = \sum_{k=0}^{2\mu_c} c_k \omega^{2k} \tag{3}$$

After certain algebraic manipulation [6], it can be shown that

$$c_k = \sum_i \binom{\mu_c}{i}\binom{\mu_c - i}{(k-i)/2}(h_c)^i(-1)^i \tag{4}$$

where $k = 0, 1, \ldots, 2\mu_c$, $i = 0, 2, 4, \ldots, k$ for even k, and $i = 1, 3, 5, \ldots, k$ for odd k. Also $c_k = 0$ for $k > 2\mu_c$.

The vector element b_k in b is given by

$$b_k = (-1)^k \binom{2r}{k}(1/\omega_0^2)^k \tag{5}$$

where $k = 1, 2, \ldots, 2r$. Also $b_k = 0$ for $k > 2r$.

Using the back substitution method [7], it can be shown that if $a_0 = 1$, $b_0 = 1$ and $c_0 = 1$, then

$$a_i = \frac{b_i}{\alpha^{2i}} - \sum_{j=0}^{i-1} a_j c_{(i-j)}, \quad i = 1, 2, \ldots, m-2$$

As the matrix M contains the unknowns h_c and α, this set of simultaneous equations cannot yet be solved for $a_1, a_2, \ldots, a_{m-2}$. An iterative procedure is therefore necessary. Before doing so, another function which is also obtained from the process of equating the coefficients of ω^{2k} mentioned earlier e.g.

$$F(h_c, \alpha, a) = \alpha^{2(m-1)}[c_{m-1}a_0 + c_{m-2}a_1 + \cdots + c_1 a_{m-2}] - b_{m-1} \tag{6}$$

will be used as one of the search indicator.

The search problem is decomposed into a linear subproblem to find $(a_1, a_2, \ldots, a_{m-2})$ and a nonlinear subproblem for (h_c, α) (see Fig. 1). For a given pair of (α, h_c), solving eqn. 2 always gives $(a_1, a_2, \ldots, a_{m-2})$ which in turn

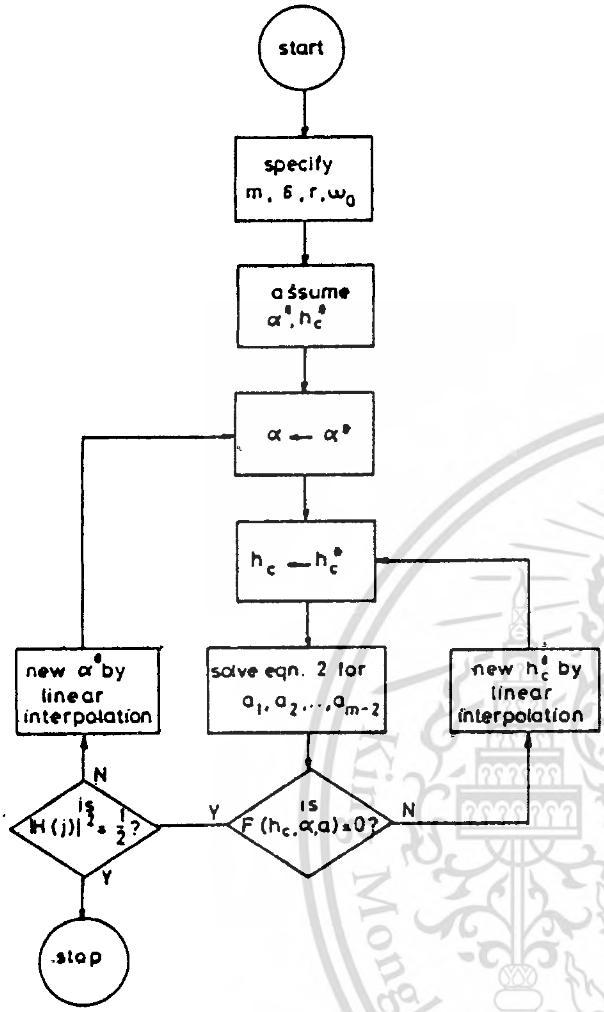

Fig. 1 *Flow chart of computer program*

provides the location of the noncritical poles. However, h_c, together with the indicator $F(h_c, \alpha, \boldsymbol{a})$, has to be searched for the appropriate location of the critical pole pair. α is to be searched for to give $|H(j)|^2 = 1/2$.

The iterative process is made simpler by *a priori* knowledge [1] that $0 < \alpha \leqslant 1$ and also $-2 \leqslant h_c < 2$. The interpolation can be accelerated by using the well known search techniques such as the binary search or Fibonacci search [8].

3 Design examples

Fig. 2 illustrates the magnitude responses for the case of $m = 10$, $\delta = 2$, $r = 1$ at various ω_0. Table 1 also shows an

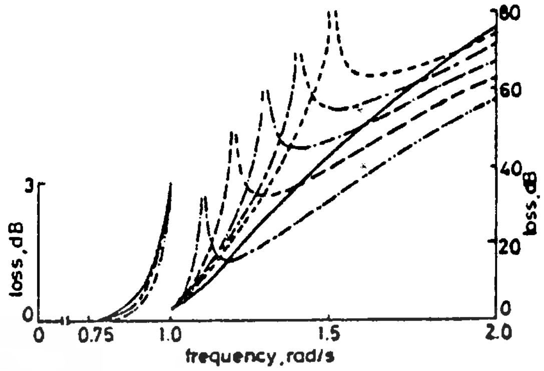

Fig. 2 *Responses for $m = 10$, $\mu_c = 2$, $r = 1$*

$\omega_0 = 1.1$
$\omega_0 = 1.2$
$\omega_0 = 1.3$
$\omega_0 = 1.4$
$\omega_0 = 1.5$
$\omega_0 = \infty$

example of variation in Q_c for the case of $r = 1$ and $\omega_0 = 1.5$ to compare with those of the all-pole case (Table 1 of Reference 1). It is obvious that Q_c decreases with increasing δ, hence the enhancement in sensitivity performances.

4 Conclusions

It has been described how to obtain a maximally flat *RC* active filter with sharp cutoff and also low sensitivity. The concepts are to include a single or multiple pair of $j\omega$-axis zero pair and a multiple pair of critical complex pole. The mathematics are nonlinear in nature but *a priori* knowledge leads us to an iterative process using a computer. Examples are demonstrated that Q-factor of the critical pole pair decreases with increasing in the degree of multiplicity. Although only lowpass filters are used in the explanation, the results can be readily applied to bandpass, highpass and bandstop using the frequency transformation techniques.

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Table 1: Values of Q_c for $r = 1$, $\omega_0 = 1.5$, $m = 3, 4, \ldots, 13$ and $\delta = 0, 2, \ldots, 10$ ($\mu_c = 1, 2, \ldots, 6$)

	μ_c	1	2	3	4	5	6
m	δ	0	2	4	6	8	10
3		1.437787	1.190172	1.080217	1.017534	0.976481	0.947216
4		1.764964	1.421839	1.262877	1.169642	1.107609	1.063188
5		2.076960	1.652514	1.444443	1.320379	1.237413	1.177670
6		2.399348	1.881662	1.624610	1.469925	1.365939	1.290846
7		2.721235	2.109493	1.803579	1.618389	1.493434	1.402999
8		3.042507	2.336286	1.981586	1.765981	1.620107	1.514351
9		3.363229	2.562268	2.158830	1.912877	1.746123	1.625066
10		3.683501	2.787622	2.335464	2.059208	1.871608	1.735268
11		4.003406	3.012482	2.511612	2.205083	1.996658	1.845048
12		4.323025	3.236954	2.687366	2.350581	2.121350	1.954478
13		4.642414	3.461113	2.862799	2.495768	2.245740	2.063616

tive discussion with Prof. T. Yangisawa of the Tokyo Institute of Technology is also gratefully acknowledged.

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Recognition of printed Thai characters using a matching method

C. Kimpan, B.Eng., M.S.E.E., A.Itoh, D.Eng. and Prof. K. Kawanishi, D.Eng.

Indexing terms: *Computer applications, Thai character recognition*

Abstract: As there are many similar characters in printed Thai, and in order to get a high recognition rate, the recognition system is separated into two stages. In the rough classification stage, fine features and noise are ignored by blurring. The blurred characters are separated into some cluster domains. The clustering criterion used is based on selection of the patterns by measuring the similarity coefficient. The Karhunen-Loève expansion is applied to get a standard pattern of each category. In the fine classification stage, subpåttern matching is used to discriminate between the characters.

List of symbols

f_x	= unknown pattern
f_{ns}	= standard pattern
$S(f_{ns}, f_x)$	= similarity of the characters
T_n	= threshold value
$\|f\|$	= norm of f
$f_1, f_2, \ldots, f_n$	= observed vector
V	= covariance matrix
λ_s	= eigenvalue
ρ_s	= eigenvector

1 Introduction

1.1 Character-pattern recognition method

A character-pattern recognition method is generally divided into matching and structure analysis. The matching method is an effective tool for the solution of problems in which the patterns include noise but exclude rotating shapes, whereas structure analysis is employed in the opposite case. As there exists noise, arising from an imperfect paper surface and interruption in the line of printed characters, a matching method is usually used. For the case of hand-written characters, their shapes are rotated according to the writer, and structure analysis is usually used.

In this paper, we propose the recognition of printed Thai characters by using a matching method. Since there are many similar characters in printed Thai, the recognition system is separated into two stages. In the rough classification stage, a blurring technique is applied to ignore fine features. The eigenvector of maximum eigenvalue of the Karhunen-Loève expansion is equivalent to a standard pattern of each category. A subpattern matching method is used in the fine classification stage. The results of experiments are presented.

1.2 Thai characters and experimental data

Thai characters [1], as shown in Fig. 1, consists of 42 consonants, 17 vowels, 7 voice tones, 10 Thai numerals and 10 Arabic numerals. A Thai language sentence, such as that in Fig. 2, is composed of consonants, vowels and voice tones, on different levels. The vertical level can be divided into four parts as shown in Fig. 3. However, as can be seen by inspection, on occasion, depending on the printing machine (see Fig. 4), a vowel can be situated above two consonants.

All Thai characters and some sentences of Thai language are printed on ordinary paper and then fed to be input of a facsimile optical character reader. This machine automatically changes the grey level of each character into sets of hexadecimal code. Each character is divided into a matrix of 128×64 pixels as shown in Fig. 5. Each point of these matrices is represented by four bits of binary or one hexadecimal number corresponding to its grey level. These binary data are recorded on magnetic tape as experimental data. The recording format is shown in Fig. 6. To give further experimental data, the hexadecimal value for each point in the matrix is converted to one binary bit, i.e. 0 in hexadecimal becomes 0 in binary but other hexadecimal values become 1 in binary. Therefore, 0 represents a white level, and 1 represents black level. However, in some cases when noise elimination is necessary, 1, 2, 3, 4 and 5 in hexadecimal may become 0 in binary, and other hexadecimal values become 1 in binary.

2 Sequence of recognition method

A flowchart, shown in Fig. 7, shows the sequence of recognition of printed Thai characters. A pattern of each

Fig. 1 *Thai character set*

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character is pulled out from the sample Thai character sentence (as shown in Fig. 2) by a technique in which the

นับตั้งแต่สมาคมส่งเสริมเทคโนโลยี (ไทย-ญี่ปุ่น) ได้ก่อตั้งขึ้น

เมื่อปี พ.ศ. 2516 เป็นต้นมา

Fig. 2 *Example of Thai language sentence*

Translation: Since Thai-Japan technology promotion organisation has been in B.E. 2516 ....

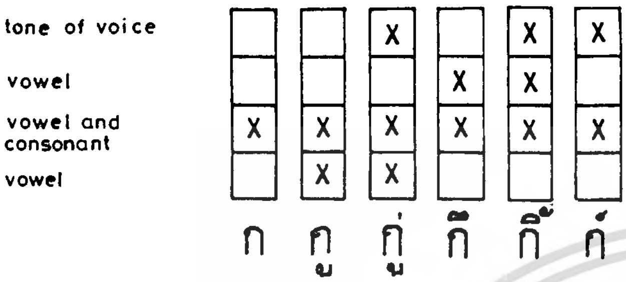


Fig. 3 *Composition of Thai character word or sentence*

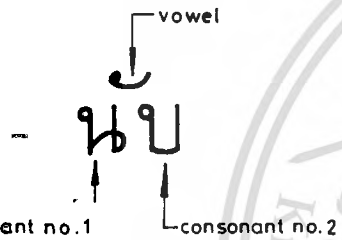


Fig. 4 *Example of Thai word*

Vowel is situated above both consonant no. 1 and no. 2

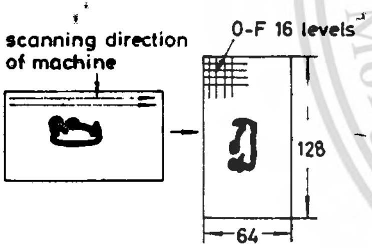


Fig. 5 *Bit pattern of a character*

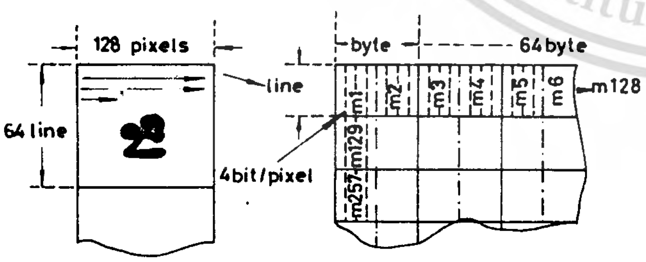


Fig. 6 *Recording format of magnetic tape*

features are extracted by one-directional scanning, performed in the vertical and horizontal directions, respectively.

The pulled out pattern is reduced to eliminate the noise and to keep the storage area of the computer as small as possible. A procedure of reduction keeping only one of four neighbouring 1's is used, i.e. if the binary values $q(i, j)$, $q(i, j+1)$, $q(i+1, j)$ and $q(i+1, j+1)$ are neighbours, the result of reduction is the integer part of

$$\{q(i, j) + q(i, j+1) + q(i+1, j) + q(i+1, j+1)\}/4 + 0.5$$

In the rough classification stage, a reduced pattern is matched with a standard pattern to cluster the characters.

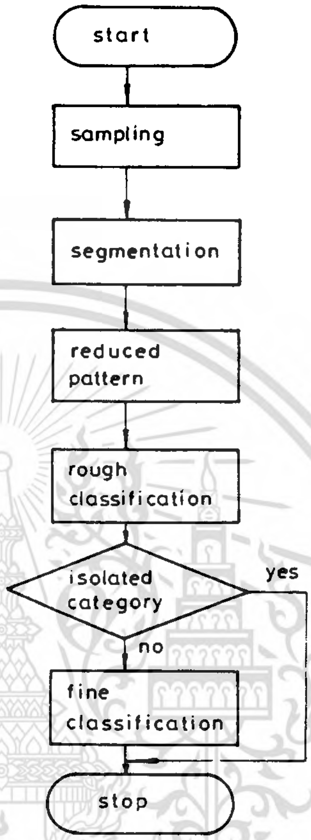


Fig. 7 *Sequence of recognition method*

Let f_{ns} be a standard pattern, f_x be the unknown pattern and $S(f_{ns}, f_x)$ be the similarity of the characters [2]. The definition of $S(f_{ns}, f_x)$ is

$$S(f_{ns}, f_x) = \frac{(f_{ns} \cdot f_x)}{\|f_{ns}\| \, \|f_x\|} \tag{1}$$

where the scalar product $(f_{ns} \cdot f_x)$ is represented by

$$(f_{ns} \cdot f_x) = \sum_i \sum_j (f_{ns(i,j)} \cdot f_{x(i,j)}) \tag{2}$$

and the norm $\|f\|$ is

$$\|f\| = \sqrt{(f \cdot f)} \tag{3}$$

The clustering criterion is based on the following equation:

$$S(f_{ns}, f_x) \geqslant T_n \tag{4}$$

Since T_n is the criterion decision value (threshold value) of a group of order n, if eqn. 4 is satisfied, f_x is a member of a group of order n. Some unknowns can be recognised in the result of rough classification. The member of clustering characters will be discriminated in the fine classification stage.

In the fine classification stage, the fine features of each character in a category are investigated. Explicit pieces are matched with the pieces of unknown pattern to recognise the characters.

3 Segmentation

Because the construction of a Thai word, as shown in Fig. 3, is different from an English word, we cannot separate the characters by scanning only in the vertical direction. In order to separate characters from the sentence, it is necessary to scan in both vertical and horizontal directions. The procedure of segmentation as shown in Fig. 8 is as follows:

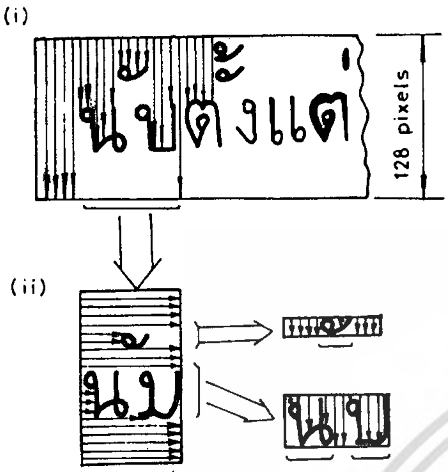


Fig. 8 *A method of segmentation*

(i) scan along the pattern of the sentence in a vertical direction to find the space, and pull out the data between two spaces

(ii) scan again along the data from item (i) in a horizontal direction, the pattern of each character is to be found between two spaces

(iii) in particular cases, if the pattern of the character is extremely large, it is necessary to scan to find the space once more in a horizontal direction along that pattern.

The procedure (ii) is very difficult to use with a pattern such as that in Fig. 9, as scanning cannot be hoirzontal only. In such a case, as scanning traverses the pattern in a horizontal direction and encounters 1's data, the scanning must then scan the vertical direction above or below that datum to find 0's data before returning again to horizontal scanning. So a combination of vertical and horizontal scanning is necessary in this case to separate one character from the other. If this fails, then it is treated as another standard pattern for future matching with unknown patterns.

4 Reduced pattern

A blurring method [2] is illustrated in Fig. 10. The bit pattern is scanned in a horizontal direction from the first

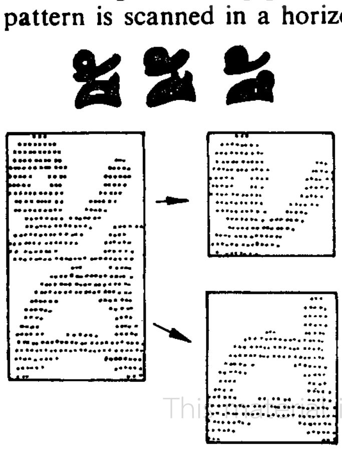


Fig. 9 *Special case of pattern in which procedure (ii) is very difficult to use*

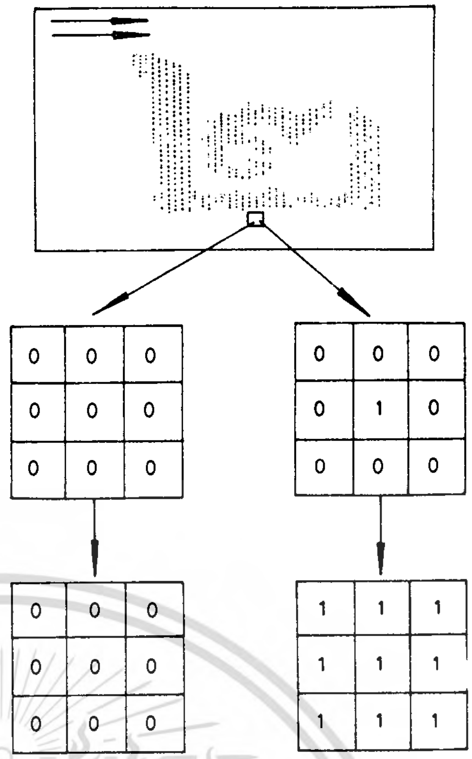


Fig. 10 *Blurring method*

to the last row, as 1's data are encountered, the neighbouring data are also converted to 1. The blurring method ascribes the name 'blurred level 1' as scanning is completed once on the pattern, in the same way 'blurred level 2' is the name of a pattern which is blurred twice and so on. As the blurred level is increased, the fine features of a character are decreased, and noise is eliminated. In order to improve the blurring method, the pattern of a character is reduced by reducing the grey level (for example, the grey levels represented by 0001, 0010 are reduced to 0000 etc.) before performing the blurring. The similarity of blurring characters (original data) and improved blurring characters (reduced data) is shown in Table 1. The similarity of

Table 1: Example of similarity (characters 1 and 2 of Fig. 1)

Blurred level	0	1	2	3	4	5
Original data	0.72	0.82	0.87	0.91	0.92	0.94
Reduced data	0.75	0.89	0.94	0.95	0.95	0.95

improved blurring characters at blurred level 2 becomes the same as the similarity of blurring characters at blurred level 5, indicating that the recognition time is reduced if the improved blurring method is used. The percentage of rough classification of improved blurring characters as blurred level 2 is shown in Table 2. t indicates the order of reduced grey level, i.e. $t = 4$ means that the grey level represented by 0001, 0010 and 0011 is reduced to white level

Table 2: Percentage of rough classification of improved blurring characters

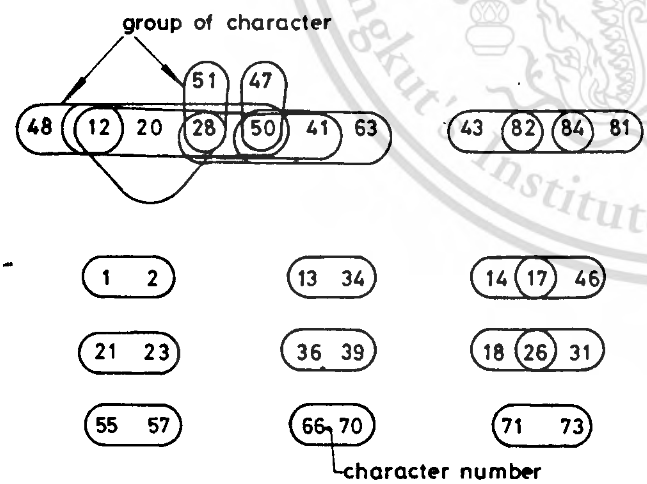
Binary threshold	$t = 0$	$t = 4$	$t = 6$
Correct %	94.0	97.6	94.0
Reject %	6.0	2.4	6.0

(0000). To improve the rejection of characters, we can proceed in two ways: firstly by adjusting blurred level, secondly by adjusting the standard pattern for matching with pattern of a category is complicated. Adjusted members in each category are transformed to a unique appropriate pattern by the Karhunen-Loève expansion.

Table 3: Examples of similarity with blurring level 5

ก 11	ฅ 12	ฆ 13	ข 14	บ 15	จ 16	ซ 17	ฌ 18	ฉ 19	ค 20	
1.00	0.93	0.85	0.86	0.75	0.83	0.85	0.78	0.83	0.90	ก 11
	1.00	0.88	0.88	0.82	0.86	0.86	0.83	0.88	0.96	ฅ 12
		1.00	0.87	0.75	0.80	0.88	0.91	0.88	0.96	ฆ 13
			1.00	0.80	0.88	0.95	0.81	0.87	0.88	ข 14
				1.00	0.85	0.77	0.72	0.79	0.82	ง 15
					1.00	0.85	0.76	0.84	0.86	จ 16
						1.00	0.82	0.84	0.88	ซ 17
							1.00	0.90	0.86	ฌ 18
								1.00	0.89	ฉ 19
									1.00	ค 20

an unknown pattern. We will introduce next a new procedure to construct the standard pattern, i.e. Karhunen-Loève expansion (K-L expansion).



threshold value = 0.94 blurring level = 5

Fig. 11 *Example of grouping characters*
Numbers refer to character numbers given in Fig. 1

5 Karhunen-Loève expansion

If we let one character be a standard pattern and other characters be unknown patterns, then the similarity value is computed using eqn. 1. Table 3 shows examples of similarity. The result of clustering characters [1, 2], using eqn. 4, is also shown in Fig. 11. Because some characters belong to the members of many groups, to determine the standard

The Karhunen-Loève expansion is a method that presents the expansion of a random vector in the eigenvectors of the covariance matrix. In the context of pattern recognition, the coefficients $f_1, f_2, \ldots, f_n$ are viewed as a feature representing the observed vector.

The covariance matrix is defined by

$$V = \left(\sum_i f_i f_i'\right) \Big/ n \tag{5}$$

An eigenvalue (λ_s) and eigenvector (ρ_s) of these matrices are determined from the equation

$$V\rho_s = \lambda_s \rho_s \tag{6}$$

The eigenvector of the maximum eigenvalue is a first principal component (equivalent to standard pattern of each category).

Fig. 12*a* shows a category no. 4 pattern, and Fig. 12*b* shows an eigenvalue of category no. 4 which is transformed by the Karhunen-Loève expansion. Fig. 12*c* shows a standard pattern of category no. 4.

6 Result of rough classification

The similarity of every character is computed. Any character which has a similarity greater than or equal to the threshold value of category *n* is to be the member of category *n*. The result of rough classification of characters is shown in Fig. 13A. In categories which have only one member, this stage is the final recognition as shown in Fig. 13B.

The characters in each category of rough classification are transformed by the Karhunen-Loève expansion to function as a standard pattern of each category. The learning (original) pattern of characters are matched with standard patterns of each category to test the correction members. In other words, a rotation of $\pm 5^{\circ}$ of the learning pattern of characters are also matched with standard patterns. The results of testing are shown in Table 4, the percentage of rejection and error are high. To improve rejection and error, the rotations by $\pm 5^{\circ}$ of the learning pattern of characters are included to act as the new standard patterns. The results of testing with new standard patterns are shown in Table 5.

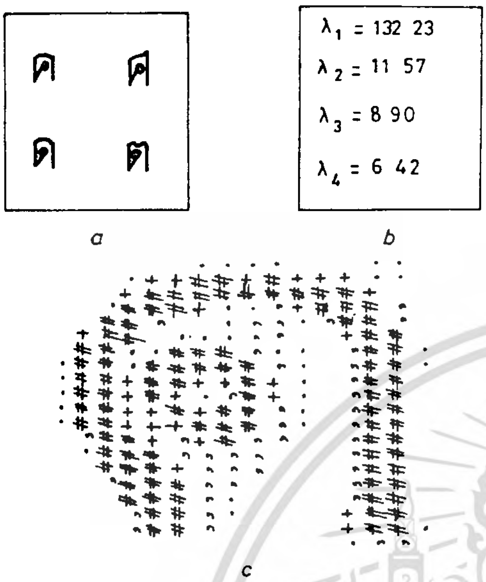


Fig. 12 *Category no. 4*
a Pattern of category no. 4 (see Fig. 13*a*)
b Eigenvalue of category no. 4
c Standard pattern of category no. 4

7 Fine classification

The fine features of each character in a category are investigated. The explicit pieces shown in Fig. 14 are used as standard patterns for matching pieces of unknown pattern from the same positions. Which piece is determined depends on the particular category of which the unknown pattern is a member. The position measured from the centre of the pattern and the dimension of explicit pieces are selected by visual inspection. The number of explicit pieces for each character are varied with respect to the categories, for example, Fig. 14*a* uses only one piece, Fig. 14*d* uses two pieces etc. To get the best fine classification, in some categories we tried several times to change the position, dimension and the number of explicit pieces. The results of fine classification are shown in Table 6, the percentage of recognition correction of learning pattern and of the rotation by $+5^{\circ}$ is 100%, only that of the rotation by -5° is less than 100%.

8 Conclusion

The percentage of recogntion correction of printed Thai characters by using the matching method as described

Table 4: Result of testing for rough classification

Pattern	Correct %	Reject %	Error %
Learning	100	—	—
Rotation (-5°)	94.0	4.8	1.2
Rotation ($+5^{\circ}$)	94.0	6.0	—

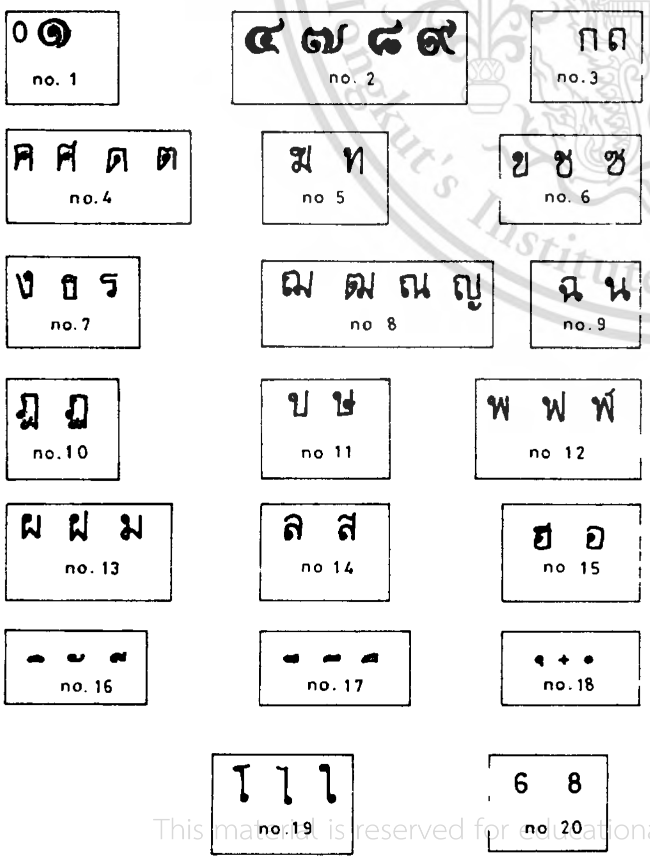


Fig. 13A *Category numbers of rough classification*

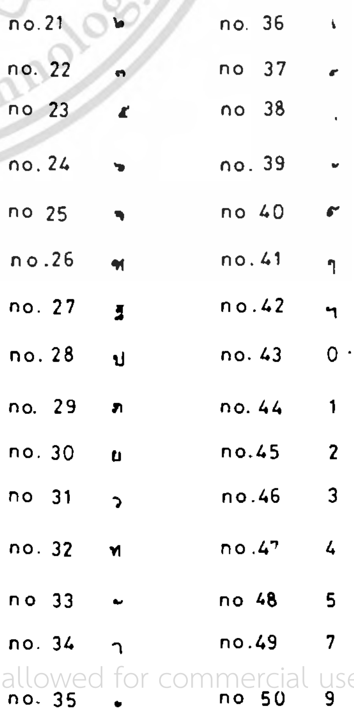


Fig. 13B *Final recognition for one-member categories*

Table 5: Result of testing for classification

Rotation deg.	Correct %	Reject %	Error %
0	100	—	—
−5	98.8	—	1.2
+5	98.8	—	1.2

Karhunan-Loève expansion including rotation pattern

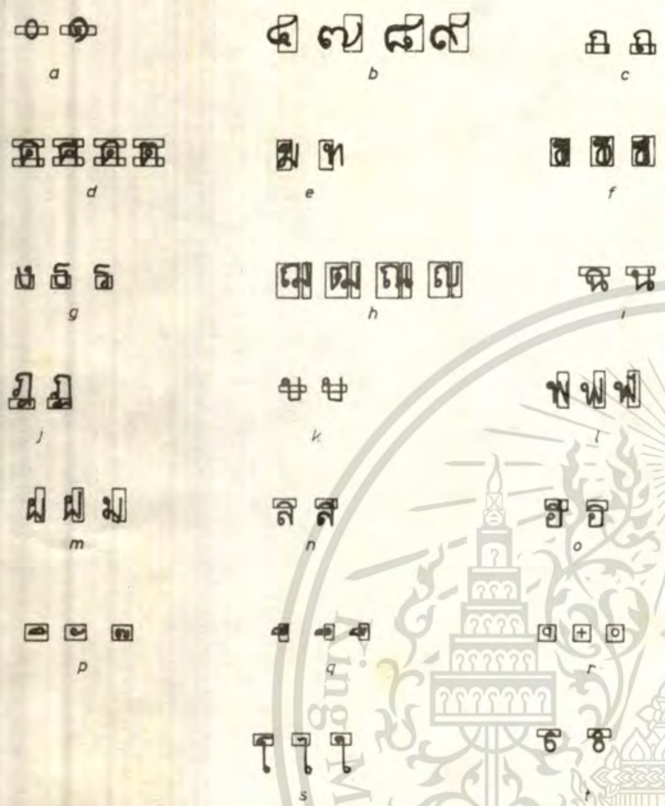


Fig. 14 *Pattern for subpattern matching*

Table 6: Result of fine classification

Pattern	Correct %	Reject %	Error %
Learning	100	—	—
Rotation (−5°)	98.2	1.2	—
Rotation (−5°)	100	—	—

in the preceding text obtained 100% success in both rough classification and fine classification of learning-pattern input data. In case of rotation input data, the percentage of correction is high enough for character recognition. In the future, different kinds of font characters will be used as input data for both rough classification and fine classification. Other methods will be used for the recognition of printed Thai characters to keep the recognition time as short as possible.

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Invited paper

Printed Thai character recognition using topological properties method

CHOM KIMPAN†

This paper describes a printed Thai character recognition method which uses topological properties. The recognition system is divided into two stages. In the rough classification stage, a thinning technique is applied to eliminate noise and fine features. The thinned characters are separated into cluster domains by using a linear decision function based on the height and width of each character. In the fine classification stage, the 'connected numbers' of points in the character pattern are computed. Topological properties are then used to distinguish between the characters.

List of symbols

NC(4)	Connected number 4
NC(8)	Connected number 8
S_i	Set of space
$d_i(x)$	A linear decision function
$(x_1, x_2, \ldots, x_n, 1)'$	Augmented pattern
$(w_1, w_2, \ldots, w_{n+1})'$	Weight vectors
$\omega_1, \omega_2, \ldots, \omega_M$	Labels of the classes
$F(i, j)$	Pattern of a character
I_x, I_y	Coordinates of the centre of gravity of a character
Q_1, Q_2, Q_3, Q_4	Labels of quadrants
P1, P2, P3	Labels of properties

1. Introduction

With advances in information processing systems, the need to deal with data or documents containing Thai characters has been increasing rapidly. An optical character recognition (OCR) machine is therefore preferred as a fast and labour-saving device. In order to develop such a machine, this paper describes a printed Thai character recognition method using topological properties. The method can be regarded as an alternative technique to the previous results given in Kimpan *et al.* (1983).

The Thai characters and experimental data in this paper are the same as those used in the earlier paper (see Appendix 1).

The sequence of the recognition method is shown in Fig. 1. The sampling and segmentation procedures are also the same as those used in the matching method—

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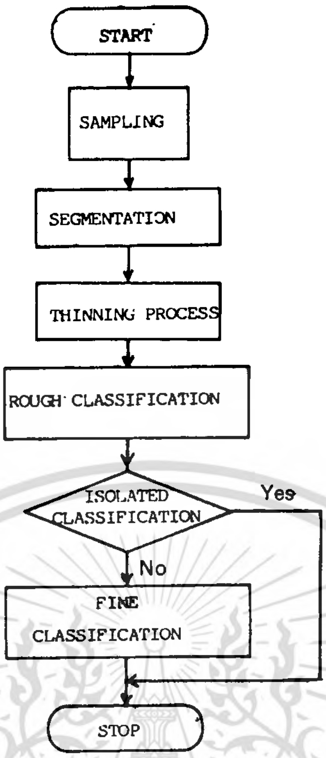


Figure 1. Sequence of recognition method.

see Appendix 2. After each character is separated from the sentence, the noise and non-essential elements are eliminated by thinning to leave only skeletal pixels. In the rough classification stage the thinned characters are separated into cluster domains by using a linear decision function based on the height and width of each character. Then, in the fine classification stage, the 'connected numbers' of the points in the pattern of the character are computed. Topological properties are then used to distinguish between characters.

2. Algorithm for the 'connected number' (Rosenfeld 1970, Yokoi *et al.* 1973)

The 'connected number' describes a relationship between the pixels in the character pattern. It may be computed from the following algorithm.

Let x_0 be a point of a digital picture that is to be analysed. Neighbouring points are x_1, x_2, x_3, x_4, x_5, x_6, x_7 and x_8 as shown in Fig. 2. The connected numbers

x_4	x_3	x_2
x_5	x_0	x_1
x_6	x_7	x_8

Figure 2. Connectivity structure.

NC(4)=4				NC(4)=0			
NC(8)=0		1		NC(8)=4	1		1
	1	1	1			1	
		1			1		1
		(a)				(b)	

Figure 3. Intersection strokes.

NC(4) and NC(8) can be computed from the following equations

$$\mathrm{NC}(4) = \sum_{k \in s_i} (x_k - x_k x_{k+1} x_{k+2}) \tag{1}$$

$$\mathrm{NC}(8) = \sum_{k \in s_i} (\bar{x}_k - \bar{x}_k \bar{x}_{k+1} \bar{x}_{k+2}) \tag{2}$$

where $s_i = 1, 3, 5, 7$ for $k \leqslant 8$; and for $k \geqslant 9 \rightarrow k = k - 8$.

For example, in Fig. 3(*a*) we have

$$\begin{aligned} \mathrm{NC}(4) &= x_1 + x_3 + x_5 + x_7 - x_1 x_2 x_3 - x_3 x_4 x_5 - x_5 x_6 x_7 - x_7 x_8 x_1 \\ &= 1 + 1 + 1 + 1 - 1 \,.\, 0 \,.\, 1 - 1 \,.\, 0 \,.\, 1 \,.\, -1 \,.\, 0 \,.\, 1 - 1 \,.\, 0 \,.\, 1 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \mathrm{NC}(8) &= \bar{x}_1 + \bar{x}_3 + \bar{x}_5 + \bar{x}_7 - \bar{x}_1 \bar{x}_2 \bar{x}_3 - \bar{x}_3 \bar{x}_4 \bar{x}_5 - \bar{x}_5 \bar{x}_6 \bar{x}_7 - \bar{x}_7 \bar{x}_8 \bar{x}_1 \\ &= 0 + 0 + 0 + 0 - 0 \,.\, 1 \,.\, 0 - 0 \,.\, 1 \,.\, 0 - 0 \,.\, 1 \,.\, 0 - 0 \,.\, 1 \,.\, 0 \\ &= 0 \end{aligned}$$

Similarly, from Fig. 3(*b*) we obtain

$$\begin{aligned} \mathrm{NC}(8) &= 0 + 0 + 0 + 0 - 0 \,.\, 1 \,.\, 0 - 0 \,.\, 1 \,.\, 0 - 0 \,.\, 1 \,.\, 0 - 0 \,.\, 1 \,.\, 0 \\ &= 0 \end{aligned}$$

$$\begin{aligned} \mathrm{NC}(8) &= 1 + 1 + 1 + 1 - 1 \,.\, 0 \,.\, 1 - 1 \,.\, 0 \,.\, 1 - 1 \,.\, 0 \,.\, 1 - 1 \,.\, 0 \,.\, 1 \\ &= 4 \end{aligned}$$

3. **The thinning algorithm** (Stefanelli and Rosenfeld 1971)

To eliminate noise and improve the features, to make it easier to measure the heights and widths of characters, the characters must be thinned. The thinning algorithm used in this paper is an improvement on J. D. Hilditch's algorithm, as follows:

```
BEGIN
F(I,J) IS POINTS IN PATTERN, SET BORDERS OF PATTERN
  FOR ALL PIXELS DO:
  FOR ALL POINTS IN PATTERN DO:
  BEGIN
  IF PIXELS EQUALS 0 THEN LABEL WITH 4
    THAT IS TO BE SET IN TERMS OF PROPERTIES,
    NEIGHBOURING POINTS F(I,J) ARE A(1) TO A(8)
      FOR A(II) EQUALS A(1) TO A(8) DO:
        BEGIN
```

```
9           IF A(II) IS -1 THEN
              A(II) = 1
              C(II) = 0
10          ELSE
              IF A(II)< = 0 THEN
                A(II) = 0
11            ELSE
                MOVE A(II) TO C(II)
12            END
13          END IF
14        MOVE A(1) TO A(9) AND C(1) TO C(9)
15        U = 4 - (A(1) + A(3) + A(5) + A(7))
16        BEGIN
17          IF U = 0 THEN LABEL WITH 4
18            FOR A(II) EQUALS A(1) TO A(8) DO:
19              BEGIN
20              IF A(II) = 0 THEN LABEL WITH 18
21              IF A(II) = 1 THEN LABEL WITH 18
                  V = V + 1
                  W = W + 1
22              END IF
23            IF V = < 1 THEN LABEL WITH 4
24            IF W = 0 THEN LABEL WITH 4
                CALL SUB-CONNEC (A,X)
25            END IF
26            IF X = 0 THEN LABEL WITH 4
27              IF A(3) = -1 THEN
                  A(3) = 0
                  CALL SUB-CONNEC (A,X)
28            IF X = 0 THEN LABEL WITH 4
               A(3) = 1
29            END IF;
30          ELSE
31            IF A(5) = -1 THEN TO LABEL 33
                A(5) = 0
                CALL SUB-CONNEC (A,X)
32              IF X = 0 THEN LABEL 3
33                F(I,J), = -1
34                R = R + 1
35              END IF;
36            END IF;
37          END IF;
38        END IF;
39      END IF;
40    END IF;
41  END

     SUB-CONNEC (A,X):
1  BEGIN
```

```
FOR A(II) EQUALS A(1) TO A(8) DO:
B(II) = (1 − A(II*2 − 1))*MAXO(A(II*2),A(II,2 + 1))
X = B(1) + B(2) + B(3) + B(4)
RETURN
```

An example of a bit pattern of a character is shown in Fig. 4(*a*), and the result of application of the thinning algorithm is shown in Fig. 4(*b*).

4. **Linear decision functions** (Tou and Gonzalez 1972)

At the rough classification stage, because of the widths and heights of printed Thai characters are different from each other, the thinned characters can be separated into cluster domains by using linear decision functions based on height and width. A technique in which the bit pattern is scanned in both the horizontal and vertical directions is used.

The width of a character in pixels is the distance between the left-most and the right-most columns containing 1's; and the height of a character in pixels is the distance between the upper-most and the lower-most rows containing 1's (see Fig. 4(*b*)). Let LN be the left-most column, LM the right-most column, CN the upper-most row, CM the lower-most row; and let x_1 be the width and x_2 the height. Then the width and the height of the thinned character are expressed by the following equation

$$x_1 = \mathrm{LM} - \mathrm{LN} + 1 \tag{3}$$

$$x_2 = \mathrm{CM} - \mathrm{CN} + 1 \tag{4}$$

If x_1 and x_2 are the coordinates of the plane, we can plot a graph of all printed Thai characters as shown in Fig. 5. A linear decision function to cluster the characters is of the form

$$d(x) = w_1 x_1 + w_2 x_2 + \ldots + w_n x_n + w_{n+1} = 0 \tag{5}$$

where the w are parameters and the x are coordinate variables. Equation (5) can be also expressed in the form

$$d(X) = W'X = 0 \tag{6}$$

where $X = (x_1, x_2, \ldots, x_n, 1)'$ and $W = (w_1, w_2, \ldots, w_n, w_{n+1})'$ are called the augmented pattern and weight vectors, respectively. In the two-class case, a decision function $d(X)$ is assumed to have the property

$$d(X) = W'X \begin{cases} >0 & \text{if} \quad X \in \omega_1 \\ <0 & \text{if} \quad X \in \omega_2 \end{cases} \tag{7}$$

where ω_1 and ω_2 are labels of the class. When we have more than two classes, denoted by $\omega_1, \omega_2, \ldots, \omega_M$, we must consider the multiclass case. For example, if each pattern class is separable from the other classes by a single decision surface, in this case there are M decision functions with the property

$$d_i(X) = W_i'X = \begin{cases} >0 & \text{if} \quad X \in \omega_i \\ <0 & \text{otherwise} \end{cases} \quad i = 1, 2, \ldots, M \tag{8}$$

where $W_i = (w_{i1}, w_{i2}, \ldots, w_{in}, w_{i,n+1})'$ is the weight vector associated with the ith decision function.

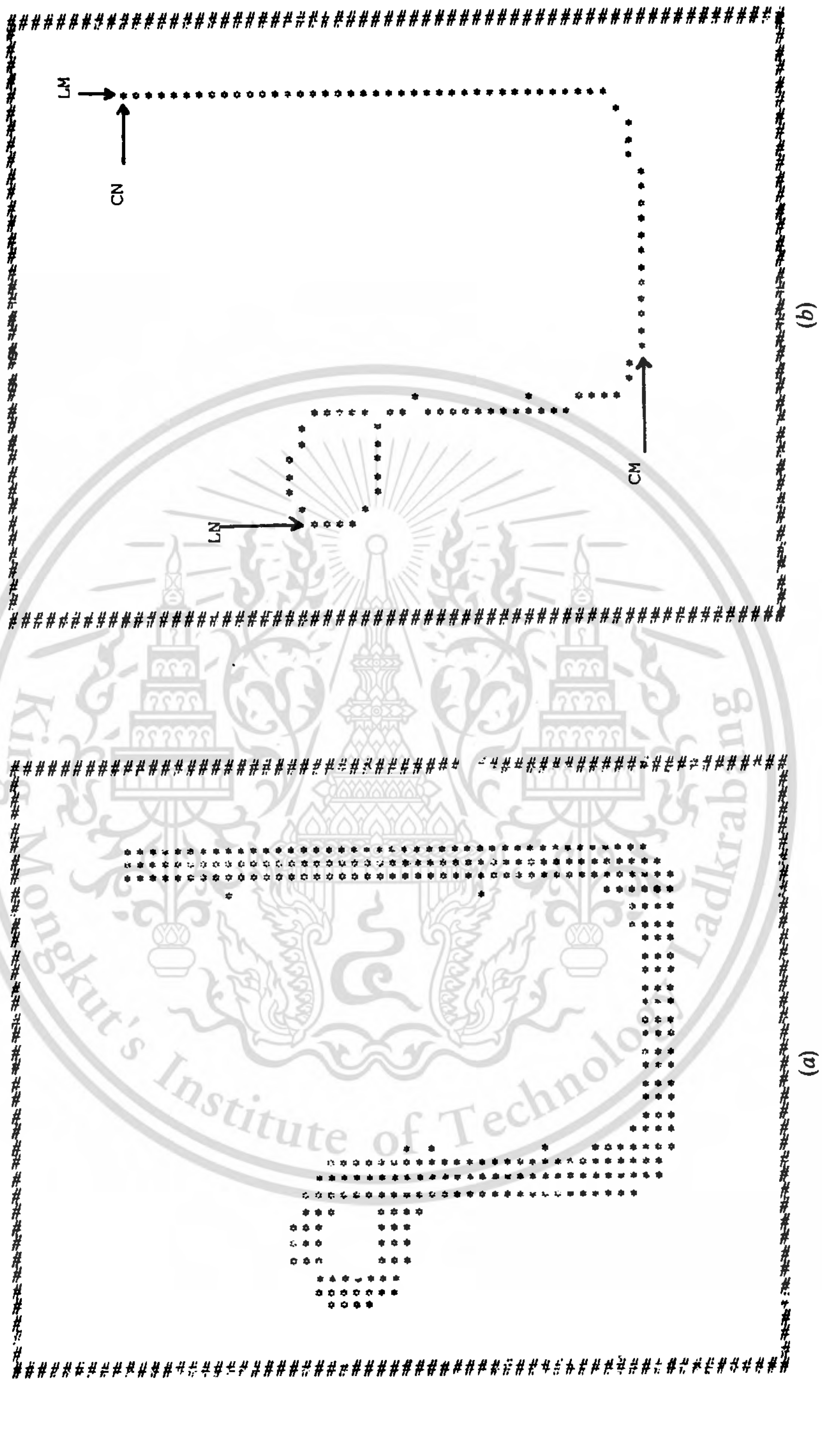


Figure 4. (*a*) Bit pattern of character. (*b*) Result of thinning algorithm.

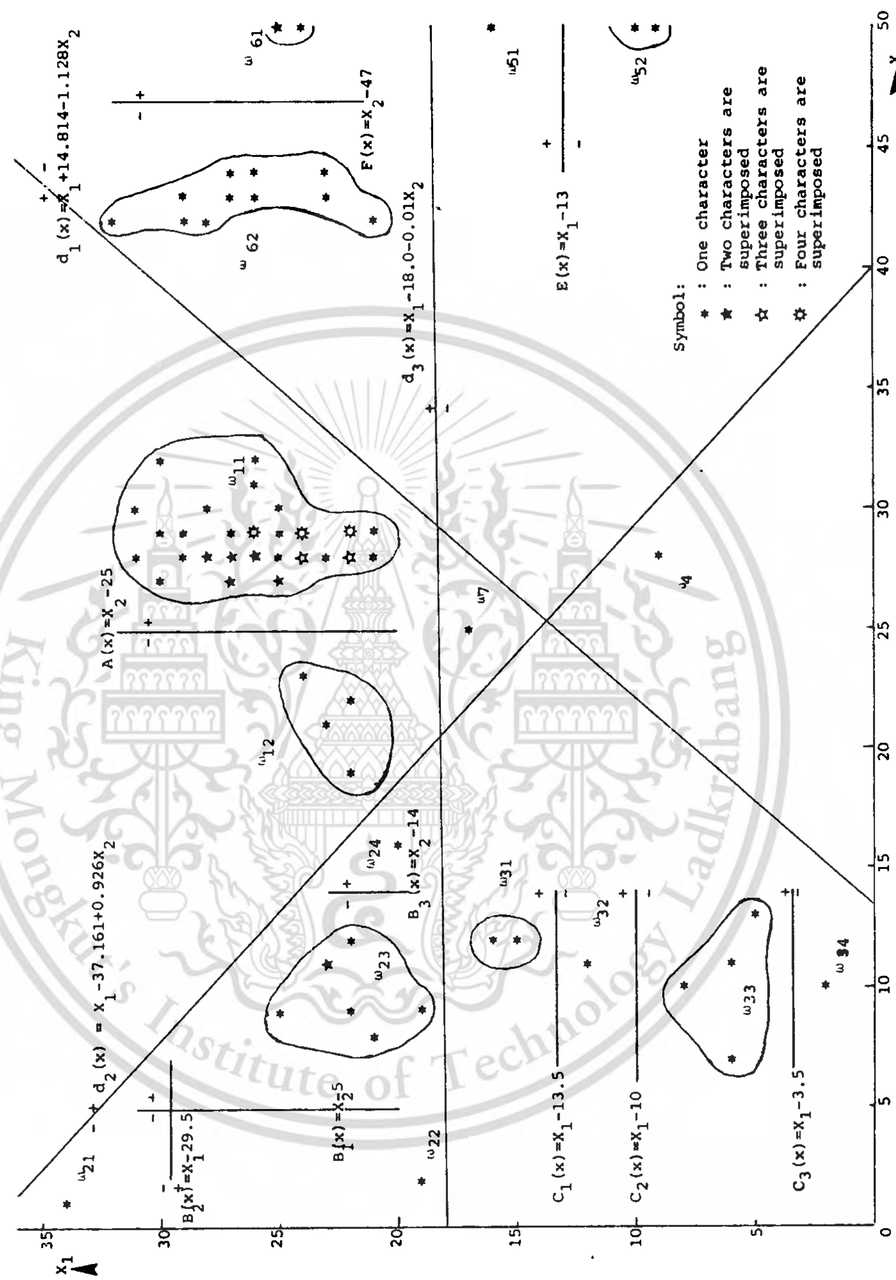


Figure 5. Linear decision function.

In our case we have seven main classes and the decision functions are as follows

$$d_1(x) = x_1 + 14{\cdot}814 - 1{\cdot}128x_2 \tag{9}$$

$$d_2(x) = x_1 - 37{\cdot}161 + 0{\cdot}926x_2 \tag{10}$$

$$d_3(x) = x_1 - 18{\cdot}0 - 0{\cdot}01x_2 \tag{11}$$

The boundary condition and members of each class are as shown in Table 1.

We can divide the main class into sub-classes by performing new decision functions, as follows

$$A(x) = x_2 - 25 \quad \text{for class } \omega_1 \tag{12}$$

$$\left.\begin{aligned} B_1(x) &= x_2 - 5 \\ B_2(x) &= x_1 - 29{\cdot}5 \\ B_3(x) &= x_2 - 14 \end{aligned}\right\} \text{for class } \omega_2 \tag{13}$$

$$\left.\begin{aligned} C_1(x) &= x_1 - 13{\cdot}5 \\ C_2(x) &= x_1 - 10 \\ C_3(x) &= x_1 - 3{\cdot}5 \end{aligned}\right\} \text{for class } \omega_3 \tag{14}$$

Main class	Condition	member
ω_1	$d_1(x) > 0$ $d_2(x) < 0$ $d_3(x) > 0$	ก ข ค ฆ ง จ ฉ ช ซ ฌ ฑ ฒ ณ ด ต ถ ท ธ น บ ผ พ ภ ม ย ร ล ว ส ศ ษ ห ฬ อ ฮ ะ า ฯ แ [illegible] ๑ ๒ ๓ ๔ ๕ ๖ ๗ ๘ ๙ ๐
ω_2	$d_1(x) > 0$ $d_2(x) < 0$ $d_3(x) < 0$	– — ็ ั ิ ี ึ ื ้ ๋
ω_3	$d_1(x) > 0$ $d_2(x) < 0$ $d_3(x) < 0$	ุ ู ์ ๊ ่ ํ ' "
ω_4	$d_1(x) < 0$ $d_2(x) < 0$ $d_3(x) < 0$	เ
ω_5	$d_1(x) < 0$ $d_2(x) > 0$ $d_3(x) < 0$	/ ( )
ω_6	$d_1(x) < 0$ $d_2(x) > 0$ $d_3(x) > 0$	ไ ใ โ ๆ ำ % ป ฝ ญ ฎ ฏ ฐ ฟ ฤ ฦ
ω_7	$d_1(x) > 0$ $d_2(x) > 0$ $d_3(x) < 0$	?

Table 1. Main classes.

Main class	condition	member
ω_{11}	$A(x)>0$	กขคฆงจฉชซฌฑฒณดตถท ธนบผพภมยรลวศษสหฟอ ฮ ะาฯแ๒๔๕ ๖๗๘๙
ω_{12}	$A(x)<0$	[illegible] ๑ ๓ ๐
ω_{21}	$B_1(x)<0$ $B_2(x)>0$ $B_3(x)<0$	—
ω_{22}	$B_1(x)<0$ $B_2(x)<0$ $B_3(x)<0$	-
ω_{23}	$B_1(x)>0$ $B_2(x)<0$ $B_3(x)<0$	
ω_{24}	$B_1(x)>0$ $B_2(x)<0$ $B_3(x)>0$	
ω_{31}	$C_1(x)>0$ $C_2(x)>0$ $C_3(x)>0$	
ω_{32}	$C_1(x)<0$ $C_2(x)>0$ $C_3(x)>0$	
ω_{33}	$C_1(x)<0$ $C_2(x)<0$ $C_3(x)>0$	
ω_{34}	$C_1(x)<0$ $C_2(x)<0$ $C_3(x)<0$	
ω_{51}	$E(x)>0$	/
ω_{52}	$E(x)<0$	( )
ω_{61}	$F(x)>0$	ไ ใ โ
ω_{62}	$F(x)<0$	ๆ ป % ำ ญ ฎ ฏ ฐ ฬ ฤ ฦ

Table 2. Sub-classes.

$$E(x) = x_1 - 13 \qquad \text{for class } \omega_5 \tag{15}$$

$$F(x) = x_2 - 47 \qquad \text{for class } \omega_6 \tag{16}$$

The boundary conditions and members of each sub-class are as shown in Table 2.

The decision functions $d_i(x)$ are performed by using linear regression formula as follows

$$a = \bar{y} - b\bar{x} \tag{17}$$

where $\bar{y} = \sum x_1/N$ and $\bar{x} = \sum x_2/N$; and

$$b = \frac{N \sum x_2 x_1 - \sum x_2 \sum x_1}{N \sum x_2^2 - (\sum x_2)^2} \tag{18}$$

where N is the number of points. a and b are then substituted into the following equation

$$d_i(x) = x_1 - a - bx_2 \tag{19}$$

For example, we can find $d_1(x)$ by the following procedure. Select the point on the surface of classes from Fig. 5; hence

$$x_2 : 25, 32, 28, 42, \quad \sum x_2 = 127$$

$$x_1 : 17, 26, \ \ 9, 32, \quad \sum x_1 = 84$$

$$\bar{x} = \frac{\sum x_2}{N} = \frac{127}{4} = 31{\cdot}75$$

$$\bar{y} = \frac{\sum x_1}{N} = \frac{84}{4} = 21$$

$$\begin{aligned}
\sum x_2^2 &= 25^2 + 32^2 + 28^2 + 42^2 \\
&= 625 + 1024 + 784 + 1764 \\
&= 4197 \\
\sum x_2 x_1 &= (25 \times 17) + (32 \times 26) + (28 \times 9) + (42 \times 32) \\
&= 425 + 832 + 252 + 1344 \\
&= 2853 \\
b &= \frac{N \sum x_2 x_1 - \sum x_2 \sum x_1}{N \sum x_2^2 - (\sum x_2)^2} \\
&= \frac{4(2853) - 127(84)}{4(4197) - (127)^2} \\
&= \frac{11412 - 10668}{16788 - 16129} = \frac{744}{659} \\
b &= 1{\cdot}128 \\
a &= \bar{y} - b\bar{x} \\
&= 21 - 1{\cdot}128(31{\cdot}75) \\
&= 21 - 35{\cdot}814 \\
&= -14{\cdot}814 \\
d_1(x) &= x_1 + 14{\cdot}814 - 1{\cdot}128x_2
\end{aligned}$$

5. Topological properties of characters

At the fine classification stage, the topological properties of characters which are represented in the form of numerical digits allow the separation of various aspects

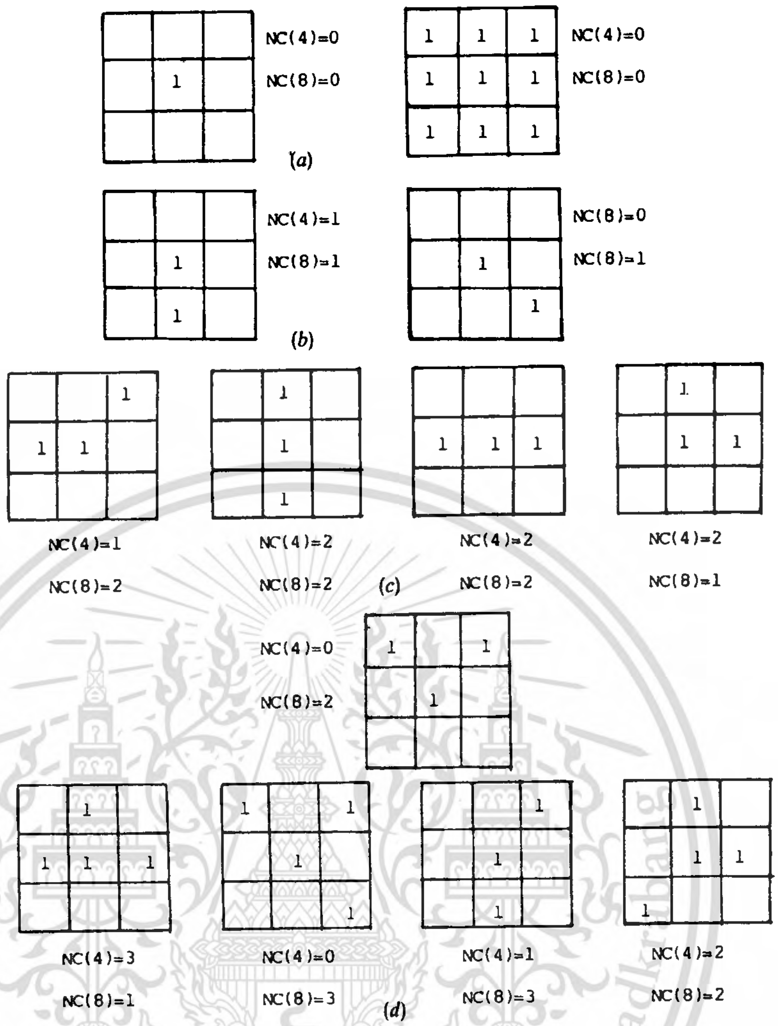


Figure 6. (*a*) NC(4) and NC(8) of independent or inner strokes. (*b*) NC(4) and NC(8) of end strokes. (*c*) NC(4) and NC(8) of connecting strokes. (*d*) NC(4) and NC(8) of branch strokes.

such as end strokes, intersection strokes, connecting strokes, independent or inner strokes. All of these properties can be used to discriminate between characters. By using eqns. (1) and (2) to compute NC(4) and NC(8) of these properties, the results can be as shown in Fig. 6. The results are summarized in Table 3.

Let 0 represent inner or independent strokes, 1 represent end strokes, 2 represent connecting strokes, 3 represent branch strokes, and 4 represent intersection strokes. Therefore by using a computer program to compute NC(4) and NC(8) of each point in the patterns of thinned Thai characters, we can convert the patterns into the form of topological properties, as shown in Fig. 7. Let P be the topological properties of a character; then the conversion rules are as follows

If NC(4) = 0 and NC(8) = 0, then

$$P = 0 \tag{20}$$

Connectivity	inner and independent strokes	end strokes		connecting strokes					branch strokes				intersection strokes		note :
NC4	0	1	0	1	2	2	2	0	3	0	1	2	4	0	the value of inner strokes=0, end strokes=1, connecting strokes=2, branch strokes=3, intersection strokes=4
NC8	0	1	1	2	2	2	1	2	1	3	3	2	0	4	

Table 3. Summary of topological properties.

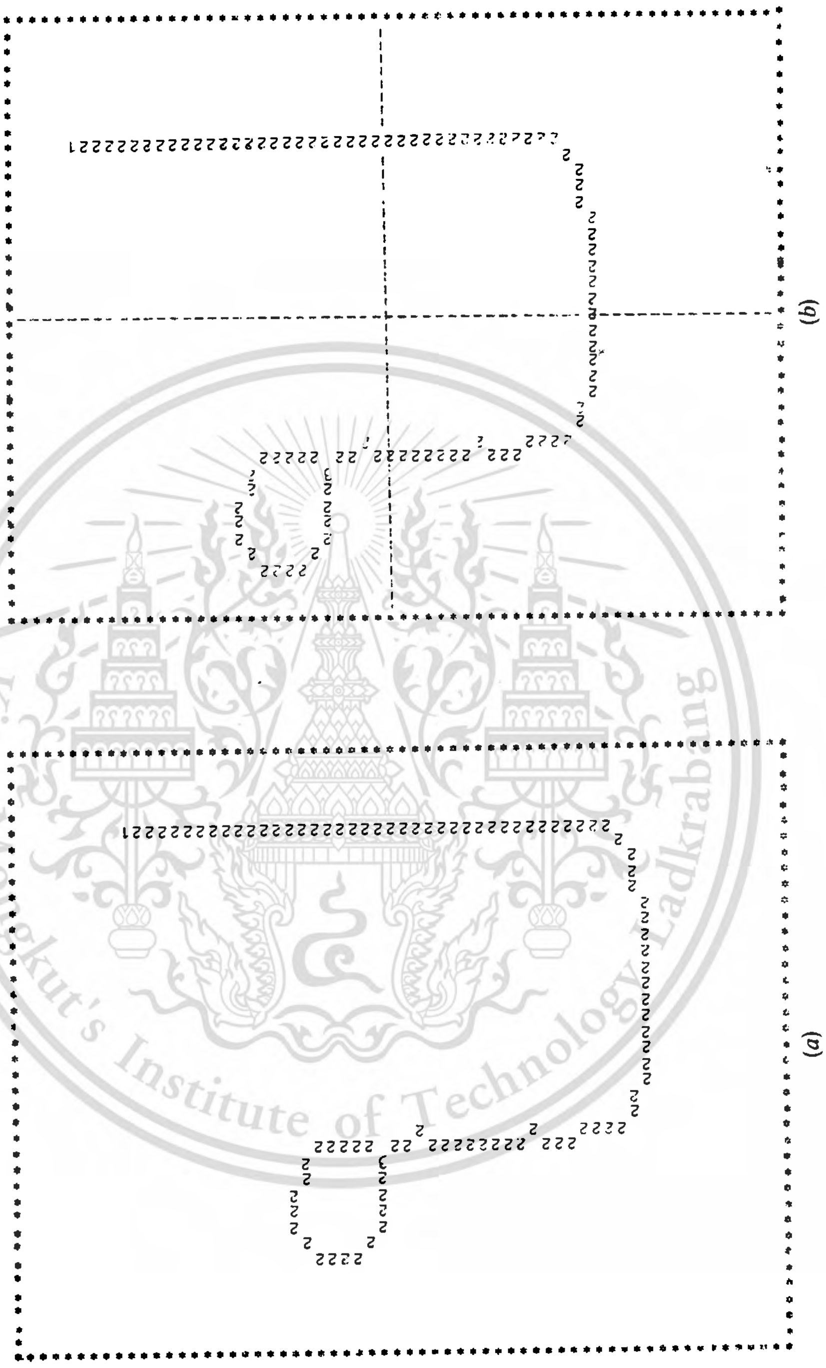


Figure 7. (*a*) Topological properties of a character in the form of the connected number. (*b*) Quadrant of a character.

If NC(4) = 1 and NC(8) = 1, or NC(4) = 0 and NC(8) = 1, then

$$P = 1 \tag{21}$$

If NC(4) = 1 and NC(8) = 2, or NC(4) = 2 and NC(8) = 2, or NC(4) = 2 and NC(8) = 1, or NC(4) = 0 and NC(8) = 2, then

$$P = 2 \tag{22}$$

If NC(4) = 2 and NC(8) = 1, or NC(4) = 0 and NC(8) = 3, or NC(4) = 1 and NC(8) = 3, then

$$P = 3 \tag{23}$$

If NC(4) = 4 and NC(8) = 0, or NC(4) = 0 and NC(8) = 4, then

$$P = 4 \tag{24}$$

Because intersection strokes rarely occur in printed Thai characters, they may be expressed in branch strokes by converting the points that have NC(4) = 4 and NC(8) = 0 or NC(4) = 0 and NC(8) = 4 into 3.

6. Decision table

The properties of each character in each class of the operation at the fine classification stage can be shown in the form of a decision table, as shown in Table 4. To design the decision table, we first compute the centre of gravity of the pattern of each character by the following equation

$$I_x = \frac{\sum_j \sum_i iF(i,j)}{\sum_j \sum_i F(i,j)} \tag{25}$$

$$I_y = \frac{\sum_i \sum_j jF(i,j)}{\sum_i \sum_j F(i,j)} \tag{26}$$

where $F(i, j)$ is the pattern of the character and I_x and I_y are the coordinates of the centre of gravity. Secondly, we divide the pattern of each character into four quadrants by using the centre of gravity as the centre of the quadrant. However, before making the quadrant, the centre of gravity of a character must also be the centre of the frame of a character pattern. Then, thirdly, we can list the number which represents the topological properties on each quadrant into the decision table.

If we let one property have the weight of 1 in each quadrant, we can compute the weight of the character by adding together the weights for all four quadrants. The character weights can be used to cluster the characters in the sub-class into sub-groups. Table 4 also shows the value of $d_i(x)$. When the properties of characters are all the same in every quadrant, these $d_i(x)$ can be used to discriminate characters; for example sub-class ω_{62} as shown in Fig. 8.

In the case shown in Fig. 9, since neither the properties nor $d_i(x)$ can be used to distinguish between these two characters, a special method must be used. The bit pattern of the thinned character is scanned in a vertical direction through its centre of gravity from the top of the pattern until 1's data are encountered. It is then back-stepped by one bit position, and the bit pattern is then scanned in a horizontal

Subclass	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{11}	า	5	-	2	-	1	2	-	-	2	-	1	2	-	5.2	10	3.5
	ฯ	5	-	2	-	-	2	3	-	2	-	1	2	-	7.2	12	6.5
	ก	6	-	2	-	-	2	-	1	2	-	1	2	-	4.0	11	3.5
	ข	6	1	2	-	-	2	3	-	2	-	-	2	-	5.9	15	6.5
	ง	6	-	2	3	1	2	-	-	2	-	-	2	-	4.0	11	3.5
	๒	6	-	2	-	1	2	3	-	2	-	-	2	-	10.2	15	8.6
	ป·	6	1	2	-	-	2	3	-	2	-	-	2	-	8.0	15	7.5
	ว	6	-	2	-	1	2	-	-	2	-	-	2	3	6.0	13	5.5
	ภ	6	-	2	-	-	2	-	-	2	3	1	2	-	7.0	14	6.5
	อ	6	-	2	-	1	2	-	-	2	3	-	2	-	6.0	13	5.5
	ย	6	1	2	-	-	2	3	-	2	-	-	2	-	5.2	10	3.6
	ะ	6	1	2	-	-	2	-	-	2	-	-	2	3	4.2	9	2.6
	๔	7	1	2	-	-	2	-	-	2	-	1	2	3	10.2	15	8.6
	๖	7	-	2	-	1	2	3	-	2	-	-	2	3	9.2	14	7.6
	จ	7	-	2	-	1	2	-	-	2	3	-	2	3	7.2	12	5.6

Table 4. Decision table.

Subclass	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{11}	ฆ	7	1	2	-	-	2	3	-	2	3	-	2	-	11.2	16	9.6
	ช	7	1	2	3	-	2	3	-	2	-	-	2	-	8.9	18	9.5
	ษ	7	1	2	3	-	2	3		2	-	-	2		12.0	19	11.5
	ธ	7	1	2	-	1	2	3	-	2	-	-	2	-	4.0	11	3.5
	ม	7	1	2	-	-	2	3	-	2	3	-	2	-	8.0	15	7.5
	ร	7	1	2		1	2	3	-	2	-	-	2	3	3.0	10	2.5
	ฮ	7	1	2	3		2	-	-	2	3	-	2	-	6.0	13	5.5
	ซ	7	1	2	-		2	3	-	2	-	-	2	-	8.6	22	11.4
	ค	7	-	2	-	-	2	-	1	2	3	1	2	-	6.0	13	5.5
	ด	7	-	2	-	-	2	-	1	2	3	1	2	-	7.2	12	5.6
	ต	7		2			2	-	1	2	3	1	2	-	8.2	13	6.6
	๕	8	1	2	3	-	2	-	-	2	3	1	2	3	8.0	15	7.5
	๘	8	1	2	-	-	2	-	1	2	3	-	2	3	11.3	12	6.6
	๙	8	1	2	-	-	2	3	-	2	3	1	2	-	9.3	12	6.6
	ฉ	8	-	2	-	1	2	-	1	2	3	-	2	3	8.0	15	7.5

Table 4. Continued

Subclass	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{11}	ฑ	8	-	2	-	-	2	3	1	2	3	1	2	-	14.3	17	11.6
	ท	8	-	2	-	-	2	3	1	2	3	1	2	-	9.2	14	7.6
	ล	8	-	2	-	1	2	-	-	2	3	1	2	3	6.2	11	4.6
	ถ	8	-	2	-	1	2	3	-	2	3	1	2	-	4.0	11	3.5
	แ	8	1	2	-	1	2	-	-	2	3	-	2	3	7.2	12	5.6
	ห	9	-	2	3	-	2	3	1	2	3	1	2	-	11.3	14	8.6
	ศ	9	1	2	3	-	2	-	1	2	3	1	2	-	4.6	18	7.4
	น	9	1	2	-	-	2	3	1	2	3	1	2	3	11.2	16	9.6
	ฒ	9	1	2	-	-	2	-	1	2	3	1	2	3	14.2	19	12.6
	ณ	10	1	2	-	1	2	3	-	2	3	1	2	3	11.7	18	10.5
	ผ	10	1	2	-	-	2	3	1	2	3	1	2	3	5.2	10	3.6
	ส	10	1	2	3	1	2	-	-	2	3	1	2	3	5.8	17	7.4
	๗	10	1	2	-	1	2	3	-	2	3	1	2	3	11.3	14	8.6
	ฌ	11	1	2	-	1	2	3	1	2	3	1	2	3	12.2	17	10.6
	ฬ	11	1	2	3	-	2	3	1	2	3	1	2	3	11.9	21	12.5

Table 4. Continued

Subclass	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{11}	ฟ	11	1	2	3	-	2	3	1	2	3	1	2	3	9.0	16	8.5
ω_{12}	O		-	2	-	-	2	-	-	2	-	-	2	-	14.1	4	4.9
	๑		-	2	-	-	2	-	-	2	3	-	2	-	11.9	4	3.9
	[illegible]		1	2	-	-	2	-	-	2	3	1	2	--	12.8	7	5.8
	[illegible]		1	2	3	-	2	-	-	2	3	1	2	-	15.3	1	4.0
ω_{21}	—		-	-	-	-	-	-	-	-	-	-	-	-	47.68	-5	16.9
ω_{22}	–		-	2	-	1	2	-	-	-	-	-	-	-	31.55	-19	1.9
ω_{23}	[illegible]	7	1	2	-		2	-	1	2	3	-	2	-	23.65	-12	1.5
	[illegible]	6	-	2	-	-	2	-	1	2	3	-	2	-	26.87	-11	3.6
	[illegible]	7	1	2	-	-	2	-	1	2	3	-	2	-	23.65	-12	1.5
	[illegible]	7	1	2	3	-	2	-	-	2	3	-	2	-	25.39	-6	5.4
	[illegible]	8	1	2	-	-	2	-	-	2	3	1	2	3	26.65	-9	4.4
	[illegible]	8	-	2	3	-	2	-	1	2	3	-	2	3	25.39	-6	5.4
	[illegible]	10	1	2	3	-	2	-	1	2	3	1	2	3	23.26	-6	4.4
ω_{24}	[illegible]		1	2	-	-	2	-	1	2	3	-	2	-	16.75	-6	5.4

Table 4. Continued

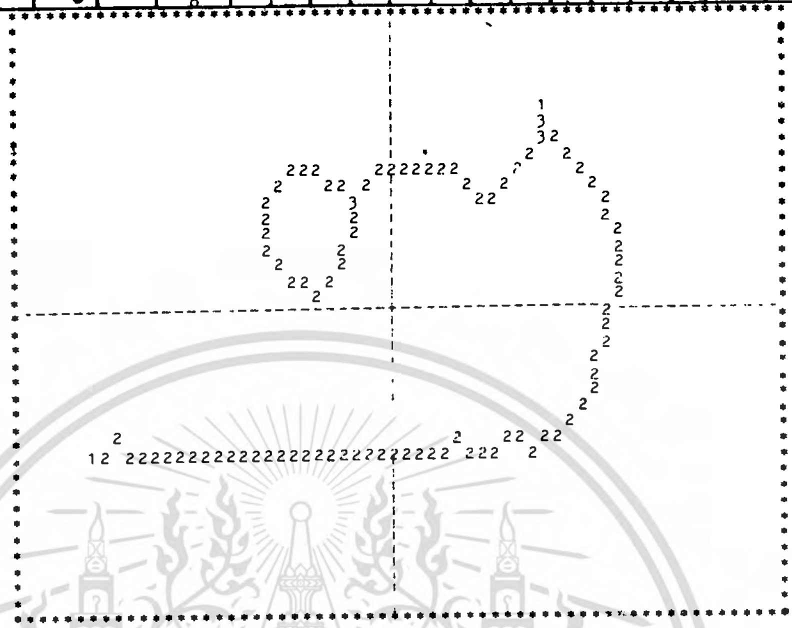
Subclass	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{31}	ู		1	2	–	–	2	–	–	2	3	–	2	–	17.26	-12	-1.6
	ุ		1	2	–	–	2	–	–	2	–	–	2	–	16.26	-13	-2.6
ω_{32}	ฺ		1	2	–	–	–	–	–	–	–	1	2	–	5.52	-28	-15
ω_{33}	้		–	–	–	–	–	–	–	–	–	1	2	–	8.39	-23	-11.5
	๋		1	2	–	1	–	–	1	–	–	1	–	–	11.52	-22	-9.5
	ํ		1	2	–	1	–	–	1	–	–	1	–	–	12.91	-27	-11.3
	่		1	2	–	–	–	–	1	2	–	–	2	–	5.13	-22	-12.6
ω_{34}	״		1	2	–	1	2	–	1	2	–	1	2	–	14.39	-17	-5.5
ω_{51}	/		1	2	–	–	–	–	1	2	–	–	–	–	-29.0	29	-3.6
ω_{52}	(		1	2	–	–	2	–	–	2	–	1	2	–	-39.4	25	-10.8
	)		–	2	–	1	2	–	1	2	–	–	2	–	-40.6	28	-9.9
ω_{61}			–	2	–	1	2	3	–	–	–	–	2	3	-16.6	33	3.5
			–	2	–	–	2	3	–	–	–	–	2	3	-19.8	36	4.4
			1	2	–	1	?	3	–	–	–	–	2	3	-17.7	36	5.4

Table 4. Continued

Subclass	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
ω_{62}	ๆ	5	-	2	-	-	2	3	-	-	-	1	2	-	-11.5	23	1.9
	ป	6	1	2	-	-	2	3	-	2	-	-	2	-	-4.5	30	8.9
	%	6	1	2	-	-	2	-	1	2	-	-	2	-	-0.5	34	12.9
	ำ	6	-	2	-	-	2	-	1	2	-	1	2	-	-8.8	30	6.8
	ฎ	7	-	2	-	-	2	3	-	2	3	1	2	-	-6.7	30	7.8
	ญ	8	1	2	-	-	2	-	-	2	3	1	2	3	-4.7	32	9.8
	ฏ	8	-	2	-	-	2	3	-	2	3	1	2	3	-7.8	31	7.8
	ถ	8	-	2	-	1	2	3	-	2	3	1	2	-	-10.7	26	3.8
	ภ	8	-	2	-	1	2	3	-	2	3	1	2	-	-7.7	29	6.8
	ฟ	11	1	2	3	-	2	3	1	2	3	1	2	3	-3.5	31	9.9
	ฐ	12	1	2	3	1	2	3	1	2	3	1	2	3	-11.8	27	3.8

Table 4. Continued

Subclass	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
	[illegible]	8	-	2	-	1	2	3	-	2	3	1	2	-	-10.7	26	3.8
	[illegible]	8	-	2	-	1	2	3	-	2	3	1	2	-	-7.7	29	6.8



(*a*)

Subclass	Characters	Weight	Quadrant												$d_1(x)$	$d_2(x)$	$d_3(x)$
			Q_1			Q_2			Q_3			Q_4					
			Properties														
			P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3			
	[illegible]	8	-	2	-	1	2	3	-	2	3	1	2	-	-10.7	26	3.8
	[illegible]	8	-	2	-	1	2	3	-	2	3	1	2	-	-7.7	29	6.8

(*b*)

Figure 8. (*a*) and (*b*) Distinguishing the characters by $d_i(x)$.

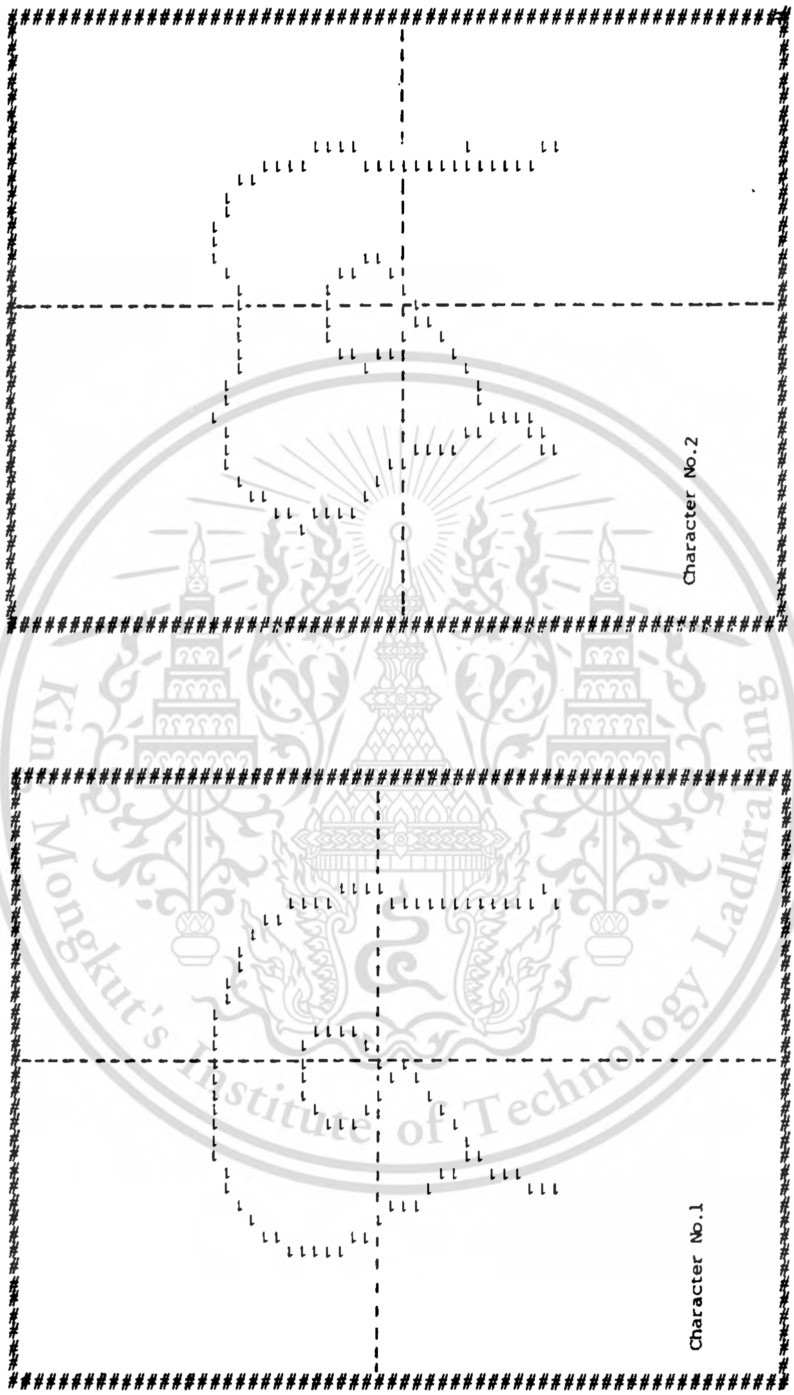


Figure 9. Distinguishing the characters by a special method.

direction. If 1's data are encountered, this indicates character no. 2; otherwise it is character no. 1.

7. Fine classification procedure

We can perform the fine classification procedure by inspection of the decision table. After the unknown input data have passed the rough classification procedure as described in the previous section, the sub-class of this input data is identified, and the fine classification procedure is started at that sub-class by inspecting the weight of the character. The properties of the character are then inspected starting from quadrants 1, 2, 3 and 4 respectively as, for an example, shown by the flow chart of sub-class ω_{62} in Fig. 10.

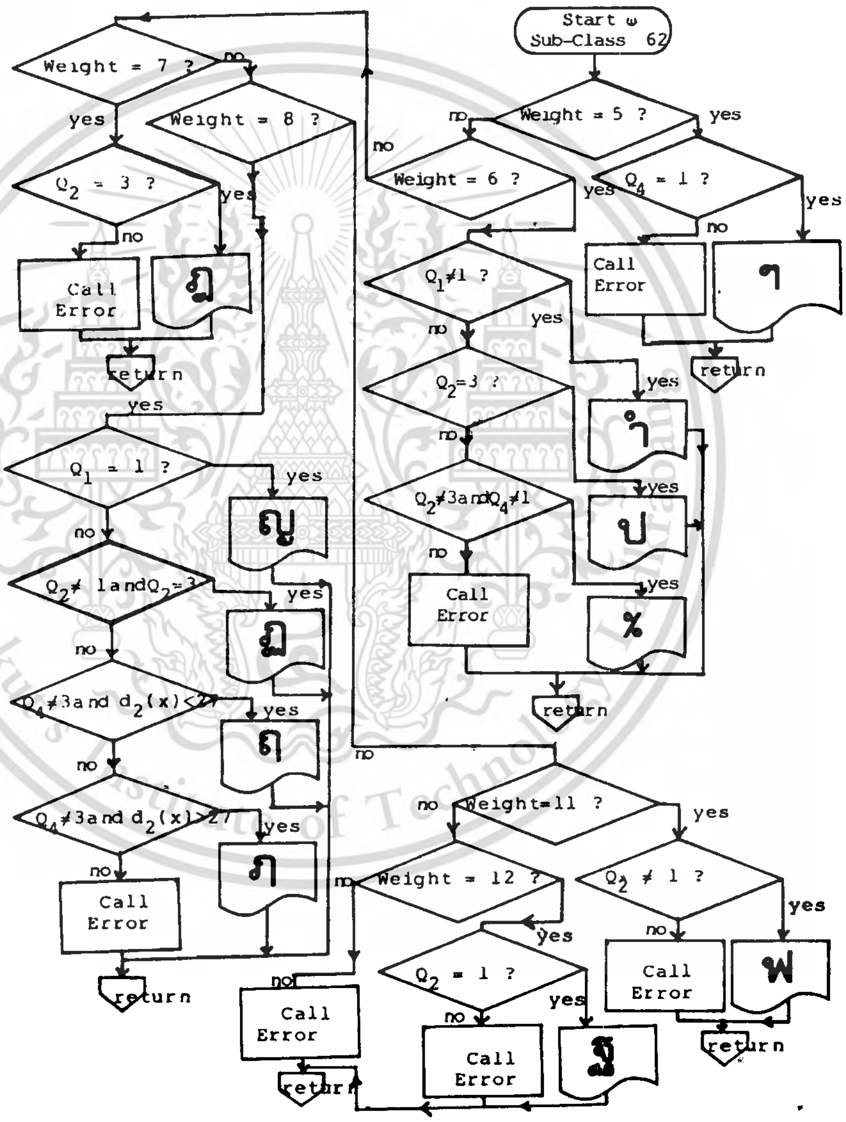
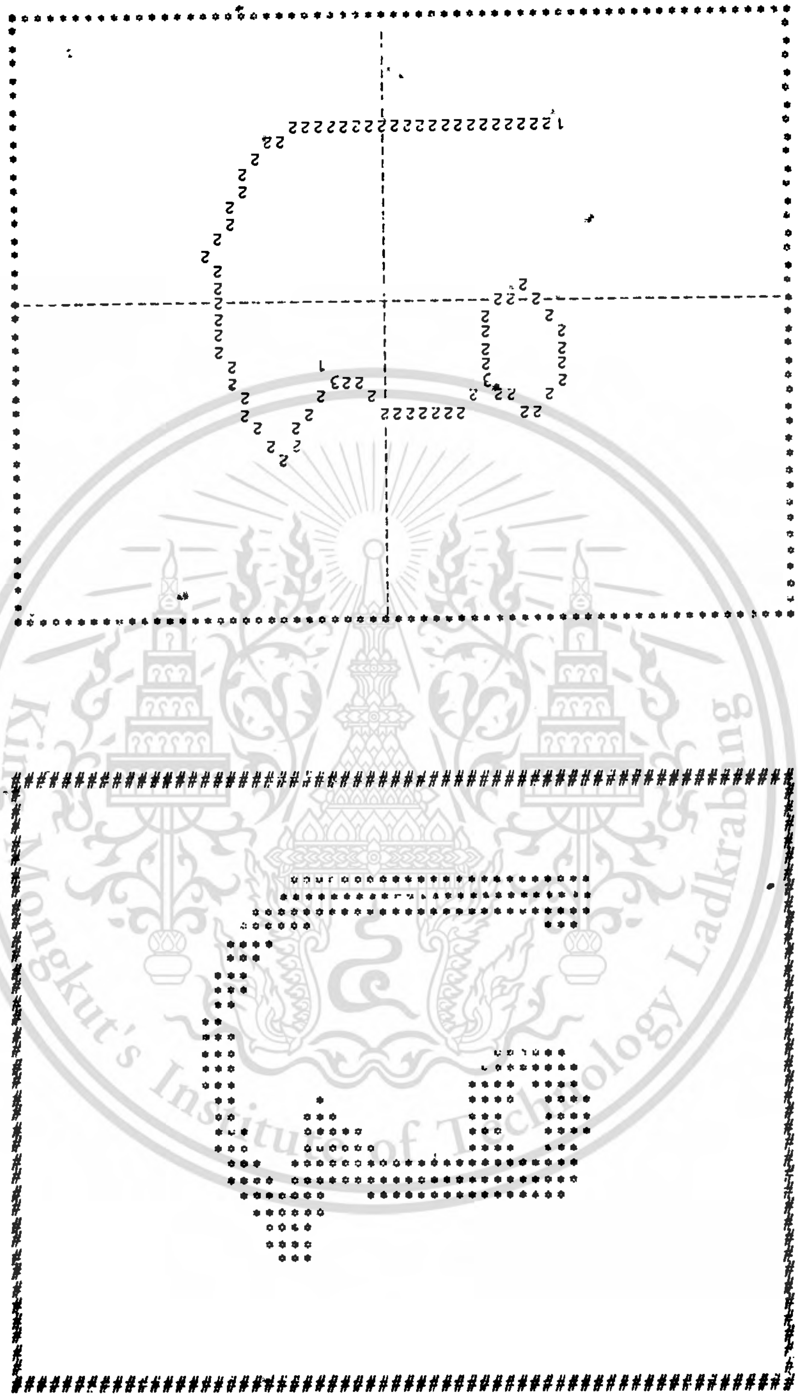
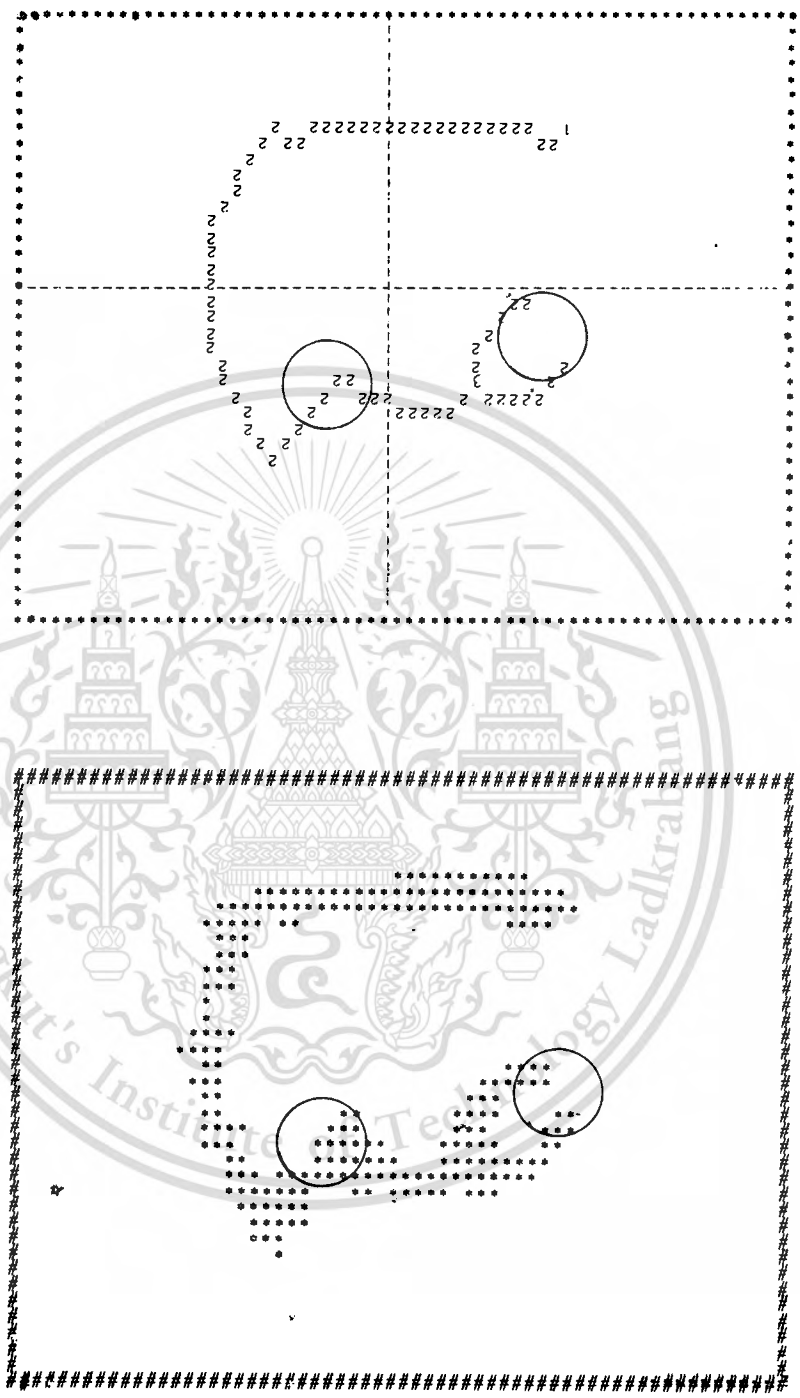


Figure 10. Procedure of the fine classification stage.



(b)

(a)



(c) (d)

Figure 11. (*a*) Normal pattern. (*b*) Normal properties. (*c*) Abnormal pattern. (*d*) Abnormal properties.

8. Conclusions

Completely successful recognition of printed Thai characters by using topological properties has been achieved for both rough classification and fine classification of learning-pattern input data. With corrupted line pattern input data, as shown in Fig. 11, the percentage of correction is still high enough for character recognition.

The weak point of this method is at the fine classification stage. If a corrupted line occurs in the pattern input data, the skeletal shape of the character will be slightly distorted after the thinning process, and hence the properties will be also distorted in some quadrant (Fig. 11). Consequently there will be an error in the weight of the character, and the cluster of this character will also be in error. To avoid this we should not use weight to cluster the character, and discard the properties in that quadrant.

Another case that we must pay careful attention to is the small circle called the head of the character. If the head of the character is broken, the properties indicating the head at that quadrant will disappear, and the cluster of that character will definitely give an error.

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Appendix 1. Thai character sets

CONSONANT

กขคฆงจฉชซฌญฎฏฐฑฒณดต
ถทธนบปผฝพฟภมยรลวศษส
หฬอฮ

VOWEL

ะ า ั ิ ี ึ ื ุ ู เ แ โ ใ ไ

TONE OF VOICE

่ ้ ๊ ๋

SPECIAL SYMBOL

ฯ ๆ ฤ ฦ ็ ์ ํ ( ) % ? — - / " ำ

THAI NUMERAL

๑๒๓๔๕๖๗๘๙๐

Appendix 2. Sampling and segmentation procedure