รายงานการวิจัย

การคำนวณเชิงตัวเลขของตัวแบบเชิงคณิตศาสตร์สำหรับการแพร่กระจายของควันจากปล่องควันของโรงงานอุตสาหกรรมในเขตเมือง

Numerical Computation of the Mathematical Model for Smoke Dispersion from Chimneys of Industrial Plant in Urban Area

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สาขาวิชาคณิตศาสตร์ คณะวิทยาศาสตร์ สถาบันเทคโนโลยีพระจอมเกล้าเจ้าคุณทหารลาดกระบัง

ได้รับทุนสนับสนุนงานวิจัยจากเงินรายได้ ประจำปีงบประมาณ 2553

คณะวิทยาศาสตร์

สถาบันเทคโนโลยีพระจอมเกล้าเจ้าคุณทหารลาดกระบัง

กิตติกรรมประกาศ

ขอขอบคุณคณะวิทยาศาสตร์ สถาบันเทคโนโลยีพระจอมเกล้าเจ้าคุณทหารลาดกระบัง ที่ให้การสนับสนุนทุนวิจัย ประเภทส่งเสริมนักวิจัยด้วยเงินรายได้คณะวิทยาศาสตร์ ประจำปีงบประมาณ 2553 จนงานวิจัยนี้สำเร็จลุล่วงไปได้ด้วยดี

รายละเอียดเกี่ยวกับโครงการ

ชื่อโครงการวิจัย

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บทคัดย่อ

การแพร่กระจายของควันเป็นหนึ่งในสาเหตุหลักของปัญหามลภาวะทางอากาศที่เกิดจากโรงงานอุตสาหกรรม การขวางการไหลของอากาศโดยสิ่งปลูกสร้างเป็นปัจจัยสำคัญอันหนึ่งของปัญหามลภาวะดังกล่าว โดยงานวิจัยนี้ผู้วิจัยต้องการสร้างตัวแบบเชิงคณิตศาสตร์เพื่อการจำลองการแพร่กระจายของควันที่ถูกปล่อยจากแหล่งกำเนิดสองแหล่ง ซึ่งถูกขัดขวางการไหลโดยสิ่งปลูกสร้าง โดยสมการก่อกำเนิดของตัวแบบคือสมการการแพร่ในบรรยากาศ วิธีขั้นเศษส่วนและวิธีไฟไนต์เอลิเมนต์ถูกนำมาประยุกต์ใช้เพื่อหาผลเฉลยโดยประมาณของสมการดังกล่าว

Abstract

The smoke discharging is a principle reason of air pollution problem from industrial plants. The flow obstruction of building is also an important factor of air quality problem. In this research, we simulate a mathematical model of the smoke dispersion over the building from two sources. The governing equation of the model is atmospheric diffusion equation. The fractional step method and finite element method are used to find the approximate solution of the equation.

สารบัญ

CHAPTER I

A Numerical Treatment of the Mathematical Model for Smoke Dispersion from Two Sources using Fractional Step Method

The smoke discharging is a principle reason of air pollution problem from industrial plants. The flow obstruction of building is also an important factor of air quality simulation. In this research, we simulate a mathematical model of the smoke dispersion over the building from two sources. The governing equation of the model is atmospheric diffusion equation. The fractional step method is used to find the approximate solution of the equation. The simulation with numerical results is also demonstrated.

1 Introduction

The air pollution problem in Thailand is still important for quality of life. This research is to study the air pollution problem by using the mathematical model; the atmospheric diffusion equation.

In [2], they used the finite difference method in the air pollution model of two dimensional spaces with single point source. In [3], the air pollution problem in three dimensional spaces with multiple sources presented. The initial conditions in the domains were assumed to be zero everywhere without obstacles.

In this research, we investigate the behavior of air pollutant release into the atmosphere with obstacle domain in two dimensional spaces. We assume that there are a large buildings inside the consideration area and the source of discharging pollutant are two point sources. The atmospheric diffusion equation is used to predict the behavior of the dispersion of air pollutant in the domain. The fractional step, Carlson and Crank-Nicolson methods are used to approximate the concentration of smoke dispersion.

1 The Governing Equation

We introduced the well-known atmospheric diffusion equation

$$\frac{\partial c}{\partial t} + \vec{v} \cdot \nabla c = \nabla \cdot (\bar{k} \nabla c) + s \qquad (1)$$

For convenience, an equation (1) can be written into the form

$$\frac{\partial c}{\partial t} + v_x \frac{\partial c}{\partial x} + v_y \frac{\partial c}{\partial y} = k_x \frac{\partial^2 c}{\partial x^2} + k_y \frac{\partial^2 c}{\partial y^2} + \frac{\partial}{\partial z}\left(k_z \frac{\partial c}{\partial z}\right) + s \tag{2}$$

where c is the air pollutant concentration at point (x, y, z) at time t (the unit of c is kg/m^3), k_x, k_y are diffusion coefficients in x-direction and y-direction respectively, v_x, v_y are the flow velocity in x-direction and y-direction respectively and s is the rate of change of substance concentration due to sources.

Suppose that the horizontal advection dominates the horizontal diffusion by the wind and the vertical diffusion dominates the vertical advection by the wind. The horizontal advection in $y-$direction is negligible. We have a cross section along the $y-$axis at the plane of obstacle. With is assumptions the equation (2) can be written as

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = \frac{\partial}{\partial z}\left(k_z \frac{\partial c}{\partial z}\right) + s, \tag{3}$$

where $v_x = u$. Equation (3) can be normalized to the non-dimensional form and changes all variables to the capital letters as follow

$$\frac{\partial C}{\partial T} + U \frac{\partial C}{\partial X} = \frac{\partial}{\partial Z}\left(K_Z \frac{\partial C}{\partial Z}\right) + S. \tag{4}$$

By the assumption as [2] and [3], the initial condition is taken to be zero concentration of air pollutant everywhere in the domain. We can see that

$$C(X, Z, 0) = 0 \qquad \text{for all } X > 0, \quad 0 < Z < H, \tag{5}$$

when H is the height of the inversion layer. The boundary conditions in the $X-$direction are assume to be zero.

$$C(X_W, Z, T) = C(X_E, Z, T) = 0 \qquad \text{for all } Z, \quad \text{for all } T > 0, \tag{6}$$

when X_W and X_E are the abscissas of the western and eastern. The flux of pollutant is assumed to be zero at the inversion layer H and at the ground which is cross section along the obstacle $Y-$plane.

$$\frac{\partial C}{\partial Z}(X, H, T) = \frac{\partial C}{\partial Z}(X, \Gamma, t) = 0, \tag{7}$$

where Γ is a line on the ground which is cross section along the obstacle $Y-$plane.

2 Numerical Techniques

The Fractional step method [6] is used to separate equation (4) into 3 stages as the following

$$\frac{\partial C'}{\partial T} = S = \sum_{r=1}^{N} Q_r \delta(X - X_r)\delta(Z - Z_r) \tag{8}$$

$$C'(X, Z, T_n) = C(X, Z, T_n) \tag{9}$$

$$\frac{\partial C''}{\partial T} = -U\frac{\partial C''}{\partial X} \tag{10}$$

$$C''(X, Z, T_n) = C'(X, Z, T_n + \Delta T) \tag{11}$$

$$\frac{\partial C'''}{\partial T} = \frac{\partial}{\partial Z}\left(K_Z \frac{\partial C'''}{\partial Z}\right) \tag{12}$$

$$C'''(X, Z, T_n) = C''(X, Z, T_n + \Delta T) \tag{13}$$

In succession, the auxiliary terms in each stage are denoted by prime ($'$) that is C', C'' and C''' respectively. Equations (8) and (9) are the emission stage, the concentration at the source is assumed to be a $\delta-$function. Solving the emission part by using the basic finite difference method. The advection stage in $X-$direction was presented in equation (10) and (11). The Carlson finite difference method [5] is used to find the numerical solution in this stage. Equations (12) and (13) are the diffusion stage in $Z-$direction. We use the Crank-Nicolson [1] approximation which is unconditionally stable and consistence to find the numerical solution.

3 Numerical Experiment

The atmospheric diffusion equation of (4) with appropriate parameter values for the tropical area were taken from Pasquill stability reference class A [4]. The parameters are given by $\bar{k}_z = 45 \quad m^2/sec$ and the velocity $\bar{u} = 3 \quad m/sec$. The sources are assumed at $z = h_s = 15$ m. above the ground. In this research, we assumed there are two point sources with different emission rate are $q_1 = 70$ gram/sec and $q_2 = 100$ gram/sec. Suppose there are two buildings with different heights away from the second source at 300 m.(at X=40 to 52) and 600 m. at (X=60 to 72) respectively. The wind velocity are assume to be a constant form

Table 1: The numerical solution along the $X-$axis at different time step ΔT and $\Delta T = 0.25$. The wind velocity is $U = 1$.

T/X	$X = 0$	$X = 20$	$X = 40$	$X = 60$	$X = 80$	$X = 100$
$8\Delta T$	0.114572577	0.163675113	6.27483E-10	6.27483E-10	0	0
$16\Delta T$	0.122116864	0.17445424	6.83287E-07	6.83287E-07	0	0
$32\Delta T$	0.12275511	0.175929991	0.000416183	0.000416183	1.63233E-10	0
$64\Delta T$	0.12275997	0.197436007	0.021093864	0.021093864	2.23685E-05	1.18347E-10
$128\Delta T$	0.12275997	0.21275123	0.051584559	0.051584559	0.015943851	0.000283978
$256\Delta T$	0.12275997	0.212762767	0.060053871	0.060053871	0.046648859	0.032173093
$512\Delta T$	0.12275997	0.212762767	0.060053885	0.060053885	0.047524938	0.041955905

Table 2: The numerical solution along the $X-$axis at different time step ΔT and $\Delta T = 0.25$. The wind velocity is $U(Z) = Z^{(0.2)}$.

T/X	$X = 0$	$X = 20$	$X = 40$	$X = 60$	$X = 80$	$X = 100$
$8\Delta T$	0.042462698	0.060660997	0	0	0	0
$16\Delta T$	0.042557854	0.060796935	0	0	0	0
$32\Delta T$	0.0425581	0.064172487	0.002671623	0.002671623	0	0
$64\Delta T$	0.0425581	0.090988389	0.02746385	0.02746385	0.000628333	1E-15
$128\Delta T$	0.0425581	0.091007278	0.04393871	0.04393871	0.030707281	0.013133237
$256\Delta T$	0.0425581	0.091007278	0.043938764	0.043938764	0.03337808	0.030258134
$512\Delta T$	0.0425581	0.091007278	0.043938764	0.043938764	0.03337808	0.030258264

$U = 1$ and variable form $U(Z) = Z^{(0.2)}$. The numerical results for the smoke concentrations are shown in the Tables (1-2) and Figures (1-2).

4 Conclusion

The concentrations of smoke released from two point sources in the domain with two obstacles of flow are calculated. The computed approximate results are obtained using fractional step method and MatLab code, with respected to the transformed atmospheric diffusion equation.

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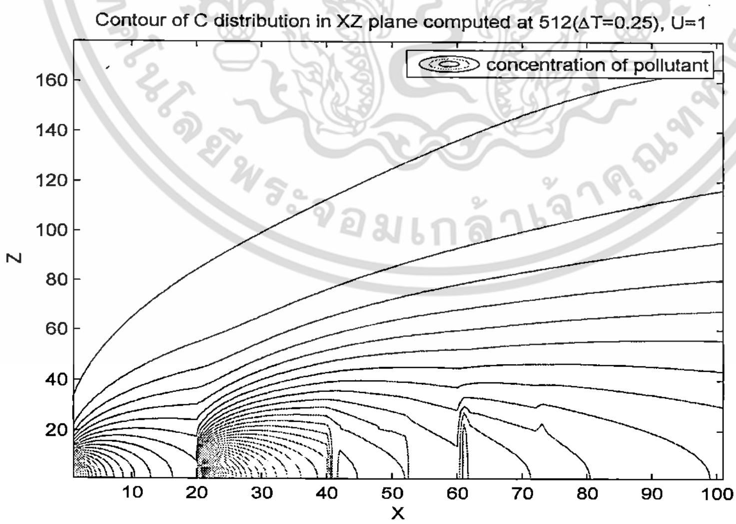

Figure 1: The contour graph along XZ plane at $512\Delta T$ and $\Delta T = 0.25$. The wind velocity is $U = 1$.

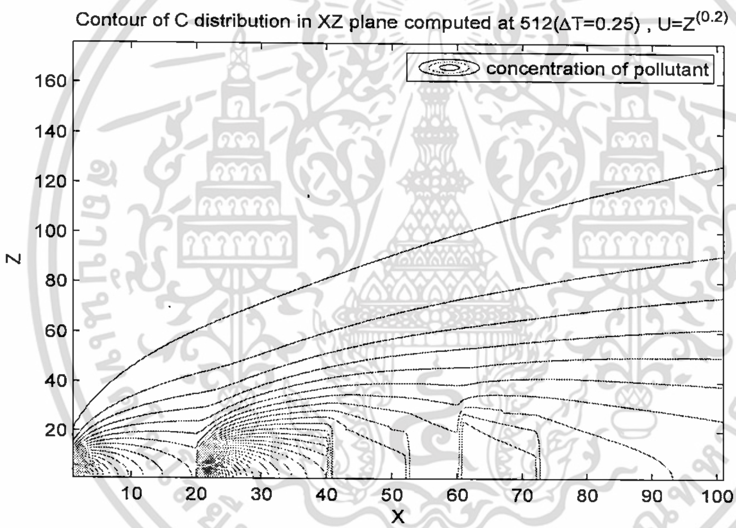

Figure 2: The contour graph along XZ plane at $512\Delta T$ and $\Delta T = 0.25$. The wind velocity is $U(Z) = Z^{(0.2)}$.

CHAPTER II

A Finite Element Solution of the Mathematical Model for Smoke Dispersion from Two Sources

Smoke discharging is a main reason of air pollution problem from industrial plants. The obstruction of air flow by building to affect with the air pollutant discharge. In this research, a mathematical model of the smoke dispersion from two sources and one source with a structural obstacle is considered. The governing equation of the model is an isothermal mass transfer model in a viscous fluid. The finite element method is used to approximate the solutions of the model. The triangular linear elements have been used for discretise the domain, and time integration has been carried out by semi-implicit finite difference method. The simulations of smoke dispersion in cases of one chimney and two chimneys are presented. The maximum calculated smoke concentration of both cases are compared. It is then used to make the decision for smoke discharging and air pollutant control problems on industrial area.

1 Introduction

The air pollution problem is still important for human health. This research is consider the air pollution problem using the mass transfer model. In [8], a nonlinear mathematical model is proposed and analyzed to study the removal of gaseous pollutants and particulate matters from the atmosphere of a city by precipitation. The three pollution indexes (SO_2, NO_2 and PM10) and the daily air pollution indexes (APIs) of Shanghai in China are analyzed by rescaled range analysis (R/S), detrended fluctuation analysis (DFA) and spectral analysis is presented in [4]. In [1], they study the air flow using a lubricated system consisting of two bodies in proximity. The Poincare compactification to the dynamical system to get a complete qualitative analysis of the global flow is applied in [11].

In [10], [12] and [5], they used the finite difference method in the air pollution model of two dimensional spaces with single point source. In [6], the air pollution problem in three dimensional spaces. The initial conditions in the domains were assumed to be zero everywhere without structural obstacles. In [7] and [13], the fractional steps method, Carlson method and Crank-Nicolson method [2]

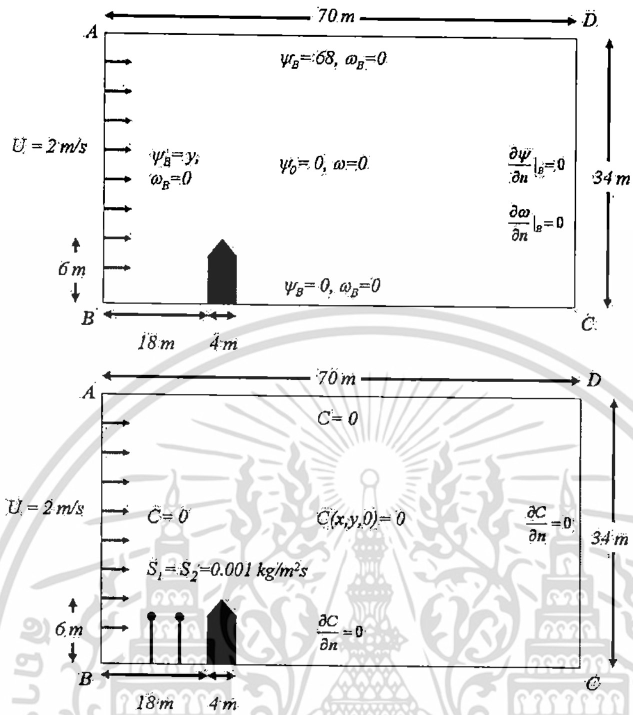

Figure 1: The initial and boundary conditions of the problem

are used to approximate the concentration of smoke dispersion. The diffusion of dust particles from a point source above ground level is considered in [3].

In this research, we investigate the behavior of smoke release into the atmosphere with a structural obstacle in two-dimensional spaces and one-dimensional in time using the finite element method and finite difference method, respectively. The structural obstacle and chimneys are added in the simulation. The simulation attempts to predict the behavior of the dispersion of smoke in the problem.

2 The Governing Equation

2.1 The mass transport model

Consider the equations describing the smoke dispersion in terms of the stream-function, vorticity and convection-diffusion equations as follow [9],

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2} = -\omega, \quad (1)$$

$$\frac{\partial\omega}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y} = \nu\nabla^2\omega, \quad (2)$$

$$\frac{\partial C}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y} = \eta\nabla^2 C+S, \quad (3)$$

where ν is a kinematic viscosity, ω is a vorticity, ψ is a stream function, C is a concentration, η is a diffusion coefficient and S is intensity of mass source.

2.2 The boundary conditions

The boundary conditions of the streamfunction equation Eq.(1) are

$$\psi=\psi_B, \quad (4)$$

which is specified on the boundary Γ_ψ, and

$$\frac{\partial\psi}{\partial n}=-V_s, \quad (5)$$

with tangential flow velocity V_s specified on the rest of the boundary Γ_s. Here n denotes the outward unit normal to the boundary. Boundary conditions on the vorticity Eq.(2) are

$$\omega=\omega_B, \quad (6)$$

which is specified on the boundary Γ_ω, and

$$\frac{\partial\omega}{\partial n}=\chi_n, \quad (7)$$

with the value of the normal derivative specified on the other part of the boundary Γ_χ. The boundary conditions of the convective-diffusion equation Eq.(3) are

$$C = C_B \text{ on } \Gamma_C, \quad (8)$$

$$-\frac{\partial C}{\partial n} = j_B \text{ on } \Gamma_j. \quad (9)$$

Since boundary is a non-absorbing, we can put $j_B=0$.

3 The Numerical Technique

3.1 The finite element discretisation

The Galerkin finite element method to discretising of the streamfunction equation Eq.(1), the vorticity transport equation Eq.(2), and the convection-diffusion equation Eq.(3) will be used. Let W_ψ, W_ω and W_C be weighting functions. The finite element formulation with the weighted residual forms of these equations:

$$\int_\Omega W_\psi(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2})d\Omega + \int_\Omega W_\psi \omega d\Omega = 0, \tag{10}$$

$$\int_\Omega W_\omega \frac{\partial\omega}{\partial t} d\Omega + \int_\Omega W_\omega(\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y})d\Omega - \int_\Omega W_\omega \nu \nabla^2 \omega d\Omega = 0, \tag{11}$$

$$\int_\Omega W_C \frac{\partial C}{\partial t} d\Omega + \int_\Omega W_C(\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y})d\Omega - \int_\Omega W_C \eta \nabla^2 C d\Omega - \int_\Omega W_C S d\Omega = 0. \tag{12}$$

Integration by parts of terms concerning the Laplacian, we get

$$\int_\Omega(\frac{\partial W_\psi}{\partial x} + \frac{\partial W_\psi}{\partial y})(\frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial y})d\Omega + \int_\Omega W_\psi \omega d\Omega - \int_{\Gamma_s} W_\psi \frac{\partial\psi}{\partial n} d\Gamma = 0, \tag{13}$$

$$\int_\Omega W_\omega \frac{\partial\omega}{\partial t} d\Omega + \int_\Omega W_\omega(\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y})d\Omega + \int_\Omega \nu \nabla W_\omega \nabla \omega d\Omega - \int_{\Gamma_x} \nu W_\omega \frac{\partial\omega}{\partial n} d\Gamma = 0, \tag{14}$$

$$\int_\Omega W_C \frac{\partial C}{\partial t} d\Omega + \int_\Omega W_C(\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y})d\Omega + \int_\Omega \eta \nabla W_C \nabla C d\Omega - \int_\Omega W_C S d\Omega - \int_{\Gamma_j} j_B W_C d\Gamma = 0. \tag{15}$$

The domain Ω is divided into triangular elements with local node numbers 1, 2 and 3 with nodal coordinates $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3), respectively. The unknown streamfunction, vorticity and concentration are linearly interpolated as follows,

$$\psi = \sum_{\alpha=1}^{3} \phi_\alpha \psi_\alpha, \qquad W_\psi = \sum_{\alpha=1}^{3} \phi_\alpha W_{\psi_\alpha}, \tag{16}$$

$$\omega = \sum_{\alpha=1}^{3} \phi_\alpha \omega_\alpha, \qquad W_\omega = \sum_{\alpha=1}^{3} \phi_\alpha W_{\omega_\alpha}, \tag{17}$$

$$C = \sum_{\alpha=1}^{3} \phi_\alpha C_\alpha, \qquad W_C = \sum_{\alpha=1}^{3} \phi_\alpha W_{C_\alpha}, \tag{18}$$

where linear interpolation functions, $\phi_\alpha = \frac{1}{2\Delta^e}(a_\alpha + b_\alpha x + c_\alpha y)$ with area of a triangular element e is $\Delta^e = \frac{b_2c_3 - b_3c_2}{2}$. For $\psi_\alpha, \omega_\alpha, C_\alpha$ are nodal values of the corresponding unknown, and $W_{\psi_\alpha}, W_{\omega_\alpha}, W_{C_\alpha}$ are their arbitrary variations.

Substituting Eqs.(16-18) into Eqs.(13-15), it is obtain that

$$\sum_{\beta=1}^{3} D^e_{\alpha\beta}\psi_\beta - \sum_{\beta=1}^{3} M^e_{\alpha\beta}\omega_\beta - \Gamma^e_{s\alpha} = 0, \tag{19}$$

$$\sum_{\beta=1}^{3} M^e_{\alpha\beta}\dot{\omega}_\beta + \sum_{\beta=1}^{3} A^e_{\alpha\beta}\omega_\beta + \nu\sum_{\beta=1}^{3} D^e_{\alpha\beta}\omega_\beta - \Gamma^e_{\chi\alpha} = 0, \tag{20}$$

$$\sum_{\beta=1}^{3} M^e_{\alpha\beta}\dot{C}_\beta + \sum_{\beta=1}^{3} A^e_{\alpha\beta}C_\beta + \eta\sum_{\beta=1}^{3} D^e_{\alpha\beta}C_\beta - S^e_\alpha + \Gamma^e_{j\alpha} = 0, \tag{21}$$

for each $\alpha = 1, 2, 3$, where the coefficients are given by

$$M^e_{\alpha\beta} = \int_e \phi_\alpha\phi_\beta d\Omega = \frac{\Delta^e}{12}(1 + \delta_{\alpha\beta}), \tag{22}$$

$$D^e_{\alpha\beta} = \int_e (\frac{\partial\phi_\alpha}{\partial x}\frac{\partial\phi_\beta}{\partial x} + \frac{\partial\phi_\alpha}{\partial y}\frac{\partial\phi_\beta}{\partial y})d\Omega = \frac{1}{4\Delta^e}(b_\alpha b_\beta + c_\alpha c_\beta), \tag{23}$$

$$\begin{aligned} A^e_{\alpha\beta} &= \int_e \phi_\alpha(\sum_{\gamma=1}^{3}\frac{\partial\phi_\gamma}{\partial y}\psi_\gamma\frac{\partial\phi_\beta}{\partial x} - \sum_{\gamma=1}^{3}\frac{\partial\phi_\gamma}{\partial x}\phi_\gamma\frac{\partial\phi_\beta}{\partial y})d\Omega, \\ &= \frac{1}{12\Delta^e}\sum_{\gamma=1}^{3}(c_\gamma b_\beta - b_\gamma c_\beta)\psi_\gamma, \end{aligned} \tag{24}$$

$$S^e_\alpha = \int_e \phi_\alpha(\sum_{\gamma=1}^{3}\phi_\gamma S_\gamma)d\Omega = \frac{1}{12\Delta^e}(S_\alpha + \sum_{\gamma=1}^{3} S_\gamma), \tag{25}$$

$$\Gamma^e_{s\alpha} = \int_{\Gamma^e_s} \phi_\alpha\frac{\partial\psi}{\partial n}d\Gamma = -\frac{V_s}{2}|\Gamma^e_s|\begin{pmatrix}0\\1\\1\end{pmatrix}, \tag{26}$$

$$\Gamma^e_{\chi\alpha} = \int_{\Gamma^e_\chi} \nu\phi_\alpha\frac{\partial\omega}{\partial n}d\Gamma = \nu\frac{\chi_n}{2}|\Gamma^e_\chi|\begin{pmatrix}0\\1\\1\end{pmatrix}, \tag{27}$$

$$\Gamma^e_{j\alpha} = \int_{\Gamma^e_j} \phi_\alpha j_B d\Gamma = \frac{j_B}{2}|\Gamma^e_j|\begin{pmatrix}0\\1\\1\end{pmatrix}, \tag{28}$$

with $\delta_{\alpha\beta}$ is the Kronecker delta function, $\Gamma^e_i = \Gamma_i \cap \partial e$ where ∂e is a boundary of an element e and $|\Gamma^e_i|$ is a length of adjacent nodes on a boundary element, for all

$i = s, \chi, j$.

Assembling all element equations over domain Ω, the total equations becomes,

$$[D]\{\psi\} - [M]\{\omega\} - \{\Gamma_s\} = \{0\}, \tag{29}$$

$$[M]\{\dot{\omega}\} + [A(\psi)]\{\omega\} + \nu[D]\{\omega\} - \{\Gamma_\chi\} = \{0\}, \tag{30}$$

$$[M]\{\dot{C}\} + [A(\psi)]\{C\} + \eta[D]\{C\} + \{S\} + \{\Gamma_j\} = \{0\}, \tag{31}$$

where $[D], [M], [A]$ denote the total matrices, $\{\psi\}, \{\omega\}, \{C\}$ are the nodal unknown column vectors.

3.2 Time integration

We interprets Eqs.(29-31) as a nonlinear system of first-order ordinary differential equations for unknown $\{\psi\}, \{\omega\}$ and $\{C\}$. Consider the discretisation of the total equations with respect to time, the semi-implicit scheme to the initial value problem is applied. The time derivatives of the nodal vorticity and concentration are approximated by

$$\dot{\omega}_\beta = \frac{d\omega_\beta}{dt} \simeq \frac{\omega_\beta^{n+1} - \omega_\beta^n}{\Delta t}, \tag{32}$$

$$\dot{C}_\beta = \frac{dC_\beta}{dt} \simeq \frac{C_\beta^{n+1} - C_\beta^n}{\Delta t}, \tag{33}$$

where ω_β^n and C_β^n denote the nodal vorticity and concentration, respectively. For a time level t_n is defined by $t_{n+1} = t_n + \Delta t$ with the time increment Δt for each $n = 0, 1, 2, \ldots$ Substituting Eqs.(32-33) into Eqs.(29-31) at time level t_{n+1}, it follows that

$$[D]\{\psi^{n+1}\} = [M]\{\omega^n\} + \{\Gamma_s\}, \tag{34}$$

$$\frac{1}{\Delta t}[M]\{\omega^{n+1}\} + \nu[D]\{\omega^{n+1}\} = \frac{1}{\Delta t}[M]\{\omega^n\} - [A(\psi^{n+1})]\{\omega^n\} - \{\Gamma_\chi^{n+1}\}, \tag{35}$$

$$\frac{1}{\Delta t}[M]\{C^{n+1}\} + \eta[D]\{C^{n+1}\} = \frac{1}{\Delta t}[M]\{C^n\} - [A(\psi^{n+1})]\{C^n\} + \{S^{n+1}\} - \{\Gamma_j^{n+1}\}. \tag{36}$$

for all $n = 1, 2, 3, \ldots$

4 Numerical Experiment

4.1 Topography of a domain

The dispersion of smoke from 2 chimneys into the atmosphere as shown in Fig.1 will be considered. Let C (kg/m^3) be the concentration of the substances in smoke. There are two chimney pots are located 7 m high above the ground. The wind at mean velocity $U = 2$ m/sec is assumed to flow horizontally from left to right. Downstream from the chimneys is located a structural obstacle with cross section of 6 m high and 4 m wide. In our assumption, we assume that the flow does not depend on the concentration.

Physical constants for the wind are; kinematic viscosity of the air $\nu = 1.45 \times 10^{-5}$ m^2/sec, diffusion coefficient of the substance is $\eta = 4.58 \times 10^{-5}$ m^2/sec, gravity acceleration $g = 9.81$ m/sec^2. Since the structural obstacle building height $L = 6$ m, the Reynolds and Peclet numbers for the problem are $Re = UL/\nu \simeq 827{,}586$ and $Pe = \nu/\eta \simeq 0.9178$

4.2 The initial and boundary conditions

The initial and boundary conditions are indicated in Fig.1. The initial condition, the atmosphere is assumed to be motionless. The boundary conditions for $t > 0$ are

$$C = 0 \text{ at } 0 \leq x \leq 70, y = 34, \tag{37}$$

$$C = 0 \text{ at } x = 0, 0 \leq y \leq 34, \tag{38}$$

$$\frac{\partial C}{\partial n} = 0 \text{ at } 0 \leq x \leq 70, y = 0, \tag{39}$$

$$\frac{\partial C}{\partial n} = 0 \text{ at } x = 70, 0 \leq y \leq 34. \tag{40}$$

4.3 Finite element solutions

Assume the condition that the concentration contours perpendicularly intersect the efflux boundary CD on the right-hand side. The time increment $\Delta t = 0.1$ sec. is used. The triangular finite element mesh with 190 elements and 120 nodes is presented in Fig. 2 .

Consider the problem in 2 cases: 1 source and 2 sources. In 2 sources

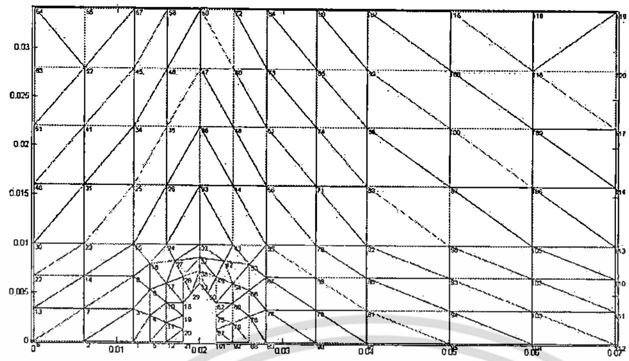

Figure 2: The triangular finite element mesh with 190 elements and 120 nodes

Table 1: The maximum of smoke concentration in each time levels

Element number of point sources	S_1 (kg/m^{2}sec)	S_2 (kg/m^{2}sec)	Time (sec)	Maximum of smoke concentration (mg/m^{2}sec)
E7, E10	0.001	0.001	4	1.3789
E7	0.002	0	4	1.4358
E7, E10	0.001	0.001	40	2.9256
E7	0.002	0	40	3.2673
E7, E10	0.001	0.001	400	3.9370
E7	0.002	0	400	3.7763
E7, E10	0.001	0.001	800	3.9181
E7	0.002	0	800	3.6947

problem, the intensities of smoke discharging are $S_1 = 0.001$ kg/m^{2}sec and $S_2 = 0.001$ kg/m^{2}sec at element numbers 7th and 10th, respectively. The case of 1 source, the intensity of smoke has been increased to be 2 folds of the first case, $S_1 = 0.002$ kg/m^{2}sec only at element numbers 7th.

The approximation of smoke concentration at 4, 200 and 800 sec of both cases are summarized in Fig.3 - Fig.5. The maximum of smoke concentration at each time levels are given in Table 1.

5 Discussion and Conclusions

In this research, two cases of smoke dispersion with a structural obstacle structure from 1 and 2 chimneys discharge into the atmosphere are presented. The smoke concentration measurement is simulated using the isothermal mass transfer in a viscous fluid. The diffusion of smoke dispersion is coupled to the equations of vis-

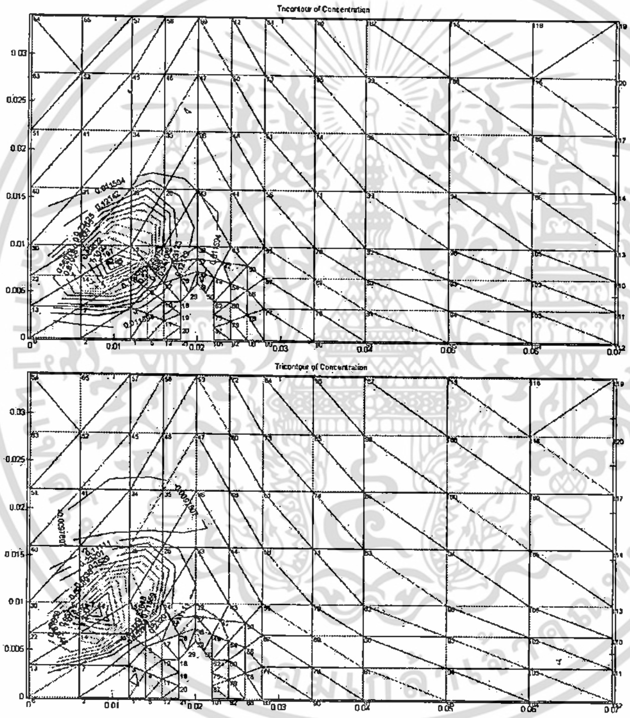

Figure 3: The finite element approximations of smoke concentration in 2 cases: 2 points source and 1 point source at 4 sec

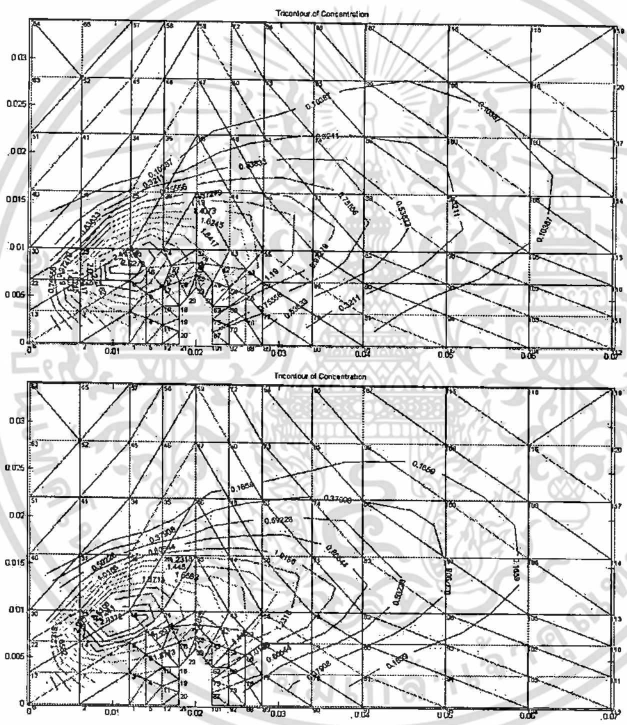

Figure 4: The finite element approximations of smoke concentration in 2 cases: 2 points source and 1 point source at 3 min 20 sec

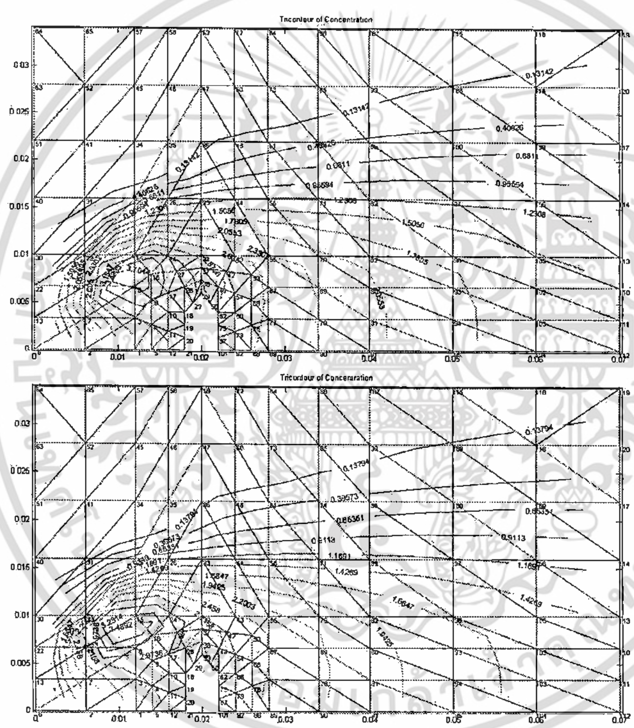

Figure 5: The finite element approximations of smoke concentration in 2 cases: 2 points source and 1 point source at 13 min 20 sec

cous fluid flow.

A model of type presented here, it is possible to determined velocity and vorticity of the air, and smoke concentration at every point of industrial area with simple structural obstacle. For the discretisation of the domain, triangular linear elements have been used. Time integration has been carried out by means of semi-implicit method.

The numerical example is supposed that concentration of smoke of 1 source is greater than 2 point sources. From the calculation results, it is obtained that the overall concentration by the case of 2 points discharging is lower than 1 point discharging in a short time. However, if we consider the maximum smoke concentration in a larger time, we found that the case of 1 source will give lower concentration than case of 2 sources. This mean that the case of 1 point source of concentration could be a better choice in this consideration problem.

For the real-world application, more complicate structural obstacle structures and several point sources should be consider. It is possible to take another indexes of air quality or concentration of toxic as the smoke concentration in this model.

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บทความวิจัยที่ได้รับการตอบรับและที่กำลังอยู่ระหว่างการพิจารณาเพื่อลงตีพิมพ์

A Numerical Treatment of the Mathematical Model for Smoke Dispersion from Two Sources

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Abstract

The smoke discharging is a principle reason of air pollution problem from industrial plants. The flow obstruction of building is also an important factor of air quality simulation. In this research, we simulate a mathematical model of the smoke dispersion over the building from two sources. The governing equation of the model is atmospheric diffusion equation. The fractional step method is used to find the approximate solution of the equation. The simulation with numerical results is also demonstrated.

Keywords: air pollution, smoke dispersion, atmospheric diffusion equation, fractional step method.

1 Introduction

The air pollution problem in Thailand is still important for quality of life. This research is to study the air pollution problem by using the mathematical model; the atmospheric diffusion equation.

In [2], they used the finite difference method in the air pollution model of two dimensional spaces with single point source. In [3], the air pollution problem in three dimensional spaces with multiple sources presented. The initial conditions in the domains were assumed to be zero everywhere without obstacles.

In this research, we investigate the behavior of air pollutant release into the atmosphere with obstacle domain in two dimensional spaces. We assume that there are a large buildings inside the consideration area and the source of discharging pollutant are two point sources. The atmospheric diffusion equation is used to predict the behavior of the dispersion of air pollutant in the domain. The fractional step, Carlson and Crank-Nicolson methods are used to approximate the concentration of smoke dispersion.

2 The Governing Equation

We introduced the well-known atmospheric diffusion equation

$$\frac{\partial c}{\partial t} + \vec{v} \cdot \nabla c = \nabla \cdot (\bar{k} \nabla c) + s \tag{1}$$

For convenience, an equation (1) can be written into the form

$$\frac{\partial c}{\partial t} + v_x \frac{\partial c}{\partial x} + v_y \frac{\partial c}{\partial y} = k_x \frac{\partial^2 c}{\partial x^2} + k_y \frac{\partial^2 c}{\partial y^2} + \frac{\partial}{\partial z}\left(k_z \frac{\partial c}{\partial z}\right) + s \tag{2}$$

where c is the air pollutant concentration at point (x, y, z) at time t (the unit of c is kg/m^3), k_x, k_y are diffusion coefficients in x-direction and y-direction respectively, v_x, v_y are the flow velocity in x-direction and y-direction respectively and s is the rate of change of substance concentration due to sources.

Suppose that the horizontal advection dominates the horizontal diffusion by the wind and the vertical diffusion dominates the vertical advection by the wind. The horizontal advection in $y-$direction is negligible. We have a cross section along the $y-$axis at the plane of obstacle. With is assumptions the equation (2) can be written as

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = \frac{\partial}{\partial z}\left(k_z\frac{\partial c}{\partial z}\right) + s, \tag{3}$$

where $v_x = u$. Equation (3) can be normalized to the non-dimensional form and changes all variables to the capital letters as follow

$$\frac{\partial C}{\partial T} + U\frac{\partial C}{\partial X} = \frac{\partial}{\partial Z}\left(K_Z\frac{\partial C}{\partial Z}\right) + S. \tag{4}$$

By the assumption as [2] and [3], the initial condition is taken to be zero concentration of air pollutant everywhere in the domain. We can see that

$$C(X, Z, 0) = 0 \qquad \text{for all } X > 0, \quad 0 < Z < H, \tag{5}$$

when H is the height of the inversion layer. The boundary conditions in the $X-$direction are assume to be zero.

$$C(X_W, Z, T) = C(X_E, Z, T) = 0 \qquad \text{for all } Z, \qquad \text{for all } T > 0, \tag{6}$$

when X_W and X_E are the abscissas of the western and eastern. The flux of pollutant is assumed to be zero at the inversion layer H and at the ground which is cross section along the obstacle $Y-$plane.

$$\frac{\partial C}{\partial Z}(X, H, T) = \frac{\partial C}{\partial Z}(X, \Gamma, t) = 0, \tag{7}$$

where Γ is a line on the ground which is cross section along the obstacle $Y-$plane.

3 Numerical Techniques

The Fractional step method [6] is used to separate equation (4) into 3 stages as the following

$$\frac{\partial C'}{\partial T} = S = \sum_{r=1}^{N} Q_r\delta(X - X_r)\delta(Z - Z_r) \tag{8}$$

$$C'(X, Z, T_n) = C(X, Z, T_n) \tag{9}$$

$$\frac{\partial C''}{\partial T} = -U\frac{\partial C''}{\partial X} \tag{10}$$

$$C''(X, Z, T_n) = C'(X, Z, T_n + \Delta T) \tag{11}$$

$$\frac{\partial C'''}{\partial T} = \frac{\partial}{\partial Z}\left(K_Z\frac{\partial C'''}{\partial Z}\right) \tag{12}$$

$$C'''(X, Z, T_n) = C''(X, Z, T_n + \Delta T) \tag{13}$$

In succession, the auxiliary terms in each stage are denoted by prime (′) that is C', C'' and C''' respectively. Equations (8) and (9) are the emission stage, the concentration at the source is assumed to be a $\delta-$function. Solving the emission part by using the basic finite difference method. The advection stage in $X-$direction was presented in equation (10) and (11). The Carlson finite difference method [5] is used to find the numerical solution in this stage. Equations (12) and (13) are the diffusion stage in $Z-$direction. We use the Crank-Nicolson [1] approximation which is unconditionally stable and consistence to find the numerical solution.

Table 1: The numerical solution along the $X-$axis at different time step ΔT and $\Delta T = 0.25$. The wind velocity is $U = 1$.

T/X	$X = 0$	$X = 20$	$X = 40$	$X = 60$	$X = 80$	$X = 100$
$8\Delta T$	0.114572577	0.163675113	6.27483E-10	6.27483E-10	0	0
$16\Delta T$	0.122116864	0.17445424	6.83287E-07	6.83287E-07	0	0
$32\Delta T$	0.12275511	0.175929991	0.000416183	0.000416183	1.63233E-10	0
$64\Delta T$	0.12275997	0.197436007	0.021093864	0.021093864	2.23685E-05	1.18347E-10
$128\Delta T$	0.12275997	0.21275123	0.051584559	0.051584559	0.015943851	0.000283978
$256\Delta T$	0.12275997	0.212762767	0.060053871	0.060053871	0.046648859	0.032173093
$512\Delta T$	0.12275997	0.212762767	0.060053885	0.060053885	0.047524938	0.041955905

Table 2: The numerical solution along the $X-$axis at different time step ΔT and $\Delta T = 0.25$. The wind velocity is $U(Z) = Z^{(0.2)}$.

T/X	$X = 0$	$X = 20$	$X = 40$	$X = 60$	$X = 80$	$X = 100$
$8\Delta T$	0.042462698	0.060660997	0	0	0	0
$16\Delta T$	0.042557854	0.060796935	0	0	0	0
$32\Delta T$	0.0425581	0.064172487	0.002671623	0.002671623	0	0
$64\Delta T$	0.0425581	0.090988389	0.02746385	0.02746385	0.000628333	1E-15
$128\Delta T$	0.0425581	0.091007278	0.04393871	0.04393871	0.030707281	0.013133237
$256\Delta T$	0.0425581	0.091007278	0.043938764	0.043938764	0.03337808	0.030258134
$512\Delta T$	0.0425581	0.091007278	0.043938764	0.043938764	0.03337808	0.030258264

4 Numerical Experiment

The atmospheric diffusion equation of (4) with appropriate parameter values for the tropical area were taken from Pasquill stability reference class A [4]. The parameters are given by $\bar{k}_z = 45 \quad m^2/sec$ and the velocity $\bar{u} = 3 \quad m/sec.$ The sources are assumed at $z = h_s = 15$ m. above the ground. In this research, we assumed there are two point sources with different emission rate are $q_1 = 70$ gram/sec and $q_2 = 100$ gram/sec. Suppose there are two buildings with different heights away from the second source at 300 m.(at X=40 to 52) and 600 m. at (X=60 to 72) respectively. The wind velocity are assume to be a constant form $U = 1$ and variable form $U(Z) = Z^{(0.2)}$. The numerical results for the smoke concentrations are shown in the Tables (1-2) and Figures (1-2).

5 Conclusion

The concentrations of smoke released from two point sources in the domain with two obstacles of flow are calculated. The computed approximate results are obtained using fractional step method and MatLab code, with respected to the transformed atmospheric diffusion equation.

Acknowledgment The first author would like to thanks the Faculty of Science, Ubon Ratchathani University for providing the financial support for this research.

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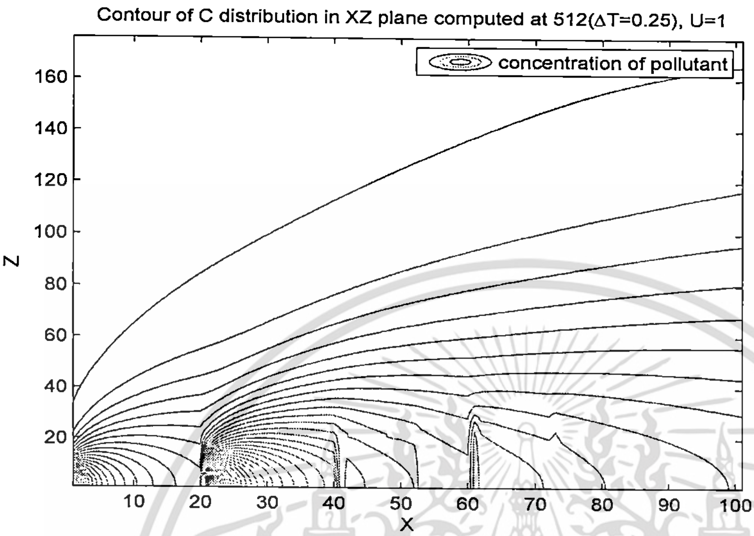
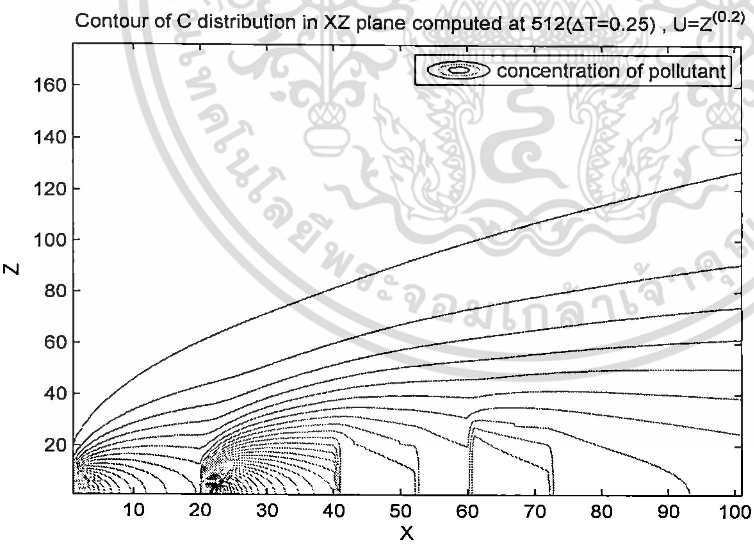


Figure 1: The contour graph along XZ plane at $512\Delta T$ and $\Delta T = 0.25$. The wind velocity is $U = 1$.



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A NUMERICAL TREATMENT OF THE MATHEMATICAL MODEL FOR SMOKE DISPERSION FROM TWO SOURCES

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Abstract

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T/X	$X=0$	$X=20$	$X=40$	$X=60$	$X=80$	$X=100$
$8\Delta T$	0.114572577	0.163675113	6.27483E-10	6.27483E-10	0	0
$16\Delta T$	0.122116864	0.17445424	6.83287E-07	6.83287E-07	0	0
$32\Delta T$	0.12275511	0.175929991	0.000416183	0.000416183	1.63233E-10	0
$64\Delta T$	0.12275997	0.197436007	0.021093864	0.021093864	2.23685E-05	1.18347E-10
$128\Delta T$	0.12275997	0.21275123	0.051584559	0.051584559	0.015943851	0.000283978
$256\Delta T$	0.12275997	0.212762767	0.060053871	0.060053871	0.046648859	0.032173093
$512\Delta T$	0.12275997	0.212762767	0.060053885	0.060053885	0.047524938	0.041955905

Table 2: The numerical solution along the $X-$axis at different time step ΔT and $\Delta T = 0.25$. The wind velocity is $U(Z) = Z^{(0.2)}$.

T/X	$X=0$	$X=20$	$X=40$	$X=60$	$X=80$	$X=100$
$8\Delta T$	0.042462698	0.060660997	0	0	0	0
$16\Delta T$	0.042557854	0.060796935	0	0	0	0
$32\Delta T$	0.0425581	0.064172487	0.002671623	0.002671623	0	0
$64\Delta T$	0.0425581	0.090988389	0.02746385	0.02746385	0.000628333	1E-15
$128\Delta T$	0.0425581	0.091007278	0.04393871	0.04393871	0.030707281	0.013133237
$256\Delta T$	0.0425581	0.091007278	0.043938764	0.043938764	0.03337808	0.030258134
$512\Delta T$	0.0425581	0.091007278	0.043938764	0.043938764	0.03337808	0.030258264

3 Numerical Experiment

The atmospheric diffusion equation of (4) with appropriate parameter values for the tropical area were taken from Pasquill stability reference class A [4]. The parameters are given by $\bar{k}_z = 45 \quad m^2/sec$ and the velocity $\bar{u} = 3 \quad m/sec$. The sources are assumed at $z = h_s = 15$ m. above the ground. In this research, we assumed there are two point sources with different emission rate are $q_1 = 70$ gram/sec and $q_2 = 100$ gram/sec. Suppose there are two buildings with different heights away from the second source at 300 m.(at X=40 to 52) and 600 m. at (X=60 to 72) respectively. The wind velocity are assume to be a constant form $U = 1$ and variable form $U(Z) = Z^{(0.2)}$. The numerical results for the smoke concentrations are shown in the Tables (1-2) and Figures (1-2).

4 Conclusion

The concentrations of smoke released from two point sources in the domain with two obstacles of flow are calculated. The computed approximate results are obtained using fractional step method and MatLab code, with respected to the transformed atmospheric diffusion equation.

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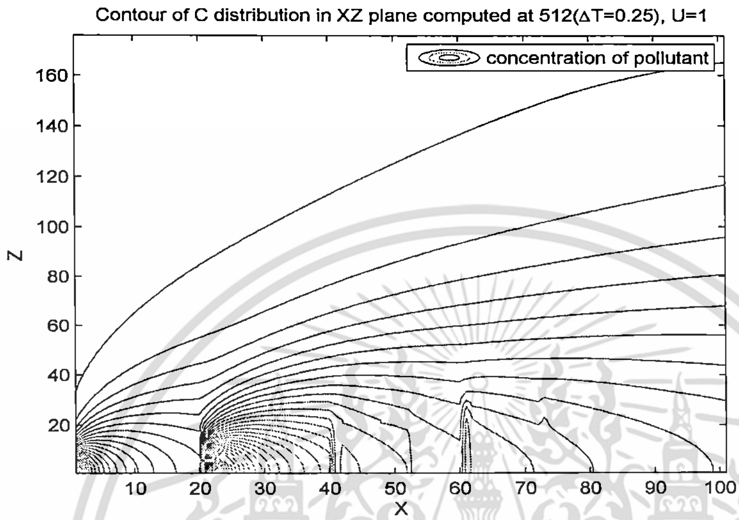

Figure 1: The contour graph along XZ plane at $512\Delta T$ and $\Delta T = 0.25$. The wind velocity is $U = 1$.

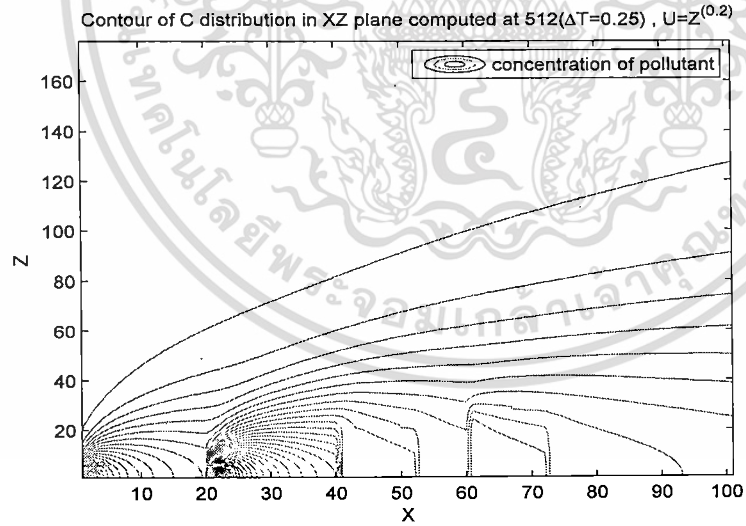

Figure 2: The contour graph along XZ plane at $512\Delta T$ and $\Delta T = 0.25$. The wind velocity is $U(Z) = Z^{(0.2)}$.

A Finite Element Solution of the Mathematical Model for Smoke Dispersion from Two Sources

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Abstract

Smoke discharging is a main reason of air pollution problem from industrial plants. The obstruction of air flow by building to affect with the air pollutant discharge. In this research, a mathematical model of the smoke dispersion from two sources and one source with a structural obstacle is considered. The governing equation of the model is an isothermal mass transfer model in a viscous fluid. The finite element method is used to approximate the solutions of the model. The triangular linear elements have been used for discretise the domain, and time integration has been carried out by semi-implicit finite difference method. The simulations of smoke dispersion in cases of one chimney and two chimneys are presented. The maximum calculated smoke concentration of both cases are compared. It is then used to make the decision for smoke discharging and air pollutant control problems on industrial area.

Key words: Air pollution/ Smoke dispersion/ Finite element method/ Stream function/ Vorticity equation/ Convection-diffusion equation/ Semi-implicit method
Mathematics Subject Classification: 65M60, 65M06, 76D05

1. Introduction

The air pollution problem is still important for human health. This research is consider the air pollution problem using the mass transfer model. In

[8], a nonlinear mathematical model is proposed and analyzed to study the removal of gaseous pollutants and particulate matters from the atmosphere of a city by precipitation. The three pollution indexes (SO_2, NO_2 and PM10) and the daily air pollution indexes (APIs) of Shanghai in China are analyzed by rescaled range analysis (R/S), detrended fluctuation analysis (DFA) and spectral analysis is presented in [4]. In [1], they study the air flow using a lubricated system consisting of two bodies in proximity. The Poincare compactification to the dynamical system to get a complete qualitative analysis of the global flow is applied in [11].

In [10], [12] and [5], they used the finite difference method in the air pollution model of two dimensional spaces with single point source. In [6], the air pollution problem in three dimensional spaces. The initial conditions in the domains were assumed to be zero everywhere without structural obstacles. In [7] and [13], the fractional steps method, Carlson method and Crank-Nicolson method [2] are used to approximate the concentration of smoke dispersion. The diffusion of dust particles from a point source above ground level is considered in [3].

In this research, we investigate the behavior of smoke release into the atmosphere with a structural obstacle in two-dimensional spaces and one-dimensional in time using the finite element method and finite difference method, respectively. The structural obstacle and chimneys are added in the simulation. The simulation attempts to predict the behavior of the dispersion of smoke in the problem.

2. The Governing Equation

2.1. The mass transport model

Consider the equations describing the smoke dispersion in terms of the streamfunction, vorticity and convection-diffusion equations as follow [9],

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega, \tag{1}$$

$$\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} = \nu \nabla^2 \omega, \tag{2}$$

$$\frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y} = \eta \nabla^2 C + S, \tag{3}$$

where ν is a kinematic viscosity, ω is a vorticity, ψ is a stream function, C is a concentration, η is a diffusion coefficient and S is intensity of mass source.

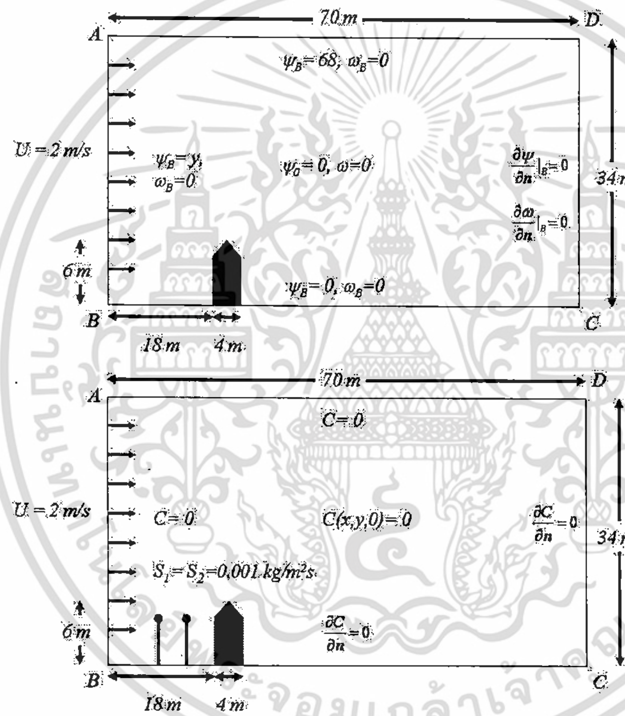

Figure 1: The initial and boundary conditions of the problem

56 *2.2. The boundary conditions*

The boundary conditions of the streamfunction equation Eq.(1) are

$$\psi = \psi_B, \tag{4}$$

which is specified on the boundary Γ_ψ, and

$$\frac{\partial \psi}{\partial n} = -V_s, \tag{5}$$

with tangential flow velocity V_s specified on the rest of the boundary Γ_s. Here n denotes the outward unit normal to the boundary. Boundary conditions on the vorticity Eq.(2) are

$$\omega = \omega_B, \tag{6}$$

which is specified on the boundary Γ_ω, and

$$\frac{\partial \omega}{\partial n} = \chi_n, \tag{7}$$

with the value of the normal derivative specified on the other part of the boundary Γ_χ. The boundary conditions of the convective-diffusion equation Eq.(3) are

$$C = C_B \text{ on } \Gamma_C, \tag{8}$$

$$-\frac{\partial C}{\partial n} = j_B \text{ on } \Gamma_j. \tag{9}$$

57 Since boundary is a non-absorbing, we can put $j_B = 0$.

58 **3. The Numerical Technique**

59 *3.1. The finite element discretisation*

The Galerkin finite element method to discretising of the streamfunction equation Eq.(1), the vorticity transport equation Eq.(2), and the convection-diffusion equation Eq.(3) will be used. Let W_ψ, W_ω and W_C be weighting functions. The finite element formulation with the weighted residual forms of these equations:

$$\int_\Omega W_\psi(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2})d\Omega+\int_\Omega W_\psi\omega d\Omega=0, \quad (10)$$

$$\int_\Omega W_\omega\frac{\partial\omega}{\partial t}d\Omega+\int_\Omega W_\omega(\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y})d\Omega-\int_\Omega W_\omega\nu\nabla^2\omega d\Omega=0, \quad (11)$$

$$\int_\Omega W_C\frac{\partial C}{\partial t}d\Omega+\int_\Omega W_C(\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y})d\Omega-\int_\Omega W_C\eta\nabla^2 Cd\Omega$$
$$-\int_\Omega W_C Sd\Omega=0. \quad (12)$$

Integration by parts of terms concerning the Laplacian, we get

$$\int_\Omega(\frac{\partial W_\psi}{\partial x}+\frac{\partial W_\psi}{\partial y})(\frac{\partial\psi}{\partial x}+\frac{\partial\psi}{\partial y})d\Omega+\int_\Omega W_\psi\omega d\Omega-\int_{\Gamma_s}W_\psi\frac{\partial\psi}{\partial n}d\Gamma=0, \quad (13)$$

$$\int_\Omega W_\omega\frac{\partial\omega}{\partial t}d\Omega+\int_\Omega W_\omega(\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y})d\Omega+\int_\Omega\nu\nabla W_\omega\nabla\omega d\Omega$$
$$-\int_{\Gamma_\chi}\nu W_\omega\frac{\partial\omega}{\partial n}d\Gamma=0, \quad (14)$$

$$\int_\Omega W_C\frac{\partial C}{\partial t}d\Omega+\int_\Omega W_C(\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y})d\Omega+\int_\Omega\eta\nabla W_C\nabla Cd\Omega$$
$$-\int_\Omega W_C Sd\Omega-\int_{\Gamma_j}j_B W_C d\Gamma=0. \quad (15)$$

The domain Ω is divided into triangular elements with local node numbers 1, 2 and 3 with nodal coordinates (x_1, y_1), (x_2, y_2) and (x_3, y_3), respectively. The unknown streamfunction, vorticity and concentration are linearly interpolated as follows,

$$\psi=\sum_{\alpha=1}^{3}\phi_\alpha\psi_\alpha, \quad W_\psi=\sum_{\alpha=1}^{3}\phi_\alpha W_{\psi_\alpha}, \quad (16)$$

$$\omega=\sum_{\alpha=1}^{3}\phi_\alpha\omega_\alpha, \quad W_\omega=\sum_{\alpha=1}^{3}\phi_\alpha W_{\omega_\alpha}, \quad (17)$$

$$C=\sum_{\alpha=1}^{3}\phi_\alpha C_\alpha, \quad W_C=\sum_{\alpha=1}^{3}\phi_\alpha W_{C_\alpha}, \quad (18)$$

where linear interpolation functions, $\phi_\alpha = \frac{1}{2\Delta^e}(a_\alpha + b_\alpha x + c_\alpha y)$ with area of a triangular element e is $\Delta^e = \frac{b_2c_3 - b_3c_2}{2}$. For $\psi_\alpha, \omega_\alpha, C_\alpha$ are nodal values of the corresponding unknown, and $W_{\psi_\alpha}, W_{\omega_\alpha}, W_{C_\alpha}$ are their arbitrary variations.

Substituting Eqs.(16-18) into Eqs.(13-15), it is obtain that

$$\sum_{\beta=1}^{3} D^e_{\alpha\beta}\psi_\beta - \sum_{\beta=1}^{3} M^e_{\alpha\beta}\omega_\beta - \Gamma^e_{s\alpha} = 0, \tag{19}$$

$$\sum_{\beta=1}^{3} M^e_{\alpha\beta}\dot{\omega}_\beta + \sum_{\beta=1}^{3} A^e_{\alpha\beta}\omega_\beta + \nu\sum_{\beta=1}^{3} D^e_{\alpha\beta}\omega_\beta - \Gamma^e_{\chi\alpha} = 0, \tag{20}$$

$$\sum_{\beta=1}^{3} M^e_{\alpha\beta}\dot{C}_\beta + \sum_{\beta=1}^{3} A^e_{\alpha\beta}C_\beta + \eta\sum_{\beta=1}^{3} D^e_{\alpha\beta}C_\beta - S^e_\alpha + \Gamma^e_{j\alpha} = 0, \tag{21}$$

for each $\alpha = 1, 2, 3$, where the coefficients are given by

$$M^e_{\alpha\beta} = \int_e \phi_\alpha\phi_\beta d\Omega = \frac{\Delta^e}{12}(1 + \delta_{\alpha\beta}), \tag{22}$$

$$D^e_{\alpha\beta} = \int_e \left(\frac{\partial\phi_\alpha}{\partial x}\frac{\partial\phi_\beta}{\partial x} + \frac{\partial\phi_\alpha}{\partial y}\frac{\partial\phi_\beta}{\partial y}\right)d\Omega = \frac{1}{4\Delta^e}(b_\alpha b_\beta + c_\alpha c_\beta), \tag{23}$$

$$\begin{aligned} A^e_{\alpha\beta} &= \int_e \phi_\alpha\left(\sum_{\gamma=1}^{3}\frac{\partial\phi_\gamma}{\partial y}\psi_\gamma\frac{\partial\phi_\beta}{\partial x} - \sum_{\gamma=1}^{3}\frac{\partial\phi_\gamma}{\partial x}\phi_\gamma\frac{\partial\phi_\beta}{\partial y}\right)d\Omega, \\ &= \frac{1}{12\Delta^e}\sum_{\gamma=1}^{3}(c_\gamma b_\beta - b_\gamma c_\beta)\psi_\gamma, \end{aligned} \tag{24}$$

$$S^e_\alpha = \int_e \phi_\alpha\left(\sum_{\gamma=1}^{3}\phi_\gamma S_\gamma\right)d\Omega = \frac{1}{12\Delta^e}\left(S_\alpha + \sum_{\gamma=1}^{3} S_\gamma\right), \tag{25}$$

$$\Gamma^e_{s\alpha} = \int_{\Gamma^e_s} \phi_\alpha\frac{\partial\psi}{\partial n}d\Gamma = -\frac{V_s}{2}|\Gamma^e_s|\begin{pmatrix}0\\1\\1\end{pmatrix}, \tag{26}$$

$$\Gamma^e_{\chi\alpha} = \int_{\Gamma^e_\chi} \nu\phi_\alpha\frac{\partial\omega}{\partial n}d\Gamma = \nu\frac{\chi_n}{2}|\Gamma^e_\chi|\begin{pmatrix}0\\1\\1\end{pmatrix}, \tag{27}$$

$$\Gamma^e_{j\alpha} = \int_{\Gamma^e_j} \phi_\alpha j_B d\Gamma = \frac{j_B}{2}|\Gamma^e_j|\begin{pmatrix}0\\1\\1\end{pmatrix}, \tag{28}$$

with $\delta_{\alpha\beta}$ is the Kronecker delta function, $\Gamma_i^e = \Gamma_i \cap \partial e$ where ∂e is a boundary of an element e and $|\Gamma_i^e|$ is a length of adjacent nodes on a boundary element, for all $i = s, \chi, j$.

Assembling all element equations over domain Ω, the total equations becomes,

$$
\begin{aligned}
[D]\{\psi\} - [M]\{\omega\} - \{\Gamma_s\} &= \{0\}, &(29)\\
[M]\{\dot{\omega}\} + [A(\psi)]\{\omega\} + \nu[D]\{\omega\} - \{\Gamma_\chi\} &= \{0\}, &(30)\\
[M]\{\dot{C}\} + [A(\psi)]\{C\} + \eta[D]\{C\} + \{S\} + \{\Gamma_j\} &= \{0\}, &(31)
\end{aligned}
$$

where $[D], [M], [A]$ denote the total matrices, $\{\psi\}, \{\omega\}, \{C\}$ are the nodal unknown column vectors.

3.2. Time integration

We interprets Eqs.(29-31) as a nonlinear system of first-order ordinary differential equations for unknown $\{\psi\}, \{\omega\}$ and $\{C\}$. Consider the discretisation of the total equations with respect to time, the semi-implicit scheme to the initial value problem is applied. The time derivatives of the nodal vorticity and concentration are approximated by

$$
\begin{aligned}
\dot{\omega}_\beta = \frac{d\omega_\beta}{dt} &\simeq \frac{\omega_\beta^{n+1} - \omega_\beta^n}{\Delta t}, &(32)\\
\dot{C}_\beta = \frac{dC_\beta}{dt} &\simeq \frac{C_\beta^{n+1} - C_\beta^n}{\Delta t}, &(33)
\end{aligned}
$$

where ω_β^n and C_β^n denote the nodal vorticity and concentration, respectively. For a time level t_n is defined by $t_{n+1} = t_n + \Delta t$ with the time increment Δt for each $n = 0, 1, 2, \ldots$ Substituting Eqs.(32-33) into Eqs.(29-31) at time level t_{n+1}, it follows that

$$
\begin{aligned}
[D]\{\psi^{n+1}\} &= [M]\{\omega^n\} + \{\Gamma_s\}, &(34)\\
\frac{1}{\Delta t}[M]\{\omega^{n+1}\} + \nu[D]\{\omega^{n+1}\} &= \frac{1}{\Delta t}[M]\{\omega^n\} - [A(\psi^{n+1})]\{\omega^n\}\\
&\quad -\{\Gamma_\chi^{n+1}\}, &(35)\\
\frac{1}{\Delta t}[M]\{C^{n+1}\} + \eta[D]\{C^{n+1}\} &= \frac{1}{\Delta t}[M]\{C^n\} - [A(\psi^{n+1})]\{C^n\}\\
&\quad +\{S^{n+1}\} - \{\Gamma_j^{n+1}\}. &(36)
\end{aligned}
$$

for all $n = 1, 2, 3, \ldots$

4. Numerical Experiment

4.1. Topography of a domain

The dispersion of smoke from 2 chimneys into the atmosphere as shown in Fig.1 will be considered. Let C (kg/m^3) be the concentration of the substances in smoke. There are two chimney pots are located 7 m high above the ground. The wind at mean velocity $U = 2$ m/sec is assumed to flow horizontally from left to right. Downstream from the chimneys is located a structural obstacle with cross section of 6 m high and 4 m wide. In our assumption, we assume that the flow does not depend on the concentration.

Physical constants for the wind are; kinematic viscosity of the air $\nu = 1.45 \times 10^{-5}$ m^2/sec, diffusion coefficient of the substance is $\eta = 4.58 \times 10^{-5}$ m^2/sec, gravity acceleration $g = 9.81$ m/sec^2. Since the structural obstacle building height $L = 6$ m, the Reynolds and Peclet numbers for the problem are $Re = UL/\nu \simeq 827{,}586$ and $Pe = \nu/\eta \simeq 0.9178$

4.2. The initial and boundary conditions

The initial and boundary conditions are indicated in Fig.1. The initial condition, the atmosphere is assumed to be motionless. The boundary conditions for $t > 0$ are

$$C = 0 \text{ at } 0 \leq x \leq 70, y = 34, \quad (37)$$

$$C = 0 \text{ at } x = 0, 0 \leq y \leq 34, \quad (38)$$

$$\frac{\partial C}{\partial n} = 0 \text{ at } 0 \leq x \leq 70, y = 0, \quad (39)$$

$$\frac{\partial C}{\partial n} = 0 \text{ at } x = 70, 0 \leq y \leq 34. \quad (40)$$

4.3. Finite element solutions

Assume the condition that the concentration contours perpendicularly intersect the efflux boundary CD on the right-hand side. The time increment $\Delta t = 0.1$ sec. is used. The triangular finite element mesh with 190 elements and 120 nodes is presented in Fig. 2 .

Consider the problem in 2 cases: 1 source and 2 sources. In 2 sources problem, the intensities of smoke discharging are $S_1 = 0.001$ kg/m^2sec and $S_2 = 0.001$ kg/m^2sec at element numbers 7th and 10th, respectively. The case of 1 source, the intensity of smoke has been increased to be 2 folds of

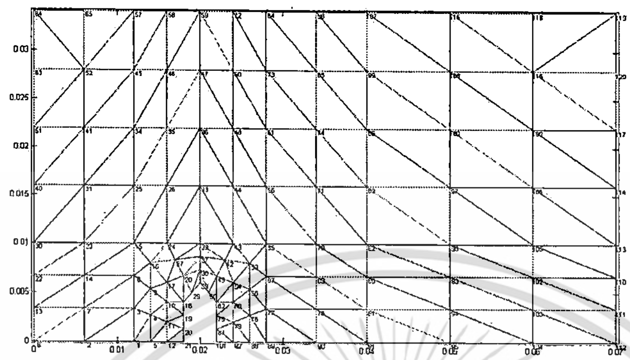

Figure 2: The triangular finite element mesh with 190 elements and 120 nodes

Table 1: The maximum of smoke concentration in each time levels

Element number of point sources	S_1 (kg/m^{2}sec)	S_2 (kg/m^{2}sec)	Time (sec)	Maximum of smoke concentration (mg/m^{2}sec)
E7, E10	0.001	0.001	4	1.3789
E7	0.002	0	4	1.4358
E7, E10	0.001	0.001	40	2.9256
E7	0.002	0	40	3.2673
E7, E10	0.001	0.001	400	3.9370
E7	0.002	0	400	3.7763
E7, E10	0.001	0.001	800	3.9181
E7	0.002	0	800	3.6947

the first case, $S_1 = 0.002$ kg/m^{2}sec only at element numbers 7th.

The approximation of smoke concentration at 4, 200 and 800 sec of both cases are summarized in Fig.3 - Fig.5. The maximum of smoke concentration at each time levels are given in Table 1.

5. Discussion and Conclusions

In this research, two cases of smoke dispersion with a structural obstacle structure from 1 and 2 chimneys discharge into the atmosphere are presented. The smoke concentration measurement is simulated using the isothermal mass transfer in a viscous fluid. The diffusion of smoke dispersion is coupled to the equations of viscous fluid flow.

A model of type presented here, it is possible to determined velocity and vorticity of the air, and smoke concentration at every point of industrial area with simple structural obstacle. For the discretisation of the domain, triangular linear elements have been used. Time integration has been carried out

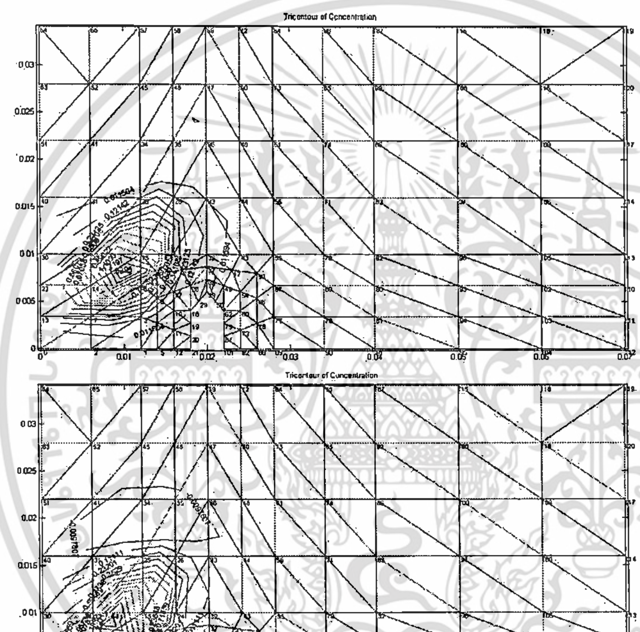

Figure 3: The finite element approximations of smoke concentration in 2 cases: 2 points source and 1 point source at 4 sec

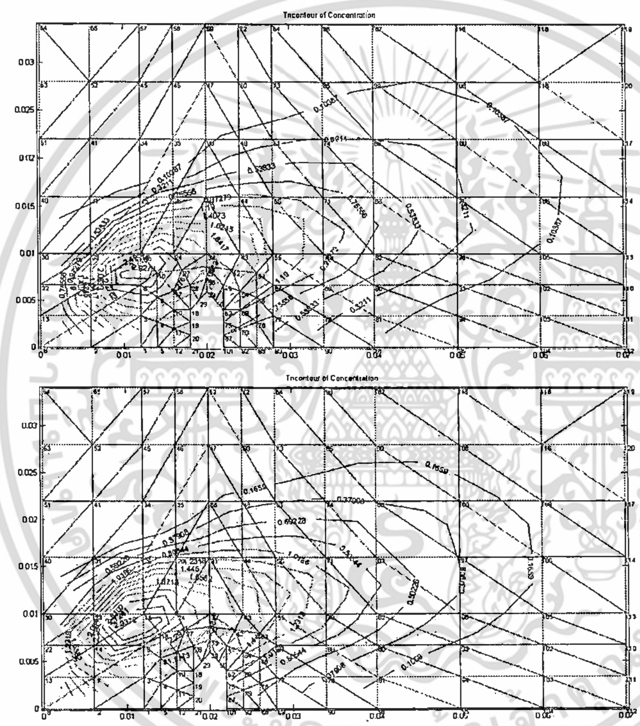

Figure 4: The finite element approximations of smoke concentration in 2 cases: 2 points source and 1 point source at 3 min 20 sec

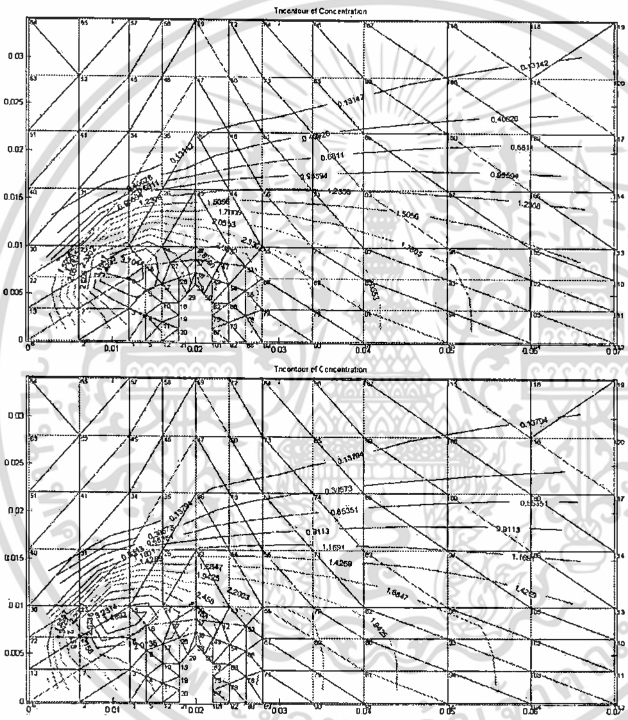

Figure 5: The finite element approximations of smoke concentration in 2 cases: 2 points source and 1 point source at 13 min 20 sec

by means of semi-implicit method.

The numerical example is supposed that concentration of smoke of 1 source is greater than 2 point sources. From the calculation results, it is obtained that the overall concentration by the case of 2 points discharging is lower than 1 point discharging in a short time. However, if we consider the maximum smoke concentration in a larger time, we found that the case of 1 source will give lower concentration than case of 2 sources. This mean that the case of 1 point source of concentration could be a better choice in this consideration problem.

For the real-world application, more complicate structural obstacle structures and several point sources should be consider. It is possible to take another indexes of air quality or concentration of toxic as the smoke concentration in this model.

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